Improvisational Specification of Design Spaces

by

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Abstract

As a mathematical abstraction and as a model for automated problem solving, the classical notion of a design space has proven convenient for the sciences and the engineering disciplines over the past sixty years. This is also true for models of calculating in design. These models, however, assume implicitly that a design space, one of possible compositions of shapes, is invented as a one-off final description or specification of designs, and that the individual who designs or composes has nothing new to contribute in the calculations other than to search and select among available possibilities. I suggest that this is limiting for the visual fields, such as architecture and applied arts. In this thesis, I articulate a novel, improvisational theory of design spaces. I describe a model of calculating an improvisational specification that complements the proposed theory and reconciles classical analytical approaches with the open-ended creative practice of improvisation.

The proposed specification is based on the shape grammar formalism and the associated algebras of shapes due to their unique treatment of shapes as raw, unanalyzed pictorial entities. The model is made of two calculating procedures: a. compositional rules are applied on shapes and their parts distinguished in observation, b. backwards descriptive rules specify the composition in terms of topological decompositions of shapes. Improvisation moves forward through compositional rules applied perceptually on shapes, while the design space in which composition happens is specified backwards by studying how shape decompositions map continuously from one rule application to another. I describe the differences between the proposed improvisational specification and classical specifications, which are defined in terms of symbols rather than spatial, pictorial entities. I outline important extensions to the proposed formalism and conclude by proposing improvisation as an alternative umbrella concept that presents opportunities to expand classical conceptualizations of design spaces from exclusive engagement with analysis as a form of preliminary projection of results before calculating.

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I dedicate this thesis to my family.

Contents

I Introduction

The notion of a design space comes from a tradition of knowledge accumulated in the pursuit of universality, mathematical rigor, control and prediction of how artifacts are conceived as abstractions in the mind of their inventors. While there exist many names that refer to the very same idea – decision spaces, problem spaces, configuration spaces, feature spaces, conformational spaces – they are not much different in what they stand for. Design spaces are universal formal devices for synthesis or composition of designs for buildings, systems, machines, proteins, artworks or other kind. Perhaps this fact escapes attention for two reasons. One, design spaces are so deeply ingrained in every aspect of design and calculating that they are taken for granted; we see this in the academic pursuit of automated design synthesis methods[1]. Two, it is a formal construction that is used in various forms according to the discipline of application.

For those who have not been aware of the existence of such a device regardless of their expertise in formal design methods or their computer implementations, by the end of this thesis, I will have provided an extensive overview of central concepts associ-

1. The term *automated design synthesis* is a placeholder for methodologies and implementations of automatic creation of designs by a digital computer. See for example, Mitchell W. J. "Techniques of automated design in architecture: a survey and evaluation" (*Computers and Urban Society* 1 (1), 963-980, 1975), Eastman, C. M. "Automatic composition in design" (In *Proceedings of the 1998 NSF Grantee Workshop on Design Theory and Methodology*, pp. 158-172. Springer-Verlag, 1989) and more recently in Cagan et al. "A framework for computational design synthesis: model and applications" (*Journal of Computing and Information Science in Engineering* 5 (3), 171-181, 2005).

ated with the term design space. For those who have substantial exposure to the notion of a design space, by the end of this thesis, I will have articulated a critique of some classical ideas, which I believe are still forcefully pressed upon contemporary research in design and calculating. At the same time, I will have provided an alternative theory of design spaces that better suits composition in architecture and the visual arts along with a model of calculating in support of the theory.

While design spaces are promoted as compositional devices, if there is anything that they offer this is precisely their analytical power given in their specification. What is the description of the space I deal with, what rules to apply to find designs, how to start, to what extent of the space rule applications reach. As a consequence of this, the notion of a design space undoubtedly serves the engineering disciplines and the sciences because it enables prediction by preliminary projection of results. After all, design space formalisms designate containers of possible options for a designer to pick according to specified ends. However, as a model of calculating, the idea that choices and actions are bounded in a space of possibilities before action starts, is an idea that necessarily ignores the productive processes involved during creative work. Design spaces operationalize the work of designers, composers, or makers[2] as having preconceived desirable outcomes and ways of evaluating those outcomes – as a kind of problem solving[3].

To some extent, this concern has found its way in present research. Opinions vary as to what exactly needs to be amended from past and present design space formalisms to make them become part of an ongoing creative process and not mere conceptual constructions specified independently of the peculiarities and insights involved in the process of design[4]. But present research never questions whether the notion of a design space itself is problematic and needs to be conceptualized differently. This is what I pursue in this thesis. I propose to consider the notion of a design space in situations wherein there is hardly any evidence if such a conceptual construction is applicable in the first place. What is the role of a design space in improvisation?

Improvisation will sound familiar to those who have engaged in one way or another with some form of artistic practice, for exam-

2. I often put these three words together. When I am not, it is only for avoiding repetition. In general, throughout this thesis I consider designers, composers or makers in the broadest sense of the word, as individuals or collective of individuals who 'create.'

3. This will not sound a problem for many people. In fact, problem solving will sound as if it is synonymous to design itself. In chapter 2, I offer a more clear picture of which aspects of this characterization I find problematic.

4. The literature is vast and largely overlapping. In chapter 2, two directions are distinguished in terms of how design spaces can be represented or explored differently.

ple as musicians, as stage actors or as part of an audience watching a performance of either kind. For others, improvisation may be the type of human behavior that belongs to the very far, mystical side of artistic creativity, and the habit of only the most gifted individual. Yet for others it might have nothing to do with the production of artistic work, but with the ability to respond spontaneously to local stimuli, for example when moving one's hand fast to catch a falling object or when coming up with the right response to an unexpected question during a conversation. These descriptions and uses of the word improvisation are common, but they are just very few of the possible descriptions and uses we encounter in real everyday life. Improvisation is everywhere, not only in design and creative composition but in all aspects of human life; an opinion greatly supported from many sides and for a variety of reasons, from the social sciences and anthropology, to organizational theory and philosophy, to computer science and artificial intelligence[5].

In improvisation, foresight has no place while the present embraces the momentary, the ephemeral, and the unexpected. What is the role of a design space then during the invention of an improvised work of art or design? I propose that present research misses an improvisational theory of design spaces; a computational view of the creative production of artifacts not in terms of results, not in terms of representations of containers of possibilities latently existing as abstractions, but by way of active, on the spot, perceptual interpretations of materials of composition[6]. In this thesis, I articulate one such theory and I describe a model of calculating that complements the theory.

In considering the connection between improvisation and the notion of a design space, we encounter the antithetic qualities of creation and analysis. In improvisation, creation is ongoing, in the making, it happens in the now. It appears at least contradictory to ask for the specification of that which is created prior to action. How do you specify the elements that go together in a composition before improvisation starts? When using design spaces, analysis – how things in the design space look like and what operations are allowed with them – is primary; it happens before creation (calculating) starts. In improvisation, analysis is secondary; if it exists, then its role is to retrospectively explain rather than define a prior world of rules and regulations to be explored in combinations.

5. Agre, P. E. "The dynamic structures of everyday life" (MIT, *AI Laboratory, Technical Report* 1085, 1987), Weick, "Improvisation as a mindset for organizational analysis" (*Organization Science* 9 (5), 543-555, 1998), Preston, B. *A philosophy of material culture* (Routledge, 2013), and Ingold, T. & Hallam, E. "Creativity and cultural improvisation: an introduction" (In *Creativity and Cultural Improvisation*, ed. E. Hallam & T. Ingold, pp. 1-24, Berg, 2007).

6. The term *materials of composition* refers to the things you create with. For example, an architect composes with visual shapes, physical tools or physical models, a painter composes with visual shapes and colors, a musician composes with the sounds and instruments, a dancer or an actor composes with gestures, speech and movement.

My purpose in developing an alternative theory of design spaces is to reconcile this antithesis between composition as an improvisation, which happens in the moment, on the spot, with classical analytical approaches that ask for the specification of the compositional elements you work with. There are two parts to this end. One part, describes what makes up the specification of that which is composed in an improvisation, which I call an improvisational specification - this is the theory part. The second part, develops a computational basis for an improvisational specification - this is the formal part, and supports the theory from a computational standpoint.

This thesis is developed in four chapters. In Chapter 2, *A design lies within some space*, I start by examining the present notion of a design space, which I call classical. In particular, from the protocol studies on human problem solving of the 1950s and 1960s I trace assumptions and motivations that underlie the idea that any act of synthesis or composition requires a space of alternative designs prior to their actual calculation - the description or specification of a design space comes in advance of calculating the entities in the space. Through examples, I show how this classical notion of a design space fails to provide insight into the momentary, improvisatory aspects of creative work when designers or composers engage actively, perceptually with materials of composition - here shapes and their parts. I describe present approaches towards devising alternative models of design spaces with a view towards making them more suitable for describing and supporting the creative process, I emphasize their limitations and I argue for a new approach.

In Chapter 3, *Improvisation, Recording, Calculating: How to Make it Different*, I synthesize literature from language dictionaries, musicological dictionaries, aesthetics of improvisational performance, material culture studies, and empirical analysis of improvisational musical practices. I distinguish characteristics of the process of invention of an improvised work of art or design and propose them as the basis for an improvisational theory of design spaces. Specifically, in improvisation there is no script implied in advance that demarcates possible choices and actions; the compositional structure is found in the process, on the spot. The self – the designer, composer, or maker – is an active perceiver of the compositional elements, and by his or her own participation in the calculations, aesthetically interprets those elements in open ended ways. I propose that during the invention of an improvised work, the notion of a design space has value only if we see it retrospectively, that is to say, as an analytical device for interrogating or explaining what happened up until some moment of interest, rather than defining a world of future possibilities. I argue for an improvisational specification, which is ongoing, in the making, calculated by the individual or collective of individuals who compose in real-time.

In Chapter 4, *Improvisational Specification: How to Set it up Formally*, I develop a model of calculating an improvisational specification. The proposed model characterizes design spaces as outputs of a creative process that proceeds through compositional rules specified on the fly according to how shapes are interpreted momentarily and perceptually. The proposed model is based on the shape grammar formalism and the associated shape algebras[7]. Algebra $\mathbf{U}_{ij}$ for visual shapes defined in terms of a partial (embedding) relation, a Boolean part and a Group part is presented. Compositional rules are defined in terms of schemas[8] that operate with shapes from $\mathbf{U}_{ij}$. An improvisational specification is found as a consequence of a rule application. In particular, while compositional rules apply freely on visual shapes to move a composition forward, the necessary symbolic descriptions that satisfy their application in each state, is found backwards. A backwards specification maps the necessary atoms from one state to another, which are consequences of the rule applied. Rules and mappings are presented for retrospectively interrogating the history of computations in order to satisfy continuity between the state descriptions involved in a visual composition. I present an application to two-dimensional visual composition with shapes to illustrate the proposed approach. In an improvisational specification, while composition moves forward the ongoing design space in which composition happens is specified backwards after every rule application or intermittently. The specification itself remains ongoing, in the making, open to new rule applications.

In Chapter 5, *Taking it Further*, I describe the differences between the proposed improvisational specification and classical specifications, which are defined in terms of symbols rather than spatial,

7. Stiny, G. & Gips, J. "Shape grammars and the generative specification of painting and sculpture" (In *Information Processing* 71, ed. O. Petrocelli, pp. 125-135, Auerbach) and Stiny, G. *Pictorial and formal aspects of shape and shape grammars* (Birkhauser, 1975).

8. Stiny, G. *Shape: talking about seeing and doing* (The MIT Press, 2006).

pictorial entities. In particular, a classical specification requires rule applications to be continuous and reversible, while continuity and reversibility in an improvisational specification, are things to specify retrospectively with appropriate rules for interrogating the history of computations. I further describe how the current formalism can be extended with more work on the type of compositional rules used, for example additive and subtractive, different rules and mappings for retrospectively specifying design spaces for a composition, and other kinds of algebras for shapes with properties, such as color or material, and even algebras for physical objects.

In concluding this thesis, I invite present work on design spaces to consider the analogy of improvisation as a novel umbrella concept that presents opportunities to expand classical conceptualizations from exclusive engagement with analysis as a form of preliminary projection of results before calculating. The proposed theory and its associated model of calculating introduce a novel characterization of design spaces that, to the extent of my knowledge, has not been considered in previous scholarly research.

The broader implications of viewing design spaces as not preconditions for creation or invention of designs but as constructions that emerge from an ongoing process, are outlined at the end of this thesis. In particular, I shortly discuss the role of the proposed formalism in describing the perceptual aspects of the process of invention of an improvised work of art or design. I then argue that design spaces, or in other terms, languages of designs[9] can be seen as outputs of a creative process. An idea that freshly revisits central concepts in the tradition of design theory initiated by the shape grammar formalism. Last, I touch upon the notion of learning from prior 'stored' evidence by comparing the motivations and assumptions underlying contemporary models of learning, to the proposed formalism, which is based on a memoryless model of calculating, namely, shape grammars. These directions present opportunities to revisit central themes in the formal, computational explanation of innovation and creativity and extend present design theory in important ways.

9. This definition of a language of designs may be somewhat frugal. But it is so only for the sake of the argument. For a more comprehensive coverage of languages of design see Knight, T. *Transformations in Design: A Formal Approach to Stylistic Change and Innovation in the Visual Arts* (Cambridge University Press, 1994). A very early paper that describes the definition of languages of designs in terms of constructive rules is given in Stiny, G. Kindergarten grammars: designing with Froebel's building gifts (*Environment and Planning B: Planning and Design*, 7, 409-462, 1980). The precursor to the idea of a language of designs - although with important differences - are phrase structure grammars. In particular, see Chomsky, N. *Syntactic Structures* (Walter de Gruyter, 1957).

II A design lies within some space

And how will you inquire, Socrates, into that which you know not? What will you put forth as the subject of inquiry? And if you find what you want, how will you ever know that this is what you did not know?
– Plato, in Meno

10. De Groot, A. *Thought and choice in chess* (The Hague, Mouton, 1965), Newell, A. "On the analysis of human problem solving protocols" (*Technical Report, Carnegie Institute of Technology*, June 27, 1966), Simon, H. A. *The sciences of the artificial* (The MIT Press, 1969), Eastman, C. M. "On the analysis of intuitive design processes" (In *Emerging Methods in Environmental Design and Planning*, ed. G. T. Moore, pp. 21-37, MIT Press, 1970)

11. Newell, A., Shaw, J. C., and Simon, H. A. "Elements of a theory of human problem solving" (*Psychological Review* 65 (3), 151-166, 1958).

12. Ibid., 151.

The united efforts of psychologists and computer scientists in the middle of the previous century introduced the notion of a design space as a preliminary element of any process of creation or invention. We read about this in the problem solving literature and in the protocol studies of the 1950s and 1960s on individuals who play chess, prove theorems in geometry or logic, solve puzzles, solve codes in cryptography, solve architectural plans[10]. Allan Newell, Cliff Shaw, and Herbert Simon were particularly interested in a theory of reasoning that explains creation not in just one of these task instances but in all of them – they viewed creation as a process of higher-level reasoning that is required for devising solutions to any kind of real world problem[11]. The novelty of their theory of course, which they called a general theory of human problem solving, lied in the fact that it introduced a calculable form of reasoning; one that specified or described problem solving behavior in terms of elementary information processes particularly suited for computer programs[12]:

> An explanation of an observed behavior of the organism is provided by a program of primitive information processes that generates this behavior.

Very much close to what thought - psychologists also pursued about the same time if not earlier[13].

Within a general theory of problem solving, creation requires a design space of possibilities always preliminary to the their actual calculation. To be more specific, consider as an example the Logic Theorist (LT), or even the General Problem Solver (GPS), which were programs that Newell, Shaw and Simon created to tackle problems in elementary calculus and theorem proving respectively[14]. Both programs took an a priori specification of a design space in terms of a set of initial 'true' hypotheses or assertions about a particular problem, and a set of operators for deriving new 'true' hypotheses or assertions from given ones – much like all later incarnations of rule based deduction systems. Once the specification was set, the actual calculations, the ones that produce one hypothesis after another until resolution, were the responsibility of an automated plan of admissible actions, namely, a search for the right sequence of operator applications (means) that lead to the desired solution of the problem (ends)[15]:

> We give it a list of expressions (axioms and previously proved theorems) that it may take as "given" for the task at hand. These are stored in LT's memory. We present LT with another expression and instruct it to discover a proof for this expression. From this point, the computer is *on its own* [italics mine].

The design space itself is an abstraction; a conceptual construction that contains all possible solutions to a problem latently lying in abstract symbolic formats. Calculations plan how these possibilities are revealed; they provide an ordered way of searching and selecting the solution(s) that satisfy goals specified in advance.

Now the literature of artificial intelligence is rich with variations of this standard problem solving theme and its calculable properties. It is also rich with alternative proposals to the evidently persisting means ends view of problem solving. For example, forward state space search methods[16], and more recently, an architecture capable of performing a much broader array of planning strategies with means ends analysis, forward chaining, and other fa-

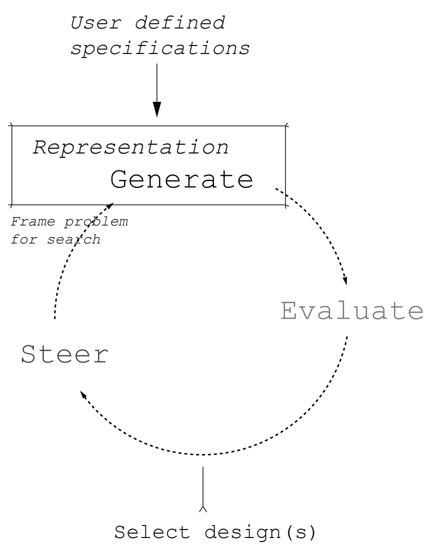
13. For example, see the extensive work of De Groot, A. *Thought and choice in chess* (The Hague, Mouton, 1965) who studied thought strategies of chess players, and the work of Bruner, J. S., Goodnow, J. & Austin, G. *A study of thinking* (Willey, 1956) on concept formation.

14. Newell, A. & Simon, H. A. *Human problem solving* (Prentice-Hall, 1972)

15. Newell, A., Shaw, J. C., and Simon, H. A. "Elements of a theory of human problem solving" (*Psychological Review* 65 (3), 151-166, 1958, 154).

16. Hoffman, J. & Nebel, B. "The FF planning system: fast plan generation through heuristic search (*Journal of Artificial Intelligence Research* 14, 253-302).

miliar search techniques as special cases[17]. More or less, attention gradually changed from conducting protocol studies on humans to find clues about problem solving, to finding better techniques for planning (problem solving) focused specifically on how computer programs problem solve independently of human origin. But nevertheless, the idea that design is problem solving, and that computers can automatically problem solve, provided almost exclusively the foundations for further research on computable models of the design process. Names, such as 'automated composition,' 'automated engineering synthesis,' 'computational design synthesis' are long standing placeholders for methodologies of automated creation of designs regardless of the particular domain of application. The following diagram captures the kinds of classifications researchers in design studies have been using over the past sixty years in their attempts to capture, systematize and above all describe as much as possible the process of design, independently of the design field.



The words that I chose to use in this diagram are the minimal ones I find useful to talk about classical concepts in calculating that are persisting to this day in the design and computation scene. They are certainly the ones most strongly associated with the notion of a design space and they have been used extensively in the literature[18]. They also talk back directly to the work of Newell, Simon and Shaw and in particular in their 'elementary information processes' of the human mind involved when solving problems[19]:

17. Langley, P., Pearce, C., Bai, Y., Worsfold, C. & Barley, M. "Variations on a theory of problem solving" (In *Proceedings of the Fourth Annual Conference on Advances in Cognitive Systems*, 2016).

18. Mitchell, W. J. "Techniques of automated design in architecture: a survey and evaluation" (*Computers and Urban Society* 1 (1), 963-980, 1975), Finger, S. & Dixon, J. R. "A review of research in mechanical engineering design. Part I: descriptive, prescriptive, and computer-based models" (*Research in Engineering Design* 1 (1), 51-67, 1989) and Cagan, J., Campbell, M. I., Finger, S., & Tomiyama, T. "A framework for computational design synthesis: model and applications" (*Journal of Computing and Information Science in Engineering* 5 (3), 171-181, 2005).

19. Newell, A., Shaw, J. C., and Simon, H. A. "Elements of a theory of human problem solving" (*Psychological Review* 65 (3), 151-166, 1958, 152).

