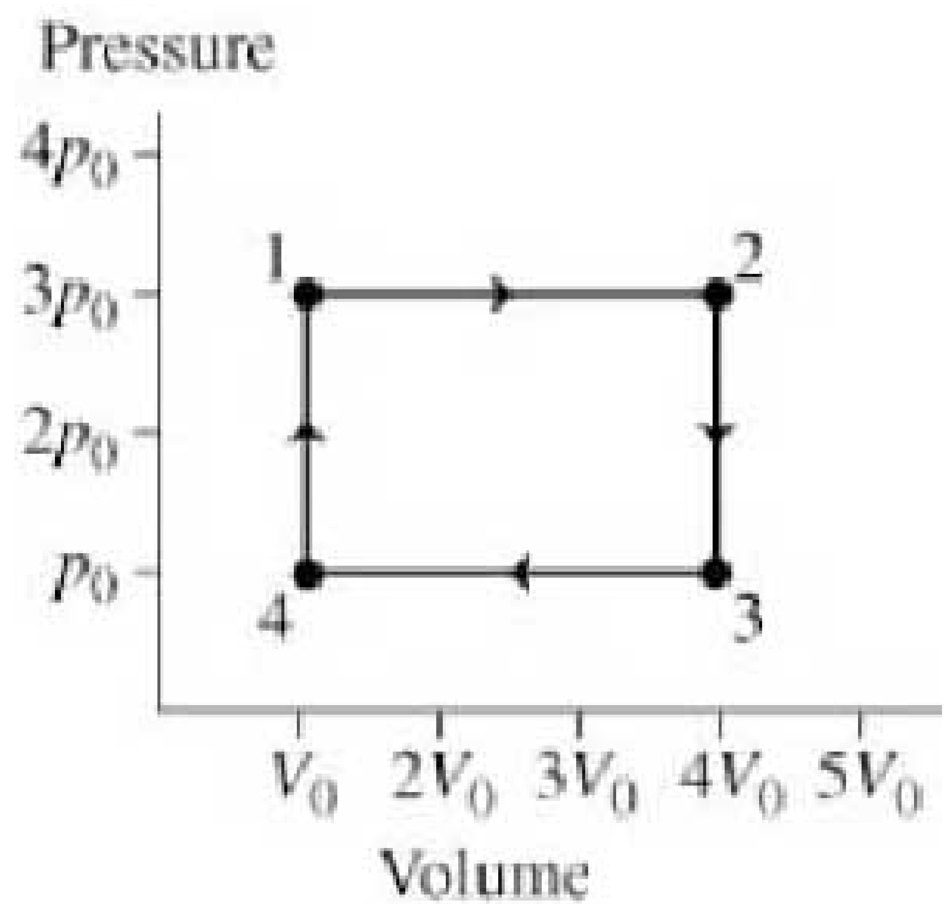
Thermodynamic Cycles

Wed. Dec. 1, 2004

PRS: Work in p-V plane:

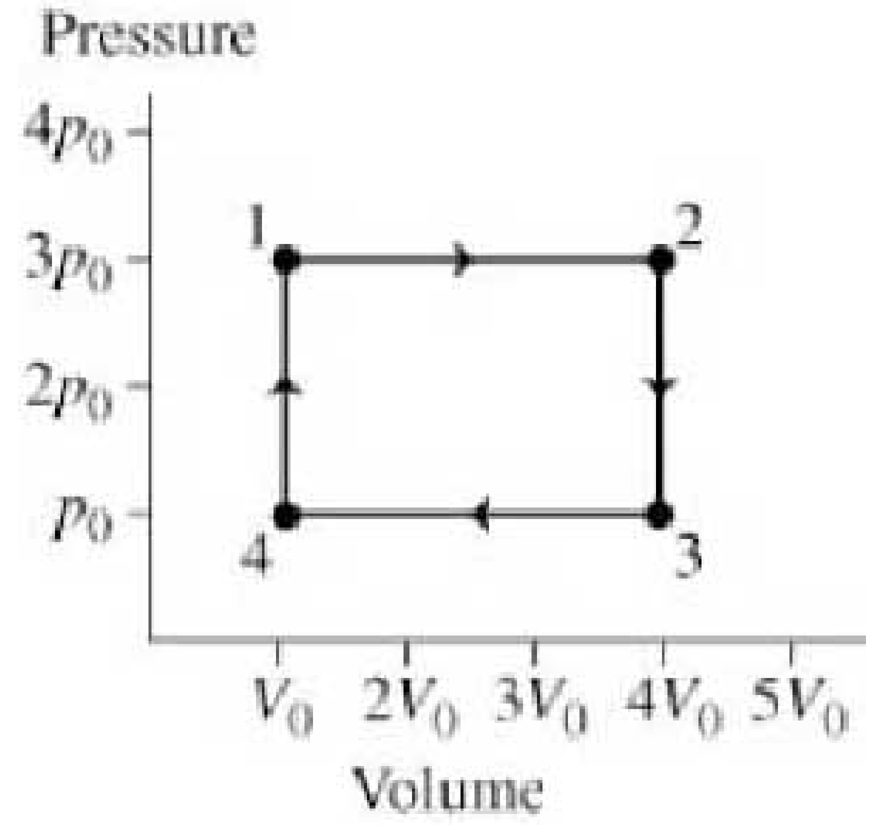
In the cycle shown what is the work done by the system going from state 4 to state 2 clockwise along the arrowed path?



1. $12\ p_0 V_0$
2. $9\ p_0 V_0$
3. $4\ p_0 V_0$
4. $3\ p_0 V_0$
5. $-12\ p_0 V_0$
6. $-9\ p_0 V_0$
7. $-4\ p_0 V_0$
8. $-3\ p_0 V_0$
9. None of above

PRS: Work in p-V plane:

