HW Solutions # 8 - 8.01 MIT - Prof. Kowalski

Momentum and Collisions.

1) *8.55*

Please review section 8.6 rocket Propulsion in the book.
a) The average thrust is impulse devided by time:

$$F_{ave} = \frac{J}{\Delta t} \tag{1}$$

So the ration of the average thrust to maximum thrust is:

$$\frac{F_{ave}}{F_{max}} = \frac{\frac{J}{\Delta t}}{F_{max}} = \frac{10}{13.3 \times 1.7} = 0.442 \tag{2}$$

b) Using the average force in equation (8.38):

$$v_{ex} = \frac{F\Delta t}{\Delta m} = \frac{J}{\Delta m} = \frac{10}{0.0125} = 800 m/s \tag{3}$$

c) Using the result of part b in equation (8.40) - the sole equation of "rocket science":

$$v - v_0 = v_{ex} \ln(\frac{m_0}{m}) \tag{4}$$

With $m = m_0 - \Delta m$ and $v_0 = 0$ we have:

$$v = v_{ex} \ln(\frac{m_0}{m_0 - \Delta m}) = 800 \ln(\frac{0.0258}{0.258 - 0.0125}) = 530 m/s \tag{5}$$

2) ***8.73***

`Please refer to figure 8.41 p.322.`

Using energy method including work:

$$K_{L_1} + U_{L_1} + W_{other} = E_{L_1} + W_{other} = E_{L_2} = K_{L_2} + U_{L_2} \quad (6)$$

$$\frac{1}{2}mv_{L_1}^2 + mgy_{L_1} + W_{other} = \frac{1}{2}mv_{L_2}^2 + mgy_{L_2} \quad (7)$$

I will measure the gravitation potential energy with respect to the horizontal line passing the bottom of the bowl.No non-conservative force is present so $W_{other} = 0$.

$$v_{L_1} = 0 \qquad y_{L_1} = R \qquad y_{L_2} = 0 \quad (8)$$

$$0 + mgR = \frac{1}{2}mv_{L_2}^2 + 0 \Rightarrow v_{L_2} = \sqrt{2gR} \quad (9)$$

At the bottom, due to momentum conservation law total momentum before and after sticking together is the same:

$$\sum \overrightarrow{\mathbf{P}} = \sum \overrightarrow{\mathbf{P}}' \quad (10)$$

Where I used " $'$ " to denote the momentum *just after* the collision. The momentum at the bottom of the bowl is horizontal so we need only the component of the above vector equation in horizontal direction:
So we have

$$mv_{L_2} + 0 = (m+m)v' = 2mv' \Rightarrow v' = \frac{v_{L_2}}{2} \quad (11)$$

v' is the velocity of the total mass 2m of the two boxes. Use again the energy conservation equations (6) and (7) for " $'$ " and " $''$ " where " $''$ " is the highest point they reach ($v'' = 0$):

$$\frac{1}{2}(m+m)v'^{\,2} + 0 = 0 + (m+m)gy'' \Rightarrow y'' = \frac{v'^{\,2}}{2g} \quad (12)$$

Combining (9) and (11) with (12):

$$\boxed{y'' = \frac{(\sqrt{2gR})^2}{4(2g)} = \frac{R}{4}}$$

Sensible: The height varies quadratically with the velocity, and is independent of mass. Therefore halving the velocity decreases the height by 4.

3) ***8.70***

`Please refer to figure 8.39 p.322.`

Notations:
Bullet mass: m
Bullet velocity: v (is an unknown, to be found from Δx)
Block mass: M
Block and bullet velocity together after the collision: V
Compression length: Δx
0.750 N : F
0.250 cm: d

Writing momentum conservation

$$\sum \overrightarrow{\mathbf{P}} = \sum \overrightarrow{\mathbf{P}}' \tag{13}$$

in horizontal direction:

$$mv + 0 = (m + M)V \tag{14}$$

$$\boxed{v = (1 + \frac{M}{m})V}$$

The energy of the system (block and bullet) *just after* the collision is:

$$E_1 = \frac{1}{2}(m + M)V^2 \tag{15}$$

The energy when at its *maximum* compression:

$$E_2 = \frac{1}{2}k\Delta x^2 \tag{16}$$

No nonconservative force is present **after** the collision so $W_{other} = 0$ after the collision and $E_1 = E_2$:

$$\frac{1}{2}(m+M)V^2 = \frac{1}{2}k\Delta x^2 \tag{17}$$

$$\boxed{V = \sqrt{\frac{k}{m+M}}\Delta x}$$

From Newton's law

$$k = \frac{F}{d} \tag{18}$$

Plugging in the numbers given in the problem:

$$\boxed{V = 2.60\ m/s}$$

Using the first boxed equation and the above result you'll get:

$$\boxed{v = 325\ m/s}$$

4) ***8.99***

Denote the emitted neutron whose y-velocity is positive by the subscript 1 and the emitted neutron that moves in $-$y-direction by the subscript 2. Using conservation of momentum $\sum \overrightarrow{\mathbf{P}} = \sum \overrightarrow{\mathbf{P}}'$ in the x and y directions, and neglecting the common factor of mass of a neutron,

$$v_0 = v' \cos 10^{\circ} + v_1 \cos 45^{\circ} + v_2 \cos 30^{\circ} \tag{19}$$

$$0 = v' \sin 10^{\circ} + v_1 \sin 45^{\circ} - v_2 \sin 30^{\circ} \tag{20}$$

Where here $v' = 2/3v_0$.

