Problem Set 3: Newton's Laws of Motion, Motion: Force, Mass, and Acceleration, Vectors in Physics Solutions

Problem 1: (*Relative Inertial Frames*)

You and a friend are in an elevator that is rising at constant but unknown velocity v_0. You are curious to know what speed the elevator is moving. You each happen to be carrying altimeters. You get an idea and drop your keys from a height h above the floor of the elevator. You hold one altimeter at the point where you dropped the keys and your friend places the other one on the floor. You check your altimeter when you release the keys and find the keys are a height d above sea level. Your friend checks the other altimeter when the keys hit the floor and finds the keys are the same height d above sea level. Assume that the gravitational constant is g. You will analyze this problem from two different reference frames. The first reference frame moves with the elevator. The second reference frame is fixed to the ground.

   a) As seen from a reference frame moving with elevator, how long does it take for the keys to hit the floor?

   b) As seen from a reference frame fixed to the ground, using your result from part a), find an expression for the speed of the elevator in terms of the height h.

   c) As seen from a reference frame fixed to the ground, what maximum height above sea level did the keys reach?
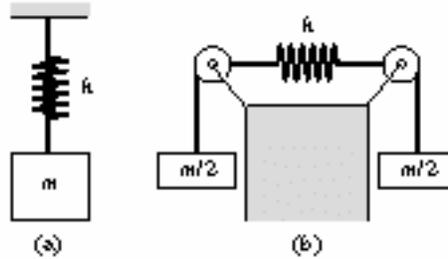
Solution

a) As in your reference frame you are dropping the keys from rest at an height h above the floor it will take a time $t = \sqrt{\frac{2h}{g}}$ for them to reach the floor.

   b) we know that in the time t the keys were in free fall the elevator covers a distance h so the speed of the elevator is: $v = \sqrt{\frac{hg}{2}}$

   c) In this reference frame the keys go up for a time $\frac{t}{2}$ before falling after that they fall for $s = \frac{gt^2}{8} = \frac{h}{4}$ the maximum height of the keys is so: $d + \frac{h}{4}$

Problem 2 Concept Question: (*Hooke's Law*)

A body of mass m is suspended from a spring with spring constant k in configuration (a) and the spring is stretched $0.1m$. If two identical bodies of mass $m/2$ are suspended from a spring with the same spring constant k in configuration (b), how much will the spring stretch? Explain your answer.
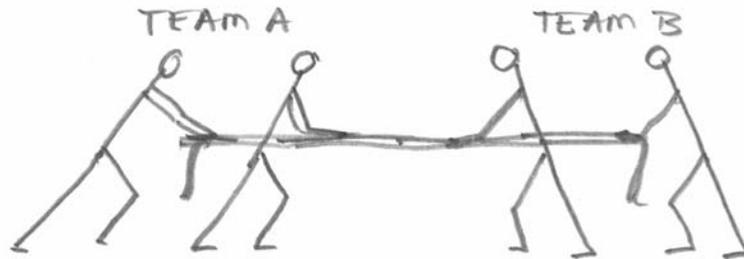

Solution

The tension exerted on the string in the second situation is half with respect to the first. The answer is so 5cm.

If you have problems substitute one of the two masses with a fixed hook; does the spring change its lenght ?

Problem 3: Free Body Diagrams

Two teams, A and B, are competing in a tug-of-war. Team A is stronger, but neither team is moving because of friction.

a) Draw free-body diagrams for team A, for team B, and for the rope.

b) Both the following statements are true; explain them. Two teams having a tug of war must always pull equally hard on one another. The team that pushes harder against the ground wins.

Solution:
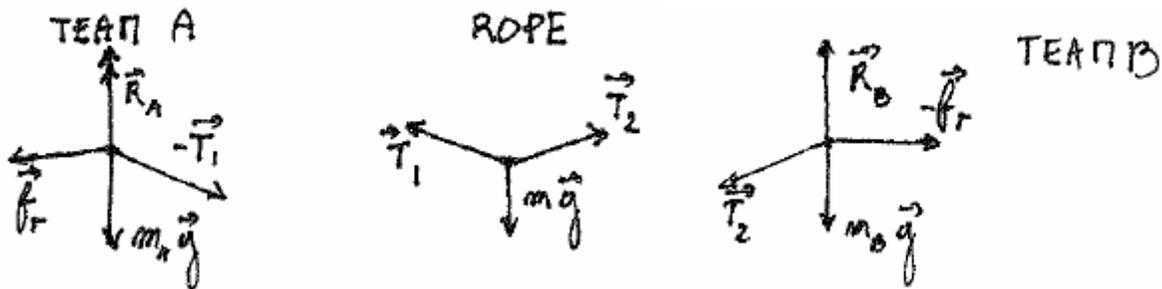
b)

The two teams have to pull the rope equally hard otherwise this will accelerate.

The horizontal component of the force the rope applies on a team must be equilibrated by the friction force between the ground and the team. Thus pushing more against the ground allows one team to pull the rope harder.

Problem 4: *Second Law and projectile motion*

An object with mass $m = 2.0\,kg$ slides down a roof a distance 6.0 m on a roof that is inclined at an angle of $3.0\times10^{1}\,deg$ to the horizontal. The roof has a coefficient of kinetic friction $\mu_k = 2.0\times10^{-1}$. The edge of the bottom of the roof is at a height $h = 9.0\,m$ above the ground. What is the horizontal displacement from the edge of the roof to the point where the object hits the ground?