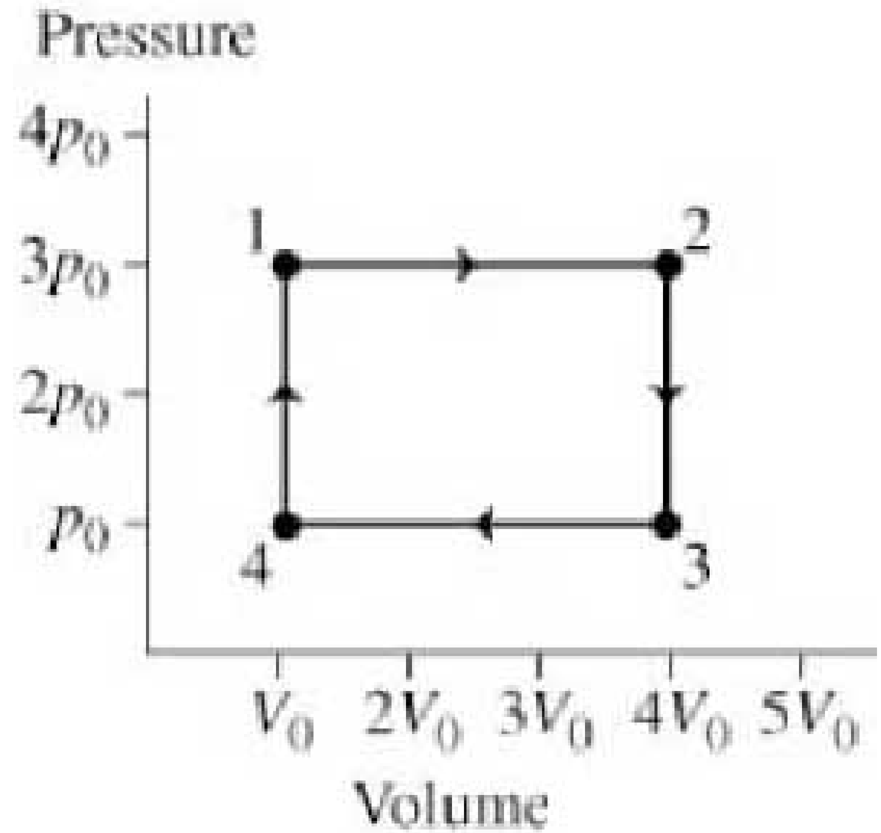
In the cycle shown what is the work done by the system going from state 2 to state 4 clockwise along the arrowed path?



1. 12 $p_0 V_0$
2. 9 $p_0 V_0$
3. 4 $p_0 V_0$
4. 3 $p_0 V_0$
5. -12 $p_0 V_0$
6. -9 $p_0 V_0$
7. -4 $p_0 V_0$
8. -3 $p_0 V_0$
9. None of above

PRS: Work in p-V plane:

In the cycle shown what is the total work done by the system starting from state 4 and going all around the loop clockwise as shown?



1. $8\ p_0 V_0$
2. $6\ p_0 V_0$
3. $3\ p_0 V_0$
4. $2\ p_0 V_0$
5. $-8\ p_0 V_0$
6. $-6\ p_0 V_0$
7. $-3\ p_0 V_0$
8. $-2\ p_0 V_0$
9. None of above
10. 0.0

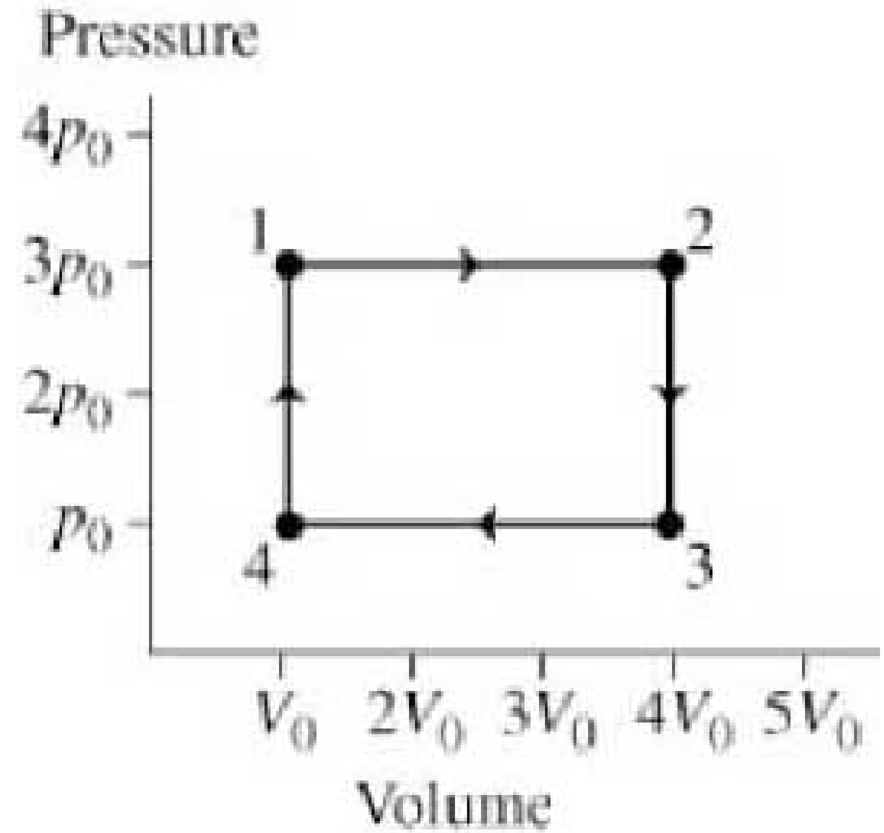
PRS: Total Work

How is it possible for the system to return to state 4 and yet do net work?

1. Not all the state variables return to their original state
2. Net heat is added even though the system returns to its original state
3. The process is not reversible
4. This loop would in practice take forever to occur
5. None of above

PRS: Work in p-V plane:

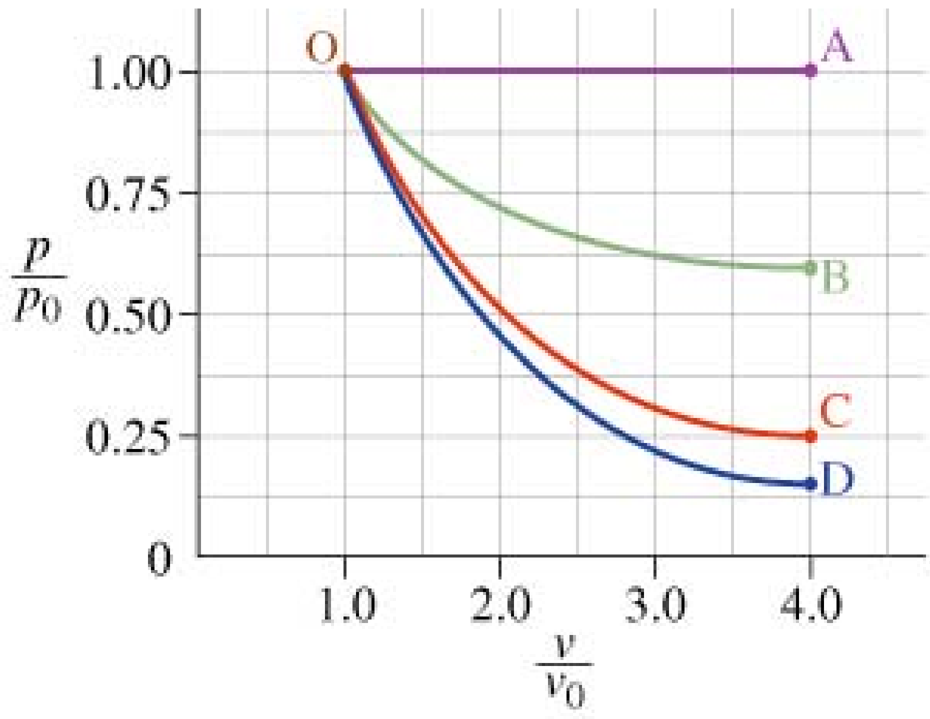
In the cycle shown what is the total work done by the system starting from state 4 and going around the loop ***counter***clockwise?



