Kinematics in Two Dimensions

8.01t

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Vector Description of Motion

- Position $\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$
- Velocity $\vec{\mathbf{v}}(t) = \dfrac{dx(t)}{dt}\hat{\mathbf{i}} + \dfrac{dy(t)}{dt}\hat{\mathbf{j}} \equiv v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}$
- Acceleration $\vec{\mathbf{a}}(t) = \dfrac{dv_x(t)}{dt}\hat{\mathbf{i}} + \dfrac{dv_y(t)}{dt}\hat{\mathbf{j}} \equiv a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}}$

Projectile Motion

- Ignore air resistance

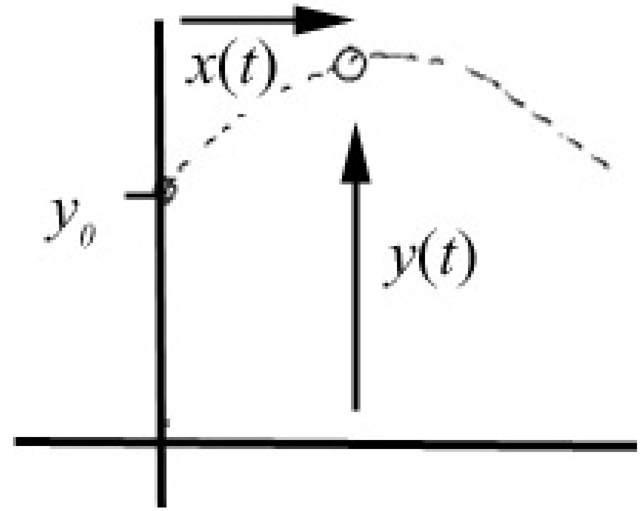
- Gravitational Force Law

$$\vec{\mathbf{F}}_{grav} = -m_{grav} g \hat{\mathbf{j}}$$

- Newton's Second Law

$$F_y^{total} = m_{in} a_y$$

$$F_x^{total} = m_{in} a_x$$



Equations of Motion

- y-component: $-m_{grav} g = m_{in} a_y$
- x-component: $0 = m_{in} a_x$

- Principle of Equivalence: $m_{grav} = m_{in}$

- Components of Acceleration:

$$a_y = -g \qquad\qquad a_x = 0$$

$$g = 9.8\, m - s^{-2}$$

Kinematic Equations:

- Acceleration y-component:

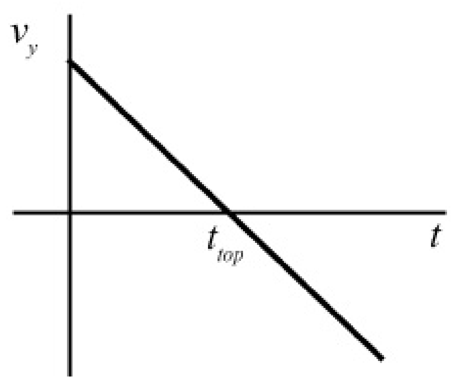
$$a_y = -g$$

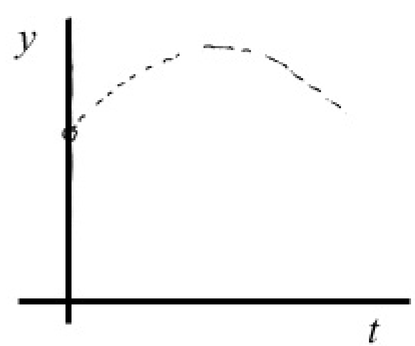
$$v_y(t) = v_{y,0} - gt$$

- Velocity y-component:

$$y(t) = y_0 + v_{y,0}t - \frac{1}{2}gt^2$$

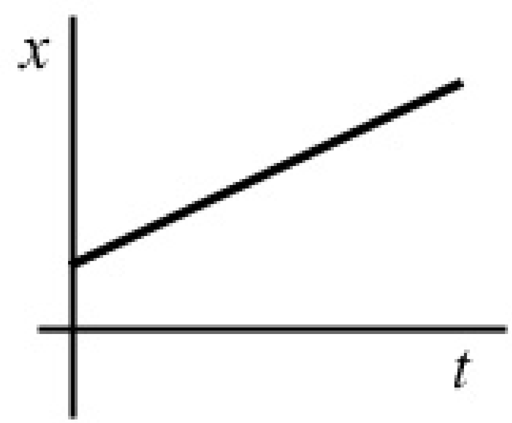
- Position y-component:





Kinematic Equations:

- Acceleration x-component: $$a_x = 0$$
- Velocity x-component: $$v_x(t) = v_{x,0}$$
- Position x-component: $$x(t) = x_0 + v_{x,0}t$$



Initial Conditions

- Initial position $\vec{r}_0 = x_0\hat{i} + y_0\hat{j}$

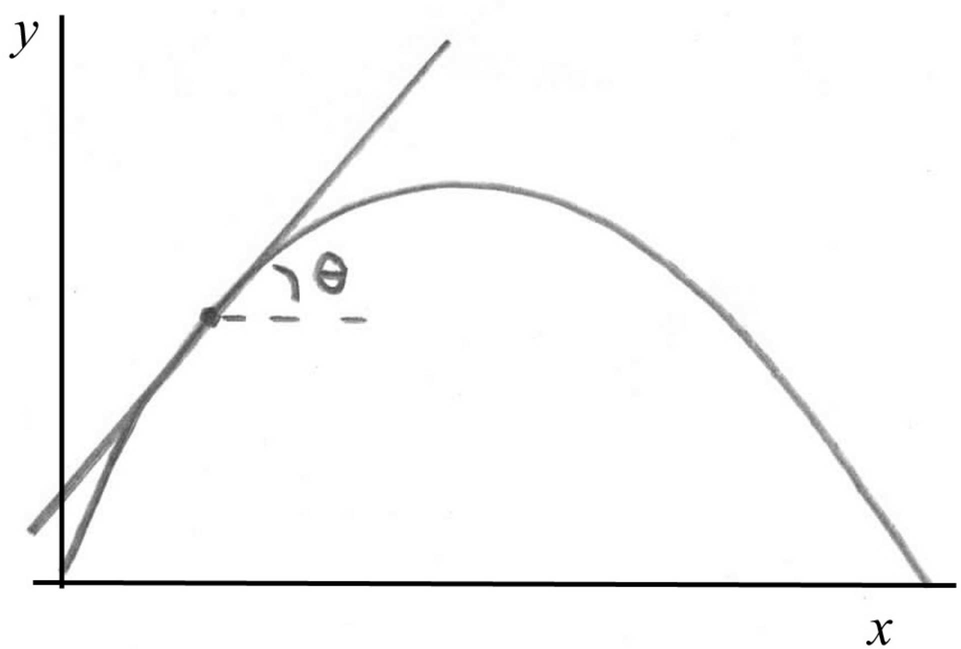
- depends on choice of origin

Initial Conditions

- Initial velocity $$\vec{\mathbf{v}}_0(t) = v_{x,o}\,\hat{\mathbf{i}} + v_{y,o}\,\hat{\mathbf{j}}$$
- with components: $$v_{x,0} = v_0 \cos\theta_0$$ $$v_{y,0} = v_0 \sin\theta_0$$
- initial speed is the magnitude of the initial velocity $$v_0 = (v_{x,0}^2 + v_{y,0}^2)^{1/2}$$
- with direction $$\theta_0 = \tan^{-1}\left(\frac{v_{y,0}}{v_{x,0}}\right)$$

