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AUTOMATIC TRACKING CONTROL OF AIRCRAFT

By

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S.B., S.M., Massachusetts Institute of Technology

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Signature of Author . . . Signature redacted . . . .

Department of Aeronautical Engineering April 2, 1947

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Committee on Graduate Students . . Signature redacted . . . .

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Aero
Thesis
1947

April 2, 1947

Professor Joseph S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Professor Newell:

In accordance with the regulations of the faculty, I hereby submit a thesis entitled "Automatic Tracking Control of Aircraft" in partial fulfillment of the requirements for the degree of Doctor of Science in Aeronautical Engineering.

Sincerely,

Signature Redacted

James B. Rea

BIOGRAPHICAL SKETCH

James B. Rea was born in Honolulu, Hawaii, on April 2, 1916. He attended primary schools in Honolulu, graduating from Punahou School in 1933. He then studied Sugar Technology at the University of Hawaii until 1934, when he transferred to the University of California at Berkeley to study Music and Mathematics.

In 1938 he transferred to the Massachusetts Institute of Technology to study Aeronautical Engineering. In 1939 he was appointed Research Assistant in Aeronautical Engineering. In 1940 he was awarded the Sloan Automotive Engineering Fellowship, and was elected to Tau Beta Pi. As a member of the Honors Group in Aeronautical Engineering he received the degrees of S.B. and S.M. simultaneously in the spring of 1941.

He was then hired by Pan American Airways as an Engineering-pilot, later becoming Flight Captain. He flew scheduled flights over the Atlantic, the Pacific, and South American routes.

In the fall of 1943 he left Pan American Airways to work as a Research Test-Pilot and Design Specialist for Consolidated Vultee Aircraft Corporation.

In the fall of 1945, he was appointed Consolidated Vultee Fellow, and was given a leave of absence to return to the Massachusetts Institute of Technology as a candidate for the degree of Doctor of Science in Aeronautical Engineering. In the summer of 1946 he was appointed Research Associate in Aeronautical Engineering.

A B S T R A C T

The primary object of the work described in this thesis was to develop equations suitable for predicting the performance of a system for automatic tracking control of aircraft, and to use these equations to determine the requirements and analyze the performance of a system for automatic tracking control of the A-26 Airplane. The secondary object was to develop equations suitable for predicting the steady-state response of aircraft to forced sinusoidal motion of controls, and to use these equations as a basis for indicating flight-test methods for determining aircraft dynamic stability coefficients.

Equations suitable for predicting the performance of a system for automatic tracking control of aircraft were developed, and applied to a system for automatic tracking control of the A-26 Airplane. These equations were analyzed by the M.I.T. Rockefeller Differential Analyzer to determine the servo stability coefficients required for practical solution time of the overall system.

The basic problem given to the Differential Analyzer was as follows: Two airplanes were assumed to be flying level, each trimmed at 300 mph TAS, one behind the other. The rear (tracking) airplane was assumed to be an A-26 Airplane, flying at 10,000 ft. density altitude, while the forward (target) airplane was assumed to be separated from the tracking airplane by an initial range of 1000 yards. In addition, the target airplane was assumed to be 150 ft. above and 150 ft. to the right of the tracking airplane, so that the initial line of sight from the tracking airplane would have an elevation component of +50 mils as well as a deflection component of +50 mils. It was then assumed that the automatic control system was suddenly turned on, thus applying a step-function input to the radar system.

Analysis of the longitudinal loop of the overall system indicated

the following:

Solution time to establish pure pursuit course $\cong$ 1 second.
Solution time to establish aerodynamic lead pursuit course $\cong$ 2 seconds.
A stability number $\cong .6$ was found to be best.

The effect on solution time of neglecting certain of the aircraft stability coefficients was also found. For instance the effect of neglecting z_u was negligible, whereas the effect of neglecting $m_{\dot{w}}$ was not.

Equations suitable for predicting the steady-state response of aircraft to forced sinusoidal motion of controls, were developed. These equations were used to compute the steady-state response of the A-26 Airplane to forced sinusoidal motion of its controls. The steady-state computed response agreed very well with steady-state flight-test response for the cases investigated. The response equations were also used to indicate flight-test methods for determining aircraft stability coefficients. The damping coefficient and the spring constant for the short-period component of the transient longitudinal response of the A-26 Airplane were computed theoretically, and determined from flight-test data by the circle-diagram method. The computed and flight-test values agreed very well.

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To the members of the Instrumentation Laboratory at the Massachusetts Institute of Technology for their cooperation, and especially to Miss A. Boissevain and Mrs. J. Fisher for help with mathematical analyses, and to Miss S. Knight and Miss G. Sloan for help with computations. To the Ralph C. Coxhead Corporation, and especially to Mrs. A. Sakakeeny, for typing the manuscript. To Mr. Earle Payne and staff for reproducing the manuscript, and especially to Mr. R. Hofmann for preparing the figures. To my wife for proofreading and compiling the manuscript.

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To the members of the Differential Analyzer staff at the Massachusetts Institute of Technology, and especially to Mr. B. Svihel and Mr. John Lof, for solving the automatic tracking control network equations.

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AUTOMATIC TRACKING CONTROL OF AIRCRAFT

OBJECT

The primary object of the work described in this thesis was to develop equations suitable for predicting the performance of a system for automatic tracking control of aircraft, and to use these equations to determine the requirements and analyze the performance of a system for automatic tracking control of the A-26 Airplane.

The object was divided into two parts. Part One dealt with the overall tracking control system, and Part Two dealt with one component of the overall system, namely the aircraft itself.

Part One
Overall Tracking Control System

1. To determine the requirements of a system for automatic tracking control of aircraft.
2. To develop equations suitable for predicting the overall performance of a system for automatic tracking control of aircraft.
3. To determine the component characteristics required for practical solution time of a system for automatic tracking control of the A-26 Airplane.

Part Two
Aircraft

1. To develop equations for computing the steady-state response of aircraft to forced sinusoidal motion of controls. To compute the steady-state response of

the A-26 Airplane to forced sinusoidal motion of its controls.

2. To develop equations for computing the transient response of aircraft to step-functions, for fixed controls, and for cases when the aircraft is equipped with an automatic pilot which uses displacement, derivative, and integral pitch control. To compute the transient response of the A-26 Airplane to step-functions, for fixed controls, and for the case when the airplane is equipped with an automatic pilot which uses ideal displacement pitch control.

3. To investigate flight-test methods for determining aircraft dynamic stability coefficients.

INTRODUCTION

Research on guided missles has been emphasized by our government during and since World War II. During the War, it was imperative to develop in as short a time as possible a missle which could track a target automatically.

Many missles were built and tested; and the testing technique was as follows: When the automatic tracking performance of a missle proved to be unstable, a readjustment was made in the tracking control system of the next missle to be fired. Since each missle fired was destroyed (except in a few cases where automatic parachute recovery could be used), the experimental techniques were necessarily one of trial and error. This technique was of course extremely costly.

Since the War, additional programs for research on automatic tracking have been started. These programs include research on automatic interception tracking, automatic fire-control tracking with fixed guns, automatic fire-control tracking with movable guns, and automatic stellar-inertial tracking. The program (Army contract W33-038-ac-14327) with which the author is currently associated, is primarily concerned with research on automatic interception tracking and automatic fire-control tracking with fixed guns. But the method of approach in this program is basically very different from that in the missle program. It is based on the premise that if first an *airplane* could be made to track a target automatically, it would then be comparatively simple to make a missle track automatically. The theory and techniques established while developing a system for suitable automatic tracking control of an airplane, could be extended and used for design of a system for automatic tracking control of a missle. Also, the extension of the theory would be more reliable if there were dependable correlation between theoretical and flight-test values of aircraft dynamic stability coefficients.

The object of the work described in this thesis was based upon the foregoing reasoning and method of approach. The primary object was to develop equations suitable for predicting the performance of a system for automatic tracking control of aircraft, and to use these equations to determine the requirements and analyze the performance of a system for automatic tracking control of the A-26 Airplane. This phase of the analysis is discussed in Part One of the Text.

The object and text were each divided into two main parts. Part One dealt with the overall tracking control system; and, because of the need for correlation between theoretical and flight-test values of aircraft dynamic stability coefficients, Part Two dealt primarily with the aircraft itself. In Part One the transient step-function response of the overall system was studied, whereas in Part Two the steady-state response of the aircraft itself, to sinusoidal forcing functions, was studied.

There are two basic methods for analyzing the characteristics of a system capable of dynamic response. One method deals with the transient response, while the other deals with the steady-state response. The transient method is preferable for the analysis of a system which is under-critically damped (oscillatory), whereas the steady-state method is preferable for the analysis of a system which is over-critically damped (non-oscillatory). These two basic methods are commonly used throughout the field of Engineering.

In a system for automatic tracking control of aircraft, the aircraft itself has six degrees of freedom. Of these six, the degrees of freedom which have by far the greatest effect on the solution time of the overall tracking control system, are those which are primarily connected with the short-period components of the aircraft transient response. These components are always very highly damped. Thus, it is most desirable to analyze the characteristics of the aircraft itself (Part Two) by the steady-state method. The transient method (also shown in Part Two) is of course very valuable for determining the solution time of the overall tracking control system, especially when a Differential Analyzer, or a simulator, is not available. It is also valuable for analysis of those degrees of freedom of the aircraft which are primarily connected with the long-period components of the aircraft transient response. However, these degrees of freedom have a negligible effect on the solution time of the overall tracking control system. (For proof of this, see results of analyses by Rockefeller Differential Analyzer in Section A of Appendix G. In Section A it is shown that the degree of freedom primarily associated with the long-period of the aircraft transient longitudinal motion has a negligible effect on solution time of the overall tracking control system).

Flight-test methods for determining aircraft dynamic stability coefficients are discussed in Part Two. It is important that these methods be developed because of the need for correlation between theoretical and flight-test values of aircraft stability coefficients, especially for those coefficients which control the short-period components of aircraft transient response.

PART I

OVERALL TRACKING CONTROL SYSTEM

Part I

SECTION I

REQUIREMENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

A. Primary Components and Degrees of Freedom

A system for automatic tracking control of aircraft should include the following primary components:

1. Tracker
2. Range Finder
3. Computer
4. Aircraft Control Servos
5. Aircraft itself

The tracker and range finder are usually combined in a radar unit mounted at the nose of the aircraft; the computer is usually a gyroscopic mechanism; and the aircraft control servos are usually electric motors (see Figure I for mechanical arrangement of components).

The radar unit has two basic functions. They are: (a) to track the line of sight, and (b) to determine the target range. The radar antenna assembly (Figure II) tracks the line of sight, and it also sends and receives the range signal. Two angular degrees of freedom are required to track the line of sight, one in elevation and one in deflection. Each degree of freedom is controlled by a radar servo motor, and each motor receives its control signal from the tracking line-line of sight comparator. The comparator is usually a dish mounted on the inner radar gimbal. The target range is determined automatically by a separate system in the radar unit. This system is essentially a timing circuit which detects range by measuring the time required for the range signal to go to the target and return.

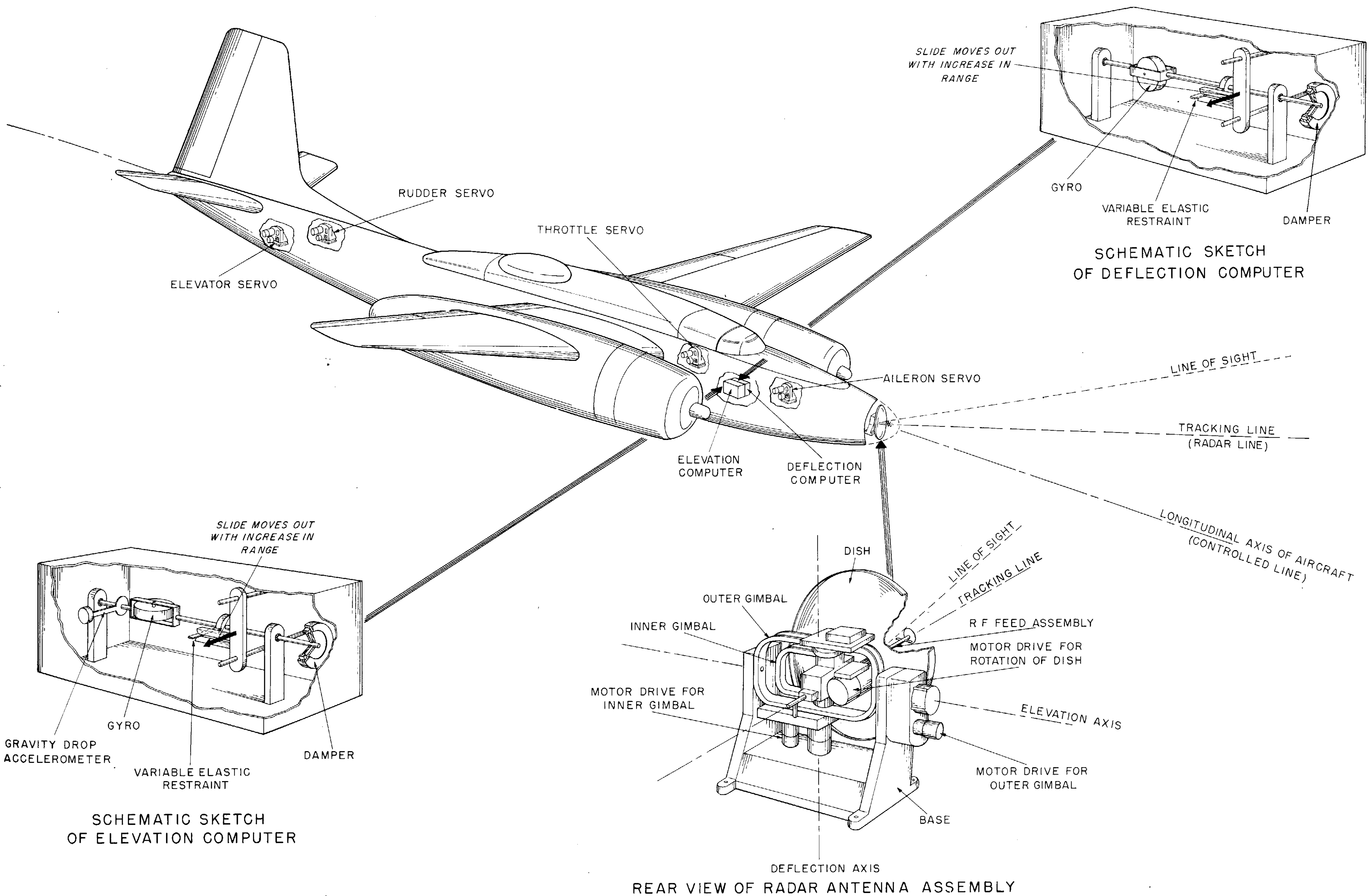

FIGURE I

MECHANICAL ARRANGEMENT OF COMPONENTS IN A SYSTEM FOR AUTOMATIC TRACKING CONTROL FOR AIRCRAFT

The computer (Figure III) has but one basic function in a system for fixed-gun fire-control, namely to indicate the required (correct) prediction angle between the line of sight and the gun line. The computer is usually composed of two units, each with its own degree of freedom. One unit is called the deflection computer, and the other is called the elevation computer. The deflection computer indicates the deflection component of the prediction angle, and the elevation computer indicates the elevation component. Each unit is essentially a gyroscopic mechanism which receives angular velocity as the primary input and produces a component of the prediction angle as the output. A third unit is sometimes included to account for cross-roll. This unit is also a gyroscopic mechanism. Details of the gyroscopic computer are discussed in reference 3.

There are usually four aircraft control servos, one each for the rudder, the elevator, the ailerons, and the throttle. The function of each servo is to control the position of the particular aircraft control with which it is associated. The servo shown in Figure IV is basically an electric motor (Minneapolis-Honneywell C-1A Servo), the characteristics of which are discussed in references 8 and 9. Each aircraft control system has one fundamental degree of freedom (neglecting cable stretch), however, the throttle control system is only cut in when there is a sudden large change in the elevation component of the forcing function, or when there is sufficient change in indicated airspeed. Thus, for small changes in forcing function or indicated airspeed the aircraft control system has only three primary degrees of freedom.

The aircraft itself has six degrees of freedom, three linear and three angular. The aircraft used as the tracking aircraft in the analyses presented in this thesis was the Douglas A-26 Airplane (Figure V and Figure VI).

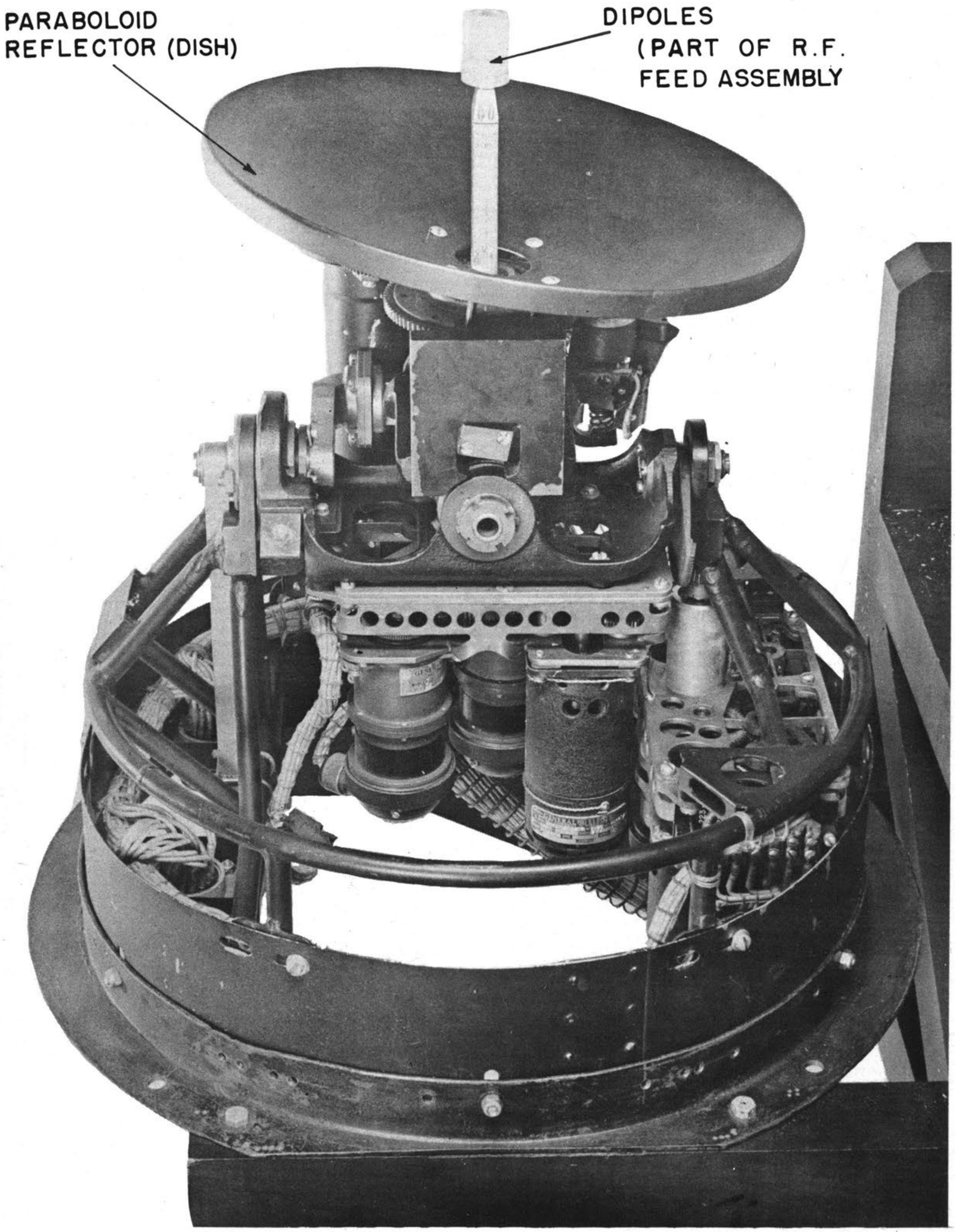

FIGURE II
ANTENNA ASSEMBLY OF G.E. AN/APG-3 (XA-1) RADAR UNIT

Thus, we see that the automatic tracking control system has thirteen primary degrees of freedom, two in the radar, two in the computer, three in the aircraft controls, and six in the aircraft itself.

B. Functional Requirements

A functional diagram of the tight loop system for automatic tracking control of aircraft is shown in Figure VII. In this system the radar has its own "tight loop", in the sense that the input signal to the radar servos comes directly from the tracking line-line of sight comparator, and the radar angle signal is merely a by-product which does not directly control the radar servos.

The inputs to the computer are discussed in Reference 3 and indicated in Figure VII. Also, schematic sketches of the two primary computer units are shown in Figure I. The output from the computer is the computer prediction angle signal.

The computer prediction angle signal adds to the radar angle signal to produce the forcing function signal for the aircraft control servo units. (Each signal is made up of deflection and elevation components, as shown in Figure VIII).

The proper displacement, derivative, and integral control signals are then produced and fed into the aircraft control servo units which cause movement of the controls. The resulting aerodynamic forces then control the motion of the aircraft, which in turn affects the radar and the computer. This completes the functional loop.

C. Safety, Anticipation, and Correct-Flight Controls

A block diagram of the entire automatic tracking control system, including safety controls, anticipation controls, and correct-flight controls, is shown in Figure VIII.

The safety controls are the usual ones required by a pilot.

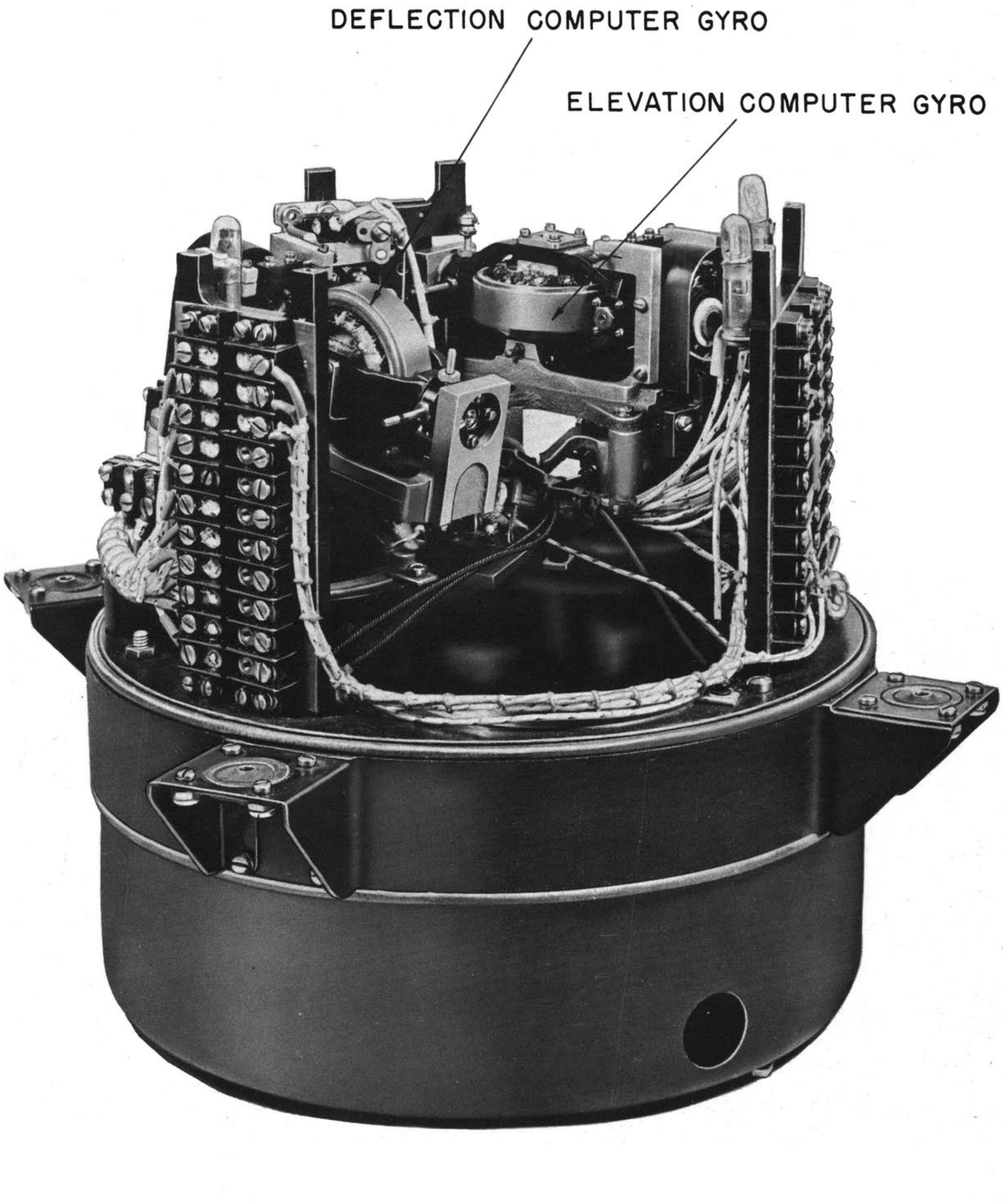

FIGURE III
TCS - I COMPUTER

Examples are as follows:

(a) When the angle of attack gets too large, the angle of attack — elevator cut-out safety control system produces a signal which reduces the elevator servo input control signal.

(b) When the normal acceleration gets too large, the normal acceleration — elevator cut-out safety control system produces a signal which reduces the elevator servo input control signal.

(c) When the absolute value of the indicated airspeed increment gets too large, the indicated airspeed — throttle cut-in safety control system produces a signal which controls the throttle servo in a manner which limits the indicated airspeed increment.

Thus, with proper design the safety controls in an automatic tracking control system can be made to simulate the safety controls used by a human pilot.

An example of an anticipation control is shown by the anticipation throttle control, shown in Figure VIII. To understand the purpose of this control let us analyze a typical case encountered by a pilot in tracking a target. Let us assume that the relative target motion necessitates a sudden large change in elevator angle to keep the tracking aircraft on the target. A large change in elevator angle would immediately cause a large change in indicated airspeed, and this would remind the pilot to change his throttle setting. (This is the control sequence followed by the indicated airspeed — throttle cut-in safety control system.) But, an experienced pilot would not wait until the change in indicated airspeed reminded him to change his throttle setting; he would anticipate this and immediately adjust his throttle as he changed the elevator angle. The anticipation throttle

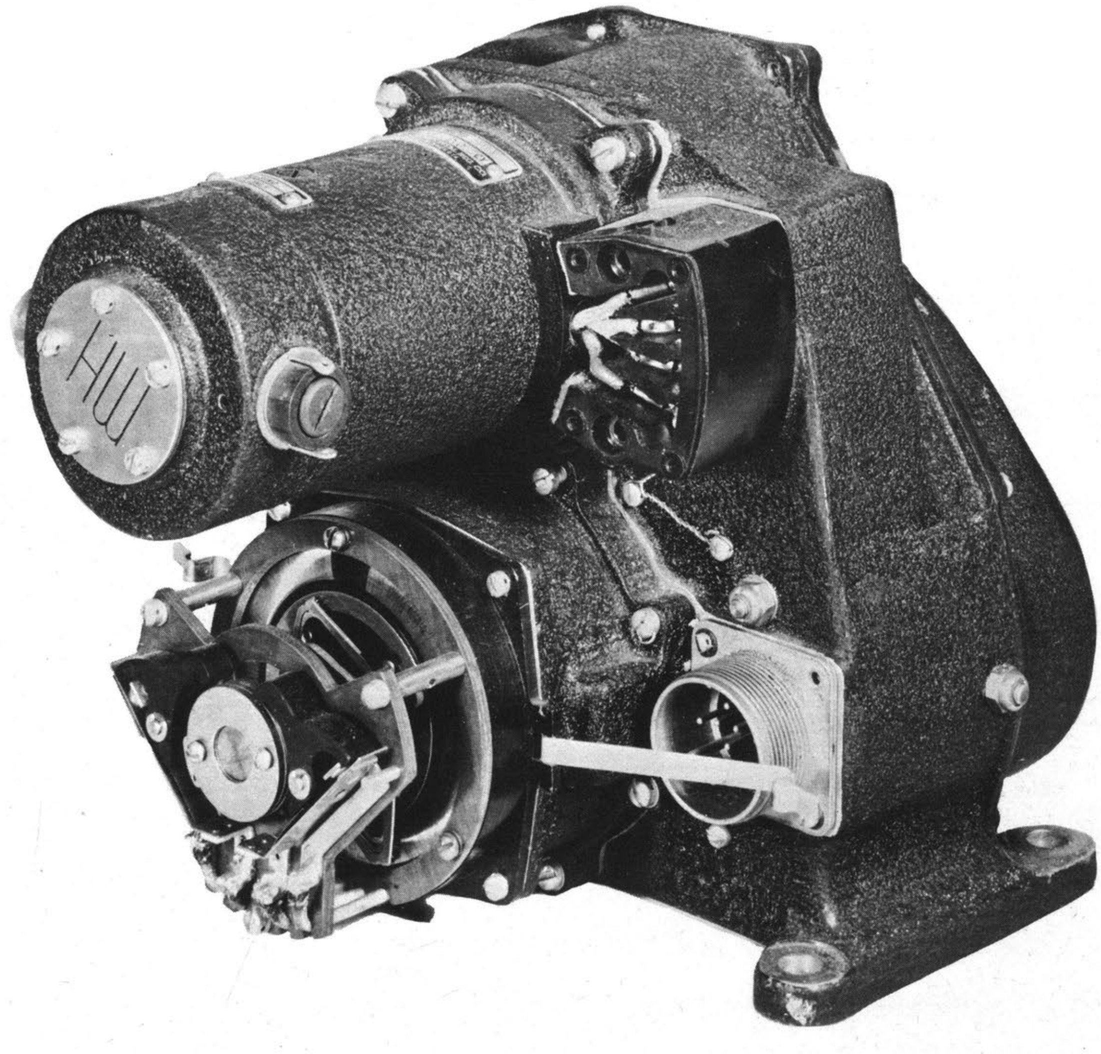

FIGURE IV
MINNEAPOLIS HONEYWELL C-IA SERVO UNIT AND BALANCE POTENTIOMETER

control shown in Figure VIII does just this. When the absolute value of F_E gets too large it immediately changes the throttle setting instead of waiting for the resulting change in airspeed to control the throttle. Of course, other anticipation controls will be necessary in an automatic tracking control system, and the required action of these controls can best be determined by analyzing the actions of an experienced human pilot.

The correct-flight controls consist of the yaw vane and the lateral accelerometer, also represented in Figure VIII. For automatic interception tracking it is essential that the aerodynamic yaw be kept as small as possible. This can be done in two ways. One is to use the yaw vane signal as the basic correction signal, and the other is to use the lateral accelerometer signal as the basic correction signal. Again let us analyze the actions of a pilot in order to see what the requirements are. Let us assume that a pilot is flying in an airplane with an open cockpit, and he feels a wind on his left cheek caused by left aerodynamic yaw. He can correct for this by applying right aileron and left rudder, and thus reduce the aerodynamic yaw without ever glancing at his lateral accelerometer (or ball-bank indicator). Now let us assume he is flying in a closed cockpit without an aerodynamic yaw indicator, and again he encounters a wind from his left. This would cause the ball in the ball-bank indicator to move to his left, and to correct for this, again he would apply right aileron and left rudder. It can thus be seen that the yaw vane and the lateral accelerometer produce corrections which are in the same direction, i.e., they do not work against each other. But if there were a sudden change in angle of bank alone, the lateral accelerometer would be the first to sense the correction because it would immediately pick up the component of the acceleration of gravity, whereas the

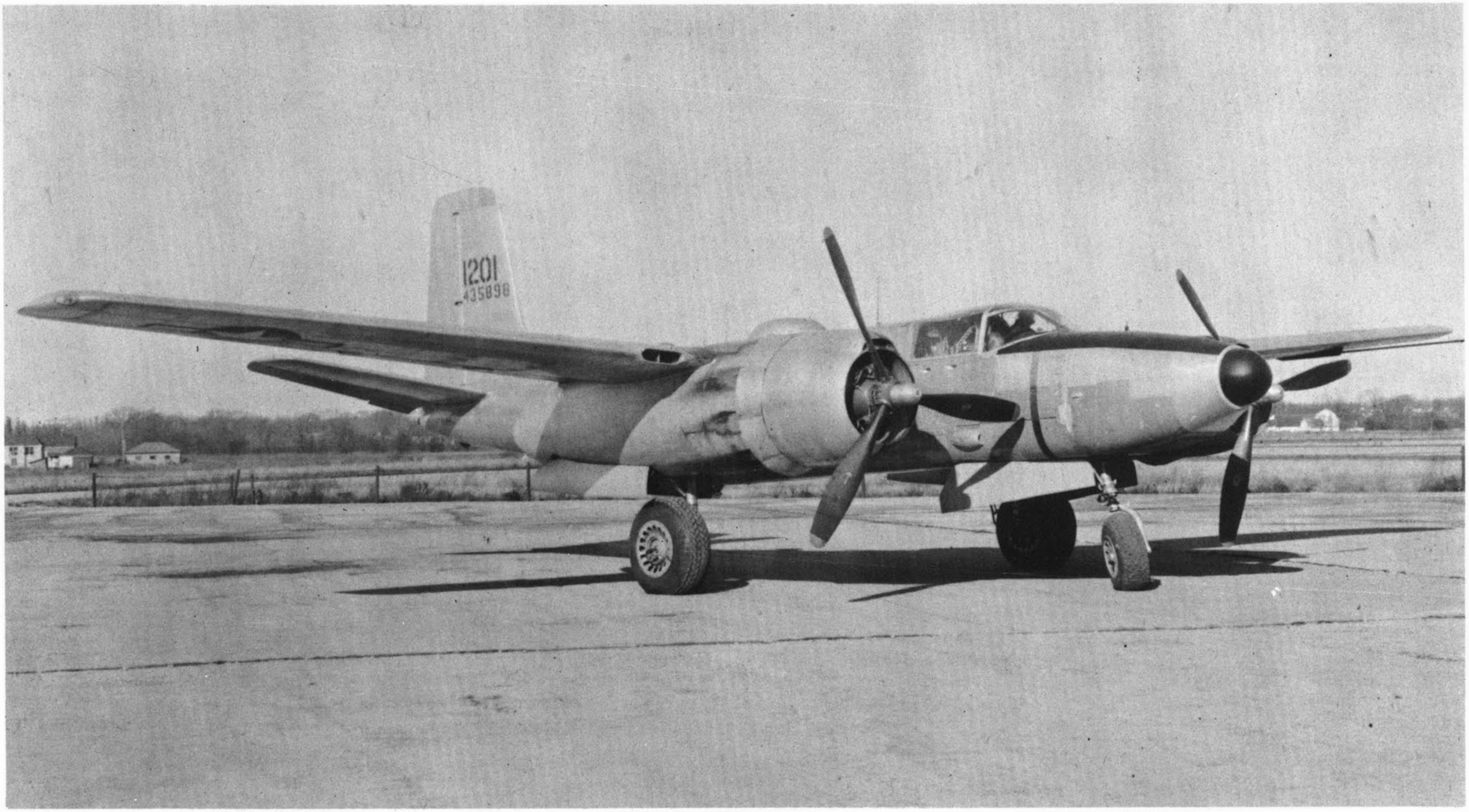

FIGURE V
A - 26 AIRPLANE 3/4 FRONT VIEW

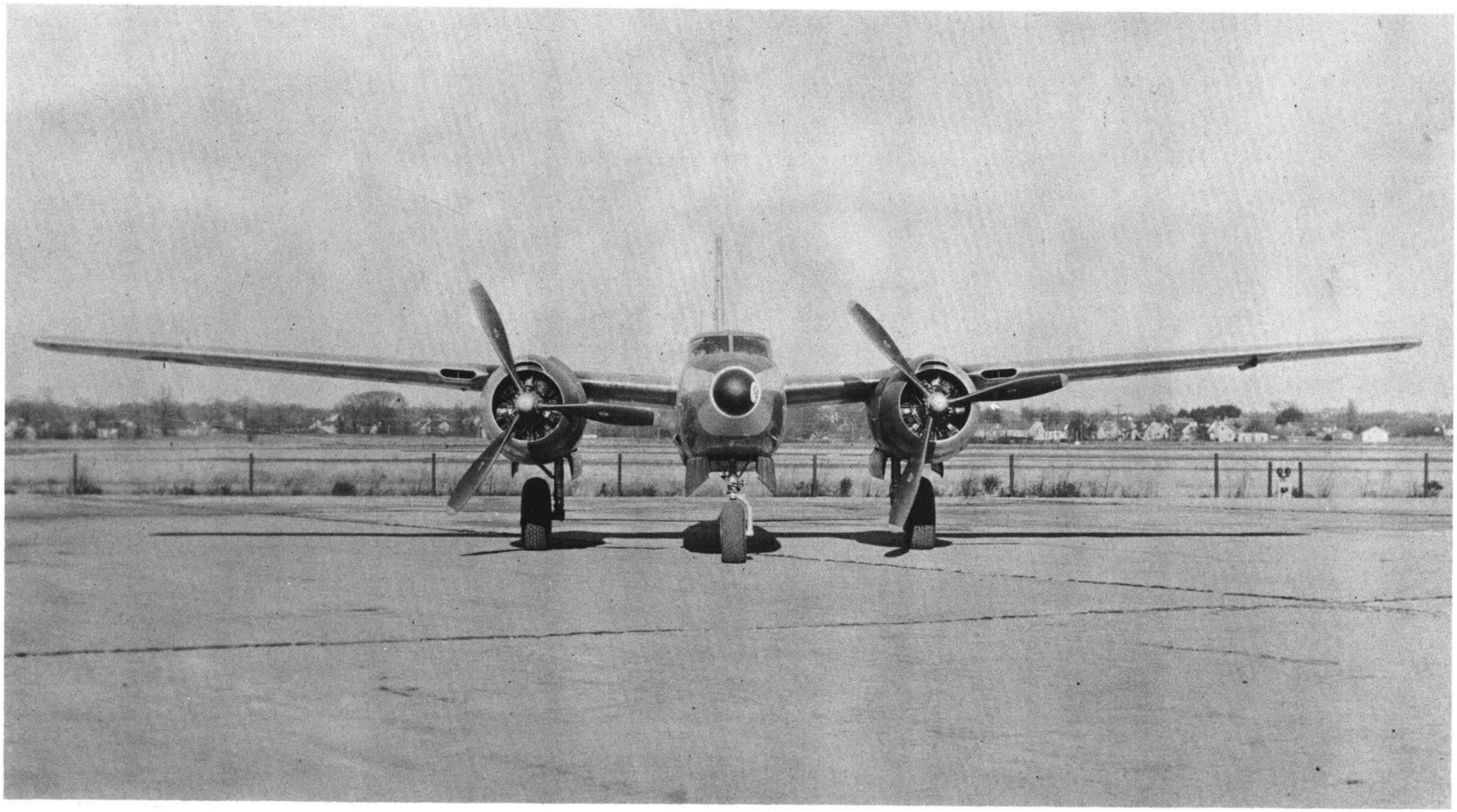

FIGURE VI
A-26 AIRPLANE, FRONT VIEW

yaw vane would produce a correction signal only after a side slip had been developed. On the other hand, a very sensitive yaw vane would be the first to sense the correction if there were a sudden lateral gust. Thus, the author feels that, although either system should work by itself, it might be best to have both in operation from the point of view of reducing lag. Just as a good pilot would make a correction for lateral acceleration (or aerodynamic yaw) by moving both the rudder and the ailerons, so a good automatic system should send the correct-flight correction signals to both the rudder and the ailerons.

It will be noted in Figure VIII that the yaw vane system uses integral control, while the lateral accelerometer system does not. As shown by the examples discussed above, these systems will not oppose each other when the usual correct-flight corrections are required. But if for some unforseen reason an unbalanced yawing moment were developed during flight, as could happen with partial loss of power in a multi-engine aircraft, it would be impossible to keep the tracking aircraft on the target, to hold zero aerodynamic yaw, and to maintain zero lateral acceleration (ball in center), all at the same time. In this case integral control in the yaw vane system, and not in the lateral accelerometer system, would permit the yaw vane to overpower the accelerometer. There are, of course, tracking control problems which would require just the opposite combination, i.e., integral control in the lateral accelerometer system and none in the yaw vane system.

D. Techniques for Controlling Aircraft by Instrument Indications

There are many techniques for controlling an aircfaft by instrument indications. An airline instrument pilot would probably tell you to maintain the indicated airspeed with the elevator and to use

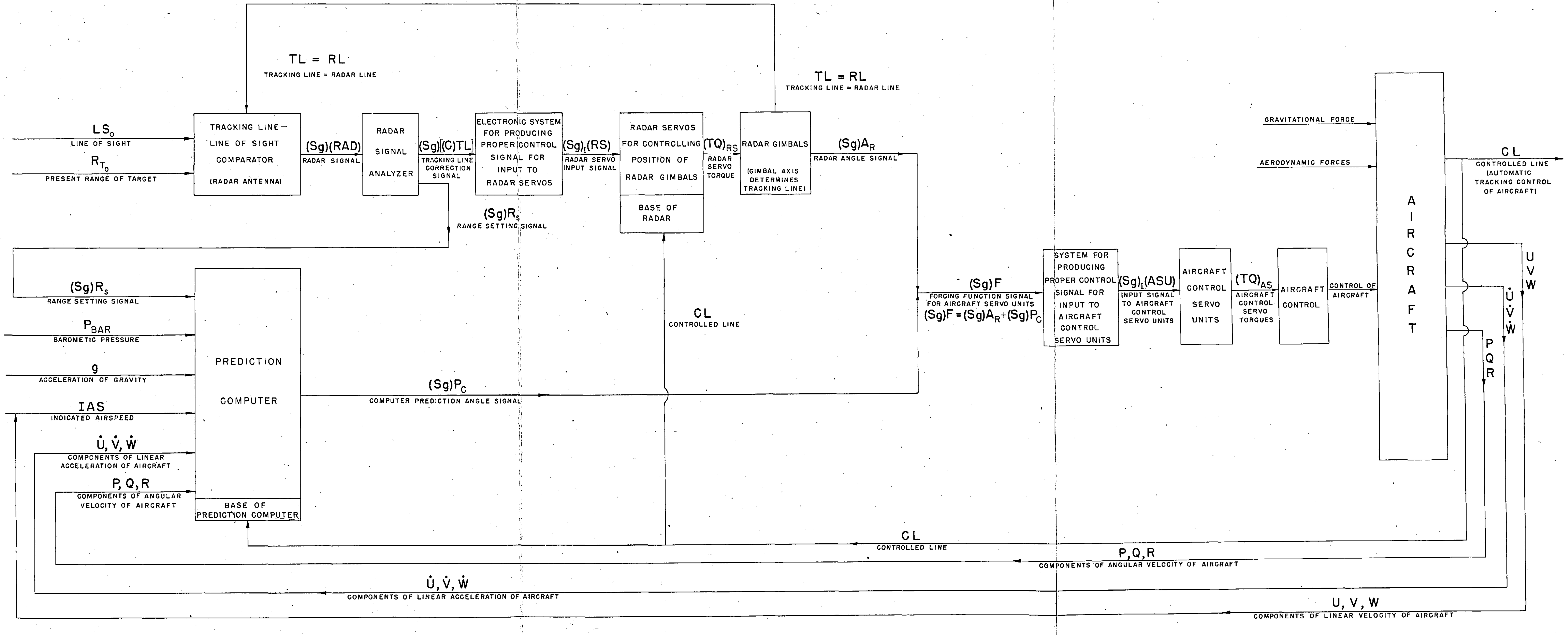

FIG. VII
FUNCTIONAL DIAGRAM OF SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT
TIGHT—LOOP SYSTEM

the throttle to adjust the rate of climb. This is the usual procedure used in airline instrument flying. But a military pilot does not use this technique when manually tracking a target; he uses his elevator (with rudder and ailerons) to keep on the target, and uses his throttle to correct for, or prevent, changes in indicated airspeed. Thus a control system designed for automatic airline instrument flying should be basically very different from one designed for automatic tracking. The latter system (Figure VIII) is the one analyzed in this thesis.

E. Kinds of Control Required

It will be noted that displacement, derivative, and integral control are used throughout the system shown in Figure VIII. The kinds of control required can best be determined by studying the control requirements of a human pilot. When a pilot learns to fly, one of the first tasks given him by his instructor is to fly straight and level at a given point on the horizon. The instructor may pick a point off to one side, and ask him to acquire the proper heading. The initial correction which the pilot applies is a function of the initial displacement angle which he sees. But he soon learns that pure displacement control will only cause him to over-control, and that a certain amount of anticipation is required. In a sense, derivative control is analogous to anticipation on the part of the pilot, because it tends to prevent too high a rate of target closure, and thus reduces the tendency to over-control. In some cases a pilot will find that he must use a steady-state control force to stay on the target. This can be simulated in an automatic tracking control system by the use of integral control. Based upon this reasoning, displacement, derivative, and integral control were chosen for the kinds of control required throughout the system shown in Figure VIII.

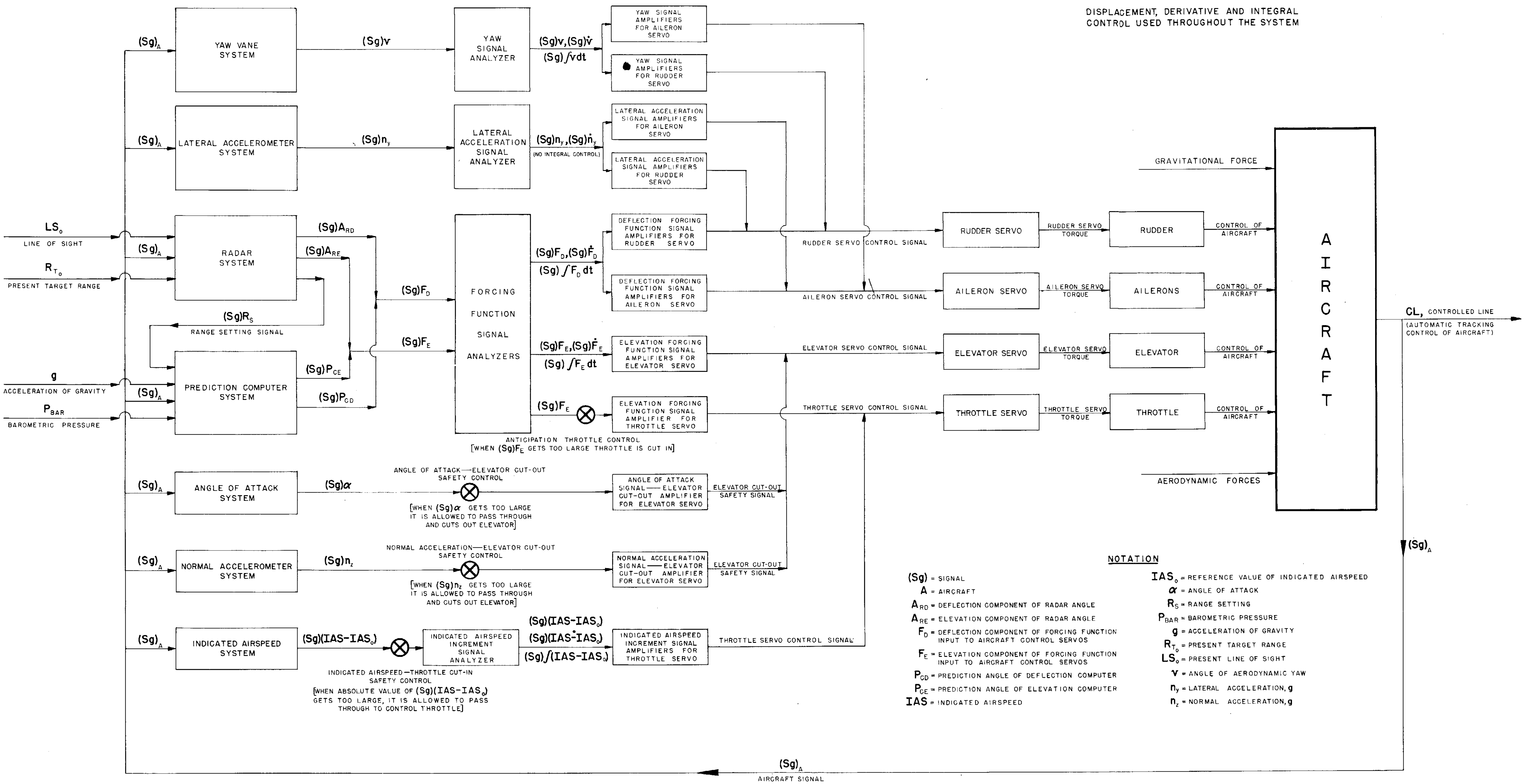

FIG. VIII
BLOCK DIAGRAM OF SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

F. *Importance of Studying Human Pilot Reactions and Control Functions*

The author would like to stress the importance of looking to the reactions and control functions of the human pilot when seeking to determine the primary requirements of an automatic tracking control system. A simple example showing the importance of this kind of reasoning is as follows: Let us assume that in order to track a given target a pilot finds he must establish and maintain a steady rate of turn, and in doing so he also finds that opposite aileron is required. The underlying theory explaining why opposite aileron is required is, of course, very simple. Because the outside wing goes faster than the inside wing it experiences more lift and so produces an unbalanced rolling moment. Thus opposite aileron is needed in the turn to counteract this unbalanced moment. A designer of an automatic tracking control system might then, based upon this theory, decide to use angular velocity about the aircraft normal axis as a correction signal to produce the opposite aileron required. But a pilot does not have to know this theory to successfully track the target, and further, he does not wait until he has fully established the turn before he applies opposite aileron. In effect, an experienced pilot observes the incremental tracking errors, and although he may not realize it, he integrates them while establishing the turn, and ends up with just the right amount of opposite aileron force to hold him in the turn. This should immediately suggest to the designer the use of integral control, using as the basic input to the aileron servo the deflection component of the forcing function signal. Even though integral control may increase the solution time somewhat, the author recommends that it be used. Besides its simplicity, one of its chief advantages is that it acts to reduce the resultant steady-state error, positive or negative, regardless of the origin of the components of the error.

Thus a study of the control functions and the reactions of the human pilot often provides the simplest way of determining the control requirements in an automatic tracking control system. The detailed engineering theory explaining why an error has occurred often suggests a method of correction which is much too complicated.

Part I

SECTION II

EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

The equations of motion for the primary components of a system for automatic tracking control of aircraft are derived in Appendix A.

As described in the foregoing section, there are thirteen fundamental degrees of freedom in the automatic tracking control system represented in Figure VIII. There are six degrees of freedom for the aircraft (three linear and three angular), three degrees of freedom for the aircraft controls (rudder, ailerons, and elevator), two degrees of freedom for the computer (one for the elevation computer, and one for the deflection computer), and two degrees of freedom for the radar unit (one in angular deflection and one in angular elevation). There are of course many other less important degrees of freedom in the overall system, as for example, the degrees of freedom represented by cable stretch in each of the servo systems. But for purpose of analysis, these minor degrees of freedom will be neglected. There are also intermittent degrees of freedom represented by the safety control and anticipation control systems. These degrees of freedom are only introduced into the overall system for short periods of time, and only when the safety control limits are exceeded. The correct-flight control system, which is made up of the yaw vane system and the lateral accelerometer system, also contains additional intermittent degrees of freedom which come into play when the aircraft encounters sudden gusts. Also, if the amplification control factors for the deflection component of the forcing function, F_D, are not in the proper proportions for control of rudder and ailerons, then slip or skid will occur. In this case the correct-flight control system would be operating all the time instead of intermittently, in order to keep the aerodynamic yaw at a minimum. However, in the analyses which follow it will be assumed that the amplification control factors are adjusted properly, and it will be assumed that the safety limits are not exceeded, so that only the fundamental degrees of freedom need be considered. The thirteen fundamental degrees of freedom for the overall system may be divided into six longitudinal and seven lateral degrees of freedom.

Just as there are the classical stability coefficients for the six degrees of freedom represented by the aircraft, so there are analogous stability coefficients for each of the other seven degrees of freedom in the overall system.

The longitudinal equations of motion for the aircraft are derived in Section A-1 in Appendix A; and the usual classical assumptions of small changes, linear superposition, etc., are made.

The equation of motion for the elevator-servo system is derived in Section A-2 of Appendix A. In this derivation the effect of cable stretch is neglected, and the entire elevator-servo system is referred, mathematically, to the elevator hinge. Hinge moment stability coefficients for the elevator-servo system are shown to exist.

The equation of motion for the elevation computer is derived in Section A-3 of Appendix A. This equation is derived by equating torques about the elevation computer shaft. Just as there are stability coefficients for other parts of the overall system, so there are analogous stability coefficients in the equation of motion for the elevation computer. These stability coefficients are derived in Section A-3 of Appendix A, and equivalent expressions for them, using the accepted fire-control notation, are given by equations 46, 51, 53, and 57 in Section A-3 of Appendix B.

Reliable calibration data for the two fundamental degrees of freedom in the radar system were not available to the author at the time of writing this thesis. Thus, in the analyses presented, it was assumed that the tracking line (same as the radar line) was always coincident with the line of sight. This was equivalent to assuming that the radar angle, A_R, was always equal to the angle of the line of sight, A_{LS}. Thus the equation of motion of the line of sight, referred to the longitudinal axis of the aircraft, represents the equation of motion for an ideal radar system. An analysis based upon these assumptions would certainly determine the potentialities of the overall system.

The elevation equation of motion for ideal radar is derived in Section A-4 of Appendix A. This equation is only valid for the special case when the target is flying level at a constant speed equal to the initial trim-speed of the tracking aircfaft (see Section A in Appendix G for detailed discussion of the basic tracking problem analyzed in this thesis).

Analogous equations of motion for the lateral case are derived in Sections B-1, B-2, B-3, B-4, and B-5 of Appendix A; and a summary of the equations of motion for the overall system is given at the end of Appendix A.

It should be noted that each equation of motion is completely nondimensional, and that in each case a characteristic time associated with its own part of the overall system was used as a basis for nondimensionalizing. These equations, of course, can not be solved simultaneously unless they are all nondimensionalized on a common time basis.

A summary of the equations of motion for the primary components of a system for automatic tracking control of the A-26 airplane is given in Appendix E.

Part I

SECTION III

STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

Formulae for the stability coefficients for a system for automatic tracking control of aircraft are derived in Appendix B. In this Appendix formulae for the usual classical stability coefficients for aircraft are derived, as well as formulae for the hinge moment stability coefficients for the servo control systems, and formulae for the prediction moment stability coefficients for units of the computer. Analogous formulae for gimbal moment stability coefficients can be derived for an actual radar system, but these derivations will not be included in this thesis, since perfect radar tracking is assumed in the analyses which follow. The gimbal moment stability coefficients for an actual radar unit are analogous to the hinge moment stability coefficients in the servo control systems, and analogous to the pitching, yawing, and rolling moment stability coefficients for the aircraft itself. It can be shown that the gimbal moment stability coefficients, for each fundamental degree of freedom in the radar unit, are functions of the natural undamped frequency, damping ratio, and certain sensitivities associated with the particular radar degree of freedom being considered. (Reliable calibration data for the two fundamental degrees of freedom in the radar system were not available to the author at the time of writing this thesis, but they should be available in the near future).

The techniques of dynamic flight-testing are rapidly being improved. Thus, in the very near future there should be available a complete correlation between actual and theoretical stability coefficients for aircraft.

Stability coefficients for a system for automatic tracking control of the A-26 Airplane are computed in Appendix D. These computations are based on the formulae developed in Appendix B and the basic constants presented at the end of the text (preceding Appendix A)..

It may be of interest to note that there are also stability coefficients associated with the forcing function input to each servo control system; see pages B-17, B-37, and B-42 in Appendix B.

A summary of the formulae for the stability coefficients for a system for automatic tracking control of aircraft is given at the end of Appendix B; and a summary of the computed values of the stability coefficients for a system for automatic tracking control of the A-26 airplane is given at the end of Appendix D. These computed values are not necessarily the best values to be used in the system. Recommended values of servo control characteristics, determined from the results of analyses by the Rockefeller Differential Analyzer, are presented in Appendix G.

Part I

SECTION IV

NETWORK EQUATIONS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT. APPLICATION TO A-26 AIRPLANE

The network equations for a system for automatic tracking control of aircraft consist of the equations of motion for the primary components of the system (derived in Appendix A), and the tracking error angle equations. The tracking error angle equations are merely definitions of the elevation and deflection components of the tracking error angle (see equations 4 and 5 in Appendix C).

In the process of nondimensionalizing the equations of motion developed in Appendix A, each equation was made nondimensional by using a characteristic time associated with the particular degree of freedom represented by the equation. Thus, before the network equations can be solved simultaneously they must be based upon a unit of time which is common to the entire system. Any unit of time could be used, but it seemed logical to choose a unit of time equal to the time required for the tracking aircraft to fly the initial target range at a speed equal to its initial speed. It was argued that if the automatic tracking control network could not reach a solution in the time required to fly the initial range of the target, it would certainly not be worth while from the fire-control point of view. On this basis the network solution time, measured in nondimensional time-ratio units, would have to be much less than unity before the system could be considered practical.

A summary of the network equations for a system for automatic tracking control of aircraft is given at the end of Appendix C; and a summary of the network equations for a system for automatic tracking control of the A-26 airplane is given at the end of Appendix F. The servo coefficients used in the equations at the end of Appendix F are not the most desirable values. Recommended values determined from analyses by the Rockefeller Differential Analyzer, are computed in Appendix G and discussed in Section V, Part One, of the text.

Part I

SECTION V

SOLUTION TIME OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE, USING DIFFERENTIAL ANALYZER TO SOLVE NETWORK EQUATIONS

The network equations for a system for automatic tracking control of the A-26 Airplane are summarized on pages F-2 and F-3 in Appendix F. The values of $\frac{\partial M_{SAEM}}{\partial F_E}$, $\frac{\partial M_{SARM}}{\partial F_D}$, and $\frac{\partial M_{SAAM}}{\partial F_D}$ given on page F-2 were used only to demonstrate the method of analysis. These values do not represent the final values indicated by the Differential Analyzer to be desirable. The recommended values are given in the following pages of this section.

The basic problem solved by the Differential Analyzer is discussed in Appendix G. Appendix G is composed of two sections. The first deals with the longitudinal loop of the automatic tracking control system, shown in Figure VIII, and the second deals with the lateral loop.

The analysis of the longitudinal loop of the automatic tracking control system shown in Figure VIII indicated that

1. The solution time required to establish a pure pursuit course, for the problem investigated, was $\cong$ 1 second.
2. The solution time required to establish an aerodynamic lead pursuit course, for the problem investigated, was $\cong$ 2 seconds.

3. The stability number of the elevation computer should be $\cong$ .6.

4. The maximum stick force required to establish a pure pursuit course in 1 second, for the case investigated, was $\cong$ 50 lbs.

5. The elevator servo characteristics should be as follows:

$$\omega_{n_{SAE}} \cong 4.17 \, \frac{\text{radians}}{\text{second}}$$

$$\zeta_{SAE} \cong 7.03$$

   In addition, it is the author's opinion that the ratio of the elevator servo stiffness to the elevator aerodynamic stiffness, measured at the elevator hinge, should not be less than 6 for positive positioning of the elevator.

6. The effect of aerodynamic forces on the elevator servo system need not be considered when the elevator servo characteristics recommended above are used.

7. The force balance along the longitudinal axis of the aircraft need not be considered, i.e., the long-period of the aircraft had negligible effect on solution time.

8. Change in longitudinal speed had a negligible effect on the ideal radar equation, for the problem investigated, and so it had a negligible effect on solution time.

9. The effect on solution time of downwash lag, $m_{\dot{w}}$, was appreciable.

10. The effect on solution time of the combination $(\theta - w)$ in the ideal radar equation was very small.

11. The input control characteristics for the elevator-servo should be as follows:

$$\frac{\partial M_{SAEM}}{\partial F_E} = -75 \frac{\text{ft-lbs}}{\text{miliradian}} .$$

$$\frac{\partial M_{SAEM}}{\partial \dot{F}_E} = -36 \frac{\text{ft-lbs}}{\text{miliradian/second}} .$$

12. The computer should not be uncaged until the tracking error angle has been decreased to at least ± 5 mils.

Analysis of the lateral loop of the automatic tracking control system shown in Figure VIII, indicated that

1. The fire-control solution time for gun-fire $\cong$ 1.3 seconds, using the control amplification factors shown in 4.

2. The rudder servo characteristics should be

$$\omega_{n_{SAR}} = 4.17 \frac{\text{radians}}{\text{second}} .$$

$$\zeta_{SAR} = 13.2 .$$

In addition, it is the author's opinion that the ratio of the rudder servo stiffness to the rudder aerodynamic stiffness, measured at the rudder hinge, should not be less than 6 for positive positioning of the rudder.

3. A fire control solution for gun fire, for small corrections in deflection angle, can be accomplished quickest by apply-

ing rudder and opposite ailerons, rather than by coördinating ailerons with rudder to change the flight path by "rolling in" and then "rolling out". (This of course would require velocity jump correction in the deflection computer).

4. The input control characteristics for the rudder servo should be as follows:

$$\frac{\partial M_{SARM}}{\partial F_D} = 131 \frac{\text{ft-lbs}}{\text{miliradian}},$$

$$\frac{\partial M_{SARM}}{\partial \dot{F}_D} = 157 \frac{\text{ft-lbs}}{\text{miliradian/second}},$$

and $$\frac{\partial M_{SARM}}{\partial \int F_D dt} = 89 \frac{\text{ft-lbs}}{(\text{miliradian})(\text{second})}.$$

It should be noted that M_{SAEM}, M_{SARM}, and M_{SAAM} are *effective* input moments at the servo axes, applied by the servo motors; they are not the *net* moments applied by the servo motors. The net moment for each servo motor may be obtained by subtracting the stiffness, damping, and inertia moments from the effective input moment. The net moment applied by the servo motor is of course equal to the negative value of the cable moment at the servo axis; see equation 41, section A-2, Appendix A.

Part I

SECTION VI

CONCLUSIONS

The conclusions derived from analysis of the overall tracking control system represented in Figure VIII, are as follows:

1. The solution time of the overall system is low enough to be practical. Solution times for the basic problem analyzed were as follows:

	Solution time
(a) Establishment of pure longitudinal pursuit course	1 second
(b) Establishment of longitudinal aerodynamic lead pursuit course	2 seconds
(c) Establishment of lateral fire-control course for gun fire, cross controls, without aerodynamic lead	1.3 seconds

Time did not permit a complete study of the lateral loop of the automatic tracking control system. However, based upon the work done, the author estimates the following:

	Estimated Solution time
(d) Establishment of lateral fire-control course for gun fire, cross-controls, with proper aerodynamic lead	2.5 seconds
(e) Establishment of pure lateral pursuit course, coördinating controls	3 seconds

2. The stability number for the elevation computer should be about .6 instead of the usual value of .2 found to be satisfactory

for most human pilots. The stability number required for the deflection computer was not investigated.

3. A fire-control solution for gun fire, for small corrections in deflection angle, can be accomplished quickest by cross-controlling rudder and ailerons. However, in this case velocity jump correction becomes a very necessary input to the deflection computer.
4. The values of natural frequency, damping ratio, and spring constant required for the control servos indicate that a major redesign of the C1-A Minneapolis Servo Pilot is required before it can be used.
5. There is need for flight-testing to determine values of aircraft dynamic stability coefficients.
6. There is need for radar tracking calibration data.
7. There is need for a *non-linear* simulator, large enough to accommodate, at the same time, all of the thirteen primary degrees of freedom in an automatic tracking control system. It is important that this simulator be able to use displacement, derivative, and integral control; and it is also important that it be able to plot automatically, and simultaneously, at least four of the dependent variables as output.
8. Displacement, derivative, and integral control will be required in the system for automatic tracking control of aircraft shown in Figure VIII.

9. Safety controls, anticipation controls, and correct-flight controls should be considered in the design of a system for automatic tracking control of aircraft.

10. Recommendations for further study

    (a) The longitudinal and lateral analyses should be extended to include actual radar equations of motion, instead of ideal radar equations of motion, as soon as radar tracking calibration data becomes available.

    (b) The lateral loop of the automatic tracking control system should be analyzed more thoroughly, especially for the case of interception tracking (coördinating rudder and ailerons).

PART II

AIRCRAFT

Part II

SECTION I

STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROLS. APPLICATION TO A-26 AIRPLANE.

A. STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROLS

The equations for steady-state response of aircraft to forced sinusoidal motion of controls are derived in Appendices H, J, and Q. Equations for steady-state response of aircraft to forced sinusoidal motion of control surfaces are derived in Appendix H. Equations for steady-state response of aircraft to forced sinusoidal variation in output signal from the oscillator which drives the elevator servo, with ideal displacement, integral, and derivative pitch control in automatic pilot, are derived in Appendix J. Equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator are derived in Appendix Q. Operational calculus is used for all derivations in these appendices.

Steady-state response equations for seven different cases are derived in Appendix H. These cases are outlined in the Table of Contents in Appendix H.

Only the longitudinal steady-state response equations were derived in Appendix J. However, the same type of analysis could be used to derive equations for the steady-state response of aircraft to forced sinusoidal variation in the output signal from the oscillator which drives the rudder servo, from the oscillator which drives the aileron servo, or from the oscillator which drives the throttle servo. This type of analysis could also be used for any reasonable combination, such as rudder and ailerons, or throttle and elevator. The response equations thus obtained would completely determine the effect of an ideal automatic pilot on the steady-state response of an aircraft to forced sinusoidal variation of input forcing functions to the automatic pilot. In this connection, it should be noted that an automatic pilot has no effect on the steady-state response of an aircraft to forced sinusoidal motion of its control surfaces. In other words, the response of an aircraft to the sinusoidal motion of its control surfaces is, of course, not dependent on the factors which caused the motion of its control surfaces.

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator (see Appendix Q) were derived for three fundamental cases, namely, (a) zero change in longitudinal velocity, (b) zero change in angle of attack, and (c) zero change in pitch angle. These three cases are directly analogous to those for lateral response of aircraft to rudder and ailerons, derived in sections B-3-a, B-3-b, and B-3-c in Appendix H.

B. ANALYSES FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF CONTROLS

Analyses for steady-state response of the A-26 Airplane to forced sinusoidal motion of its controls, are presented in Appendices L, N, and R. Analyses for steady-state response of the A-26 Airplane to forced sinusoidal motion of its control surfaces are presented in Appendix L. An analysis for the steady-state response of the A-26 Airplane to forced sinusoidal variation in the output signal from the oscillator which drives the elevator servo, with ideal displacement pitch control in automatic pilot, is presented in Appendix N. Analyses for steady-state response of the A-26 Airplane to forced sinusoidal motion of throttle and elevator are presented in Appendix R.

Figures L-1 through L-10 show the computed longitudinal steady-state response of the A-26 Airplane to forced sinusoidal motion of its elevator, for the following two cases:

I. Conventional analysis, assuming m_u and z_θ negligible.

II. Conventional analysis, assuming m_u, z_θ, and z_u negligible.

These figures show that neglecting the additional term in z_u has a negligible effect on change in angle of attack and change in pitch angle. However, z_u does have an appreciable effect on change in longitudinal velocity per airplane trim speed, in the low frequency range, as shown by Figures L-1, L-4, and L-8. Thus, Figures L-1 through L-10 prove that, especially in the high frequency range, the effect of change in longitudinal velocity may be neglected when computing the response of conventional aircraft to forced sinusoidal motion of elevator.

Figures L-11 through L-16 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of rudder, with fixed ailerons. Note that there is a definite resonance frequency for $\mu_{v\delta_r}$ and $\mu_{\psi\delta_r}$, as shown in Figure L-12. Also note that at high rudder frequency the response is comparatively small, especially for ϕ/δ_{r_a} shown in Figure L-16.

Figures L-17 through L-22 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of ailerons, with fixed rudder. Note that there is a definite resonance frequency for $\mu_{v\delta_a}$, as shown in Figure L-18. Also note that, at high aileron frequency, the response is comparatively small, especially for v/δ_{a_α} and ψ/δ_{a_α} shown in Figure L-22.

Figures L-23 through L-28 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of rudder and ailerons, adjusted for zero change in aerodynamic yaw. Note that the required $\mu_{\delta_a\delta_r}$ is high, and increases with aileron-rudder frequency, as shown in Figure L-24.

Figures L-29 through L-34 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of rudder and ailerons, adjusted for zero change in geometric yaw.

Figures L-35 through L-40 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of rudder and ailerons, adjusted for zero change in angle of bank.

In a few cases the curves of computed steady-state response in Appendix L do not agree with the steady-state response expected by the author. This could be because of error in basic values* used to compute

* Reference 6.

stability coefficients, because of certain effects* not included in the analyses, because of error in derivation of equations, or because of error in expected response. Flight test data on lateral response were not available at the time of writing this thesis, but these data should be available in the near future. (The Flight Research Department of the Cornell Aeronautical Laboratory was, at the time of writing this thesis, testing a B-25 Airplane for lateral response).

Figures N-1 through N-7 show the effect of ideal displacement pitch control in the automatic pilot on the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of the elevator, using elevator motion without pitch control as a basis for comparison. As explained in Appendices J and N, these curves may also be thought of as curves which show the effect of ideal displacement pitch control in the automatic pilot on the computed steady-state response of the A-26 Airplane to forced sinusoidal variation in output signal from the oscillator which drives the elevator servo.

Figures R-1 through R-6 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of throttle and elevator, adjusted for zero change in longitudinal velocity. Note that at high values of elevator-throttle frequency the required value of $\mu_{t_x \delta_e}$ is comparatively small, as shown by Figures R-2 and R-6.

Figures R-7 through R-11 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of throttle and elevator, adjusted for zero change in angle of attack. Note that the required value of $\mu_{t_x \delta_e}$ increases rapidly with elevator-throttle fre-

* For instance, the effect of adverse yaw due to aileron deflection was not included in the analyses because of lack of data.

quency, as shown in Figure R-8. This means that at high values of elevator-throttle frequency the amplitude of the elevator oscillation must be kept very low in order that the required thrust (t_x, nondimensional, defined on page Q-1) does not exceed the available thrust.

Figures R-12 through R-17 show the computed steady-state response of the A-26 Airplane to forced sinusoidal motion of throttle and elevator, adjusted for zero change in pitch angle. Note that the required value of $\mu_{t_x \delta_e}$ increases *very* rapidly with elevator-throttle frequency, as shown in Figure R-13. This agrees with the fact that most airplanes require very little change in elevator angle with change in power, to maintain zero change in pitch angle.

Because of lack of data, the analysis for steady-state response of the A-26 Airplane to forced sinusoidal oscillation of throttle* alone, has not been computed.

Another longitudinal analysis that might be investigated is that for sinusoidal motion of throttle and elevator, adjusted for zero change in flight path, i.e., for zero change in $(\theta - w)$. The corresponding analysis in the lateral case would be for sinusoidal motion of ailerons and rudder, adjusted for zero change in $(\psi + v)$, but this would be rather difficult from the flight-test point of view.

These and other similar theoretical investigations should point the way to new flight-test techniques for determining aircraft dynamic stability coefficients.

* Oscillation of throttle is meant to imply oscillation of power, and may include oscillation of engine r.p.m. if required.

Part II

SECTION II

FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO STEP FUNCTIONS. APPLICATION TO A-26 AIRPLANE.

A. FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO STEP-FUNCTIONS.

The equations for fixed-control transient response of aircraft to longitudinal and lateral step-functions are derived in Appendix I. The method used is straight-forward and simple, and is easy to visualize physically because the complex plane is not used in the basic equations (equations 15-a, 16-a, 17-a, 75, 76, and 77 in Appendix I). Laplace Transforms may also be used, but the physical interpretation of each step of the analysis, using Laplace Transforms, is not as easy to follow.

B. FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO STEP-FUNCTIONS

1. Analysis for Fixed-Control Transient Response of A-26 Airplane to Longitudinal Step Function

An analysis of the fixed-control transient response of the A-26 Airplane to a longitudinal step-function is presented in Section A of Appendix M. The boundary conditions were established by assuming that the A-26 Airplane was flying level at 10,000 feet density altitude, and at a true airspeed of 300 m.p.h. It was further assumed that the airplane encountered a gust which caused a sudden positive rotation in pitch of 50 miliradians, without appreciable change in flight-path or speed. Thus, the assumed boundary conditions for this case were $u_o = 0$, $w_o = .05$, $\theta_o = .05$, and $\dot{\theta}_o = 0$. The results of this analysis are shown in Figures M-1 through M-6. Figures M-1, M-3 and M-5 represent the short-period components of the resulting motion; and Figures M-2, M-4 and M-6 represent the long-period components. It will be noted that the short period components are over-critically damped, and the long-period components are under-critically damped.

2. Analysis for Fixed-Control Transient Response of A-26 Airplane to Lateral Step-Function

An analysis of the fixed-control transient response of the A-26 Airplane to a lateral step function is presented in Section B of Appendix M. The boundary conditions were established by assuming that the A-26 Airplane was flying level at 10,000 feet density altitude, and at a true air-speed of 300 mph. It was further assumed that the air-plane encountered a gust which caused a sudden positive rotation in roll of 50 miliradians, without appreciably changing the flight path or speed. Thus, the assumed boundary conditions for this case were $\psi_o = 0$, $\dot{\psi}_o = 0$, $v_o = 0$, $\phi_o = .05$, and $\dot{\phi}_o = 0$. The results of this analysis are shown in Figures M-7 through M-10. The highly damped subsidence components are shown in Figure M-7; the spiral stability components are shown in Figure M-8; and the Dutch roll components are shown in Figure M-9. The sum of the subsidence, spiral stability, Dutch roll, and constant components represents the total response, shown in Figure M-10.

Part II

SECTION III

TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION WHEN AIRCRAFT IS EQUIPPED WITH AUTOMATIC PILOT WHICH USES DISPLACEMENT, INTEGRAL, AND DERIVATIVE PITCH CONTROL. APPLICATION TO A-26 AIRPLANE, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

A. TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION WHEN AIRCRAFT IS EQUIPPED WITH AUTOMATIC PILOT WHICH USES DISPLACEMENT, INTEGRAL, AND DERIVATIVE PITCH CONTROL.

Equations for the transient response of aircraft to a longitudinal step-function, with *ideal* displacement, integral, and derivative pitch control in the automatic pilot, are derived in section A of Appendix K. The term *ideal* is used in the sense that functions of change in pitch angle are assumed to cause pitching moments on the aircraft without any servo lag. This is, in effect, what an ideal servo pilot would do. Thus there are only three primary degrees of freedom, as represented by equations 1, 2, and 3 in Section A of Appendix K.

Equations for the transient response of aircraft to a longitudinal step-function, with *actual* displacement, integral, and derivative pitch control in the automatic pilot, are derived in section B of Appendix K. The term *actual* is used in the sense that functions of change in pitch angle are assumed to cause hinge moments in the elevator-servo system, as would be the case in an actual automatic pilot. Thus, there are four primary degrees of freedom -- three degrees of freedom for the aircraft itself, and one degree for the elevator-servo system. These four degrees of freedom are represented by equations 12, 13, 14, and 15 in Section B of Appendix K.

B. TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

Based upon the equations derived in Section A of Appendix K, an analysis for the transient response of the A-26 Airplane to a longitudinal step-function, with ideal displacement pitch control in the automatic pilot, was made. This analysis is presented in Appendix O.

The time history of the change in longitudinal velocity, per airplane trim speed, for $k_\theta = 0$, is shown in Figure O-1. Also shown in Figure O-1 is the corresponding time history with no automatic pilot control, i.e., with fixed controls ($k_\theta = 0$).

The time history of the change in angle of attack for $k_\theta = 0$ is shown in Figure O-2. Also shown in Figure O-2 is the corresponding time history with no automatic pilot control, i.e., with fixed controls ($k_\theta = 0$).

The time history of change in pitch angle for $k_\theta = 0$ is shown in Figure O-3. Also shown in Figure O-3 is the corresponding time history with no automatic pilot control, i.e., with fixed controls ($k_\theta = 0$).

The same type of analysis could be made with any combination of *ideal* displacement, integral, and derivative pitch control, using the basic equations derived in Section A of Appendix K. Also, the same type of analysis could be made with any combination of *actual* displacement, integral, and derivative pitch control, using the basic equations derived in Section B of Appendix K.

Part II

SECTION IV

METHODS FOR DETERMINING DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROLS. APPLICATION TO A-26 AIRPLANE.

A. METHODS FOR DETERMINING DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT, FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES.

Methods for determining dynamic stability coefficients for aircraft, from steady-state flight-test response of aircraft to forced sinusoidal motion of control surfaces, are presented in Appendix P. Included in Appendix P is a table of contents for the methods investigated.

Although the method shown in Section A-1 is theoretically feasible, it is the author's opinion that it could not be used with present-day techniques in dynamic flight testing, because of the large inaccuracy-spread in the flight-test data.

The method shown in Section A-2 of Appendix P, neglects the effect of change in longitudinal velocity in the equations of motion for change in normal force and change in pitching moment. This method is related to the circle-diagram method discussed in Appendix T and in reference 1. It is the author's opinion that the circle-diagram method is very practical because it is a simple way of determining the damping coefficient and the spring constant for the *short period* component of the transient longitudinal response of an aircraft. It also has the advantage that the values of damping coefficient and spring constant determined from the faired circle are dependent on all, instead of just a few, of the flight-test points.

It is also the author's opinion that the method discussed in Section B-1 in Appendix P is theoretically feasible, but not practical from the flight-test point of view.

The methods discussed in Sections B-3-a, B-3-b, and B-3-c in Appendix P are based, respectively, on the equations for aircraft response to sinusoidal motion of rudder and ailerons, derived in sections B-3-a, B-3-b, and B-3-c, in Appendix H. The method discussed in Section B-3-a of Appendix P is based on rudder-aileron adjustment for zero aerodynamic yaw; the method discussed in section B-3-b is based on rudder-aileron adjustment for zero geometric yaw; and the method discussed in Section B-3-c is based on rudder-aileron adjustment for zero angle of bank. Each of these methods should be adaptable to the circle-diagram technique discussed in Appendix T and in reference 1. It is the author's opinion that these three methods are very practical.

B. METHODS FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO

FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR.

Methods for determining longitudinal dynamic stability coefficients for aircraft from steady-state flight-test response of aircraft to forced sinusoidal motion of throttle and elevator are presented in Sections A, B, and C in Appendix S.

The method presented in Section A is based on throttle-elevator adjustment for zero change in longitudinal speed, and is therefore essentially the same as the method discussed in Section A-2 of Appendix P, where the effect of change in longitudinal speed was neglected. However, this throttle-elevator method should be more adaptable in the low frequency range than the method discussed in Section A-2 of Appendix P.

The method presented in Section B of Appendix S is based on throttle-elevator adjustment for zero change in angle of attack. Reliable angle of attack meters have recently been developed, so this method should be easily adaptable to flight-test instrumentation.

The method presented in Section C of Appendix S is based on throttle-elevator adjustment for zero change in pitch angle. This method is also easily adaptable to flight-test instrumentation.

