Experiment 08: Physical Pendulum

8.01t

Nov 10, 2004

Goals

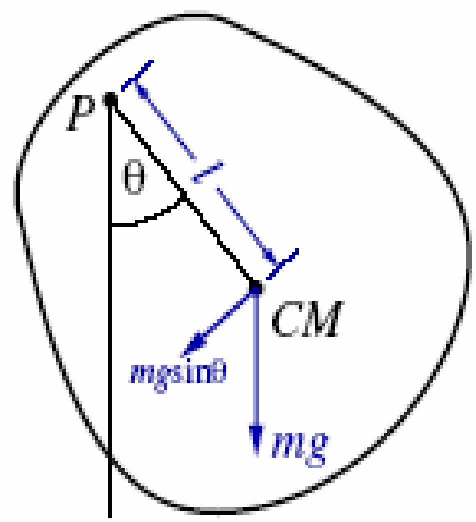
- Investigate the oscillation of a real (physical) pendulum and compare to an ideal (point mass) pendulum.

- Angular frequency calculation:

$$\vec{\tau} = I\vec{\alpha} \qquad\qquad -lmg\sin\theta = I\frac{d^2\theta}{dt^2}$$

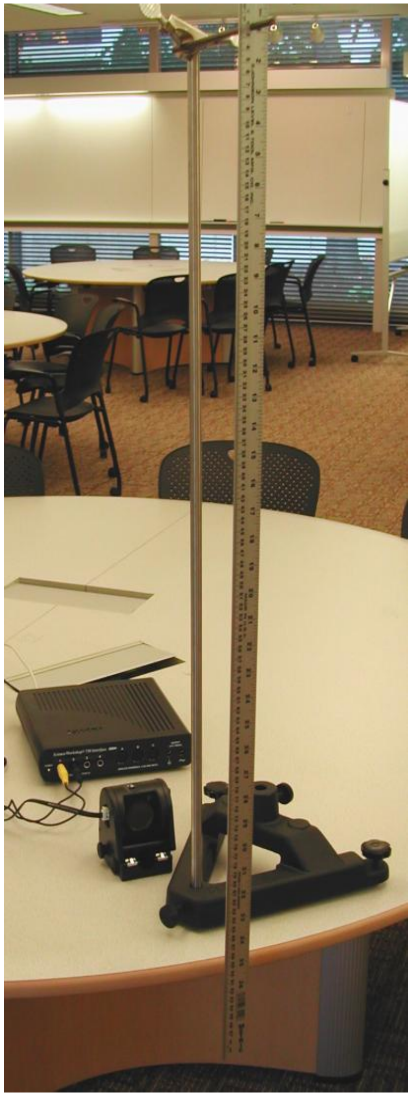
$$\frac{d^2\theta}{dt^2} + \frac{lmg}{I}\sin\theta = 0$$

$$\text{With } \sin\theta \simeq \theta \;:\; \omega = \sqrt{\frac{lmg}{I}} \qquad\qquad \frac{d^2\theta}{dt^2} + \frac{lmg}{I}\theta = 0$$



- Practice calculating moments of inertia, using them, and solving the $\tau = I\,\alpha$ equation of motion.

