Particle speeds up / slows down } velocity changes

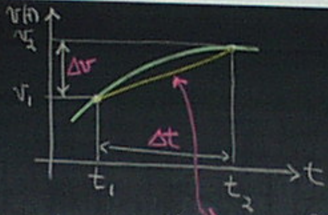
⇓

acceleration

Acceleration = Rate of change of velocity!

1-D ≡ Δv ; magnitude

2D/3D ≡ mag & direction



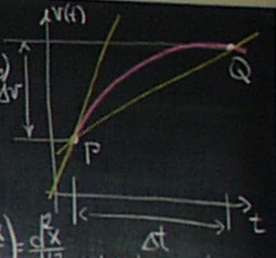
Average: $\bar{a}$

$$a_{av} = \bar{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \quad \frac{[L]}{[T^2]} = \frac{v(t+\Delta t) - v(t)}{\Delta t} \equiv$$ Slope of line connecting points (v_1, t_1) + (v_2, t_2)

Acceleration: Instantaneous

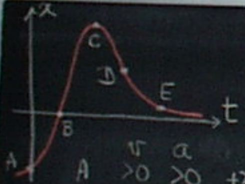
$$a(t) = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \frac{dv}{dt} \text{ (calculus)}$$

For Point P: $a(t)_P$ = slope of tangent line through P.

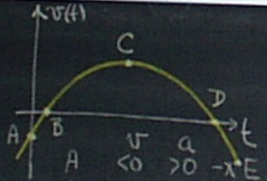


$$v(t) = \frac{dx}{dt} \qquad a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

2nd derivative



	v	a	
A	>0	>0	+x
B	>0	0	+x
C	0	<0	
D	<0	0	−x
E	<0	>0	−x



	v	a	
A	<0	>0	−x
B	0	>0	
C	>0	0	+x
D	0	<0	
E	<0	<0	−x

Example: $v(t) = \frac{1}{2}\beta t^2$

What is $\bar{a}$ betw. $t_1 = 1$ and $t_2 = 3$ s?

$\Delta t = t_2 - t_1 = 2$ s

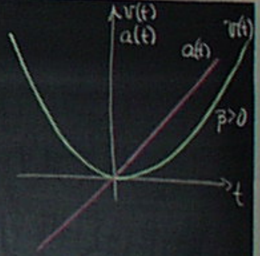
$$v(t+\Delta t) = \frac{1}{2}\beta(t+\Delta t)^2 = \frac{1}{2}\beta t^2 + \beta t \Delta t + \frac{1}{2}\beta \Delta t^2$$

$$\bar{a} = \frac{v(t+\Delta t) - v(t)}{\Delta t} = \beta t + \frac{1}{2}\beta \Delta t$$

$$\bar{a} = \beta(1) + \frac{\beta}{2} 2 = 2\beta \text{ m/s}^2$$

$$a(t) = \frac{dv}{dt} = \beta t$$

$$\left.\begin{aligned} a(1) &= \beta \\ a(3) &= 3\beta \end{aligned}\right\} \quad \bar{a} \equiv 2\beta$$



Lecture 3, Blackboard #1

Particle speeds up / slows down } velocity changes

⇓

acceleration

Acceleration = Rate of change of velocity!

1-D ≡ Δv ; magnitude

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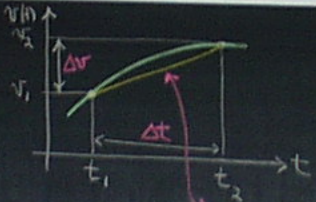

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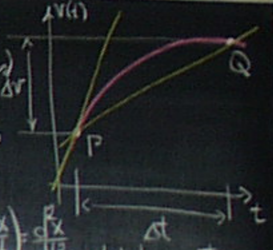
≡ Slope of line connecting points (v_1, t_1) + (v_2, t_2)

Acceleration: Instantaneous

$$a(t) = \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t} = \frac{dv}{dt} \text{ (calculus)}$$

For Point P: $a(t)_P$ = Slope of tangent line through P.



$$v(t) = \frac{dx}{dt} \qquad a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

2nd derivative

Constant Acceleration: Special Case

- Important class of problems.

$a(t) = a \equiv$ constant

$a(t) = \frac{dv}{dt} = a$ (const.)