We have 2 equations with 2 unknowns (v_1 and v_2) you can combine them to get v_1 and v_2.
Specifically you can use $\sin 45^{\circ} = \cos 45^{\circ}$, these two equations can be subtracted to eliminate v_1, and rearrangement gives:

$$v_0(1 - (2/3)cos10^{\circ} + (2/3)\sin 10^{\circ}) = v_2(\cos 30^{\circ} + \sin 30^{\circ}) \quad (21)$$

from which $v_2 = 1.01 \times 10^3$ m/s substitution of this into either of the momentum relations gives $v_1 = 221 m/s$.
All that is known is that there is no z component of momentum, and so only the ratio of speeds can be determined:

$$m_{Ba}v_{Ba} - m_{Kr}v_{Kr} = 0 \Rightarrow v_{Kr} = \frac{m_{Ba}}{m_{Kr}}v_{Ba} \quad (22)$$

We don't know what the v_{Ba} is. However if we know the the released energy (it would be etermined from the difeerence of the masses of these nuclei using the formula $E = \Delta mc^2$) you can set up the **energy conservation equation** . Combined with $v_{Kr} = \frac{m_{Ba}}{m_{Kr}}v_{Ba}$ we have 2 equations and two unknowns (we have already found v_1 and v_2 from momentum conservation) and in principle you can solve for v_{Kr} and v_{Ba}.

5) ***8.106***

`Please review section 8.6 rocket Propulsion in the book.`

a)

Consider system of plane + chunk of stationary air of Δm, immediately in front of the propeller. Use coordinate system in which air is initially at rest:

$$P_1 = m_P v_P + dm(0) = m_P v_P \quad (23)$$

After passing through the propeller the air chunk has:

$$v_{air} = v_P - v_{ex} < 0 \quad (24)$$

(Analogous to v_{fuel} in the book section 8.6). Here:

$$P_2 = m_P(v_P + dv_P) + dm v_{air} \tag{25}$$

We ignore external force (e.g. air drag)in the system . So $P_2 = P_1$ and hence:

$$m_P v_P = m_P(v_P + dv_P) + dm v_{air} \tag{26}$$

$$m_{_P} \frac{dv_P}{dt} = \frac{dm_{_P}}{dt}(v_P - v_{ex}) \tag{27}$$

This process accelerates the plane, and from an engineering perspective we can regard the air as generating a force on the plane:

$$\boxed{F_{net} = m_{_P} \frac{dv_P}{dt} = (v_{_P} - v_{ex})\frac{dm_{_P}}{dt}}$$

b) With the numbers given in the problem, the velocity that the propeller imparts to the air is:

$$v_{air} = v_{_P} - v_{ex} = \frac{F_{net}}{\frac{dm}{dt}} = \frac{1300\ N}{-150\ kg/s} = -8.66\ m/s = -31\ km/h \tag{28}$$

c) Neglecting turbulence we have:

$$P_{_{into\ prop}} = P_{_{plane}} + P_{_{on\ air}} = \overrightarrow{\mathbf{F}}_{_{net}}.\overrightarrow{\mathbf{v}}_{_{plane}} - \frac{1}{2}\frac{dm}{dt}(v_{_P} - v_{ex})^2 \tag{29}$$

(Note that $\frac{1}{2}\frac{dm}{dt} < 0$)

Simplify $-\frac{1}{2}\frac{dm}{dt}(v_{_P} - v_{ex})^2$:

$$-\frac{1}{2}\frac{dm}{dt}(v - v_{ex})^2 = -\frac{1}{2}\left[\frac{dm}{dt}(v_{_P} - v_{ex})\right](v_{_P} - v_{ex}) = -\frac{1}{2}F_{_{net}} v_{air} \tag{30}$$

(Note that $v_{air} < 0$)

The efficiency ϵ is:

$$\epsilon = \frac{P_{plane}}{P_{into\ prop}} = \frac{F_{net} v_P}{F_{net} v_P - \frac{1}{2} F_{net} v_{air}} = \frac{1}{1 - \frac{v_{air}}{2 v_P}} \tag{31}$$

For the numbers given in the problem:

$$\epsilon = \frac{1}{1 - \frac{-31}{2 \times 130}} = 89\% \tag{32}$$

d) If the diameter of the propeller were halved, the area would be 1/4 so does the dm/dt would be one fourth. We want to have the same net force and from part a you see that $v_{air} = v_P - v_{ex}$ should be multiplied by 4. This will increase the denominator, so doing this will decrease the efficiency ϵ:

$$\epsilon = \frac{1}{1 - \frac{-31 \times 4}{2 \times 130}} = 68\% \tag{33}$$

It's better to make them bigger. You can't make it too big though because turbulence will become more and more important.