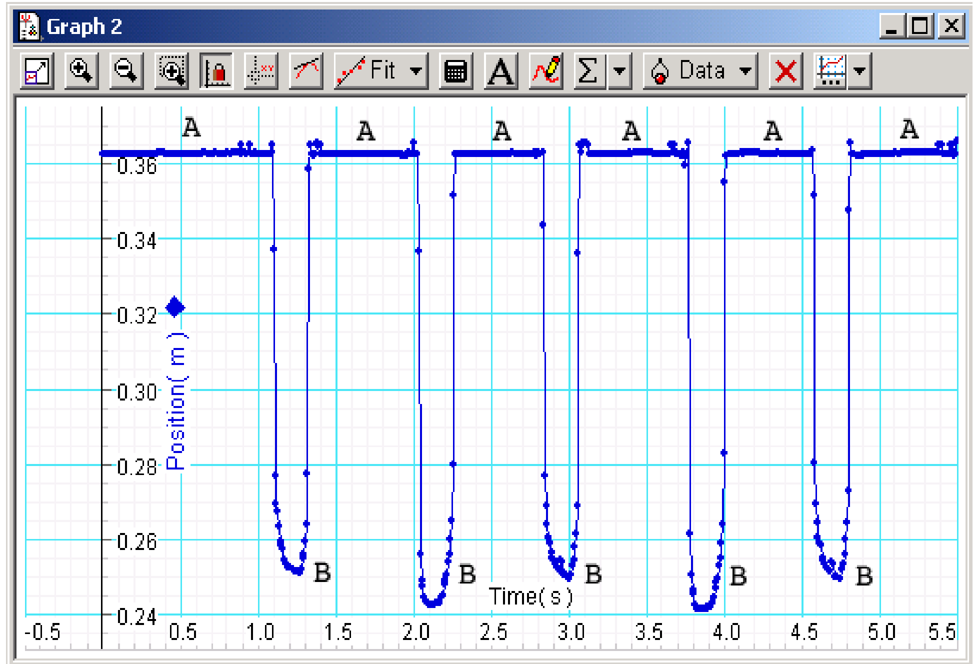
Equipment setup



- Suspend 1m ruler so it can swing over edge of table.
- Measure the period of oscillation with the DataStudio motion sensor.
- Set motion sensor on narrow beam, aim it to just miss support rod and hit ruler about 25 cm away.
- Place a chair about 40-50 cm from motion sensor to intercept ultrasound beam when ruler swings out of beam.

Understanding the graphs

Position vs. time data from the motion sensor.



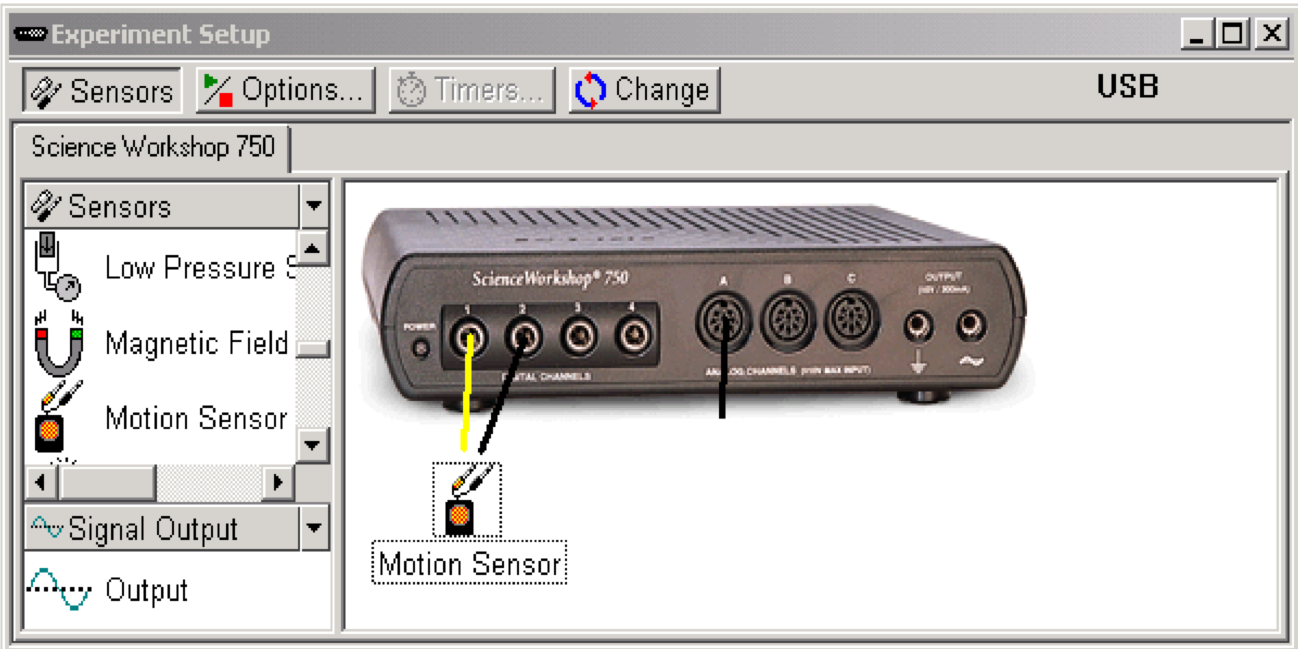
What is happening:

1. Along the top plateaus marked by A?
2. At the downward peaks marked by B?

How do you use this graph to find the period of oscillation of the pendulum?

Starting DataStudio

- ❑ Create a new experiment.
- ❑ Plug motion sensor into the 750 and
- ❑ drag their icons to inputs in the Setup window.



- ❑ Double-click the Motion Sensor icon, set trigger rate to 120.
- ❑ Plot position vs. time.

Ruler pendulum

Click 

Delayed Start = None.   Automatic Stop = 10 sec.

Pull ruler aside and release it to swing at the same time you start DataStudio.
Measure periods filling in the table below.