> In general, the processes that compose the program are familiar from everyday experience and from research on human problem solving: searching for possible solutions, generating these possibilities out of other elements, and evaluating partial solutions and cues.

The very first step towards problem solving is the specification or description of a design space in which we automatically search for solutions. The next steps in my diagram, evaluate, select and steer are essentially abstract labels that people use to talk about control structures that guide how search proceeds with respect to utility functions. Control structures steer search towards certain designs that have desired properties. Note that the concepts of evaluation and steering are direct relatives of another important concept of contemporary importance, which is computational optimization. Optimization will not be covered in this chapter, nor in any other part of this thesis. Remember also that things are much more complicated in practice than any information processing input-output diagram will ever capture. For the interested reader, the Design Methods movement of the 1960s is founded on the same belief; that the process of design while intuitive, tacit and based on experience and apprenticeship can be systematically captured and verbalized with words and models like analysis, synthesis, cyclical, branching and others, and ultimately taught in schools of architecture and engineering[20].

I use the term 'design space' here a little too generously. Many names exist for the same idea. Conformational spaces are used in computational biology for designing protein structures based on their conformational features[21]. Configuration spaces and feature spaces are other names used, for example in the synthesis of machines from predefined mechanical parts[22], the synthesis of kinematic chains[23], the synthesis or simulation of vehicle paths[24]. Newell, Shaw and Simon, used the general name 'problem spaces' due to its clearly universal character. Decision spaces or parameter spaces are names that refer to geometric and multi-dimensional Cartesian design spaces that underlie almost exclusively the field of design optimization and they originate in economics and operations research. John Gero and Anthony Radford explain it well[25]:

> Imagine, that everything about the building is fixed except two design variables [window height, shade projection]... We can represent our two design variables as axes in a design (or decision) space, where each point in the space will represent a particular combination of design decisions.

20. A well known source for the Design Methods movement is Jones, C. J. and Thornley, D. (Eds.) *The conference on design methods: papers presented at the conference on systematic and intuitive methods in engineering, industrial design, architecture, and communications* (London, Pergamon Press, 1962). Also, a short but comprehensive summary of key ideas linked with models that describe what designers do in practice in parallel with computer based models of design processes like parametric design, configuration design and constraint-based design is Finger, S. & Dixon, J. R. "A review of research in mechanical engineering design. Part I: descriptive, prescriptive, and computer-based models" (*Research in Engineering Design* 1 (1), 51-67, 1989).

21. For example, see the formalism given in Parisien, M., Major, F. & Peitsch, M. "A protein conformational search space defined by secondary structure contacts" (In *Proceedings of Annual Pacific Symposium on Biocomputing*, 425-436, 1998).

22. Brown, K. N., McMahon, C. A. & Sims Williams, J. H. "Features, aka the semantics of a formal language of manufacturing" (*Research in Engineering Design* 7 (3), 151-172, 1995).

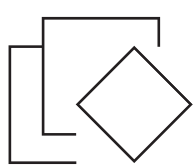
23. Subramanian, D. & Wang, C. "Kinematic synthesis with configuration spaces" (*Research in Engineering Design* 7 (3), 193-213, 1995).

Regardless if designing architectural plans, protein structures or machines, the meaning of a design space is common to all; in all cases a design space implies a container of possibilities latently existing as abstractions. But the use of the term design space in place of other more discipline specific terms is maybe even more appropriate. It serves the vision for formal grounding of all professional fields concerned with design – a vision expressed in the still seminal *Sciences of the Artificial* by Herbert Simon. According to Simon's perspective, architects, industrial designers, mechanical engineers, artists are not the only ones who are concerned with design. "Schools of engineering, as well as schools of architecture, business, education, law, and medicine, are all centrally concerned with the process of design"[26]. The notion of a design space – and the term itself – embodies this vision because it enables universality, mathematical rigor, control and prediction of how artifacts are conceived as abstractions in the mind of their inventors, indeed, across professions and domains of interest.

To illusrate how design spaces are specified I will present the following short example. In particular, the important step towards formulating a design space is its representation because it captures the entities in the space. A representation describes in a symbolic format the way entities in the space look like, namely, the entities we want to design. A compact way to present a description of an entity in a representation is the simple couple {*s*, **D**}, where entity *s* is represented with a description **D**, which specifies what aspects of *s* we are interested in, what aspects we can control and how. **D** answers questions such as 'What is the description of *s*?' 'What is it?' 'What is it made of?' As the dictionary puts it[27], a description is:

> A statement that tells you how something or someone looks, sounds etc.: words that describe something or someone.

To illustrate how {*s*, **D**} works, consider *s* to be the following picture



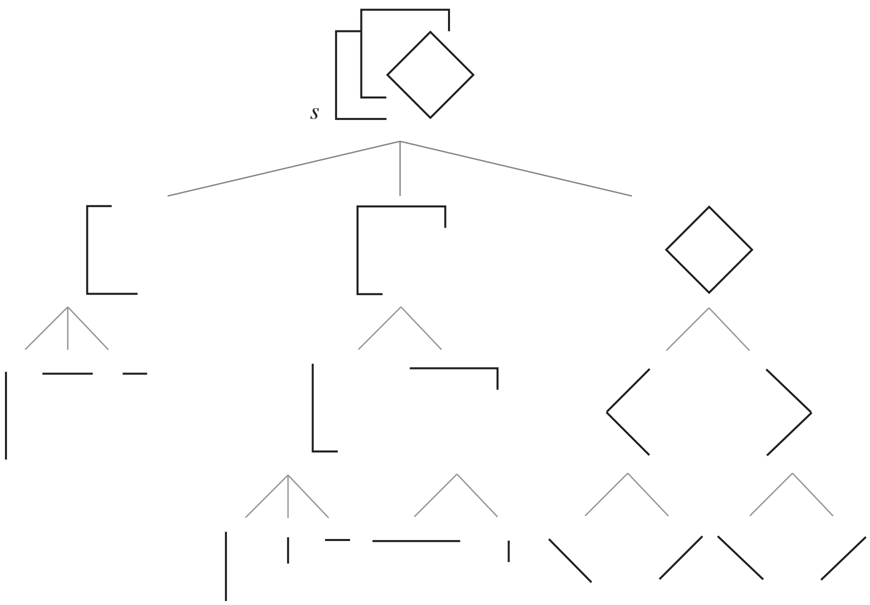
24. Lozano-Perez, T. "Spatial planning: a configuration space approach" (*IEEE Transactions on Computers* C-32, 1983).

25. Radford, D. & Gero, J. Design by optimization in architecture, building, and construction (John Wiley & Sons, 1988, 8).

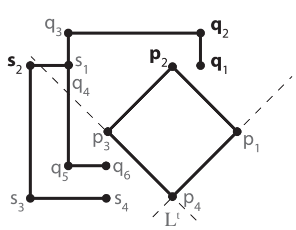
26. Simon, H. A. *The sciences of the artificial* (The MIT Press, 1969, 111)

27. Merriam-Webster Dictionary.

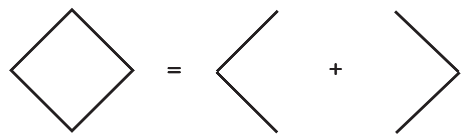
which is a pictorial entity, a shape. One way to describe *s* is to use a hierarchy. A hierarchy is a descriptive device that defines an ordered structure; it tells how elements are put together to make up an entity and therefore a hierarchy essentially defines a decomposition of an entity *s* into primitive elements the sum of which equals *s*. The following figure shows one decomposition of s into a hierarchy of primitive parts



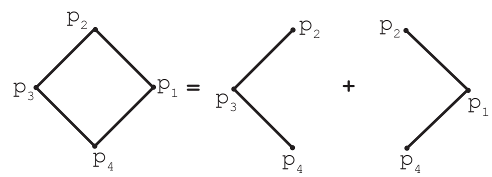
Further, I will assign names to the endpoints of each primitive linear element shown at the bottom of the above hierarchy like so



The collection of these linear elements is now represented as three groups of points: {**P**, **A**}, a set of points $\{\{<p_1, p_2>, <p_1, p_4>\}, \{<p_3, p_4>, <p_2, p_3>\}\}$, coupled with the set $\{p_4, L^t\}$ that is made of a label L^t attached to the point p_4 and denotes a frame of reference for point set representation; {**S**}, the set $\{<s_1, s_2>, <s_2, s_3>, <s_3, s_4>\}$; {**Q**}, the set $\{\{<q_1, q_2>, <q_2, q_3>\},\{<q_3, q_4>, <q_4, q_5>, <q_5, q_6>\}\}$. With this representation scheme, a linear element is represented with two endpoints between the symbols <>, a shape is represented with collections of endpoints inside the curly brackets {}. For example, the shape



is essentially represented with the point set {**P**, **A**}



All computer graphics applications operate on sets of points that describe shapes. Essentially, operations with shapes are constrained to those points and their possible movements in a two or three dimensional Cartesian space. The final description then for shape *s* according to the above representation scheme **D** is

$\{\, \{\mathbf{P}, \mathbf{A}\}, \{\mathbf{S}\}, \{\mathbf{Q}\} \,\} = \{\, \{\{\langle p_1, p_2\rangle, \langle p_1, p_4\rangle\}, \{\langle p_3, p_4\rangle, \langle p_2, p_3\rangle\}, \{p_4, L^t\}\}, \{\langle s_1, s_2\rangle, \langle s_2, s_3\rangle, \langle s_3, s_4\rangle\}, \{\{\langle q_1, q_2\rangle, \langle q_2, q_3\rangle\}, \{\langle q_3, q_4\rangle, \langle q_4, q_5\rangle, \langle q_5, q_6\rangle\}\} \,\}$.

The highlighted symbols $\mathbf{q}_1$, $\mathbf{q}_2$, $\mathbf{s}_2$ and $\mathbf{p}_2$ are the design variables I distinguished for this particular example; they determine how *s* can be modified in the two dimensional plane. In general, variables take either continuous or discrete values depending on what we want to represent in a design. If a variable represents the face of a die then it takes discrete values from the closed set [1... 6]. In our case, we are dealing with continuous values in the two dimensional euclidean space E^2 and they take the coordinates of two-dimensional points of the form (x, y). The coordinates are defined with respect to the fixed label L^t. Depending on the degrees of freedom we need, we can either use both dimensions of E^2, or choose to fix one axis and keep the other variable, for example

$\mathbf{p}_2 \longleftarrow (x_1, y_1)$
$\mathbf{s}_{2y} \longleftarrow y_2$
$\mathbf{q}_{1x} \longleftarrow x_3$
$\mathbf{q}_2 \longleftarrow (x_4, y_4)$

Here x_i and y_i are real valued numbers. A description **D** can represent constraints on the variables used in the representation. These are defined in order to describe restrictions on the possible values of each variable as well as other kinds of spatial relations between them. Specifically, in this example I assign a lower and an upper numerical bound to each variable's range of values like so

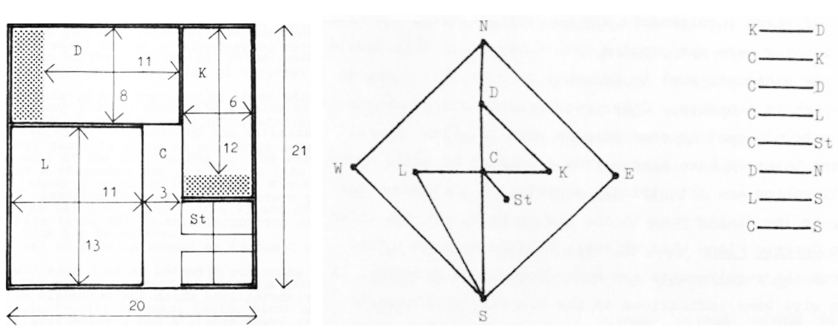
$$-1 \leq p_{2x} \leq 1 \qquad -2 \leq s_{2y} \leq 0.5 \qquad -0.5 \leq q_{1x} \leq 0.5$$
$$-1 \leq p_{2y} \leq 1 \qquad -2 \leq q_{2x} \leq 2$$
$$-2 \leq q_{2y} \leq 2$$

and spatial constraints between individual axis. For example, I force equality and perpendicularity constraints between certain elements, such as

$$q_4 == s_1$$
$$s_{2y} == s_{1y}$$
$$\langle p_1, p_4 \rangle \perp \langle p_4, p_3 \rangle$$
$$\langle s_4, s_3 \rangle \perp \langle s_3, s_2 \rangle$$
$$\langle q_6, q_5 \rangle \perp \langle q_5, q_3 \rangle$$

Constraints of this nature limit the range of possible designs in the search space. They are a standard way of controlling a search process towards designs that satisfy requirements.

The formal devices I use describe or specify a design space explicitly. In a sense, a specification is about commitments we make about possible designs prior to calculating them; how potential designs look like, their constituent atomic parts, the way they are put together, and the utilities they are supposed to satisfy. This can be seen in other types of design space specifications. In particular, very often we see applications of universal, graph-theoretical descriptions for generating architectural plan layouts. Consider for example the following figures[28]



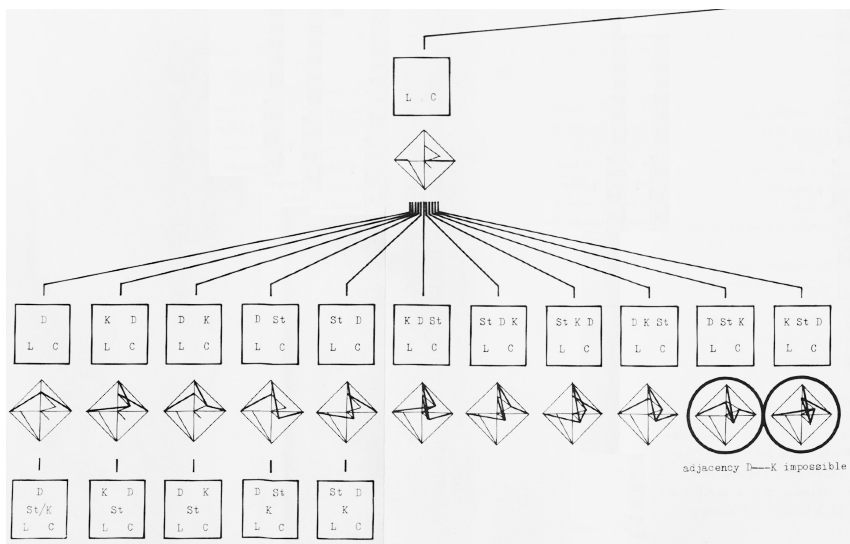
28. Steadman, P. "The automatic generation of minimum-standard house plans" (*Land Use and Built Form Studies, Working Paper No. 23.*, 1970).

The left figure shows an architectural plan; labels D, K, L, C, St stand for individual rooms and uses, such as K for kitchen, St for stair-case, L for living room. The right figure is a graph theoretical representation of the plan on the left; nodes are the rooms of the plan and the edges show required adjacencies between rooms. The edges express requirements, such as “the living room be adjacent to some outside area to allow natural lighting,” or that “the bathroom have access from a landing or hall”[29]. The shapes in the plan are constrained to minimal allowable dimensions and resultant minimal areas. For example L must be 11 x 13 units or 143 square units of area[30]. This plan is one allowable solution based on these requirements given in advance in the problem statement. What the graph offers is the exhaustive enumeration of topologically distinct planar solutions; given the specification of the problem in terms of dimensional and graph theoretical constraints, a design space of all possible allowable plans can be created. The following image shows a small part of the generated design space[31].

29. Ibid., 7.

30. Ibid., 29.

31. Ibid., 34.



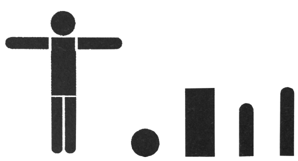
In theory, every design in this space of plan layouts can be enumerated by hand[32]. But the difficulty of keeping track of all constraints in the graph and the numerous resultant partial tree decompositions makes the use of computers the right choice as we often see in the literature[33].

If not by graphs, then another famous path is the specification of a design space in terms of a vocabulary of compositional units given

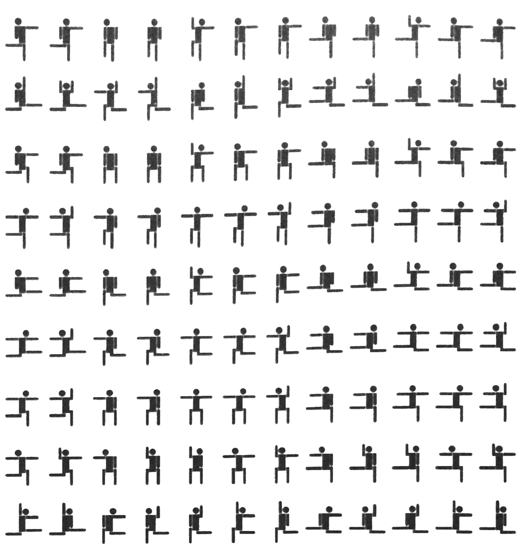
32. One of the very first examples of using graphs for representing all possible plan layouts can be found in Levin, P. H., "Use of graphs to decide the optimum layout of buildings" (*The Architect’s Journal Information Library* 7, 809-815, 1964).

33. Mitchell, W. J. "Techniques of automated design in architecture: a survey and evaluation" (*Computers and Urban Society* 1 (1), 963–980, 1975), Woodbury, R. F. "Searching for designs: paradigm and practice" (*Building and Enviroment* 26 (1), 61-73, 1991), Akin, O. & Sen, R. "Navigation within a structured search space in layout problems" (*Environment and Planning B: Planning and Design*, 23 (4), 421–442, 1996).

in advance and the generation of designs in the design space by combining these units in ways given with compositional rules. In formal languages and spatial systems, this design space is said to be constructed by a grammar. Each design in the design space, or in the language defined by the grammar, can be reached constructively by applying rule applications on compositional units specified in the grammar. For example, Christopher Carlson and Robert Woodbury propose a production system for graphic design that operates on a set or vocabulary of primitive shapes, which they call a structure grammar[34]. Each shape in the vocabulary is treated as a symbolic object; its integrity is preserved because it cannot be decomposed in any other way. Each structure (design) in the design space calculated by their grammar is an admissible combination of mutually exclusive primitive shapes. In particular, consider the following images[35]



The human's graphical figure (left) is one structure in the design space. This structure is parsed into fixed primitive shapes the sum of which forms the structure (right). The following image[36] shows a small part of the design space generated by enumerating all admissible combinations of these primitive shapes according to compositional rules. Both primitive shapes and rules are specified at the outset of calculating structures in the design space.



34. Carlson, C. N. & Woodbury, R. F. "Structure grammars and their application to design" (In *Intelligent Computer Aided Design*, Ed. D. C. Brown, M. B. Waldron & H. Yoshikawa, pp.107-128. North-Holland, 1992).

35. Ibid.

36. Ibid.

Set grammars[37] are a much earlier formalism for defining designs in a design space in this constructive manner, that is starting with one primitive shape defined in the vocabulary and combining (adding to it or subtracting from it) it with the rest of the available primitive shapes according to specified compositional rules.

The above examples are not much different from what Newell, Shaw and Simon were after. In particular, LT's and GPS's 'hypotheses or assertions' are simply replaced by symbolic representations of primitive shapes, 'kits of parts' for architectural design, graphic design or structural design, or in other examples, mechanical parts for the design of machines or electrical circuits; 'search,' namely problem solving, stands as a synonym of 'design,' 'synthesis,' 'invention,' or more generally, 'creation' with the notion of a design space standing as preliminary and necessary to all of them. In any case, I would like to point out that the notion of a design space clearly presses a kind of foresight before invention; for creation to happen designers are required to answer questions, such as 'What kind of entities do I work with?' 'What is their description?' 'What operations are allowed with those descriptions?' 'What goals should my calculations satisfy?' Analysis precedes calculating and calculating amounts to planned action.

At the same time, however, what is exactly the insight we gain into the creative process of synthesis or composition? Where do design spaces come from? How do we invent their constituent elements in advance? For example, Newell, Shaw and Simon provided ready-made design spaces themselves – they gave precise specifications in terms of operators and already proven assertions in mathematics and logic, which they extracted from books, such as the *Principia Mathematica* by Whitehead and Bertrand Russel - and then they simply programmed computers to problem solve in them[38]:

> the theory says little about the selection and construction of problem spaces; primarily because experience so far has been mostly with problem solving systems in which the investigators invented the problem spaces themselves.