Orbit equation

$$y(t) = -\frac{1}{2}\frac{g}{v_{x,0}^2}x(t)^2 + \frac{v_{y,0}}{v_{x,0}}x(t)$$



slope of the curve

$y(t)$ vs. $x(t)$

at any point determines the direction of the velocity

$$\theta = \tan^{-1}(\frac{dy}{dx})$$

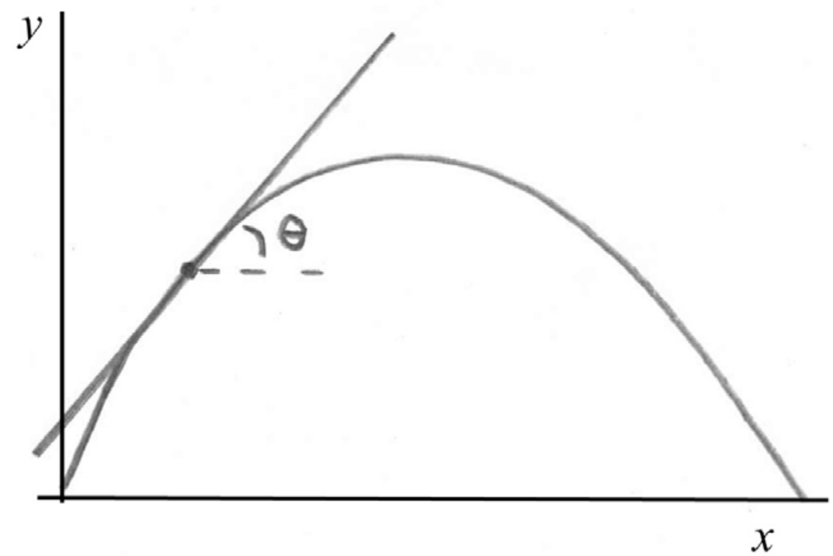
with $x_0 = 0$ $y_0 = 0$

Derivation

$$x(t) = v_{x,0}t$$

$$t = \frac{x(t)}{v_{x,0}}$$

$$y(t) = v_{y,0}t - \frac{1}{2}gt^2$$



- equation for a parabola

$$y(t) = \frac{v_{y,0}}{v_{x,0}}x(t) - \frac{1}{2}\frac{g}{{v_{x,0}}^2}x(t)^2$$

Experiment 2: Projectile Motion

- Initial Velocity

$$v_0 = \sqrt{\frac{g x(t)^2}{2\cos^2\theta_0\left(\tan\theta_0 x(t) - y(t)\right)}}$$

- Gravitational Constant

$$g = \frac{{v_0}^2}{x(t)^2}\left(2\cos^2\theta_0\left(\tan\theta_0 x(t) - y(t)\right)\right)$$

Experiment 2: Projectile Motion

Reminder on projectile motion

- Horizontal motion (x) has no acceleration.
- Vertical motion (y) has acceleration –*g*.
- Horizontal and vertical motion may be treated separately and the results combined to find, for example, the trajectory or path.
- Use the kinematic equations for x and y motion:

$$x(t) = x_0 + v_0 t \cos\theta$$

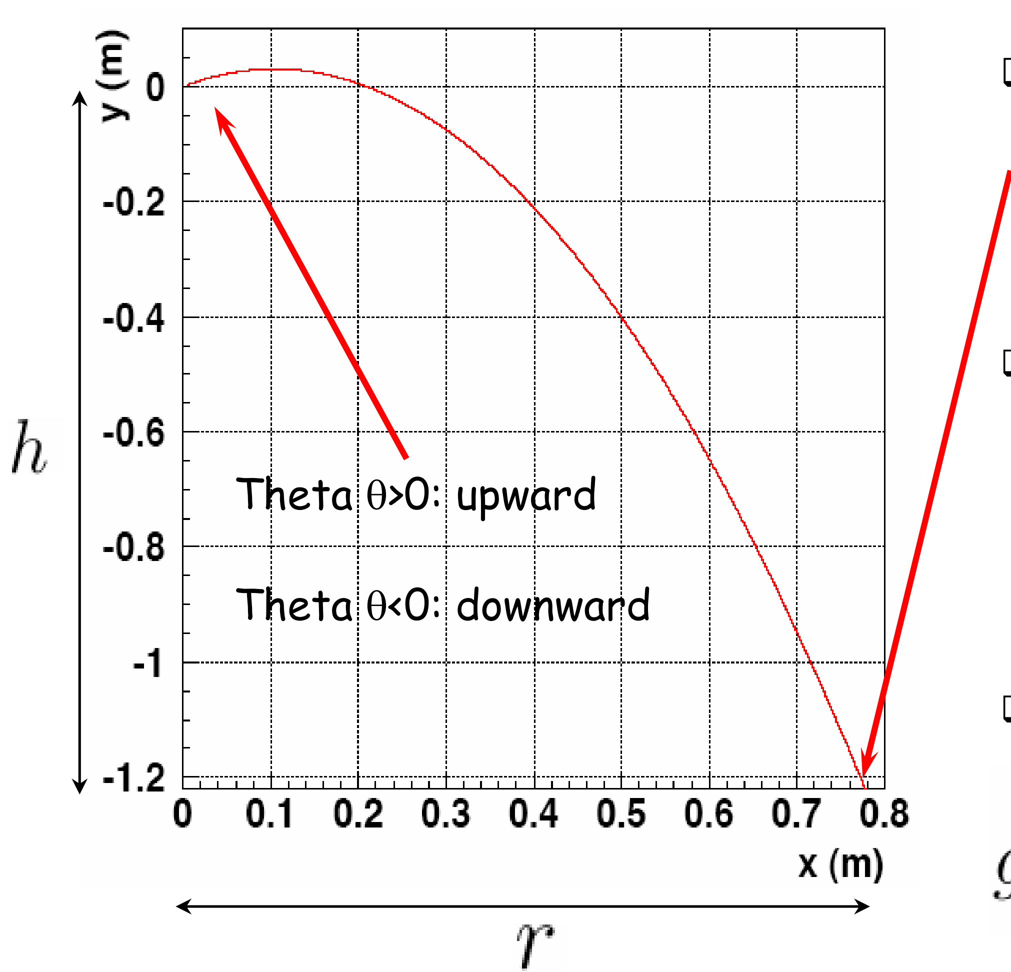
$$y(t) = y_0 + v_0 t \sin\theta - \frac{1}{2} g t^2$$

Experimental setup

- Coordinate system

$$x_0 = 0 \qquad y_0 = 0$$

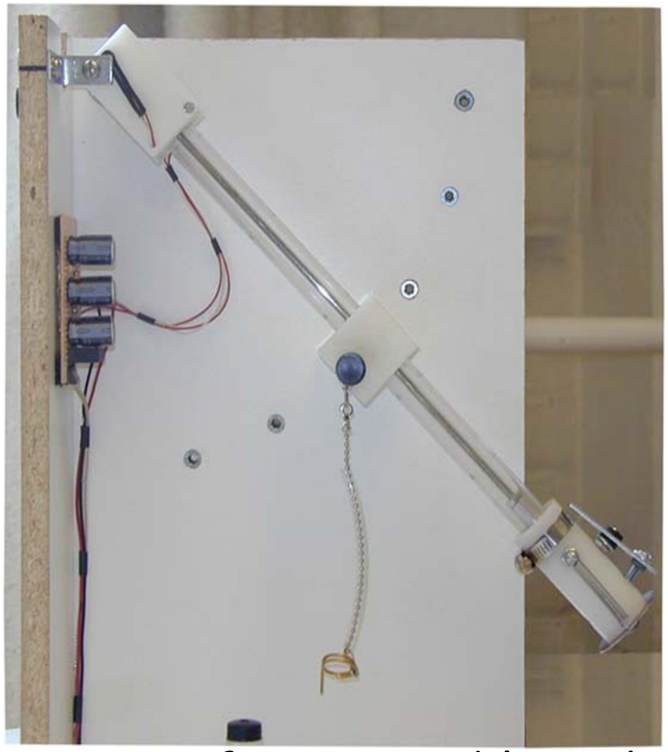
$$y(x) = x \tan\theta - \frac{g}{2v_0^2 \cos^2\theta} x^2$$