1. $8\ p_0 V_0$
2. $6\ p_0 V_0$
3. $3\ p_0 V_0$
4. $2\ p_0 V_0$
5. $-8\ p_0 V_0$
6. $-6\ p_0 V_0$
7. $-3\ p_0 V_0$
8. $-2\ p_0 V_0$
9. None of above
10. 0.0

PRS: Coolest Expansion

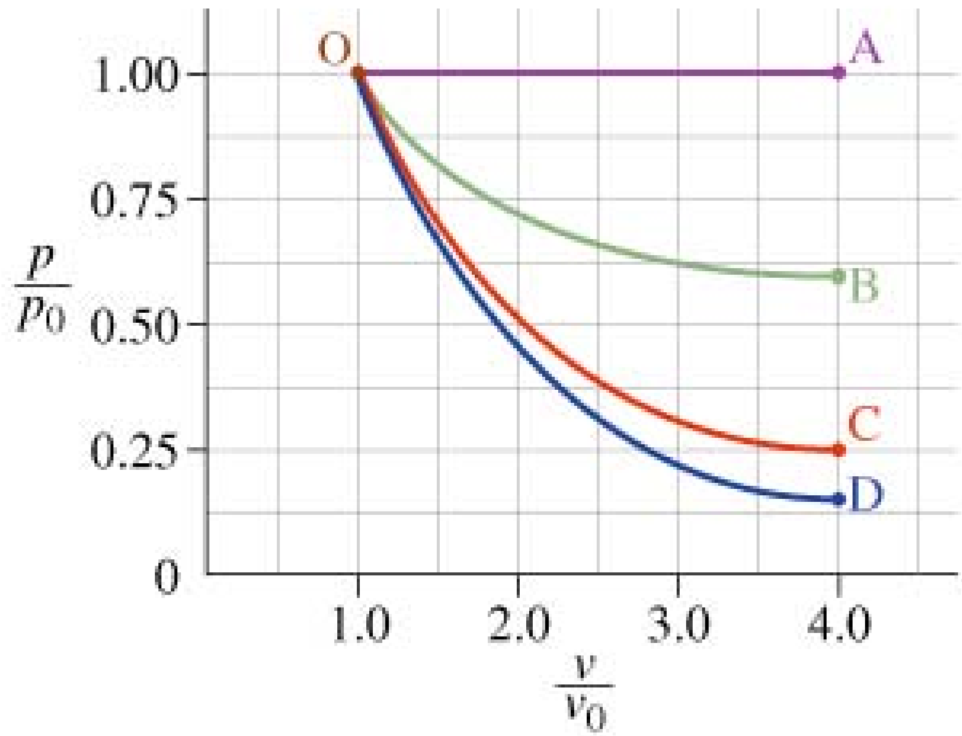
A gas can be expanded along any of the curves shown from state O to the labeled final states. Along which path will the final temperature be the lowest?



1. A
2. B
3. C
4. D
5. Need more information

PRS: Most Added Heat

A gas can be expanded along any of the curves shown from state O to the labeled final states. Along which path must the added heat be the highest?



1. A
2. B
3. C
4. D
5. Need more information

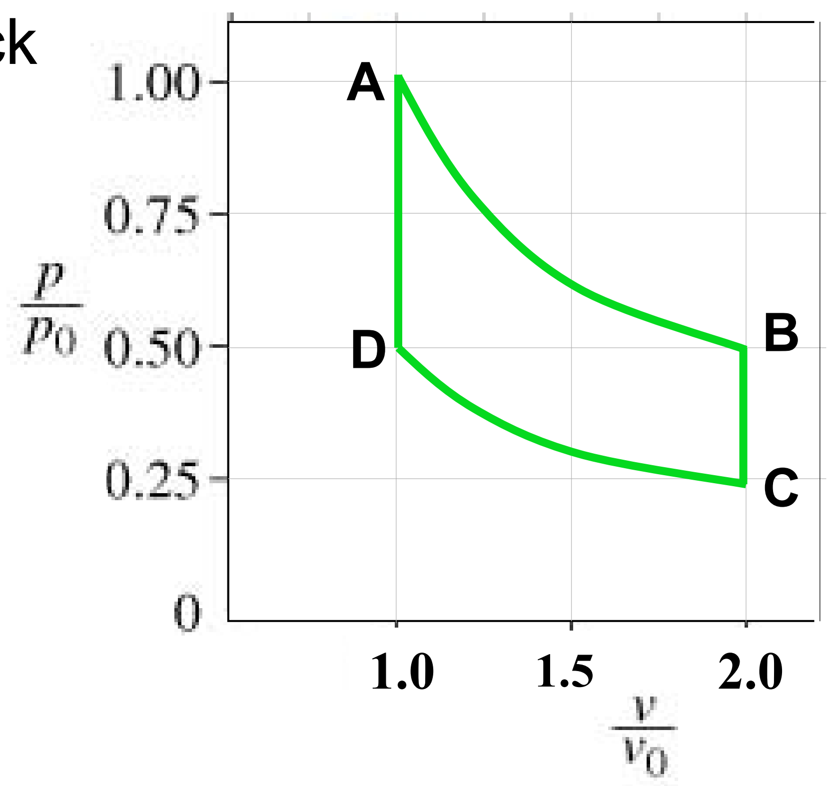
Thermodynamic cycles

A thermodynamic cycle is any process that brings a system back to its original state.

The cycle involves a path in state space over which various processes may act.

Addition/Removal of heat and work are typical processes.

