Jitters, Jumps, and Peso Problems in Foreign Exchange

by

Leo Kropywiansky
A.B., Economics
Dartmouth College, 1990

Submitted to the Department of Economics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy in Economics

at the

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Chapter 1 shows that option-implied jump expectations are economically significant, and that they have a strong relationship with measures of the deviation of the current nominal exchange rate from a "fundamental" rate such as a PPP target. Jump expectations have a significant relationship with the U.S.-foreign interest differential for the $/DM rate but not the $/Yen. Jump expectations have no significant relationship with government intervention in foreign exchange markets, or with traditional measures of exchange rate overvaluation such as the trade deficit or the government budget deficit.

Chapter 2 uses option-implied jump expectations to determine whether such expectations lead to a "peso problem" bias in tests of exchange rate pricing models. There is little evidence for "peso problem" biases for the $/Yen and $/DM in the 1984-1993 period.

Chapter 3 considers how exchange rate jumps are related to U.S. intervention activity. Strong evidence is found that the U.S. "leans against the wind" to offset small, "non-jump" movements in the exchange rate, and that such intervention is effective. However, there is no strong evidence that "jump" movements affect or are affected by intervention activity.

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0.1 Introduction

The dollar exchange rate in the post-1973 era of floating is characterized by occasional large "jumps," movements which take place over the course of a month, week, or even day, of a magnitude which would have been unimaginable to denizens of the foreign exchange market during the Bretton Woods period. Jumps are the rare daily returns of 3%, 4%, or even 5% which take place in response to extraordinary news, shifts in fiscal and monetary policy, or are the result of bursting speculative bubbles. At the weekly frequency, movements as large as 8% are sometimes observed, while over a month or two months a nominal revaluation of 20% is possible. Such large movements can make or break a speculator over a short period of time. More importantly, in a world of sticky goods prices, jumps in the nominal exchange rate lead to large overnight *real* revaluations, with concomitant large effects on the relative competitiveness of nations. This dissertation examines the nature of jumps in the $/DM and $/Yen exchange rates during 1984-1993 and the related question of how market participants form their expectations of such jumps. Particular advantage is taken of recent advances in the empirical option-pricing literature, which show that it is possible to use prices of options to infer the market's expectation of a jump in the price of the underlying asset. Newly released Federal Reserve Board data on daily U.S. foreign exchange intervention are also exploited to shed light on the question of how intervention is related to jumps and jump expectations.

In chapter 1, the prices of foreign exchange options are used to infer, on a month-by-month basis, the market expectation of a "jump" in the underlying exchange rate. It is shown that monthly jump expectations for the $/DM and $/Yen were economically significant during 1984-1993 and highly variable from month to month. Expectations of jump depreciations in the $/DM are found to be positively related to the the one-month U.S.-German interest differential, evidence that the differential widens in compensation for the possibility of a "crash" in the dollar. By contrast, the $/Yen jump expectations seem to have no significant relationship with the U.S.-Japan interest differential. Both the $/DM and $/Yen jump expectations are found to be strongly *regressive*, with fears of dollar jump depreciation increasing when the dollar strays away from a "fundamental" target level such as the purchasing power parity level. Somewhat surprisingly, option-implied fears of jump depreciation are found to have no strong relationship with current government intervention against the dollar, or with traditional measures of overvaluation such as the government budget deficit and the trade deficit. For both the \$/DM and \$/Yen exchange rates, option-implied expectations of exchange rate volatility are shown to be good predictors of *ex post* volatility. Option-implied jump expectations seem to have some power in explaining *ex post* jumps in the \$/DM exchange rate, but not the \$/Yen rate.

Chapter 2 uses option-implied jump expectations to address the "peso problem" as a possible cause of the forward discount bias. When the one-month U.S. interest rate is above the German interest rate, one would expect that, on average, the dollar should depreciate over the coming month. The overwhelming empirical evidence is that the dollar does not depreciate one-for-one with the current interest differential. For some currencies and subperiods it even appreciates on average. The "peso problem" explanation suggests that market participants may be expecting a large depreciation of the dollar which is so rare it does not occur in small samples. Thus, a continued appreciation of the dollar in the face of a positive U.S.-German interest differential is possible, and reflects an unrealized fear of a sudden "crash" in the dollar. Until recently, the peso problem explanation of the forward discount bias has been difficult to refute, since it involves the essentially unobservable jump expectations of market participants. This chapter uses option-implied jump expectations to conduct an exchange rate pricing test which is resistant to the peso problem. For the \$/DM rate, a strong positive relationship between the one-month jump expectation and the one-month interest differential implies that, during sub-periods when jumps are expected but do not occur, the "peso problem" can account for some of the forward discount bias. However, for the 1984-1993 period as a whole, the expected jumps seem to have occured "often enough," so that the peso problem is not a problem for the period as a whole. For the \$/Yen exchange rate, the evidence for a peso problem explanation of the forward discount is even weaker. There is no significant relationship between the \$/Yen expected jump depreciation and the U.S.-Japan interest differential.

Chapter 3 examines the relationship between jumps in the exchange rate and U.S. intervention in support of the dollar. It is shown that the U.S. "leans against the wind" in response to small "non-jump" movements in the exchange rate, buying dollars in reponse to dollar depreciations and selling dollars in response to appreciations. Moreover, it seems that such intervention has a statistically significant effect at the daily and weekly frequency, although the effect operates with a lag. There seems to be no clear relationship between intervention and large "jump" movements in the dollar exchange rate. That is, intervention is neither more nor less likely to occur after a jump has occurred, and jumps are neither more not less likely to occur after an intervention. This result must be interpreted in light of a possible simultaneity bias. It is possible that "leaning against the wind" in response to very large "jump" movements is futile and hence should not be practiced. However, it may also be the case that, during the time of the intervention in support of the dollar, some exogenous factor is operating to depreciate the dollar, so that the net effect of the intervention and exogenous factor "wash out," leaving the probability of a jump unchanged. Thus, in the absence of the intervention in support of the dollar, a jump depreciation *would* have been more likely. The overall conclusions of this chapter are in the spirit of other recent studies on intervention, which find that intervention, even if sterilized, can have some limited effect on the exchange rate, especially at daily and weekly horizons.

Chapter 1

On the Nature of Jitters in Foreign Exchange Markets: Evidence From Option Prices

1.1 Introduction

Mounting empirical evidence suggests that exchange rates in the post-1970's period of generalized floating are best characterized by a mixed jump-diffusion process.[1] In the jump-diffusion model, exchange rates are susceptible to occasional large movements, or "jumps," caused by the arrival of extraordinary information about market fundamentals or perhaps by the bursting of speculative bubbles. Between such discontinuous jumps the exchange rate takes small continuous ("diffusion") movements with the arrival of ordinary day-to-day information. At any given time, participants in the spot and options foreign exchange markets hold some before-the-fact opinions about the behavior of the exchange rate over the coming month. In making directional bets and/or hedging decisions, it is critical for them to form an expectation of the day-to-day diffusion volatility of the exchange rate as well as the possible risk of a large unidirectional movement, or "crash."

As Bates [6], [4], [5], [6] has shown, it is technically possible to use a theoretical

[1]See Jorion [25], Akgiray and Booth [2], and Tucker and Pond [32], as well as chapter 3 of this dissertation. These studies find that a jump-diffusion model dominates various forms of the pure diffusion model in explaining the behavior of the major trading currencies during the post-1974 floating rate period.

option-pricing formula and observed options prices to "back out" the option market's implied expectations of future volatility and crashes.[2] Suppose that the exchange rate process is well-approximated by a jump-diffusion. Then, if prices currently paid for options are rational, they will reflect the market's subjective assessment of (i) the probability of a crash, (ii) the size of the crash if it occurs, and (iii) the day-to-day diffusion volatility over the life of the options.

In this paper we find the option-implied diffusion volatility and jump expectation at monthly frequency for the $/DM and $/Yen exchange rates during 1984-1992. We then examine how the option-implied expectations are formed, how they are related across currencies, and whether or not they are good predictors of *ex post* jumps and volatility. This work relates to that of Frankel and Froot [13], who use *survey* data on exchange rate expectations to ask how such expectations are formed and whether they are rational. Option-implied expectation estimates are superior to survey data in that they reflect actual bets made by market participants. However, two caveats are in order. The option-implied approach suffers the drawback that only the jump frequency, jump size, and volatility expectations may be recovered, and not the *mean* movement of the exchange rate.[3] Moreover, as Wei and Frankel [29] point out in their study of volatilities in FX markets, the option-implied approach depends critically on the correctness of the assumed option-pricing model, including the particular form assumed for the underlying exchange rate process.

In section 1.2, the option-pricing formula, the options data, and the technique for estimating the option-implied parameters are described. Section 1.3 describes the monthly time series of option-implied jump expectations and volatilities. It is shown that, while expected diffusion volatilities tend to persist from month to month, jump expectations are quite mercurial. Jump fears in the current month have little

[2]This practice is the jump-diffusion analogue to the well-worn tradition of finding implied volatilities using the Black-Scholes formula.

[3]This is of course related to the well-known "risk-neutral" option pricing results. Under appropriate conditions, options may be priced as if the process followed by the underlying asset is the one which would hold in a market of risk-neutral participants. Hence, the only information which can be recovered from options prices is information regarding the "risk-neutral" exchange rate process, which has mean drift equal to the US-Foreign interest differential.

to do with jump fears in the previous two months. Section 1.4 describes how the option-implied "jitters" are related across the \$/DM and \$/Yen exchange rates. The expected diffusion volatilities for the two currencies track one another quite closely. The jump expectations, which likely reflect country-specific factors, are also positively related, though not as closely as the diffusion volatilities are. Option-implied jump expectations and volatility expectations are more variable for the \$/Yen than for the \$/DM exchange rate. Section 1.5 turns to an examination of what information the options market uses in forming its volatility and jump expectations. Section 1.5.1 discusses the relationship between jump expectations and the interest differential in the context of a trading strategy known as "buying the rich currency and selling the poor." It shown that there is a strong positive relationship between the U.S.-German interest differential and fears of jump depreciation. However, this relationship does not hold for the \$/Yen exchange rate. In section 1.5.2, it is shown that jump expectations have a strong regressive component: when the current exchange rate is strong relative to some "fundamental" level, expectations of a jump depreciation back towards the fundamental increase. There is also weak evidence of "bandwagon" expectations for the \$/DM exchange rate, with large exchange rate movements in one month leading to increased jump expectations in the following month. There is no evidence of bandwagon effects for the \$/Yen exchange rate. Section 1.5.3 considers the possibility that current foreign exchange intervention, trade deficits, and government deficits may have an effect on expectations of a jump in the exchange rate in the future, perhaps by affecting expectations of a future shift in policy. The effects of intervention and the twin deficits are found to be negligible. Finally, in section 1.6 we consider whether option-implied expectations are good predictors of *ex post* exchange rate behavior. For the \$/DM rate, it is shown that the option-implied *total* volatility, including volatility due to diffusion and jumps, gives an unbiased forecast of *ex post* total volatility. While the \$/Yen option-implied total volatility is not an unbiased predictor, it nonetheless carries information about the future in the sense that it can predict the *direction* if not the magnitude of changes in *ex post* volatility. Tests of whether option-implied jump expectations can predict *ex* *post* jumps are shown to depend critically on which one-day returns in the sample period are considered "jumps" and which are not. If we consider movements of about 2.25 standard deviations larger than the mean daily movement to be "jumps," then it is the case that beginning-of-the-month option-implied jump expectations carry statistically significant information about subsequent jumps in the \$/DM exchange rate, but not the \$/Yen exchange rate.

1.2 Estimation of Option-Implied Parameters

The options data used are from the Philadelphia Stock Exchange (PHLX) and consist of transactions data on foreign currency options traded between January 1984 and December 1992. Each observation consists of the price paid for the option, the Telerate spot exchange rate at the time of the transaction, and the term to maturity and strike price of the option. The options data for June through November of 1985 cannot be used due to severe errors in reporting during that period.

To invert for jump–diffusion parameters, we will need to specify an option pricing formula that is used by the market to price options in the presence of jumps. We assume that, between now and the time the option expires, options traders take the exchange rate to follow the jump-diffusion process given below

$$\frac{dS}{S} = [\mu - \lambda\kappa]dt + \sigma dz(t) + \kappa dq(t) \tag{1.1}$$

where $z(t)$ is a Wiener process and $q(t)$ is a poisson process with jump intensity λ and where κ is a non–stochastic jump size. We assume traders input their current belief of λ, κ, and σ when pricing options. The instantaneous expected rate of return of this process will be μ. Conditional on no jumps occurring however, the expected rate of return will be $\mu - \lambda\kappa$. The expected depreciation due to jumps is $\lambda\kappa$. To price the option, we follow Merton [28] and assume that the jump risk is not priced–i.e. that it can be diversified away.[4]

[4]If the jump risk is not diversifiable, the option can still be priced. However, the option price will depend on a risk-neutralized jump frequency λ^* and not the true distributional frequency λ.

Because the majority of currency option transactions on the PHLX are of American style options (96% of the Deutschemark option transactions are American—almost all transactions were American prior to 1991), we chose to work with the American options in order that we do not run out of observations in many of the years prior to 1991. A drawback of using American options is that there exists no analytic closed form formula for the price of an American option. We rely on an analytic approximation proposed by Bates [6] which extends the work of Barone-Adesi and Whaley [5] and MacMillan [27] to the case where the underlying price process follows a jump-diffusion process.

As pointed out by Bates [6], theoretical pricing of PHLX options must also take account of delivery lags. According to PHLX contractual agreements, an American option is settled in 5 days after expiration and 7 days (5 business days) after early exercise. Given these contractual specifications and the process described above, one can arrive at the following approximation to the American put or call price:

$$\Psi_A(pc) = \begin{cases} \Psi_E(pc) + \left(\frac{S}{S^*}\right)^q \left[pc \cdot \left(e^{-r^*\tau_2} S^* - e^{-r\tau_2} K\right) - \Psi_E(pc)\right] & \text{if } S < S^* \\ pc \cdot \left(e^{-r^*\tau_2} S^* - e^{-r\tau_2} K\right) & \text{if } S > S^* \end{cases} \tag{1.2}$$

where pc takes the value 1 if the option is a call and -1 if it is a put, S is the value of the underlying exchange rate, K is the strike price, τ_2 is the delivery lag after early exercise, and S^* is the level of the exchange rate at which it is optimal to exercise early and hence satisfies the following equation:[5]

$$S^* = \text{argmax}_{S^*} \left(\frac{S}{S^*}\right)^q \left[pc \cdot \left(e^{-r^*\tau_2} S^* - e^{-r\tau_2} K\right) - \Psi_E(pc)\right] \tag{1.3}$$

while the parameter of curvature q satisfies:

$$\frac{1}{2}\sigma^2 q^2 + (r - r^* - \frac{1}{2}\sigma^2 - \lambda\kappa)q - \frac{r}{1 - e^{r\tau}} + \lambda(1+\kappa)^q - \lambda = 0 \tag{1.4}$$

[5]The first order condition of this maximization is equivalent to the smooth pasting condition derived in Bates.

and $\Psi_E(pc)$ is the corresponding European option price as derived in Merton [28] and Jones [20]:

$$\Psi_E(pc) = e^{-r(\tau+\tau_1)} \sum_{n=0}^{\infty} P(n) \cdot pc \cdot \left[S e^{(r-r^*)\tau_1 + b_n \tau} \mathrm{N}(pc \cdot d_{1n}) - K \mathrm{N}(pc \cdot d_{2n}) \right] \qquad (1.5)$$

where:

$$P(n) = \frac{e^{-\lambda\tau}(\lambda\tau)^n}{n!} \qquad (1.6)$$

and τ_1 is the delivery lag after expiration, N(.) denotes the cumulative normal distribution and:

$$b_n = r - r^* - \lambda\kappa + n \log(1+\kappa)/\tau \qquad (1.7)$$

$$\begin{aligned} d_{1n} &= \frac{\log(S/K) + (r - r^*)\tau_1 + (b_n + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \\ d_{2n} &= \frac{\log(S/K) + (r - r^*)\tau_1 + (b_n - \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \end{aligned} \qquad (1.8)$$

A set of observed option prices $\{y_i\}_{i=1}^{M}$ will in all likelihood not fit the parametric specification. A set of three options along with the price of the underlying asset and the other observable inputs to the option pricing function will uniquely determine $\hat{\lambda}, \hat{\kappa}$ and $\hat{\sigma}$. Any other set of three options will probably produce another set of $\hat{\lambda}, \hat{\kappa}$ and $\hat{\sigma}$. This is a problem common to all option pricing models. The likely reasons for this is that (i) the assumed model is misspecified, (ii) some options are thinly traded and trades do not take place at "true values", (iii) trades take place at bid and ask prices instead of at the "true value," (iv) investors make errors in evaluating the "true value" of the options. All four reasons are likely play a role. We will assume that our specification is approximately correct and that the majority of the errors come from factors (ii), (iii) and (iv). We further assume that the deviations of the observed prices from the "true values" can be expressed in terms of a mean zero random variable that is independent of the arguments entering the option pricing formula.

By taking a set of options traded during a given day, we can "invert" these options to find the implicit parameters used by option traders during that day.[6] A consistent estimate of λ, κ and σ is obtained by the following non-linear least squares regression:[7]

$$\{\hat{\lambda}, \hat{\kappa}, \hat{\sigma}\} = \text{argmin}_{\lambda,\kappa,\sigma} \sum_{i=1}^{M} (y_i - \Psi_i(pc))^2 \tag{1.9}$$

Due to the over-abundance of data in many months, we construct a sub-sample of the data as follows. Our objective was to pick one trading day in a given month and take options trading in that day that had the same expiration date. We also wanted at least 50 options. We start with the first trading day of a given month and find the expiration date that was less than 120 days away with the most options. If that set contained at least 50 options we used that set. Otherwise we proceeded to the next trading day.

1.3 Option-Implied Estimates

The results of the monthly non-linear least squares regressions along with the asymptotic approximations to the standard errors are reported in Tables 2.5 through 2.9 appended to chapter 2 of this dissertation, while the monthly time series for the jump expectation of the \$/DM exchange rate $\lambda\kappa$ is shown in Figure 1-1. The jump expectation is quoted in annualized terms. Hence, 0.05, or 5 per cent, corresponds to a expected depreciation of 0.42 per cent over a single month (λ is measured in a scale of probability per unit of time while κ has no time dimension). Although the estimates for $\lambda\kappa$ are of a fairly reasonable order of magnitude, the estimates of the individual components λ and κ are somewhat less believable. In the space of λ and κ (for a fixed value of total variance[8]) there is an approximate hyperbolic ridge in the least squares objective function—the objective function is peaked for one value of $\lambda \times \kappa$.[9]

[6]The inversion process is conceptually straightforward but computationally challenging. See the appendix to chapter 2 of this dissertation for a discussion of the computational issues.

[7]Consistency is achieved as the number of options $M \to \infty$.

[8]The objective function is maximized for one value of total variance, ν, almost regardless of the other parameters.

[9]The estimates for κ can sometimes be quite large especially when λ is small. This is not surprising since when λ is very small, κ does not much influence the pricing function. This is the

The individual estimates of λ and κ are not well estimated because the peak of the ridge is not very well identified—the objective function takes on similar value for all points on the ridge.[10]

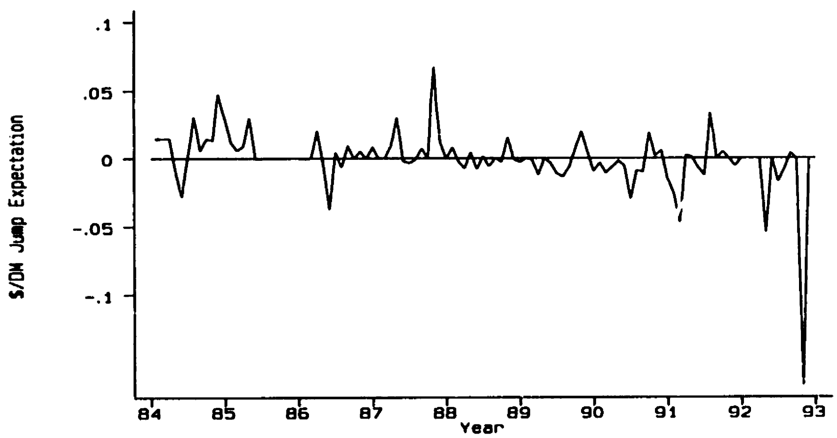

Figure 1-1: Jump Expectations $\lambda\kappa$ for the DM

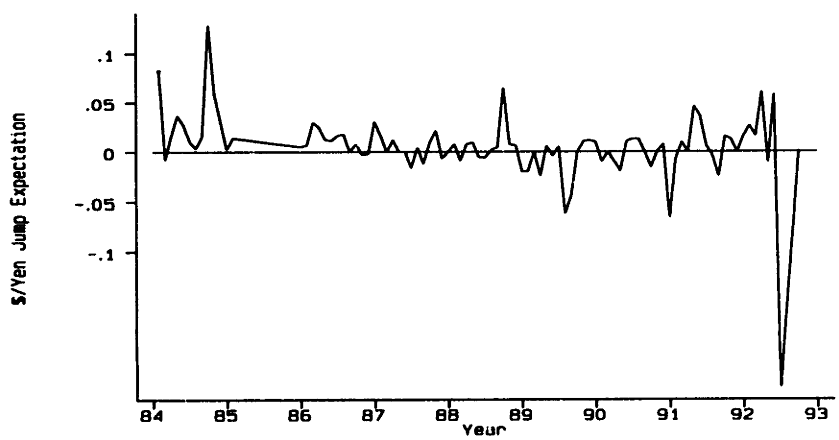

Figure 1-2: Jump Expectations $\lambda\kappa$ for the Yen

The estimated jump expectations are of an economically reasonable magnitude and, moreover, the time series of $\hat{\lambda}\hat{\kappa}$ seems to agree with the qualitative stories told about the dollar in the 1980's and 1990's. During the mid-1980's when the dollar was thought to be overvalued it was indeed the case that participants in the options mar-

reason why the standard errors can be very large for κ when λ is small.

[10]There were also two months out of the total 114 for which the non-linear least squares procedure did not converge. For the yen there was one month that did not converge.

ket were expecting dollar depreciations due to jumps.[11] See, for example, the 1985 studies of Krugman [22] and Marris [25], who argued that the strong 1984-85 appreciation of the dollar, inexplicable by fundamentals, was evidence of an unsustainable "speculative bubble" in the exchange rate. These results are in general accord with those of Bates [4],[5] who also found substantial expected jump depreciations for his sample of 1984–87 using CME options on Deutschemark futures.

Interestingly, occasional fears of dollar depreciation due to jumps persisted until the late 1980's while the dollar was on a steady downward path during this entire period. This accords with the "hard landing" fears expressed by then-chairman of the Federal Reserve Paul Volcker. It was widely perceived in 1985-87 that the dollar needed to depreciate, but there was fear that this depreciation might come in the form of a sudden crash, a concomitant shift from dollar assets, and a sharp rise in interest rates.

January 1990 marks the start of a new period during which there are many months when the market was calm, with essentially no expected depreciation due to jumps. These calm months are interspersed with months when there are fairly dramatic expected appreciations. During this later period, the economic problems associated with the reunification of Germany became apparent. Market participants claimed that the tight monetary policy implemented by the German government could not be sustained given the severe recession in the former East Germany. It was believed the Bundesbank would eventually lower its rate and consequently send the Deutschemark tumbling.[12]

The two months with the most dramatic jump expectations are October 1987, when a crash occurred in the U.S. stock market, and October 1992, the period of the ERM crisis. In October 1987 there was a 6.7% expected dollar depreciation while in October 1992 there was a 16.7% expected Deutschemark depreciation. In both cases,

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[11]It is unfortunate that the PHLX options price data are completely contaminated and hence unusable for the months June–November 1985 which includes the post-Plaza Accord crash of the dollar. The expected depreciation for April 1985 is large, perhaps indicating anticipation of the eventual decision of the G-5 to bring the dollar down.

[12]See Dornbusch [14].

the currency of the country in crisis was expected to jump down.

Figure 1-2 displays the jump expectations in the $/Yen rate while Tables 2.10 through 2.14 appended to chapter 2 of this dissertation report the estimates and asymptotic standard errors. There are some similarities here to the $/DM case. In the period from 1984 to 1985 there are fears of dollar depreciations from jumps. Again, it is possible that market participants feared the coordinated G-5 intervention that ultimately took place in late 1985. After this period, from 1986 to late 1988 there remained fears of a dollar depreciation although much smaller than what was expected earlier. From 1989 to late 1990 there are months with dollar expected depreciations interspersed with months of large expected dollar appreciations. After 1990, any similarity to the $/DM jump expectations end. From 1991 to mid 1992, there are consistent fears of dollar depreciation. Starting in mid-1992 the options trading on the yen became so thin on the PHLX that for many months there was not a single day where more than 50 options with the same maturity were traded. From July 1992 to December 1992, 10 of the 18 months did not have enough options. It is not surprising in this case that for the months for which we did have enough data that the estimates were so erratic. Perhaps the trading was so thin in the markets that options traded far from their "true" value.

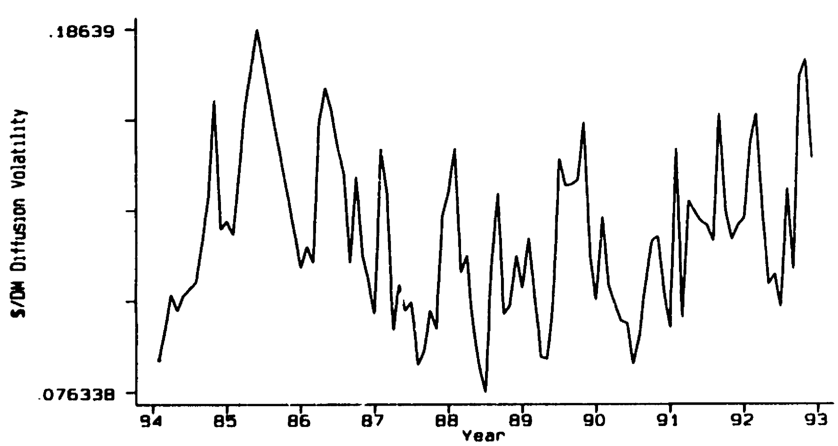

Figure 1-3: Diffusion Volatility for the DM

Figure 1-3 gives the monthly time series of σ, the expected diffusion volatility. The volatility figure, which is annualized, varies from a high of about 18 per cent

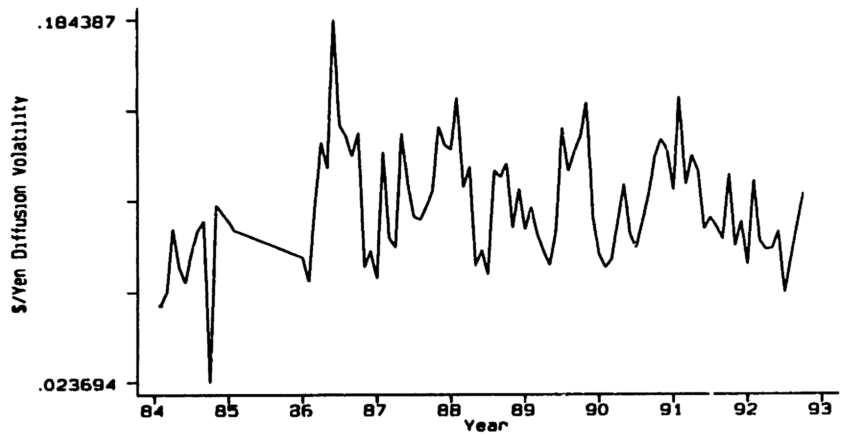

Figure 1-4: Diffusion Volatility for the Yen

during the summer of 1985 to lows which are below 10 per cent during many months in the sample. The option implied estimates for the yen's continuous volatility are plotted in Figure 1-4. The continuous volatility in the yen falls to a low of 0.023 in September 1984. Because λ and κ are large during this month the total volatility including the jump component works out to a more realistic number of 0.092. This *total* volatility, inclusive of the jump component, is given by $\nu = (\sigma^2 + \lambda\gamma^2)^{\frac{1}{2}}$, where $\gamma = log(1 + \kappa)$. The total volatility is plotted on the same axis with the diffusion volatility for each of the currencies in Figures 1-5 and 1-6. With few exceptions, the total volatility tracks the diffusion volatility quite closely.

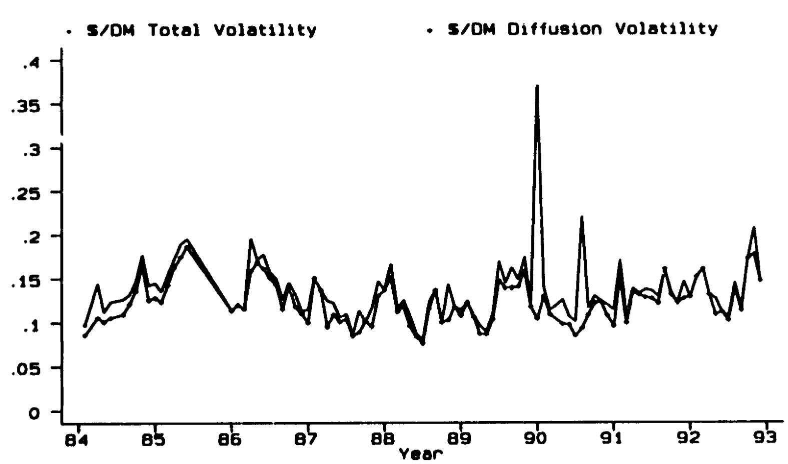

Figure 1-5: Diffusion Volatility and Total Volatility for the DM

Figures 1-7 and 1-8 report the fraction $f = \frac{\lambda\gamma^2}{\nu^2}$ of expected total variance which

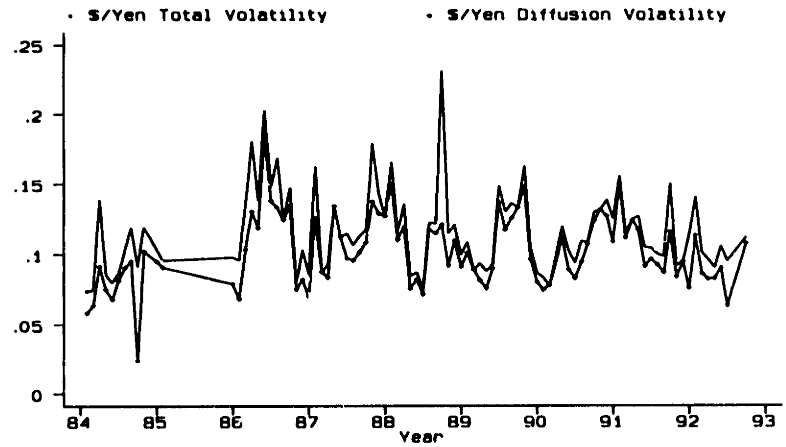

Figure 1-6: Diffusion Volatility and Total Volatility for the Yen

is accounted for by the expected jump variance $\lambda\gamma^2$. The fraction f is typically less than 0.25, though there are months of exceptionally large jump expectations when this fraction is greater than 0.5.

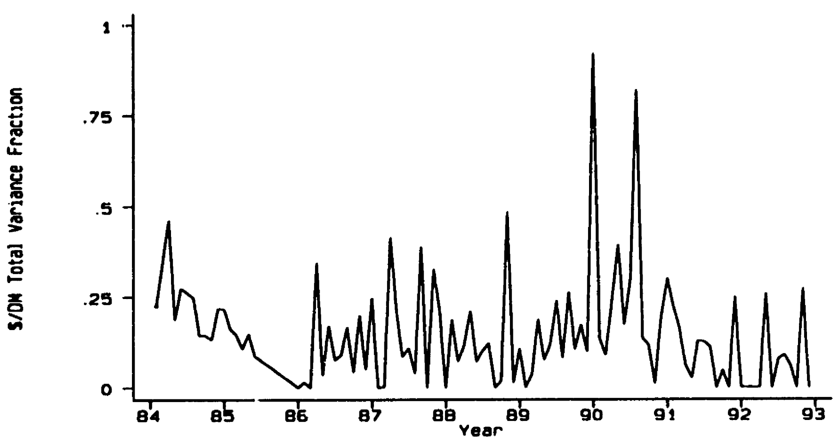

Figure 1-7: Jump Variance as Fraction of Total Variance for the DM

We turn now to the issue of the *persistence* of the option-implied jitters. Consider the regressions, reported in Table 1.1, of the various option-implied jitters on their own one- and two-month lags. The estimates reveal that, for both the \$/DM and \$/Yen exchange rates, the expected diffusion volatility persists from month to month: large expected volatility is associated with large expected volatilities during the two preceding months. By contrast, the option-implied jump expectations are quite mercurial. For both currencies the autoregressive coefficients are insignificantly

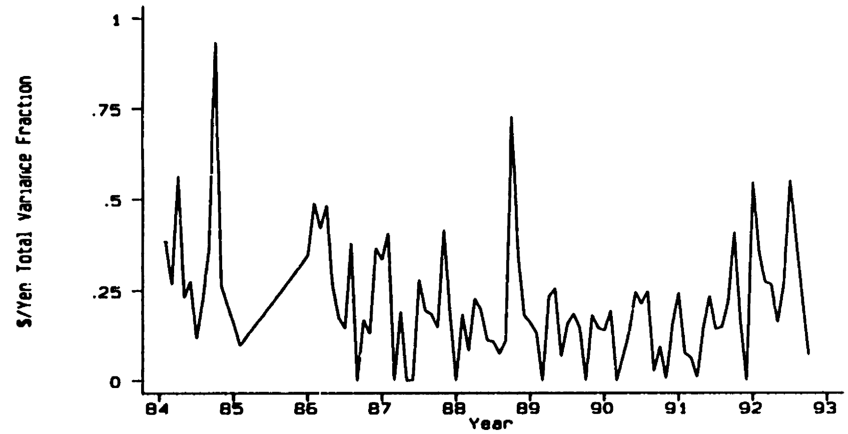

Figure 1-8: Jump Variance as Fraction of Total Variance for the Yen

different from zero, indicating that this month's jump fears are their own in the sense that they have little to do with those in the preceding two months. For total volatility, including both the diffusion and jump component, the estimates indicate a high degree of persistence for the \$/Yen rate and a weaker persistence for the \$/DM rate.

1.4 Cross-Country Comparison

In this section we consider how volatilities and jump expectations are related across the \$/DM and \$/Yen exchange rates. Figures 1-9, 1-10, and 1-11 plot the jump expectations, diffusion volatilities, and total volatilities for the two currencies on common axes.

The diffusion volatilities for the two currencies seem to track one another closely, while the relationship between the jump expectations is less clear, particularly for the period after 1990, which exhibited large expected jump depreciations for the \$/Yen and "calm" months or expected jump *appreciations* for the \$/DM exchange rate. Table 1.2 reports the results of OLS regressions in which each of three option-implied \$/DM option-implied measures was regressed on the corresponding \$/Yen measure. The diffusion volatilities are indeed positively related, with a highly significant slope coefficient of 0.42, indicating that the \$/DM volatilities do not vary as much as do those for the \$/Yen rate. The jump expectations are also positively related,

Table 1.1: AR(2) Estimates for Option-Implied Measures

DM	α	β_1	β_2
Diffusion Volatility	0.0540 (4.436)**	0.5360 (5.192)**	0.0293 (0.285)
Jump Expectation	-0.0009 (-0.3880)	0.0937 (0.912)	0.0563 (0.549)
Total Volatility	0.0956 (5.208)**	0.1478 (1.442)	0.1558 (1.529)
Yen	α	β_1	β_2
Diffusion Volatility	0.0487 (4.236)**	0.3325 (3.140)**	0.1944 (1.844)*
Jump Expectation	0.0033 (0.818)	0.0293 (0.274)	0.1708 (1.158)
Total Volatility	0.0682 (4.613)**	0.2022 (1.935)*	0.2167 (2.092)**

Regression of option-implied quantity X_t on its one- and two-month lags:

$$X_t = \alpha + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \epsilon_t \tag{1.10}$$

Asymptotic t-statistics in parentheses. (*), (**) denote significance at 90% and 95% levels.