In each of the three throttle-elevator methods discussed in Appendix S, thrust measurements need not be taken unless the stability coefficients x_u, x_w, and x_θ are required. In each case the elevator may be used as the reference forcing function. Also, the circle diagram technique is adaptable to each of these methods. It is the author's opinion that the throttle-elevator methods are very practical.

C. DETERMINATION OF DAMPING COEFFICIENT AND SPRING CONSTANT FOR SHORT-PERIOD COMPONENT OF LONGITUDINAL MOTION OF A-26 AIRPLANE, FROM FLIGHT-TEST DATA, USING CIRCLE DIAGRAM METHOD. COMPARISON OF COMPUTED RESPONSE WITH FLIGHT-TEST RESPONSE, FOR A-26 AIRPLANE

The development of theory for the circle-diagram method is presented in Section A, part 1, in Appendix T.* Mr. Ira M. Ross, Manager of the Flight-Research Department of the Cornell Aeronautical Research Laboratory in Buffalo, is primarily responsible for the development of this excellent method. This method is analogous to the basic methods of circuit analysis used by electrical engineers, and it is also analogous to the methods of analysis used by servo-mechanism engineers.

Flight-test data** for the A-26 Airplane are presented in Figures T-1 and T-2. Figure T-3 shows the circle diagram which was computed from the flight-test data given in Figure T-1. The values of damping coefficient, b, and spring constant k, computed from the theoretical stability coefficients, are as follows:

$$b_{theor.} = 25.23, \text{ non-dimensional.}$$

$$k_{theor.} = 356.18, \text{ non-dimensional.}$$

The corresponding flight-test values are as follows:

$$b_{flight\text{-}test} = 25.5, \text{ non-dimensional.}$$

$$k_{flight\text{-}test} = 345.96, \text{ non-dimensional.}$$

* This method is also discussed in Reference 1.
**Taken from Reference 10.

It will be noted that the comparison between theoretical and flight-test values for b and k is very good.

A comparison of computed response with flight-test response, for the A-26 Airplane, is shown in Figure T-2. This comparison is also good.

Part II

SECTION V

CONCLUSIONS

Based upon the theoretical and flight-test analyses discussed in Sections I, II, III, and IV, in Part II, the author concludes the following:

1. Operation Calculus is exceptionally useful for the study of aircraft transient and steady-state response.
2. The three most practical methods of analysis for determining lateral dynamic stability coefficients are based upon the following flight-test techniques:
    a. Oscillation of rudder and ailerons, adjusted for zero aerodynamic yaw.
    b. Oscillation of rudder and ailerons, adjusted for zero geometric yaw.
    c. Oscillation of rudder and ailerons, adjusted for zero angle of bank.
3. The three most practical methods of analysis for determining longitudinal dynamic stability coefficients are based upon the following flight-test techniques:
    a. Oscillation of throttle and elevator, adjusted for zero change in longitudinal velocity.
    b. Oscillation of throttle and elevator adjusted for zero change in angle of attack.

c. Oscillation of throttle and elevator, adjusted for zero change in pitch angle.

4. The circle-diagram method discussed in Section IV of Part II is exceptionally useful for determining the dynamic response characteristics of aircraft.

5. The author recommends the method of analysis for transient response of aircraft discussed in Section II of Part II, and demonstrated in Appendices M and O. This method is very simple, and easy to visualize physically, because the complex plane is not used. It is not as easy to follow the physical interpretation of each step of the analysis if Laplace Transforms are used.

6. Analysis of the steady-state response of aircraft to forced sinusoidal motion of controls provides the most practical method for determining short-period response characteristics, because the short-period response is always very highly (near-critically) damped. On the other hand, analysis of the transient response of aircraft to a step function imposed by the control surfaces, provides the most practical method for determining long-period response characteristics, because the long-period response is always very slowly (under-critically) damped.

7. The effect of an automatic pilot on the transient response of an aircraft to a longitudinal step-function, can be found by the methods described in Section III of Part II.

8. The steady-state response of aircraft to forced sinusoidal variation in output signal from the oscillator which drives the control servo, when the aircraft is equipped with an automatic pilot which uses ideal displacement, derivative, and integral pitch control, can be determined by the methods described in Section I of Part II (also see Appendix J).

9. The effect of change in longitudinal velocity may be neglected when computing the response of conventional aircraft to forced sinusoidal motion of elevator, especially in the high frequency range.

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NOTATION AND CONVENTIONS

FOR

AUTOMATIC TRACKING CONTROL

OF AIRCRAFT

NOTATIONS AND CONVENTIONS

REFERENCE SYSTEM:

In the analyses which follow, one set of reference axes has been used for all components in the tracking control network. The reference axes chosen were the conventional aircraft body axes, fixed in the aircraft, which are used in most problems in aircraft stability and control.

The line of sight is usually used as the fire-control reference, but in this case the author felt that it would be more convenient to solve the fire control problem by referring all equations of motion to aircraft body axes, since it was desirable to have only one set of reference axes for the entire network. The controlled line (same as the gun line for fixed gun fighters) has been assumed to be coincident with the longitudinal axis of the aircraft, and, in accordance with conventional practice for problems in stability and control, the longitudinal axis of the aircraft has been assumed to be coincident with the initial steady state wind direction.

TYPES OF TRACKING:

1. Pure Pursuit Course.

   A pure pursuit course is a course followed by an aircraft when its gun line is pointed directly at the target. In this case no prediction computer is required. It may be thought of as a "camera course", or an aerodynamic lead pursuit course for an aircraft capable of firing a bullet which travels at infinite speed.

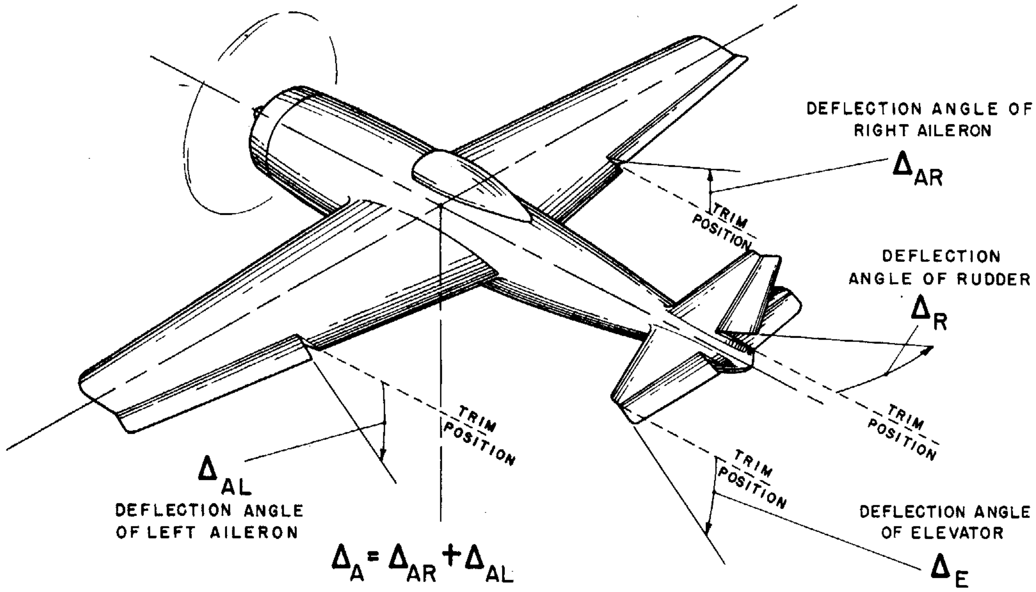
2. Aerodynamic Lead Pursuit Course.

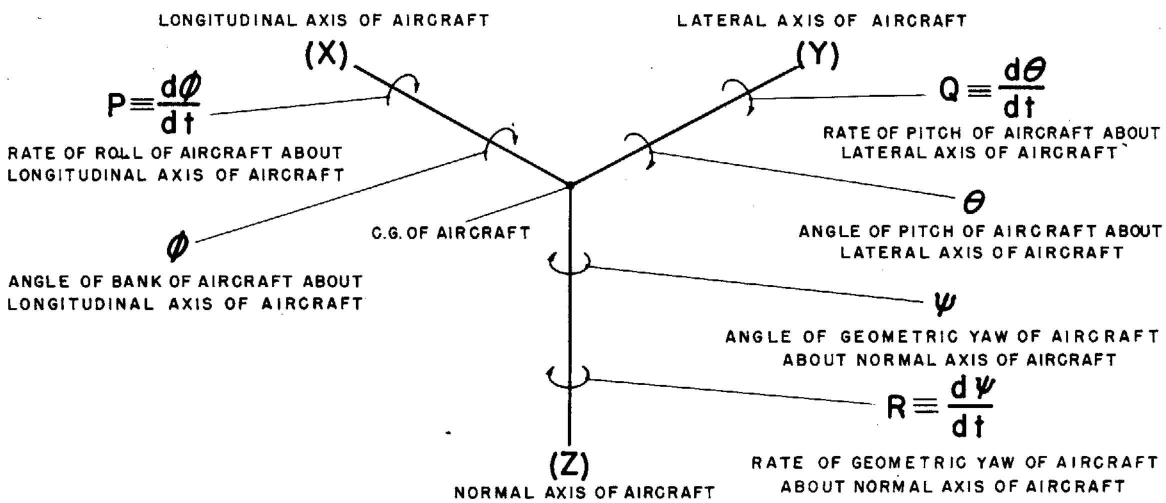
An aerodynamic lead pursuit course is a course followed by an aircraft when its gun line leads the line of sight by an angle equal to the correct prediction angle of the gun line. At any point in the course the gun could be fired, and the future position of the target and the projectile would be the same. (A prediction computer is required)

3. Collision (or Interception) Course.

A collision (or Interception) course is a course followed by an aircraft when its longitudinal axis leads the line of sight by an angle equal to the prediction angle required for collosion with the target in the shortest possible time.

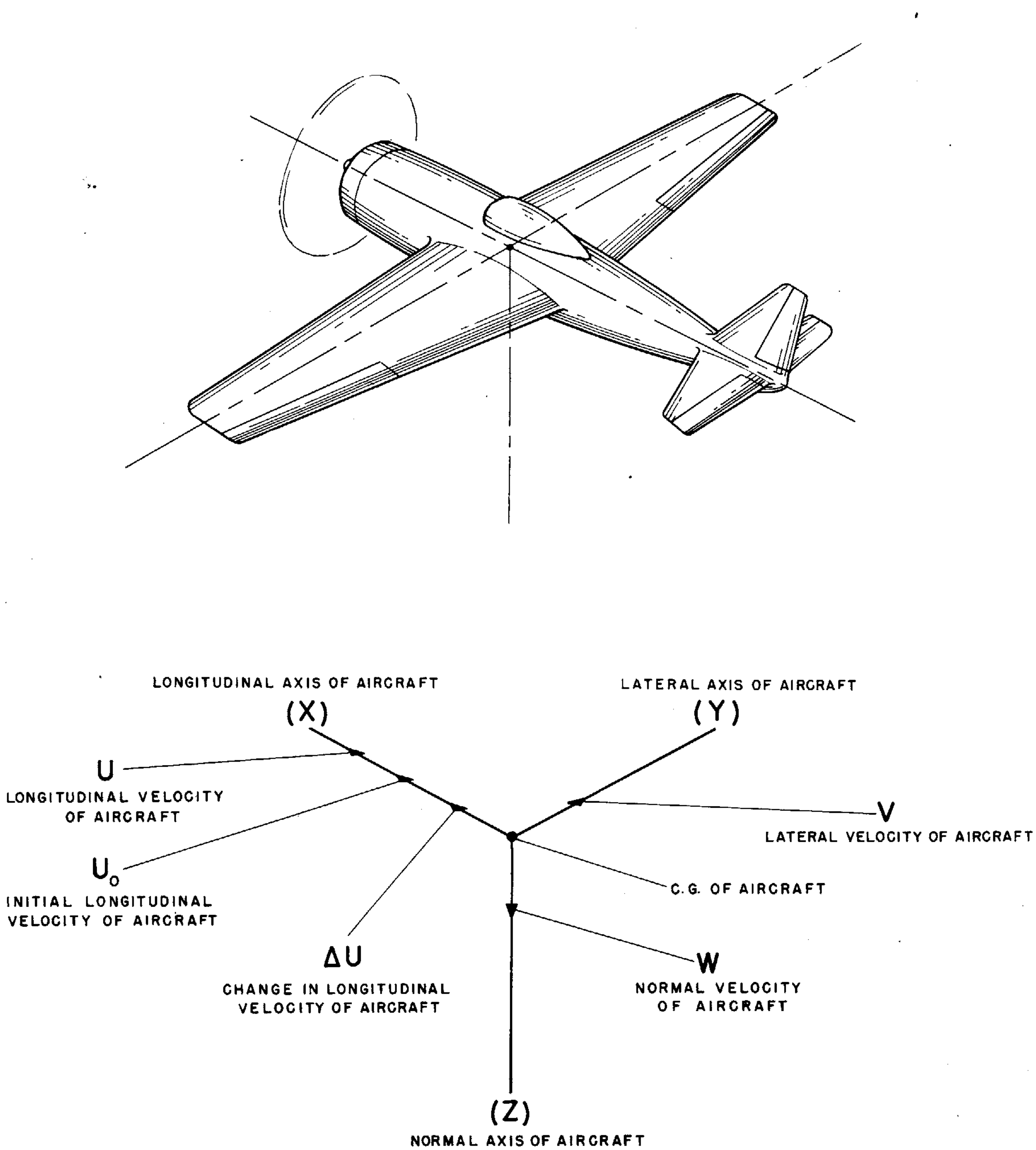
Figures IX through XV demonstrate the notation and conventions which are defined in the glossary of symbols. The concept of prediction line, shown in Figure XIV, is not the same as the concept of Tracking Line for a disturbed sight computer. The concept of prediction line may be understood by visualizing the line established by a prediction indicator in the cockpit.





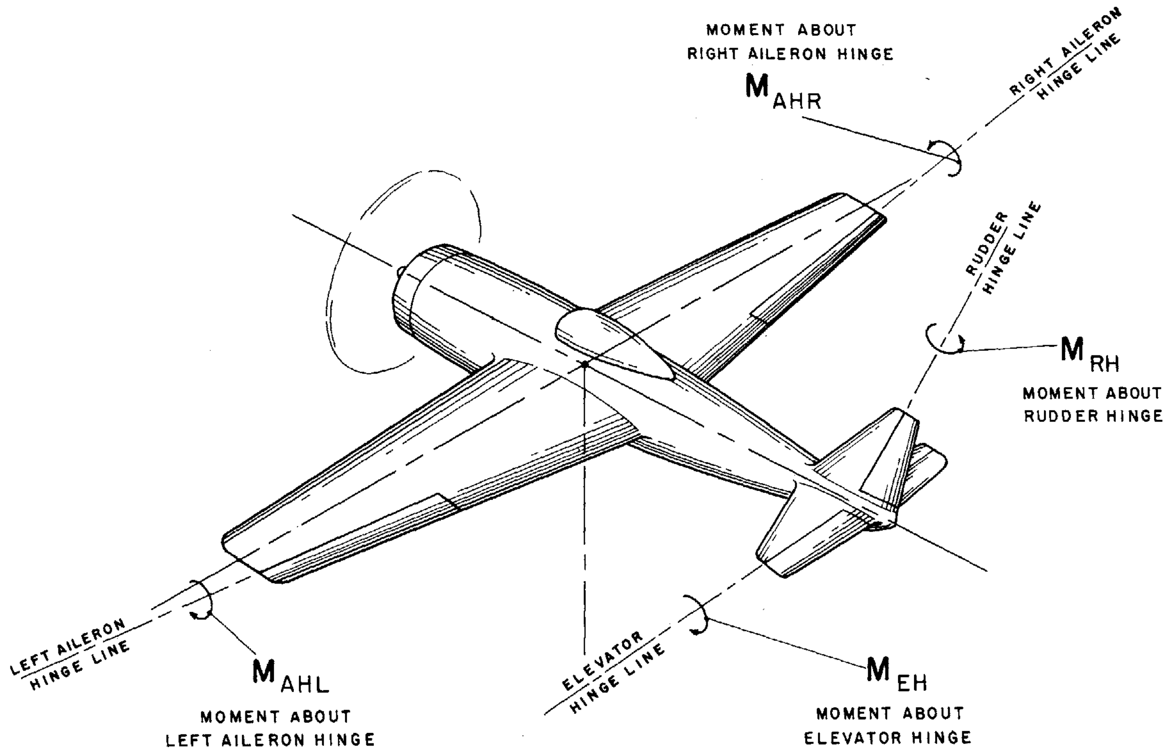
NOTE: ALL CONVENTIONS SHOWN IN POSITIVE SENSE

FIGURE IX
ANGLE AND ANGULAR VELOCITY NOTATION AND CONVENTIONS FOR AIRCRAFT

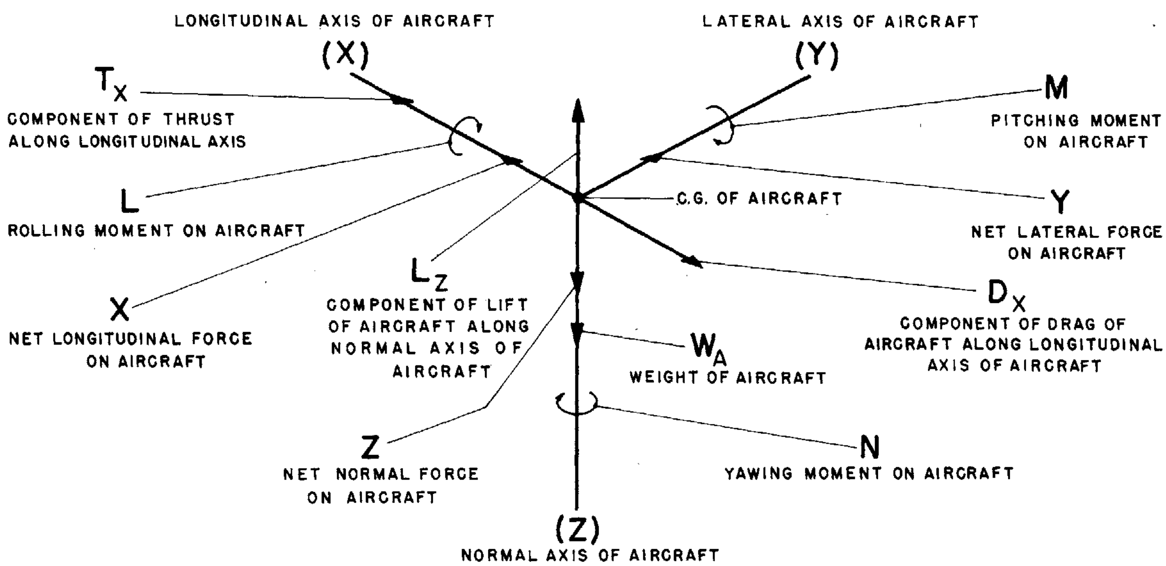


NOTE: ALL CONVENTIONS SHOWN IN POSITIVE SENSE.
AIRCRAFT SHOWN IN EQUILIBRIUM POSITION.
W_0 AND V_0 ARE EQUAL TO ZERO.

FIGURE X
LINEAR VELOCITY NOTATION AND CONVENTIONS FOR AIRCRAFT

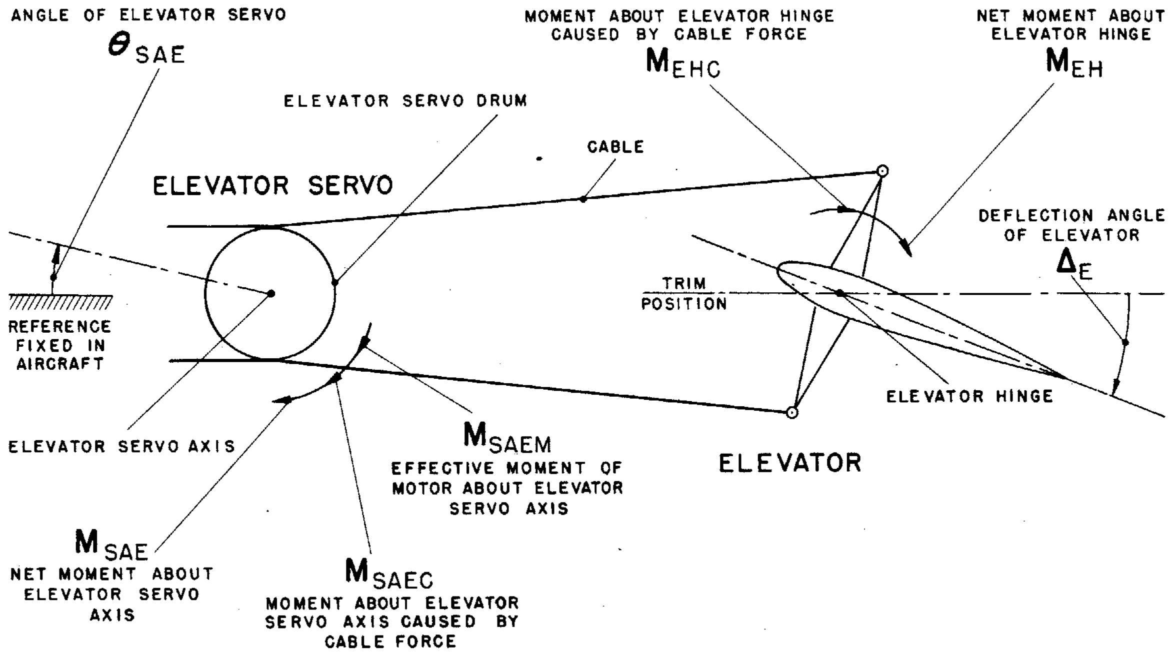


$$M_{AH} = M_{AHR} + M_{AHL}$$

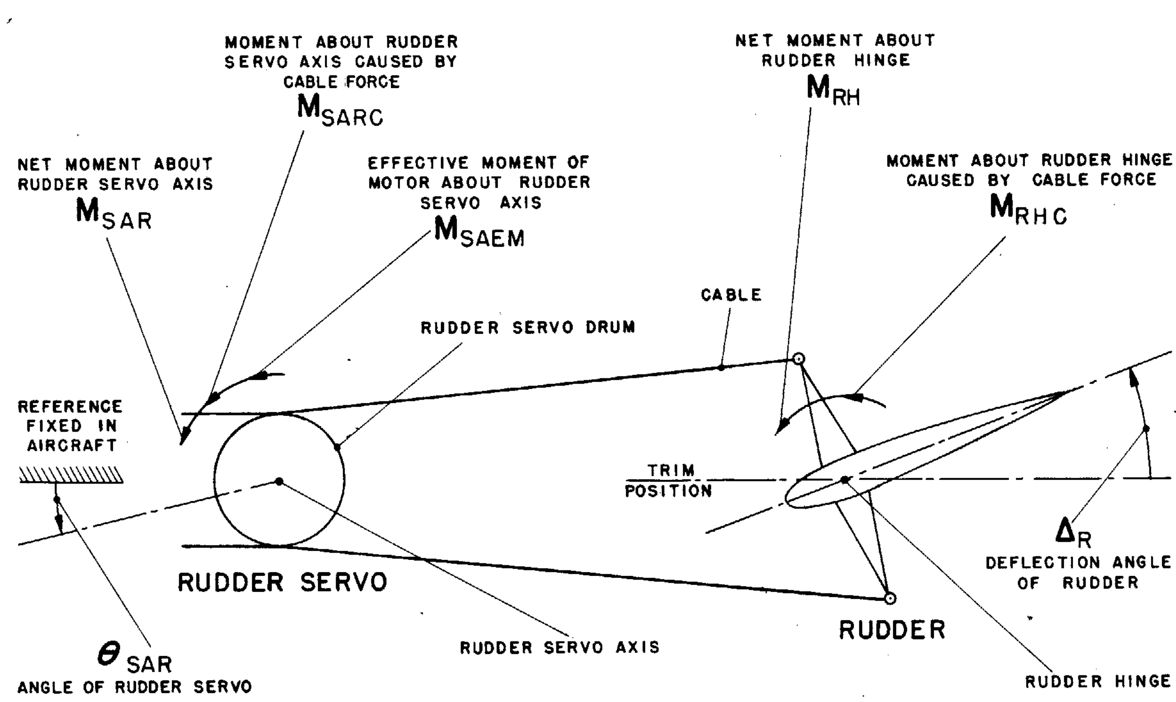


NOTE: ALL CONVENTIONS SHOWN IN POSITIVE SENSE.
AIRCRAFT IS SHOWN IN EQUILIBRIUM POSITION.

FIGURE XI
MOMENT AND FORCE NOTATION AND CONVENTIONS FOR AIRCRAFT



ELEVATOR – SERVO SYSTEM



RUDDER SERVO SYSTEM

NOTE: ALL CONVENTIONS SHOWN IN POSITIVE SENSE

FIGURE XII

SCHEMATIC SKETCHES OF ELEVATOR – SERVO AND RUDDER – SERVO SYSTEMS

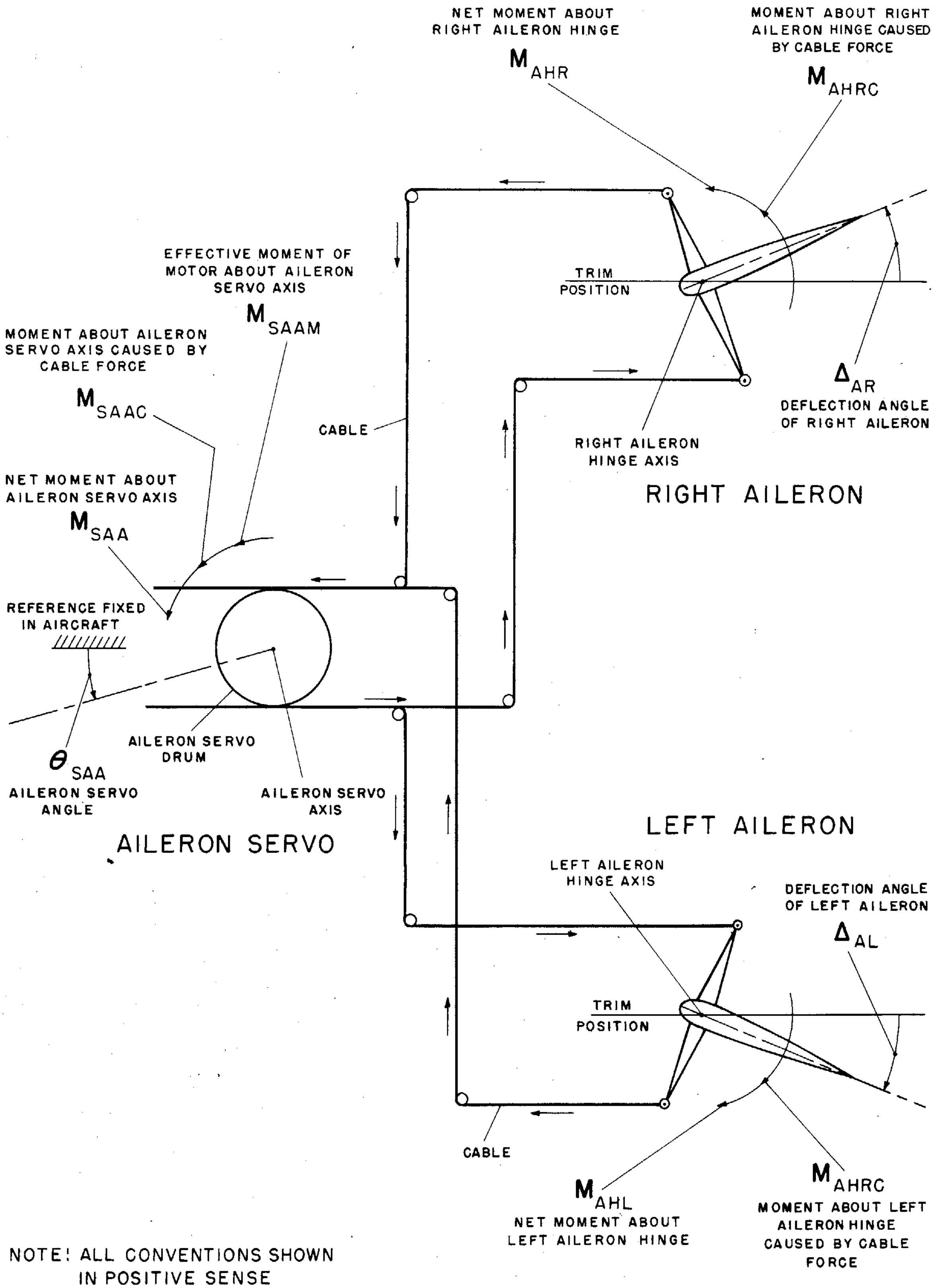
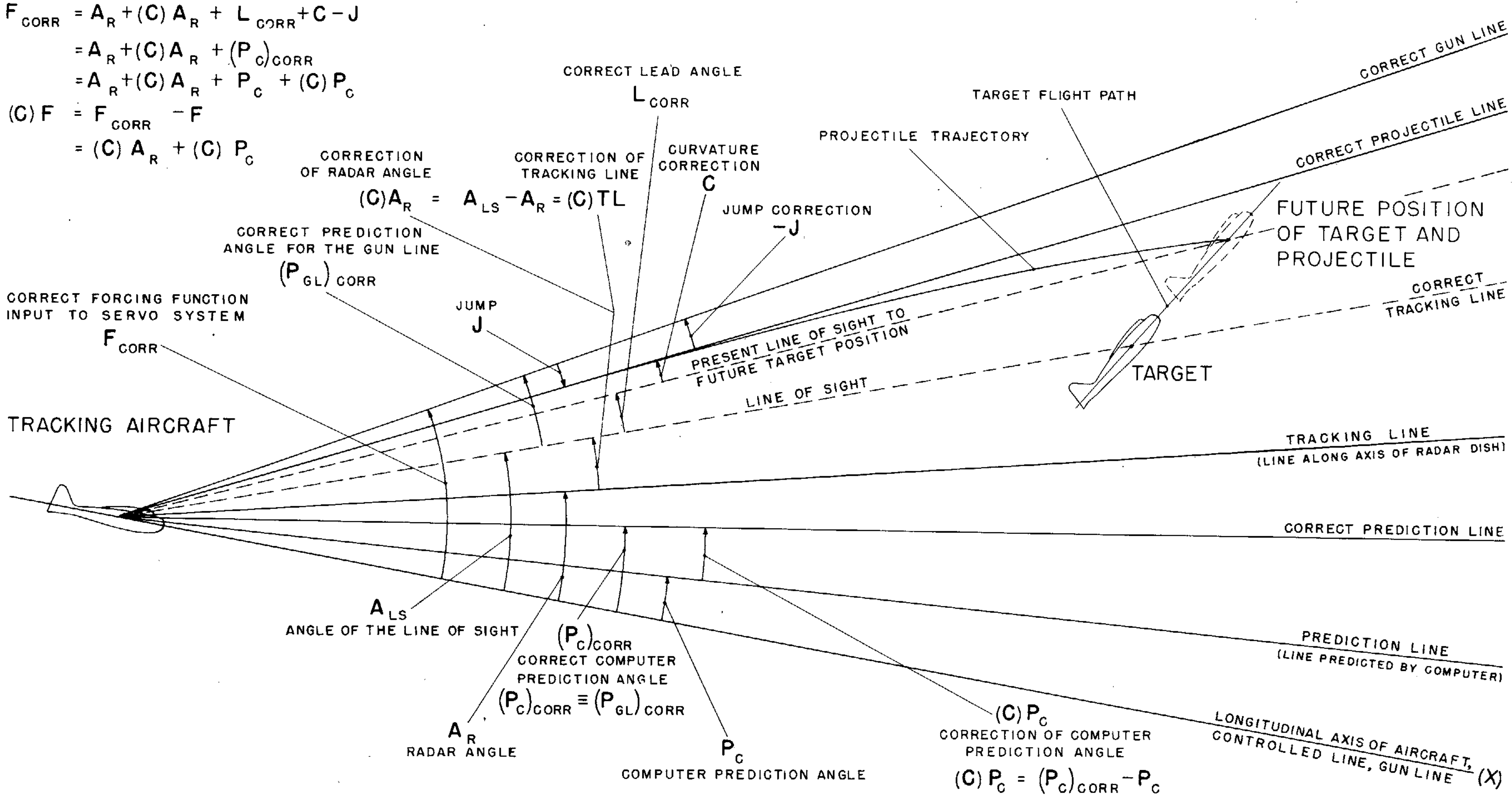


FIGURE XIII
SCHEMATIC SKETCH OF AILERON-SERVO SYSTEM

$$F = A_R + P_C$$

= FORCING FUNCTION INPUT TO SERVO SYSTEM

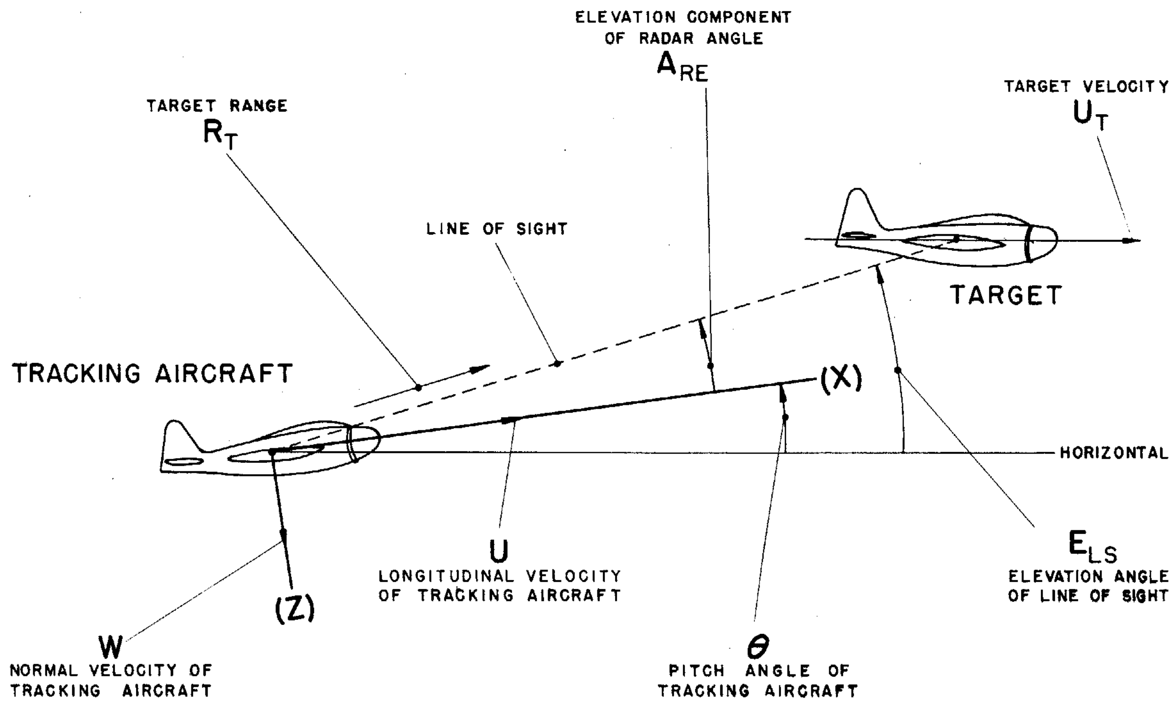
$$F_{CORR} = A_R + (C)A_R + L_{CORR} + C - J$$
$$= A_R + (C)A_R + (P_C)_{CORR}$$
$$= A_R + (C)A_R + P_C + (C)P_C$$
$$(C)F = F_{CORR} - F$$
$$= (C)A_R + (C)P_C$$



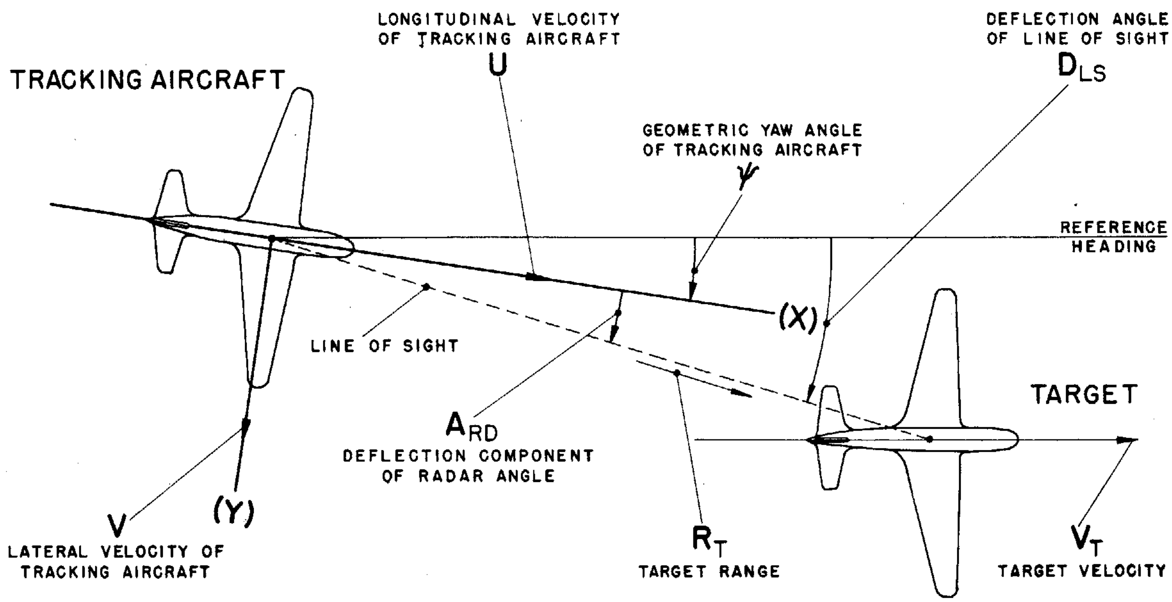
NOTE: ALL CONVENTIONS SHOWN IN POSITIVE SENSE.

FIGURE XIV
TRACKING NOTATION AND CONVENTIONS FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT
(TIGHT LOOP SYSTEM)

Because of lack of radar calibration data, perfect radar tracking has been assumed. This means that $(C)A_R \equiv 0$, and $A_R \equiv A_{LS}$. Thus, in the figures which follow, no distinction has been made between A_{LS} and A_R, and the symbol A_R has been used for both symbols.



ELEVATION TRACKING



DEFLECTION TRACKING

NOTE: ALL CONVENTIONS SHOWN
IN POSITIVE SENSE

FIGURE XV

TRACKING NOTATION AND CONVENTIONS FOR AIRCRAFT

GLOSSARY OF SYMBOLS

In general, the notation and conventions used in this thesis are those used in the field of aircraft stability and control. Upper case letters are used for dimensional terms, and lower case letters are used for non-dimensional terms, except where standardized notation takes preference. A bar over a symbol indicates that the symbol is complex.

I. ENGLISH SYMBOLS

A . . . . Angle, also stands for Aircraft or aileron if used as a subscript.

A_{CSE} . . . . Angle about shaft of elevation computer, measured from reference position.

A_{CSD} . . . . Angle about shaft of deflection computer, measured from reference position.

A_R, a_r . . . . Angle of radar, measured from longitudinal axis of aircraft to tracking line ($a_r \equiv A_R$, $\dot{a}_r = T_A\dot{A}_R$, etc).

A_{RE} (or A_{RD}) . . . . . . Elevation (or deflection) component of A_R.

A_{LS}, a_{ls} . . . . Angle of Line of sight, measured from longitudinal axis of aircraft to line of sight. When (C)A_R is zero (perfect radar), $A_{LS} = A_R$.

a_{re} (or a_{rd}) . . . . . . Elevation (or deflection) component of a_r.

a . . . . Linear acceleration of aircraft.

a_e, a_a, a_r . . . . Displacement control amplification factors, for elevator-, aileron-, and rudder-servo systems, respectively.

A_u, A_w, A_θ, A_v, A_ψ and A_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

a_1, a_2, a_3, etc., ... Coefficients in characteristic equations.

b . . . . Span of wing of Aircraft. (In Appendix H, Section A-2, the symbol "b" is also used for the nondimensional damping coefficient for the short period motion of the aircraft because this is consistent with the established notation in dynamic flight-testing (reference 1). However, the symbol C is used for damping coefficient in all other sections of this thesis.)

b_e, b_a, b_r . . . . Derivative control amplification factors, for elevator-, aileron-, and rudder-servo systems, respectively.

b_1, b_2, b_3, etc. . . . . Coefficients in characteristic equations.

BHP . . . . Brake horsepower.

B_u, B_w, B_θ, B_v, B_ψ and B_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

C . . . . Curvature correction angle.

C_L . . . . Coefficient of lift for aircraft.

C_L' . . . . Coefficient of lift for horizontal tail of aircraft.

C_L'' . . . . Coefficient of lift for vertical tail of aircraft.

C_{L_o} . . . . Initial value of C_L.

C_1, C_2, C_3, C_4, C_5, C_6, etc., are defined by expressions in the appendices, and are used only for mathematical convenience.

$C_{D_{p_{min}}}$ . . . . Coefficient of parasite drag for aircraft, at angle of attack for zero lift.

c . . . . Chord of wing of Aircraft.

c_e, c_a, c_r . . . . Integral control amplification factors, for elevator-, aileron-, and rudder-servo systems, respectively.

C_D . . . . Coefficient of drag for aircraft.

C_m . . . . Coefficient of pitching moment for aircraft.

C_l . . . . Coefficient of rolling moment for aircraft.

C_n . . . . Coefficient of yawing moment for aircraft.

C_X . . . . Coefficient of force along longitudinal axis of aircraft.

C_Y . . . . Coefficient of force along lateral axis of aircraft.

C_Z . . . . Coefficient of force along normal axis of aircraft.

C_{D_o} . . . . Initial value of C_D.

C_u, C_w, C_θ, C_v, C_ψ and C_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

CL . . . . Controlled Line, assumed to be along longitudinal axis of aircraft for work in this thesis.

C_{EAHL} . . . . Chord of elevator aft of hinge line.

C_{RAHL} . . . . Chord of rudder aft of hinge line.

C_{AAHL} . . . . Chord of aileron aft of hinge line.

C_{RH} . . . . Hinge moment coefficient for rudder.

C_{EH} . . . . Hinge moment coefficient for elevator.

C_{AH} . . . . Hinge moment coefficient for aileron.

c_A . . . . Chord of ailerons.

C_{CSE} . . . . Damping coefficient about shaft of elevation computer.

C_{CSD} . . . . Damping coefficient about shaft of deflection computer.

$(C_{EH})_{EFF}$ . . . . Effective damping coefficient of elevator-servo system about elevator hinge.

$(C_{RH})_{EFF}$ . . . . .Effective damping coefficient of rudder-servo system about rudder hinge.

$(C_{AH})_{EFF}$ . . . . .Effective damping coefficient of aileron-servo system about aileron hinges.

$(C_{SAE})_{EFF}$. . . . . .Effective damping coefficient of elevator servo, about elevator servo axis.

$(C_{SAR})_{EFF}$. . . . . .Effective damping coefficient of rudder servo, about rudder servo axis.

$(C_{SAA})_{EFF}$. . . . . .Effective damping coefficient of aileron servo, about aileron servo axis.

$(C)A_R$ . . . . .Correction of Radar Angle.

D . . . . .Dimensional operator, $\frac{d}{dt}$.

d . . . . .Nondimensional operator, $\frac{d}{d\gamma}$. Also means "differential of".

D^2 . . . . Dimensional operator, $\frac{d^2}{dt^2}$.

d^2 . . . . Nondimensional operator, $\frac{d^2}{d\gamma^2}$.

d_{ce} . . . . Nondimensional operator for elevation computer, $= T_{CSE}D$.

d_{cd} . . . . Nondimensional operator for deflection computer, $= T_{CSD}D$.

d_f . . . . Nondimensional flight operator, $= T_{AF_o}D$, $= \frac{d}{d\gamma_f}$.

D_{LS} . . . . Deflection angle of line of sight, measured from reference azimuth heading to line of sight, in top horizontal view.

D . . . . Drag of Aircraft.

D_u, D_w, D_θ, D_v, D_ψ and D_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

D_X, d_x . . . . Component of Drag of Aircraft along longitudinal axis of aircraft.

d_1, d_2, d_3, d_4, d_5, etc., represent roots to charactertistic equations.

E_{LS} . . . . Elevation Angle of line of sight, measured from horizontal to line of sight, in elevation view.

e . . . . Oswald's efficiency factor, Also base of natural logarithums.

E_u, E_w, E_θ, E_v, E_ψ and E_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

F . . . . stands for "function of". Also stands for tracking error angle, which is the forcing function input to aircraft servo controls.

F_{CORR} . . . . Correct value of F.

F_E, f_e . . . . Elevation component of F.

F_D, f_d . . . . Deflection component of F.

$\frac{F_E}{D}$. . . . . $= \int F_E dt$.

$\frac{f_e}{d}$. . . . . $= \int f_e d\gamma$.

$\frac{F_D}{D}$. . . . . $= \int F_D dt$.

$\frac{fd}{d}$. . . . . $= \int f_d d\gamma$.

f_m . . . . $=$ mechanism f-number.

F_u, F_w, F_θ, F_v, F_ψ and F_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

GL . . . . Gun Line, line established by longitudinal axis of gun barrel,

g . . . . Acceleration of gravity.

G_1, G_2, G_u, G_w, G_θ, G_v, G_ψ and G_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

H_{GE} . . . . Angular momentum of rotor of elevation gyro, measured about spin axis of elevation gyro.

H_{GD} . . . . Angular momentum of rotor of deflection gyro, measured about spin axis of elevation gyro.

H_1, H_2, H_u, H_w, H_θ, H_v, H_ψ, and H_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

I_X . . . . Moment of inertia of aircraft about longitudinal axis of aircraft.

I_Y . . . . Moment of inertia of aircraft about lateral axis of aircraft.

I_Z . . . . Moment of inertia of aircraft about normal axis of aircraft.

I_{EH}. . . . . Moment of inertia of elevator about elevator hinge.

$(I_{EH})_{EFF}$ . . . . Effective moment of inertia of elevator-servo system about elevator hinge.

I_{RH} . . . . Moment of inertia of rudder about rudder hinge.

$(I_{RH})_{EFF}$ . . . . Effective moment of inertia of rudder-servo system about rudder hinge.

I_{AHR} . . . . Moment of inertia of right aileron about right aileron hinge.

I_{AHL} . . . . Moment of inertia of left aileron about left aileron hinge.

I_{AH} . . . . $I_{AHR} + I_{AHL}$.

$(I_{AH})_{EFF}$ . . . . Effective moment of inertia of aileron-servo system referred to aileron hinges.

$(I_{SAE})_{EFF}$ . . . . Effective moment of inertia of elevator servo about elevator servo axis.

$(I_{SAR})_{EFF}$ . . . . Effective moment of inertia of rudder servo about rudder servo axis.

$(I_{SAA})_{EFF}$ . . . . Effective moment of inertia of aileron servo about aileron servo axis.

$(I_{CSE})_{EFF}$ . . . . Effective moment of inertia of rotating system of elevation computer, about elevation computer shaft.

$(I_{CSD})_{EFF}$ . . . . Effective moment of inertia of rotating system of deflection computer, about deflection computer shaft.

I_{GD} . . . . Inclination of output axis of deflection gyro.

IAS . . . . Indicated airspeed.

IAS_o . . . . Initial or reference value of IAS.

i . . . . $= \sqrt{-1}$

$\mathcal{I}$ . . . . Imaginary part of.

I_u, I_w, I_θ, I_v, I_ψ and I_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

J . . . . Jump Angle.

-J . . . . Jump correction Angle.

J_1, J_2......, J_u, J_w, J_θ, J_v, J_ψ and J_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

k . . . . Nondimensional stiffness coefficient for the short period motion of the aircraft.

K_{SAE} . . . . Stiffness coefficient of the elevator servo about the elevator servo axis.

K_{SAR} . . . . Stiffness coefficient of the rudder servo about the rudder servo axis.

K_{SAA} . . . . Stiffness coefficient of the aileron servo about the aileron servo axis.

$(K_{EH})_{EFF}$ . . . . Effective stiffness coefficient of the elevator-servo system about the elevator hinge.

$(K_{RH})_{EFF}$ . . . . Effective stiffness coefficient of the rudder-servo system about the rudder hinge.

$(K_{AH})_{EFF}$ . . . . Effective stiffness coefficient of the aileron-servo system about the aileron hinges.

K_{CSE} . . . . Stiffness coefficient of rotating system of elevation computer, about computer shaft of elevation computer.

K_{CSD} . . . . Stiffness coefficient of rotating system of deflection computer, about computer shaft of deflection computer.

k_q . . . . $= \frac{1}{T_A}(\frac{\partial \Delta_E}{\partial Q}) = \frac{\partial \delta_e}{\partial q}$, derivative pitch control amplification factor in automatic pilot.

k_θ . . . . $= \frac{\partial \Delta_E}{\partial \theta} = \frac{\partial \delta_e}{\partial \theta}$, displacement pitch control amplification factor in automatic pilot.

$\frac{k_\theta}{d}$ . . . . $= T_A(\frac{\partial \Delta_E}{\partial \int \theta dt}) = \frac{\partial \delta_e}{\partial \int \theta d\gamma}$, integral pitch control amplification factor in automatic pilot.

k_1, k_2, k_u, k_w, k_θ, k_v, k_ψ and k_ϕ are defined by expressions in the appendices, and are used only for mathematical convenience.

L . . . . Lead angle. Also stands for net rolling moment on aircraft, about longitudinal axis of aircraft. Also used for lift of aircraft. Also used for function defined by Equation 11 in Appendix P.

L_{CORR} . . . . Correct lead angle.

l . . . . Rolling moment stability coefficient for aircraft, about longitudinal axis of aircraft.

L_Z, l_z . . . . Component of lift of aircraft along normal axis of aircraft.

L_o . . . . Initial value of L.

ΔL . . . . Change in rolling moment on aircraft about longitudinal axis of aircraft. Also used for change in lift of aircraft.

$LA_{(first\ symbol)(second\ symbol)}$

Lead angle between the quantity represented by the (first symbol) and the quantity represented by the (second symbol).

L_{AGE} . . . . Distance from gyro axis of elevation computer to center of gravity of suspended accelerometer mass.

L_{ACSE} . . . . Effective distance from computer shaft of elevation computer to center of gravity of suspended accelerometer mass,

$$= L_{AGE}\left(\frac{\partial M_{CSE}}{\partial M_{GE}}\right).$$

$L_{\Delta U}$, l_u . . . . (Change in longitudinal speed - change in rolling moment) stability coefficient for aircraft.

$L_{\Delta V}$, l_v . . . . (Change in lateral velocity - change in rolling moment) stability coefficient for aircraft.

L_R, l_r . . . . (Change in rate of yaw - change in rolling moment) stability coefficient for aircraft.

L_P, l_p . . . . (Change in rate of roll - change in rolling moment) stability coefficient for aircraft.

L_{Δ_A}, l_{δ_a} . . . . (Change in aileron angle - change in rolling moment) stability coefficient for aircraft.

L_1, L_2, L_3, L_u, L_w, L_θ, etc. are defined by expressions in the appendices, and are used only for mathematical convenience.

LS_o . . . . .Present line of sight.

M . . . . .Net Pitching moment on aircraft, about lateral axis of aircraft.

m . . . . .Pitching moment stability coefficient for aircraft, about lateral axis of aircraft.

M' . . . . Pitching moment on aircraft about lateral axis of aircraft, caused by horizontal tail.

ΔM . . . . Change in M.

M_o . . . . Initial value of M.

M_A . . . . Mass of aircraft.

$M_{\Delta U}$, m_u . . . . (Change in longitudinal velocity - change in pitching moment) stability coefficient for aircraft.

$M_{\Delta W}$, m_w . . . . (Change in normal velocity - change in Pitching moment) stability coefficient for aircraft. Can also be thought of as (change in angle of attack - change in pitching moment) stability coefficient for aircraft.

$M_{\Delta \dot{W}}$, $m_{\dot{w}}$ . . . . (Lag of wing downwash - change in pitching moment) stability coefficient for aircraft.

M_Q, m_q . . . . (Change in rate of pitch - change in pitching moment) stability coefficient for aircraft, caused by aerodynamic damping.

M_{Δ_E}, m_{δ_e} . . . . (Change in elevator angle - change in pitching moment) stability coefficient for aircraft.

M_θ, m_θ . . . . (Change in pitch angle - change in pitching moment) stability coefficient for aircraft, caused by ideal displacement pitch control in automatic pilot.

M_{QC}, m_{qc} . . . . (Change in rate of pitch - change in pitching moment) stability coefficient for aircraft, caused by ideal derivative pitch control in automatic pilot.

$M_{\frac{\theta}{D}}$, $m_{\frac{\theta}{d}}$ . . . . (Change in integral of pitch angle - change in pitching moment) stability coefficient for aircraft, caused by ideal integral pitch control in automatic pilot.

$M_{\dot{\Delta}_E}$, $m_{\dot{\delta}_e}$ . . . . (Change in rate of change of elevator angle - change in pitching moment) stability coefficient for aircraft.

M_{EH} . . . . Net moment about elevator hinge.

m_{eh} . . . . Elevator hinge moment stability coefficient for elevator-servo system.

M_{EH_o} . . . . Initial value of M_{EH}.

M_{EHC} . . . . Moment about elevator hinge caused by cable force.

$M_{EH_{\Delta E}}$, $m_{eh_{\delta_e}}$ . . . . . . (Change in elevator angle - change in elevator hinge moment) stability coefficient for elevator-servo system.

$M_{EH_{\Delta \dot{E}}}$, $m_{eh_{\dot{\delta}_e}}$ . . . . . . (Change in rate of change of elevator angle - change in elevator hinge moment) stability coefficient for elevator-servo system.

M_{EH_θ}, m_{eh_θ} . . . . (Change in pitch angle - change in elevator hinge moment) stability coefficient for elevator-servo system, caused by displacement pitch control in automatic pilot.

$M_{EH_{\Delta W}}$, m_{eh_w} . . . .(Change in normal velocity - change in elevator hinge moment) stability coefficient for elevator-servo system.

$M_{EH_{\Delta \dot{W}}}$, $m_{eh_{\dot{w}}}$ . . . . (Lag of wing downwash - change in elevator hinge moment) stability coefficient for elevator-servo system.

$M_{EH_{\Delta U}}$, m_{eh_u} . . . . (Change in longitudinal velocity - change in elevator hinge moment) stability coefficient for elevator-servo system.

M_{EH_Q}, m_{eh_q} . . . . (Change in rate of pitch - change in elevator hinge moment) stability coefficient for elevator-servo system, caused by aerodynamic damping.

$M_{EH_{QC}}$, $m_{eh_{qc}}$ . . . . . . (Change in rate of pitch-change in elevator hinge moment) stability coefficient for elevator-servo system, caused by derivative pitch control in automatic pilot.

$M_{EH_{\frac{\theta}{D}}}$, $m_{eh_{\frac{\theta}{d}}}$ . . . . (Change in integral of pitch angle - change in elevator hinge moment) stability coefficient for elevator-servo system, caused by integral pitch control in automatic pilot.

$M_{EH_{F_E}}$, $m_{eh_{f_e}}$ . . . . (Change in elevation component of tracking error angle - change in elevator hinge moment) stability coefficient for elevator-servo system.

$M_{EH_{\dot{F}_E}}$, $m_{eh_{\dot{f}_e}}$ . . . . (Change in rate of change of elevation component of tracking error angle - change in elevator hinge moment) stability coefficient for elevator-servo system.

$M_{EH_{\frac{F_E}{D}}}$, $m_{eh_{\frac{f_e}{d}}}$ . . . . (Change in integral of elevation component of tracking error angle - change in elevator hinge moment) stability coefficient for elevator-servo system.

M_{SAE} . . . . Net moment about servo axis of elevator servo.

M_{SAE_o} . . . . Initial value of M_{SAE}.

M_{SAEC} . . . . Moment about servo axis of elevator servo, caused by cable force.

M_{SAEM} . . . . Effective moment of motor about servo axis of elevator servo, caused by change, rate of change, integral, etc., of elevation component of tracking error angle.

M_{AE} . . . . Mass of accelerometer in elevation computer.

M_{CSE} . . . . Moment about computer shaft of elevation computer.

M_{GE} . . . . Moment about gyro axis of elevation computer, caused by suspended accelerometer mass.

m_{ce} . . . . Prediction moment stability coefficient for elevation computer.

m_{ce_w} . . . . (Change in normal velocity - change in prediction moment) stability coefficient for elevation computer. Can also be thought of as (change in angle of attack - change in prediction moment) stability coefficient for elevation computer.

$m_{ce_{\dot{w}}}$ . . . . (Change in rate of change of normal velocity - change in prediction moment) stability coefficient for elevation computer.

$m_{ce_{\theta}}$ . . . . (Change in pitch angle - change in prediction moment) stability coefficient for elevation computer.

$m_{ce_{q}}$ . . . . (Change in rate of pitch - change in prediction moment) stability coefficient for elevation computer.

M_{RH} . . . . Net moment about rudder hinge.

m_{rh} . . . . Rudder hinge moment stability coefficient for rudder-servo system.

$M_{RH_{o}}$ . . . . Initial value of M_{RH}.

M_{RHC} . . . . Moment about rudder hinge caused by cable force.

$M_{RH_{\Delta R}}$, $m_{rh_{\delta_r}}$ . . . . (Change in rudder angle - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH_{\dot{\Delta} R}}$, $m_{rh_{\dot{\delta}_r}}$ . . . . (Change in rate of change of rudder angle - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH_{\Delta V}}$, $m_{rh_{v}}$ . . . . (Change in lateral velocity - change in rudder hinge moment) stability coefficient for rudder-servo system. Can also be thought of as (change in aerodynamic yaw - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH_{\Delta U}}$, $m_{rh_{u}}$ . . . . (Change in longitudinal velocity - change in rudder hinge-moment) stability coefficient for rudder-servo system.

$M_{RH_{R}}$, $m_{rh_{r}}$ . . . . (Change in rate of yaw - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH_{F_D}}$, $m_{rh_{f_d}}$ . . . . (Change in deflection component of tracking error angle - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH\dot{F}_D}$, $m_{rh\dot{f}_d}$ . . . . (Change in rate of change of deflection component of tracking error angle - change in rudder hinge moment) stability coefficient for rudder-servo system.

$M_{RH\frac{F_D}{D}}$, $m_{rh\frac{f_d}{d}}$ . . . . (Change in integral of deflection component of tracking error angle - change in rudder hinge moment) stability coefficient for rudder-servo system.

M_{SAR} . . . . Net moment about servo axis of rudder servo.

M_{SAR_o} . . . . Initial value of M_{SAR}.

M_{SARC} . . . . Moment about servo axis of rudder servo, caused by cable force.

M_{SARM} . . . . Effective moment of motor about servo axis of rudder servo, caused by change, rate of change, integral, etc., of deflection component of tracking error angle.

M_{CSD} . . . . Moment about computer shaft of deflection computer.

m_{cd} . . . . Prediction moment stability coefficient of deflection computer.

m_{cd_r} . . . . (Change in rate of yaw - change in prediction moment) stability coefficient for deflection computer.

m_{cd_p} . . . . (Change in rate of roll - change in prediction moment) stability coefficient for deflection computer.

m_{cd_v} . . . . (Change in lateral velocity - change in prediction moment) stability coefficient for deflection computer. Can also be thought of as (change in aerodynamic yaw - change in prediction moment) stability coefficient for deflection computer.

M_{AHR} . . . . Net moment about right aileron hinge.

M_{AHR_o} . . . . Initial value of M_{AHR}.

M_{AHL} . . . . Net moment about left aileron hinge.

M_{AHL_o} . . . . Initial value of M_{AHL}.

M_{AH} . . . . $= M_{AHR} + M_{AHL}$.

M_{AH_o} . . . . Initial value of M_{AH}.

m_{ahr} . . . . Right aileron hinge moment stability coefficient for aileron-servo system.

m_{ahl} . . . . Left aileron hinge moment stability coefficient for aileron-servo system.

m_{ah} . . . . $= m_{ahr} + m_{ahl}$.

M_{AHRC} . . . . Moment about right aileron hinge caused by cable force.

M_{AHLC} . . . . Moment about left aileron hinge caused by cable force.

M_{AHC} . . . . $= M_{AHRC} + M_{AHLC}$.

$M_{AH_{\Delta_A}}$, $m_{ah_{\delta_a}}$ . . . . . . (Change in aileron angle - change in aileron hinge moment) stability coefficient for aileron-servo system.

$M_{AH_{\dot{\Delta}_A}}$, $m_{ah_{\dot{\delta}_a}}$ . . . . . . (Change in rate of change of aileron angle - change in aileron hinge moment) stability coefficient for aileron-servo system.

M_{AH_p}, m_{ah_p} . . . . (Change in rate of roll - change in aileron hinge moment) stability coefficient for aileron-servo system.

$M_{AH_{F_D}}$, $m_{ah_{f_d}}$ . . . . . . (Change in deflection component of tracking error angle - change in aileron hinge moment) stability coefficient for aileron-servo system.

$M_{AH_{\dot{F}_D}}$, $m_{ah_{\dot{f}_d}}$ . . . . . . (Change in rate of change of deflection component of tracking error angle - change in aileron hinge moment) stability coefficient for aileron-servo system.

$M_{AH_{\frac{F_D}{D}}}$, $m_{ah_{\frac{f_d}{d}}}$ . . . . (Change in integral of deflection component of tracking error angle - change in aileron hinge moment) stability coefficient for aileron-servo system.

M_{SAA} . . . . Net moment about servo axis of aileron servo.

M_{SAA_o} . . . . Initial value of M_{SAA}.

M_{SAAC} . . . . Moment about servo axis of aileron servo, caused by cable force.

M_{SAAM} . . . . Effective moment of motor about servo axis of aileron servo, caused by change, rate of change, integral, etc., of deflection component of tracking error angle.

M_u, M_w, M_θ, etc. . . . . . are defined by expressions in the appendices, and are used only for mathematical convenience.

N . . . . Net yawing moment on aircraft, about normal axis of aircraft.

N_o . . . . Initial value of N.

ΔN . . . . Change in N.

n . . . . Yawing moment stability coefficient about normal axis of aircraft. (also see next page)

N″ . . . . Yawing moment on aircraft about normal axis of aircraft, caused by vertical tail.

$N_{\Delta U}$, n_u . . . . (Change in longitudinal velocity - change in yawing moment) stability coefficient for aircraft.

$N_{\Delta V}$, n_v . . . . (Change in lateral velocity - change in yawing moment) stability coefficient for aircraft. Can also be thought of as (change in aerodynamic yaw - change in yawing moment) stability coefficient for aircraft.

N_R, n_r . . . . (Change in rate of yaw - change in yawing moment) stability coefficient for aircraft.

N_P, n_p . . . . (Change in rate of roll - change in yawing moment) stability coefficient for aircraft.

$N_{\Delta R}$, n_{δ_r} . . . . (Change in rudder angle - change in yawing moment) stability coefficient for aircraft.

n_z . . . . Number of g's acceleration of aircraft along normal axis of aircraft, positive downward.

n . . . . Number of g's acceleration of aircraft along normal axis of aircraft, positive upward, (measured by normal accelerometer, just 180 degrees out of phase with n_z).

n_y . . . . Number of g's acceleration of aircraft along lateral axis of aircraft, positive to the right.

n_x . . . . Number of g's acceleration of aircraft along longitudinal axis of aircraft, positive forward.

N_u, N_w, N_θ, etc. . . . . . are defined by expressions in the appendices, and are used only for mathematical convenience.

O_u, O_w, O_θ, etc. . . . . . are defined by expressions in the appendices, and are used only for mathematical convenience.

P, p . . . . Rate of roll of aircraft about longitudinal axis of aircraft.

P_o . . . . Initial value of P.

ΔP . . . . Change in P.

P_C . . . . Computer prediction angle.

$(P_C)_{CORR}$ . . . . Correct value of P_C.

$(C)P_C$ . . . . Correction in P_C.

P_{CE}, p_{ce} . . . . Prediction angle of elevation computer.

P_{CD}, p_{cd} . . . . Prediction angle of deflection computer.

$(P_{GL})_{CORR}$ . . . . Correct prediction angle for the gun line.

P_1, P_2. . . . ., P_u, P_w, P_θ, etc., are defined by expressions in the appendices, and are used only for mathematical convenience.

P_{BAR} . . . . Barametric pressure.

Q, q . . . . Rate of pitch of aircraft about lateral axis of aircraft.

Q_o . . . . Initial value of Q.

ΔQ . . . . Change in Q.

Q_{SAE} . . . . Rate of change of angle of elevator servo about elevator servo axis.

Q_{SAR} . . . . Rate of change of angle of rudder servo about rudder servo axis.

Q_{SAA} . . . . Rate of change of angle of aileron servo about aileron servo axis.

Q_1, Q_2, etc. . . . . are defined by expressions in the appendices, and are used only for mathematical convenience.

R, r . . . . Rate of yaw of aircraft about normal axis of aircraft. Also used for aspect ratio of aircraft.

R_o . . . . Initial value of R.

ΔR . . . . Change in R.

RL . . . . Radar Line (same as Tracking Line when radar is doing the tracking).

R_T, r_t . . . . Range of target.

R_{T_o}, r_{t_o} . . . . Initial value of range of target, (present range).

r_1, r_2, r_3, r_4 . . . . Radii of movable parts in aileron control system.

S . . . . Wing area of aircraft.

S′ . . . . Horizontal tail area of aircraft.

S″ . . . . Vertical tail area of aircraft.

S_E . . . . Area of elevator.

S_R . . . . Area of rudder.

S_A . . . . Area of ailerons, both.

S_{EAHL} . . . . Area of elevator aft of hinge line.

S_{RAHL} . . . . Area of rudder aft of hinge line.

S_{AAHL} . . . . Area of ailerons aft of hinge line, both ailerons.

$S_{p(aP)_e}$ . . . . (Linear acceleration - prediction angle) sensitivity of the elevation prediction system.

$S_{p(WP)_e}$ . . . . (Angular velocity - prediction angle) sensitivity of the elevation prediction system.

$S_{p(WP)_d}$ . . . . (Angular velocity - prediction angle) sensitivity of the deflection prediction system.

$S_{p(\Delta W-P)_e}$ . . . . (Change in normal velocity-prediction angle) sensitivity of the elevation prediction system.

(Sg) . . . . Signal.

$(Sg)_A$ . . . . Aircraft Signal

$(Sg)v$ . . . . Aerodynamic Yaw Angle Signal.

$(Sg)n_y$ . . . . Lateral acceleration signal.

$(Sg)A_R$ . . . . Radar angle signal.

$(Sg)A_{RD}$ . . . . Signal of deflection component of radar angle.

$(Sg)A_{RE}$ . . . . Signal of elevation component of radar angle.

$(Sg)R_s$ . . . . Range setting signal.

$(Sg)P_C$ . . . . Computer prediction angle signal.

$(Sg)P_{CD}$ . . . . Signal of deflection component of prediction angle of computer.

$(Sg)P_{CE}$ . . . . Signal of elevation component of prediction angle of computer.

$(Sg)F$ . . . . Forcing function input signal for aircraft servo units. (tracking error angle signal)

$(Sg)F_D$ . . . . Signal of deflection component of forcing function input to aircraft servo-units. (deflection tracking error angle signal).

$(Sg)F_E$ . . . . Signal of elevation component of forcing function input to aircraft servo units. (elevation tracking error angle signal)

$(Sg)\alpha$ . . . . Angle of attack signal.

$(Sg)n_Z$ . . . . Normal acceleration signal.

$(Sg)(IAS - IAS_o)$ . . . . (Change in indicated airspeed) signal.

(Sg)(RAD) . . . . Radar Signal.

(Sg)[(C)TL] . . . . Tracking line correction signal.

$(Sg)_I(RS)$ . . . . Radar servo input signal.

$(Sg)_I(ASU)$ . . . . Input signal to aircraft servo units.

$S_{p(\Delta V-P)_d}$ . . . . (Change in lateral velocity - prediction angle) sensitivity of the deflection prediction system.

$S_{p(w-P)_e} \doteq S_{p(\alpha-P)_e}$ . . . (Change in angle of attack - prediction angle) sensitivity of the elevation prediction system.

$S_{p(v-P)_d}$ . . . . (Change in aerodynamic yaw - prediction angle) sensitivity of the deflection prediction system.

$(SN)_e$ . . . . Stability number of the elevation computer.

$(SN)_d$ . . . . Stability number of the deflection computer.

t . . . . time.

T_S . . . . Characteristic time of automatic tracking control system.

T_{AF_o} . . . . Initial time of flight of aircraft, equal to initial target range divided by initial speed of aircraft.

T_A . . . . Characteristic time of aircraft.

T_{CSE} . . . . Characteristic time of elevation computer. (CS stands for computer shaft.)

T_{CSD} . . . . Characteristic time of deflection computer. (CS stands for computer shaft.)

T . . . . Thrust on aircraft.

T_o . . . . Initial value of T.

T_X, t_x . . . . Component of thrust on aircraft along longitudinal axis of aircraft.

TL . . . . Tracking line.

$(TQ)_{AS}$ . . . . Aircraft Servo torque, or torques.

$(TQ)_{RS}$ . . . . Radar servo torque.

U . . . . Longitudinal velocity of aircraft.

U_o, u_o . . . . Initial value of U, u.

ΔU, u . . . . Change in longitudinal velocity of aircraft.

U_T . . . . Velocity of target.

U_{T_o} . . . . Initial value of U_T.

u_1, u_2, u_3, u_4 . . . . . Starting values for components of u_o, used in transient analyses.

V . . . . Lateral velocity of aircraft.

V_o, v_o . . . . Initial value of V, v.

ΔV, v . . . . Change in lateral velocity of aircraft.

v_1, v_2, v_3, v_4, v_5 . . . Starting values for components of v_o, used in transient analyses.

W . . . . Normal velocity of aircraft.

W_o, w_o . . . . Initial value of W, w.

ΔW, w . . . . Change in normal velocity of aircraft.

W_A . . . . Weight of aircraft.

w_1, w_2, w_3, w_4 . . . . . Starting values for components of w_o, used in transient analyses.

X . . . . Net longitudinal force on aircraft.

X_o . . . . Initial value of X.

ΔX . . . . Change in X.

x . . . . Distance along longitudinal axis of aircraft, positive forward.

$X_{\Delta U}$, x_u . . . . (Change in longitudinal velocity - change in longitudinal force) stability coefficient for aircraft.

$X_{\Delta W}$, x_w . . . . (Change in normal velocity - change in longitudinal force) stability coefficient for aircraft. Can also be thought of as (change in angle of attack - change in longitudinal force) stability coefficient for aircraft.

X_θ, x_θ . . . . (Change in pitch angle - change in longitudinal force) stability coefficient for aircraft.

X_Q, x_q . . . . (Change in rate of pitch - change in longitudinal force) stability coefficient for aircraft.

X_{Δ_E}, x_{δ_e} . . . . (Change in elevator angle - change in longitudinal force) stability coefficient for aircraft.

$X_{\dot{\Delta}_E}$, $x_{\dot{\delta}_e}$ . . . . (Change in rate of change of elevator angle - change in longitudinal force) stability coefficient for aircraft.

$X_{\Delta\dot{W}}$, $x_{\dot{w}}$ . . . . (Lag of wing downwash - change in longitu- force) stability coefficient for aircraft.

X_{T_X}, x_{t_x} . . . . (Change in component of thrust along longitudinal axis - change in longitudinal force) stability coefficient for aircraft, x_{t_x} equal to unity by definition.

Y . . . . Net lateral force on aircraft.

Y_o . . . . Initial value of Y.

ΔY . . . . Change in Y.

y . . . . Distance along lateral axis of aircraft, positive out along right wing.

$Y_{\Delta V}$, y_v . . . . (Change in lateral velocity - change in lateral force) stability coefficient for aircraft. Can also be thought of as (change in aerodynamic yaw - change in lateral force) stability coefficient for aircraft.

Y_ϕ, y_ϕ . . . . (Change in angle of bank - change in lateral force) stability coefficient for aircraft.

Y_{Δ_R}, y_{δ_r} . . . . (Change in rudder angle - change in lateral force) stability coefficient for aircraft.

$Y_{\dot{\Delta}_R}$, $y_{\dot{\delta}_r}$ . . . . (Change in rate of change of rudder angle - change in lateral force) stability coefficient for aircraft.

Y_P, y_p . . . . (Change in rate of roll - change in lateral force) stability coefficient for aircraft.

Y_R, y_r . . . . (Change in rate of yaw - change in lateral force) stability coefficient for aircraft.

Z . . . . Net normal force on aircraft.

Z_o . . . . Initial value of Z.

ΔZ . . . . Change in Z.

z . . . . Distance along normal axis of aircraft, positive downward.

$Z_{\Delta U}$, z_u . . . . (Change in longitudinal velocity - change in normal force) stability coefficient for aircraft.

$Z_{\Delta W}$, z_w . . . . (Change in normal velocity - change in normal force) stability coefficient for aircraft. Can also be thought of as (change in angle of attack - change in normal force) stability coefficient for aircraft.

Z_θ, z_θ . . . . (Change in pitch angle - change in normal force) stability coefficient for aircraft.

Z_Q, z_q . . . . (Change in rate of pitch - change in normal force) stability coefficient for aircraft.

Z_{Δ_E}, z_{δ_e} . . . . (Change in elevator angle - change in normal force) stability coefficient for aircraft.

$Z_{\dot{\Delta}_E}$, $z_{\dot{\delta}_e}$ . . . . (Change in rate of change of elevator angle - change in normal force) stability coefficient for aircraft.

II GREEK SYMBOLS

α . . . . Angle of attack of aircraft.

α_o . . . . Initial value of α.

$\Delta\alpha$ . . . . Change in α.

α' . . . . Angle of attack of horizontal tail of aircraft.

α'_o . . . . Initial value of α'. Also equal to angle of incidence of horizontal tail of aircraft.

$\Delta\alpha'$ . . . . Change in α'.

α'' . . . . Angle of attack of vertical tail.

α''_o . . . . Initial value of α''.

$\Delta\alpha''$ . . . . Change in α''.

α_1, α_2, α_3, α_4, etc. . . . Phase angles defined in Appendices. Also used for real parts of complex roots in transient analyses.

β_1, β_2 . . . . Phase angles defined in Appendices. Also used for real coefficients in imaginary parts of complex roots in transient analyses.

γ . . . . Nondimensional time ratio, based upon characteristic time of aircraft.

γ_f . . . . Nondimensional time ratio, based upon initial time of flight of aircraft.

Δ_E, δ_e . . . . Deflection angle of elevator.

Δ_{E_R}, δ_{e_r} . . . . Resultant value of Δ_E, δ_e.

Δ_{E_o}, δ_{e_o} . . . . Initial value of Δ_E, δ_e.

$\Delta\Delta_E$ . . . . Change in Δ_E.

Δ_{E_a}, δ_{e_a} . . . . Amplitude of Δ_E, δ_e.

Δ_{AR}, δ_{ar} . . . . Deflection angle of right aileron.

Δ_{AL}, δ_{al} . . . . Deflection angle of left aileron.

Δ_A, δ_a . . . . Total aileron deflection angle, $= \Delta_{AR} + \Delta_{AL}$, $= \delta_{ar} + \delta_{al}$.

Δ_{A_a}, δ_{a_a} . . . . Amplitude of Δ_A, δ_a.

Δ_{A_o} . . . . Initial value of Δ_A.

$\Delta\Delta_A$ . . . . Change in Δ_A.

Δ_R, δ_r . . . . Reflection angle of rudder.

Δ_{R_o} . . . . Initial value of Δ_R.

$\Delta\Delta_R$ . . . . Change in Δ_R.

Δ_{R_a}, δ_{r_a} . . . . Amplitude of Δ_R, δ_r.

Δ_1, Δ_2. . . ., Δ_{24}, Δ_{u1}. . . ., $\Delta_{\theta 1}$. . . ., Δ_{w1}. . . ., Determinants defined in Appendices.

ϵ . . . . Angle of downwash from wing. Also used for real coefficient in imaginary part of complex root in transient analysis.

ϵ_o . . . . Initial value of ϵ.

$\Delta\epsilon$ . . . . Change in ϵ.

ζ_{SAE} . . . . Damping ratio of elevator servo about elevator servo axis.

ζ_{SAR} . . . . Damping ratio of rudder servo about rudder servo axis.

ζ_{SAA}	Damping ratio of aileron servo about aileron servo axis.
η_p	Efficiency of propeller.
η'	Aerodynamic efficiency of horizontal tail of aircraft.
η''	Aerodynamic efficiency of vertical tail of aircraft.
θ, θ	Pitch angle of aircraft, about lateral axis of aircraft.
θ_o	Initial value of θ.
$\Delta\theta$	Change in θ.
θ_{SAE}	Angle of elevator servo, about elevator servo axis.
θ_{SAR}	Angle of rudder servo, about rudder servo axis.
θ_{SAA}	Angle of aileron servo, about aileron servo axis.
θ_1, θ_2, θ_3, θ_4	Starting values for components of θ_o, used in transient analyses.
λ_1, λ_2, λ_3, λ_4	Phase angles defined in appendices.
μ_c	Aircraft longitudinal relative density, based upon wing chord.
μ_b	Aircraft lateral relative density, based upon wing span.
μ [first symbol] [second symbol]	Amplitude response ratio, amplitude of quantity represented by (first symbol) divided by amplitude of quantity represented by (second symbol).
ξ	Defined by expression in Appendix P, used only for mathematical convenience.
π	3.1416
ρ	Mass density of air.

σ . . . . Real part of complex root in transient analyses.

σ_1, σ_2, σ_3 . . . . Phase angles defined in appendices.

ϕ, ϕ . . . . Angle of roll of aircfaft about longitudinal axis of aircraft.

ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5 . . . Phase angles defined in Appendices.

ψ, ψ . . . . Angle of geometric yaw of aircraft about normal axis of aircraft.

ψ_1, ψ_2, etc. . . . . Phase angles defined in appendices.

ψ_1, ψ_2, etc. . . . . Starting values for components of ψ_o, used in transient analyses.

Ω_E, ω_e . . . . Frequency of elevator oscillation.

Ω_R, ω_r . . . . Frequency of rudder oscillation.

Ω_A, ω_a . . . . Frequency of aileron oscillation.

Ω_{AR}, ω_{ar} . . . . Frequency of aileron-rudder oscillation.

Ω_{E-TH}, ω_{e-th} . . . . . . Frequency of elevator-throttle oscillation.

Ω_{T_X}, ω_{t_x} . . . . Frequency of longitudinal thrust oscillation, which results from throttle oscillation.

$\omega_{n_{SAE}}$ . . . . Undamped natural frequency of elevator servo about elevator servo axis.

$\omega_{n_{SAR}}$ . . . . Undamped natural frequency of rudder servo about rudder servo axis.

$\omega_{n_{SAA}}$ . . . . Undamped natural frequency of aileron servo about aileron servo axis.