Often the objective is to get work from heat or vice versa, as in a *heat engine.*



Find W and Q for Each Leg

- First Law Applies:

$$Q_{BA} = W_{BA} + \Delta U = W_{BA} + U_B - U_A$$

- Think in p-V space, then work is

$$W_{BA} = \int pdV = \int p(V)dV$$

- If Ideal Gas:

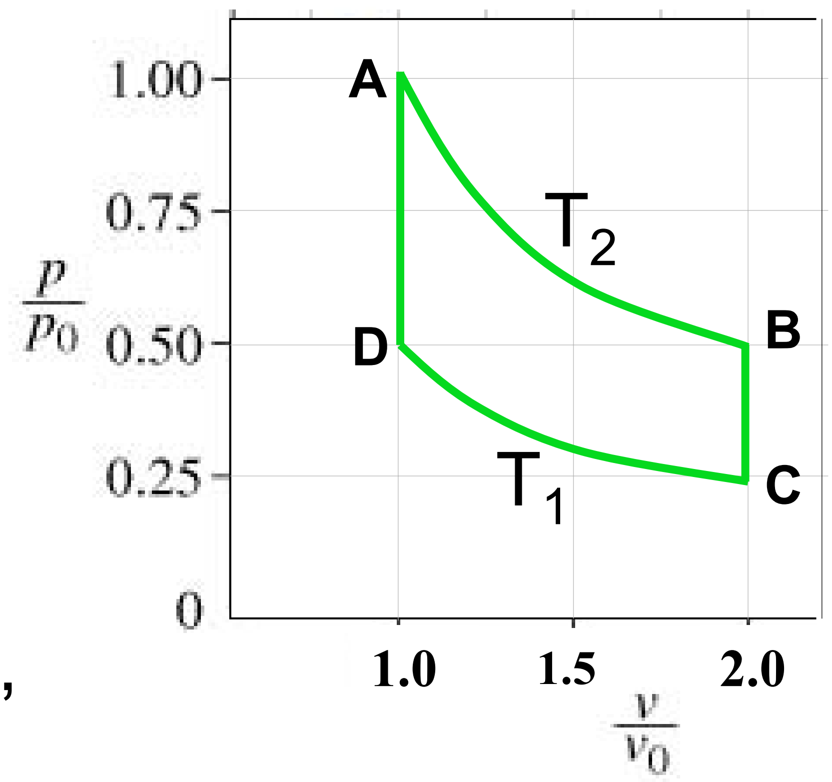
$$p(V)V = NkT \quad \text{Satisfied everywhere}$$

$$U_A = 3/2NkT_A \quad \text{Internal Energy (monatomic)}$$

- Generally find Heat from First Law

Isothermal Cycle

Consider the closed cycle shown here consisting of two isothermal processes, at temperatures T_2 and T_1 .

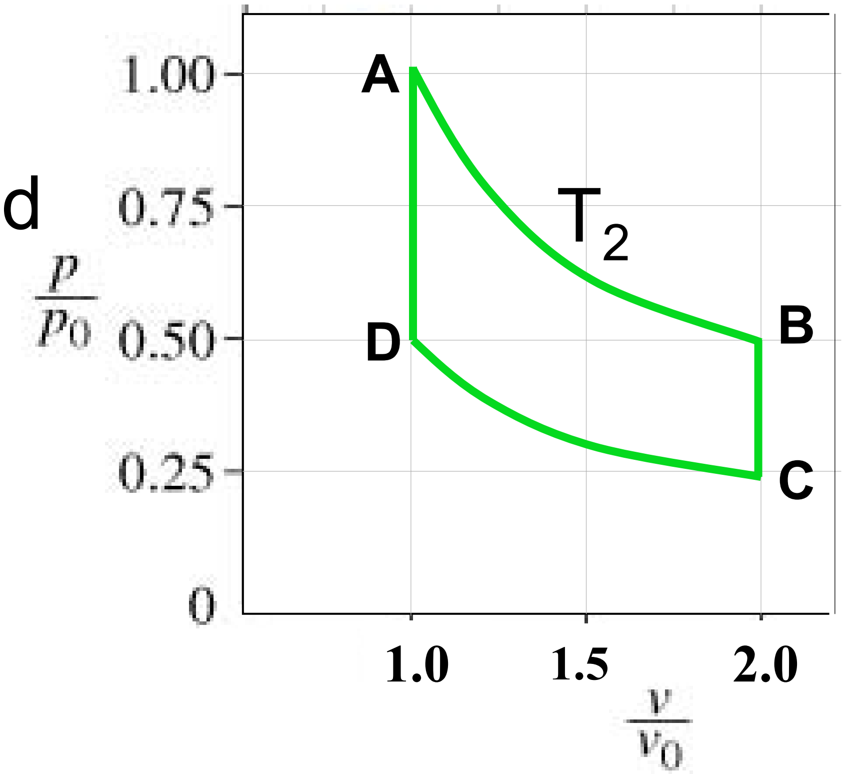


The work on the AB path is more than that on the DC path, so if we go around clockwise (e.g. ABCDA) this will be a heat engine that does positive work on the outside world

Work on path A>B

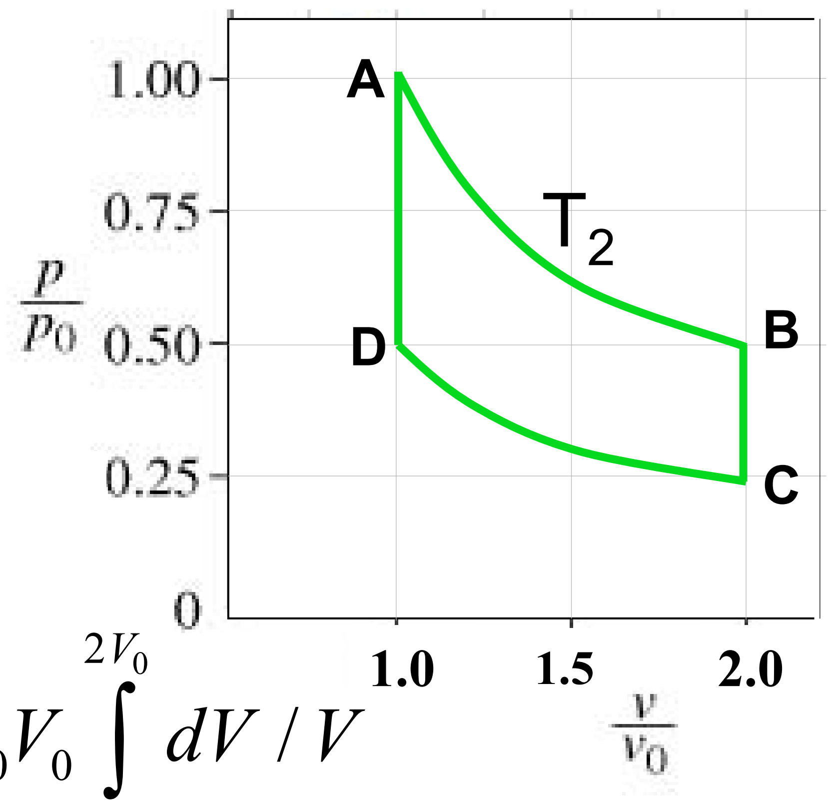
On the first part of the thermodynamic cycle the system goes from A to B and is maintained at T_2.