When we are designing artifacts in architecture or other associated fields of design, such as house plans or graphic illustrations, where do the compositional parts we pick originate and how do we predict which ones work best each time to formulate fixed design

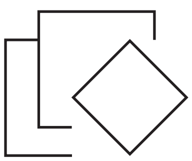
37. Stiny, G. "Spatial relations and grammars" (*Environment and Planning B: Planning and Design* 9 (1), 113-114, 1982).

38. Newell, A. "On the analysis of human problem solving protocols" (*Technical Report, Carnegie Institute of Technology*, June 27, 1966, 9).

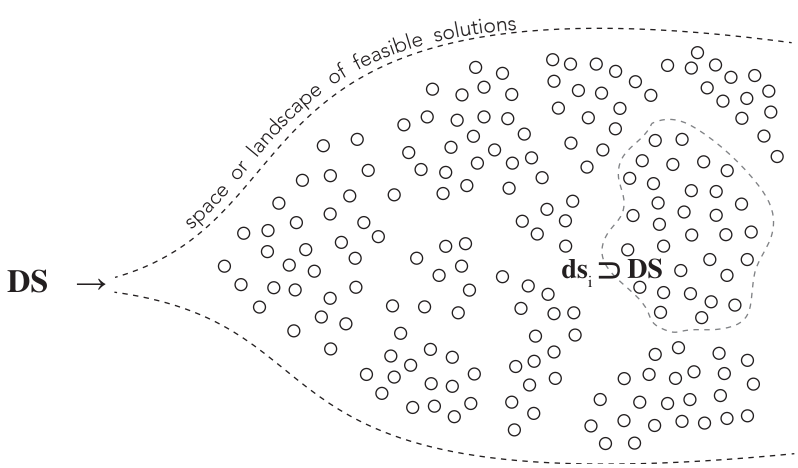
spaces? Maybe painting or architectural design sound the natural areas for asking such questions due to their long standing pursuit of original invention, but Franz Reuleaux, the 19th century's German pioneer of a science of machines expresses this concern very clearly in his *Kinematics of Machinery*[39]:

> The mathematical investigations referred to bring the whole apparatus of a great science to the examination of the properties of a given machine... What is left unexamined is... the question: How did the mechanism, or the elements of which it is composed, originate? What laws govern its building up? Is it indeed formed according to any laws whatever? Or have we simply to accept as data what invention gives us, the analysis of what is thus obtained being the only scientific problem left – as in the case of natural history?

I will go back to the example I presented earlier where the picture



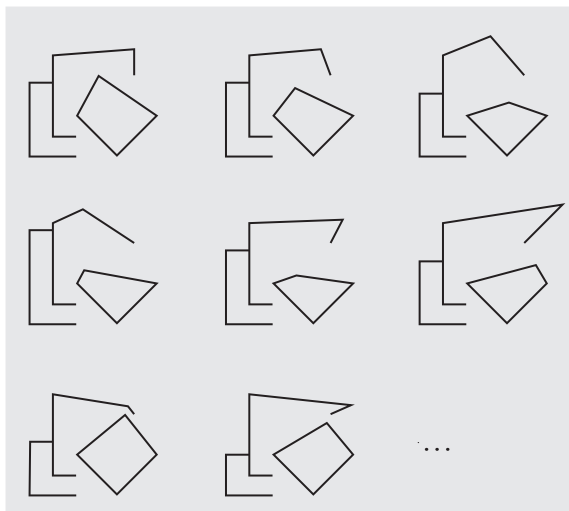
was considered a shape *s* decomposed in a hierarchy of primitive linear elements. The specifications I gave in terms of design variables, bounds and constraints imply a design space **DS** of possible designs. Due to the nature of the specification, the design space itself is a multi dimensional Cartesian space, which is geometric and continuous. Each axis of the space constitutes a design variable among the ones defined earlier. Now each individual design in the space is simply one point or a vector. Because this space is continuous and infinite - there are infinite combinations of the values that these design variables take - the only way to see what kind of designs are in the space is to sample some subspace of it[40].



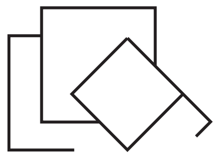
39. Reuleaux, F. *The kinematics of machinery: outlines of a theory of machines* (Macmillan and Co., 1876).

40. See Radford, D. & Gero, J. *Design by optimization in architecture, building, and construction* (John Wiley & Sons, 1988) for how numerical techniques of sampling work at the technical level, which are not the focus of this thesis.

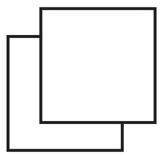
The above image is only a diagrammatic depiction of this concept. What is important to note here is that once the specification of this design space is set, then the only thing left is to sample it by some automated method; the actual space with the available designs is 'out there' ready to be extracted and interrogated. It is as if the design space, once specified, becomes itself the object of scientific inquiry, of hidden mysteries that only through analysis we can explore and understand. As an illustration, consider the notation $\mathbf{ds}_i$ that denotes a sampled portion of the generated design space **DS**. This portion contains designs, such as the ones shown in the following figure

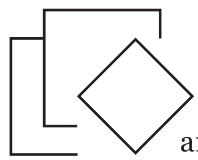


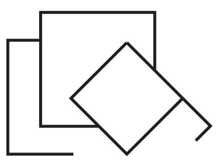
While design spaces are promoted as compositional devices, I believe that if there is anything they offer then this is precisely their analytical power, given in their specification. At least as they are understood and formalized in this classical way, which I presently describe. When constituent elements of designs are decided in advance, what appears later to be synthesis – when we search their possible combinations – is the fruit of well thought analysis. The model says nothing about where those parts come from. Nor it says anything about the role of the human participant – the designer, composer, or maker in the broadest sense of the word – in picking or redefining those parts in the process of designing or composing, other than the fact that he or she must remain fidelitous to a one-off, final specification given as a 'signature' in advance. This undermines many important aspects of how composition works in art and design. Imagine that the following picture



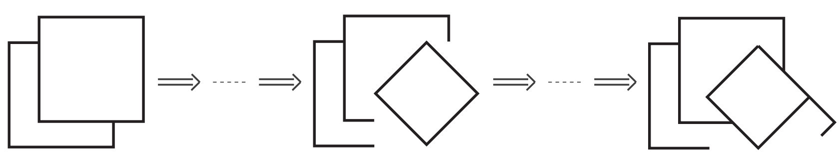
is some fixed moment during a composition that had started some time ago. The composition started with shape *a*



it includes the famous *s* from this chapter  and after a finite number of steps, by applying certain operations, the starting shape *a* results in

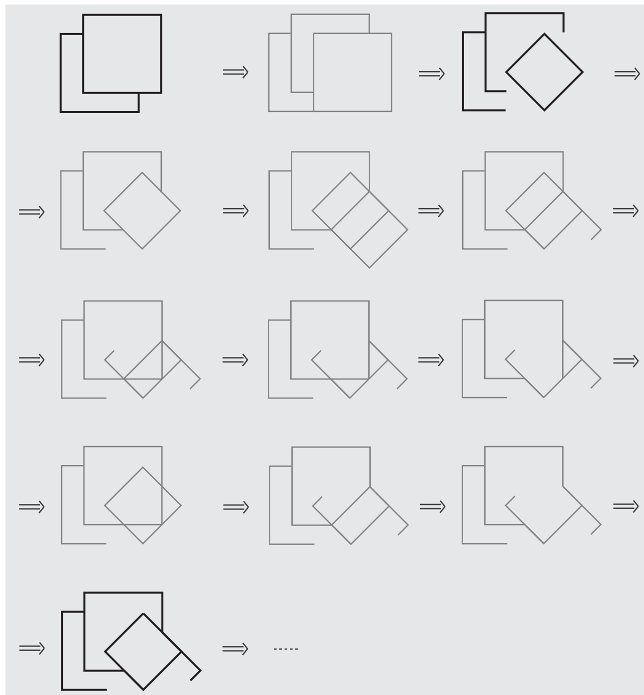


which I will call *b*. What are the calculations that take you from *a* to *b* through *s*? Are they unique? What kinds of shapes are there in between *a*, *s*, and *b*?

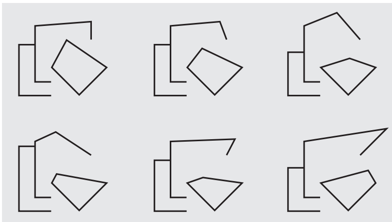


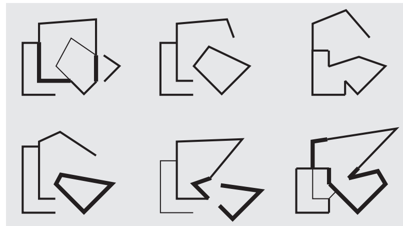
My main concern here is that a classical approach towards pecifying design spaces requires from a designer or composer to answer those questions prior to calculating these designs. A classical specification requires thinking in advance of calculating; it requires to know from before hand what needs to happen in the future. If we look the three shapes it is easy to tell what is missing from one but exists in the other, for example the shape that looks like a rotated Greek Γ in *b* does not exist in *s* and *s* contains some parts of the squares in *a* and it includes an extra 45° rotated square. If the goal was to make *a* look like *s* or *b* - meaning, if we had to formulate this composition as a goal-directed situation - then it is easy to do from this episcopal position - now that everything is there, given from before hand. But this not really how things work during composition. We merely know what we want before we start; instead we discover opportunities and surprises as we go.

What if these designs are part of the following sequence of compositional steps?



Is this sequence of pictures a unique way to derive *a*, *s* and *b*? Surely not, and *b* is not the end of the story (calculations). The point is that during composition, the things we want to distinguish and operate with are momentary, ephemeral, and they change according to the active interpretation of their appearance from an individual, namely, a designer or composer. An automated search procedure essentially is indifferent to the momentary interpreation of the designs in a design space from the person who composes or designs. In composition, I believe, it is not possible to know from before hand how things will turn out; interpretations will vary, and as long as a human interpreter does the composition, then they will vary a lot. It is also plausible to ask if we can do the same in a classical design space. The answer is that the only way we can observe and operate with designs in a design space is by being fidelitous to the description we gave at the outset of calculating the designs; the bygone moment we decided how things look like in the space, namely, in the representation. In the example with shape *s*, which was described in terms of a hierarchy it is not possible for example to get designs such as the following ones (right)



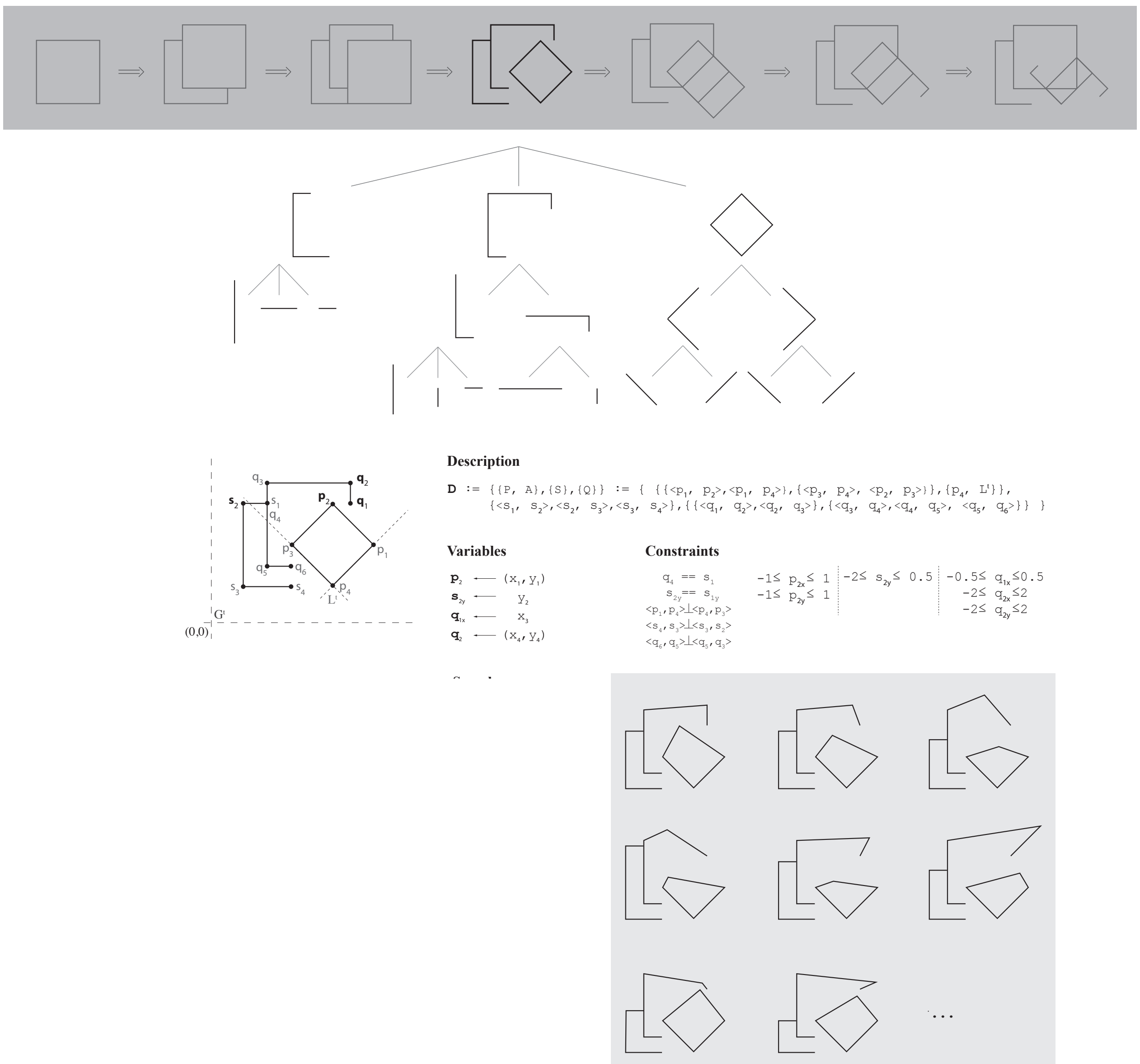


Where do these things come from? The candidate answer is that they come from imagination, because imagination is present when we operate with materials of composition in the present, actively, by manipulating how they look like. The designs are part of my own contribution to the appearance of the things I find as the composition proceeds in the present; they cannot be part of a one-off description of some past. Therefore, the nature of descriptions is that they are final; once instantiated at the outset of calculations they mark a future of possible designs based only on what they can offer.

Similarly, not much can be done if a house plan is represented in a graph theoretical manner. All designs enumerated in the design space must be interpreted according to their underlying graphs. A designer has essentially no actual participation in operating with how they are generated, how they look like, in changing their constituent compositional parts actively. In fact, most applications use graphs because they are neat for representing functional relationships between parts in a layout; the actual appearance of layouts is independent of their graph representations – "the graph is completely independent of the physical shapes or sizes of the rooms" [41]. Moreover, when we fix the compositional units in a vocabulary the generated designs cannot be interpreted any further as units hold distinct identity in the compositional structure of a design. Maybe as a model of calculating, a design space is a convenient analytical device for the engineering disciplines and the sciences because it enables control and prediction by preliminary projection of results. But as a model of calculating offers no real insight into what goes on in the creative process other than operationalizing decisions and actions as having preconceived desirable outcomes and ways of evaluating those outcomes - as a kind of problem solving. For composition, this assumes that one already

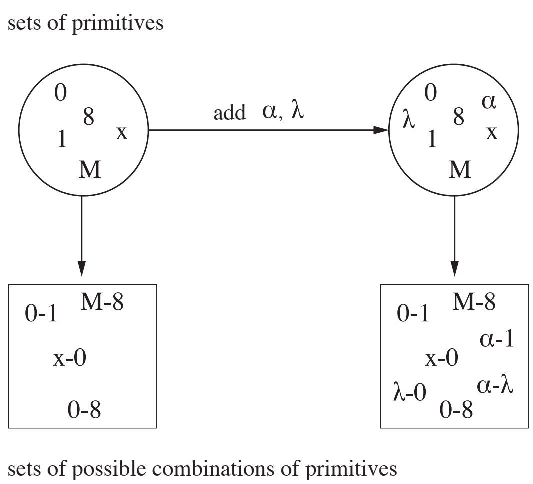
41. Levin, P. H. "Use of graphs to decide the optimum layout of buildings" (*The Architect's Journal Information Library* 7, 809-815, 1964, 809).

knows what he or she is looking for. A classical design space is where the engineering comes in and the scientific analysis of spatial *s*'s into symbolc **D**'s. I therefore believe that a classical specification implies a frozen fragment of a much more fluid, pictorial (spatial) process of composition. The following figure should be illustrative of the metaphor.



To some extent, the concerns I express here have found their way in present research. Opinions vary as to what exactly needs to be amended from past and present design space formalisms to make them become part of an ongoing creative process and not mere conceptual constructions specified independently of the peculiarities and insights involved in the creative process. On one end, research focuses on results, namely, on end-designs and how to represent them differently. On the other end, research focuses on alternative models of exploration in a design space.

John Gero[42] proposes that in the process of design, exploration of alternative solutions is not search in a fixed state space given a priori. But exploration is the process by which state spaces are modified or new state spaces are defined. The way he proposes to do so is by creating new symbols out of prior ones by way of addition, substitution or evolutionary combination. The state spaces that result from these operations are automatically created and they are mutually exclusive; symbols relate with one another only with identity relations. Further, operations may be applied to different non-parametric, combinatorial state spaces in parallel with the ultimate goal of expanding the set of available designs and thus increasing chances for finding designs with better performance (or behavior). The same kind of additive, substitutive and combinatorial approach towards changing the symbols that define a state space at a given moment in time can be found in Peter Cariani[43] and in the context of self- steering percept-action devices. He argues that a creative process involves discovery of new symbols that do not exist in the current state space, which he calls creative emergence and he explains it with the following diagram



In particular, the introduction of new symbols increases the ex-

42. Gero, J. S. "Towards a model of exploration in computer-aided design" (In *Formal Design Methods for Computer-Aided Design*, Ed. J. S. Gero & N. Tyugu, pp. 315-336, North-Holland, 1993).

43. Cariani, P. "Creating new informational primitives in mind and machines" (In Computers and Creativity, Ed. J. McCormack & M. d'Inverno, pp. 383-417. Springer Berlin Heidelberg, 2012).

isting set of primitives (designs). Thus, the new set of primitives increases the set of possible combinations of those primitives. Symbolic primitives defined in sets are - as in the example with the human's graphical figure - mutually exclusive; they cannot be parced into something other than what they a priori stand for even when they are combined and recombined exhaustively with al other available primitives. Maher et al.[44] suggest that a problem definition – the means and the objectives – and its corresponding design or solution space should be both amenable to alteration in the process of design. They describe an automated evolutionary model that considers both as two co-evolving search spaces; the latter's selected individuals redefining the former through time under the guidance of utility functions. Cagan et al.[45] argue that the "richness and sophistication" of end designs in a design space depends on their descriptions chosen in advance of synthesis. In their future directions of research, they point towards better reasoning for how possibilities in design spaces are represented in advance – indeed towards increased foresight before invention.

Other efforts concentrate on how designers can have more contribution to how search proceeds in a specified design space. These originate in the area of design optimization and may allow a designer, for example to adjust the number of discrete solutions generated at each step of search during optimization[46], or to visually select through an end- user interface the design solutions that will act as parents for offsprings within an evolutionary optimization framework[47]. They do not offer a model of the creative process but more application oriented techniques in which a designer interacts with designs according to pre fixed parameters pertinent to the particular choice of optimization method.

The above attempts, as well as many others, attempt to tackle very important questions pertinent to the notion of a design space; questions essential towards making them more appropriate models of calculating for explaining or supporting a creative process. However, even in these cases: design spaces – altered ones or new ones – are still considered as having concrete fixtures, as containers of projected results; change of a design space is understood as going from one fixed state space to another by addition of mutually exclusive symbols – by way of increasing their available combinations; design spaces are still automatically calculated by artificial

44. Maher, ML, Poon J. & Boulanger, S. "Formalizing design exploration as co-evolution: a combined gene approach" (In *Advances in Formal Design Methods for CAD,* Ed. J. S. Gero & F. Sudweeks, pp. 3-30, Springer US, 1996).

45. Cagan, J., Campbell, M. I., Finger, S., & Tomiyama, T. "A framework for computational design synthesis: model and applications" (*Journal of Computing and Information Science in Engineering* 5 (3), 171-181, 2005).