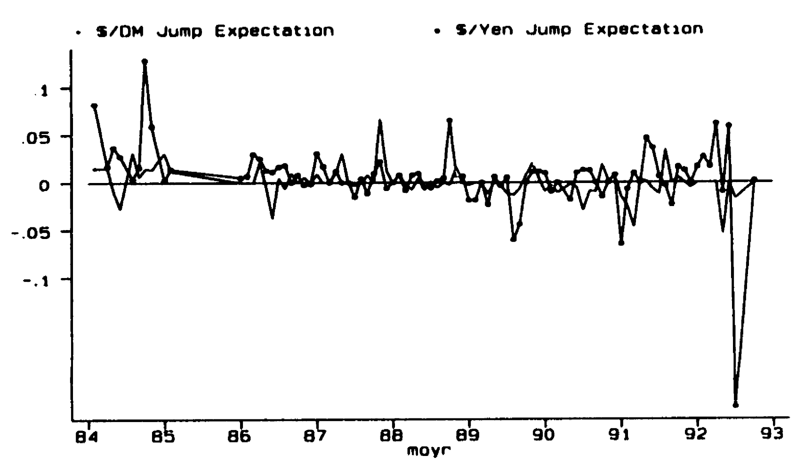

Figure 1-9: Jump Expectation for Yen and DM Rates

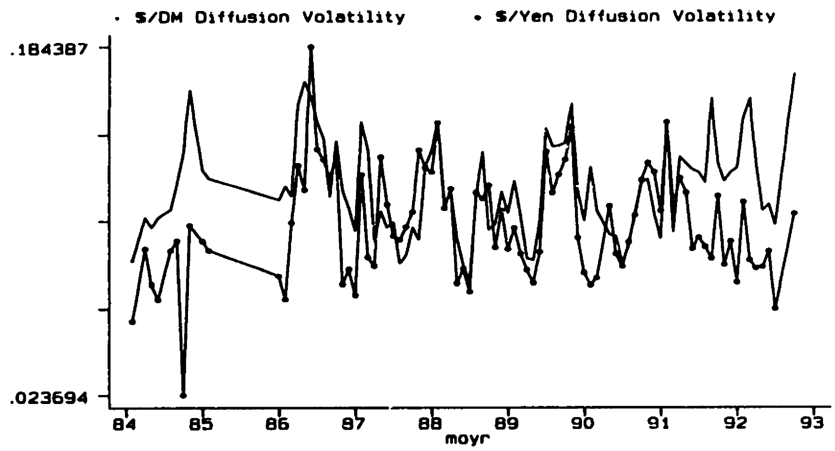

Figure 1-10: Diffusion Volatility for Yen and DM Rates

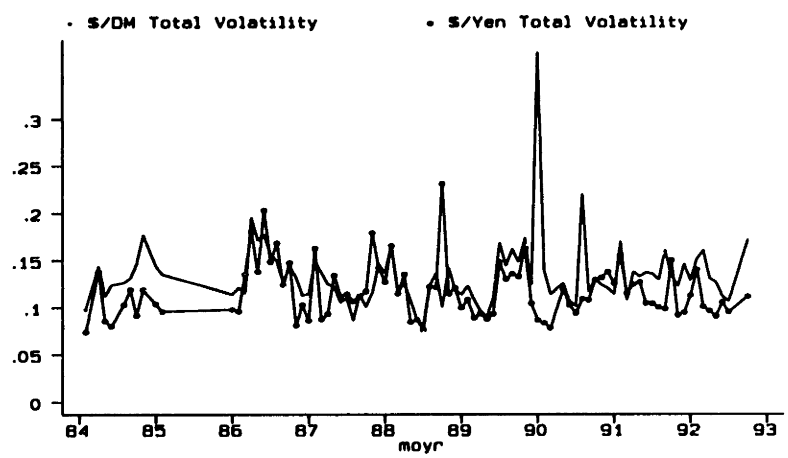

Figure 1-11: Total Volatility for Yen and DM Rates

Table 1.2: Cross-Currency OLS Regressions

	Constant	Slope
Diffusion Volatility	0.0759	0.4329
	(8.665)	(5.234)
Jump Expectation	-0.00041	0.0940
	(-0.254)	(2.108)
Total Volatility	0.1017	0.2749
	(6.735)	(2.201)

OLS regression of \$/DM option-implied measure on the corresponding \$/Yen option-implied measure. Asymptotic t-statistics in parentheses. T=92 observations on the months for which there were sufficient options data to estimate both the \$/DM and the \$/Yen option-implied parameters.

though less significantly so. This is not surprising, given that the directional risk represented by jumps may sometimes be due to factors specific to Japan or Germany, factors which may well work in opposite directions at times. A slope coefficient of 0.09 indicates that \$/DM jump expectations are less variable than the corresponding \$/Yen jump expectations. Finally, and not surprisingly, the expected total volatility due to both diffusion and jump movements is also positively and significantly related across currencies, with greater variability of this measure for the \$/Yen rate.[13]

1.5 How Are Jump Expectations Formed?

1.5.1 Buying the Rich Currency, Selling the Poor

The intuition of uncovered interest parity suggests that, when the 30-day U.S.-German Eurocurrency differential is positive, that is, when dollar deposits pay a higher interest rate than DM deposits, then the dollar should on average depreciate over those 30 days. This must occur to make risk-neutral investors indifferent between deposits denominated in the two different currencies. The empirical evidence points to an overwhelming rejection of uncovered interest parity, both across time and

[13]These results are robust to the exclusion of the outliers associated with the October 1987 crash and the September 1992 ERM crisis.

across various currencies.[14] The currency of the high interest rate country does *not* depreciate one-for-one with the interest differential, and in fact often *appreciates.*[15] This result suggests the profitability of the following strategy: borrow money from a bank in the low interest rate country (the "poor" currency, in the parlance of foreign exchange traders), and place that money in a bank account denominated in the high interest ("rich") currency. By covered interest parity, this strategy can also be implemented by buying the rich currency in the forward market and selling the poor currency. See Choie [9] for a practitioner's account of the profitability of this strategy. See also Backus, Gregory and Telmer [4], who find evidence that the strategy gives superior returns, even on a risk-adjusted basis, in the sense of having a higher Sharpe ratio than a passive strategy of holding an equity portfolio.

It is possible that the returns to a strategy of "buying the rich, selling the poor" are illusory in the following sense. Suppose that uncovered interest parity holds, so that the U.S.-foreign Eurocurrency differential always equals expected depreciation of the dollar. Suppose further that a major portion of the expected depreciation is due to a small probability of a very large crash in the high interest rate currency. Then, because the probability of the crash is small, it is possible that over a limited time horizon the crash does not occur, creating an illusion of "excess returns" to the trading strategy. However, if the strategy is still being followed when the crash occurs, profits accrued earlier may well be wiped out.[16] If the returns to a "buying the rich, selling the poor" strategy are driven by *ex ante* fears of a large depreciation in the high-interest currency, then at a minimum it should be the case that expectations of jump depreciations are *positively* correlated with the interest differential.[17] That is, the

[14]Frankel and Froot [14] summarize the evidence on this point.

[15]As Lewis [22] notes in her survey of this so-called "forward discount bias," the failure of uncovered interest parity is difficult to account for even when risk aversion of investors is allowed.

[16]W.T. Baily notes, based on discussions with traders and researchers at Goldman Sachs, Merrill Lynch, and Salomon Brothers, that such a strategies have actually been implemented by traders. The "buying the rich" strategy would have worked particularly well during the 1980-85 period when the dollar appreciated strongly while trading at a forward discount. However, the strategy would have been disastrous if followed during the sharp post-Plaza Accord depreciation of the dollar.

[17]This is closely related to the question of whether "peso problem" jump fears can explain the forward discount bias. As chapter 2 of this dissertation discusses in more detail, positive correlation of jump depreciation fears with the interest differential is a necessary though not sufficient condition

U.S. interest rate should exceed the foreign interest rate precisely when the markets fear a large crash in the dollar. Option-implied jump expectations permit a direct examination of the hypothesis that jump fears move with the interest differential.[18]

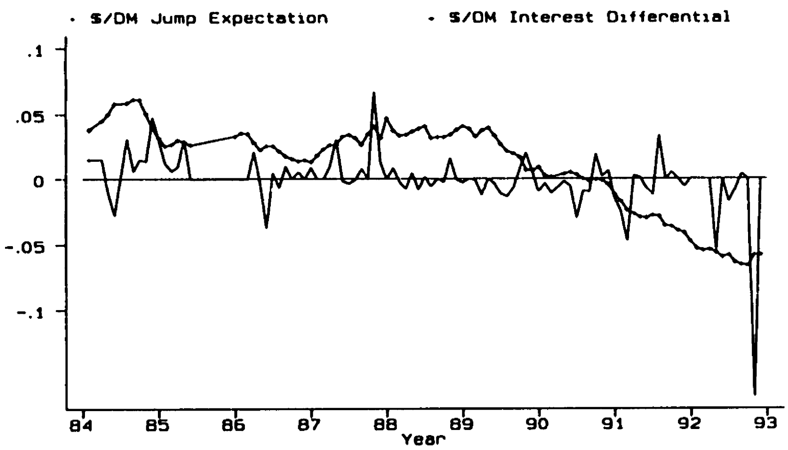

Figure 1-12: $\lambda\kappa$ for the DM & U.S.-German Interest Differential

The broad picture presented by the $/DM option-implied jump expectations is one of expected depreciation due to jumps during 1984–1989 and expected appreciation thereafter. As Figure 1-12 shows, this pattern of expected jump depreciation tracks the behavior of the U.S.-German one month Eurocurrency interest differential. As Table 1.3 reports, a linear regression of the jump expectation on the interest differential yields a highly significant slope coefficient of 0.22. This positive relationship between jump depreciation and the interest differential is robust to exclusion of the months corresponding to the 1987 stock market crash and the 1992 ERM crisis. Even excluding these months, the slope coefficient is 0.115 and significant, indicating that when the interest differential widens by one hundred basis points, jump fears widen by about 11.5 basis points.

Figure 1-13 displays the time series of the yen jump expectation together with the U.S.-Japan one month interest differential. The $/Yen rate presents a stark contrast to the case of the $/DM rate. The jump differential is not at all correlated with the

for a "peso problem" explanation of the forward discount bias.

[18]The work of Frankel and Froot [13] shows that survey expectations of dollar depreciation vary closely with the interest differential. However, the survey expectations do not allow for a decomposition of expected depreciation into a "jump" and "non-jump" component.

Table 1.3: Regressions of Jump Expectation on Interest Differential

	Constant	Slope	T
DM	-0.0028 (-1.225)	0.2218 (3.449)	99
DM •	-0.0009 (-0.557)	0.1150 (2.674)	97
Yen	0.0025 (0.494)	0.1627 (0.945)	91
Yen ••	0.0081 (2.235)	0.0071 (0.057)	90

OLS regression of option-implied jump expectation on the 30-day Eurocurrency interest differential. Asymptotic t-statistics in parentheses. Run on a sample of T months during 1984-92, excluding June-November 1985 and months with thin trading in options. (•) indicates regression run excluding the October 1987 Crash and September 1992 ERM crisis. (••) indicates regression run excluding the June 1992 outlier in the $/Yen jump expectation.

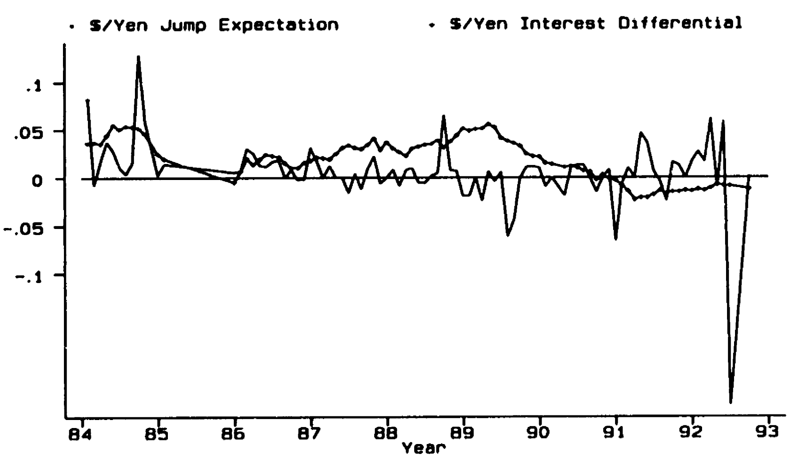

Figure 1-13: $\lambda\kappa$ for the Yen & U.S.–Japan Interest Differential

interest differential. A regression of the jump expectation on the interest differential yields a slope coefficient which is not significantly different from zero. Thus, the statement that crashes are feared in the high interest rate currency does not hold universally, although it may hold for some currencies such as the Deutschemark.

1.5.2 Jump Expectations: Regressive or Bandwagon?

Floating exchange rates tend to spend long periods of time away from "target" or "fundamental" levels, such as those indicated by purchasing power parity. Using survey data on exchange rate expectations, Frankel and Froot [13] suggest that expectations have a *regressive* component in the sense that, when the current nominal exchange rate is away from its target, an eventual return to the target level is expected. [19] Another possibility is that of "bandwagon" expectations, in which market participants extrapolate the most recent trend: large depreciations of the exchange rate in the previous months lead to expectations of further depreciation. It is interesting to consider whether option-implied jump expectations are regressive or bandwagon. To consider the matter of regressivity, two "target" levels of the exchange rate are defined. The first is a purchasing power parity (PPP) level. This PPP level for the \$/DM and \$/Yen exchange rates was constructed by (i) setting the parity level of the exchange rate equal to the nominal rate at the time of the Louvre Accord in February of 1987, when the G-7 agreed that exchange rates should be stabilized "at or near their present levels," and (ii) updating the PPP level by the change in the appropriate bilateral Consumer Price Index ratio.[20] The second target level considered is the one reported by Funabashi [15] in his study of G-7 exchange rate policy. There are no publicly announced targets for the dollar exchange rate. Nonetheless, during 1985-89,

[19]See Takagi [27] for a survey of the large and growing literature on exchange rate survey expectations and their formation. See also Ito [18] for evidence from "micro" survey data on yen exchange rate expectations.

[20]Precisely, PPP is the nominal \$/DM exchange rate $\overline{S_t}$ which solves $\frac{S_t P^*}{P} = k$, where P and $P*$ are the U.S. and German price levels and k is some constant. Once k has been chosen, changes in the PPP level are governed solely by changes in the ratio $\frac{P}{P^*}$. CPI's were used to calculate this ratio for the results reported, but the general results in this paper are robust to the use of other price indexes.

the G-7 had certain target levels in mind and intervened in the FX market when the dollar drifted from these levels. See Frankel and Dominguez [10] for evidence that deviations of the exchange rate from the implicit targets have power in explaining intervention activity. The 1985-87 Funabashi target for the DM/$ is 2.6, agreed on by the G-5 in the Plaza Accord in September of 1985. From February 1987 onward, the target is 1.825 DM to the dollar, as set in the Louvre Accord, with the exception of the brief period from October 1987 to June 1988, when dollar weakness following the U.S. stock market crash necessitated a revision of the level to 1.7 DM/$. For the Yen/$, the 1985-87 target is the 200 Yen level deemed acceptable to Japan at the time of the Plaza Accord, while from 1987 onwards it is the 153.5 Yen level agreed on in the Louvre Accord. Figures 1-14 and 1-15 show the nominal exchange rates, PPP targets, and Funabashi targets for both the $/DM and $/Yen exchange rates. The figures indicate considerable overvaluation of the dollar during the early part of the 1984-1992 period and fluctuations around the targets thereafter.

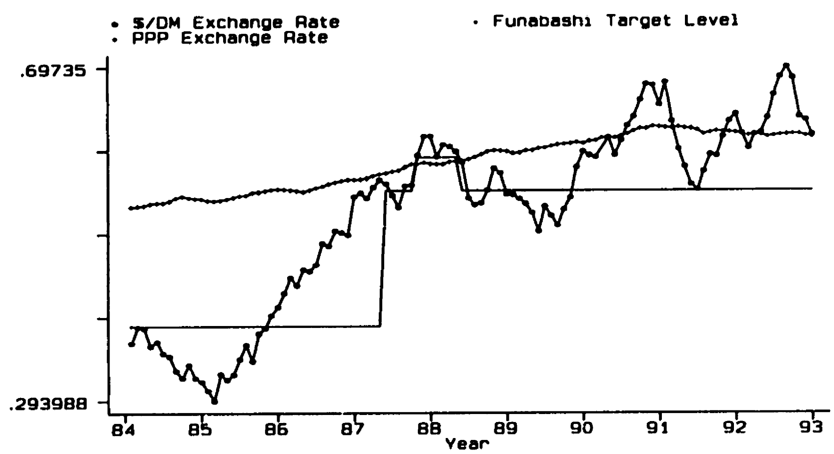

Figure 1-14: $/DM Nominal, PPP Target, and Funabashi Target Exchange Rates

The first two rows of each of Tables 1.4 and 1.5 report the results of regressing the option-implied jump expectation on each of the two measures of deviation from target levels: $PPPDEV_t = log(\frac{S_t}{S_t^{ppp}})$ and $FDEV_t = log(\frac{S_t}{S_t^{Fun}})$, where S_t^{ppp} and S_t^{Fun} are the PPP and Funabashi target levels. For both currencies and for both measures of the target level, the slope coefficient is negative and significant at the 95% level. This indicates, quite plausibly, that the stronger is the dollar relative to the target level,

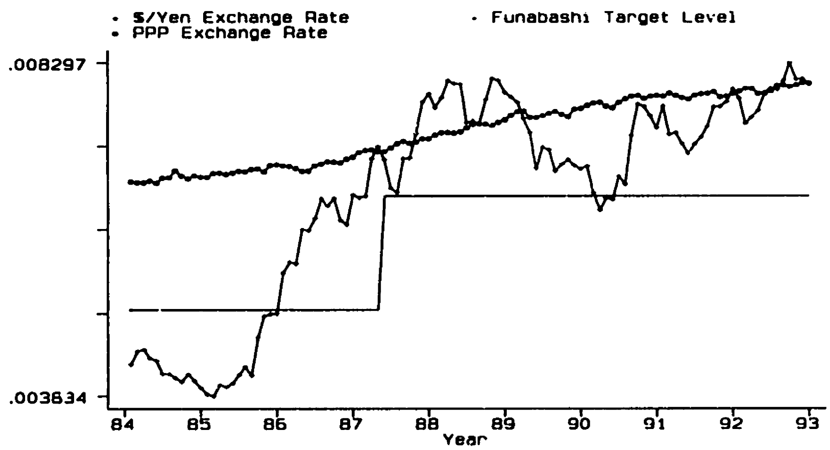

Figure 1-15: $/JY Nominal, PPP Target, and Funabashi Target Exchange Rates

the greater is the expectation of a dollar jump depreciation. Thus, participants in the options market seem to have in mind a model of the world in which the exchange rate may drift away from fundamental levels, but when it does so there is an increased expectation of a sudden jump back in the direction of the fundamental.

The possibility of bandwagon expectations was also considered. It may be that expectations of jump depreciations in the coming month are caused by nervousness over large depreciations in the previous month. Tables 1.4 and 1.5 report the results of regressions of the jump expectations on ΔS_{t-1}, the exchange rate depreciation in the previous month. For the DM, the coefficient is positive, indicating that depreciation in the previous month leads to expectations of further depreciation, although the effect is not statistically significant. For the Yen, the coefficient is negative, though not statistically significant. The results of hybrid regressions which include both regressive and bandwagon terms are also reported. In these hybrid regressions, the regressive term retains its significant and negative effect for both currencies. For the DM, the ΔS_{t-1} has a positive and nearly significant coefficient, which may be taken as weak evidence of bandwagon effects. For the Yen, the bandwagon term is negative and insignificant in the hybrid regressions.

The bottom halves of Tables 1.4 and 1.5 repeat all of the foregoing regressions, excluding the outliers associated with the 1987 stock market crash, the 1992 ERM crisis, and the large outlier in the $/Yen jump expectation in June 1992. The results

Table 1.4: Regression of Jump Expectations on Regressive and Bandwagon Terms

	Constant	PPPDEV	FDEV	ΔS_{t-1}	T
DM	-0.0047 (-1.758)	-0.0359 (-2.768)**			99
	0.0014 (0.561)		-0.0345 (-2.006)**		99
	-0.0011 (-0.478)			0.0672 (0.998)	98
	-0.0055 (-2.001)**	-0.0386 (-2.908)**		0.0988 (1.497)	98
	0.0014 (0.536)		-0.0423 (-2.357)**	0.1137 (1.656)	98
DM •	-0.0058 (-2.263)**	-0.03903 (-3.163)**			97
	0.0018 (1.095)		-0.0243 (-2.213)**		97
	-0.0000 (-0.011)			0.0364 (0.846)	96
	-0.0038 (-2.221)**	-0.0328 (-3.987)**		0.0641 (1.579)	96
	0.0017 (1.020)		-0.0288 (-2.511)**	0.0687 (1.568)	96

(•) indicates regressions run excluding the October 1987 Crash and September 1992 ERM crisis. Asymptotic t-statistics in parentheses. OLS regression of option-implied jump expectation $PPPDEV_t = log\left(\frac{S_t}{S_t^{PPP}}\right)$, where S_t is the current nominal exchange rate in dollars per unit of foreign currency, and S_t^{PPP} is the current "equilibrium" exchange rate indicated by purchasing power parity. Analogously, $FDEV_t$ is the month t percentage deviation of the nominal exchange rate from the politically determined "target" levels reported by Funabashi [15]. ΔS_{t-1} is the percentage change in the nominal exchange rate in the preceding month. Run on a sample of T months during 1984-92, excluding June-November 1985 and months with thin trading in options. (*), (**) indicate significance at 90% and 95% levels.

Table 1.5: Regression of Jump Expectations on Regressive and Bandwagon Terms

	Constant	PPPDEV	FDEV	ΔS_{t-1}	T
Yen	-0.0014 (-0.330)	-0.0650 (-2.925)**			91
	0.0120 (2.567)**		-0.0633 (-2.262)**		91
	0.0050 (1.324)			-0.0667 (-0.585)	90
	-0.0010 (-0.226)	-0.0561 (-2.443)**		-0.0247 (-0.220)	90
	0.0103 (2 149)**		-0.0521 (-1.771)	-0.0133 (-0.114)	90
Yen ••	0.0024 (0.767)	-0.0541 (-3.481)**			90
	0.0132 (3.975)**		-0.0488 (-2.460)**		90
	0.0074 (2.800)**			-0.0024 (-0.030)	89
	0.0024 (0.771)	-0.0468 (-2.967)**		0.0315 (0.408)	89
	0.0115 (3.473)**		-0.0407 (-1.996)**	0.0386 (0.476)	89

(••) indicates regressions run excluding the June 1992 outlier in the $/Yen jump expectation. (*), (**) indicate significance at 90% and 95% levels.

on regressive and bandwagon effects are robust to these exclusions.

1.5.3 Foreign Exchange Intervention and the Twin Deficits

There is a literature which suggests that government intervention in foreign exchange markets may affect expectations of future exchange rates by "signalling" information about future monetary policy. For example, it may be the case that current interventions in support of the dollar credibly indicate future tightening of monetary policy. If so, current intervention may lead to expectations of dollar appreciation even if the intervention is small and/or sterilized by sales of domestic bonds. Kaminsky and Lewis [27] and Ghosh [22] give some evidence that interventions signal future monetary policy. Using survey data on exchange rate expectations, Dominguez and Frankel [11] show that current interventions in support of the dollar are associated with expected dollar appreciation. In this section we examine whether intervention affects option-implied jump expectations in the same way. More generally, if jumps in the exchange rate are caused by shifts in fiscal, monetary, or exchange rate policy, then variables which help predict such shifts should affect current jump expectations.[21] This leads us to also consider the effect of the so-called "twin deficits:" the government budget and the trade balance. Market participants are likely to use information about the twin deficits in forming expectations about the exchange rate. Large U.S. deficits may signal the need for an eventual large depreciation of the dollar.

Table 1.6 reports the results of regressing jump expectations on intervention and the twin deficits. The variable INT measures intervention by the Federal Reserve Board during the month previous to the day that the jump expectation is formed. INT is in terms of millions of dollars of either DM or Yen *bought* in support of the dollar. TRADE is the appropriate bilateral trade surplus (exports minus imports) during the previous month, while GDEF is the government budget surplus. Both surpluses are measured in billions of dollars.

Somewhat surprisingly, the intervention and twin deficit measures seem to have

[21] See Borenzstein [1], who shows that trade surpluses help explain "jump expectations" in the 1980-84 dollar as measured by the excess foreign exchange return $\Delta S_t - (r_t - r_t^*)$.

Table 1.6: Jumps, Intervention and the Twin Deficits

	Constant	INT	TRADE	GDEF	PPPDEV	ΔS_{t-1}	T
DM	-0.0010 (-0.419)	-0.0000015 (-0.347)					97
	-0.0043 (-0.686)		-0.000003 (-0.839)				98
	-0.0012 (-0.403)			-0.00003 (-0.263)			98
	-0.0054 (-0.780)		-0.000005 (-0.674)	-0.00004 (-0.385)			98
	-0.0091 (-1.306)	-0.000002 (-0.438)	-0.000003 (-0.434)	-0.00005 (-0.500)	-0.0379 (-2.780)**	0.1036 (1.456)	97
Yen	0.0047 (1.288)	-0.0000174 (-0.003)					90
	0.0212 (1.111)		0.000004 (0.837)				91
	0.0047 (0.967)			-0.00005 (-0.282)			91
	0.0204 (1.006)		0.000004 (0.797)	-0.00002 (-0.145)			91
	-0.0162 (-0.697)	0.000057 (0.081)	-0.00003 (-0.653)	-0.000036 (-0.216)	-0.0641 (-2.441)**	-0.0163 (-0.131)	90

Asymptotic t-statistics in parentheses. OLS regression of jump expectation on various measures from previous month. INT is Federal Reserve Board data on intervention in previous month, measured as millions of dollars worth of foreign currency (DM or Yen) bought. TRADE is U.S. bilateral trade surplus with Germany or Japan, billions of dollars, and monthly U.S. government surplus, billions of dollars. PPPDEV is percent deviation of current exchange rate from its PPP level, and ΔS_{t-1} the exchange depreciation during the previous month. Run on a sample of T months during 1984-92, excluding June-November 1985 and months with thin trading in options. (**) denotes significance at 95% level. The results in this table are robust to the exclusion of the 1987 crash, 1992 ERM crisis, and the June 1992 outlier in the $/Yen jump expectations.

little power in explaining jump expectations. This is true in simple and multiple regressions and for both the \$/DM and \$/Yen exchange rates. The coefficients are generally negative, which accords with intuition: dollar purchases, U.S. trade surpluses, and U.S. government surpluses should be associated with expected jump *appreciation* of the dollar. However, the coefficients are all insignificantly different from zero. This remains true when intervention and the twin deficits are included in the hybrid expectations formation regression of section 1.5.2 above. As before, the single variable most strongly associated with jump expectations is the deviation of the exchange rate from its fundamental level. There is again weak evidence of bandwagon effects for the \$/DM but not for the \$/Yen.[22] It must be concluded that, while intervention and deficits may affect expectations of the exchange rate, as reported by Dominguez and Frankel [11], options traders do not seem to believe that such intervention affects the probability or likely size of a *jump* movement in the dollar. This does not rule out the possibility that intervention affects smaller "drift" movements in the exchange rate.[23]

1.6 Are Option-Implied Jitters Justified?

Having examined option-implied jitters and the information the market uses in forming them, we now consider whether such jitters are justified *ex post*. Do option-implied jump expectations help predict jumps? Does option-implied volatility provide an unbiased forecast of actual future volatility in the exchange rate? This latter question was considered by Wei and Frankel [29] for the simple Black-Scholes diffusion case with no jumps in the underlying exchange rate. In this section, their methodology is used to study the jump-diffusion case.

First consider the option-implied forecast of the total volatility $\nu = (\sigma^2 + \lambda\gamma^2)^{\frac{1}{2}}$ which is due to both jumps and diffusion movements. The option-implied estimate $\hat{\nu}$ is extracted from options trading on a given day at the beginning of the month. The

[22]The results in this section are all robust to exclusion of the 1987 crash, 1992 ERM crisis, and the June 1992 outlier in the \$/Yen jump expectations.

[23]See chapter 3 of this dissertation for evidence that, at daily and weekly frequency, intervention may have effects on drift movements but not jumps.

realized volatility ν^s is calculated by taking daily exchange returns over the coming month and calculating their sample standard deviation.[24] Rationality of the option-implied forecast can be tested by running the following regression of realized volatility on the option-implied volatility:

$$\nu_t^s = a + b\hat{\nu}_t + \eta_t \tag{1.11}$$

Under the null hypothesis of rationality of the option-implied forecast, the coefficient b is equal to one. A strict interpretation of "rationality" would also include $a = 0$ in the null hypothesis, but we allow for the possibility that the forecast is biased by a constant term. The results of this regression, using both regular asymptotic standard errors as well as White heteroskedasticity-robust standard errors, are reported in Table 1.7. For the \$/DM exchange rate using the full sample, the hypothesis of unbiasedness ($b = 1$) can be rejected with 95% confidence regardless of whether regular or White standard errors are used. The coefficient a is also significantly different from zero. However, this rejection of unbiasedness is driven by a single outlier in the data: the large 37% total volatility forecast in December of 1989 which can be seen in Figure 1-5. The third and fourth rows of Table 1.7 repeat the \$/DM unbiasedness regressions excluding this single data point. The results now indicate that the hypothesis of unbiasedness *cannot* be rejected, as b is insignificantly different from one. It is also the case that the intercept a is insignificant from zero at the 95% level. We may take this as evidence that, for the \$/DM rate, the option-implied total volatility $\hat{\nu}$ provides a rational forecast of *ex post* volatility over the coming month. The last rows of Table 1.7 report the results of unbiasedness regressions for the \$/Yen rate. For the Yen, we can reject the null hypothesis of $b = 1$ with 95% confidence, although the forecasts are still valuable in the sense that b is significantly different from zero. Hence, the option implied ν for the \$/Yen rate is valuable in forecasting the direction if not the precise magnitude of the *ex post* volatility. The point estimate b is less than 1, which indicates that the option-implied total volatility is excessively

[24]The daily data are business daily from the Bank for International Settlements.

variable as a forecast of realized volatility. This is in accord with the results of Wei and Frankel [29] for the pure diffusion (no jumps) case.

The foregoing unbiasedness regressions provide a quite stringent test of the rationality of option-implied forecasts. A looser criterion is the Henriksson and Merton [17] sign test. This test can be used to determine whether the forecast change in volatility is in the *right direction*, regardless of its magnitude. Define the option-implied forecast of the change in volatility as $\Delta\hat{\nu}_t = \hat{\nu}_{t+1} - \nu_t^s$, and the actual *ex post* change in volatility as $\Delta\nu_t^s = \nu_{t+1}^s - \nu_t^s$, where again $\hat{\nu}_t$ is the option-implied volatility extracted from options prices at the *beginning* of month t and ν_t^s is the sample volatility calculated using daily returns *during* month t. Define the probabilities

$$p_1(t) = Prob[\Delta\hat{\nu}_t > 0 | \Delta\nu_t^s > 0] \tag{1.12}$$

$$p_2(t) = Prob[\Delta\hat{\nu}_t \leq 0 | \Delta\nu_t^s \leq 0] \tag{1.13}$$

These give the probabilities of the option-implied forecast being correct, given the direction of the *ex post* volatility change. Forecasts which are always wrong will be such that $p_1 + p_2 = 0$, while forecasts which are always correct will have $p_1 + p_2 = 2$.[25] We would like to examine the null hypothesis of $p_1 + p_2 = 1$, which represents a situation in which the option-implied total volatility has *no value* in forecasting the direction of the change in *ex post* volatility.[26] Define the following quantities: N_1 is the number of observations such that $\Delta\nu_t^s \leq 0$, and N_2 is the number of observations such that $\Delta\nu_t^s > 0$. Then $N = N_1 + N_2$ is the total number of observations. Let n_1 be the number of forecasts which are correct, given that $\Delta\nu_t^s \leq 0$, while n_2 is the number of incorrect forecasts given that $\Delta\nu_t^s > 0$. Hence, $n = n_1 + n_2$ is the total number of times in the sample that $\Delta\nu_t^s$ is forecast to be less than or equal to zero. It can be shown that, under the null hypothesis of valueless forecasts, n_1 will be distributed as

[25]As Henrikkson and Merton point out, forecasts which are systematically incorrect in the sense of $p_1 + p_2 < 1$ have value if one simply reverses their direction. The pundit who is always wrong is worth as much as the one who is always correct.

[26]An example of a valueless forecast is the random walk, which always forecasts the current volatility as the next month's volatility. For the random walk forecast, $p_1 = 0$, $p_2 = 1$, and $p_1 + p_2 = 1$.

a hypergeometric independent of p_1 and p_2. Hence, n_1 can be used as the statistic in a test of the null hypothesis. Moreover, a normal approximation to the distribution of n_1 is appropriate, with mean and variance given by

$$E(n_1) = \frac{nN_1}{N} \tag{1.14}$$

$$Var(n_1) = \frac{n_1 N_1 (N - N_1)(N - n)}{N^2(N-1)} \tag{1.15}$$

Table 1.8 reports various statistics of this test as applied to the \$/DM and \$/Yen option-implied forecasts. For the DM, the option-implied $\Delta\nu_t^s$ correctly forecast a fall in volatility 38 of 49 times, while it correctly forecast a rise in volatility 34 of 49 times. The final column reports the t-statistic in the test of the null hypothesis the option-implied total volatility has no value in explaining the direction of change in *ex post* volatility. For both currencies, we can strongly reject the null, indicating that the option-implied total volatility is valuable in forecasting the direction of change of volatility in the subsequent month.

Using option-implied parameters to predict the jump component of the exchange rate is problematic in several respects. First, although the jump expectation estimate $\hat{\lambda}\hat{\kappa}$ is well-identified and of an economically reasonable order of magnitude, the individual estimates of λ and κ are not. Hence, although it would be desirable to have estimates of the individual parameters, we are confined to using information about the size and sign of the jump expectation in predicting *ex post* jumps. Second, in evaluating the predictive power of the jump expectations, we must of course identify which portion of the *ex post* monthly return is due to "jump" movements and which is due to diffusion movements. The following *ad hoc* but intuitively plausible approach to identifying jumps is adopted.[27] The option-implied jump expectation is extracted using option trading on a given day at the beginning of the month. The realized daily exchange rate returns during that month are used to estimate the volatility $\hat{\sigma}$ and drift $\hat{\mu}$ of the process for the month. A daily return is deemed to have a jump

[27]See chapter 3 of this dissertation for a more formal approach to jump identification which uses maximum likelihood estimates of the parameters of the jump-diffusion process.

