Potential Energy and Conservation of Energy

8.01t

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Conservative Forces

Definition: Conservative Work

Whenever the work done by a force in moving an object from an initial point to a final point is independent of the path, then the force is called a conservative force

Conservative Forces

- motion of a object under the influence of a gravitational force near the surface of the earth

- work done by gravity only depends on the change in the vertical position,
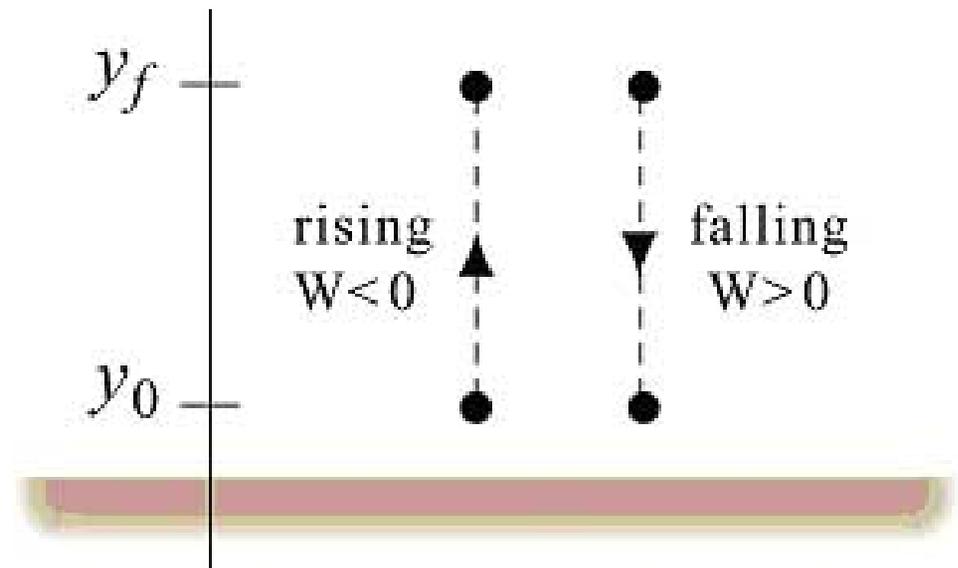
$$W_g = F_{g,y} \Delta y = -mg \Delta y$$

Change in Potential Energy

Definition: Change in Potential Energy

The change in potential energy of a body associated with a conservative force is the negative of the work done by the conservative force in moving the body along any path connecting the initial position to the final position.

$$\Delta U_{system} = -W_c = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$$

Conservation of Mechanical Energy
for Conservative Forces

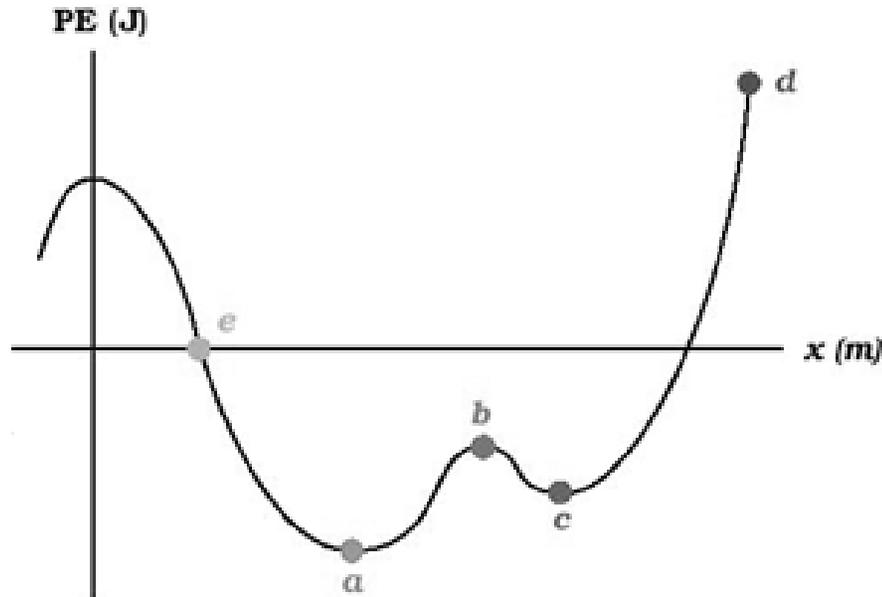- Change in Potential Energy is the work done by the conservative force