This is an isothermal process.



WORK on ISOTHERMAL path A>B

$$W_{BA} = \oint_{path} p\,dV = \oint_{path} p(V)\,dV$$



p(V) is determined from the perfect gas law,
Knowing temperature constant

$$p(V) = NkT_2 / V = p_0 V_0 / V$$

$$W_{BA} = \int_{V_0}^{2V_0} (p_0 V_0 / V)\,dV = p_0 V_0 \int_{V_0}^{2V_0} dV / V$$

$$\text{General Isothermal Process: } W_{BA}^{isothermal} = p_0 V_0 \ln(V_B / V_A)$$

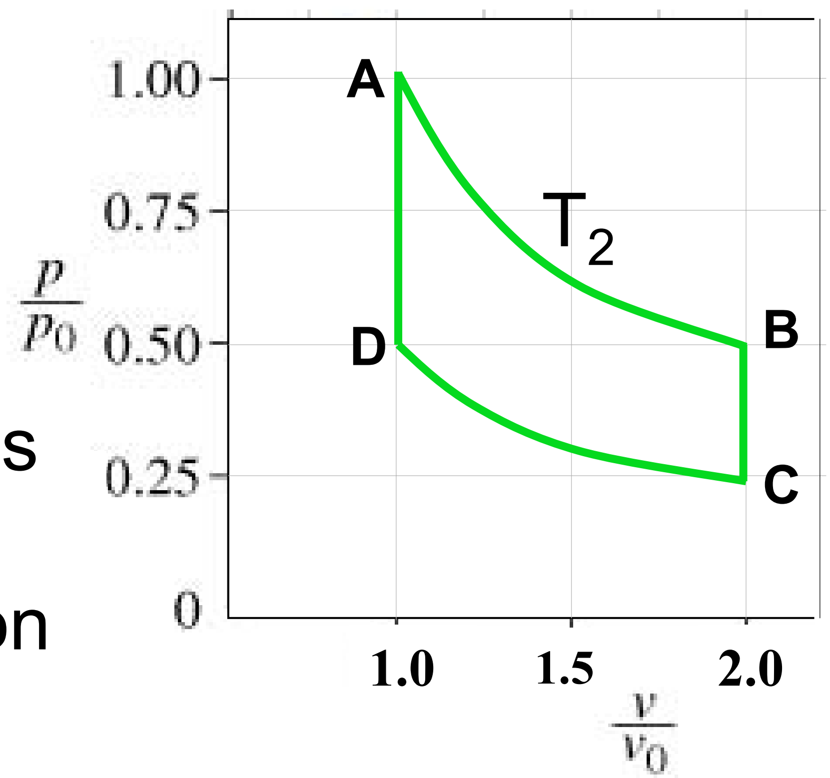
$$W_{BA}^{isothermal} = p_0 V_0 \ln(2)$$

HEAT on ISOTHERMAL path A>B

We know the work, how do we find the heat?

FIRST LAW!

$$Q_{BA} = W_{BA} + U_B - U_A$$



The change in internal energy is determined from State A&B

Internal Energy depends only on the temperature, which is T_2

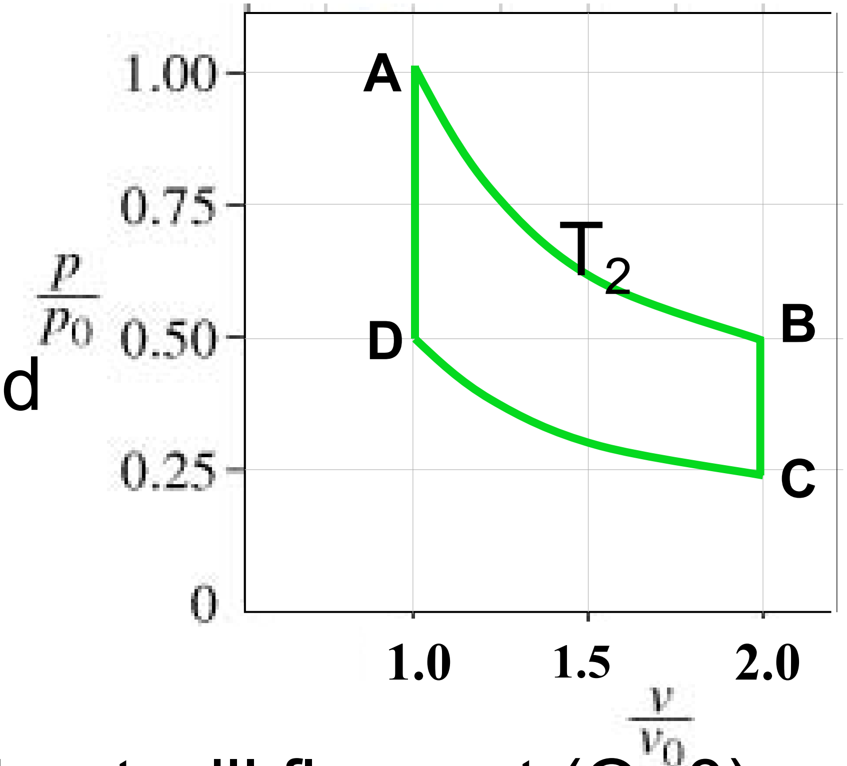
Hence $\Delta U = 0$.

$$\text{Iso thermal Process}: \; Q_{BA}^{isothermal} = W_{BA}^{isothermal} = p_0 V_0 \ln(V_B / V_A)$$

$$Q_{BA}^{isothermal} = W_{BA}^{isothermal} = p_0 V_0 \ln(2)$$

Reversibility

On the path between A and B. Which way does the system traverse this?



If the piston is allowed to expand then the gas will cool a bit and heat will flow in.

