Special Relativity

Review: Newtonian Mechanics

Galilean Transformation

- Motion is relative
- Velocity/acceleration depend on frame.
- Ship velocity $\vec{V} = \frac{d\vec{R}}{dt}$ rel. to ground
- Systems coincide at $t = 0$.

Ground: x, y, z, t

Ship: x', y', z', t'

Time is absolute

$v \ll c$; $t = t'$

$\vec{r}, \vec{r}'$: Position vectors!

$\vec{r} = \vec{r}' + \vec{R}$ ①

$\vec{R} = \vec{V}t$ ②

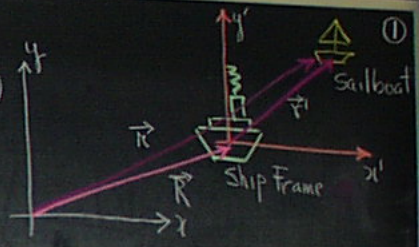
$\vec{r} = \vec{r}' + \vec{V}t$ ③

$\vec{r}' = \vec{r} - \vec{V}t$ ④

Special Case: $\vec{V} = V\hat{\imath}$ (along x-axis)

$\vec{r}' = \vec{r} - Vt\,\hat{\imath}$

$$\left.\begin{aligned} x' &= x - Vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \text{Galilean Trans. Eq's.}$$



①

Velocities

Eq④ $\vec{v}' = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t)$

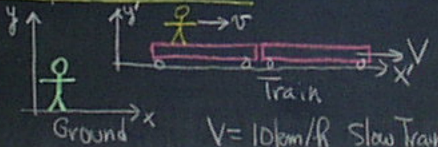
$\vec{v}' = \frac{d\vec{r}}{dt} - \vec{V} = \vec{v} - \vec{V}$

$\vec{v} = \vec{v}' + \vec{V}$

For $\vec{V} = V\hat{\imath}$

$$\left.\begin{aligned} v_x' &= v_x - V \\ v_y' &= v_y \\ v_z' &= v_z \end{aligned}\right\} \text{Vel. Trans.}$$

Example: Man on Train



$V = 10\,\text{km/h}$ Slow Train

$v = 5\,\text{km/h}$ Walking

$v_x = v_x' + V$

$= 5 + 10 = 15\,\text{km/h}$

Rel. to ground!

1h: Train $\Delta x' = 15\,\text{km}$

1h: Man $\Delta x_M' = 5\,\text{km}$

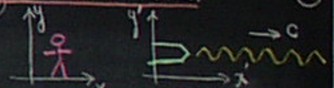
$\Delta x = 15 + 5 = 20\,\text{km}$

$v = \frac{20\,\text{km}}{1\,\text{h}} = \frac{20\,\text{km}}{\text{h}}$

Makes good sense!!

Example: Light Beam and Rocket

②



$V = \frac{3}{4}c$

$v_x = v_x' + V$

$= c + \frac{3}{4}c = \frac{7}{4}c$! Almost 2×Light Speed.

If $v_x' = -c$ Light opp. direction

$v_x = -c + \frac{3}{4}c = \frac{c}{4}$ Light almost stopped!!

Serious Problem!!

Lecture 32 Blackboard #1

Velocities

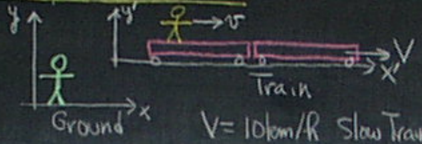
Eq(4) $\vec{v}' = \frac{d\vec{r}'}{dt} = \frac{d}{dt}(\vec{r} - \vec{V}t)$

$\vec{v}' = \frac{d\vec{r}}{dt} - \vec{V} = \vec{v} - \vec{V}$

$\vec{v} = \vec{v}' + \vec{V}$

For $\vec{V} = V\hat{i}$ $\quad \left.\begin{array}{l} v_x' = v_x - V \\ v_y' = v_y \\ v_z' = v_z \end{array}\right\}$ Vel. Trans.

