Short Questions

1. Suppose that output is given by

$$Y(t) = F\left(\int_0^t Q(\tau) I(\tau) d\tau, L(t)\right),$$

where $L(t)$ is labor used at time t, $I(\tau)$ is investment made at time τ and $Q(\tau)$ is the quality of that investment (there is no depreciation). F exhibits constant returns to scale. Suppose that we compute the capital stock as $K(t) = \int_0^t I(\tau) d\tau$ and perform growth accounting. What will we find? Suppose instead that a statistician computes quality adjusted capital stock. What would growth accounting imply in that case? Which estimate is more informative about the role of "technology" in economic growth?

2. "The large divergence between countries in the 19th century is evidence in support of the endogenous growth models." True or false?

3. Suppose that

$$Y(t) = \exp(g_A t) F(\exp(g_K t) K(t), \exp(g_L t) L(t)),$$

where F exhibits constant returns to scale. Suppose that $\dot{L}(t)/L(t) = n$ and $\dot{K}(t) = sY(t)$. Suppose also that F is not Cobb-Douglas (more specifically, suppose the share of labor changes if the effective capital-labor ratio $\frac{\exp(g_K(t))K(t)}{\exp(g_L(t))L(t)}$ changes). Show that balanced growth, where output grows at a constant rate, is only possible if $g_K = g_A = 0$.

Long Questions

1. Consider an infinite-horizon economy that admits a representative household with preferences at time 0 given by

$$\int_0^\infty \exp(-\rho t) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.$$

Population is given by $L(t)$ and grows at the constant rate n. Labor is supplied inelastically. The unique final good is produced with the production function

$$Y(t) = \frac{1}{1-\beta}\left[\int_0^{N(t)} x(\nu, t)^{1-\beta} d\nu\right] L_E(t)^\beta,$$

where $\beta \in (0,1)$, $x(\nu, t)$ denotes intermediate goods of type ν used in final good production at time t, $L_E(t)$ denotes the labor allocated to the production sector at time t and $N(t)$ is the number of intermediate good types available at time t. Once a particular type of intermediate good is invented, it can be produced by using ψ units of final good. The innovation possibilities frontier of the economy is

$$\dot{N}(t) = \eta (N(t))^{\phi} L_R(t)$$

where $\phi \leq 1$ and $L_R(t)$ is labor allocated to R&D at time t. The resource constraint for labor is $L_E(t) + L_R(t) \leq L(t)$, and the resource constraint for the final good is $C(t) + X(t) \leq Y(t)$, where $X(t)$ is spending on intermediate goods. There is free entry into research and a firm that invents a new intermediate good type receives a perpetual patent and becomes the monopolist producer of that good. However, a competitive fringe of producers can copy any intermediate good, produce it at the marginal cost $\gamma\psi$, where $\gamma \in \left[1, (1-\beta)^{-1}\right]$, and supply it to the market. The economy starts with $N(0) > 0$ intermediate goods at time $t = 0$.

For parts a-d, assume that $\phi = 1$ and $n = 0$.

(a) Define the equilibrium and balanced growth path (BGP) allocations.

(b) Characterize the BGP equilibrium. Does the equilibrium have transitional dynamics?

(c) Analyze the effects of an increase in the degree of competition, captured by a decline in γ. Show that greater competition reduces growth. Provide an intuition for this result. Is γ a good inverse measure of competition? How would you enrich the model so as to have a more realistic or theoretically more appealing model of competitive pressure on innovating firms?

(d) Now suppose $\phi < 1$ and $n > 0$ and repeat part b. Does the BGP growth rate depend on γ? Why is this?

2. Consider a variant of the neoclassical economy with preferences given by

$$\int_0^{\infty} \exp(-\rho t) \frac{c(t)^{1-\theta} - 1}{1-\theta},$$

and a large number of firms, with each firm j having access to the constant returns to scale production function $Y_j(t) = F(K_j(t), A(t) L_j(t))$ (also assume the Inada conditions hold). Suppose that $A(t) = \left[\sum_j K_j(t)\right]^{\phi}$, where $\phi < 1$. Capital and labor markets are competitive, and labor supply is constant at L. Define a steady-state equilibrium and characterize it. Is the steady-state equilibrium (saddle-path) stable? Why is there no growth in this economy? Is the equilibrium Pareto optimal?