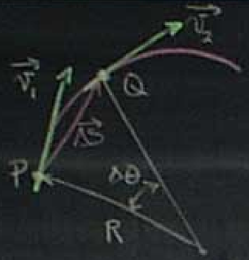
Uniform Circular Motion

Constant speed

Circle of radius R

Velocity changes in magnitude / direction

⇒ acceleration



$\vec a = \frac{d\vec v}{dt}$ Speed = $|v|$ = constant

$|\vec v_1| = |\vec v_2| = v =$ const.

$\Delta\vec v = \vec v_2 - \vec v_1$ Diff in vel for Δt

If Δt is small

$|\Delta\vec v| = v\,\Delta\theta$

$v\Delta\theta$ → exact as $\Delta t \to 0$ (1)

$a = \frac{|\Delta v|}{\Delta t} \sim v\left(\frac{\Delta\theta}{\Delta t}\right)$ $\frac{\Delta\theta}{\Delta t} = ?$ rad/s.

In time Δt, particle moves $(v\Delta t)$ dist.

This must equal $R\,\Delta\theta$ — Angle turned; Radius of Circle

$\therefore R(\Delta\theta) = v(\Delta t)$

$\frac{\Delta\theta}{\Delta t} = \frac{v}{R} = \omega =$ constant. ω = angular velocity (rad/s)

$a_R = a_\perp = v\left(\frac{v}{R}\right) = \frac{v^2}{R} = \omega^2 R$ (m/s²)

Direction of $\vec a$?

$\vec a = \frac{d\vec v}{dt}$

In limit as $\Delta t \to 0$

$\Delta\vec v \perp \vec v$

$\therefore \vec a \perp \vec v$ everywhere!!

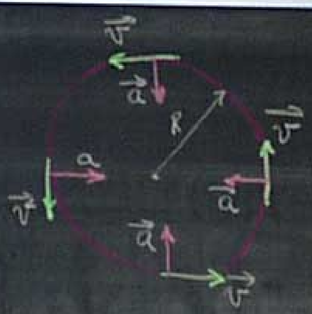
$\vec v \perp \vec R$: Circle

$\therefore \vec a$ is along $\vec R$

$\vec a$ = centripetal acceleration

$v = \frac{2\pi R}{\tau}$ ⟵ Circumference; ⟵ Period

$a_\perp = a_R = \frac{4\pi^2 R}{\tau^2}$ m/s²



Example: Carnival Ride   (2)

$\tau = 4$ s $a_\perp = ?$

$R = 5$ m

$v = \frac{2\pi R}{\tau} = \frac{2\pi(5)}{4} = 7.85$ m/s

$a_\perp = \frac{v^2}{R} = \frac{7.85^2}{5} = 12.3$ m/s²

$\frac{a_\perp}{g} = \frac{12.3}{9.81} = 1.25$ gees!

Lecture 7, Blackboard #1

$a_R = a_\perp = v\left(\frac{v}{R}\right) = \frac{v^2}{R} = \omega^2 R \ (m/s^2)$

Direction of $\vec{a}$?

$\vec{a} = \frac{d\vec{v}}{dt}$

In limit as $\Delta t \to 0$

$\vec{\Delta v} \perp \vec{v}$

$\therefore \vec{a} \perp \vec{v}$ everywhere!!

$\vec{v} \perp \vec{R}$ Circle

$\therefore \vec{a}$ is along $\vec{R}$

$\vec{a}$ = centripetal acceleration

$v = \frac{2\pi R}{\tau}$ ⟵ Circumference, ⟵ Period

$a_\perp = a_R = \frac{4\pi^2 R}{\tau^2} \ m/s^2$

Example Carousel Ride

③

$\tau = 4\,s$

$R = 5\,m$ $\quad a_\perp = ?$

$v = \frac{2\pi R}{\tau} = \frac{2\pi(5)}{4} = 7.85\ m/s$

$a_\perp = \frac{v^2}{R} = \frac{(7.85)^2}{5} = 12.3\ m/s^2$

$\frac{a_\perp}{g} = \frac{12.3}{9.8} = 1.25\ g\text{'s}!$

Example Rotating Earth

- At equator

$a_\perp = \frac{v^2}{R} = \omega^2 R$

$\omega = \frac{2\pi}{24 \times 3600}$ $\quad R = 6.38 \times 10^6\ m$

$a_\perp = \left(\frac{2\pi}{24 \times 3600}\right)^2 \times 6.38 \times 10^6$

$= 0.0337\ m/s^2$

Constant $\vec{a}$?

a) Free-Fall Problems

$\frac{d\vec{a}}{dt} = 0 \quad \vec{a} = \text{const}$ mag. dir.

b) Uniform Circular Motion

$\vec{a}$ mag. const.

direction – NO!!