Table 1.7: Rationality of Option-Implied Total Volatility ν

	a	b	T
DM	0.0766 (4.4264)	0.4843 (3.816)** [-4.064]	98
DM (White)	0.0766 (2.405)**	0.4843 (2.865)** [-2.5606]**	98
DM •	0.0350 (1.570)	0.8013 (4.939)** [-0.361]	97
DM • (White)	0.0350 (1.498)	0.8013 (4.451)** [-1.1039]	97
Yen	0.0559 (3.252)**	0.5315 (3.701)** [-3.263]**	90
Yen (White)	0.0559 (2.832)**	0.5315 (3.125)** [-2.7540]**	90

Brackets [ ] hold asymptotic t-statistics in the test of the hypothesis that b is different from one. Parentheses ( ) hold asymptotic t-statistics in test of the hypothesis that either a or b is different from zero. (•) indicates regressions run excluding the December 1989 outlier in the \$/DM option-implied total volatility. (*), (**) indicate significance at 90% and 95% levels. Rows labeled (White) calculate standard errors using the heteroskedasticity-robust method of White.

Table 1.8: Henriksson-Merton Test of $\Delta\hat{\nu}$ Forecast

	$\#(\Delta\nu_t^s \leq 0)$	# Correct	$\#(\Delta\nu_t^s > 0)$	# Correct	t-Stat
DM	49	38	49	34	5.478
Yen	54	36	36	28	5.279

Final column is the t-statistic in the test of the null hypothesis that the option-implied forecasts are valueless ($p_1 + p_2 = 1$). T=98 total observations for the \$/DM, T = 90 for the \$/Yen.

component if

$$|\Delta S_t^{daily} - (\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)| > f * \hat{\sigma} \tag{1.16}$$

where f, the "filter level," is a number greater than or equal to zero. This is akin to a classical hypothesis test of whether a particular one day move came from a pure diffusion process or not. A detrended one day movement more than f standard deviations away from zero is deemed to have a jump component. The movements $\Delta S_t^{daily} - (\hat{\mu} - \frac{1}{2}\hat{\sigma}^2)$ that were determined to be "jumps" are summed to obtain the jump component of the *monthly* return.

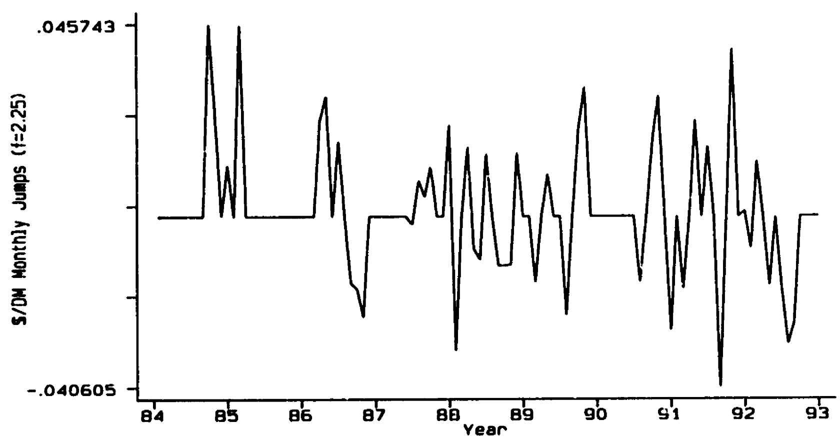

Figure 1-16: \$/DM Monthly Jump Movement (f = 2.25)

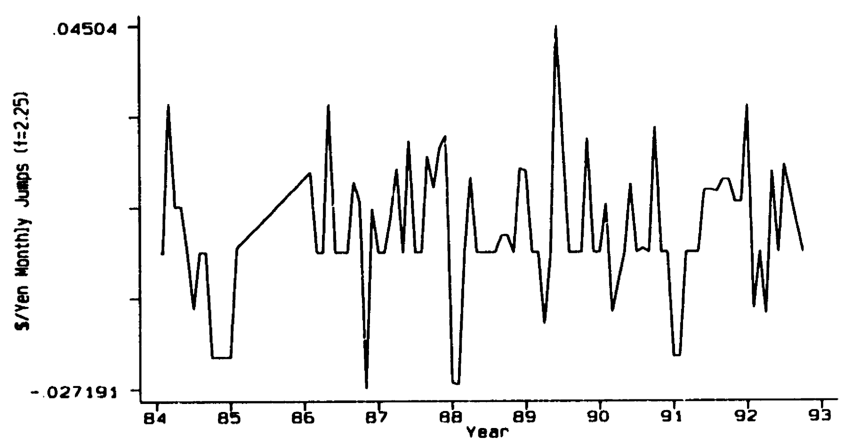

Figure 1-17: \$/Yen Monthly Jump Movement (f = 2.25)

Figures 1-16 and 1-17 show the time series of monthly jump movements at filter level f = 2.25. Tables 1.9 and 1.10 report the results of regressing the monthly *ex post* jump movements on the option-implied jump expectation. Several values of the filter level f were considered. For the \$/DM rate, the slope coefficients at each of the filter levels are positive, indicating that expected jump depreciation and and subsequent jump depreciation are positively related. However, the slope coefficients are not significantly different from zero at the 95% level. The lower half of Table 1.9 reports the results of the regressions when the two data points corresponding to the '87 market crash and '92 ERM crisis are excluded. This exclusion renders the slope coefficients larger and more significant. The slope coefficient corresponding to a filter level of 2.25 is significant at the 95% level. The coefficients for higher filter levels are not as significant. Perhaps it is the case that filter levels greater than 2.25 are too "stingy," and do not identify as jumps some daily returns which are in fact jumps and which are predicted by the option-implied jump expectations. If $f = 2.25$ is the correct filter in the sense that daily detrended movements which are 2.25 standard deviations from zero are truly "jumps," we may conclude that option-implied jump expectations have some power in predicting jumps during the subsequent months.

For the \$/Yen rate, there is less evidence in favor of the hypothesis that jump expectations help predict jumps. At some filter levels it is the case that the slope coefficient is negative, indicating a negative relationship between the expected jump depreciation and subsequent jump depreciation. For $f = 2.75$, the negative coefficient is even significant. Excluding two large outliers in the jump expectation for September 1984 and June 1992 renders the slope coefficients insignificant for all filter levels, though some of the slope coefficients are still negative. It must be concluded that, for the \$/Yen rate, there is little evidence that jump expectations can predict subsequent jumps in the exchange rate.

1.7 Conclusion

This paper has examined option-implied jump expectations and volatility expectations for the dollar exchange rate during 1984-1992. Option-implied expectations of jumps in the dollar exchange rate have been economically significant at various

Table 1.9: Regression of Monthly Jump Movement on Jump Expectation $\lambda\kappa$

DM	Intercept	Slope	T
$f = 0$	0.0033	0.3537	98
	(0.937)	(2.354)*	
$f = 2.0$	-0.0007	0.0521	98
	(-0.422)	(0.712)	
$f = 2.25$	0.0012	0.1007	98
	(0.792)	(1.579)	
$f = 2.5$	0.0003	0.0411	98
	(0.244)	(0.808)	
$f = 2.75$	-0.0005	0.0421	98
	(-0.480)	(0.937)	
DM •	Intercept	Slope	T
$f = 0$	0.0031	0.1193	96
	(0.878)	(0.501)	
$f = 2.0$	-0.0008	0.1339	96
	(-0.469)	(1.150)	
$f = 2.25$	0.0011	0.2467	96
	(0.709)	(2.466)*	
$f = 2.5$	0.0002	0.1015	96
	(0.195)	(1.256)	
$f = 2.75$	-0.0006	0.1077	96
	(-0.542)	(1.513)	

Asymptotic t-statistics in parentheses. (•) indicates regressions run excluding months of the 1987 stock market crash and 1992 ERM crisis. (*) indicates significance at 95% level.

Table 1.10: Regression of Monthly Jump Movement on Jump Expectation $\lambda\kappa$

Yen	Intercept	Slope	T
$f = 0$	0.0048	0.0629	90
	(1.313)	(0.621)	
$f = 2.0$	-0.0007	0.0521	90
	(-0.422)	(0.712)	
$f = 2.25$	0.0054	0.1007	90
	(3.671)*	(-1.510)	
$f = 2.5$	0.0029	-0.0677	90
	(2.258)*	(-1.909)	
$f = 2.75$	0.0027	-0.0817	90
	(2.421)*	(-2.637)*	
Yen ••	Intercept	Slope	T
$f = 0$	0.0037	0.2495	88
	(0.982)	(1.446)	
$f = 2.0$	0.0053	-0.0356	88
	(3.432)*	(-0.540)	
$f = 2.25$	0.0044	-0.0276	88
	(2.838)	(-0.422)	
$f = 2.5$	0.0002	0.1015	88
	(0.195)	(1.256)	
$f = 2.75$	0.0028	-0.0193	88
	(2.040)*	(-0.335)	

Asymptotic t-statistics in parentheses. (••) indicates regressions run excluding the June 1992 and September 1984 outliers in the $/Yen option-implied jump expectations. (*) indicates significance at 95% level.

times during the 1984-1992 period, and seem to agree with the qualitative stories told about appreciation and depreciation fears of the market during that period. Jump expectations seem to be quite mercurial, with little persistence from month to month. Expected diffusion volatilities are relatively more persistent, with this month's expected volatility depending strongly on that in the previous two months. A comparison of the \$/DM and \$/Yen rates shows that diffusion volatilities for the two currencies track quite closely. Jump expectations also have a positive relationship across currencies, although this relationship is not as significant statistically as for the diffusion volatility. For both jump expectations and diffusion volatility, the \$/Yen series are more variable than the corresponding \$/DM series.

On examining the relationship of jump expectations to various measures in the information set of traders, it was found that for the \$/DM, but not the \$/Yen, jump depreciation expectations are strongly correlated with the U.S.-German interest differential. This suggests that, at least for the DM, the returns to a strategy of "buying the rich currency and selling the poor," are perceived as being in compensation for the possibility of a large dollar depreciation. Both the \$/DM and \$/Yen jump expectations are strongly related to the distance of the current nominal exchange rate from fundamental "target" levels. This suggests option traders have a view of the world in mind such that the exchange rate drifts away from fundamentals for long periods of time, but is expected to return to the fundamental level with a sudden "jump." For the \$/DM rate, there is also weak evidence of bandwagon expectations, with large depreciations in one month leading to expected jump depreciation in the following month. There is no evidence of bandwagon effects for the Yen. For both the DM and the Yen, jump expectations are unrelated to classic measures of overvaluation like the government budget deficit and the trade deficit. Nor is it the case that current U.S. foreign exchange intervention affects jump expectations.

Finally, option-implied predictions of *ex post* total volatility in the exchange rate are shown to unbiased for the \$/DM rate. They are biased for the \$/Yen, but still carry information in the sense of being able to predict the *direction* of change in total volatility. Option-implied jump expectations are shown to have some power in predicting subsequent jumps in the \$/DM exchange rate, but not the \$/Yen rate.

In general, the \$/Yen option-implied expectations are more difficult to explain than are those for the \$/DM exchange rate in the sense that they have weaker relationships to variables in the *ex ante* information set of traders. The Yen expectations are also more variable than those for the DM, and have less power in explaining subsequent movements in the exchange rate. This may indicate some deep differences in how traders explain the behavior of the two currencies. However, it may also be due to the fact that the PHLX Yen options are more thinly traded than are those for the DM. Low volume could mean that the market in Yen options processes information less efficiently, and trades take place at prices far away from their "true" values. This would of course cause the option-implied \$/Yen expectations to be "noisier" than those for the \$/DM exchange rate.

Bibliography

[1] E. R. Borenzstein "Alternative Hypotheses About the Excess Return on Dollar Assets, 1980-84," IMF Staff Papers 1986, pp. 29-59.

[2] V. Akgiray and G.G. Booth, "Mixed Diffusion–Jump Process Modeling of Exchange Rate Movements," *Review of Economics and Statistics* **70**, November 1988, 638–647.

[3] D. Backus, A. Gregory, and C. Telmer, "Accounting for Forward Rates in Markets for Foreign Currency," *Journal of Finance*, **48**, December 1993, 1887–1908.

[4] Barone–Adesi, Giovanni and Whaley, Robert E., "Efficient Analytic Approximation of American Option Values," *Journal of Finance*, **42**, June 1987, 301–320.

[5] D. Bates, "The Crash of 87: Was It Expected? The Evidence from Option Markets," *Journal of Finance*, July 1991, 1009–1044.

[6] D. Bates, "Crashes, Options, and International Asset Substitutability," Princeton University Ph.D. dissertation, January 1989.

[7] D. Bates, "The Crash Premium: Option Pricing Under Asymmetric Processes, with Applications to Options on Deutschemark Futures," Working Paper 36–88, University of Pennsylvania, Rodney L. White Center, 1988.

[8] D. Bates, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in PHLX Deutschemark Options," National Bureau of Economic Research Working Paper No. 4596, December 1993.

[9] K.S. Choie "Currency Exchange Rate Forecast and the Interest Rate Differential," *Journal of Portfolio Analysis and Management*, Winter 1993, 58-64.

[10] K.M. Dominguez and J.A. Frankel, *Does Foreign Exchange Intervention Work?* Washington: Institute for International Economics, 1993.

[11] K.M. Dominguez and J.A. Frankel, "Foreign Exchange Intervention: An Empirical Assessment," in J. Frankel, ed., *On Exchange Rates*, Cambridge: MIT Press 1993.

[12] R. Dornbusch, "Why the Mighty Mark is a Sitting Duck," *Business Week* **30**, March 7, 1994.

[13] J. Frankel and K. Froot, "Using Survey Data to Test Standard Propositions Regarding Exchange Rate Expectations," *American Economic Review*, March 1987, 133-153.

[14] J. Frankel and K. Froot, "The Forward Discount Bias: Is it an Exchange Risk Premium?" *Quarterly Journal of Economics*, February 1989, 139-161.

[15] Y. Funabashi, *Managing the Dollar: From the Plaza to the Louvre* Washington: Institute for International Economics, 1989.

[16] A.R. Ghosh "Is It Signalling? Exchange Intervention and the Dollar-DM Rate," *Journal of International Economics* **32**, 1992, 201-220.

[17] R. Henriksson and R.C. Merton, "On Market timing and Investment Performance. II. Statistical Procedures for Evaluating Forecasting Skills," *Journal of Business* **54**, 513-534.

[18] T. Ito, "Foreign Exchange Rate Expectations: Micro Survey Data," *American Economic Review* **80**, June 1990, 434-449.

[19] P. Jorion, "On Jump Processes in the Foreign Exchange and Stock Markets," *Review of Financial Studies* **1**, 1988, 427–445.

[20] E.P. Jones, "Options Arbitrage and Strategy with Large Price Changes," *Journal of Financial Economics*, **13**, 1984, 91–113.

[21] G. Kaminsky and K.K. Lewis, "Does Foreign Exchange Intervention Signal Future Monetary Policy?" Finance and Economics Discussion Series No. 93-1. Washington: Federal Reserve board, 1993.

[22] P. Krugman, "Is the Strong Dollar Sustainable?" in *The U.S. Dollar Recent Developments, Outlook, and Policy Options*, Kansas City: Federal Reserve Bank of Kansas City, 1985.

[23] K.K. Lewis "Puzzles in International Financial Markets," forthcoming in *Handbook of International Economics*, 1195.

[24] MacMillan, L.W., "Analytic Approximation for the American Put," *Advances in Futures and Options Research*, **1**, 1986, 119–139.

[25] S. Marris, "Deficits and the Dollar: The World Economy at Risk," *Policy Analyses in International Economics* **8**, Washington: Institute for International Economics.

[26] R. C. Merton, "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, **3**, 125–144.

[27] S. Takagi, "Exchange Rate Expectations: A Survey of Survey Studies," *IMF Staff Papers*, March 1991, 156-183.

[28] A.L. Tucker and L. Pond, "The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes," *Review of Economics and Statistics* **70**, November 1988, 638–647.

[29] S. Wei and J. Frankel, "Are the Forecasts of Exchange Rate Volatility Implicit in Options Prices Excessively Variable?" NBER Working Paper 3910, November 1991.

Chapter 2

A Peso Problem Resistant Test of Foreign Exchange Pricing Which Uses Option-Implied Jump Expectations (joint with W.T. Baily)

2.1 Introduction

Asset pricing theories typically give testable relationships between an asset's *expected* return and some set of explanatory variables. For example, in many models of exchange rate determination the expected rate of depreciation of a currency is related to the current interest rate differential between the domestic and foreign countries, or, equivalently, to the current forward discount. Because expectations of an asset's return are generally not observable, empirical tests use *ex post* returns and assume "rational expectations": that is, that the *ex post* return is equal to its expectation plus a conditionally mean zero error term. This practice is susceptible to the "peso problem" critique. When markets expect a large price movement, or a jump, but only with low probability, it will be difficult to obtain a "typical" sample. A very large sample may be needed before enough jumps occur to make the sample average representative of the true expectation. In the extreme case when no jumps are observed in the sample, *ex post* returns will clearly be unsuitable proxies for expected returns.

The problem is quite general, and could arise in empirical tests of stock pricing or the interest rate term structure.[1] However, because exchange rates seem to be particularly susceptible to occasional large jumps, peso problems are most likely to arise in the context of international asset pricing models. The term "peso problem" has its roots in a puzzle of the early 1970's, when the Mexican currency traded at a forward discount despite a policy of fixed exchange rates. The puzzle of an unchanging exchange rate and a persistent forward discount is resolved by noting that forward market speculators were expecting an eventual large devaluation of the peso, which finally occurred in 1976.[2]

Peso problems may be just as likely to occur under floating exchange rate regimes as they are under fixed rates. It has long been speculated that floating exchange rates, like other assets, may be subject to "bubbles" which persist for some time before eventually bursting in dramatic fashion. It is also true that the post-1970's regime of floating is best characterized as a "dirty float," in which exchange markets are subject to occasional large interventions by national governments. Thus, it is *a priori* plausible that either bubbles or expected interventions could give rise to expectations of exchange rate jumps. Note that bursting bubbles and interventions need not be mutually exclusive events. The so-called "dollar problem" of the 1980's could be interpreted as a bubble which was eliminated by intervention. During 1980-85, the dollar appreciated steadily against each of the major currencies while trading at a forward discount. The dollar's rise was reversed in the fall of 1985 when the G-5 countries staged a coordinated open market operation. It is plausible that the pre-1985 forward discount reflected the market's expectation that the dollar's rise would eventually end, either of its own accord or through intervention. More recently, there has been speculation of a peso problem in the Deutschemark. Some believe that the Bundesbank may eventually lower interest rates to stimulate the German economy

[1]Reitz [29] suggests that rare but catastrophic falls in aggregate consumption could explain the equity premium in stock returns. Lewis [22] gives a discussion of peso problems and the term structure.

[2]Lizondo [25] gives a detailed account of the behavior of the forward discount before the Mexican devaluation.

and thereby send the Deutschemark tumbling.[3]

The anecdotal evidence on the existence of occasional large jumps in foreign exchange markets is confirmed by the formal empirical analyses of Jorion [25], Akgiray and Booth [2], and Tucker and Pond [32]. These studies find that a jump-diffusion model dominates various forms of the pure diffusion model in explaining the behavior of the major trading currencies during the post-1974 floating rate period.[4] There is also an empirical option pricing literature which provides further evidence that jumps are important in currency markets. For example, Bodurtha and Courtadon [9] find that the simple Black-Scholes option pricing formula makes systematic errors when used to price out-of-the-money foreign exchange options, and that those errors are consistent with the market expectations of large jumps. In a series of papers, Bates [6], [4], [5], [6] shows that options prices can be used to uncover market expectations of jumps and that such expectations are significant for the dollar-DM exchange rate in the early 1980's.[5]

We should re-iterate that the econometric difficulties which attend the peso problem only arise when jumps are of low frequency. When this is the case, there is a high probability that a given sample of exchange rate returns will be unrepresentative in the sense that it contains too few jumps. Exchange rate pricing tests which use *ex post* depreciation in place of expected depreciation may be misleading. In this paper we build on the suggestion of Bates [4] that *option-implied* jump expectations might be used to conduct exchange rate pricing tests which are resistant to the peso problem.

In order to illustrate the basic point of how large, infrequent exchange rate jumps might lead to misleading econometric inferences, we first develop the peso-problem-

[3]See for example Dornbusch [14].

[4]Jorion's result holds even when he allows for conditional heteroskedasticity in the diffusion process. Akgiray and Booth and Tucker and Pond compare the jump-diffusion model to mixture-of-normals and stable Paretian models.

[5]In [4], [5], [6], Bates shows that there was significant downward skewness in the dollar-DM exchange rate during the "dollar problem" period of the early 1980's, and that this skewness is consistent with the presence of jump expectations. In [6], he examines the behavior of options on S&P 500 futures during the time leading up to the '87 crash and concludes that the crash was expected.

resistant exchange rate pricing test in the risk-neutral, or "uncovered interest parity" (UIP) context. Besides being the simplest example to consider, UIP is of interest because it is the subject of a large empirical literature. The chief finding of this literature is the so-called "forward discount bias:" when *ex post* exchange depreciation is regressed on the forward discount, or equivalently the home-foreign interest differential, the regression coefficient is typically less than one and often significantly less than zero. This runs counter to the intuition that a discount currency should depreciate on average.

2.1.1 The Uncovered Interest Parity Example

Suppose that the dollar-DM spot exchange rate S satisfies:

$$\Delta S_t = (r_t^{US} - r_t^{DM}) + (I_t - p_t)\kappa_t + \epsilon_t \tag{2.1}$$

where ΔS_t is the one month percentage depreciation, $r_t^{US} - r_t^{DM}$ is the one-month U.S.-German interest differential, I_t is a Bernoulli random variable which takes the value 1 if a jump occurs during the month and 0 otherwise, p_t is the probability of the jump occurring, κ_t is the size of the jump given that it occurs, and ϵ_t is a normally distributed random variable independent of I_t such that $\text{E}[\epsilon_t | r_t^{US} - r_t^{DM}] = 0$. The variables κ_t and p_t are allowed to be functions of the interest differential. We define the expectational error as $\eta_t \equiv (I_t - p_t)\kappa_t + \epsilon_t$. Because $\text{E}[\eta_t | r_t^{US} - r_t^{DM}] = 0$, it follows that the exchange rate satisfies the UIP hypothesis: $\text{E}[\Delta S_t | r_t^{US} - r_t^{DM}] = r_t^{US} - r_t^{DM}$.

The econometrician does not know the true exchange rate process, and would like to *test* whether UIP holds or not. UIP is a hypothesis about expected depreciation, which is not normally observable.[6] However, by assuming rational expectations, one can write down a testable equation in terms of *ex post* depreciation:

$$\Delta S_t = \beta(r_t^{US} - r_t^{DM}) + \eta_t \tag{2.2}$$

[6] See, however, Frankel and Froot [17], who use survey data to obtain a measure of expected exchange rate depreciation and use this measure in UIP regressions.

An estimate of β near 1 will be taken as evidence in favor of UIP. The peso problem arises because η_t, which is the sum of a Bernoulli and a normal random variable, is ill-behaved in small samples. For small p_t and large κ_t, that is, rare and large jumps, η_t will be skewed. As Krasker [21] points out, under skewness of η_t the sampling distribution of the β estimate will be slow in converging to normality as the sample size grows. This will result in misleading inferences, and may frequently cause a rejection of the hypothesis $\beta = 1$ when it is in fact true.

To see how this model fits with the empirical findings, we make further specializations. Assume that κ_t is positively correlated with $r_t^{US} - r_t^{DM}$. This says, for example, that if the currency is on average expected to appreciate (e.g. $r_t^{US} - r_t^{DM} > 0$), then it will appreciate the most when the jump actually occurs (i.e. $I_t\kappa_t > 0$). The direction of the jump, in other words, coincides with the sign of the expected return of the currency. This conforms with the common perception that the jump component forms a large part of the expected currency movement. If one reverses the direction of causality from κ_t to $r_t^{US} - r_t^{DM}$, this is equivalent to saying that when the Deutschemark is expected to take an upward jump, investors will in equilibrium demand a higher return on dollar assets. Now suppose that our sample contains too few jumps; say, for simplicity, that there are no jumps. Then $\eta_t = -p_t\kappa_t + \epsilon_t$. Thus if jumps are expected but do not occur in-sample, the interest rate differential and the error term η_t will be negatively correlated, causing a downward bias in the estimate of β in (2.2). This could account for the finding, universal in the empirical UIP literature, that β is less than 1 and frequently significantly less than zero.

2.1.2 A Test Immune to the Peso Problem

If p_t and κ_t could be observed or estimated, one could include the jump expectation in a modified UIP regression, in effect replacing the ill-behaved error term η_t with an error term which is well-behaved in small samples. This can be achieved by separating the exchange rate movement into a "jump" and "continuous" component in the following manner:

$$\Delta S_t = \Delta_J S_t + \Delta_C S_t \tag{2.3}$$

where:

$$\begin{aligned} \Delta_J S_t &= I_t \kappa_t \\ \Delta_C S_t &= r_t^{US} - r_t^{DM} - p_t \kappa_t + \epsilon_t \end{aligned}$$

During a "peso problem" period most of the observed one-month depreciations will be of the continuous type $\Delta_C S_t$, that is, composed of the interest differential, an extra drift term $-p_t \kappa_t$ due to the expected but unrealized jump, and the mean zero error term. This suggests that, if $p_t \kappa_t$ could be observed, one could test UIP by running the following regression:

$$\Delta_C S_t + p_t \kappa_t = \beta(r_t^{US} - r_t^{DM}) + \epsilon_t \tag{2.4}$$

and testing whether β is equal to 1. This test will be robust to the peso problem since it does not depend on how many jumps occurred in the sample. Note that the regression only involves the continuous component of the exchange rate movement. For a month in which no jump occurs, the continuous component equals the observed exchange rate movement ($\Delta S_t = \Delta_C S_t$). For months in which a jump *does* occur, it is necessary to "de-jump" the observed movement by subtracting the jump size κ_t. Thus, to implement (2.4) it will be necessary to identify those months, if any, during which a jump actually occurs. The expected movement due to jumps $p_t \kappa_t$, which we have moved to the left-hand side of the regression, is of course not literally observable. However, one might consider the possibility of *estimating* the jump expectation.

As Bates [6] points out, estimating jump expectation parameters from historical data is a difficult, perhaps impossible task, since (i) the parameters are time-varying, and (ii) even if the jump process is time-invariant, the jumps may be rare. In that case, even a long sample of past data may contain very few observations with jumps. Our research builds on the insight of Bates that option prices can be used to recover the otherwise unobservable jump expectation.

Theoretical option-pricing formulas exist which apply when the underlying asset follows a jump-diffusion process.[7]. The option price is shown to depend on the size and frequency of jumps. Thus, one can use a theoretical pricing formula together with observed option prices to uncover the expectations of jump size and frequency which are held by market participants.[8] Knowledge of the jump expectations will allow us to perform the test of exchange rate dynamics presented in (2.4) which is resistant to the peso problem

The example above assumed risk-neutrality in order to highlight the point that the peso problem can arise even in the absence of a risk premium. In the most general case, expected exch ge rate depreciation will be related not only to the interest differential but also to an exchange risk premium. Indeed, as Fama [15] shows, even in the absence of a peso problem the empirical anomaly of $\beta < 1$ could be explained by a time-varying risk premium which is highly variable and negatively correlated with the interest differential.[9] Below we develop a continuous time general equilibrium model of international asset pricing with risk-averse investors. We allow for undiversifiable jump risk in the economy and PPP deviations which cause agents to be heterogeneous across national boundaries. We then show how one can use the *equivalent martingale pricing measure* or *risk-neutralized* parameters recovered from option prices in conjunction with the observed path of the exchange rate to conduct an empirical test of the exchange rate dynamics implied by our model.

Our theoretical analysis shows that it is exactly the equivalent martingale parameters and not the true distributional ones that are needed to test the final empirical implications analogous to (2.4). Thus even if one could estimate the true distributional parameters of the jump process, one would then have to make assumptions to convert that into the equivalent martingale parameters. The approach of recovering

[7]Merton [28] and Jones [20] develop European option formulas for the jump-diffusion case

[8]This is the jump-diffusion analogue to the familiar practice of using the Black-Scholes formula to find option-implied volatilities.

[9]There are many other possible explanations of the forward discount bias which our model is unable to nest. For example, Lewis [23] shows that if there are periodic monetary policy regime shifts which agents learn about rationally but slowly, it is possible that there will be exchange rate forecasting errors for a period of time, thereby causing a forward discount bias. Backus, Gregory, and Telmer [4] show that the forward discount can be explained in the context of a model with habit persistence. Lewis [24] gives a very thorough and recent survey of competing explanations of the forward discount bias.

the equivalent martingale parameters from option prices thus turns out to be "just right".

To our knowledge, this will be the first empirical test of an equilibrium model of exchange rates which explicitly allows jumps, and the first such test which incorporates options price data to isolate jump expectations.[10] In section 2.2 we develop the continuous-time equilibrium model of international asset pricing when assets are subject to jumps. Section 2.3 describes how one can recover monthly jump expectations from prices of currency options on the Deutschemark and Yen. We find that, during the 1984-1993 period, there were many months with economically significant jump expectations. Moreover, for the dollar-DM rate, there was a strong positive correlation between expected jump depreciation and the interest differential. This implies that, *during periods when few or no jumps occur*, the jump expectations will account for at least a portion of the forward discount bias. However, as we show in section 2.4, it *cannot* be concluded that jump expectations cause bias in exchange rate tests unless we are certain that we are dealing with a sample period in which few or no jumps occur. This requires us to identify exactly which exchange rate movements are jumps and which are not. Another way to state this is that our peso problem resistant test must be performed using *ex post* exchange depreciations which have been "de-jumped." The results of the test will be sensitive to the the method used to de-jump the data. We report results for several methods of de-jumping and conclude that, for de-jumping methods which are *a priori* reasonable, it *cannot* be concluded that jump expectations lead to bias in exchange pricing tests. Thus, although jump expectations are economically significant for the dollar-DM rate in 1984-1993, it seems that "enough" jumps occurred in the sample, so that the "peso problem" is not a problem for this period for the DM. For the dollar-Yen rate, there is no significant relationship between jump expectations and the interest differential. Thus, unsurprisingly, we also conclude that the "peso problem" cannot be taken as an explanation of the

[10]Lyons [26] uses Black-Scholes implied volatilities as proxies for time varying risk premia in a test of a risk aversion modified UIP Our approach is different in that we explicitly allow for jumps. A recent working paper by Bates [10] also uses option-implied jump expectations to shed light on the peso problem.

substantial forward discount bias in the \$/Yen rate.

2.2 The International Economy

We consider a world of $L+1$ countries and currencies $\{0, 1, ...L\}$. The currency of country 0 will serve as the measurement currency. There are n nominally risky assets (e.g.. corporate debt and equity) in positive supply whose dynamics in terms of the measurement currency is given by the jump diffusion equation:

$$\begin{aligned} \frac{d\eta_i(t)}{\eta_i(t)} &= [\alpha_i(Z,t) - \lambda(Z,t)\kappa_i(Z,t)]\,dt \\ &\quad + \underline{G}_i(Z,t)dw(t) + \kappa_i(Z,t)dq(t), \end{aligned} \tag{2.5}$$

where $w(t)$ is an $(n+k)$ dimensional standard Wiener process in $\Re^{n+k}$, $q(t)$ is a one dimensional standard Poisson process in $\Re$ with jump intensity $\lambda(Z,t)$ whose increments are independent of the increments of $w(t)$, Z is a k-dimensional vector of state variables, $\alpha_i(Z,t)$ is a bounded function taking values in $\Re$, $\underline{G}_i(Z,t)$ is a bounded function taking values in $\Re^{n+k}$ while $\kappa_i(Z,t) > -1$ is a *non-random* function taking values in $\Re$.

Defining the matrix $\underline{\underline{G}}$, as the $n \times (n+k)$ dimensional matrix whose i-th row is $\underline{G}_i(Z,t)$, we further assume that $\underline{\underline{GG'}}$ is positive definite. For further reference we define $\underline{\alpha}$ as the n dimensional vector such that $\underline{\alpha}_i = \alpha_i(Z,t)$ and define $\underline{K}$ similarly[11].

The evolution of assets given in (2.5) follows the approach taken in Ahn and Thompson [2]. The state variable Z is a k-dimensional vector process that evolves according to:

$$dZ(t) = [\mu_Z(Z,t)dt - \lambda(Z,t)\Delta_Z(Z,t)]\,dt + \underline{\underline{V}}_Z(Z,t)dw(t) + \Delta_Z(Z,t)dq(t) \tag{2.6}$$

where $\mu(Z,t)$ and $\Delta_Z(Z,t)$ are bounded function taking values in $\Re$ and $\underline{\underline{V}}_Z(Z,t)$ is a bounded $k \times (n+k)$ matrix valued function.

[11]In this paper, we will use $\underline{A}$ to denote a vector or vector valued function and $\underline{\underline{A}}$ to denote the corresponding matrix or matrix valued function that results from vertically stacking $\underline{A}_i$'s.

We consider a world with a single consumption good. There is a representative agent l from each country. An agent from country l can purchase one unit of consumption for $P_l(t)$ units of the country 0 currency, where $P_l(t)$ is a stochastic process in $\Re$ that evolves according to the dynamics:

$$\frac{dP_l(t)}{P_l(t)} = [\mu_l - \lambda(Z,t)\kappa_{P_l}]\,dt + \underline{\sigma}_{P_l}dw(t) + \kappa_{P_l}dq(t) \tag{2.7}$$

where μ_l and $\kappa_{P_l} > -1$ are constants in $\Re$ and $\underline{\sigma}_{P_l}$ is a constant in $\Re^{n+k}$. Allowing different countries to purchase consumption at different prices allows us to capture deviations from purchasing power parity.

In addition to the equity assets, there are also assets which are in zero net supply, such as bank deposits denominated in various currencies, forward contracts, and option contracts. Our objective will be to characterize the dynamics of such zero net supply securities, which will also follow jump-diffusion processes due to the generalized Itô theorem for jump-diffusions. Given this fact, we can write down the price processes for these assets as follows:

$$\frac{dF_i(t)}{F_i(t)} = (\beta_i - \lambda(Z,t)\kappa_{F_i})dt + \underline{H}_i dw(t) + \kappa_{F_i}dq(t) \tag{2.8}$$

where β_i and κ_{F_i} take values in $\Re$ and $\underline{H}_i$ is in $\Re^{n+k}$. The value of β_i, κ_{F_i} and $\underline{H}_i$ at each point in time will be endogenously determined in equilibrium It is important to note that we are simply providing notation for entities that we will later examine in detail. We are not exogenously specifying the price processes since we allow $\beta_i(t)$ and $\kappa_{F_i}(t)$ and $\underline{H}_i(t)$ to be random functions of time. We will be particularly interested in the price process of various currencies in terms of the measurement currency and the value of a bank account in the different countries again in terms of the measurement currency. We develop further notation for these assets. The spot exchange rate l, measured in units of currency 0 per unit of currency l, evolves according to:

$$\frac{dS_l(t)}{S_l(t)} = [\theta_l(t) - \lambda(Z,t)\kappa_{e_l}]\,dt + \underline{V}_{e_l}(t)dw(t) + \kappa_{e_l}(t)dq(t) \tag{2.9}$$

An individual can also invest in an instantaneously risk-less bank account in country 0 whose value B_0 evolves according to:

$$dB_0(t) = r_0(t)B_0(t)dt \tag{2.10}$$

The country l bank account B_l *in units of the country-l currency* evolves similarly to the equation above except with rate of growth equal to $r_l(t)$. In units of the country 0 currency, this implies that:

$$\frac{dB_l(t)}{B_l(t)} = [r_l(t) + \theta_l(t) - \lambda\kappa_l(t)]\, dt + \underline{V}_{e_l} dw(t) + \kappa_{e_l} dq(t) \tag{2.11}$$

Note that the diffusion and jump increments of the country l bank account are the same as those of that country's exchange rate. In this model exchange rates are simply the rates at which returns on country l assets are translated into other currencies for purposes of consumption. Thus, by determining the equilibrium drift on the country l bank account measured in currency 0, we are in effect also determining the equilibrium drift θ_l of the exchange rate. For simplicity of notation, we will simply refer to r_0 as r for the rest of this paper. We will also treat the country 0 bank account separately from the rest of the zero net supply assets and will not include it among the assets referred to as F.