46. Fogg, D. C. "Heuristically guided enumeration: a framework for multi-criteria para metric design" (In *Intelligent Computer Aided Design*, Ed. D. C. Brown, M. B. Waldron & H. Yoshikawa, pp. 55-79. North-Holland, 1992).

47. Mueller, C. T. & Ochsendorf, J. A. "Combining structural performance and designer preferences in evolutionary design space exploration" (*Automation in Construction*, 52, 70–82, 2015).

systems while the human participant has no actual contribution to the process of their making in favor of productive gains in automation; design spaces are still specified as abstractions, as spaces where symbolic descriptions of designs latently lie. What to do differently then?

To start, I believe that the notion of a design space itself is problematic. Consider its meaning and use in situations where it is impossible and of no real value to know from before hand the compositional elements and the rules they are put together to make a work of art or design, that is, in situations where foresight has no place. In particular, what is the notion of a design space in the creation of an improvised work of art or design? In improvisation, creation happens in the now, with the active, perceptual, on the spot engagement of improvisers with materials of composition, such as sound in the case of musical composition, speech and bodily acts in the case of theatrical improvisation, shapes and their meanings in the case of painting and visual composition. Improvisation thus sees creation as ongoing, in the making and in fact unseparable from the individual or collective of individuals creating a work of art or design in the now, with what is at hand. Classical design spaces require specifications given in advance of creation.

But how do you decide the compositional elements or how to stick them together before improvisation starts? In improvisation it is precisely the ephemeral and on the spot improvisatory actions of a human participant – a composer, a designer, an inventor, a maker – that give shape to the materials of composition, that create or invent designs as they happen. In their exclusive engagement with planning and anticipation of final products from an abstract world already made, design space formalisms neglect that there is more to the productive processes underlying creative work; there is the unexpected, that which happens now as a practical effect of an action in the real world. I propose that we miss an improvisational theory of design spaces. A computational view of the processes that give rise to artifacts not in terms of the ends they aim for, not in terms of results, but by way of the processes underlying their formation; computation needs to characterize design spaces as themselves originating from the creative process, not defining it. In the next chapter, I synthesize one such theory.

III Improvisation, Recording, Describing

How to Make it Different

There is no script for social and cultural life. People have to work it out as they go along. In a word, they have to improvise.
– Tim Ingold and Elizabeth Hallam in *Creativity and Cultural Improvisa tion*

Improvisation has a meaning and use that is anything else but fixed and permanent. Consider the myriad different ways in which musicologist Lawrence Gushee[48] describes musical improvisation: extemporization, 'blue' faking, interpolate, vamp, routineer, hot, dirt, fill in, break, ending, space filler, ride, windjam. When used as a form of characterization, the word improvisation may refer to the design of a product, a work of art, that happens in a perfunctory, hasty, impetuous way, to an oral poet that performs a piece without having previously written the piece, to a musically analphabetic person who 'plays by ear,' and to supernumenary other things. Improvisation in its practical use in language has many meanings depending on the situation. This makes it difficult for anyone who wants to find descriptions of improvisation pertinent to the study of a particular topic. Here I make no attempt to enumerate the possible meanings and uses of improvisation, but I note only the distinctions that are more relevant to support my views pertinent to the study of the specification of design spaces.

48. Gushee, L. "Improvisation and related terms in middle-period jazz" (In *Musical Improvisation: Art, Education, and Society. 2nd ed.*, Ed. G. Solis & B. Nettl, pp. 263-280. University of Illinois Press., 2009).

The word 'improvisation' in human language is variously applied. It appears in the English language toward the end of the 18th century, but there are earlier versions of the word in Italian and French. The *Oxford English Dictionary*[49] includes a brief historical overview of the word

1. The action or fact of composing or performing music, poetry, drama, etc., spontaneously, or without preparation; this method of performance.
1777 H. L. Thrale Diary Nov.–Dec. in Thraliana (1942) I. 209 Baretti and I were talking one Day of the Art of Improvisation: Johnson says he, can do it as well as any Italian of us all if he pleases.
1811 Scott Don Roderick Introd. ix. 72 (note) The flexibility of the Italian and Spanish languages..renders these countries distinguished for the talent of improvisation.

2. The action or fact of doing anything spontaneously, without preparation, or on the spur of the moment; the action of responding to circumstances or making do with what is available; an instance of this. Also: the result of this; something produced or created in this manner.
1874 J. A. Symonds Sketches Italy & Greece (1898) I. xi. 214 The terra-cotta decorations..have all the spontaneity of improvisation.
1944 Pop. Mech. Feb. 146/2 With machine shops often unavailable, and with tools and parts often missing, field servicemen are past masters at the art of improvisation.

In particular, the Latin-derived *improvviso* appears around the 13th century and means a thing or event that is unexpected, sudden, unforeseen (from *im* + *prōvīsus*, which means provided or foreseen) and unprepared. In French, the word *impromptu*, also Latin-derived, appears in 17th century and means much the same as the more contemporary versions of improvisation. The word primarily emphasized the act of composing or performing on the spur of the moment without premediation or previous thought:

A. (adv.) Without preparation or premeditation; off-hand, on the spur of the moment; extempore.
1669 Lady Chaworth in 12th Rep. Royal Comm. Hist. MSS (1890) App. v. 11 Mr. Elliot..desired Mr. Titus to make some verses..which he did thus, impromptu [etc.].

B. (n.) Something composed or uttered without preparation or premeditation; an extemporaneous composition or performance; an improvisation. Also, a musical composition having the character of an improvisation.
1683 D. A. Whole Art Converse 44 We must deal plainly and seriously with such men, waving all in promptu's and subtilities.
1693 Dryden Disc. conc. Satire in tr. Juvenal Satires p. xxii, They were made extempore, and were, as the French call them, Impromptus.

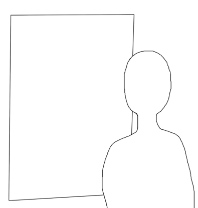
49. Oxford Dictionaries, "Oxford Dictionary, 3rd ed." (Oxford University Press, 2015).

50. Ibid.

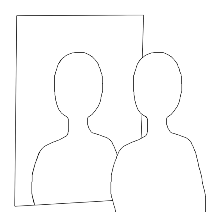
C. (adj.) Composed or uttered without preparation or premeditation; improvised; invented, produced, etc. on the spur of the moment and without previous thought.
1789 H. L. Piozzi Observ. Journey France I. 240 Who would risque the making impromptu poems at Paris

The above sources already suggest two styles of description for improvisation. As a method of composition or performance, improvisation is spontaneous, it is done without premediation or previous thought of plan; an improvisational composition is extemporaneous, produced or invented “on the spur of the moment.” As a mode of behavior or action, it is doing anything spontaneously, off-hand, responding to a present circumstance or making with what is available at hand. So improvisation, on the one hand, refers to the process of making a composition, a work of art, music, sculpture, painting or design, and on the other hand, it refers to a broader characterization of human behavior or action.

The Greek equivalent of the word improvisation is the word *αυτοσχεδιασμός*. The word is a composite noun. The first component is *αυτ(ο)-*, which comes from *εαυτό* and means ‘self,’ and the second component is *-σχεδιασμός*, which means – not surprisingly – ‘design.’ Composite words in Greek language, especially verbs that suggest an action and come with first component the word ‘self’ (αυτ(ο)-) are reflexive verbs[51]. The action returns back to the one who performs the action. Just as when you look into a mirror



you perform an action, 'looking',



that results to you. In the case of improvisation, the verb ‘to improvise’ (αυτοσχεδιάζω) suggests the following important idea. That an improvisational method of working, making, designing, or doing in any way whatsoever, includes the ‘self’ – the person who makes, designs, performs, or composes – as a constant participant in the development of a composition from beginning to end.

Moving on from descriptions of improvisation in language dictionaries to musicological dictionaries, for example the *New Grove Dictionary of Music and Musicians*[52], we find the following, more

51. Babiniotis, G. "The dictionary of Modern Greek. 2nd ed." (Lexicology Center Ltd., 2002).

52. Sadie et al. Ed. *The new Grove dictionary of music and musicians, 2nd ed*. (Maxmillan Pub & Grove's Dictionaries).

elaborate description of musical improvisation:

> [Improvisation is] The creation of a musical work, or the final form of a musical work, as it is being performed. It may involve the work's immediate composition by its performers, or the elaboration or adjustment of an existing framework, or anything in between. To some extent every performance involves elements of improvisation, although its degree varies according to period and place, and to some extent every improvisation rests on a series of conventions or implicit rules. The term 'extemporization' is used more or less interchangeably with 'improvisation.'

According to this description, improvisation should not only refer to a process of creating a musical composition, but also to the result of the composition. It suggests that form is not something preconceived and latently existing before composition starts, but decisive acts occur during the process. In simple terms, the creative process constitutes the result or the product of an improvisational composition - "the product is the process itself"[53]. To see why this is so, simply consider what it takes to specify what happens on stage when observing an oral poet performing, or a theatrical ensemble playing an improvised piece, or a jazz ensemble improvising in real time. How do we know the compositional elements in all these cases? How and when do we specify them? There is no past script that can help us, nor there is any oracle that specifies what will happen in the future. What is left is momentary creation and inevitably retrospective explanation; to tell the pieces that go together in improvisation is to stop creation and start specifying the events that gave birth to that we observe now. A description of improvisation I find particularly relevant to this point comes from William Harris[54]. In a a vein of poetic prose, he argues that:

> Improvisation might be metaphorically described as the act of stepping out of the fixed and fossilized world of the Past, standing for a moment on a tight-rope Wire representing the moment of the Present, while preparing to test the waters of the Future with an exploratory toe.

In a similar vein of poetic prose, the poet David Lawrence talks about the poetry of the present[55]. In his own words,

> The poetry of the beginning and the poetry of the end must have that exquisite finality, perfection which belongs to the far off. It is in the realm of all that is perfect. It is of the nature of all that is complete and consummate... But there is another kind of poetry: the poetry of that which is at hand: the immediate present. In the immediate present there is no perfection, no consummation, nothing finished.

53. Sawyer, R. K. "Improvisation and the creative process: Dewey, Collingwood, and the Aesthetics of spontaneity" (*The Journal of Aesthetics and Art Criticism* 58 (2), 149-161, 2000, 153).

54. Harris, W. "Improvisation: the new spirit in the arts" (n.d.). Available online at: http://community.middlebury.edu/~harris/improvisation.html

55. Lawrence, D. H. "The poetry of the present" (1919). Available online at: http://www.poetryfoundation.org/learning/essay/237874

Lawrence does not refer to improvisation in this quote, nor anywhere else in the original text of *The Poetry of the Present*[56]. Regardless, I will consider his text as in itself a description of an improvisational way of making poetry or a work of art. What is of interest to me is that the outcome of improvisation does not originate from a fixed specification or script conceived in the past. Nor it belongs to a perfectly planned future. But past and future are compressed in the present. The outcome of improvisation along with the process that produces this outcome are things that take shape in the now.

There is, however, something important to consider here. In the description of improvisation in the musicological dictionary I quoted a little earlier, it is written "to some extent every performance involves *elements* [italics mine] of improvisation, although its *degree varies* [italics mine] according to period and place." What this sentence tells us is that the characterization of a work of art or an improvisational composition using a one-shot, binary classification of the form "improvised"-"not improvised" will probably lead towards wrong paths. Because, in musical improvisation there are grades of improvisation, elements that may or may not be improvisatory, and even if they are, the degree to which they are and their type will still vary. If a composition for example is written in notation, then improvisation refers to departures from the text by following implied conventions of performance until completion. In other examples, a fixed musical underlying score may allow greater freedom for improvisation, as in the 16th century dance music or jazz using memorized harmonic schemes[57]. Structure is not excluded and neither is imagination. This point can be found in the following short but excellent excerpt on various cultures of musical improvisation[58].

> In the improvisation of a fugue the difficulty is in adhering to the predetermined form; in the *kalpana svara* of Karnatak music it is the juxtaposition of rhythmic patterns that depart from but return to the *tāla*; in Iranian music it is the maintenance of a balance between quoting memorized material and moving too far beyond it; in South Slavonic epics it is keeping to a textual line structure while alternating memorized themes with commentary. In most instances audiences evaluate improvisations by their balancing of obligatory features against imaginative departures from them.

A musical performance is composed by schemes and rules, mem-

56. Ibid.

57. Nettl, B. et al. "Improvisation" (In *The new Grove dictionary of music and mucisians. 2nd ed*., Ed. S. Sadie et al., Maxmillan Pub & Grove's Dictionaries, 2009).

58. Ibid.

orized harmonic schemes, transitions of chords and scales, which are obligatory elements but nevertheless their choice and use in action is not scripted but open to real-time imaginative judgment by the one or the many who participate in the composition.

As a method of composition, improvisation is performative, on the spot and it emphasizes momentary actions in the real, material world, as well, as the practical effects of those actions. Everyday conversation is an improvisational performance. Language is based on a fixed lexical part, the alphabet and the grammar and the various syntax rules. But the verbal composition of words and sentences during everyday conversation is a real time, social act; we are in not control of whom we are going to meet or what this person is going to say. We respond instantly and momentarily depending on how things evolve in real time. Improvisation as a public verbal performance can be found in linguistic anthropology and in ethnographic studies of speaking in a variety of cultures[59]. In the work of Rosalind Thomas, we find that the ancient oral poet was supposed to be spontaneous and improvisatory because the poem was performed publicly, and on the spot without the aid of a written counterpart[60].

The same thing holds when we observe jazz musicians who collaborate on stage or a theatrical ensemble performing an improv act. These are both cases of ephemeral performances – just like conversation – where the compositional structure is not permanent but open to momentary, imaginative interpretations. The use of formulae – of rules – is not contradictory at all to this. Musicians and dancers may improvise following certain rules of composition but the practical effects of their application is not planned or specified at the outset of composition; rules are given but structure is found as a result of choice and judgment in response to a present circumstance. Having rules that tell you how to do things isn't such a bad idea when you are free to redirect your work and your goals, "as options emerge in a stream of surprises"[61]. What matters is that the use and application of rules and normative devices and the effects they have on the production and perception of an artwork is not scripted or planned before composition starts; planning and execution are one and the same.

It should be by now evident that a final and fixed description of im-

59. Sawyer, R. K. "Improvisation and the creative process: Dewey, Collingwood, and the Aesthetics of spontaneity" (*The Journal of Aesthetics and Art Criticism* 58 (2), 149-161, 2000).

60. Thomas, R. *Literacy and orality in ancient Greece* (Cambridge University Press, 1992).

61. Stiny, G. "What rule(s) should I use?" (*Nexus Network Journal* 13 (1), 15–47, 2011, 34).

provisation and its use is amenable to failure. It means everything that I described so far, but it might also refer to a plethora of other things. The meaning of improvisation we want to emphasize, will be different from one situation to another, and this makes it difficult for anyone who wants to find a description of improvisation that will fit all cases; but this is also the interesting thing with descriptions, you can invent them once and embed them anew in different disguise. "The fact that the meanings of words change, not only from age to age, but from context to context, is certainly interesting; but it is interesting solely because it is a nuisance" [62].

To conclude this discussion on the etymological characteristics, description and use of the word improvisation, I will make a short summary of the arguments thus far: *(a)* in improvisation, the specification of the thing to be composed is not thought, set or scripted before composition starts, but found in non-scriptable steps during composition; *(b)* the characterization of a product, a work of art or composition as improvised or not must allow for a plurality of intermediate meanings; *(c)* there are rules and structure, but there is also imagination and individual momentary choice to allow you to escape from a given structure; *(d)* improvisation emphasizes the process of making a work of art or design and the aesthetic contribution of a human participant or collective of participants in it; *(e)* improvisation refers to a broader characterization of behavior or action of individuals participating in the invention of an improvised work; *(f)* to behave or act in improvisatory ways is to behave or act in the present, with what is at hand.

The etymological characteristics, description and use of the word improvisation presented thus far capture dimensions of improvisation, which I believe are important to study in relation to the specification of design spaces. The connection between improvisation and the specification of design spaces can be established through the following question. If in improvisation, composition is done in the present, then how and when do we know the specification of the 'design space' in which composition happens? To remind one more time, a specification defines a design space of latently existing designs; designs are discovered provided that rule productions apply on shapes with descriptions made of atomic parts specified from before hand. A specification then provides an analysis, an explanation of how creation (calculating) takes

62. Barfield, O. *Poetic diction: a study in meaning* (Wesleyan University Press, 1928).

place. In a classical design space, however, analysis is primary and it takes place before calculations start. In improvisation creation happens in the now and the only way to know what took place in an improvisation is to freeze the process and retrospectively interrogate its compositional structure. To this end, we are faced with at least two challenges. First, how do we inquiry into the structure of an 'ongoing' process? Second, what is a design space for improvisation, or more generally, how can we interrogate an improvised process and describe it from a computational point of view?

Present scholarly studies on improvisational practices, focus on specific types of improvisatory work and devise empirical methods of inquiry into the creative process of improvisation to describe what professional improvisers do in practice. For example, philosopher Beth Preston[63] studies improvisation in the context of songwriting. Her empirical study aims to discover strategies that professional songwriters follow, individually or collaboratively, in the making of the lyrics for a musical piece. She offers an account of improvisation during songwriting that stems from analysis of interviews with songwriters that she records on audiotape and later transcribes in accordance to ethnographic and qualitative procedures. The extensively referenced ethnographic fieldwork of Paul Berliner[64] examines the artistry behind jazz improvisation. The author documents how jazz musicians transcribe, analyze and practice recorded pieces of jazz masters. His explanations are based on a few years of writings on field observations, informal meetings and interviews with artists, and analytical transcriptions of jazz recordings. Eitan Wilf [65] focuses on jazz improvisation as well, but examines how professional improvisers acquire embodied practical mastery and how this mastery can be taught. By drawing on an ethnographic fieldwork he conducted in a postsecondary jazz school in the United States, he describes how educators of jazz improvisation inculcate their students with formalized procedural means that are nevertheless inadequate for imbuing musical creativity and novelty in students unless they emphasize, develop and train their embodied playing habits.

The psychologist, who undertakes the same task, namely, to expose the processes underlying the invention of an improvised work, encounters the impossibility of a person to think and report (analyze) at the same time. Methodological devices for inquiring into

63. Preston, B. *Philosophy of material culture: action, function, and mind* (Routledge, 2013).

64. Berliner, P. F. *Thinking in jazz: the infinite art of improvisation* (The University of Chicago Press, 1994).

65. Wilf, E. "Swinging within the iron cage: Modernity, creativity and embodied practice in American postsecondary jazz education" (*American Ethnologist* 37 (3), 563–582, 2010).

the psychology of the creative process exist and they are specific to what psychologists call protocol analysis. In particular, retrospection and thinking aloud. Retrospection, a form of verbal analysis of a composition from the composer when it is over, is preferred over thinking aloud because the latter inevitably interferes with the artist's real-time creative process as opposed to the former. In the general literature of experimental psychology it has been criticized for its incompleteness, that is to say, the existence of gaps in a person's reasoning process as it is being developed[66]. Nevertheless, it is the path taken in the empirical study of Mendonca & Wallace[67], where the compositional elements of a jazz duo's improvising performance, are provided by the very improvisers themselves retrospectively after listening to an audiotape and looking at a videotape of their complete performance. Magerko et al.[67] follow the same method in their study of theatrical improv – a kind of theatrical acting in which a group of actors coordinate between each other in real-time following a set of game rules in addition to suggestions from the audience. A group of stage actors, after performing a given scene, are asked individually to look a video capture of their performance and describe verbally what they were thinking in the moment each action took place. The experimenters in both works aim to decipher a coherent coordination and concatenation of actions over time by formulating the conditions (recordings, interviews, instructions to improvisers) by which improvisation can become describable (analyzable) both by the experimenters and the improvisers.

The above empirical studies show that analysis comes inevitably secondary to creation as the two cannot happen at the same time. To tell the pieces that go together in composition is to bring closure to its ongoing making; it is to stop improvising and start explaining. Nor can analysis become preliminary to creation because it diminishes the very essence of an improvisation, that is, to be in the moment and to be entangled in temporal and on the spot actions that make composition move forward in open ended ways. Analysis prescribes and predicts possible movements in a specified world; improvisation instead speaks about forward movement in a world in formation[68]. At the same time, it seems that the very act of looking backwards to the creative process to pick out compositional parts is a form of retrospective analytical judgment; an attempt to imbue a sense of coherency and coordinated action in

66. De Groot, A. *Thought and choice in chess* (The Hague, Mouton, 1965)

67. Mendonca, D. & Wallace, W. A. "Cognition in jazz improvisation: an exploratory study" (In *Proceedings of the 26th Annual Meeting of the Cognitive Science Society*, Chicago, Illinois, 2004).