BASIC CONSTANTS FOR AN

AUTOMATIC TRACKING CONTROL SYSTEM

FOR THE A-26 AIRPLANE

BASIC CONSTANTS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE

I A-26 AIRPLANE CONSTANTS

	Reference
$I_{EH} = 2.62$ slugs-ft^2	6
$I_{AH} = \frac{2(1900)}{(32.2)(144)} = .81953$ slugs-ft^2	6
$I_{RH} = \frac{11,400}{(32.2)(144)} = 2.4586$ slugs-ft^2	6
$W_A = 30,000$ lbs	Assumed
$S = 540$ ft^2	6
$c = 8.14$ ft	6
$C_{D_{p_{min}}} = .0244$	Assumed
$R = 9.07$ (aspect ratio)	6
$\frac{\partial C_L}{\partial \alpha} = .085$/degree, $= 4.8701$/radian	6
$e = .8$ (Oswald's efficiency)	Assumed
$I_Y = 48,531$ slugs-ft^2	6
$I_X = 66.903$ slugs-ft^2	6
$I_Z = 111,555$ slugs-ft^2	6

Fixed control Neutral Point = 36%, high power, at 26% c.g., based upon flight-test data for A-26 Airplane, Reference

recommended by Cornell Aeronautical Laboratory.

Thus, $\frac{\partial C_m}{\partial C_L} = -.10$ at c.g. = 26%, or $\frac{\partial C_m}{\partial \alpha} = -.0085$/degree, = -.48701/radian.

$(\frac{\partial C_L}{\partial \alpha})' = .0679$/degree, = 3.8904/radian 6

$\frac{d\epsilon}{d\alpha} = .41$ . . . . Assumed

$S' = 116.1$ ft^2 6

$l = 30.1$ ft. 6

$\eta' = .9$, power on, high power 6

$\frac{\partial C_m}{\partial \Delta_E} = -.020$/degree, = -1.146/radian 6

$\frac{\partial C_{EH}}{\partial \alpha'} = -.005$/degree, = -.2865 6

$S_{EAHL} = 22.36$ ft^2 6

$c_{EAHL} = 1.27$ ft 6

$S_E = 32.66$ ft^2 6

$c_E = 1.85$ ft 6

$\frac{\partial C_{EH}}{\partial \dot{\Delta}_E} = -.0056523$ sec at 300 mph TAS 7

$\frac{\partial C_{EH}}{\partial \Delta_E} = -.009$/degree, = -.5157/radian 6

$b = 70$ ft 6

$S'' = 71.35$ ft^2 6

Reference

$(\frac{\partial C_L}{\partial \alpha})'' = \frac{\partial C_L''}{\partial v} = .0455/\text{degree}, = 2.607/\text{radian}$ 6

$\eta'' = .9$ . . . . Assumed

$\frac{\partial C_n}{\partial v} = .0011/\text{degree}$, high power, stick fixed.

$= .0630/\text{radian}$, high power, stick fixed. 6

$\frac{\partial C_l}{\partial v} = -.00184/\text{degree}, = -.1054/\text{radian}$ 6

$\frac{\partial C_Y}{\partial v} = -.65/\text{radian}$ . . . . Assumed

$\frac{\partial C_n}{\partial \Delta_R} = .0016/\text{degree}, = .0917/\text{radian}$ 6

$\frac{\partial C_l}{\partial \Delta_A} = .16/\text{radian}$ of total aileron angle 6

Rectangular wing plan-form assumed.

$S_{RAHL} = 23\ ft^2$ 6

$S_R = 30\ ft^2$ 6

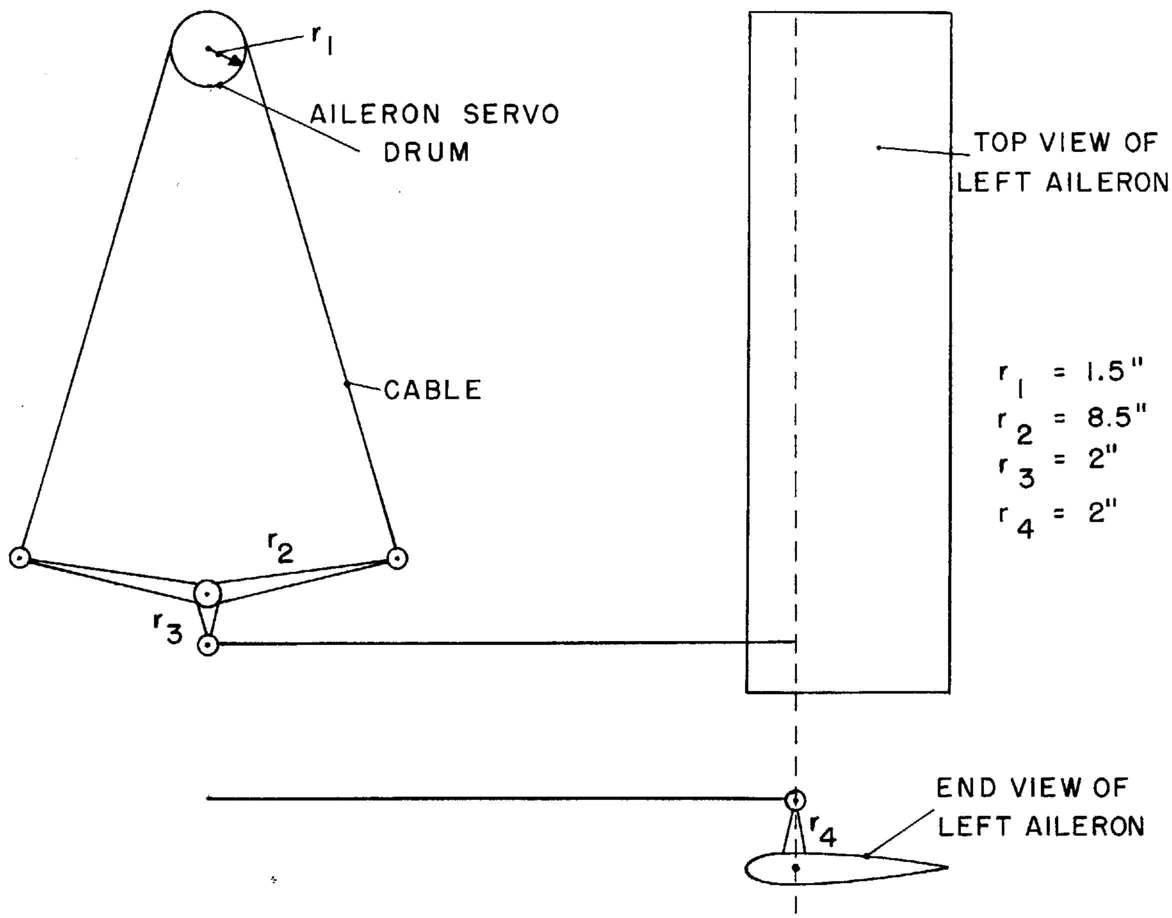
$C_{RAHL} = 2.3750$ ft. 6

$C_R = 3.0833$ ft. 6

$\frac{\partial C_{RH}}{\partial v} = -.0015/\text{degree}, = -.085944/\text{radian}$ 6

$I_{AHR} = I_{AHL} = .40977\ \text{slugs-ft}^2$ 6

$\frac{\partial C_{RH}}{\partial \dot{\Delta}_R} = -2.2548 \times 10^{-3}$ sec. 7



SCHEMATIC SKETCH SHOWING LEVER RATIOS ASSOCIATED WITH AILERON CONTROL SYSTEM FOR A-26 AIRPLANE

	Reference
$\frac{\partial C_{RH}}{\partial \Delta_R}$ = -.0040/degree, = -.22918/radian	6
$\frac{\partial C_{AH}}{\partial \Delta_A}$ = -.0031/degree, = -.17762/radian	6
c_{AAHL} = 1.2083 ft.	6
S_{AAHL} = 18.2 ft^2	6
c_A = 1.6083 ft	6
S_A = 36.5 ft^2	6
$\frac{\partial C_{AH}}{\partial \dot{\Delta}_A}$ = assumed negligible	7

Diameter of elevator Horn = 12.6 inches; diameter of rudder horn = 20.5 inches, measurements taken from scaled drawings for control system of A-26 Airplane.
Experimental tests showed that 82 lbs stick force produces 1490 in-lbs elevator hinge moment.

II SERVO CONSTANTS | Reference

$\zeta_{SAE} = \zeta_{SAR} = \zeta_{SAA} = 1.1$ 8

$\omega_{n_{SAE}} = \omega_{n_{SAR}} = \omega_{n_{SAA}} = 5.1$ radians/sec 8

$(C_{SAE})_{EFF} = (C_{SAR})_{EFF} = (C_{SAA})_{EFF} = 1078.4 \frac{\text{ft-lbs}}{\text{radians/sec.}}$ . . 8

$K_{SAE} = K_{SAR} = K_{SAA} = 2500$ ft-lbs/rad 8

$(I_{SAE})_{EFF} = (I_{SAR})_{EFF} = (I_{SAA})_{EFF} = 96.117$ slugs-ft^2 . . . 8

Diameter of drums for elevator, rudder and aileron servos were measured and found to be 3 inches. Thus,

$$\frac{\partial\theta_{SAE}}{\partial\Delta_E} = \frac{\text{Diameter of elevator horn}}{\text{Diameter of elevator servo drums}} = \frac{12.6}{3} = 4.2$$

$$\frac{\partial\theta_{SAR}}{\partial\Delta_R} = \frac{\text{Diameter of rudder horn}}{\text{Diameter of rudder servo drum}} = \frac{20.5}{3} = 6.8333.$$

$$\frac{\partial\theta_{SAA}}{\partial\Delta_A} = \left(\frac{1}{2}\right)\left(\frac{r_4}{r_3}\right)\left(\frac{r_2}{r_1}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{2}\right)\left(\frac{8.5}{1.5}\right) = 2.833$$

III COMPUTER CONSTANTS

For 300 mph TAS, at 10,000 ft. density altitude, target range 1000 yds:

$S_{p(WP)_e} = 1.1751$ sec. 3

$S_{p(aP)_e} = 1.8247 \times 10^{-4}$ sec^2/ft 3

$(SN)_e = .2$ . . . . Assumed

$(SN)_d = .2$ . . . . Assumed

$m_{ce_w} = 0$ . . . . Assumed

	Reference
$m_{cd_v} = 0$	Assumed
$I_{GD} = 10^{\circ}$	3
$S_{P(WP)_d} = 1.1751$ sec.	3

IV PERFECT RADAR ASSUMED

(Tracking inaccuracy assumed negligible.)

A P P E N D I X   A

DERIVATION OF EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

APPENDIX A

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APPENDIX A

DERIVATION OF EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

A. LONGITUDINAL EQUATIONS OF MOTION

A-1: DERIVATION OF LONGITUDINAL EQUATIONS OF MOTION FOR AIRCRAFT

Euler's equations of motion for a moving body with principal body-axes as reference, are:

$$L = I_X\dot{P} - (I_Y - I_Z)QR, \tag{1}$$

$$M = I_Y\dot{Q} - (I_Z - I_X)RP, \tag{2}$$

$$N = I_Z\dot{R} - (I_X - I_Y)PQ, \tag{3}$$

$$X = M_A(\dot{U} + WQ - VR), \tag{4}$$

$$Y = M_A(\dot{V} + UR - WP) \text{ and} \tag{5}$$

$$Z = M_A(\dot{W} + VP - UQ). \tag{6}$$

For pure longitudinal motion,

$$P,\ R,\ V,\ \dot{P},\ \dot{R} \text{ and } \dot{V} \equiv 0.$$

Thus, equations 1, 3 and 5 become:

$$L = 0, \tag{1-a}$$

$$N = 0 \text{ and} \tag{3-a}$$

$$Y = 0. \tag{5-a}$$

And equations 2, 4 and 6 become:

$$M = I_Y\dot{Q}, \tag{2-a}$$

$$X = M_A(\dot{U} + WQ) \text{ and} \tag{4-a}$$

$$Z = M_A(\dot{W} - UQ). \tag{6-a}$$

Functional Dependence:

Let $M = F(W, \dot{W}, U, Q, \Delta_E)$, (7)

$X = F(W, \theta, U)$ and (8)

$Z = F(W, \theta, U, \Delta_E)$. (9)

Other effects will be neglected.

With reference to initial conditions:

$$\begin{aligned} M &= M_o + \Delta M, & \Delta_E &= \Delta_{E_o} + \Delta\Delta_E, \\ X &= X_o + \Delta X, & \dot{U} &= \dot{U}_o + \Delta\dot{U}, \\ Z &= Z_o + \Delta Z, & \dot{W} &= \dot{W}_o + \Delta\dot{W}, \\ U &= U_o + \Delta U, & Q &= Q_o + \Delta Q \text{ and} \\ W &= W_o + \Delta W, & \dot{Q} &= \dot{Q}_o + \Delta\dot{Q}. \\ \theta &= \theta_o + \Delta\theta, \end{aligned} \tag{10}$$

Initial conditions for steady-state horizontal flight:

$$\begin{aligned} &M_o = 0, & &\Delta_{E_o} = 0, \\ &X_o = 0, & &\dot{U}_o = 0, \\ &Z_o = 0, & &\dot{W}_o = 0, \\ &U_o \text{ large}, & &Q_o = 0, \text{ and} \\ &W_o \text{ negligibly small}, & &\dot{Q}_o = 0. \\ &\theta_o = 0, \end{aligned} \tag{11}$$

Thus, for departures from steady-state horizontal flight:

$$\begin{aligned} M &= \Delta M, & \Delta_E &= \Delta\Delta_E, \\ X &= \Delta X, & \dot{U} &= \Delta\dot{U}, \\ Z &= \Delta Z, & \dot{W} &= \Delta\dot{W}, \end{aligned} \tag{12}$$

$U = U_o + \Delta U,$ $\quad Q = \Delta Q$ and

$W \cong \Delta W,$ $\quad \dot{Q} = \Delta\dot{Q},$

$\theta = \Delta\theta,$

Neglecting second order terms, equations 2-a, 4-a and 6-a become:

$$M = I_Y\dot{Q} \tag{2-a}$$

$$X = M_A\Delta\dot{U} \tag{4-b}$$

$$Z = M_A(\Delta\dot{W} - U_oQ) \tag{6-b}$$

Assuming that all changes are small, and using the principle of linear superposition, equations 7, 8 and 9 may be expanded as follows:

$$M = M_o + (\frac{\partial M}{\partial\Delta U})\Delta U + (\frac{\partial M}{\partial\Delta W})\Delta W + (\frac{\partial M}{\partial\Delta\dot{W}})\Delta\dot{W} + (\frac{\partial M}{\partial Q})Q + (\frac{\partial M}{\partial\Delta_E})\Delta_E, \tag{13}$$

$$X = X_o + (\frac{\partial X}{\partial\Delta U})\Delta U + (\frac{\partial X}{\partial\Delta W})\Delta W + (\frac{\partial X}{\partial\theta})\theta \text{ and} \tag{14}$$

$$Z = Z_o + (\frac{\partial Z}{\partial\Delta U})\Delta U + (\frac{\partial Z}{\partial\Delta W})\Delta W + (\frac{\partial Z}{\partial\theta})\theta + (\frac{\partial Z}{\partial\Delta_E})\Delta_E, \tag{15}$$

where

$$M_o,\ X_o, \text{ and } Z_o = 0, \text{ as previously stated.} \tag{16}$$

We may define:

$$X_{\Delta U} = (\frac{1}{M_A})(\frac{\partial X}{\partial\Delta U}),\ X_{\Delta W} = (\frac{1}{M_A})(\frac{\partial X}{\partial\Delta W}),\ X_\theta = (\frac{1}{M_A})(\frac{\partial X}{\partial\theta}),$$

$$Z_{\Delta U} = (\frac{1}{M_A})(\frac{\partial Z}{\partial\Delta U}),\ Z_{\Delta W} = (\frac{1}{M_A})(\frac{\partial Z}{\partial\Delta W}),\ Z_\theta = (\frac{1}{M_A})(\frac{\partial Z}{\partial\theta}),$$

$$Z_{\Delta_E} = (\frac{1}{M_A})(\frac{\partial Z}{\partial\Delta_E}),\ M_{\Delta U} = (\frac{1}{I_Y})(\frac{\partial M}{\partial\Delta U}),\ M_{\Delta W} = (\frac{1}{I_Y})(\frac{\partial M}{\partial\Delta W}),$$

$$M_Q = \left(\frac{1}{I_Y}\right)\left(\frac{\partial M}{\partial Q}\right), \; M_{\Delta_E} = \left(\frac{1}{I_Y}\right)\left(\frac{\partial M}{\partial \Delta_E}\right), \text{ and } M_{\Delta\dot{W}} = \left(\frac{1}{I_Y}\right)\left(\frac{\partial M}{\partial \Delta\dot{W}}\right).$$

Then the dimensional longitudinal equations of motion are:

$$\Delta\dot{U} = X_{\Delta U}\Delta U + X_{\Delta W}\Delta W + X_\theta \theta, \tag{17}$$

$$\Delta\dot{W} = Z_{\Delta U}\Delta U + Z_{\Delta W}\Delta W + Z_\theta\theta + U_o Q + Z_{\Delta_E}\Delta_E \tag{18}$$

and $$\dot{Q} = M_{\Delta U}\Delta U + M_{\Delta W}\Delta W + M_{\Delta\dot{W}}\Delta\dot{W} + M_Q Q + M_{\Delta_E}\Delta_E. \tag{19}$$

If we let $D = \frac{d}{dt}$, then $Q = D\theta$ and $\dot{Q} = D^2\theta$.

Thus, in operator form, the dimensional longitudinal equations of motion are:

$$(-D + X_{\Delta U})\Delta U + (X_{\Delta W})\Delta W + (X_\theta)\theta = 0, \tag{20}$$

$$(Z_{\Delta U})\Delta U + (-D + Z_{\Delta W})\Delta W + (DU_o + Z_\theta)\theta + (Z_{\Delta_E})\Delta_E = 0 \tag{21}$$

and $$(M_{\Delta U})\Delta U + (DM_{\Delta\dot{W}} + M_{\Delta W})\Delta W + (-D^2 + DM_Q)\theta + (M_{\Delta_E})\Delta_E = 0. \tag{22}$$

Non-dimensionalizing:

Let:

$$t = T_A\gamma,$$

$$dt = T_A d\gamma,$$

$$T_A = \frac{U_o C_{L_o}}{g} = \frac{M_A}{\frac{\rho}{2} S U_o}$$

$$x_u = T_A X_{\Delta U},$$

$$x_w = T_A X_{\Delta W},$$

Let:

$$d = T_A D,$$

$$m_{\dot{w}} = c M_{\Delta\dot{W}},$$

$$z_u = T_A Z_{\Delta U},$$

$$z_w = T_A Z_{\Delta W},$$

$$z_\theta = T_A Z_\theta\left(\frac{1}{U_o}\right),$$

$$z_{\delta_e} = T_A Z_{\Delta_E}\left(\frac{1}{U_o}\right),$$

$$x_\theta = T_A X_\theta \left(\frac{1}{U_o}\right),$$

$$m_u = cT_A M_{\Delta U},$$

$$m_w = cT_A M_{\Delta W},$$

$$m_q = T_A M_Q,$$

$$m_{\delta_e} = T_A^2 M_{\Delta_E},$$

$$\delta_e = \Delta_E$$

and $\dot{\delta}_e = T_A \dot{\Delta}_E.$

$$q = T_A Q = d\theta$$

$$u = \frac{\Delta U}{U_o},$$

$$w = \frac{\Delta W}{U_o},$$

$$\dot{w} = \frac{T_A}{U_o}\Delta\dot{W},$$

$$\mu_c = \frac{T_A U_o}{c} = \frac{U_o^2 C_{L_o}}{gc} = \frac{M_A}{\frac{\rho}{2}Sc},$$

$\theta = \theta$ and

$$\dot{\theta} = T_A \dot{\theta}$$

Then the dimensional longitudinal equations of motion become:

$$\left[-\left(\frac{d}{T_A}\right) + \left(\frac{x_u}{T_A}\right)\right] uU_o + \left(\frac{x_w}{T_A}\right) wU_o + \left(\frac{x_\theta}{T_A}\right) U_o \theta = 0, \tag{23}$$

$$\left(\frac{z_u}{T_A}\right) uU_o + \left[-\left(\frac{d}{T_A}\right) + \left(\frac{z_w}{T_A}\right)\right] wU_o + \left[\left(\frac{d}{T_A}\right) U_o + \left(\frac{z_\theta}{T_A}\right) U_o\right]\theta \tag{24}$$

$$+ \left[\left(\frac{U_o}{T_A}\right) z_{\delta_e}\right]\delta_e = 0$$

and

$$\left(\frac{m_u}{cT_A}\right) uU_o + \left[\left(\frac{d}{T_A}\right)\left(\frac{m_{\dot{w}}}{c}\right) + \left(\frac{m_w}{cT_A}\right)\right] wU_o + \left[-\left(\frac{d^2}{T_A^2}\right) + \left(\frac{d}{T_A}\right)\left(\frac{m_q}{T_A}\right)\right]\theta \tag{25}$$

$$+ \left(\frac{m_{\delta_e}}{T_A^2}\right)\delta_e = 0.$$

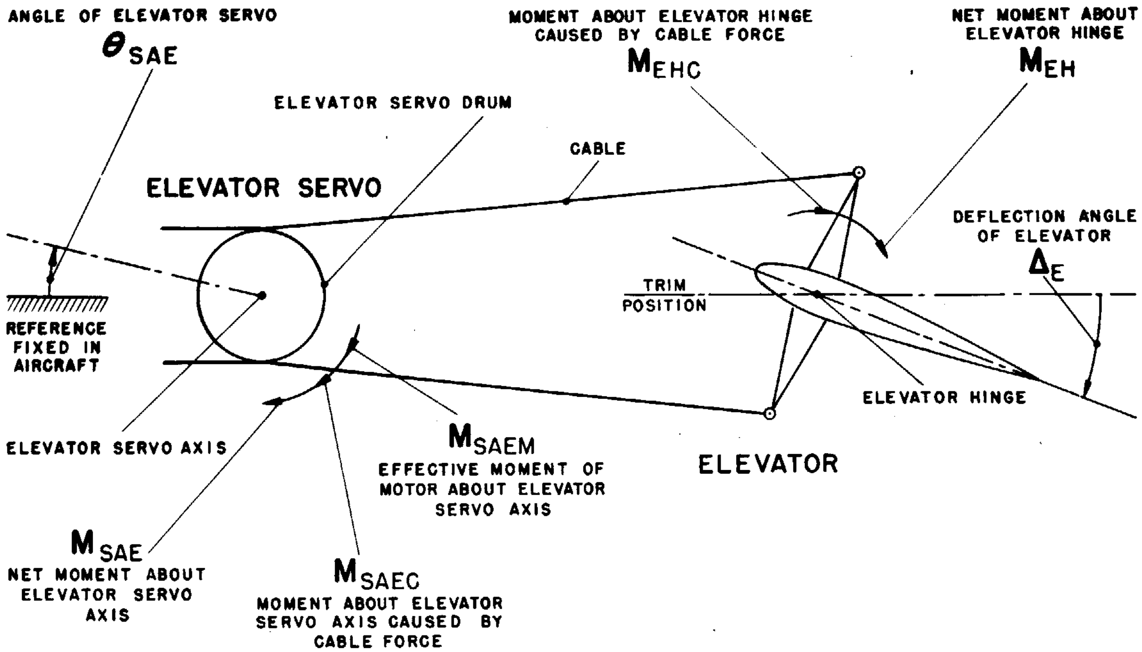
If we multiply equations 23 and 24 by T_A/U_o, and multiply equation 25 by T_A^2 we obtain longitudinal equations of motion which are completely non-dimensional:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta = 0, \tag{26}$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta + (z_{\delta_e})\delta_e = 0 \tag{27}$$

$$\text{and}\quad (\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta + (m_{\delta_e})\delta_e = 0. \tag{28}$$

A-2: DERIVATION OF EQUATION OF MOTION FOR ELEVATOR-SERVO SYSTEM



ELEVATOR-SERVO SYSTEM

From Newton's Law:

$$M_{EH} = I_{EH}\ddot{\Delta}_E \tag{29}$$

$$\text{and}\quad M_{SAE} = (I_{SAE})_{EFF}\dot{Q}_{SAE}. \tag{30}$$

Functional Dependence:

$$\text{Let } M_{EH} = F(W, \dot{W}, U, Q, \Delta_E, \dot{\Delta}_E, M_{EHC}) \tag{31}$$

$$\text{and } M_{SAE} = F(\theta_{SAE}, Q_{SAE}, M_{SAEM}, M_{SAEC}). \tag{32}$$

Other effects will be neglected.

Assuming all changes small, and using the principle of linear superposition, we have:

$$M_{EH} = M_{EH_o} + (\frac{\partial M_{EH}}{\partial \Delta W})\Delta W + (\frac{\partial M_{EH}}{\partial \Delta \dot{W}})\Delta \dot{W} + (\frac{\partial M_{EH}}{\partial \Delta U})\Delta U + (\frac{\partial M_{EH}}{\partial Q})Q + (\frac{\partial M_{EH}}{\partial \Delta_E})\Delta E$$

$$+ (\frac{\partial M_{EH}}{\partial \dot{\Delta}_E})\dot{\Delta}_E + M_{EHC}, \tag{33}$$

$$\text{and } M_{SAE} = M_{SAE_o} + (\frac{\partial M_{SAE}}{\partial \theta_{SAE}})\theta_{SAE} + (\frac{\partial M_{SAE}}{\partial Q_{SAE}})Q_{SAE} + M_{SAEM} + M_{SAEC}. \tag{34}$$

Elevator servo trim conditions:

$$M_{EH_o} = M_{SAE_o} \equiv 0. \tag{35}$$

Also, if we neglect cable stretch, we have:

$$-M_{EHC} = M_{SAEC}(\frac{\partial M_{EHC}}{\partial M_{SAEC}}) = M_{SAEC}(\frac{\partial \theta_{SAE}}{\partial \Delta_E}), \tag{36}$$

because

$$\frac{\partial M_{EHC}}{\partial M_{SAEC}} = \frac{\partial \theta_{SAE}}{\partial \Delta_E}. \tag{37}$$

Further,

$$\Delta_E = (\frac{\partial \Delta_E}{\partial \theta_{SAE}})\theta_{SAE}, \quad \dot{\Delta}_E = (\frac{\partial \Delta_E}{\partial \theta_{SAE}})Q_{SAE}, \quad \ddot{\Delta}_E = (\frac{\partial \Delta_E}{\partial \theta_{SAE}})\dot{Q}_{SAE} \tag{38}$$

and

$$M_{SAEM} = (\frac{\partial M_{SAEM}}{\partial F_E})F_E + (\frac{\partial M_{SAEM}}{\partial \dot{F}_E})\dot{F}_E + (\frac{\partial M_{SAEM}}{\partial \int F_E dt})\int F_E dt + \ldots \tag{39}$$

Combining equations 29, 33 and 35 we have:

$$M_{EHC} = I_{EH}\ddot{\Delta}_E - (\frac{\partial M_{EH}}{\partial \dot{\Delta}_E})\dot{\Delta}_E - (\frac{\partial M_{EH}}{\partial \Delta_E})\Delta_E - (\frac{\partial M_{EH}}{\partial \Delta W})\Delta W - (\frac{\partial M_{EH}}{\partial \Delta \dot{W}})\Delta \dot{W}$$

$$- (\frac{\partial M_{EH}}{\partial \Delta U})\Delta U - (\frac{\partial M_{EH}}{\partial Q})Q. \qquad (40)$$

Combining equations 30, 34 and 35 we have:

$$M_{SAEM} = -M_{SAEC} + (I_{SAE})_{EFF}\dot{Q}_{SAE} - (\frac{\partial M_{SAE}}{\partial Q_{SAE}})Q_{SAE} - (\frac{\partial M_{SAE}}{\partial \theta_{SAE}})\theta_{SAE}. \qquad (41)$$

Putting equations 36, 38, 39 and 40 in equation 41, we have:

$$[I_{EH} + (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (I_{SAE})_{EFF}]\ddot{\Delta}_E + [-(\frac{\partial M_{EH}}{\partial \dot{\Delta}_E}) - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (\frac{\partial M_{SAE}}{\partial Q_{SAE}})]\dot{\Delta}_E$$

$$+ [-(\frac{\partial M_{EH}}{\partial \Delta_E}) - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (\frac{\partial M_{SAE}}{\partial \theta_{SAE}})]\Delta_E - (\frac{\partial M_{EH}}{\partial \Delta W})\Delta W - (\frac{\partial M_{EH}}{\partial \Delta \dot{W}})\Delta \dot{W}$$

$$- (\frac{\partial M_{EH}}{\partial \Delta U})\Delta U - (\frac{\partial M_{EH}}{\partial Q})Q = [(\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \dot{F}_E})]\dot{F}_E + [(\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial F_E})]F_E$$

$$+ [(\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \int F_E dt})]\int F_E dt + \ldots. \qquad (42)$$

We may define

$$(I_{EH})_{EFF} = I_{EH} + (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (I_{SAE})_{EFF}, \qquad (43)$$

$$(C_{EH})_{EFF} = -(\frac{\partial M_{EH}}{\partial \dot{\Delta}_E}) - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (\frac{\partial M_{SAE}}{\partial Q_{SAE}}) \qquad (44)$$

and $(K_{EH})_{EFF} = -(\frac{\partial M_{EH}}{\partial \Delta_E}) - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (\frac{\partial M_{SAE}}{\partial \theta_{SAE}})$. (45)

Note that, $-(\frac{\partial M_{SAE}}{\partial Q_{SAE}}) \equiv (C_{SAE})_{EFF}$ and

$$-(\frac{\partial M_{SAE}}{\partial \theta_{SAE}}) \equiv K_{SAE}.$$

Then equation 42 becomes:

$$[(I_{EH})_{EFF}D^2 + (C_{EH})_{EFF}D + (K_{EH})_{EFF}]\Delta_E + [-(\frac{\partial M_{EH}}{\partial \Delta \dot{W}})D - (\frac{\partial M_{EH}}{\partial \Delta W})]\Delta W$$

$$- (\frac{\partial M_{EH}}{\partial \Delta U})\Delta U - (\frac{\partial M_{EH}}{\partial Q})D\theta = [(\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \dot{F}_E})D + (\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial F_E})$$

$$+ (\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \int F_E dt})(\frac{1}{D}) + \ldots.]F_E. \quad (42\text{-a})$$

We may define

$$M_{EH_{\dot{\Delta}_E}} = -[\frac{(C_{EH})_{EFF}}{(I_{EH})_{EFF}}], \quad M_{EH_{\Delta_E}} = -[\frac{(K_{EH})_{EFF}}{(I_{EH})_{EFF}}],$$

$$M_{EH_{\Delta W}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial \Delta W}), \quad M_{EH_{\Delta \dot{W}}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial \Delta \dot{W}}),$$

$$M_{EH_{\Delta U}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial \Delta U}),$$

$$M_{EH_Q} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial Q}), \quad M_{EH_{F_E}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial F_E}),$$

$$M_{EH_{\dot{F}_E}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \dot{F}_E}) \text{ and}$$

$$M_{EH_{\int F_E dt}} = [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \int F_E dt}).$$

Then equation 42-a becomes:

$$(D^2 - M_{EH_{\dot{\Delta}_E}} D - M_{EH_{\Delta_E}})\Delta_E + [-M_{EH_{\Delta\dot{W}}} D - M_{EH_{\Delta W}}]\Delta W - M_{EH_{\Delta U}}\Delta U$$

$$-M_{EH_Q} D\theta = [M_{EH_{\dot{F}_E}} D + M_{EH_{F_E}} + M_{EH_{\int F_E dt}}(\frac{1}{D}) + \ldots] F_E. \quad (42\text{-b})$$

Non-dimensionalizing:

Let $m_{eh_{\dot{\delta}_e}} = T_A M_{EH_{\dot{\Delta}_E}}$, $m_{eh_{\delta_e}} = T_A{}^2 M_{EH_{\Delta_E}}$, $m_{eh_{\dot{w}}} = c M_{EH_{\Delta\dot{W}}}$,

$m_{eh_w} = cT_A M_{EH_{\Delta W}}$, $m_{eh_u} = cT_A M_{EH_{\Delta U}}$, $m_{eh_q} = T_A M_{EH_Q}$,

$m_{eh_{\dot{f}_e}} = T_A M_{EH_{\dot{F}_E}}$, $m_{eh_{f_e}} = T_A{}^2 M_{EH_{F_E}}$, $m_{eh_{\frac{f_e}{d}}} = T_A{}^3 M_{EH_{\frac{F_E}{D}}}$,

$\delta_e \equiv \Delta_E$, $\dot{\delta}_e = T_A \dot{\Delta}_E$, $\ddot{\delta}_e = T_A{}^2 \ddot{\Delta}_E$, $\theta \equiv \theta$, $u = \frac{\Delta U}{U_o}$,

$d = T_A D = \frac{d}{d\gamma}$, $w = \frac{\Delta W}{U_o}$, $\dot{w} = \frac{T_A}{U_o}\Delta\dot{W}$, $\mu_c = \frac{T_A U_o}{c} = \frac{U_o{}^2 C_{L_o}}{gc} = \frac{M_A}{\frac{\rho}{2}Sc}$,

$$T_A = \frac{M_A}{\frac{\rho}{2}SU_o} = \frac{U_o C_{L_o}}{g}, \quad t = T_A\gamma, \quad q = T_A Q = d\theta, \quad \int f_e d\gamma = \frac{f_e}{d},$$

$$\int F_E dt = \frac{F_E}{D}, \text{ etc.}$$

Then equation 42-b becomes:

$$[(\frac{d^2}{T_A{}^2}) - (\frac{m_{eh_{\dot{\delta}_e}}}{T_A})(\frac{d}{T_A}) - (\frac{m_{eh_{\delta_e}}}{T_A{}^2})]\delta_e + [-(\frac{m_{eh_{\dot{w}}}}{c})(\frac{d}{T_A}) - (\frac{m_{eh_w}}{cT_A})]U_o w$$

$$-(\frac{m_{eh_u}}{cT_A})U_o u - (\frac{m_{eh_q}}{T_A})(\frac{d}{T_A})\theta = [(\frac{m_{eh_{\dot{f}_e}}}{T_A})(\frac{d}{T_A}) + (\frac{m_{eh_{f_e}}}{T_A{}^2})$$

$$+ (\frac{m_{eh_{\frac{f_e}{d}}}}{T_A{}^3})(\frac{T_A}{d}) + \ldots]f_e. \tag{42-c}$$

After multiplying by $T_A{}^2$, equation 42-c becomes completely non-dimensional:

$$(-\mu_c m_{eh_u})u + (-\mu_c m_{eh_{\dot{w}}} d - \mu_c m_{eh_w})w + (-m_{eh_q} d)\theta$$

$$+ (d^2 - m_{eh_{\dot{\delta}_e}} d - m_{eh_{\delta_e}})\delta_e$$

$$- [m_{eh_{\dot{f}_e}} d + m_{eh_{f_e}} + m_{eh_{\frac{f_e}{d}}}(\frac{1}{d}) + \ldots]f_e = 0. \tag{46}$$

A-3: DERIVATION OF EQUATION OF MOTION FOR ELEVATION COMPUTER

Using Newton's Law, we equate torques about the elevation computer shaft as follows:

$$-(I_{CSE})_{EFF}\ddot{A}_{CSE} - C_{CSE}\dot{A}_{CSE} - K_{CSE}A_{CSE} = M_{AE}L_{ACSE}\Delta\dot{W} + (\frac{\partial M_{CSE}}{\partial \Delta W})\Delta W$$

$$+ U_o Q M_{AE} L_{ACSE} + (\frac{\partial M_{CSE}}{\partial M_{GE}}) H_{GE} Q + M_{AE} L_{ACSE}\, g \cos\theta = 0. \quad (47)$$

For successful operation, the damping in a computer must be very high. Thus, the inertia torque may be neglected. Then, if we divide equation 47 by $-K_{CSE}$, and define $T_{CSE} = \frac{C_{CSE}}{K_{CSE}}$, and let $P_{CE} = (\frac{\partial P_{CE}}{\partial A_{CSE}})A_{CSE}$, and let $\dot{P}_{CE} = (\frac{\partial P_{CE}}{\partial A_{CSE}})\dot{A}_{CSE}$, we have:

$$T_{CSE}\dot{P}_{CE} + P_{CE} + (\frac{\partial P_{CE}}{\partial A_{CSE}})(\frac{M_{AE}L_{ACSE}}{K_{CSE}})\Delta\dot{W} - (\frac{\partial P_{CE}}{\partial A_{CSE}})(\frac{\partial M_{CSE}}{\partial \Delta W})(\frac{1}{K_{CSE}})\Delta W$$

$$-[(\frac{1}{K_{CSE}})(\frac{\partial P_{CE}}{\partial A_{CSE}})][(\frac{\partial M_{CSE}}{\partial M_{GE}})H_{GE} + U_o M_{AE} L_{ACSE}]Q$$

$$-(\frac{1}{K_{CSE}})(\frac{\partial P_{CE}}{\partial A_{CSE}})M_{AE}L_{ACSE}\, g\cos\theta = 0, \quad (48)$$

where $L_{ACSE} = L_{AGE}(\frac{\partial M_{CSE}}{\partial M_{GE}})$.

Non-dimensionalizing:

Let $p_{ce} \equiv P_{CE}$, $\dot{p}_{ce} = T_{CSE}\dot{P}_{CE}$, $w = \frac{\Delta W}{U_o}$, $\dot{w} = (\frac{T_{CSE}}{U_o})\Delta\dot{W}$, $q = T_{CSE}Q$,

$d_{ce} = T_{CSE}D$, $D = \frac{d}{dt}$, $\theta \equiv \theta$, $\dot{\theta} \equiv T_{CSE}\dot{\theta}$

Then equation 48 becomes:

$$\dot{p}_{ce} + p_{ce} + \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}U_o}{C_{CSE}}\right)\dot{w} - \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial \Delta w}\right)\left(\frac{U_o}{K_{CSE}}\right)w$$

$$- \left[\left(\frac{1}{C_{CSE}}\right)\left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\right]\left[\left(\frac{\partial M_{CSE}}{\partial M_{GE}}\right)H_{GE} + U_o M_{AE} L_{ACSE}\right]q$$

$$- \left[\left(\frac{1}{K_{CSE}}\right)\left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)M_{AE}L_{ACSE}g\right] \cos\theta = 0. \qquad (49)$$

We may define

$$m_{ce_{\dot{w}}} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}U_o}{C_{CSE}}\right), \qquad (50)$$

$$m_{ce_q} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{1}{C_{CSE}}\right)\left[\left(\frac{\partial M_{CSE}}{\partial M_{GE}}\right)H_{GE} + U_o M_{AE} L_{ACSE}\right], \qquad (51)$$

$$m_{ce_\theta} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}g}{K_{CSE}}\right) \qquad (52)$$

and $m_{ce_w} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial \Delta W}\right)\left(\frac{U_o}{K_{CSE}}\right).$ (53)

Note: For proper prediction $\frac{\partial M_{CSE}}{\partial \Delta W}$ must be negative for positive ΔW.

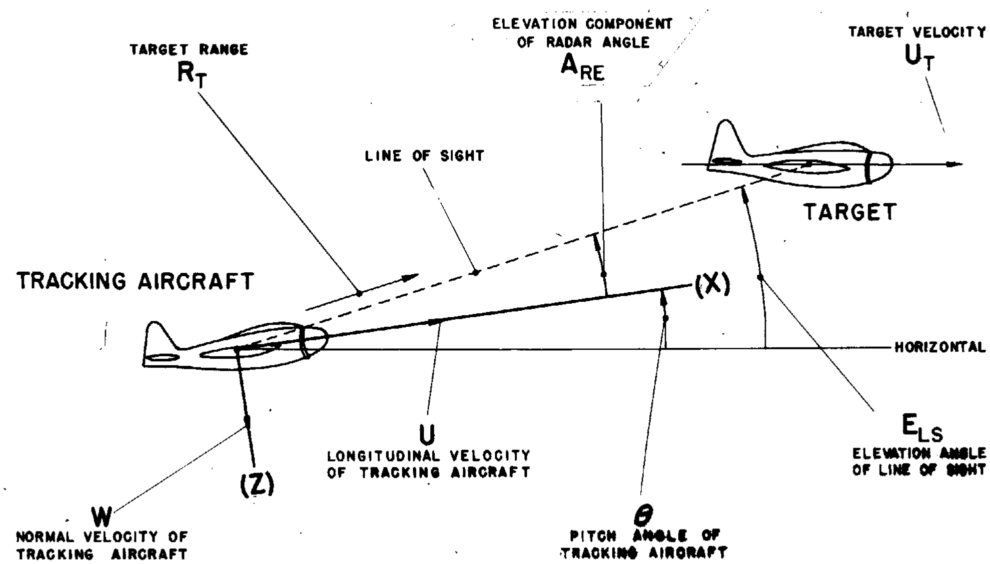
Then, equation 49 becomes:

$$(d_{ce}m_{ce_{\dot{w}}} + m_{ce_w})w - (d_{ce}m_{ce_q})\theta - m_{ce_\theta}\cos\theta + (d_{ce} + 1)p_{ce} = 0. \qquad (54)$$

And, for θ small, $\cos\theta \cong 1$. Thus, the linearized equation for the elevation computer is:

$$(d_{ce}m_{ce_{\dot{w}}} + m_{ce_w})w - (d_{ce}m_{ce_q})\theta + (d_{ce} + 1)p_{ce} - m_{ce_\theta} = 0. \quad (55)$$

A-4: DERIVATION OF ELEVATION EQUATION OF MOTION FOR IDEAL RADAR



ELEVATION TRACKING

For small angles, the effect of P is of second order and will be neglected.

Thus, for ideal radar:

$$E_{LS} = \dot{A}_{RE} + Q = A_{RE}\left[\frac{(U_o+\Delta U)}{R_T}\right] + \left(\frac{\Delta W}{R_T}\right) - \left(\frac{U_T}{R_T}\right)(A_{RE} + \theta). \quad (56)$$

When $U_T = U_{T_o} = U_o$, we have:

$$R_T \cong R_{T_o} - \int_0^t \Delta U dt \quad (57)$$

Thus, equation 56 becomes:

$$\dot{A}_{RE} - A_{RE}\left(\frac{\Delta U}{R_{T_o} - \int_0^t \Delta U \, dt}\right) - \left(\frac{\Delta W}{R_{T_o} - \int_0^t \Delta U \, dt}\right)$$

$$+ Q + \left(\frac{U_o \theta}{R_{T_o} - \int_0^t \Delta U \, dt}\right) = 0. \tag{58}$$

Non-dimensionalizing:

$$T_{AF_o} = \frac{R_{T_o}}{U_o}, \quad a_{re} \equiv A_{RE}, \quad \dot{a}_{re} = T_{AF_o}\dot{A}_{RE}, \quad q = T_{AF_o}Q,$$

$$u = \frac{\Delta U}{U_o}, \quad w = \frac{\Delta W}{U_o}, \quad t = T_{AF_o}\gamma_f, \quad dt = T_{AF_o} d\gamma_f, \quad \theta \equiv \theta,$$

and $\dot{\theta} = T_{AF_o}\dot{\theta}$.

Then equation 58 becomes:

$$\dot{a}_{re} - a_{re}\left(\frac{u}{1 - \int_0^{\gamma_f} u d\gamma_f}\right) - \left(\frac{w}{1 - \int_0^{\gamma_f} u d\gamma_f}\right) + q$$

$$+ \left(\frac{1}{1 - \int_0^{\gamma_f} u d\gamma_f}\right)\theta = 0. \tag{59}$$

We may define

$$d_f = \frac{d}{d\gamma_f}. \tag{60}$$

Then equation 59 becomes:

$$\{d_f - [\frac{u}{1 - (\frac{u}{d_f})}]\}a_{re} - [\frac{1}{1 - (\frac{u}{d_f})}]w + \{d_f + [\frac{1}{1 - (\frac{u}{d_f})}]\}\theta = 0. \quad (61)$$

Neglecting the second order term with the product ua_{re}, we have:

$$d_f a_{re} - [\frac{w}{1 - (\frac{u}{d_f})}] + \{d_f + [\frac{1}{1 - (\frac{u}{d_f})}]\}\theta = 0, \quad (62)$$

Note that

$$r_{t_o} = \frac{R_{T_o}}{R_{T_o}} \equiv 1. \quad (63)$$

B. LATERAL EQUATIONS OF MOTION:

B-1: DERIVATION OF LATERAL EQUATIONS OF MOTION FOR AIRCRAFT

Neglecting terms of second order in equations 1, 3 and 5, the basic lateral equations of motion become:

$$L = I_X \dot{P}, \quad (64)$$

$$N = I_Z \dot{R} \text{ and} \quad (65)$$

$$Y = M_A(\Delta\dot{V} + U_o R). \quad (66)$$

Functional Dependence:

$$\text{Let } L = F(V, R, P, \Delta_A), \quad (67)$$

$$N = F(V, R, P, \Delta_R) \text{ and} \quad (68)$$

$$Y = F(V, \phi, \Delta_R). \quad (69)$$

Other effects will be neglected.

Assuming that all changes are small, and using the principle of linear superposition we have:

$$L = L_o + (\frac{\partial L}{\partial \Delta V})\Delta V + (\frac{\partial L}{\partial R})R + (\frac{\partial L}{\partial P})P + (\frac{\partial L}{\partial \Delta_A})\Delta_A, \tag{70}$$

$$N = N_o + (\frac{\partial N}{\partial \Delta V})\Delta V + (\frac{\partial N}{\partial R})R + (\frac{\partial N}{\partial P})P + (\frac{\partial N}{\partial \Delta_R})\Delta_R \tag{71}$$

and $$Y = Y_o + (\frac{\partial Y}{\partial \Delta V})\Delta V + (\frac{\partial Y}{\partial \phi})\phi + (\frac{\partial Y}{\partial \Delta_R})\Delta_R. \tag{72}$$

Initial Trim conditions:

$$L_o = N_o = Y_o \equiv 0.$$

We may define:

$$L_{\Delta V} \equiv (\frac{1}{I_X})(\frac{\partial L}{\partial \Delta V}),\ N_{\Delta V} = (\frac{1}{I_Z})(\frac{\partial N}{\partial \Delta V}),\ Y_{\Delta V} = (\frac{1}{M_A})(\frac{\partial Y}{\partial \Delta V}),$$

$$L_R = (\frac{1}{I_X})(\frac{\partial L}{\partial R}),\ N_R = (\frac{1}{I_Z})(\frac{\partial N}{\partial R}),\ Y_\phi = (\frac{1}{M_A})(\frac{\partial Y}{\partial \phi}),$$

$$L_P = (\frac{1}{I_X})(\frac{\partial L}{\partial P}),\ N_P = (\frac{1}{I_Z})(\frac{\partial N}{\partial P}),\ Y_{\Delta_R} = (\frac{1}{M_A})(\frac{\partial Y}{\partial \Delta_R}),$$

$$L_{\Delta_A} = (\frac{1}{I_X})(\frac{\partial L}{\partial \Delta_A})$$

and $$N_{\Delta_R} = (\frac{1}{I_Z})(\frac{\partial N}{\partial \Delta_R}).$$

The equations of motion now become:

$$\dot{P} = L_{\Delta V}\Delta V + L_R R + L_P P + L_{\Delta_A}\Delta_A. \tag{73}$$

$$\dot{R} = N_{\Delta V}\Delta V + N_R R + N_P P + N_{\Delta R}\Delta_R, \tag{74}$$

and $$\Delta\dot{V} = Y_{\Delta V}\Delta V - U_o R + Y_\phi \phi + Y_{\Delta_R}\Delta_R. \tag{75}$$

If we define $D = \frac{d}{dt}$, then $P = D\phi$, $\dot{P} = D^2\phi$, $R = D\psi$, $\dot{R} = D^2\psi$ and

$\Delta\dot{V} = D\Delta V$.

Thus, equations 73, 74 and 75 become:

$$(-L_{\Delta V})\Delta V - (L_R D)\psi + (D^2 - L_P D)\phi - (L\Delta_A)\Delta A = 0, \tag{76}$$

$$(-N_{\Delta V})\Delta V + (D^2 - N_R D)\psi + (-N_P D)\phi - (N_{\Delta_R})\Delta_R = 0 \tag{77}$$

and $$(D-Y_{\Delta V})\Delta V + (U_o D)\psi - (Y_\phi)\phi + (-Y_{\Delta_R})\Delta_R = 0. \tag{78}$$

Non-dimensionalizing:

Let: $v = \frac{\Delta V}{U_o}$,

$\gamma = \frac{t}{T_A}$,

$l_v = T_A b L_{\Delta V}$,

$l_r = T_A L_R$,

$l_p = T_A L_P$,

$n_v = T_A b N_{\Delta V}$,

$\mu_b = \frac{M_A}{\frac{\rho}{2}Sb} = \frac{T_A U_o}{b} = \frac{U_o^2 C_{L_o}}{gb}$,

$y_v = T_A Y_{\Delta V}$,

$y_\phi = (\frac{T_A}{U_o}) Y_\phi$,

$r = T_A R = d\psi$,

$p = T_A P = d\phi$,

$$n_r = T_A N_R,$$

$$n_p = T_A N_P,$$

$$n_{\delta_r} = T_A^2 N_{\Delta_R},$$

$$d = T_A D,$$

$$T_A = \frac{U_o C_{L_o}}{g} = \frac{M_A}{\frac{\rho}{2} S U_o},$$

$$\delta_r \equiv \Delta_R,$$

$$\dot{\delta}_r \equiv T_A \dot{\Delta}_R,$$

$$\delta_a \equiv \Delta_A,$$

$$\dot{\delta}_a \equiv T_A \dot{\Delta}_A,$$

$$\psi \equiv \psi,$$

$$\dot{\psi} \equiv T_A \dot{\psi},$$

$$\phi \equiv \phi,$$

$$\dot{\phi} \equiv T_A \dot{\phi},$$

$$l_{\delta_a} = T_A^2 L_{\Delta_A} \text{ and}$$

$$y_{\delta_r} = \left(\frac{T_A}{U_o}\right) Y_{\Delta_R}.$$

Substituting into equations 76, 77 and 78, we have:

$$\left(\frac{-l_v}{T_A b}\right) U_o v + \left[\left(\frac{-l_r}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\psi + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{l_p}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\phi$$

$$- \left(\frac{l_{\delta_a}}{T_A^2}\right)\delta_a = 0, \qquad (79)$$

$$\left(\frac{-n_v}{T_A b}\right) U_o v + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{n_r}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\psi + \left[\left(\frac{-n_p}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\phi$$

$$- \left(\frac{n_{\delta_r}}{T_A^2}\right)\delta_r = 0 \qquad (80)$$

and $[(\frac{d}{T_A}) - (\frac{y_v}{T_A})]U_o v + (\frac{d}{T_A})U_o\psi + [(-y\phi)(\frac{U_o}{T_A})]\phi + [(-y_{\delta_r})(\frac{U_o}{T_A})]\delta_r = 0.$ (81)

If we multiply equations 79 and 80 by T_A^2, and multiply equation 81 by T_A/U_o, we have:

$$(-\mu_b l_v)v - (l_r d)\psi + (d^2 - l_p d)\phi - (l_{\delta_a})\delta_a = 0, \quad (82)$$

$$-(\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi - (n_{\delta_r})\delta_r = 0 \quad (83)$$

and $(d - y_v)v + d\psi - (y_\phi)\phi + (-y_{\delta_r})\delta_r = 0.$ (84)

B-2: DERIVATION OF EQUATION OF MOTION FOR RUDDER-SERVO SYSTEM:

By analogy, using equation 42, we have:

$$[I_{RH} + (\frac{\partial\theta_{SAR}}{\Delta_R})^2(I_{SAR})_{EFF}]\ddot{\Delta}_R + [-(\frac{\partial M_{RH}}{\partial\dot{\Delta}_R}) - (\frac{\partial\theta_{SAR}}{\partial\Delta_R})^2(\frac{\partial M_{SAR}}{\partial Q_{SAR}})]\dot{\Delta}_R$$

$$+ [-(\frac{\partial M_{RH}}{\partial\Delta_R}) - (\frac{\partial\theta_{SAR}}{\partial\Delta_R})^2(\frac{\partial M_{SAR}}{\partial\theta_{SAR}})]\Delta_R - (\frac{\partial M_{RH}}{\partial\Delta V})\Delta V - (\frac{\partial M_{RH}}{\partial_R})R$$

$$- (\frac{\partial M_{RH}}{\partial\Delta U})\Delta U = [(\frac{\partial\theta_{SAR}}{\partial\Delta_R})(\frac{\partial M_{SARM}}{\partial\dot{F}_D})]\dot{F}_D + [(\frac{\partial\theta_{SAR}}{\partial\Delta_R})(\frac{\partial M_{SARM}}{\partial F_D})]F_D$$

$$+ [(\frac{\partial\theta_{SAR}}{\partial\Delta_R})(\frac{\partial M_{SARM}}{\partial\int F_D dt})]\int F_D dt + \ldots . \quad (85)$$

We may define

$$(I_{RH})_{EFF} = I_{RH} + \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)^2 (I_{SAR})_{EFF}, \qquad (86)$$

$$(C_{RH})_{EFF} = -\left(\frac{\partial M_{RH}}{\partial\dot{\Delta}_R}\right) - \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)^2 \left(\frac{\partial M_{SAR}}{\partial Q_{SAR}}\right) \qquad (87)$$

and $$(K_{RH})_{EFF} = -\left(\frac{\partial M_{RH}}{\partial\Delta_R}\right) - \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)^2 \left(\frac{\partial M_{SAR}}{\partial\theta_{SAR}}\right). \qquad (88)$$

Then equation 85 becomes:

$$[(I_{RH})_{EFF}D^2 + (C_{RH})_{EFF}D + (K_{RH})_{EFF}]\Delta_R - \left(\frac{\partial M_{RH}}{\partial\Delta V}\right)\Delta V$$

$$- \left(\frac{\partial M_{RH}}{\partial\Delta U}\right)\Delta U - \left(\frac{\partial M_{RH}}{\partial R}\right)D\psi = \left[\left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial\dot{F}_D}\right)D + \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial F_D}\right)\right.$$

$$\left. + \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial\int F_D dt}\right)\left(\frac{1}{D}\right) + \ldots\right]F_D \qquad (89)$$

Note that

$$-\left(\frac{\partial M_{SAR}}{\partial Q_{SAR}}\right) \equiv (C_{SAR})_{EFF} \text{ and}$$

$$-\left(\frac{\partial M_{SAR}}{\partial\theta_{SAR}}\right) \equiv K_{SAR}.$$

We may define:

$$M_{RH\dot{\Delta}_R} = -\left[\frac{(C_{RH})_{EFF}}{(I_{RH})_{EFF}}\right], \quad M_{RH\Delta_R} = -\left[\frac{(K_{RH})_{EFF}}{(I_{RH})_{EFF}}\right],$$

$$M_{RH_{\Delta V}} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial M_{RH}}{\partial \Delta V}), \quad M_{RH_{\Delta U}} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial M_{RH}}{\partial \Delta U}),$$

$$M_{RH_R} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial M_{RH}}{\partial R}), \quad M_{RH_{F_D}} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial F_D}),$$

$$M_{RH_{\dot{F}_D}} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial \dot{F}_D}) \text{ and}$$

$$M_{RH_{\int F_D dt}} = [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial \int F_D dt}).$$

Thus, equation 89 becomes:

$$(D^2 - M_{RH_{\dot{\Delta}_R}} D - M_{RH_{\Delta_R}})\Delta_R - M_{RH_{\Delta V}}\Delta V - M_{RH_{\Delta U}}\Delta U - M_{RH_R}D\psi$$

$$= [M_{RH_{\dot{F}_D}} D + M_{RH_{F_D}} + M_{RH_{\int F_D dt}} (\frac{1}{D}) + \ldots.]F_D. \tag{90}$$

Non-dimensionalizing:

Let $m_{rh_{\dot{\delta}_r}} = T_A M_{RH_{\dot{\Delta}_R}}$,

$m_{rh_{\delta_r}} = T_A{}^2 M_{RH_{\Delta_R}}$,

$m_{rh_v} = bT_A M_{RH_{\Delta V}}$,

$m_{rh_u} = bT_A M_{RH_{\Delta U}}$,

$m_{rh_r} = T_A M_{RH_R}$,

$$\mu_b = \frac{T_A U_o}{b} = \frac{U_o{}^2 C_{L_o}}{gb} = \frac{M_A}{\frac{\rho}{2}Sb},$$

$$T_A = \frac{M_A}{\frac{\rho}{2}SU_o} = \frac{U_o C_{L_o}}{g},$$

$t = T_A\gamma$,

$r = T_A R$,

$$\int f_d d\gamma = \frac{f_d}{d},$$

$$m_{rh_{\dot{f}_d}} = T_A M_{RH_{\dot{F}_D}}\ ,$$

$$m_{rh_{f_d}} = T_A{}^2 M_{RH_{F_D}},$$

$$m_{rh_{\frac{f_d}{d}}} = T_A{}^3 M_{RH_{\frac{F_D}{D}}}\ ,$$

$$u = \frac{\Delta U}{U_o},$$

$$d = T_A D = \frac{d}{d\gamma},$$

$$v = \frac{\Delta V}{U_o},$$

$$\int F_D dt = \frac{F_D}{D},$$

$$\delta_r \equiv \Delta_R,$$

$$\dot{\delta}_r = T_A \dot{\Delta}_R,$$

$$f_d \equiv F_D,$$

$$\dot{f}_d = T_A \dot{F}_D,$$

$$\psi \equiv \psi,$$

$$\dot{\psi} = T_A \dot{\psi},\ \text{etc.}$$

Thus, equation 90 becomes:

$$\left[\left(\frac{d^2}{T_A{}^2}\right) - \left(\frac{m_{rh_{\dot{\delta}_r}}}{T_A}\right)\left(\frac{d}{T_A}\right) - \left(\frac{m_{rh_{\delta_r}}}{T_A{}^2}\right)\right]\delta_r - \left(\frac{m_{rh_v}}{bT_A}\right)U_o v$$

$$- \left(\frac{m_{rh_u}}{bT_A}\right)U_o u - \left[\left(\frac{m_{rh_r}}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\psi = \left[\left(\frac{m_{rh_{\dot{f}_d}}}{T_A}\right)\left(\frac{d}{T_A}\right) + \left(\frac{m_{rh_{f_d}}}{T_A{}^2}\right)$$

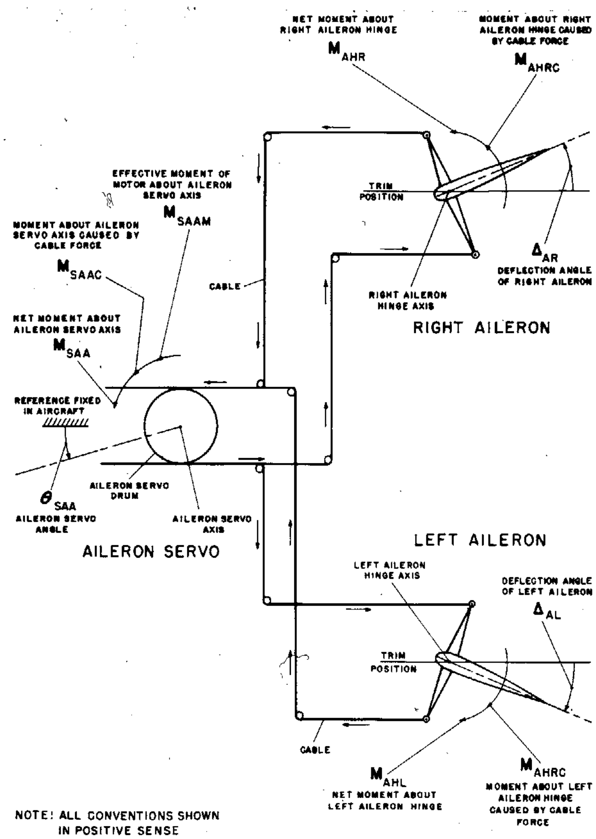
$$+ \left(\frac{m_{rh_{\frac{f_d}{d}}}}{T_A{}^3}\right)\left(\frac{T_A}{d}\right) + \ldots\right] f_d. \qquad (91)$$

Multiplying equation 91 by $T_A{}^2$ we have:

$$(-\mu_b m_{rh_u})u + (-\mu_b m_{rh_v})v - (m_{rh_r} d)\psi + (d^2 - m_{rh_{\dot{\delta}_r}} d - m_{rh_{\delta_r}})\delta_r$$

$$- [m_{rh\dot{f}_d} d + m_{rhf_d} + m_{rh\frac{f_d}{d}} (\frac{1}{d}) + \ldots] f_d = 0. \qquad (92)$$

B-3: DERIVATION OF EQUATION OF MOTION FOR AILERON-SERVO SYSTEM



SCHEMATIC SKETCH OF AILERON-SERVO SYSTEM

By definition,

$$\Delta_A = \Delta_{AR} + \Delta_{AL}, \qquad (93)$$

where $+\Delta_A$ causes right wing to go down,

and $+M_{SAA}$ causes right wing to go down.

Non-linear effects caused by differential ailerons will be neglected, i. e., it will be assumed that $\frac{\partial \Delta_{AL}}{\partial \Delta_{AR}} \cong 1$.

From Newton's Law,

$$M_{AHR} = I_{AHR}\ddot{\Delta}_{AR}, \tag{94}$$

$$M_{AHL} = I_{AHL}\ddot{\Delta}_{AL} \tag{95}$$

and $$M_{SAA} = (I_{SAA})_{EFF}\dot{Q}_{SAA}. \tag{96}$$

Functional Dependence:

Let $$M_{AHR} = F(U,\ W,\ Q,\ P,\ \Delta_{AR},\ \dot{\Delta}_{AR},\ M_{AHRC}), \tag{97}$$

$$M_{AHL} = F(U,\ W,\ Q,\ P,\ \Delta_{AL},\ \dot{\Delta}_{AL},\ M_{AHLC}), \tag{98}$$

and $$M_{SAA} = F(\theta_{SAA},\ Q_{SAA},\ M_{SAAM},\ M_{SAAC}). \tag{99}$$

Assuming all changes are small, and using the principle of linear superposition, we have:

$$M_{AHR} = M_{AHR_o} + \left(\frac{\partial M_{AHR}}{\partial \Delta U}\right)\Delta U + \left(\frac{\partial M_{AHR}}{\partial \Delta W}\right)\Delta W + \left(\frac{\partial M_{AHR}}{\partial Q}\right)Q$$

$$+ \left(\frac{\partial M_{AHR}}{\partial P}\right)P + \left(\frac{\partial M_{AHR}}{\partial \Delta_{AR}}\right)\Delta_{AR} + \left(\frac{\partial M_{AHR}}{\partial \dot{\Delta}_{AR}}\right)\dot{\Delta}_{AR} + M_{AHRC}, \tag{100}$$

$$M_{AHL} = M_{AHL_o} + \left(\frac{\partial M_{AHL}}{\partial \Delta U}\right)\Delta U + \left(\frac{\partial M_{AHL}}{\partial \Delta W}\right)\Delta W + \left(\frac{\partial M_{AHL}}{\partial Q}\right)Q$$

$$+ \left(\frac{\partial M_{AHL}}{\partial P}\right)P + \left(\frac{\partial M_{AHL}}{\partial \Delta_{AL}}\right)\Delta_{AL} + \left(\frac{\partial M_{AHL}}{\partial \dot{\Delta}_{AL}}\right)\dot{\Delta}_{AL} + M_{AHLC}, \tag{101}$$

and $$M_{SAA} = M_{SAA_o} + \left(\frac{\partial M_{SAA}}{\partial \theta_{SAA}}\right)\theta_{SAA} + \left(\frac{\partial M_{SAA}}{\partial Q_{SAA}}\right)Q_{SAA} + M_{SAAM} + M_{SAAC}. \quad (102)$$

Trim Conditions:

$$M_{AHR_o} = M_{AHL_o} = M_{SAA_o} \equiv 0. \quad (103)$$

Also, neglecting cable stretch, we have:

$$-M_{SAAC} = M_{AHRC}\left(\frac{\partial \Delta_{AR}}{\partial \theta_{SAA}}\right) + M_{AHLC}\left(\frac{\partial \Delta_{AL}}{\partial \theta_{SAA}}\right), \quad (104)$$

and $$\Delta_{AR} = \left(\frac{\partial \Delta_{AR}}{\partial \theta_{SAA}}\right)\theta_{SAA}, \; \dot{\Delta}_{AR} = \left(\frac{\partial \Delta_{AR}}{\partial \theta_{SAA}}\right)Q_{SAA}$$

and $$\ddot{\Delta}_{AR} = \left(\frac{\partial \Delta_{AR}}{\partial \theta_{SAA}}\right)\dot{Q}_{SAA}. \quad (105)$$

Analogously,

$$\Delta_{AL} = \left(\frac{\partial \Delta_{AL}}{\partial \theta_{SAA}}\right)\theta_{SAA}, \; \dot{\Delta}_{AL} = \left(\frac{\partial \Delta_{AL}}{\partial \theta_{SAA}}\right)Q_{SAA}$$

and $$\ddot{\Delta}_{AL} = \left(\frac{\partial \Delta_{AL}}{\partial \theta_{SAA}}\right)\dot{Q}_{SAA}. \quad (106)$$

Now, neglecting non-linear effects, $\frac{\partial \Delta_{AL}}{\partial \Delta_{AR}} \cong 1$, Thus, using equations 105, 106 and 93 we have:

$$\Delta_{AR} = \Delta_{AL} = \frac{\Delta_A}{2}, \; \dot{\Delta}_{AR} = \dot{\Delta}_{AL} = \frac{\dot{\Delta}_A}{2} \text{ and } \ddot{\Delta}_{AR} = \ddot{\Delta}_{AL} = \frac{\ddot{\Delta}_A}{2}. \quad (107)$$

Putting equation 103 in equations 100 and 101, and then putting equation 100 in equation 94, and equation 101 in equation 95, and then adding [equation 94 multiplied by $(\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})$] to [equation 95 multiplied by $(\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})$],

we have:

$$\left\{\frac{\partial[(\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})M_{AHR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})M_{AHL}]}{\partial\Delta U}\right\}\Delta U$$

$$+ \left\{\frac{\partial[(\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})M_{AHR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})M_{AHL}]}{\partial\Delta W}\right\}\Delta W$$

$$+ \left\{\frac{\partial[(\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})M_{AHR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})M_{AHL}]}{\partial Q}\right\}Q$$

$$+ \left\{\frac{\partial[(\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})M_{AHR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})M_{AHL}]}{\partial P}\right\}P$$

$$+ (\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})(\frac{\partial M_{AHR}}{\partial\Delta_{AR}})\Delta_{AR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})(\frac{\partial M_{AHL}}{\partial\Delta_{AL}})\Delta_{AL} + (\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})(\frac{\partial M_{AHR}}{\partial\dot{\Delta}_{AR}})\dot{\Delta}_{AR}$$

$$+ (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})(\frac{\partial M_{AHL}}{\partial\dot{\Delta}_{AL}})\dot{\Delta}_{AL} + (\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})M_{AHRC} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})M_{AHLC}$$

$$= (\frac{\partial\Delta_{AR}}{\partial\theta_{SAA}})I_{AHR}\ddot{\Delta}_{AR} + (\frac{\partial\Delta_{AL}}{\partial\theta_{SAA}})I_{AHL}\ddot{\Delta}_{AL}. \qquad (108)$$

Putting equations 103 and 104 in equation 102, and then putting equation 102 in equation 96, we have:

$$(\frac{\partial M_{SAA}}{\partial \theta_{SAA}})\theta_{SAA} + (\frac{\partial M_{SAA}}{\partial Q_{SAA}})Q_{SAA} + M_{SAAM} - M_{AHRC}(\frac{\partial \Delta_{AR}}{\partial \theta_{SAA}}) - M_{AHLC}(\frac{\partial \Delta_{AL}}{\partial \theta_{SAA}})$$

$$= (I_{SAA})_{EFF}\dot{Q}_{SAA}. \qquad (109)$$

Now, we define $M_{AH} = M_{AHR} + M_{AHL}$ (110)

$$\text{and, } M_{SAAM} = (\frac{\partial M_{SAAM}}{\partial \dot{F}_D})\dot{F}_D + (\frac{\partial M_{SAAM}}{\partial F_D})F_D + (\frac{\partial M_{SAAM}}{\partial \int F_D dt})\int F_D dt + \ldots . \qquad (111)$$

Now, putting equations 107 and 110 in equation 108, and then adding the resulting equation to equation 109, we have:

$$(\frac{1}{2})(\frac{\partial \Delta_A}{\partial \theta_{SAA}})[(\frac{\partial M_{AH}}{\partial \Delta U})\Delta U + (\frac{\partial M_{AH}}{\partial \Delta W})\Delta W + (\frac{\partial M_{AH}}{\partial Q})Q + (\frac{\partial M_{AH}}{\partial P})P$$

$$+ (\frac{\partial M_{AH}}{\partial \Delta_A})\Delta_A + (\frac{\partial M_{AH}}{\partial \dot{\Delta}_A})\dot{\Delta}_A] + (\frac{\partial M_{SAA}}{\partial \theta_{SAA}})\theta_{SAA} + (\frac{\partial M_{SAA}}{\partial Q_{SAA}})Q_{SAA}$$

$$+ M_{SAAM} = (\frac{1}{4})(\frac{\partial \Delta_A}{\partial \theta_{SAA}})(I_{AHR} + I_{AHL})\ddot{\Delta}_A + (I_{SAA})_{EFF}\dot{Q}_{SAA}. \qquad (112)$$

Putting equations 105, 106 and 107 in equation (112), and then multiplying the resulting equation by $4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})$, and rearranging, we have:

$$2[-(\frac{\partial M_{AH}}{\partial \Delta U})\Delta U - (\frac{\partial M_{AH}}{\partial \Delta W})\Delta W - (\frac{\partial M_{AH}}{\partial Q})Q - (\frac{\partial M_{AH}}{\partial P})P]$$

$$- [2(\frac{\partial M_{AH}}{\partial \Delta_A}) + 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 (\frac{\partial M_{SAA}}{\partial \theta_{SAA}})]\Delta_A - [2(\frac{\partial M_{AH}}{\partial \dot{\Delta}_A})$$

$$+ 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 (\frac{\partial M_{SAA}}{\partial Q_{SAA}})]\dot{\Delta}_A + [I_{AHR} + I_{AHL} + 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 (I_{SAA})_{EFF}]\ddot{\Delta}_A$$

$$= 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})M_{SAAM}. \qquad (113)$$

We may define

$$I_{AH} = I_{AHR} + I_{AHL}, \qquad (114)$$

$$(K_{AH})_{EFF} = -2(\frac{\partial M_{AH}}{\partial \Delta_A}) - 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 (\frac{\partial M_{SAA}}{\partial \theta_{SAA}}), \qquad (115)$$

$$(C_{AH})_{EFF} = -2(\frac{\partial M_{AH}}{\partial \dot{\Delta}_A}) - 4(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 (\frac{\partial M_{SAA}}{\partial Q_{SAA}}), \qquad (116)$$

and $$(I_{AH})_{EFF} = 4(I_{SAA})_{EFF}(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 + I_{AH}. \qquad (117)$$

Note that $-(\frac{\partial M_{SAA}}{\partial \theta_{SAA}}) = K_{SAA}$ and $-(\frac{\partial M_{SAA}}{\partial Q_{SAA}}) = (C_{SAA})_{EFF}$.

For most aircraft,

$$(\frac{\partial M_{AHR}}{\partial \Delta U})\Delta U = -(\frac{\partial M_{AHL}}{\partial \Delta U})\Delta U, \quad (\frac{\partial M_{AHR}}{\partial \Delta W})\Delta W = -(\frac{\partial M_{AHL}}{\partial \Delta W})\Delta W \quad \text{and}$$

$$(\frac{\partial M_{AHR}}{\partial Q})Q = -(\frac{\partial M_{AHL}}{\partial Q})Q.$$

$$\text{Thus, } \left(\frac{\partial M_{AH}}{\partial \Delta U}\right)\Delta U = 0, \quad \left(\frac{\partial M_{AH}}{\partial \Delta W}\right)\Delta W = 0 \text{ and } \left(\frac{\partial M_{AH}}{\partial Q}\right)Q = 0. \tag{118}$$

Putting equations 114, 115, 116, 117 and 118 in equation 113, we have:

$$[(I_{AH})_{EFF}D^2 + (C_{AH})_{EFF}D + (K_{AH})_{EFF}]\Delta_A - 2\left(\frac{\partial M_{AH}}{\partial P}\right)P$$

$$= [4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial \dot{F}_D}\right)D + 4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial F_D}\right)$$

$$+ 4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial \int F_D dt}\right)\left(\frac{1}{D}\right) + \ldots]F_D. \tag{119}$$

We may define

$$M_{AH_{\dot{\Delta}_A}} = -[\frac{(C_{AH})_{EFF}}{(I_{AH})_{EFF}}], \quad M_{AH_{\Delta_A}} = -[\frac{(K_{AH})_{EFF}}{(I_{AH})_{EFF}}],$$

$$M_{AH_P} = [\frac{2}{(I_{AH})_{EFF}}]\left(\frac{\partial M_{AH}}{\partial P}\right), \quad M_{AH_{\dot{F}_D}} = [\frac{4}{(I_{AH})_{EFF}}]\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial \dot{F}_D}\right),$$

$$M_{AH_{F_D}} = [\frac{4}{(I_{AH})_{EFF}}]\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial F_D}\right),$$

$$M_{AH_{\int F_D dt}} = [\frac{4}{(I_{AH})_{EFF}}]\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial \int F_D dt}\right), \quad \int F_D dt = \frac{F_D}{D}, \text{ and } P = D\phi.$$

Thus, equation 119 becomes:

$$-M_{AH_P}D\phi + (D^2 - M_{AH_{\dot{\Delta}_A}}D - M_{AH_{\Delta_A}})\Delta_A =$$

$$[M_{AH_{\dot{F}_D}}D + M_{AH_{F_D}} + M_{AH_{\frac{F_D}{D}}}\left(\frac{1}{D}\right) + \ldots]F_D. \tag{120}$$

Non-dimensionalizing:

$$m_{ah_p} = T_A M_{AH_P}, \qquad t = T_A \gamma,$$

$$m_{ah_{\dot{\delta}_a}} = T_A M_{AH_{\dot{\Delta}_A}}, \qquad p = T_A P = d\phi,$$

$$m_{ah_{\delta_a}} = T_A^2 M_{AH_{\Delta_A}}, \qquad \delta_a \equiv \Delta_A,$$

$$m_{ah_{\dot{f}_d}} = T_A M_{AH_{\dot{F}_D}}, \qquad \dot{\delta}_a \equiv T_A \dot{\Delta}_A,$$

$$m_{ah_{f_d}} = T_A^2 M_{AH_{F_D}}, \qquad f_d \equiv F_D,$$

$$m_{ah_{\frac{f_d}{d}}} = T_A^3 M_{AH_{\frac{F_D}{D}}}, \qquad \dot{f}_d = T_A \dot{F}_D,$$

$$d = T_A D = \frac{d}{d\gamma}, \qquad \frac{f_d}{d} = \left(\frac{1}{T_A}\right)\left(\frac{F_D}{D}\right),$$

$$\mu_b = \frac{T_A U_o}{b} = \frac{U_o^2 C_{L_o}}{gb} = \frac{M_A}{\frac{\rho}{2} Sb}, \qquad \phi \equiv \phi.$$

$$T_A = \frac{M_A}{\frac{\rho}{2} S U_o} = \frac{U_o C_{L_o}}{g}, \qquad \dot{\phi} = T_A \dot{\phi}.$$

Substituting in equation 120, we have:

$$-[(\frac{m_{ah_p}}{T_A})(\frac{d}{T_A})]\phi + [(\frac{d^2}{T_A^2}) - (\frac{m_{ah_{\dot{\delta}_a}}}{T_A})(\frac{d}{T_A}) - (\frac{m_{ah_{\delta_a}}}{T_A^2})]\delta_a$$

$$= [(\frac{m_{ah_{\dot{f}_d}}}{T_A})(\frac{d}{T_A}) + (\frac{m_{ah_{f_d}}}{T_A^2}) + (\frac{m_{ah_{\frac{f_d}{d}}}}{T_A^3})(\frac{T_A}{d}) + \ldots]f_d. \qquad (121)$$

After multiplying equation 121 by T_A^2 we have:

$$(-m_{ah_p}d)\phi + (d^2 - m_{ah_{\dot{\delta}_a}}d - m_{ah_{\delta_a}})\delta_a =$$

$$[m_{ah_{\dot{f}_d}}d + m_{ah_{f_d}} + m_{ah_{\frac{f_d}{d}}}(\frac{1}{d}) + \ldots]f_d. \qquad (122)$$

B-4: DERIVATION OF EQUATION OF MOTION FOR DEFLECTION COMPUTER

Using Newton's Law, we equate torques about the deflection computer shaft as follows:

$$-(I_{CSD})_{EFF}\ddot{A}_{CSD} - C_{CSD}\dot{A}_{CSD} - K_{CSD}A_{CSD} + (\frac{\partial M_{CSD}}{\partial \Delta V})\Delta V$$

$$+ (\frac{\partial M_{CSD}}{\partial M_{GD}})[R(\cos I_{GD}) - P(\sin I_{GD})]H_{GD} = 0. \qquad (123)$$

For successful operation, the damping in a computer must be very high. Thus, the inertia torque may be neglected. Then, if we divide equation 123 by $-K_{CSD}$, and define $T_{CSD} = \frac{C_{CSD}}{K_{CSD}}$, and let

$$P_{CD} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right) A_{CSD}, \text{ and let } \dot{P}_{CD} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\dot{A}_{CSD}, \text{ we have:}$$

$$T_{CSD}\dot{P}_{CD} + P_{CD} + \left[-\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)\left(\frac{1}{K_{CSD}}\right)\right]\Delta V$$

$$- \left(\frac{1}{K_{CSD}}\right)\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)(H_{GD})[R(\cos I_{GD}) - P(\sin I_{GD})] = 0. \quad (124)$$

Non-dimensionalizing:

Let

$\dot{p}_{cd} = T_{CSD}\dot{P}_{CD}$,

$D = \frac{d}{dt}$,

$p_{cd} \equiv P_{CD}$,

$d_{cd} = T_{CSD}D$,

$v = \frac{\Delta V}{U_o}$,

$R = \dot{\psi}$,

$r = \dot{\psi} = T_{CSD}R$,

$P = \dot{\phi}$.