$\therefore v(t)$ = st. line

$\bar{a} = a = \frac{v(t) - v_0}{t - 0}$ $\quad v_0 \equiv$ velocity at $t=0$

$v>0 \longrightarrow +x$ } motion

$v<0 \longrightarrow -x$ }

Solve: $\boxed{v(t) = v_0 + at}$ ①

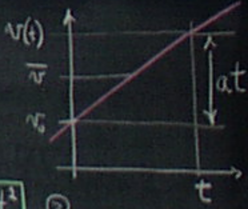
Let particle be at x_0 at $t = 0$

At time t: $x(t) = x_0 + \bar{v} t$

$v(t)$ is a st. line

$\therefore \bar{v} = \frac{1}{2}[v_0 + v(t)] = \frac{1}{2}[v_0 + v_0 + at]$

$\bar{v} = v_0 + \frac{at}{2}$



$\therefore \boxed{x(t) = x_0 + v_0 t + \frac{1}{2}at^2}$ ②

Pos. at $t=0$

change in pos. due to initial v.

change in pos. due to acceleration.

Lecture 3, Blackboard #2

From ① $t = \frac{v - v_0}{a}$

Sub in ② $x(t) - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2}a\left(\frac{v - v_0}{a}\right)^2$

Solve: $\boxed{v^2 - v_0^2 = 2a(x - x_0)}$ ③

We had: $\bar{v} = \frac{x - x_0}{t - 0} = \frac{v + v_0}{2}$

$\therefore \boxed{x - x_0 = \left(\frac{v + v_0}{2}\right)t}$ ④

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$

$\frac{dx}{dt} = v(t) = v_0 + at$

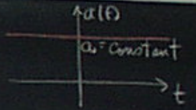
$\frac{d^2x}{dt^2} = \frac{dv}{dt} = a$

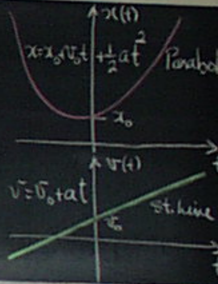
$\Downarrow$

Calculus

Const. a Problems

EQ.	Missing
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0





Example:

How long does it take a car to travel 30m if it accelerates from rest at a rate $a = 2\,m/s^2$

Know

$x_0 = 0$

$v_0 = 0$

$a = 2\,m/s^2$

$x = 30\,m$

Want

$t = ?$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$30 = 0 + 0(t) + \frac{1}{2} \times 2t^2$

$\therefore t = \sqrt{30}$

$= 5.5\,s$

Acceleration of Gravity: $a = g = 9.8\,m/s^2$

Objects in Free-Fall near earth's surface.

Greeks: Aristotle (384 - 322) BC.

- Heavier objects fall faster / Philosophy.

Galileo: (1564 - 1642)

- Expts + Observations
- All objects near earth accel. at same rate.

Very precise results

Diff: $1 \times 10^{-10} - 1 \times 10^{-12}$

g depends on altitude latitude and longitude

- earth is not round.
- finite radius.

Lecture 3, Blackboard #3

From ① $t = \frac{v - v_0}{a}$

Sub in ② $x(t) - x_0 = v_0\left(\frac{v - v_0}{a}\right) + \frac{1}{2} a \left(\frac{v - v_0}{a}\right)^2$

Solve: $\boxed{v^2 - v_0^2 = 2a(x - x_0)}$ ③

We had: $\bar{v} = \frac{x - x_0}{t - 0} = \frac{v + v_0}{2}$

$\therefore \boxed{x - x_0 = \left(\frac{v + v_0}{2}\right) t}$ ④

$x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$

$\frac{dx}{dt} = v(t) = v_0 + at$

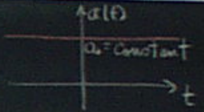
$\frac{d^2x}{dt^2} = \frac{dv}{dt} = a$

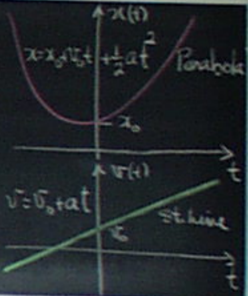
$\Downarrow$

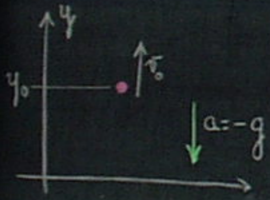
Calculus

Const. a Problems

EQ.	Missing
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v) t$	a
$x - x_0 = vt - \frac{1}{2} a t^2$	v_0







$y_0, v_0 \Rightarrow t = 0$

position velocity

Gravity: $a = -g$

$v = v_0 - gt$

$y = y_0 + v_0 t - \frac{1}{2} g t^2$

$v^2 - v_0^2 = -2g(y - y_0)$

$y = y_0 + vt + \frac{1}{2} g t$

"Gees"

Acceleration sometimes measured in units of "gees".