Example: Man on Train



V = 10 km/h Slow Train

v = 5 km/h Walking

$v_x = v_x' + V$
$= 5 + 10 = 15$ km/h
Rel to ground!

1h: Train $\Delta x' = 15$ km

1h: Man $\Delta x_M' = 5$ km

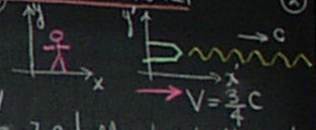
$\Delta x = 15 + 5 = 20$ km

$v = \frac{20\,km}{1h} = \frac{20\,km}{h}$

Makes good sense!!

Example: Light Beam and Rocket

②



$v_x = v_x' + V$
$= c + \frac{3}{4}c = \frac{7}{4}c$! Almost 2x light speed.

If $v_x' = -c$ Light opp direction

$v_x = -c + \frac{3}{4}c = -\frac{c}{4}$ light almost stopped!!

Serious Problem!!

Galilean Accelerations

$\vec{a}' = \frac{d\vec{v}'}{dt'} = \frac{d\vec{v}'}{dt} = \frac{d}{dt}(\vec{v} - \vec{V})$

$\vec{a}' = \frac{d\vec{v}}{dt} = \vec{a}$

Acceleration is absolute.

$\left.\begin{array}{l} a_x' = a_x \\ a_y' = a_y \\ a_z' = a_z \end{array}\right\}$ True because $\frac{dV}{dt} = 0$

Otherwise Not!!

If $v \simeq c$
Analysis is Incorrect!!
Will Show!

Light: Theory of Waves

- Young (1801-1804)
- Wave phenomenon
- Interference Effects
- Diffraction
- Polarization

③

Light Wave ⟷ Sound Wave.

Sound needs medium (fluid) to propagate.

Maybe light needs a medium.

— luminiferous ether

Speed through medium should be independant of velocity of source.

Lecture 32 Blackboard #2

Stellar Aberration

Bradley 1726

- Use parallax to measure distance to the nearest star.
- Know diameter of earth's orbit.

$$\varphi = \frac{2R}{L} = \frac{2\times1.5\times10^{11}}{9.46\times10^{15}} = 0.34\times10^{-4}\ \text{rad}$$

$L = 1$ light-year

$\varphi = 6.5''$ arc.

$\varphi = 0.65''$ $[L = 10\ \text{Ly}]$

Observations showed

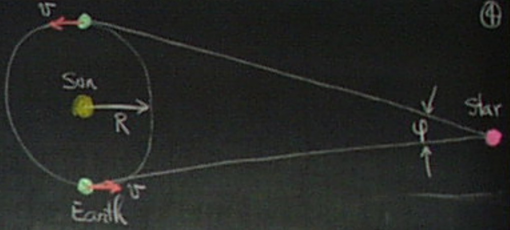
$(\varphi)_{exp} = 41''$. Much Larger

φ: Correlated to earth's velocity and not just position.

Light Speed = c

Earth Velocity = v

$$v = \frac{2\pi\times1.5\times10^{11}}{365\times24\times3600} = 30\ \text{km/s}$$



④

Explain Aberration

- Star directly overhead
  If earth at rest light goes st. down.
- Earth moving: v
  Need to tilt telescope to keep light from hitting sides!!

Let Δt = time to travel down telescope $\Delta t = l/c$

ΔS = distance scope moves in Δt

$\Delta S = v\,\Delta t$.

Angle of tilt.

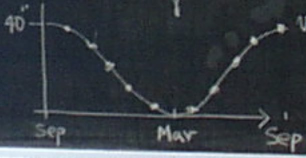
$$\tan\alpha = \frac{\Delta S}{\Delta l} = \frac{v\Delta t}{c\Delta t} = \frac{v}{c}$$

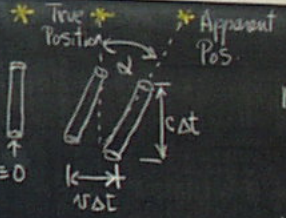
For earth: $\alpha = 20.5''$ arc.