If the piston is pushed in (W<0)
then the gas will heat a bit and heat will flow out (Q<0)

If you are patient, this process is reversible - it can run either way with the opposite heat transfer and work,
all at the temperature T_2

HEAT on ISOTHERMAL path C>D

p(V) is determined by T_1

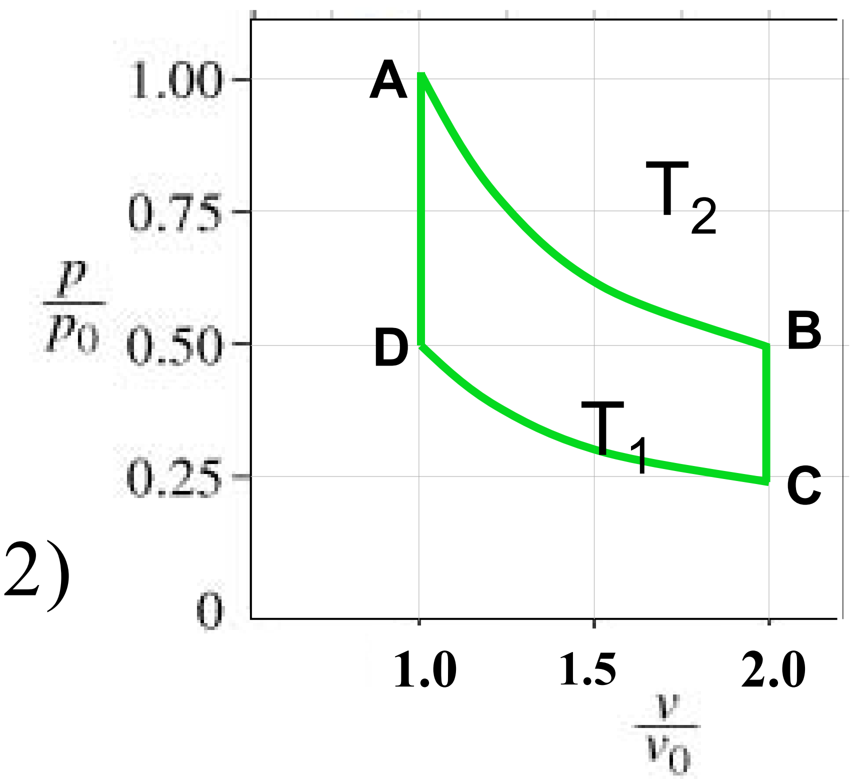
$$p(V) = NkT_1 / V = p_0 V_0 (T_1 / T_2) / V$$

$$W_{DC} = \int_{2V_0}^{V_0} p(V)\ dV = p_0 V_0 (T_1 / T_2) \ln(V_0 / 2V_0)$$

$$Q_{DC} = W_{DC} + U_D - U_C$$

In an Isothermal process with perfect gas $\Delta U = 0$.

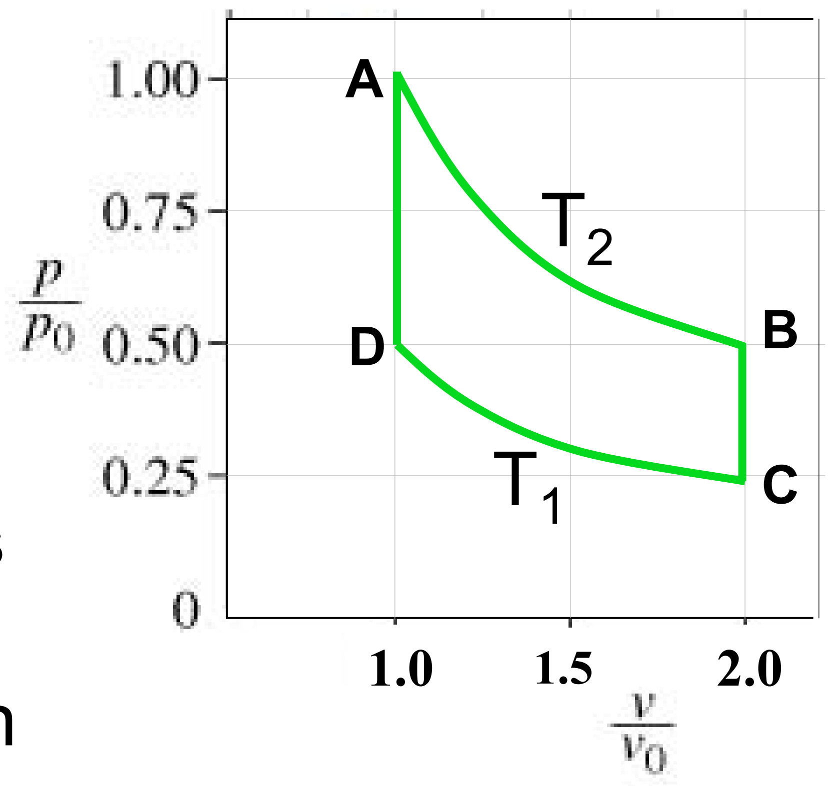
$$Q_{DC} = W_{DC} = -p_0 V_0 (T_1 / T_2) \ln(2)$$



Q and W at constant Volume

Now find the work and heat on the leg B to C

Note that the volume does not change (called *isochoric*)



$$W_{CB} = \int_{\text{path}} p\,dV = 0$$

The change in internal energy is determined from State A&B

Internal Energy depends only on the (changing) temperature

So $\Delta U = U_C - U_B = 3/2\ Nk(T_1 - T_2)$

$$Q_{CB} = W_{CB} + \Delta U = \frac{3}{2} Nk(T_1 - T_2) = p_0 V_0 \frac{T_1 - T_2}{T_2} < 0$$

W and Q for Isothermal Cycle

Now find the total work and heat for the whole cycle
Add the W on each leg & Q at each Temperature

