Quantifying Transient Uncertainty in the BEAVRS Benchmark Using Time Series Analysis Methods

Quantifying Transient Uncertainty in the BEAVRS Benchmark Using Time Series Analysis Methods

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INTRODUCTION

Advances in computation have brought about significant improvements in creating fast-running high-fidelity simulations of nuclear cores. The BEAVRS benchmark [1] is a highly-detailed PWR specification with two cycles of measured operational data used to validate high-fidelity core analysis methods. This PWR depletion benchmark captures the fine details of the LWR fuel assemblies, burnable absorbers, in-core fission detectors, core loading and shuffling patterns. Specifically, 58 of the 193 assemblies contain in-core detectors with measurements taken over 61 axial positions every month. These detectors are U-235 fission chambers with slightly varying mass of U-235. The collected signals are normalized on a given assembly permitting full core comparisons. The fuel layout for cycle 1 and instrument tube locations for the reactor are given in figures 1 and 2 respectively. Through a series of data processing and comparisons, it was shown [2] that axially integrated radial maps of reaction rates were in close agreement between provided detector data and calculated data.

More recently, the focus of the BEAVRS project has been on quantifying uncertainty to further assess the validity of aforementioned results. A close investigation of sources of error shows that uncertainties from operational nuclear data arise primarily from data measurements and processing. A summary of concrete values for such uncertainties is listed in Table I.

Source of Uncertainty	Type of Uncertainty	Uncertainty
Detector Count Rates	Measurement	1.0%
Background Signal	Measurement	~0%
Gain Factor	Measurement	~0%
Core Power Factor	Measurement	<0.1%
Interpolation	Post-processing	1%-13%
Axial Realignment	Post-processing	10%-15%
Radial Integration	Post-processing	1.5%

TABLE I. All sources of error in the BEAVRS benchmark related to measurement and post-processing uncertainties[3]

While the errors in axial realignment and interpolation seem significant, these sources affect only a small subset of data points near grid spacers and do not persist throughout the entire core. Instead, measurement uncertainty from detector count rates and radial integration dominate the uncertainty in measured data are determined as the most significant sources of error.

This uncertainty analysis treated each given burnup step as independent of comparable data at neighboring burnup steps. More recent work has been targeted at understanding how reaction rates vary over time, in order to determine whether calculated reaction rates follow any observable trends. This abstract hones in on transient uncertainty quantification, in order to compare observed data against models for transient behavior. The first section of the paper uses linear regression tools to fit operational data, while the latter portion of the paper explores more complex simulation tools to fit operational data. Ultimately, the BEAVRS benchmark aims to serve as a true non-proprietary international benchmark for the validation of high-fidelity tools.

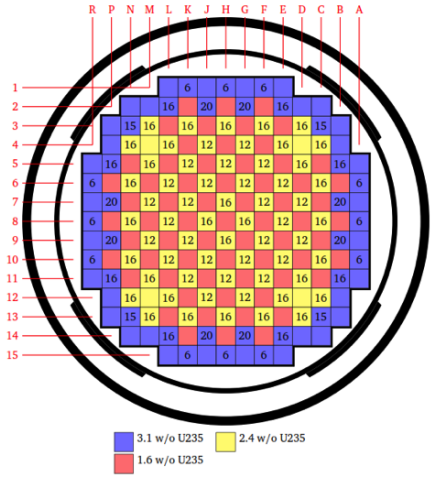

Fig. 1. Fuel Layout for cycle 1 - Numbers indicate burnable absorber pins [1]

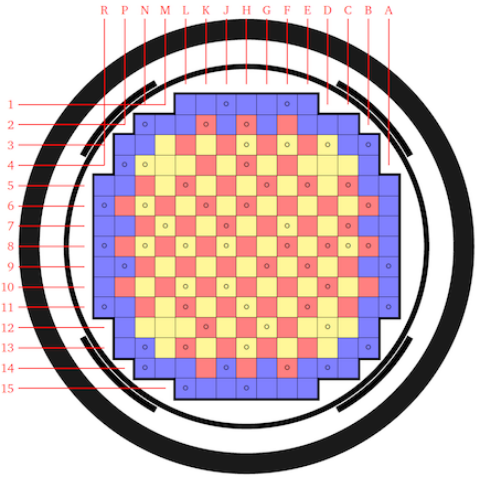

Fig. 2. Instrument tube locations [1]