68. Ingold, T. & Hallam, E. "Creativity and cultural improvisation: an introduction" (In *Creativity and Cultural Improvisation,* Ed. E. Hallam & T. Ingold, pp. 1-24. Berg, 2007).

the creative process in order to make it describable. But this point raises precisely the fact that composition in improvisation is ongoing, alive and unpredictable – it just happens. Any sense of plan and continuous causal action are distinguishable elements of improvisation only in retrospect[69]. Therefore, the specification of that which is composed in the moment cannot exist before composition starts; a specification is not detached from the very participation of the artists, designers, makers, or composers since the compositional elements are unknown unless they take action. This very property of improvisation to characterize human creative action by way of its processes and by way of its momentary perceptions created on the spot is what makes the analogy of improvisation an excitingly novel alternative to consider for the study of the notion of a design space.

In considering the relation between improvisation and the classical notion of design spaces, we encounter one more time the antithetic qualities of creation and analysis. Classical design spaces become operational with foresight – analysis and prediction well in advance of creation (calculating). But in improvisation, if there is to be any notion of a design space in which individuals engage with things in the now, then I believe that this design space should be considered as one of indeterminate and open ended nature; a design space for improvisation is 'ongoing' and 'in the making,' rather than one latently out there before composition (calculating) starts. To get the specification of this ongoing design space, which I call an improvisational specification, I propose that creation (composition) and analysis (specification) should be considered as complementary elements of the very same process; creation must remain open to new aesthetic judgments by the individual or collective of individuals who design or compose, while analysis should come retrospectively in order to describe the momentary, perceptual actions that make composition move forward in open ended ways.

In the next chapter, I present a computational basis for an improvisational specification. I illustrate my approach in the context of visual composition with two-dimensional shapes. Shapes underlie most design space formalisms for purposes of composition in architecture and other associated areas of design. Most commonly, shapes come with an exquisite finality in their descriptions;

69. Preston, B. *Philosophy of material culture: action, function, and mind* (Routledge, 2013).

this is what computer applications for design synthesis are after, supported by a long tradition of research in solid modeling and general Computer Aided Design (CAD) research and practice. The reasons for defining shape descriptions in precise symbolic formats are given elsewhere in detail[70]. Here I am concerned with how calculating can be like improvising with shapes and their various meanings. How do we specify the appearance, the shape or figure of 'things' in a design space that itself has no fixed and definite description? How can we compose with shapes that are unanalyzed and allow freedom of interpretation as an improvisation unfolds, and at the same time, ask for the specification of the space in which we are operating? At the technical level, we need a formal device for composition, for calculating with shapes with no fixed descriptions. In parallel to this, we need a method that simultaneously records the composition and retrospectively describes what is done.

70. Requicha, A. G. "Representations for rigid solids: theory, methods, and systems" (*ACM Computing Surveys* (CSUR) 12 (4), 437-464, 1980).

71. Whitehead, A. N. *A treatise on universal algebra, with applications* (Cambridge University Press, 1980, 29).

72. The ideas for an algebra of shapes are first developed in the following series of publications: Stiny, G. "The algebras of design" (*Research in Engineering Design* 2 (3), 171-181, 1991), Stiny, G. "Weights" (*Environment and Planning B: Planning and Design* 19 (4), 413-430, 1992) and Stiny, G. "Boolean algebras for shapes and individuals" (*Environment and Planning B: Planning and Design* 20 (3), 359-362, 1993).

73. Stiny, G. & Gips, J. "Shape grammars and the generative specification of painting and sculpture" (In *Information Processing* 71, ed. O. Petrocelli, pp. 125-135, Auerbach) and Stiny, G. *Pictorial and formal aspects of shape and shape grammars* (Birkhauser, 1975).

74. Stiny, G. *Shape: talking about seeing and doing* (The MIT Press, 2006).

III Improvisational Specification

How to Set it Up Formally

The history of mathematics is rich with ideas for calculating with numbers and quantities. But calculating can be done not only with numerical entities; there can be algebras of “nonnumerical genus,” as Alfred Whitehead puts it in his 1898 treatise on a universal algebra[71]. In improvisation, the designer, composer or maker in the broadest sense of the word, shapes the materials of composition in real time and on the spot. To enable this in the context of calculating, we need to define an algebra of the entities we work with. An algebra allows us to calculate directly with the entities of interest and not through some abstract symbols that stand for themselves. Therefore, the properties of the materials of composition we are interested in using during composition can be defined formally by developing the necessary operations and rules in an algebra. For the case of an improvisational specification for visual composition, we need algebras of shapes that enable us to calculate directly with shapes. In the literature of shape computation, the construction of the term ‘algebra of shapes’[72] has been central in the development of the formal mathematical basis of the shape grammar formalism[73] or visual calculating[74], now with a history counting more than forty years. Note that the term ‘shape computation’ points unavoidably to the realm of geometric modeling. The differences between an algebra of shapes and geometric modeling are not touched here in a meaningful systematic way. Instead, the interested reader is advised to refer to the works of Rudi Stouffs[75] and Chris Earl[76].

The shape grammar formalism uniquely treats shapes as raw, unanalyzed visual entities and of calculating as bereft of symbols. These two ideas are stressed in the latest monograph on the subject given by George Stiny[77]. To see shapes as unanalyzed, visual entities is to see shapes as real, spatial things with no inherent description. But descriptions (meanings) can be embedded with compositional rules defined by the individual, the designer or composer, who performs calculations. With respect to this property of shapes as pictorial, unanalyzed entities I believe the question most relevant to an improvisational specification is when and how do you determine descriptions for shapes. The answer from the point of view of the information processing tradition, as we already saw, is that you do it before calculations start. The answer from the shape grammars perspective is that you – yourself – are doing it throughout a calculation. In the first case, shapes come with predefined symbolic descriptions. In the second case, shapes acquire descriptions as you go – as you design or draw; your observations, rules and choices are free to apply as (visual) opportunities arise. This latter view considers calculating as a practice that needs no fixed set of symbols or vocabulary to be rightful. It also enables us to treat creation as a forward movement indifferent to analysis because the materials of composition remain open to new interpretations, to new descriptions and meanings created in the moment. At the same time, the formalism offers all the tools needed to retrospectively interrogate, analyze and describe the design space in which creation happens; in allows us to reconcile creation and analysis as two complementary and in fact unseparable aspects of the very same creative process.

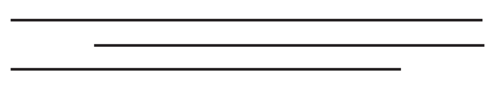
For an improvisational specification, there are two parts, which I will present here in detail. One part is a process of composition that is improvisatory and proceeds with rule applications on shapes defined in algebras of shapes to generate visually a work of art or design. The second part is a process of specification, or to put it in a manner that is more relevant to improvisation, a process of recording, which describes the ongoing improvisatory act in a

75. Stouffs, R. *The algebra of shapes* (Ph.D. Thesis. Carnegie Mellon University, 1994).

76. Earl, C. F. "Shape boundaries" (*Environment and Planning B: Planning and Design* 24 (5), 669-687, 1997).

77. Stiny, G. *Shape: talking about seeing and doing* (The MIT Press, 2006).

symbolic, analytic format. I illustrate this reciprocal relationship between creation and analysis through an example of a two dimensional visual composition.
An algebra for shapes is a special kind of algebraic structure, the shape lattice **U**, which is based on the part relation (≤), closed under algebraic operations of sum and product, and the Euclidean transformations. A pictorial *s* or a shape in **U** is a finite arrangement of shapes that are also from **U**. If letter *i* denotes the dimension of a shape, then $i = 0, 1, 2$, and 3 for points, lines, planes and solids respectively. If letter *j* denotes the dimension of the space that a shape exists, then *j* is either greater than or equal to *i*. Notation $\mathbf{U}_{ij}$ refers to *i*-dimensional shapes in a *j*-dimensional space, with $i \leq j$. Except for points, all other pictorial entities hold a spatial description; lines are made of lines, planes are made of planes, and solids are made of solids. In the context of visual composition, $\mathbf{U}_{ij}$ is restricted to shapes in a three-dimensional space or $\mathbf{U}_{i3}$. But $\mathbf{U}_{ij}$ may extend to higher dimensional hyperplanes, although the details for this are not covered here. A shape in $\mathbf{U}_{ij}$ is represented by its set of maximal shapes. For example, a line of finite length is made of a finite set of smaller or equal lines ordered with a subshape or part relation (≤). This means that any line in this set is always embedded, or synonymously, is always a subshape of another line of equal or greater length, while the sum of all these lines in the set equals the original line such as in the following figure



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This principle of ordering subshapes in terms of an embedding relation applies to planar regions, solids and even to higher dimensional hyperplanes, but it does not apply to points. “A point is a figure which cannot be divided into parts”[78]. The maximal representation of shapes of dimension $i > 0$ is a purely visual account of shapes, in the sense that it allows for partitioning shapes in any way of interest to the moment of observation, or to the task at hand.

Shape lattice $\mathbf{U}_{ij}$ comes with binary operations of sum +, and product ·, for performing arithmetic with shapes. For any two overlapping shapes $x, y \ni \mathbf{U}_{ij}$, the shape $x + y$ is the smallest shape that

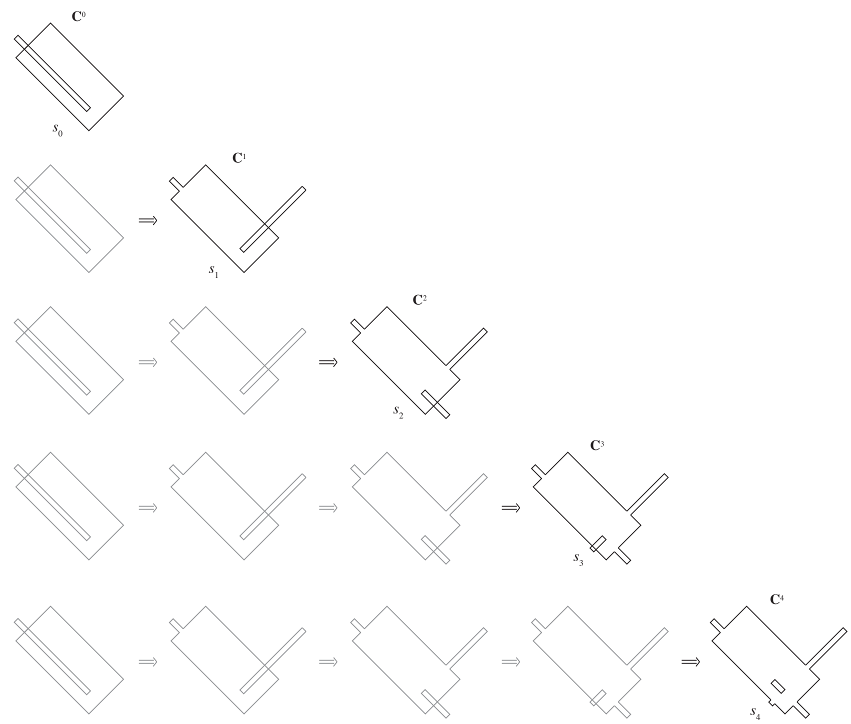
78. Alberti, L. B. 1976 *On Painting*. Greenwood Press.

contains both x and y as parts, while the shape $x \cdot y$ is the largest shape that is part of both x and y. If $x \leq y$, then $x + y = y$ and $x \cdot y = x$, that imply the embedding (partial order) relation between the two shapes. Further, $\mathbf{U}_{ij}$ is equipped with a bottom element, which is the empty shape denoted with 0, and it is part of any other shape. The empty shape allows for defining the relative complement of a shape with respect to another shape, which in turn allows for defining the operations of difference – and symmetric difference ⊗. In particular, the difference $x - y$ between shapes x and y is the relative complement of shape $x \cdot y$ in the closed interval [0, x], or $0 \leq x \cdot y \leq x$. The (relative) complement for a shape is unique because $\mathbf{U}_{ij}$ is distributive[79]. Intuitively, the difference $x - y$ corresponds to erasing the parts of x that are shared by y ($y - x$ is defined in an analogous way). Last, the symmetric difference $x \otimes y$ (or $y \otimes x$) is the sum of $x - y$ and $y - x$. Shape lattice $\mathbf{U}_{ij}$ and the operations between shapes introduced, designate an algebraic structure $\langle \mathbf{U}_{ij}, (+, \cdot, 0); \leq \rangle$, which is a relatively complemented, distributive lattice with bottom element 0, closed under the algebraic operations of sum and product, structured with a partial order. This lattice designates a generalized Boolean algebra[80] and it is independent of any underlying geometric representation for shapes.

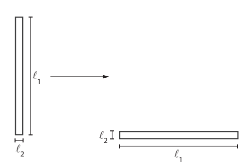
Shape algebra $\mathbf{U}_{ij}$ is associated with the shape grammar formalism. Since their invention, shape grammars as other production systems, such as phrase structure grammars and Markov algorithms, are defined with production rules of the form $a \rightarrow b$, where $a, b \ni \mathbf{U}_{ij}$. Composition proceeds by applying rules on shapes to produce other shapes. For a rule to apply, we need to have a 'match' between a given shape and the left hand side of a rule. Formally, if s is a shape then if there exists a transformation t such that $\mathbf{t}(a) \leq s$, then the rule applies to s under transformation $\mathbf{t}$ to produce the new shape $[s - \mathbf{t}(a)] + \mathbf{t}(b)$. A repeated application of rules is called a derivation, and is noted as $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2 \Rightarrow \dots \Rightarrow \mathbf{C}_n$, where $\mathbf{C}^k$ is the k^{th} – step in a derivation of finite length, with $k = 0 \dots n$. To start, consider the following five step derivation that starts from shape s_0 in $\mathbf{C}_0$ and ends in shape s_4 in $\mathbf{C}_4$ (opposite page)

79. Stouffs, R. *The algebra of shapes* (Ph.D. Thesis. Carnegie Mellon University, 1994), and Krstic, D. "Constructing algebras of design" (*Environment and Planning B: Planning and Design* 26 (1), 45-57, 1999).

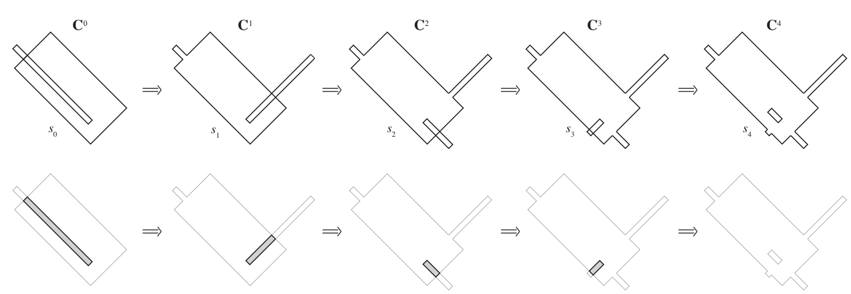
80. Stone, M. H. "The theory of representation for Boolean algebras" (*Transactions of the American Mathematical Society* 40 (1), 37-111, 1936), Krstic, D. "Constructing algebras of design" (*Environment and Planning B: Planning and Design* 26 (1), 45-57, 1999).



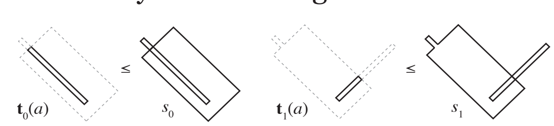
The composition is defined for shapes in $\mathbf{U}_{12}$ and proceeds by sequential application of the following parametric rule

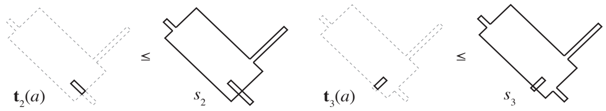


As the rule applies to a shape to generate a new shape, the previous states of the composition are kept as a trace of the actions performed. The rule is applied to distinguished or emergent shapes

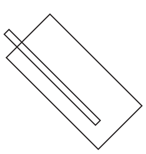


that satisfy the following relations of the form $\mathbf{t}_k(a) \leq s_k$, $k = 0 \ldots 3$

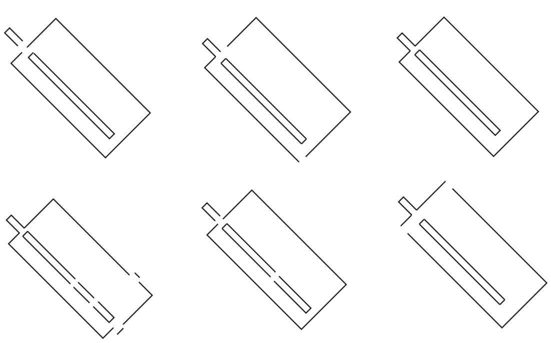




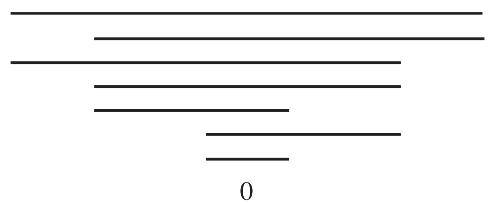
Here transformation $\mathbf{t}_k$ is Euclidean, in particular an isometry augmented with scale. More general homogeneous transformations can be defined to serve a different purpose. Now that we have a sequence of improvisatory actions and resultant shapes or designs we can start interrogating this sequence and see how we can specify a design space that explains this composition. We already know what rule was involved to generate the shapes and the initial shape s_o in $\mathbf{C}_o$. What is left unexamined is the description of each shape after every rule application. To have a specification of this composition we need to derive the necessary shape descriptions that explain this composition and the rules involved. At minimum, we need to make sure that the descriptions support the recognition of the shape in the left hand side of the rule used during composition. To do so we need some more information on shape descriptions. Compartitions or decompositions show how shapes are cut up into parts as rules apply in a calculation; they are themselves descriptions of shapes. There can be infinitely many decompositions for a shape and each of them may serve a different purpose at different point in time, even during the same composition. Consider the shape that started the composition in our example



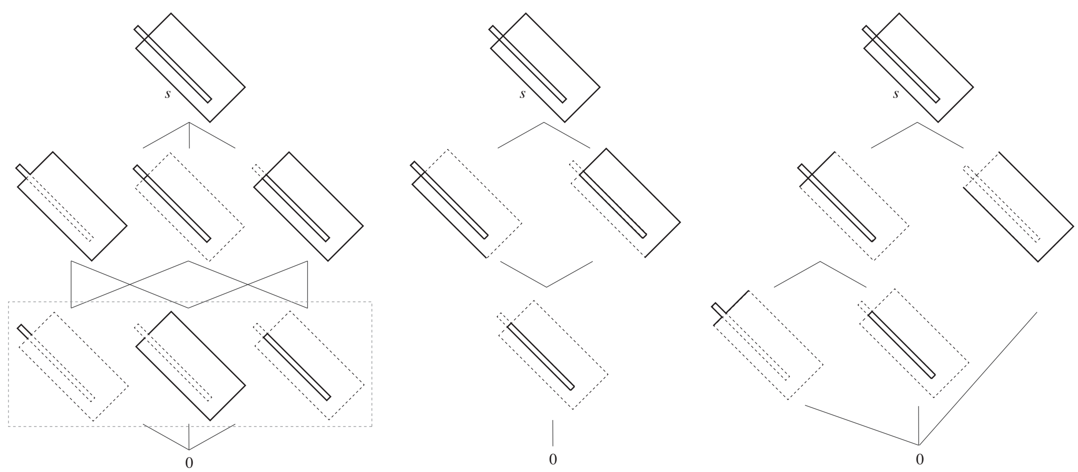
where there is no apparent structure assigned, that is to say, we do not know how the shape is cut up into smaller pieces or better we do not know yet how to cut it up. Many possibilities arise just by simply looking at the above picture



Formally, decompositions can be defined in terms of hierarchies and topologies. We already saw how hierarchies work - an entity is parced into a fixed set of primitives that preserve their integrity when combined. For example, if *a* and *b* are two primitive shapes then if they are part of a hierarchy that represents a shape then $a \cdot b = 0$. A topology is defined differently. Consider the following line and its subshapes



In a topology, each shape is a subshape of another shape in the topology. Notice that the line itself along with every sum and product of each subshape with all other subshapes in the topology, are in the topology. The following figure shows three ways of decomposing the same shape

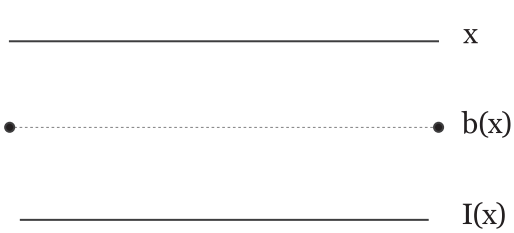


where the first two are topological decompositions or topologies and the third is a hierarchy with the empty shape 0 added at the bottom of each decomposition to make them lattices. These diagrams are called lattice diagrams or Hasse diagrams and represent pictorially how entities are structured in an algebra in terms of parts defined in the same algebra. More formally then, a topology **T** for a shape *s* satisfies the following conditions:

(1) the empty shape 0 and *s* itself are both in **T**;

(2) if shapes $x, y \ni \mathbf{T}$ then their sum $x + y$ is also in $\mathbf{T}$;
(3) (*i*) for every $x \ni \mathbf{T}$ there exists a smallest shape $z \ni \mathbf{T}$ that includes *s* as a part, that is, $x \leq z$. (*ii*) This implies that for any $x, y \ni \mathbf{T}$ the product $x \cdot y$ is also in $\mathbf{T}$.