$$\Delta K_{system} = W_c = -\Delta U_{system}$$

- Change in Mechanical Energy is zero

$$\Delta E_{mechanical} \equiv \Delta K_{system} + \Delta U_{system} = 0$$

PRS Question: Energy Curve

Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy.
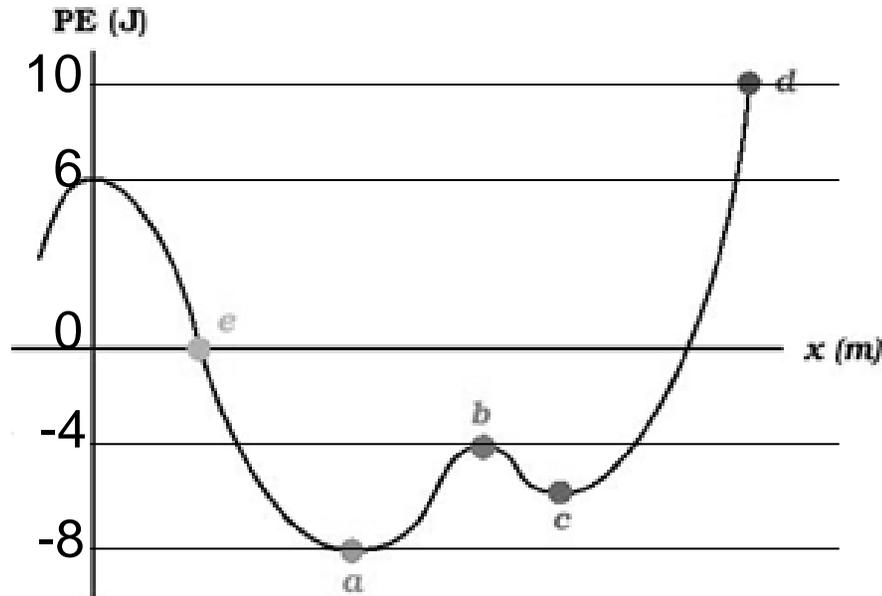

If a particle travels through the entire region of space shown in the diagram, at which point is the particle's velocity a maximum?

1.   a

2.   b

3.   c

4.   d

5.   e

PRS : Energy Curve - E$_{min}$

Consider the following sketch of potential energy for a particle as a function of position. There are no dissipative forces or internal sources of energy.

What is the minimum total mechanical energy that the particle can have if you know that it has traveled over the entire region of X shown?

1. -8
2. 6
3. 10
4. It depends on direction of travel
5. Can't say - Potential Energy uncertain by a constant

Force and Potential Energy Difference

- Potential energy difference: one dimensional motion
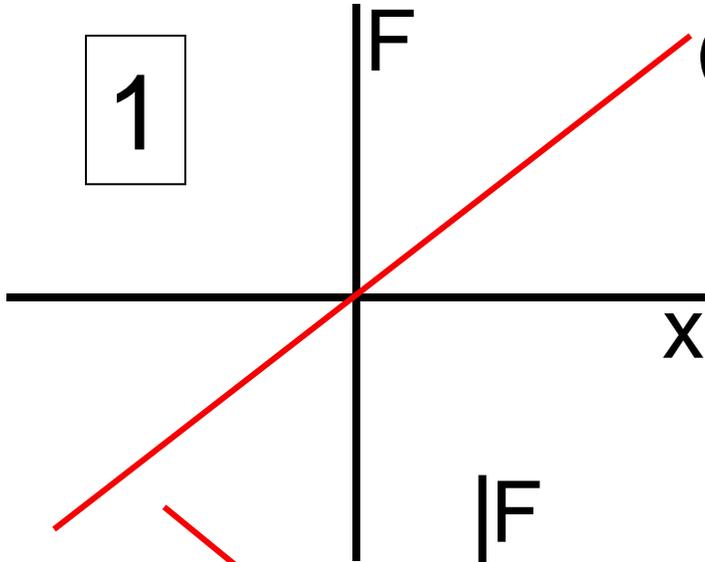
$$U(x) = U(x_0) - \int_A^B F_x \, dx$$