LINEAR REGRESSION MODELS

Models for time series analysis fit under two broad categories - stationary and non-stationary models. Stationary models assume that statistical properties such as mean, variance, and autocorrelation of the underlying distribution do not vary over time, while non-stationary models have statistical properties that are time-dependent. Written mathematically, a stationary model can be expressed as:

$$y_t = \mu + \varepsilon_t \tag{1}$$

and a non-stationary model can be expressed as:

$$y_t = \mu_t + \varepsilon_t, \tag{2}$$

where y_t is the regressed variable, ε_t can be viewed as the residual between the regressed and observed value, and μ is time-independent, while μ_t is time dependent.[4] For a nonstationary linear regression, μ_t can be expressed as

$$\mu_t = \beta_0 + \beta_1 t \tag{3}$$

For a linear regression model to be valid, it needs to meet four requirements[5]:

1. Residuals of regressed values and measured values are independent of each other with respect to time.
2. Residuals of regressed values and measured values are homoscedastic.
3. Regressed linear lines exhibit a high value of R^2
4. Residuals of regressed values and measured values exhibit a normal distribution centered around a 0 mean.

Homoscedacity refers to the fact that residuals converge to a constant value, while R^2 is a measure of goodness of fit of a linear model to the data, and is a value between 0 and 1 that describes the percentage of variable variance that is explained by a the model. A higher value of R^2 implies that a the model is indeed a good fit for the data, and is defined mathematically, for a linear model, as[5]:

$$R^2 = 1 - \frac{\sum_T \varepsilon_t^2}{\sum_T (y_t - \bar{y})^2}, \tag{4}$$

where $\bar{y}$ is the mean observed value. Out of the four aforementioned conditions, the first two conditions can be shown as valid for BEAVRS data using statistical tests, while the third and fourth conditions will be explored more closely in the next section.

Even if these four conditions are true, however, the biggest challenge with using a non-stationary model such as a linear regression model is to prove that the time trend persists for all time periods, and is not merely a transitory phenomenon. However, for certain ranges of burnups, linear models are quite an accurate fit for the data. In fact, as a fist-order approximation, it has been shown that at steady-state operations reactivity and hence reaction rates, follow a linear trend with burnup[6].

FITTING LINEAR REGRESSION MODELS TO BEAVRS DATA

This section attempts to explore how well linear regression models work with BEAVRS data. We begin by looking at the eighth-core symmetric radial map of reaction rates at Hot Zero Power Conditions.

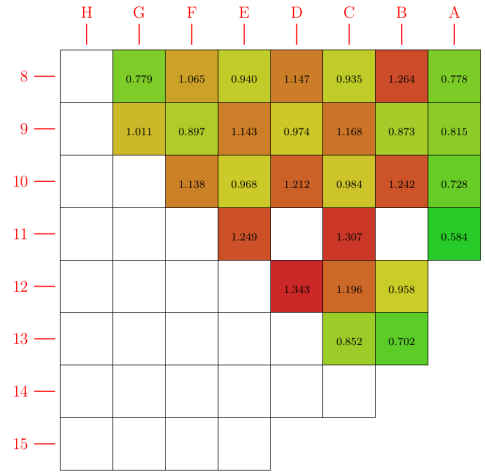

Fig. 3. Octant-symmetric radial map of normalized reaction rates at 0.0 MWd/kg

Each assembly exhibits some trend over burnup, and we wish to overlay a linear model that minimizes total residual. Figure 4 shows BEAVRS data compared to a linear fit for Assembly B9

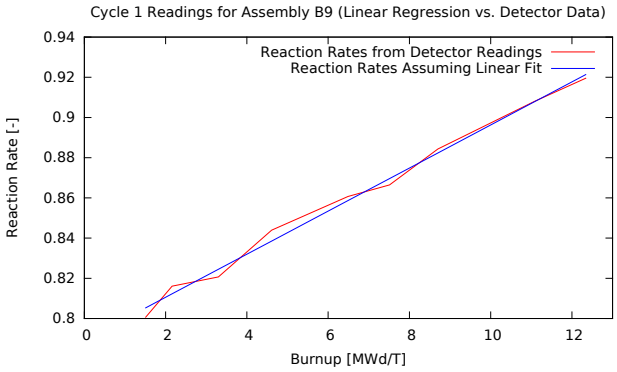

Fig. 4. Measured and linearly regressed reaction rates vs. Burnup for Assembly B9

With this plot, both R^2 and RMS values between BEAVRS and linear fit can be calculated. For this specific assembly, R^2 is equal to 99.3% and RMS is 0.3%. Figures 5 and 6 summarize both of these values as radial map for Cycle 1 data and Cycle 2 data respectively, where the top number corresponds to RMS error, while the bottom number corresponds to R^2 goodness of fit.