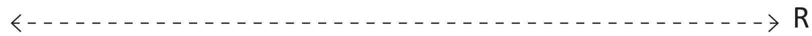
These conditions overlap with the conditions that are required for topologies defined on sets of points with the exception that condition 3 (*i*) is specially formulated for shapes that contain parts, which are subshapes and not sets of points (for sets of points, 3 *ii*. is the standard condition[81]). But whereas topologies for point sets can be infinite, topologies for shapes are required to be finite as they describe entities of material and spatial extent, and not abstract mathematical objects. Notice how the conditions (1) through (3) are satisfied only by the first two lattices shown earlier. A more general study of decompositions of shapes and their various algebraic properties can be found in the work of Djordje Krstic[82]. Lattice diagrams, however, are not necessary or the only means for representing topologies. For example, the symbol set ({a, b, c}, {a, c}, {a, b}, {a}, ∅) is one other way of representing a topology, namely, a topology for the set {a, b, c}, where every curly bracket in the set is one subset of the topology. But lattice diagrams are preferred for their pictorial nature and hence they are used exclusively throughout this chapter. The shape descriptions involved in the computation $\mathbf{C}_0 \Rightarrow \ldots \Rightarrow \mathbf{C}_4$ of our example will be defined in terms of topologies for shapes. Before I present the precise methods to do so there is another property of topologies that needs to be explained. Either describing spatial entities like shapes or symbolic entities like sets, a topology comes equipped with the topological closure operator. The closure operator describes the closing or closure of some part in a larger whole. The concept of a closure can be illustrated in a variety of ways. Suppose x is a line in $\mathbf{U}_{12}$ it has a boundary b(x) defined in $\mathbf{U}_{02}$ (end-points are emphasized for illustration), that is a set of two endpoints and an interior I(x), which is the largest open subset of x



81. Willard, S. *General topology* (Addison-Wesley Publishing Company, 1970).

82. Krstic, D. *Decompositions of shapes* (Ph.D. Thesis. University of California, Los Angeles, 1996).

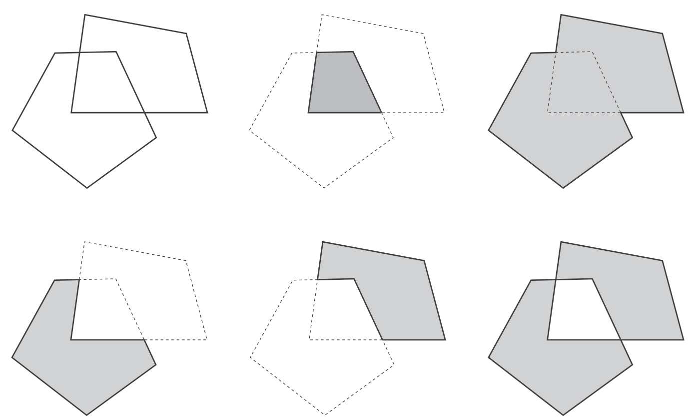
Clearly x = I(x) + b(x), which is the union of the interior and the boundary of x. Further consider x as being some part on the familiar line of real numbers



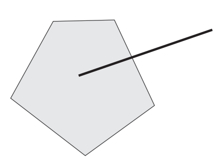
and replace the symbol x with an open interval (a b) somewhere on the number line

Its closure, the closed set [a b], is obtained simply by adding to the open set its two endpoints {a, b}, which is essentially the smallest closed set that contains (a, b)

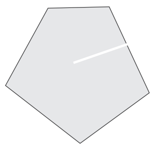
Spatial analysis methods for shapes, such as those included in a computational geometry library are equipped with set-theoretic operations



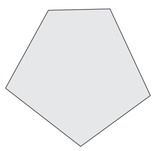
the results of which are point-sets. In some cases, the operation between two shapes of different dimension, such as the difference between two shapes where one is a line and the other a plane



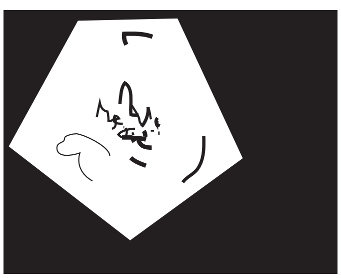
will return an open set



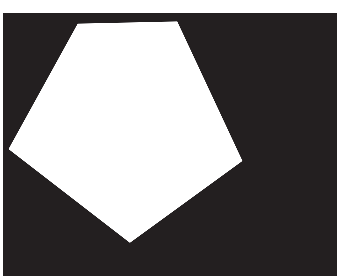
which is undesirable for CAD systems, which primarily operate on closed sets and their boundaries. A closure heals such degenerate cases and returns the closure of the point-set theoretic result



that is the smallest closed set that contains the shape we need. The concept of healing can be found in morphological image processing, in particular when thresholding an image. A thresholded image is a black and white version of the original image that might have some undesired artifacts in interior regions of a seemingly closed shape like so



By closing the image we are essentially geting back the closure of the interior region as a solid version healed from cracks



Formally, the closure operator of a shape s associates to every part $x \ni \mathbf{T}(s)$ its closure $\Gamma(x)$, which is the smallest shape in $\mathbf{T}(s)$ that includes x as a part. In lattice theory a closure operation of a lattice, such as the one given for $\mathbf{T}(s)$ that comes with bottom element the empty shape, is defined as a mapping from a shape to itself

Γ: $s \rightarrow s$ that essentially associates every part x of shape s with the smallest shape in the topology that includes x as a part. More specifically, the topological closure operator satisfies the following properties:

(1) $\Gamma(0) = 0$;
(2) $x \leq \Gamma(x)$;
(3) $\Gamma(\Gamma(x)) = \Gamma(x)$;
(4) $\Gamma(x) = x$, implies x is closed;
(5) $\Gamma(x + y) = \Gamma(x) + \Gamma(y)$, and it follows that $x \leq y$ implies $\Gamma(x) \leq \Gamma(y)$;

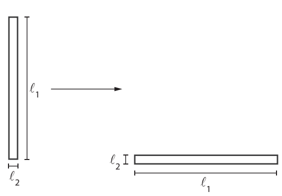
where $x, y \ni \mathbf{T}(s)$. Given a mapping Γ: $s \rightarrow s$ of a shape s into itself satisfying (1) through (5), if we obtain the closed subshapes in s using (4) the result is a topology $\mathbf{T}$ on s whose closure operation is Γ. In later paragraphs, I use the more specialized notation Γ_k to refer to the topological closure operator of shape s_k at step C_k in a shape grammar derivation. A topology for a shape s is a description for the shape. And since, as we said earlier, each shape can have any number of subshapes in its description, the following question appears immediately and is particularly related to composition, 'How do we pick topologies?'

Topologies are picked according to the rules applied in an improvised composition by the designer or composer who calculates. A rule applies to make a new design out of a previous one, a topology records this rule application in an analytical way; it shows what descriptions are needed to satisfy and support the rule application. In doing so, a topology explains the computation, it shows what necessary atoms are needed for a rule to apply in specific state to derive a new state. The study of how rules map from one state to another in a computation is formalized next in terms of the continuity of the shape descriptions implied in the states. More specifically, I concentrate on descriptions of shapes, namely, topologies that map continuously to one another in a composition. The term 'continuous' is used in the same way the term homomorphism is used in abstract algebra to compare two groups for structural similarities. Intuitively, in going from s_k to s_{k+1} a homomorphism between the two determines how much of the structure in $\mathbf{T}(s_k)$ is preserved in the structure of $\mathbf{T}(s_{k+1})$. Suppose that a rule $a \rightarrow b$, where $a, b \ni \mathbf{U}_{ij}$, applies to shape s_k under transformation $\mathbf{t}_k$ to produce the new shape $s_{k+1} = [s - \mathbf{t}_k(a)] + \mathbf{t}_k(b)$, where $k = 0 \ldots n$ and $s_k, s_{k+1} \ni \mathbf{U}_{ij}$. And suppose futher that the topologies of the shapes s_k and s_{k+1} are equipped with the topological closure operators Γ_k and Γ_{k+1}, satisfying (1) through (5). Then the rule application is continuous whenever[83]:

(i) $\mathbf{t}_k(a)$ is closed in $T(s_k)$, or $\Gamma_k(\mathbf{t}_k(a)) = \mathbf{t}_k(a)$, and
(ii) the rule implies a homomorphism h: $s_k \rightarrow s_{k+1}$ where $h(s_k) = s_{k+1}$, that preserves the closure operations in the corresponding topologies, and the Boolean operations defined in their shape algebras.

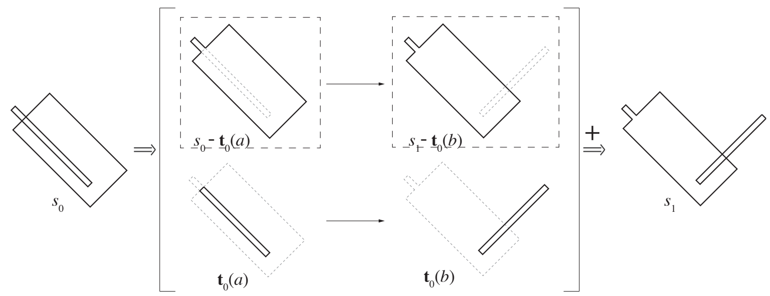
The first condition makes sure that a rule can actually apply and it is obligatory for every topology involved in a continuous calculation. The second condition makes sure that every rule application corresponds to a continuous mapping between the closed parts marked by the closure operation in one topology, with the closed parts marked by the closure operation in the other topology.

In general, the kind of homomorphism we pick to study calculations with shape grammars depends on the rules involved and the kinds of spatial alterations they induce. For the specific example I use in this subsection, rule $a \rightarrow b$ refers exclusively to the parametric rule defined earlier

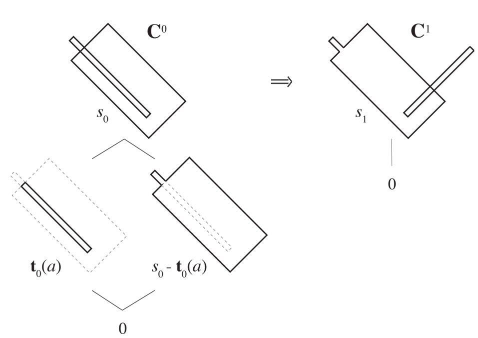


This rule considers a and b as two independent pieces with one merely replacing the other under the application of a rotational transformation (this transformation should not be confused with the transformation needed to register a shape in every step, namely $\mathbf{t}(a)$). It is therefore enough to choose a mapping that considers at each step only what $\mathbf{t}(a)$ alters in a shape, in particular the mapping h: $s \rightarrow s - \mathbf{t}(a)$. As rules apply one after the other h shows which parts of a shape are preserved from one step to another. For example, the following figure shows that when a rule $a \rightarrow b$ applies to shape s_0 under transformation $\mathbf{t}_0$ to produce shape s_1 then shape $s_0 - \mathbf{t}_0(a) \leq s_0$ is preserved and equals the shape $s_1 - \mathbf{t}_0(b) \leq s_1$.

83. Stiny, G. "Shape rules: closure, continuity, and emergence" (*Environment and Planning B: Planning and Design* 21 (1), 49-78, 1994).

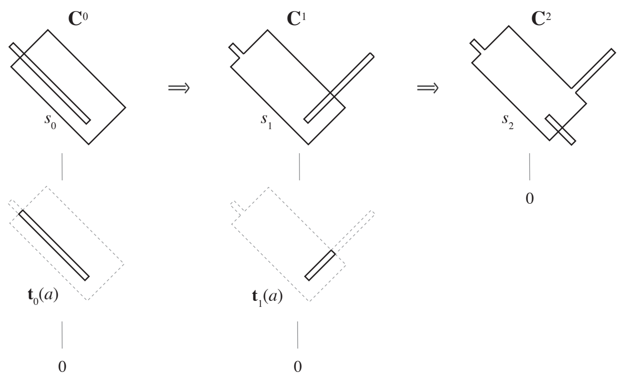


Mapping h is a description of how s_o maps to some common part in s_1, namely the part $s'_1 = s_1 - \mathbf{t}_o(b) = s_o - \mathbf{t}_o(a)$. Notice how s'_1 becomes via h a closed part in the topologies of both s_o and s_1. Therefore, no part closed in s_1 (current shape) before $\mathbf{t}_o(b)$ is added implies a closed part not already recognized in the topology of s_o (previous shape). With respect to this observation, we can specify the general principle that for a rule to be continuous emergent shapes must look as if they are anticipated; they have to be closed in the description (topology) of both shapes participating in a single rule application. In addition, for a mapping h to be a homomorphism it must preserve the algebraic operations defined in the algebras it tries to connect. Formally then, mapping h must satisfy the equality $h[\Gamma_k(x)] = \Gamma_{k+1}[h(x)]$. In Stiny[84], the right hand side of this equality is restricted to the parts of $s'_{k+1} \leq s_{k+1}$, that is, to the parts that remain unchanged in going from s_k and s_{k+1}, like so $h[\Gamma_k(x)] = s'_{k+1} \cdot \Gamma_{k+1}[h(x)]$, thus implying $h[\Gamma_k(x)] \leq \Gamma_{k+1}[h(x)]$, for any part $x \leq s_k$. By connecting the two topological closures via h, we expose the necessary and sufficient similarity between the shape topologies involved so that a rule implies a continuous map from one to another. We are now ready to show how all these formal devices connect together to provide us a strong mechanism for interrogating the computations made in the algebra $\mathbf{U}_{12}$ and specify descriptions for the shapes involved. Consider as a start the step $\mathbf{C}_o \Rightarrow \mathbf{C}_1$

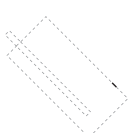


84. Ibid.

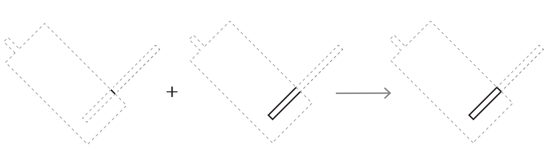
that takes shape s_0 to s_1 under transformation $\mathbf{t}_0$. For the rule application to be continuous we need at minimum to close the obligatory part $\mathbf{t}_0(a)$ in $\mathbf{T}(s_0)$, and optionally its complement $s_0 - \mathbf{t}_0(a)$. We can keep s_1 undivided at this point as having the trivial topology where the empty shape and the shape itself are the only members. However, as more rules are applied, simply by recognizing $\mathbf{t}(a)$ is not enough to preserve continuity in rule applications. For example, the topologies in this figure



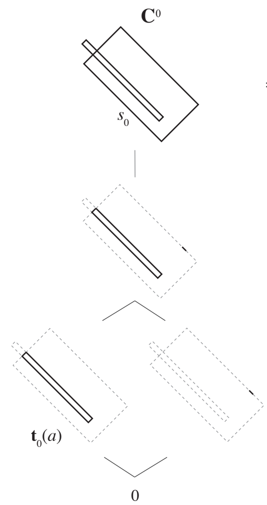
for the derivation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2$ are not continuous (the complement of $\mathbf{t}(a)$ is ignored for purposes of simplicity). The reason is that the rule in step $\mathbf{C}_1$ applies to an emergent rectangle, which is not implied in any way in the description of the precedent shape, the topology of the shape s_0 in $\mathbf{C}_0$. For the rule to apply continuously in the calculation $\mathbf{C}_1 \Rightarrow \mathbf{C}_2$ the compartitions of shape s_0 must be such so that they anticipate the emergent rectangle $\mathbf{t}_1(a)$ used in $\mathbf{C}_1$. We have to therefore close this part



in the topology of s_0 so that when the rule applies under transformation $\mathbf{t}_0$, this part contributes to the formation of the required emergent rectangle like so

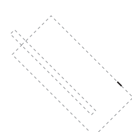


The following figure shows how the topology of shape s_0 is extended so that continuity is established in the derivation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2$.



How did we decide which part to pick for making the rule application continuous? There exists an intuitive answer, and a formal counterpart.

In order to make the step $\mathbf{C}_1 \Rightarrow \mathbf{C}_2$ continuous we picked a topology for s_0 that anticipates the emergent rectangle we use in $\mathbf{C}_1$. In particular, first we freely distinguish $\mathbf{t}_1(a)$ in shape s_1 to use in our composition, and applied the rule to get the new shape s_2 in $\mathbf{C}_2$. To support this observation and action we then went backwards to shape s_0 in $\mathbf{C}_0$ and calculated the intersection between $\mathbf{t}_1(a)$ and the complement $s_0 - \mathbf{t}_0(a)$. We thus augmented the topology of s_0 with the shape that results from the operation $[s_0 - \mathbf{t}_0(a)] \cdot \mathbf{t}_1(a)$, shown pictorially



A simple formula supports this informal verbal explanation. The formula connects closure operators $\Gamma_k(s_k)$ and $\Gamma_{k+1}(s_{k+1})$ for any two shapes s_k and s_{k+1} using equality $h[\Gamma_k(x)] = s'_{k+1} \cdot \Gamma_{k+1}[h(x)]$ for any part $x \leq s_k$, which is the necessary condition for homomorphism between topologies $\mathbf{T}(s_k)$ and $\mathbf{T}(s_{k+1})$[85]. By expanding both sides of the equality using the mapping h we defined earlier, we get $\Gamma_k(x) - \mathbf{t}_k(a) = s'_{k+1} \cdot \Gamma_{k+1}[x - \mathbf{t}_k(a)] \Leftrightarrow \Gamma_k(x) = \mathbf{t}_k(a) + s'_{k+1} \cdot \Gamma_{k+1}[x - \mathbf{t}_k(a)]$, for any nonempty part $x \leq s_k$ and $\Gamma_k(0) = 0$. In our case, $s'_{k+1} = s_{k+1} - \mathbf{t}_k(b) = s_k - \mathbf{t}_k(a)$.

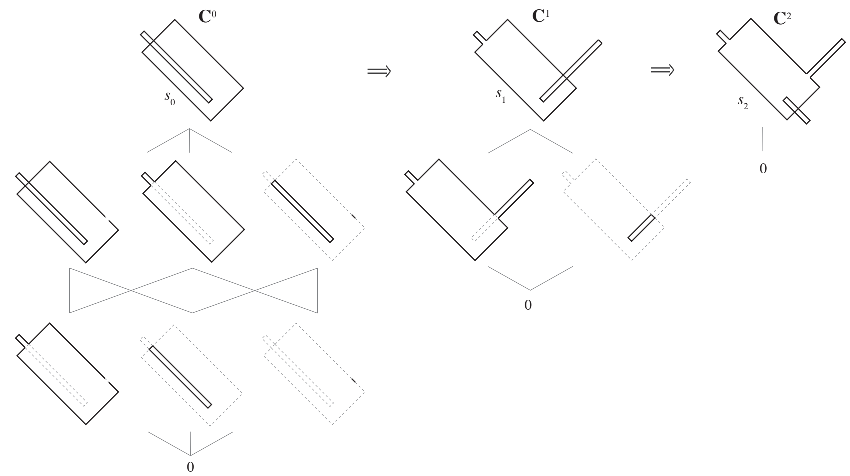
85. See Willard, S. *General topology* (Addison-Wesley Publishing Company, 1970) and Davey, B. A. & Priestley, H. A. *Introduction to lattices and order* (Cambridge University Press, 1990) for the standard formulation in topology, as well as, Stiny, G. "Shape rules: closure, continuity, and emergence" (*Environment and Planning B: Planning and Design* 21 (1), 49-78, 1994) for the equivalent formulation for shapes.