Displace ment	θ_0	Period
0.10 m	0.10	
0.25 m	0.25	
0.50 m	0.52	

Modified ruler pendulum

Clip a 50g brass weight to the ruler at positions in table in order to change the moment of inertia. (Clip is 8.6 g.) Measure the period of oscillation filling in the table below:

Displacement	Weight	Position	Period
0.20 m	58.6 g	0.25 m	
0.20 m	58.6 g	0.50 m	
0.20 m	58.6 g	0.90 m	

Angular Momentum and Fixed Axis Rotation

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Dynamics: Translational and Rotational Motion

Translational Dynamics

- Total Force $\vec{\mathbf{F}}_{ext}^{total}$
- Momentum of a System $\vec{\mathbf{p}}^{total}$
- Dynamics of Translation $\vec{\mathbf{F}}_{ext}^{total} = \dfrac{d\vec{\mathbf{p}}^{total}}{dt}$

Rotational Dynamics of point mass about S

- Torque $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{F}}_m$
- Angular Momentum about S $\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}}$
- Dynamics of Rotation $\vec{\boldsymbol{\tau}}_S^{total} = \dfrac{d\vec{\mathbf{L}}_S}{dt}$

Angular Velocity Vector and Angular Acceleration Vector for Fixed Axis Rotation

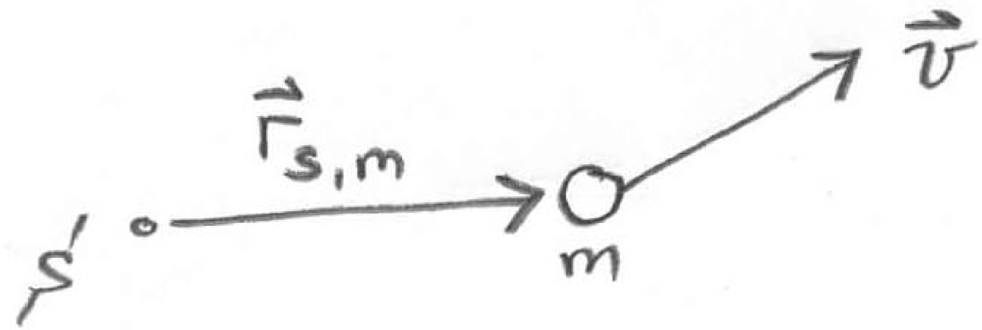
- Fixed axis of rotation: z-axis

- Angular velocity vector $$\vec{\boldsymbol{\omega}} = \frac{d\theta}{dt}\hat{\mathbf{k}}$$

- Angular acceleration vector $$\vec{\boldsymbol{\alpha}} = \frac{d^2\theta}{dt^2}\hat{\mathbf{k}}$$

Angular Momentum of a Point Particle

- point particle of mass m moving with a velocity $\vec{\mathbf{v}}$
- momentum $\vec{\mathbf{p}} = m\vec{\mathbf{v}}$



- Fix a point S
- vector $\vec{\mathbf{r}}_{S,m}$ from the point to the location of the object
- angular momentum about the point S

$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}}$$

Cross Product: Angular Momentum of a Point Particle

Magnitude: $\left|\vec{\mathbf{L}}_S\right| = \left|\vec{\mathbf{r}}_{S,m}\right|\left|\vec{\mathbf{p}}\right|\sin\theta$

a) moment arm

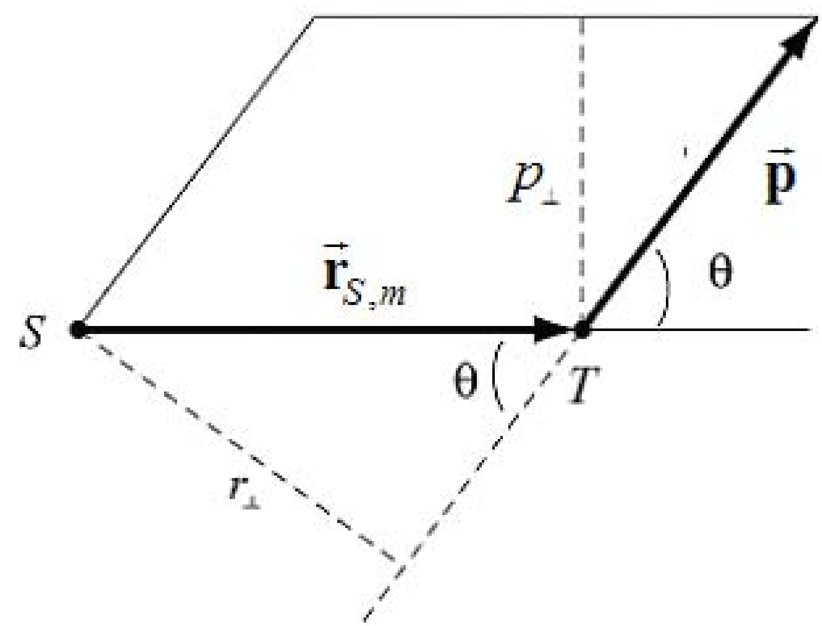
$$r_\perp = \left|\vec{\mathbf{r}}_{S,m}\right|\sin\theta$$