$p = \dot{\phi} = T_{CSD}P$,

Then equation 124 becomes:

$$\dot{p}_{cd} + p_{cd} + \left[-\left(\frac{P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)\left(\frac{U_o}{K_{CSD}}\right)\right]v$$

$$- \left[\left(\frac{1}{C_{CSD}}\right)\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)H_{GD}\right][\dot{\psi}(\cos I_{GD}) - \dot{\phi}(\sin I_{GD})] = 0. \quad (125)$$

We may define

$$m_{cd_v} = -\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)\left(\frac{U_o}{K_{CSD}}\right), \quad (126)$$

[for proper prediction $\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)$ must be negative for positive ΔV]

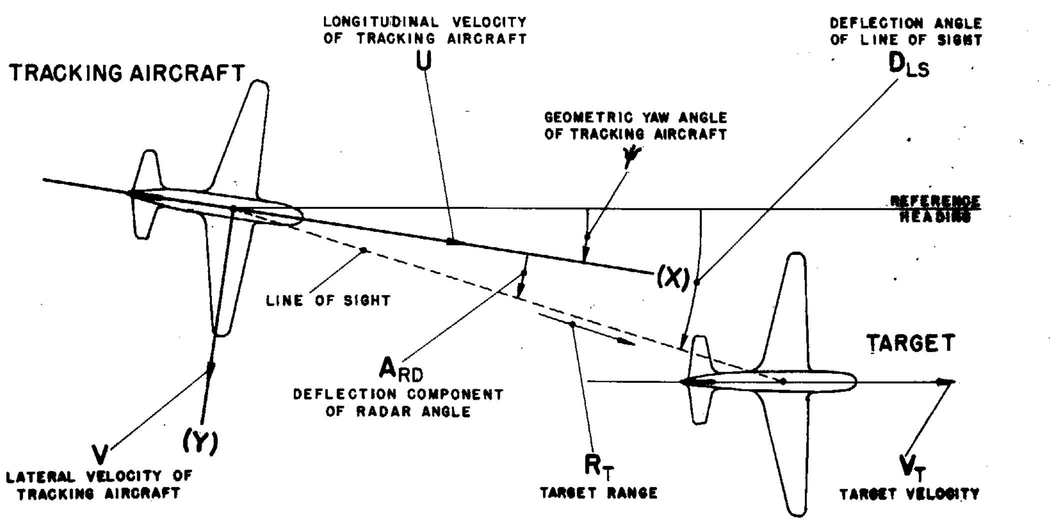
$$m_{cd_r} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{1}{C_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)(H_{GD})(\cos I_{GD}) \tag{127}$$

and
$$m_{cd_p} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{1}{C_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)(H_{GD})(\sin I_{GD}). \tag{128}$$

Then equation 125 becomes:

$$(m_{cd_v})v - (d_{cd}m_{cd_r})\psi + (d_{cd}m_{cd_p})\phi + (d_{cd} + 1)p_{cd} = 0. \tag{129}$$

B-5: DERIVATION OF DEFLECTION EQUATION OF MOTION FOR IDEAL RADAR



DEFLECTION TRACKING

For small angles, the effect of P is of second order and will be neglected.

Thus, for ideal radar:

$$\dot{D}_{LS} = \dot{A}_{RD} + R = A_{RD}\left[\frac{(U_o + \Delta U)}{R_T}\right] - \left(\frac{\Delta V}{R_T}\right)$$

$$- \left(\frac{U_T}{R_T}\right)(A_{RD} + \psi). \tag{130}$$

Equation 130 is analogous to equation 56.

When $U_T = U_{T_o} = U_o$, we have:

$$R_T \cong R_{T_o} - \int_0^t \Delta U dt. \tag{57}$$

Thus, equation 130 becomes:

$$\dot{A}_{RD} - A_{RD}\left(\frac{\Delta U}{R_{T_o} - \int_0^t \Delta U dt}\right) + \left(\frac{\Delta V}{R_{T_o} - \int_0^t \Delta U dt}\right)$$

$$+ R + \left(\frac{U_o \psi}{R_{T_o} - \int_0^t \Delta U dt}\right) = 0. \tag{131}$$

Non-dimensionalizing:

Let

$$u = \frac{\Delta U}{U_o},$$

$$T_{AF_o} = \frac{R_{T_o}}{U_o}, \qquad v = \frac{\Delta V}{U_o},$$

$$a_{rd} \equiv A_{RD}, \qquad t = T_{AF_o}\gamma_f,$$

$$\dot{a}_{rd} = T_{AF_o}\dot{A}_{RD}, \qquad dt = T_{AF_o}d\gamma_f,$$

$$\dot{\psi} = T_{AF_o}R, = T_{AF_o}\dot{\Psi} \qquad \psi \equiv \psi.$$

Then equation 131 becomes:

$$\dot{a}_{rd} - a_{rd}\left(\frac{u}{1 - \int_0^{\gamma_f} ud\gamma_f}\right) + \left(\frac{v}{1 - \int_0^{\gamma_f} ud\gamma_f}\right)$$

$$+ \dot{\psi} + \left(\frac{\psi}{1 - \int_0^{\gamma_f} ud\gamma_f}\right) = 0. \tag{132}$$

If we define $d_f = \frac{d}{d\gamma_f}$, then equation 132 becomes:

$$\{d_f - [\frac{u}{1 - (\frac{u}{d_f})}]\}a_{rd} + [\frac{1}{1 - (\frac{u}{d_f})}]v$$

$$+ \{d_f + [\frac{1}{1 - (\frac{u}{d_f})}]\}\psi = 0. \tag{133}$$

Neglecting the second order term with the product ua_{rd}, we have:

$$d_f a_{rd} + [\frac{v}{1 - (\frac{u}{d_f})}] + \{d_f + [\frac{1}{1 - (\frac{u}{d_f})}]\}\psi = 0. \tag{134}$$

C. SUMMARY OF EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

	Longitudinal Equations of Motion	Equation Reference No.
AIRCRAFT LONGITUDINAL EQUATIONS OF MOTION	$(-d + x_u)u + (x_w)w + (x_\theta)\theta = 0$	(26)
	$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta + (z_{\delta_e})\delta_e = 0$	(27)
	$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta + (m_{\delta_e})\delta_e = 0$	(28)
ELEVATOR-SERVO SYSTEM EQUATION OF MOTION	$(-\mu_c m_{eh_u})u + (-\mu_c m_{eh_{\dot{w}}} d - \mu_c m_{eh_w})w + (-m_{eh_q} d)\theta$ $+ (d^2 - m_{eh_{\dot{\delta}_e}} d - m_{eh_{\delta_e}})\delta_e$ $= [m_{eh_{\dot{f}_e}} d + m_{eh_{f_e}} + m_{eh_{\frac{f_e}{d}}}(\frac{1}{d}) + \dots]f_e$	(46)
ELEVATION COMPUTER EQUATION OF MOTION	$(d_{ce} m_{ce_{\dot{w}}} + m_{ce_w})w - (d_{ce} m_{ce_q})\theta + (d_{ce} + 1)p_{ce} - m_{ce_\theta} = 0$	(55)

IDEAL RADAR ELEVATION EQUATION OF MOTION

$$d_f a_{re} - \left[\frac{w}{1 - (\frac{u}{d_f})}\right] + \left\{d_f + \left[\frac{1}{1 - (\frac{u}{d_f})}\right]\right\}\theta = 0 \qquad (62)$$

Lateral Equations of Motion

AIRCRAFT LATERAL EQUATIONS OF MOTION

$$(-\mu_b l_v)v - (l_r d)\psi + (d^2 - l_p d)\phi - (l_{\delta_a})\delta_a = 0 \qquad (82)$$

$$-(\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi - (n_{\delta_r})\delta_r = 0 \qquad (83)$$

$$(d - y_v)v + d\psi - (y_\phi)\phi + (-y_{\delta_r})\delta_r = 0 \qquad (84)$$

RUDDER-SERVO SYSTEM EQUATION OF MOTION

$$(-\mu_b m_{rh_u})u + (-\mu_b m_{rh_v})v - (m_{rh_r} d)\psi$$

$$+ (d^2 - m_{rh_{\dot{\delta}_r}} d - m_{rh_{\delta_r}})\delta_r$$

$$= [m_{rh_{\dot{f}_d}} d + m_{rh_{f_d}} + m_{rh_{\frac{f_d}{d}}}(\frac{1}{d}) + \ldots]f_d \qquad (92)$$

AILERON-SERVO SYSTEM EQUATION OF MOTION

$$(-m_{ah_p} d)\phi + (d^2 - m_{ah_{\dot{\delta}_a}} d - m_{ah_{\delta_a}})\delta_a = [m_{ah_{\dot{f}_d}} d + m_{ah_{f_d}} + m_{ah_{\frac{f_d}{d}}}(\frac{1}{d}) + \dots]f_d \qquad (122)$$

DEFLECTION COMPUTER EQUATION OF MOTION

$$(m_{cd_v})v - (d_{cd}m_{cd_r})\psi + (d_{cd}m_{cd_p})\phi + (d_{cd} + 1)p_{cd} = 0 \qquad (129)$$

IDEAL RADAR DEFLECTION EQUATION OF MOTION

$$d_f a_{rd} + [\frac{v}{1 - (\frac{u}{d_f})}] + \{d_f + [\frac{1}{1 - (\frac{u}{d_f})}]\}\psi = 0 \qquad (134)$$

APPENDIX B

DERIVATION OF FORMULAE FOR STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

APPENDIX B

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APPENDIX B

DERIVATION OF FORMULAE FOR STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

A. LONGITUDINAL FORMULAE:

A-1: DERIVATION OF FORMULAE FOR LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT:

(a) (Change in Longitudinal Velocity - Change in Longitudinal Force) Stability Coefficient for Aircraft, x_u:

From Section A-1, Appendix A,

$$x_u = T_A X_{\Delta U} = \left(\frac{T_A}{M_A}\right)\left(\frac{\partial X}{\partial \Delta U}\right). \tag{1}$$

$$T_A = \frac{M_A}{\frac{\rho}{2} S U_o} \text{ and } \mu_c = \frac{T_A U_o}{c}, \tag{2}$$

$$X \cong T_X - D_X - M_A g \sin\theta.$$

1) For power off, $T_X \cong 0$ and $D_X \cong D$, where

$$D = \left(\frac{1}{2}\right)\rho U^2 C_D S.$$

Thus, $X = -\left(\frac{1}{2}\right)\rho U^2 C_D S - M_A g \sin\theta.$

$$U = U_o + \Delta U.$$

Thus, $\frac{\partial X}{\partial \Delta U} = -\rho C_D S U.$

So, $x_u = \left(\frac{M_A}{M_A \frac{\rho}{2} S U_o}\right)(-\rho C_D S U) = -2C_D\left(\frac{U}{U_o}\right)$

For $\frac{U}{U_o} \cong 1$, and $C_D \cong C_{D_o}$, we have:

$x_u = -2C_{D_o}$, power off. (3)

2) For power on, T varies inversely with U.

$D_X \cong D = (\frac{1}{2})\rho U^2 C_D S$, and $T_X \cong T$.

If we assume that changes in η_p and BHP are negligible for small oscillations, then $T_o U_o = TU$, or $T = \frac{T_o U_o}{U}$. Thus,

$$\frac{\partial X}{\partial \Delta U} \cong \frac{\partial T}{\partial \Delta U} - \frac{\partial D}{\partial \Delta U}, \text{ where}$$

$$\frac{\partial T}{\partial \Delta U} = -\frac{T_o U_o}{U^2} \text{ and}$$

$$\frac{\partial D}{\partial \Delta U} = \rho C_D S U.$$

Also,

$T_o \cong D_o = (\frac{1}{2})\rho C_{D_o} S U_o^2$, for $\theta_o = 0$.

Thus, $$\frac{\partial X}{\partial \Delta U} = (\frac{1}{2})\rho C_{D_o} S U_o^2 (\frac{U_o}{U^2}) - \rho C_D S U$$

and $$x_u = \frac{M_A}{M_A \frac{\rho}{2} S U_o} [-(\frac{1}{2})\rho C_{D_o} S (\frac{U_o^3}{U^2}) - \rho C_D S U],$$

or $$x_u = -C_{D_o}(\frac{U_o}{U})^2 - 2C_D(\frac{U}{U_o}).$$

For $\frac{U}{U_o} \cong 1$, and $C_D \cong C_{D_o}$, we have:

$x_u = -3C_{D_o}$, power on. (4)

(b) (Change in Normal Velocity - Change in Longitudinal Force)

Stability Coefficient for Aircraft, x_w:

From section A-1, Appendix A,

$$x_w = T_A X_{\Delta W} = \left(\frac{T_A}{M_A}\right)\left(\frac{\partial X}{\partial \Delta W}\right) \tag{5}$$

$X \cong T_X - D_X - M_A g \sin\theta$, and $D_X = -\left(\frac{\rho}{2}\right) C_X S U^2.$

Thus, $\frac{\partial X}{\partial \Delta W} = \frac{\partial T_X}{\partial \Delta W} - \frac{\partial D_X}{\partial \Delta W}.$

If we assume $\frac{\partial T_X}{\partial \Delta W}$ is negligible for ΔW small, then $\frac{\partial X}{\partial \Delta W} = -\frac{\partial D_X}{\partial \Delta W}$. But

$$\frac{\partial D_X}{\partial \Delta W} = -\left(\frac{1}{2}\right)\rho S U^2\left(\frac{\partial C_X}{\partial \Delta W}\right)$$

because $\frac{\partial U}{\partial \Delta W} \equiv 0$. Also,

$\Delta\alpha \cong \frac{\Delta W}{U}$ for ΔW small, so $\frac{\partial \Delta W}{\partial \Delta\alpha} = U$.

Then $\frac{\partial X}{\partial \Delta W} = \left(\frac{1}{2}\right)\rho S U\left(\frac{\partial C_X}{\partial \Delta\alpha}\right)$. Further,

$C_X = -C_D \cos\alpha + C_L \sin\alpha$, where

$\alpha = \alpha_0 + \Delta\alpha$. Thus,

$$\frac{\partial C_X}{\partial \alpha} = \frac{\partial C_X}{\partial \Delta\alpha} = -\left(\frac{\partial C_D}{\partial \alpha}\right)\cos\alpha + C_D \sin\alpha + \left(\frac{\partial C_L}{\partial \alpha}\right)\sin\alpha + C_L \cos\alpha.$$

But $\cos \alpha \cong 1$, and $\sin \alpha \cong 0$, for ΔW small.

Thus, $\frac{\partial C_X}{\partial \alpha} = -(\frac{\partial C_D}{\partial \alpha}) + C_L$.

$$C_D = C_{D_{p_{min}}} + \frac{C_L^2}{e\pi R}, \text{ so}$$

$$\frac{\partial C_D}{\partial \alpha} = (\frac{2}{e\pi R})\ C_L\ (\frac{\partial C_L}{\partial \alpha}).$$

Thus,

$$x_w = [\frac{M_A}{M_A \frac{\rho}{2} S U_o}]\ [\frac{\rho}{2} S U]\ [-(\frac{2}{e\pi R}) C_L (\frac{\partial C_L}{\partial \alpha}) + C_L],$$

$$= (\frac{U}{U_o}) C_L [1 - (\frac{2}{e\pi R})(\frac{\partial C_L}{\partial \alpha})].$$

For $\frac{U}{U_o} \cong 1$, and $C_L \cong C_{L_o}$ we have:

$$x_w = C_{L_o} [1 - (\frac{2}{e\pi R})(\frac{\partial C_L}{\partial \alpha})]. \qquad (6)$$

(c) (Change in Pitch Angle - Change in Longitudinal Force) Stability Coefficient for Aircraft, x_θ:

From section A-1, Appendix A,

$$x_\theta = (\frac{T_A}{U_o}) X_\theta = (\frac{T_A}{U_o})(\frac{1}{M_A})(\frac{\partial X}{\partial \theta}). \qquad (7)$$

$$X \cong T_X - D_X - M_A g \sin \theta.$$

$$\frac{\partial T_X}{\partial\theta} = 0, \text{ and } \frac{\partial D_X}{\partial\theta} = 0. \quad \text{Thus,}$$

$$\frac{\partial X}{\partial\theta} = -M_A g \cos\theta. \quad \text{But } M_A g = W_A = (\frac{1}{2})\rho U_o^2 C_{L_o} S.$$

Thus,

$$x_\theta = (\frac{M_A}{\frac{\rho}{2} S U_o{}^2})(\frac{1}{M_A})[-(\frac{\rho}{2})U_o^2 C_{L_o} S \cos\theta] = -C_{L_o} \cos\theta.$$

For θ small, $\cos\theta \cong 1$. Thus,

$$x_\theta = -C_{L_o}. \tag{8}$$

(d) (Change in Longitudinal Velocity - Change in Normal Force) Stability Coefficient for Aircraft, z_u:

From Section A-1, Appendix A,

$$z_u = T_A Z_{\Delta U} = (\frac{T_A}{M_A})(\frac{\partial Z}{\partial \Delta U}). \tag{9}$$

$$Z = C_Z S \frac{\rho}{2} U^2 + W_A \cos\theta, \text{ and } C_Z \cong -C_L. \quad \text{Thus,}$$

$$Z = -C_L S \frac{\rho}{2} U^2 + W_A \cos\theta, \text{ where } U = U_o + \Delta U.$$

$$\frac{\partial Z}{\partial \Delta U} = \frac{\partial Z}{\partial U} = -(C_L S \frac{\rho}{2}) 2U. \quad \text{Thus,}$$

$$z_u = (\frac{M_A}{M_A \frac{\rho}{2} S U_o})(-C_L S \frac{\rho}{2}) 2U = -2C_L (\frac{U}{U_o}).$$

For $C_L \cong C_{L_o}$ and $\frac{U}{U_o} \cong 1$, we have:

$$z_u = -2C_{L_o}. \tag{10}$$

(e) (Change in Normal Velocity - Change in Normal Force) Stability Coefficient for Aircraft, z_w:

From Section A-1, Appendix A,

$$z_w = T_A Z_{\Delta W} = \left(\frac{T_A}{M_A}\right)\left(\frac{\partial Z}{\partial \Delta W}\right). \tag{11}$$

$Z = C_Z S\frac{\rho}{2}U^2 + W_A \cos\theta$, and $C_Z \cong -C_L$. Thus,

$Z = -C_L S\frac{\rho}{2}U^2 + W_A \cos\theta$, where $U = U_o + \Delta U$,

Thus,

$\frac{\partial Z}{\partial \Delta W} = -S\frac{\rho}{2}U^2\left(\frac{\partial C_L}{\partial \Delta W}\right)$, because $\frac{\partial U}{\partial \Delta W} = 0$. Also,

$\frac{\partial Z}{\partial \Delta W} = -S\frac{\rho}{2}U^2\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial \Delta W}\right)$ and $\Delta W \cong U\Delta\alpha$:

$\alpha = \alpha_o + \Delta\alpha$, so $\frac{\partial \alpha}{\partial \Delta W} = \frac{\partial \Delta\alpha}{\partial \Delta W} = \frac{1}{U}$. Thus,

$\frac{\partial Z}{\partial \Delta W} = -S\frac{\rho}{2}U\left(\frac{\partial C_L}{\partial \alpha}\right)$ and

$$z_w = \left(\frac{M_A}{M_A\frac{\rho}{2}SU_o}\right)\left[-S\frac{\rho}{2}U\left(\frac{\partial C_L}{\partial \alpha}\right)\right] = -\left(\frac{U}{U_o}\right)\left(\frac{\partial C_L}{\partial \alpha}\right).$$

For $\frac{U}{U_o} \cong 1$,

$$z_w = -\left(\frac{\partial C_L}{\partial \alpha}\right). \tag{12}$$

(f) (Change in Pitch Angle - Change in Normal Force) Stability Coefficient for Aircraft, z_θ:

From Section A-1, Appendix A,

$$z_\theta = \left(\frac{T_A}{U_o}\right) Z_\theta = \left(\frac{T_A}{U_o}\right)\left(\frac{1}{M_A}\right)\left(\frac{\partial Z}{\partial \theta}\right). \tag{13}$$

$Z \cong W_A \cos\theta - L_Z$, where $L_Z \cong W_A$. Thus,

$Z \cong W_A(\cos\theta - 1)$.

$\frac{\partial Z}{\partial \theta} = -W_A \sin\theta$, where $W_A = M_A g \cong L$

$= \left(\frac{1}{2}\right)\rho U^2 C_L S$.

$z_\theta = \left(\frac{M_A}{\frac{\rho}{2} S U_o^2}\right)\left(\frac{1}{M_A}\right)\left[-\left(\frac{\rho}{2}\right)U^2 C_L S \sin\theta\right], =$

$= -\left(\frac{U}{U_o}\right)^2 C_L \sin\theta$.

For $\theta_o = 0$, $\sin\theta \cong \sin\theta_o = 0$. Thus,

$$z_\theta = 0 \text{ for } \theta_o = 0. \tag{14}$$

(g) (Change in Longitudinal Velocity - Change in Pitching Moment) Stability Coefficient for Aircraft, m_u:

From Section A-1, Appendix A,

$$m_u = cT_A M_{\Delta U} = cT_A\left(\frac{1}{I_Y}\right)\left(\frac{\partial M}{\partial \Delta U}\right). \tag{15}$$

$\frac{\partial M}{\partial \Delta U}$ depends on:

1. Change in thrust with change in speed of aircraft, when thrust line does not go through the center of gravity of the aircraft,
2. Change in downwash from wing, with change in speed of aircraft,
3. Change in equilibrium position of elevator, when elevator is not statically balanced (free controls), etc.

These derivations will not be included, since m_u will be considered negligible in the analyses which follow.

(h) (Change in Normal Velocity - Change in Pitching Moment) Stability Coefficient for Aircraft, m_w:

From Section A-1, Appendix A,

$$m_w = cT_A M_{\Delta W} = cT_A \left(\frac{1}{I_Y}\right)\left(\frac{\partial M}{\partial \Delta W}\right), \tag{16}$$

$M = C_m c S \frac{\rho}{2} U^2$. Thus,

$$\frac{\partial M}{\partial \Delta W} = cS\frac{\rho}{2}U^2\left(\frac{\partial C_m}{\partial \Delta W}\right), \text{ because } \frac{\partial U}{\partial \Delta W} = 0.$$

$$\frac{\partial C_m}{\partial \Delta W} = \left(\frac{\partial C_m}{\partial \alpha}\right)\left(\frac{\partial \alpha}{\partial \Delta W}\right), \text{ and } \Delta W \cong U\Delta\alpha.$$

$\alpha = \alpha_0 + \Delta\alpha$, so $\dfrac{\partial\alpha}{\partial\Delta W} = \dfrac{\partial\Delta\alpha}{\partial\Delta W} = \dfrac{1}{U}$. Thus

$$m_W = c\left(\frac{M_A}{\frac{\rho}{2}SU_0}\right)\left(\frac{1}{I_Y}\right)\left(cS\frac{\rho}{2}U^2\right)\left(\frac{\partial C_m}{\partial\alpha}\right)\left(\frac{1}{U}\right),$$

$$= \left(\frac{U}{U_0}\right)\left(\frac{c^2M_A}{I_Y}\right)\left(\frac{\partial C_m}{\partial\alpha}\right).$$

For $\dfrac{U}{U_0} \cong 1$,

$$m_W = \left(\frac{M_Ac^2}{I_Y}\right)\left(\frac{\partial C_m}{\partial\alpha}\right). \tag{17}$$

(i) (Lag of Wing Downwash - Change in Pitching Moment) Stability Coefficient for Aircraft, $m_{\dot{W}}$:

From Section A-1, Appendix A,

$$m_{\dot{W}} = cM_{\Delta\dot{W}} = \left(\frac{c}{I_Y}\right)\left(\frac{\partial M}{\partial\Delta\dot{W}}\right). \tag{18}$$

$$M = M' = \left(\frac{\partial C_L}{\partial\alpha}\right)'(\Delta\alpha')\left(\frac{\rho}{2}\right)S'\eta' lU^2, \text{ so}$$

$$\frac{\partial M}{\partial\Delta\dot{W}} = \left(\frac{\partial C_L}{\partial\alpha}\right)'\left(\frac{\rho}{2}\right)S'\eta' lU^2\left(\frac{\partial\Delta\alpha'}{\partial\Delta\dot{W}}\right). \text{ Also}$$

$\alpha' = \alpha - \epsilon + \alpha_0'$. Thus,

$\Delta\alpha' = \Delta\alpha - \Delta\epsilon$, for α_0' = constant.

Now $\Delta\alpha$ due to $\Delta\dot{W} = 0$ at $t = 0$, so $\Delta\alpha'$ due to $\Delta\dot{W}$ is equal to $-\Delta\epsilon$ due to $\Delta\dot{W}$. Thus,

$$\Delta\epsilon_{\left[\substack{\text{due to } \Delta\dot{W} \\ \text{at } t = 0}\right]} = \left(\frac{\partial\epsilon}{\partial\alpha}\right)\left(\Delta\alpha \text{ which occured during time } \frac{l}{U}\right),$$

$$= \frac{\partial\epsilon}{\partial\alpha}\left(\frac{l\Delta\dot{W}}{U^2}\right).$$

so,

$$m_{\dot{w}} = -c(\frac{1}{I_Y})(\frac{\partial C_L}{\partial \alpha})'(\frac{\rho}{2})S'\eta' 1U^2(\frac{\partial \epsilon}{\partial \alpha})(\frac{1}{U^2}), \text{ or}$$

$$m_{\dot{w}} = -(\frac{\partial C_L}{\partial \alpha})' (\frac{\partial \epsilon}{\partial \alpha})(\frac{\frac{\rho}{2}S' 1^2 c\eta'}{I_Y}) \tag{19}$$

(j) (Change in Rate of Pitch - Change in Pitching Moment)

Stability Coefficient for Aircraft, m_q:

From Section A-1, Appendix A,

$$m_q = T_A M_Q = (\frac{T_A}{I_Y})(\frac{\partial M}{\partial Q}) \tag{20}$$

$$M = M' = -C_L' 1S'\frac{\rho}{2}U^2\eta' \text{ and } C_L' = (\frac{\partial C_L}{\partial \alpha})' (\frac{\partial \alpha'}{\partial Q})Q, \text{ so}$$

$$(\frac{\partial M}{\partial Q}) = -1S'\frac{\rho}{2}\eta' U^2(\frac{\partial C_L}{\partial \alpha})' (\frac{\partial \alpha'}{\partial Q}).$$

$$\alpha' = \frac{1Q}{U}, \text{ so } \frac{\partial \alpha'}{\partial Q} = \frac{1}{U}.$$

Thus,

$$m_q = [\frac{M_A}{I_Y\frac{\rho}{2}SU_o}] [-1S'\frac{\rho}{2}\eta' U^2(\frac{\partial C_L}{\partial \alpha})' (\frac{1}{U})],$$

$$= -(\frac{U}{U_o})(\frac{\partial C_L}{\partial \alpha})' (\frac{S'}{S})(\frac{1^2 M_A}{I_Y})\eta'.$$

For $\frac{U}{U_o} \cong 1$, and using a factor of $\frac{5}{4}$ to allow for fuselage, wing and nacelle damping, we have:

$$m_q = -\left(\frac{5}{4}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)'\left(\frac{S'}{S}\right)\left(\frac{\eta' M_A l^2}{I_Y}\right). \qquad (21)$$

(k) (Change in Elevator Angle - Change in Pitching Moment) Stability Coefficient for Aircraft, m_{δ_e}:

From Section A-1, Appendix A,

$$m_{\delta_e} = T_A{}^2 M_{\Delta_E} = \left(\frac{T_A{}^2}{I_Y}\right)\left(\frac{\partial M}{\partial \Delta_E}\right). \qquad (22)$$

$$M = M_o + \left(\frac{\partial C_m}{\partial \Delta_E}\right)\Delta_E c S \frac{\rho}{2} U^2, \text{ where}$$

M_o = moment when $\Delta_E = 0$,

= 0, for Δ_E measured from trim position of elevator.

Thus,

$$\frac{\partial M}{\partial \Delta_E} = \left(\frac{\partial C_m}{\partial \Delta_E}\right) c S \frac{\rho}{2} U^2. \quad \text{Also } \mu_c = \frac{M_A}{\frac{\rho}{2} S c}$$

and $T_A = \dfrac{M_A}{\frac{\rho}{2} S U_o}$.

so,

$$m_{\delta_e} = \left(\frac{T_A{}^2}{I_Y}\right) c S \frac{\rho}{2} U_o{}^2 \left(\frac{\partial C_m}{\partial \Delta_E}\right). \qquad (23)$$

or,

$$m_{\delta_e} = \mu_c^2 \left(\frac{\frac{\rho}{2} S c^3}{I_Y}\right)\left(\frac{\partial C_m}{\partial \Delta_E}\right). \quad (24)$$

(l) (Change in Elevator Angle - Change in Normal Force) Stability Coefficient for Aircraft, z_{δ_e}:

From Section A-1, Appendix A,

$$z_{\delta_e} = T_A Z_{\Delta_E}\left(\frac{1}{U_o}\right) = \left(\frac{T_A}{U_o}\right)\left(\frac{1}{M_A}\right)\frac{\partial Z}{\partial \Delta_E}. \quad (25)$$

$$Z = \frac{M}{l} = \left(\frac{\partial C_m}{\partial \Delta_E}\right)\Delta_E\left(\frac{c}{l}\right)\left(\frac{\rho}{2}\right)U^2 S.$$

$$\text{Thus, } z_{\delta_e} = \left(\frac{M_A}{\frac{\rho}{2} S U_o^2}\right)\left(\frac{1}{M_A}\right)\left(\frac{\partial C_m}{\partial \Delta_E}\right)\left(\frac{c}{l}\right)\left(\frac{\rho}{2}\right)U^2 S,$$

$$= \left(\frac{U}{U_o}\right)^2\left(\frac{\partial C_M}{\partial \Delta_E}\right)\left(\frac{c}{l}\right).$$

For $\frac{U}{U_o} \cong 1$, we have:

$$z_{\delta_e} = \left(\frac{\partial C_m}{\partial \Delta_E}\right)\left(\frac{c}{l}\right). \quad (26)$$

A-2: DERIVATION OF FORMULAE FOR ELEVATOR HINGE MOMENT STABILITY COEFFICIENTS FOR ELEVATOR-SERVO SYSTEM.

(a) (Change in Longitudinal Velocity - Change in Elevator Hinge Moment) Stability Coefficient For Elevator-Servo System, m_{eh_u}:

From Section A-2, Appendix A,

$$m_{eh_u} = cT_A M_{EH_{\Delta U}} = cT_A [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial \Delta U}). \qquad (27)$$

$\frac{\partial M_{EH}}{\partial \Delta U}$ depends on:

1. Change in downwash from wing, with change in speed of aircraft,
2. Initial trim position of elevator,
3. Change in equilibrium of elevator when it is not statically balanced (free controls), etc.

These derivations will not be included, since m_{eh_u} will be considered negligible in the analyses which follow.

(b) (Change in Normal Velocity - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, m_{eh_w}:

From Section A-2, Appendix A,

$$m_{eh_w} = cT_A M_{EH_{\Delta W}} = cT_A [\frac{1}{(I_{EH})_{EFF}}](\frac{\partial M_{EH}}{\partial \Delta W}). \qquad (28)$$

$$M_{EH} = (\frac{\partial C_{EH}}{\partial \alpha'})(\Delta\alpha')c_{EAHL}S_{EAHL}\frac{\rho}{2}U^2, \text{ and}$$

$$\Delta\alpha' = \frac{\Delta W}{U}. \quad \text{Thus, } \frac{\partial M_{EH}}{\partial \Delta W} = (\frac{\partial C_{EH}}{\partial \alpha'})c_{EAHL}S_{EAHL}\frac{\rho}{2}U.$$

so, for $\frac{U}{U_o} \cong 1$,

$$m_{eh_w} = \left(\frac{S_{EAHL}}{S}\right)\left[\frac{cc_{EAHL}M_A}{(I_{EH})_{EFF}}\right]\left(\frac{\partial C_{EH}}{\partial \alpha'}\right). \qquad (29)$$

(c) (Lag of Wing Downwash - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{\dot{w}}}$:

From Section A-2, Appendix A,

$$m_{eh_{\dot{w}}} = cM_{EH_{\Delta\dot{W}}} = \left[\frac{c}{(I_{EH})_{EFF}}\right]\left(\frac{\partial M_{EH}}{\partial \Delta\dot{W}}\right) \qquad (30)$$

By analogy, using equation 19, we see that

$$m_{eh_{\dot{w}}} = \left(\frac{\partial C_{EH}}{\partial \alpha'}\right)\left(\frac{\partial \epsilon}{\partial \alpha}\right)\left[\frac{\frac{\rho}{2}S_{EAHL}l c_{EAHL}c}{(I_{EH})_{EFF}}\right]. \qquad (31)$$

Note: $\frac{\partial C_{EH}}{\partial \alpha'}$ is negative for positive α'. η' is included in the definition of C_{EH} so it is not needed in equation 31.

(d) (Change in Rate of Pitch - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, m_{eh_q}:

From Section A-2, Appendix A,

$$m_{eh_q} = T_A M_{EH_Q} = T_A\left[\frac{1}{(I_{EH})_{EFF}}\right]\left(\frac{\partial M_{EH}}{\partial Q}\right). \qquad (32)$$

$$M_{EH} = (\frac{\partial C_{EH}}{\partial \alpha'})(\Delta\alpha')c_{EAHL}S_{EAHL}\frac{\rho}{2}U^2, \text{ and}$$

$$\Delta\alpha' = \frac{lQ}{U}. \quad \text{Thus,}$$

$$\frac{\partial M_{EH}}{\partial Q} = (\frac{\partial C_{EH}}{\partial \alpha'})lc_{EAHL}S_{EAHL}\frac{\rho}{2}U.$$

Thus, for $\frac{U}{U_o} \cong 1$, we have:

$$m_{eh_q} = (\frac{S_{EAHL}}{S})[\frac{lc_{EAHL}M_A}{(I_{EH})_{EFF}}](\frac{\partial C_{EH}}{\partial \alpha'}). \tag{33}$$

(e) (Change in Elevator Angle - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{\delta_e}}$:

From Section A-2, Appendix A,

$$m_{eh_{\delta_e}} = T_A{}^2M_{EH_{\Delta_E}} = [\frac{T_A{}^2}{(I_{EH})_{EFF}}][(\frac{\partial M_{EH}}{\partial \Delta_E}) + (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2(\frac{\partial M_{SAE}}{\partial \theta_{SAE}})]. \tag{34}$$

$$M_{EH} = (\frac{\partial C_{EH}}{\partial \Delta_E})\Delta_E c_{EAHL}S_{EAHL}\frac{\rho}{2}U^2, \text{ so}$$

$$\frac{\partial M_{EH}}{\partial \Delta_E} = (\frac{\partial C_{EH}}{\partial \Delta_E})c_{EAHL}S_{EAHL}\frac{\rho}{2}U^2. \quad \delta_e \equiv \Delta_E, \ \dot{\delta}_e = T_A\dot{\Delta}_E,$$

and $-(\frac{\partial M_{SAE}}{\partial \theta_{SAE}}) \equiv K_{SAE}$. Thus, for $\frac{U}{U_o} \cong 1$,

$$m_{eh_{\delta_e}} = [\frac{T_A^2}{(I_{EH})_{EFF}}][(\frac{\partial C_{EH}}{\partial \Delta_E}) c_{EAHL} S_{EAHL} \frac{\rho}{2} U_o^2$$

$$-(\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 K_{SAE}]. \qquad (35)$$

(f) (Change in Rate of Change of Elevator Angle - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{\dot{\delta}_e}}$:

From Section A-2, Appendix A,

$$m_{eh_{\dot{\delta}_e}} = T_A M_{EH_{\dot{\Delta}_E}} = [\frac{T_A}{(I_{EH})_{EFF}}][(\frac{\partial M_{EH}}{\partial \dot{\Delta}_E})$$

$$+(\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (\frac{\partial M_{SAE}}{\partial Q_{SAE}})]. \qquad (36)$$

$M_{EH} = (\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}) \dot{\Delta}_E c_E S_E \frac{\rho}{2} U^2$, so

$\frac{\partial M_{EH}}{\partial \dot{\Delta}_E} = (\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}) c_E S_E \frac{\rho}{2} U^2$. Also $-(\frac{\partial M_{SAE}}{\partial Q_{SAE}})$

$\equiv (C_{SAE})_{EFF}$. For $\frac{U}{U_o} \cong 1$,

$$m_{eh_{\dot{\delta}_e}} = \left[\frac{T_A}{(I_{EH})_{EFF}}\right]\left[\left(\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}\right) c_E S_E \frac{\rho}{2} U_o{}^2\right.$$

$$\left. - \left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)^2 (C_{SAE})_{EFF}\right]. \qquad (37)$$

(g) (Change in Elevation Component of Tracking Error Angle - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{f_e}}$:

From Section A-2, Appendix A,

$$m_{eh_{f_e}} = \left[\frac{T_A{}^2}{(I_{EH})_{EFF}}\right]\left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial F_E}\right). \qquad (38)$$

(h) (Change in Rate of Change of Elevation Component of Tracking Error Angle - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{\dot{f}_e}}$:

From Section A-2, Appendix A,

$$m_{eh_{\dot{f}_e}} = \left[\frac{T_A}{(I_{EH})_{EFF}}\right]\left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial \dot{F}_E}\right). \qquad (39)$$

(i) (Change in Integral of Elevation Component of Tracking Error Angle - Change in Elevator Hinge Moment) Stability Coefficient for Elevator-Servo System, $m_{eh_{\frac{f_e}{d}}}$:

From Section A-2, Appendix A,

$$m_{eh_{\frac{f_e}{d}}} = \left[\frac{T_A^3}{(I_{EH})_{EFF}}\right]\left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial\int F_E dt}\right). \tag{40}$$

Note: From Section A-2, Appendix A,

$$(I_{EH})_{EFF} = I_{EH} + \left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)^2 (I_{SAE})_{EFF}. \tag{41}$$

From reference 7,

$$\frac{\partial C_{EH}}{\partial\dot{\Delta}_E} = \left(\frac{1}{U_o}\right)\left[-\left(\frac{\partial C_{EH}}{\partial D\Delta_E}\right)_A - \left(\frac{\partial C_L}{\partial\alpha}\right)'\left(\frac{\partial C_{EH}}{\partial D\Delta_E}\right)_B\right], \tag{42}$$

where $\left(\frac{\partial C_{EH}}{\partial D\Delta_E}\right)_A$ and $\left(\frac{\partial C_{EH}}{\partial D\Delta_E}\right)_B$ are positive for positive $D\Delta_E$. As defined in reference 7, $D\Delta_E \equiv \left(\frac{1}{U_o}\right)\dot{\Delta}_E$. This is not the same as $D\Delta_E \equiv \dot{\Delta}_E$ defined in this thesis.

A-3: DERIVATION OF FORMULAE FOR PREDICTION MOMENT STABILITY COEFFICIENTS FOR ELEVATION COMPUTER.

(a) (Change in Rate of Change of Normal Velocity - Change in Prediction Moment) Stability Coefficient for Elevation Computer, $m_{ce_{\dot{w}}}$:

From equation 50, Appendix A,

$$m_{ce_{\dot{w}}} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}U_o}{C_{CSE}}\right). \tag{43}$$

In accordance with the ideas and notation presented in reference 3, we may represent the "linear acceleration-prediction angle sensitivity of the elevation prediction system" by $S_{p(aP)_e}$. Then using the notion in this thesis,

$$S_{p(aP)_e} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}}{K_{CSE}}\right). \tag{44}$$

Since $T_{CSE} = \dfrac{C_{CSE}}{K_{CSE}}$, we have: (45)

$$m_{ce_{\dot{w}}} = \left(\frac{U_o}{T_{CSE}}\right)S_{p(aP)_e}. \tag{46}$$

(b) (Change in Rate of Pitch - Change in Prediction Moment) Stability Coefficient for Elevation Computer, m_{ce_q}:

From equation 51, Appendix A,

$$m_{ce_q} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{1}{C_{CSE}}\right)\left[\left(\frac{\partial M_{CSE}}{\partial M_{GE}}\right)H_{GE} + U_o M_{AE}L_{ACSE}\right]. \tag{47}$$

Again in accordance with the ideas and notation presented in reference 3, we may represent the "angular velocity-prediction angle sensitivity of the elevation prediction system" by $S_{p(WP)_e}$. Then, using the notation of this thesis, we have:

$$S_{p(WP)_e} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial M_{GE}}\right)\left(\frac{H_{GE}}{K_{CSE}}\right). \tag{48}$$

The symbol W in equation 48 stands for angular velocity (as defined in reference 3); It could be changed to the symbol Q to be consistent with the notation in this thesis, but the author has preferred to keep the established notation in reference 3 even though it means that one symbol has two meanings.

In further accordance with reference 3, the "stability number of the elevation system" may be defined by the equation:

$$(SN)_e = [\frac{T_{CSE}}{S_{p(WP)_e}(1 + f_m)}] - 1, \tag{49}$$

and the "mechanism f-number" may be defined by the equation:

$$f_m = [\frac{U_o S_{p(aP)_e}}{S_{p(WP)_e}}]; = [\frac{U_o M_{AE} L_{ACSE}}{H_{GE}(\frac{\partial M_{CSE}}{\partial M_{GE}})}], \tag{50}$$

$$= (\frac{U_o M_{AE} L_{AGE}}{H_{GE}}).$$

Thus,

$$m_{ce_q} = [\frac{1}{T_{CSE}}][S_{p(WP)_e} + U_o S_{p(aP)_e}]$$

$$= [\frac{S_{p(WP)_e}(1 + f_m)}{T_{CSE}}];$$

or,

$$m_{ce_q} = \frac{1}{1 + (SN)_e}. \tag{51}$$

(c) (Change in Pitch Angle - Change in Prediction Moment) Stability Coefficient for Elevation Computer, m_{ce_θ}:

From equation 52, Appendix A,

$$m_{ce_\theta} = \left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{M_{AE}L_{ACSE}g}{K_{CSE}}\right). \tag{52}$$

From equations 44 and 52, above, we have:

$$m_{ce_\theta} = gS_{p(aP)_e} \tag{53}$$

(d) (Change in Normal Velocity - Change in Prediction Moment) Stability Coefficient for Elevation Computer, m_{ce_w}:

From equation 53, Appendix A,

$$m_{ce_w} = -\left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial \Delta W}\right)\left(\frac{U_o}{K_{CSE}}\right). \tag{54}$$

We may define the "change in normal velocity-prediction angle sensitivity of the elevation prediction system" by the relation:

$$S_{p(\Delta W-P)_e} = -\left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial \Delta W}\right)\left(\frac{1}{K_{CSE}}\right), \tag{55}$$

and we may define the "change in angle of attack - prediction angle sensitivity of the elevation prediction system" by the relation:

$$S_{p(w-P)_e} = -\left(\frac{\partial P_{CE}}{\partial A_{CSE}}\right)\left(\frac{\partial M_{CSE}}{\partial \Delta W}\right)\left(\frac{U_o}{K_{CSE}}\right). \quad (56)$$

Then $m_{ce_w} = U_o S_{p(\Delta W-P)_e} = S_{p(w-P)_e}$. (57)

Note that

$$T_{CSE} = [1 + (SN)_e][S_{p(WP)_e} + U_o S_{p(aP)_e}]:$$

B. LATERAL FORMULAE:

B-1: DERIVATION OF FORMULAE FOR LATERAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT:

(a) (Change in Lateral Velocity - Change in Rolling Moment) Stability Coefficient for Aircraft, l_v:

From Section B-1, Appendix A,

$$l_v = T_A b L_{\Delta V} = T_A b\left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial \Delta V}\right). \quad (58)$$

$$v \equiv \frac{\Delta V}{U_o}, \text{ so}$$

$$\frac{\partial L}{\partial \Delta V} = \left(\frac{\partial L}{\partial v}\right)\left(\frac{1}{U_o}\right). \quad \text{Also,}$$

$$L = \left(\frac{\partial C_l}{\partial \Delta V}\right)(\Delta V)U^2 S\left(\frac{\rho}{2}\right)b, \text{ so}$$

$$\frac{\partial L}{\partial \Delta V} = \left(\frac{\partial C_l}{\partial \Delta V}\right)U^2 S\left(\frac{\rho}{2}\right)b = \left(\frac{\partial C_l}{\partial v}\right)\left(\frac{1}{U_o}\right)U^2 S\left(\frac{\rho}{2}\right)b. \quad \text{Thus,}$$

$$l_v = \left(\frac{M_A}{\frac{\rho}{2}SU_o}\right) b \left(\frac{1}{I_X}\right) \left(\frac{\partial C_l}{\partial v}\right) \left(\frac{U^2}{U_o}\right) S \left(\frac{\rho}{2}\right) b.$$

For $\frac{U}{U_o} \cong 1$, we have:

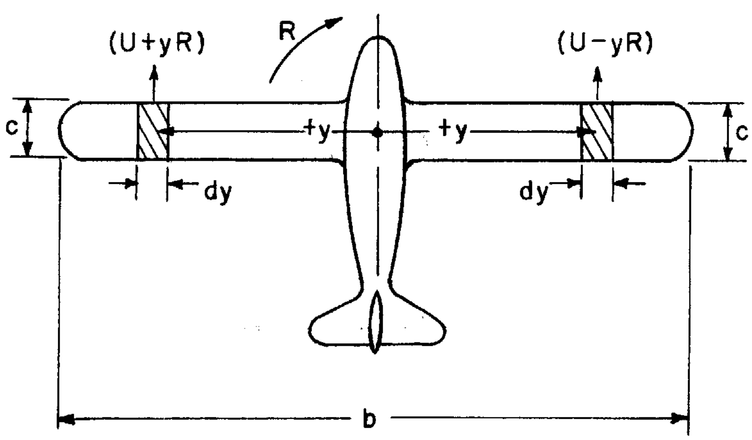
$$l_v = \left(\frac{\partial C_l}{\partial v}\right) \left(\frac{b^2 M_A}{I_X}\right). \qquad (59)$$

(b) (Change in Rate of Yaw - Change in Rolling Moment)

Stability Coefficient for Aircraft, l_r:

From Section B-1, Appendix A,

$$l_r = T_A L_R = T_A \left(\frac{1}{I_X}\right) \left(\frac{\partial L}{\partial R}\right). \qquad (60)$$



TOP VIEW OF AIRCRAFT

Note:

In this case for mathematical facility +y is used along left and right wings, instead of right wing only.

For rectangular plan-form of wing, we have:

$$L = \int_0^{b/2} yC_Lc\ dy\ (\frac{\rho}{2})[(U + yR)^2 - U^2]$$

(Left Wing)

$$- \int_0^{b/2} yC_Lc\ dy\ (\frac{\rho}{2})[(U - yR)^2 - U^2],$$

(Right Wing)

$$= \int_0^{b/2} C_Lc(\frac{\rho}{2})(4Uy^2R)dy = C_Lc(\frac{\rho}{2})2UR(\frac{b^3}{12}).$$ Thus,

$$\frac{\partial L}{\partial R} = 2C_L(\frac{\rho}{2})U(\frac{b^2S}{12})$$ because $S = bc$ for rectangular wing plan-form.

Thus,

$$l_r = (\frac{M_A}{\frac{\rho}{2}SU_o})[2C_L(\frac{\rho}{2})U(\frac{b^2S}{12})](\frac{1}{I_X}).$$

For $C_L \cong C_{L_o}$, and for $\frac{U}{U_o} \cong 1$, we have:

$$l_r = C_{L_o}(\frac{M_Ab^2}{6I_X}) \quad \text{for rectangular plan-form} \qquad (61)$$

of wing.

Analogously,

$$l_r = C_{L_o}(\frac{M_Ab^2}{8\bar{I}_X}) \quad \text{for elliptical plan-form} \qquad (62)$$

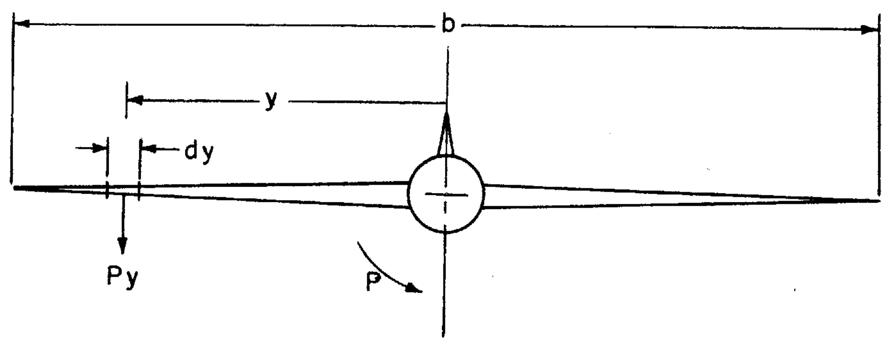
of wing.

(c) (Change in Rate of Roll - Change in Rolling Moment)

Stability Coefficient for Aircraft, l_p:

From Section B-1, Appendix A,

$$l_p = T_A L_P = T_A\left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial P}\right). \tag{63}$$



FRONT VIEW OF AIRCRAFT

For rectangular plan-form of wing:

$$L = -2\int_{o}^{b/2} \left(\frac{Py}{U}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)(dy\ c)U^2\left(\frac{\rho}{2}\right)y,$$

$$= -2cU\left(\frac{\rho}{2}\right)P\left(\frac{\partial C_L}{\partial \alpha}\right)\int_{o}^{b/2} y^2\ dy,$$

$$= -cU\left(\frac{\rho}{2}\right)P\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{b^3}{12}\right).$$

Thus,

$$\frac{\partial L}{\partial P} = -cU\left(\frac{\rho}{2}\right)\left(\frac{b^3}{12}\right)\left(\frac{\partial C_L}{\partial \alpha}\right).$$

Since S = bc for rectangular plan-form of wing,

then

$$l_p = \left(\frac{M_A}{\frac{\rho}{2}SU_o}\right)\left[-U\left(\frac{\rho}{2}\right)\left(\frac{b^2S}{12}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{1}{I_X}\right). \quad \text{For } \frac{U}{U_o} \cong 1,$$

we have:

$$l_p = -\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{b^2 M_A}{12 I_X}\right) \text{ for rectangular plan-form} \quad (64).$$

of wing.

Analogously,

$$l_p = -\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{b^2 M_A}{16 I_X}\right) \text{ for elliptical plan-form} \quad (65)$$

of wing.

(d) (Change in Aileron Angle - Change in Rolling Moment) Stability Coefficient for Aircraft, l_{δ_a}:

From Section B-1, Appendix A,

$$l_{\delta_a} = T_A{}^2 L_{\Delta_A} = T_A{}^2\left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial \Delta_A}\right). \quad (66)$$

$$L = \left(\frac{\partial C_l}{\partial \Delta_A}\right)\Delta_A U^2 S\left(\frac{\rho}{2}\right)b. \quad \text{So}$$

$$\frac{\partial L}{\partial \Delta_A} = \left(\frac{\partial C_l}{\partial \Delta_A}\right)U^2 S\left(\frac{\rho}{2}\right)b.$$

For $\frac{U}{U_o} \cong 1$, we have:

$$l_{\delta_a} = \left(\frac{T_A^{\,2}}{I_X}\right) U_o^{\,2} S\left(\frac{\rho}{2}\right) b \left(\frac{\partial C_l}{\partial \Delta_A}\right), \tag{67}$$

or,

$$l_{\delta_a} = \left(\frac{M_A^{\,2} b}{\frac{\rho}{2} S I_X}\right)\left(\frac{\partial C_l}{\partial \Delta_A}\right). \tag{68}$$

Note: Δ_A = sum of aileron deflections.

(e) (Change in Lateral Velocity - Change in Yawing Moment) Stability Coefficient for Aircraft, n_v:

From Section B-1, Appendix A,

$$n_v = T_A b N_{\Delta V} = T_A b \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial \Delta V}\right). \tag{69}$$

$$N = \left(\frac{\partial C_n}{\partial \Delta V}\right) \Delta V \; Sb\left(\frac{\rho}{2}\right) U^2, \text{ so}$$

$$\frac{\partial N}{\partial \Delta V} = \left(\frac{\partial C_n}{\partial \Delta V}\right) Sb\left(\frac{\rho}{2}\right) U^2. \quad v \equiv \frac{\Delta V}{U_o}, \text{ thus}$$

$$n_v = \left(\frac{M_A}{\frac{\rho}{2} S U_o}\right) b \left(\frac{1}{I_Z}\right)\left(\frac{1}{U_o}\right)\left(\frac{\partial C_n}{\partial v}\right) Sb\left(\frac{\rho}{2}\right) U^2.$$

For $\frac{U}{U_o} \cong 1$,

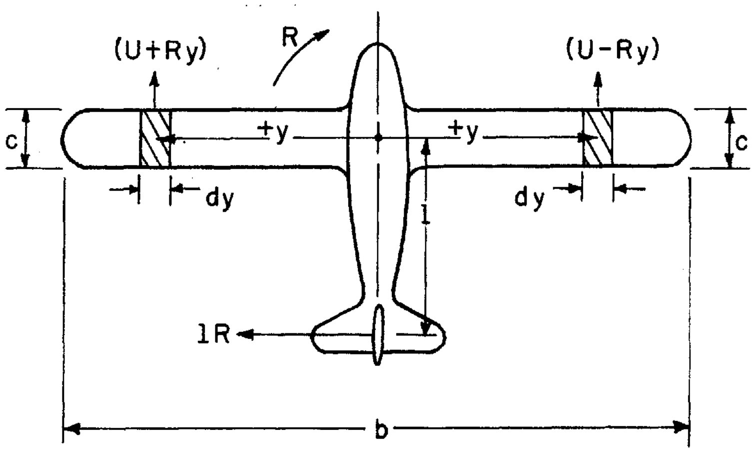
$$n_v = \left(\frac{\partial C_n}{\partial v}\right)\left(\frac{M_A b^2}{I_Z}\right). \tag{70}$$

(f) (Change in Rate of Yaw - Change in Yawing Moment)

Stability Coefficient for Aircraft, n_r:

From Section B-1, Appendix A,

$$n_r = T_A N_R = T_A \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial R}\right). \tag{71}$$



TOP VIEW OF AIRCRAFT

Note:

In this case for mathematical facility +y is used along left and right wings, instead of right wing only.

For rectangular plan-form of wing,

$$N = -\left(\frac{lR}{U_o}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)'' S''\left(\frac{\rho}{2}\right)U^2 \eta'' l$$

(Vertical Tail)

$$- \int_0^{b/2} c\, dy\, C_D y \left(\frac{\rho}{2}\right)[(U + Ry)^2 - U^2]$$

(Left Wing)

$$+ \int_0^{b/2} c\, dy\, C_D y \left(\frac{\rho}{2}\right)[(U - Ry)^2 - U^2],$$

(Right Wing)

$$= -\left(\frac{lR}{U}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)'' S''\left(\frac{\rho}{2}\right)U^2 \eta'' l - 4UcC_D\left(\frac{\rho}{2}\right)R\left(\frac{b^3}{24}\right).$$

Thus,

$$\frac{\partial N}{\partial R} = -(\frac{l^2}{U})(\frac{\partial C_L}{\partial \alpha})'' S''(\frac{\rho}{2})U^2 \eta'' - 2UC_D(\frac{\rho}{2})(\frac{b^3 c}{12}).$$

S = bc for rectangular plan-form of wing. Thus, for $\frac{U}{U_o} \cong 1$, and for $C_D \cong C_{D_o}$, and using a factor of $5/4$ to account for fuselage and nacelle damping, we have:

$$n_r = -(\frac{5}{4})\eta''(\frac{S''}{S})(\frac{\partial C_L}{\partial \alpha})''(\frac{M_A l^2}{I_Z}) - (\frac{C_{D_o}}{6})(\frac{M_A b^2}{I_Z}), \qquad (72)$$

for rectangular plan-form of wing.

Analogously,

$$n_r = -(\frac{5}{4})\eta''(\frac{S''}{S})(\frac{\partial C_L}{\partial \alpha})''(\frac{M_A l^2}{I_Z}) - (\frac{C_{D_o}}{8})(\frac{M_A b^2}{I_Z}), \qquad (73)$$

for elliptical plan-form of wing.

(g) (Change in Rate of Roll - Change in Yawing Moment) Stability Coefficient for Aircraft, n_p:

From Section B-1, Appendix A,

$$n_p = T_A N_P = T_A(\frac{1}{I_Z})(\frac{\partial N}{\partial P}). \qquad (74)$$

For rectangular plan-form of wing:

$$N = -2\int_0^{b/2} (\frac{\partial C_X}{\partial \alpha})(\frac{Py}{U})c\ dy\ y(\frac{\rho}{2})U^2,$$

$$= -2PcU\left(\frac{\rho}{2}\right)\left(\frac{\partial C_X}{\partial \alpha}\right)\left(\frac{b^3}{24}\right), \text{ and } S = bc.$$

Thus,

$$\frac{\partial N}{\partial P} = -U\left(\frac{\rho}{2}\right)\left(\frac{\partial C_X}{\partial \alpha}\right)\left(\frac{b^2 S}{12}\right), \quad \text{where}$$

$$\frac{\partial C_X}{\partial \alpha} \cong \left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]C_L, \text{ as shown in derivation}$$

for equation 6.

So, for $\frac{U}{U_o} \cong 1$, and $C_L \cong C_{L_o}$, we have:

$$n_p = -C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{b^2 M_A}{12 I_Z}\right), \tag{75}$$

for rectangular plan-form of wing.

$$n_p = -C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{b^2 M_A}{16 I_Z}\right), \tag{76}$$

for elliptical plan-form of wing.

(h) (Change in Rudder Angle - Change in Yawing Moment) Stability Coefficient for Aircraft, n_{δ_r}:

From Section B-1, Appendix A,

$$n_{\delta_r} = T_A{}^2 N_{\Delta_R} = T_A{}^2\left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial \Delta_R}\right). \tag{77}$$

$$N = \left(\frac{\partial C_n}{\partial \Delta_R}\right)\Delta_R b S\left(\frac{\rho}{2}\right)U^2, \text{ so}$$

$$\frac{\partial N}{\partial \Delta_R} = (\frac{\partial C_n}{\partial \Delta_R}) bS(\frac{\rho}{2})U^2. \quad \text{For} \frac{U}{U_o} \cong 1, \text{ we have:}$$

$$n_{\delta_r} = (\frac{T_A{}^2}{I_Z}) bS(\frac{\rho}{2})U_o{}^2(\frac{\partial C_n}{\partial \Delta_R}), \tag{78}$$

$$\text{or} \quad n_{\delta_r} = (\frac{M_A{}^2 b}{\frac{\rho}{2} S I_Z})(\frac{\partial C_n}{\partial \Delta_R}). \tag{79}$$

(i) (Change in Lateral Velocity - Change in Lateral Force)

Stability Coefficient for Aircraft, y_v:

From Section B-1, Appendix A,

$$y_v = T_A Y_{\Delta V} = T_A(\frac{1}{M_A})(\frac{\partial Y}{\partial \Delta V}). \tag{80}$$

$$Y = (\frac{\partial C_Y}{\partial \Delta V})\Delta V(\frac{\rho}{2})SU^2. \quad \text{Thus,}$$

$$\frac{\partial Y}{\partial \Delta V} = (\frac{\partial C_Y}{\partial \Delta V})(\frac{\rho}{2})SU^2. \quad \text{Since } v \equiv \frac{\Delta V}{U_o},$$

then, for $\frac{U}{U_o} \cong 1$, we have:

$$y_v = \frac{\partial C_Y}{\partial v}. \tag{81}$$

(j) (Change in Angle of Bank - Change in Lateral Force)

Stability Coefficient for Aircraft, y_ϕ:

From Section B-1, Appendix A,

$$y_\phi = (\frac{T_A}{U_o})Y_\phi = (\frac{T_A}{U_o})(\frac{1}{M_A})(\frac{\partial Y}{\partial \phi}).$$ Due to angle of bank

alone,

$Y = W_A \sin \phi$, so

$$\frac{\partial Y}{\partial \phi} = W_A \cos \phi \cong W_A$$ for ϕ small. Thus,

$$y_\phi = (\frac{T_A}{U_o})(\frac{W_A}{M_A}) = (\frac{T_A g}{U_o}).$$

or

$$y_\phi = C_{L_o}. \qquad (83)$$

(k) (Change in Rudder Angle - Change in Lateral Force)

Stability Coefficient for Aircraft, y_{δ_r}:

From Section B-1, Appendix A,

$$y_{\delta_r} = (\frac{T_A}{U_o})Y_{\Delta_R} = (\frac{T_A}{U_o})(\frac{1}{M_A})(\frac{\partial Y}{\partial \Delta_R}). \qquad (84)$$

$$Y = -(\frac{N}{l}) = -(\frac{\partial C_n}{\partial \Delta_R})\Delta_R bS(\frac{\rho}{2})U^2(\frac{1}{l}),$$ so

$$\frac{\partial Y}{\partial \Delta_R} = -(\frac{\partial C_n}{\partial \Delta_R})bS(\frac{\rho}{2})U^2(\frac{1}{l}).$$ Thus,

$$y_{\delta_r} = (\frac{M_A}{\frac{\rho}{2}SU_o^2})(\frac{1}{M_A})[-(\frac{\partial C_n}{\partial \Delta_R})(\frac{b}{l})S(\frac{\rho}{2})U^2].$$

For $\frac{U}{U_o} \cong 1$, we have:

$$y_{\delta_r} = -(\frac{b}{l})(\frac{\partial C_n}{\partial \Delta_R}). \quad (85)$$

Note: The effects of n_u and l_u will be considered negligible.

B-2: DERIVATION OF FORMULAE FOR RUDDER HINGE MOMENT STABILITY COEFFICIENTS FOR RUDDER-SERVO SYSTEM:

(a) (Change in Longitudinal Velocity - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, m_{rh_u}:

From Section B-2, Appendix A,

$$m_{rh_u} = bT_A M_{RH_{\Delta U}} = bT_A [\frac{1}{(I_{RH})_{EFF}}](\frac{\partial M_{RH}}{\partial \Delta U}). \quad (86)$$

$\frac{\partial M_{RH}}{\partial \Delta U}$ depends on:

1. Change in flow of air over rudder when effect of propeller wash is important,
2. Initial trim position of rudder,
3. Change in equilibrium of rudder when it is not statically balanced (free controls), etc.

These derivations will not be included in this thesis, since m_{rh_u} will be considered negligible in the analyses which follow.

(b) (Change in Lateral Velocity - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, m_{rh_v}:

From Section B-2, Appendix A,

$$m_{rh_v} = bT_A M_{RH_{\Delta V}} = bT_A \left[\frac{1}{(I_{RH})_{EFF}}\right]\left(\frac{\partial M_{RH}}{\partial \Delta V}\right). \quad (87)$$

$$M_{RH} = \left(\frac{\partial C_{RH}}{\partial \Delta V}\right)(\Delta V) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2, \text{ and}$$

$$v = \frac{\Delta V}{U_o}. \quad \text{Thus, } \frac{\partial M_{RH}}{\partial \Delta V} = \left(\frac{\partial C_{RH}}{\partial v}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2 \left(\frac{1}{U_o}\right).$$

Then,

$$m_{rh_v} = b\left(\frac{M_A}{\frac{\rho}{2} S U_o}\right)\left[\frac{1}{(I_{RH})_{EFF}}\right]\left(\frac{\partial C_{RH}}{\partial v}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2 \left(\frac{1}{U_o}\right)$$

For $\frac{U}{U_o} \cong 1$,

$$m_{rh_v} = \left(\frac{S_{RAHL}}{S}\right)\left[\frac{b c_{RAHL} M_A}{(I_{RH})_{EFF}}\right]\left(\frac{\partial C_{RH}}{\partial v}\right). \quad (88)$$

(c) (Change in Rate of Yaw - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, m_{rh_r}:

From Section B-2, Appendix A,

$$m_{rh_r} = T_A M_{RH_R} = T_A \left[\frac{1}{(I_{RH})_{EFF}}\right] \left(\frac{\partial M_{RH}}{\partial R}\right). \qquad (89)$$

$M_{RH} = \left(\frac{\partial C_{RH}}{\partial R}\right) R c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2.$ Thus,

$\frac{\partial M_{RH}}{\partial R} = \left(\frac{\partial C_{RH}}{\partial R}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2.$ But

$\frac{\partial C_{RH}}{\partial R} = -\left(\frac{\partial C_{RH}}{\partial v}\right)\left(\frac{1}{U}\right).$ Then,

$$m_{rh_r} = \left(\frac{M_A}{\frac{\rho}{2} S U_o}\right) \left[\frac{1}{(I_{RH})_{EFF}}\right] \left(-\frac{\partial C_{RH}}{\partial v}\right) \left(\frac{1}{U}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2.$$

For $\frac{U}{U_o} \cong 1$, we have:

$$m_{rh_r} = -\left(\frac{S_{RAHL}}{S}\right) \left[\frac{1 c_{RAHL} M_A}{(I_{RH})_{EFF}}\right] \left(\frac{\partial C_{RH}}{\partial v}\right). \qquad (90)$$

(d) (Change in Rudder Angle - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, $m_{rh_{\delta_r}}$:

From Section B-2, Appendix A,

$$m_{rh_{\delta_r}} = T_A{}^2 M_{RH_{\Delta_R}} = T_A{}^2 \left[\frac{1}{(I_{RH})_{EFF}}\right] \left[\left(\frac{\partial M_{RH}}{\partial \Delta_R}\right) + \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 \left(\frac{\partial M_{SAR}}{\partial \theta_{SAR}}\right)\right]. \qquad (91)$$

$$M_{RH} = \left(\frac{\partial C_{RH}}{\partial \Delta_R}\right) \Delta_R c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2, \text{ so}$$

$$\frac{\partial M_{RH}}{\partial \Delta_R} = \left(\frac{\partial C_{RH}}{\partial \Delta_R}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U^2.$$

$-\left(\frac{\partial M_{SAR}}{\partial \theta_{SAR}}\right) \equiv K_{SAR}$. Thus, for $\frac{U}{U_o} \cong 1$, we have:

$$m_{rh_{\delta_r}} = \left[\frac{T_A^{\,2}}{(I_{RH})_{EFF}}\right] \left[\left(\frac{\partial C_{RH}}{\partial \Delta_R}\right) c_{RAHL} S_{RAHL} \left(\frac{\rho}{2}\right) U_o^2 - \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 K_{SAR}\right]. \qquad (92)$$

(e) (Change in Rate of change of Rudder Angle - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, $m_{rh_{\dot{\delta}_r}}$:

From Section B-2, Appendix A,

$$m_{rh_{\dot{\delta}_r}} = T_A M_{RH_{\dot{\Delta}_R}} = \left[\frac{T_A}{(I_{RH})_{EFF}}\right] \left[\left(\frac{\partial M_{RH}}{\partial \dot{\Delta}_R}\right) + \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 \left(\frac{\partial M_{SAR}}{\partial Q_{SAR}}\right)\right]. \qquad (93)$$

$$M_{RH} = \left(\frac{\partial C_{RH}}{\partial \dot{\Delta}_R}\right) \dot{\Delta}_R c_R S_R \left(\frac{\rho}{2}\right) U^2, \text{ so}$$

$$\frac{\partial M_{RH}}{\partial \dot{\Delta}_R} = \left(\frac{\partial C_{RH}}{\partial \dot{\Delta}_R}\right) c_R S_R \left(\frac{\rho}{2}\right) U^2. \quad \text{Also,} \; -\left(\frac{\partial M_{SAR}}{\partial Q_{SAR}}\right)$$

$$= (C_{SAR})_{EFF}.$$

Thus, for $\frac{U}{U_o} \cong 1$,

$$m_{rh_{\dot{\delta}_r}} = \left[\frac{T_A}{(I_{RH})_{EFF}}\right]\left[\left(\frac{\partial C_{RH}}{\partial \dot{\Delta}_R}\right) c_R S_R \left(\frac{\rho}{2}\right) U_o^2\right.$$

$$\left. - \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 (C_{SAR})_{EFF}\right]. \qquad (94)$$

(f) (Change in Deflection Component of Tracking Error Angle - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, $m_{rh_{f_d}}$:

From Section B-2, Appendix A,

$$m_{rh_{f_d}} = \left[\frac{T_A^2}{(I_{RH})_{EFF}}\right]\left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial F_D}\right). \qquad (95)$$

(g) (Change in Rate of change of Deflection Component of Tracking Error Angle - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, $m_{rh_{\dot{f}_d}}$:

From Section B-2, Appendix A,

$$m_{rh_{\dot{f}_d}} = \left[\frac{T_A}{(I_{RH})_{EFF}}\right]\left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial\dot{F}_D}\right). \qquad (96)$$

(h) (Change in Integral of Deflection Component of Tracking Error Angle - Change in Rudder Hinge Moment) Stability Coefficient for Rudder-Servo System, $m_{rh_{\frac{f_d}{d}}}$:

From Section B-2, Appendix A,

$$m_{rh_{\frac{f_d}{d}}} = \left[\frac{T_A^{\,3}}{(I_{RH})_{EFF}}\right]\left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial\int F_D dt}\right). \qquad (97)$$

Note: From Section B-2, Appendix A,

$$(I_{RH})_{EFF} = I_{RH} + \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)^2 (I_{SAR})_{EFF} \qquad (98)$$

From reference 7,

$$\frac{\partial C_{RH}}{\partial\dot{\Delta}_R} = \left(\frac{1}{U_o}\right)\left[-\left(\frac{\partial C_{RH}}{\partial D\Delta_R}\right)_A - \left(\frac{\partial C_L}{\partial\alpha}\right)'' \left(\frac{\partial C_{RH}}{\partial D\Delta_R}\right)_B\right], \qquad (99)$$

where $\left(\frac{\partial C_{RH}}{\partial D\Delta_R}\right)_A$ and $\left(\frac{\partial C_{RH}}{\partial D\Delta_R}\right)_B$ are positive for positive $D\Delta_R$.

As defined in reference 7, $D\Delta_R \equiv \left(\frac{1}{U_o}\right)\dot{\Delta}_R$. This is not the same as $D\Delta_R \equiv \dot{\Delta}_R$ defined in this thesis.

B-3: DERIVATION OF FORMULAE FOR AILERON HINGE MOMENT STABILITY COEFFICIENTS FOR AILERON-SERVO SYSTEM

(a) (Change in Rate of Roll - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System, m_{ah_p}:

From Section B-3, Appendix A,

$$m_{ah_p} = T_A M_{AH_p} = [\frac{2T_A}{(I_{AH})_{EFF}}](\frac{\partial M_{AH}}{\partial P}). \qquad (100)$$

A derivation for $(\frac{\partial M_{AH}}{\partial P})$ will not be included in this thesis, since m_{ah_p} will be considered negligible in the analyses which follow.

(b) (Change in Aileron Angle - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System, $m_{ah_{\delta_a}}$:

From Section B-3, Appendix A,

$$m_{ah_{\delta_a}} = T_A^2 M_{AH_{\Delta_A}} = -[\frac{T_A^2 (K_{AH})_{EFF}}{(I_{AH})_{EFF}}],$$

$$= [\frac{T_A^2}{(I_{AH})_{EFF}}][2(\frac{\partial M_{AH}}{\partial \Delta_A}) + 4\,(\frac{\partial \theta_{SAA}}{\partial \Delta_A})^2 \frac{\partial M_{SAA}}{\partial \theta_{SAA}}]. \qquad (101)$$

$$M_{AH} = \left(\frac{\partial C_{AH}}{\partial \Delta_A}\right)\Delta_A c_{AAHL} S_{AAHL} \left(\frac{\rho}{2}\right) U^2. \quad \text{Thus,}$$

$$\frac{\partial M_{AH}}{\partial \Delta_A} = \left(\frac{\partial C_{AH}}{\partial \Delta_A}\right) c_{AAHL} S_{AAHL} \left(\frac{\rho}{2}\right) U^2.$$

$$-\left(\frac{\partial M_{SAA}}{\partial \theta_{SAA}}\right) \equiv K_{SAA}. \quad \text{Thus, for } \frac{U}{U_o} \cong 1, \text{ we have:}$$

$$m_{ah_{\delta_a}} = \left[\frac{T_A{}^2}{(I_{AH})_{EFF}}\right]\left[2\left(\frac{\partial C_{AH}}{\partial \Delta_A}\right) c_{AAHL} S_{AAHL} \left(\frac{\rho}{2}\right) U_o{}^2 - 4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)^2 K_{SAA}\right]. \qquad (102)$$

(c) (Change in Rate of Change of Aileron Angle - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System $m_{ah_{\dot{\delta}_a}}$:

From Section B-3, Appendix A,

$$m_{ah_{\dot{\delta}_a}} = T_A M_{AH_{\dot{\Delta}_A}} = -T_A \left[\frac{(C_{AH})_{EFF}}{(I_{AH})_{EFF}}\right],$$

$$= \left[\frac{T_A}{(I_{AH})_{EFF}}\right]\left[2\left(\frac{\partial M_{AH}}{\partial \dot{\Delta}_A}\right) + 4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)^2 \left(\frac{\partial M_{SAA}}{\partial Q_{SAA}}\right)\right] \qquad (103)$$

$$M_{AH} = \left(\frac{\partial C_{AH}}{\partial \dot{\Delta}_A}\right)\dot{\Delta}_A c_A S_A \left(\frac{\rho}{2}\right) U^2. \quad \text{Thus,}$$

$$\frac{\partial M_{AH}}{\partial \dot{\Delta}_A} = \left(\frac{\partial C_{AH}}{\partial \dot{\Delta}_A}\right) c_A S_A \left(\frac{\rho}{2}\right) U^2.$$

$$-\left(\frac{\partial M_{SAA}}{\partial Q_{SAA}}\right) \equiv (C_{SAA})_{EFF}.$$ Thus, for $\frac{U}{U_o} \cong 1$, we have:

$$m_{ah_{\dot{\delta}_a}} = \left[\frac{T_A}{(I_{AH})_{EFF}}\right]\left[2\left(\frac{\partial C_{AH}}{\partial \dot{\Delta}_A}\right) c_A S_A \left(\frac{\rho}{2}\right) U_o^2 - 4\left(\frac{\partial \theta_{SAA}}{\partial \Delta_A}\right)^2 (C_{SAA})_{EFF}\right]. \qquad (104)$$

Note: From reference 7,

$$\frac{\partial C_{AH}}{\partial \dot{\Delta}_A} \cong 2\left(\frac{\partial C_{AHR}}{\partial \dot{\Delta}_{AR}}\right) = \left(\frac{2}{U_o}\right)\left[-\left(\frac{\partial C_{AHR}}{\partial D\Delta_{AR}}\right)_A - \left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{\partial C_{AHR}}{\partial D\Delta_{AR}}\right)_B\right],$$

where $\left(\frac{\partial C_{AHR}}{\partial D\Delta_{AR}}\right)_A$ and $\left(\frac{\partial C_{AHR}}{\partial D\Delta_{AR}}\right)_B$ are positive for positive $D\Delta_{AR}$.

As defined in reference 7, $D\Delta_{AR} \equiv \left(\frac{1}{U_o}\right)\dot{\Delta}_{AR}$. This is not the same as $D\Delta_{AR} \equiv \dot{\Delta}_{AR}$ defined in this thesis.
Also, $\frac{\partial C_{AH}}{\partial \dot{\Delta}_A} \cong 2\left(\frac{\partial C_{AHL}}{\partial \dot{\Delta}_{AL}}\right) = 2\left(\frac{\partial C_{AHR}}{\partial \dot{\Delta}_{AR}}\right)$ and $c_{AH} \cong c_{AHR}, \cong c_{AHL}$.

(d) (Change in Deflection Component of Tracking Error Angle - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System, $m_{ah_{f_d}}$:

From Section B-3, Appendix A,

$$m_{ah_{f_d}} = \left[\frac{4T_A{}^2}{(I_{AH})_{EFF}}\right]\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial F_D}\right). \tag{105}$$

(e) (Change in Rate of Change of Deflection Component of Tracking Error Angle - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System, $m_{ah_{\dot{f}_d}}$:

From Section B-3, Appendix A,

$$m_{ah_{\dot{f}_d}} = \left[\frac{4T_A}{(I_{AH})_{EFF}}\right]\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial \dot{F}_D}\right). \tag{106}$$

(f) (Change in Integral of Deflection Component of Tracking Error Angle - Change in Aileron Hinge Moment) Stability Coefficient for Aileron-Servo System, $m_{ah_{\frac{f_d}{d}}}$:

From Section B-3, Appendix A,

$$m_{ah_{\frac{f_d}{d}}} = \left[\frac{4T_A{}^3}{(I_{AH})_{EFF}}\right]\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial\int F_D dt}\right). \tag{107}$$

Note: From Section B-3, Appendix A,

$$(I_{AH})_{EFF} = 4(I_{SAA})_{EFF}\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)^2 + I_{AH}, \text{ and} \tag{108}$$

$$I_{AH} = I_{AHR} + I_{AHL}. \tag{109}$$

B-4: DERIVATION OF FORMULAE FOR PREDICTION MOMENT STABILITY COEFFICIENTS FOR DEFLECTION COMPUTER

(a) (Change in Rate of Yaw - Change in Prediction Moment) Stability Coefficient for Deflection Computer, m_{cd_r}:

From equation 127, Appendix A,

$$m_{cd_r} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{1}{C_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)H_{GD} \cos I_{GD} \qquad (110)$$

In accordance with the ideas and notation presented in reference 3, we may represent the "angular velocity-prediction angle sensitivity of the deflection prediction system" by $S_{p(WP)_d}$. Then, using the notation in this thesis, we have:

$$S_{p(WP)_d} = \left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial M_{GD}}\right)\left(\frac{H_{GD}}{K_{CSD}}\right). \qquad (111)$$

The symbol W in equation 111 stands for angular velocity (as defined in reference 3). It could be changed to the symbol Q to be consistent with the notation in this thesis, but the author has preferred to keep the established notation in reference 3, even though it means that one symbol has two meanings. Also in accordance with reference 3, the "stability number of the deflection system" may be defined by the equation:

$$(SN)_d = \left[\frac{T_{CSD}}{S_{p(WP)_d} \cos I_{GD}}\right] - 1. \qquad (112)$$

Thus,

$$m_{cd_r} = [\frac{1}{T_{CSD}}][S_{p(WP)_d}]\cos I_{GD}, \text{ or}$$

$$m_{cd_r} = [\frac{1}{1 + (SN)_d}]. \quad (113)$$

(b) (Change in Rate of Roll - Change in Prediction Moment) Stability Coefficient for Deflection Computer, m_{cd_p}:

From equation 128, Appendix A,

$$m_{cd_p} = (\frac{\partial P_{CD}}{\partial A_{CSD}})(\frac{1}{C_{CSD}})(\frac{\partial M_{CSD}}{\partial M_{GD}})H_{GD} \sin I_{GD}. \quad (114)$$

Combining equations 111, 112 and 114, we have:

$$m_{cd_p} = [\frac{1}{1 + (SN)_d}] \tan I_{GD}. \quad (115)$$

(c) (Change in Lateral Velocity - Change in Prediction Moment) Stability Coefficient for Deflection Computer, m_{cd_v}:

From equation 126, Appendix A,

$$m_{cd_v} = -(\frac{\partial P_{CD}}{\partial A_{CSD}})(\frac{\partial M_{CSD}}{\partial \Delta V})(\frac{U_o}{K_{CSD}}) \quad (116)$$

We may define the "change in lateral velocity - prediction angle sensitivity of the deflection prediction system" by the relation:

$$S_{p(\Delta V-P)_d} = -\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)\left(\frac{1}{K_{CSD}}\right), \quad (117)$$

and we may define the "change in aerodynamic yaw-prediction angle sensitivity of the deflection prediction system" by the relation:

$$S_{p(v-P)_d} = -\left(\frac{\partial P_{CD}}{\partial A_{CSD}}\right)\left(\frac{\partial M_{CSD}}{\partial \Delta V}\right)\left(\frac{U_o}{K_{CSD}}\right), \quad (118)$$

Then

$$m_{cd_v} = U_o S_{p(\Delta V-P)_d} = S_{p(v-P)_d}. \quad (119)$$

Note that

$$T_{CSD} = [1 + (SN)_d][S_{p(WP)_d}]\cos I_{GD}.$$

C. SUMMARY OF FORMULAE FOR TRACKING CONTROL STABILITY COEFFICIENTS FOR AIRCRAFT:

1. LONGITUDINAL FORMULAE

(a) Formulae for Longitudinal Dynamic Stability Coefficients for Aircraft:

1. $x_u = -2C_{D_o}$, power off. (3)

$= -3C_{D_o}$, power on. (4)

2. $x_w = C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]$. (6)

3. $x_\theta = -C_{L_o}$. (8)

4. $z_u = -2C_{L_o}$. (10)

5. $z_w = -(\frac{\partial C_L}{\partial \alpha})$ (12)

6. $z_\theta = 0$ for $\theta_o = 0$. (14)

7. (m_u assumed negligible) (15)

8. $m_w = (\frac{M_A C^2}{I_Y})(\frac{\partial C_M}{\partial \alpha})$. (17)

9. $m_{\dot{w}} = -(\frac{\partial C_L}{\partial \alpha})'(\frac{\partial \epsilon}{\partial \alpha})(\frac{\frac{\rho}{2}S'l^2 c\eta'}{I_Y})$. (19)

10. $m_q = -(\frac{5}{4})(\frac{\partial C_L}{\partial \alpha})'(\frac{S'}{S})(\frac{\eta' M_A l^2}{I_Y})$. (21)

11. $m_{\delta_e} = (\frac{T_A{}^2}{I_Y})cS\frac{\rho}{2}U_o{}^2(\frac{\partial C_m}{\partial \Delta_E}), = \mu_c^2(\frac{\frac{\rho}{2}Sc^3}{I_Y})(\frac{\partial C_m}{\partial \Delta_E})$. (23), (24)

12. $z_{\delta_e} = (\frac{\partial C_m}{\partial \Delta_e})(\frac{c}{l})$. (26)

where $T_A = (\frac{M_A}{\frac{\rho}{2}SU_o}), = \frac{U_o C_{L_o}}{g}$; (2)

and $\mu_c = \frac{M_A}{\frac{\rho}{2}Sc}, = \frac{T_A U_o}{c}, = \frac{U_o{}^2 C_{L_o}}{gc}$. (2)

(b) Formulae for Elevator Hinge Moment Stability Coefficients for Elevator-Servo System:

1. (m_{eh_u} assumed negligible).