$\frac{d\vec{a}}{dt} \neq 0 \quad \vec{a} \neq \text{const}!$ Eq's NG!

Non-Uniform Circular Motion

$a_\perp = a_R = \frac{v^2}{R} = \omega^2 R$ always!

$a_\parallel = a_t = \lim_{\Delta t \to 0} \frac{\Delta v_t}{\Delta t} = \frac{dv_t}{dt}$

$a = \sqrt{a_\perp^2 + a_\parallel^2}$

Cylindrical Coordinates

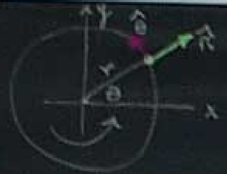
- useful for circular motion

$\vec{a} = \vec{a}_{\parallel} + \vec{a}_{\perp}$

$= \frac{d|v|}{dt}\hat{\theta} - \frac{v^2}{R}\hat{r}$

$\hat{r}, \hat{\theta}$: Move with particle.

...Vary with time.



$\hat{r}$: Unit vector along $\vec{r}$

$\hat{\theta}$: Unit vector along θ

Centrifugal Acceleration

- Acceleration away from center.
- Forget it quickly!!!

Ground Ref Frame

$a_{\perp}$ = centripetal away from apple!

Car Reference Frame ④

centrifugal accel away from door!

Fictitious Forces!!

Accel Coord System

Non-Inertial Reference Frame

Relativity of Motion: 1D

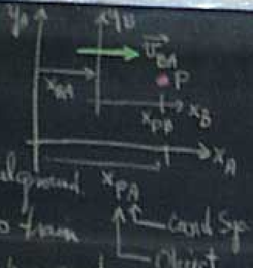
A. Ground } Point P - On Train

B. Train

$x_{PA} = x_{PB} + x_{BA}$

$\frac{dx_{PA}}{dt} = \frac{dx_{PB}}{dt} + \frac{dx_{BA}}{dt}$

$v_{PA} = v_{PB} + v_{BA}$



v_{BA}: vel of train rel ground

v_{PB}: vel of P rel to train

v_{PA}: vel of P seen from grnd.

Coord Sys

Object

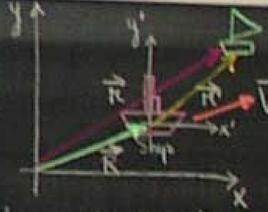
Relative Motion General 3D ⑤

- Motion is relative
- Vel/Accel depend on frame?
- Ground Ref F. (x, y, z, t)
- Ship Ref F. (x', y', z', t')

$t' = t$: Time is Absolute

$\vec{r}$: Pos. Vector of sailboat rel. to Ground

$\vec{r}'$: Pos. Vector of sailboat rel Ship.



Sailboat

$\vec{V}$ = const. vel of ship rel to gnd

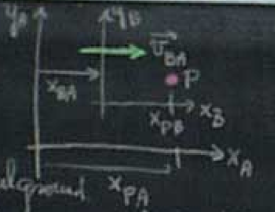
$\vec{r} = \vec{r}' + \vec{R}$ ①

For ... length is Absolute!!

Lecture 7, Blackboard #4

Relativity of Motion: 1D

A. Ground } Point P - On Train
B. Train }



$x_{PA} = x_{PB} + x_{BA}$

$\frac{dx_{PA}}{dt} = \frac{dx_{PB}}{dt} + \frac{dx_{BA}}{dt}$

$v_{PA} = v_{PB} + v_{BA}$

v_{BA}: vel of train rel ground

v_{PB}: vel of P rel to train

v_{PA}: vel of P seen from grnd.

x_{PA} — A: Coord Sys. — P: Object

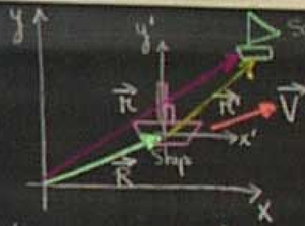
Relative Motion General 3D

- Motion is relative
- Vel/Accel depend on frame?
- Ground Ref F. (x, y, z, t)
- Ship Ref. F. (x', y', z', t')

$t' = t$: $v \ll c$ Time is Absolute

$\vec{r}$: Posit. Vector of sailboat rel. to Ground

$\vec{r}'$: Pos. Vector of sailboat rel Ship.