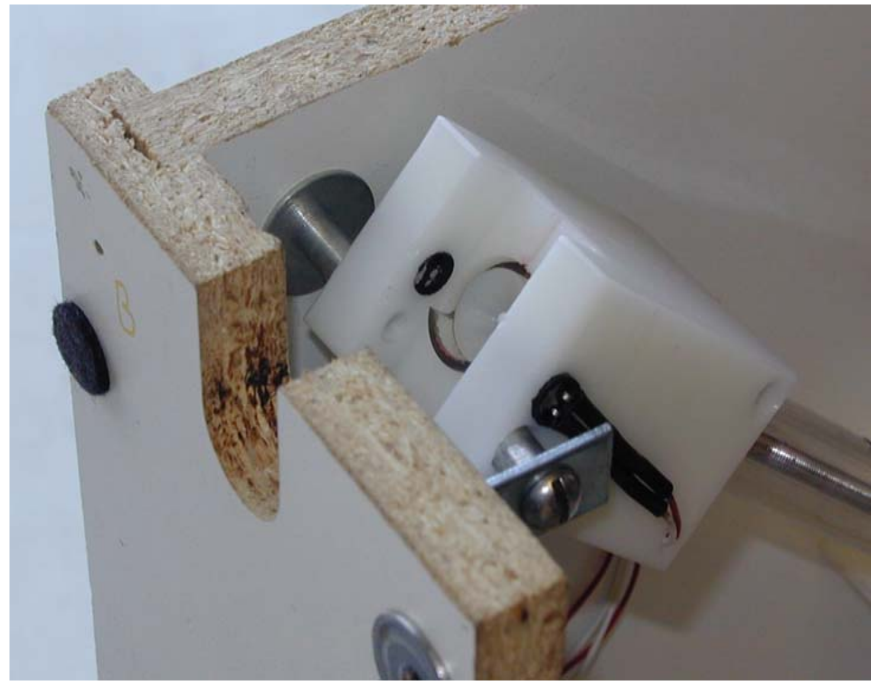
- Impact point:
  - Height: h
  - Horizontal displacement: r
- With chosen coordinate system:
  - Height: y=-h
  - Horizontal displacement: x=r
- Solve above equation for *g*:

$$g = \frac{2v_0^2 \cos^2\theta}{r^2} \left[r \tan\theta + h \right]$$

Experimental setup



Set up for upward launch



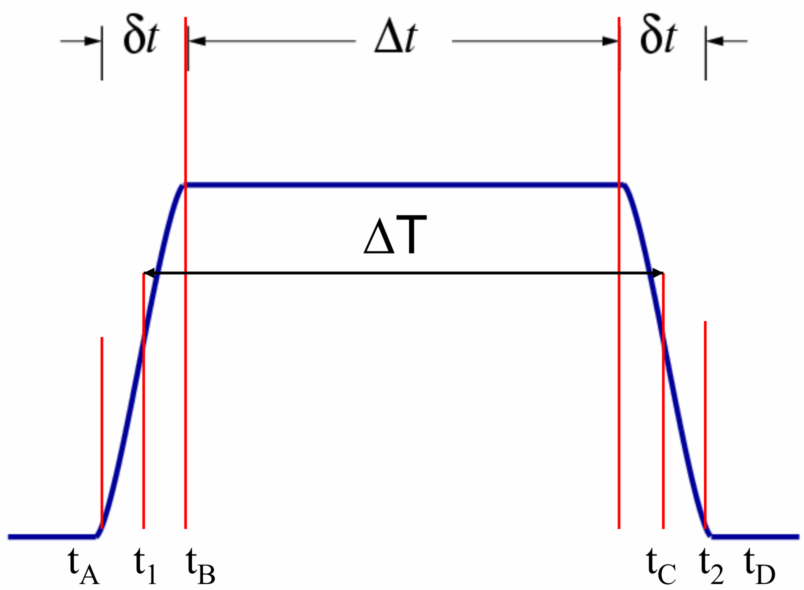
The output end with photogate

Connect voltage probe from 750 interface to red & black terminals.

Connect 12VAC supply to the apparatus; the LED should light up.