$a(\text{gees}) = \left(\frac{a}{g}\right)$ dimensionless

$a = a(\text{gees})\, g$

1 gee $\Rightarrow a = g$

2 gees $\Rightarrow a = 2g$

Lecture 3, Blackboard #4

Example:

Ball thrown upward with initial velocity of 25 m/s.

a) How long to reach max. height?

b) How high does it go?

c) What is velocity when it hits ground?

d) What is time for total trip?

$$\left.\begin{aligned} y_0 &= 0 \\ v_0 &= 25\,\text{m/s} \\ a &= -g \end{aligned}\right\} t=0$$

$$y(t) = v_0 t - \tfrac{1}{2} g t^2$$

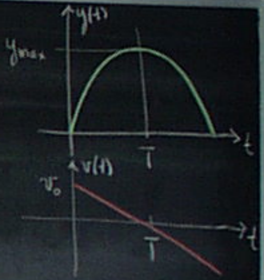
$$v(t) = v_0 - gt$$

What defines max. height?

At $t=T$; $v(T) \equiv 0$

$$v(T) = 0 = v_0 - gT$$

$$T = \frac{v_0}{g} = \frac{25\ \text{m/s}}{9.81\ \text{m/s}^2} = 2.55\,\text{s}$$



$$v^2 - v_0^2 = -2g(y - y_0)$$

At $v(T)=0$ $\quad y(T) = y_{max}$

$$0 - v_0^2 = -2g(y_{max} - 0)$$

$$y_{max} = \frac{v_0^2}{2g} = \frac{(25)^2}{2 \times 9.81} = 31.9\,\text{m.}$$

When ball returns: $y=0$

$$v^2 - v_0^2 = -2g(\underbrace{y}_{y=0} - 0)$$

$$\therefore v^2 = v_0^2$$

$$v = \pm 25\,\text{m/s}$$

$$y = 0 + v_0 t - \tfrac{1}{2} g t^2$$

$$0 = v_0 t - \tfrac{1}{2} g t^2 \quad (y=0 \text{ on return})$$

Solve $t=0$ (start)

and $t = \dfrac{2v_0}{g} = 2T$

Lecture 3, Blackboard #5

Example:

Ball thrown upward with initial velocity of 25 m/s.

a) How long to reach max. height?
b) How high does it go?
c) What is velocity when it hits ground?
d) What is time for total trip?

$$\left.\begin{aligned} y_0 &= 0 \\ v_0 &= 25\,\text{m/s} \\ a &= -g \end{aligned}\right\} t=0$$

$$y(t) = v_0 t - \tfrac{1}{2} g t^2$$

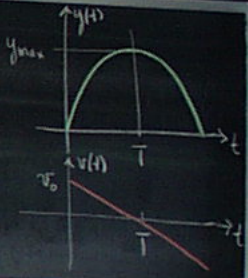
$$v(t) = v_0 - g t$$

What defines max. height?

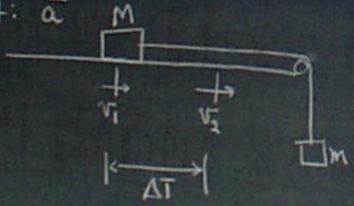
At $t = T$; $v(T) \equiv 0$

$$v(T) = 0 = v_0 - gT$$

$$T = \frac{v_0}{g} = \frac{25\ \text{m/s}}{9.81\ \text{m/s}^2} = 2.55\,\text{s}$$



Expt: $\bar{a}$



$M = 206\,\text{g}$

$m = 10\,\text{g}$

$$v_1 = \frac{10\,\text{cm}}{T_1}$$

$$v_2 = \frac{10\,\text{cm}}{T_2}$$

$$a_{exp} = \frac{\Delta v}{\Delta T} = \frac{v_2 - v_1}{\Delta T} =$$

$$a_{Theory} = g\,\frac{m}{M+m}$$

$$= 9.81\,\frac{10}{206+10} = 45.4\,\frac{\text{cm}}{\text{s}^2}$$

Forces