In 6 months velocity reverses

Star traces out a cone with

$2\alpha = 41''$ of arc.





Exp: Perfect Agreement!

Ether not dragged with earth!

⑤

Explain Aberration

- Star direcly overhead.
- If earth at rest light goes st. down.
- Earth moving : v
- Need to tilt telescope to keep light from hitting sides!!

Let Δt = time to travel down telescope $\Delta t = l/c$

Δs = distance scope moves in Δt

$\Delta s = v\,\Delta t$.

Angle of tilt.

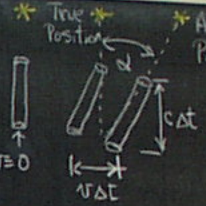
$$\tan\alpha = \frac{\Delta s}{\Delta l} = \frac{v\Delta t}{c\Delta t} = \frac{v}{c}$$

For earth: $\alpha = 20.5''$ arc.

In 6 months velocity reverses

Star traces out a cone with

$2\alpha = 41''$ of arc.



Exp: Perfect Agreement!

Ether not dragged with earth!

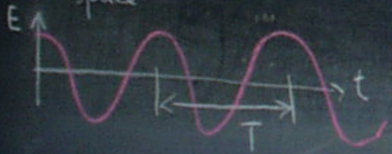
⑤

Michelson-Morley

- Sound : medium = air
- Light : medium = ether

} Wave Propagation

↳ electromagnetic wave disturbance oscillates and travels through space.



f : frequency of oscillation Hz

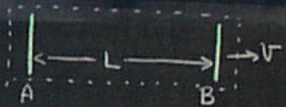
T : Period of oscillation

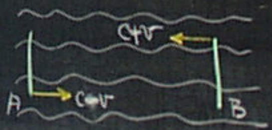
λ = wavelength

$c = f\lambda = 3\times10^8$ m/s.

Assume light moves though ether filling all space.

Earth moves with velocity, v





Round Trip A→B→A

Separation = L

Time: A→B $t_1 = \frac{l}{c-v}$

B→A $t_2 = \frac{l}{c+v}$

If $v=0$ $t_0 = l/c$

Effect of earth's motion delays return of light.

$$\Delta t = t_1 + t_2 - 2t_0$$

$$= \frac{l}{c-v} + \frac{l}{c+v} - \frac{2l}{c}$$

$$= \frac{l}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c} - 2\right]$$

⑥

$$\Delta t = \frac{2\ell}{c}\left[\frac{1}{1-v^2/c^2} - 1\right]$$
$$\simeq \frac{2\ell}{c}\,\frac{v^2}{c^2}$$

For $v/c = 10^{-4}$

$\ell = 1\,m$

$\Delta t = 7\times 10^{-17}\,s$

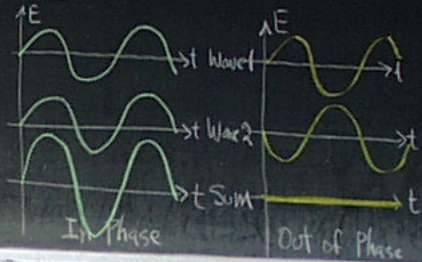
Impossible to Measure!!

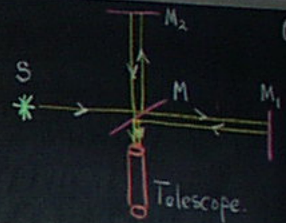
Michelson-Morley Experiment (1882)

- Mirror M → 2 beams.
- M → M1 → Observer
- M → M2 → Observer
- No delay → bright fringe.
- If $\Delta t = T/2$ one half cycle.

cancellation → dark fringe.