Force is then the derivative of the potential energy
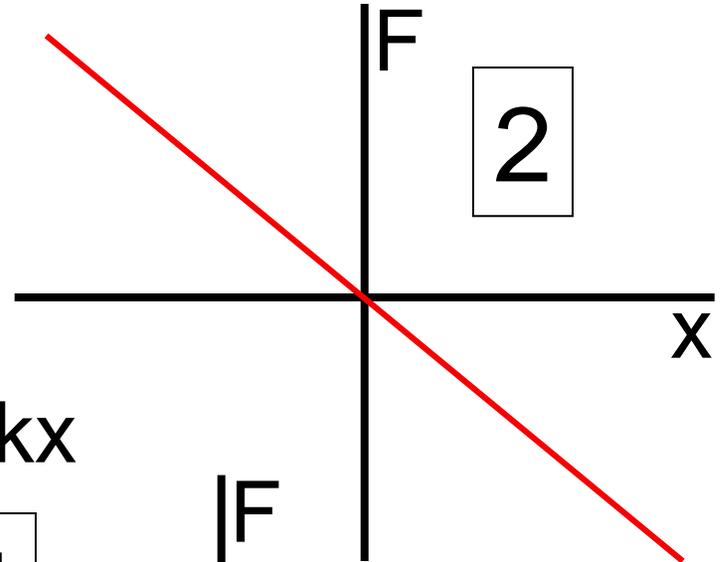
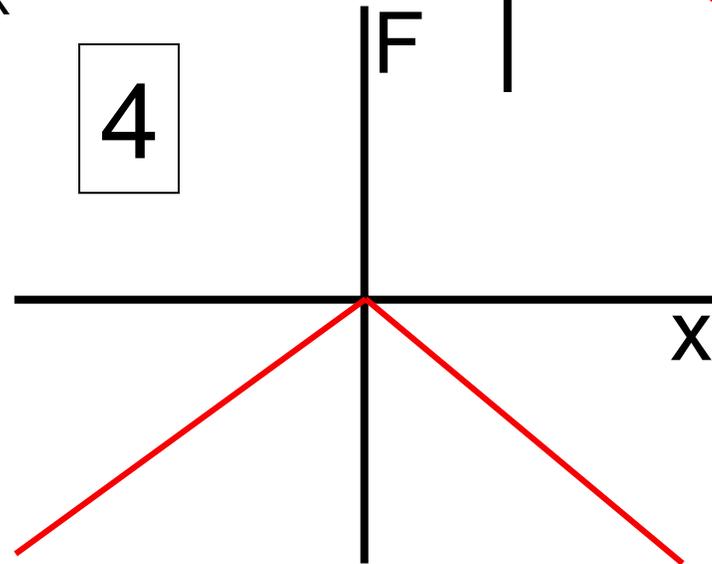$$F_x = -\frac{dU}{dx}$$