The idea that something emergent is anticipated may sound somehow paradoxical. But it is so, only if you understand anticipation as a necessary plan of your results before action, that is as a precondition for calculating. Something different is suggested here. Composition is improvisatory and goes forward through free seeing; rules apply to distinguished parts because of the embedding property of the shape algebra. The act of assigning descriptions (topologies) to shapes that make emergent parts look anticipated and the whole improvisation continuous and consistent is a backwards, analytical process. Descriptions are not planned before composition starts, but specified retrospectively to explain or record in a symbolic format a forward, improvisatory composition. They "record what I've done without limiting what I can do... the past is reconfigured in the present, while the future is always open"[86]. But the interesting thing with descriptions is that they can be devised in multiple ways to tell the same story (computation), without one being more right than the other. In the way designers or architecture students talk about what was put into the making of their developing work of composition; each time, depending on what they want to observe and what to ignore, as they look backwards to their process thus far. And since in the visual fields we deal with spatial, pictorial entities, as we look backwards to them, there will be always something new to see and to distinguish.

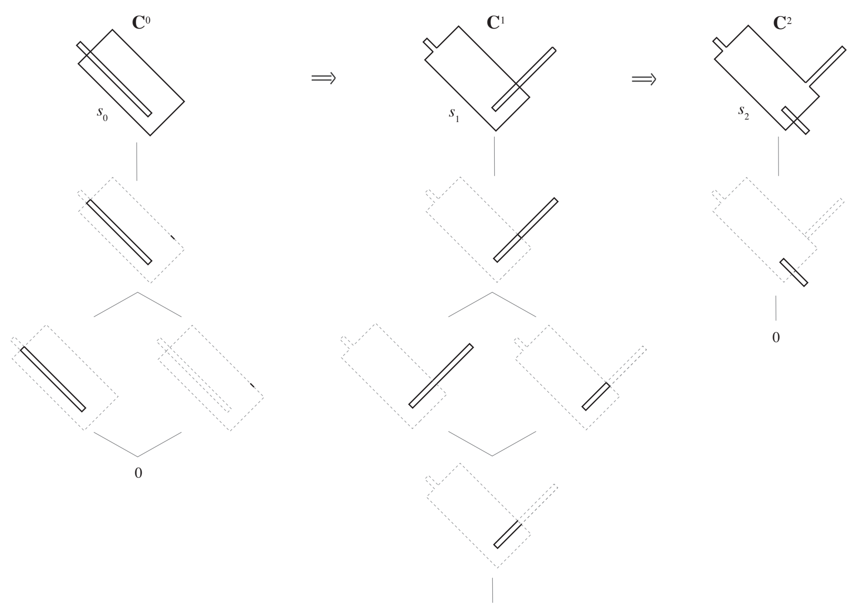
The very property of looking backwards to the materials of composition thus far and distinguishing certain compositional elements over others is formalized next through the components in the definition of a production rule. In particular, so far the attention has been solely on the part $\mathbf{t}(a)$. In the tradition initiated by post production systems and in a shape grammar, spatial changes can be caused by the two other members of a production rule, namely $s - \mathbf{t}(a)$ and $\mathbf{t}(b)$, which are included in the standard formula $[s - \mathbf{t}(a)] + \mathbf{t}(b)$. We can focus on how these parts alter shapes to obtain different topologies for the same calculation, thus reading the same composition in more than one ways. The following figures show two other ways of obtaining topologies for the derivation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2$ which are homomorphic with one another. In the first series, at step $\mathbf{C}_0$ we close shape $\mathbf{t}_k(a)$ in $\mathbf{T}(s_0)$ in addition to shapes $s_0 - \mathbf{t}_k(a)$ and $[s_0 - \mathbf{t}_0(a)] \cdot \mathbf{t}_1(a)$. The lattice structure of $\mathbf{T}(s_0)$ designates a Boolean algebra with all bottom parts combining in algebraic sums and products in all possible ways, thus partitioning

86. Stiny, G. *Shape: talking about seeing and doing* (The MIT Press, 2006, 287).

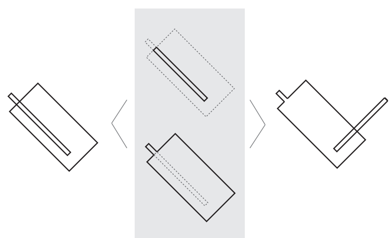
s_k exhaustively into a set of subshapes, each with its complement closed in $\mathbf{T}(s_k)$. The final shape is kept again monolithic as having the trivial topology.



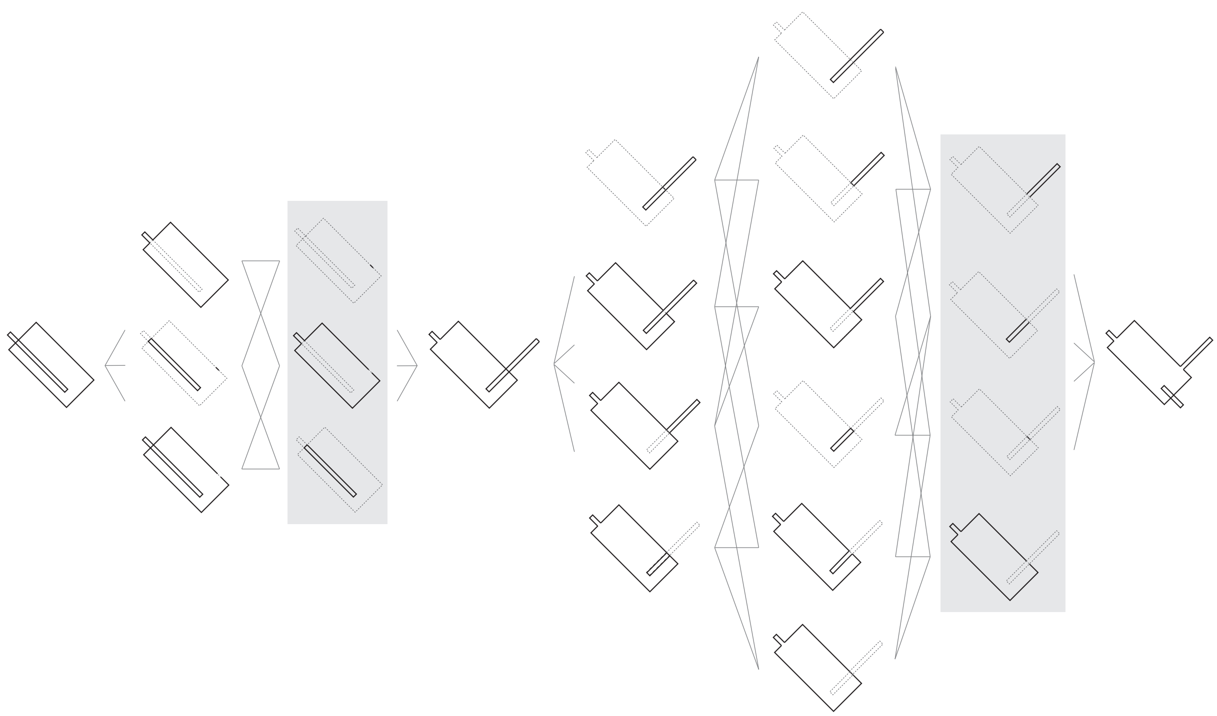
We can explain this new series of topologies with a formula that connects the closure operators $\Gamma_k(s_k)$ and $\Gamma_{k+1}(s_k+1)$ for any two shapes s_k and s_{k+1}. In particular, $\Gamma_k(x) = [s_k - \mathbf{t}_k(a)] \cdot \Gamma_{k+1}(x)$, for any part $x \leq s_k - \mathbf{t}_k(\mathrm{a})$. In the second series, topologies are obtained by closing $\mathbf{t}_k(a)$ in $\mathbf{T}(s_k)$ as usually, and $\mathbf{t}_k(b)$ in $\mathbf{T}(s_{k+1})$. The closure operator $\Gamma_k(x)$ was defined for all previous cases in terms of closure operator $\Gamma_{k+1}(x)$. In this case, we define in a similar way the closure operator $\Gamma_{k+1}(x)$ in terms of $\Gamma_k(x)$ like so, $\Gamma_{k+1}(x) = \mathbf{t}_k(b) + [s_k - \mathbf{t}_k(a)] \cdot \Gamma_k[x - \mathbf{t}_k(b)]$, for any nonempty part $x \leq s_{k+1}$.



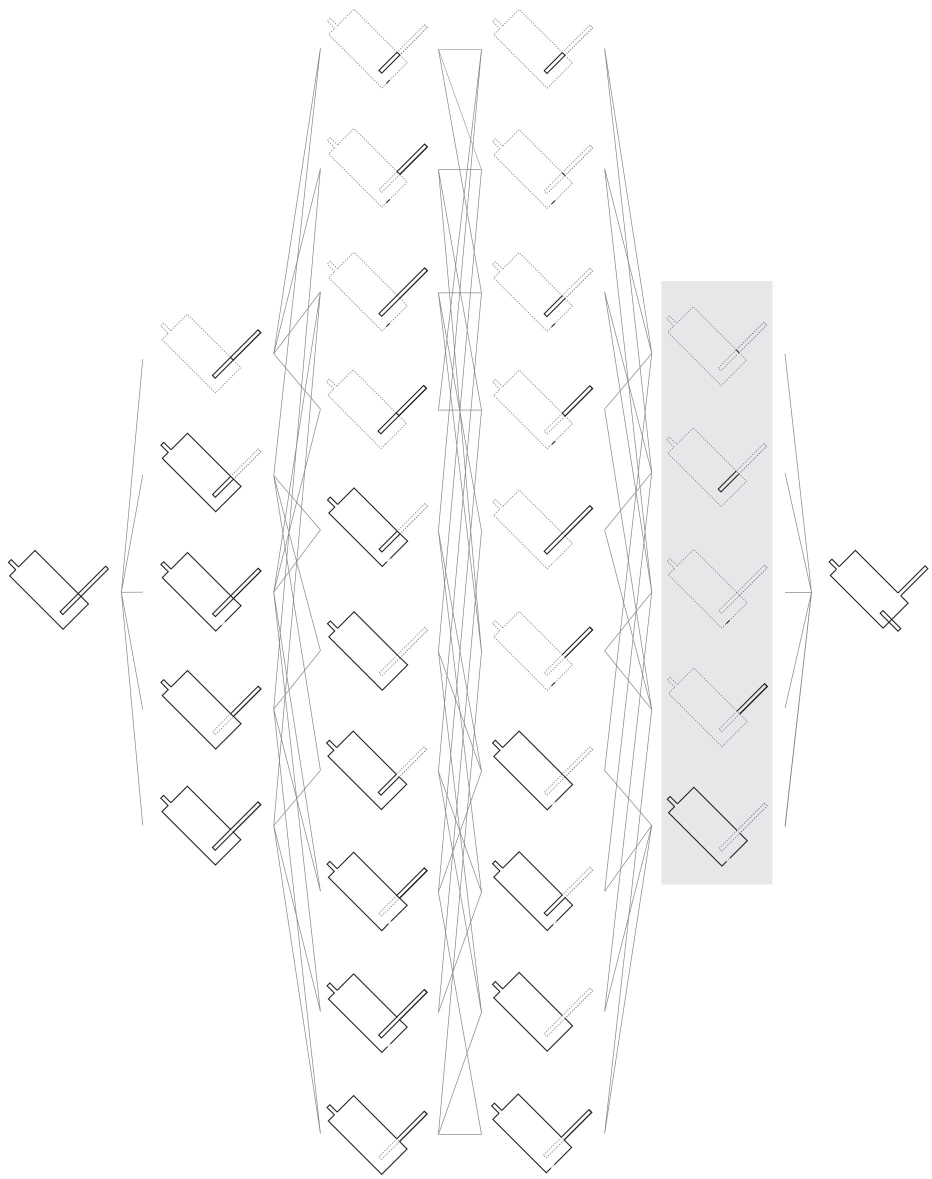
In the next series of topologies, the individual studies presented above are put together to obtain a complete specification of continuous descriptions for the derivation $\mathbf{C}_0 \Rightarrow ... \Rightarrow \mathbf{C}_4$. Specifically, a new series of topologies is obtained by closing both $\mathbf{t}_k(a)$ and $s_k - \mathbf{t}_k(a)$ in $\mathbf{T}(s_k)$ as usually, and $\mathbf{t}_k(b)$ in $\mathbf{T}(s_{k+1})$. This makes the lattices to grow up significantly as the next series show. Topologies are specified iteratively, rule after rule, by distinguishing atoms first for calculation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1$

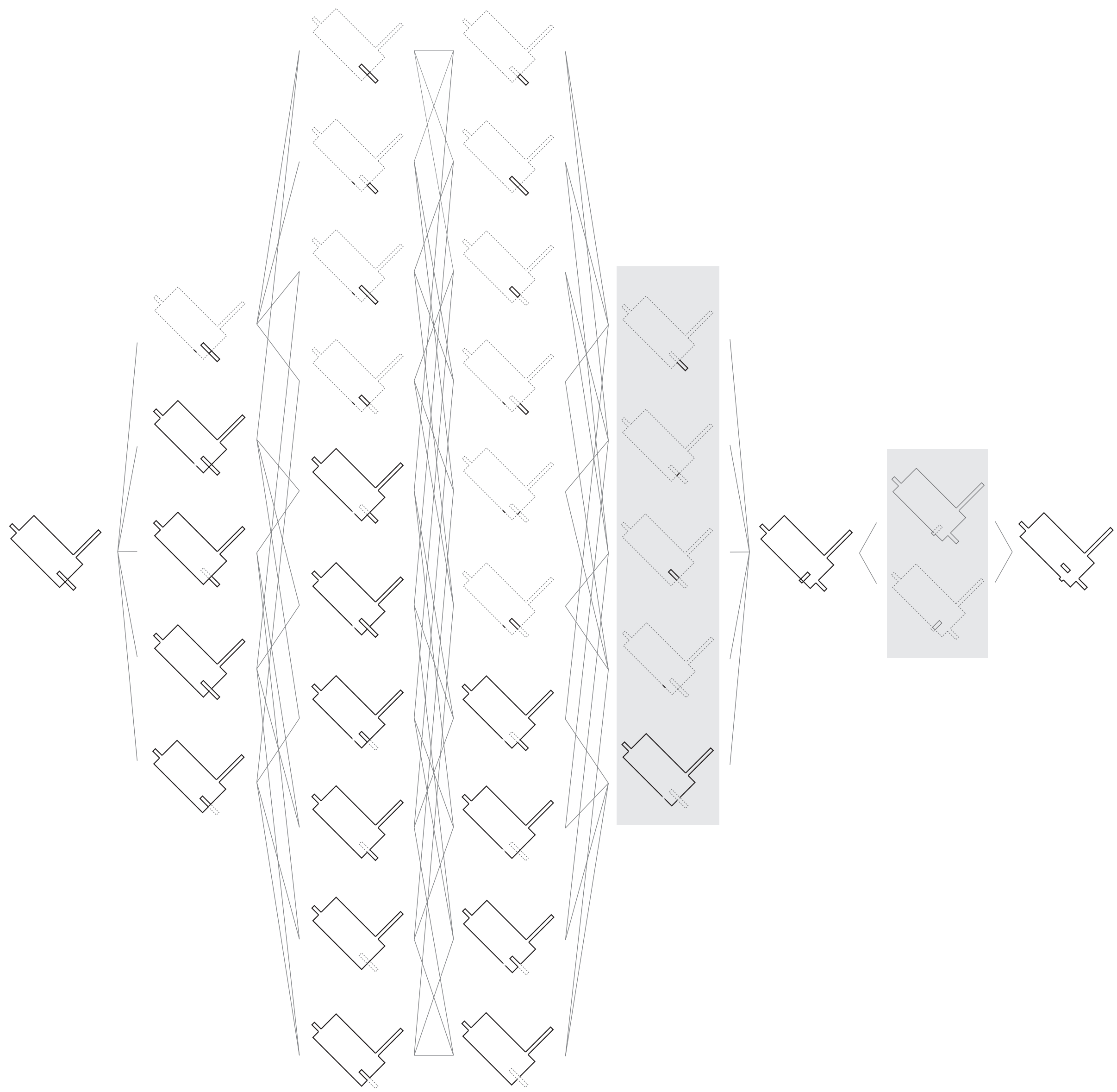


then for calculation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2$

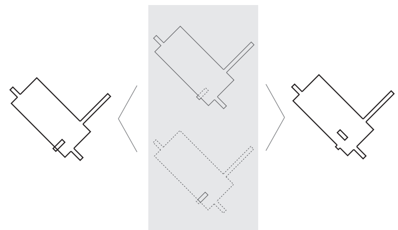


then for calculation $\mathbf{C}_1 \Rightarrow \mathbf{C}_2 \Rightarrow \mathbf{C}_3$ (continues in the next two pages).





and last for calculation $\mathbf{C}_3 \Rightarrow \mathbf{C}_4$

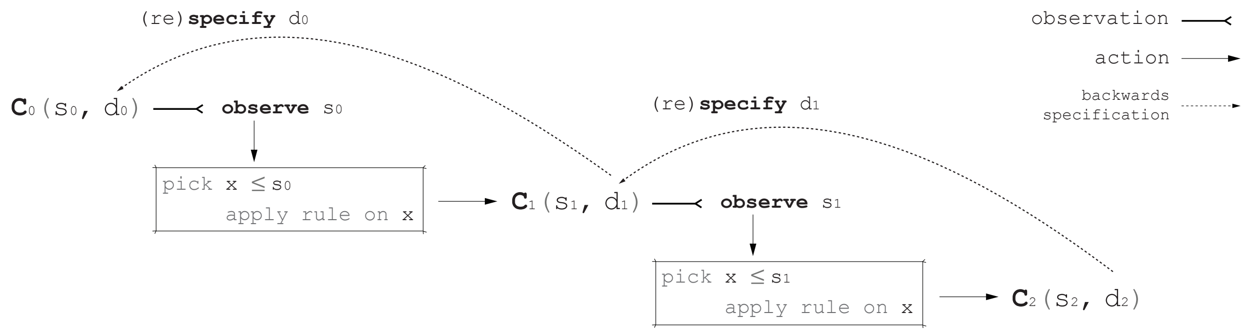


Again in this new series of topologies closure operators $\Gamma_k(s_k)$ and $\Gamma_{k+1}(s_k+1)$ for any two shapes s_k ands_{k+1}, are connected with formulas:

(i) $\Gamma_k(x) = [s_k - \mathbf{t}_k(a)] \cdot \Gamma_{k+1}(x)$, for any part $x \leq s_k - \mathbf{t}_k(a)$;
or $\Gamma_k(x) = \mathbf{t}_k(a) + [s_k - \mathbf{t}_k(a)] \cdot \Gamma_{k+1}[\mathrm{x} - \mathbf{t}_k(a)]$, otherwise;
(ii) $\Gamma_{k+1}(x) = \mathbf{t}_k(b) + [s_k - \mathbf{t}_k(a)] \cdot \Gamma_k[x - \mathbf{t}_k(b)]$, for any nonempty part $x \leq s_{k+1}$.

Notice further, that in the last series of topologies each individual topology is a Boolean algebra. Each topology is a powerset of its atoms, which are closed under addition, multiplication and complement, and each designated Boolean algebra is atomic. A Boolean algebra **B** is atomic whenever any shape can be constructed by the union of a discrete set of parts or atoms, that is, for any shape $x \in \mathbf{B}$, $x = \bigvee\{p_i \in \mathbf{Pt}(\mathbf{B}) \mid 0 \longleftarrow p_i \leq x, i = 1, 2...\}$ (the symbol $\longleftarrow$ means that p 'covers' 0). Atoms, highlighted in grey boxes, are the distinguished individuals that stand for the symbolic counterparts of shapes. They are not fixed before composition starts, but found in the course of applying rules as their byproduct and not precondition.