2.2.1 Individual Optimization

We first consider the problem of an individual investor. An agent from country l seeks to maximize

$$\mathrm{E}\left[\int_0^\infty e^{-\rho t} U_l(C_l(t))dt\right], \tag{2.12}$$

subject to conditions given below, where $C_l(t)$, is flow consumption. The proportion of wealth invested by agent l in the positive net supply assets will be denoted a_l, where a_l is an N-vector, while the proportion in the zero net supply assets will be denoted b_l, where b_l is a k vector. Using 1_M to denote an M-vector of ones, the remaining proportion of wealth $1 - a_l' 1_N - b_l' 1_\kappa$ (possibly negative) will be put into the country

0 bank account. Thus the nominal wealth of agent l, W_l° follows the jump diffusion process:

$$\begin{aligned} dW_l^\circ &= \Big[\left(r + a_l'\left[\underline{\alpha} - r\underline{1}_N\right] + b_l'\left[\underline{\beta} - r\underline{1}_\kappa\right]\right) W_l^\circ \\ &\qquad -C_l P_l - \lambda\left(W_l^\circ\left(a_l'\underline{\kappa}_\eta + b_l'\underline{\kappa}_F\right)\right)\Big]\, dt \\ &\quad + \left[W_l^\circ\left(a_l'\underline{\underline{G}} + b_l'\underline{\underline{H}}\right)\right] dw(t) + \left[W_l^\circ\left(a_l'\underline{\kappa}_\eta + b_l'\underline{\kappa}_F\right)\right] dq(t) \\ &\equiv \left(\mu_{W_l^\circ} - \lambda\kappa_{W_l^\circ}\right) W_l^\circ dt + W_l^\circ \sigma_{W_l^\circ} dw(t) + W_l^\circ \kappa_{W_l^\circ} dq(t) \end{aligned} \tag{2.13}$$

where:

$$\begin{aligned} \mu_{W_l^\circ} &= \left(r + a_l'\left[\underline{\alpha} - r\underline{1}_N\right] + b_l'\left[\underline{\beta} - r\underline{1}_\kappa\right]\right) - C_l P_l / W_l^\circ \\ \sigma_{W_l^\circ} &= a_l'\underline{\underline{G}} + b_l'\underline{\underline{H}} \\ \kappa_{W_l^\circ} &= a_l'\underline{\kappa}_\eta + b_l'\underline{\kappa}_F \end{aligned} \tag{2.14}$$

We will solve the consumers' optimization problem, and characterize the resulting equilibrium using the Bellman optimality principle. The agent selects a trading strategy $\{a_l(t), b_l(t)\}$ and consumption plan $\{C_l(t)\}$ so as to maximize (2.12). We make the standard assumption that the agent restricts attention to the class of admissible feedback controls.[12] At this point, we can make a simplifying re-normalization by expressing the wealth process in units of the consumption good. By a simple application of Itô's lemma, we find that the wealth of agent l in units of the consumption good W_l evolves according to:

$$\begin{aligned} dW_l &= W_l\left(\mu_{W_l^\circ} - \mu_{P_l} - \lambda\left(\kappa_{W_l^\circ} - \kappa_{P_l}\right) + \sigma_{P_l}'\sigma_{P_l} - \sigma_{W_l^\circ}'\sigma_P\right) dt \\ &\quad + W_l\left(\sigma_{W_l^\circ} - \sigma_{P_l}\right) dw(t) + \left(\frac{\kappa_{W_l^\circ} - \kappa_{P_l}}{1 + \kappa_{P_l}}\right) dq(t) \\ &= W_l\left(\mu_{W_l} - \lambda\kappa_{W_l}\right) dt + W_l \sigma_{W_l} dw(t) + W_l \kappa_{W_l} dq(t) \end{aligned} \tag{2.15}$$

[12]See Fleming and Rishel [16].

where:

$$\begin{aligned}
\mu_{W_l} &= \mu_{W_l^\circ} - \mu_{P_l} - \lambda\left(\kappa_{W_l^\circ} - \kappa_{P_l}\right) + \sigma_{P_l}'\sigma_{P_l} - \sigma_{W_l^\circ}'\sigma_P \\
\sigma_{W_l} &= \sigma_{W_l^\circ} - \sigma_{P_l} \\
\kappa_{W_L} &= \frac{\kappa_{W_l^\circ} - \kappa_{P_l}}{1+\kappa_{P_l}}
\end{aligned} \tag{2.16}$$

Because we have assumed in (2.7) that the price of consumption for each country (in units of the country 0 currency) follows a geometric jump diffusion, the real wealth process now becomes a Markov system. This is checked by verifying that the parameters given in (2.16) can all be written as at most a function of time and W_l. Thus W_l becomes a sufficient statistic for W_l° and P_l for the individual solving the investment consumption problem. The variables that affect the maximum attainable value of (2.12) at any given point in time, however, include not only W_l, and Z, but also W_k for all $k \neq l$ as well. This is because one investor's wealth will affect the equilibrium rate of return required of the zero net supply assets and consequently will affect the objective function of other investors.[13] For notational simplicity, we define Y_l as the vertically stacked vector of Z and $\{W_k\}_{k\neq l}$. The elements of this vector are in essence the relevant non-wealth state variables for agent l. Further define μ_{Y_l} as the vertically stacked vector of μ_Z and $W_k\mu_{W_k}$ for $k \neq l$, σ_Y as the vertically stacked matrix of σ_Z and $W_k\sigma_{W_k}$ for $k \neq l$, and Δ_Z as the vertically stacked vector of Δ_Z and $W_k\kappa_{W_k}$ for $k \neq l$.

Define the maximum attainable value of (2.12) as $J^l(W_0, ..., W_L, Z, t)$. Under these conditions, a necessary condition for optimality of the controls is given by the Bellman equation:

$$\max_{\{a_l, b_l, C_l\}} \left[\mathcal{L}J^l + U_l(C_l)\right] + J_t^l = 0 \tag{2.17}$$

where $\mathcal{L}$ is the differential generator associated with the processes $\{\{W_l\}_{l=0}^L, Z\}$ so

[13] Imagine in a world with decreasing relative risk aversion that all investors except one became rich. Then the rate of return offered on the zero net supply assets will tend to fall. The poor investor would be even worse off because the rate of return on these assets has fallen.

that:

$$\begin{aligned}\mathcal{L}(J) &= [\mu_{W_l} - \lambda\kappa_{W_l}]\, W_l J^l_{W_l} + [\mu_{Y_l} - \lambda\Delta_{Y_l}]\, J_{Y_l} \\ &+ \frac{1}{2}\left[W_l^2 \sigma_{W_l}\sigma'_{W_l} J^l_{W_l W_l} + 2\sigma_{W_l}\sigma'_{Y_l} J_{W_l Y_l} + tr\left(\sigma_{Y_l}\sigma'_{Y_l} J_{Y_l Y_l}\right)\right] \\ &+ \lambda\left[J(W_l(1+\kappa_{W_l}), Y_l + \Delta Y_l, t) - J(W_l, Y_l, t)\right] \end{aligned} \tag{2.18}$$

We assume that the solution J^l is twice continuously differentiable and concave in W_l and once differentiable in t. The first order conditions resulting from the maximization in (2.17) are:

$$J^l{}_W = U^{l\prime}(C_l) \tag{2.19}$$

$$\begin{aligned}\underline{\alpha} - r\underline{1}_N &= R_l\left(\underline{\underline{G}}\underline{\underline{G}}' a_l + \underline{\underline{G}}\underline{\underline{H}}' b_l\right) \\ &+ (1 - R_l)\,\underline{\underline{G}}\sigma'_{P_l} - \underline{\underline{G}}\sigma_{Y_l}\Gamma_l \\ &- \lambda\kappa_\eta\left[\left(\frac{1}{1+\kappa_{P_l}}\right)\left(\frac{J^+_{W_l}}{J_{W_l}}\right) - 1\right]\end{aligned} \tag{2.20}$$

$$\begin{aligned}\underline{\beta} - r\underline{1}_K &= R_l\left(\underline{\underline{H}}\underline{\underline{H}}' b_l + \underline{\underline{H}}\underline{\underline{G}}' a_l\right) \\ &+ (1 - R_l)\,\underline{\underline{H}}\sigma'_{P_l} + \underline{\underline{H}}\sigma_{Y_l}\Gamma_l \\ &- \lambda\kappa_F\left[\left(\frac{1}{1+\kappa_{P_l}}\right)\left(\frac{J^+_{W_l}}{J_{W_l}}\right) - 1\right]\end{aligned} \tag{2.21}$$

where:

$$R_l = -\frac{J^l_{W_l W_l} W_l}{J_{W_l}} \tag{2.22}$$

$$\Gamma_l = \frac{J^l_{W_l Y_l}}{J_{W_l}} \tag{2.23}$$

$$\frac{J^+_{W_l}}{J_{W_l}} = \frac{J^l_{W_l}(W_l(1+\kappa_{W_l}), Y + \Delta_{Y_l}, t)}{J^l_{W_l}(W_l, Y_l, t)} \tag{2.24}$$

Before aggregating the above quantities, it is worth making a few remarks about the implications for asset pricing. The first line of equations (2.20) and (2.21) to the right of the equals sign are the risk premiums that result due to direct diffusion wealth risk. The second line reflects the premiums that result due to individuals desire to hedge against the diffusion risk of the consumption price fluctuating as well as other state variables. The third line represents the premium resulting from jump risk. Just as with the diffusion part of price risk, the premium does not disappear in the case of risk neutrality ($\frac{J^+_{W_l}}{J_{W_l}} = 1$) unless $\kappa_{P_l} = 0$ (which corresponds in the diffusion case to where $\sigma_{P_l} = 0$).

2.2.2 World Equilibrium

World equilibrium is characterized by the conditions:

$$\underline{1}'_N \left(\sum_{l=0}^{L} W_l a_l \right) = \sum_{l=0}^{L} W_l \tag{2.25}$$

$$\sum_{l=0}^{L} W_l b_l = \underline{0}_K \tag{2.26}$$

This simply says that all the world wealth must be stored in the positive net supply assets and that the net position in zero net supply assets must be zero. It is important to note that this does not imply $b_l = 0$. It is possible that some countries will hold a positive or negative amount of a zero net supply asset–one country can be a net borrower, like the U.S., or a net lender, like Taiwan.

Our objective will be to derive a testable version of (2.21). We follow Adler and Dumas [1] and take a weighted sum of (2.21) over l using the weights:

$$\pi_l = \frac{\frac{W_l}{R_l}}{\sum_{k=0}^{L} \frac{W_k}{R_k}} \tag{2.27}$$

It will also be useful to define $\overline{R}$ as the aggregate world relative risk aversion:

$$\overline{R} = \left(\sum_{l=0}^{L} \overline{W_l} \frac{1}{R_l} \right)^{-1} \tag{2.28}$$

where:

$$\overline{W_l} = \frac{W_l}{\sum_{k=0}^{L} W_k} \tag{2.29}$$

By multiplying (2.21) by π_l and summing over l, the b_l terms disappear because of (2.26) and we obtain:

Lemma 1: *The rate of return of the zero net supply asset in the international economy can be expressed as:*

$$\begin{aligned}\beta - r\underline{1}_K &= \overline{R}\underline{\underline{H}}\sum_{l=0}^{L}\underline{\underline{G}}'a_l\overline{W_l} + \underline{\underline{H}}\left[1 - \overline{R}\right]\frac{\sum_{l=0}^{L}(1 - R_l^{-1})W_l\sigma_{P_l}}{\sum_{l=0}^{L}(1 - R_l^{-1})W_l} \\ &+ \underline{\underline{H}}\sum_{l=0}^{L}\pi_l\sigma_{Y_l}\Gamma_l - \lambda\kappa_F\left[\left(\sum_{l=0}^{L}\pi_l\left(\frac{1}{1+\kappa_{P_l}}\right)\frac{J^+_{W_l}}{J_{W_l}}\right) - 1\right]\end{aligned} \tag{2.30}$$

We can also obtain a familiar result from Cox, Ingersoll and Ross [13], Ahn and Thompson [2] and Bates [4] in our international jump-diffusion context:

Lemma 2: *The rates of return on all securities in this economy that can be written in the form $F(S, W_0, ...W_l, Z, t)$ can be represented as:*

$$(\beta_F - r\underline{1}_K)\,F = [\Phi_S, \Phi_{W_0}, ..., \Phi_{W_L}, \Phi_Z]\,[F_S, F_{W_0}, ..., F_{W_L}, F_Z]' - \lambda\Delta F[\delta - 1] \tag{2.31}$$

where:

$$\begin{aligned}\Phi_S &= S\sigma_S\Psi \\ \Phi_l &= W_l[a_l'\underline{\underline{G}} + b_l'\underline{\underline{H}}]\Psi \\ \Phi_Z &= \sigma_Z\Psi \\ \Psi &= \overline{R}\sum_{l=0}^{L}\underline{\underline{G}}'a_l\overline{W_l} + \left[1 - \overline{R}\right]\frac{\sum_{l=0}^{L}(1 - R_l^{-1})W_l\sigma_{P_l}}{\sum_{l=0}^{L}(1 - R_l^{-1})W_l} + \sum_{l=0}^{L}\pi_l\sigma_{Y_l}\Gamma_l \\ \Delta F &= F\left(S(1+\kappa_S), W_0(1+\kappa_{W_0}), ..., W_L(1+\kappa_{W_L}), Z + \Delta_Z, t\right) \\ &\quad - F(S, W_0, ...W_l, Z, t) \\ \delta &= \sum_{l=0}^{L}\pi_l\left(\frac{1}{1+\kappa_{P_l}}\right)\frac{J^+_{W_l}}{J_{W_l}}\end{aligned} \tag{2.32}$$

By applying Itô's lemma to $F(S, W_0, ..., W_l, Z, t)$ and equating the resulting drift of F with that obtained in Lemma 2, we find the differential-difference equation that must be satisfied by these assets:

$$
\begin{aligned}
rF &= F_t + F_S S[r - \lambda\delta\kappa_S] + \sum_{l=0}^{L} F_{W_l} W_l [r - C_l/W_l - \lambda\delta\kappa_{W_l}] \\
&\quad + [\mu_z - \Phi_Z - \lambda\Delta_Z] F_Z + (1/2)\nabla\nabla_{S,W_0,\ldots,W_l,Z}(F) \\
&\quad + \lambda\delta\Delta F
\end{aligned} \tag{2.33}
$$

where $\nabla\nabla_X(F)$ is short-hand for the second order terms in the Itô expansion of F.

By using an argument along the lines of Merton [28, Appendix] and Bates [4] this differential-difference equation allows us to determine the process that prices will follow under the equivalent martingale pricing measure (cf. Harrison and Kreps [18]).

Lemma 3: *The processes under the equivalent martingale pricing measure Q evolves according to:*

$$dS = S[r - \lambda^*\kappa_S]dt + S\sigma_S dw^*(t) + S\kappa_S dq(t) \tag{2.34}$$

$$dW_l = W_l[r - C_l/W_l - \lambda^*\kappa_{W_l}]dt + W_l\sigma_W dw^*(t) + W_l\kappa_{W_l} dq(t) \tag{2.35}$$

$$dZ = [\mu_Z - \Phi_Z - \lambda\Delta Z]dt + \sigma_Z dw^*(t) + \Delta_Z dq(t) \tag{2.36}$$

where under Q, $w^*(t)$ is a Wiener process and $q(t)$ is Poisson process that increases by 1 with probability $\lambda^* \equiv \lambda\delta$ per unit time.

This result will greatly facilitate our empirical implementation since the only parameters we can directly recover from option prices are the parameters of the equivalent martingale pricing measure. Combining Lemmas 1, 2, and 3 we obtain:

Theorem: *The pure diffusion part of the price process has an expected rate of return which by definition is equal to $\beta - \lambda\kappa_F$. In equilibrium, this can be written as:*

$$
\begin{aligned}
\beta - \lambda\kappa_F &= r\underline{1}_K + \overline{R}\underline{\underline{H}}\sum_{l=0}^{L}\underline{\underline{G}}' a_l \overline{W_l} + \underline{\underline{H}}\left[1 - \overline{R}\right]\frac{\sum_{l=0}^{L}(1 - R_l^{-1})W_l\sigma_{P_l}}{\sum_{l=0}^{L}(1 - R_l^{-1})W_l} \\
&\quad + \underline{\underline{H}}\sum_{l=0}^{L}\pi_l\sigma_{Y_l}\Gamma_l - \lambda^*\kappa_F
\end{aligned} \tag{2.37}
$$

where λ^* is the Poisson jump probability of the equivalent martingale pricing measure *given in Lemma 3. This can also be written in the following form:*

$$\begin{aligned}\beta_i - \lambda\kappa_{F_i} = r \;\; &+ \;\; \overline{R}\,\mathrm{Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dF_i}{F_i}\right) \\ &+ \;\; \left[1-\overline{R}\right]\sum_{l=0}^{L}\left(\frac{(1-R_l^{-1})W_l}{\sum_{k=0}^{L}(1-R_k^{-1})W_k}\right)\,\mathrm{Cov}_C\left(\frac{dP_l}{P_l}, \frac{dF_i}{F_i}\right) \\ &+ \;\; \sum_{l=0}^{L}\pi_l\,\mathrm{Cov}_C\left(I_{Y_l}^{-1}dY_l, \frac{dF_i}{F_i}\right)\Gamma_l - \lambda^*\kappa_{F_i} \qquad (2.38)\end{aligned}$$

where W_{world} is world wealth and $Cov_C(\frac{dX}{X}, \frac{dY}{Y})$ is the covariance of the pure diffusion component of X and Y.

2.2.3 Some Testable Cases

The asset pricing models presented so far will be difficult to implement empirically unless we make further assumptions on the utility function. Our first step will be to assume that all agents have the same utility functions reflecting constant relative risk aversion or $U_l(C) = \frac{C^{1-\gamma}-1}{1-\gamma}$ for all l. In this case, it is straightforward to verify that $J(W,Y) = f(Y)U(W) + g(Y)$ so that $R_l = \gamma$. In this case (2.37) reduces to:

$$\begin{aligned}\beta_i - \lambda\kappa_{F_i} = r \;\; &+ \;\; \gamma\,\mathrm{Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dF_i}{F_i}\right) + [1-\gamma]\sum_{l=0}^{L}\overline{W_l}\,\mathrm{Cov}_C\left(\frac{dP_l}{P_l}, \frac{dF_i}{F_i}\right) \\ &+ \;\; \sum_{l=0}^{L}\overline{W_l}\,\mathrm{Cov}_C\left(I_{Y_l}^{-1}dY_l, \frac{dF_i}{F_i}\right)\Gamma_l(Y_l) - \lambda^*\kappa_{F_i} \qquad (2.39)\end{aligned}$$

Applying the above result to the case where the asset in question is the country l bank account, by (2.11) we can rewrite the rate of return on the pure diffusion part of the currency exchange as:

$$\begin{aligned}\theta_{S_l} - \lambda\kappa_l = r - r_l \;\; &+ \;\; \gamma\,\mathrm{Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dS_l}{S_l}\right) + [1-\gamma]\sum_{l=0}^{L}\overline{W_l}\,\mathrm{Cov}_C\left(\frac{dP_l}{P_l}, \frac{dS_l}{S_l}\right) \\ &+ \;\; \sum_{l=0}^{L}\overline{W_l}\,\mathrm{Cov}_C\left(I_{Y_l}^{-1}dY_l, \frac{dS_l}{S_l}\right)\Gamma_l(Y_l) - \lambda^*\kappa_l \qquad (2.40)\end{aligned}$$

In the case where $\gamma = 1$, or in other words $U(C) = \ln(C)$, it can be verified that

$J(W,Y) = \ln(W)/\rho + g(Y)$, so that:

$$\beta_i - \lambda\kappa_{F_i} = r + \text{ Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dF_i}{F_i}\right) - \lambda^*\kappa_{F_i} \tag{2.41}$$

and for currencies:

$$\theta_{S_l} - \lambda\kappa_l = r - r_l + \text{ Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dS_l}{S_l}\right) - \lambda^*\kappa_l \tag{2.42}$$

Note that (2.42) gives an expression for equilibrium expected depreciation along a continuous path of the exchange rate, that is, the sort of path which is actually observed during a peso problem period. Hence, it is natural to use this expression for expected depreciation *given that no jumps occur* to conduct exchange rate pricing tests which are robust to the peso problem. The right hand side of (2.42) depends on the risk-neutral parameters of the jump process but not the true distributional parameters. This is convenient since it is precisely the risk-neutral parameters which can be recovered from observed option prices.

2.3 Empirical Implementation

2.3.1 One–Month Exchange Rate Dynamics

When log-utility of investors is assumed, the instantaneous expected depreciation of the country l exchange rate S_l, measured in units of currency 0 per unit of currency l, is given by (2.42). Thus, dropping the subscript l for simplicity, defining $r^* \equiv r_l$ and $\phi \equiv \text{Cov}_C\left(\frac{dW_{world}}{W_{world}}, \frac{dS}{S}\right)$, we can write the instantaneous dynamics of the exchange rate as

$$\frac{dS}{S} \;=\; [r - r^* + \phi - \lambda^*\kappa]dt + \sigma(t)dz(t) + \kappa(t)dq(t) \tag{2.43}$$

Note that we have also made the substitution $\underline{V}_e(t)dw(t) \equiv \sigma(t)dz(t)$, where σ is scalar and $z(t)$ is a one-dimensional standard Wiener process.

Since instantaneous returns are not observed, we must integrate (2.43) over some discrete time interval to derive an empirically testable equation. In order to do the integration, we shall make further assumptions about how the parameters of the exchange rate process evolve over time. Suppose that there is a time interval $[t_0, T]$ which has been divided into N subintervals $[t_0, t_1], [t_1, t_2], ...[t_{N-1}, T]$, each of length $\tau = t_{i+1} - t_i$, where τ is taken to be one month. The parameters σ, λ, and κ are assumed to be *constant* over the course of the month, as is the interest differential. Because we wish to concentrate on the "peso problem" effect of a time-varying jump expectation, the diffusion risk premium ϕ is assumed constant over the entire interval $[t_0, T]$. Given these assumptions about the parameters, we can integrate to find the log exchange rate depreciation over month i:

$$\begin{aligned} \Delta S_i \equiv \log\left[\frac{S(t_{i+1})}{S(t_i)}\right] &= [(r_i - r_i^*) + \phi - \frac{1}{2}\sigma_i^2 - \lambda_i^*\kappa_i]\tau - \\ &+ \sigma_i[z(t_{i+1}) - z(t_i)] + n_i \log(1 + \kappa_i) \end{aligned} \quad (2.44)$$

The subscript i indicates that although the parameters and interest rates are assumed constant over a given month, it is nonetheless possible that they may vary from month to month. This allows us to capture possible non-stationarities in the exchange rate process.

We can decompose the one-month depreciation into continuous and discontinuous components in the same way that we did for the risk-neutral example of section 2.1:

$$\Delta S_i = \Delta_C S_i + \Delta_J S_i \quad (2.45)$$

where

$$\begin{aligned} \Delta_C S_i &= [r_i - r_i^* + \phi - \frac{1}{2}\sigma_i^2 - \lambda_i^*\kappa_i]\tau + \sigma_i[z(t_{i+1}) - z(t_i)] \\ \Delta_J S_i &= n_i \log(1 + \kappa_i) \end{aligned}$$

If jumps have low probability, during most months no jumps will occur ($n_i = 0$), so that the observed depreciation ΔS_i is equal to the continuous component $\Delta_C S_i$.

The continuous depreciations will be made up of (i) the interest differential, (ii) the mean zero Wiener error term, and (iii) an extra drift term $d_i \equiv [\phi - \frac{1}{2}\sigma_i^2 - \lambda_i^* \kappa_i]\tau$. The quantity ϕ is the premium investors demand for bearing the diffusion risk due to normal daily movement of the exchange rate. Similarly, $-\lambda_i^* \kappa_i$ is the extra drift demanded by investors along the no-jump path in compensation for the possibility of occasional large jumps. The final term $\frac{1}{2}\sigma_i^2$ is due to Ito's lemma or Jensen's inequality and arises when we continuously compound the instantaneous exchange depreciation over a discrete interval.[14] It follows that during a period with few or no jumps, regressions of *ex post* depreciation on the interest differential will be subject to a bias due to omission of the term d_i. If d_i is *negatively* correlated with the interest differential, the regression coefficient will be biased *downward*, as is commonly found in the empirical literature. This suggests that, if the term d_i can be estimated, we can eliminate or at least reduce the bias by running a regression of the following form:

$$\Delta_C S_i + [\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2]\tau = \phi\tau + \beta(r_i - r_i^*)\tau + \epsilon_i \tag{2.46}$$

Again, the diffusion risk premium term ϕ is treated as time-invariant and hence is part of the intercept in the regression.[15]

Below we present evidence that, for the \$/DM exchange rate, the option-implied "peso problem" term $\lambda_i^* \kappa_i$ indeed has significant positive correlation with the interest differential. This does *not* hold for the \$/Yen rate, however, which shows a slight negative correlation of jump expectations with the interest differential. The option-implied Jensen–Ito term $\frac{1}{2}\sigma_i^2$ is shown to be small and have slight negative correlation

[14]The Jensen-Ito term will typically be very small, perhaps 50 annualized basis points. Nonetheless, as Bekaert and Hodrick [11] point out, this term is likely to be time-varying and correlated with the interest differential in some way, and hence is *a priori* a potential source of bias in forward discount regressions. They account for this possible bias using a GARCH model for σ_i^2 and Monte Carlo simulation and find that little if any bias can be attributed to the Ito–Jensen term. Our conclusion using the option-implied σ_i^2 is essentially the same.

[15]A time-varying diffusion risk premium ϕ_i could in principle be estimated using daily data on a world equity index along with daily exchange rate data. We do not pursue this because (i) we wish to concentrate on the time-varying "peso problem" term, and (ii) previous attempts to explain the forward discount bias using risk premia such as this, i.e., covariances with world wealth, have typically found that the resulting risk premia do not vary enough to explain the forward discount. See, for example, Lewis [24] for a survey of evidence on this point.

with the interest differential for both the DM and the Yen, so that it cannot account for any of the forward discount bias. Note that the exchange depreciation $\Delta_C S_i$ on the left hand side of the regression is the continuous depreciation, which is equal to observed depreciation during most months. However, if the sample contains a few months during which jumps actually occur, the monthly return must be "de-jumped" in some way. Details of this de-jumping are treated in section 2.4.

Because $\frac{1}{2}\sigma_i^2$, and $\lambda_i^* \kappa_i$ are *estimated* quantities, we have moved them to the left-hand side of the regression to avoid the error-in-variables problem. Thus, the regression error term ϵ_i will be made up of estimation error as well as the normal white noise exhibited by the exchange rate along its continuous, or "no-jump" path.

We emphasize once again that to conduct the test in (2.46) we do not need an estimate of the true jump frequency λ_i, but rather an estimate of the "risk-neutralized" jump frequency λ_i^*. This is fortuitous because it is precisely the risk-neutralized parameters which can be estimated from options prices. Under our assumption of a non-stochastic jump size κ_i, the risk-neutral jump size is equal to the actual jump size. It is also the case that the risk-neutral σ_i is equal to the true σ_i, since it is an "almost sure" property of the exchange rate path and hence is preserved under the change to the equivalent martingale measure. In the next section we discuss the method used to estimate the option-implied parameters. In Section 2.3.3 we examine the properties of the option-implied parameters and in Section 2.4 we discuss the final result of running the peso-problem-resistant test of exchange rate dynamics given by (2.46).

2.3.2 Estimation of Option-Implied Parameters

The options data used are from the Philadelphia Stock Exchange (PHLX) and consist of transactions data on foreign currency options traded between January 1984 and December 1993. Options on the Deutschemark and Yen were considered because they are the most heavily traded. PHLX currency options are written on the underlying cash market on the respective foreign currency—not the futures market as is the case in the Chicago markets. Each observation consists of the pri[illegible] for the option, the

Telerate spot exchange rate at the time of the transaction, and the term to maturity and strike price of the option. The options data for June through November of 1985 cannot be used due to severe errors in reporting during that period.

Because the majority of currency option transactions on the PHLX are of American style options (96% of the Deutschemark option transactions are American—almost all transactions were American prior to 1991), we chose to work with the American options in order that we do not run out of observations in many of the years prior to 1991. A drawback of using American options is that there exists no analytic closed form formula for the price of an American option. One must either use a finite difference backward marching scheme or rely on an analytic approximation of the American option price. Because the non-linear least squares regression we use to invert the options prices is an iterative method, relying on finite difference methods would make the whole procedure prohibitively time consuming. Instead, we rely on an analytic approximation proposed by Bates [6] which extends the work of Barone–Adesi and Whaley [5] and MacMillan [27] to the case where the underlying asset price follows a jump-diffusion process.

As pointed out by Bates [6], theoretical pricing of PHLX options must also take account of delivery lags. According to PHLX contractual agreements, an American option is settled in 5 days after expiration and 7 days (5 business days) after early exercise. Given these contractual specifications and the equilibrium exchange rate dynamics described by (2.43), one can arrive at the following approximation to the American put or call price:

$$\Psi_A(pc) = \begin{cases} \Psi_E(pc) + \left(\frac{S}{S^*}\right)^q \left[pc \cdot \left(e^{-r^*\tau_2}S^* - e^{-r\tau_2}K\right) - \Psi_E(pc)\right] & \text{if } S < S^* \\ pc \cdot \left(e^{-r^*\tau_2}S^* - e^{-r\tau_2}K\right) & \text{if } S > S^* \end{cases} \tag{2.47}$$

where pc takes the value 1 if the option is a call and -1 if it is a put, S is the value of the underlying exchange rate, K is the strike price, τ_2 is the delivery lag after early exercise, and S^* is the level of the exchange rate at which it is optimal to exercise early and hence satisfies the following equation:[16]

$$S^* = \text{argmax}_{S^*} \left(\frac{S}{S^*}\right)^q \left[pc \cdot \left(e^{-r^*\tau_2} S^* - e^{-r\tau_2} K\right) - \Psi_E(pc)\right] \tag{2.48}$$

while the parameter of curvature q satisfies:

$$\frac{1}{2}\sigma^2 q^2 + (r - r^* - \frac{1}{2}\sigma^2 - \lambda^*\kappa)q - \frac{r}{1 - e^{r\tau}} + \lambda^*(1+\kappa)^q - \lambda^* = 0 \tag{2.49}$$

and $\Psi_E(pc)$ is the corresponding European option price, as shown in Merton [28] and Jones [20]:

$$\Psi_E(pc) = e^{-r(\tau+\tau_1)} \sum_{n=0}^{\infty} P(n) \cdot pc \cdot \left[S e^{(r-r^*)\tau_1 + b_n\tau} \text{N}(pc \cdot d_{1n}) - K\text{N}(pc \cdot d_{2n})\right] \tag{2.50}$$

where:

$$P(n) = \frac{e^{-\lambda^*\tau}(\lambda^*\tau)^n}{n!} \tag{2.51}$$

and τ_1 is the deli ery lag after expiration, N(.) denotes the cumulative normal distribution and:

$$b_n = r - r^* - \lambda^*\kappa + n\log(1+\kappa)/\tau \tag{2.52}$$

$$\begin{aligned} d_{1n} &= \frac{\log(S/K) + (r - r^*)\tau_1 + (b_n + \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \\ d_{2n} &= \frac{\log(S/K) + (r - r^*)\tau_1 + (b_n - \sigma^2/2)\tau}{\sigma\sqrt{\tau}} \end{aligned} \tag{2.53}$$

A set of observed option prices $\{y_i\}_{i=1}^M$ will in all likelihood not fit the parametric specification. A set of three options along with the price of the underlying asset and the othe observable inputs to the option pricing function will uniquely determine $\hat{\lambda}^*$, $\hat{\kappa}$ and $\hat{\sigma}$. Any other set of three options will probably produce another set of $\hat{\lambda}^*$, $\hat{\kappa}$ and $\hat{\sigma}$. This is a problem common to all option pricing models. The likely reasons for this is that (i) the assumed model is misspecified, (ii) some options are thinly traded

[16] The first order condition of this maximization is equivalent to the smooth pasting condition derived in Bates.

and trades do not take place at "true values", (iii) trades take place at bid and ask prices instead of at the "true value," (iv) investors make errors in evaluating the "true value" of the options. All four reasons are likely play a role. We will assume that our specification is approximately correct and that the majority of the errors come from factors (ii), (iii) and (iv). We further assume that the deviations of the observed prices from the "true values" can be expressed in terms of a mean zero random variable that is independent of the arguments entering the option pricing formula.

By taking a set of options traded during a given day, we can "invert" these options to find the implicit parameters used by option traders during that day.[17] A consistent estimate of λ^*, κ and σ is obtained by the following non-linear least squares regression:[18]

$$\{\hat{\lambda^*}, \hat{\kappa}, \hat{\sigma}\} = \text{argmin}_{\lambda^*,\kappa,\sigma} \sum_{i=1}^{M} (y_i - \Psi_i(pc))^2 \tag{2.54}$$

Due to the over–abundance of data in many months, we construct a sub-sample of the data as follows. Our objective was to pick one trading day in a given month and take options trading in that day that all had the same expiration date. We also wanted at least 50 options. We start with the first trading day of a given month and find the expiration date that was less than 120 days away with the most options. If that set contained at least 50 options we used that set. Otherwise we proceeded to the next trading day.

2.3.3 Jump Expectation Estimates

For the \$/DM rate, the results of the monthly non-linear least squares regressions along with the asymptotic approximations to the standard errors are reported in Tables 2.5 through 2.9 appended to the end of this chapter, while the monthly time series for the jump expectation of the \$/DM exchange rate $\lambda^*\kappa$ is shown in Figure 2-1. Although the estimates for $\lambda^*\kappa$ are of a fairly reasonable order of magnitude, the estimates of the individual components λ^* and κ are somewhat less believable. In

[17]While conceptually straightforward, the process of inverting for the option-implied parameters is computationally challenging. The appendix describes some of the computational techniques used.
[18]Consistency is achieved as the number of options $M \rightarrow \infty$.

the space of λ^* and κ (for a fixed value of total variance[19]) there is an approximate hyperbolic ridge in the least squares objective function—the objective function is peaked for one value of $\lambda^* \times \kappa$.[20] The individual estimates of λ^* and κ, however, are not well estimated because the peak of the ridge is not very well identified—the objective function takes on similar value for all points on the ridge.[21]

The jump expectation is quoted in annualized terms. Hence, 0.05, or 5 per cent, corresponds to a expected depreciation of 0.425 per cent over a single month (λ^* is measured in a scale of probability per unit of time while κ has no time dimension). except for the case in which investors are risk-neutral, this is *not* the depreciation $\lambda\kappa$ due to jumps which is expected by market participants. Rather it is the equivalent martingale or risk-neutral jump expectation $\lambda^*\kappa$, and as such may be interpreted as the extra drift which the market demands along a no-jump path *in compensation for* the possibility of jump depreciation with expected value $\lambda\kappa$. Nonetheless, it will likely be the case that the true jump expectation and the equivalent martingale jump expectation are highly correlated, so that we may think of one as a proxy for the other. It is certainly true that the estimated equivalent martingale jump expectation is of an economically reasonable magnitude.

