

Motion with a Constant Force

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$$\vec{F} = m\vec{a}$$

[2-nd Law]

If $\vec{F} = \text{constant}$
Then $\vec{a} = \text{constant}$.

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

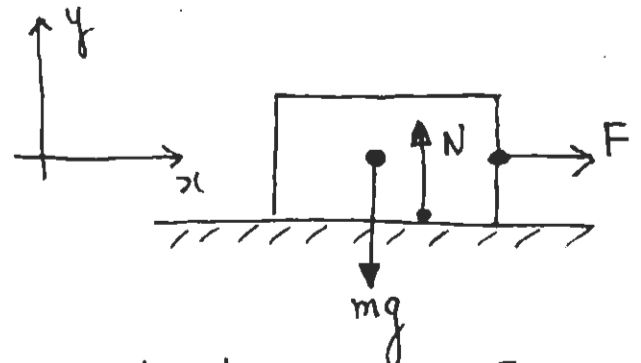
$$\sum F_z = ma_z$$

} Rectangular Reference Axes.

Forces

- Tension
- Gravity
- Normal / Contact
- Friction
- Spring.

Example



$$\begin{aligned} \text{y-axis: } N - mg &= ma_y = 0 \\ N &= mg \end{aligned}$$

[No acceleration along y]

$$\begin{aligned} \text{x-axis: } F &= ma_x \\ \therefore a_x &= \frac{F}{m} \end{aligned}$$

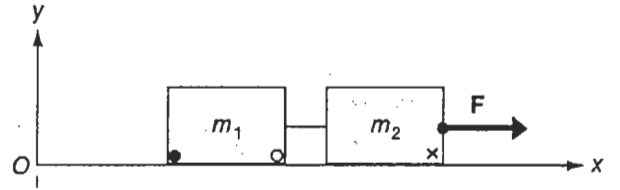
Problem-Solving Strategy

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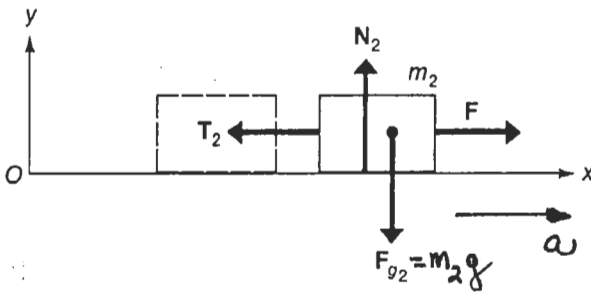
1. Draw a diagram indicating all key features in the problem.
2. Draw one or more free-body diagrams for the objects. For the chosen object include all the forces acting on it. Do not include any internal forces. Do not include any forces exerted by the body on some other body.
3. Select a coordinate system and show it in the free-body diagram. Determine components of the forces with reference to these axes. When the direction of acceleration is known in advance — choose that direction as $+x$ -axis. Can choose different reference frame for each body. All must be 'inertial'.
4. If there are geometrical relationships between two or more bodies — relate these algebraically !!
5. Write down Newton's Eq. of Motion for each body and solve for unknowns.
$$\vec{F} = m\vec{a}$$
$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$
$$\sum F_z = ma_z$$
6. Check special cases and extreme values of quantities, compare with intuitive expectations. Does the result make sense?

Example

- Frictionless plane
- Massless string



Mass m_2 :



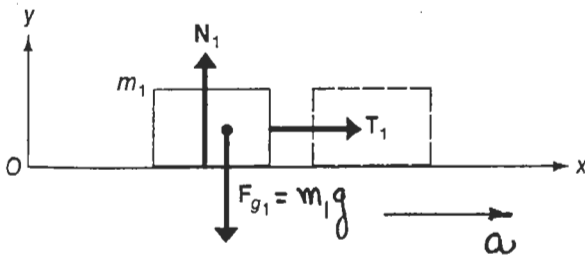
$$F - T_2 = m_2 a_2$$

x-axis ①

$$N_2 - m_2 g = 0$$

y-axis ②

Mass m_1 :



$$T_1 = m_1 a_1$$

x-axis ③

$$N_1 - m_1 g = 0$$

y-axis ④

Since bodies constrained to move together $a_1 = a_2 = a$
 Ideal string $T_1 = -T_2 = T$