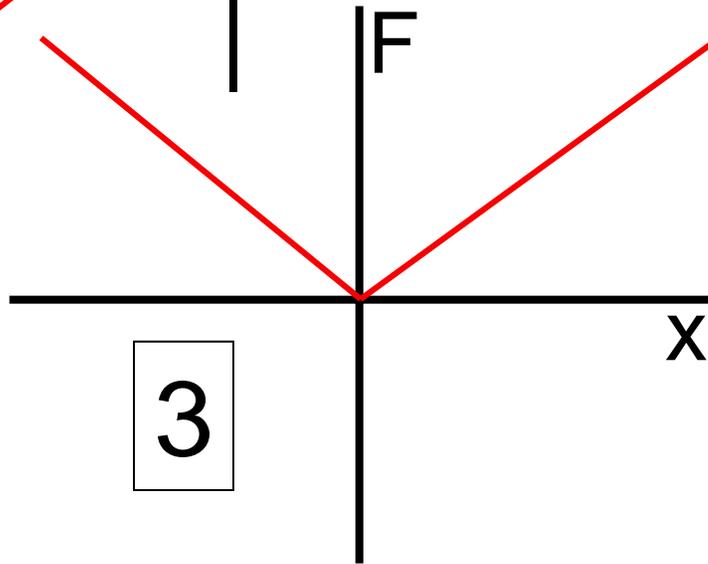# Potential Energy and Force: Graphs

1

F

x

2

F

x

Hint: $F_x = -kx$

3

F

x

4

F

x

Which Graph Represents Force of Spring?

Change in Potential Energy: Constant Gravity

Force: $\vec{\mathbf{F}}_{grav} = m\vec{\mathbf{g}} = F_{grav,y}\hat{\mathbf{j}} = -mg\hat{\mathbf{j}}$

Work: $W_{gravity} = F_{gravity,y}\Delta y = -mg\Delta y$

Potential Energy: $\Delta U_{system} = -W_{gravity} = mg\Delta y = mgy_f - mgy_0$

Choice of Zero Point: Whatever "ground" is convenient

PRS Question

A basketball dropped (from rest) from a height of 1 meter strikes the earth and returns to a height of 1 meter. For which choice of system is energy conserved at all times?

1. the basketball by itself
2. the earth by itself
3. the basketball plus the earth
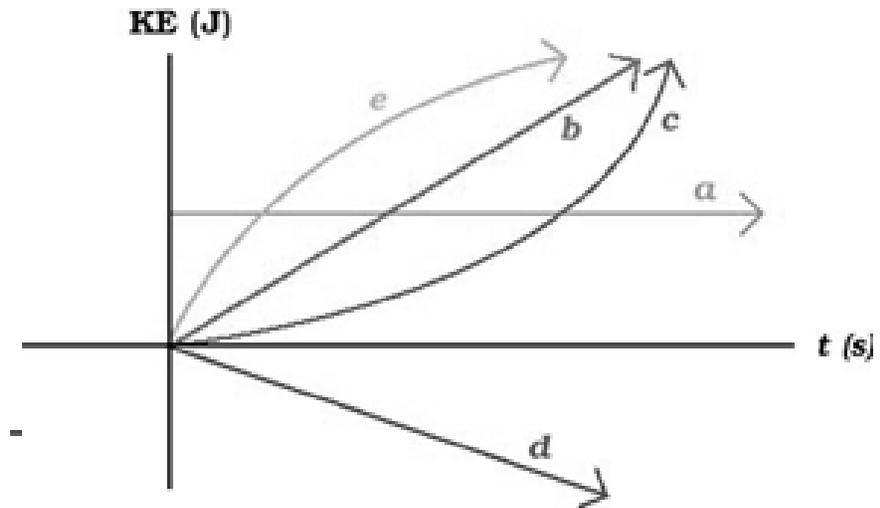4. all of the above

PRS Question

A 1kg ball falls from a height of 1 meter. The work done on the earth-ball system is:

1. 0 J
2. 10 J
3. - 10 J
4. 20 J
5. -20 J

PRS Question

An object is dropped to the earth from a height of 10m. Which of the following sketches best represent the kinetic energy of the object as it approaches the earth (neglect friction).

1. a
2. b
3. c
4. d
5. e

PRS Question

You lift a ball at constant velocity from a height h_i to a greater height h_f. Considering the ball and the earth together as the system, which of the following statements is true?

1. The potential energy of the system increases.
2. The kinetic energy of the system decreases.
3. The earth does negative work on the system.
4. You do negative work on the system.
5. The source energy of the ball increases.
6. Two of the above.
7. None of the above.