$$\left|\vec{\mathbf{L}}_S\right| = r_\perp\left|\vec{\mathbf{p}}\right|$$

b) Perpendicular momentum

$$p_\perp = \left|\vec{\mathbf{p}}\right|\sin\theta$$

$$\left|\vec{\mathbf{L}}_S\right| = \left|\vec{\mathbf{r}}_{S,T}\right| p_\perp$$

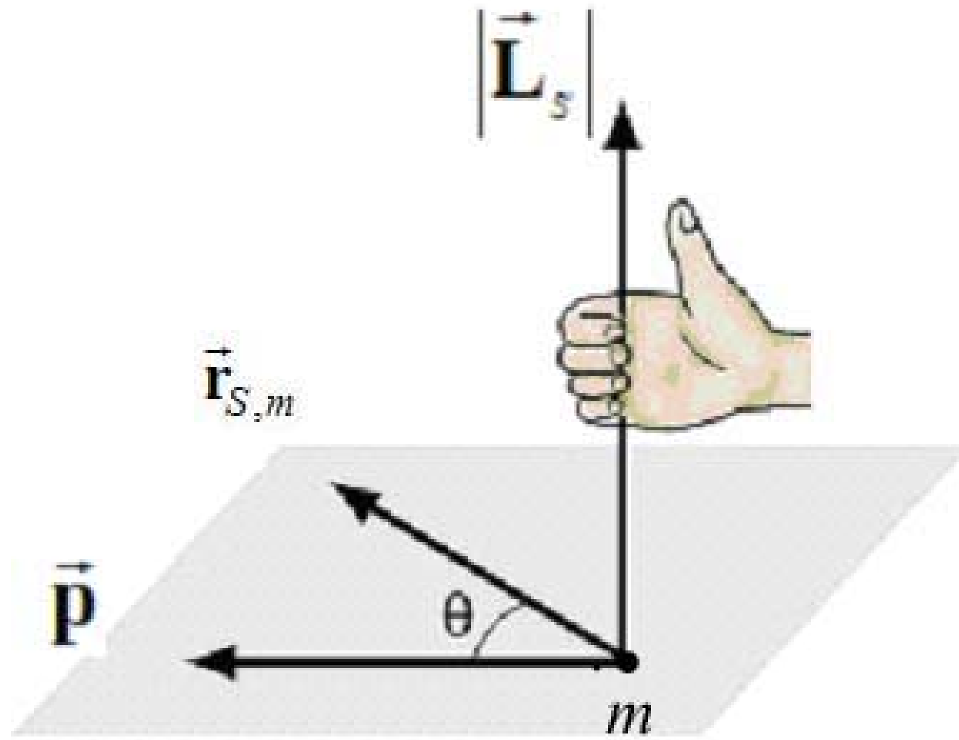


$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}}$$

Cross Product: Angular Momentum of a Point Particle

Direction

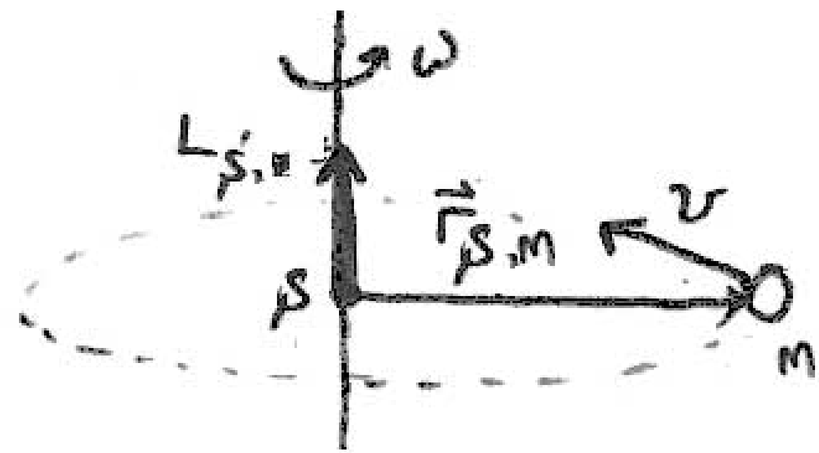
Right Hand Rule



Angular Momentum for Fixed Axis Rotation

- Fixed axis of rotation: z-axis
- Angular velocity

$$\vec{\mathbf{v}} = \vec{\mathbf{r}} \times \vec{\boldsymbol{\omega}} = r\omega\hat{\mathbf{k}}$$