Add ① + ③ $F = (m_1 + m_2) a$

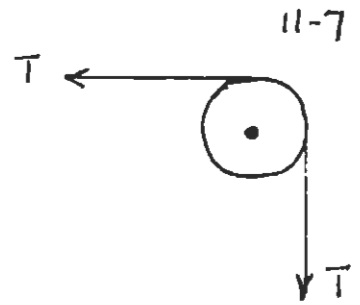
$$a = \frac{F}{m_1 + m_2}$$

From ③ $T = m_1 a = \frac{m_1 F}{m_1 + m_2}$

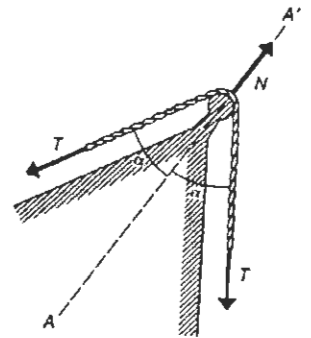
Ideal Pulleys/Pegs

- Are used to change the direction of force exerted by string.
- If string and pulley are both massless, the tension is the same on either side of pulley.
- If not massless then not true.
- Ideal pulley has zero mass and is also frictionless.

Normal (contact) force lies on a line which bisects the angle between the ends of the string.

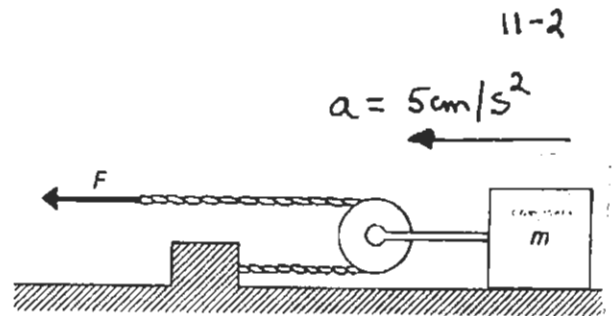


Free-body diagram for an ideal (massless) string that passes over an ideal (frictionless) peg. The magnitude of the contact force is $N = 2T \cos \alpha$.



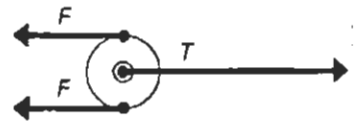
Block + Pulley

- Frictionless surface
- Massless pulley
- Massless rope
- $m = 300 \text{ kg}$

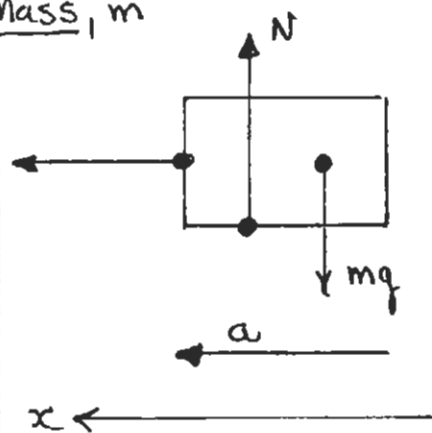


Pulley

$$T - 2F = 0$$
$$T = 2F$$



Mass, m



$$T = ma$$
$$2F = ma$$
$$a = \frac{2F}{m}$$

$$N - mg = 0$$

$$F = \frac{ma}{2} = \frac{300 \times .05}{2}$$
$$= 7.5 \text{ N}$$

Pulley system provides mechanical advantage.
In this case a factor of 2.