2. $$m_{eh_w} = (\frac{S_{EAHL}}{S})[\frac{c c_{EAHL} M_A}{(I_{EH})_{EFF}}](\frac{\partial C_{EH}}{\partial \alpha'}). \qquad (29)$$

3. $$m_{eh_{\dot{w}}} = (\frac{\partial C_{EH}}{\partial \alpha'})(\frac{\partial \epsilon}{\partial \alpha})[\frac{\frac{\rho}{2} S_{EAHL} l c_{EAHL} c}{(I_{EH})_{EFF}}]. \qquad (31)$$

4. $$m_{eh_q} = (\frac{S_{EAHL}}{S})[\frac{l c_{EAHL} M_A}{(I_{EH})_{EFF}}](\frac{\partial C_{EH}}{\partial \alpha'}). \qquad (33)$$

5. $$m_{eh_{\delta_e}} = [\frac{T_A^2}{(I_{EH})_{EFF}}][(\frac{\partial C_{EH}}{\partial \Delta_E}) c_{EAHL} S_{EAHL} (\frac{\rho}{2}) U_o^2 - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 K_{SAE}]. \qquad (35)$$

6. $$m_{eh_{\dot{\delta}_e}} = [\frac{T_A}{(I_{EH})_{EFF}}][(\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}) c_E S_E (\frac{\rho}{2}) U_o^2 - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (C_{SAE})_{EFF}]. \qquad (37)$$

7. $$m_{eh_{f_e}} = [\frac{T_A^2}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial F_E}). \qquad (38)$$

8. $$m_{eh_{\dot{f}_e}} = [\frac{T_A}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial \dot{F}_E}). \qquad (39)$$

9. $m_{eh_{\frac{f_e}{d}}} = [\frac{T_A^3}{(I_{EH})_{EFF}}](\frac{\partial\theta_{SAE}}{\partial\Delta_E})(\frac{\partial M_{SAEM}}{\partial\int F_E dt}).$ (40)

where $T_A = \frac{M_A}{\frac{\rho}{2}SU_o} = \frac{U_o C_{L_o}}{g}$, (2)

and $(I_{EH})_{EFF} = I_{EH} + (\frac{\partial\theta_{SAE}}{\partial\Delta_E})^2 (I_{SAE})_{EFF}.$ (41)

(c) Formulae for Prediction Moment Stability Coefficients for Elevation Computer:

1. $m_{ce_{\dot{w}}} = (\frac{U_o}{T_{CSE}})S_{p(aP)_e}.$ (46)

2. $m_{ce_q} = [\frac{1}{1 + (SN)_e}].$ (51)

3. $m_{ce_\theta} = gS_{p(aP)_e}.$ (53)

4. $m_{ce_w} = U_o S_{p(\Delta W-P)_e}, = S_{p(w-P)_e}.$ (57)

Note that $T_{CSE} = [1 + (SN)_e][S_{p(WP)_e} + U_o S_{p(aP)_e}]$.

2. LATERAL FORMULAE:

(a) Formulae for Lateral Dynamic Stability Coefficients for Aircraft:

1. $$l_v = \left(\frac{\partial C_l}{\partial v}\right)\left(\frac{M_A b^2}{I_X}\right) \qquad (59)$$

2. $$l_r = C_{L_o}\left(\frac{M_A b^2}{6I_X}\right), \text{ for rectangular plan-form of wing.} \qquad (61)$$

$$= C_{L_o}\left(\frac{M_A b^2}{8I_X}\right), \text{ for elliptical plan-form of wing.} \qquad (62)$$

3. $$l_p = -\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{M_A b^2}{12I_X}\right), \text{ for rectangular plan-form of wing.} \qquad (64)$$

$$= -\left(\frac{\partial C_L}{\partial \alpha}\right)\left(\frac{M_A b^2}{16I_X}\right), \text{ for elliptical plan-form of wing.} \qquad (65)$$

4. $$l_{\delta_a} = \left(\frac{T_A^2}{I_X}\right)U_o^2 S\left(\frac{\rho}{2}\right)b\left(\frac{\partial C_l}{\partial \Delta_A}\right), = \left(\frac{M_A^2 b}{\frac{\rho}{2}SI_X}\right)\left(\frac{\partial C_l}{\partial \Delta_A}\right). \qquad (67), (68)$$

5. $$n_v = \left(\frac{\partial C_n}{\partial v}\right)\left(\frac{M_A b^2}{I_Z}\right). \qquad (70)$$

6. $$n_r = -\left(\frac{5}{4}\right)\eta''\left(\frac{S''}{S}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)''\left(\frac{M_A l^2}{I_Z}\right) - \left(\frac{C_{D_o}}{6}\right)\left(\frac{M_A b^2}{I_Z}\right) \qquad (72)$$

for rectangular plan-form of wing.

$$n_r = -\left(\frac{5}{4}\right)\eta''\left(\frac{S''}{S}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)''\left(\frac{M_A l^2}{I_Z}\right) - \left(\frac{C_{D_o}}{8}\right)\left(\frac{M_A b^2}{I_Z}\right), \tag{73}$$

for elliptical plan-form of wing.

7. $$n_p = -C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{M_A b^2}{12 I_Z}\right), \tag{75}$$

for rectangular plan-form of wing.

$$= -C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{M_A b^2}{16 I_Z}\right), \tag{76}$$

for elliptical plan-form of wing.

8. $$n_{\delta_r} = \left(\frac{T_A{}^2}{I_Z}\right) bS\left(\frac{\rho}{2}\right)U_o{}^2\left(\frac{\partial C_n}{\partial \Delta_R}\right), = \left(\frac{M_A{}^2 b}{\frac{\rho}{2} S I_Z}\right)\left(\frac{\partial C_n}{\partial \Delta_R}\right). \quad (78),\ (79)$$

9. $$y_v = \left(\frac{\partial C_Y}{\partial v}\right). \tag{81}$$

10. $$y_\phi = C_{L_o}. \tag{83}$$

11. $$y_{\delta_r} = -\left(\frac{b}{l}\right)\left(\frac{\partial C_n}{\partial \Delta_R}\right), \tag{85}$$

where $$T_A = \frac{U_o C_{L_o}}{g}, = \frac{M_A}{\frac{\rho}{2} S U_o}, \tag{2}$$

(b) Formulae for Rudder Hinge Moment Stability Coefficients for Rudder-Servo System.

1. (m_{rh_u} assumed negligible). (86)

2. $$m_{rh_v} = \left(\frac{S_{RAHL}}{S}\right)\left[\frac{bc_{RAHL}M_A}{(I_{RH})_{EFF}}\right]\left(\frac{\partial C_{RH}}{\partial v}\right). \qquad (88)$$

3. $$m_{rh_r} = -\left(\frac{S_{RAHL}}{S}\right)\left[\frac{lc_{RAHL}M_A}{(I_{RH})_{EFF}}\right]\left(\frac{\partial C_{RH}}{\partial v}\right). \qquad (90)$$

4. $$m_{rh_{\delta_r}} = \left[\frac{T_A^{\,2}}{(I_{RH})_{EFF}}\right]\left[\left(\frac{\partial C_{RH}}{\partial \Delta_R}\right)c_{RAHL}S_{RAHL}\left(\frac{\rho}{2}\right)U_o^{\,2} - \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 K_{SAR}\right]. \qquad (92)$$

5. $$m_{rh_{\dot{\delta}_r}} = \left[\frac{T_A}{(I_{RH})_{EFF}}\right]\left[\left(\frac{\partial C_{RH}}{\partial \dot{\Delta}_R}\right)c_R S_R\left(\frac{\rho}{2}\right)U_o^{\,2} - \left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)^2 (C_{SAR})_{EFF}\right]. \qquad (94)$$

6. $$m_{rh_{f_d}} = \left[\frac{T_A^{\,2}}{(I_{RH})_{EFF}}\right]\left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial F_D}\right). \qquad (95)$$

7. $$m_{rh_{\dot{f}_d}} = \left[\frac{T_A}{(I_{RH})_{EFF}}\right]\left(\frac{\partial \theta_{SAR}}{\partial \Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial \dot{F}_D}\right). \qquad (96)$$

8. $$m_{rh_{\frac{f_d}{d}}} = \left[\frac{T_A^{3}}{(I_{RH})_{EFF}}\right]\left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)\left(\frac{\partial M_{SARM}}{\partial\int F_D dt}\right). \quad (97)$$

where $$(I_{RH})_{EFF} = I_{RH} + \left(\frac{\partial\theta_{SAR}}{\partial\Delta_R}\right)^2 (I_{SAR})_{EFF}. \quad (98)$$

and $$T_A = \left(\frac{M_A}{\frac{\rho}{2}SU_o}\right), = \frac{U_o C_{L_o}}{g}. \quad (2)$$

(c) Formulae for Aileron Hinge Moment Stability Coefficients for Aileron-Servo System.

1. (m_{ah_p} assumed negligible). (100)

2. $$m_{ah_{\delta_a}} = \left[\frac{T_A^{2}}{(I_{AH})_{EFF}}\right]\left[2\left(\frac{\partial C_{AH}}{\partial\Delta_A}\right)c_{AAHL}S_{AAHL}\left(\frac{\rho}{2}\right)U_o^2 - 4\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)^2 K_{SAA}\right]. \quad (102)$$

3. $$m_{ah_{\dot\delta_a}} = \left[\frac{T_A}{(I_{AH})_{EFF}}\right]\left[2\left(\frac{\partial C_{AH}}{\partial\dot\Delta_A}\right)c_A S_A\left(\frac{\rho}{2}\right)U_o^2 - 4\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)^2 (C_{SAA})_{EFF}\right]. \quad (104)$$

4. $$m_{ah_{f_d}} = \left[\frac{4T_A^{2}}{(I_{AH})_{EFF}}\right]\left(\frac{\partial\theta_{SAA}}{\partial\Delta_A}\right)\left(\frac{\partial M_{SAAM}}{\partial F_D}\right). \quad (105)$$

5. $m_{ah_{\dot{f}_d}} = [\frac{4T_A}{(I_{AH})_{EFF}}](\frac{\partial\theta_{SAA}}{\partial\Delta_A})(\frac{\partial M_{SAAM}}{\partial\dot{F}_D}).$ (106)

6. $m_{ah_{\frac{f_d}{d}}} = [\frac{4T_A^3}{(I_{AH})_{EFF}}](\frac{\partial\theta_{SAA}}{\partial\Delta_A})(\frac{\partial M_{SAAM}}{\partial\int F_D dt}).$ (107)

where $(I_{AH})_{EFF} = 4(I_{SAA})_{EFF}(\frac{\partial\theta_{SAA}}{\partial\Delta_A})^2 + I_{AH}.$ (108)

and $I_{AH} = I_{AHR} + I_{AHL}.$ (109)

(d) Formulae for Prediction Moment Stability Coefficients for Deflection Computer:

1. $m_{cd_r} = [\frac{1}{1 + (SN)_d}].$ (113)

2. $m_{cd_p} = [\frac{1}{1 + (SN)_d}]\tan I_{GD}.$ (115)

3. $m_{cd_v} = U_o S_{p(\Delta V-P)_d} = S_{p(v-P)_d}.$ (119)

Note that

$T_{CSD} = [1 + (SN)_d][S_{p(WP)_d}]\cos I_{GD}.$

APPENDIX C

DEVELOPMENT OF NETWORK EQUATIONS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR AIRCRAFT

APPENDIX C

DEVELOPMENT OF NETWORK EQUATIONS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR AIRCRAFT.

The network equations consist of the equations of motion for the primary components of the system, and the tracking error angle equations.

The equations of motion for the primary components were derived in Appendix A. Each equation was made non-dimensional by using its own characteristic time. But before the network equations can be solved simultaneously they must be based upon a unit of time which is common to the entire system. Any unit of time could be used but, for reasons presented in the fore-going text, a system characteristic time equal to the initial time of flight of the aircraft will be chosen. Thus, let

$$T_S = T_{AF_o}, \tag{1}$$

where T_S is the system characteristic time, and T_{AF_o} is the initial time of flight of the aircraft. We may define the non-dimensional flight operator by

$$d_f = T_{AF_o} D. \tag{2}$$

Then

$$d_f = \left(\frac{R_{T_o}}{U_o}\right) D = \left(\frac{T_{AF_o}}{T_A}\right) d = \left(\frac{T_{AF_o}}{T_{CSE}}\right) d_{ce} = \left(\frac{T_{AF_o}}{T_{CSD}}\right) d_{cd}. \tag{3}$$

The tracking error angle equations are merely definitions of the elevation and deflection components of the tracking error angle:

$$F_E = A_{RE} + P_{CE} \tag{4}$$

$$F_D = A_{RD} + P_{CD} \tag{5}$$

Using the non-dimensional notation used in this thesis, we have:

$$f_e = a_{re} + p_{ce} \tag{4-a}$$

$$f_d = a_{rd} + p_{cd} \tag{5-a}$$

where

$$f_e \equiv F_E,$$

$$\dot{f}_e = T_S \dot{F}_E = T_{AF_o}{}^{*} \dot{F}_E, \text{ etc.}$$

The network equations, based upon the initial time of flight of the aircraft, may be found by combining equations 3, 4-a and 5-a with the equations of motion summarized at the end of Appendix A.

A summary of the network equations for the case when m_u, z_θ, m_{eh_u}, m_{rh_u}, and $m_{ah_p} = 0$, is given on pages C-3, C-4 and C-5.

SUMMARY OF NETWORK EQUATIONS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF AIRCRAFT

Longitudinal Network Equations

For m_u, z_θ, m_{eh_u}, m_{rh_u} and $m_{ah_p} = 0$

AIRCRAFT LONGITUDINAL EQUATIONS OF MOTION

$$[d_f - (\frac{T_{AF_o}}{T_A}) x_u] u - [(\frac{T_{AF_o}}{T_A}) x_w] w - [(\frac{T_{AF_o}}{T_A}) x_\theta] \theta = 0.$$

$$[-(\frac{T_{AF_o}}{T_A}) z_u] u + [d_f - (\frac{T_{AF_o}}{T_A}) z_w] w - d_f \theta - [(\frac{T_{AF_o}}{T_A}) z_{\delta_e}] \delta_e = 0.$$

$$[-d_f (\frac{T_{AF_o}}{T_A}) \mu_c m_{\dot{w}} - (\frac{T_{AF_o}}{T_A})^2 \mu_c m_w] w + [d_f^2 - d_f (\frac{T_{AF_o}}{T_A}) m_q] \theta - [(\frac{T_{AF_o}}{T_A})^2 m_{\delta_e}] \delta_e = 0.$$

ELEVATOR-SERVO SYSTEM EQUATION OF MOTION

$$[-d_f (\frac{T_{AF_o}}{T_A}) \mu_c m_{eh_{\dot{w}}} - (\frac{T_{AF_o}}{T_A})^2 \mu_c m_{eh_w}] w - d_f [(\frac{T_{AF_o}}{T_A}) m_{eh_q}] \theta + [d_f^2 - d_f (\frac{T_{AF_o}}{T_A}) m_{eh_{\dot{\delta}_e}}$$

$$- (\frac{T_{AF_o}}{T_A})^2 m_{eh_{\delta_e}}] \delta_e = [d_f (\frac{T_{AF_o}}{T_A}) m_{eh_{\dot{f}_e}} + (\frac{T_{AF_o}}{T_A})^2 m_{eh_{f_e}}$$

$$+ (\frac{1}{d_f}) (\frac{T_{AF_o}}{T_A})^3 m_{eh_{\frac{f_e}{d}}} + \dots] f_e.$$

ELEVATION COMPUTER EQUATION OF MOTION

$$[d_f m_{ce_{\dot{w}}} + (\frac{T_{AF_o}}{T_{CSE}}) m_{ce_w}] w - (d_f m_{ce_q}) \theta + [d_f + (\frac{T_{AF_o}}{T_{CSE}})] p_{ce} - (\frac{T_{AF_o}}{T_{CSE}}) m_{ce_\theta} = 0.$$

IDEAL RADAR ELEVATION EQUATION OF MOTION

$$d_f a_{re} - \left[\frac{w}{1-(\frac{u}{d_f})}\right] + \left\{d_f + \left[\frac{1}{1-(\frac{u}{d_f})}\right]\right\}\theta = 0.$$

ELEVATION TRACKING ERROR ANGLE EQUATION

$$f_e = a_{re} + p_{ce}.$$

Lateral Network Equations

AIRCRAFT LATERAL EQUATIONS OF MOTION

$$\left\{\begin{array}{l} [-\mu_b l_v(\frac{T_{AF_o}}{T_A})^2]v - [l_r(\frac{T_{AF_o}}{T_A})d_f]\psi + [d_f^2 - l_p(\frac{T_{AF_o}}{T_A})d_f]\phi - [l_{\delta_a}(\frac{T_{AF_o}}{T_A})^2]\delta_a = 0. \\ [-\mu_b n_v(\frac{T_{AF_o}}{T_A})^2]v + [d_f^2 - n_r(\frac{T_{AF_o}}{T_A})d_f]\psi - [n_p(\frac{T_{AF_o}}{T_A})d_f]\phi - [n_{\delta_r}(\frac{T_{AF_o}}{T_A})^2]\delta_r = 0. \\ [d_f - (\frac{T_{AF_o}}{T_A})y_v]v + d_f\psi - [y_\phi(\frac{T_{AF_o}}{T_A})]\phi - [y_{\delta_r}(\frac{T_{AF_o}}{T_A})]\delta_r = 0. \end{array}\right.$$

RUDDER-SERVO SYSTEM EQUATIONS OF MOTION

$$\left\{\begin{array}{l} [-\mu_b m_{rh_v}(\frac{T_{AF_o}}{T_A})^2]v - [m_{rh_r}(\frac{T_{AF_o}}{T_A})d_f]\psi + [d_f^2 - m_{rh_{\dot{\delta}_r}}(\frac{T_{AF_o}}{T_A})d_f - m_{rh_{\delta_r}}(\frac{T_{AF_o}}{T_A})^2]\delta_r \\ \quad = [d_f(\frac{T_{AF_o}}{T_A})m_{rh_{\dot{f}_d}} + (\frac{T_{AF_o}}{T_A})^2 m_{rh_{f_d}} + m_{rh_{\frac{f_d}{d}}}(\frac{T_{AF_o}}{T_A})^3(\frac{1}{d_f}) + \dots]f_d. \end{array}\right.$$

AILERON-SERVO SYSTEM EQUATIONS OF MOTION

$$\begin{cases} [d_f^{\,2} - m_{ah_{\dot{\delta}_a}} (\frac{T_{AF_o}}{T_A}) d_f - m_{ah_{\delta_a}} (\frac{T_{AF_o}}{T_A})^2] \delta_a = [d_f (\frac{T_{AF_o}}{T_A}) m_{ah_{\dot{f}_d}} + (\frac{T_{AF_o}}{T_A})^2 m_{ah_{f_d}} \\ \quad + m_{ah_{\frac{f_d}{d}}} (\frac{T_{AF_o}}{T_A})^3 (\frac{1}{d_f}) + \cdots] f_d. \end{cases}$$

DEFLECTION COMPUTER EQUATION OF MOTION

$$[(\frac{T_{AF_o}}{T_{CSD}}) m_{cd_v}] v - (d_f m_{cd_r}) \psi + (d_f m_{cd_p}) \phi + [d_f + (\frac{T_{AF_o}}{T_{CSD}})] p_{cd} = 0.$$

IDEAL RADAR DEFLECTION EQUATION OF MOTION

$$d_f\, a_{rd} + [\frac{v}{1-(\frac{u}{d_f})}] + \{d_f + [\frac{1}{1-(\frac{u}{d_f})}]\} \psi = 0.$$

ELEVATION TRACKING ERROR ANGLE EQUATION

$$f_d = a_{rd} + p_{cd}.$$

APPENDIX D

DETERMINATION OF STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE

APPENDIX D

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APPENDIX D

DETERMINATION OF STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE

A. LONGITUDINAL STABILITY COEFFICIENTS:

1. DETERMINATION OF THEORETICAL LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR THE A-26 AIRPLANE:

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen as a basis for the analyses which follow. Using the formulae derived in section A-1, Appendix B, and the data given at the end of the text, the theoretical longitudinal dynamic stability coefficients for the A-26 Airplane were computed as follows:

(a) Preliminary Aerodynamic Computations:

$$(1)\quad T_A = \frac{M_A}{\frac{\rho}{2} S U_o} = \frac{17.83(\frac{W_A}{S})}{\sigma U_{o_{mph}},\text{ true}}$$

$$= \frac{(17.83)(30,000)}{(540)(300)(.7384)}$$

$$= 4.4716 \text{ sec.}$$

$$(2)\quad \mu_c = 26.12 \frac{(\frac{W_A}{S})}{\sigma c_{ft}}$$

$$= \frac{(26.12)(30,000)}{(540)(.7384)(8.14)}$$

$$= 241.43$$

(3) $\frac{1}{2}\rho U_o^2 = \frac{\sigma U_o{}^2{}_{mph,\,true}}{391}$

$= \frac{(.7384)(300)^2}{391}$

$= 169.964\ {}^{lbs}/_{ft^2}$

(4) $C_{L_o} = \frac{(\frac{W_A}{S})}{\frac{1}{2}\rho U_o^2}$

$= \frac{(30,000)}{(540)(169.964)}$

$= .32687$

(5) $C_{D_o} = C_{D_{p_{min}}} + \frac{C_{L_o}^2}{e\pi R}$

$= .0244 + \frac{(.32687)^2}{(.8)(9.07)(3.1416)}$

$= .029087$

(b) Longitudinal Dynamic Stability Coefficients:

(1) $x_u = -3C_{D_o}$, power on.

$= -3(.029087)$

$= -.087261$

(2) $x_w = C_{L_o}\ [1 - (\frac{2}{e\pi R})(\frac{\partial C_L}{\partial \alpha})]$

$= .32687\ [1 - \frac{2(4.8701)}{(.8)(9.07)(3.1416)}]$

$= .18720$

(3) $x_\theta = -C_{L_o}$

$= -.32687$

(4) $z_u = -2C_{L_o}$

$= -2(.32687)$

$= -.65374$

(5) $z_w = -\left(\frac{\partial C_L}{\partial \alpha}\right)$

$= -4.8701$

(6) $z_\theta = 0$ for $\theta_o = 0.$

(7) m_u assumed negligible.

(8) $m_w = \left(\frac{M_A c^2}{I_Y}\right)\left(\frac{\partial C_m}{\partial \alpha}\right)$

$$= \frac{(30{,}000)(8.14)^2(-.48701)}{(32.2)(48{,}531)}$$

$= -.61949$

(9) $m_{\dot{w}} = -\left(\frac{\partial C_L}{\partial \alpha}\right)'\left(\frac{\partial \epsilon}{\partial \alpha}\right)\left(\frac{\frac{\rho}{2} S' l^2 c \eta'}{I_Y}\right)$

$= -(3.8904)(.41)$ multiplied by

$$\left[\frac{(.002378)(.7384)(116.1)(30.1)^2(8.14)(.9)}{(2)(48{,}531)}\right]$$

$= -.022336$

(10) $m_q = -\frac{5}{4}\left(\frac{\partial C_L}{\partial \alpha}\right)'\left(\frac{S'}{S}\right)\left(\frac{\eta' M_A l^2}{I_Y}\right)$

$$= -\left(\frac{5}{4}\right)(3.8904)\left(\frac{116.1}{540}\right)\left[\frac{(.9)(30{,}000)(30.1)^2}{(32.2)(48{,}531)}\right]$$

$$= -16.3668$$

$$(11)\quad m_{\delta_e} = \left(\frac{\partial C_m}{\partial \Delta_E}\right)\mu_c^2\left(\frac{\frac{\rho}{2} Sc^3}{I_Y}\right)$$

$$= (-1.146)(241.43)^2 \text{ multiplied by}$$

$$\left[\frac{(.7384)(.002378)(540)(8.14)^3}{(2)(48{,}531)}\right]$$

$$= -351.95$$

$$(12)\quad z_{\delta_e} = \left(\frac{\partial C_m}{\partial \Delta_E}\right)\left(\frac{c}{l}\right)$$

$$= (-1.146)\left(\frac{8.14}{30.1}\right)$$

$$= -.3099$$

2. DETERMINATION OF ELEVATOR HINGE MOMENT STABILITY COEFFICIENTS FOR AN ELEVATOR-SERVO SYSTEM FOR THE A-26 AIRPLANE:

Using the formulae derived in section A-2, Appendix B, and the data given at the end of the text, the Elevator Hinge Moment Stability Coefficients were computed. A basis of 300 mph true airspeed at 10,000 feet density altitude was used.

(a) Preliminary Computations:

$$(1)\quad (I_{SAE})_{EFF} = \frac{K_{SAE}}{\omega^2_{n_{SAE}}}$$

$$= \frac{2500}{(5.1)^2}$$

$$= 96.117 \text{ slugs-ft}^2$$

(2) $(C_{SAE})_{EFF} = 2\zeta_{SAE}\omega_{n_{SAE}}(I_{SAE})_{EFF}$

$$= (2)(1.1)(5.1)(96.117)$$

$$= 1078.4 \frac{\text{ft-lbs}}{\text{radian/sec.}}$$

(3) $(I_{EH})_{EFF} = I_{EH} + \left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)^2 (I_{SAE})_{EFF}$

$$= 2.62 + (4.2)^2(96.117)$$

$$= 1698.1 \text{ slugs-ft}^2$$

(b) Elevator Hinge Moment Stability Coefficients:

(1) m_{eh_u} assumed negligible.

(2) $m_{eh_w} = \left(\frac{S_{EAHL}}{S}\right)\left[\frac{c c_{EAHL} M_A}{(I_{EH})_{EFF}}\right]\left(\frac{\partial C_{EH}}{\partial\alpha'}\right)$

$$= \left(\frac{22.36}{540}\right)\left[\frac{(8.14)(1.27)(30,000)}{(32.2)(1698.1)}\right](-.2865)$$

$$= -.067287$$

(3) $m_{eh_{\dot{w}}} = \left(\frac{\partial C_{EH}}{\partial\alpha'}\right)\left(\frac{\partial\epsilon}{\partial\alpha}\right)\left[\frac{\frac{\rho}{2} S_{EAHL} l c_{EAHL} c}{(I_{EH})_{EFF}}\right]$

$= (-.2865)(.41)$ multiplied by

$$\left[\frac{(.7384)(.002378)(22.36)(8.14)(1.27)(30.1)}{(2)(1698.1)}\right]$$

$$= -.00042254$$

(4) $m_{eh_q} = \left(\frac{\partial C_{EH}}{\partial\alpha'}\right)\left[\frac{l c_{EAHL} M_A}{(I_{EH})_{EFF}}\right]\left(\frac{S_{EAHL}}{S}\right)$

$$= (-.2865)\left[\frac{(30.1)(1.27)(30,00)}{(32.2)(1698.1)}\right]\left(\frac{22.36}{540}\right)$$

$$= -.24881$$

(5) $$m_{eh_{\delta_e}} = [\frac{T_A^2}{(I_{EH})_{EFF}}][(\frac{\partial C_{EH}}{\partial \Delta_E}) c_{EAHL} S_{EAHL} \frac{\rho}{2} U_o^2$$

$$-(\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 K_{SAE}]$$

$$= \frac{(4.4716)^2}{1698.1} \text{ multiplied by}$$

$$[\frac{(-.5157)(1.27)(22.36)(.002378)(.7384)(440)^2}{(2)}$$

$$-(4.2)^2(2500)]$$

$$= -548.53$$

(6) $$m_{eh_{\dot{\delta}_e}} = [\frac{T_A}{(I_{EH})_{EFF}}][(\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}) c_E S_E \frac{\rho}{2} U_o^2 - (\frac{\partial \theta_{SAE}}{\partial \Delta_E})^2 (C_{SAE})_{EFF}]$$

$$= \frac{4.4716}{1698.1} \text{ multiplied by}$$

$$[(-.0056523)(1.85)(32.66)(.002378)(\frac{1}{2})(.7384)(440)^2$$

$$-(4.2)^2(1078.4)]$$

$$= -50.247$$

(7) $$m_{eh_{f_e}} = [\frac{T_A^2}{(I_{EH})_{EFF}}](\frac{\partial \theta_{SAE}}{\partial \Delta_E})(\frac{\partial M_{SAEM}}{\partial F_E})$$

$$= (4.4716)^2 \frac{1}{1698.1} (4.2)(-100)$$

$$= -4.9456$$

$$(8)\quad m_{eh_{f_e}} = \left[\frac{T_A}{(I_{EH})_{EFF}}\right]\left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial f_E}\right)$$

$$= \frac{4.4716}{1698.1}(4.2)(\text{undecided yet})$$

$$= \text{undecided yet.}$$

$$(9)\quad m_{eh_{\frac{f_e}{d}}} = \left[\frac{T_A{}^3}{(I_{EH})_{EFF}}\right]\left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial\int F_E dt}\right)$$

$$= \frac{(4.4716)^3}{1698.1}(4.2)\ (\text{undecided yet})$$

$$= \text{undecided yet.}$$

3. DETERMINATION OF PREDICTION MOMENT STABILITY COEFFICIENTS FOR AN ELEVATION COMPUTER FOR THE A-26 AIRPLANE

For a range of 1000 yards, at 10,000 feet density altitude, and for a .50 caliber gun, the following values for $S_{p(aP)_e}$ and $S_{p(WP)_e}$ were taken from reference 3:

$$S_{p(aP)_e} = 1.8247 \times 10^{-4}\ \frac{sec^2}{ft}, \text{ and}$$

$$S_{p(WP)_e} = 1.1751 \text{ sec.}$$

A stability number of .2 has been found to be satisfactory for manual tracking, so it was used as an initial trial value in this analysis for automatic tracking.

Thus, $T_{CSE} = [1 + (SN)_e][S_{p(WP)_e} + U_o S_{p(aP)_e}]$

$$= [1 + .2][1.1751 + (440)(1.8247 \times 10^{-4})]$$

$$= 1.5065 \text{ sec.}$$

Then, from Section A-3, Appendix B,

(a) $m_{ce_{\dot{w}}} = (\frac{U_o}{T_{CSE}}) S_p (aP)_e$

$= (\frac{440}{1.5065})(1.8247 \times 10^{-4})$

$= .053294$

(b) $m_{ce_q} = \frac{1}{1 + (SN)_e}$

$= \frac{1}{1 + .2}$

$= .83333$

(c) $m_{ce_\theta} = g \times S_p (aP)_e$

$= (32.2)(1.8247 \times 10^{-4})$

$= .0058755$

(d) m_{ce_w} was neglected in this analysis.

B. LATERAL STABILITY COEFFICIENTS:

1. DETERMINATION OF THEORETICAL LATERAL DYNAMIC STABILITY COEFFICIENTS FOR THE A-26 AIRPLANE:

Using a basis of 300 mph true airspeed at 10,000 feet density altitude, and the formulae derived in section B-1, Appendix B, and the data given at the end of the text, the theoretical lateral dynamic stability coefficients for the A-26 Airplane were computed as follows:

(a) Preliminary Aerodynamic Computations:

(1) T_A = 4.4716 sec (previously computed)

(2) $\mu_b = \frac{M_A}{\frac{\rho}{2}Sb}$

$= \frac{(30,000)(2)}{(32.2)(540)(70)(.002378)(.7384)}$

$= 28.074$

(3) C_{L_o} = .32687 (previously computed)

(4) C_{D_o} = .029087 (previously computed)

(b) Lateral Dynamic Stability Coefficients:

(1) $l_v = (\frac{\partial C_l}{\partial v})(\frac{M_A b^2}{I_X})$

$= (-.1054)[\frac{(30,000)(70)^2}{(32.2)(66,903)}]$

$= -7.19205$

(2) $l_r = C_{L_o}(\frac{M_A b^2}{6I_X})$, for rectangular plan-form of wing.

$= (.32687)[\frac{(30,000)(70)^2}{(32.2)(6)(66,903)}]$

$= 3.7174$

(3) $l_p = -(\frac{\partial C_L}{\partial \alpha})(\frac{M_A b^2}{12I_X})$, for rectangular plan-form of of wing.

$= -(4.8701)[\frac{(30,000)(70)^2}{(32.2)(12)(66,903)}]$

$= -27.693$

$$(4)\quad l_{\delta_a} = \left(\frac{M_A{}^2 b}{\frac{\rho}{2} S I_X}\right)\left(\frac{\partial C_l}{\partial \Delta_A}\right)$$

$$= \frac{(30{,}000)^2(70)(2)(.16)}{(32.2)^2(.7384)(.002378)(540)(66{,}903)}$$

$$= 306.52$$

$$(5)\quad n_v = \left(\frac{\partial C_n}{\partial v}\right)\left(\frac{M_A b^2}{I_Z}\right)$$

$$= (.0630)\left[\frac{(30{,}000)(70)^2}{(32.2)(111{,}555)}\right]$$

$$= 2.5782$$

$$(6)\quad n_r = -\left(\frac{5}{4}\right)\eta''\left(\frac{S''}{S}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)''\left(\frac{M_A l^2}{I_Z}\right) - \left(\frac{C_{D_o}}{6}\right)\left(\frac{M_A b^2}{I_Z}\right)$$

for rectangular plan-form of wing.

$$= -\frac{5}{4}(.9)\left(\frac{71.35}{540}\right)(2.607)\left[\frac{(30{,}000)(30.1)^2}{(32.2)(111{,}555)}\right]$$

$$- \frac{.029087}{6}\left[\frac{(30{,}000)(70)^2}{(32.2)(111{,}555)}\right]$$

$$= -3.1307$$

$$(7)\quad n_p = -C_{L_o}\left[1 - \left(\frac{2}{e\pi R}\right)\left(\frac{\partial C_L}{\partial \alpha}\right)\right]\left(\frac{M_A b^2}{12 I_Z}\right)$$

for rectangular plan-form of wing.

$= -$ (.32687) multiplied by

$$\left[1 - \frac{(2)(4.8701)}{(.8)(3.1416)(9.07)}\right]\left[\frac{(70)^2(30{,}000)}{(32.2)(12)(111{,}555)}\right]$$

$$= -.63842$$

(8) $n_{\delta_r} = \left(\frac{M_A{}^2 b}{\frac{\rho}{2} S I_Z}\right)\left(\frac{\partial C_n}{\partial \Delta_R}\right)$

$$= \frac{(30{,}000)^2(70)(2)(.0917)}{(32.2)^2(540)(111{,}555)(.7384)(.002378)}$$

$= 105.35$

(9) $y_v = \left(\frac{\partial C_Y}{\partial v}\right)$

$= -.65$

(10) $y_\phi = C_{L_o}$

$= .32687$

(11) $y_{\delta_r} = -\left(\frac{b}{1}\right)\left(\frac{\partial C_n}{\partial \Delta_R}\right)$

$$= -\frac{(70)(.0917)}{(30.1)}$$

$= -.2133$

2. DETERMINATION OF RUDDER HINGE MOMENT STABILITY COEFFICIENTS FOR A RUDDER-SERVO SYSTEM FOR THE A-26 AIRPLANE:

Using the formulae derived in section B-2, Appendix B, and the data given at the end of the text, the Rudder Hinge Moment Stability Coefficients were computed. Here also, a basis of 300 mph true airspeed, at 10,000 feet density altitude, was used.

(a) Preliminary Computations

(1) $(I_{SAR})_{EFF} = \frac{K_{SAR}}{\omega_{n_{SAR}}^2}$

$$= \frac{2500}{(5.1)^2}$$

$$= 96.117 \text{ slugs-ft}^2$$

(2) $(C_{SAR})_{EFF} = 2\zeta_{SAR}\omega_{n_{SAR}}(I_{SAR})_{EFF}$

$$= (2)(1.1)(5.1)(96.117)$$

$$= 1078.4 \frac{\text{ft-lbs}}{\text{radian/sec}}$$

(3) $(I_{RH})_{EFF} = I_{RH} + (\frac{\partial\theta_{SAR}}{\partial\Delta_R})^2(I_{SAR})_{EFF}$

$$= \frac{11,400}{(32.2)(144)} + (6.8333)^2(96.117)$$

$$= 4490.59 \text{ slugs-ft}^2$$

(b) Rudder Hinge Moment Stability Coefficients

(1) m_{rh_u} assumed negligible

(2) $m_{rh_v} = (\frac{S_{RAHL}}{S})[\frac{bc_{RAHL}M_A}{(I_{RH})_{EFF}}]\frac{\partial C_{RH}}{\partial v}$

$$= (\frac{23}{540})[\frac{(70)(28.5)(30,000)}{(12)(32.2)(4490.59)}](-.085944)$$

$$= -.12626$$

(3) $m_{rh_r} = -(\frac{S_{RAHL}}{S})[\frac{lc_{RAHL}M_A}{(I_{RH})_{EFF}}]\frac{\partial C_{RH}}{\partial v}$

$$= -(\frac{23}{540})[\frac{(30.1)(28.5)(30,000)}{(12)(32.2)(4490.59)}](-.085944)$$

$$= .054293$$

(4) $m_{rh_{\delta_r}} = [\frac{T_A^2}{(I_{RH})_{EFF}}][(\frac{\partial C_{RH}}{\partial\Delta_R})c_{RAHL}S_{RAHL}(\frac{\rho}{2})U_o^2 - (\frac{\partial\theta_{SAR}}{\partial\Delta_R})^2 K_{SAR}]$

$$= \frac{(4.4716)^2}{(4490.59)} \text{ multiplied by}$$

$$[\frac{(-.22918)(28.5)(23)(.002378)(.7384)(440)^2}{(12)(2)}$$

$$-(6.8333)^2(2500)]$$

$$= -529.238$$

(5) $$m_{rh_{\dot{\delta}_r}} = [\frac{T_A}{(I_{RH})_{EFF}}][(\frac{\partial C_{RH}}{\partial \dot{\Delta}_R})c_R S_R(\frac{\rho}{2})U_o^2 - (\frac{\partial \theta_{SAR}}{\partial \Delta_R})^2(C_{SAR})_{EFF}]$$

$$= \frac{(4.4716)}{(4490.59)} \text{ multiplied by}$$

$$[\frac{(-2.2548\times 10^{-3})(37)(30)(440)^2(.002378)(.7384)}{(12)(2)}$$

$$-(6.8333)^2(1078.4)]$$

$$= -50.177$$

(6) $$m_{rh_{f_d}} = [\frac{T_A^2}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial F_D})$$

$$= \frac{(4.4716)^2}{(4490.59)}(6.8333)(300)$$

$$= 9.1279$$

(7) $$m_{rh_{\dot{f}_d}} = [\frac{T_A}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial \dot{F}_D})$$

$$= \frac{4.4716}{4490.59}(6.8333)(\text{Undecided yet})$$

= undecided yet

(8) $$m_{rh_{\frac{f_d}{d}}} = [\frac{T_A^3}{(I_{RH})_{EFF}}](\frac{\partial \theta_{SAR}}{\partial \Delta_R})(\frac{\partial M_{SARM}}{\partial \int F_D dt})$$

$$= \frac{(4.4716)^3}{4490.59} (6.8333)(\text{undecided yet})$$

$$= \text{undecided yet}$$

3. DETERMINATION OF AILERON HINGE MOMENT STABILITY COEFFICIENTS FOR AN AILERON-SERVO SYSTEM FOR THE A-26 AIRPLANE:

Using the formulae derived in section B-3, Appendix B, and the basic data given at the end of the text, Aileron Hinge Moment Stability Coefficients were computed. Here also, a basis of 300 mph true airspeed at 10,000 feet density altitude was used.

(a) Preliminary Computations

(1) $$(I_{SAA})_{EFF} = \frac{K_{SAA}}{\omega^2_{n_{SAA}}}$$

$$= \frac{2500}{(5.1)^2}$$

$$= 96.117 \text{ slugs-ft}^2$$

(2) $$(C_{SAA})_{EFF} = 2\zeta_{SAA}\omega_{n_{SAA}}(I_{SAA})_{EFF}$$

$$= (2)(1.1)(5.1)(96.117)$$

$$= 1078.4 \frac{\text{ft-lbs}}{\text{radian/sec.}}$$

(3) $$I_{AH} = I_{AHR} + I_{AHL}$$

$$= \frac{1900}{32.2 \times 144} + \frac{1900}{32.2 \times 144}$$

$= .81953$ slugs-ft^2

(4) $(I_{AH})_{EFF} = 4(I_{SAA})_{EFF}(\frac{\partial\theta_{SAA}}{\partial\Delta_A})^2 + I_{AH}$

$= 4(96.117)(2.833)^2 + .81953$

$= 3086.5$ slugs-ft^2

(b) Aileron Hinge Moment Stability Coefficients:

(1) m_{ah_p} assumed negligible

(2) $m_{ah_{\delta_a}} = [\frac{T_A{}^2}{(I_{AH})_{EFF}}]$ multiplied by

$[2(\frac{\partial C_{AH}}{\partial\Delta_A})c_{AAHL}S_{AAHL}(\frac{\rho}{2})U_o{}^2 - 4(\frac{\partial\theta_{SAA}}{\partial\Delta_A})^2 K_{SAA}]$

$= \frac{(4.4716)^2}{(3086.5)}$ multiplied by

$[\frac{(2)(-.0031)(180)(14.5)(18.2)(.7384)(.002378)(440)^2}{(3.1416)(12)(2)}$

$- 4(2.833)^2(2500)]$

$= -528.537$

(3) $m_{ah_{\dot{\delta}_a}} = [\frac{T_A}{(I_{AH})_{EFF}}]\,[\underbrace{2(\frac{\partial C_{AH}}{\partial\dot{\Delta}_A})c_A S_A \frac{\rho}{2} U_o{}^2}_{\text{negligible}} - 4(\frac{\partial\theta_{SAA}}{\partial\Delta_A})^2 (C_{SAA})_{EFF}]$

$= \frac{4.4716}{3086.5}\quad[\qquad\qquad - 4(2.833)^2(1078.4)]$

$= -50.156$

(4) $m_{ah_{f_d}} = [\frac{4T_A{}^2}{(I_{AH})_{EFF}}](\frac{\partial\theta_{SAA}}{\partial\Delta_A})(\frac{\partial M_{SAAM}}{\partial F_D})$

$$= \frac{4(4.4716)^2}{(3086.5} (2.833)(50)$$

$$= 3.6706$$

(5) $$m_{ah\dot{f}_d} = [\frac{4T_A}{(I_{AH})_{EFF}}](\frac{\partial\theta_{SAA}}{\partial\Delta_A})(\frac{\partial M_{SAAM}}{\partial\dot{F}_D})$$

$$= \frac{4(4.4716)}{(3086.5)} (2.833)(\text{undecided yet})$$

$$= \text{undecided yet}$$

(6) $$m_{ah\frac{f_d}{d}} = [\frac{4T_A^{\ 3}}{(I_{AH})_{EFF}}](\frac{\partial\theta_{SAA}}{\partial\Delta_A})(\frac{\partial M_{SAAM}}{\partial\int F_D dt})$$

$$= \frac{4(4.4716)^3}{(3086.5)} (2.833)(\text{undecided yet})$$

$$= \text{undecided yet}$$

4. DETERMINATION OF PREDICTION MOMENT STABILITY COEFFICIENTS FOR A DEFLECTION COMPUTER FOR THE A-26 AIRPLANE:

For a range of 1000 yards, at 10,000 feet density altitude, and for a .50 caliber gun, the following value of $S_{p(WP)_d}$ was taken from reference 3:

$$S_{p(WP)_d} = 1.1751 \text{ sec.}$$

A stability number of .2 has been found to be satisfactory for manual tracking, so it was used as an initial trial value in this analysis for automatic tracking.

Thus,

$$T_{CSD} = [1 + (SN)_d][S_{p(WP)_d}] \cos I_{GD}$$

$= (1 + .2)(1.1751)(\cos 10^{\circ})$

$= 1.3887$ sec.

Then, from Section B-4, Appendix B,

(a) $m_{cd_r} = \frac{1}{1 + (SN)_d}$

$= \frac{1}{1 + .2}$

$= .83333$

(b) $m_{cd_p} = [\frac{1}{1 + (SN)_d}] \tan I_{GD}$

$= [\frac{1}{1 + .2}](\tan 10^{\circ})$

$= .14694$

(c) m_{cd_v} was neglected in this analysis.

C. SUMMARY OF STABILITY COEFFICIENTS FOR A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE:

1. LONGITUDINAL STABILITY COEFFICIENTS:

(a) Theoretical Longitudinal Dynamic Stability Coefficients for the A-26 Airplane:

For $U_o = 300$ mph true airspeed at 10,000 feet density altitude,

$T_A = 4.4716$ sec

$\mu_c = 241.43$

$C_{L_o} = .32687$

$C_{D_o} = .029087$

$x_u = -.087261$

$x_w = .18720$

$x_\theta = -.32687$

$z_u = -.65374$

$z_w = -4.8701$

$z_\theta = 0$ (for $\theta_o = 0$)

m_u assumed negligible

$m_w = -.61949$

$m_{\dot{w}} = -.022336$

$m_q = -16.3668$

$m_{\delta_e} = -351.95$

$z_{\delta_e} = -.3099$

(b) Elevator Hinge Moment Stability Coefficients for an Elevator-Servo System for the A-26 Airplane:

For $U_o = 300$ mph true airspeed at 10,000 feet density altitude,

$T_A = 4.4716$ sec

m_{eh_u} assumed negligible

$m_{eh_w} = -.067287$

$m_{eh_{\dot{w}}} = -.00042254$

$m_{eh_q} = -.24881$

$m_{eh_{\delta_e}} = -548.53$

$m_{eh_{\dot{\delta}_e}} = -50.247$

$m_{eh_{f_e}} = -4.9456$

$m_{eh_{\dot{f}_e}} =$ undecided yet

$m_{eh_{\frac{f_e}{d}}} =$ undecided yet

(c) Prediction Moment Stability Coefficients for an Elevation Computer for the A-26 Airplane:

For a range of 1000 yards, at 10,000 feet density altitude, and for a .50 caliber gun,

$T_{CSE} = 1.5065$ sec.

$m_{ce_{\dot{w}}} = .053294$

$m_{ce_q} = .83333$

$m_{ce_\theta} = .0058755$

m_{ce_w} was neglected in this analysis

2. LATERAL STABILITY COEFFICIENTS:

(a) Theoretical Lateral Dynamic Stability Coefficients for the A-26 Airplane:

For $U_o = 300$ mph true airspeed at 10,000 feet density altitude,

$T_A = 4.4716$ sec.

$\mu_b = 28.074$

$C_{L_o} = .32687$

$C_{D_o} = .029087$

$l_v = -7.19205$

$l_r = 3.7174$

$l_p = -27.693$

$l_{\delta_a} = 306.52$

$n_v = 2.5782$

$n_r = -3.1307$

$n_p = -.63842$

$n_{\delta_r} = 105.35$

$y_v = -.65$

$y_\phi = .32687$

$y_{\delta_r} = -.2133$

(b) Rudder Hinge Moment Stability Coefficients for a Rudder-Servo System for the A-26 Airplane:

For U_o = 300 mph true airspeed at 10,000 feet density altitude:

$T_A = 4.4716$ sec

m_{rh_u} assumed negligible

$m_{rh_v} = -.12626$

$m_{rh_r} = .054293$

$m_{rh_{\delta_r}} = -529.238$

$m_{rh_{\dot{\delta}_r}} = -50.177$

$m_{rh_{f_d}} = 9.1279$

$m_{rh_{\dot{f}_d}} =$ undecided yet

$m_{rh_{\frac{f_d}{d}}} =$ undecided yet

(c) Aileron Hinge Moment Stability Coefficients for an Aileron-Servo System for the A-26 Airplane:

For $U_o = 300$ mph true airspeed at 10,000 feet density altitude:

$T_A = 4.4716$ sec

m_{ah_p} assumed negligible

$m_{ah_{\delta_a}} = -528.537$

$m_{ah_{\dot{\delta}_a}} = -50.156$

$m_{ah_{f_d}} = 3.6706$

$m_{ah_{\dot{f}_d}} =$ undecided yet

$m_{ah_{\frac{f_d}{d}}} =$ undecided yet

(d) Prediction Moment Stability Coefficients for a Deflection Computer for the A-26 Airplane:

For a range of 1,000 yards, at 10,000 feet density altitude, and for a .50 caliber gun,

$T_{CSD} = 1.3887$ sec.

$m_{cd_r} = .83333$

$m_{cd_p} = .14694$

m_{cd_v} was neglected in this analysis.

A P P E N D I X   E

EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE

APPENDIX E

EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE.

The equations of motion were derived in Appendix A, and the formulae for the stability coefficients used in these equations were derived in Appendix B. The stability coefficients for a system for automatic tracking control of A-26 Airplane were determined in Appendix D.

If we put the stability coefficients determined in Appendix D, in the equations of motion derived in Appendix A, we obtain the equations of motion for a system for automatic tracking control of the A-26 Airplane.

A summary of the equations of motion, for the case when m_u, z_θ, m_{eh_u}, m_{rh_u}, and $m_{ah_p} = 0$, with displacement control only, is presented on pages E-2 and E-3.

SUMMARY OF EQUATIONS OF MOTION FOR THE PRIMARY COMPONENTS OF A SYSTEM FOR AUTOMATIC TRACKING CONTROL OF THE A-26 AIRPLANE

Displacement Control Only.

For m_u, z_θ, m_{eh_u}, m_{rh_u} and m_{ah_p} negligible.

For $\frac{\partial M_{SAEM}}{\partial F_E} = -100 \frac{\text{ft lbs}}{\text{radian}}$, $\frac{\partial M_{SARM}}{\partial F_D} = 300 \frac{\text{ft-lbs}}{\text{radian}}$, and

$$\frac{\partial M_{SAAM}}{\partial F_D} = 50 \frac{\text{ft-lbs}}{\text{radian}}.$$

For m_{ce_w} and $m_{cd_v} = 0$. $U_o = 300$ mph true airspeed, at 10,000 feet density altitude. Target Range = 1000 yards. .50 caliber gun. $(SN)_e = (SN)_d = .2$

Longitudinal Equations of Motion

A-26 AIRPLANE LONGITUDINAL EQUATIONS OF MOTION

$$\begin{cases} (d + .087261)u - .18720w + .32687\theta = 0. \\ .65374u + (d + 4.8701)w - d\theta + .3099\delta_e = 0. \\ (5.3684d + 149.56)w + (d^2 + 16.3668d)\theta + 351.95\delta_e = 0. \end{cases}$$

ELEVATOR-SERVO SYSTEM EQUATION OF MOTION

$$(.102014d + 16.2434)w + .24881d\theta + (d^2 + 50.247d + 548.53)\delta_e = -4.9456f_e.$$

ELEVATION COMPUTER EQUATION OF MOTION

$$.053294d_{ce}w - .83333d_{ce}\theta + (d_{ce} + 1)p_{ce} - .0058755 = 0.$$

IDEAL RADAR ELEVATION EQUATION OF MOTION

$$d_f a_{re} - \left[\frac{w}{1-\left(\frac{u}{d_f}\right)}\right] + \left\{d_f + \left[\frac{1}{1-\left(\frac{u}{d_f}\right)}\right]\right\}\theta = 0.$$

Lateral Equations of Motion

A-26 AIRPLANE LATERAL EQUATIONS OF MOTION

$$\begin{cases} 201.9096v - 3.7174d\psi + (d^2 + 27.693d)\phi - 306.52\delta_a = 0. \\ -72.3804v + (d^2 + 3.1307d)\psi + .63842d\phi - 105.35\delta_r = 0. \\ (d + .65)v + d\psi - .32687\phi + .2133\delta_r = 0. \end{cases}$$

RUDDER-SERVO SYSTEM EQUATION OF MOTION

$$3.54462v - .054293d\psi + (d^2 + 50.177d + 529.238)\delta_r = 9.1279f_d.$$

AILERON-SERVO SYSTEM EQUATION OF MOTION

$$(d^2 + 50.156d + 528.537)\delta_a = 3.6706f_d.$$

DEFLECTION COMPUTER EQUATION OF MOTION

$$- .83333d_{cd}\psi + .14694d_{cd}\phi + (d_{cd} + 1)p_{cd} = 0.$$

IDEAL RADAR DEFLECTION EQUATION OF MOTION

$$d_f a_{rd} + \left[\frac{v}{1-\left(\frac{u}{d_f}\right)}\right] + \left\{d_f + \left[\frac{1}{1-\left(\frac{u}{d_f}\right)}\right]\right\}\psi = 0.$$

APPENDIX F

NETWORK EQUATIONS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE, BASED UPON A SYSTEM CHARACTERISTIC TIME EQUAL TO THE INITIAL TIME OF FLIGHT OF THE AIRPLANE

APPENDIX F

NETWORK EQUATIONS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE, BASED UPON A SYSTEM CHARACTERISTIC TIME EQUAL TO THE INITIAL TIME OF FLIGHT OF THE AIRPLANE

The network equations were developed in Appendix C, and the formulae for the stability coefficients used in these equations were derived in Appendix B. The stability coefficients for a system for automatic tracking control of the A-26 Airplane were determined in Appendix D.

If we put the stability coefficients determined in Appendix D, in the network equations developed in Appendix C, using the proper characteristic time ratios as indicated, we obtain the network equations of a system for Automatic Tracking Control of the A-26 Airplane.

A summary of the network equations, for the case when m_u, z_θ, m_{eh_u}, m_{rh_u} and $m_{ah_p} = 0$, and for $T_S = T_{AF_o}$, with displacement control only, is presented on pages F-2 and F-3.

SUMMARY OF NETWORK EQUATIONS FOR AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE, BASED UPON A SYSTEM CHARACTERISTIC TIME EQUAL TO THE INITIAL TIME OF FLIGHT OF THE AIRPLANE

Displacement Control Only

For m_u, z_θ, m_{eh_u}, m_{rh_u} and m_{ah_p} negligible.

For $\dfrac{\partial M_{SAEM}}{\partial F_E} = -100 \dfrac{\text{ft-lbs}}{\text{radian}}$, $\dfrac{\partial M_{SARM}}{\partial F_D} = 300 \dfrac{\text{ft-lbs}}{\text{radian}}$, and $\dfrac{\partial M_{SAAM}}{\partial F_D} = 50 \dfrac{\text{ft-lbs}}{\text{radian}}$.

For m_{ce_w} and $m_{cd_v} = 0$. $U_o = 300$ mph true airspeed, at 10,000 feet density altitude. Target Range = 1000 yards. .50 caliber gun. $(SN)_e = (SN)_d = .2$

Longitudinal Network Equations

A-26 AIRPLANE LONGITUDINAL EQUATIONS OF MOTION

$$\begin{cases} (d_f + .13306)u - .28544w + .49841\theta = 0. \\ .99682u + (d_f + 7.4259)w - d_f\theta + .47254\delta_e = 0. \\ (8.1857d_f + 347.73)w + (d_f^2 + 24.956d_f)\theta + 818.28\delta_e = 0. \end{cases}$$

ELEVATOR-SERVO SYSTEM EQUATION OF MOTION

$$(.15555d_f + 37.770)w + .37939d_f\theta + (d_f^2 + 76.617d_f + 1275.3)\delta_e = -11.499f_e.$$

ELEVATION COMPUTER EQUATION OF MOTION

$$.053294d_f w - .83333d_f\theta + (d_f + 4.5259)p_{ce} - .026592 = 0.$$

IDEAL RADAR ELEVATION EQUATION OF MOTION

$$d_f a_{re} - \left[\frac{w}{1-(\frac{u}{d_f})}\right] + \left\{d_f + \left[\frac{1}{1-(\frac{u}{d_f})}\right]\right\}\theta = 0.$$

ELEVATION TRACKING ERROR ANGLE EQUATION	$f_e = a_{re} + p_{ce}.$

Lateral Network Equations

A-26 AIRPLANE LATERAL EQUATIONS OF MOTION	$469.44v - 5.6682 d_f \psi + (d_f^2 + 42.226 d_f)\phi - 712.66 \delta_a = 0.$ $-168.28v + (d_f^2 + 4.7737 d_f)\psi + .97346 d_f \phi - 244.94 \delta_r = 0.$ $(d_f + .99112)v + d_f \psi - .49841\phi + .32524 \delta_r = 0.$
RUDDER-SERVO SYSTEM EQUATION OF MOTION	$8.2412v - .082786 d_f \psi + (d_f^2 + 76.510 d_f + 1230.5)\delta_r = 21.222 f_d.$
AILERON-SERVO SYSTEM EQUATION OF MOTION	$(d_f^2 + 76.478 d_f + 1228.8)\delta_a = 8.5341 f_d.$
DEFLECTION COMPUTER EQUATION OF MOTION	$-.8333 d_f \psi + .14694 d_f \phi + (d_f + 4.9098) p_{cd} = 0.$
IDEAL RADAR DEFLECTION EQUATION OF MOTION	$d_f a_{rd} + \left[\frac{v}{1-\left(\frac{u}{d_f}\right)}\right] + \left\{d_f + \left[\frac{1}{1-\left(\frac{u}{d_f}\right)}\right]\right\}\psi = 0.$
ELEVATION TRACKING ERROR ANGLE EQUATION	$f_d = a_{rd} + p_{cd}.$

APPENDIX G

DETERMINATION OF SOLUTION TIME OF AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE, USING DIFFERENTIAL ANALYZER TO SOLVE NETWORK EQUATIONS

APPENDIX G

DETERMINATION OF SOLUTION TIME OF AN AUTOMATIC TRACKING CONTROL SYSTEM FOR THE A-26 AIRPLANE, USING DIFFERENTIAL ANALYZER TO SOLVE NETWORK EQUATIONS.

The network equations for an automatic tracking control system for the A-26 Airplane, based upon a system characteristic time equal to the initial time of flight of the airplane, were summarized on pages F-2 and F-3 in Appendix F. The values of $\frac{\partial M_{SAEM}}{\partial F_E}$, $\frac{\partial M_{SARM}}{\partial F_D}$ and $\frac{\partial M_{SAAM}}{\partial F_D}$, (-100, 300 and 50 $\frac{\text{ft-lbs}}{\text{radian}}$, respectively), given on Page F-2, were used only to demonstrate the method of analysis. These values do not represent the final values indicated by the Differential Analyzer to be desirable.

The basic problem given to the Differential Analyzer was as follows: Two airplanes were assumed to be flying level, each trimmed at 300 mph TAS, one behind the other. The rear (tracking) airplane was assumed to be an A-26 airplane, flying at 10,000 ft. density altitude, while the forward (target) airplane was assumed to be separated from the tracking airplane by an initial range of 1000 yards. In addition, the target airplane was assumed to be 150 ft. above and 150 ft. to the right of the tracking airplane, so that the initial line of sight from the tracking airplane would have an elevation component of +50 mils as well as a deflection component of +50 mils. The automatic tracking control system was then assumed to be suddenly turned on, thus applying a step-function input to the radar system. It was further assumed that there was perfect radar tracking, i.e., it was assumed that the radar angle (A_R) was *always* equal to the angle of the line of sight (A_{LS}). For initial elevation and deflection component angles as small as + 50 mils it was reasonable to assume that the ensuing cross-roll terms developed during the solution time would be of second order and so could be neglected. In other words, except for the effect of change in speed in the ideal radar equation of motion, it was apparent that the lateral network equations were independent of the longitudinal network equations. Thus it was decided to solve the longitudinal equations first, and later to solve the lateral equations by using as an independent input the change in speed resulting from the longitudinal equations.

A. SOLUTION OF LONGITUDINAL NETWORK EQUATIONS

The Differential Analyzer circuit representing the equations of the entire longitudinal network is shown in Figure G-1-a. It was realized that some of the terms in this circuit would have only a small effect on the solution time, but in order to prove just how small the effect was, it was decided to include all of the terms until the control coefficients had been adjusted to give a reasonable solution time, and then to drop one term at a time and note its effect on solution time.

The first objective was to obtain the solution time required to establish a pure pursuit course (with computer caged). After many trial runs it was found that a solution time of about .25 (in terms of initial time of flight of tracking airplane) could be obtained with the following coefficients:

$$a_e = -\left(\frac{T_{AF_o}}{T_A}\right)^2 m_{eh_{f_e}} = 6{,}324.45 \text{ ....(displacement control coefficient)}$$

$$b_e = -\left(\frac{T_{AF_o}}{T_A}\right) m_{eh_{\dot{f}_e}} = 505.92 \text{ ....(derivative control coefficient)}$$

$$c_e = -\left(\frac{T_{AF_o}}{T_A}\right)^3 m_{eh_{\frac{f_d}{d}}} = 0 \text{ ....(integral control coefficient)}$$

$$\left(\frac{T_{AF_o}}{T_A}\right)^2 (-m_{eh_{\delta_e}}) = 800 \text{ ....(servo spring coefficient)}$$

$$\left(\frac{T_{AF_o}}{T_A}\right)(-m_{eh_{\dot{\delta}_e}}) = 200 \text{ ....(servo damping coefficient)}$$

The nondimensional time history for a_{re} based upon the coefficients listed above, is shown by the solid line in Figure G-2, and the corresponding tabulated values are shown in Table G-1. Table G-2 gives the tabulated values for the time history obtained by neglecting the term in $m_{eh_{\dot{w}}}$. This effect was so small that it could not be shown in Figure G-2.

Table G-3 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$ and m_{eh_q}. Again the effect was so small that it could not be shown in Figure G-2. Table G-4 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$, m_{eh_q}, and $\int u d\gamma$ in the ideal radar equation of motion. Again the effect was too small to be shown in Figure G-2.

Table G-5 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$, m_{eh_q}, $\int ud\gamma$, and m_{eh_w}. Here too the effect was too small to be shown in Figure G-2.

Table G-6 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$, m_{eh_q}, $\int ud\gamma$, m_{eh_w} and $m_{\dot{w}}$. In this case the effect of neglecting the additional term in $m_{\dot{w}}$ was appreciable, as shown by the dashed curve in Figure G-2.

Table G-7 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$, m_{eh_q}, $\int ud\gamma$, m_{eh_w}, and in addition the combination $(\theta - w)$ in the ideal radar equation of motion. The effect of neglecting the $(\theta - w)$ combination is shown by the dotted line in Figure G-2. Although this effect was small, it was decided to keep the $(\theta - w)$ combination in the circuit.

Table G-8 gives the tabulated values for the time history obtained by neglecting the terms in $m_{eh_{\dot{w}}}$, m_{eh_q}, $\int ud\gamma$, m_{eh_w} and z_u. Again the effect was too small to be shown in Figure G-2. In this case the variation in u shown in Table G-8 had no effect on solution time since it was merely a by-product of the network. This meant that only two degrees of freedom for the airplane needed to be considered, i.e., the long period of the airplane did not affect the solution time in any appreciable way. A simplified Differential Analyzer circuit representing the longitudinal network equations is shown in Figure G-1-b.

After a few more trial runs it was found that a pure pursuit course solution time of .15 (in terms of the initial time of flight of the tracking airplane) could be obtained with the following coefficients:

$$a_e = -\left(\frac{T_{AF_o}}{T_A}\right)^2 m_{eh_{f_e}} = 8,624.5$$ ....(displacement control coefficient)

$$b_e = -\left(\frac{T_{AF_o}}{T_A}\right) m_{eh_{\dot{f}_e}} = 607.104$$ ....(derivative control coefficient)

$$c_e = -\left(\frac{T_{AF_o}}{T_A}\right)^3 m_{eh_{\frac{f_d}{d}}} = 0$$ ....(integral control coefficient)

$$\left(\frac{T_{AF_o}}{T_A}\right)^2 (-m_{eh_{\delta_e}}) = 800$$ ....(servo spring coefficient)

$$\left(\frac{T_{AF_o}}{T_A}\right)(-m_{eh_{\dot{\delta}_e}}) = 400$$ ....(servo damping coefficient)

The nondimensional time history for a_{re}, based upon the coefficients listed above, is shown by the solid line in Figure G-3, and the corresponding tabulated values are shown in tables G-9-a and G-9-b.

It was then decided to introduce the computer into the circuit at the first point where a_{re} was equal to ±5 mils. The effect of stability number, SN_e, is shown by the curves for $f_e = a_{re} + p_{ce}$, which are shown dotted in Figure G-3. The corresponding curves for p_{ce} are shown in Figure G-4, and the tabulated values for these runs are shown in tables G-10, G-11, G-12 and G-13 (for SN_e = 0, .2, .4 and .6, respectively). The correct value for p_{ce}, equal to +5.7 mils, is shown as a dotted line in Figure G-4, (value from Ref. 3, V.I, p-299). Thus it appears that the over-all solution time required to establish an aerodynamic lead pursuit course, in this case, is about .3, or .3 x 6.8182 ≅ *2 seconds.*

All Differential Analyzer runs shown in this thesis were checked by substituting the tabulated values in the original equations.

In order to determine and check the order of magnitude of the maximum stick force required to establish a pure pursuit course in approximately one second, the following two formulae, based upon the equations presented in section A-2 of Appendix A, for the elevator hinge moment caused by the cable, were developed:

$$M_{EHC} = \left(\frac{I_{EH}}{T_{AF_o}^2}\right)\ddot{\delta}_e + \left[-\left(\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}\right)c_E S_E \frac{\rho}{2} U_o^2\right]\frac{\dot{\delta}_e}{T_{AF_o}}$$

$$+\left[-\left(\frac{\partial C_{EH}}{\partial \Delta_E}\right)c_{EAHL}S_{EAHL}\frac{\rho}{2}U_o^2\right]\delta_e$$

$$-\left[\frac{\rho}{2}U_o^2\, S_{EAHL}c_{EAHL}\left(\frac{\partial C_{EH}}{\partial \alpha'}\right)\right] w$$

$$-\left[\left(\frac{\partial C_{EH}}{\partial \alpha'}\right)\left(\frac{\partial \epsilon}{\partial \alpha}\right)\left(\frac{\rho}{2}U_o^2\right)\left(\frac{1}{U_o T_{AF_o}}\right)c_{EAHL}S_{EAHL}\right]\dot{w}$$

$$-\left[\left(\frac{\partial C_{EH}}{\partial \alpha'}\right)\left(\frac{\rho}{2}U_o^2\right)\left(\frac{1}{U_o T_{AF_o}}\right)S_{EAHL}c_{EAHL}\right]\dot{\theta} \qquad (1)$$

and

$$M_{EHC} = \left(\frac{I_{EH}}{T_{AF_o}^2}\right)\ddot{\delta}_e + \left[-\left(\frac{\partial C_{EH}}{\partial \dot{\Delta}_E}\right)c_E S_E \frac{\rho}{2} U_o^2\right]\frac{\dot{\delta}_e}{T_{AF_o}}$$

$$+\left[-\left(\frac{\partial C_{EH}}{\partial \Delta_E}\right)c_{EAHL} S_{EAHL} \frac{\rho}{2} U_o^2\right]\delta_e$$

$$+\left[\frac{(I_{EH})_{EFF}}{T_{AF_o}^2}\right]\left[m_{eh_{\dot{f}_e}}\left(\frac{T_{AF_o}}{T_A}\right)\dot{f}_e + m_{eh_{f_e}}\left(\frac{T_{AF_o}}{T_A}\right)^2 f_e\right.$$

$$+ m_{eh_{\frac{f_e}{d}}}\left(\frac{T_{AF_o}}{T_A}\right)^3 \int f_e d\gamma - \ddot{\delta}_e + (m_{eh_{\dot{\delta}_e}})\left(\frac{T_{AF_o}}{T_A}\right)\dot{\delta}_e$$

$$\left. + (m_{eh_{\delta_e}})\left(\frac{T_{AF_o}}{T_A}\right)^2 \delta_e\right] \qquad (2)$$

The tabulated values given in table G-9-a were used to compute M_{EHC}, using equations 1 and 2. These equations would give identical results if all the tabulated values were of the same degree of accuracy. However, computations based upon equation 2 are dependent on small differences in very large values, and so are not as reliable as those based upon equation 1. It was found that the maximum value for M_{EHC}, from the data in table G-9-a, was:

$M_{EHC_{max}}$ = -66.485 ft-lbs, using equation 1.

= -88.551 ft-lbs, using equation 2.

This maximum value occured just after the start of the run, as would be expected, at a value of $\gamma_f \cong .04000$.

Since 82 lbs stick force causes 1490 in-lb elevator hinge moment, it means that the maximum stick force needed to establish a pure pursuit course in one second is somewhere between 44 and 58 lbs, or say about *50 lbs*, which seems reasonable.

The elevator-servo input control characteristics corresponding to the required value of $a_e = 8{,}624.5$ and $b_e = 607.104$ are:

$$\frac{\partial M_{SAEM}}{\partial F_E} = -75 \frac{\text{ft-lbs}}{\text{miliradian}} \text{ and } \frac{\partial M_{SAEM}}{\partial \dot{F}_E} = -36 \frac{\text{ft-lbs}}{\left(\frac{\text{miliradian}}{\text{sec.}}\right)}.$$

Based upon the values of $\left(\frac{T_{AF_o}}{T_A}\right)^2(-m_{eh_{\delta_e}}) = 800$, and $\left(\frac{T_{AF_o}}{T_A}\right)(-m_{eh_{\dot{\delta}_e}}) = 400$, which were found to be desirable, as shown by the results tabulated in Tables G-9-a and G-9-b, and plotted in Figure G-3, the required elevator servo characteristics may be computed as follows:

Since $(I_{EH})_{EFF} = I_{EH} + \left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)^2(I_{SAE})_{EFF}$,

$$\cong \left(\frac{\partial\theta_{SAE}}{\partial\Delta_E}\right)^2(I_{SAE})_{EFF},$$

and since that part of the total stiffness in the elevator-servo system which is due to the aerodynamic forces is small compared with the stiffness caused by the elevator servo itself, we may estimate the required value of $\omega_{n_{SAE}}$ by the following formula:

$$\omega_{n_{SAE}} \cong \frac{\sqrt{\left(\frac{T_{AF_o}}{T_A}\right)^2(-m_{eh_{\delta_e}})}}{T_{AF_o}},$$

$$\cong \frac{\sqrt{800}}{6.8182}, \cong \underline{4.17} \frac{\text{radians}}{\text{sec.}}$$

And by similar reasoning,

$$\zeta_{SAE} \cong \frac{\left(\frac{T_{AF_o}}{T_A}\right)(-m_{eh\dot{\delta}_e})}{(2)(T_{AF_o})(\omega_{n_{SAE}})},$$

$$\cong \frac{400}{(2)(6.8182)(4.17)}, \cong \underline{7.03}$$

We may summarize the information presented in this section as follows: The analysis of the longitudinal loop of the automatic tracking control system described in this thesis has indicated that

1. The effect of aerodynamic forces on the elevator servo system need not be considered.
2. The long period associated with the aircraft need not be considered, i. e. change in longitudinal speed may be neglected.
3. Change in longitudinal speed has a negligible effect on the ideal radar equation.
4. The effect of downwash lag, $m_{\dot{w}}$, is appreciable.
5. The effect of the combination $(\theta - w)$ in the ideal radar equation is very small.
6. The solution time required to establish a pure pursuit course, for the case investigated, was $\cong$ 1 sec.
7. The solution time required to establish an aerodynamic lead pursuit course, for the case investigated, was $\cong$ 2 sec.
8. The stability number for the elevation computer should be $\cong .6$

9. The maximum stick force required to establish a pure pursuit course in 1 second, for the case investigated, was $\cong$ 50 lbs.

10. The elevator servo characteristics should be

$\omega_{n_{SAE}} \cong 4.17 \dfrac{\text{radians}}{\text{sec}}$, and $\zeta_{SAE} \cong 7.03$.

11. The input control characteristics for the elevator-servo should be:

$$\frac{\partial M_{SAEM}}{\partial F_E} = -75 \frac{\text{ft-lbs}}{\text{miliradian}}$$

$$\frac{\partial M_{SAEM}}{\partial \dot{F}_E} = -36 \frac{\text{ft-lbs}}{\left(\frac{\text{miliradian}}{\text{sec.}}\right)}$$

12. The computer should not be uncaged until the tracking error angle has been decreased to at least ±5 mils.

B. SOLUTION OF LATERAL NETWORK EQUATIONS

Based upon the information summarized at the end of section A, the lateral network equations presented on page F-3 in Appendix F were simplified and changed as follows:

$$469.44v - 5.6682\, d_f\psi + (d_f^2 + 42.226\, d_f)\phi - 712.66\, \delta_a = 0 \quad (3)$$

$$-168.28v + (d_f^2 + 4.7737\, d_f)\psi + .97346\, d_f\phi - 244.94\, \delta_r = 0 \quad (4)$$

$$(d_f + .99112)v + d_f\psi - .49841\phi + .32524\, \delta_r = 0 \quad (5)$$

$$(d_f^2 + 400d_f + 800)\delta_r - a_r f_d - b_r \dot{f}_d = 0 \quad (6)$$

$$(d_f^2 + 400d_f + 800)\delta_a - a_a f_d - b_a \dot{f}_d = 0 \quad (7)$$

$$d_f(a_{rd} + \psi) + (v + \psi) = 0 \quad (8)$$

$$-.8333\ d_f\psi + .14694\ d_f\phi + [d_f + 4.9098]p_{cd} = 0, \text{ for } SN_d = .2, \quad (9)$$

$$f_d = \alpha_{rd} + p_{cd} \quad (10)$$

The displacement control coefficients, a_r, a_a, and the derivative control coefficients, b_r, b_a, were first to be determined with the computer caged ($SN_d = \infty$). The Differential Analyzer circuit for the simplified lateral network equations, with computer caged, is shown in Figure G-5. For the first runs, the rudder-servo system, and aileron-servo system, spring and damping coefficients were kept the same as those for the elevator-servo system (400 for spring and 800 for damping were found to be desirable in the longitudinal analysis). However, after a few runs it was found that more damping would be required in the rudder-servo system. It was also found that smooth application of rudder and ailerons, keeping $v = 0$, could not be obtained with constant values of a_r, b_r, a_a and b_a, and that when a_r and b_r were adjusted to values high enough to give a practical fire-control solution time, large positive variations in a_a and b_a had very little immediate effect on aerodynamic yaw, Physically this meant that the flight path could not be changed appreciably in the time required for a practical fire-control solution time. In this connection it should be noted that a pilot can correct for a *small* deflection angle much faster by using rudder alone than by coördinating rudder and ailerons to "roll in" and then "roll out". This of course assumes that he is merely solving the fire-control problem, and is not trying to change his flight path appreciably. But if he uses rudder alone the resulting aerodynamic yaw does not reach a steady-state value, and so the required rudder force does not reach a steady-state value. However, if the proper amount of opposite aileron is used the rudder force required *will* reach a steady-state value. It is the author's belief that small fire-control corrections in deflection angle can be made quickest by cross-controlling rudder and ailerons, i.e., using rudder with opposite ailerons, and using integral control to produce a steady-state rudder force to counteract the resulting steady-state aerodynamic yaw. This technique, of course, would not be used when establishing an interception course. Figure G-6 shows the time history of the deflection component of the ideal radar angle which resulted when the following amplification control factors were used:

$a_r = \left(\frac{T_{AF_o}}{T_A}\right)^2 m_{rh_{f_d}} = 9{,}300$ ....(rudder displacement control amplification factor)

$b_r = \left(\frac{T_{AF_o}}{T_A}\right) m_{rh_{\dot{f}_d}} = 1625$ ....(rudder derivative control amplification factor)

$c_r = \left(\frac{T_{AF_o}}{T_A}\right)^3 m_{rh_{\frac{f_d}{d}}} = 43{,}000$ ....(rudder integral control amplification factor)

$a_a = \left(\frac{T_{AF_o}}{T_A}\right)^2 m_{ah_{f_d}} = 0$ ....(aileron displacement control amplification factor)

$$b_a = \left(\frac{T_{AF_o}}{T_A}\right) m_{ah_{\dot{f}_d}} = 0$$ ....(aileron derivative control amplification factor)

$$c_a = \left(\frac{T_{AF_o}}{T_A}\right)^3 m_{ah_{\frac{f_d}{d}}} = 0$$ ....(aileron integral control amplification factor)

$$\left(\frac{T_{AF_o}}{T_A}\right)^2 (-m_{rh_{\delta_r}}) = 800$$ ....(servo spring amplification factor for rudder system)

$$\left(\frac{T_{AF_o}}{T_A}\right)(-m_{rh_{\dot{\delta}_r}}) = 750$$ ....(servo damping amplification factor for rudder system)

The corresponding differential analyzer data for Figure G-6 are given in Table G-14. Higher values of a_r, b_r and c_r only required a higher value of rudder damping to keep the rudder from reversing too soon. Thus the higher values resulted in no appreciable gain in solution time. It will be noted that the Differential Analyzer run shown in Figure G-6 was accomplished by using displacement, derivative, and integral control in the rudder servo, with no forcing function input to the aileron servo. As a result the aerodynamic yaw did not reach a steady-state value as it would have if opposite aileron (using negative values of a_a, b_a and c_a) had been used. If this run had been continued further it would have been found that as the aerodynamic yaw decreased, less integral control would be required, and so the deflection component of the radar angle would end up with a negative bias. This of course could be prevented by applying opposite aileron angle during the run. The fire-control solution time for the lateral run shown in Figure G-6 was $\gamma_f \cong .19$, or (.19) (6.8182) $\cong$ *1.3 seconds*. It is the author's belief that if sufficient opposite aileron had been used to produce a constant steady-state aerodynamic yaw, there would have been very little effect on solution time; and the solution time which would have been obtained by coördinating rudder and ailerons (with $v \cong 0$) would have been at least 3 or 4 times as long.