- light on each path twice

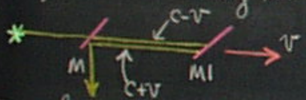




⑦

Expt can measure $\lambda/100$!

- Align earth v along M1



$$T_1 = \frac{\ell}{c-v} + \frac{\ell}{c+v}$$
$$= \frac{\ell}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c}\right]$$

$$T_1 = \frac{2\ell}{c}\left[\frac{1}{1-v^2/c^2}\right]$$
$$\simeq \frac{2\ell}{c}\left[1+v^2/c^2\right]$$

ℓ = length of each arm.

M → M2 → M also delayed.

M2 is moving to right velocity v

Light travels angular path.

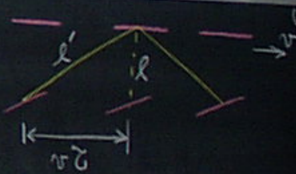
Let τ = time M → M2.

$$\ell' = (\ell^2 + v^2\tau^2)^{1/2}$$
$$\ell' = c\tau$$
$$\tau = \frac{(\ell^2+v^2\tau^2)^{1/2}}{c}$$

$$\tau^2 = \frac{\ell^2}{c^2} + \frac{v^2}{c^2}\tau^2$$
$$\tau = \frac{\ell}{c}\,\frac{1}{\sqrt{1-v^2/c^2}}$$

For complete trip

$$T_2 = 2\tau = \frac{2\ell}{c}\,\frac{1}{\sqrt{1-v^2/c^2}}$$

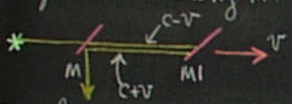


⑧

8

Exp can measure $\lambda/100$!

·Align earth v along M1



$$T_1 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{l}{c}\left[\frac{1}{1-v/c} + \frac{1}{1+v/c}\right]$$

$$T_1 = \frac{2l}{c}\left[\frac{1}{1-v^2/c^2}\right]$$

$$\simeq \frac{2l}{c}\left[1 + v^2/c^2\right]$$

l = length of each arm.

M → M2 → M also delayed.

M2 is moving to right velocity v

Light travels angular path.

Let τ = time M → M2.

$$l' = (l^2 + v^2\tau^2)^{1/2}$$

$$l' = c\tau$$

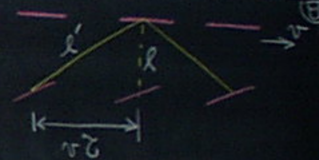
$$\tau = \frac{(l^2 + v^2\tau^2)^{1/2}}{c}$$

$$\tau^2 = \frac{l^2}{c^2} + \frac{v^2}{c^2}\tau^2$$

$$\tau = \frac{l}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$

For complete trip

$$T_2 = 2\tau = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}}$$



9

$$T_2 \simeq \frac{2l}{c}\left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)$$

$$\Delta T = T_1 - T_2 = \frac{l}{c}\frac{v^2}{c^2}$$

Apparatus has no zero.

Arms not same length.

Line up M1 → $\vec{v}$

Then rotate so M2 → $\vec{v}$

Change in delay is $2\Delta T$

$\Delta T \to \frac{\lambda}{2}$ shifts pattern by 1-fringe.

$2\Delta T$ shifts N fringes.

$$N = \frac{2\Delta T}{\lambda/c} = \frac{2l}{\lambda}\frac{v^2}{c^2}$$

$l = 10\,m$

$\lambda = 5.5\times10^{-7}\,m$ (Yellow Light)

$\frac{v}{c} = 10^{-4}$

$$N = \frac{2\times10\times(10^{-4})^2}{5.5\times10^{-7}}$$

$= 0.36$ Fringes.

·1887: No shift observed!!

Maybe ether carried by earth.

Then no shift.

Stellar Aberration says ether stays put!

Big Problem

Special Relativity!!