- angular momentum about the point S

$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}} \quad = \vec{\mathbf{r}}_S \times m\vec{\mathbf{v}}$$

- z-component of the angular momentum about S,

$$\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times m\ \vec{\mathbf{v}} = rmv\ \hat{\mathbf{k}} = rmr\omega\hat{\mathbf{k}} = mr^2\omega\hat{\mathbf{k}}$$

Fixed Axis Rotation

- Angular Momentum about z-axis

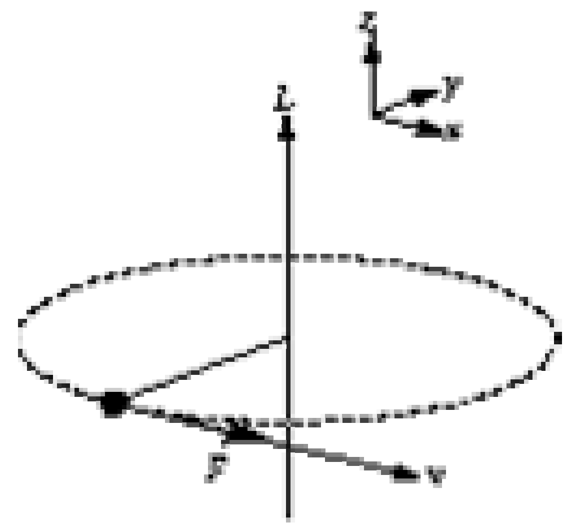
$$L_{S,z}^{total} = mr^2\omega = I_S\omega$$

- Rotational Dynamics

$$\tau_{S,z} = \frac{dL_{S,z}^{total}}{dt} = I_S\frac{d\omega}{dt} = I_S\alpha$$

PRS Question

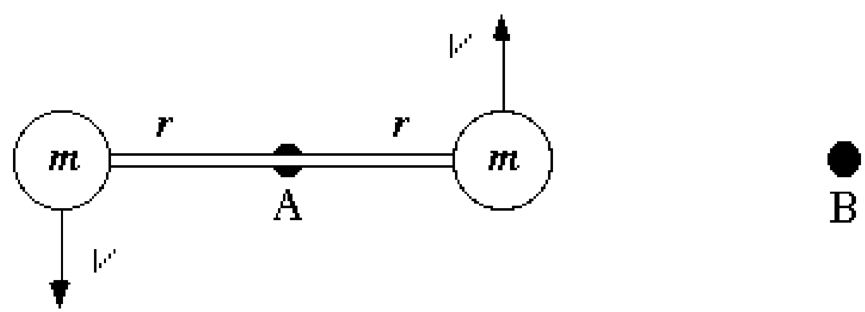
A person spins a tennis ball on a string in a horizontal circle (so that the axis of rotation is vertical). At the point indicated below, the ball is given a sharp blow in the forward direction. This causes a change in angular momentum dL in the



1. x direction
2. y direction
3. z direction

PRS Question

A dumbbell is rotating about its center as shown. Compared to the dumbbell's angular momentum about its center, its angular momentum about point B is



1. bigger.
2. the same.
3. smaller.

Time Derivative of Angular Momentum for a Point Particle

Time derivative of the angular momentum about S:

$$\frac{d\vec{\mathbf{L}}_S^{total}}{dt} = \frac{d}{dt}\left(\vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}}\right)$$

Product rule $$\frac{d\vec{\mathbf{L}}_S^{total}}{dt} = \frac{d}{dt}\left(\vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{p}}\right) = \frac{d\vec{\mathbf{r}}_{S,m}}{dt} \times \vec{\mathbf{p}} + \vec{\mathbf{r}}_{S,m_i} \times \frac{d}{dt}\vec{\mathbf{p}}$$

Key Fact: $$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}_{S,m}}{dt} \Rightarrow \frac{d\vec{\mathbf{r}}_{S,m}}{dt} \times m\vec{\mathbf{v}} = \vec{\mathbf{v}} \times m\vec{\mathbf{v}} = \vec{\mathbf{0}}$$