Equations analogous to equations 1 and 2 could be developed to determine the rudder (or aileron) hinge moments from the Differential Analyzer data.

The rudder-servo input control characteristics corresponding to the required values of a_r = 9,300, b_r = 1,625, and c_r = 43,000 are:

$$\frac{\partial M_{SARM}}{\partial F_D} = 131 \frac{\text{ft-lbs}}{\text{miliradian}},$$

$$\frac{\partial M_{SARM}}{\partial \dot{F}_D} = 157 \frac{\text{ft-lbs}}{(\text{miliradians/sec})} \text{ and}$$

$$\frac{\partial M_{SARM}}{\partial \int F_D dt} = 89 \frac{\text{ft-lbs}}{(\text{miliradian})(\text{sec.})}.$$

Based upon the values of $\left(\frac{T_{AF_o}}{T_A}\right)^2(-m_{rh_{\delta_r}}) = 800$, and $\left(\frac{T_{AF_o}}{T_A}\right)(-m_{rh_{\dot{\delta}_r}})$ = 750, which were found to be desirable, as shown by the results tabulated in Table G-14, and plotted in Figure G-6, the required rudder servo characteristics may be computed as follows:

$$\omega_{n_{SAR}} \cong \frac{\sqrt{\left(\frac{T_{AF_o}}{T_A}\right)^2 (-m_{rh_{\delta_r}})}}{T_{AF_o}}$$

$$\cong \frac{\sqrt{800}}{6.8182} \cong 4.17 \ \frac{\text{radians}}{\text{sec.}}$$

$$\zeta_{SAR} \cong \frac{\left(\frac{T_{AF_o}}{T_A}\right)(-m_{rh_{\dot{\delta}_r}})}{(2)(T_{AF_o})(\omega_{n_{SAR}})} \quad ,$$

$$\cong \frac{750}{(2)(6.8182)(4.17)} \cong 13.2$$

Time did not permit a complete analysis of the lateral equations. However we may summarize the results of this investigation as follows: The analysis of the lateral loop of the automatic tracking control system described in this thesis has indicated that

1. The fire-control solution time for gun fire is $\cong$ 1.3 seconds for the control amplification factors chosen.
2. A fire-control solution for gun fire, for small corrections in deflection angle, can be accomplished quickest by applying rudder and opposite ailerons, rather than by coordinating ailerons with rudder to change the flight path by "rolling in" and then "rolling out". (This of course would require velocity jump correction in the deflection computer).

3. The input control characteristics for the rudder-servo should be:

$$\frac{\partial M_{SARM}}{\partial F_D} = 131 \frac{\text{ft-lbs}}{\text{miliradian}}$$

$$\frac{\partial M_{SARM}}{\partial \dot{F}_D} = 157 \frac{\text{ft-lbs}}{(\text{miliradians/sec})}$$

$$\frac{\partial M_{SARM}}{\partial \int F_D dt} = 89 \frac{\text{ft-lbs}}{(\text{miliradian})(\text{sec})}$$

4. The rudder servo characteristics should be:

$$\omega_{n_{SAR}} = 4.17 \frac{\text{radians}}{\text{sec.}}, \text{ and}$$

$$\zeta_{SAR} = 13.2$$

It is the author's opinion that the ratio of the rudder servo stiffness to the rudder aerodynamic stiffness, measured at the rudder hinge, should not be less than 6 for positive positioning of the rudder. (This factor should also apply for positive positioning of the elevator).

It should be noted that M_{SAEM}, M_{SARM}, and M_{SAAM} are *effective* input moments at the servo axes applied by the servo motors; they are not the *net* moments applied by the servo motors. The net moment for each servo motor may be obtained by subtracting the stiffness, damping, and inertia moments from the effective input moment. The net moment applied by the servo motor is of course just the negative of the cable moment at the servo axis, see equation 41, Section A-2, Appendix A.

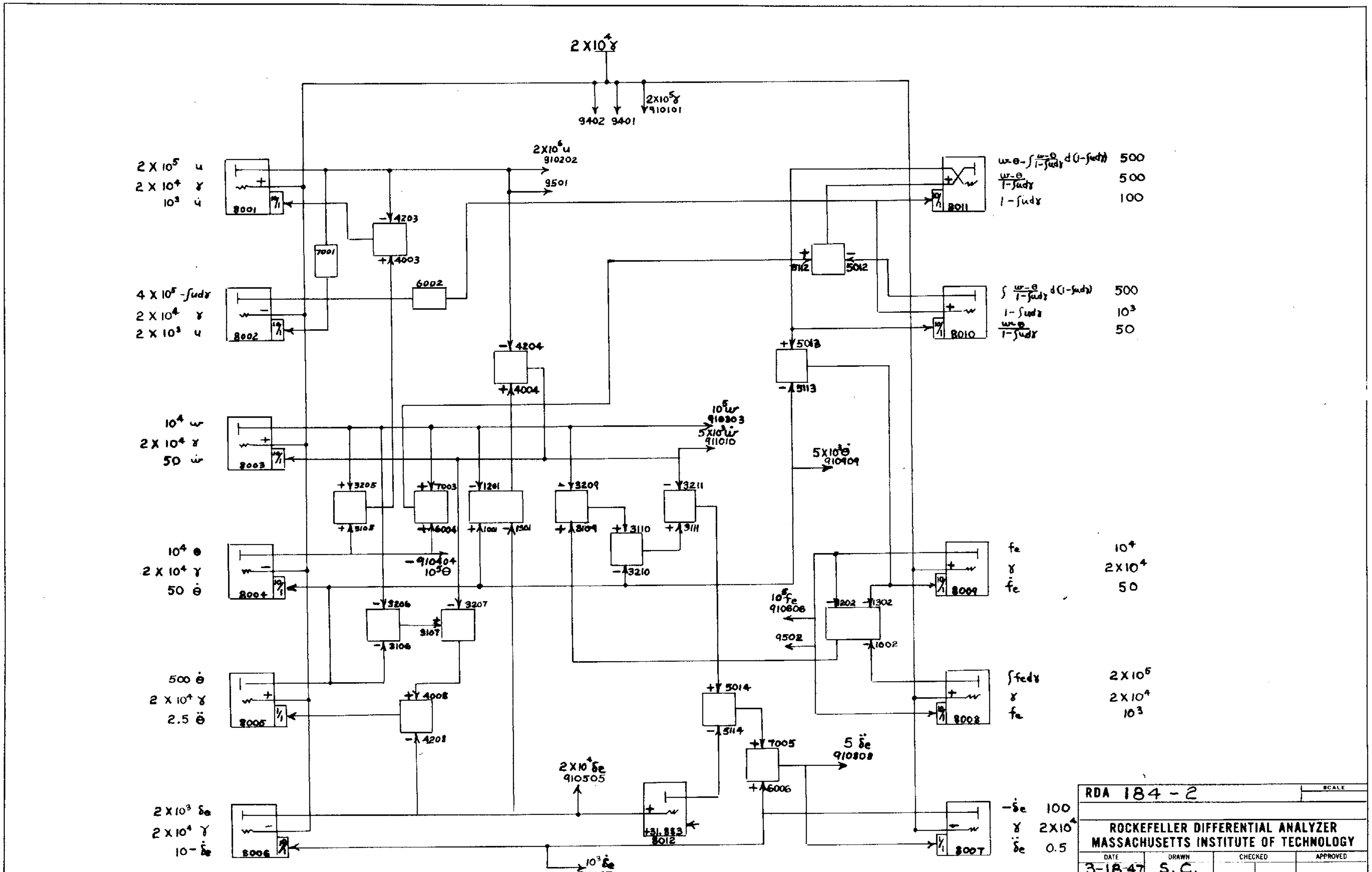


FIGURE G-1-a, DIFFERENTIAL ANALYZER CIRCUIT FOR COMPLETE LONGITUDINAL NETWORK EQUATIONS

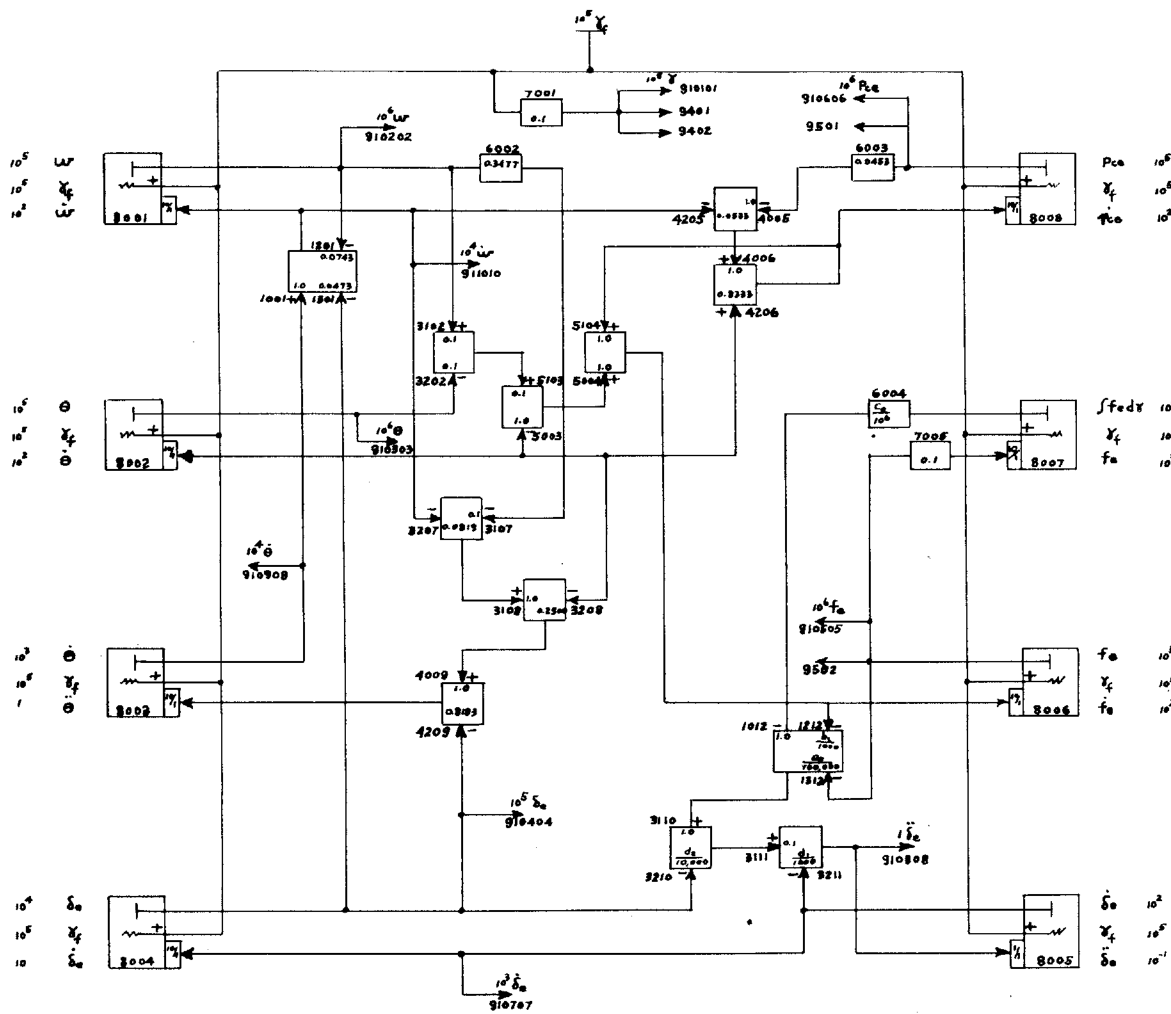
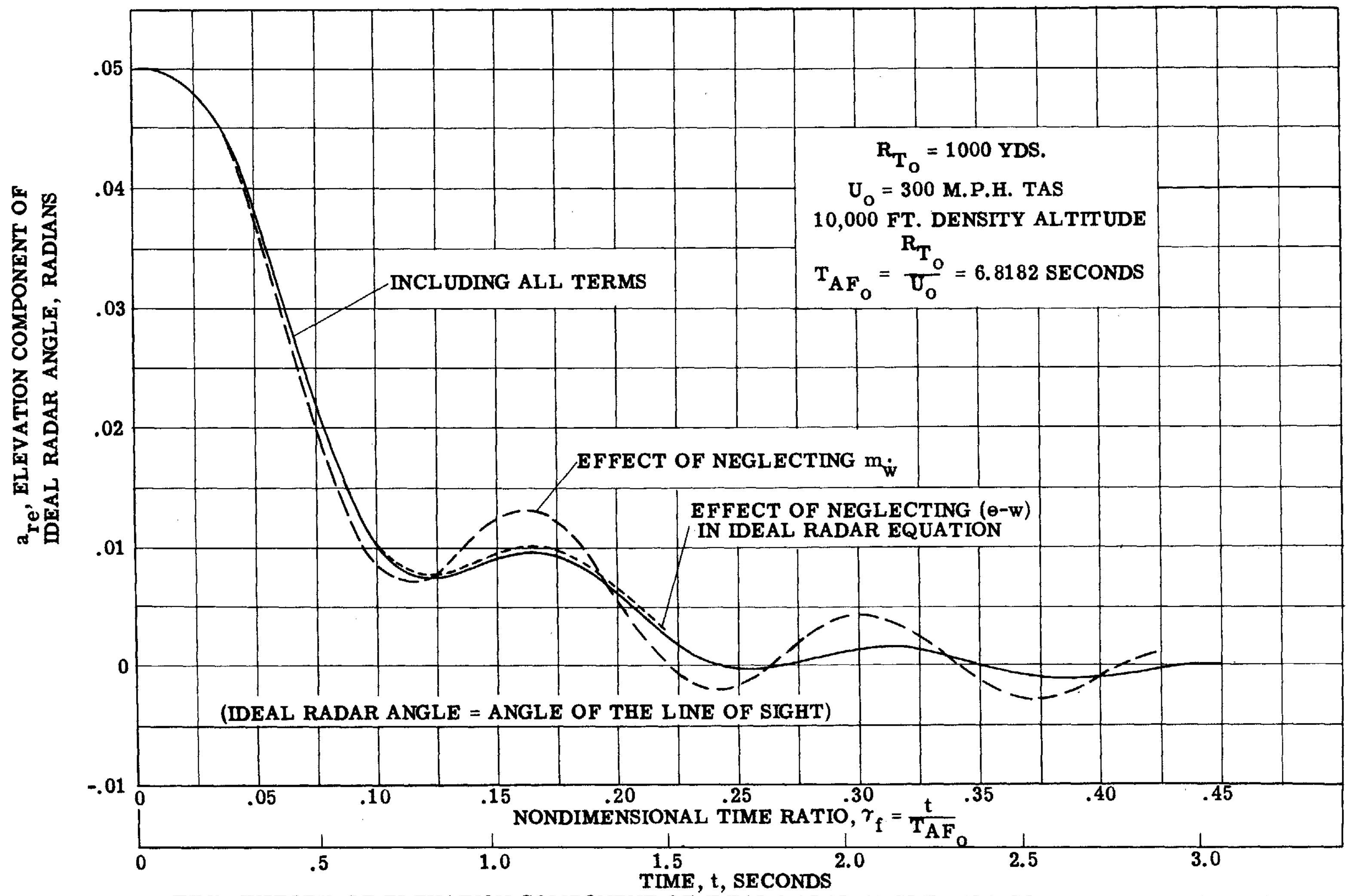


FIGURE G-1-b

DIFFERENTIAL ANALYZER CIRCUIT FOR SIMPLIFIED LONGITUDINAL NETWORK EQUATIONS

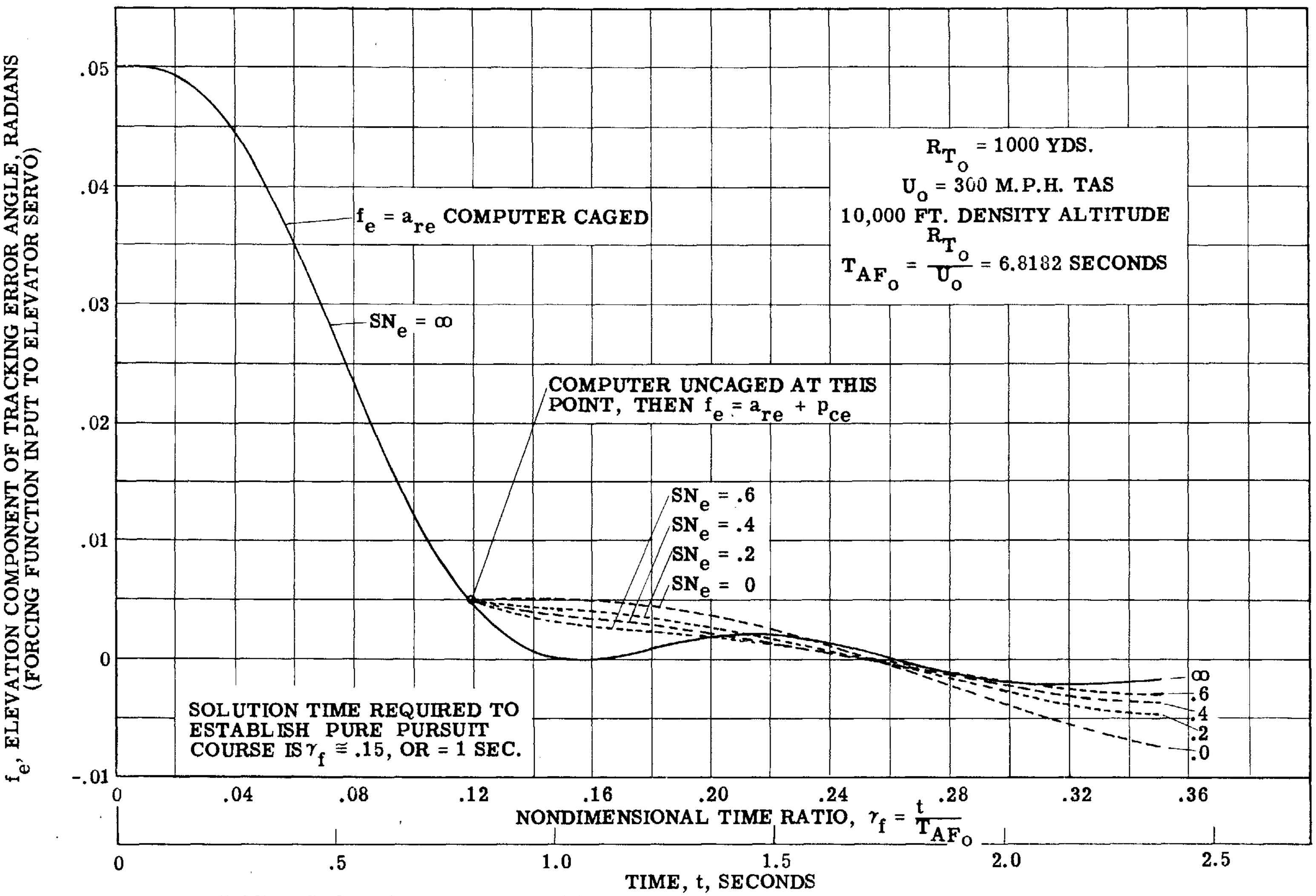
RDA	184-3		SCALE
Aut. Track. Cont. For A-26 Airplane			
ROCKEFELLER DIFFERENTIAL ANALYZER			
MASSACHUSETTS INSTITUTE OF TECHNOLOGY			
DATE	DRAWN	CHECKED	APPROVED
3/10/47	SC		

CHANGES	DATE	BY	DATE	BY



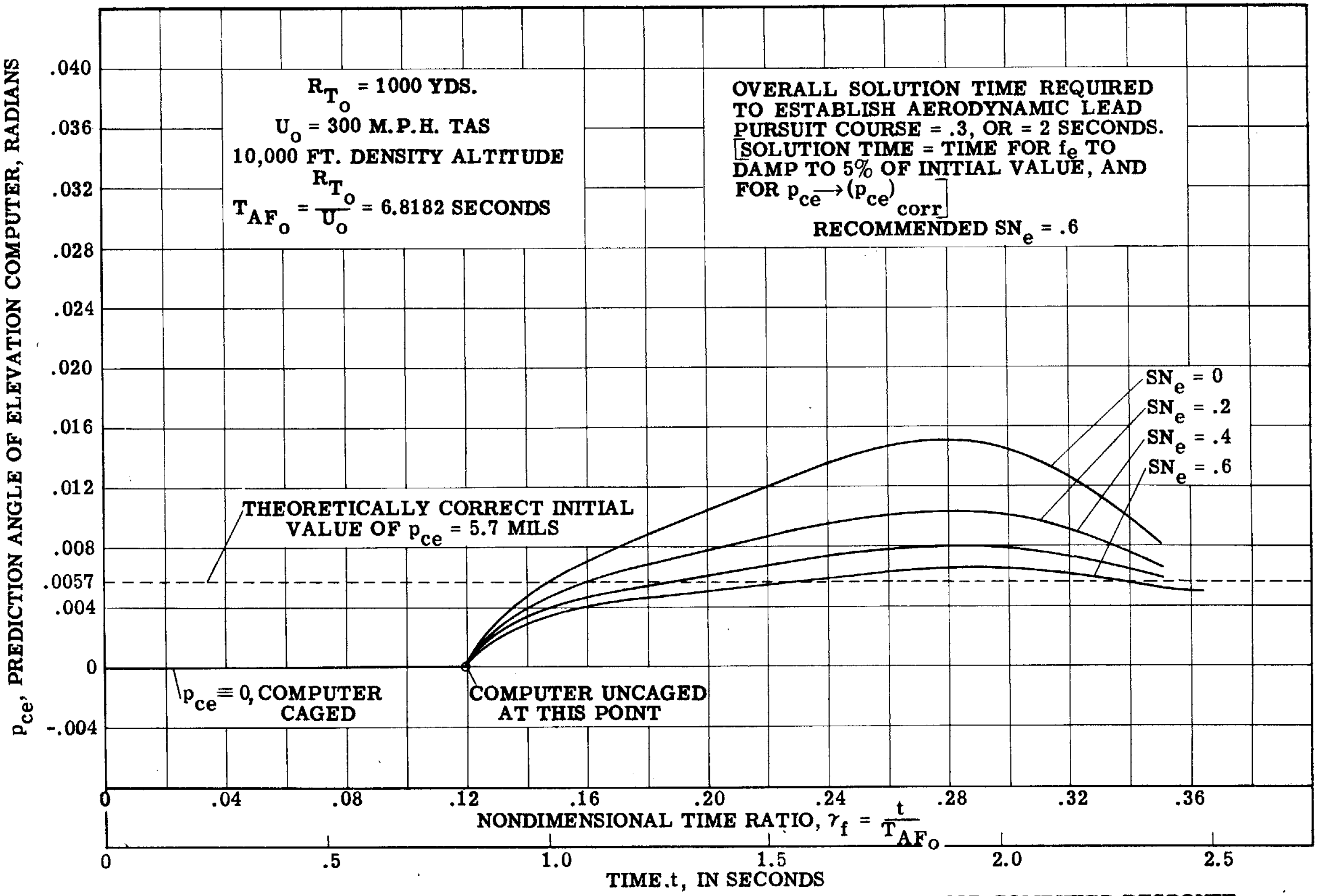
TIME HISTORY OF ELEVATION COMPONENT OF IDEAL RADAR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO STEP-FUNCTION IMPOSED ON AUTOMATIC TRACKING CONTROL SYSTEM, EFFECTS OF NEGLECTING CERTAIN STABILITY COEFFICIENTS, ANALYSIS BY DIFFERENTIAL ANALYZER

FIGURE G-2



TIME HISTORY OF ELEVATION COMPONENT OF TRACKING ERROR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO STEP-FUNCTION IMPOSED ON AUTOMATIC TRACKING CONTROL SYSTEM, COMPUTER CAGED UNTIL $f_e = \pm 5$ MILIRADIANS, ANALYSIS BY DIFFERENTIAL ANALYZER

FIGURE G-3



TIME HISTORY OF PREDICTION ANGLE OF ELEVATION COMPUTER, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO STEP-FUNCTION IMPOSED ON AUTOMATIC TRACKING CONTROL SYSTEM, COMPUTER CAGED UNTIL $f_e = \pm$ 5 MILIRADIANS, ANALYSIS BY DIFFERENTIAL ANALYZER

FIGURE G-4

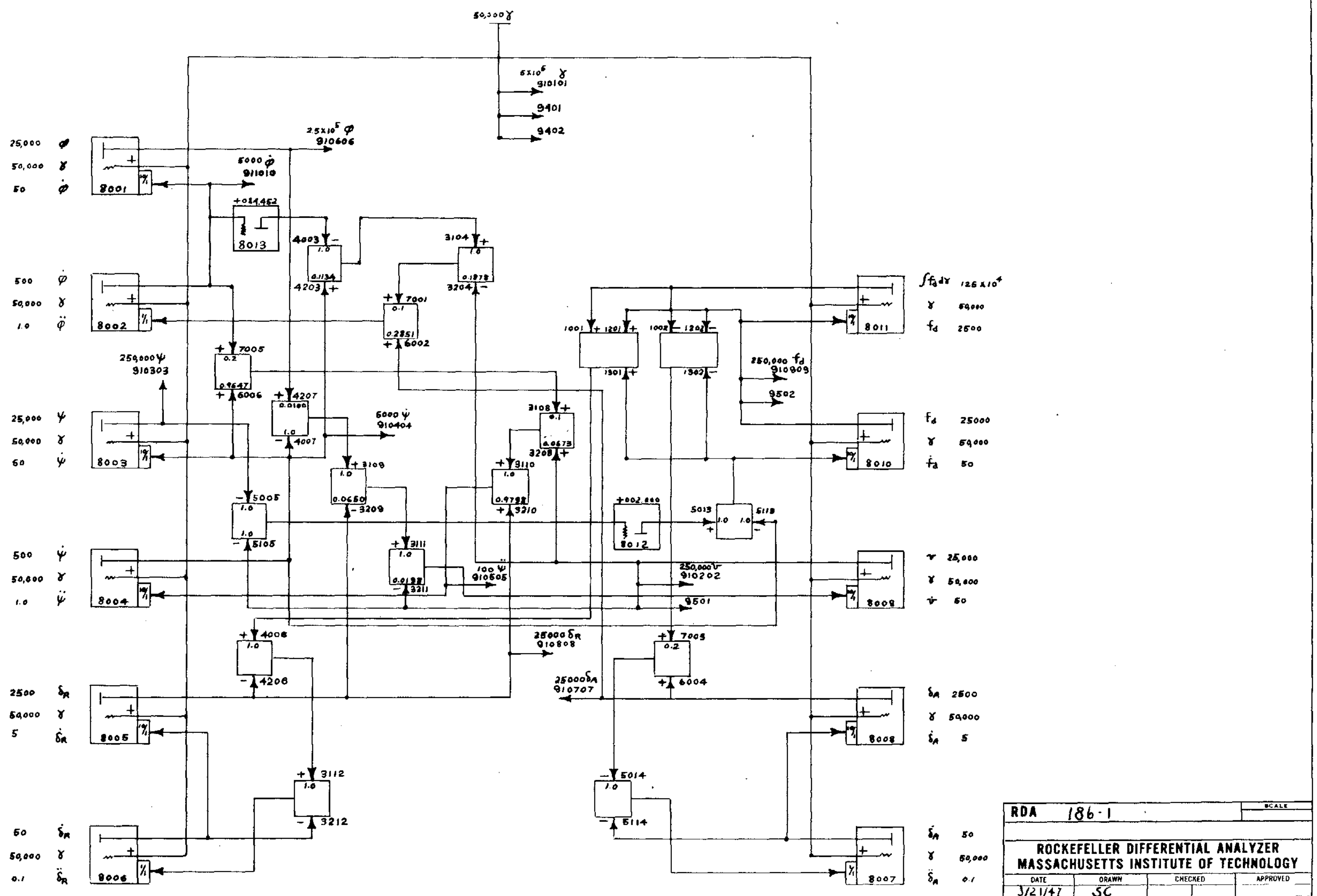
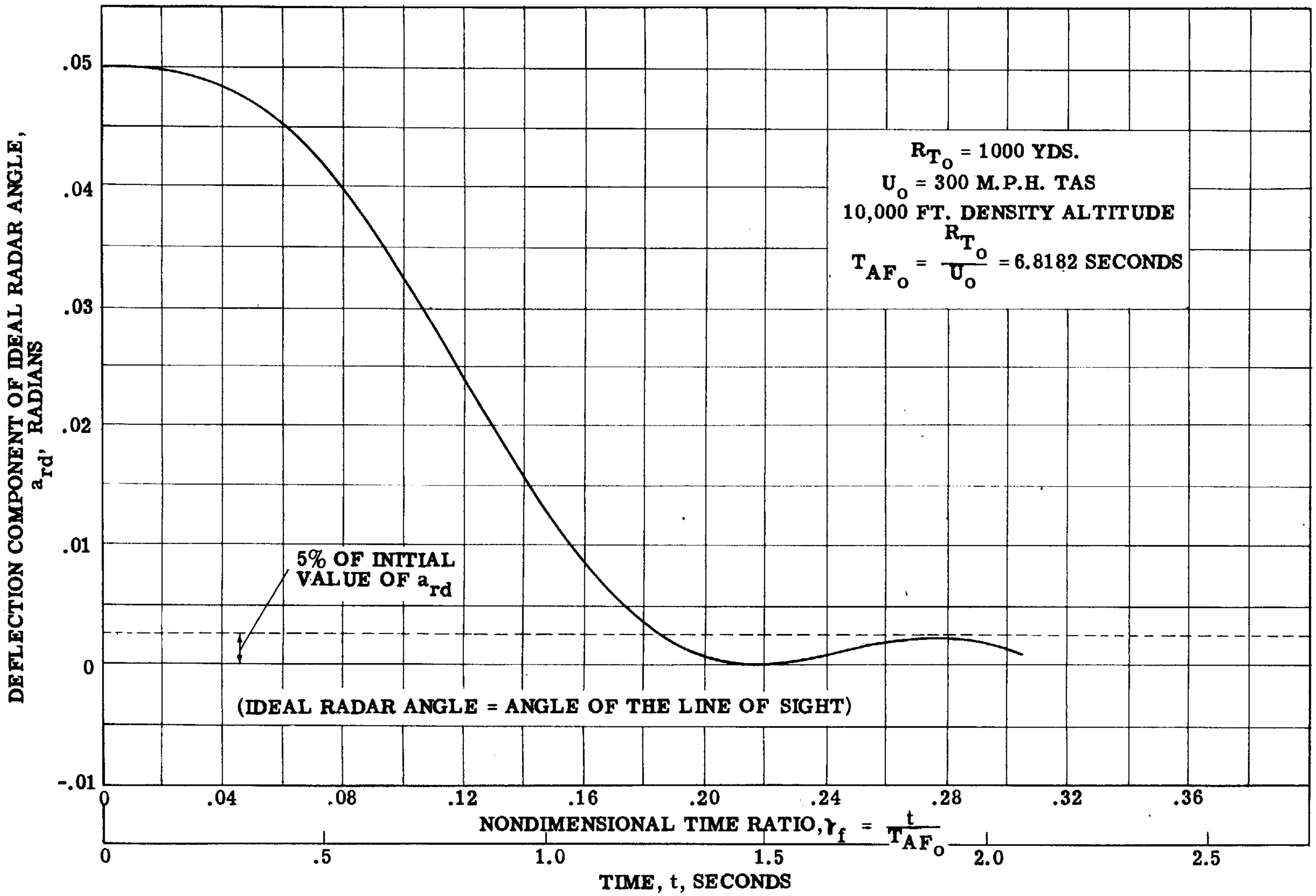


FIGURE G-5, DIFFERENTIAL ANALYZER CIRCUIT FOR SIMPLIFIED LATERAL NETWORK EQUATIONS



TIME HISTORY OF DEFLECTION COMPONENT OF IDEAL RADAR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO STEP-FUNCTION IMPOSED ON AUTOMATIC TRACKING CONTROL SYSTEM, ANALYSIS BY DIFFERENTIAL ANALYZER

FIGURE G-6

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 19 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 12:40

a_e		b_e		c_e	SN_e	p_{ce}
6324.45		505.92		0	∞	0

TABLE G-1

(Establishing a Pure Pursuit Course (No Computer))

000000
000000
000000
0000000
-000003
-00002
-158111
049997
000000
000000
1.00000

	$2\times10^5\gamma_t$	$2\times10^6 u$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5\alpha_{re} = 10^5 f_e$	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
[illegible]	00000	00000	00000	00000	00000	05000	00000	-01581	00000	00000
020000	02000	00000	00008	00008	-00178	04993	-01325	-00153	00126	00144
	04000	-00001	00083	00078	-00448	04922	-01283	00140	00641	00663
	16000	-00007	00288	00282	-00667	04718	-00884	00245	01426	01397
	08000	-00026	00641	00651	-00792	04349	-00355	00274	02252	02107
	10000	-00066	01116	01172	-00809	03828	00174	00247	02911	02592
	12000	-00134	01654	01794	-00728	03204	00602	00178	03258	02730
	14000	-00235	02182	02450	-00578	02547	00864	00084	03234	02492
	16000	-00373	02629	03064	-00395	01929	00933	-00012	02860	01931
	18000	-00545	02941	03577	-00217	01411	00825	-00091	02227	01162
	20000	-00748	03089	03947	-00074	01034	00586	-00141	01464	00327
	22000	-00976	03077	04163	00014	00808	00280	-00157	00710	-00432
	24000	-01223	02930	04239	00040	00719	-00024	-00141	00084	-01005
	26000	-01483	02692	04209	00009	00733	-00268	-00099	-00332	-01329
	28000	-01751	02416	04122	-00060	00805	-00415	-00045	-00503	-01393
	30000	-02025	02149	04022	-00149	00887	-00450	00011	-00459	-01234
	32000	-02303	01932	03950	-00233	00940	-00381	00057	-00242	-00925
	34000	-02587	01783	03931	-00295	00938	-00238	00084	00065	-00556
	36000	-02378	01697	03975	-00325	00871	-00018	00093	00377	-00211
	38000	-03179	01692	04078	-00318	00746	00119	00082	00625	00043
	40000	-03491	01715	04218	-00259	00581	00259	00057	00759	00164
	42000	06184	01748	04372	-00218	00401	00341	00025	00762	00149
	44000	-04154	01767	04514	-00147	00232	00356	-00008	00645	00015
	46000	-04504	01750	04624	-00079	00095	00311	-00034	00435	-00194
	48000	-04865	01688	04685	-00025	00003	00222	-00050	00185	-00426
	50000	-05233	01601	04698	00010	-00040	00112	-00056	-00060	-00635
	52000	-05608	01438	04665	00022	-00039	00005	-00049	-00261	-00783
	54000	-05985	01273	04599	00014	-00006	-00080	-00034	-00388	-00847
	56000	-06364	01105	04516	-00008	00044	-00131	-00014	-00430	-00824
-018735	58000	-06744	00949	04433	-00036	00093	-00141	00005	-00396	-00727
-012878	60000	-07123	00817	04361	-00063	00129	-00115	00021	-00306	-00584
-001712	62000	-07503	00717	04312	-00081	00143	-00064	00030	-0018	-00425
-002024	64000	-07883	00647	04285	-00088	00132	-00001	00032	-00070	-00280
002386	66000	-08264	00602	04280	-00081	00100	00060	00028	00021	-00172
000167	68000	-08647	00574	04290	-00063	00053	00105	00018	00068	-00116
001106	70000	-09031	00552	04305	-00039	00002	00129	00006	00066	-00112
049998	72000	-09419	00527	04315	-00012	-00045	00128	-00006	00022	-00149
000067	74000	-09807	00491	04312	00013	-00081	00106	-00013	-00051	-00210
-001959	76000	-10197	00442	04293	00032	-00101	00073	-00019	-00136	-00279
100106	78000	-10586	00380	04257	00044	-00105	00022	-00020	-00218	-00338
020000	80000	-10975	00308	04206	00047	-00092	-00007	-00017	-00281	-00374
	82000	-11361	00233	04146	00043	-00071	-00037	-00011	-00316	-00380
	84000	-11746	00159	04082	00033	-00045	-00052	-00002	-00319	-00353
	86000	-12127	00093	04019	00023	-00022	-00051	00004	-00293	-00300
	88000	-12506	00039	03965	00013	-00006	-00037	00009	-00251	-00236
	90000	-12881	-00001	03919	00008	00000	-00016	00011	-00202	-00171

$$\left(\frac{T_{AF_0}}{T_A}\right)\left(-m_{eh_{\dot{\delta}_e}}\right) = 200, \quad \left(\frac{T_{AF_0}}{T_A}\right)^2\left(-m_{eh_{\delta_e}}\right) = 800$$

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000000
000000
000001
-000004
-@0001
-158111
049998
000000
000000
100000
020000

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 20 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 4:30

a_e	b_e	c_e	SN_e	p_{oe}
6324.45	505.92	0	∞	0

TABLE G-2

(Establishing A Pure Pursuit Course)

(No Computer)

$2\times10^5 r_f$	$2\times10^6 u$	$10^5 w$	$10^5 \theta$	$2\times10^4 \delta_e$	$10^5 a_{pe} = 10^5 f_e$	$10^3 \dot{\delta}_e$	$5 \ddot{\delta}_e$	$5\times10^3 \dot{\theta}$	$5\times10^3 \dot{w}$
00000	00000	00000	00000	00000	05000	00000	-01581	@0000	00000
02000	00000	-00002	00006	-00176	04984	-01325	-00157	00121	00139
04000	-00001	00081	00075	-00446	04924	-01285	00136	00632	00655
@6000	-00007	00284	00277	-00666	04722	-00888	00242	01414	01388
08000	-00026	00634	00644	-00792	04355	-00359	00271	02239	02098
10000	-00065	01098	01161	-00809	03837	00168	00245	02899	02584
12000	-00133	01645	01782	-00729	03215	00597	00176	03249	02725
14000	-00234	02172	02435	-00580	02558	00860	00083	03227	02490
16000	-00370	02619	03049	-00397	0 941	00931	-00013	02856	01932
18000	-00542	02931	03561	-00219	01424	00823	-00092	02226	01165
20000	-00744	03081	03930	-00076	01046	00585	-00143	0147	00332
22000	-00971	03069	04147	00012	00819	00280	-00159	00712	-00426
24000	-01217	02923	04224	00038	00730	-00024	-00142	00086	-01000
26000	-01476	02686	04195	00008	00744	-00268	-00101	-00331	-01324
28000	-01743	02410	04108	-00062	00815	-00415	-00048	-00507	-01390
30000	-02016	02145	04008	-00150	00897	-00450	00009	-00459	-01232
32000	-02293	01928	03935	-00234	00949	-00383	00055	-00243	-00924

-@14021
-002294
-009242
-002422
013759
003816
005643
049999
001412
-001005
100005
020000

Same as Run 19, except that $m_{eh_{\dot{w}}}$ assumed negligible.

The output table, and the above data show that $m_{eh_{\dot{w}}}$ is negligible.

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 21 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 5:05

a_e	b_e	c_e	SN_e	p_{ce}
6324.45	505.92	0	∞	0

TABLE G-3
(Establishing A Pure Pursuit Course)
(No Computer)

000000
000000
000000
-000002
-000002
000001
-158111
049997
000000

	$2\times10^5 \gamma_f$	$2\times10^6 u$	$10^5 w$	$10^5 \theta$	$2\times10^4 \delta_e$	$10^5 a_{re} = 10^5 f_e$	$10^3 \dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3 \dot{\theta}$	$5\times10^3 \dot{w}+10$
000000										
100000	00000	00000	00000	00000	00000	05000	00000	-01581	00000	00010
XXXXXXX	02000	00000	00009	00007	-00178	04993	-01325	-00155	00125	00154
	04000	00000	00083	00077	-00448	04923	-01283	00140	00640	00672
0200000	06000	-00007	00288	00281	-00668	04719	-00883	00244	01425	01407
	08000	-00025	00641	00650	-00792	04350	-00354	00275	02251	02118
	10000	-00065	01115	01170	-00809	03829	00176	00247	02910	02502
	12000	-00133	01654	01793	-00728	03206	00604	00176	0326	02740
	14000	-00235	02182	02449	-00578	02548	00865	00083	03233	02502
	16000	-00372	02629	03063	-00395	01930	00934	-00012	02858	01940
	18000	-00544	02940	03574	-00216	01413	00825	-00092	02224	01170
	20000	-00747	03089	03944	-00073	01036	00585	-00142	01462	00336
	22000	-00975	03076	04159	00014	00810	00279	-00157	00708	-00423
	24000	-01222	02928	04235	00039	00722	-00025	-00141	00083	-00995
	26000	-01481	02691	04205	00008	00736	-00279	-00099	-00332	-01318

-013256
-001485
-013284
-003320
-014712
002700
-009870
049998
002558
-000762
100002
020000

Same as run 20 except that m_{eh_q} assumed negligible ($q \equiv \dot{\theta}$)

The output table and the above data show that m_{eh_q} <u>is</u> negligible.

000000
000000
000000
000000
-000002
000000
-158111
049998
000000

M. I. T. DIFFERENTIAL ANALYZER #2					
Run No. 22	RDA No. 184	Ed. No. 2			
Date 2/26/47		Time 5:40			
a_e		b_e	c_e	SN_e	R_{ce}
6324.45		505.92	0	∞	0

TABLE G-4

(Establishing a Pure Pursuit Course)

(no computer)

	$2\times10^5\gamma$	$2\times10^6 u$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5 a_{re} = 10^5 f_e$	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
000000	00000 f	00000	00000	00000	00000	05000	-00000	-01581	00000	00000
100000	02000	-00000	00008	00008	-00177	04993	-01325	-00154	00126	00145
020000	04000	-00001	00083	00079	-00447	04922	-012 2	00140	00642	00664
	06000	-00007	00288	00283	-00667	04718	-00882	00245	01427	01399
	08000	-00026	00641	00652	-00791	04349	-00353	00275	02253	02109
	10000	-00065	01116	01173	-00808	03828	00177	00247	02912	02593
	12000	-00134	01655	01796	-00727	03204	00605	00177	03258	02731
	14000	-00235	02184	02452	-00577	02546	00866	00082	03233	02491
	16000	-00373	02630	03066	-00393	01919	00935	-00012	02858	01929
	18000	-00545	02941	03578	-00215	01412	00825	-00091	02224	01159
	20000	-00748	03090	03948	-00072	01034	00585	-00141	01461	-00325
	22000	-00976	03077	04163	00015	00809	00279	-00158	00707	-00434
	24000	-01223	02929	04239	00041	00721	-00025	-00141	00083	-01006
	26000	-01483	02691	04210	00009	00735	-00269	-00099	-00332	-01328
	28000	-01751	02415	04123	-00061	00807	-00415	-00044	-00505	-01390
	30000	-02025	02150	04024	-00149	00888	-00449	00011	-00455	-01230
	31001	-02164	02034	03983	-00193	00920	-00425	00036	-00362	-01088

-013959
-002165
-010887
-003615
011103
004261
003587
049999
002640
-000972
100000
020000

Same as run 21 except that $\int u d\sigma$ assumed negligible.
The output table and the above data show that $\int u d\sigma$ has negligible effect on ideal radar equation.

000000
000000
000000
000001
-000003
-000001
-158111
049997
000000
000000
100000
020000

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. ~~22~~ 23 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 6:10

a_e		b_e		c_e	SN_e	ρ_{ce}
6324.45		505.92		0	∞	0

TABLE G-5

(Establishing a Pure Pursuit Course)

(no computer)

$2\times10^5\gamma_f$	$2\times10^6 u$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5 f_e$ ($10^5 a_{re}$)	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
00000	00000	00000	00000	00000	05000	00000	-01581	00000	00000
02000	00000	00008	00006	-00177	04993	-01326	-00154	00126	00145
04000	-00001	00083	00077	-00447	04922	-01283	00140	0064	00663
06000	-00007	00237	00281	-00667	04719	-00883	00245	01426	01398
08000	-00026	00640	00650	-00792	04350	-00353	00275	02252	02108
10000	-00065	01115	01171	-00808	03829	00177	00247	02910	02592
12000	-00133	01654	01794	-00727	03205	00605	00177	03257	02729
14000	-00235	02182	02449	-00576	02549	00867	00083	03230	02490
16000	-00372	02628	03062	-00393	01931	00935	-00013	02855	01926
18000	-00544	02938	03573	-00214	01415	00816	-00092	02219	01156
20000	-00747	03086	03941	-00070	01040	00585	-00143	01455	00320
22000	-00975	03071	04156	00017	00816	00278	-00158	00699	-00440
24000	-01221	02922	04230	00042	00730	-00027	-00142	00074	-01013
26000	-01481	02683	04199	00011	00746	-00272	-00100	-00340	-01334
28000	-01743	02426	04110	-00060	00819	-00419	-00045	-00513	-01395
30000	-02022	02140	04009	-00149	00902	-00453	00012	-00460	-01232
32000	-02299	01923	03937	-00234	00955	-00383	00057	-00240	-00920

-014044
-002299
-000206
-023977
013992
003829
005632
049999
001389
-001005
100000
020000

Same as run 22 except that meh_w assumed negligible.

The output table and the above data show that meh_w is negligible.

M. I. T. DIFFERENTIAL ANALYZER #2					
Run No. 24	RDA No. 184	Ed. No. 2			
Date 2/26/47	Time 8:15				
a_e	b_e	c_e	SN_e	P_{ce}	
6324.45	505.92	0	∞	0	

TABLE G-6
(Establishing A Pure Pursuit Course)
(no computer)

000000
000000
000000
-000002
-000003
-000001
-158111
050001
000000

000000	$2\times10^5\gamma_t$	$2\times10^6\mu$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5 f_e$ ($10^5 q_{re}$)	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
100000	00000	00000	00000	00000	00000	05000	00000	-01581	00000	00000
020000	02000	00000	00008	00007	-00177	04993	-013 4	-00153	00130	00147
	04000	-00002	00086	00079	-00446	04919	-012 4	00147	00670	00691
	06000	-00008	00301	00295	-00662	04704	-00850	00262	01517	01484
	08000	-00028	00678	00690	-00775	04308	-00277	00299	02430	02269
	10000	-00071	01192	01254	-00771	03734	00303	00271	03165	02814
	12000	-00144	01777	01932	-00661	03065	00769	00188	0354	02960
	14000	-00254	02346	02640	-00476	02354	01037	00076	03469	02655
	16000	-00403	02812	03289	-00261	01700	01069	-00044	0296	01948
	18000	-00587	03107	03802	-00062	01181	00876	-00143	02126	00981
	20000	-00803	03199	04129	00080	00846	00520	-00205	01136	-00059
	22000	-01042	03092	04258	00141	00706	00089	-00218	00183	-00979
	24000	-01295	02826	04215	00118	00735	-00320	-00184	-00560	-01620
	26000	-01556	02468	04056	00022	00879	-00622	-00113	-00975	-01891
	28000	-01818	02095	03850	-00119	01068	-00761	-00024	-01020	-01780
	30000	-02080	01777	03670	-00270	01230	-00720	00064	-00731	-01354
	32000	-02343	01566	03573	-00396	01307	-00521	00131	-00210	-00739
	34000	-02610	01484	03591	-00471	01268	-00218	00166	00400	-00089
	36000	-02886	01523	03729	-00480	01109	00119	00165	00954	00452
	38000	-03174	01649	03960	-00425	00855	00416	00128	01326	00771
	40000	-03481	01812	04241	-00320	00550	00614	00068	01443	00816
	42000	-03805	01957	04519	-00187	00248	00679	-00003	01288	00593
	44000	-04149	02036	04741	-00056	-00001	00607	-00067	00908	00169
	46000	-04508	02019	04873	00049	-00161	00424	-00112	00392	-00353
	48000	-04878	01897	04896	00110	-00213	00187	-00131	-00150	-00855
	50000	-05254	01685	04818	00120	-00166	-00079	-00121	-00608	-01235
	52000	-05632	01416	04663	00082	-00044	-00288	-00086	-00900	-01421
	54000	-06008	01131	04471	00010	00115	-00413	-00037	-00981	-01388
	56000	-06380	00874	04284	-00076	00268	-00433	00017	-00855	-01155
	58000	-06748	00678	04139	-00155	00378	-00352	00063	-0056	-00786
	60000	-07112	00563	04062	-00210	00421	-00194	00091	-0019	-00366
	62000	-07477	00529	04059	-00230	00388	-00002	00098	00168	00017
	64000	-07843	00562	04123	-00210	00289	00185	00084	00452	00287
-018799	66000	-08215	00623	04230	-00158	00146	00325	00053	00596	00400
-011707	68000	-08594	00710	04350	-00084	-00010	00392	00014	00580	00347
-005220	70000	-08980	00763	04451	-00005	-00149	00379	-00026	00416	00156
-005508	72000	-09371	00767	04509	00063	-00243	00292	-00057	00149	-00121
009317	74000	-09766	00714	04508	00109	-00281	00157	-00074	-00160	-00414
002367	76000	-10163	00605	04445	00126	-00257	00005	-00075	-00444	-00659
002804	78000	-10558	00457	04334	00113	-00185	-00133	-00060	-00648	-00804
049999	80000	-10948	00292	04194	00076	-00084	-00228	-00033	-00734	-00825
003553	82000	-11332	00135	04049	00026	00023	-00264	-00002	-00695	-00721
-001956	84000	-11711	00010	03922	-00025	00109	-00237	00028	-00550	-00522

100001
020000

Same as run 23 except that $m_{\dot{w}}$ assumed negligible — The output table and the above data show that effect of $m_{\dot{w}}$ is <u>not</u> negligible on solution time.

TABLE G-7

(Establishing a ~~Pure~~ Pure Pursuit Course)

(no computer)

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 25 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 9:20

a_e		b_e		c_e	SN_e	p_{ce}
6324.45		505.92		0	∞	0

000000
000000
000000
000000
-000004
-00001
-158111
049998
000000
000000
100000
[illegible]
020000

$2\times10^5\gamma_f$	$2\times10^6 u$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5 a_{re}$ / $10^5 f_e$	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
00000	00000	00000	00000	00000	05000	00000	-01581	00000	00000
02000	00000	00009	00007	-00177	04993	-01325	-00154	00126	00145
04000	-00001	00084	00078	-00447	04922	-01282	00139	0064	00663
06000	-00007	00288	00282	-00666	04718	-00881	00245	01426	01398
08000	-00026	006 1	00651	-00791	04349	-00352	00275	02252	02107
10000	-00066	01116	01171	-00807	03828	00177	00247	02910	02591
12000	-00134	01654	01793	-00727	03206	00534	00175	03256	02728
14000	-00235	02182	02448	-00577	02551	00862	00080	03230	02489
16000	-00373	02629	03062	-00394	01937	00927	-00014	02857	01929
18000	-00545	02940	03574	-00217	01425	00814	-00094	02226	01162
20000	-00748	03089	03945	-00077	01055	00570	-00144	01469	00334
22000	-00976	03079	04162	00007	00837	00260	-00160	00724	-00416
24000	-01223	02936	04243	00028	00756	-00046	-00142	00112	-00978
26000	-01483	02705	04222	-00007	00777	-00290	-00098	-00287	-01287
28000	-01752	02439	04145	-00081	00854	-00435	-00044	-00444	-01335
30000	-02027	02186	04059	-00173	00939	-00465	00013	-00376	-01162
32000	-02307	01984	04006	-00260	00993	-00392	00058	-00141	-00841
34000	-02594	01853	04008	-00324	00990	-0024	00087	00179	-00463
36000	-02890	01796	04077	-00355	00922	-00060	00094	00502	-00116
38000	-03196	01800	04204	-00348	00795	00119	00082	00756	00137
40000	-03516	01841	04371	-00309	00628	00259	00056	00892	00254
42000	-03850	01892	04552	-00248	00447	00338	00023	00894	00231
44000	-04198	01926	04721	-00178	00278	00350	-00011	00774	00091

-017826
-004200
000902
007738
-008599
-003496
-001065
050000
-007744
-001400
100000
020000

Same as run no. 23 except that $(\theta-w)$ was neglected in the ideal radar equation.

It was found that neglecting $(\theta-w)$ increased solution time only very slightly. But since this was an easy term to carry along, it was decided to include it in subsequent analyses.

000000
000000
000000
000000
-000004
000001
-158111
049997
000000
000000
100000
020000

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 26 RDA No. 184 Ed. No. 2

Date 2/26/47 Time 10:45

a_e		b_e		c_e	SN_e	p_{ce}
6324.45		505.92		0		0

TABLE G-8 (Establishing a Pure Pursuit Course) (no computer)

$2\times10^5\gamma_f$	$2\times10^6 u$	$10^5 w$	$10^5\theta$	$2\times10^4\delta_e$	$10^5 f_e$ ($10^5 q_{re}$)	$10^3\dot{\delta}_e$	$5\ddot{\delta}_e$	$5\times10^3\dot{\theta}$	$5\times10^3\dot{w}$
00000	00000	00000	00000	00000	05000	00000	-01581	00000	00000
00000	00000	00000	00000	00000	05000	00000	-01581	-00000	00000
02000	00000	00008	00007	-00177	04993	-01326	-00155	00126	00143
04000	-00001	00083	00078	-00447	04922	-0128	00140	00641	00662
06000	-00007	00288	00282	-00666	04719	-00883	00245	01426	01397
08000	-00026	00641	00651	-00791	04350	-00354	00275	0225	02106
10000	-00066	01116	01171	-00807	03829	00176	00247	02910	02590
12000	-00134	01654	01794	-00726	03206	00605	00176	03256	02727
14000	-00235	02181	02448	-00576	02549	00866	00083	03230	02488
16000	-00373	02627	03062	-00392	01932	00935	-00013	02854	01924
18000	-00545	02937	03572	-00213	01416	00826	-00093	02219	01152
20000	-00748	03084	03940	-00070	01040	00585	-00143	01454	00316
22000	-00976	03070	04155	00017	00817	00278	-00159	00699	-00443
24000	-01222	02920	04229	00042	00731	-00028	-00142	00074	-01016
26000	-01481	02680	04197	00011	00746	-00273	-00100	-00340	-01338
28000	-01749	02403	04108	-00060	00820	-00419	-00045	-00512	-01399
30000	-02022	02136	04007	-00148	00902	-00453	00011	-00459	-01236
32000	-02300	01919	03936	-00233	00955	-00383	00057	-00239	-00925
34000	-02583	01771	03917	-00295	00952	-00238	00085	00071	-00552
36000	-02874	01696	03963	-00325	00884	-00056	00093	00385	-00207
38000	-03174	01682	04066	-00318	00758	00122	00082	00634	00046
40000	-03486	01705	04209	-00278	00591	00263	00057	00767	00166
42000	-03810	01739	04364	-00215	00410	00344	00024	00768	00147
44000	-04148	01757	04507	-00144	00240	00358	-00008	00647	00011
46000	-04498	01739	04616	-00075	00102	00332	-00036	00436	-00201

-017832
-004498
-002008
004359
-011994
-003122
-003580
049999
-005798
-001436
100000
020000

Same as run 23, except Z_u assumed negligible. It was negligible, so only 2 degrees of freedom for the airplane need be considered, i.e. the "long period" of the airplane does not enter the picture in any appreciable way. The variation in u shown above is merely a by-product of the system, and has no effect on solution time.

000000
000000
000000
000002
-043121
000000
050001
026592

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 29 RDA No. 184 Ed. No. 3

Date 2/27/47 Time 5:40

a_e	b_e	c_e	SN_e	p_{ce}
8,624.25	607.104	0	∞	0

TABLE G-9-a

(Establishing a Pure Pursuit Course (No Computer))

$10^5 \gamma_f$	$10^6 w$	$10^6 \theta$	$10^5 s_e$	$10^6 f_e$ / $10^6 a_{re}$	$10^6 p_{ce}$	$10^3 \dot{s}_e$	$\ddot{s}_e$	$10^4 \dot{\theta}$	$10^4 \dot{w}$
00000	00000	00000	00000	50000	00000	00000	-00431	00000	00000
00500	00007	00006	-00306	49994	00000	-00927	-00056	00046	00060
01000	00084	00074	-00806	49927	00000	-01026	00000	00249	00280
01500	00308	00282	-01311	49719	00000	-0097	00012	00609	00648
02000	00744	00703	-01785	49298	00000	-00907	00018	01094	01123
02500	01437	01391	-02214	48612	00000	-00806	00021	01667	01665
03000	02408	02379	-02589	47625	00000	-00689	00024	02292	02236
03500	03666	03685	-02902	46318	00000	-00561	00026	02936	02801
04000	05198	05312	-03150	44692	00000	-00427	00027	03568	03330
04500	06980	07245	-03330	42757	00000	-00292	00027	04161	03800
05000	08978	09461	-03442	40530	00000	-00158	00026	04693	04188
05500	11146	11924	-03491	38074	00000	-00031	00024	05146	04482
06000	13438	14590	-03476	35403	00000	00087	00022	05506	04672
06500	15796	17411	-03405	32575	00000	00193	00020	05763	04750
07000	18168	20335	-03285	29643	00000	00284	00017	05913	04719
07500	20497	23306	-03123	26698	00000	00362	00013	05957	04582
08000	22732	26273	-02926	23675	00000	00423	00010	05895	04344
08500	24826	29183	-02703	20745	00000	00466	00007	05735	04018
09000	26737	31992	-02462	17913	00000	00493	00003	05485	03615
09500	28430	34655	-02212	15221	00000	00506	00000	05158	03150
10000	29879	37138	-01960	12704	00000	00503	-00002	04765	02638
10500	31063	39411	-01711	10392	00000	00488	-00004	04321	02093
11000	31970	41452	-01475	08306	00000	00461	-00006	03839	01534
11500	32596	43246	-01253	06462	00000	00426	-00008	03335	00973
12000	32945	44785	-01051	04866	00000	00383	-00009	02823	00424
12500	33026	46068	-00872	03520	00000	0034	-00010	02314	-00099
13000	32853	47101	-00717	02419	00000	00284	-00010	01821	-00586
13500	32448	47893	-00589	01553	00000	00232	-00010	01355	-01028
14000	31834	48461	-00487	00906	00000	00179	-00010	00923	-01419
14500	31040	48823	-00410	00457	00000	00129	-00010	00534	-01753
15000	30092	49002	-00355	00186	00000	00081	-00009	00191	-02028
15500	29022	49023	-00323	00068	00000	00038	-00008	-00102	-02243
16000	27861	48908	-00314	00078	00000	00000	-00007	-00343	-02398
16500	26636	48686	-00323	00192	00000	-00032	-00006	-00531	-02495
17000	25376	48384	-00348	00381	00000	-00058	-00004	-00667	-02537
17500	24108	48026	-00385	00622	00000	-00078	-0001	-00755	-02528
18000	22855	47637	-00429	00910	00000	-00090	-00002	-00796	-02475
18500	21638	47236	-00479	01165	00000	-00097	-00001	-00798	-02383
19000	20476	46844	-00530	01427	00000	-00098	00000	-00764	-02260
19500	19381	46475	-00580	01661	00000	-00093	00001	-00701	-02114
20000	18363	46145	-00627	01856	00000	-0007	00001	-00616	-01951
20499	17430	45862	-00669	01998	00000	-00072	00002	-00514	-01778
20999	16585	45632	-00704	02085	00000	-00056	00003	-00402	-01601
21499	15829	45460	-00730	02110	00000	-00038	00003	-00284	-01426
21999	15157	45345	-00746	02073	00000	-00019	00003	-00168	-01259
22499	14567	45285	-00751	01977	00000	00000	00003	-00057	-01104
22999	14050	45282	-00746	01825	00000	00020	00003	00045	-00964
23499	13598	45328	-00730	01622	00000	00039	00003	00133	-00843

M. I. T. DIFFERENTIAL ANALYZER #2					
Run No. 29 continued	RDA No. 184	Ed. No. 3			
Date 2/27/47		Time 5:40			
a_e	b_e		c_e	SN_e	ρ_{ce}
8,624.25	407.104		0	∞	0

TABLE G-9-b
(Establishing a Pure Pursuit Course)
(No computer)

$10^5 \gamma_f$	$10^6 w$	$10^6 \theta$	$10^5 s_e$	$10^6 f_e$ ($10^6 a_{ce}$)	$10^6 \rho_{ce}$	$10^3 \dot{s}_e$	$\ddot{s}_e$	$10^4 \dot{\theta}$	$10^4 \dot{w}$
23999	13202	45416	-00705	01378	00000	00056	00003	00205	-00742
24499	12851	45533	-00670	01099	00000	00072	00002	00260	-00663
24999	12534	45673	-00628	00795	00000	00084	00002	00297	-00605
25499	12241	45827	-00580	00475	00000	00094	00001	00315	-00567
25999	11962	45985	-00528	00148	00000	00103	00001	00316	-00548
26499	11687	46140	-00472	-00178	00000	00108	00000	00299	-00548
26999	11410	46283	-00415	-00493	00000	00111	00000	00267	-00561
27499	11124	46406	-00356	-00791	00000	00111	00000	00222	-00588
27999	10821	46505	-00299	-01064	00000	00109	-00001	00166	-00624
28499	10499	46575	-00242	-01291	00000	00104	-00001	00101	-00668
28999	10153	46610	-00188	-01524	00000	00099	-00001	00030	-00716
29499	09783	46605	-00137	-01704	00000	00092	-00002	-00045	-00765
29999	09388	46559	-00091	-01847	00000	00083	-00002	-00121	-00815
30499	08969	46475	-00049	-01953	00000	00074	-00002	-00198	-00862
30999	08527	46357	-00012	-02024	00000	00064	-00002	-00272	-00905
31499	08066	46203	00020	-02062	00000	00054	-00002	-00342	-00942
31999	07587	46015	00046	-02068	00000	00044	-00002	-00405	-00971
32499	07096	45797	00068	-02042	00000	00034	-00002	-00462	-00992
32999	06597	45553	00084	-01991	00000	00026	-00002	-00510	-01004
33499	06094	45286	00095	-01920	00000	00018	-00002	-00550	-01007
33999	05591	45003	00103	-01833	00000	00012	-00001	-00580	-01001
34499	05093	44706	00108	-01734	00000	00006	-00001	-00602	-00986

-009863
-006028
-000336
000064
-000085
002064
-001736
-003636

$$\left(\frac{T_{AF_0}}{T_A}\right)\left(-m_{eh_{\dot{s}_e}}\right) = 400$$

$$\left(\frac{T_{AF_0}}{T_A}\right)^2\left(-m_{eh_{\dot{s}_e}}\right) = 800$$

This was best combination so far

ST = .15, or = (.15)(6.8182) = 1.023 seconds

005069
029045
-010283
003920
-020186
001700
004998
031916

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 38 RDA No. 184 Ed. No. 3

Date 2/28/47 Time 10:35

a_e		b_e		c_e	SN_e	
8624.	25	607.104		0	0.0	

TABLE G-10
(Establishing an Aerodynamic Lead Pursuit Course)

$10^5 \gamma_t$	$10^6 w$	$10^6 \theta$	$10^5 \delta_e$	$10^6 f_e$	$10^6 \phi_{ce}$	$10^3 \dot{\delta}_e$	$\ddot{\delta}_e$	$10^4 \dot{\theta}$	$10^4 \dot{w}$
11923	32958	44640	-01083	05000	00000	00392	-00202	02905	-00597
12000	32994	44860	-01059	05012	00241	00258	-00149	02826	00425
12500	33080	46149	-01036	05078	01660	-00058	-00020	02345	-00064
13000	32947	47218	-01081	05118	02836	-00095	-00001	01946	-00450
13500	32645	48109	-01131	05132	03816	-00093	00000	01630	-00741
14000	32218	48860	-01179	05121	04638	-00085	00000	01387	-00951
14500	31704	49504	-01223	05089	05341	-0007	00000	01203	-01094
15000	31132	50071	-01263	05058	05950	-00070	00001	01070	-01183
15500	30528	50581	-01299	04968	06479	-00062	00001	00979	-01227
16000	29910	51055	-01331	04880	06979	-00054	00001	00924	-01235
16500	29296	51508	-01359	04775	07435	-00045	00001	00896	-01216
17000	28696	51954	-01382	04650	07871	-0003	00001	00890	-01177
17500	28119	52400	-01402	04507	08294	-00028	00001	00900	-01122
18000	27573	52855	-01416	04347	08713	-00019	00001	00924	-01057
18500	27061	53324	-01425	04167	09132	-00010	00001	00956	-00987
19000	26584	53810	-01428	03968	09555	00000	00001	00991	-00916
19500	26142	54314	-01427	03751	09982	00011	00001	01027	-00847
20000	25734	54835	-01420	03516	10412	00021	00001	01062	-00783
20500	25356	55373	-01407	03262	10846	00032	00001	01093	-00725
21000	25005	55926	-01387	02990	11281	0004	00001	01118	-00674
21500	24678	56489	-01362	02700	11711	00054	00001	01135	-00634
22000	24367	57058	-01331	02394	12177	00065	00001	01142	-00605
22500	24068	57628	-01293	02070	12551	00075	00001	01139	-00587
23000	23775	58195	-01251	01731	12949	00087	00001	01126	-00581
23500	23483	58751	-01202	01378	13326	00098	00001	01100	-00587
24000	23185	59292	-01149	01009	13680	00109	00001	01064	-00604
24500	22876	59812	-01090	00629	14002	00120	00001	01016	-00632
25000	22550	60305	-01026	00237	14291	00130	00001	00956	-00670
25500	22203	60765	-00959	-00165	14541	00140	00001	00885	-00718
26000	21829	61188	-00887	-00575	14750	00150	00001	00804	-00776
26500	21425	61567	-00810	-00993	14911	00160	00001	00712	-00841
27000	20986	61897	-00727	-01416	15022	00169	00001	00609	-00915
27500	20508	62174	-00642	-01842	15081	00178	00001	00497	-00996
28000	19988	62392	-00552	-02271	15082	00186	00001	00376	-01083
28500	19423	62547	-00458	-02699	15023	0019	00000	00246	-01175
29000	18812	62638	-00361	-03125	14904	00200	00000	00109	-01271
29500	18150	62657	-00260	-03548	14721	00205	00000	-00036	-01372
30000	17438	62597	-00157	-03964	14472	00211	00000	-00189	-014 7
30500	16673	62462	-00051	-04373	14157	00215	00000	-00347	-01583
31000	15854	62247	00058	-04772	13775	00219	00000	-00511	-01691
31500	14981	61950	00167	-05160	13324	00222	00000	-00680	-01800
32000	14053	61567	00278	-05533	12804	00224	-00001	-00852	-01909
32500	13072	61097	00388	-05890	12216	00224	-00001	-01027	-02016
33000	12038	60540	00499	-06230	11561	00224	-00001	-01203	-02120
33500	10952	59894	00610	-06549	10840	0022	-00001	-01380	-02222
34000	09816	59160	00721	-06847	10053	00221	-00001	-01557	-02319
34500	08633	58337	00829	-07122	09204	00218	-00002	-01732	-02412
35000	07405	57428	00937	-07372	08294	00214	-00002	-01905	-02499

-024992
-019051
-003449
002137
-000082
-004727
-007372
-018772
014040

005068
029045
-010285
003920
-017955
-003616
005000
026597

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 39 RDA No. 184 Ed. No. 3

Date 3/14/47 Time 12:50

a_e	b_e	c_e	SN_e
8624.25	607.104	0	0.2

TABLE G-11

(Establishing an Aerodynamic Lead Pursuit Course)

$10^5 \gamma_f$	$10^6 w$	$10^6 \theta$	$10^5 \delta_e$	$10^4 f_e$	$10^6 R_{ce}$	$10^3 \dot{\delta}_e$	$\ddot{\delta}_e$	$10^4 \dot{\theta}$	$10^4 \dot{w}$
11923	32958	44640	-01083	05000	00000	00392	-00180	02905	00507
11938	32965	44684	-01078	04995	00041	00366	-00170	02889	00491
12077	33023	45074	-01040	04945	00399	00185	-00098	02747	00343
12577	33070	46326	-01017	04702	01550	-00028	-00014	02272	-00138
13000	32938	47213	-01029	0458	02371	-00056	-00004	01929	-00470
13500	32620	48089	-01048	04524	03188	-00063	-00001	01589	-00785
14000	32165	48812	-01068	04400	03868	-00063	-00001	01314	-01025
14500	31605	49413	-01088	04283	04437	-00062	00000	01096	-01201
15000	30971	49916	-01107	0416	04918	-00060	00000	00926	-01323
15500	30288	50345	-01124	04053	05329	-00057	00000	00797	-01401
16000	29576	50718	-01138	03934	05687	-00053	00000	00703	-01441
16500	28851	51053	-01149	03808	05995	-00048	00000	00638	-01452
17000	28126	51360	-01157	03676	06293	-00043	00000	00595	-01441
17500	27412	51651	-01163	03534	06561	-00037	00000	00571	-01412
18000	26716	51934	-01167	03382	06816	-00030	00000	00562	-01369
18500	26043	52215	-01169	0321	07061	-00024	00000	00564	-01317
19000	25399	52500	-01166	03043	07303	-00016	00000	00574	-01258
19500	24784	52790	-01158	02854	07543	-00008	00000	00599	-01198
20000	24199	53089	-01146	02653	07783	-00010	00000	00606	-01139
20500	23644	53395	-01130	02439	08023	00007	00000	00622	-01082
21000	23115	53709	-01109	02212	08262	00016	00000	00636	-01029
21500	22612	54029	-01084	01973	08500	00024	00000	00646	-00983
[illegible]78	22536	54080	-01080						
22000	22130	54354	-01055	01724	08734	00032	00000	00651	-00943
22500	21667	54680	-01022	01464	08964	00040	00000	00671	-00912
23000	21217	55005	-00985	01194	09185	00045	00000	-----	-00888
23500	20777	55323	-00944	00916	09396	00056	00000	00627	-00872
24000	20342	55632	-00899	00631	09595	0006	00000	00604	-00866
24500	19909	55926	-00851	00341	09777	00070	00000	00572	-00867
25000	19473	56203	-00799	00046	09940	00076	00000	00532	-00877
25500	19030	56459	-00744	-00252	10083	00083	00000	00484	-00895
26000	18577	56689	-00686	-00551	10201	00088	00000	00428	-00920
26500	18109	56889	-00625	-00850	10292	00094	00000	00365	-00952
27000	17624	57054	-00562	-01148	10355	00098	00000	00294	-00989
27500	17118	57182	-00497	-01443	10387	00102	00000	00217	-01032
28000	16590	57270	-00431	-01733	10386	00106	00000	00133	-01079
28500	16038	57317	-00364	-02018	10350	00109	00000	00045	-01130
29000	15460	57316	-00295	-0229	10279	00111	00000	-00048	-01182
29500	14854	57266	-00226	-02565	10173	00113	00000	-00144	-01237
30000	14221	57169	-00156	-02824	10030	00115	00000	-00243	-01293
30500	13560	57022	-00086	-03073	09851	00116	00000	-00346	-01349
31000	12871	56824	-00015	-03309	09635	00116	-00001	-00430	-01406
31500	12155	56573	00056	-03532	09381	00115	-00001	-00555	-01461
32000	11409	56268	00126	-03741	09091	00114	-00001	-00662	-01515
32500	10638	55911	00196	-03935	08766	00113	-00001	-00768	-01568
33000	09842	55501	00265	-04113	08405	00110	-00001	-00874	-01617
33500	09021	55038	00332	-04274	08010	00107	-00001	-00978	-01664
34000	08177	54524	00399	-04418	07583	00104	-00002	-01080	-01707
34500	07314	53958	00463	-0454	07124	00100	-00002	-01181	-01746

at $\gamma_f \times 10^5 = 56500$, lead screws read as follows:

.009144 ×10⁵
.000355 ×10⁵
.001837 ×10⁵
−.000878 ×10⁵
.000053 ×10⁵
.002417 ×10⁵
.002123 ×10⁵

005069
029045
-010286
003920
-014652
-007421
005000
022797

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 40 RDA No. 184 Ed. No. 3

Date 3/14/47 Time 3:30

a_e		b_e		c_e	SN_e	
8624.25		607.104		0	0.4	

TABLE G-12
(Establishing an Aerodynamic Lead Pursuit Course)

	$10^5\gamma_f$	$10^6 w$	$10^6\theta$	$10^5\delta_e$	$10^6 f_e$	$10^6\rho_{ce}$	$10^3\dot{\delta}_e$	$\ddot{\delta}_e$	$10^4\dot{\theta}$	$10^4\dot{w}$
	11923	32958	44640	-01083	05000	00000	00392	-00146	02905	00507
	12000	32994	44860	-01057	04944	00173	00294	-00108	02826	00424
	12500	33079	56149	-00987	04610	01190	00053	-00016	02336	-00075
	13000	32932	47207	-00971	04329	02035	00012	-00003	01909	-00493
	13500	32598	48069	-00967	0409	02733	00001	-00001	01546	-00830
	14000	32113	48764	-00967	03890	03307	-00006	-00001	01244	-01097
	14500	31512	49322	-00971	03718	03769	-00011	00000	00996	-01299
	15000	30823	49768	-00977	03567	04166	-00014	00000	00797	-01448
	15500	30072	50126	00985	03431	04486	-00016	00000	00640	-01548
	16000	29281	50415	-00993	03307	04753	-00016	00000	00520	-01608
	16500	28468	50652	-01001	03188	04979	-00015	00001	00472	-01636
	17000	27649	50852	-01008	03070	05175	-00013	00001	00371	-01637
-014997	17500	26835	51027	-01013	02950	05349	-00010	00001	00372	-01615
-012293	18000	26036	51188	-01017	02825	05508	-00006	00001	00311	-01576
-001007	18500	25260	51342	-01018	02695	05659	-00001	00002	00305	-01524
000682	19000	24512	51497	-01017	02555	05806	00004	00002	00309	-01464
-000105	19500	23796	51655	-01012	02405	05952	00011	00002	00322	-01399
000117	20000	23113	51821	-01004	02244	06098	0001	00002	00340	-01331
-003644	20500	22465	51997	-00993	02002	06247	00024	00002	00360	-01263
-007557	21000	21849	52182	-00978	01889	06399	00031	00002	004 0	-01198
	21500	21265	52377	-00959	01694	06554	00038	00002	00399	-01136
	22000	20711	52580	-00937	0 488	06711	00045	00002	00415	-01080
	22500	20184	52791	-00911	01273	06868	00052	00002	0426	-01031
	23000	19679	53006	-00882	01048	07024	00059	00002	00472	-00989
	23500	19193	53222	-00848	00815	07177	00066	00002	00473	-00954
	24000	18722	53437	-00812	00567	07326	00072	00002	00426	-00928
	24500	18263	53648	-00772	0034	07467	00078	00002	00412	-00909
	25000	17812	53850	-00730	00084	07600	0008	00002	00391	-00899
	25500	17363	54038	-00683	-00166	07721	00089	00002	00362	-00896
	26000	16914	54210	-00634	-00417	07827	00094	00002	00326	-00901
	26500	16460	54362	-00583	-00667	07918	00098	00002	00282	-00914
	27000	15999	54491	-00531	-00915	07992	00101	00001	00272	-00932
	27500	15526	54594	-00476	-01157	08045	00105	00001	00175	-00957
	28000	15041	54667	-00420	-01395	08077	00107	00001	00112	-00986
	28500	14540	54708	-00363	-01626	08086	00109	00001	00045	-01019
	29000	14022	54714	-00304	-01849	08069	00110	00001	-00028	-01055
	29500	13484	54678	-00246	-02061	08027	0010	00001	-00104	-01094
	30000	12927	54606	-00187	-02264	07959	00111	00001	-00182	-01135
	30500	12350	54494	-00129	-02455	07866	00111	00000	-00264	-01176
	31000	11751	54342	-00071	-02633	07747	00110	00000	-00347	-01217
	31500	11132	54147	-00014	-02799	07602	00108	00000	-00431	-01258
	32000	10493	53911	00042	-02950	07430	0010	00000	-00515	-01297
	32500	09835	53632	00097	-03086	07233	00104	00000	-00599	-01334
	33000	09159	53313	00150	-03208	07012	00101	00000	-00681	-01368
	33500	08467	52953	00202	-03315	06766	00098	00000	-00761	-01399
	34000	07760	52553	00252	-03406	06498	00095	00000	-00839	-01428
	34500	07040	52114	00300	-03483	06208	00091	00000	-00913	-01451
	35000	06309	51639	00346	-03544	05899	00087	00000	-00985	-01470
	35500	05570	51130	00389	-03591	05572	00083	00000	-01052	-01485
	36000	04824	50587	00430	-03625	05228	00078	00000	-01116	-01495
	36500	04075	50014	00468	-03641	04869	0007	-00001	-01174	-01500
	37000	03325	49414	00505	-03643	04497	00068	-00001	-01228	-01500

005069
029045
-010286
003920
-012920
-010267
005000
019948

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 41 RDA No. 184 Ed. No. 3

Date 3/14/47 Time 4:10

a_e		b_e		c_e	SN_e	
8624.25		607.104		0	0.6	

TABLE G-13 (Establishing an Aerodynamic Lead Pursuit Course)

$10^5 r_f$	$10^6 w$	$10^6 \theta$	$10^5 \delta_e$	$10^6 f_e$	$10^6 p_{ce}$	$10^3 \dot{\delta}_e$	$\ddot{\delta}_e$	$10^4 \dot{\theta}$	$10^4 \dot{w}$
11923	32958	44640	-01083	05000	00000	00392	-00129	02905	00507
12000	32994	44861	-01056	04921	00151	0031	-00096	02816	00425
12490	33078	46149	-00971	04462	01042	00089	-00015	02334	-00077
13000	32928	47204	-00939	04081	01784	00046	-00004	01898	-00504
13500	32585	48057	-00918	03766	02395	00030	-00002	01522	-00855
14000	32084	48737	-00905	03507	02895	00018	-00002	01203	-01137
14500	31457	49270	-00896	03293	03300	00009	-00001	00937	-01358
14990	30735	49683	-00893	03117	03628	00002	-00001	00718	-01523
15500	29942	49996	-00892	02970	03891	-00003	-00001	00542	-01641
16000	29101	50231	-00895	02844	04104	-00007	00000	00404	-01716
16500	28231	50406	-00899	02734	04277	-00008	00000	00299	-01755
16990	27349	50536	-00904	02632	04419	-0000	00000	00224	-01766
17500	26469	50635	-00908	02537	04540	-00008	00001	00173	-01751
18000	25600	50714	-00911	02441	04645	-00006	00001	001 2	-01717
18500	24753	50782	-00914	02341	04741	-0000	00001	00129	-01667
18990	23934	50847	-00914	02235	04833	00001	00001	00129	-01606
19500	23148	50915	-00912	02121	04924	00006	00001	00140	-01537
20000	22297	50991	-00907	01997	05016	00011	00001	00157	-01464
20500	21683	51076	-00899	01862	05112	00017	00001	00180	-01389
20990	21007	51174	-00888	01717	05212	00023	00002	00204	-01315
21500	20368	51284	-00873	01559	05317	00030	00002	00229	-01243
22000	19762	51406	-00855	01389	05426	00036	00002	00252	-01176
22500	19189	51538	-00833	01209	05538	00043	00002	00271	-01115
22990	18646	51679	-00808	01018	05652	00049	00002	00287	-01061
23500	18127	51826	-00780	00820	05768	00056	00002	00296	-01014
24000	17630	51976	-00749	00613	05883	00062	00002	00300	-00974
24500	17150	52127	-00714	00401	05995	00067	00002	00298	-00943
24990	16685	52275	-00676	00185	06103	00073	00002	00288	-00920
25500	16229	52416	-00636	-00034	06205	00078	00001	00272	-00904
26000	15779	52546	-00594	-00255	06299	00082	00001	00248	-00896
26500	15331	52664	-00550	-00474	06383	00076	00001	00218	-00895
26990	14882	52765	-00503	-00691	06455	00089	00001	00181	-00901
27500	14429	52846	-00455	-00904	06514	00091	00001	00138	-00912
28000	13968	52904	-00405	-01112	06558	00093	00001	00089	-00929
28500	13498	52938	-00355	-01313	06584	00094	00001	00035	-00951
28990	13017	52941	-00304	-01504	06594	0009	00001	-00023	-00976
29500	12522	52913	-00253	-01688	06584	00095	00001	-00085	-01003
30000	12013	52853	-00202	-01940	06555	00095	00000	-00150	-01033
30500	11488	52760	-00151	-02022	06506	00094	00000	-00217	-01064
31000	10948	52633	-00102	-02170	06438	00093	00000	-002 6	-01095
31490	10393	52473	-00053	-02306	06351	00091	00000	-00356	-01125
32000	09823	52278	-00006	-02429	06245	00089	00000	-00426	-01155
32500	09238	52049	00040	-02538	06120	00086	00000	-00494	-01182
33000	08641	51786	00083	-02632	05976	00083	00000	-00561	-01207
33490	08031	51490	00125	-02714	05815	00080	00000	-00626	-01228
34000	07412	51162	00165	-02791	05637	00077	00000	-00688	-01246
34500	06785	50803	00203	-0283	05444	00073	00000	-00747	-01261
35000	06152	50416	00240	-02890	05237	00069	00000	-00803	-01271
35490	05514	50002	00275	-02911	05016	00065	00000	-00855	-01278
36000	04874	49562	00308	-02929	04784	00062	00000	-00904	-01281
36500	04234	49099	00339	-02932	04541	00058	00000	-00949	-01280
37000	03595	48614	00369	-02925	04289	00054	00000	-00989	-01275
37500	02959	48110	00396	-02904	04028	00049	00000	-01026	-01265

000000
000000
000000
000001
-00001
046495
000000
000000
-000001
000000
125000
002000
084452

000341

M. I. T. DIFFERENTIAL ANALYZER #2

Run No. 28 RDA No. 186 Ed. No. 1

Date 3/19/47 Time 6:10

a_a	b_a	c_a	a_r	b_r	c_r
0	0	0	9300	1625	43,000

TABLE G-14

DATA FOR ESTABLISHING FIRE CONTROL CORRECTIONS

5×10^5 Υ_t	250,000 v	250,000 ψ	5,000 $\dot\psi$	100 $\ddot\psi$	250,000 ϕ	25,000 δ_a	25,000 δ_r	250,000 $f_d = a_{rd}$	5,000 $\dot\phi$
00000	00000	00000	00000	00000	00000	00000	00000	12500	00000
05000	-00007	00004	00029	00131	00001	00000	00138	12495	00000
10000	-00053	00043	00130	00273	00003	00000	00295	12456	00005
15000	-00169	00149	00298	00395	00011	00000	00445	12351	00021
20000	-00385	00372	00521	00490	00029	00000	00578	12148	00054
25000	-00724	00678	00782	00550	00068	00000	00690	11824	00109
30000	-01200	01140	0 065	00573	00142	00000	00775	11363	00190
35000	-01814	01746	01349	00557	00262	00000	00831	10760	00297
40000	-02560	02490	0 615	00501	00443	00000	00853	10018	00432
45000	06578	033 9	0 843	00408	00698	00000	0084	09152	00593
50000	-04374	04330	02016	00285	01040	00000	00804	08184	00778
55000	-05384	05371	02124	00142	0 479	00000	00741	07146	00982
60000	-06413	06446	02157	-00011	02023	00000	00663	06071	01196
65000	-07422	07518	02111	-00164	02675	00000	00577	05000	01414
70000	-08374	08548	01993	-00307	03437	00000	00489	03969	01630
75000	-09235	09502	01808	-00431	04303	00000	00408	03014	01834
80000	-09973	10348	01566	-00529	05267	00000	00339	02165	02019
85000	-10566	11063	0 284	-00593	16317	00000	00290	01446	02178
90000	-11000	11630	00979	-00620	07440	00000	00266	00874	02308
95000	-11270	12043	00670	-00607	08620	00000	00270	00454	02408
100000	-11382	12304	00379	-00554	09841	00000	00304	00184	02473
105000	-11351	12428	00122	-00467	11087	00000	00367	00051	02505
110000	-11200	12436	-00085	-00355	12341	00000	00450	00030	02506
115000	-10960	12354	-00230	-00224	13591	00000	00553	00099	02480
120000	-10664	12218	-00307	-00083	14823	00000	00668	00221	02437
125000	-10348	120 0	-00314	00059	26031	00000	00789	00362	02384
130000	-10047	11916	-00251	00190	17211	00000	00908	00488	02326
135000	-09794	11819	-00126	00304	18362	00000	01018	00565	02270
140000	-09617	11796	00049	00393	19487	00000	01112	00565	02224
145000	-09535	11875	00261	00451	20593	00000	01185	00466	02193
1[illegible]6000	-09531	11903	00307	00459	20813	00000	01197	00432	02189

021892
-00231
003066
004584
001065
-01063
000000
000000
000127
-03544
004324
001999
084452

$$-m_{rh_{\dot{\delta}_r}}\left(\frac{T_{AF_0}}{T_A}\right) = 750$$

$$-m_{rh_{\delta_r}}\left(\frac{T_{AF_0}}{T_A}\right)^2 = 800$$

APPENDIX H

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

APPENDIX H

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APPENDIX H

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

A. LONGITUDINAL EQUATIONS:

A-1: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION

According to Euler's Equations of motion for a moving body with principal body-axes as reference, we have:

$$L = I_X\dot{P} - (I_Y - I_Z)QR, \tag{1}$$

$$M = I_Y\dot{Q} - (I_Z - I_X)RP, \tag{2}$$

$$N = I_Z\dot{R} - (I_X - I_Y)PQ, \tag{3}$$

$$X = M_A(\dot{U} + WQ - VR), \tag{4}$$

$$Y = M_A(\dot{V} + UR - WP), \tag{5}$$

and $Z = M_A(\dot{W} + VP - UQ).$ (6)

If the airplane is restricted to longitudinal motion,

$P,\ R,\ V,\ \dot{P},\ \dot{R}$ and $\dot{V} \equiv 0.$

Thus, equations 1, 3 and 5 reduce to

$$L = N = Y \equiv 0, \tag{7}$$

and equations 2, 4 and 6 reduce to

$$M = I_Y\dot{Q} \tag{8}$$

$$X = M_A(\dot{U} + WQ) \tag{9}$$

and $Z = M_A(\dot{W} - UQ)$ (10)

Using the initial conditions for steady-state horizontal flight, given by equations 11, Section A-1, Appendix A, and neglecting second order terms, equations 8, 9 and 10 (above) become:

$$M = I_Y\dot{Q} \quad (8)$$

$$X = M_A\Delta\dot{U} \quad (11)$$

$$Z = M_A(\Delta\dot{W} - U_0Q) \quad (12)$$

Functional Dependence:

Let $M = F(W, \dot{W}, U, Q, \Delta_E)$, (13)

$X = F(W, \theta, U)$ (14)

and $Z = F(W, \theta, U)$. (15)

Other effects, such as X_Q, Z_Q, X_{Δ_E}, Z_{Δ_E}, $X_{\dot{\Delta}_E}$, $Z_{\dot{\Delta}_E}$, $M_{\dot{\Delta}_E}$, $X_{\Delta\dot{W}}$, etc., will be neglected.

