Motion with a Constant Force

$\vec{F} = m\vec{a}$ (2-nd Law)

If $\vec{F}$ = constant,

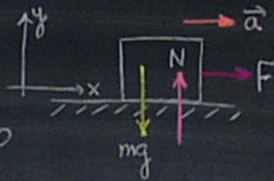
Then $\vec{a}$ = constant

$\Sigma F_x = ma_x$

$\Sigma F_y = ma_y$

$\Sigma F_z = ma_z$

Example



y-axis:

$N - mg = ma_y \equiv 0$

$N = mg$

x-axis:

$F = ma_x \Longrightarrow a_x = \frac{F}{m}$

Problem Solving Recipe

①

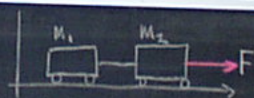
1. Draw a diagram showing key features of problem
2. Draw one or more free-body diagrams for the objects. For each object show forces acting on it only. Do not include internal forces. Do not include forces it exerts on other bodies.
3. Choose a coord. system. If $\vec{a}$ is known choose that direction along one axis.

Determine force components along axes.

4. Look for geometrical constraints. Write an equation for constraint.
5. Write $\vec{F} = m\vec{a}$ for each body.
6. Solve eq'ts
7. Check special cases, units, signs and compare with expectations.

Example

②



Body m_2: $F - T_2 = m_2 a_2$ ① x-axis

$N_2 - m_2 g = 0$ ② y-axis

Body m_1: $T_1 = m_1 a_1$ ③ x-axis

$N_1 - m_1 g = 0$ ④ y-axis

Bodies move together: constraint

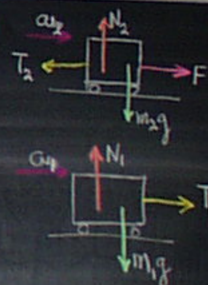
$a_1 = a_2$

Ideal string: No mass

$\vec{T}_1 = -\vec{T}_2 = T$

① + ③ $F = (m_1 + m_2) a$

$a = \frac{F}{m_1 + m_2}$ $\quad T = m_1 a = \frac{m_1}{m_1 + m_2} F$



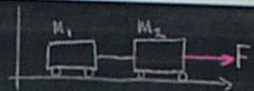
Lecture 10, Blackboard #1

②

Determine force components along axes.

4. Look for geometrical constraints. Write an equation for constraint.
5. Write $\vec{F} = m\vec{a}$ for each body.
6. Solve eq't's.
7. Check special cases, units, signs and compare with expectations.

Example



Body m_2: $F - T_2 = m_2 a_2$ ① x-axis

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$N_1 - m_1 g = 0$ ④ y-axis

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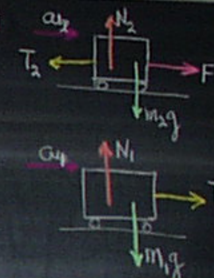


③

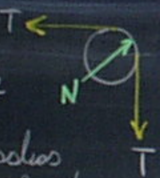
Ideal Pulley/Pegs

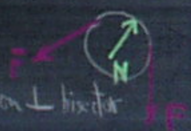
. change direction of force exerted by string.

. If string and pulley massless magnitude of T same at both ends.

. Ideal pulley massless

No friction.





N is on ⊥ bisector

Block + Pulley

Pulley: $T - 2F = 0$

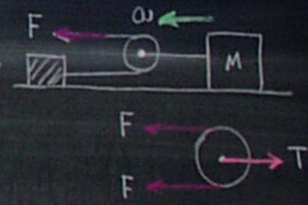
$T = 2F$ [Mech. Advantage 2x]

Mass m: $T = ma$ $\quad N - mg = 0$

$2F = ma$

$a = \frac{2F}{m}$ $\quad F = \frac{ma}{2} = \frac{300 \times .05}{2}$

$= 7.5\,N$

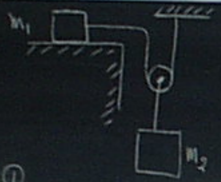


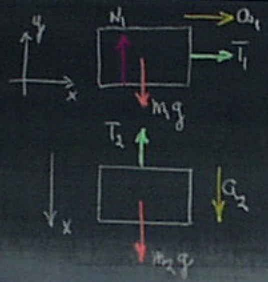
$M = 300\,kg$

$a = 5\,cm/s^2$

Lecture 10, Blackboard #2

Example:
- Frictionless
- Massless pulley
- Massless String





Mass m_1: $T_1 = m_1 a_1$ ①

$N_1 - m_1 g = 0$ ②

Mass m_2: $m_2 g - T_2 = m_2 a_2$ ③

Pulley: $2T_1 - T_2 = 0$ ④

Constraint: When m_1 move distance x_1
m_2 move distance x_2

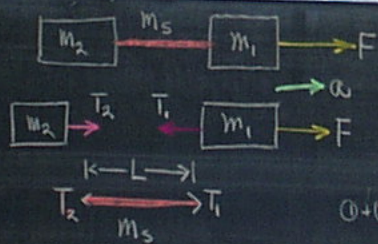
$x_2 = \frac{1}{2}x_1 \Rightarrow \therefore a_2 = \frac{1}{2}a_1 \quad \frac{d^2x_2}{dt^2} = \frac{1}{2}\frac{d^2x_1}{dt^2}$

Solve: $a_2 = \left(\frac{m_2}{4m_1 + m_2}\right)g$

$a_1 = 2a_2$

④

Strings / Ideal / Mass?
- Tension only
- No push or compression (rod)
- If massless, tension the same everywhere.
- No stretching
- Always aligns with force.