Result:

$$\frac{d\vec{\mathbf{L}}_S^{total}}{dt} = \vec{\mathbf{r}}_{S,m_i} \times \frac{d}{dt}\vec{\mathbf{p}} = \vec{\mathbf{r}}_{S,m} \times \vec{\mathbf{F}} = \vec{\boldsymbol{\tau}}_S$$

Torque and the Time Derivative of Angular Momentum for a Point Particle

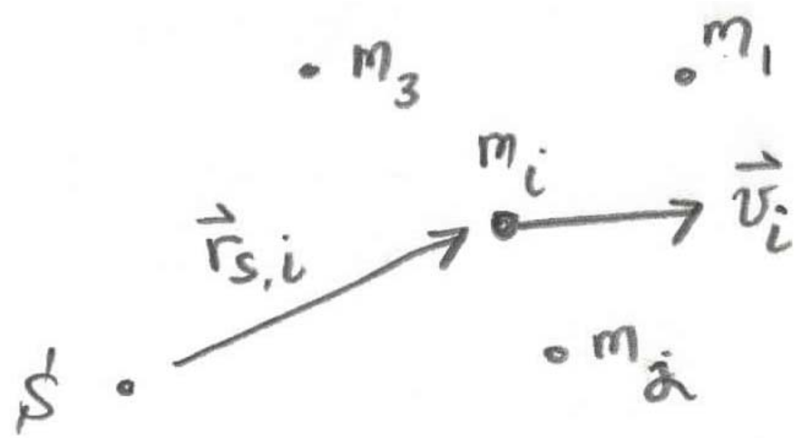
Torque about a point S is equal to the time derivative of the angular momentum about S .

$$\vec{\boldsymbol{\tau}}_S^{total} = \frac{d\vec{\mathbf{L}}_S}{dt}$$

Angular Momentum for a System of Particles

- Treat each particle separately

$$\vec{\mathbf{L}}_{S,i} = \vec{\mathbf{r}}_{S,m_i} \times \vec{\mathbf{p}}_i$$



- Total Angular Momentum for System about S

$$\vec{\mathbf{L}}_S^{total} = \sum_{i=1}^{i=N} \vec{\mathbf{L}}_{S,i} = \sum_{i=1}^{i=N} \vec{\mathbf{r}}_{S,m_i} \times \vec{\mathbf{p}}_i$$

Angular Momentum and Torque for a System of Particles

- Total torque about S is the time derivative of angular momentum about S

$$\frac{d\vec{\mathbf{L}}_S^{total}}{dt} = \sum_{i=1}^{i=N} \frac{d\vec{\mathbf{L}}_{S,i}}{dt} = \sum_{i=1}^{i=N} \vec{\mathbf{r}}_{S,m_i} \times \vec{\mathbf{F}}_i = \sum_{i=1}^{i=N} \vec{\boldsymbol{\tau}}_{S,i} = \vec{\boldsymbol{\tau}}_S{}^{total}$$

Angular Momentum of a Rigid Body for Fixed Axis Rotation

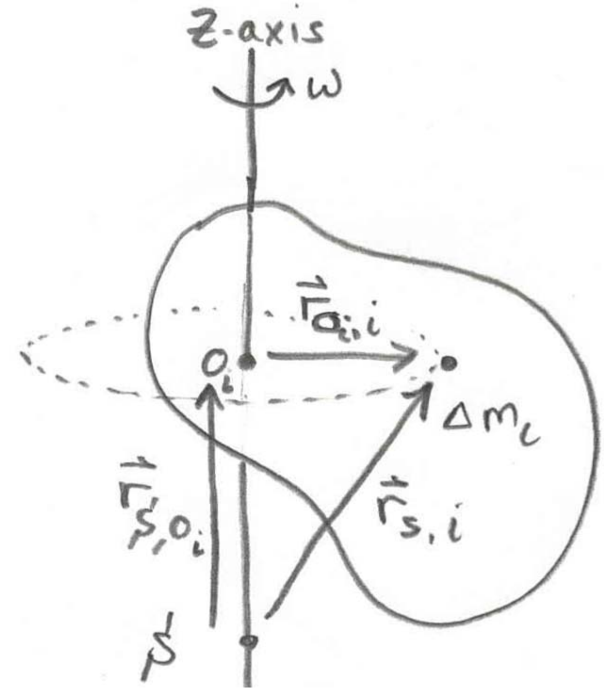
- Fixed axis of rotation: z-axis

- angular momentum about the point S

$$\vec{\mathbf{L}}_{S,i} = \vec{\mathbf{r}}_{S,i} \times \vec{\mathbf{p}}_i = \vec{\mathbf{r}}_{S,i} \times \Delta m_i \vec{\mathbf{v}}_i$$