Assuming that all changes are small, and using the principle of linear superposition, we have:

$$M = M_0 + \left(\frac{\partial M}{\partial \Delta U}\right)\Delta U + \left(\frac{\partial M}{\partial \Delta W}\right)\Delta W + \left(\frac{\partial M}{\partial \Delta \dot{W}}\right)\Delta\dot{W} + \left(\frac{\partial M}{\partial Q}\right)Q + \left(\frac{\partial M}{\partial \Delta_E}\right)\Delta_E, \quad (16)$$

$$X = X_0 + \left(\frac{\partial X}{\partial \Delta U}\right)\Delta U + \left(\frac{\partial X}{\partial \Delta W}\right)\Delta W + \left(\frac{\partial X}{\partial \theta}\right)\theta \quad (17)$$

and

$$Z = Z_0 + \left(\frac{\partial Z}{\partial \Delta U}\right)\Delta U + \left(\frac{\partial Z}{\partial \Delta W}\right)\Delta W + \left(\frac{\partial Z}{\partial \theta}\right)\theta. \quad (18)$$

Trim Conditions:

Assume $M_0 = X_0 = Z_0 \equiv 0$, before the forced oscillation is started.

Forced Oscillation:

$$\Delta_E = \Delta_{E_a} \sin \Omega_E t = \Delta_{E_a} \mathcal{I} e^{i\Omega_E t}. \quad (19)$$

We may define:

$$X_{\Delta U} = (\frac{1}{M_A})(\frac{\partial X}{\partial \Delta U}),\quad X_{\Delta W} = (\frac{1}{M_A})(\frac{\partial X}{\partial \Delta W}),\quad X_\theta = (\frac{1}{M_A})(\frac{\partial X}{\partial \theta}),$$

$$Z_{\Delta U} = (\frac{1}{M_A})(\frac{\partial Z}{\partial \Delta U}),\quad Z_{\Delta W} = (\frac{1}{M_A})(\frac{\partial Z}{\partial \Delta W}),\quad Z_\theta = (\frac{1}{M_A})(\frac{\partial Z}{\partial \theta}),$$

$$M_{\Delta U} = (\frac{1}{I_Y})(\frac{\partial M}{\partial \Delta U}),\quad M_{\Delta W} = (\frac{1}{I_Y})(\frac{\partial M}{\partial \Delta W}),\quad M_Q = (\frac{1}{I_Y})(\frac{\partial M}{\partial Q}),$$

$$M_{\Delta_E} = (\frac{1}{I_Y})(\frac{\partial M}{\partial \Delta_E}) \text{ and } M_{\Delta \dot{W}} = (\frac{1}{I_Y})(\frac{\partial M}{\partial \Delta \dot{W}}).$$

The equations of motion now become:

$$\Delta\dot{U} = X_{\Delta U}\Delta U + X_{\Delta W}\Delta W + X_\theta \theta, \tag{20}$$

$$\Delta\dot{W} = Z_{\Delta U}\Delta U + Z_{\Delta W}\Delta W + Z_\theta \theta + U_o Q \tag{21}$$

$$\text{and } \dot{Q} = M_{\Delta U}\Delta U + M_{\Delta W}\Delta W + M_{\Delta\dot{W}}\Delta\dot{W} + M_Q Q + M_{\Delta_E}\Delta_E. \tag{22}$$

By definition:

$$D = \frac{d}{dt} \text{ so } Q = D\theta \text{ and } \dot{Q} = D^2\theta.$$

Thus, the equations of motion reduce to:

$$(-D + X_{\Delta U})\Delta U + (X_{\Delta W})\Delta W + (X_\theta)\theta = 0, \tag{23}$$

$$(Z_{\Delta U})\Delta U + (-D + Z_{\Delta W})\Delta W + (DU_o + Z_\theta)\theta = 0 \tag{24}$$

$$\text{and } (M_{\Delta U})\Delta U + (DM_{\Delta\dot{W}} + M_{\Delta W})\Delta W + (-D^2 + DM_Q)\theta$$

$$= -M_{\Delta_E}\Delta_{E_a}\mathcal{J}e^{i\Omega_E t}. \tag{25}$$

Non-dimensionalizing:

By definition, let:

$$\gamma = \frac{t}{T_A},$$

$$T_A = \frac{U_o C_{L_o}}{g} = \frac{M_A}{\frac{\rho}{2}SU_o},$$

$$x_u = T_A X_{\Delta U},$$

$$x_w = T_A X_{\Delta W},$$

$$x_\theta = T_A X_\theta(\frac{1}{U_o}),$$

$$m_u = cT_A M_{\Delta U},$$

$m_w = cT_A M_{\Delta W}$,

$m_q = T_A M_Q$,

$\delta_e = \delta_{e_a} \sin \omega_e \gamma$,

$m_{\dot{w}} = cM_{\Delta \dot{W}}$,

$m_{\delta_e} = T_A{}^2 M_{\Delta_E}$,

$d = T_A D$,

$u = \dfrac{\Delta U}{U_o}$,

$w = \dfrac{\Delta W}{U_o}$,

$\dot{w} = (\dfrac{T_A}{U_o}) \Delta \dot{W}$,

$\omega_e = T_A \Omega_E$,

$\mu_c = \dfrac{T_A U_o}{c} = \dfrac{U_o C_{L_o}}{gc} = \dfrac{M_A}{\frac{\rho}{2} Sc}$,

$z_u = T_A Z_{\Delta U}$,

$z_w = T_A Z_{\Delta W}$,

$z_\theta = T_A Z_\theta (\dfrac{1}{U_o})$,

$q = T_A Q$,

$\delta_{e_a} \equiv \Delta_{E_a}$,

$\delta_e = \Delta_E$,

$\dot{\delta}_e = T_A \dot{\Delta}_E$.

After substituting in the equations of motion we have:

$$[-(\frac{d}{T_A}) + (\frac{x_u}{T_A})] uU_o + (\frac{x_w}{T_A}) wU_o + (\frac{x_\theta}{T_A}) U_o \theta = 0, \tag{26}$$

$$(\frac{z_u}{T_A}) uU_o + [-(\frac{d}{T_A}) + (\frac{z_w}{T_A})] wU_o + [(\frac{d}{T_A}) U_o + (\frac{z_\theta}{T_A}) U_o] \theta = 0 \tag{27}$$

and

$$(\frac{m_u}{cT_A}) uU_o + [(\frac{d}{T_A})(\frac{m_{\dot{w}}}{c}) + (\frac{m_w}{cT_A})] wU_o + [-(\frac{d^2}{T_A{}^2}) + (\frac{d}{T_A})(\frac{m_q}{T_A})] \theta$$

$$= - (\frac{m_{\delta_e}}{T_A{}^2}) \delta_{e_a} \mathcal{I} e^{i\omega_e \gamma}. \tag{28}$$

If we multiply equations 26 and 27 by T_{A/U_o} , and multiply equation 28 by $T_A{}^2$, we have:

$$(-d + x_u) u + (x_w) w + (x_\theta) \theta = 0, \tag{29}$$

$$(z_u) u + (-d + z_w) w + (d + z_\theta) \theta = 0 \text{ and} \tag{30}$$

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta = -m_{\delta_e}\delta_{e_a} \mathcal{d} e^{i\omega_e\gamma}. \quad (31)$$

Let $u = \mathcal{d}\bar{u}_a e^{i\omega_e\gamma}$, (32)

$w = \mathcal{d}\bar{w}_a e^{i\omega_e\gamma}$ and (33)

$\theta = \mathcal{d}\bar{\theta}_a e^{i\omega_e\gamma}$, (34)

where $\bar{u}_a$, $\bar{w}_a$ and $\bar{\theta}_a$ are complex amplitudes.

Putting equations 32, 33 and 34 in equations 29, 30 and 31, respectively, we have:

$$(-i\omega_e + x_u)\bar{u}_a + (x_w)\bar{w}_a + (x_\theta)\bar{\theta}_a = 0, \quad (29\text{-a})$$

$$(z_u)\bar{u}_a + (-i\omega_e + z_w)\bar{w}_a + (i\omega_e + z_\theta)\bar{\theta}_a = 0 \text{ and} \quad (30\text{-a})$$

$$(\mu_c m_u)\bar{u}_a + (i\omega_e\mu_c m_{\dot{w}} + \mu_c m_w)\bar{w}_a + (\omega_e^2 + i\omega_e m_q)\bar{\theta}_a = -m_{\delta_e}\delta_{e_a}. \quad (31\text{-a})$$

By determinants,

$$\bar{w}_a = \frac{\Delta_1}{\Delta_2}, \quad \bar{\theta}_a = \frac{\Delta_3}{\Delta_2} \text{ and } \bar{u}_a = \frac{\Delta_4}{\Delta_2}, \quad (35)$$

where

$$\Delta_1 = \begin{vmatrix} (-i\omega_e + x_u) & (0) & (x_\theta) \\ (z_u) & (0) & (i\omega_e + z_\theta) \\ (\mu_c m_u) & (-m_{\delta_e}\delta_{e_a}) & (\omega_e^2 + i\omega_e m_q) \end{vmatrix} \quad (36)$$

Expanding, we have:

$$\Delta_1 = -m_{\delta_e}\delta_{e_a}\sqrt{(C_1 - \omega_e^2)^2 + (\omega_e C_2)^2}\, e^{i\phi_1}, \quad (37)$$

where $C_1 = x_\theta z_u - x_u z_\theta$, (38)

$C_2 = z_\theta - x_u$ and (39)

$$\phi_1 = \tan^{-1}\left(\frac{\omega_e C_2}{C_1 - \omega_e^2}\right). \quad (40)$$

Also,

$$\Delta_2 = \begin{vmatrix} (-i\omega_e + x_u) & (x_w) & (x_\theta) \\ (z_u) & (-i\omega_e + z_w) & (i\omega_e + z_\theta) \\ (\mu_c m_u) & (i\omega_e \mu_c m_{\dot{w}} + \mu_c m_w) & (\omega_e^2 + i\omega_e m_q) \end{vmatrix} \quad (41)$$

Expanding, we have:

$$\Delta_2 = \sqrt{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2(\omega_e^2 C_5 - C_6)^2}\; e^{i\phi_2}, \quad (42)$$

where

$$C_3 = (x_u z_w - x_w z_u) + (x_u + z_w)m_q + \mu_c m_{\dot{w}}(x_u - z_\theta)$$

$$-\mu_c m_w, \quad (43)$$

$$C_4 = \mu_c m_w(x_u z_\theta - x_\theta z_u) + \mu_c m_u(x_\theta z_w - x_w z_\theta), \quad (44)$$

$$C_5 = -x_u - z_w - m_q - \mu_c m_{\dot{w}}, \quad (45)$$

$$C_6 = (x_w z_u - x_u z_w)m_q + (x_u - z_\theta)\mu_c m_w$$

$$+ (x_u z_\theta - x_\theta z_u)\mu_c m_{\dot{w}} - (x_w + x_\theta)\mu_c m_u \quad (46)$$

and $\phi_2 = \tan^{-1}\left[\dfrac{\omega_e(\omega_e^2 C_5 - C_6)}{-\omega_e^4 + \omega_e^2 C_3 - C_4}\right].$ (47)

Also,

$$\Delta_3 = \begin{vmatrix} (-i\omega_e + x_u) & (x_w) & (0) \\ (z_u) & (-i\omega_e + z_w) & (0) \\ (\mu_c m_u) & (i\omega_e \mu_c m_{\dot{w}} + \mu_c m_w) & (-m_{\delta_e}\delta_{e_a}) \end{vmatrix} \quad (48)$$

Expanding, we have

$$\Delta_3 = -m_{\delta_e} \delta_{e_a} \sqrt{(K_1 - \omega_e^2)^2 + (\omega_e K_2)^2}\, e^{i\psi_1}, \tag{49}$$

where

$$K_1 = x_u z_w - x_w z_u, \tag{50}$$

$$K_2 = -x_u - z_w \text{ and} \tag{51}$$

$$\psi_1 = \tan^{-1} \left(\frac{\omega_e K_2}{K_1 - \omega_e^2}\right). \tag{52}$$

And,

$$\Delta_4 = \begin{vmatrix} (0) & (x_w) & (x_\theta) \\ (0) & (-i\omega_e + z_w) & (i\omega_e + z_\theta) \\ (-m_{\delta_e} \delta_{e_a}) & (i\omega_e \mu_c m_{\dot{w}} + \mu_c m_w) & (\omega_e^2 + i\omega_e m_q) \end{vmatrix} \tag{53}$$

Expanding, we have

$$\Delta_4 = -m_{\delta_e} \delta_{e_a} \sqrt{(-H_1)^2 + (-\omega_e H_2)^2}\, e^{i\beta_1}, \tag{54}$$

where $H_1 = x_\theta z_w - x_w z_\theta,$ (55)

$$H_2 = -x_w - x_\theta \text{ and} \tag{56}$$

$$\beta_1 = \tan^{-1} \left(\frac{-\omega_e H_2}{-H_1}\right). \tag{57}$$

Now, from equations 32, 33, 34, 35, 37, 42, and 54, we have:

$$u = \mu_{u\delta_e} \delta_{e_a} \sin (\omega_e \gamma + LA_{u\delta_e}), \tag{58}$$

$$w = \mu_{w\delta_e} \delta_{e_a} \sin (\omega_e \gamma + LA_{w\delta_e}) \text{ and} \tag{59}$$

$$\theta = \mu_{\theta\delta_e}\delta_{e_a} \sin(\omega_e\gamma + LA_{\theta\delta_e}), \tag{60}$$

where the amplitude response ratios are:

$$\mu_{u\delta_e} = \frac{u_a}{\delta_{e_a}} = -m_{\delta_e}\sqrt{\frac{(-H_1)^2 + (-\omega_e H_2)^2}{(-\omega_e{}^4 + \omega_e{}^2C_3 - C_4)^2 + \omega_e{}^2(\omega_e{}^2C_5 - C_6)^2}}, \tag{61}$$

$$\mu_{w\delta_e} = \frac{w_a}{\delta_{e_a}} = -m_{\delta_e}\sqrt{\frac{(C_1 - \omega_e{}^2)^2 + (\omega_e C_2)^2}{(-\omega_e{}^4 + \omega_e{}^2C_3 - C_4)^2 + \omega_e{}^2(\omega_e{}^2C_5 - C_6)^2}}, \tag{62}$$

and

$$\mu_{\theta\delta_e} = \frac{\theta_a}{\delta_{e_a}} = -m_{\delta_e}\sqrt{\frac{(K_1 - \omega_e{}^2)^2 + (\omega_e K_2)^2}{(-\omega_e{}^4 + \omega_e{}^2C_3 - C_4)^2 + \omega_e{}^2(\omega_e{}^2C_5 - C_6)^2}}, \tag{63}$$

and the lead angles are:

$$LA_{u\delta_e} = \beta_1 - \phi_2, \tag{64}$$

$$LA_{w\delta_e} = \phi_1 - \phi_2 \text{ and} \tag{65}$$

$$LA_{\theta\delta_e} = \psi_1 - \phi_2. \tag{66}$$

Differentiating equations 58, 59 and 60, and rearranging, we have:

$$\dot{u} = \mu_{\dot{u}\delta_e}\delta_{e_a} \sin(\omega_e\gamma + LA_{\dot{u}\delta_e}), \tag{67}$$

$$\dot{w} = \mu_{\dot{w}\delta_e}\delta_{e_a} \sin(\omega_e\gamma + LA_{\dot{w}\delta_e}) \text{ and} \tag{68}$$

$$q = \mu_{q\delta_e}\delta_{e_a} \sin(\omega_e\gamma + LA_{q\delta_e}), \tag{69}$$

where

$$\mu_{\dot{u}\delta_e} = \omega_e\mu_{u\delta_e}, \qquad LA_{\dot{u}\delta_e} = LA_{u\delta_e} + \frac{\pi}{2}, \tag{70}$$

$$\mu_{\dot{w}\delta_e} = \omega_e \mu_{w\delta_e}, \qquad LA_{\dot{w}\delta_e} = LA_{w\delta_e} + \frac{\pi}{2}, \tag{71}$$

$$\mu_{q\delta_e} = \omega_e \mu_{\theta\delta_e} \quad \text{and} \quad LA_{q\delta_e} = LA_{\theta\delta_e} + \frac{\pi}{2}. \tag{72}$$

From equations 59, 60, 62, 63, 68, 69, 71 and 72, we see that

$$\mu_{q\dot{w}} = \frac{q_\alpha}{\dot{w}_\alpha} = \mu_{\theta w} = \frac{\theta_\alpha}{w_\alpha} = \sqrt{\frac{(K_1 - \omega_e{}^2)^2 + (\omega_e K_2)^2}{(C_1 - \omega_e{}^2)^2 + (\omega_e C_2)^2}}\,. \tag{73}$$

A-2: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL-FORCE AND PITCHING-MOMENT

When the effect of change in longitudinal velocity is neglected in the equations of motion for normal force and pitching moment, equations 30 and 31 reduce to:

$$(-d + z_w)w + (d + z_\theta)\theta = 0 \text{ and} \tag{74}$$

$$(d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta = -m_{\delta_e}\delta_{e_\alpha}\mathcal{J}e^{i\omega_e\gamma}. \tag{75}$$

Substituting equations 33 and 34 in equations 74 and 75, we have:

$$(-i\omega_e + z_w)\overline{w}_\alpha + (i\omega_e + z_\theta)\overline{\theta}_\alpha = 0 \text{ and} \tag{76}$$

$$(i\omega_e\mu_c m_{\dot{w}} + \mu_c m_w)\overline{w}_\alpha + (\omega_e{}^2 + i\omega_e m_q)\overline{\theta}_\alpha = -m_{\delta_e}\delta_{e_\alpha}. \tag{77}$$

Solving equations 76 and 77, simultaneously, for $\overline{w}_\alpha$ and $\overline{\theta}_\alpha$, and putting these values in equations 33 and 34, we have:

$$w = \mu_{w\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{w\delta_e}) \text{ and} \tag{78}$$

$$\theta = \mu_{\theta\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{\theta\delta_e}), \tag{79}$$

where, for $z_\theta = 0$:

$$\mu_{w\delta_e} = \frac{w_a}{\delta_{e_a}} = -\frac{m_{\delta_e}}{\sqrt{(-\omega_e b)^2 + (k - \omega_e^2)^2}} \tag{80}$$

$$\mu_{\theta\delta_e} = \frac{\theta_a}{\delta_{e_a}} = -\frac{m_{\delta_e}}{\omega_e}\sqrt{\frac{(z_w)^2 + (-\omega_e)^2}{(-\omega_e b)^2 + (k - \omega_e^2)^2}} \quad , \tag{81}$$

$$LA_{w\delta_e} = \frac{3\pi}{2} - \phi_2, \tag{82}$$

$$LA_{\theta\delta_e} = \psi_1 - \phi_2, \tag{83}$$

$$b = -z_w - m_q - \mu_c m_{\dot{w}} \ , \tag{84}$$

$$k = -\mu_c m_w + z_w m_q, \tag{85}$$

$$\phi_2 = \tan^{-1}\left(\frac{k - \omega_e^2}{-\omega_e b}\right) \text{ and} \tag{86}$$

$$\psi_1 = \tan^{-1}\left(\frac{-\omega_e}{z_w}\right). \tag{87}$$

Now, substituting equations 78 and 79 in equation 29, we have:

$$u = \mu_{u\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{u\delta_e}), \tag{88}$$

where

$$\mu_{u\delta_e} = \frac{u_a}{\delta_{e_a}} = \sqrt{\frac{A^2 + B^2}{(x_u)^2 + (-\omega_e)^2}} \ , \tag{89}$$

$$A = -x_{\theta}\mu_{\theta\delta_e} \cos \beta_1 - x_w\mu_{w\delta_e} \cos \beta_2, \tag{90}$$

$$B = -x_{\theta}\mu_{\theta\delta_e} \sin \beta_1 - x_w\mu_{w\delta_e} \sin \beta_2, \tag{91}$$

$$LA_{u\delta_e} = \tan^{-1}\left(\frac{B}{A}\right), \tag{92}$$

$$\beta_1 = \psi_1 - \phi_2 - \alpha_1, \tag{93}$$

$$\beta_2 = \frac{\pi}{2} - \phi_2 - \alpha_1 \text{ and} \tag{94}$$

$$\alpha_1 = \tan^{-1} \left(\frac{-\omega_e}{x_u}\right). \tag{95}$$

Differentiating equations 78, 79 and 88, and rearranging, we have:

$$\dot{w} = \mu_{\dot{w}\delta_e}\delta_{e_a} \sin (\omega_e\gamma + LA_{\dot{w}\delta_e}), \tag{96}$$

$$q = \mu_{q\delta_e}\delta_{e_a} \sin (\omega_e\gamma + LA_{q\delta_e}) \text{ and} \tag{97}$$

$$\dot{u} = \mu_{\dot{u}\delta_e}\delta_{e_a} \sin (\omega_e\gamma + LA_{\dot{u}\delta_e}), \tag{98}$$

where

$$\mu_{\dot{w}\delta_e} = \omega_e\mu_{w\delta_e}, \qquad LA_{\dot{w}\delta_e} = LA_{w\delta_e} + \frac{\pi}{2}, \tag{99}$$

$$\mu_{q\delta_e} = \omega_e\mu_{\theta\delta_e}, \qquad LA_{q\delta_e} = LA_{\theta\delta_e} + \frac{\pi}{2}, \tag{100}$$

$$\mu_{\dot{u}\delta_e} = \omega_e\mu_{u\delta_e} \text{ and} \qquad LA_{\dot{u}\delta_e} = LA_{u\delta_e} + \frac{\pi}{2}. \tag{101}$$

From equations 78, 79, 80, 81, 96, 97, 99 and 100 we see that

$$\mu_{q\dot{w}} = \frac{q_a}{\dot{w}_a} = \mu_{\theta w} = \frac{\theta_a}{w_a} = \sqrt{\left(\frac{z_w}{\omega_e}\right)^2 + 1}. \tag{102}$$

Combining equations 82, 86, 87 and 100, we see that

$$LA_{q\delta_e} = \tan^{-1} \left[\frac{-\omega_e(k-\omega_e^2) - z_w\omega_e b}{+z_w(k-\omega_e^2) - \omega_e^2 b}\right] \quad (100\text{-a})$$

and

$$LA_{w\delta_e} = \tan^{-1} \left[\frac{-b\omega_e}{k-\omega_e^2}\right]. \quad (82\text{-a})$$

B. LATERAL EQUATIONS:

B-1: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS.

If the airplane is restricted to lateral motion,

$Q = \dot{Q} = \Delta W = \Delta\dot{W} = \Delta U = \Delta\dot{U} \equiv 0.$

Thus, equations 1, 2, 3, 4, 5 and 6 reduce to:

$$L = I_X\dot{P}, \quad (103)$$

$$M = -(I_Z - I_X)RP, \quad (104)$$

$$N = I_Z\dot{R}, \quad (105)$$

$$X = -M_A VR, \quad (106)$$

$$Y = M_A(\dot{V} + UR), \text{ and} \quad (107)$$

$$Z = M_A VP. \quad (108)$$

With reference to initial conditions for steady-state horizontal flight, and neglecting second order terms, we have:

$$L = I_X\dot{P} \quad (103)$$

$$N = I_Z\dot{R} \text{ and} \quad (105)$$

$$Y = M_A(\Delta\dot{V} + U_oR). \quad (107)$$

Functional Dependence:

Let $L = F(V, R, P)$, (108-a)

$N = F(V, R, P, \Delta_R)$ and (109)

$Y = F(V, \phi)$. (110)

Other effects such as Y_{Δ_R}, Y_P, Y_R, etc., will be neglected.

Assuming that all changes are small, and using the principle of linear superposition, we have:

$$L = L_o + \left(\frac{\partial L}{\partial \Delta V}\right)\Delta V + \left(\frac{\partial L}{\partial R}\right)R + \left(\frac{\partial L}{\partial P}\right)P, \qquad (111)$$

$$N = N_o + \left(\frac{\partial N}{\partial \Delta V}\right)\Delta V + \left(\frac{\partial N}{\partial R}\right)R + \left(\frac{\partial N}{\partial P}\right)P + \left(\frac{\partial N}{\partial \Delta_R}\right)\Delta_R \qquad (112)$$

and

$$Y = Y_o + \left(\frac{\partial Y}{\partial \Delta V}\right)\Delta V + \left(\frac{\partial Y}{\partial \phi}\right)\phi. \qquad (113)$$

Trim Conditions:

Assume $L_o = N_o = Y_o \equiv 0$, before forced oscillation is started.

Forced Oscillation:

$$\Delta_R = \Delta_{R_a} \sin \Omega_R t = \mathcal{I}\Delta_{R_a} e^{i\Omega_R t}; \qquad (114)$$

We may define:

$$L_{\Delta V} = \left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial \Delta V}\right), \quad N_{\Delta V} = \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial \Delta V}\right), \quad Y_{\Delta V} = \left(\frac{1}{M_A}\right)\left(\frac{\partial Y}{\partial \Delta V}\right),$$

$$L_R = \left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial R}\right), \quad N_R = \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial R}\right), \quad Y_\phi = \left(\frac{1}{M_A}\right)\left(\frac{\partial Y}{\partial \phi}\right),$$

$$L_P = \left(\frac{1}{I_X}\right)\left(\frac{\partial L}{\partial P}\right), \quad N_P = \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial P}\right) \text{ and } \quad N_{\Delta_R} = \left(\frac{1}{I_Z}\right)\left(\frac{\partial N}{\partial \Delta_R}\right).$$

The equations of motion now become:

$$\dot{P} = L_{\Delta V}\Delta V + L_R R + L_P P, \qquad (115)$$

$$\dot{R} = N_{\Delta V}\Delta V + N_R R + N_P P + N_{\Delta_R}\Delta_R \text{ and} \qquad (116)$$

$$\Delta\dot{V} = Y_{\Delta V}\Delta V - U_o R + Y_\phi \phi. \qquad (117)$$

By definition,

$$D = \frac{d}{dt}, \text{ so } P = D\phi, \ \dot{P} = D^2\phi, \ R = D\psi, \ \dot{R} = D^2\psi \text{ and } \Delta\dot{V} = D\Delta V.$$

Thus, equations 115, 116 and 117 reduce to:

$$(-L_{\Delta V})\Delta V - (L_R D)\psi + (D^2 - L_P D)\phi = 0, \qquad (118)$$

$$(-N_{\Delta V})\Delta V + (D^2 - N_R D)\psi + (-N_P D)\phi = N_{\Delta_R}\Delta_{R_\alpha} \sin \Omega_R t \qquad (119)$$

and $(D - Y_{\Delta V})\Delta V + (U_o D)\psi + (-Y_\phi)\phi = 0.$ (120)

Non-dimensionalizing:

Let $v = \dfrac{\Delta V}{U_o}$, $\quad d = T_A D$,

$T_A = \dfrac{U_o C_{L_o}}{g} = \dfrac{M_A}{\frac{\rho}{2} S U_o}$, $\quad \gamma = \dfrac{t}{T_A}$,

$\omega_r = T_A \Omega_R$, $\quad \mu_b = \dfrac{M_A}{\frac{\rho}{2} S b} = \dfrac{T_A U_o}{b} = \dfrac{U_o^2 C_{L_o}}{g b}$,

$l_v = T_A b L_{\Delta V}$, $\quad y_v = T_A Y_{\Delta V}$,

$l_r = T_A L_R$, $\quad y_\phi = \left(\dfrac{T_A}{U_o}\right) Y_\phi$,

$l_p = T_A L_P$, $\quad r = T_A R = d\psi = \dot{\psi}$,

$n_v = T_A b N_{\Delta V}$, $\quad p = T_A P = d\phi = \dot{\phi}$,

$n_r = T_A N_R$, $\quad \delta_{r_\alpha} \equiv \Delta_{R_\alpha}$,

$n_p = T_A N_P$, $\quad \delta_r \equiv \Delta_R$,

$n_{\delta_r} = T_A^2 N_{\Delta_R}$, $\quad$ and $\delta_r = \delta_{r_\alpha} \sin \omega_r \gamma$. (114-a)

Substituting in equations 118, 119 and 120, we have:

$$\left(\frac{-l_v}{T_A b}\right)U_o v + \left(\frac{-l_r}{T_A}\right)\left(\frac{d}{T_A}\right)\psi + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{l_p}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\phi = 0, \tag{121}$$

$$\left(\frac{-n_v}{T_A b}\right)U_o v + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{n_r}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\psi + \left(\frac{-n_p}{T_A}\right)\left(\frac{d}{T_A}\right)\phi = \left(\frac{n_{\delta_r}}{T_A^2}\right)\delta_{r_a} \sin \omega_r\gamma \tag{122}$$

and

$$\left[\left(\frac{d}{T_A}\right) - \left(\frac{y_v}{T_A}\right)\right]U_o v + U_o\left(\frac{d}{T_A}\right)\psi + (-y_\phi)\left(\frac{U_o}{T_A}\right)\phi = 0. \tag{123}$$

If we multiply equations 121 and 122 by T_A^2, and multiply equation 123 by T_A/U_o, we have:

$$(-\mu_b l_v)v + (-l_r d)\psi + (d^2 - l_p d)\phi = 0, \tag{124}$$

$$(-\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi = N_{\delta_r} \mathcal{J} \delta_{r_a} e^{i\omega_r\gamma} \tag{125}$$

$$\text{and } (d - y_v)v + d\psi + (-y_\phi)\phi = 0. \tag{126}$$

$$\text{Let } v = \mathcal{J}\bar{v}_a e^{i\omega_r\gamma}, \tag{127}$$

$$\psi = \mathcal{J}\bar{\psi}_a e^{i\omega_r\gamma} \text{ and} \tag{128}$$

$$\phi = \mathcal{J}\bar{\phi}_a e^{i\omega_r\gamma}, \tag{129}$$

where $\bar{v}_a$, $\bar{\psi}_a$ and $\bar{\phi}_a$ are complex amplitudes.

Putting equations 127, 128, and 129 in equations 124, 125 and 126, respectively, we have:

$$(-\mu_b l_v)\bar{v}_a + (-l_r i\omega_r)\bar{\psi}_a + (-\omega_r^2 - l_p i\omega_r)\bar{\phi}_a = 0, \tag{130}$$

$$(-\mu_b n_v)\bar{v}_a + (-\omega_r^2 - n_r i\omega_r)\bar{\psi}_a + (-n_p i\omega_r)\bar{\phi}_a = n_{\delta_r}\delta_{r_a} \tag{131}$$

$$\text{and } (i\omega_r - y_v)\bar{v}_a + (i\omega_r)\bar{\psi}_a + (-y_\phi)\bar{\phi}_a = 0. \tag{132}$$

By determinants,

$$\bar{v}_a = \frac{\Delta_5}{\Delta_6}, \quad \bar{\psi}_a = \frac{\Delta_7}{\Delta_6} \text{ and } \bar{\phi}_a = \frac{\Delta_8}{\Delta_6}, \tag{133}$$

where

$$\Delta_5 = \begin{vmatrix} (0) & (-l_r i\omega_r) & (-\omega_r^2 - l_p i\omega_r) \\ (n_{\delta_r}\delta_{r_a}) & (-\omega_r^2 - n_r i\omega_r) & (-n_p i\omega_r) \\ (0) & (i\omega_r) & (-y_\phi) \end{vmatrix} \tag{134}$$

Expanding, we have:

$$\Delta_5 = n_{\delta_r}\delta_{r_a}\omega_r \sqrt{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}\; e^{i\alpha_1}, \tag{135}$$

where

$$C_1 = -l_p, \tag{136}$$

$$C_2 = l_r y_\phi \text{ and} \tag{137}$$

$$\alpha_1 = \tan^{-1}\left(\frac{-\omega_r^2 - C_2}{-\omega_r C_1}\right). \tag{138}$$

Also,

$$\Delta_6 = \begin{vmatrix} (-\mu_b l_v) & (-l_r i\omega_r) & (-\omega_r^2 - l_p i\omega_r) \\ (-\mu_b n_v) & (-\omega_r^2 - n_r i\omega_r) & (-n_p i\omega_r) \\ (i\omega_r - y_v) & (i\omega_r) & (-y_\phi) \end{vmatrix} . \tag{139}$$

Expanding, we have:

$$\Delta_6 = \omega_r \times \sqrt{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}\; e^{i\alpha_2}, \tag{140}$$

where

$$C_3 = y_v(n_p l_r - n_r l_p) + \mu_b(l_v n_p - l_p n_v - l_v y_\phi), \tag{141}$$

$$C_4 = -l_p - y_v - n_r, \tag{142}$$

$$C_5 = y_\phi(l_v n_r - l_r n_v)\mu_b, \tag{143}$$

$$C_6 = (l_p n_r - l_r n_p) + y_v(n_r + l_p) + \mu_b n_v \tag{144}$$

and $$\alpha_2 = \tan^{-1}\left(\frac{-C_5 + \omega_r^2 C_6 - \omega_r^4}{\omega_r C_3 - \omega_r^3 C_4}\right). \tag{145}$$

Also,

$$\Delta_7 = \begin{vmatrix} (-\mu_b l_v) & (0) & (-\omega_r^2 - l_p i\omega_r) \\ (-\mu_b n_v) & (n_{\delta_r}\delta_{r_a}) & (-n_p i\omega_r) \\ (i\omega_r - y_v) & (0) & (-y_\phi) \end{vmatrix} . \tag{146}$$

Expanding, we have:

$$\Delta_7 = n_{\delta_r}\delta_{r_a}\sqrt{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1K_2 + \omega_r^2)^2}\; e^{i\alpha_3}, \tag{147}$$

where

$$K_1 = -l_v y_\phi \mu_b, \tag{148}$$

$$C_1 = -l_p, \text{ as defined previously,} \tag{136}$$

$$K_2 = -y_v \text{ and} \tag{149}$$

$$\alpha_3 = \tan^{-1}\left[\frac{\omega_r(-C_1K_2 + \omega_r^2)}{-K_1 + \omega_r^2(K_2 + C_1)}\right]. \tag{150}$$

And,

$$\Delta_8 = \begin{vmatrix} (-\mu_b l_v) & (l_r i\omega_r) & (0) \\ (-\mu_b n_v) & (-\omega_r^2 - n_r i\omega_r) & (n_{\delta_r}\delta_{r_a}) \\ (i\omega_r - y_v) & (i\omega_r) & (0) \end{vmatrix} . \quad (151)$$

Expanding, we have:

$$\Delta_8 = n_{\delta_r}\delta_{r_a} \;\; \omega_r \sqrt{(\omega_r H_1)^2 + (-H_2)^2} \;\; e^{i\alpha_4}, \quad (152)$$

where

$$H_1 = l_r, \quad (153)$$

$$H_2 = -(y_v l_r + \mu_b l_v) \text{ and} \quad (154)$$

$$\alpha_4 = \tan^{-1} \left(\frac{-H_2}{\omega_r H_1}\right). \quad (155)$$

Now, from equations 127, 128, 129, 133, 135, 140, 147 and 152 we have:

$$v = \mu_{v\delta_r}\delta_{r_a} \sin (\omega_r \gamma + LA_{v\delta_r}), \quad (156)$$

$$\psi = \mu_{\psi\delta_r}\delta_{r_a} \sin (\omega_r \gamma + LA_{\psi\delta_r}) \text{ and} \quad (157)$$

$$\phi = \mu_{\phi\delta_r}\delta_{r_a} \sin (\omega_r \gamma + LA_{\phi\delta_r}), \quad (158)$$

where the amplitude response ratios are:

$$\mu_{v\delta_r} = n_{\delta_r} \sqrt{\frac{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}} \;, \quad (159)$$

$$\mu_{\psi\delta_r} = \left(\frac{n_{\delta_r}}{\omega_r}\right) \sqrt{\frac{[-K_1 + \omega_r^2 (K_2 + C_1)]^2 + \omega_r^2 (-C_1 K_2 + \omega_r^2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}} \quad (160)$$

and $\mu_{\phi\delta_r} = n_{\delta_r} \sqrt{\frac{(\omega_r H_1)^2 + (-H_2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}}$, (161)

and the lead angles are:

$$LA_{v\delta_r} = \alpha_1 - \alpha_2, \quad (162)$$

$$LA_{\psi\delta_r} = \alpha_3 - \alpha_2 \text{ and} \quad (163)$$

$$LA_{\phi\delta_r} = \alpha_4 - \alpha_2. \quad (164)$$

Differentiating equations 156, 157 and 158, and rearranging, we have:

$$\dot{v} = \mu_{\dot{v}\delta_r} \delta_{r_a} \sin(\omega_r \gamma + LA_{\dot{v}\delta_r}), \quad (165)$$

$$\dot{\psi} = \mu_{\dot{\psi}\delta_r} \delta_{r_a} \sin(\omega_r \gamma + LA_{\dot{\psi}\delta_r}) \text{ and} \quad (166)$$

$$\dot{\phi} = \mu_{\dot{\phi}\delta_r} \delta_{r_a} \sin(\omega_r \gamma + LA_{\dot{\phi}\delta_r}), \quad (167)$$

where

$$\mu_{\dot{v}\delta_r} = \omega_r \mu_{v\delta_r}, \qquad LA_{\dot{v}\delta_r} = LA_{v\delta_r} + \frac{\pi}{2}, \quad (168)$$

$$\mu_{\dot{\psi}\delta_r} = \omega_r \mu_{\psi\delta_r}, \qquad LA_{\dot{\psi}\delta_r} = LA_{\psi\delta_r} + \frac{\pi}{2}, \quad (169)$$

$$\mu_{\dot{\phi}\delta_r} = \omega_r \mu_{\phi\delta_r} \text{ and} \qquad LA_{\dot{\phi}\delta_r} = LA_{\phi\delta_r} + \frac{\pi}{2}. \quad (170)$$

From equations 156, 157, 159, 160, 165, 166, 168 and 169 we see that

$$\frac{\dot{v}_a}{\dot{\psi}_a} = \frac{v_a}{\psi_a} = \frac{\mu_{\dot{v}\delta_r}}{\mu_{\dot{\psi}\delta_r}} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}}$$

$$= \omega_r \sqrt{\frac{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1 K_2 + \omega_r^2)^2}}. \quad (171)$$

B-2: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER

The basic equations for Lateral motion are given by equations 103, 105 and 107.

Functional dependence:

$$\text{Let } L = F(V, R, P, \Delta_A), \tag{172}$$

$$N = F(V, R, P) \text{ and} \tag{173}$$

$$Y = F(V, \phi). \tag{110}$$

Other effects such as N_{Δ_A}, Y_P, Y_R, etc., will be neglected. Assuming that all changes are small, and using the principle of linear superposition, we have:

$$L = L_o + (\frac{\partial L}{\partial \Delta V})\Delta V + (\frac{\partial L}{\partial R})R + (\frac{\partial L}{\partial P})P + (\frac{\partial L}{\partial \Delta_A})\Delta_A, \tag{174}$$

$$N = N_o + (\frac{\partial N}{\partial \Delta V})\Delta V + (\frac{\partial N}{\partial R})R + (\frac{\partial N}{\partial P})P \text{ and} \tag{175}$$

$$Y = Y_o + (\frac{\partial Y}{\partial \Delta V})\Delta V + (\frac{\partial Y}{\partial \phi})\phi. \tag{113}$$

Trim Conditions:

Assume $L_o = N_o = Y_o \equiv 0$, before forced oscillation is started.

Forced Oscillation:

$$\Delta_A = \Delta_{A_a} \sin \Omega_A t = \mathcal{I}\Delta_{A_a} e^{i\Omega_A t}. \tag{176}$$

We may define

$$L_{\Delta V} = (\frac{1}{I_X})(\frac{\partial L}{\partial \Delta V}), \quad N_{\Delta V} = (\frac{1}{I_Z})(\frac{\partial N}{\partial \Delta V}), \quad Y_{\Delta V} = (\frac{1}{M_A})(\frac{\partial Y}{\partial \Delta V}),$$

$$L_R = (\frac{1}{I_X})(\frac{\partial L}{\partial R}), \quad N_R = (\frac{1}{I_Z})(\frac{\partial N}{\partial R}), \quad Y_\phi = (\frac{1}{M_A})(\frac{\partial Y}{\partial \phi}),$$

$$L_P = (\frac{1}{I_X})(\frac{\partial L}{\partial P}), \quad N_P = (\frac{1}{I_Z})(\frac{\partial N}{\partial P}) \text{ and } L_{\Delta_A} = (\frac{1}{I_X})(\frac{\partial L}{\partial \Delta_A}).$$

The equations of motion now become:

$$\dot{P} = L_{\Delta V}\Delta V + L_R R + L_P P + L_{\Delta_A}\Delta_A, \tag{177}$$

$$\dot{R} = N_{\Delta V}\Delta V + N_R R + N_P P \text{ and} \tag{178}$$

$$\Delta\dot{V} = Y_{\Delta V}\Delta V - U_o R + Y_\phi \phi. \tag{117}$$

By definition:

$$D = \frac{d}{dt}, \text{ so } P = D\phi,\ \dot{P} = D^2\phi,\ R = D\psi,\ \dot{R} = D^2\psi \text{ and } \Delta\dot{V} = D\Delta V.$$

Thus, equations 177, 178 and 117 reduce to:

$$(-L_{\Delta V})\Delta V - (L_R D)\psi + (D^2 - L_P D)\phi = L_{\Delta_A}\Delta_{A_a} \sin \Omega_A t, \tag{179}$$

$$(-N_{\Delta V})\Delta V + (D^2 - N_R D)\psi + (-N_P D)\phi = 0 \text{ and} \tag{180}$$

$$(D - Y_{\Delta V})\Delta V + (U_o D)\psi + (-Y_\phi)\phi = 0. \tag{120}$$

Non-dimensionalizing:

Let

$$v = \frac{\Delta V}{U_o}, \qquad d = T_A D,$$

$$\gamma = \frac{t}{T_A}, \qquad \omega_a = T_A \Omega_A,$$

$$l_v = T_A b L_{\Delta V}, \qquad y_v = T_A Y_{\Delta V},$$

$$l_r = T_A L_R = l_{\dot{\psi}}, \qquad y_\phi = \left(\frac{T_A}{U_o}\right) Y_\phi,$$

$$l_p = T_A L_P = l_{\dot{\phi}}, \qquad r = T_A R = d_\psi = \dot{\psi},$$

$$n_v = T_A b N_{\Delta V}, \qquad p = T_A P = d_\phi = \dot{\phi},$$

$$n_r = T_A N_R = n_{\dot{\psi}}, \qquad T_A = \frac{U_o C_{L_o}}{g} = \frac{M_A}{\frac{\rho}{2} S U_o},$$

$$n_p = T_A N_P = n_{\dot{\phi}}, \qquad \mu_b = \frac{M_A}{\frac{\rho}{2} S b} = \frac{T_A U_o}{b} = \frac{U_o^2 C_{L_o}}{g b},$$

$$\delta_{a_\alpha} \equiv \Delta_{A_\alpha}, \qquad l_{\delta_a} = T_A^2 L_{\Delta_A} \text{ and}$$

$$\delta_a \equiv \Delta_A, \qquad \delta_a = \delta_{a_\alpha} \sin \omega_a \gamma. \tag{176-a}$$

Substituting in equations 179, 180, and 120, we have:

$$\left(\frac{-l_v}{T_A b}\right) U_o v + \left(\frac{-l_r}{T_A}\right)\left(\frac{d}{T_A}\right)\psi + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{l_p}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\phi = \left(\frac{l_{\delta_a}}{T_A^2}\right)\delta_{a_\alpha} \sin \omega_a \gamma. \tag{181}$$

$$\left(\frac{-n_v}{T_A b}\right) U_o v + \left[\left(\frac{d^2}{T_A^2}\right) - \left(\frac{n_r}{T_A}\right)\left(\frac{d}{T_A}\right)\right]\psi + \left(\frac{-n_p}{T_A}\right)\left(\frac{d}{T_A}\right)\phi = 0 \text{ and} \tag{182}$$

$$\left[\left(\frac{d}{T_A}\right) - \left(\frac{y_v}{T_A}\right)\right] U_o v + U_o\left(\frac{d}{T_A}\right)\psi + (-y_\phi)\left(\frac{U_o}{T_A}\right)\phi = 0. \tag{123}$$

If we multiply equations 181 and 182 by T_A^2, and multiply equation 123 by $\frac{T_A}{U_o}$, we have:

$$(-\mu_b l_v)v + (-l_r d)\psi + (d^2 - l_p d)\phi = l_{\delta_a} \mathscr{A} \delta_{a_\alpha} e^{i\omega_a \gamma}, \tag{183}$$

$$(-\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi = 0 \text{ and} \tag{184}$$

$$(d - y_v)v + d\psi + (-y_\phi)\phi = 0. \tag{126}$$

Let $v = \mathscr{A}\bar{v}_\alpha e^{i\omega_a \gamma}$, (185)

$\psi = \mathscr{A}\bar{\psi}_\alpha e^{i\omega_a \gamma}$ and (186)

$\phi = \mathscr{A}\bar{\phi}_\alpha e^{i\omega_a \gamma}$, (187)

where $\bar{v}_\alpha$, $\bar{\psi}_\alpha$ and $\bar{\phi}_\alpha$ are complex amplitudes.

Putting equations 185, 186 and 187 in equations 183, 184 and 126, respectively, we have:

$$(-\mu_b l_v)\bar{v}_\alpha + (-l_r i\omega_a)\bar{\psi}_\alpha + (-\omega_a^2 - l_p i\omega_a)\bar{\phi}_\alpha = l_{\delta_a}\delta_{a_\alpha} , \tag{188}$$

$$(-\mu_b n_v)\bar{v}_\alpha + (-\omega_a^2 - n_r i\omega_a)\bar{\psi}_\alpha + (-n_p i\omega_a)\bar{\phi}_\alpha = 0 \text{ and} \tag{189}$$

$$(i\omega_a - y_v)\bar{v}_\alpha + (i\omega_a)\bar{\psi}_\alpha + (-y_\phi)\bar{\phi}_\alpha = 0. \tag{132}$$

By determinants:

$$\bar{v}_\alpha = (\frac{\Delta_9}{\Delta_{10}}),\ \bar{\psi}_\alpha = (\frac{\Delta_{11}}{\Delta_{10}}) \text{ and } \bar{\phi}_\alpha = (\frac{\Delta_{12}}{\Delta_{10}}), \tag{190}$$

where

$$\Delta_9 = \begin{vmatrix} (l_{\delta_a}\delta_{a_\alpha}) & (-l_r i\omega_a) & (-\omega_a^2 - l_p i\omega_a) \\ (0) & (-\omega_a^2 - n_r i\omega_a) & (-n_p i\omega_a) \\ (0) & (i\omega_a) & (-y_\phi) \end{vmatrix} . \tag{191}$$

Expanding, we have:

$$\Delta_9 = l_{\delta_a}\delta_{a_\alpha}\omega_a \sqrt{(\omega_a G_1)^2 + (-G_2)^2}\, e^{i\lambda_1}, \tag{192}$$

where

$$G_1 = y_\phi - n_p, \tag{193}$$

$$G_2 = -\, y_\phi n_r \text{ and} \tag{194}$$

$$\lambda_1 = \tan^{-1} (\frac{-G_2}{\omega_a G_1}). \tag{195}$$

Also,

$$\Delta_{10} = \begin{vmatrix} (-\mu_b l_v) & (-l_r i\omega_a) & (-\omega_a^2 - l_p i\omega_a) \\ (-\mu_b n_v) & (-\omega_a^2 - n_r i\omega_a) & (-n_p i\omega_a) \\ (i\omega_a - y_v) & (i\omega_a) & (-y_\phi) \end{vmatrix} . \tag{196}$$

Expanding, we have:

$$\Delta_{10} = \omega_a \sqrt{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}\, e^{i\lambda_2}, \tag{197}$$

where C_3, C_4, C_5 and C_6 are given by equations 141, 142, 143 and 144, respectively, and

$$\lambda_2 = \tan^{-1} \left(\frac{-C_5 + \omega_a^2 C_6 - \omega_a^4}{\omega_a C_3 - \omega_a^3 C_4}\right). \tag{198}$$

And,

$$\Delta_{11} = \begin{vmatrix} (-\mu_b l_v) & (l_{\delta_a} \delta_{a_a}) & (-\omega_a^2 - l_p i\omega_a) \\ (-\mu_b n_v) & (0) & (-n_p i\omega_a) \\ (i\omega_a - y_v) & (0) & (-y_\phi) \end{vmatrix} . \tag{199}$$

Expanding, we have:

$$\Delta_{11} = l_{\delta_a} \delta_{a_a} \sqrt{(-J_1 - \omega_a^2 J_2)^2 + (\omega_a J_3)^2}\, e^{i\lambda_3}, \tag{200}$$

where,

$$J_1 = \mu_b n_v y_\phi = n_v\left(\frac{T_A^2 g}{b}\right), \tag{201}$$

$$J_2 = -n_p, \tag{202}$$

$$J_3 = n_p y_v \text{ and} \tag{203}$$

$$\lambda_3 = \tan^{-1} \left(\frac{\omega_a J_3}{-J_1 \omega_a^2 J_2}\right). \tag{204}$$

And,

$$\Delta_{12} = \begin{vmatrix} (-\mu_b l_v) & (-l_r i\omega_a) & (l_{\delta_a} \delta_{a_a}) \\ (-\mu_b n_v) & (-\omega_a^2 - n_r i\omega_a) & (0) \\ (i\omega_a - y_v) & (i\omega_a) & (0) \end{vmatrix} . \tag{205}$$

Expanding, we have:

$$\Delta_{12} = l_{\delta_a} \delta_{a_\alpha} \omega_a \sqrt{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}\; e^{i\lambda_4}, \qquad (206)$$

where

$$J_4 = -n_r - y_v, \qquad (207)$$

$$J_5 = n_r y_v + \mu_b n_v \text{ and} \qquad (208)$$

$$\lambda_4 = \tan^{-1}\left(\frac{-J_5 + \omega_a^2}{\omega_a J_4}\right). \qquad (209)$$

Now, from equations 185, 186, 187, 190, 192, 197, 200 and 206 we have:

$$v = \mu_{v\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{v\delta_a}), \qquad (210)$$

$$\psi = \mu_{\psi\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{\psi\delta_a}) \text{ and} \qquad (211)$$

$$\phi = \mu_{\phi\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{\phi\delta_a}), \qquad (212)$$

where the amplitude response ratios are:

$$\mu_{v\delta_a} = \frac{v_\alpha}{\delta_{a_\alpha}} = l_{\delta_a} \sqrt{\frac{(\omega_a G_1)^2 + (-G_2)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}}, \qquad (213)$$

$$\mu_{\psi\delta_a} = \frac{\psi_\alpha}{\delta_{a_\alpha}} = \left(\frac{l_{\delta_a}}{\omega_a}\right) \sqrt{\frac{(-J_1 - \omega_a^2 J_2)^2 + (\omega_a J_3)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}}, \qquad (214)$$

and

$$\mu_{\phi\delta_a} = \frac{\phi_\alpha}{\delta_{a_\alpha}} = l_{\delta_a} \sqrt{\frac{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}}, \qquad (215)$$

and the lead angles are:

$$LA_{v\delta_a} = \lambda_1 - \lambda_2, \qquad (216)$$

$$LA_{\psi\delta_a} = \lambda_3 - \lambda_2 \quad \text{and} \qquad (217)$$

$$LA_{\phi\delta_a} = \lambda_4 - \lambda_2 \,. \qquad (218)$$

Differentiating equations 210, 211, and 212, and rearranging, we have:

$$\dot{v} = \mu_{\dot{v}\delta_a}\delta_{a_\alpha} \sin\,(\omega_a\gamma + LA_{\dot{v}\delta_a}), \qquad (219)$$

$$\dot{\psi} = \mu_{\dot{\psi}\delta_a}\delta_{a_\alpha} \sin\,(\omega_a\gamma + LA_{\dot{\psi}\delta_a}) \quad \text{and} \qquad (220)$$

$$\dot{\phi} = \mu_{\dot{\phi}\delta_a}\delta_{a_\alpha} \sin\,(\omega_a\gamma + LA_{\dot{\phi}\delta_a}), \qquad (221)$$

where

$$\mu_{\dot{v}\delta_a} = \omega_a\mu_{v\delta_a}\,, \qquad LA_{\dot{v}\delta_a} = LA_{v\delta_a} + \frac{\pi}{2}\,, \qquad (222)$$

$$\mu_{\dot{\psi}\delta_a} = \omega_a\mu_{\psi\delta_a}\,, \qquad LA_{\dot{\psi}\delta_a} = LA_{\psi\delta_a} + \frac{\pi}{2}\,, \qquad (223)$$

$$\mu_{\dot{\phi}\delta_a} = \omega_a\mu_{\phi\delta_a}\,; \text{ and} \qquad LA_{\dot{\phi}\delta_a} = LA_{\phi\delta_a} + \frac{\pi}{2}\,. \qquad (224)$$

From equations 210, 212, 213, 215, 219, 221, 222 and 224, we see that

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{v_\alpha}{\phi_\alpha} = \frac{\mu_{\dot{v}\delta_a}}{\mu_{\dot{\phi}\delta_a}} = \frac{\mu_{v\delta_a}}{\mu_{\phi\delta_a}} = \sqrt{\frac{(\omega_a G_1)^2 + (-G_2)^2}{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}}\,. \qquad (225)$$

B-3-a: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW

When the ailerons and the rudder are forced to move at the same frequency,

$$\omega_a = \omega_r = \omega_{ar} \cdot \tag{226}$$

We may assume that the forced motion of the rudder is known:

$$\delta_r = \mathcal{R}\delta_{r_a} e^{i\omega_{ar}\gamma}, \tag{227}$$

where δ_{r_a} is real.

We may also assume that the forced motion of the ailerons is unknown:

$$\delta_a = \mathcal{R}\bar{\delta}_{a_a} e^{i\omega_{ar}\gamma}, \tag{228}$$

where $\bar{\delta}_{a_a}$ is complex, and is to be determined.

From equations 124, 125, 126, 183 and 184, we see that the general equations describing the airplane response to a combination of forced sinusoidal motion of rudder and ailerons are:

$$(-\mu_b l_v)v + (-l_r d)\psi + (d^2 - l_p d)\phi = l_{\delta_a}\delta_a , \tag{229}$$

$$(-\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi = n_{\delta_r}\delta_r \tag{230}$$

and $(d - y_v)v + d\psi + (-y_\phi)\phi = 0.$ (126)

The problem is to mathematically adjust δ_a so that $v = 0.$*

To do this, we put $v = 0$ in equations 200, 201 and 97, and use equations 227 and 228, as well as

$$\psi = \mathcal{R}\bar{\psi}_a e^{i\omega_{ar}\gamma} \text{ and} \tag{231}$$

$$\phi = \mathcal{R}\bar{\phi}_a e^{i\omega_{ar}\gamma} . \tag{232}$$

* Either rudder motion or aileron motion can be adjusted mathematically to give $v = 0$, but for purpose of illustration aileron motion will be adjusted.

Note that $\bar{\psi}_a$ and $\bar{\phi}_a$ are complex unknown amplitudes. Thus we have:

$$(-l_r i\omega_{ar})\bar{\psi}_a + (-\omega_{ar}^2 - l_p i\omega_{ar})\bar{\phi}_a + (-l_{\delta_a})\bar{\delta}_{a_a} = 0, \quad (233)$$

$$(-\omega_{ar}^2 - n_r i\omega_{ar})\bar{\psi}_a + (-n_p i\omega_{ar})\bar{\phi}_a = n_{\delta_r}\delta_{r_a} \quad \text{and} \quad (234)$$

$$(i\omega_{ar})\bar{\psi}_a + (-y_\phi)\bar{\phi}_a = 0. \quad (235)$$

By determinants,

$$\bar{\delta}_{a_a} = \frac{\Delta_{13}}{\Delta_{14}}, \quad \bar{\phi}_a = \frac{\Delta_{15}}{\Delta_{14}} \text{ and } \bar{\psi}_a = \frac{\Delta_{16}}{\Delta_{14}}, \quad (236)$$

where

$$\Delta_{13} = \begin{vmatrix} (-l_r i\omega_{ar}) & (-\omega_{ar}^2 - l_p i\omega_{ar}) & (0) \\ (-\omega_{ar}^2 - n_r i\omega_{ar}) & (-n_p i\omega_{ar}) & (n_{\delta_r}\delta_{r_a}) \\ (i\omega_{ar}) & (-y_\phi) & (0) \end{vmatrix}. \quad (237)$$

Expanding, we have:

$$\Delta_{13} = n_{\delta_r}\delta_{r_a}\omega_{ar}\sqrt{(l_p\omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}\, e^{i\phi_3}, \quad (238)$$

$$\text{where } \phi_3 = \tan^{-1}\left(\frac{-\omega_{ar}^2 - l_r y_\phi}{l_p\omega_{ar}}\right). \quad (239)$$

Also,

$$\Delta_{14} = \begin{vmatrix} (-l_r i\omega_{ar}) & (-\omega_{ar}^2 - l_p i\omega_{ar}) & (-l_{\delta_a}) \\ (-\omega_{ar}^2 - n_r i\omega_{ar}) & (-n_p i\omega_{ar}) & (0) \\ (i\omega_{ar}) & (-y_\phi) & (0) \end{vmatrix}. \quad (240)$$

Expanding, we have:

$$\Delta_{14} = l_{\delta_a}\omega_{ar}\sqrt{(n_p - y_\phi)^2\omega_{ar}^2 + (-n_r y_\phi)^2}\quad e^{i\phi_4}, \quad (241)$$

$$\text{where } \phi_4 = \tan^{-1}\left[\frac{-n_r y_\phi}{(n_p - y_\phi)\omega_{ar}}\right]. \quad (242)$$

And,

$$\Delta_{15} = \begin{vmatrix} (-l_r i\omega_{ar}) & (0) & (-l_{\delta_a}) \\ (-\omega_{ar}^2 - n_r i\omega_{ar}) & (n_{\delta_r}\delta_{r_\alpha}) & (0) \\ (i\omega_{ar}) & (0) & (0) \end{vmatrix} \qquad (243)$$

Expanding, we have:

$$\Delta_{15} = l_{\delta_a}\omega_{ar} n_{\delta_r}\delta_{r_\alpha} e^{i\phi_5}, \qquad (244)$$

$$\text{where } \phi_5 = \tan^{-1}\frac{+1}{0} = \tan^{-1}(+\infty) = \frac{\pi}{2}. \qquad (245)$$

And,

$$\Delta_{16} = \begin{vmatrix} (0) & (-\omega_{ar}^2 - l_p i\omega_{ar}) & (-l_{\delta_a}) \\ (n_{\delta_r}\delta_{r_\alpha}) & (-n_p i\omega_{ar}) & (0) \\ (0) & (-y_\phi) & (0) \end{vmatrix} \qquad (246)$$

Expanding, we have:

$$\Delta_{16} = n_{\delta_r}\delta_{r_\alpha} l_{\delta_a} y_\phi \,. \qquad (247)$$

Now, from equations 228, 231, 232, 236, 238, 241, 244 and 247, we have:

$$\delta_a = \mu_{\delta_a\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\delta_a\delta_r}), \qquad (248)$$

$$\phi = \mu_{\phi\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\phi\delta_r}) \text{ and} \qquad (249)$$

$$\psi = \mu_{\psi\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\psi\delta_r}), \qquad (250)$$

where

$$\mu_{\delta_a \delta_r} = \frac{\delta_{a_\alpha}}{\delta_{r_\alpha}} = \left(\frac{n_{\delta_r}}{l_{\delta_a}}\right) \sqrt{\frac{(l_p \omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}}, \qquad (251)$$

$$\mu_{\phi \delta_r} = \frac{\phi_\alpha}{\delta_{r_\alpha}} = \frac{n_{\delta_r}}{\sqrt{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}} \quad \text{and} \qquad (252)$$

$$\mu_{\psi \delta_r} = \frac{\psi_\alpha}{\delta_{r_\alpha}} = \frac{n_{\delta_r} y_\phi}{\omega_{ar} \sqrt{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}} . \qquad (253)$$

Also,

$$LA_{\delta_a \delta_r} = \phi_3 - \phi_4, \qquad LA_{\delta_r \delta_a} = \phi_4 - \phi_3 , \qquad (254)$$

$$LA_{\phi \delta_r} = \frac{\pi}{2} - \phi_4 , \qquad LA_{\phi \delta_a} = \frac{\pi}{2} - \phi_3, \qquad (255)$$

$$LA_{\psi \delta_r} = -\phi_4 \quad \text{and} \quad LA_{\psi \delta_a} = -\phi_3. \qquad (256)$$

Differentiating equations 248, 249 and 250, and rearranging, we have:

$$\dot{\delta}_\alpha = \mu_{\dot{\delta}_a \delta_r} \delta_{r_\alpha} \sin (\omega_{ar} \gamma + LA_{\dot{\delta}_a \delta_r}), \qquad (257)$$

$$\dot{\phi} = \mu_{\dot{\phi} \delta_r} \delta_{r_\alpha} \sin (\omega_{ar} \gamma + LA_{\dot{\phi} \delta_r}) \quad \text{and} \qquad (258)$$

$$\dot{\psi} = \mu_{\dot{\psi} \delta_r} \delta_{r_\alpha} \sin (\omega_{ar} \gamma + LA_{\dot{\psi} \delta_r}), \qquad (259)$$

where

$$\mu_{\dot{\delta}_a \delta_r} = \omega_{ar} \mu_{\delta_a \delta_r} , \qquad LA_{\dot{\delta}_a \delta_r} = LA_{\delta_a \delta_r} + \frac{\pi}{2}, \qquad (260)$$

$$\mu_{\dot{\phi}\delta_r} = \omega_{ar}\mu_{\phi\delta_r}\,, \qquad LA_{\dot{\phi}\delta_r} = LA_{\phi\delta_r} + \frac{\pi}{2}\,, \tag{261}$$

$$\mu_{\dot{\psi}\delta_r} = \omega_{ar}\mu_{\psi\delta_r} \quad \text{and} \quad LA_{\dot{\psi}\delta_r} = LA_{\psi\delta_r} + \frac{\pi}{2}\,. \tag{262}$$

From equations 249, 250, 252, 253, 258, 259, 261 and 262, we see that

$$\frac{\dot{\phi}_\alpha}{\dot{\psi}_\alpha} = \frac{\phi_\alpha}{\psi_\alpha} = \frac{\mu_{\dot{\phi}\delta_r}}{\mu_{\dot{\psi}\delta_r}} = \frac{\mu_{\phi\delta_r}}{\mu_{\psi\delta_r}} = \frac{\omega_{ar}}{y_\phi}\,. \tag{263}$$

And from equations 248, 249, 251, 252, 257, 258, 260 and 261, we see that

$$\mu_{\phi\delta_a} = \frac{\dot{\phi}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{\phi_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{\phi}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{\phi\delta_r}}{\mu_{\delta_a\delta_r}} = \frac{l_{\delta_a}}{\sqrt{(l_p\omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}} \tag{264}$$

Analogously,

$$\mu_{\psi\delta_a} = \frac{\dot{\psi}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{\psi_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{\psi}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{\psi\delta_r}}{\mu_{\delta_a\delta_r}} = \frac{l_{\delta_a} y_\phi}{\omega_{ar}\sqrt{(l_p\omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}}\,. \tag{265}$$

B-3-b: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW

Equations 226, 227, 228, 229, 230, and 126 are valid for this case. The problem is to mathematically adjust δ_a so that $\psi = 0$*. To do this we put $\psi = 0$ in equations 229, 230 and 126, and use equations 227, 228 and 232, as well

* Either rudder motion or aileron motion can be adjusted mathematically to give $\psi = 0$, but for purpose of illustration aileron motion will be adjusted.

as

$$v = \mathcal{J}\bar{v}_{\alpha} e^{i\omega_{ar}\gamma}. \tag{266}$$

Note that $\bar{v}_{\alpha}$ is a complex amplitude to be determined. Thus we have:

$$(-\mu_b l_v)\bar{v}_{\alpha} + (-\omega_{ar}^2 - l_p i\omega_{ar})\bar{\phi}_{\alpha} + (-l_{\delta_a})\bar{\delta}_{a_{\alpha}} = 0, \tag{267}$$

$$(-\mu_b n_v)\bar{v}_{\alpha} + (-n_p i\omega_{ar})\bar{\phi}_{\alpha} = n_{\delta_r}\delta_{r_{\alpha}} \quad \text{and} \tag{268}$$

$$(i\omega_{ar} - y_v)\bar{v}_{\alpha} + (-y_{\phi})\bar{\phi}_{\alpha} = 0. \tag{269}$$

By determinants,

$$\bar{\delta}_{a_{\alpha}} = \frac{\Delta_{17}}{\Delta_{18}}, \quad \bar{\phi}_{\alpha} = \frac{\Delta_{19}}{\Delta_{18}} \quad \text{and} \quad \bar{v}_{\alpha} = \frac{\Delta_{20}}{\Delta_{18}}, \tag{270}$$

where

$$\Delta_{17} = \begin{vmatrix} (-\mu_b l_v) & (-\omega_{ar}^2 - l_p i\omega_{ar}) & (0) \\ (-\mu_b n_v) & (-n_p i\omega_{ar}) & (n_{\delta_r}\delta_{r_{\alpha}}) \\ (i\omega_{ar} - y_v) & (-y_{\phi}) & (0) \end{vmatrix}. \tag{271}$$

Expanding, we have:

$$\Delta_{17} = n_{\delta_r}\delta_{r_{\alpha}} \sqrt{(P_1 - \omega_{ar}^2 P_2)^2 + \omega_{ar}^2(P_3 - \omega_{ar}^2)^2}\; e^{i\psi_2}, \tag{272}$$

where

$$P_1 = -l_v y_{\phi}\mu_b, \tag{273}$$

$$P_2 = -(y_v + l_p), \tag{274}$$

$$P_3 = l_p y_v \quad \text{and} \tag{275}$$

$$\psi_2 = \tan^{-1}\left[\frac{\omega_{ar}(P_3 - \omega_{ar}^2)}{P_1 - \omega_{ar}^2 P_2}\right]. \tag{276}$$

Also,

$$\Delta_{18} = \begin{vmatrix} (-\mu_b l_v) & (-\omega^2_{ar} - l_p i\omega_{ar}) & (-l_{\delta_a}) \\ (-\mu_b n_v) & (-n_p i\omega_{ar}) & (0) \\ (i\omega_{ar} - y_v) & (-y_\phi) & (0) \end{vmatrix}. \tag{277}$$

Expanding, we have:

$$\Delta_{18} = l_{\delta_a} \sqrt{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2} \quad e^{i\psi_3}, \tag{278}$$

where

$$P_4 = n_v y_\phi \mu_b, \tag{279}$$

$$P_5 = -n_p, \tag{280}$$

$$P_6 = n_p y_v \quad \text{and} \tag{281}$$

$$\psi_3 = \tan^{-1}\left(\frac{\omega_{ar}P_6}{-P_4 - P_5\omega_{ar}^2}\right). \tag{282}$$

And,

$$\Delta_{19} = \begin{vmatrix} (-\mu_b l_v) & (0) & (-l_{\delta_a}) \\ (-\mu_b n_v) & (n_{\delta_r}\delta_{r_a}) & (0) \\ (i\omega_{ar} - y_v) & (0) & (0) \end{vmatrix}. \tag{283}$$

Expanding, we have:

$$\Delta_{19} = l_{\delta_a} n_{\delta_r} \delta_{r_a} \sqrt{(-y_v)^2 + (\omega_{ar})^2} \; e^{i\psi_4}, \tag{284}$$

where

$$\psi_4 = \tan^{-1}\left(\frac{\omega_{ar}}{-y_v}\right). \tag{285}$$

And,

$$\Delta_{20} = \begin{vmatrix} (0) & (-\omega_{ar}^2 - l_p i\omega_{ar}) & (-l_{\delta_a}) \\ (n_{\delta_r}\delta_{r_\alpha}) & (-n_p i\omega_{ar}) & (0) \\ (0) & (-y_\phi) & (0) \end{vmatrix}. \tag{286}$$

Expanding, we have:

$$\Delta_{20} = n_{\delta_r}\delta_{r_\alpha} l_{\delta_a} y_\phi. \tag{287}$$

Now, from equations 228, 232, 266, 270, 272, 278, 284 and 287, we have:

$$\delta_a = \mu_{\delta_a\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\delta_a\delta_r}), \tag{288}$$

$$\phi = \mu_{\phi\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\phi\delta_r}) \text{ and} \tag{289}$$

$$v = \mu_{v\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{v\delta_r}), \tag{290}$$

where

$$\mu_{\delta_a\delta_r} = \frac{\delta_{a_\alpha}}{\delta_{r_\alpha}} = \left(\frac{n_{\delta_r}}{l_{\delta_a}}\right) \frac{\sqrt{(P_1 - \omega_{ar}^2 P_2)^2 + \omega_{ar}^2 (P_3 - \omega_{ar}^2)^2}}{\sqrt{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}}, \tag{291}$$

$$\mu_{\phi\delta_r} = \frac{\phi_\alpha}{\delta_{r_\alpha}} = n_{\delta_r} \frac{\sqrt{(-y_v)^2 + (\omega_{ar})^2}}{\sqrt{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}} \text{ and} \tag{292}$$

$$\mu_{v\delta_r} = \frac{v_\alpha}{\delta_{r_\alpha}} = \frac{n_{\delta_r} y_\phi}{\sqrt{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}}. \tag{293}$$

Also,

$$LA_{\delta_a\delta_r} = \psi_2 - \psi_3, \qquad LA_{\delta_r\delta_a} = \psi_3 - \psi_2, \tag{294}$$

$$LA_{\phi\delta_r} = \psi_4 - \psi_3, \qquad LA_{\phi\delta_a} = \psi_4 - \psi_2, \tag{295}$$

$$LA_{v\delta_r} = -\psi_3 \quad \text{and} \quad LA_{v\delta_a} = -\psi_2. \tag{296}$$

Differentiating equations 288, 289, and 290, and rearranging, we have:

$$\dot{\delta}_a = \mu_{\dot{\delta}_a\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\dot{\delta}_a\delta_r}), \tag{297}$$

$$\dot{\phi} = \mu_{\dot{\phi}\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\dot{\phi}\delta_r}) \text{ and} \tag{298}$$

$$\dot{v} = \mu_{\dot{v}\delta_r}\delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\dot{v}\delta_r}), \tag{299}$$

where

$$\mu_{\dot{\delta}_a\delta_r} = \omega_{ar}\mu_{\delta_a\delta_r}, \qquad LA_{\dot{\delta}_a\delta_r} = LA_{\delta_a\delta_r} + \frac{\pi}{2}, \tag{300}$$

$$\mu_{\dot{\phi}\delta_r} = \omega_{ar}\mu_{\phi\delta_r}, \qquad LA_{\dot{\phi}\delta_r} = LA_{\phi\delta_r} + \frac{\pi}{2}, \tag{301}$$

$$\mu_{\dot{v}\delta_r} = \omega_{ar}\mu_{v\delta_r}, \quad \text{and} \quad LA_{\dot{v}\delta_r} = LA_{v\delta_r} + \frac{\pi}{2}. \tag{302}$$

From equations 289, 290, 292, 293, 298, 299, 301 and 302, we see that

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{v_\alpha}{\phi_\alpha} = \frac{\mu_{\dot{v}\delta_r}}{\mu_{\dot{\phi}\delta_r}} = \frac{\mu_{v\delta_r}}{\mu_{\phi\delta_r}} = \frac{y_\phi}{-\sqrt{(-y_v)^2 + (\omega_{ar})^2}}. \tag{303}$$

And, from equations 288, 289, 291, 292, 297, 298, 300 and 301, we see that

$$\mu_{\phi\delta_a} = \frac{\dot{\phi}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{\phi_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{\phi}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{\phi\delta_r}}{\mu_{\delta_a\delta_r}} = l_{\delta_a}\frac{(-y_v)^2 + (\omega_{ar})^2}{\sqrt{(P_1-\omega_{ar}^2P_2)^2+\omega_{ar}^2(P_3-\omega_{ar}^2)^2}}. \tag{304}$$

Analogously,

$$\mu_{v\delta_a} = \frac{\dot{v}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{v_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{v}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{v\delta_r}}{\mu_{\delta_a\delta_r}} = \frac{l_{\delta_a} y_\phi}{\sqrt{(P_1-\omega_{ar}^2 P_2)^2+\omega_{ar}^2(P_3-\omega_{ar}^2)^2}} \quad (305)$$

B-3-C: DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK

Equations 226, 227, 228, 229, 230 and 126 are valid for this case. The problem is to mathematically adjust δ_a so that $\phi = 0$.* To do this we put $\phi = 0$ in equations 229, 230 and 126, and use equations 227, 228, 231 and 266. Thus, we have:

$$(-\mu_b l_v)\bar{v}_\alpha + (-l_r i\omega_{ar})\bar{\psi}_\alpha + (-l_{\delta_a})\bar{\delta}_{a_\alpha} = 0, \quad (306)$$

$$(-\mu_b n_v)\bar{v}_\alpha + (-\omega_{ar}^2 - n_r i\omega_{ar})\bar{\psi}_\alpha = n_{\delta_r}\delta_{r_\alpha} \text{ and} \quad (307)$$

$$(i\omega_{ar}-y_v)\bar{v}_\alpha + (i\omega_{ar})\bar{\psi}_\alpha = 0. \quad (308)$$

By determinants,

$$\bar{\delta}_{a_\alpha} = \frac{\Delta_{21}}{\Delta_{22}},\ \bar{\psi}_\alpha = \frac{\Delta_{23}}{\Delta_{22}} \text{ and } \bar{v}_\alpha = \frac{\Delta_{24}}{\Delta_{22}}. \quad (309)$$

where

$$\Delta_{21} = \begin{vmatrix} (-\mu_b l_v) & (-l_r i\omega_{ar}) & (0) \\ (-\mu_b n_v) & (-\omega_{ar}^2 - n_r i\omega_{ar}) & (n_{\delta_r}\delta_{r_\alpha}) \\ (i\omega_{ar}-y_v) & (i\omega_{ar}) & (0) \end{vmatrix} \quad (310)$$

* Either rudder motion or aileron motion can be adjusted mathematically to give $\phi = 0$, but for purpose of illustration aileron motion will be adjusted.