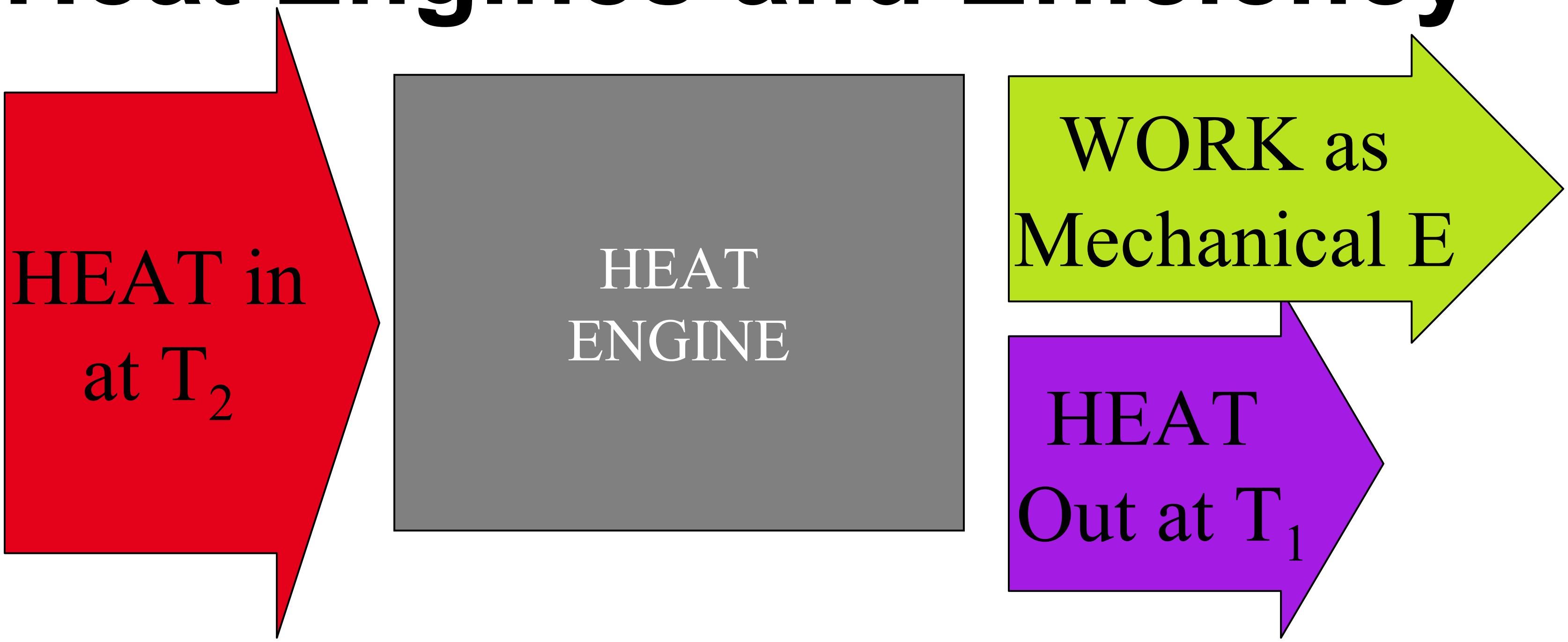
Leg	W	Q
A to B	$p_0V_0\ln(2)$	$p_0V_0\ln(2)$
B to C	0	$p_0V_0(T_1 - T_2)/T_2$
C to D	$-p_0V_0\ln(2)T_1/T_2$	$-p_0V_0\ln(2)T_1/T_2$
D to A	0	$p_0V_0(T_2 - T_1)/T_2$

$$W_{cycle} = p_0V_0(1 - T_1/T_2)\ln(2)$$

$$Q(\text{at}\quad T_2) = p_0V_0\ln(2) + p_0V_0(T_2 - T_1)/T_2$$

$$Q(\text{at}\quad T_1) = -p_0V_0\ln(2)T_1/T_2 - p_0V_0(T_2 - T_1)/T_2$$

Heat Engines and Efficiency



Energy Conserved $Q(\text{in at } T_2) = W_{cycle} + Q(\text{out at } T_1)$

Heat Engine Efficiency: $$\varepsilon = \frac{W_{cycle}}{Q(\text{in at } T_2)}$$

Efficiency of Isothermal Cycle

Heat Engine Efficiency: $\varepsilon = \dfrac{W_{cycle}}{Q(\text{in at } T_2)}$

$$\varepsilon = \frac{p_0 V_0 (1 - T_1 / T_2) \ln(2)}{p_0 V_0 \ln(2) + p_0 V_0 (T_2 - T_1) / T_2} = \frac{(T_2 - T_1) \ln(2)}{T_2 \ln(2) + (T_2 - T_1)}$$

This is quite low - even if T_2 is twice T_1 , ε=0.29

Maximum Thermodynamic Efficiency

The Isothermal Cycle had a volume ratio change of only a factor of two, leading to the ln(2) term.

$$\varepsilon = \frac{(T_2 - T_1)\ln(2)}{T_2 \ln(2) + (T_2 - T_1)}$$

We can increase the efficiency by making the volume ratio [I.e. ln(2)] arbitrarily large.

$$\varepsilon = \frac{T_2 - T_1}{T_2} = \frac{\Delta T}{T_2}$$

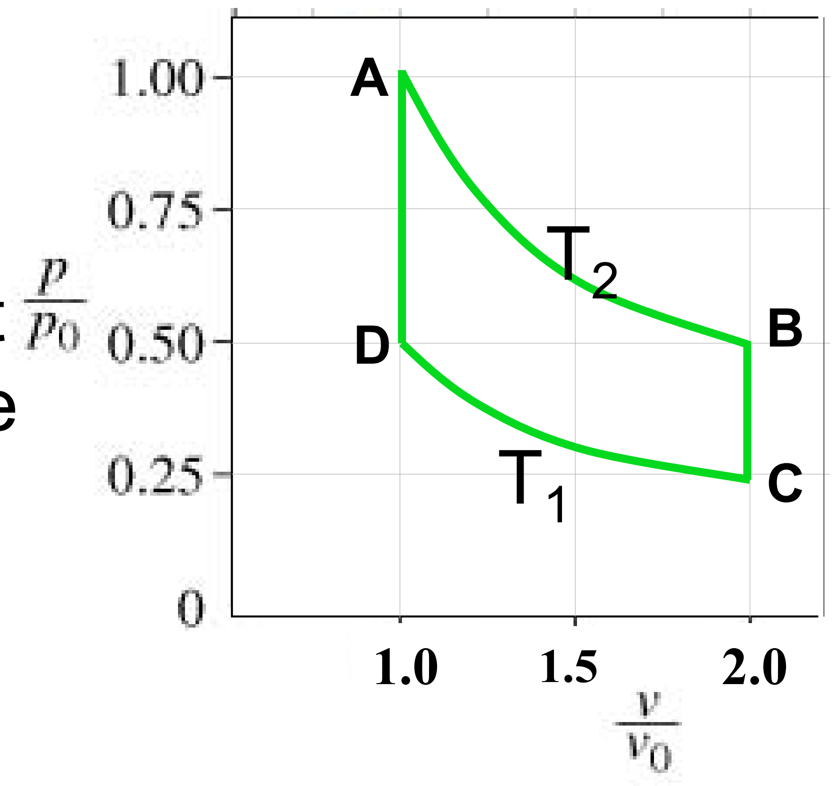
This is the largest possible efficiency possible with **any** heat engine and is often called the **Carnot** efficiency

Reversibility of Cycle

We showed that any leg of this cycle is irreversible.