Tied together: common a.

Mass m_1: $F - T_1 = m_1 a$ ①

Mass m_2: $T_2 = m_2 a$ ②

String: $T_1 - T_2 = m_s a$ ③

①+②+③ $F = (m_1 + m_2 + m_s)a$

$T_1 = F - m_1 a = (m_2 + m_s)a$

$T_2 = m_2 a$

$T_1 \neq T_2$ string has mass.

If $m_s = 0$ $T_1 = T_2$!!

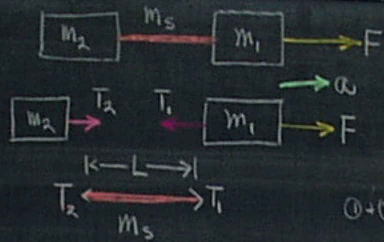
How does tension change along the string?

⑤

Lecture 10, Blackboard #3

Strings / Ideal/Mass?

- Tension only
- No push or compression (rod)
- If massless, tension the same everywhere.
- No stretching.
- Always aligns with force.



Tied together: common a.

Mass m_1: $F - T_1 = m_1 a$ ①

Mass m_2: $T_2 = m_2 a$ ②

String: $T_1 - T_2 = m_s a$ ③

①+②+③ $F = (m_1 + m_2 + m_s)a$

$T_1 = F - m_1 a = (m_2 + m_s)a$

$T_2 = m_2 a$

$T_1 \neq T_2$ string has mass.

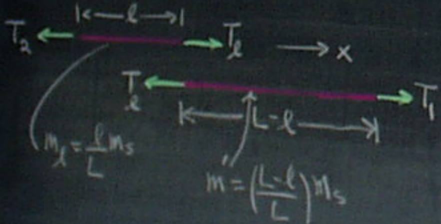
If $m_s = 0$ $T_1 = T_2$!!

How does tension change along the string?

⑤

T_ℓ : Tension at location ℓ

ℓ : length from LHS.



$T_1 - T_\ell = \left(\frac{L-\ell}{L}\right) m_s a$

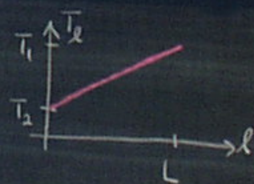
$T_\ell - T_2 = \frac{\ell}{L} m_s a$

Sub. for a

$$T_\ell = T_2 + \frac{\ell}{L} \frac{m_s F}{m_1 + m_2 + m_s}$$

$\ell = 0$ $T_\ell = T_2$

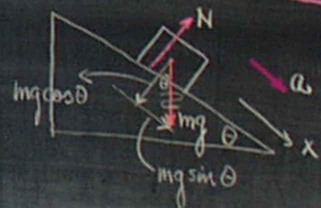
$\ell = L$ $T_\ell = T_2 + m_s a = T_1$



If $m_s \ll m_1$
$m_s \ll m_2$

$T_\ell \sim T_1 \sim T_2$

Example - Inclined Plane



- No friction
- Gravity only

$mg\sin\theta = ma$

$a = g\sin\theta$

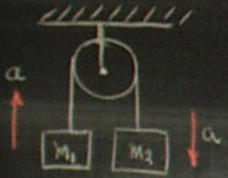
$N - mg\cos\theta = 0$

$N = mg\cos\theta$

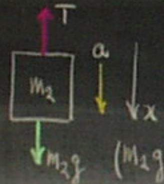
⑥

Lecture 10, Blackboard #4

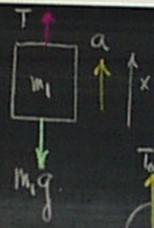
Attwood's Machine



Massless pulley
Massless string



$(m_2 g - T) = m_2 a$ ①



$T - m_1 g = m_1 a$ ②

①+② $(m_2 - m_1)g = (m_2 + m_1)a$

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g$$

Pulley: $T_0 - 2T = 0$

$T_0 = 2T$

$m_1 = 550g$

$m_2 = 560g$

$S = \frac{1}{2} a t^2$

$$T = m_1(a+g) = m_1\left[\frac{m_2 - m_1}{m_2 + m_1} + g\right]$$ ⑦

$$= \left(\frac{2 m_1 m_2}{m_2 + m_1}\right) g$$

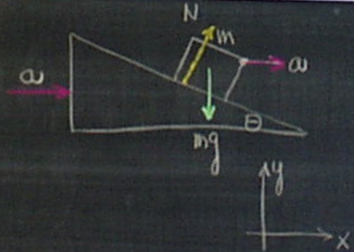
$a_{exp} = \frac{2S}{t^2} = \frac{2 \times 1m}{}$