Solution:

First we have to determine the speed of the object when it leaves the roof. The friction force on the object directed opposite to the direction of motion is $|F_f| = \mu_k mg \cos(\theta)$. Opposite to that the gravitational force $|F_g| = mg \sin(\theta)$ is pulling the object downwards. The acceleration of the object along the direction of the roof is then $a = g(\sin(\theta) - \mu_k \cos(\theta))$. The object will slide for a time $t = \sqrt{\frac{2d}{a}}$ reaching the speed $v_0 = at = \sqrt{2dg(\sin(\theta) - \mu_k \cos(\theta))}$.

The time of fall from the edge of the roof is then given by solving:

$\frac{1}{2}gt_f^2 + v_0 \sin(\theta)t_f = h$ that is $t_f = \frac{1}{g}(\sqrt{v_0^2 \sin^2(\theta) + 2gh} - v_0 \sin(\theta))$.

The horizonal displacement is then: $l = v_0 \cos(\theta)t_f = \frac{v_0 \cos(\theta)}{g}(\sqrt{v_0^2 \sin^2(\theta) + 2gh} - v_0 \sin(\theta))$. Plugging numerical values gives: $l = 5.7\ m$

Problem 5: (*Experiment 4 Circular Motion Pre-Lab Question*)

Consider a spring with negligible mass that has an unstretched length $l_0 = 8.8 \times 10^{-2} m$. A body of mass, $m_1 = 1.5 \times 10^{-1} kg$, is suspended from one end of the spring. The other end of the spring is fixed. After a series of oscillations has died down, the new stretched length of the spring is $l = 9.8 \times 10^{-2} m$.
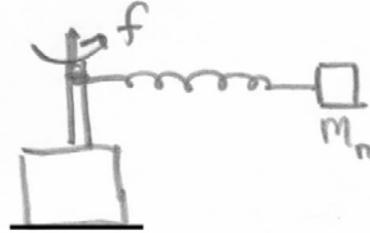
part a)	parts b) - d)

a) Assume that the spring satisfies Hooke's Law when it is stretched. What is the spring constant?

The body is then removed and one end of the spring is attached to the central axis of a motor. The axis of the motor points along the vertical direction. A small ball of mass, $m_n = 3.0 \times 10^{-3} kg$, is then attached to the other end of the spring. The motor rotates at a constant frequency, $f = 2.0 \times 10^1 Hz$.

b) How long does it take the ball to complete one rotation?

c) What is the angular frequency of the ball in radians per sec?

Neglect the gravitational force exerted on the ball. Assume that the ball and spring rotate in a horizontal plane.

d) What is the radius of the circular motion of the ball?

Solution

a) Using Hooke's law we get $k = \frac{m_1 g}{\Delta l} = 147 \; Nm^{-1}$
b) the revolution time is $\Delta t = f^{-1} = 5 \; 10^{-2} s$
c) $\omega = 2\pi f = 126 \; s^{-1}$
d) The centripetal force has magnitude $|F| = m\omega^2 R$.
Also using Hooke's law $|F| = k(R - l_0)$.
So that $R = \frac{k l_0}{k - m\omega^2} = 13 \; cm$

Problem 6: *Experiment 3: Analysis*

Suppose the function

$$y = A \frac{9170 \, z}{z^4 + 500 \, z^2 + 5000}$$

with

$$z = x + B$$

fits the force law you measured in an experiment to determine the force between two magnets, where y is the force and x is the gap between the magnets, measured in mm. The parameters obtained from the fit are $A = 0.60$ N and B = 3 mm. Now suppose you approximate the force with a different law

$$y = A_1 e^{-Cx} = A_1 e^{-C(z-B)}$$

a) Find the parameters A_1 and C so that this function will give the same force as the first one when $z = 5$ mm and $z = 15$ mm.

b) As you saw in the notes for Experiment 03, both of these functions fit the measurements equally well (in the sense that the RMS error is the same for both) when the force was measured over the approximate range $5 \le z \le 20$ that you were able to measure. If you were able to measure the force out to $z = 30$, do you think it would be possible to decide which of the two functions was a better representation of the force law? (Use the parameters you have from above, and explain your answer.)

Solution:

1. First, evaluate for the first function $y(5) = 1.518$ N, $y(15) = 0.4909$ N. For the second function $y(5) = A_1 e^{-2C}$, and $y(15) = A_1 e^{-12C}$. Here are 2 equations in 2 unknowns. If you take the ratio of them you get $1.518/0.4909 = 3.092 = e^{10C}$. Thus $C = (1/10)\ln(3.092) = 0.1129$ mm^{-1}. Substituting in either $y(5)$ or $y(15)$ for the second function gives the result $A_1 = 1.903$ N.

2. For the second part, evaluate these two functions when $z = 30$. The first function is 0.1305 N (or 5.3 pennies), while the second one is 0.0903 N (or 3.7 pennies). The first is nearly 1.5 times greater than the second, and the difference is 0.04 N (or 1.6 pennies). If you could measure accurately enough you should be able to detect a difference of nearly 50%. But, a better way to look at this is to assume a force of 0.1305 N; we know this would give $z = 30$ (or a gap of 27 mm) with the first force law. With the same 0.1305 N force, the second force law would give $z = 26.7$ mm (gap = 23.7 mm) and the difference of about 3mm should be readily detectable. Thus the answer is yes.