Example

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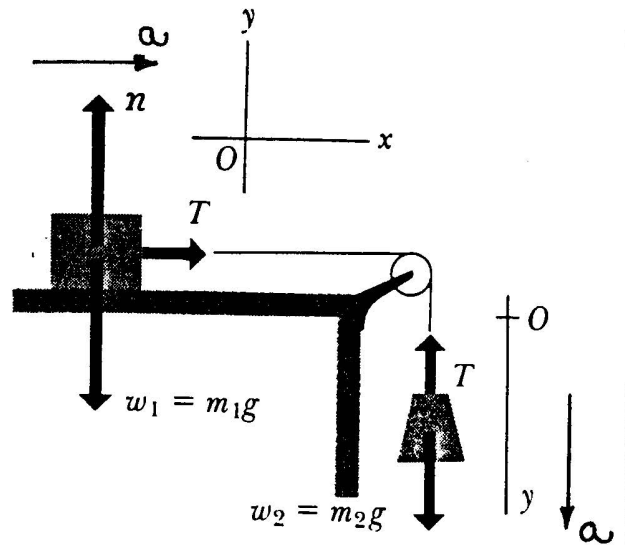
Mass m_2 :

$$m_2 g - T = m_2 a \quad (1)$$

Mass m_1 :

$$T = m_1 a \quad (2)$$

$$N - m_1 g = 0 \quad (3)$$



Force diagram for block on a frictionless horizontal surface and for the hanging block.

Accelerations are equal since masses are connected.

Add (1) + (2)

$$m_2 g = (m_1 + m_2) a$$

$$a = \left(\frac{m_2}{m_1 + m_2} \right) g$$

Example - Inclined Plane

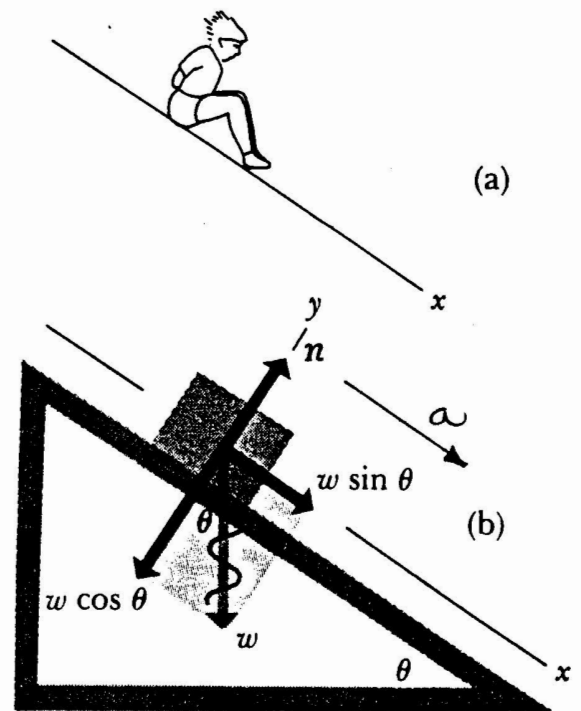
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$$w = mg$$