The time series of $\hat{\lambda}^*\hat{\kappa}$ seems to agree with the qualitative stories told about the dollar in the 1980's and 1990's. During the mid-1980's when the dollar was thought to be overvalued it was indeed the case that participants in the options market were expecting dollar depreciations due to jumps. These results are in general accord with those of Bates [4],[5] who also found substantial expected jump depreciations for his sample of 1984–87 using CME options on Deutschemark futures.[22] Interestingly,

[19]The objective function is maximized for one value of total variance, ν, almost regardless of the other parameters.

[20]The estimates for κ can sometimes be quite large especially when λ^* is small. This is not surprising since when λ^* is very small, κ does not much influence the pricing function. This is the reason why the standard errors can be very large for κ when λ^* is small.

[21]There were also two months out of the total 114 for which the non–linear least squares procedure did not converge. For the yen there was one month that did not converge.

[22]It is unfortunate that the PHLX options price data are completely contaminated and hence unusable for the months June–November 1985 which includes the post-Plaza Accord crash of the dollar. The expected depreciation for April 1985 is large, perhaps indicating anticipation of the eventual decision of the G-5 to bring the dollar down.

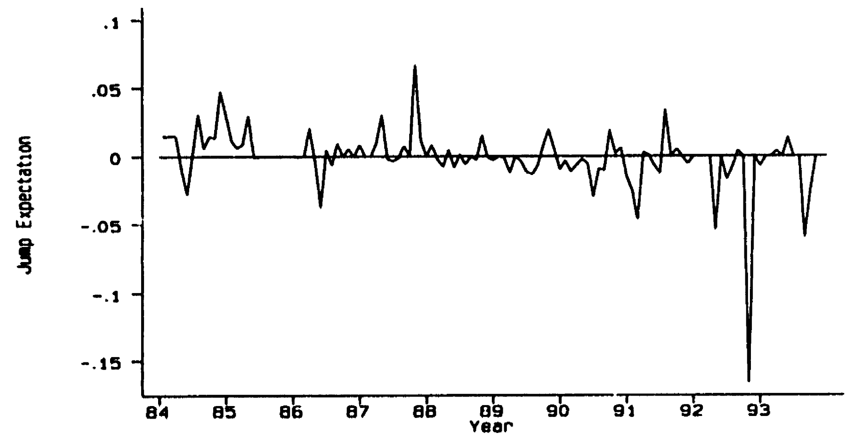

Figure 2-1: Jump Expectations $\lambda^*\kappa$ for the DM

occasional fears of dollar depreciation due to jumps persisted until the late 1980's, even though the dollar was on a steady downward path during this entire period. January 1990 marks the start of a new period during which there are many months when the market was calm, with essentially no expected depreciation due to jumps. These calm months are interspersed with months when there are fairly dramatic expected appreciations.[23] This later period coincides with the time that the economic problems associated with the reunification of Germany became apparent. Market observers claimed that the tight monetary policy implemented by the German government could not be sustained given the severe recession in the former East Germany. It was believed the Bundesbank would eventually have to lower its rate and would consequently send the Deutschemark tumbling.[24].

It is interesting to note that the two months with the most dramatic jump expectations are October 1987, when a crash occurred in the U.S. stock market, and October 1992, the period of the ERM crisis. In October 1987 there was a 6.7% expected

[23]Bates [6] estimates option-implied exchange rate parameters under an assumption of time-invariance of the parameters over the entire period 1984-1991 and concludes that there is no statistically significant jump component in the dollar-DM exchange rate. There are at least two ways to reconcile this result with ours and with Bates' results. First, because Bates assumes time-invariance of the parameters in [6], the fact that the jump expectations shown in Figure 2-1 are large and time-varying may be masked. Second. Bates allows for stochastic volatility of the continuous component of exchange rates while we do not, so that we may be imputing skewness of the exchange rate process to the jump component, when it is in fact due to stochastic volatility of the continuous component.

[24]See Dornbusch [14]

dollar depreciation while in October 1992 there was a 16.7% expected Deutschemark depreciation. In both cases, the currency of the country in crisis was expected to jump down.

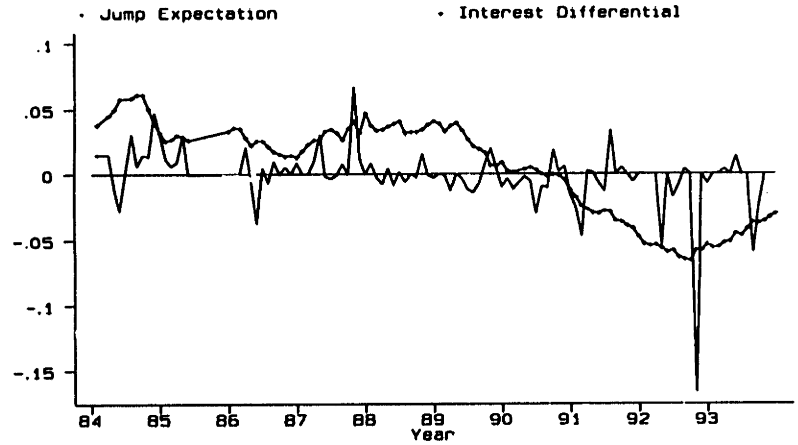

Figure 2-2: $\lambda^{*}\kappa$ for the DM & U.S.–German Interest Differential

The broad picture presented by the option-implied jump expectations is one of expected depreciation due to jumps during 1984–1989 and expected appreciation thereafter. As Figure 2-2 shows, this pattern of expected jump depreciation closely tracks the behavior of the U.S.–German one month interest differential. During the earlier period of expected depreciations, the interest differential is positive (i.e., a dollar forward *discount*), while in the latter period of expected appreciations the differential is negative. The correlation between the interest differential and the jump expectation is 0.30. As the first row of Table 2.1 indicates, a linear regression of the jump expectation on the interest differential yields a statistically significant slope coefficient of 0.19, indicating that when the interest differential widens by 100 annualized basis points, fears of jump depreciation increase by 19 basis points.[25]

As was pointed out in section 2.3.1, during a period when jumps are expected but few or none occur, regressions of *ex post* depreciation on the interest differential will be biased downward if the differential and the extra drift term d_i are negatively correlated. Since $-\lambda_i^{*}\kappa_i$ figures prominently in the drift term d_i, the fact that, for the

[25]Excluding the two outliers October 1992 and October 1987 does not change this significant positive relationship between the jump expectation and the interest differential. Throwing out the outliers gives a slope coefficient of 0.11 with an OLS standard error of 0.04.

Table 2.1: Option-Implied Parameters and the Interest Differential: \$/DM

LHS Variable	Constant	Slope
$\lambda_i^* \kappa_i$	-0.00196	0.19038
	(0.00209)	(.05666)
$\frac{1}{2}\sigma_i^2$	0.00782	-0.01384
	(0.00028)	(0.00767)
$\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2$	0.00586	0.17655
	(0.00210)	(0.05672)

Standard errors in parentheses. OLS regressions of the jump expectation, the Jensen-Ito term, and the sum of the two on the one-month U.S.-German Eurocurrency interest differential. Run on a sample of N=111 months during the 1984-1993 period, excluding June-November 1985 and months with thin trading in options.

\$/DM rate, $\lambda_i^* \kappa_i$ is strongly positively correlated with the interest differential means that the "peso problem" will be responsible for some of the forward discount bias *if* few or no jumps occur in-sample. Of course this does not necessarily mean that there will be too few jumps in the sample. In section 2.4 we will consider evidence which suggests that while jump expectations were economically significant during 1984-1993, "enough" jumps occurred in the \$/DM sample, so that the peso problem is not an explanation of the forward discount bias in this period.

Figure 2-3 gives the monthly time series of the option-implied continuous volatility σ_i. The volatility figure, which is annualized, varies from a high of about 18 per cent during the summer of 1985 to lows which are below 10 per cent during many months in the sample. The correlation of the Ito–Jensen term $\frac{1}{2}\sigma_i^2$ with the interest differential is small and negative, about -0.17, so that this term is unlikely to help explain the forward discount bias. This accords with the conclusion of Bekaert and Hodrick [11], who use GARCH estimation and Monte Carlo methods to simulate a time-varying $\frac{1}{2}\sigma_i^2$ and find that it has no explanatory power in forward discount regressions. Of course, as the regression (2.46) shows, it is the relationship of the composite term $\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2$ to the interest differential which is of ultimate interest. The final row of Table 2.1 shows that the positive relationship of the jump expectation to the interest differential dominates, as the composite term has a significant and positive slope coefficient in a regression on the differential.

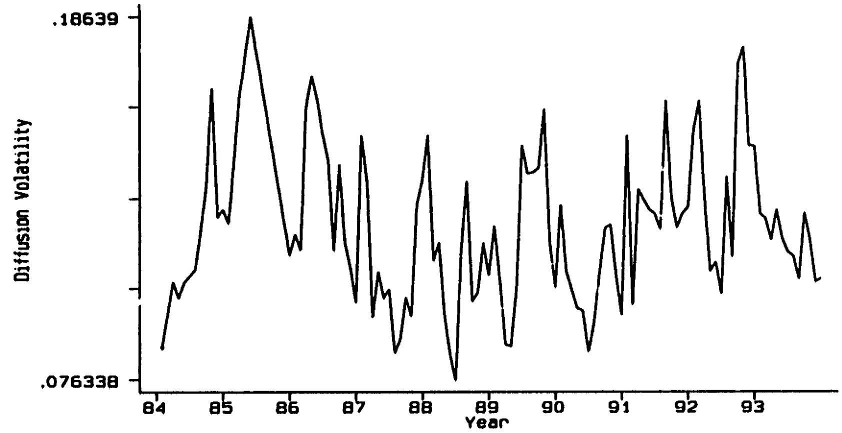

Figure 2-3: Diffusion Volatility for the DM

Turning to the $/Yen rate, Figure 2-4 displays the jump expectations while Tables 2.10 through 2.14 appended to this chapter show the estimates and standard errors. There are some similarities here to the $/DM case. In the period from 1984 to 1985 there are fears of dollar depreciations from jumps. Again it is possible that market participants feared the coordinated attempt by the G-5 to bring down the dollar that ultimately took place in late 1985. After this period, from 1986 to late 1988 there remained fears of a dollar depreciation although much smaller than what was expected earlier. From 1989 to late 1990 there are months with dollar expected depreciations interspersed with months of large expected dollar appreciations. After 1990, any similarity to the DM–dollar jump expectations end. From 1991 to mid 1992, there are consistent fears of dollar depreciation. Starting in mid–1992, trading in Yen options became so thin on the PHLX that for many months there was not a single day during which more than 50 options with the same maturity were traded. From July 1992 to December 1993, 10 of the 18 months did not have enough options. Given the thin-ness of trading, it is not surprising the estimates are so erratic even in months which meet our minimal standards for inclusion in the sample. It may be that, with such small trading volume, options traded at prices far from their "true" value.

Figure 2-5 displays the time series of the yen jump expectation together with the

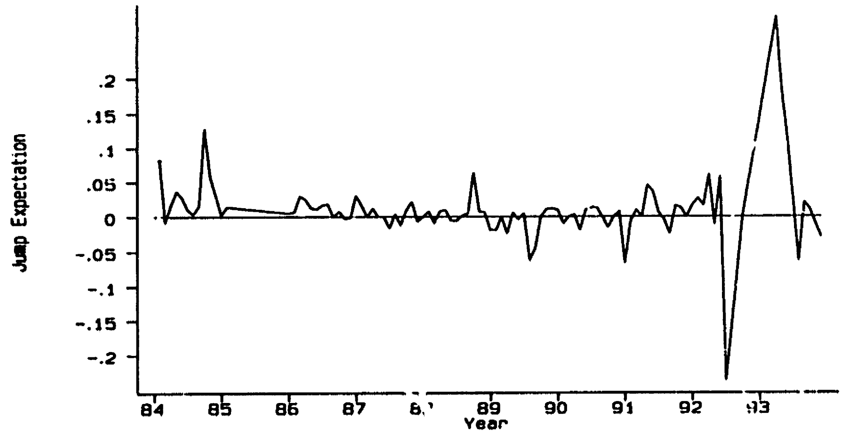

Figure 2-4: Jump Expectations $\lambda^*\kappa$ for the Yen

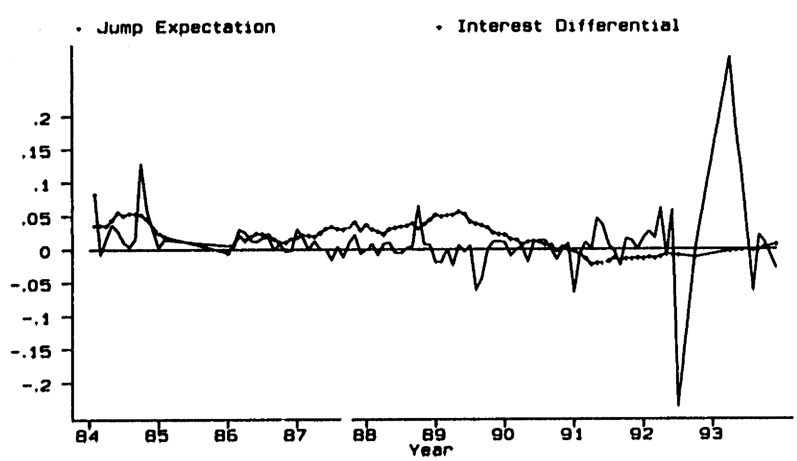

Figure 2-5: $\lambda^*\kappa$ for the Yen & U.S.–Japan Interest Differential

Table 2.2: Option-Implied Parameters and the Interest Differential: \$/Yen

LHS Variable	Constant	Slope
$\lambda_i^* \kappa_i$	0.01150	-0.05844
	(0.00649)	(0.23259)
$\frac{1}{2}\sigma_i^2$	0.00621	-0.02013
	(0.00041)	(0.01477)
$\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2$	0.01771	-0.07854
	(0.00652)	(0.23376)

Standard errors in parentheses. OLS regressions of the jump expectation, the Jensen-Ito term, and the sum of the two on the one-month U.S.-Japan Eurocurrency interest differential. Run on a sample of N=99 months during the 1984-1993 period, excluding June-November 1985 and months with thin trading in options.

U.S.-Japan one month interest differential. The \$/Yen rate presents a stark contrast to the case of the \$/DM rate. The jump expectation has a weak *negative* relationship with the interest differential. The correlation between the interest differential and the jump expectation is -0.03. The first row of Table 2.2 shows that a regression of the jump expectation on the interest differential yields a slope coefficient of -0.06 and an OLS standard error of 0.23.[26] Hence, we must conclude that, even during periods when few or no jumps occur, the "peso problem" jump expectation term cannot explain the forward discount in the \$/Yen rate.

The option implied estimates for the yen's continuous volatility are plotted in Figure 2-6. As is the case for the DM, the Jensen-Ito term $\frac{1}{2}\sigma_i^2$ has a negative though insignificant relationship with the interest differential. The regression in the middle row of Table 2.2 shows a slope coefficient of -0.020 with standard error of 0.015. The composite term $\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2$ also has a negative but weak relation to the interest differential, and hence can have no power in explaining the forward discount bias.

[26] This weak negative relationship holds even when the suspect estimates from post-1992 are excluded. In this case, the correlation is -0.0012 and the regression coefficient is -0.0013 with an OLS standard error of 0.12.

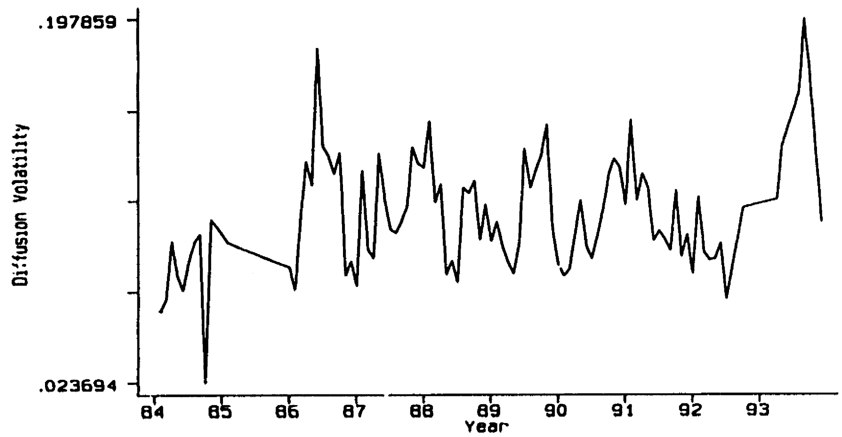

Figure 2-6: Diffusion Volatility for the Yen

2.4 A Peso Problem Resistant Test

The results of the previous section suggest that the peso problem may account for some of the forward discount bias in the \$/DM exchange rate, provided that few jumps occur in sample, but that it cannot account for any of the bias in the \$/Yen exchange rate.

The first row of Table 2.3 reports the results of a simple UIP regression of monthly *ex post* exchange depreciations on the one-month U.S.-German Eurocurrency interest differential and a constant. The estimation period is 1984-1993. Because we will be comparing the results from this benchmark regression to those of the peso problem resistant test, those months in 1985 for which options data are mismeasured (June-October) have been excluded. The benchmark is typical of forward discount regressions in that the interest differential coefficient $\beta = 0.245$ is less than one, although this is not statistically significant.

The peso problem resistant test involves running the following regression:

$$\Delta_C S_i + \lambda_i^* \kappa_i \tau_i + \frac{1}{2}\sigma_i^2 \tau = \alpha + \phi\tau + \beta(r_i - r_i^*)\tau + \epsilon_i \qquad (2.55)$$

The test is motivated by the observation that during peso problem periods most or all of the realized exchange depreciations are of the continuous type $\Delta_C S_t$, so that it is natural to conduct exchange pricing tests which address the dynamics of the

Table 2.3: Peso Problem Resistant Regression: $/DM Exchange Rate

	Jumps	MJD	MJA	$\alpha + \phi\tau$	β
Benchmark	-	-	-	0.00237	0.24500
				(0.00337)	(1.0802)
f = 8	0	-	-	0.00286	0.40179
				(0.00340)	(1.0914)
f = 5	1	0.0454	-	0.00247	0.32715
				(0.00347)	(1.1148)
f = 4	3	0.0454	0.0250	0.00306	0.02130
				(0.00352)	(1.1310)
f = 3.5	9	0.0149	0.0142	0.00372	-0.2512
				(0.00353)	(1.1355)
f = 3.25	20	0.0149	0.0137	0.00440	-0.31226
				(0.00348)	(1.1170)

Standard errors in parentheses. OLS regressions run on a sample of N=111 months during the 1984-1993 period, excluding June-November 1985 and months with thin trading in options. The Benchmark regression in the first row is the standard UIP regression of *ex post* monthly depreciation on the one-month U.S.-German Eurocurrency interest differential. The rest of the rows in the table are the peso problem resistant regressions performed with monthly depreciations de-jumped at filter level f. The column "Jumps" reports how many of 2,316 business daily returns in the sample were deemed to be jumps at filter level f. MJD and MJA report the smallest daily depreciation and the smallest daily appreciation deemed to have a "jump" component at filter level f.

exchange rate along the continuous path. While conceptually straightforward, the decomposition into continuous and jump components may be difficult to implement during periods when it is not clear whether jumps occurred or not. Jumps must be identified and extracted before the regression (2.55) can be run, since it involves only the continuous one month depreciations $\Delta_C S_i$.

We use daily exchange rate data from the Bank for International Settlements to identify jumps.[27] An ideal jump filter would use information about all of the important parameters of the exchange rate process, namely σ_i, λ_i, and κ_i, to identify which daily movements are most likely to be jumps and which are not. Unfortunately, while we have an option-implied σ_i, we have only an estimate of the equivalent martingale jump frequency λ_i^* and not the true distributional frequency λ_i. Further, as was discussed in section 2.3.3, although the estimate of the product $\lambda_i^* \kappa_i$ is well-identified and of an economically reasonable order of magnitude, the estimates of the individual parameter κ_i are somewhat unbelievable. There are many months in which κ_i is well in excess of 10 per cent in absolute value. Keep in mind that κ_i is the *instantaneous* per cent depreciation if a jump occurs, and that there were no one-day depreciations greater than 4.5 per cent in the entire 1984-1993 sample. Hence, using the option-implied κ_i in a dejumping exercise will give misleading results. In the absence of plausible estimates of λ_i and κ_i, we opted for the following compromise. The exchange rate depreciation for a given day was termed a "jump" if it satisfied the criterion

$$| \Delta_S^{Daily} - (r_i - r_i^*)\tau_d + (\frac{1}{2}\sigma_i^2 + \lambda_i^* \kappa_i)\tau_d | > \ f\sigma_i\sqrt{\tau_d} \qquad (2.56)$$

where τ_d is one day during the business week and three days for weekend depreciations which take place between mid-day Friday and mid-day Monday. The integer i indexes the month in which the given daily exchange depreciation took place. The quantity on the left hand side of the inequality is the daily exchange rate appreciation in

[27] The higher the frequency of the data, the easier it will be to identify jumps. In the limiting case of continuous time sample paths of the exchange rate, once could presumably distinguish the discontinuous jump movements from the continuous Wiener increments by inspection.

excess of the drift implied by our model.[28] The filter level f is a positive number. For $f = 2$, a daily depreciation will be called a jump if in absolute value it is two option-implied standard deviations away from zero, the mean of the continuous excess return. Thus, while we are not using any information about the individual jump process parameters κ_i and λ_i, our filter is nonetheless reasonable in the sense that it identifies daily movements which are unlikely to have come solely from the continuous component. Once the jumps have been identified, a de-jumped monthly return $\Delta_C S_i$ can be calculated. The assumption of a geometric jump-diffusion process allows us to add up the daily log exchange rate differences to obtain the monthly depreciation, excluding the excess return for any days in which jumps occurred.

The second row of Table 2.3 reports the result of regression (2.55) using filter level $f = 8$, or 8 option-implied standard deviations from zero. At this filter level there are no jumps in the sample, so that $\Delta_C S_i = \Delta S_i$ for all months. The interest differential coefficient is $\beta = 0.4018$, which is closer to one than is the case for the benchmark regression, although the estimate is not significantly different from either zero or one. Thus, if we believe that the option-implied jump expectations reflect fears of jumps so large that in fact none occurred during the 1984-1993 sample, we could conclude that the "peso problem" is responsible for a portion of the downward bias in the point estimate of β. Note that the option-implied estimates of κ_i do little to disabuse us of the idea that the expected jumps were of catastrophic size. The majority of the estimated κ_i's are in excess of 0.10 in absolute value, indicating *instantaneous* appreciations or depreciations in excess of 10 per cent if the jump occurs. This is at odds with the fact that were no daily movements in the 1984-1993 sample greater than 4.5 per cent.

The remaining rows of Table 8 give the results of regression (2.55) for progressively lower values of the filter level f. For each value of f the table reports how many of 2316 business daily depreciations in the sample were deemed to be jumps. The

[28]The drift is included for the sake of correctness. However, at daily frequency the drift will not be very important relative to the the continuous Wiener innovation, since the latter is of order $o(\sqrt{t})$ and the drift is of order $o(t)$. Note that the filter (2.56) implicitly sets the constant diffusion risk premium ϕ to be equal to zero.

columns MJD and MJA report the smallest daily depreciation and the smallest daily appreciation which were deemed to be jumps at filter level f. For example, there was only one daily movement in the sample more than 5 option-implied standard deviations away from zero. For $f = 5$, the β coefficient is still larger than in the benchmark regression, though not dramatically so.

For f of 4 and lower, the estimated β coefficient is *smaller* than in the benchmark regression. Hence, if it is the case that a filter level of 4 or smaller is "correct" in the sense that it identifies movements which are truly jumps, then the peso problem cannot account for any of the downward bias in the coefficient β. While this can hardly be the last word on the matter, it seems *a priori* reasonable to consider movements which are 3.25 option-implied standard deviations from zero as "jumps." If this is the case, we must conclude that, while jump expectations were significant during 1984-1993, "enough" jumps actually occurred, so that the peso problem is not a problem for this sample. This does not rule out the existence of sub-periods where jumps were expected but did not occur. Moreover, the demonstrated strong positive correlation of expected \$/DM jump depreciation with the interest differential indicates that the peso problem will indeed account for some of the forward discount bias during such sub-periods.

The \$/Yen rate poses an even greater challenge to the UIP intuition than does the \$/DM, in the sense that the benchmark β coefficient is in fact *negative*, and nearly significantly so, as shown in Table 2.4. Unfortunately, it has already been shown that, for the \$/Yen exchange rate, the composite option-implied term $\lambda_i^* \kappa_i + \frac{1}{2}\sigma_i^2$ has a slight negative correlation with the interest differential and hence will be unable to explain any of the substantial forward discount bias. This is true independent of the method used to de-jump the monthly returns. Table 2.4 confirms this. For each of the filter levels between $f = 3.25$ and $f = 8$ considered, the peso problem resistant β is in fact more *negative* than the benchmark value of $\beta = -3.0117$. In most of these cases the coefficient β is significantly different from one and very near to being significantly different from zero.

Table 2.4: Peso Problem Resistant Regression: $/Yen Exchange Rate

	Jumps	MJD	MJA	$\alpha + \phi\tau$	β
Benchmark	-	-	-	0.00962 (0.00423)	-3.0117 (1.7944)
f = 8	0	-	-	0.01118 (0.00434)	-3.1247 (1.8408)
f = 5	4	0.031	0.029	0.01172 (0.00440)	-3.7527 (1.8655)
f = 4	13	0.010	0.017	0.01055 (0.0041)	-3.0820 (1.7171)
f = 3.5	24	0.005	0.004	0.00995 (0.00399)	-3.1236 (1.6916)
f = 3.25	27	0.005	0.004	0.00970 (0.00393)	-3.2180 (1.6671)

Standard errors in parentheses. OLS regressions run on a sample of N=99 months during the 1984-1993 period, excluding June-November 1985 and months with thin trading in options. The Benchmark regression in the first row is the standard UIP regression of *ex post* monthly depreciation on the one-month U.S.-Japan Eurocurrency interest differential. The rest of the rows in the table are the peso problem resistant regressions performed with monthly depreciations de-jumped at filter level f. The column "Jumps" reports how many of 1,988 business daily returns in the sample were deemed to be jumps at filter level f. MJD and MJA report the smallest daily depreciation and the smallest daily appreciation deemed to have a "jump" component at filter level f.

2.5 Conclusion

The most interesting and robust empirical finding in this paper is the strong positive correlation between the monthly option-implied jump expectation and the one-month U.S.-German interest differential. This positive correlation implies that, during sample periods when few or no jumps occur, regressions of *ex post* depreciation on the interest differential will have coefficients biased downward. However, one cannot conclude that the peso problem causes bias in a regression over the entire 1984-1993 sample unless it is the case that too few jumps occurred during this sample. Moreover, the positive relationship between expected jump depreciation and the interest differential does not hold universally, as the \$/Yen case shows.

Because the final results of the peso problem resistant test are sensitive to the de-jumping method, it is inevitable that prior beliefs about what is a jump and what is not must enter our analysis. If one believes that option-implied jump expectations referred to events which were so rare that none or almost none occurred during 1984-1993, then the peso problem resistant regressions for the \$/DM exchange rate can be taken as evidence that the peso problem was a reality in this period. However, it may be more reasonable to identify daily movements 3.25 option-implied standard deviations from zero as "jumps." This leads to the conclusion that "enough" jumps occurred in the 1984-1993 period to render the peso problem a non-problem for the \$/DM exchange rate. For the case of the the \$/Yen rate, the peso problem must be deemed a non-problem regardless of the method of de-jumping which is used.

Appendix: Computational Considerations

In theory, the inversion process is straightforward given the setup above. The actual implementation of this procedure however presents many computational challenges and consumed most of our time. We spend some time here on the problems that came up and the method we used to tackle them. Some of the techniques we present should be useful even to options researchers who are not planning on inverting for jump diffusion parameters.

First, one must come to terms with the problem of calculating the European jump diffusion option price $\Psi_E(.)$ which is expressed in closed form only as an infinite series. This European formula is an integral part of the analytic approximations used for the American formula. The standard approach used to calculate this formula is to simply determine a cut off level for the number of terms in the sum based on the parameters. We found that such a method would require a very large number of terms and would make the inversion process extremely time consuming. We noticed, however, that the number of terms required for accuracy was much less for the puts compared to the calls when κ was positive. The reverse was true when κ was negative. One can take advantage of this to reduce the computation time for all options using put–call parity. We will return to this shortly. The reason why fewer terms are needed for the puts relative to the calls when $\kappa > 0$ can be understood by taking the simplified case of a pure jump process. The put price in this case (ignoring delivery lags) would be:

$$e^{-r\tau} \sum_{n=0}^{\infty} P(n) \left(K - e^{-\lambda^* \kappa\tau} S_0 (1+\kappa)^n\right)^+ \tag{2.57}$$

Each term of the sum inside the $(.)^+$ reflects the final payoff conditional on n jumps occurring. If κ is positive $1+\kappa$ is greater than one so that $(1+\kappa)^n$ is increasing in n—or in other words, each jump is pushing the option out of the money. So for sufficiently large M, $\left(K - e^{-\lambda^* \kappa\tau} S_0(1+\kappa)^n\right)^+$ will be equal to 0 for all $n > M$. It is also clear that deeper in the money puts require more terms since more jumps are needed until the put goes out of the money. On the other hand, the call price is equal to:

$$e^{-r\tau} \sum_{n=0}^{\infty} P(n) \left(e^{-\lambda^* \kappa\tau} S_0 (1+\kappa)^n - K\right)^+ \tag{2.58}$$

Thus when $\kappa > 0$, each jump pushes the option deeper *into* the money. This means that the payoff is increasing in n. The infinite sum will still converge since the probabilities are dying out at the rate of $1/n!$, but many more terms will be required before the product is insignificant. With the put and the call, the price can be written:

$$\sum_{n=0}^{\infty} h_n i_n \tag{2.59}$$

where h_n is the product of the probability and $e^{-r\tau}$ and i_n is the payoff at expiration. When κ is positive, h_n and i_n are both decreasing for the call while for the put although h_n is decreasing, i_n is increasing. So clearly, calculating the put in this case requires much fewer terms. A similar logic can be applied to show that the call is faster to converge when κ is negative. A similar argument holds for the case when there is diffusion on top of the jump process.

So to calculate a call price when $\kappa > 0$, one can first calculate the corresponding put price and use put-call parity to find the call price. When $\kappa < 0$ one can do the reverse. The put-call parity relation with proportional dividends r^* and delivery lag τ_1 can be written:

$$C(S,K) + e^{-r(\tau+\tau_1)}K = P(S,K) + e^{-r^*(\tau+\tau_1)}S \tag{2.60}$$

We then selected the number of terms to equal the minimum of 8 and $6 * \lambda^* * \tau$ plus 8 times the moneyness in excess of 0.05.[29] This procedure gave us values for the option price that were accurate to within 1×10^{-10} for all parameter values that we tried and typically required only 8 to 10 terms. We found that this technique had its greatest positive impact when we used the same basic technique to calculate the partial derivatives of the pricing formulas with respect to the jump diffusion parameters–a necessary step when actually doing the non-linear least squares minimization.

Second, to calculate the analytic approximation to the American price one must solve for S^* using equation 2.48. Again in theory this procedure is straightforward, but in practice requires great care. The second derivative of the objective function of S^*, (2.48), can be very erratic and necessitates the use of a bisection algorithm in combination with Newton-Raphson algorithm in solving for S^*. The first derivative is not even monotonic. In other words, the objective function (2.48) is not well approximated by a quadratic function. If one started with the standard Newton Raphson procedure without the bisection algorithm the procedure would not converge in many cases. The problem is especially acute for shorter term options.

Third, to do the actual parameter inversion, we use the Levenberg–Marquardt procedure that takes advantage of the approximate local quadratic property of the objective function around the maxima in a non-linear least squares regression.[30] This procedure however, requires that we be able to calculate the derivative of the function $\Psi_A(.)$ with respect to the parameters. Whether one takes this derivative analytically or calculates it numerically, one can save considerable time by noting the following. The partial derivative of Ψ_A with respect to the parameters is the sum of the derivative of Ψ_E with respect to the parameter and the partial derivative of the American premium, the second piece in the formula for Ψ_A, with respect to the parameters. The derivative of

[29]In other words, we added more terms if the option was more than five percent in the money. We did not take away terms if it was less than five percent in the money.

[30]To stem any possible confusion, note that here we are referring to the approximate quadratic property of the sum of squared residual objective function and not the objective function for S^* which we were referring to in the above paragraph.

the American premium, call it AP, with respect to the generic parameter β is:

$$\frac{\partial AP}{\partial \beta} = \left.\frac{\partial AP}{\partial \beta}\right|_{\text{Keeping } s^* \text{ fixed}} + \frac{\partial AP}{\partial S^*}\frac{\partial S^*}{\partial \beta} \tag{2.61}$$

All the terms involved are straightforward to calculate except the very last one, $\frac{\partial S^*}{\partial \beta}$. However, one finds that this term need not be calculated because $\frac{\partial AP}{\partial S^*}$ is equal to zero since S^* is the maximizer of AP. Thus:

$$\frac{\partial AP}{\partial \beta} = \left.\frac{\partial AP}{\partial \beta}\right|_{\text{Keeping } s^* \text{ fixed}} \tag{2.62}$$

This is a consequence of the envelope theorem. This fact greatly simplifies the calculations of the derivative.

Fourth, one can drastically reduce the required computing time by noting as did Bates[6] that S^* for a fixed τ,r and r^* is homogeneous in K. In other words, given the same τ,r and r^*, if S_1^* is the solution to the objective function (2.48) with K equal to K_1, then S_2^* for K equal to K_2 is equal to $S_1^* \frac{K_2}{K_1}$. Since we are using only one day's worth of data and one maturity for each given month, τ,r, and r^* will all be the same. So we only need to calculate one S^* for the puts and one S^* for the calls for each set of parameters that we try.

Fifth, we use the innovation of Bates [6] and minimize the least squares objective function by searching over the transformed variables:

$$\{\beta_1, \beta_2, \beta_3\} = \{\log(\nu), \mathrm{N}^{-1}(f), \log(1+\kappa)\} \tag{2.63}$$

where:

$$\begin{aligned} \nu &= \lambda^*(\log(1+\kappa))^2 + \sigma^2 \\ f &= \lambda^*(\log(1+\kappa))^2/\nu \end{aligned}$$

These last two parameters ν and f are the total variance of the process and the proportion of the variance coming from jumps respectively. This effectively reduces the dimension of the search because the estimate of ν takes on roughly the same value regardless of the other two parameters. Because the objective function has multiple minima, six starting guesses were used in each monthly regression to ensure that the global minimum was found.

Lastly, all the programming was coded in Fortran to make the procedures as fast as possible. We initially coded the whole procedure in the matrix programming language Matlab, but found that one set of regressions for one country would take a month to run.