Change in Potential Energy: Inverse Square Gravity

- Force:

$$\vec{\mathbf{F}}_{m_1,m_2} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

- Work done:

$$W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{r_0}^{r_f} \left(-\frac{Gm_1m_2}{r^2}\right)dr = \frac{Gm_1m_2}{r}\bigg|_{r_0}^{r_f} = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

- Potential Energy Change:

$$\Delta U_{gravity} \equiv -W_{gravity} = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

- Zero Point:

$$U_{gravity}(\mathrm{r}_0 = \infty) \equiv 0$$

- Potential Energy Function

$$U_{gravity}(\mathrm{r}) = -\frac{Gm_1m_2}{r}$$

Change in Potential Energy: Spring Force

- Force:

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} = -kx\hat{\mathbf{i}}$$

- Work done:

$$W = \int_{x=x_0}^{x=x_f} (-kx)\,dx = -\frac{1}{2}k\left(x_f^2 - x_0^2\right)$$

- Potential Energy Change:

$$\Delta U_{spring} \equiv -W_{spring} = \frac{1}{2}k\left(x_f^2 - x_0^2\right)$$

- Zero Point:

$$U_{spring}(x = 0) \equiv 0$$

- Potential Energy Function

$$U_{spring}(x) = \frac{1}{2}kx^2$$

PRS Question

- **1.** In part (a) of the figure, an air track cart attached to a spring rests on the track at the position $x_{equilibrium}$ and the spring is relaxed. In (b), the cart is pulled to the position x_{start} and released. It then oscillates about $x_{equilibrium}$. Which graph correctly represents the potential energy of the spring as a function of the position of the cart?
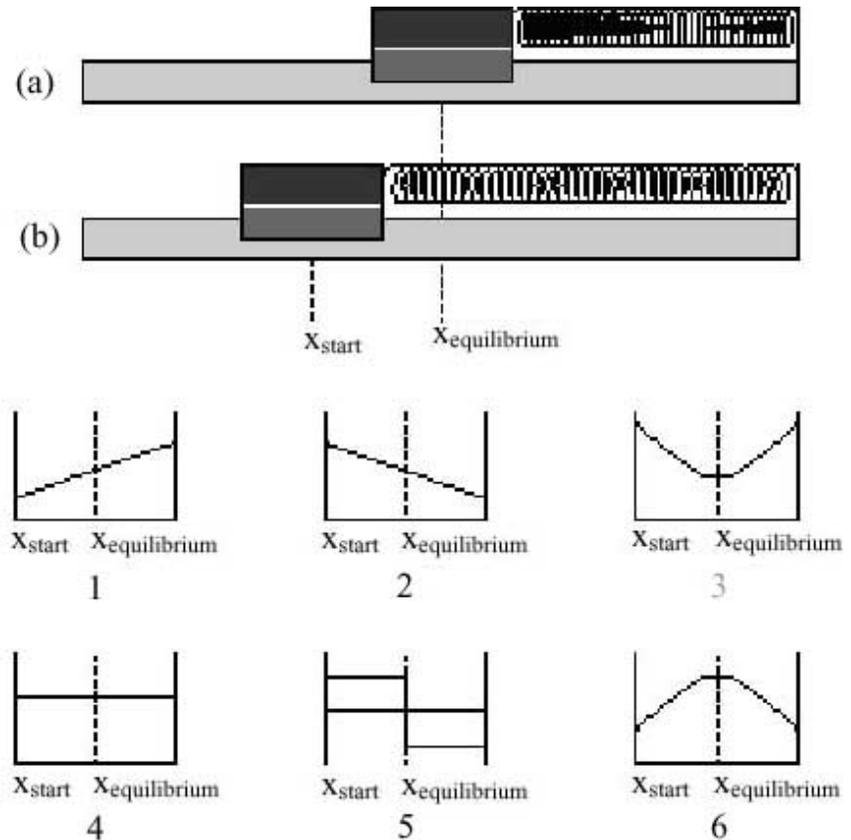
Non-Conservative Forces