$$mg \sin \theta = ma$$

$$N - mg \cos \theta = 0$$

$$a = g \sin \theta$$

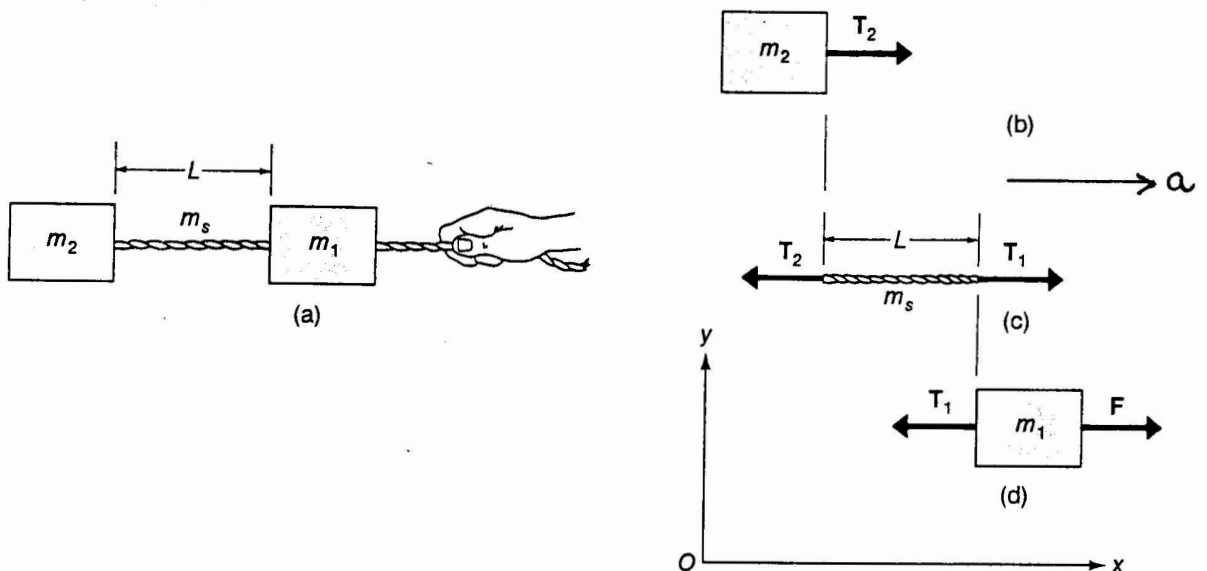


- (a) A body on a frictionless inclined plane.
(b) Free-body diagram.

Ideal Strings

- Can only exert a pull (tension)
no push (compression)
- massless: tension is the same everywhere
- mass $\neq 0$: consider like any other body with mass
- string has no internal resistance and aligns itself with the applied force.
- assume no stretching = constant length.

Example



- Tied together, all acceleration equal: a

$$\text{Mass } m_1 : F - T_1 = m_1 a \quad (1)$$

$$\text{Mass } m_2 : T_2 = m_2 a \quad (2)$$

$$\text{String} : T_1 - T_2 = m_s a \quad (3)$$

$$\textcircled{1} + \textcircled{2} + \textcircled{3} \quad F = (m_1 + m_2 + m_s) a$$

$$\begin{aligned} \bar{T}_1 &= F - m_1 a \\ &= (m_2 + m_s) a \end{aligned}$$

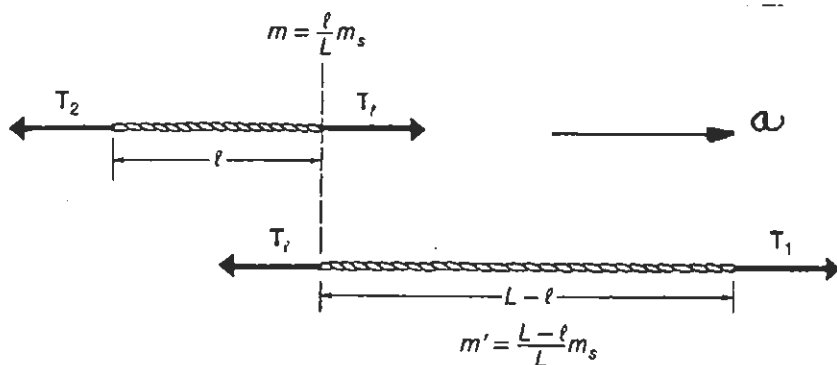
$$\bar{T}_2 = m_2 a$$

$$\bar{T}_1 \neq \bar{T}_2$$

Q: How does tension vary along string?

l : length of string from LHS.

T : Tension at location l .



The connecting string shown

is divided into lengths l and $L - l$.