Table 2.5: Deutschemark Option Implied Parameters: 1984-1985

	$\hat{\lambda^*\kappa}$	$\hat{\lambda^*}$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1984: 1	.01484 (.00669)	.08719 (.07636)	.17016 (.07920)	.08613 (.00234)	90
1984: 2	Did Not Converge	Did Not Converge	Did Not Converge	Did Not Converge	186
1984: 3	.01498 (.00193)	.00476 (.01064)	3.14405 (6.68264)	.10575 (.00082)	232
1984: 4	-.00960 (.00315)	.04837 (.02140)	-.19847 (.02693)	.10107 (.00144)	98
1984: 5	-.02795 (.00804)	.21475 (.11855)	-.13017 (.03535)	.10552 (.00183)	74
1984: 6	Did Not Converge	Did Not Converge	Did Not Converge	Did Not Converge	50
1984: 7	.03088 (.00855)	.20895 (.10361)	.14778 (.03362)	.10959 (.00270)	66
1984: 8	.00617 (.00296)	.00863 (.02291)	.71437 (1.55910)	.12175 (.00120)	92
1984: 9	.01465 (.00464)	.05411 (.03883)	.27080 (.10986)	.13588 (.00163)	200
1984:10	.01362 (.00179)	.00027 (.00029)	51.30559 (61.07282)	.16473 (.00092)	180
1984:11	.04755 (.02218)	.46536 (.40677)	.10219 (.04227)	.12575 (.00408)	106
1984:12	.03054 (.00863)	.17536 (.08457)	.17418 (.03569)	.12805 (.00292)	70
1985: 1	.01180 (.00265)	.03493 (.02530)	.33777 (.17165)	.12397 (.00103)	178
1985: 2	.00613 (.00146)	.00059 (.00107)	10.46990 (17.15566)	.14278 (.00097)	161
1985: 3	.00925 (.00264)	.00023 (.00119)	39.69226 (194.66454)	.16270 (.00148)	108
1985: 4	.03016 (.01891)	.14128 (.18842)	.21344 (.15230)	.17445 (.00392)	208
1985: 5	-.00040 (.00249)	.00042 (.00058)	-.93878 (6.23759)	.18639 (.00112)	159
1985: 6	NA	NA	NA	NA	NA
1985: 7	NA	NA	NA	NA	NA
1985: 8	NA	NA	NA	NA	NA
1985: 9	NA	NA	NA	NA	NA
1985:10	NA	NA	NA	NA	NA
1985:11	NA	NA	NA	NA	NA
1985:12	.00000 (.00163)	.00000 (.00299)	-.56572 (>999.0)	.11402 (.00129)	52

Non-linear least squares regression results. The PHLX options data from June to November 1985 is completely contaminated and unusable.

Table 2.6: Deutschemark Option Implied Parameters: 1986-1987

	$\hat{\lambda}^*\hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1986: 1	-.00043 (.00380)	.00135 (.01353)	-.31532 (.34790)	.12024 (.00200)	53
1986: 2	.00000 (.00278)	.00000 (.00350)	-.79293 (39.26578)	.11568 (.00162)	72
1986: 3	.02069 (.01839)	.00781 (.23834)	2.64869 (78.56580)	.15837 (.00297)	80
1986: 4	-.00403 (.00774)	.01936 (.05842)	-.20814 (.24846)	.16844 (.00244)	54
1986: 5	-.03729 (.01867)	.29739 (.29031)	-.12540 (.06140)	.16162 (.00293)	186
1986: 6	.00451 (.01234)	.00598 (.10801)	.75401 (11.57873)	.15083 (.00318)	66
1986: 7	-.00642 (.00282)	.02706 (.02083)	-.23729 (.08263)	.14274 (.00109)	142
1986: 8	.00953 (.02321)	.02432 (.48734)	.39158 (6.91109)	.11554 (.00375)	104
1986: 9	-.00023 (.00323)	.00027 (.00479)	-.84269 (3.07188)	.14148 (.00116)	74
1986:10	.00531 (.00137)	.00134 (.00324)	3.95616 (9.00849)	.11797 (.00066)	135
1986:11	-.00024 (.00149)	.00032 (.00068)	-.76092 (4.30859)	.11013 (.00086)	120
1986:12	.00849 (.00722)	.01332 (.08385)	.63757 (3.48390)	.09958 (.00221)	54
1987: 1	.00000 (.00135)	.00000 (.00143)	-.94836 (>999.0)	.15024 (.00100)	114
1987: 2	-.00001 (.00371)	.00001 (.00547)	-.83040 (79.36450)	.13676 (.00158)	68
1987: 3	.01001 (.01198)	.00317 (.26659)	3.15459 (261.41756)	.09536 (.00232)	64
1987: 4	.03031 (.02227)	.26422 (.42957)	.11470 (.10268)	.10875 (.00415)	74
1987: 5	-.00180 (.00221)	.00514 (.00778)	-.35018 (.10603)	.10106 (.00094)	64
1987: 6	-.00349 (.00473)	.01275 (.02132)	-.27387 (.09612)	.10354 (.00138)	106
1987: 7	-.00092 (.01066)	.00372 (.04996)	-.24731 (.46636)	.08463 (.00340)	96
1987: 8	.00772 (.01006)	.00183 (.24168)	4.22075 (552.29952)	.08862 (.00196)	59
1987: 9	.00000 (.00227)	.00000 (.00541)	-.43907 (177.24395)	.10107 (.00081)	85
1987:10	.06731 (.03825)	.96552 (.90560)	.06972 (.02595)	.09548 (.00660)	207
1987:11	.01234 (.01199)	.02030 (.19690)	.60778 (5.31675)	.12996 (.00248)	160
1987:12	.00000 (.00349)	.00000 (.00407)	-.87268 (550.46510)	.13706 (.00177)	56

Non-linear least squares regression results.

Table 2.7: Deutschemark Option Implied Parameters: 1988-1989

	$\lambda^* \hat{\kappa}$	λ^*	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1988: 1	.00830 (.01296)	.00359 (.16189)	2.30816 (100.44408)	.15023 (.00305)	75
1988: 2	-.00222 (.00584)	.00707 (.02299)	-.31387 (.20471)	.11247 (.00157)	124
1988: 3	-.00750 (.00351)	.03821 (.02674)	-.19636 (.04943)	.11763 (.00097)	54
1988: 4	.00451 (.00991)	.00325 (.16842)	1.38643 (68.78617)	.09590 (.00253)	68
1988: 5	-.00795 (.01452)	.12253 (.37374)	-.06487 (.08213)	.08375 (.00242)	64
1988: 6	.00141 (.01726)	.00151 (.43298)	.93783 (258.00673)	.07634 (.00349)	156
1988: 7	-.00573 (.00249)	.02338 (.01414)	-.24522 (.04644)	.11556 (.00105)	114
1988: 8	.00000 (.00271)	.00000 (.00335)	-.82776 (150.01432)	.13646 (.00101)	80
1988: 9	-.00240 (.02759)	.03317 (.58711)	-.07240 (.45082)	.09998 (.00487)	60
1988:10	.01577 (.01625)	.00641 (.37136)	2.46070 (140.05107)	.10255 (.00269)	112
1988:11	-.00065 (.00357)	.00271 (.01715)	-.23887 (.19786)	.11758 (.00117)	81
1988:12	-.00276 (.00389)	.00816 (.01395)	-.33858 (.10774)	.10791 (.00114)	80
1989: 1	-.00002 (.00571)	.00003 (.01053)	-.57049 (10.05064)	.12280 (.00142)	63
1989: 2	-.00113 (.00486)	.00412 (.02036)	-.27480 (.18302)	.10615 (.00124)	102
1989: 3	-.01204 (.00548)	.09562 (.06818)	-.12589 (.03608)	.08693 (.00163)	120
1989: 4	-.00004 (.00103)	.00004 (.00104)	-.98091 (.33548)	.08637 (.00052)	84
1989: 5	-.00345 (.00274)	.01170 (.01142)	-.29524 (.06005)	.10332 (.00084)	118
1989: 6	-.01138 (.00446)	.02998 (.01884)	-.37967 (.09755)	.14726 (.00120)	106
1989: 7	-.01358 (.01911)	.12088 (.32964)	-.11238 (.15023)	.13891 (.00280)	56
1989: 8	-.00694 (.00375)	.01366 (.01040)	-.50775 (.12397)	.13919 (.00110)	104
1989: 9	.00766 (.01267)	.01693 (.18861)	.45231 (4.29296)	.14053 (.00222)	160
1989:10	.01990 (.01263)	.05569 (.17253)	.35743 (.88607)	.15820 (.00195)	208
1989:11	.00514 (.00326)	.01170 (.03926)	.43905 (1.21001)	.11777 (.00078)	111
1989:12	-.00956 (.00926)	.00984 (.01054)	-.97222 (.11016)	.10422 (.00319)	62

Non-linear least squares regression results.

Table 2.8: Deutschemark Option Implied Parameters: 1990-1991

	$\hat{\lambda}^* \hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1990: 1	-.00338	.00756	-.44744	.12941	171
	(.00289)	(.00897)	(.16802)	(.00080)	
1990: 2	-.01102	.11507	-.09578	.10894	57
	(.02397)	(.51201)	(.21991)	(.00430)	
1990: 3	.14049	1.62874	.08626	.07188	104
	(.04097)	(1.23116)	(.04114)	(.00827)	
1990: 4	-.00205	.00260	-.78637	.09786	50
	(.00191)	(.00132)	(.90770)	(.00060)	
1990: 5	-.00536	.01969	-.27206	.09711	90
	(.00344)	(.02479)	(.18080)	(.00108)	
1990: 6	-.02970	.31751	-.09353	.08474	124
	(.01229)	(.25365)	(.03609)	(.00281)	
1990: 7	-.00939	.01104	-.85019	.09320	63
	(.00258)	(.00454)	(.13549)	(.00091)	
1990: 8	-.01010	.06434	-.15697	.10858	79
	(.00575)	(.09625)	(.15066)	(.00120)	
1990: 9	.01908	.16466	.11587	.12206	132
	(.02156)	(.42718)	(.17017)	(.00346)	
1990:10	.00195	.01902	.10265	.12339	57
	(.02960)	(.52756)	(1.29871)	(.00469)	
1990:11	.00613	.00667	.91910	.10769	87
	(.00268)	(.00820)	(.81890)	(.00075)	
1990:12	-.01488	.07115	-.20918	.09591	56
	(.02426)	(.65924)	(1.60055)	(.00358)	
1991: 1	-.02524	.12348	-.20445	.15027	118
	(.01286)	(.18296)	(.20026)	(.00248)	
1991: 2	-.04675	1.20746	-.03872	.09900	165
	(.07526)	(3.05838)	(.03600)	(.00749)	
1991: 3	.00254	.00296	.85849	.13413	304
	(.00241)	(.00386)	(.34841)	(.00103)	
1991: 4	.00099	.00100	.99304	.13096	149
	(.00336)	(.00425)	(.88923)	(.00115)	
1991: 5	-.00687	.02617	-.26240	.12813	184
	(.00949)	(.16277)	(1.27403)	(.00186)	
1991: 6	-.01243	.07889	-.15756	.12677	73
	(.00865)	(.13996)	(.17925)	(.00178)	
1991: 7	.03408	.58149	.05860	.12208	101
	(.06329)	(1.76487)	(.06938)	(.00693)	
1991: 8	.00000	.00000	3.59082	.16069	233
	(.00198)	(.00062)	(756.39840)	(.00083)	
1991: 9	.00516	.02604	.19819	.13138	91
	(.00497)	(.03771)	(.10481)	(.00114)	
1991:10	.00002	.00001	1.96951	.12261	152
	(.00398)	(.00229)	(47.61608)	(.00127)	
1991:11	-.00556	.01114	-.49923	.12680	206
	(.00775)	(.11378)	(4.43558)	(.00190)	
1991:12	.00002	.00001	3.92008	.12889	87
	(.01118)	(.00330)	(350.78826)	(.00296)	

Non-linear least squares regression results.

Table 2.9: Deutschemark Option Implied Parameters: 1992-1993

	$\lambda^* \hat{\kappa}$	λ^*	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1992: 1	.00003 (.00256)	.00003 (.00304)	.94545 (10.21535)	.15158 (.00101)	126
1992: 2	.00000 (.00272)	.00000 (.00126)	2.36471 (>999.0)	.16069 (.00096)	286
1992: 3	.00000 (.00198)	.00000 (.00199)	1.06865 (>999.0)	.13121 (.00069)	151
1992: 4	-.05395 (.06067)	.74915 (1.47157)	-.07202 (.06089)	.10904 (.00918)	122
1992: 5	.00000 (.00501)	.00000 (.00758)	.71729 (656.56762)	.11186 (.00142)	127
1992: 6	-.01702 (.05288)	.34640 (1.71758)	-.04913 (.09185)	.10228 (.00650)	62
1992: 7	-.00791 (.01211)	.04123 (.18133)	-.19183 (.55535)	.13790 (.00251)	151
1992: 8	.00397 (.00363)	.01467 (.01797)	.27059 (.08844)	.11351 (.00109)	68
1992: 9	.00000 (.00319)	.00000 (.00140)	2.49346 (>999.0)	.17208 (.00157)	482
1992:10	-.16642 (.25648)	2.52705 (6.07613)	-.06585 (.05708)	.17717 (.02382)	83
1992:11	.00000 (.00300)	.00000 (.00258)	1.27870 (333.24275)	.14722 (.00132)	121
1992:12	-.00691 (.01379)	.01103 (.21915)	-.62631 (11.20695)	.14696 (.00248)	74
1993: 1	.00000 (.00329)	.00000 (.00194)	1.87472 (>999.0)	.12674 (.00154)	89
1993: 2	.00021 (.00554)	.00010 (.00296)	2.19512 (9.96525)	.12529 (.00215)	138
1993: 3	.00374 (.00320)	.01450 (.01610)	.25814 (.07016)	.11871 (.00101)	75
1993: 4	.00000 (.00365)	.00000 (.00182)	2.18133 (>999.0)	.12791 (.00122)	77
1993: 5	.01398 (.00732)	.10291 (.09498)	.13582 (.05920)	.11908 (.00171)	71
1993: 6	.00000 (.00255)	.00000 (.00149)	1.85153 (>999.0)	.11491 (.00112)	69
1993: 7	.00000 (.00292)	.00000 (.00198)	1.59380 (>999.0)	.11363 (.00094)	95
1993: 8	-.05959 (.34878)	2.05367 (18.75430)	-.02902 (.09534)	.10670 (.02281)	116
1993: 9	-.02495 (.02896)	.19716 (.48754)	-.12654 (.17134)	.12706 (.00612)	75
1993:10	.00003 (.00382)	.00001 (.00243)	1.78686 (34.71422)	.11858 (.00132)	82
1993:11	.00000 (.00289)	.00000 (.00133)	2.35403 (>999.0)	.10579 (.00246)	53
1993:12	.00000 (.00273)	.00000 (.00252)	1.16309 (>999.0)	.10678 (.00105)	74

Non-linear least squares regression results.

Table 2.10: Yen Option Implied Parameters: 1984-1985

	$\hat{\lambda}^* \hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1984: 1	.08212 (.13291)	3.15364 (7.79196)	.02604 (.02223)	.05810 (.01428)	56
1984: 2	-.00742 (.00262)	.04428 (.02426)	-.16765 (.05233)	.06398 (.00148)	56
1984: 3	.01694 (.00593)	.00312 (.01137)	5.42420 (18.81884)	.09149 (.00355)	144
1984: 4	.03663 (.03501)	.74849 (1.17786)	.04894 (.03048)	.07520 (.00534)	72
1984: 5	.02739 (.01553)	.39900 (.42303)	.06864 (.03512)	.06815 (.00334)	68
1984: 6	.01047 (.01165)	.11341 (.22428)	.09232 (.08069)	.08142 (.00265)	72
1984: 7	.00370 (.00472)	.00081 (.05402)	4.56073 (298.09819)	.09071 (.00177)	52
1984: 8	.01572 (.01365)	.03255 (.24142)	.48312 (3.17482)	.09501 (.00330)	56
1984: 9	.12838 (.00364)	1.97365 (.12750)	.06505 (.00241)	.02369 (.00599)	58
1984:10	.05890 (.07608)	.87614 (1.84122)	.06722 (.05477)	.10204 (.01177)	57
1984:11	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1984:12	.00266 (.00254)	.00078 (.00597)	3.41530 (23.13685)	.09489 (.00101)	66
1985: 1	.01407 (.02233)	.20630 (.53834)	.06819 (.07006)	.09069 (.00383)	54
1985: 2	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1985: 3	Did Not Converge	Did Not Converge	Did Not Converge	Did Not Converge	56
1985: 4	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1985: 5	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1985: 6	NA	NA	NA	NA	NA
1985: 7	NA	NA	NA	NA	NA
1985: 8	NA	NA	NA	NA	NA
1985: 9	NA	NA	NA	NA	NA
1985:10	NA	NA	NA	NA	NA
1985:11	NA	NA	NA	NA	NA
1985:12	.00520 (.00224)	.00088 (.00242)	5.94526 (14.62967)	.07909 (.00111)	73

Non-linear least squares regression results. The PHLX options data from June to November 1985 is completely contaminated and unusable.

Table 2.11: Yen Option Implied Parameters: 1986-1987

	$\hat{\lambda}^*\hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1986: 1	.00690 (.00269)	.00179 (.00892)	3.85742 (18.26340)	.06842 (.00130)	67
1986: 2	.02962 (.02348)	.08305 (.54621)	.35659 (2.06382)	.10324 (.00367)	183
1986: 3	.02500 (.00693)	.00990 (.05928)	2.52690 (14.77603)	.13005 (.00205)	86
1986: 4	.01261 (.01688)	.01837 (.30525)	.68611 (10.49026)	.11860 (.00295)	61
1986: 5	.01135 (.00834)	.00425 (.06009)	2.66946 (35.90711)	.18439 (.00192)	118
1986: 6	.01659 (.01960)	.06838 (.32138)	.24255 (.85664)	.13759 (.00336)	70
1986: 7	.01798 (.01115)	.00943 (.13700)	1.90733 (26.56515)	.13295 (.00239)	84
1986: 8	.00000 (.00055)	.00000 (.00084)	-.67848 (640.98433)	.12405 (.00051)	94
1986: 9	.00786 (.01193)	.00838 (.15237)	.93775 (15.66880)	.13427 (.00186)	59
1986:10	-.00255 (.00337)	.01015 (.01746)	-.25093 (.12168)	.07521 (.00115)	50
1986:11	-.00198 (.00363)	.00289 (.00860)	-.68369 (.84117)	.08195 (.00125)	81
1986:12	.03041 (.02630)	.34378 (.52862)	.08845 (.06020)	.07022 (.00674)	66
1987: 1	.01696 (.00434)	.00298 (.02693)	5.69500 (50.13680)	.12558 (.00138)	67
1987: 2	.00000 (.00641)	.00001 (.01365)	-.49415 (56.30940)	.08749 (.00321)	51
1987: 3	.01201 (.00950)	.07528 (.13427)	.15953 (.15891)	.08354 (.00309)	63
1987: 4	.00000 (.00237)	.00000 (.00312)	-.77584 (>999.0)	.13402 (.00128)	71
1987: 5	.00000 (.00540)	.00001 (.00734)	-.75898 (29.84215)	.11185 (.00171)	57
1987: 6	-.01554 (.00773)	.08151 (.06710)	-.19066 (.07090)	.09712 (.00214)	95
1987: 7	.00384 (.00802)	.00232 (.12980)	1.65217 (88.93422)	.09550 (.00199)	87
1987: 8	-.01149 (.00454)	.06805 (.04984)	-.16882 (.06261)	.10124 (.00135)	114
1987: 9	.00954 (.00927)	.03510 (.10919)	.27164 (.58256)	.10827 (.00296)	61
1987:10	.02150 (.03350)	.00953 (.59411)	2.25621 (137.15446)	.13709 (.00531)	97
1987:11	-.00629 (.00385)	.01649 (.01440)	-.38148 (.11520)	.12894 (.00146)	71
1987:12	.00000 (.00217)	.00000 (.00321)	-.69553 (201.59369)	.12691 (.00123)	87

Non-linear least squares regression results.

Table 2.12: Yen Option Implied Parameters: 1988-1989

	$\lambda^* \hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1988: 1	.00786 (.01487)	.00219 (.20083)	3.59400 (323.39196)	.14967 (.00317)	116
1988: 2	-.00839 (.00725)	.07202 (.10621)	-.11656 (.07669)	.11009 (.00159)	61
1988: 3	.00784 (.01467)	.00614 (.24201)	1.27671 (47.95901)	.11899 (.00230)	58
1988: 4	.00954 (.01054)	.05471 (.25390)	.17431 (.61913)	.07569 (.00256)	70
1988: 5	-.00521 (.00303)	.03709 (.03411)	-.14055 (.05042)	.08210 (.00071)	97
1988: 6	-.00588 (.00365)	.06010 (.06369)	-.09777 (.04470)	.07180 (.00094)	59
1988: 7	.00173 (.00729)	.00054 (.07713)	3.22901 (450.48254)	.11724 (.00192)	52
1988: 8	.00473 (.01371)	.00877 (.25469)	.53974 (14.13426)	.11456 (.00256)	123
1988: 9	.06478 (.08370)	.03314 (3.88565)	1.95459 (226.64914)	.12064 (.00599)	96
1988:10	.00773 (.03057)	.00394 (.82853)	1.96253 (405.14342)	.09211 (.00491)	86
1988:11	.00658 (.00722)	.00933 (.12720)	.70476 (8.84578)	.10909 (.00149)	95
1988:12	-.01910 (.01099)	.24786 (.25040)	-.07705 (.03447)	.09135 (.00175)	88
1989: 1	-.01900 (.01699)	.25313 (.39895)	-.07506 (.05172)	.10074 (.00238)	68
1989: 2	.00000 (.00059)	.00000 (.00139)	-.44272 (>999.0)	.08915 (.00040)	117
1989: 3	-.02360 (.01414)	.29758 (.31677)	-.07931 (.03721)	.08171 (.00294)	94
1989: 4	.00598 (.00629)	.01189 (.18072)	.50306 (7.12420)	.07589 (.00115)	64
1989: 5	-.00374 (.00333)	.02721 (.03179)	-.13745 (.04029)	.08998 (.00099)	63
1989: 6	.00534 (.03231)	.00125 (.53078)	4.28192 (>999.0)	.13639 (.00596)	85
1989: 7	-.06112 (.09608)	1.24579 (2.91522)	-.04906 (.03774)	.11725 (.01090)	111
1989: 8	-.04394 (.06987)	.75621 (1.93430)	-.05811 (.05651)	.12563 (.00748)	58
1989: 9	-.00001 (.00490)	.00001 (.00750)	-.67954 (15.19774)	.13282 (.00125)	87
1989:10	.01087 (.02830)	.01270 (.42155)	.85611 (26.21976)	.14775 (.00415)	78
1989:11	.01156 (.00225)	.07254 (.03561)	.15942 (.04911)	.09635 (.00075)	67
1989:12	.00997 (.01308)	.08570 (.20233)	.11638 (.12369)	.08039 (.00469)	57

Non-linear least squares regression results.

Table 2.13: Yen Option Implied Parameters: 1990-1991

	$\lambda^{*}\hat{\kappa}$	λ^{*}	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1990: 1	-.00950 (.00343)	.07633 (.05173)	-.12446 (.04025)	.07486 (.00094)	100
1990: 2	.00000 (.00027)	.00000 (.00031)	-.87911 (>999.0)	.07794 (.00039)	69
1990: 3	.00346 (.00068)	.00634 (.00400)	.54601 (.27202)	.09514 (.00050)	65
1990: 4	-.01859 (.05183)	.19674 (1.25051)	-.09451 (.33883)	.11107 (.00598)	56
1990: 5	.01043 (.01193)	.03152 (.26171)	.33088 (2.37447)	.08906 (.00229)	86
1990: 6	.01308 (.00928)	.07740 (.23832)	.16893 (.40099)	.08321 (.00190)	56
1990: 7	.01283 (.01682)	.04299 (.40429)	.29853 (2.41862)	.09446 (.00230)	50
1990: 8	-.00010 (.00301)	.00013 (.00304)	-.78554 (4.82423)	.10660 (.00099)	108
1990: 9	-.01497 (.02092)	.15679 (.40698)	-.09550 (.11601)	.12318 (.00372)	66
1990:10	.00012 (.00243)	.00008 (.00176)	1.64992 (6.38384)	.13129 (.00108)	57
1990:11	.00798 (.00501)	.01273 (.02515)	.62663 (.88381)	.12692 (.00118)	59
1990:12	-.06525 (.05253)	1.18222 (1.48748)	-.05519 (.02558)	.10876 (.00699)	52
1991: 1	-.00701 (.00399)	.03341 (.03971)	-.20997 (.13074)	.14991 (.00111)	98
1991: 2	.00976 (.02630)	.10095 (.56309)	.09663 (.28090)	.11124 (.00365)	67
1991: 3	.00020 (.00263)	.00007 (.00116)	2.81099 (8.97184)	.12399 (.00107)	82
1991: 4	.04602 (.09270)	.84656 (2.78979)	.05436 (.06987)	.11721 (.00980)	85
1991: 5	.03627 (.02345)	.47701 (.71177)	.07605 (.06458)	.09150 (.00320)	59
1991: 6	.00619 (.00412)	.01847 (.02294)	.33513 (.20759)	.09635 (.00107)	62
1991: 7	-.00370 (.00638)	.01282 (.11647)	-.28863 (2.13481)	.09232 (.00150)	50
1991: 8	-.02388 (.01700)	.29460 (.36249)	-.08106 (.04289)	.08697 (.00350)	62
1991: 9	.01572 (.01910)	.00899 (.04433)	1.74951 (6.83855)	.11575 (.00297)	106
1991:10	.01264 (.00891)	.10296 (.17847)	.12274 (.13266)	.08400 (.00175)	50
1991:11	.00001 (.00441)	.00002 (.00774)	.62737 (27.37423)	.09461 (.00136)	110
1991:12	.01637 (.00350)	.02120 (.01537)	.77220 (.41439)	.07605 (.00090)	77

Non-linear least squares regression results.

Table 2.14: Yen Option Implied Parameters: 1992-1993

	$\hat{\lambda}^* \hat{\kappa}$	$\hat{\lambda}^*$	$\hat{\kappa}$	$\hat{\sigma}$	Obs.
1992: 1	.02642 (.00557)	.07248 (.04426)	.36449 (.16151)	.11287 (.00135)	67
1992: 2	.01677 (.01418)	.08418 (.23841)	.19923 (.41017)	.08618 (.00249)	53
1992: 3	.06118 (.04152)	1.46273 (1.60499)	.04182 (.01767)	.08236 (.00536)	60
1992: 4	-.00999 (.01806)	.08510 (.54355)	-.11742 (.53892)	.08291 (.00297)	63
1992: 5	.05827 (.08929)	1.07681 (3.22003)	.05411 (.07956)	.09023 (.01050)	78
1992: 6	-.23637 (.60844)	11.46564 (44.50849)	-.02062 (.02708)	.06379 (.05437)	79
1992: 7	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1992: 8	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1992: 9	.00147 (.01320)	.00020 (.00344)	7.36968 (60.87501)	.10741 (.00513)	91
1992:10	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1992:11	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1992:12	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1993: 1	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1993: 2	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1993: 3	.28880 (.18452)	9.98507 (11.88631)	.02892 (.01639)	.11136 (.01132)	50
1993: 4	.18450 (.08072)	3.49497 (3.53775)	.05279 (.03157)	.13775 (.00567)	73
1993: 5	.11460 (.35797)	.09908 (21.53421)	1.15672 (247.81432)	.14630 (.01511)	51
1993: 6	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1993: 7	-.06242 (.11364)	.23227 (2.84634)	-.26875 (2.81383)	.16269 (.01232)	51
1993: 8	.02087 (.09179)	.06251 (1.76734)	.33388 (7.98880)	.19786 (.00852)	52
1993: 9	.01038 (.01497)	.00430 (.13419)	2.41485 (72.12596)	.16930 (.00188)	56
1993:10	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0
1993:11	-.02878 (.01131)	.03301 (.04712)	-.87190 (.94528)	.10029 (.00157)	64
1993:12	Insufficient Data	Insufficient Data	Insufficient Data	Insufficient Data	0

Non-linear least squares regression results.

Bibliography

[1] M. Adler and B. Dumas, "International Portfolio Choice and Corporation Finance: A Synthesis," *Journal of Finance* **38**, June 1983, 925–984.

[2] C. Ahn and H. Thompson, "Jump Diffusion Processes and the Term Structure of Interest Rates," *Journal of Finance* **43**, March 1988, 155–174.

[3] V. Akgiray and G.G. Booth, "Mixed Diffusion–Jump Process Modeling of Exchange Rate Movements," *Review of Economics and Statistics* **70**, November 1988, 638–647.

[4] D. Backus, A. Gregory, and C. Telmer, "Accounting for Forward Rates in Markets for Foreign Currency," *Journal of Finance*, **48**, December 1993, 1887–1908.

[5] Barone–Adesi, Giovanni and Whaley, Robert E., "Efficient Analytic Approximation of American Option Values," *Journal of Finance*, **42**, June 1987, 301–320.

[6] D. Bates, "The Crash of 87: Was It Expected? The Evidence from Option Markets," *Journal of Finance*, July 1991, 1009–1044.

[7] D. Bates, "Crashes, Options, and International Asset Substitutability," Princeton University Ph.D. dissertation, January 1989.

[8] D. Bates, "The Crash Premium: Option Pricing Under Asymmetric Processes, with Applications to Options on Deutschemark Futures," Working Paper 36–88, University of Pennsylvania, Rodney L. White Center, 1988.

[9] D. Bates, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in PHLX Deutschemark Options," National Bureau of Economic Research Working Paper No. 4596, December 1993.

[10] D. Bates, "Distributional Abnormalities Implicit in Foreign Currency Futures Options," Wharton School Working Paper, 1995.

[11] G. Bekaert and Robert J. Hodrick, "On Biases in the Measurement of Foreign Exchange Risk Premiums," *Journal of International Money and Finance*,**12**, 115–138.

[12] J. Bodurtha and G. Courtadon, "Tests of the American Option Pricing Model in the Foreign Currency Option Market," *Journal of Financial and Quantitative Analysis*, **22**, 1987, 153–167.

[13] J. Cox, J. Ingersoll, S. Ross, "An Intertemporal General Equilibrium Model of Asset Prices," *Econometrica* **36**, 1985, 363–384.

[14] R. Dornbusch, "Why the Mighty Mark is a Sitting Duck," *Business Week* **30**, March 7, 1994.

[15] E.F. Fama, "Forward and Spot Exchange Rates," *Journal of Monetary Economics* **14**, November 1984, 319–338.

[16] W. Fleming and R. Rishel, *Deterministic and Stochastic Optimal Control*, Springer Verlag, New York, 1975.

[17] J. Frankel and K. Froot, "The Forward Discount Bias: Is it an Exchange Risk Premium?" *Quarterly Journal of Economics*, February 1989, 139-161.

[18] J. Harrison and D. Kreps, "Martingales and Multiperiod Securities Markets," *Journal of Economic Theory* **20**, 1979, 381–408.

[19] P. Jorion, "On Jump Processes in the Foreign Exchange and Stock Markets," *Review of Financial Studies* **1**, 1988, 427–445.

[20] E.P. Jones, "Options Arbitrage and Strategy with Large Price Changes," *Journal of Financial Economics*, **13**, 1984, 91–113.

[21] W. Krasker, "The 'Peso Problem' in Testing the Efficiency of Forward Exchange Markets," *Journal of Monetary Economics* **6**, 1980, 269–276.

[22] K. Lewis, "Was There a 'Peso Problem' in the U.S. Term Structure of Interest Rates: 1979–1982?," *International Economic Review*, February 1991, 159–173.

[23] K. Lewis, "Can Learning Affect Exchange-Rate Behavior? The Case of the Dollar in he Early 1980's," *Journal of Monetary Economics*, **23**, 1989, 79–100.

[24] K. Lewis, "Puzzles in International Financial Markets," forthcoming, *Handbook of International Economics*, 1995.

[25] J.S. Lizondo, "Foreign Exchange Futures Prices Under Fixed Exchange Rates," *Journal of International Economics*, **14**, 1983, 69–84.

[26] R. Lyons, "Tests of the Foreign Exchange Risk Premium Using the Expected Second Moments Implied by Option Pricing," *Journal of International Money and Finance* **7**, 1988, 91–108.

[27] MacMillan, L.W., "Analytic Approximation for the American Put," *Advances in Futures and Options Research*, **1**, 1986, 119–139.

[28] R. Merton, "Option Pricing When Underlying Stock Returns are Discontinuous," *Journal of Financial Economics*, **3**, 125–144.

[29] T. Reitz, "The Equity Risk Premium: A Solution," *Journal of Monetary Economics* **22**, 1989, 117–131.

[30] A.L. Tucker and L. Pond, "The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes," *Review of Economics and Statistics* **70**, November 1988, 638–647.

Chapter 3

Intervention and Jumps in the Dollar Exchange Rate

3.1 Introduction

Floating exchange rates take extraordinary movements, or "jumps," from time to time. In a colloquial sense, "jumps" are the infrequently observed daily foreign exchange returns of 2, 3, or even 5 percent which occur in response to extraordinary news events. The term "jump" can also be given a more precise statistical definition as a return realization which is so large that it is unlikely to have come from a stationary normal distribution. For some periods and currencies, returns exhibit leptokurtosis, indicating that outliers may be exceptionally large appreciations *or* depreciations. Early studies documenting leptokurtosis in high-frequency FX returns include Westerfield [33], and Friedman and Vandersteel [21]. Skewness has also been found in FX returns by Calderon-Rossell [11] and So [31], indicating occasional directional bias to the extreme exchange rate movements.

The nonparametric evidence of skewness and leptokurtosis has led to the estimation of parametric models which can account for these features of the data, including (i) stationary models with fat tails, such as the stable Paretian, (ii) models with time-varying moments, most notably the autoregressive conditional heteroskedasticity (ARCH) process, which as Engle [18] has shown, can exhibit leptokurtosis, and (iii) mixed jump-diffusion processes, in which exchange rate returns are the sum of a continuous component and a discontinuous "jump."

Recent work points toward a tentative consensus in favor of the mixed jump-diffusion process as a model of FX returns. Akgiray and Booth [2] and Tucker and Pond [32] find that the jump-diffusion process dominates the pure diffusion and stable Paretian models in explaining the returns of the major trading currencies during the floating rate period. Studies such as Engle and Bollerslev [19], and Hsieh [23] find that, while there are significant ARCH effects, they do not completely account for the observed discontinuities in FX returns.[1] In an important study, Jorion [25] nests the ARCH and jump specifications in a single model. He finds that the ARCH-jump model dominates the pure ARCH model in explaining floating exchange rates.[2]

The post-1973 regime of floating is best characterized as a "dirty float," in which exchange markets are subject to occasional interventions by national governments. Given the empirical regularity of exchange rate jumps, it is interesting to examine the relationship between jumps and U.S. intervention activity. Jorion [25] found that CRSP stock index returns do not exhibit the same marked jump behavior as exchange rate returns.[3] He suggested that government intervention in the foreign exchange markets (which has no analogue in the U.S. equity markets), may be responsible for jumps. Dominguez [14] notes that average daily FX volume in April 1992 was $192 billion, while the average daily intervention operation was on the order of $350 million. Although the amounts of currency actually bought and sold in interventions are small relative to the total market volume, there is a school of thought which suggests that, provided the intervention is sensitively timed, it can still have large

[1] See Bollerslev, Chou, and Kroner [10] for a survey of ARCH estimation of exchange rate returns.

[2] Option pricing studies provide further evidence that jumps are important in FX markets. For example, Bodurtha and Courtadon [9] find that the simple Black-Scholes option pricing formula makes systematic errors when used to price out-of-the-money foreign exchange options, and that those errors are consistent with market expectations of large jumps in the underlying currency. In a series of papers, Bates [4], [5], [6] ,[7] shows that option prices can be used to uncover market expectations of jumps and that such expectations are significant for the dollar-DM exchange rate in the early 1980's.