These maps illustrate that assemblies with high values of R^2 have low RMS values, and in general there is reason to believe that linear models are adequate for first-order fitting

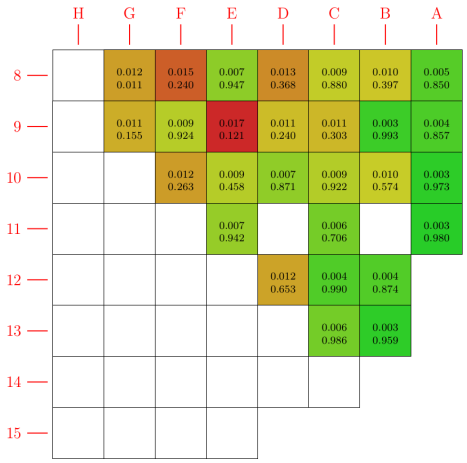

Fig. 5. RMS and R^2 values using linear regression model for each assembly over cycle 1 burnups. The top value corresponds to RMS error, while the bottom value corresponds to R^2 goodness of fit.

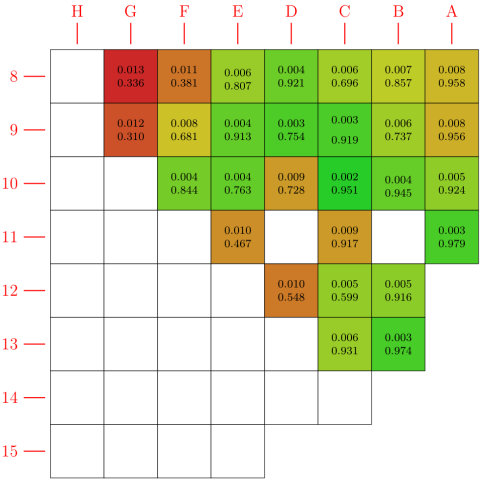

Fig. 6. RMS and R^2 values using linear regression model for each assembly over cycle 2 burnups. The top value corresponds to RMS error, while the bottom value corresponds to R^2 goodness of fit.

purposes. However, there are assemblies in Cycle 1 where a linear fit is inadequate, and a big reason for this is that the power history for Cycle 1 is erratic, thus violating the steady-state assumption of the linear reactivity model. Power histories for both cycles are given in figures 7 and 8

While figures 5 and 6 show that a linear model is an adequate model for most assemblies, another condition that needs to hold true for a linear regression model to be valid is that the residuals between regressed and actual values need to be normally distributed. Figures 9 and 10 plot the residuals for all assemblies as a histogram for both cycle 1 and 2. The figures resemble the shape of a normal distribution, and also pass the Shapiro-Wilk test for normality. Thus, the results from the linear model indicates that errors are approximately

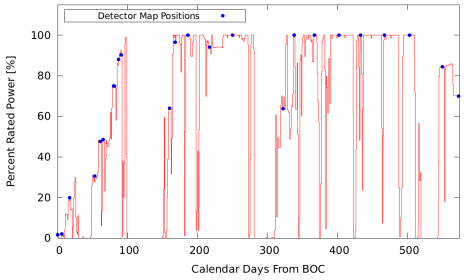

Fig. 7. Power history for Cycle 1

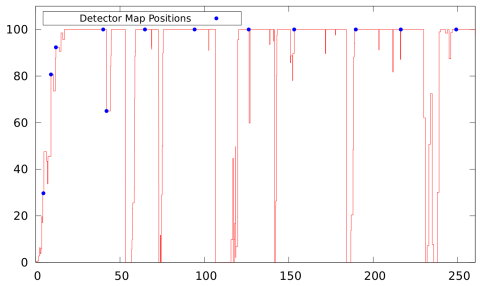

Fig. 8. Power History for Cycle 2

on the order of 0.8% for cycle 1 and 0.6% for cycle 2.