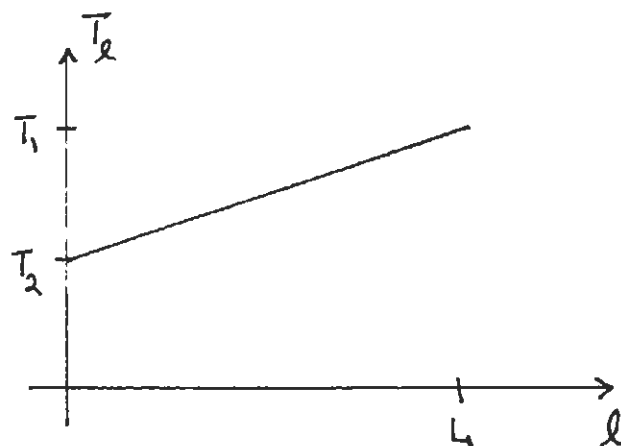
$$T_1 - T_l = \left(\frac{L-l}{L} \right) m_s a \quad \textcircled{4}$$

and

$$\frac{T_l - T_2}{L} = \frac{l}{L} m_s a \quad \textcircled{5}$$

Subs. for a . in $\textcircled{5}$

$$T_l = T_2 + \frac{l}{L} m_s \frac{F}{(m_1 + m_2 + m_s)}$$



T_l increases from $T_l = T_2$ at $l=0$, to $T_l = T_1$ at $l=L$.

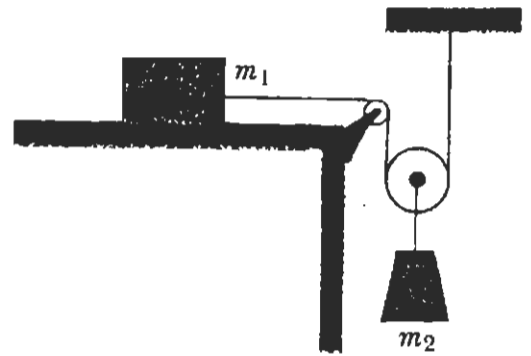
If $m_s \ll m_1$ and $m_s \ll m_2$, tensions are all equal.

$$T_l \sim T_1 \sim T_2$$

Example: Constraints

- Frictionless
- Massless string.
- Massless pulley.

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Mass m_1 :

$$T_1 = m_1 a_1 \quad (1)$$

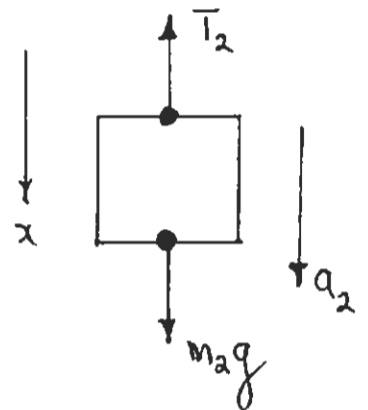
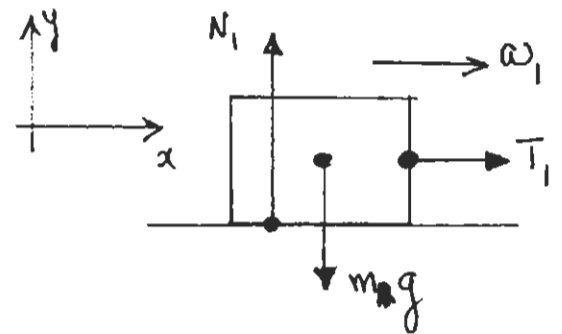
$$N - m_1 g = 0 \quad (2)$$

Mass m_2 :

$$m_2 g - T_2 = m_2 a_2 \quad (3)$$

Pulley:

$$2T_1 - T_2 = 0 \quad (4)$$



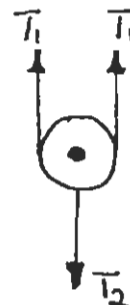
Constraint:

When m_1 moves a distance x_1 to right, the mass m_2 drops a distance

$$x_2 = x_1 / 2.$$

$$\therefore a_2 = \frac{d^2 x_2}{dt^2} = \frac{1}{2} \left(a_1 = \frac{d^2 x_1}{dt^2} \right)$$

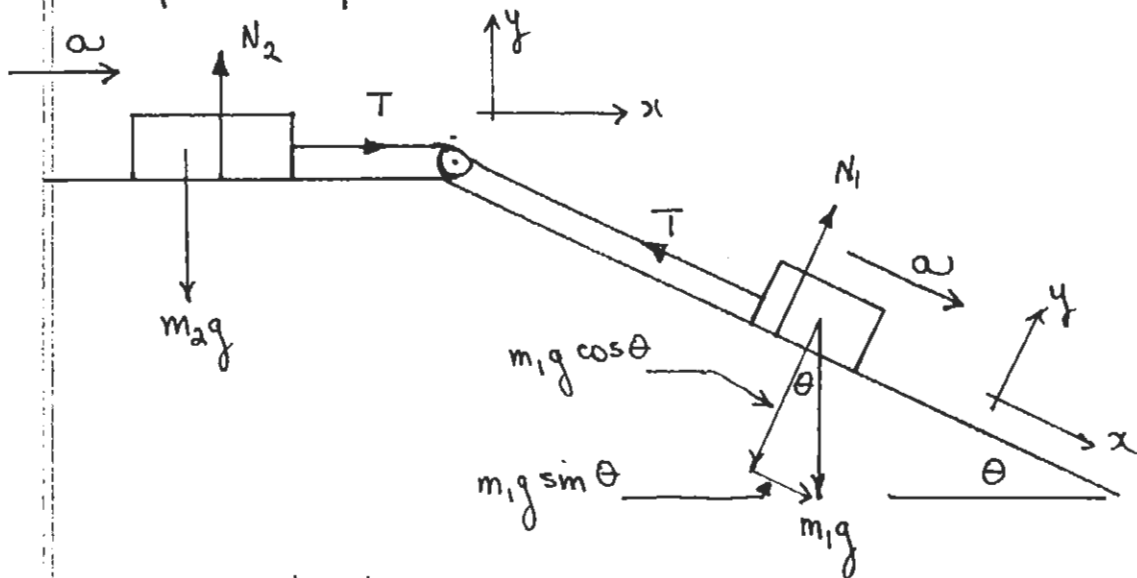
$$a_2 = \frac{1}{2} a_1$$



Solve: $a_2 = \left(\frac{m_2}{4m_1 + m_2} \right) g$ and $a_1 = 2a_2$

Example: Coupled Masses

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- Assume frictionless.
- Massless string.

Mass m_1 :

$$\text{Along } -x \quad m_1 g \sin \theta - T = m_1 a \quad (1)$$

$$\text{Along } -y \quad N_1 - m_1 g \cos \theta = 0 \quad (2)$$

Mass m_2 :

$$\text{Along } -x \quad T = m_2 a \quad (3)$$

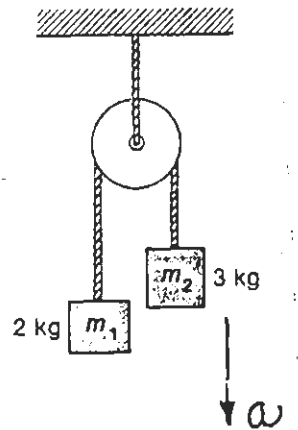
$$\text{Along } -y \quad N_2 - m_2 g = 0 \quad (4)$$

$$(1) + (3) \quad m_1 g \sin \theta = (m_1 + m_2) a$$

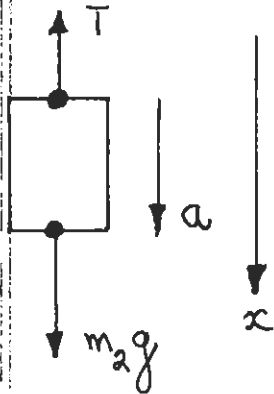
$$a = \left(\frac{m_1}{m_1 + m_2} \right) g \sin \theta$$