To summarize, in the example I presented here: *(a)* improvisation takes place through observations and actions, namely compositional rules, using two dimensional shapes defined in algebras of shapes to make compositions of shapes or designs. Following is a verbal sketch for the process of specification using arrows that stand for 'observations' and 'actions.' I omit the verbal sketches for the last two topologies and only show a verbal sketch for computation $\mathbf{C}_0 \Rightarrow \mathbf{C}_1 \Rightarrow \mathbf{C}_2$ because the rest are simple straightforward extensions.



(b) the design space in which improvisation happens is specified backwards, by retrospectively studying the continuity between shape decompositions implied in the rules; *(c)* the specification is spatial, made of structured decompositions of shapes, namely topologies, with a symbolic specification made of atoms given as a special case; *(d)* alternative ways of specifying topologies are defined according to how we read the history of computations; *(e)* the distinguished individuals – discrete parts or atoms – that allow each rule application involved in the improvisation are found in the course of applying rules but they are not their foundation.

V Taking it further

In the previous chapter, I presented a model of calculating an improvisational specification for two dimensional shapes using algebras of shapes and formal grammars. The proposed specification characterizes design spaces as continuously changing constructions while their descriptions in terms of primitive atoms become fixed and definite through a retrospective analytical interrogation. End-designs are not considered as latently existing abstractions in a specified design space, but as emerging, momentary outcomes of an ongoing process of specification. Composition proceeds forward through active momentary interpretations of spatial parts by the designer or composer whole calculates, while topological decompositions record the rule applications and the designs they demarcate in a backwards, analytical manner. Topologies explain retrospectively how the pieces of an improvisational act stick together, or more formally, how topologies map from one step of a calculation to another in a continuous fashion. This back and forth from a visual action to its symbolic specification and back again, provides a powerful framework for studying how designers and composers engage with materials of composition in real time and how designs are derived as byproducts of this engagement. In this chapter, I note differences between the proposed specification and classical ones and then I outline important extensions.

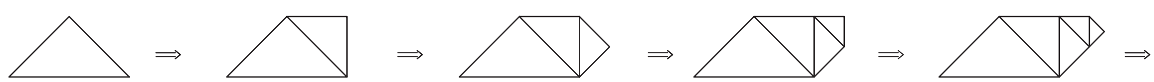
In a classical state space, each state or solution is represented as a partial tree decomposition. More specifically, consider state v, which has a description in terms of a finite set of atoms. To go from state v to a new state w, an operator or rule of the form $a \rightarrow b$ must be applicable to state v. The rule has an antecedent part a and a consequent part b, each being a set of atoms (in some formalisms, part a is made up of two lists of atoms, one being a precondition and the other an erase list[87]). To apply the rule in state v, it must be $a \subseteq v$. The result is the new state $w = v \cup b \setminus a$, which is a finite set of atoms ($\cup$ and $\setminus$ are the set-theoretic operations for addition and subtraction respectively). The application of more than one rule to a state recursively defines a tree decomposition. The rule application that takes you from v to w is continuous because there is homomorphism between the atoms in the description of both states; the description of the present state, namely, w can be explained by the description in the antecedent state v as a causal derivative of the rule applied in v, and likewise we can reverse the computation and backtrack from w to v consistently and without break. The isomorphic situation for architectural design and spatial composition is a state space of compositions or designs. Each design in the space is a finite set of mutually exclusive atomic shapes composed in sums and can be reached by a sequence of continuous rules applied on those atomic shapes recursively. Typically, a search strategy follows a sequence of rule applications automatically. Therefore, rules as well as the atomic decompositions they imply must be specified in advance.

In an improvisational specification, a designer or composer applies compositional rules to visual shapes. This slower but real-time action oriented and perceptual approach to calculating enables a designer to interpret materials constantly and devise rules that are not restricted to specified atoms, but are applied freely to any part of a shape distinguished by observation. Consider the shape decompositions and the associated atoms highlighted at the bottom of every lattice presented in the previous chapter. These decompositions can be specified either rule after rule in real-time, or by interrogating the process intermittently. In both cases, decompositions are retrospective after an improviser has taken ac-

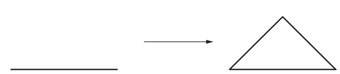
87. See for example Hoffmann, J. & Nebel, B. "The FF planning system: fast plan generation through heuristic search" (*Journal of Arti cial Intelligence Research* 14, 253-302, 2001) and Langley, P., Pearce, C., Bai, Y., Worsfold, C. & Barley, M. "Variations on a theory of problem solving" (In *Proceedings of the Fourth Annual Conference on Advances in Cognitive Systems*, 2016).

tion, that is to say, after he or she has applied compositional rules on visual shapes free of atoms. Rules as well as shape decompositions are found in the process that includes surprises (emergent shapes) and change. Hence, while a classical design space requires a priori planning of a continuous chain of actions, in improvisation, it is enough to freeze the process momentarily and retrospectively interrogate and distinguish the atomic parts and the rules that derived them to establish continuity. The opposite, to specify a prior world to be explored through causal chains of actions between analyzed states, is what I think misses the open ended, improvisatory nature of creative work in art and design. Consistency and continuity in improvisation, as well as, in composition in the visual fields, are things to tell in the aftermath.

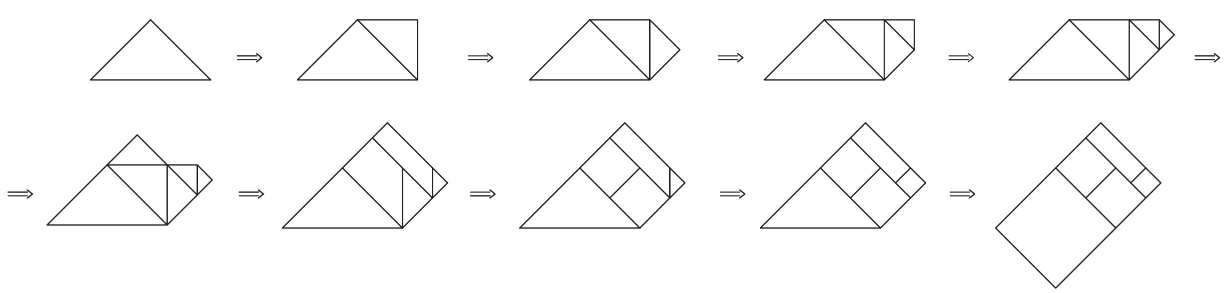
An improvisational specification with visual shapes can be studied with respect to different kinds of rules and the shape grammars they suggest, for example additive, subtractive and so on. The simple rule I used in the example of the previous chapter was for purposes of demonstration; it does not add or subtract anything from the rectangle in the left hand side of the rule. But imagine the following designs



calculated using the following additive rule that preserves the shape in the left hand side where a linear element is transformed into a triangle.



Additive rules are strictly continuous regardless of topologies because $a \leq b$ in all cases[88]. Assume now that a new series of designs is created



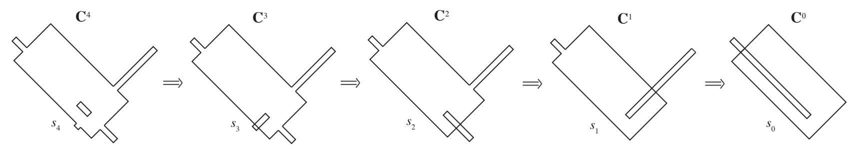
88. A formal proof for this is given in Krishnamurti, R. & Stou s, R. "Spacial change: continuity, reversibility, and emergent shapes" (Environment and Planning B: Planning and Design 24 (3), 359-384, 1997).

using the rule

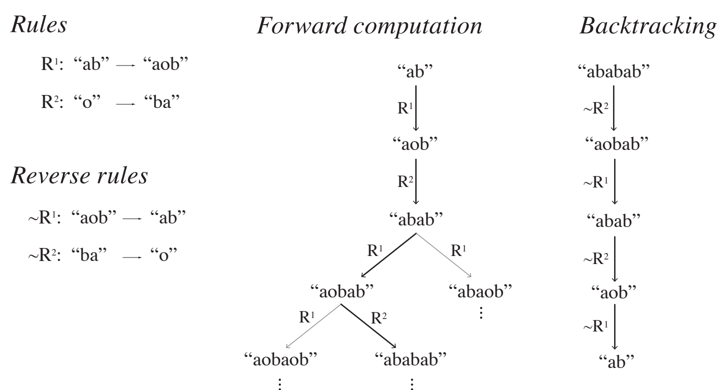
which is specified on the fly and does not preserve the shape on the left hand side. What is the specification of this design space where a triangle temporarily aims to become a truss unexpectedly transforms into a plan? It surely is an interesting dynamic situation with surprises on the way the elements interact temporally over time. But if we were not from this episcopal position – now that we know how things evolved – we would not know what is possible before we start the composition. An improvisational specification emphasizes and supports this point. Rules are inherent to the process that creates them and to their use from a designer. In this new case, regardless of the prior history of designs the compositional elements remain open to new alterations, to new rule definitions and aesthetic judgments – open to new perception.

More study on how schemas and rules work in an ongoing creative process, along with the homomorphisms implied between descriptions when explaining the process, becomes necessary for future work. More specifically, the mapping h I chose to study the continuity of the calculations in my example concentrates only on what $\mathbf{t}(a)$ alters in a shape. Many other alternative mappings exist. Mappings might take into consideration the part $a \cdot b$, which is the common part between shapes a and b in a rule $a \rightarrow b$. They may also take into consideration the part $[s - \mathbf{t}(a)] \cdot \mathbf{t}(b)$, which is the common part between $s - \mathbf{t}(a)$ and $\mathbf{t}(b)$ added back to it. By incorporating different rules and different mappings, a designer can interrogate his or her own inherent rules of composition with retrospective rules of analysis and provide not just a single explanation of an improvisatory composition but many according to individual choice. Rules and homomorphisms can show us the practical effects of observations and actions, that is to say, what improvisers perceive and do on the spot that makes the composition move forward.

Moreover, consider that a classical state space is structured with a set of continuous rule applications, which can be applied in reverse order to allow backtracking in a design space. The example I presented here is one such computation. More specifically, observe the reverse computation of $\mathbf{C}_0 \Rightarrow \ldots \Rightarrow \mathbf{C}_4$



where in each step $\mathbf{t}(b - a) \cdot s = 0$, for any shape s involved[89]. Intuitively, this condition means that no spatial elements are merging when applying the rules reversely. This is always the case in algebra $\mathbf{U}_{oj}$ for zero dimensional symbols and in all classical state spaces where designs are made of sums of atoms (zero dimensional). The following figure illustrates the case for strings.

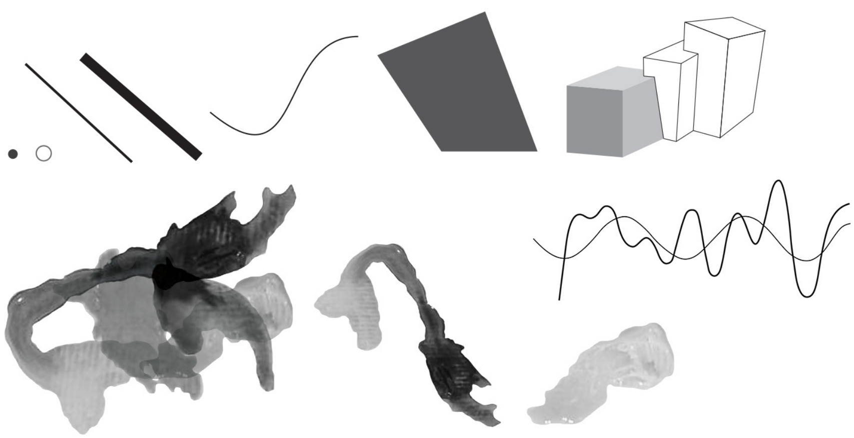


It is not always the case, however, with designs made of higher than $i = 0$ dimension, that is to say, when computations are spatial rather than symbolic. Assume that a set of continuous and reversible rules along with state descriptions that support their application together specify a single classical design space. In an improvisational specification, rules and descriptions are taken retrospectively, after spatial computations stop. We can expect that one or more design spaces pertinent to the creative task will become outputs of the same process according to whether we want to specify them in terms of continuous and reversible rules or not. It depends on the kinds of rules used, the descriptions they imply, and the mappings we pick during retrospective analysis. This provides a much more natural framework for studying how creative work happens since in the process of creation opinions are ephemeral, they change as compositional rules apply.

An improvisational specification provides a basis for studying many types of creative work by using different algebras of s – of

89. Ibid.

shapes and other things. Shapes were considered here as compositions of linear elements in a two dimensional world. There exist shapes with colors[90], or more generally weights[91], and may be used for the improvisational specification of a composition with colored shapes, such as during painting with watercolors. This would require more work in the context of shape decompositions and their continuity for weighted shapes and their algebras, which does not exist presently. Moreover, as Terry Knight[92] reminds us, there are many other kinds of composition that extend beyond architectural composition and the visual fields; composition with shapes, weighted or other, is just one kind of composition. Consider one more time s to be a raw, unanalyzed spatial thing. Examples are given in the following figure



Recently, Knight & Stiny[93] proposed to extend the shape grammar formalism to consider algebras of things that can be used for defining making grammars. Interesting avenues open in relation to the present work on design spaces, in particular, more work on how we interrogate and specify a making process computationally. Specifically, what further algebras need to be defined in order to manipulate things for making, what kind of mappings are relevant in rules for defining making actions and what descriptions are implied by those rules in terms of not only the shape of the objects of interest but their physical-material aspects. What constitutes continuity between states in a making process and what specifications of design spaces emerge from the process? Studies towards these directions extend to the realm of the physical world and shed light to even more fascinating aspects of design spaces, which are specified in the moment, they are made by a designer,

90. Knight, T. W. "Color grammars: designing with lines and colors" (*Environment and Planning B: Planning and Design* 16 (4), 417-449, 1989).

91. Stiny, G. "Weights" (*Environment and Planning B: Planning and Design* 19 (4), 413-430, 1992).

92. Knight, T. "Shapes and Other Things" (Nexus Network Journal 17 (3), 963–980, 2015).

93. Knight, T. & Stiny, G. "Making grammars: from computing with shapes to computing with things" (*Design Studies* 41 (A), 8-28, 2015).

composer, or maker in the broadest sense of the word, and consist of real, spatial things rather than symbol representations.

VI Discussion

There are traditions in design research that consider that the production of artifacts requires a well thought specification of a possible world of choices and actions in advance of calculating them. The notion of a design space embodies this vision. But in many ways, creation, synthesis or composition is more than a projection of possible results before action. My attempt in this thesis has been to introduce the analogy of improvisation to emphasize that a design space is not a conceptual construction that originates from a fixed specification, which we define independently of our participation in a creative process. But here I showed that design spaces can be calculated as part of the creative process itself; their descriptions emerge according to how designers, composers, or makers actively shape materials of composition on the spot. I therefore propose improvisation as an alternative and novel umbrella concept for including and thus expanding classical conceptualizations of design spaces. The technical mechanisms I presented here based on the shape grammar formalism and the associated shape algebras support my claim from a computational standpoint.

In concluding this thesis, I would like to briefly elaborate on a few important points that this work raises for design theory and the formal study of how creative work happens.

The analogy of improvisation as an alternative to planning and prediction should not stay limited to composition in architecture and associated areas of design. Present scholarly studies on the aesthetics of improvisational performance focus on specific types of improvisatory work and devise empirical methods of inquiry into the creative process, such as interviews, field observations, and protocol collection in order to make improvisation describable, analyzable (for examples, see Chapter 3). The possibility of using computation for the same reason is understudied. Literature that couples computation with improvisation exists but diverges in focus; either provides a calculable form of improvisation, namely a simulation of an improvisation of some kind, or explains the creative process as a derivative of the workings of the mind with psychological models that have computational support from cognitive science[94]. Using the shape grammar formalism, I showed how to describe computationally the perceptual aspects of an improvisation; what improvisers perceive and do on the spot on materials of composition – shapes and their parts – and what are the practical effects of their doings that make composition move forward. This was achieved using rules and schemas and retrospective interrogations of computations, which maybe offer ideas for how to interrogate other types of creative work, for example a musical performance, a theatrical performance, a verbal performance or other.

Further, the notion of a design space is relevant to a much broader spectrum of disciplines, cultures and discourses. Classical economics, operations research and artificial intelligence are the custodians of a classical design space, of a specified space of possible choices and actions that enables rational decision making and rightful prediction. But policy making, laws, political dialogue are among others, instances of areas demarcating spaces of choice and of possible courses of actions, that is, classical design spaces. By agreeing with Herbert Simon in his generous definition of design as "courses of action aimed at changing existing situations into preferred ones"[95] we should likewise consider these areas as in fact areas of design. But where the design and specification of those 'spaces of choice and action' happens in a contractual form

94. See for instance Johnson-Laird, P. "Jazz improvisation: a theory at the computational level" (In *Representing Musical Structure,* Ed. P. Howell, R. West & I. Cross, pp. 291-325. London Academic, 1991) and Rousseau, D. & Hayes-Roth, B. "Improvisational synthetic actors with flexible personalities" (*Knowledge Systems Laboratory, Report KSL No. 97-10*, Department of Computer Science, Stanford University, 1997).

95. Simon, H. A. *The sciences of the artificial* (The MIT Press, 1969, 111).

(specified at the outset in a symbolic format), their actual execution, the actions to which they refer to and try to demarcate from before hand, happen in the real world – the application of those rules and regulations, the computation itself, happens outside and has effects that are anything else but fixed and predictable. Therefore, new conceptualizations of the notion of the design space can be devised in order to characterize actions and choices in a real, physical world as calculating and this includes visual composition, the design and making of objects and sculptures, as well as many other levels of cultural and social practice. As a provisional remark, broader areas of research where an improvisational way of looking at design spaces may prove valuable and tremendously interesting are among others, management science and the associated areas of design thinking and creative leadership, children's pedagogy and play, verbal composition and narrative systems, as well as, policy-making, laws, social and cultural aesthetics.

Moreover, areas of important inquiry emerge when we compare the assumptions underlying contemporary models of learning, which are based on induction from prior sets of recorded evidence, to the formalism I proposed here, which is based on a memory-less model of calculating, namely, shape grammars. To be more specific, shape grammars are memory-less because first, descriptions do not reveal or preserve some kind of 'deep hidden structure' carried out in a computation, but change depending on how the 'surface' of the things we operate with is visually interpreted with schemas and rules defined on the fly; second, the history of computations involved is recorded unanalyzed, but is reconfigured in the present as we retrospectively interrogate the inherent rules that caused appearances of new evidence – here shapes and parts distinguished in the process. Contrary to this, statistical inference considers that judgments and beliefs about new evidence, at any given moment, must be in conformity with prior evidence. In this sense history, recorded evidence, namely, memory and experience, shape the present and predict the future. We see many important applications of this view in artificial systems for creativity, vision, recognition, shape synthesis, medical diagnosis among many others. At the same time, we gain no actual insight into the rules that create prior evidence; where they come from or how they were invented, but consider them as 'given' independently of origin or our own present interpretations[96]. How can we reconcile

96. For this criticism on statistical inference and its role in the modern philosophy of science, see Glymour, C. "Bayesian Ptolemaic psychology" (In *Probability and Inference: Essays in Honor of Henry E. Kyburg Jr.*, Ed. W. Harper & G. Wheeler, pp. 123-41. Kings College Publishers, 2007) and Norton, J. D. "Challenges to Bayesian confirmation theory" (In *Handbook of the Philosophy of Science, Vol 7: Philosophy of Statistics*, Ed. P. S. Bandyopadhyay & M. R. Forster, pp. 391-440. Elsevier B. V., 2011).

collisions between analysis as a form of preliminary projection of results with appearances of new evidence, which come from our creative participation by present momentary observation, in fact, free of prior structure? Can a memory-less model of calculating explain how we invent prior evidence and at the same time save the appearances of new evidence independently of priors?

Last, to iterate once more, an improvisational specification at its very basis assumes that design spaces are outputs of a creative process and not preconditions. This was formally shown here by maintaining a specification, which is ongoing and open to new rule applications. As a direct extension of this assumption, we may start considering that languages of design are outputs of the creative process itself – as byproducts of the inherent rules defined and used throughout. When composition proceeds improvisationally with compositional rules defined based on what a designer or composer distinguishes on the spot, it is enough to freeze the process thus far to save the languages of design that appeared by interrogating the history of computations. More work towards this path presents refreshing and unexplored ideas that revisit central issues in the tradition of computational design theory initiated by the shape grammar formalism. It remains to see how the technical mechanisms presented here can be tested, validated or improved as more work is conducted in the terrain of design computation.

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