FITTING DATA FROM SIMULATION CODES TO BEAVRS DATA

While a linear model can be used to fit data for reaction rates over burnup, more sophisticated models are explored in order to account for higher-order effects that cannot be captured simply by a linear fit. Simulated reaction rates from CASMO-5 lattice codes and SIMULATE-3 nodal diffusion simulator serve as an additional basis to compare with reaction rates over burnup. However, these simulations consistently overpredict reaction rates, and we attribute this persistent discrepancy as model bias. Further work is in progress to quantify an exact amount for this model bias, but as an example, figure

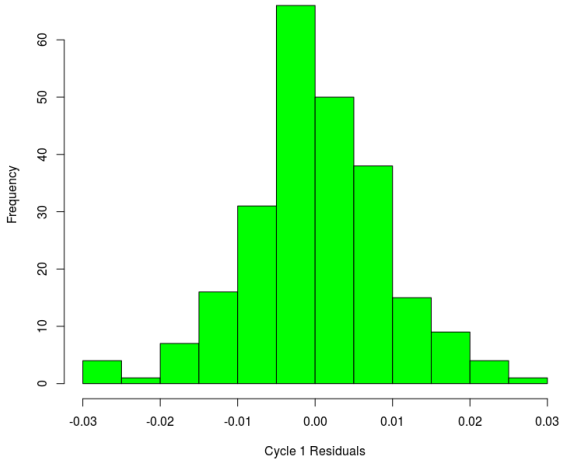

Fig. 9. Histogram of all residuals for Cycle 1

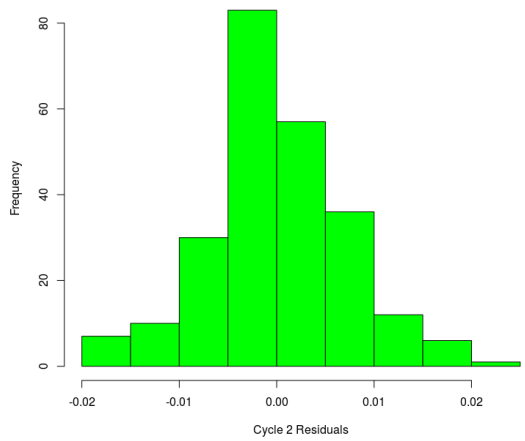

Fig. 10. Histogram of all residuals for Cycle 2

11 shows how simulated reaction rates vary over burnup for assembly B9. The blue line is the normalized reaction rate, and this curve is shifted down by a constant percentage in order to be compared to the detector rates. This percentage signifies the model bias, and now the green line can be used as an additional basis for calculating RMS values, just like the linear fit.

Once model bias is explicitly quantified for each assembly, a radial map similar to figure 5 can be generated to express RMS values between CASMO/Simulate and detector readings for each assembly. We are working towards generating radial maps similar to figures 5 and 6, and expect to find the error between CASMO/Simulate and detector data to be smaller than the error between the linear model and detector data, since CASMO/Simulate account for higher order effects occurring during reactor operations.

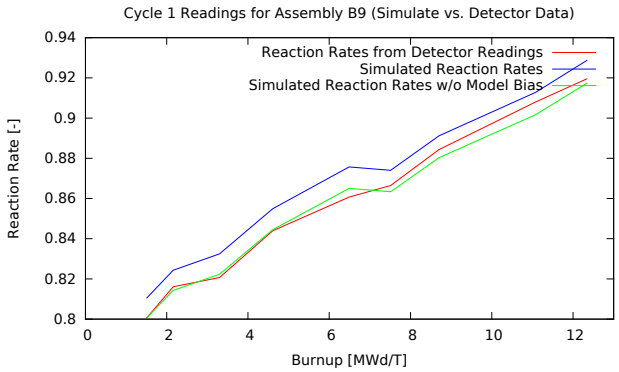

Fig. 11. Measured and simulated reaction rates vs. burnup for assembly B9. The green simulated line corrects for the model bias observed in the blue line

CONCLUSIONS

The BEAVRS benchmark has been instrumental in showing the efficacy of high fidelity modeling tools to model realistic PWR models. Recent work has been focused in quantifying the uncertainty in areas of data measurement, data processing, and simulation tools. Time series analysis methods were investigated as a means to calculate transient detector uncertainty data, and we expect to find that this uncertainty should be consistent with measurement and post-processing uncertainty that were calculated at each burnup step. For linear models, errors were on the order of 0.6% to 0.8%. We also use simulation codes to calculate RMS error to more accurately model reaction rates, and expect to find that such modeling tools will yield more accurate results than linear regression models. Future work involves looking at stationary models instead of non-stationary models as added references for fitting available data. In addition, we hope to cross reference CASMO/Simulate with other codes to verify that our results are consistent across multiple software platforms.

ACKNOWLEDGMENTS

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