[3] Note that "jump behavior" does not necessarily imply greater volatility, as measured by the standard deviation of returns. For example, the annualized standard deviations of weekly CRSP stock returns is, as noted by Jorion, about 15%, while for the weekly $/DM returns it is 10%. Nonetheless, exchange rates are better characterized by "jump behavior" in the sense that there are occasional returns which are very large *compared* to the returns in normal diffusion, or "non-jump" weeks.

effects.[4] Another possibility is that, rather than *causing* jumps, intervention happens *in response* to jumps, presumably in an attempt to offset them. This is an example of the so-called "leaning against the wind" which governments profess to practice in their FX interventions. Data on daily U.S. intervention activity, which is now being made publicly available to researchers with a one-year lag, allows a direct examination of the relationship between intervention and jumps in exchange rates. A simultaneity problem makes empirical work in this area particularly challenging. If the government intervenes in support of the dollar precisely when some separate factor is operating to depreciate the dollar, it may be difficult to discern the effect on the FX return which is due solely to intervention. Empirical results on the return-intervention relationship must be interpreted in light of this potential simultaneity.

The objectives of this paper are three-fold: (i) to affirm that "jump" behavior is important in explaining the $/DM and $/Yen exchange rates for the sample period 1980-1993, (ii) to identify jumps in the daily and weekly FX return time series, and (iii) to explore the relationship between jumps in the exchange rate and U.S. intervention activity. In section 3.2, nonparametric evidence of jumps in high frequency exchange returns is presented, and the intervention data are also summarized. In section 3.3, a two-jump-ARCH model is fitted to the exchange rate data. It is shown that jumps are important in explaining skewed and leptokurtotic behavior which cannot be accounted for by ARCH effects. The two-jump-ARCH model extends previous applications of the jump model to FX returns by allowing jumps to be either depreci-

[4]For a discussion of the fine art of timing intervention operations, see Mulford [30]. Skeptics of the efficacy of FX intervention note that such intervention is not only small but is routinely sterilized. That is, sales of foreign exchange are offset by open market purchases of domestic bonds, leaving the domestic money base unaffected. Nonetheless, there are at least two channels through which even sterilized interventions might affect the exchange rate. One, the so-called portfolio balance channel, hinges on the imperfect substitutability of domestic and foreign bonds. Under imperfect substitability, sterilized intervention operations lead to a rebalancing of portfolio shares between foreign and domestic bonds. In order for this rebalancing to take place in equilibrium, the exchange rate or the interest differential or both must change. See, for example, Dominguez and Frankel [16]. A second possible channel is the so-called expectations channel. Current exchange rates depend on expectations of future rates. Thus, intervention might affect the current exchange rate by affecting expectations, for example, expectations of future monetary policy. See Kaminsky and Lewis [27] and Ghosh [22] for evidence that such "signalling" of future monetary policy is important. Edison [17] gives a very thorough survey of the literature on intervention efficacy in general.

ations or appreciations. Section 3.4 uses a maximum likelihood technique to identify jumps in the data. This technique characterizes "jumps" as being daily movements in excess of about 1% or, at weekly frequency, movements in excess of 2%. In Section 3.5, the relationship between exchange rate movements and intervention is examined. There is strong evidence of "leaning against the wind" behavior: the U.S. intervenes in an attempt to smooth out exchange rate fluctuations, buying dollars during and after a dollar depreciation and selling dollars during and after an appreciation. This "leaning against the wind" takes place in response to small, or "non-jump" exchange rate movements, and seems to have a statistically significant effect in the desired direction at both the weekly and daily frequency, although this effect operates with a lag. There seems to be no statistically significant relationship between "jump" movements and intervention activity. The statistical insignificance of intervention in response to jumps may well be a result of simultaneity bias. For example, it may well be the case that, in the days around a dollar sale, some factor separate from the intervention operates to appreciate the dollar. Taken together, the separate factor and the intervention "wash out" so that there is no statistically significant net effect on the probability of a jump in the exchange rate. However, had the dollar sale *not occurred*, there *would* be a higher probability of a jump appreciation due to the effect of the separate factor. Thus, the conclusion that intervention in response to jumps is ineffective, and hence should not be pursued, is not necessarily warranted. The conclusions of this study are in the same spirit as those of recent work by, among others, Dominguez and Frankel [15], who also find, using similar data, that intervention can play a limited but important role in influencing the FX markets. This conclusion contrasts with earlier work from the 1980's, as typified by the Jurgensen report [26], which concluded that sterilized FX intervention has little effect on exchange rates.

3.2 Data

The sample of business daily \$/DM and \$/Yen exchange rates is from the Bank for International Settlements and covers the period from January 4, 1980 to Decmber, 31, 1993. The DM data are sampled at 1 p.m. Frankfurt time and the Yen data are Tokyo closing rates. Figure 3-1 shows the two time series. For purposes of this study, the sample has been subdivided into three regimes. Regime 1 runs until January 31, 1985, and includes the "laissez faire" period of 1982-1984 in which the U.S. pursued an explicit policy of nonintervention in the exchange markets.[5] Perhaps coincidentally, the dollar appreciated strongly against most of the major trading currencies during this time. The dollar reached a peak against the Deutschemark in February of 1985, when the accession of James Baker to the post of Treasury Secretary signalled a more activist policy with respect to the exchange rate.[6] Accordingly, Regime 2 runs from February 1, 1985 until December, 1988, and hence includes the September 22, 1985 Plaza Accord, in which the G-5 countries agreed to intervene actively to bring the dollar down from what was perceived to be an unsustainably high level. Regime 2 also includes the February 22, 1987 Louvre Accord, in which the G-5 generally agreed that the dollar had come down enough, and that it would be desirable to maintain exchange rates at their then-current levels. Regime 3 runs from January 1, 1989 until December 31, 1993.

Letting S_t denote the exchange rate at time t in terms of dollars per unit of foreign currency, returns are computed as $\Delta S_t = log\left(\frac{s_t}{s_{t-1}}\right)$, the logarithm of the business day-to-business day exchange rate ratio. Weekly returns are calculated by sampling the business daily data on the Wednesday of each week.

Tables 3.1 and 3.2 report summary statistics for the daily and weekly exchange rate returns. Keep in mind that the exchange rates are measured in dollars per unit of foreign currency, so that a positive return indicates a *depreciation* of the dollar. The returns for this sample exhibit the classic pattern of skewness and leptokurtosis noted in previous studies. Both currencies exhibit excess kurtosis over each of the three regimes in the sample. For the \$/Yen rate, there is skewness in the direction of dollar *depreciation* over the entire sample, while for the \$/DM rate there is depre-

[5]However, the Bundesbank and Bank of Japan were both active in exchange markets during this time, as is discussed below.

[6]See Destler and Henning [13], pp. 41-42 and Dominguez and Frankel [15], pp. 11-13.

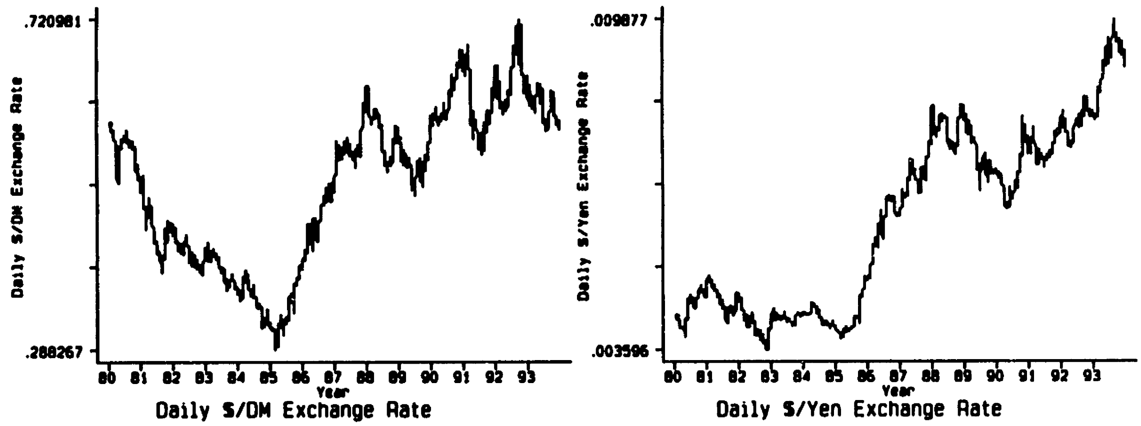

Figure 3-1: Dollar Exchange Rate against DM and Yen

ciation skewness in Regimes 1 and 2, and skewness in the direction of *appreciation* during Regimes 3. For both currencies, leptokurtosis is less for weekly than for daily returns. This suggests the presence of some ARCH-like effects at daily frequency: large movements followed by large movements *of either sign*, so that there may be some cancelling out under time aggregation. For the \$/Yen rate, skewness is greater for weekly returns than daily, suggesting that there also weeks in which jumps of the same sign tend to cluster together.

To get an idea of how large the largest "jumps" might be, note that at daily frequency the largest dollar appreciations are about 4%, while the largest depreciations are about 5%. At weekly frequency, the maximum movements are as large as 7% or 8% in either direction.

The data on U.S. intervention are from the Federal Reserve Board, and report daily operations in the Yen and DM markets by the Federal Reserve of New York, as authorized by the U.S. Treasury. The intervention data distinguish between customer and non-customer transactions. The former are transactions initiated not by the New York Fed but by a customer who would have otherwise carried out the transaction

Table 3.1: Daily Exchange Rate Returns: Summary Statistics

DM	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	-0.000479	0.000588	0.000019	0.000002
standard deviation	0.007316	0.007898	0.007822	0.007671
skewness	0.607095	0.551690	-0.285425	0.260478
kurtosis	5.94774	7.19684	4.64176	5.84896
observations	1281	979	1256	3516
minimum	-0.0266	-0.0317	-0.0406	-0.0406
maximum	0.0457	0.0575	0.0307	0.0575
Yen	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	-0.000066	0.000726	0.000084	0.000212
standard deviation	0.006405	0.006927	0.006424	0.006571
skewness	0.392483	0.614086	0.323402	0.453208
kurtosis	4.884076	7.684604	6.370940	6.380511
observations	1266	975	1237	3478
minimum	-0.0265	-0.0293	-0.0336	-0.0366
maximum	0.0314	0.0504	0.0422	0.0504

Data is from Bank for International Settlements. Business daily frequency. \$/DM rate is sampled at 1 p.m. Frankfurt time, \$/Yen at noon Tokyo time. Returns are calculated as $\Delta S_t = log\left(\frac{S_t}{S_{t-1}}\right)$ where S_t and S_{t-1} are the exchange rates in terms of dollars per unit of foreign currency in succesive periods. Note that an *increase* in S_t corresponds to a dollar depreciation.

Table 3.2: Weekly Exchange Rate Returns: Summary Statistics

DM	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	-0.002378	0.003095	0.000170	0.000022
standard deviation	0.015338	0.017042	0.017518	0.016730
skewness	0.262011	0.264306	-0.208739	0.100362
kurtosis	3.68038	3.45499	4.71049	4.09167
observations	257	195	251	703
minimum	-0.0471	-0.0389	-0.0683	-0.0683
maximum	0.0467	0.0688	0.0723	0.0723
Yen	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	-0.000257	0.003783	0.000443	0.001093
standard deviation	0.014389	0.015427	0.014865	0.014941
skewness	0.738150	0.815521	0.624087	0.727711
kurtosis	3.63642	5.54660	6.609423	5.322392
observations	254	197	247	698
minimum	-0.0307	-0.0369	-0.0414	-0.0414
maximum	0.0491	0.0739	0.0827	0.0827

Weekly data is sampled on Wednesdays from the business daily data.

in the open market. Customer transactions are sometimes referred to as "passive" interventions because they are not initiated by the Fed. Nonetheless, as Dominguez and Frankel note [15], even such "passive" interventions can signal important information to the market. This study defines intervention as "total" intervention, that is, the sum of customer and non-customer transactions. In practice, the non-customer and total measures are quite close. None of the material conclusions of this study would be affected by using non-customer rather than "total" transactions. A more important drawback of this study is that only U.S. intervention data are used. The Bank of Japan and the Bundesbank do not make their daily intervention data publicly available.[7] This is a particularly serious problem for the 1982-84 period, when the U.S. was not heavily involved in FX intervention but it is known that Germany and Japan were. For the other time periods in the 1980-1993 sample it is perhaps less problematic, since interventions are often coordinated. Thus, Bank of Japan and Bundesbank interventions are typically in the same direction as U.S. interventions.

Tables 3.3 and 3.4 summarize the daily intervention data and weekly intervention as measured by cumulative daily intervention. The intervention is measured in millions of dollars sold in support of either the DM or Yen. Note that intervention is a fairly infrequent event, occurring in the DM market in about 12% of the days in the sample, and in the Yen market for about 6% of the days in the sample. The largest daily purchases and sales are on the order of $700 or $800 million and the average purchase or sale is about $100 million. There was net *selling* of dollars over the 1980-1993 period, about $16 billion on net in support of the DM and $5.5 billion on net in support of the Yen. Looking at the weekly data, it is clear that the average weekly purchases and sales are larger than the daily, indicating that it is common for the Fed to intervene during several days in a week when it does intervene, and to intervene in the same direction during each day of that week.

Figure 3-2 shows the daily intervention along with the daily return data. The relationship between jumps and intervention is hard to discern with the naked eye.

[7]See, however, Dominguez and Frankel [15] who were allowed to use BOJ and Bundesbank data in their studies on the condition they report only summary results.

Table 3.3: Daily Dollar Sales (Millions) In Support of Foreign Currency

DM	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	4.3	3.7	5.4	4.6
mean sale	53.6	147.5	137.6	91.2
mean purchase	-84.9	-123.5	-135.7	-110.4
largest sale	295.0	797	695	797
largest purchase	-386.3	-395	-400	-400
total sales				28,811
total purchases				-12,812
sale days	181	55	80	316
purchase days	149	36	31	116
total days	1280	979	1256	3515
Yen	1/4/80- 1/31/85	2/1/85- 12/31/88	1/1/89- 12/31/93	Whole sample
mean	0.3	-7.4	9.2	1.3
mean sale	19.2	38.1	152.7	101.9
mean purchase	-27.4	-139.7	-286.2	-147.0
largest sale	50	211	555	555
largest purchase	-50.2	-720.2	-492	-720.2
total sales				14,770
total purchases				-10,287
sale days	20	41	84	145
purchase days	2	63	5	70
total days	1265	975	1237	3477

Federal Reserve Board data. Intervention measured as sum of customer and noncustomer transactions.

Table 3.4: Weekly Dollar Sales (Millions) In Support of Foreign Currency

DM	1/4/80-1/31/85	2/1/85-12/31/88	1/1/89-12/31/93	Whole sample
mean	21.6	21.0	31.3	24.9
mean sale	185.4	324.5	320.3	261
mean purchase	-170.0	-192.2	-308.2	-210.4
largest sale	800.7	1306	1480	1480
largest purchase	-706.9	-687	-986	-986
total sales				28,811
total purchases				-12,812
sale weeks	50	25	37	112
purchase weeks	22	21	13	56
total weeks	256	195	251	702
Yen	1/4/80-1/31/85	2/1/85-12/31/88	1/1/89-12/31/93	Whole sample
mean	1.3	-36.2	47.2	7.0
mean sale	34.9	77.0	335.5	212.5
mean purchase	-27.4	-292.0	-357.8	-284.6
largest sale	84	328.5	1723.5	1723.5
largest purchase	-50.2	-1712	-692	-1712
total sales				14,770
total purchases				-10,287
sale weeks	11	21	39	71
purchase weeks	2	30	4	36
total weeks	253	197	247	697

Weekly intervention measured as cumulative daily intervention from Wednesday to Thursday of each week.

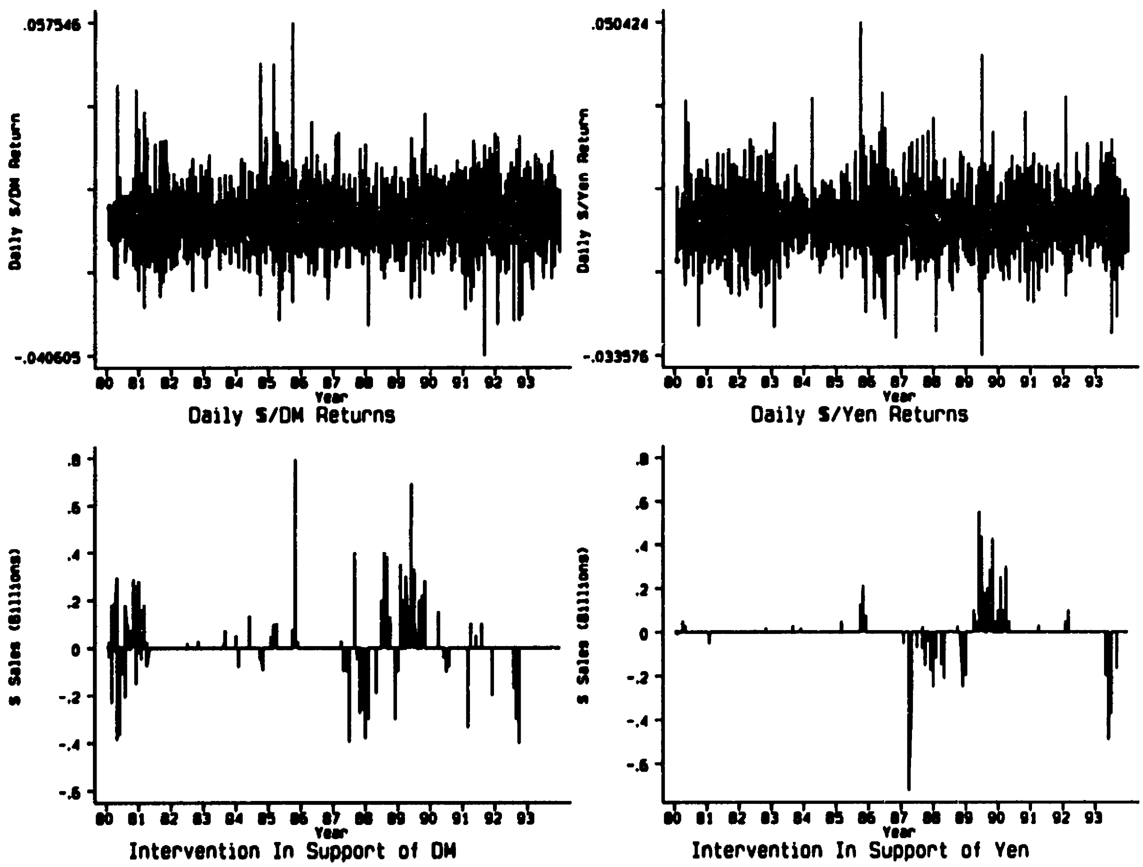

Figure 3-2: Daily Dollar Exchange Returns and U.S. Intervention

However, if we take "jumps" to be daily movements in excess of 1% (this definition is confirmed in a formal statistical sense in the section below), then it seems that there is no clear contemporaneous relationship between jumps and intervention. For example, some jumps seem to occur even when there is no intervention activity. More powerful statistical techniques will be employed in section 4 in an attempt to untangle the jump-intervention relationship.

Note that, over most periods in the sample, extreme exchange rate movements can be either appreciations or depreciations. This may be explained partly by ARCH-like "volatility clustering," that is, the tendency for large returns to be followed by large returns of either sign. However, as is shown below, ARCH cannot completely account for the extreme movements. A mixed jump-ARCH process is a promising extension. Given the empirical fact of large movements of either sign which cannot be accounted for by ARCH, it is important to allow for jumps of either sign.

3.3 A Two-Jump-ARCH Model of FX Returns

The skewness and leptokurtosis of FX returns suggests the use of a parametric model which can account for these properties. Exchange rate returns, measured in dollars per unit of foreign currency, are assumed to follow a discretized jump-diffusion process:[8]

$$\Delta S_t = \mu\tau_t + \sigma\sqrt{\tau_t}\Delta Z + \kappa\Delta J_\kappa(p\tau_t) + \theta\Delta J_\theta(q\tau_t) \tag{3.1}$$

where τ_t is the lag between successive exchange rate quotes (e.g., for business daily data, 1 day during the business week and 3 days for weekend returns), μ is the deterministic drift, σ is the diffusion volatility, and ΔZ is a standard normal random variable. The random variables ΔJ_κ and ΔJ_θ are assumed to be Bernoulli, independent of one another and of ΔZ. Jumps of type κ occur with probability p per unit time and jumps of type θ with probability q per unit time. The constants κ and θ

[8]See Ball and Torous [3], who use a Taylor expansion to show that the Bernoulli jump process may be thought of as an approximation to a continuos-time Poisson jump process. The lower is the Poisson jump frequency, the better is the Bernoulli approximation.

are the sizes of each type of jump, given that a jump of that type occurs. The diffusion volatility σ may be thought of as a measure of the exchange rate uncertainty occurring due to normal day-to-day information arrivals, while the jump sizes κ and θ correspond to extraordinary exchange rate movements due to low-probability events. The model considered here differs from earlier empirical applications of the jump-diffusion to FX returns in that it allows for two independent jump processes.[9] Hence, the model allows for the possibility that, in a given day or week, a large appreciation *or* large depreciation may occur, as Figure 3-2 suggests is the case.

Jorion [25] notes that it is important to account for the possibility of conditional heteroskedasticity in the diffusion process, since it could lead to "fat tails" even in the absence of jump processes. Accordingly, we model the diffusion volatility as an ARCH(1) process:

$$\sigma_t^2 = \alpha_0 + \alpha_1(\frac{1}{\tau_{t-1}})(\Delta S_{t-1} - E[\Delta S_{t-1}])^2 \tag{3.2}$$

The ARCH specification allows for "volatility clustering." For example, for $\alpha_1 > 0$, a large exchange rate innovation on a given day will be associated with a large movement (of either sign) on the following day. Note that the factor $(\frac{1}{\tau_{t-1}})$ accounts for the possibility that the $t-1$ and t innovations occur over time intervals of differing lengths. The model was estimated on the daily and weekly exchange returns using maximum likelihood, with log-likelihood function given by

$$\begin{aligned} -\frac{T}{2}log(2\pi) \quad + \Sigma_{t=1}^{T} log[\quad & \frac{(1 - p\tau_t - q\tau_t)}{\sigma\sqrt{\tau_t}} exp\left(\frac{-(\Delta S_t - \mu)^2}{2\sigma^2\tau_t}\right) \\ & + \frac{p\tau_t}{\sigma\sqrt{\tau_t}} exp\left(\frac{-(\Delta S_t - \mu - \kappa)^2}{2\sigma^2\tau_t}\right) \\ & + \frac{q\tau_t}{\sigma\sqrt{\tau_t}} exp\left(\frac{-(\Delta S_t - \mu - \theta)^2}{2\sigma^2\tau_t}\right)] \end{aligned} \tag{3.3}$$

[9] Aase and Guttorp [1] use a two-jump-diffusion model with no ARCH effects to study stock returns. See Jorion [25], Akgiray and Booth [2], and Tucker and Pond [32] for FX applications of one-jump models. Das [12] applies a one-jump-ARCH model in his study of interest rates, and Friedman and Laibson [20] and Jarrow and Rosenfeld [24] study stock returns with a one-jump model.

Tables 3.5 and 3.6 report the parameter estimates of the two-jump-ARCH model as well as those for a pure ARCH model, estimated under the constraints $p = 0$, $\kappa = 0$, $q = 0$, and $\theta = 0$. For the daily returns, there is evidence of ARCH effects in the DM for Regime 1, with $\alpha_1 > 0$ and significantly so. For the Yen, ARCH effects are most significant in Regimes 2 and 3. The jump probabilities and jump sizes are highly significant for most of the sub-periods. Jumps are fairly rare, occurring with probabilities of anywhere from 0.02 to 0.08 per business day. The jump sizes are about 1% in either direction, although they are somewhat larger for jump *depreciations* during Regime 2. This is a reflection of the large one-day depreciations which occurred in response to the Plaza Accord in September 1985. The final column of table 3.5 reports the likelihood ratio test statistic of the null hypothesis that jumps are irrelevant in explaining daily exchange returns: $(p = 0, \kappa = 0, q = 0, \theta = 0)$. The statistic is distributed χ^2_4, with 0.99 critical level of 13.25. Thus, the statistics in the final column present overwhelming evidence that jumps are important, even when ARCH effects are allowed for.

In order to preserve degrees of freedom, the weekly data were estimated on just two subperiods: that before the Louvre accord of February 1987, and the period after the Louvre. ARCH effects seem to be less important at weekly frequency than at daily frequency. This confirms the suspicion that there are large daily movements of conflicting sign during the course of a week, which "wash out" in aggregation to weekly frequency. For the DM, the ARCH coefficient α_1 is not significantly different from zero for any of the sub-periods. This differs from the findings of Jorion [25] who found significant ARCH effects in weekly \$/DM returns for the period 1974-1985. For the \$/Yen rate, ARCH effects are significant for the pre-Louvre period but not for the post-Louvre. Not surprisingly, the individual jump size and probability coefficients are also less significant at weekly frequency then they are at daily. However, the likelihood ratio test still indicates rejection of the hypothesis $(p = 0, \kappa = 0, q = 0, \theta = 0)$ at the 0.99 level for both subperiods and both currencies. Jump sizes are generally larger at weekly frequency, greater than 2% per week and as large as 8% per week. It is interesting to note that for the \$/DM rate in the pre-Louvre period, both κ and θ are

Table 3.5: ML Estimates: Daily Exchange Returns

DM	μ	α_0	α_1	p	κ	q	θ	Log-Like	χ^2_4
1/4/80- 1/31/85	-0.00037 (-6.766)	0.00006 (36.016)	0.02357 (12.162)					4277.1	
T=1279	-0.00001 (-1.6404)	0.00003 (21.469)	0.18714 (11.007)	0.15289 (8.175)	-0.00917 (-22.530)	0.08773 (6.673)	0.01105 (25.587)	4395.4	236.6
2/1/85- 12/31/88	0.00037 (5.350)	0.00009 (63.179)	-0.02771 (-1.203)					3146.0	
T=978	0.00105 (14.980)	0.00006 (29.089)	-0.01648 (-0.082)	0.08731 (5.150)	-0.01336 (-19.980)	0.00203 (1.415)	0.05182 (10.012)	3250.4	208.8
1/1/89- 12/31/93	0.00007 (1.183)	0.00009 (48.572)	0.04976 (1.890)					4012.0	
T=1257	-0.00020 (-3.276)	0.00004 (33.325)	-0.02279 (-2.381)	0.05567 (6.307)	-0.01567 (-34.533)	0.10828 (7.778)	0.01290 (31.571)	4196.6	369.2
Whole Sample	0.000005 (0.0146)	0.00008 (86.682)	0.12722 (10.586)					11430.0	
T=3515	0.000003 (0.078)	0.00004 (72.415)	0.09396 (10.384)	0.07939 (10.565)	-0.012349 (-42.895)	0.07454 (11.270)	0.01312 (44.014)	11804.7	750.0
Yen	μ	α_0	α_1	p	κ	q	θ	Log-Like	χ^2_4
1/4/80- 1/31/85	0.00003 (0.532)	0.00006 (52.875)	0.01183 (0.563)					4286.6	
T=1264	-0.00058 (-11.815)	0.00003 (32.188)	0.01559 (0.768)	0.02651 (3.620)	-0.01239 (-19.869)	0.07476 (7.253)	0.01385 (40.731)	4465.2	357.2
2/1/85- 12/31/88	0.00045 (7.730)	0.00007 (74.925)	0.18114 (5.360)					3205.8	
T=974	0.00036 (6.660)	0.00004 (43.677)	0.06821 (2.109)	0.02374 (3.452)	-0.01519 (-23.638)	0.02553 (3.997)	0.02112 (40.067)	3344.2	276.7
1/1/89- 12/31/93	-0.00004 (-0.866)	0.00006 (52.281)	0.19726 (6.980)					4226.5	
T=1238	-0.00027 (-5.565)	0.00003 (35.474)	0.0686 (4.711)	0.03692 (4.580)	-0.01263 (-25.898)	0.07734 (6.636)	0.01176 (29.197)	4351.9	250.1
Whole Sample	0.00012 (4.092)	0.00006 (113.064)	0.10737 (7.396)					11720.1	
T=3477	-0.00026 (-8.912)	0.00003 (92.822)	0.03076 (3.311)	0.02529 (6.543)	-0.01381 (-41.048)	0.0594 (10.813)	0.01446 (60.737)	12155.2	870.3

Business daily observations. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). First row for each sample period reports coefficients of the pure ARCH-diffusion model, second row the two-jump-ARCH -diffusion model. The final column reports the value of the LR statistic in the test of the hypothesis that jumps are irrelevant:($p = 0, \kappa = 0, q = 0, \theta = 0$). The statistic is distributed χ^2 with 4 degrees of freedom. The 0.990 and 0.995 confidence levels of the χ^2_4 are 13.25 and 14.86.

Table 3.6: ML Estimates: Weekly Exchange Returns

DM	μ	α_0	α_1	p	κ	q	θ	Log-Like	χ^2_4
Pre-Louvre	0.00006 (0.711)	0.00030 (16.135)	0.00027 (0.005)					967.3	
T=371	-0.00066 (-6.750)	0.00020 (13.110)	-0.01496 (-0.987)	0.16458 (3.768)	0.02795 (10.100)	0.00222 (0.633)	0.07073 (0.183)	976.0	17.4
Post-Louvre	0.00020 (2.200)	0.00028 (16.484)	-0.00752 (-0.106)					873.3	
T=331	-0.00005 (-0.482)	0.00021 (10.969)	-0.00036 (-0.007)	0.00912 (0.964)	-0.04918 (-1.685)	0.04610 (1.283)	0.02944 (5.990)	880.0	13.5
Whole Sample	0.00013 (2.043)	0.00029 (23.092)	-0.00504 (-0.133)					1850.4	
T=702	-0.00017 (-2.225)	0.00018 (13.887)	0.00610 (0.230)	0.03599 (1.175)	-0.02868 (-5.977)	0.09047 (3.019)	0.02771 (11.406)	1864.8	28.8
Yen	μ	α_0	α_1	p	κ	q	θ	Log-Like	χ^2_4
Pre-Louvre	0.00005 (0.557)	0.00017 (14.927)	0.27512 (3.506)					1020.0	
T=370	0.00015 (1.714)	0.00008 (7.117)	0.19964 (2.688)	0.25345 (3.437)	-0.01212 (-5.765)	0.09361 (4.112)	0.02553 (10.918)	1032.8	25.5
Post-Louvre	0.00017 (2.041)	0.00025 (28.198)	-0.01510 (-0.3873)					882.6	
T=327	0.00023 (2.528)	0.00018 (13.812)	0.00886 (0.209)	0.03499 (0.993)	-0.02720 (-5.065)	0.00313 (0.991)	0.07941 (0.046)	908.5	51.6
Whole Sample	0.00013 (2.141)	0.00022 (34.447)	0.09937 (3.328)					1907.1	
T=697	0.00018 (2.931)	0.00015 (15.822)	0.13368 (2.970)	0.06900 (2.165)	-0.01817 (-5.546)	0.01397 (2.556)	0.04762 (19.399)	1940.3	66.4

Weekly observations. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). First row for each sample period reports coefficients of the pure ARCH-diffusion model, second row the two-jump-ARCH -diffusion model. The final column reports the value of the LR statistic in the test of the hypothesis that jumps are irrelevant:($p = 0, \kappa = 0, q = 0, \theta = 0$). The statistic is distributed χ^2 with 4 degrees of freedom. The 0.990 and 0.995 confidence levels of the χ^2_4 are 13.25 and 14.86.

positive. The best-fitting model is characterized by fairly frequent jump depreciations of about 2.8%, and less frequent but larger jump depreciations of about 7%. Thus, while fitting of the two-jump-ARCH model to FX returns usually leads to κ and θ of opposite sign, this need not necessarily be the case.

3.4 Identifying Jumps

Given the demonstrated importance of jump behavior in high frequency FX returns, it is interesting to ask *which* of the daily and weekly returns are "jump" movements and which are not. In order to do this, we employ a likelihood ratio technique, akin to those used in Das [12], Aase and Guttorp [1], and Friedman and Laibson [20]. Consider the following conditional density functions of the FX return ΔS_t:

$$\begin{aligned} f_t^{NJ}(\Delta S_t, \alpha_0, \alpha_1, \mu, p, \kappa, q, \theta) &= \frac{1}{\sqrt{2\pi\sigma_t^2\tau_t}} exp\left(\frac{-(\Delta S_t-\mu)^2}{2\sigma_t^2\tau_t}\right) \\ f_t^{\kappa}(\Delta S_t, \alpha_0, \alpha_1, \mu, p, \kappa, q, \theta) &= \frac{1}{\sqrt{2\pi\sigma_t^2\tau_t}} exp\left(\frac{-(\Delta S_t-\mu-\kappa)^2}{2\sigma_t^2\tau_t}\right) \\ f_t^{\theta}(\Delta S_t, \alpha_0, \alpha_1, \mu, p, \kappa, q, \theta) &= \frac{1}{\sqrt{2\pi\sigma_t^2\tau_t}} exp\left(\frac{-(\Delta S_t-\mu-\theta)^2}{2\sigma_t^2\tau_t}\right) \end{aligned} \tag{3.4}$$

These give the density if there is no jump at time t, if there is a jump of type κ, and if there is a jump of type θ. These densities are used to form the following likelihood ratios, which can be evaluated at the parameter values $\hat{\alpha_0}, \hat{\alpha_1}, \hat{\mu}, \hat{p}, \hat{\kappa}, \hat{q}$, and $\hat{\theta}$, obtained by maximum likelihood estimation:

$$\begin{aligned} LR_t^{\kappa} &= log\left[f_t^{\kappa}(\Delta S_t, \hat{\alpha_0}, \hat{\alpha_1}, \hat{\mu}, \hat{p}, \hat{\kappa}, \hat{q}, \hat{\theta})\right] - log\left[f_t^{NJ}(\Delta S_t, \hat{\alpha_0}, \hat{\alpha_1}, \hat{\mu}, \hat{p}, \hat{\kappa}, \hat{q}, \hat{\theta})\right] \\ LR_t^{\theta} &= log\left[f_t^{\theta}(\Delta S_t, \hat{\alpha_0}, \hat{\alpha_1}, \hat{\mu}, \hat{p}, \hat{\kappa}, \hat{q}, \hat{\theta})\right] - log\left[f_t^{NJ}(\Delta S_t, \hat{\alpha_0}, \hat{\alpha_1}, \hat{\mu}, \hat{p}, \hat{\kappa}, \hat{q}, \hat{\theta})\right] \end{aligned} \tag{3.5}$$

Intuitively, the ratio LR_t^{κ} should be large on those days when the return has a jump component of type κ, and LR_t^{θ} should be large for those days most likely to have a jump of type θ. It is known that there are, on average, pT jumps of type

κ and qT jumps of type θ in a sample of size T. Hence, given maximum likelihood estimates of the jump probabilites, we identify the $\hat{p}T$ days in the sample for which LR_t^κ is largest as κ-jump days, and the $\hat{q}T$ days for which LR_t^θ is largest as θ-jump days.[10]

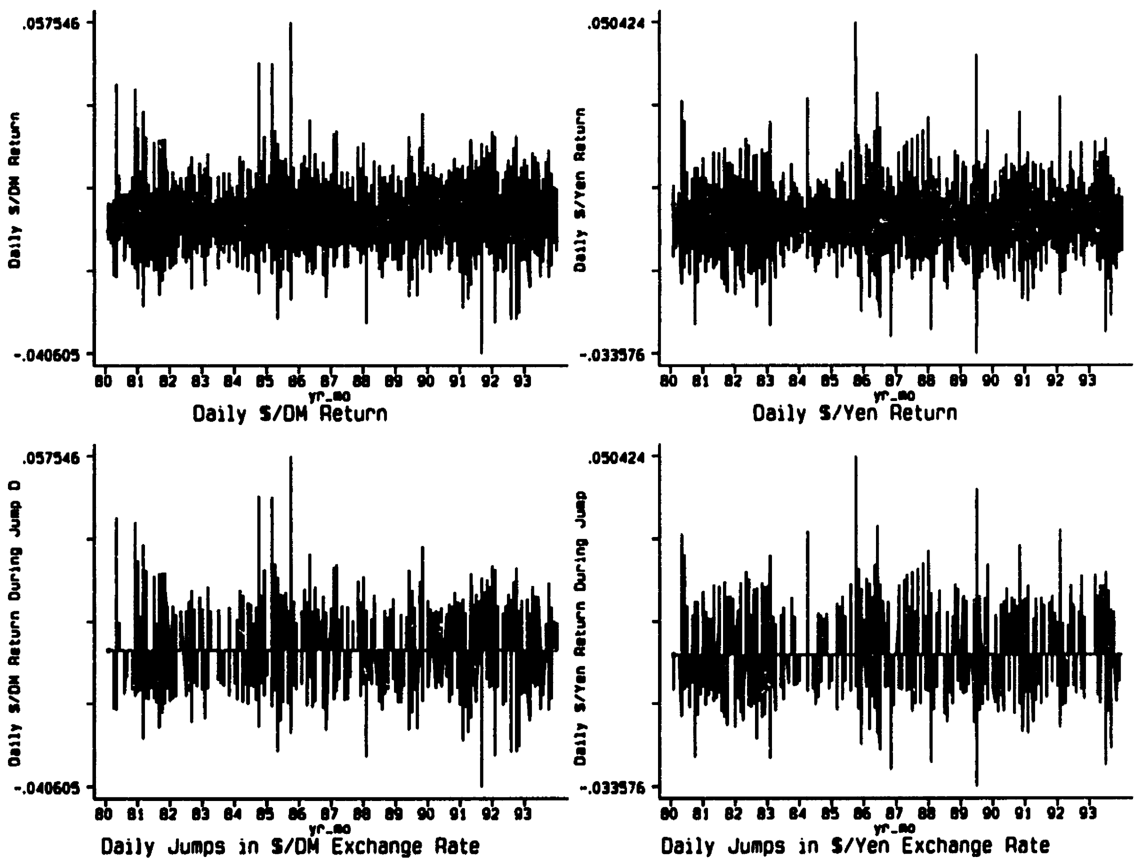

Figure 3-3: Daily Dollar Returns and Returns on Jump Days

Figure 3-3 shows graphically the result of applying the foregoing jump identification technique to the daily returns. The parameter estimates used are those obtained by estimation on the whole sample period 1980-1993. The top part of the graph shows the entire daily return series, and the lower part shows just the jumps, with the diffusion movements filtered out. For the \$/DM rate, there are κ-type jump appreciations in 279 of the 3515 days in the sample, and θ-type jump depreciations for 262 days. For the \$/Yen there are 88 jump appreciations in 3477 total days, and

[10]The real numbers $\hat{p}T$ and $\hat{q}T$ are rounded to the nearest integer.