Atwood's Machine

- Frictionless, massless pulley.
- Massless rope

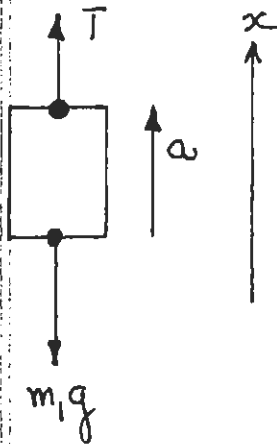


Mass m_2 :



$$(m_2 g - T) = m_2 a \quad (1)$$

Mass m_1 :



$$(T - m_1 g) = m_1 a \quad (2)$$

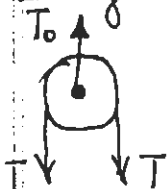
$$(1) + (2) \quad (m_2 - m_1)g = (m_2 + m_1)a$$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1} \right) g = \frac{1}{5} g \text{ (m/s}^2\text{)}$$

$$\text{From (2) } T = m_1(a + g) = m_1 \left[\frac{m_2 - m_1}{m_2 + m_1} + 1 \right] g$$

$$= \left(\frac{2m_1 m_2}{m_2 + m_1} \right) g$$

Pulley:



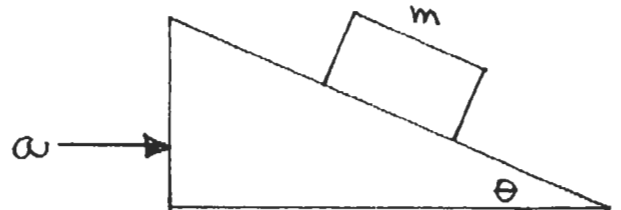
$$(T_0 - 2T) = 0$$

$$T_0 = 2T$$

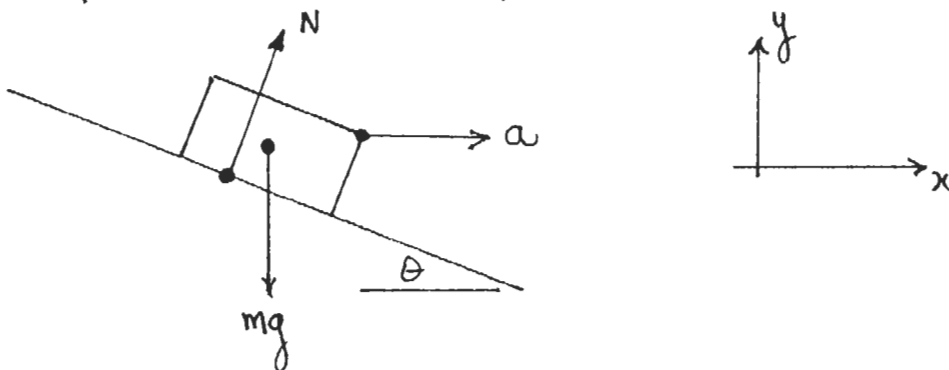
Example: Accelerated Inclined Plane

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- Block m on frictionless plane
- Wedge accelerated with ' a ' to right



Q: What is angle θ such that block does not slip up or down the plane.



Block will not slip if its acceleration equals that of wedge.

Forces along $-x$ $N \sin \theta = ma$ (1)

Forces along $-y$ $N \cos \theta - mg = 0$ (2)

From (2) $N = \frac{mg}{\cos \theta}$

$\therefore \frac{mg \sin \theta}{\cos \theta} = ma$

$a = g \tan \theta$ [Condition on a]