$\vec{V}$ = const. vel of ship rel to grnd

$\vec{r} = \vec{r}' + \vec{R}$ ①

For $v \ll c$ length is Absolute!!

⑤

Assume ship at $x=0, y=0, z=0$ at $t=0$

$\vec{R} = \vec{V} t$ ②

$\vec{r} = \vec{r}' + \vec{V} t$ ③

$\vec{r}' = \vec{r} - \vec{V} t$ ④

Special Case: $\vec{V} = V\hat{x}$ (x-axis)

$\vec{r}' = \vec{r} - Vt\hat{x}$ ⑤

$$\boxed{\begin{array}{ll} x' = x - Vt & y' = y \\ t' = t & z' = z \end{array}}$$ Galilean Transf.

Velocities

Diff Eq(4)

$$\vec{v}' = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t) = \frac{d\vec{r}}{dt} - \vec{V}$$

$\vec{v}' = \vec{v} - \vec{V}$ and $\vec{v} = \vec{v}' + \vec{V}$

For $\vec{V} = V\hat{x}$

$$\boxed{\begin{array}{l} v_x' = v_x - V \\ v_y' = v_y \\ v_z' = v_z \end{array}}$$

Acceleration

$$\vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}'}{dt} = \frac{d}{dt}(\vec{v} - \vec{V})$$

$\vec{V}$ — Const.

$$\vec{a}' = \frac{d\vec{v}}{dt} = \vec{a}$$

$\vec{a}$ is absolute rel. to an inertial frame.

If $\frac{d\vec{V}}{dt} \neq 0$. Non-Inertial Frame.

Extra terms in $\vec{a}$

Problem Solving Strategy

⑥

- Label all velocities
- Use two subscripts.
- First refers to object
- Second to reference frame where object has a given vel.

v_{AB} : velocity of A rel. to frame B.

Write Eq relating velocities

- First sub on LHS = First Sub on RHS.
- 2nd Sub on LHS = 2nd Sub on RHS in last term of exp.
- Adjacent sub, match + cancel.

$$v_{AB} = v_{AC} + v_{CB}$$

For two objects in ref. frames A, B.

$$\vec{v}_{BA} = -\vec{v}_{AB}$$

Example:

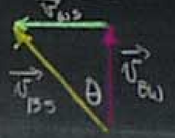
$\vec{v}_{WS}$ = 12 km/h West

$\vec{v}_{BW}$ = 20 km/h North

$\vec{v}_{BS}$ = ?

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS}$$

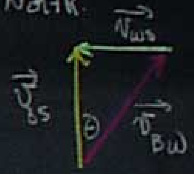
$$\tan\theta = \frac{12}{60} = 0.60$$



What is correct boat heading so boat travels st. across: i.e due North.

$$\vec{v}_{BS} = \vec{v}_{BW} + \vec{v}_{WS} \equiv \text{North.}$$

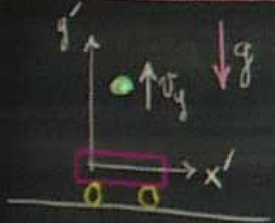
$$\sin\theta = \frac{v_{WS}}{v_{BW}} = \frac{12}{60} = 0.60$$



$\theta \sim 36.9°$ East of North / upstream

⑦

Relative Motion Cart:

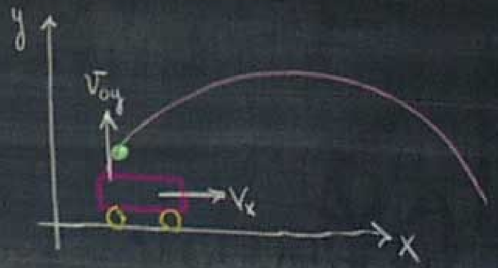


Cart Frame:

$$y' = y_0 + v_{oy}t - \tfrac{1}{2}gt^2$$

$$x' \equiv 0$$

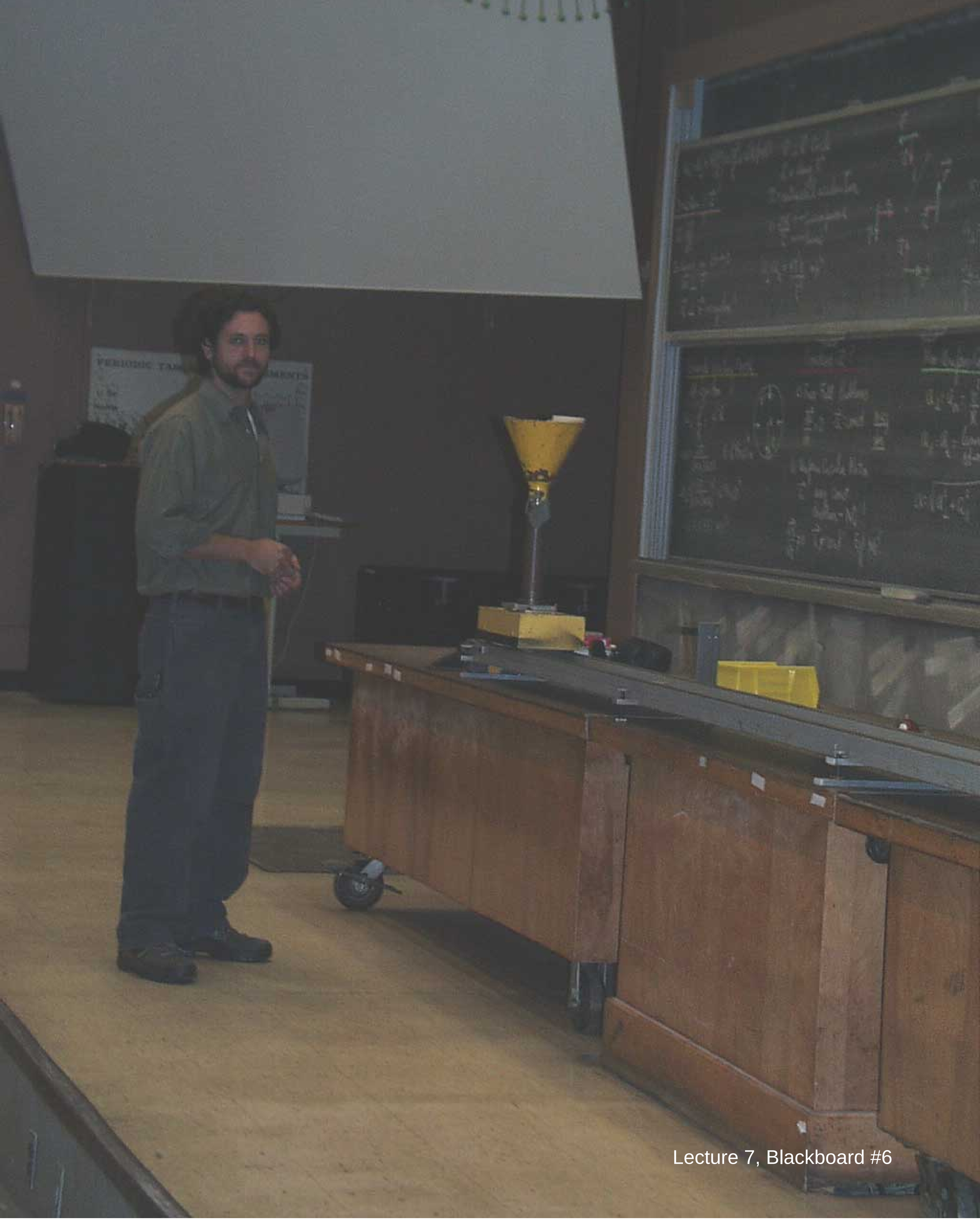
Ground Frame



$$x(t) = V_x t$$

$$y(t) = y_0 + v_{oy}t - \tfrac{1}{2}gt^2$$

⑧



Lecture 7, Blackboard #6