Therefore, the entire cycle (heat engine) could operate in reverse

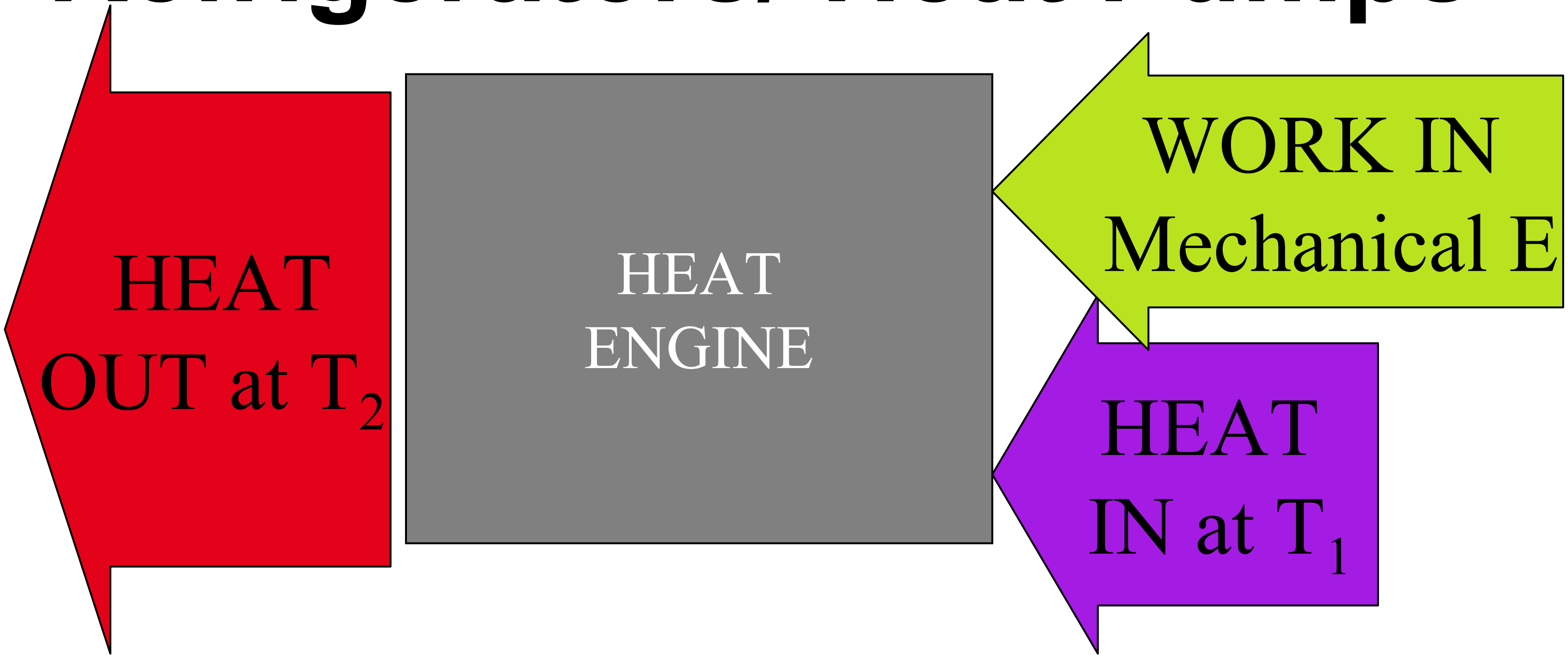
In this case the total Work and the Heat flow will be reversed.



Operated in reverse it is a refrigerator that removes heat at the lower temperature

Or a heat pump delivering heat at the temperature T_2

Refrigerators/ Heat Pumps



Energy Conserved $Q(\text{out at } T_2) = W_{in} + Q(\text{in at } T_1)$

Heat Pump Gain: $g = \dfrac{Q(\text{out at } T_2)}{W_{in}} = \dfrac{1}{\varepsilon} = \dfrac{T_2}{\Delta T} > 1$

Refrigerator Performance: $K = \dfrac{Q(\text{in at } T_1)}{W_{in}} = \dfrac{T_1}{\Delta T} > 1$

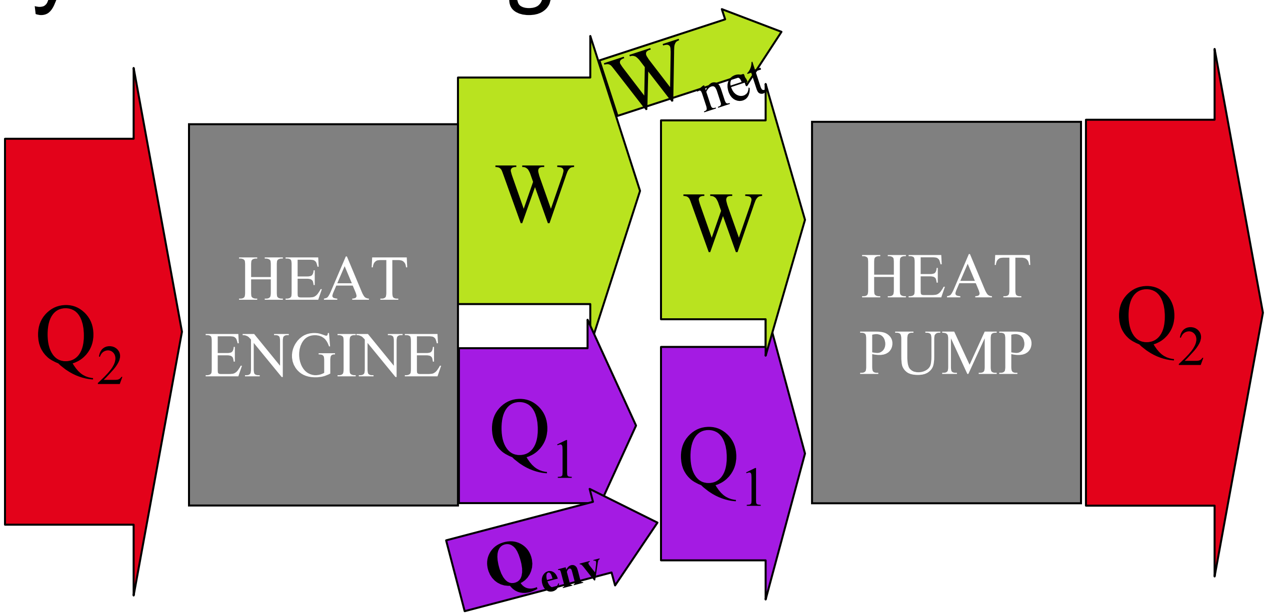
PRS: Refrigerator Light Bulb

Due to the failure of the switch that senses the closed door, the 25 watt interior light bulb in a refrigerator remains on constantly. The refrigerator maintains a temperature of -23C in the freezer and its outside coils are at 77C. What is the minimum extra power that the refrigerator will consume?

1. 10W
2. 25W
3. 35W
4. 62.5W
5. 87.5W
6. None of Above

Why Carnot is Maximum

Say a heat engine exceeded Carnot Limit



Hook it to a perfect gas heat pump with the same T_1

Net Effect: Heat Q_{env} becomes work W_{net}

Second law of Thermodynamics

No process shall have the only result that Heat is turned into Work

or

No process shall have the only result that Heat is transferred from cooler to hotter.

The second law is the formal statement of the irreversibility of Nature on a classical scale: friction can irreversibly convert mechanical energy into heat; nothing can do the reverse.