$a_{Thy} =$

Example: Accelerated Inclined Plane

- Block on frictionless plane
- Wedge accelerated 'a' to right

Q. What is angle θ such that block does not slip up or down?



Block does not slip if its acceleration equals that of wedge!! ⑧

Forces along x: $N \sin\theta = ma$ ①

Forces along y: $N \cos\theta - mg = 0$ ②

From ② $N = \frac{mg}{\cos\theta}$

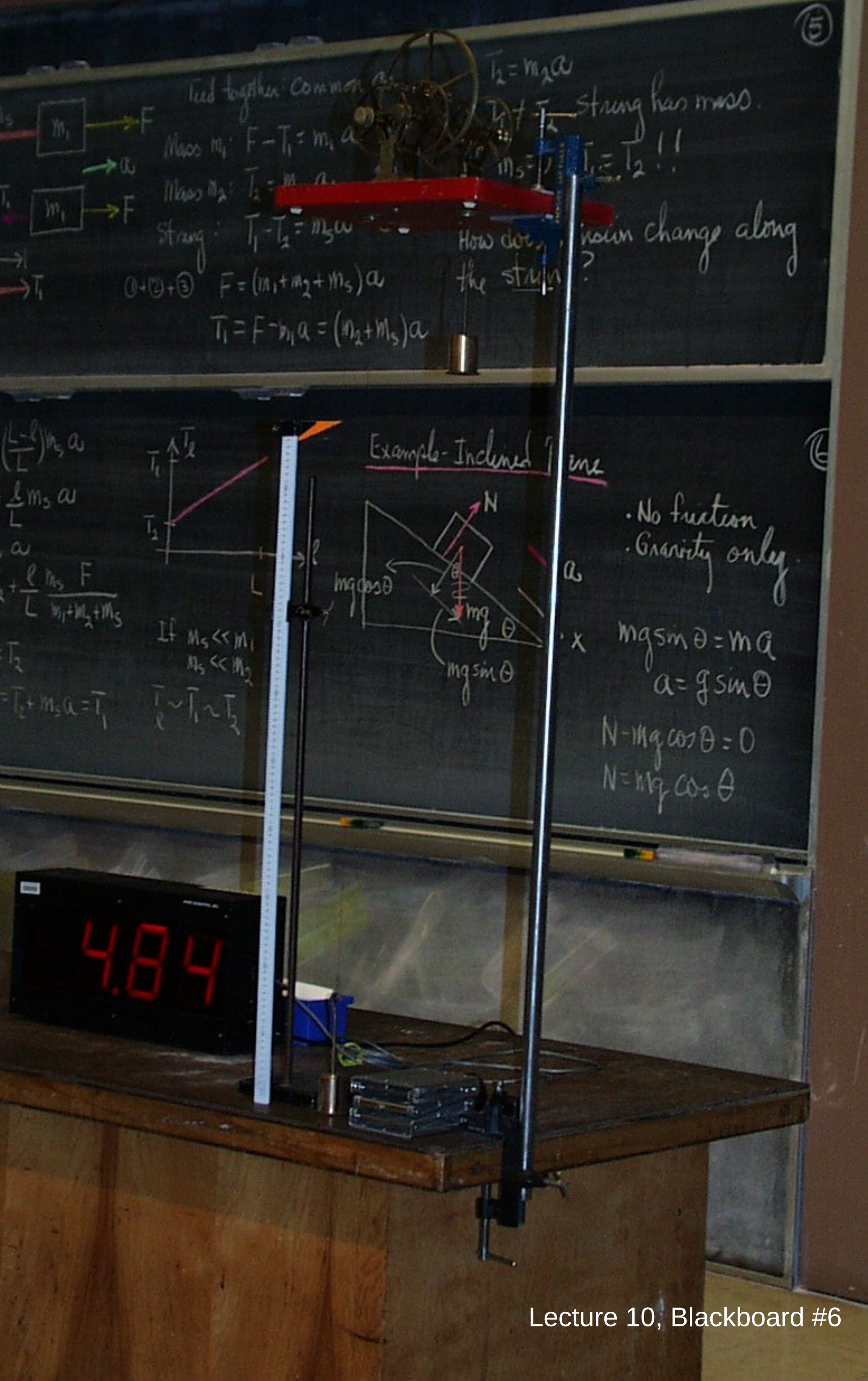
$\therefore \frac{mg}{\cos\theta} \sin\theta = ma$

$\therefore a = g \tan\theta$

If $\theta = 0$ $a = 0$

$\theta = 45^\circ$ $a = g$

Lecture 10, Blackboard #5



Lecture 10, Blackboard #6