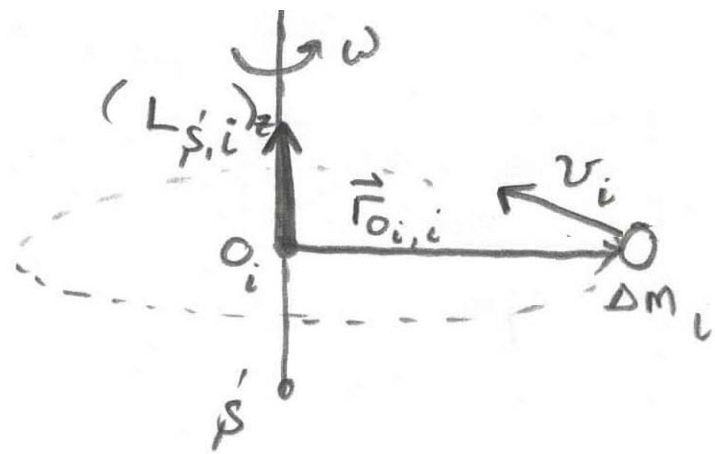
- z-component of the angular momentum about S,

$$\left(\vec{\mathbf{L}}_{S,i}\right)_z = \vec{\mathbf{r}}_{O_i,i} \times \Delta m_i \vec{\mathbf{v}}_i$$



$$\vec{\mathbf{r}}_{S,i} = \vec{\mathbf{r}}_{S,O_i} + \vec{\mathbf{r}}_{O_i,i}$$

Z-component of the Angular Momentum about S



- Mass element Δm_i
- radius of the circle $r_{\perp,i}$
- momentum $\Delta m_i v_i$
- z-component of the angular momentum about S $\left(L_{S,i}\right)_z = r_{\perp,i}\Delta m_i v_i$
- Velocity $v_i = r_{\perp,i}\omega$
- Summary: $\left(L_{S,i}\right)_z = r_{\perp,i}\Delta m_i v_i = \Delta m_i \left(r_{\perp,i}\right)^2 \omega$

Z-component of the Angular Momentum about S

- Sum over all mass elements

$$\left(L_S^{total}\right)_z = \sum_i \left(L_{S,i}\right)_z = \sum_i \Delta m_i \left(r_{\perp,i}\right)^2 \omega$$

- Continuous body

$$\left(L_S^{total}\right)_z = \int_{body} dm\left(r_\perp\right)^2 \omega$$

- Moment of Inertia

$$I_{S,z} = \int_{body} dm(r_\perp)^2$$

- Main Result

$$\left(L_S^{total}\right)_z = I_{S,z}\omega$$

Torque and Angular Momentum for Fixed Axis Rotation

- torque about S is equal to the time derivative of the angular momentum about S

$$\vec{\boldsymbol{\tau}}_S^{total} = \frac{d\vec{\mathbf{L}}_S^{total}}{dt}$$

- resolved in the z-direction

$$\left(\tau_S^{total}\right)_z = \frac{d\left(L_S^{total}\right)_z}{dt} = \frac{d\left(I_{S,z}\omega\right)}{dt} = I_{S,z}\frac{d\omega}{dt} = I_{S,z}\frac{d^2\theta}{dt^2} = I_{S,z}\alpha$$

Conservation of Angular Momentum about a Point S

- Rotational dynamics

$$\vec{\boldsymbol{\tau}}_S^{total} = \frac{d\vec{\mathbf{L}}_S^{total}}{dt}$$

- No external torques

$$\vec{\mathbf{0}} = \vec{\boldsymbol{\tau}}_S^{total} = \frac{d\vec{\mathbf{L}}_S^{total}}{dt}$$

- Change in Angular momentum is zero

$$\Delta\vec{\mathbf{L}}_S^{total} \equiv \left(\vec{\mathbf{L}}_S^{total}\right)_f - \left(\vec{\mathbf{L}}_S^{total}\right)_0 = \vec{\mathbf{0}}$$

- Angular Momentum is conserved

$$\left(\vec{\mathbf{L}}_S^{total}\right)_f = \left(\vec{\mathbf{L}}_S^{total}\right)_0$$

PRS Question

A figure skater stands on one spot on the ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her rotational inertia and her angular speed increases so that her angular momentum is conserved. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be

1. the same.
2. larger because she's rotating faster.
3. smaller because her rotational inertia is smaller.