- work done on the
  object by the force
  depends on the path
  taken by the object

path 1

x_0 x_f x

- Example: friction

$$W_{friction} = -\mu_k N \Delta x < 0$$

path 2

x_0 x_f x
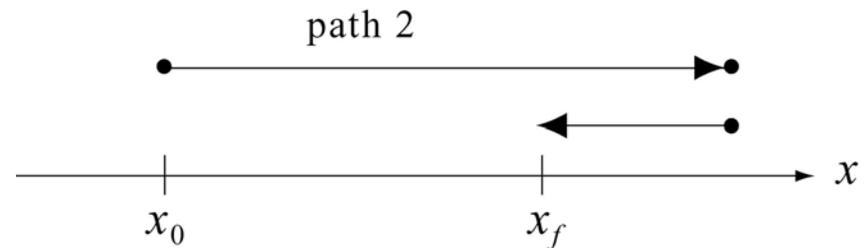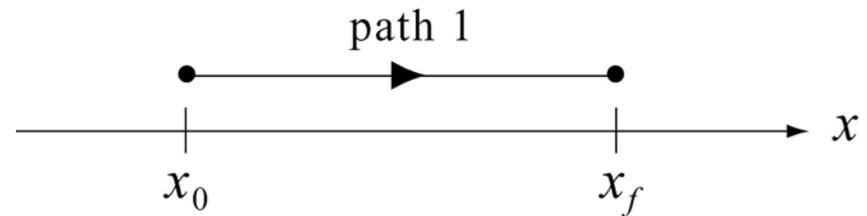
Non-Conservative Forces

Definition: Non-conservative Work

Whenever the work done by a force in moving an object from an initial point to a final point depends on the path, then the force is called a non-conservative force.

Change in Energy
for Conservative Forces and
Non-conservative Forces

- Total force
$$\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_c{}^{total} + \vec{\mathbf{F}}_{nc}{}^{total}$$

- Total work done is change in kinetic energy

$$W_{total} = \int_A^B \vec{\mathbf{F}}^{total} \cdot d\vec{\mathbf{r}} = \int_A^B \left(\vec{\mathbf{F}}_c{}^{total} + \vec{\mathbf{F}}_{nc}{}^{total} \right) \cdot d\vec{\mathbf{r}} = -\Delta U^{total} + W_{nc} = \Delta K$$

- Energy Change:
$$\Delta K + \Delta U^{total} = W_{nc}$$

PRS Question

A person lifts a 1kg cup of water 1 meter at constant velocity. Defining the system as the cup of water plus the earth, the work done on the system is:

1. 10 J
2. - 10 J
3. 0 J
4. none of the above

PRS Question

You lift a ball at constant velocity from a height hi to a greater height hf. Considering the ball to be the system, which of the following statements is true?

1. The potential energy of the system increases.
2. The kinetic energy of the system decreases.
3. The earth does negative work on the system.
4. You do negative work on the system.
5. The source energy of the ball increases.
6. Two of the above.
7. None of the above.

PRS Question

A block of inertia m is attached to a relaxed spring on an inclined plane. The block is allowed to slide down the incline, and comes to rest. The coefficient of kinetic friction of the block on the incline is μk. For which definition of the system is the change in total energy (after the block is released) zero?

1. block
2. block + spring
3. block + spring + incline
4. block + spring + incline + Earth
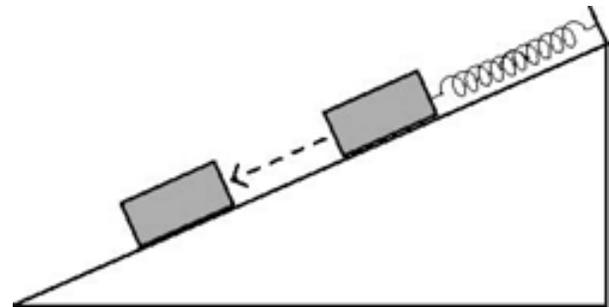
PRS Question

Suppose you want to ride your mountain bike up a steep hill. Two paths lead from the base to the top, one twice as long as the other. Compared to the average force you would exert if you took the short path, the average force you exert along the longer path is

1. four times as small.
2. three times as small.
3. half as small.
4. the same.
5. undetermined-it depends on the time taken.