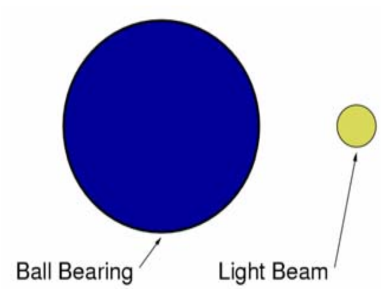
Velocity measurement

The photogate produces a pulse when the ball interrupts a beam of light to a phototransistor; the more light is blocked, the greater the pulse amplitude. If you look carefully at the output pulse, it looks approximately like this:



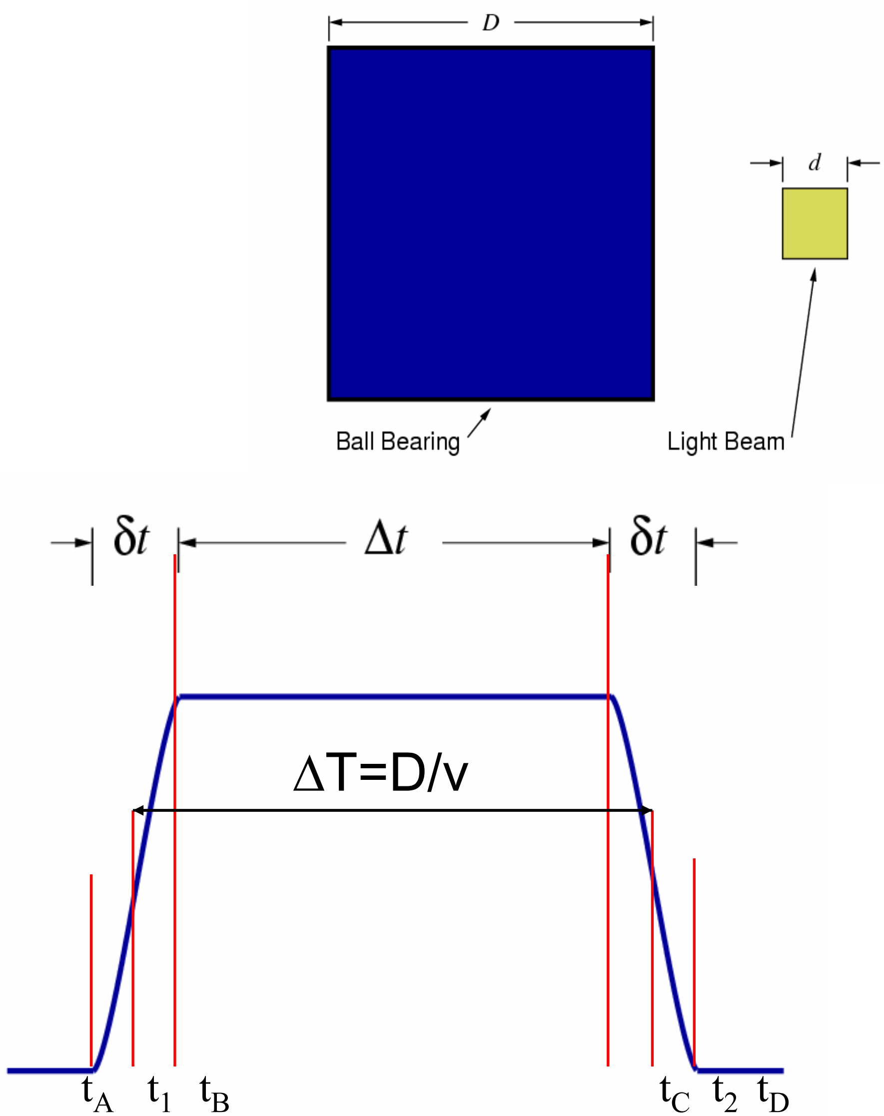
Rise & fall time: δt
Flat top lasting: Δt

To analyze the experiment we need to understand why the pulse has this shape!



δt is time the ball partly blocks the beam Δt is time it completely blocks the beam

Velocity measurement



Rise & fall time: $\delta t = d/v$
Flat top lasting: $\Delta t = (D-d/v)$

Therefore: $v = D/(\delta t + \Delta t)$

Determine $(\delta t + \Delta t)$ from Full Width at Half Maximum (FWHM)!