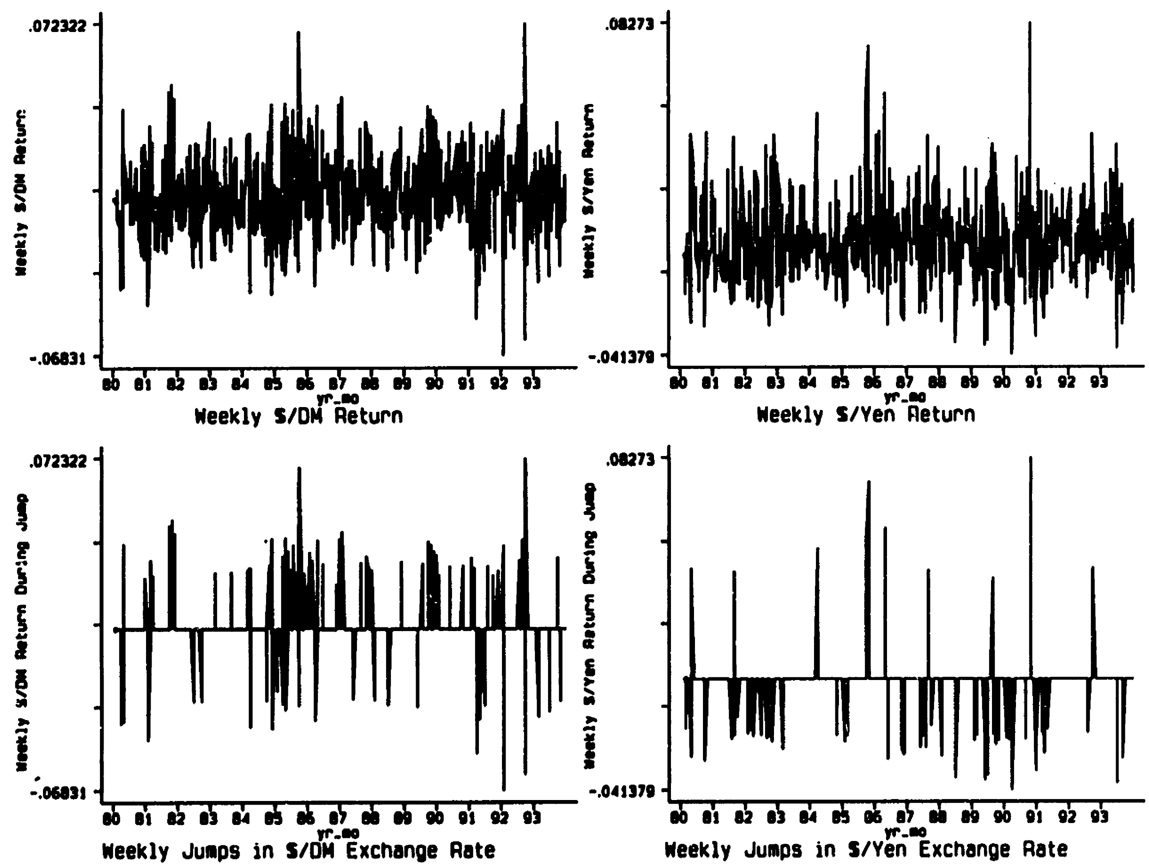

Figure 3-4: Weekly Dollar Returns and Returns in Jump Weeks

208 jump depreciations. Typical jumps for both currencies are on the order of 1% or 2%, as the ML estimates would suggest they should be, although there are also jump days with movements as large as 5%. The jumps are, generally speaking, the largest one-day exchange rate returns in the sample, although there are some fairly large one-day returns which, because they are preceded by other large returns, are imputed to the ARCH component of the diffusion.

Figure 3-4 shows the result of the jump-identification on the weekly returns. For the $/DM, there are 25 jump appreciation weeks in 702 total weeks, and 66 jump depreciations. For the $/Yen, there are 48 jump appreciations in 697 weeks, and 10 jump depreciations.

3.5 Intervention and Jumps

Now we turn to an examination of how jump behavior in FX returns is related to intervention. It is well know that central banks profess to "lean against the wind," intervening in support of a currency precisely when that currency is depreciating, presumably due to factors separate from the intervention itself. This may of course lead to a simultaneity bias in the contemporaneous relationship between intervention and the exchange rate. See, for example, Loopesko [28] for evidence of a high degree of contemporaneous interaction between exchange rates and intervention. OLS regressions of FX returns on same-day intervention reported by Dominguez and Frankel [15] reveal a statistically significant relationship, but with the wrong sign: interventions in support of the dollar are associated with dollar *depreciations.* This is strong evidence that simultaneity bias is actually present.

This simultaneity may be partially unraveled by considering the relationship of FX returns to leads and lags of intervention as well as contemporaneous intervention. Table 3.7 reports results of an OLS regression of daily FX returns on the two-day lead, one-day lead, contemporaneous, one-day lag, and two-day lag of intervention, as measured by billions of dollars *sold.* There is a statistically significant pattern which holds for both the DM and Yen rates and for both the pre- and post-Louvre peri-

Table 3.7: OLS Estimates: Daily Exchange Returns and Intervention

DM	α	β_1	β_2	β_3	β_4	β_5	T
Pre-Louvre	0.00084 (0.463)	-0.00323 (-0.632)	-0.0213 (-3.951)***	-0.0206 (-3.829)***	0.0158 (2.929)***	0.00010 (0.197)	1857
Post-Louvre	0.00010 (0.535)	-0.00667 (-1.910)*	-0.01885 (-5.147)***	-0.00144 (-0.0396)	0.00349 (0.952)	0.01160 (3.320)***	1654
Whole sample	0.00008 (0.603)	-0.00537 (-1.847)*	-0.01992 (-6.520)***	-0.00764 (-2.512)**	0.00739 (2.420)**	0.00864 (2.974)***	3511
Yen	α	β_1	β_2	β_3	β_4	β_5	T
Pre-Louvre	0.00025 (1.635)	-0.00523 (-0.826)	-0.01547 (-2.356)**	-0.00854 (-1.372)	0.01516 (2.309)**	0.00426 (0.672)	1840
Post-Louvre	0.00021 (1.302)	-0.01387 (-3.548)***	-0.01540 (-3.727)***	-0.00151 (-0.361)	0.01291 (3.125)***	0.00598 (1.529)	1633
Whole sample	0.00023 (2.072)**	-0.01095 (-3.296)***	-0.01574 (-4.159)***	-0.00421 (-1.220)	0.00139 (3.987)***	0.00588 (1.769)*	3473

Asymptotic t-statistics in parentheses. OLS regression of the form

$$\Delta S_t = \alpha + \beta_1 INT_{t+2} + \beta_2 INT_{t+1} + \beta_3 INT_t + \beta_4 INT_{t-1} + \beta_5 INT_{t-2} + \epsilon_t \qquad (3.6)$$

where INT_t measures daily intervention in terms of billions of dollars sold. (*), (**), and (***) indicate significance at the 90%, 95%, and 99% level.

ods. The coefficients β_1 and β_2 are typically significantly negative, indicating that, in the two days before a dollar sale (purchase), the dollar appreciates (depreciates). Thus, the U.S. does indeed seem to "lean against the wind" entering the market in an attempt to smooth exchange rate fluctutations of the previous two days. The contemporaneous coefficient β_3 is either significantly negative or insignificantly different from zero, depending on the currency and subperiod considered. This is of course also consistent with "leaning against the wind:" same-day dollar sales occur in the face of dollar appreciation, sometimes with little overall effect on that day's return. Finally, the coefficients β_4 and β_5 are positive and typically significantly so. This indicates that, in the two days following a dollar sale (purchase), the dollar depreciates (appreciates), as the monetary authorities would hope.

To see if the intervention-return relationship operates differently at the weekly horizon, consider the coefficients, reported in table 3.8, of an OLS regression of the weekly FX returns on one-week lead, same-week, and one-week lagged intervention. The pattern is similar, and again is robust across currencies and subperiods. The

Table 3.8: OLS Estimates: Weekly Exchange Returns and Intervention

DM	α	β_1	β_2	β_3	T
Pre-Louvre	0.00033 (0.377)	-0.00923 (-1.199)	-0.0325 (-3.752)***	0.0183 (2.348)**	370
Post-Louvre	0.00056 (0.621)	-0.01233 (-2.466)**	-0.01460 (-2.770)***	0.01567 (3.156)***	330
Whole sample	0.00039 (0.631)	-0.01154 (-2.744)***	-0.01968 (-4.350)***	0.01632 (3.883)***	700
Yen	α	β_1	β_2	β_3	T
Pre-Louvre	0.00135 (1.700)*	0.00041 (0.046)	-0.01856 (-1.772)*	0.0106 (1.186)	369
Post-Louvre	0.00110 (1.403)	-0.02085 (-3.933)***	-0.00841 (-1.375)	0.01407 (2.655)***	326
Whole sample	0.00119 (2.139)**	-0.01471 (-3.161)***	-0.01129 (-2.091)**	0.01276 (2.740)***	695

Asymptotic t-statistics in parentheses. OLS regression of the form

$$\Delta S_t = \alpha + \beta_1 INT_{t+1} + \beta_2 INT_t + \beta_3 INT_{t-1} + \epsilon_t \tag{3.7}$$

where INT_t measures weekly intervention in terms of billions of dollars sold. (*), (**), and (***) indicate significance at the 90%, 95%, and 99% level.

coefficients β_1 and β_2 are either significantly negative or insignificant from zero, which may be taken as evidence that dollar sales (purchases) are conducted in response to dollar appreciation (depreciation) in the week of the intervention and the week preceding the intervention. The coefficient β_3 is significantly positive, indicating that interventions work to move the exchange rate in the desired direction, albeit with a one week lag.

While suggestive, the foregoing OLS regressions do not allow us to distinguish whether the relationship between intervention and returns differs according to whether the return is a "jump" or not. To this end, we parameterize the two-jump-ARCH model to depend on lead, lag, and contemporaneous values of daily intervention. The drift, which represents the average FX return in "non-jump" days, is given by

$$\mu_t = \mu_0 + \mu_1 INT_{t+2} + \mu_2 INT_{t+1} + \mu_3 INT_t + \mu_4 INT_{t-1} + \mu_5 INT_{t-2} \tag{3.8}$$

where INT_t measures intervention during day t in terms of billions of dollars sold.

The jump probabilites p and q are allowed to depend on intervention in a logistic fashion:

$$p_t = \frac{1}{1 + exp\left(-\gamma_0 - \sum_{i=1}^{5} \gamma_i INT_{3-i+t}\right)} \tag{3.9}$$

$$q_t = \frac{1}{1 + exp\left(-\phi_0 - \sum_{i=1}^{5} \phi_i INT_{3-i+t}\right)} \tag{3.10}$$

The "base jump probabilites," or jump probabilities for a day with zero intervention action in an immediate five-day window, are given by $\bar{p} = \frac{1}{1+exp(-\gamma_0)}$ and $\bar{q} = \frac{1}{1+exp(-\phi_0)}$.

Tables 3.9 and 3.10 report the coefficients obtained by maximum likelihood estimation on the daily data for the pre-Louvre and post-Louvre periods. The post-Louvre period is of particular interest given the size and frequency of U.S. intervention activity during that time. The pre-Louvre period saw some U.S. intervention activity in the DM market but very little in the Yen market. The columns labeled "BJP" give the base jump probabilites for each type of jump. The jump sizes and the base jump probabilities are quite precisely estimated for both the DM and Yen. The jump sizes of about 1% or 1.5% in absolute value are comparable to those estimated in the previous sections of this paper. The κ-type and θ-type jumps take opposite signs for each of the currencies and sub-periods except the pre-Louvre $/DM, which exhibits rare jump depreciations of 3.6% and more frequent jump depreciations of about 1.8%.

The parameters of the drift process μ_t reveal a pattern similar to the one in the foregoing OLS regressions: dollar sales at day t are usually associated with dollar *appreciation* in day t and the two preceding days, while dollar sales in day t lead to dollar *depreciation* in the two days following the intervention. This pattern is more marked and more significant statistically for the post-Louvre period than it is for the pre-Louvre, which is not surprising given the relative infrequency of intervention in the earlier period, particularly for the $/Yen rate. We can conclude that the U.S. "leans against the wind" for small drift movements in the exchange rate, and that it has some success in doing so, though the effect of intervention operates with a one or two day lag. The sum $\sum_{i=1}^{5} \mu_i$, which is reported near the bottom of Tables 3.9 and

Table 3.9: Pre-Louvre Period: Daily FX Returns and Intervention

DM	Parameter	T-Stat	BJP	Yen	Parameter	T-Stat	BJP
α_0	0.000036	(29.95)**		α_0	0.000019	(24.93)**	
α_1	0.0708	(3.948)**		α_1	0.1368	(4.864)**	
μ_0	-0.00028	(-3.451)**		μ_0	-0.00022	(-2.095)*	
μ_1	-0.00248	(0.1310)		μ_1	-0.00193	(-0.3023)	
μ_2	-0.01486	(-2.916)**		μ_2	-0.01275	(-2.946)**	
μ_3	-0.01391	(-5.078)**		μ_3	-0.00160	(-0.4299)	
μ_4	0.00939	(2.286)*		μ_4	0.006771	(1.310)	
μ_5	-0.00105	(-0.3856)		μ_5	0.002372	(0.4246)	
γ_0	-5.461	(-11.17)**	0.0042	γ_0	-4.305	(-9.930)**	0.0133
γ_1	-5.661	(0.0035)		γ_1	1.361	(0.0138)	
γ_2	0.0824	(-0.3691)		γ_2	2.143	(0.0315)	
γ_3	-1.516	(1.585)		γ_3	3.534	(0.1276)	
γ_4	0.2922	(-1.196)		γ_4	-0.4942	(-0.0070)	
γ_5	-0.9924	(-0.0923)		γ_5	1.0272	(0.0163)	
κ	0.0360	(14.41)**		κ	-0.0132	(-11.14)**	
ϕ_0	-4.548	(-9.683)**	0.0105	ϕ_0	-3.3633	(-15.06)**	0.0335
ϕ_1	-4.240	(-1.376)		ϕ_1	2.484	(0.1002)	
ϕ_2	0.1132	(-0.0951)		ϕ_2	4.098	(0.3334)	
ϕ_3	-1.418	(-0.597)		ϕ_3	-3.668	(-0.4962)	
ϕ_4	0.2651	(0.4674)		ϕ_4	4.774	(0.3788)	
ϕ_5	-1.024	(0.5861)		ϕ_5	2.805	(0.1067)	
θ	0.0179	(11.91)**		θ	0.01368	(19.51)**	
$\sum_{i=1}^{5} \mu_i$	-0.0229	(-3.4621)**		$\sum_{i=1}^{5} \mu_i$	-0.0057	(-0.7055)	
Log-Like	6439.5				6722.7		
T	1859				1842		

Business daily observations. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). The column "BJP" reports the base jump probabilities $\overline{p} = \frac{1}{1+exp(-\gamma_0)}$ and $\overline{q} = \frac{1}{1+exp(-\phi_0)}$. (*) and (**) denote significance at the 95% and 99% levels.

Table 3.10: Post-Louvre Period: Daily FX Returns and Intervention

DM	Parameter	T-Stat	BJP	Yen	Parameter	T-Stat	BJP
α_0	0.000032	(26.00)**		α_0	0.000020	(23.45)**	
α_1	0.0464	(2.002)*		α_1	0.1222	(4.958)**	
μ_0	0.00007	(0.5582)		μ_0	-0.00016	(-1.434)	
μ_1	-0.00397	(-1.214)		μ_1	-0.00687	(-2.592)**	
μ_2	-0.0157	(-5.655)**		μ_2	-0.00134	(-4.518)**	
μ_3	-0.0036	(-1.441)		μ_3	-0.00500	(-1.538)	
μ_4	-0.0040	(1.573)		μ_4	0.00743	(2.750)**	
μ_5	0.0081	(3.234)**		μ_5	0.00011	(0.0507)	
γ_0	-4.817	(-12.77)**	0.008	γ_0	-5.090	(-8.266)**	0.006
γ_1	3.594	(0.2538)		γ_1	0.7626	(0.0283)	
γ_2	2.323	(0.2743)		γ_2	0.7472	(0.0265)	
γ_3	-8.174	(-2.376)*		γ_3	-0.0672	(-0.0088)	
γ_4	-2.849	(-0.4442)		γ_4	0.4407	(0.0197)	
γ_5	-1.334	(-0.1717)		γ_5	0.3586	(0.0115)	
κ	-0.0218	(-13.14)**		κ	-0.01554	(-9.840)**	
ϕ_0	-4.513	(-5.342)**	0.0108	ϕ_0	-3.593	(-14.13)**	0.0268
ϕ_1	-0.1518	(-0.0276)		ϕ_1	0.3619	(0.1026)	
ϕ_2	6.455	(1.736)		ϕ_2	1.746	(0.3780)	
ϕ_3	2.600	(0.7085)		ϕ_3	0.9004	(0.2718)	
ϕ_4	-5.746	(-0.5434)		ϕ_4	2.668	(1.388)	
ϕ_5	2.178	(0.2866)		ϕ_5	2.679	(1.027)	
θ	0.0147	(7.950)**		θ	0.01350	(15.90)**	
$\sum_{i=1}^{5} \mu_i$	-0.0112	(-2.663)**		$\sum_{i=1}^{5} \mu_i$	-0.0178	(-4.3017)**	
Log-Like	5825.8.0				5991.2		
T	1656				1635		

Business daily observations. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). The column "BJP" reports the base jump probabilities $\bar{p} = \frac{1}{1+exp(-\gamma_0)}$ and $\bar{q} = \frac{1}{1+exp(-\phi_0)}$. (*) and (**) denote significance at the 95% and 99% levels.

3.10, can be interpreted as follows. Suppose that there is a dollar sale of average size, say \$0.15 billion, in support of the DM during some day t in the post-Louvre period. Suppose further that the sale is the only intervention which occurs during the five-day window around t. Then the exchange rate drift (movement exclusive of jumps) during the five day window will differ from the base drift of $5\mu_0$ by $0.15 \times \sum_{i=1}^{5} \mu_i = 0.15 \times (-0.0012) = -0.00018$. The sum $\sum_{i=1}^{5} \mu_i$ is negative for both the DM and Yen rates, indicating that dollar sales are associated with extra drift *appreciation* over the five-day intervention window, a counterintuitive result which is again likely due to simultaneity of dollar sales with some exogenous factor which works in the direction of dollar appreciation. However, we have decomposed this five day extra drift movement into a three-day period of extra appreciation followed by a two day period in which intervention moves the exchange rate in the "desired" direction.

Now consider the estimates of γ_i and ϕ_i, which measure the effect of intervention on the jump probabilities. The estimates of γ_1 and γ_2 are not significantly different from zero for either of the currencies or sub-periods. Because of the simultaneity problem, there are several ways to interpret this result. One might say that, in the two days before a dollar sale, the dollar is neither more nor less likely to undergo a jump appreciation. Thus, we might take the coefficients as evidence that the U.S. does *not* attempt to offset large appreciations with subsequent dollar sales. This may reflect a belief on the part of the government that it is futile to attempt to offset very large movements in the exchange rate [11] Alternatively, and perhaps less plausibly, it may be the case that during the two days before the dollar sale some exogenous factor acts, *ceteris paribus*, to increase the probability of a jump appreciation, but because market participants *anticipate* the eventual dollar sale, the effect of the exogenous factor is cancelled out, so that there is no net effect on the probability of a jump appreciation. A similar argument holds for the coefficients ϕ_1 and ϕ_2, which are

[11] The futility of intervention in the face of very sharp movements in the exchange rate of course says nothing about why such movements occur. The movements may be purely irrational and eventually self-correcting, in which case a wise government would not waste FX reserves in offsetting them. Or the movements may represent the *bursting* of an irrational bubble and hence a return to "fundamentals," in which case a policy of "standing pat" would also make the most sense.

also insignificantly different from zero. This may be interpreted as evidence that the U.S. does not respond to jump depreciations with dollar purchases, or as evidence that, during times of exogenous stress, dollar purchases are anticipated by market participants and the anticipation helps calm jump depreciation fears before the fact. The contemporaneous coefficients γ_3 and ϕ_3 are insignificantly different from zero for all currencies and subperiods except γ_3 for the post-Louvre $/DM, which is significantly negative. This may be taken as evidence that, for this currency and subperiod, same-day dollar purchases actually increased the probability of a jump appreciation. This is an example of "leaning with the wind" operations which cause large movements in the exchange rate.[12] Finally, for both periods and currencies, the coefficients $\gamma_4, \gamma_5, \phi_4$, and ϕ_5 on lagged intervention are all insignificant. Again, this result may be interpreted in at least two ways: (i) intervention at date t is ineffective in calming jump fears during the following two days, or (ii) because some exogenous factor is driving jump fears during the two days after the intervention, the calming effect of the intervention "washes out" with the exogenous factor, leaving jump probabilities unchanged. If this second explanation is true, it would be dangerous to conclude that intervention in response to jumps is ineffective.

To see whether the intervention-return relationship operates differently at longer horizons, an analogous estimation was performed for the weekly data, with the drift and jump probabilites parameterized to depend on the one-week lead, contemporaneous, and one-week lagged intervention, again measured in billions of dollars sold. The results are reported in Tables 3.11 and 3.12.

As the sign and significance of the drift coefficients μ_1, μ_2, and μ_3 indicate, the relationship of intervention to small movements in the exchange rate mirrors that of the weekly OLS regression reported in Table 3.8. For both currencies in the post-Louvre period, the coefficients μ_1 and μ_2 are negative and significant, indicating that dollar purchases in week t are associated with dollar drift depreciation during that week and the preceding week. Again, for small drift movements, the exchange

[12]A large movement of this sort may result when a well-timed and well-publicized intervention is used to burst an existing speculative bubble.

Table 3.11: Pre-Louvre: Weekly Exchange Rate and Intervention

DM	Parameter	T-Stat	BJP	Yen	Parameter	T-Stat	BJP
α_0	0.00015	(11.84)**		α_0	0.00008	(8.432)**	
α_1	-0.00215	(-1.412)		α_1	0.2222	(3.421)**	
μ_0	-0.00439	(-5.277)**		μ_0	-0.00295	(-4.434)**	
μ_1	-0.00195	(-0.242)		μ_1	-0.00621	(-0.5258)	
μ_2	-0.0332	(-4.270)**		μ_2	-0.0124	(-1.266)	
μ_3	0.0156	(1.522)		μ_3	-0.00137	(-0.0106)	
γ_0	-1.554	(-4.116)**	0.016	γ_0	-4.654	(-5.508)**	0.0094
γ_1	-4.074	(-1.414)		γ_1	-2.605	(-0.1370)	
γ_2	0.4153	(0.134)		γ_2	17.27	(1.940)*	
γ_3	1.026	(0.476)		γ_3	-5.976	(-0.3215)	
κ	-0.0265	(-11.68)**		κ	0.0517	(5.331)**	
ϕ_0	-6.078	(-1.400)	0.198	ϕ_0	-1.863	(-4.111)**	0.1344
ϕ_1	1.267	(0.0128)		ϕ_1	2.594	(0.3184)	
ϕ_2	0.9505	(0.0073)		ϕ_2	-4.931	(-0.4776)	
ϕ_3	-0.4943	(-0.0029)		ϕ_3	12.74	(0.9328)	
θ	0.0751	(0.3428)		θ	0.0217	(8.597)**	
$\sum_{i=1}^{5}\mu_i$	-0.0196	(-1.626)		$\sum_{i=1}^{5}\mu_i$	-0.0200	(-1.1251)	
Log-Like	1002.4				1047.4.3		
T	371				370		

Wednesday-to-Wednesday returns. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). "BJP" reports the base jump probabilities $\overline{p} = \frac{1}{1+exp(-\gamma_0)}$ and $\overline{q} = \frac{1}{1+exp(-\phi_0)}$. (*) and (**) denote significance at the 95% and 99% levels.

Table 3.12: Post-Louvre: Weekly Exchange Rate and Intervention

DM	Parameter	T-Stat	BJP	Yen	Parameter	T-Stat	BJP
α_0	0.00019	(10.79)**		α_0	0.00012	(10.96)**	
α_1	0.00926	(0.1581)		α_1	0.00604	(0.179)	
μ_0	0.01025	(1.286)		μ_0	0.00080	(1.205)	
μ_1	-0.01263	(-2.602)**		μ_1	-0.0160	(-2.996)**	
μ_2	-0.01056	(-3.428)**		μ_2	-0.0192	(-3.182)**	
μ_3	0.01249	(2.561)*		μ_3	0.0129	(2.248)*	
γ_0	-4.3826	(-5.545)**	0.0123	γ_0	-3.636	(-1.951)*	0.0257
γ_1	1.3206	(0.0678)		γ_1	0.6194	(-0.0389)	
γ_2	-1.1453	(-0.3354)		γ_2	-0.9717	(-0.163)	
γ_3	-3.4674	(-1.419)		γ_3	-2.223	(-0.731)	
κ	-0.04862	(-6.145)**		κ	-0.0263	(-3.456)**	
ϕ_0	-5.5296	(-2.907)**	0.004	ϕ_0	-3.624	(-7.170)**	0.0260
ϕ_1	1.7044	(0.1315)		ϕ_1	-1.177	(-0.476)	
ϕ_2	0.46644	(0.0246)		ϕ_2	3.185	(1.522)	
ϕ_3	0.56670	(0.0205)		ϕ_3	-0.1824	(-0.046)	
θ	0.06013	(4.271)**		θ	0.0373	(8.329)**	
$\sum_{i=1}^{5} \mu_i$	-0.01521	(-2.4563)*		$\sum_{i=1}^{5} \mu_i$	-0.0233	(-3.386)**	
Log-Like	899.6				942.3		
T	331				327		

Wednesday-to-Wednesday returns. Asymptotic t-statistics in parentheses. Maximum likelihood estimates, with asymptotic variance-covariance matrix calculated using the method of Berndt *et als* (1974). "BJP" reports the base jump probabilities $\overline{p} = \frac{1}{1+exp(-\gamma_0)}$ and $\overline{q} = \frac{1}{1+exp(-\phi_0)}$. (*) and (**) denote significance at the 95% and 99% levels.

rate moves in the desired direction in the week following the intervention. A similar pattern holds for the pre-Louvre period, though the response of the exchange rate to intervention is not as significant as in the pre-Louvre. As is the case for the daily returns, the effects of intervention on jump probabilities, as measured by the coefficients γ_i and ϕ_i, $i = 1...5$, are in general insignificant. The same simultaneity caveats hold in interpreting this "insignificance" result at the weekly horizon as did for the daily case.

3.6 Conclusion

We have shown that daily and weekly dollar exchange rates are well-characterized by a jump-diffusion model in which regular daily drift returns of a few basis points each are interspersed with occasional "jumps" of 1% or more. The FX return data exhibit significant "jump" behavior even when ARCH effects are allowed for. The exchange rate drift movements are related to leads and lags of intervention in a systematic pattern which may be interpreted as evidence that the U.S. "leans against the wind" in response to small exchange rate changes, and that such "wind-leaning" does have some effect, with a lag, at the daily and weekly horizons. However, there seems to be no strong relationship between jump movements in the exchange rate and interventions. Although this result must be interpreted in light of a potential simultaneity problem, one possible explanation is that the U.S. does not in general respond to *large* exchange rate movements with offsetting intervention activity, because intervention in response to large movements is ineffective.

Bibliography

[1] K. Aase and P. Guttorp, "Estimation in Models for Security Prices," *Scandanavian Actuarial Journal*, 1987, 211-224.

[2] V. Akgiray and G.G. Booth, "Mixed Diffusion–Jump Process Modeling of Exchange Rate Movements," *Review of Economics and Statistics* **70**, November 1988, 638–647.

[3] C.A. Ball and W.N. Torous, "A Simplified Jump Process for Common Stock Returns," *Journal of Financial and Quantitative Analysis* **18**(1), 53-65.

[4] D. S. Bates, "Crashes, Options, and International Asset Substitutability," Princeton University Ph.D. dissertation, January 1989.

[5] D. S. Bates, "The Crash Premium: Option Pricing Under Asymmetric Processes, with Applications to Options on Deutschemark Futures," Working Paper 36–88, University of Pennsylvania, Rodney L. White Center, 1988.

[6] D. S. Bates, "Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in PHLX Deutschemark Options," National Bureau of Economic Research Working Paper No. 4596, December 1993.

[7] D. S. Bates, "Distributional Abnormalities Implict in Foreign Currency Futures Options," forthcoming, *Journal of International Money and Finance*, 1995.

[8] E. Berndt, B. Hall, R. Hall, and J. Hausman, "Estimation and Inference in Nonlinear Models," *Annals of Economic and Social Measurement*,**3/4**,1974, 653-665.

[9] J. Bodurtha and G. Courtadon, "Tests of the American Option Pricing Model in the Foreign Currency Option Market," *Journal of Financial and Quantitative Analysis*, **22**, 1987, 153-167.

[10] T. Bollerslev, R.Y. Chou, and K.F. Kroner, "ARCH Modeling in Finance: a Review of the Theory and Empirical Evidence," *Journal of Econometrics*, **52**, 1992, 5-59.

[11] J.R. Calderon-Rossell and M. Ben-Horim, "The Behavior of Foreign Exchange Rates," *Journal of International Business Studies*, Fall 1982, 99-111.

[12] S. Das, "Jump-Diffusion Processes and the Bond Markets," Harvard Business School Working Paper 95-034, 1994.

[13] I.M. Destler and C.R. Henning, *Dollar Politics: Exchange Rate Policymaking in the United States*, Washington: Institute for International Economics, 1989.

[14] K.M. Dominguez, "Does Central Bank Intervention Increase the Volatility of Foreign Exchange Rates?" National Bureau of Economic Research Working Paper No. 4532.

[15] K.M. Dominguez and J.A. Frankel, *Does Foreign Exchange Intervention work?*, Washington: Institute for International Economics, 1993.

[16] K.M. Dominguez and J.A. Frankel, "Does Foreign Exchange Intervention Matter? The Portfolio Effect," *American Economic Review*, **83**, 1993, 1356-1369.

[17] H. Edison, "The Effectiveness of Central Bank Intervention: A Survey of the Post- 1982 Literature," *Special Papers on International Economics* **18** Princeton University, 1993.

[18] R.F. Engle, "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of UK Inflation," *Econometrica*, **50**, 1982, 987-1008.

[19] R.F. Engle and T. Bollerslev, "Modelling the Persistence of Conditional Variances," *Econometric reviews*, **5**, 1986, 1-47.

[20] B. Friedman and D. Laibson, "Economics Implications of Extraordinary Movements in Stock Prices," *Brookings Papers on Economic Activity*, 2:1989, 137-189.

[21] D. Friedman and S. Vandersteel, "Short-Run Fluctuations in Foreign Exchange Rates," *Journal of International Economics*, **13** 1982, 171-186.

[22] A.R. Ghosh "Is It Signalling? Exchange Intervention and the Dollar-DM Rate," *Journal of International Economics* **32**, 1992, 201-220.

[23] D.A. Hsieh, "The Statistical Properties of Daily Foreign Exchange Rates: 1974-1983," *Journal of International Economics*, **24**,1988, 129-145.

[24] R. Jarrow and E. Rosenfeld, "Jump Risks and the Intertemporal Capital Asset Pricing Model," *Journal of Business*, **57**, 1984, 337-351.

[25] P. Jorion, "On Jump Processes in the Foreign Exchange and Stock Markets," *Review of Financial Studies* **1**, 1988, 427-445.

[26] P. Jurgensen, "Report of the Working Group on Exchange Market Intervention, Washinton: Treasury Department, 1983.

[27] G. Kaminsky and K. Lewis, "Does Foreign Exchange Intervention Signal Future Monetary Policy?" Finance and Economic Discussion Series No. 93-1. Washington: Federal Reserve Board, 1993.

[28] B. Loopesko, "Relationships Among Exchange Rates, Intervention, and Interest Rates: An Empirical Investigation," *Journal of International Money and Finance*, **3**, 1984, 257-77.

[29] J.W. McFarland, R.R. Pettit, S.K. Sung, "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement," June 1982, 693-715.

[30] D. Mulford "The G7 Strikes Back!" *The International Economy*, July/August 1991, 15-23.

[31] J.C. So, "The Distribution of Foreign Exchange Price Changes: Trading Day Effects and Risk Measurement–A Comment," *Journal of Finance*, March 1987, 181-187.

[32] A.L. Tucker and L. Pond, "The Probability Distribution of Foreign Exchange Price Changes: Tests of Candidate Processes," *Review of Economics and Statistics* **70**, November 1988, 638–647.

[33] J.M. Westerfield, "An Examination of Foreign Exchange Risk Under Fixed and Floating Rate Regimes," *Journal of International Economics* **7**, 1977, 181-200.