Conservation Principles

- Change in mechanical energy

$$W_{nc} = \Delta E_{mechanical} = \Delta K + \Delta U^{total}$$

- No non-conservative work

$$0 = W_{nc} = \Delta E_{mechanical} = \Delta K + \Delta U^{total}$$

- Change in momentum

$$\vec{\mathbf{F}}_{external}^{total} = \sum_{i=1}^{N} \frac{d\vec{\mathbf{p}}_i}{dt} = \frac{d}{dt}\vec{\mathbf{p}}^{total}$$

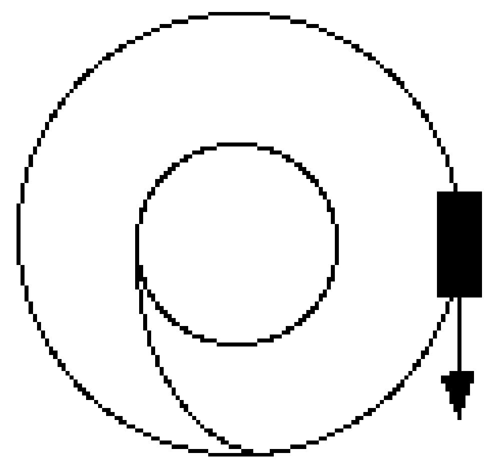
- No external forces

$$0 = \left(\vec{\mathbf{F}}_{external}^{total}\right)_x = \frac{d}{dt}\left(\vec{\mathbf{p}}^{total}\right)_x$$

$$0 = \left(\vec{\mathbf{F}}_{external}^{total}\right)_y = \frac{d}{dt}\left(\vec{\mathbf{p}}^{total}\right)_y$$

PRS Question

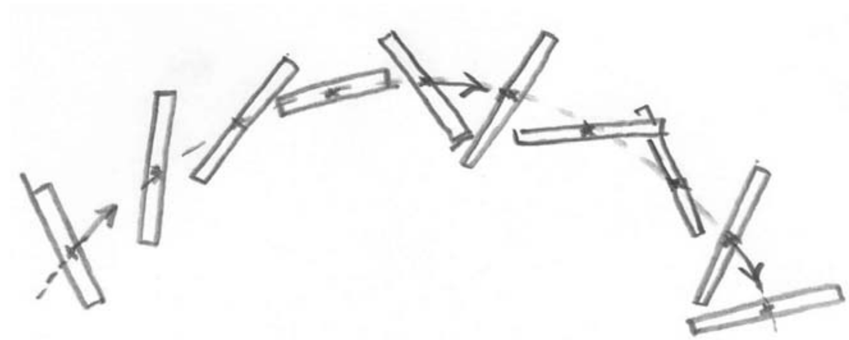
A streetcar is freely coasting (no friction) around a large circular track. It is then switched to a small circular track. When coasting on the smaller circle the streetcar's



1. mechanical energy is conserved and angular momentum about the center is conserved
2. mechanical energy is not conserved and angular momentum about the center is conserved
3. mechanical energy is not conserved and angular momentum about the center is not conserved
4. mechanical energy is conserved and angular momentum about the center is not conserved.

Total Angular Momentum about a Fixed Point

- Total for translation and rotation about point S



$$\vec{\mathbf{L}}_S^{total} = \vec{\mathbf{r}}_{S,cm} \times m_T \vec{\mathbf{v}}_{cm} + \vec{\mathbf{L}}_{cm}^{spin}$$

- Orbital angular momentum $\vec{\mathbf{L}}_S^{orbital} = \vec{\mathbf{r}}_{S,cm} \times \vec{\mathbf{p}}^{total}$

- Spin Angular Momentum for fixed axis rotation $\vec{\mathbf{L}}_{cm}^{spin} = I_{cm} \vec{\boldsymbol{\omega}}_{spin}$

Class Problem

- A meteor of mass m is approaching earth as shown on the sketch. The radius of the earth is R. The mass of the earth is m_e. Suppose the meteor has an initial speed of v_e. Assume that the meteor started very far away from the earth. Suppose the meteor just grazes the earth. The initial moment arm of the meteor (h on the sketch) is called the impact parameter. The effective scattering angle for the meteor is the area πh^2 . This is the effective target size of the earth as initially seen by the meteor.

Class Problem

a) Draw a force diagram for the forces acting on the meteor.

b) Can you find a point about which the gravitational torque of the earth's force on the meteor is zero for the entire orbit of the meteor?

c) What is the initial angular momentum and final angular momentum (when it just grazes the earth) of the meteor?

d) Apply conservation of angular momentum to find a relationship between the meteor's final velocity and the impact parameter.

e) Apply conservation of energy to find a relationship between the final velocity of the meteor and the initial velocity of the meteor.

f) Use your results in parts d) and e) to calculate the impact parameter and the effective scattering cross section.