Expanding, we have:

$$\Delta_{21} = n_{\delta_r \delta_{r_a}} \omega_{ar} \sqrt{(l_r \omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}\, e^{i\sigma_1}, \qquad (311)$$

where

$$\sigma_1 = \tan^{-1} \left(\frac{l_r y_v + \mu_b l_v}{l_r \omega_{ar}}\right). \qquad (312)$$

Also,

$$\Delta_{22} = \begin{vmatrix} (-\mu_b l_v) & (-l_r i\omega_{ar}) & (-l_{\delta_a}) \\ (-\mu_b n_v) & (-\omega_{ar}^2 - n_r i\omega_{ar}) & (0) \\ (i\omega_{ar} - y_v) & (i\omega_{ar}) & (0) \end{vmatrix} . \qquad (313)$$

Expanding, we have:

$$\Delta_{22} = l_{\delta_a} \omega_{ar} \sqrt{(-Q_1 \omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}\, e^{i\sigma_2}, \qquad (314)$$

where

$$Q_1 = - y_v - n_r, \qquad (315)$$

$$Q_2 = n_v \mu_b + n_r y_v \text{ and} \qquad (316)$$

$$\sigma_2 = \tan^{-1} \left(\frac{Q_2 - \omega_{ar}^2}{-Q_1 \omega_{ar}}\right). \qquad (317)$$

And,

$$\Delta_{23} = \begin{vmatrix} (-\mu_b l_v) & (0) & (-l_{\delta_a}) \\ (-\mu_b n_v) & (n_{\delta_r \delta_{r_a}}) & (0) \\ (i\omega_{ar} - y_v) & (0) & (0) \end{vmatrix} . \qquad (318)$$

Expanding, we have:

$$\Delta_{23} = l_{\delta_a} n_{\delta_r} \delta_{r_\alpha} \sqrt{(-y_v)^2 + (\omega_{ar})^2}\ e^{i\sigma_3}, \tag{319}$$

where

$$\sigma_3 = \tan^{-1} \left(\frac{\omega_{ar}}{-y_v}\right). \tag{320}$$

And,

$$\Delta_{24} = \begin{vmatrix} (0) & (-l_r i\omega_{ar}) & (-l_{\delta_a}) \\ (n_{\delta_r} \delta_{r_\alpha}) & (-\omega_{ar}^2 - n_r i\omega_{ar}) & (0) \\ (0) & (i\omega_{ar}) & (0) \end{vmatrix} . \tag{321}$$

Expanding, we have:

$$\Delta_{24} = l_{\delta_a} n_{\delta_r} \delta_{r_\alpha} \omega_{ar} e^{i\frac{3\pi}{2}} . \tag{322}$$

Now, from equations 228, 231, 266, 309, 311, 314, 319 and 322 we have:

$$\delta_a = \mu_{\delta_a \delta_r} \delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\delta_a \delta_r}), \tag{323}$$

$$\psi = \mu_{\psi \delta_r} \delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{\psi \delta_r}) \text{ and} \tag{324}$$

$$v = \mu_{v \delta_r} \delta_{r_\alpha} \sin(\omega_{ar}\gamma + LA_{v \delta_r}), \tag{325}$$

where

$$\mu_{\delta_a \delta_r} = \frac{\delta_{a_\alpha}}{\delta_{r_\alpha}} = \left(\frac{n_{\delta_r}}{l_{\delta_a}}\right) \sqrt{\frac{(l_r\omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}}, \tag{326}$$

$$\mu_{\psi \delta_r} = \frac{\psi_\alpha}{\delta_{r_\alpha}} = \left(\frac{n_{\delta_r}}{\omega_{ar}}\right) \sqrt{\frac{(-y_v)^2 + (\omega_{ar})^2}{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}} \text{ and} \tag{327}$$

$$\mu_{v\delta_r} = \frac{v_a}{\delta_{r_a}} = \frac{n_{\delta_r}}{\sqrt{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}} \quad . \qquad (328)$$

Also,

$$LA_{\delta_a\delta_r} = \sigma_1 - \sigma_2 \ , \qquad LA_{\delta_r\delta_a} = \sigma_2 - \sigma_1 \ , \qquad (329)$$

$$LA_{\psi\delta_r} = \sigma_3 - \sigma_2 \ , \qquad LA_{\psi\delta_a} = \sigma_3 - \sigma_1 \ , \qquad (330)$$

$$LA_{v\delta_r} = \frac{3\pi}{2} - \sigma_2 \text{ and } \qquad LA_{v\delta_a} = \frac{3\pi}{2} - \sigma_1 \ , \qquad (331)$$

Differentiating equations 323, 324 and 325, and rearranging, we have:

$$\dot{\delta}_a = \mu_{\dot{\delta}_a\delta_r}\delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\dot{\delta}_a\delta_r}), \qquad (332)$$

$$\dot{\psi} = \mu_{\dot{\psi}\delta_r}\delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\dot{\psi}\delta_r}) \text{ and} \qquad (333)$$

$$\dot{v} = \mu_{\dot{v}\delta_r}\delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\dot{v}\delta_r}), \qquad (334)$$

where

$$\mu_{\dot{\delta}_a\delta_r} = \omega_{ar}\mu_{\delta_a\delta_r} \ , \qquad LA_{\dot{\delta}_a\delta_r} = LA_{\delta_a\delta_r} + \frac{\pi}{2} \ , \qquad (335)$$

$$\mu_{\dot{\psi}\delta_r} = \omega_{ar}\mu_{\psi\delta_r} \ , \qquad LA_{\dot{\psi}\delta_r} = LA_{\psi\delta_r} + \frac{\pi}{2} \ , \qquad (336)$$

$$\mu_{\dot{v}\delta_r} = \omega_{ar}\mu_{v\delta_r} \quad \text{and} \qquad LA_{\dot{v}\delta_r} = LA_{v\delta_r} + \frac{\pi}{2} \ , \qquad (337)$$

From equations 324, 325, 327, 328, 333, 334, 336 and 337, we see that

$$\frac{\dot{v}_a}{\dot{\psi}_a} = \frac{v_a}{\psi_a} = \frac{\mu_{\dot{v}\delta_r}}{\mu_{\dot{\psi}\delta_r}} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}} = \frac{\omega_{ar}}{\sqrt{(-y_v)^2 + (\omega_{ar})^2}} \quad . \qquad (338)$$

And, from equations 323, 324, 326, 327, 332, 333, 335 and 336 we see that

$$\mu_{\psi\delta_a} = \frac{\dot{\psi}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{\psi_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{\psi}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{\psi\delta_r}}{\mu_{\delta_a\delta_r}} = \frac{l_{\delta_a}}{\omega_{ar}}\sqrt{\frac{(-y_v)^2 + (\omega_{ar})^2}{(l_r\omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}} \quad . \tag{339}$$

Analagously,

$$\mu_{v\delta_a} = \frac{\dot{v}_\alpha}{\dot{\delta}_{a_\alpha}} = \frac{v_\alpha}{\delta_{a_\alpha}} = \frac{\mu_{\dot{v}\delta_r}}{\mu_{\dot{\delta}_a\delta_r}} = \frac{\mu_{v\delta_r}}{\mu_{\delta_a\delta_r}} = \frac{l_{\delta_a}}{\sqrt{(l_r\omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}} \quad . \tag{340}$$

APPENDIX I

DERIVATION OF EQUATIONS FOR FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO STEP-FUNCTIONS

APPENDIX I

DERIVATION OF EQUATIONS FOR FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO STEP-FUNCTIONS

A. DERIVATION OF EQUATIONS FOR FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION.

With reference to equations 29, 30 and 31 in Appendix H, the longitudinal differential equations of motion for transient response are:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta = 0 \qquad (1)$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta = 0 \qquad (2)$$

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta = 0 \qquad (3)$$

The stability roots may be found from the characteristic equation, which may be represented in determinant form as follows:

$$\begin{vmatrix} (-d + x_u) & (x_w) & (x_\theta) \\ (z_u) & (-d + z_w) & (d + z_\theta) \\ (\mu_c m_u) & (d\mu_c m_{\dot{w}} + \mu_c m_w) & (-d^2 + dm_q) \end{vmatrix} = 0 \qquad (4)$$

After expanding equation 4, we have:

$$a_1 d^4 + a_2 d^3 + a_3 d^2 + a_4 d + a_5 = 0 \qquad (5)$$

where

$$a_1 = 1, \qquad (6)$$

$$a_2 = -(z_w + m_q + \mu_c m_{\dot{w}} + x_u), \qquad (7)$$

$$a_3 = x_u(z_w + m_q) + \mu_c m_{\dot{w}}(x_u - z_\theta) \tag{7}$$

$$+ z_w m_q - \mu_c m_w - x_w z_u, \tag{8}$$

$$a_4 = \mu_c m_{\dot{w}}(z_\theta x_u - z_u x_\theta) + \mu_c m_w(x_u - z_\theta) - \mu_c m_u(x_\theta + x_w)$$

$$-m_q(x_u z_w - x_w z_u), \text{ and} \tag{9}$$

$$a_5 = \mu_c m_w(x_u z_\theta - x_\theta z_u) + \mu_c m_u(x_\theta z_w - x_w z_\theta) \tag{10}$$

The general rules for quartic stability are applicable, but for a dynamically stable aircraft, the roots to equation 5 usually consist of two pairs of complex conjugates. One pair represents a highly damped "short period" oscillation, while the other represents a slowly damped "long period" oscillation. Laplace Transforms may be used to determine the actual transient motion, from the roots and boundary conditions, but the author prefers the following procedure because of its simplicity:

Let the roots be:

$$\bar{d}_1 = \alpha_1 + i\beta_1 , \tag{11}$$

$$\bar{d}_2 = \alpha_1 - i\beta_1 , \tag{12}$$

$$\bar{d}_3 = \alpha_2 + i\beta_2 \text{ and} \tag{13}$$

$$\bar{d}_4 \quad \alpha_2 - i\beta_2 . \tag{14}$$

We may assume that the transient solutions are of the form:

$$u = \bar{u}_1 e^{\bar{d}_1\gamma} + \bar{u}_2 e^{\bar{d}_2\gamma} + \bar{u}_3 e^{\bar{d}_3\gamma} + \bar{u}_4 e^{\bar{d}_4\gamma} , \tag{15}$$

$$w = \bar{w}_1 e^{\bar{d}_1\gamma} + \bar{w}_2 e^{\bar{d}_2\gamma} + \bar{w}_3 e^{\bar{d}_3\gamma} + \bar{w}_4 e^{\bar{d}_4\gamma} \quad \text{and} \tag{16}$$

$$\theta = \bar{\theta}_1 e^{\bar{d}_1\gamma} + \bar{\theta}_2 e^{\bar{d}_2\gamma} + \bar{\theta}_3 e^{\bar{d}_3\gamma} + \bar{\theta}_4 e^{\bar{d}_4\gamma} . \tag{17}$$

By De Moivre's theorem,

$$e^{\pm ix} = \cos x \pm i \sin x. \tag{18}$$

Thus, equations 15, 16 and 17 may be changed to:

$$u = e^{\alpha_1 \gamma}(A_u \cos \beta_1 \gamma + B_u \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(C_u \cos \beta_2 \gamma + D_u \sin \beta_2 \gamma), \tag{15-a}$$

$$w = e^{\alpha_1 \gamma}(A_w \cos \beta_1 \gamma + B_w \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(C_w \cos \beta_2 \gamma + D_w \sin \beta_2 \gamma) \tag{16-a}$$

and

$$\theta = e^{\alpha_1 \gamma}(A_\theta \cos \beta_1 \gamma + B_\theta \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(C_\theta \cos \beta_2 \gamma + D_\theta \sin \beta_2 \gamma). \tag{17-a}$$

where

$$\begin{aligned}
A_u &= \bar{u}_1 + \bar{u}_2, & C_u &= \bar{u}_3 + \bar{u}_4, \\
A_w &= \bar{w}_1 + \bar{w}_2, & C_w &= \bar{w}_3 + \bar{w}_4, \\
A_\theta &= \bar{\theta}_1 + \bar{\theta}_2, & C_\theta &= \bar{\theta}_3 + \bar{\theta}_4, \\
B_u &= i(\bar{u}_1 - \bar{u}_2), & D_u &= i(\bar{u}_3 - \bar{u}_4), \\
B_w &= i(\bar{w}_1 - \bar{w}_2), & D_w &= i(\bar{w}_3 - \bar{w}_4) \text{ and} \\
B_\theta &= i(\bar{\theta}_1 - \bar{\theta}_2), & D_\theta &= i(\bar{\theta}_3 - \bar{\theta}_4).
\end{aligned} \tag{19}$$

Note that equations 15-a, 16-a and 17-a are entirely real, because $\bar{u}_1$ and $\bar{u}_2$, $\bar{w}_1$ and $\bar{w}_2$, $\bar{\theta}_1$ and $\bar{\theta}_2$, are pairs of complex conjugates.

Equations 15-a, 16-a and 17-a may also be changed to:

$$u = e^{\alpha_1 \gamma} u_1 \sin(\beta_1 \gamma + LA_{u1}) + e^{\alpha_2 \gamma} u_2 \sin(\beta_2 \gamma + LA_{u2}), \tag{15-b}$$

$$w = e^{\alpha_1 \gamma} w \sin(\beta_1 \gamma + LA_{w1}) + e^{\alpha_2 \gamma} w_2 \sin(\beta_2 \gamma + LA_{w2}), \tag{16-b}$$

and

$$\theta = e^{\alpha_1 \gamma} \theta_1 \sin (\beta_1 \gamma + LA_{\theta 1}) + e^{\alpha_2 \gamma} \theta_2 \sin (\beta_2 \gamma + LA_{\theta 2}), \qquad (17\text{-b})$$

where the amplitudes are:

$$u_1 = \sqrt{A_u^2 + B_u^2}, \qquad u_2 = \sqrt{C_u^2 + D_u^2},$$

$$w_1 = \sqrt{A_w^2 + B_w^2}, \qquad w_2 = \sqrt{C_w^2 + D_w^2}, \qquad (21)$$

$$\theta_1 = \sqrt{A_\theta^2 + B_\theta^2} \quad \text{and} \quad \theta_2 = \sqrt{C_\theta^2 + D_\theta^2},$$

and the lead angles are:

$$LA_{u1} = \tan^{-1}\left(\frac{A_u}{B_u}\right), \qquad LA_{u2} = \tan^{-1}\left(\frac{C_u}{D_u}\right),$$

$$LA_{w1} = \tan^{-1}\left(\frac{A_w}{B_w}\right), \qquad LA_{w2} = \tan^{-1}\left(\frac{C_w}{D_w}\right), \qquad (22)$$

$$LA_{\theta 1} = \tan^{-1}\left(\frac{A_\theta}{B_\theta}\right) \quad \text{and} \quad LA_{\theta 2} = \tan^{-1}\left(\frac{C_\theta}{D_\theta}\right).$$

Equations 15-b, 16-b and 17-b indicate that each variable, u, w and θ, is composed of two components. One component represents the "short-period" oscillation, while the other represents the "long-period" oscillation.

The amplitude and phase of each component may be determined from the boundary conditions and the values of α_1, β_1, α_2 and β_2.

To do this we proceed as follows:

Establish the boundary conditions by assuming values of u_o, w_o, θ_o and $\dot{\theta}_o$. Then, from equations 1, 2 and 3:

$$\dot{u}_o = x_u u_o + x_w w_o + x_\theta \theta_o, \qquad (23)$$

$$\dot{w}_o = z_u u_o + z_w w_o + \dot{\theta}_o + \dot{z}_\theta \theta_o \text{ and} \tag{24}$$

$$\ddot{\theta}_o = \mu_c m_u u_o + \mu_c m_{\dot{w}} \dot{w}_o + \mu_c m_w w_o + m_q \dot{\theta}_o . \tag{25}$$

After differentiating equations 1, 2 and 3, we have:

$$\ddot{u}_o = x_u \dot{u}_o + x_w \dot{w}_o + x_\theta \dot{\theta}_o , \tag{26}$$

$$\ddot{w}_o = z_u \dot{u}_o + z_w \dot{w}_o + \ddot{\theta}_o + z_\theta \dot{\theta}_o \text{ and} \tag{27}$$

$$\dddot{\theta}_o = \mu_c m_u \dot{u}_o + \mu_c m_{\dot{w}} \ddot{w}_o + \mu_c m_w \dot{w}_o + m_q \ddot{\theta}_o . \tag{28}$$

Differentiating again, we have:

$$\dddot{u}_o = x_u \ddot{u}_o + x_w \ddot{w}_o + x_\theta \ddot{\theta}_o , \tag{29}$$

$$\dddot{w}_o = z_u \ddot{u}_o + z_w \ddot{w}_o + \dddot{\theta}_o + z_\theta \ddot{\theta}_o \text{ and} \tag{30}$$

$$\ddddot{\theta}_o = \mu_c m_u \ddot{u}_o + \mu_c m_{\dot{w}} \dddot{w}_o + \mu_c m_w \ddot{w}_o + m_q \dddot{\theta}_o , \tag{31}$$

etc., if required. In this way, the boundary conditions are completely established.

Now the amplitude and phase angle of each component of each variable may be found. To indicate the method we will consider the procedure in determining the amplitude and phase angle of each component of the variable u:

If we differentiate equation 15-a, we have:

$$\dot{u} = e^{\alpha_1 \gamma}(E_u \cos \beta_1 \gamma + F_u \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(G_u \cos \beta_2 \gamma + H_u \sin \beta_2 \gamma) \tag{32}$$

where,

$$E_u = B_u \beta_1 + A_u \alpha_1 , \tag{33}$$

$$F_u = B_u \alpha_1 - A_u \beta_1 , \tag{34}$$

$$G_u = D_u \beta_2 + C_u \alpha_2 \text{ and} \tag{35}$$

$$H_u = D_u \alpha_2 - C_u \beta_2 . \tag{36}$$

Analagously,

$$\ddot{u} = e^{\alpha_1 \gamma}(I_u \cos \beta_1 \gamma + J_u \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(K_u \cos \beta_2 \gamma + L_u \sin \beta_2 \gamma) \tag{37}$$

where

$$I_u = F_u \beta_1 + E_u \alpha_1 , \tag{38}$$

$$J_u = F_u \alpha_1 - E_u \beta_1 , \tag{39}$$

$$K_u = H_u \beta_2 + G_u \alpha_2 \quad \text{and} \tag{40}$$

$$L_u = H_u \alpha_2 - G_u \beta_2 . \tag{41}$$

And,

$$\dddot{u} = e^{\alpha_1 \gamma}(M_u \cos \beta_1 \gamma + N_u \sin \beta_1 \gamma) + e^{\alpha_2 \gamma}(O_u \cos \beta_2 \gamma + P_u \sin \beta_2 \gamma) \tag{42}$$

where

$$M_u = J_u \beta_1 + I_u \alpha_1 , \tag{43}$$

$$N_u = J_u \alpha_1 - I_u \beta_1 , \tag{44}$$

$$O_u = L_u \beta_2 + K_u \alpha_2 \quad \text{and} \tag{45}$$

$$P_u = L_u \alpha_2 - K_u \beta_2 , \tag{46}$$

etc., if required.

When $\gamma = 0$, $u = u_o$, $\dot{u} = \dot{u}_o$, $\ddot{u} = \ddot{u}_o$, $\dddot{u} = \dddot{u}_o$, $\sin \beta_1 \gamma = 0$, $\cos \beta_1 \gamma = 1$, $\sin \beta_2 \gamma = 0$, $\cos \beta_2 \gamma = 1$, $e^{\alpha_1 \gamma} = 1$ and $e^{\alpha_2 \gamma} = 1$.

Thus, from equations 15-a, 32, 37 and 42, we have:

$$u_o = A_u + C_u , \tag{47}$$

$$\dot{u}_o = E_u + G_u , \tag{48}$$

$$\ddot{u}_o = I_u + K_u \quad \text{and} \tag{49}$$

$$\dddot{u}_o = M_u + O_u . \tag{50}$$

Combining equations 33, 34, 35, 36, 38, 39, 40, 41, 43, 44, 45, 46, 47, 48, 49 and 50 we have:

$$u_o = A_u + C_u, \tag{51}$$

$$\dot{u}_o = \alpha_1 A_u + \beta_1 B_u + \alpha_2 C_u + \beta_2 D_u, \tag{52}$$

$$\ddot{u}_o = (\alpha_1^2 - \beta_1^2) A_u + (2\alpha_1\beta_1) B_u + (\alpha_2^2 - \beta_2^2) C_u + (2\alpha_2\beta_2) D_u \tag{53}$$

and

$$\dddot{u}_o = (\alpha_1^3 - 3\alpha_1\beta_1^2) A_u + (-\beta_1^3 + 3\alpha_1^2\beta_1) B_u + (\alpha_2^3 - 3\alpha_2\beta_2^2) C_u + (-\beta_2^3 + 3\alpha_2^2\beta_2) D_u \tag{54}$$

A_u, B_u, C_u and D_u may be found by determinants. The basic determinant of the coefficients in equations 51, 52, 53 and 54 is:

	A_u	B_u	C_u	D_u
u_o	1	0	1	0
$\dot{u}_o$	α_1	β_1	α_2	β_2
$\ddot{u}_o$	$\alpha_1^2 - \beta_1^2$	$2\alpha_1\beta_1$	$\alpha_2^2 - \beta_2^2$	$2\alpha_2\beta_2$
$\dddot{u}_o$	$\alpha_1^3 - 3\alpha_1\beta_1^2$	$-\beta_1^3 + 3\alpha_1^2\beta_1$	$\alpha_2^3 - 3\alpha_2\beta_2^2$	$-\beta_2^3 + 3\alpha_2^2\beta_2$

Thus,

$$A_u = \frac{\Delta_{u1}}{\Delta_{u2}}, \quad B_u = \frac{\Delta_{u3}}{\Delta_{u2}}, \quad C_u = \frac{\Delta_{u4}}{\Delta_{u2}} \text{ and } D_u = \frac{\Delta_{u5}}{\Delta_{u2}}, \tag{55}$$

where,

$$\Delta_{u1} = \begin{vmatrix} (u_o) & (0) & (1) & (0) \\ (\dot{u}_o) & (\beta_1) & (\alpha_2) & (\beta_2) \\ (\ddot{u}_o) & (2\alpha_1\beta_1) & (\alpha_2^2 - \beta_2^2) & (2\alpha_2\beta_2) \\ (\dddot{u}_o) & (-\beta_1^3 + 3\alpha_1^2\beta_1) & (\alpha_2^3 - 3\alpha_2\beta_2^2) & (-\beta_2^3 + 3\alpha_2^2\beta_2) \end{vmatrix}, \quad (56)$$

$$\Delta_{u2} = \begin{vmatrix} (1) & (0) & (1) & (0) \\ (\alpha_1) & (\beta_1) & (\alpha_2) & (\beta_2) \\ (\alpha_1^2 - \beta_1^2) & (2\alpha_1\beta_1) & (\alpha_2^2 - \beta_2^2) & (2\alpha_2\beta_2) \\ (\alpha_1^3 - 3\alpha_1\beta_1^2) & (-\beta_1^3 + 3\alpha_1^2\beta_1) & (\alpha_2^3 - 3\alpha_2\beta_2^2) & (-\beta_2^3 + 3\alpha_2^2\beta_2) \end{vmatrix}, \quad (57)$$

etc.

Once A_u, B_u, C_u and D_u have been determined, u_1, u_2, LA_{u1} and LA_{u2} may be computed from equations 21 and 22. Thus, the constants in the expression for u, given by equation 15-b, are completely determined. Equations 32, 33, 34, 35 and 36 may be used if $\dot{u}$ is required, and in the same manner, $\ddot{u}$ and $\dddot{u}$, etc., may be established by using equations 33 to 46, inclusive.

The same type of analysis can be made to determine the amplitudes and phase angles for the components of w and θ.

3. DERIVATION OF EQUATIONS FOR FIXED-CONTROL TRANSIENT RESPONSE OF AIRCRAFT TO LATERAL STEP-FUNCTION.

With reference to equations 183, 184 and 126 in Appendix H, the lateral differential equations of motion for transient response are:

$$(-\mu_b l_v)v + (-l_r d)\psi + (d^2 - l_p d)\phi = 0, \tag{58}$$

$$(-\mu_b n_v)v + (d^2 - n_r d)\psi + (-n_p d)\phi = 0 \quad \text{and} \tag{59}$$

$$(d - y_v)v + d\psi + (-y_\phi)\phi = 0. \tag{60}$$

The stability roots may be found from the characteristic equation, which may be represented in determinant form as follows:

$$\begin{vmatrix} (-\mu_b l_v) & (-l_r d) & (d^2 - l_p d) \\ (-\mu_b n_v) & (d^2 - n_r d) & (-n_p d) \\ (d - y_v) & (d) & (-y_\phi) \end{vmatrix} = 0 \tag{61}$$

After expanding equation 61, we have:

$$d(b_1 d^4 + b_2 d^3 + b_3 d^2 + b_4 d + b_5) = 0 \tag{62}$$

where

$$b_1 = 1, \tag{63}$$

$$b_2 = -(n_r + l_p + y_v), \tag{64}$$

$$b_3 = (l_p n_r - l_r n_p) + y_v(n_r + l_p) + \mu_b n_v, \tag{65}$$

$$b_4 = y_v(l_r n_p - l_p n_r) + \mu_b(l_v n_p - l_p n_v - l_v y_\phi) \quad \text{and} \tag{66}$$

$$b_5 = \mu_b y_\phi(l_v n_r - l_r n_v). \tag{67}$$

The general rules of quartic stability apply. If b_5 is negative the aircraft will be spirally unstable, but this is not too serious, as many of our present-day aircraft are spirally unstable. For most aircraft, the roots of equation 62 consist of two real roots and one pair of complex conjugates.

The complex conjugates represent the "Dutch roll", and a positive real root represents spiral instability. A comparatively large negative real root usually occurs, and this represents the highly damped "roll subsidence".

Laplace Transforms may be used to determine the actual transient motion, from the roots and boundary conditions, but the author prefers the following procedure because of its simplicity:

Let the roots be:

$$d_1 \mp 0 \tag{68}$$

$$d_2 \tag{69}$$

$$d_3 \tag{70}$$

$$\overline{d}_4 = \sigma + i\epsilon \text{ and} \tag{71}$$

$$\overline{d}_5 = \sigma - i\epsilon,$$

where d_2 and d_3 are real.

We may assume that the transient solutions are of the form:

$$v = v_1 + v_2 e^{d_2\gamma} + v_3 e^{d_3\gamma} + \overline{v}_4 e^{\overline{d}_4\gamma} + \overline{v}_5 e^{\overline{d}_5\gamma}, \tag{72}$$

$$\psi = \psi_1 + \psi_2 e^{d_2\gamma} + \psi_3 e^{d_3\gamma} + \overline{\psi}_4 e^{\overline{d}_4\gamma} + \overline{\psi}_5 e^{\overline{d}_5\gamma} \quad \text{and} \tag{73}$$

$$\phi = \phi_1 + \phi_2 e^{d_2\gamma} + \phi_3 e^{d_3\gamma} + \overline{\phi}_4 e^{\overline{d}_4\gamma} + \overline{\phi}_5 e^{\overline{d}_5\gamma} . \tag{74}$$

Using equation 18, we may change equations 72, 73 and 74 to:

$$v = v_1 + v_2 e^{d_2\gamma} + v_3 e^{d_3\gamma} + e^{\sigma\gamma}(A_v \cos \epsilon\gamma + B_v \sin \epsilon\gamma), \tag{75}$$

$$\psi = \psi_1 + \psi_2 e^{d_2\gamma} + \psi_3 e^{d_3\gamma} + e^{\sigma\gamma}(A_\psi \cos \epsilon\gamma + B_\psi \sin \epsilon\gamma), \tag{76}$$

$$\text{and } \phi = \phi_1 + \phi_2 e^{d_2\gamma} + \phi_3 e^{d_3\gamma} + e^{\sigma\gamma}(A_\phi \cos \epsilon\gamma + B_\phi \sin \epsilon\gamma), \tag{77}$$

where

$$A_v = \bar{v}_4 + \bar{v}_5, \qquad B_v = i(\bar{v}_4 - \bar{v}_5),$$

$$A_\psi = \bar{\psi}_4 + \bar{\psi}_5, \qquad B_\psi = i(\bar{\psi}_4 - \bar{\psi}_5), \tag{78}$$

$$A_\phi = \bar{\phi}_4 + \bar{\phi}_5 \quad \text{and} \quad B_\phi = i(\bar{\phi}_4 - \bar{\phi}_5).$$

Note that equations 75, 76, and 77 are entirely real, because $\bar{v}_4$ and $\bar{v}_5$, $\bar{\psi}_4$ and $\bar{\psi}_5$, $\bar{\phi}_4$ and $\bar{\phi}_5$, are pairs of complex conjugates. Equations 75, 76 and 77 may also be changed to

$$v = v_1 + v_2 e^{d_2\gamma} + v_3 e^{d_3\gamma} + e^{\sigma\gamma} v_4 \sin(\epsilon\gamma + LA_v), \tag{79}$$

$$\psi = \psi_1 + \psi_2 e^{d_2\gamma} + \psi_3 e^{d_3\gamma} + e^{\sigma\gamma} \psi_4 \sin(\epsilon\gamma + LA_\psi), \quad \text{and} \tag{80}$$

$$\phi = \phi_1 + \phi_2 e^{d_2\gamma} + \phi_3 e^{d_3\gamma} + e^{\sigma\gamma} \phi_4 \sin(\epsilon\gamma + LA_\phi), \tag{81}$$

where the amplitudes v_4, ψ_4 and ϕ_4 are, respectively:

$$v_4 = \sqrt{A_v^2 + B_v^2}, \tag{82}$$

$$\psi_4 = \sqrt{A_\psi^2 + B_\psi^2}, \tag{83}$$

$$\phi_4 = \sqrt{A_\phi^2 + B_\phi^2}, \tag{84}$$

and the lead angles are:

$$LA_v = \tan^{-1} \frac{A_v}{B_v}, \tag{85}$$

$$LA_\psi = \tan^{-1} \frac{A_\psi}{B_\psi} \quad \text{and} \tag{86}$$

$$LA_\phi = \tan^{-1} \frac{A_\phi}{B_\phi} . \tag{87}$$

The amplitudes and phase angles may be determined from the boundary conditions and the roots to equation 62. To do this we proceed as follows:

Establish the boundary conditions by assuming values of v_o, ψ_o, ϕ_o, $\dot{\psi}_o$ and $\dot{\phi}_o$. Then, from equations 58, 59 and 60:

$$\ddot{\phi}_o = \mu_b l_v v_o + l_r \dot{\psi}_o + l_p \dot{\phi}_o, \tag{88}$$

$$\ddot{\psi}_o = \mu_b n_v v_o + n_r \dot{\psi}_o + n_p \dot{\phi}_o \quad \text{and} \tag{89}$$

$$\dot{v}_o = y_v v_o - \dot{\psi}_o + y_\phi \phi_o \ . \tag{90}$$

Differentiating equations 88, 89 and 90, we have:

$$\dddot{\phi}_o = \mu_b l_v \dot{v}_o + l_r \ddot{\psi}_o + l_p \ddot{\phi}_o \ , \tag{91}$$

$$\dddot{\psi}_o = \mu_b n_v \dot{v}_o + n_r \ddot{\psi}_o + n_p \ddot{\phi}_o \quad \text{and} \tag{92}$$

$$\ddot{v}_o = y_v \dot{v}_o - \ddot{\psi}_o + y_\phi \dot{\phi}_o \ . \tag{93}$$

Differentiating again:

$$\ddddot{\phi}_o = \mu_b l_v \ddot{v}_o + l_r \dddot{\psi}_o + l_p \dddot{\phi}_o \ , \tag{94}$$

$$\ddddot{\psi}_o = \mu_b n_v \ddot{v}_o + n_r \dddot{\psi}_o + n_p \dddot{\phi}_o \ \text{and} \tag{95}$$

$$\dddot{v}_o = y_v \ddot{v}_o - \dddot{\psi}_o + y_\phi \ddot{\phi}_o \ . \tag{96}$$

Differentiating again

$$\overset{\cdot\cdot\cdot\cdot\cdot}{\phi}_o = \mu_b l_v \dddot{v}_o + l_r \ddddot{\psi}_o + l_p \ddddot{\phi}_o \ , \tag{97}$$

$$\overset{\cdot\cdot\cdot\cdot\cdot}{\psi}_o = \mu_b n_v \dddot{v}_o + n_r \ddddot{\psi}_o + n_p \ddddot{\phi}_o \ \text{and} \tag{98}$$

$$\ddddot{v}_o = y_v \dddot{v}_o - \ddddot{\psi}_o + y_\phi \dddot{\phi}_o \ , \tag{99}$$

etc., if required. In this way the boundary conditions are completely established.

In order to demonstrate the method for determining the amplitudes and phase angles, we will consider the variable v: If we differentiate equation 75 we have:

$$\dot{v} = v_2 d_2 e^{d_2\gamma} + v_3 d_3 e^{d_3\gamma} + e^{\sigma\gamma}(C_v \cos \epsilon\gamma + D_v \sin \epsilon\gamma), \quad (100)$$

where

$$C_v = B_v\epsilon + A_v\sigma \quad \text{and} \quad (101)$$

$$D_v = B_v\sigma - A_v\epsilon \ . \quad (102)$$

Analogously,

$$\ddot{v} = v_2 d_2^2 e^{d_2\gamma} + v_3 d_3^2 e^{d_3\gamma} + e^{\sigma\gamma}(E_v \cos \epsilon\gamma + F_v \sin \epsilon\gamma), \quad (103)$$

where

$$E_v = D_v\epsilon + C_v\sigma \quad \text{and} \quad (104)$$

$$F_v = D_v\sigma - C_v\epsilon. \quad (105)$$

And,

$$\dddot{v} = v_2 d_2^3 e^{d_2\gamma} + v_3 d_3^3 e^{d_3\gamma} + e^{\sigma\gamma}(G_v \cos \epsilon\gamma + H_v \sin \epsilon\gamma) \quad (106)$$

where

$$G_v = F_v\epsilon + E_v\sigma \text{ and} \quad (107)$$

$$H_v = F_v\sigma - E_v\epsilon. \quad (108)$$

And,

$$\ddddot{v} = v_2 d_2^4 e^{d_2\gamma} + v_3 d_3^4 e^{d_4\gamma} + e^{\sigma\gamma}(I_v \cos \epsilon\gamma + J_v \sin \epsilon\gamma) \quad (109)$$

where

$$I_v = H_v\epsilon + G_v\sigma \text{ and} \quad (111)$$

$$J_v = H_v\sigma - G_v\epsilon,$$

etc., if required.

When $\gamma = 0$, $v = v_o$, $\dot{v} = \dot{v}_o$, $\ddot{v} = \ddot{v}_o$, $\dddot{v} = \dddot{v}_o$, $\ddddot{v} = \ddddot{v}_o$,

$\cos \epsilon\gamma = 1$, $\sin \epsilon\gamma = 0$, $e^{d_2\gamma} = 1$ and $e^{d_3\gamma} = 1$.

Thus, from equations 75, 100, 103, 106 and 109 we have:

$$v_o = v_1 + v_2 + v_3 + A_v, \tag{112}$$

$$\dot{v}_o = v_2 d_2 + v_3 d_3 + C_v, \tag{113}$$

$$\ddot{v}_o = v_2 d_2^2 + v_3 d_3^2 + E_v, \tag{114}$$

$$\dddot{v}_o = v_2 d_2^3 + v_3 d_3^3 + G_v \text{ and} \tag{115}$$

$$\ddddot{v}_o = v_2 d_2^4 + v_3 d_3^4 + I_v. \tag{116}$$

Combining equations 101, 102, 104, 105, 107, 108, 110, 111, 112, 113, 114, 115 and 116, we have:

$$v_o = v_1 + v_2 + v_3 + A_v, \tag{117}$$

$$\dot{v}_o = v_2 d_2 + v_3 d_3 + A_v\sigma + B_v\epsilon, \tag{118}$$

$$\ddot{v}_o = v_2 d_2^2 + v_3 d_3^2 + A_v(\sigma^2 - \epsilon^2) + B_v(2\sigma\epsilon), \tag{119}$$

$$\dddot{v}_o = v_2 d_2^3 + v_3 d_3^3 + A_v(\sigma^3 - 3\sigma\epsilon^2) + B_v(-\epsilon^3 + 3\sigma^2\epsilon) \text{ and} \tag{120}$$

$$\ddddot{v}_o = v_2 d_2^4 + v_3 d_3^4 + A_v[(\sigma^2 - \epsilon^2)^2 - 4\sigma^2\epsilon^2] + B_v[4\sigma\epsilon(\sigma^2 - \epsilon^2)]. \tag{121}$$

v_1, v_2, v_3, A_v and B_v may be found by the method of determinants, in section A of this Appendix. Once they have been determined, v_4 and LA_v may be computed from equations 82 and 85 respectively. Thus the constants in the expression for v, given by equation 79, are completely determined. Equations 100, 101 and 102 may be used if $\dot{v}$ is required, and in the same manner, $\ddot{v}$, $\dddot{v}$ and $\ddddot{v}$ may be established by using equations 101 to 111 inclusive. The same type of analysis can be made to determine the amplitudes and phase angles for the components of ψ and ϕ.

APPENDIX J

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL VARIATION IN OUTPUT SIGNAL FROM OSCILLATOR WHICH DRIVES ELEVATOR SERVO, WITH IDEAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT

APPENDIX J

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL VARIATION IN OUTPUT SIGNAL FROM OSCILLATOR WHICH DRIVES ELEVATOR SERVO, WITH IDEAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT.

For ideal displacement, integral and derivative pitch control in the automatic pilot, equations 29, 30 and 31 in Appendix H may be extended as follows:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta = 0 \tag{1}$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta = 0 \tag{2}$$

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + [-d^2 + d(m_q + m_{qc}) + m_\theta + m_{\frac{\theta}{d}}\frac{1}{d}]\theta = -m_{\delta_e}\delta_{e_a}\mathcal{J}e^{i\omega_e\gamma} \tag{3}$$

where
$$m_{qc} = m_{\delta_e}k_q, \tag{4}$$

$$k_q = \frac{\partial\delta_e}{\partial q} = \left(\frac{1}{T_A}\right)\left(\frac{\partial\Delta_E}{\partial Q}\right) \tag{5}$$

$$m_\theta = m_{\delta_e}k_\theta, \tag{6}$$

$$k_\theta = \frac{\partial\delta_e}{\partial\theta} = \frac{\partial\Delta_E}{\partial\theta} \tag{7}$$

$$m_{\frac{\theta}{d}} = m_{\delta_e}k_{\frac{\theta}{d}} \text{ and} \tag{8}$$

$$k_{\frac{\theta}{d}} = \frac{\partial\delta_e}{\partial\int\theta d\gamma} = T_A\frac{\partial\Delta_E}{\partial\int\theta dt} \tag{9}$$

In this case the signal from the automatic pilot adds to the output signal from the oscillator to produce a resultant signal which drives the elevator servo. Thus, for ideal pitch control the resultant motion of the elevator is directly proportional to the resultant signal. But the resultant motion of the elevator may be thought of as being derived from two component motions, one component being directly proportional to the signal from the automatic pilot, and the other being directly proportional to the output signal from the oscillator. Analogously, the resultant moment on the aircraft caused by the resultant motion of the elevator may be thought of as being derived from two component moments, one component being directly proportional to the signal from the automatic pilot, and the other being directly proportional to the output signal from the oscillator. But since the signal from the automatic pilot is a function of the resultant variation in pitch angle, then the resultant moment on the aircraft caused by the resultant motion of the elevator may be thought of as being derived from the resultant variation in pitch angle and that component of the resultant elevator motion which is proportional to the output signal from the oscillator. In other words, the resultant moment on the aircraft caused by the resultant elevator motion may be thought of as being equal to the sum of the moment that would be produced by the elevator if there were no automatic pilot feed-back signal and an equivalent moment which is a function of the resultant change in pitch angle.

Thus the expression on the right hand side of equation 3 simulates only that component of the resultant moment which is directly proportional to the output signal from the oscillator, and which would exist if there were no automatic feed-back signal. Analogously, the elevator motion represented by a part of this expression is merely that component of the elevator motion which would exist if there were no automatic pilot feed-back signal. After equations 1, 2 and 3 are solved the resultant elevator motion may be found from the expression given by equation 21.

To solve equations 1, 2 and 3 we proceed by using operational calculus:

If we let

$$u = \mathcal{R}\bar{u}_a e^{i\omega_e\gamma}, \tag{10}$$

$$w = \mathcal{R}\bar{w}_a e^{i\omega_e\gamma} \text{ and} \tag{11}$$

$$\theta = \mathcal{R}\bar{\theta}_a e^{i\omega_e\gamma}, \tag{12}$$

where $\bar{u}_\alpha$, $\bar{w}_\alpha$ and $\bar{\theta}_\alpha$ are complex, we have:

$$(-i\omega_e + x_u)\bar{u}_\alpha + (x_w)\bar{w}_\alpha + (x_\theta)\bar{\theta}_\alpha = 0, \tag{13}$$

$$(z_u)\bar{u}_\alpha + (-i\omega_e + z_w)\bar{w}_\alpha + (i\omega_e + z_\theta)\bar{\theta}_\alpha = 0, \text{ and} \tag{14}$$

$$(\mu_c m_u)\bar{u}_\alpha + (i\omega_e\mu_c m_{\dot{w}} + \mu_c m_w)\bar{w}_\alpha + [\omega_e^2 + i\omega_e(m_q + m_{\delta_e}k_q)$$

$$+ m_{\delta_e}k_\theta + m_{\delta_e}k_{\frac{\theta}{d}}\frac{1}{i\omega_e}]\bar{\theta}_\alpha = -m_{\delta_e}\delta_{e_\alpha}. \tag{15}$$

Equations 13, 14 and 15 are extensions of equations 32, 33 and 34 in Appendix H. Determinants, may be used to solve for $\bar{u}_\alpha$, $\bar{w}_\alpha$

and $\bar{\theta}_\alpha$, and then equations 10, 11 and 12 may be used to solve for u, w and θ. "This expansion will be left to the student". When there is ideal displacement and derivative pitch control only, equation 15 reduces to:

$$(\mu_c m_u)\bar{u}_\alpha + (i\omega_e \mu_c m_{\dot{w}} + \mu_c m_w)\bar{w}_\alpha + [\omega_e^2 + i\omega_e(m_q + m_{\delta_e} k_q)$$

$$+ m_{\delta_e} k_\theta]\bar{\theta}_\alpha = -m_{\delta_e}\delta_{e_a} \qquad (16)$$

If we solve equations 13, 14 and 16, we find that the equations for steady-state response derived in Section A-1 of Appendix H are applicable, except that the expressions for C_3, C_4, C_5 and C_6, given by equations 43, 44, 45 and 46 respectively, in Section A-1 of Appendix H, must be extended as follows:

$$C_3 = (x_u z_w - x_w z_u) + (x_u + z_w)(m_q + m_{\delta_e} k_q)$$

$$+ \mu_c m_{\dot{w}}(x_u - z_\theta) - m_{\delta_e} k_\theta - \mu_c m_w, \qquad (17)$$

$$C_4 = m_{\delta_e} k_\theta (x_w z_u - x_u z_w) + \mu_c m_w (x_u z_\theta - x_\theta z_u)$$

$$+ \mu_c m_u (x_\theta z_w - x_w z_\theta), \qquad (18)$$

$$C_5 = -x_u - z_w - (m_q + m_{\delta_e} k_q) - \mu_c m_{\dot{w}}, \quad \text{and} \qquad (19)$$

$$C_6 = (x_w z_u - x_u z_w)(m_q + m_{\delta_e} k_q) + (x_u + z_w) m_{\delta_e} k_\theta$$

$$+ (x_u - z_\theta)\mu_c m_w + (x_u z_\theta - x_\theta z_u)\mu_c m_{\dot{w}} - (x_w + x_\theta)\mu_c m_u. \qquad (20)$$

The resultant elevator motion may be found from:

$$\delta_{e_r} = \delta_{e_\alpha} \sin \omega_e \gamma + k_\theta \theta + k_q q + \frac{k_\theta}{d} \int \theta d\gamma + \ldots, \qquad (21)$$

where $q = \dot{\theta}$, and θ is given by equation 60 in Appendix H. In equation 21, δ_{e_α} is the amplitude of the elevator motion which would exist if there were no automatic pilot feed-back.

APPENDIX K

DERIVATION OF EQUATIONS FOR TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL AND ACTUAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT

APPENDIX K

DERIVATION OF EQUATIONS FOR TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL AND ACTUAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT

A. DERIVATION OF EQUATIONS FOR TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION WITH IDEAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT

From equations 1 to 9 in Appendix J, we see that the differential equations for transient response to longitudinal step-function, with ideal displacement, integral and derivative pitch control, are:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta = 0, \qquad (1)$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta = 0, \text{ and} \qquad (2)$$

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + [-d^2 + d(m_q + m_{\delta_e} k_q) + m_{\delta_e} k_\theta + m_{\delta_e} \frac{k_\theta}{d}\frac{1}{d}]\theta = 0. \qquad (3)$$

The stability roots may be found from the characteristic equation, which may be represented in determinant form as follows:

$$\begin{vmatrix} (-d + x_u) & (x_w) & (x_\theta) \\ (z_u) & (-d + z_w) & (d + z_\theta) \\ (\mu_c m_u) & (d\mu_c m_{\dot{w}} + \mu_c m_w) & [-d^2 + d(m_q + m_{\delta_e} k_q) + m_{\delta_e} k_\theta + m_{\delta_e} \frac{k_\theta}{d}\frac{1}{d}] \end{vmatrix} = 0. \qquad (4)$$

After expanding equation 4 we have:

$$a_1d^5 + a_2d^4 + a_3d^3 + a_4d^2 + a_5d + a_6 = 0, \quad (5)$$

where

$$a_1 = 1, \quad (6)$$

$$a_2 = -(z_w + m_q + m_{\delta_e}k_q + \mu_c m_{\dot{w}} + x_u), \quad (7)$$

$$a_3 = x_u(z_w + m_q + m_{\delta_e}k_q + \mu_c m_{\dot{w}}) + z_w(m_q + m_{\delta_e}k_q)$$

$$-m_{\delta_e}k_\theta - z_\theta\mu_c m_{\dot{w}} - \mu_c m_w - x_w z_u, \quad (8)$$

$$a_4 = x_u[-z_w(m_q + m_{\delta_e}k_q) + m_{\delta_e}k_\theta + z_\theta\mu_c m_{\dot{w}} + \mu_c m_w]$$

$$+ x_w z_u(m_q + m_{\delta_e}k_q) - x_w\mu_c m_u + m_{\delta_e}(z_w k_\theta - \frac{k_\theta}{d})$$

$$-z_\theta\mu_c m_w - x_\theta(z_u\mu_c m_{\dot{w}} + \mu_c m_u), \quad (9)$$

$$a_5 = x_u[-m_{\delta_e}(z_w k_\theta - \frac{k_\theta}{d}) + z_\theta\mu_c m_w] + m_{\delta_e}z_w\frac{k_\theta}{d}$$

$$+ x_w(z_u m_{\delta_e}k_\theta - z_\theta\mu_c m_u) - x_\theta(z_u\mu_c m_w - z_w\mu_c m_u), \quad (10)$$

and

$$a_6 = m_{\delta_e}\frac{k_\theta}{d}(x_w z_u - x_u z_w). \quad (11)$$

The general rules for quintic stability are applicable.

Laplace Transforms may be used to determine the actual transient motion for u, w and θ, using the boundary conditions and the roots obtained from equation 5. Also, the method outlined in Appendix I may be applied. The method outlined in Appendix I is completely general, and will not be repeated for this case.

B. DERIVATION OF EQUATIONS FOR TRANSIENT RESPONSE OF AIRCRAFT TO LONGITUDINAL STEP-FUNCTION WITH ACTUAL DISPLACEMENT, INTEGRAL AND DERIVATIVE PITCH CONTROL IN AUTOMATIC PILOT

In this case there are four degrees of freedom - three for the Aircraft, and one for the elevator-servo system. The longitudinal differential equations of motion are:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta + (x_{\delta_e})\delta_e = 0, \tag{12}$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta + (z_{\delta_e})\delta_e = 0, \tag{13}$$

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta + (m_{\delta_e})\delta_e = 0, \tag{14}$$

and

$$(\mu_c m_{eh_u})u + (d\mu_c m_{eh_{\dot{w}}} + \mu_c m_{eh_w})w + [d(m_{eh_q} + m_{eh_{qc}})$$

$$+ m_{eh_\theta} + m_{eh_{\frac{\theta}{d}}} \frac{1}{d}]\theta$$

$$+ (-d^2 + dm_{eh_{\dot{\delta}_e}} + m_{eh_{\delta_e}})\delta_e = 0, \tag{15}$$

where

$$x_{\delta_e} = \left(\frac{T_A}{U_o}\right)\left(\frac{1}{M_A}\right)\frac{\partial X}{\partial \Delta_E}, \tag{16}$$

$$m_{eh_{qc}} = \left[\frac{T_A}{(I_{EH})_{EFF}}\right]\left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial Q}\right), \tag{17}$$

$$m_{eh_\theta} = \left[\frac{T_A^2}{(I_{EH})_{EFF}}\right]\left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial \theta}\right), \quad \text{and} \tag{18}$$

$$m_{eh_{\frac{\theta}{d}}} = \left[\frac{T_A^3}{(I_{EH})_{EFF}}\right]\left(\frac{\partial \theta_{SAE}}{\partial \Delta_E}\right)\left(\frac{\partial M_{SAEM}}{\partial \int \theta dt}\right) . \tag{19}$$

The remaining coefficients are defined in Appendix B.

The stability roots may be found from the characteristic equation, which may be represented in determinant form as follows:

$$\begin{vmatrix} (-d + x_u) & (x_w) & (x_\theta) & (x_{\delta_e}) \\ (z_u) & (-d + z_w) & (d + z_\theta) & (z_{\delta_e}) \\ (\mu_c m_u) & (d\mu_c m_{\dot{w}} + \mu_c m_w) & (-d^2 + dm_q) & (m_{\delta_e}) \\ (\mu_c m_{eh_u}) & (d\mu_c m_{eh_{\dot{w}}} + \mu_c m_{eh_w}) & [d(m_{eh_q} + m_{eh_{qc}}) + m_{eh_\theta} + m_{eh_{\frac{\theta}{d}}}(\frac{1}{d})] & (-d^2 + dm_{eh_{\dot{\delta}_e}} + m_{eh_{\delta_e}}) \end{vmatrix} = 0. \tag{20}$$

For the case when z_θ, m_u, m_{eh_u} and x_{δ_e} are negligible, equation 20 may be expanded to give:

$$a_1 d^7 + a_2 d^6 + a_3 d^5 + a_4 d^4 + a_5 d^3 + a_6 d^2 + a_7 d + a_8 = 0, \tag{21}$$

where

$$a_1 = 1, \tag{22}$$

$$a_2 = -(m_{eh_{\dot{\delta}_e}} + m_q + z_w + \mu_c m_{\dot{w}} + x_u), \tag{23}$$

$$a_3 = m_q m_{eh_{\dot{\delta}_e}} - m_{eh_{\delta_e}} + z_w(m_q + m_{eh_{\dot{\delta}_e}}) + \mu_c m_{\dot{w}} m_{eh_{\dot{\delta}_e}} - \mu_c m_w - \mu_c m_{eh_{\dot{w}}} z_{\delta_e} + x_u(m_{eh_{\dot{\delta}_e}} + m_q + z_w + \mu_c m_{\dot{w}}) - z_u x_w , \tag{24}$$

$$a_4 = m_q m_{eh_{\delta_e}} - m_{\delta_e}(m_{eh_q} + m_{eh_{qc}}) + z_w(m_{eh_{\delta_e}} - m_q m_{eh_{\dot{\delta}_e}})$$

$$+ \mu_c m_{\dot{w}}[m_{eh_{\delta_e}} - z_{\delta_e}(m_{eh_q} + m_{eh_{qc}})] + \mu_c m_w m_{eh_{\dot{\delta}_e}}$$

$$- \mu_c m_{eh_{\dot{w}}}(m_{\delta_e} - z_{\delta_e} m_q) - \mu_c m_{eh_w} z_{\delta_e} + x_u[-m_q m_{eh_{\dot{\delta}_e}} + m_{eh_{\delta_e}}$$

$$- z_w(m_{eh_{\dot{\delta}_e}} + m_q) - \mu_c m_{\dot{w}} m_{eh_{\dot{\delta}_e}} + \mu_c m_w + \mu_c m_{eh_{\dot{w}}} z_{\delta_e}]$$

$$+ z_u x_w(m_{eh_{\dot{\delta}_e}} + m_q) - \mu_c m_{\dot{w}} z_u x_\theta) \tag{25}$$

$$a_5 = -m_{\delta_e} m_{eh_\theta} - z_w[m_q m_{eh_{\delta_e}} - m_{\delta_e}(m_{eh_q} + m_{eh_{qc}})] - \mu_c m_{\dot{w}} z_{\delta_e} m_{eh_\theta}$$

$$+ \mu_c m_w[m_{eh_{\delta_e}} - z_{\delta_e}(m_{eh_q} + m_{eh_{qc}})] - \mu_c m_{eh_w}(m_{\delta_e} - z_{\delta_e} m_q)$$

$$+ x_u\{-m_q m_{eh_{\delta_e}} + m_{\delta_e}(m_{eh_q} + m_{eh_{qc}}) + z_w(m_q m_{eh_{\dot{\delta}_e}} - m_{eh_{\delta_e}})$$

$$+ \mu_c m_{\dot{w}}[z_{\delta_e}(m_{eh_q} + m_{eh_{qc}}) - m_{eh_{\delta_e}}] - \mu_c m_w m_{eh_{\dot{\delta}_e}}$$

$$+ \mu_c m_{eh_{\dot{w}}}(m_{\delta_e} - z_{\delta_e} m_q) + \mu_c m_{eh_w} z_{\delta_e}\} + z_u x_w(m_{eh_{\delta_e}} - m_q m_{eh_{\dot{\delta}_e}})$$

$$+ z_u x_\theta(\mu_c m_{\dot{w}} m_{eh_{\dot{\delta}_e}} - \mu_c m_w), \tag{26}$$

$$a_6 = -m_{\delta_e} m_{eh_{\frac{\theta}{d}}} + z_w m_{\delta_e} m_{eh_\theta} - \mu_c m_{\dot{w}} z_{\delta_e} m_{eh_{\frac{\theta}{d}}} - \mu_c m_w z_{\delta_e} m_{eh_\theta}$$

$$+ x_u\{m_{\delta_e} m_{eh_\theta} + z_w[m_q m_{eh_{\delta_e}} - m_{\delta_e}(m_{eh_q} + m_{eh_{qc}})]$$

$$+ \mu_c m_{\dot{w}} z_{\delta_e} m_{eh_\theta} + \mu_c m_w[z_{\delta_e}(m_{eh_q} + m_{eh_{qc}}) - m_{eh_{\delta_e}}]$$

$$+ \mu_c m_{eh_w}(m_{\delta_e} - z_{\delta_e} m_q)\} + z_u x_\theta(\mu_c m_{\dot{w}} m_{eh_{\delta_e}} + \mu_c m_w m_{eh_{\dot{\delta}_e}}$$

$$- \mu_c m_{eh_{\dot{w}}} m_{\delta_e}) + z_u x_w[-m_q m_{eh_{\delta_e}} + m_{\delta_e}(m_{eh_q} + m_{eh_{qc}})], \qquad (27)$$

$$a_7 = \underline{m_{eh_\theta}}(z_w m_{\delta_e} - \mu_c m_w z_{\delta_e}) + x_u(m_{\delta_e} \underline{m_{eh_\theta}} - z_w m_{\delta_e} m_{eh_\theta}$$
$$ \;\; d \qquad\qquad\qquad\qquad\qquad\qquad\qquad d$$

$$+ \mu_c m_{\dot{w}} z_{\delta_e} \underline{m_{eh_\theta}} + \mu_c m_w z_{\delta_e} m_{eh_\theta}) + z_u x_\theta(\mu_c m_w m_{eh_{\delta_e}}$$
$$\qquad\qquad\qquad d$$

$$- \mu_c m_{eh_{\dot{w}}} m_{\delta_e}) + z_u x_w m_{\delta_e} m_{eh_\theta} , \qquad (28)$$

and

$$a_8 = \underline{m_{eh_\theta}}(z_u x_w m_{\delta_e} + \mu_c m_w x_u z_{\delta_e} - x_u z_w m_{\delta_e}) \qquad (29)$$
$$ \;\; d$$

Laplace Transforms may be used to determine the actual transient motion for u, w, θ and δ_e, using the boundary conditions and the roots obtained from equation 21. Also, the method outlined in Appendix I may be applied, but it will not be repeated for this case.

APPENDIX L

ANALYSES FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

APPENDIX L

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APPENDIX L

ANALYSES FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

A. LONGITUDINAL RESPONSE:

A-1: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION.

With reference to section A-1 in Appendix H, a summary of the basic equations used in this analysis is as follows:

$$C_1 = x_\theta z_u - x_u z_\theta \tag{38}$$

$$C_2 = z_\theta - x_u \tag{39}$$

$$\phi_1 = \tan^{-1}\left(\frac{\omega_e C_2}{C_1 - \omega_e^2}\right) \tag{40}$$

$$C_3 = (x_u z_w - x_w z_u) + (x_u + z_w) m_q + \mu_c m_{\dot{w}} (x_u - z_\theta) - \mu_c m_w \tag{43}$$

$$C_4 = \mu_c m_w (x_u z_\theta - x_\theta z_u) + \mu_c m_u (x_\theta z_w - x_w z_\theta) \tag{44}$$

$$C_5 = -x_u - z_w - m_q - \mu_c m_{\dot{w}} \tag{45}$$

$$C_6 = (x_w z_u - x_u z_w) m_q + (x_u - z_\theta)\mu_c m_w + (x_u z_\theta - x_\theta z_u)\mu_c m_{\dot{w}} - (x_w + x_\theta)\mu_c m_u \tag{46}$$

$$\phi_2 = \tan^{-1}\left[\frac{\omega_e(\omega_e^2 C_5 - C_6)}{-\omega_e^4 + \omega_e^2 C_3 - C_4}\right] \quad (47)$$

$$K_1 = x_u z_w - x_w z_u \quad (50)$$

$$K_2 = - x_u - z_w \quad (51)$$

$$\psi_1 = \tan^{-1}\left(\frac{\omega_e K_2}{K_1 \omega_e^2}\right) \quad (52)$$

$$H_1 = x_\theta z_w - x_w z_\theta \quad (55)$$

$$H_2 = - x_w - x_\theta \quad (56)$$

$$\beta_1 = \tan^{-1}\left(\frac{-\omega_e H_2}{-H_1}\right) \quad (57)$$

$$\delta_e = \delta_{e_a} \sin \omega_e \gamma \quad (19\text{-a})$$

$$u = \mu_{u\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{u\delta_e}) \quad (58)$$

$$w = \mu_{w\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{w\delta_e}) \quad (59)$$

$$\theta = \mu_{\theta\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{\theta\delta_e}) \quad (60)$$

$$\mu_{u\delta_e} = - m_{\delta_e} \sqrt{\frac{(-H_1)^2 + (-\omega_e H_2)^2.}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2(\omega_e^2 C_5 - C_6)^2}} \quad (61)$$

$$\mu_{w\delta_e} = - m_{\delta_e} \sqrt{\frac{(C_1 - \omega_e^2)^2 + (\omega_e C_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2(\omega_e^2 C_5 - C_6)^2}} \quad (62)$$

$$\mu_{\theta\delta_e} = -m_{\delta_e} \sqrt{\frac{(K_1-\omega_e^2)^2 + (\omega_e K_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2(\omega_e^2 C_5 - C_6)^2}} \qquad (63)$$

$$LA_{u\delta_e} = \beta_1 - \phi_2 \qquad (64)$$

$$LA_{w\delta_e} = \phi_1 - \phi_2 \qquad (65)$$

$$LA_{\theta\delta_e} = \psi_1 - \phi_2 \qquad (66)$$

$$\frac{\theta_a}{w_a} = \frac{\dot{\theta}_a}{\dot{w}_a} = \sqrt{\frac{(K_1-\omega_e^2)^2 + (\omega_e K_2)^2}{(C_1-\omega_e^2)^2 + (\omega_e C_2)^2}} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}} \qquad (73)$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical longitudinal dynamic stability coefficients used in this analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec.

μ_c = 241.43

x_u = -.087261

x_w = .18720

x_θ = -.32687

z_u = -.65374

z_w = -4.8701

z_θ negligible for $\theta_o = 0$.

m_u assumed negligible for A-26 Airplane.

$m_w = -.61949$

$m_{\dot{w}} = -.022236$

$m_q = -16.3668$

$m_{\delta_e} = -351.95$

From equations 38, 39, 43, 44, 45, 46, 50, 51, 55 and 56:

$C_1 = .21369$	$K_1 = .54735$
$C_2 = .087261$	$K_2 = 4.9574$
$C_3 = 231.71$	$H_1 = 1.5919$
$C_4 = 31.960$	$H_2 = .13967$
$C_5 = 26.693$	
$C_6 = 23.157$	

Using equations 40, 47, 52, 57, 64, 65 and 66, the lead angles, $LA_{u\delta_e}$, $LA_{w\delta_e}$ and $LA_{\theta\delta_e}$, were computed for chosen values of Ω_E. These computations are shown in Table L-1, and the results are plotted in Figures L-1, L-2, and L-3.

Using equations 61, 62 and 63, the amplitude response ratios, $\mu_{u\delta_e}$, $\mu_{w\delta_e}$ and $\mu_{\theta\delta_e}$ were computed for chosen values of Ω_E. These computations are shown in Table L-2, and the results are plotted in Figures L-4, L-5 and L-6.

Equation 73 was used to compute the ratio $\frac{\theta_a}{w_a} = \frac{q_a}{\dot{w}_a} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}}$ for chosen values of Ω_E. These computations are shown in Table L-2, and the results are plotted in Figure L-7.

Equations 19-a, 58, 59 and 60 were used to compute the time history of $\frac{\delta_e}{\delta_{e_a}}$, $\frac{u}{\delta_{e_a}}$, $\frac{w}{\delta_{e_a}}$ and $\frac{\theta}{\delta_{e_a}}$, respectively.

These computations are shown in Tables L-3, L-4 and L-5.

The results are plotted in Figures L-8, L-9 and L-10.

A-2: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT.

With reference to Section A-2 in Appendix H, a summary of the basic equations used in this analysis follows:

$$b = -z_w - m_q - \mu_c m_{\dot{w}} \tag{84}$$

$$k = -\mu_c m_w + z_w m_q \tag{85}$$

$$\phi_2 = \tan^{-1}\left(\frac{k-\omega_e^2}{-\omega_e b}\right) \tag{86}$$

$$\psi_1 = \tan^{-1}\left(\frac{-\omega_e}{z_w}\right) \tag{87}$$

$$\mu_{w\delta_e} = -\frac{m_{\delta_e}}{\sqrt{(-\omega_e b)^2 + (k-\omega_e^2)^2}} \tag{80}$$

$$\mu_{\theta\delta_e} = -\frac{m_{\delta_e}}{\omega_e}\sqrt{\frac{(z_w)^2 + (-\omega_e)^2}{(-\omega_e b)^2 + (k-\omega_e^2)^2}} \tag{81}$$

$$LA_{w\delta_e} = \frac{3\pi}{2} - \phi_2 \qquad (82)$$

$$LA_{\theta\delta_e} = \psi_1 - \phi_2 \qquad (83)$$

$$w = \mu_{w\delta_e}\delta_{e_\alpha} \sin(\omega_e\gamma + LA_{w\delta_e}) \qquad (78)$$

$$\theta = \mu_{\theta\delta_e}\delta_{e_\alpha} \sin(\omega_e\gamma + LA_{\theta\delta_e}) \qquad (79)$$

$$\frac{\theta_\alpha}{w_\alpha} = \frac{q_\alpha}{\dot{w}_\alpha} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}} = \sqrt{\left(\frac{z_w}{\omega_e}\right)^2 + 1} \qquad (102)$$

$$\delta_e = \delta_{e_\alpha} \sin \omega_e\gamma \qquad (19\text{-a})$$

$$u = \mu_{u\delta_e}\delta_{e_\alpha} \sin(\omega_e\gamma + LA_{u\delta_e}) \qquad (88)$$

$$\mu_{u\delta_e} = \sqrt{\frac{A^2 + B^2}{(x_u)^2 + (-\omega_e)^2}} \qquad (89)$$

$$A = -x_\theta\mu_{\theta\delta_e} \cos \beta_1 - x_w\mu_{w\delta_e} \cos \beta_2 \qquad (90)$$

$$B = -x_\theta\mu_{\theta\delta_e} \sin \beta_1 + x_w\mu_{w\delta_e} \sin \beta_2 \qquad (91)$$

$$LA_{u\delta_e} = \tan^{-1} \left(\frac{B}{A}\right) \qquad (92)$$

$$\beta_1 = \psi_1 - \phi_2 - \alpha_1 \qquad (93)$$

$$\beta_2 = \frac{\pi}{2} - \phi_2 - \alpha_1 \qquad (94)$$

$$\alpha_1 = \tan^{-1} \left(\frac{-\omega_e}{x_u}\right) \qquad (95)$$

As in part A-1, an initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of analysis. Thus, the theoretical longitudinal dynamic stability coefficients used in this Analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec

μ_c = 241.43

x_u = -.087261

x_w = .18720

x_θ = -.32687

z_u assumed negligible

z_w = -4.8701

z_θ negligible

m_u assumed negligible

m_w = -.61949

$m_{\dot{w}}$ = -.022236

m_q = -16.3668

m_{δ_e} = -351.95

From equations 84 and 85, the values for b and k were found to be:

b = 26.605 (same as C_7 in Table L-7)

k = 229.27 (same as C_8 in Table L-7)

Using equations 86, 87, 82, 83, 90, 91, 92, 93, 94 and 95, the lead angles, $LA_{u\delta_e}$, $LA_{w\delta_e}$ and $LA_{\theta\delta_e}$, were computed for chosen values of Ω_E. These computations are shown in Table L-6, and the results are plotted in Figures L-1, L-2 and L-3.

From equations 89, 80 and 81 the amplitude response ratios, $\mu_{u\delta_e}$, $\mu_{w\delta_e}$ and $\mu_{\theta\delta_e}$, were computed for chosen values of Ω_E. These computations are shown in Tables L-6 and L-7, and the results are plotted in Figures L-4, L-5 and L-6. Equation 102 was used to compute the ratio $\frac{\theta_a}{w_a} = \frac{q_a}{\dot{w}_a} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}}$ for chosen values of Ω_E. These computations are shown in Table L-7, and the results are plotted in Figure L-7.

Equations 19-a, 88, 78 and 79 were used to compute the time history of $\frac{\delta_e}{\delta_{e_a}}$, $\frac{u}{\delta_{e_a}}$, $\frac{w}{\delta_{e_a}}$ and $\frac{\theta}{\delta_{e_a}}$, respectively. These computations are shown in Tables L-3, L-4 and L-5, and the results are plotted in Figures L-8, L-9 and L-10.

B. LATERAL RESPONSE:

B-1: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS.

With reference to Section B-1 in Appendix H, a summary of the basic equations used in this analysis follows:

$$C_1 = -l_p \quad (136)$$

$$C_2 = l_r y_\phi \quad (137)$$

$$\alpha_1 = \tan^{-1} \left(\frac{-\omega_r^2 - C_2}{-\omega_r C_1}\right) \tag{138}$$

$$C_3 = y_v(n_p l_r - n_r l_p) + \mu_b(l_v n_p - l_p n_v - l_v y_\phi) \tag{141}$$

$$C_4 = -l_p - y_v - n_r \tag{142}$$

$$C_5 = y_\phi (l_v n_r - l_r n_v)\mu_b \tag{143}$$

$$C_6 = (l_p n_r - l_r n_p) + y_v(n_r + l_p) + \mu_b n_v \tag{144}$$

$$\alpha_2 = \tan^{-1} \left(\frac{-C_5 + \omega_r^2 C_6 - \omega_r^4}{\omega_r C_3 - \omega_r^3 C_4}\right) \tag{145}$$

$$K_1 = -l_v y_\phi \mu_b \tag{148}$$

$$K_2 = -y_v \tag{149}$$

$$\alpha_3 = \tan^{-1} \left[\frac{\omega_r(-C_1 K_2 + \omega_r^2)}{-K_1 + \omega_r^2(K_2 + C_1)}\right] \tag{150}$$

$$H_1 = l_r \tag{153}$$

$$H_2 = -(y_v l_r + \mu_b l_v) \tag{154}$$

$$\alpha_4 = \tan^{-1} \left(\frac{-H_2}{\omega_r H_1}\right) \tag{155}$$

$$\delta_r = \delta_{r_a} \sin \omega_r \gamma \tag{114-a}$$

$$v = \mu_{v\delta_r} \delta_{r_a} \sin(\omega_r \gamma + LA_{v\delta_r}) \tag{156}$$

$$\psi = \mu_{\psi\delta_r} \delta_{r_a} \sin(\omega_r \gamma + LA_{\psi\delta_r}) \tag{157}$$

$$\phi = \mu_{\phi\delta_r}\delta_{r_a}\sin(\omega_r\gamma + LA_{\phi\delta_r}) \tag{158}$$

$$\mu_{v\delta_r} = n_{\delta_r}\sqrt{\frac{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}} \tag{159}$$

$$\mu_{\psi\delta_r} = \left(\frac{n_{\delta_r}}{\omega_r}\right)\sqrt{\frac{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1K_2 + \omega_r^2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}} \tag{160}$$

$$\mu_{\phi\delta_r} = n_{\delta_r}\sqrt{\frac{(\omega_r H_1)^2 + (-H_2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}} \tag{161}$$

$$LA_{v\delta_r} = \alpha_1 - \alpha_2 \tag{162}$$

$$LA_{\psi\delta_r} = \alpha_3 - \alpha_2 \tag{163}$$

$$LA_{\phi\delta_r} = \alpha_4 - \alpha_2 \tag{164}$$

$$\frac{v_a}{\psi_a} = \frac{\dot{v}_a}{r_a} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}} = \omega_r\sqrt{\frac{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1K_2 + \omega_r^2)^2}} \tag{171}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical lateral dynamic stability coefficients used in this analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec

μ_b = 28.074

$l_v = -7.19205$

$l_r = 3.7174$

$l_p = -27.693$

$n_v = 2.5782$

$n_r = -3.1307$

$n_p = -.63842$

$n_{\delta_r} = 105.35$

$y_v = -.65$

$y_\phi = .32687$

From equations 136, 137, 141, 142, 143, 144, 148, 149, 153 and 152:

$C_1 = 27.693$

$C_2 = 1.21510$

$C_3 = 2257.2$

$C_4 = 31.474$

$C_5 = 118.67$

$C_6 = 181.49$

$K_1 = 65.998$

$K_2 = .65$

$H_1 = 3.7174$

$H_2 = 204.33$

Using equations 138, 145, 150, 155, 162, 163 and 164, the lead angles, $LA_{v\delta_r}$, $LA_{\psi\delta_r}$, $LA_{\phi\delta_r}$, were computed for chosen values of Ω_R. These computations are shown in Table L-9, and the results are plotted in Figure L-11.

Using equations 159, 160 and 161, the amplitude response ratios, $\mu_{v\delta_r}$, $\mu_{\psi\delta_r}$ and $\mu_{\phi\delta_r}$, were computed for chosen values of Ω_R.

These computations are shown in Table L-8, and the results are plotted in Figure L-12. Equation 171 was used to compute the ratio $\frac{v_\alpha}{\psi_\alpha} = \frac{\dot{v}_\alpha}{r_\alpha} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}}$ for chosen values of Ω_R. These computations are shown in Table L-8, and the results are plotted in Figure L-13.

Equations 114-a, 156, 157 and 158 were used to compute the time history of $\frac{\delta_r}{\delta_{r_\alpha}}$, $\frac{v}{\delta_{r_\alpha}}$, $\frac{\psi}{\delta_{r_\alpha}}$ and $\frac{\phi}{\delta_{r_\alpha}}$, respectively. These computations are shown in Tables L-10 and L-11, and the results are plotted in Figures L-14, L-15 and L-16.

-2: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER.

With reference to Section B-2 in Appendix H, a summary of the basic equations used in this analysis follows:

$$G_1 = y_\phi - n_p \tag{193}$$

$$G_2 = -y_\phi n_r \tag{194}$$

$$\lambda_1 = \tan^{-1}\left(\frac{-G_2}{\omega_a G_1}\right) \tag{195}$$

$$C_3 = y_v(n_p l_r - n_r l_p) + \mu_b(l_v n_p - l_p n_v - l_v y_\phi) \tag{141}$$

$$C_4 = -l_p - y_v - n_r \tag{142}$$

$$C_5 = y_\phi(l_v n_r - l_r n_v)\mu_b \tag{143}$$

$$C_6 = (l_p n_r - l_r n_p) + y_v(n_r + l_p) + \mu_b n_v \tag{144}$$

$$\lambda_2 = \tan^{-1}\left(\frac{-C_5 + \omega_a^2 C_6 - \omega_a^4}{\omega_a C_3 - \omega_a^3 C_4}\right) \tag{198}$$

$$J_1 = n_v y_\phi \mu_b \tag{201}$$

$$J_2 = - n_p \tag{202}$$

$$J_3 = n_p y_v \tag{203}$$

$$\lambda_3 = \tan^{-1}\left(\frac{\omega_a J_3}{-J_1 - \omega_a^2 J_2}\right) \tag{204}$$

$$J_4 = -n_r - y_v \tag{207}$$

$$J_5 = n_r y_v + \mu_b n_v \tag{208}$$

$$\lambda_4 = \tan^{-1}\left(\frac{-J_5 + \omega_a^2}{\omega_a J_4}\right) \tag{209}$$

$$\delta_a = \delta_{a_\alpha} \sin \omega_a \gamma \tag{176-a}$$

$$v = \mu_{v\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{v\delta_a}) \tag{210}$$

$$\psi = \mu_{\psi\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{\psi\delta_a}) \tag{211}$$

$$\phi = \mu_{\phi\delta_a} \delta_{a_\alpha} \sin(\omega_a \gamma + LA_{\phi\delta_a}) \tag{212}$$

$$\mu_{v\delta_a} = l_{\delta_a} \sqrt{\frac{(\omega_a G_1)^2 + (-G_2)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}} \tag{213}$$

$$\mu_{\psi\delta_a} = \frac{l_{\delta_a}}{\omega_a} \sqrt{\frac{(-J_1 - \omega_a^2 J_2)^2 + (\omega_a J_3)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}} \tag{214}$$

$$\mu_{\phi\delta_a} = l_{\delta_a} \sqrt{\frac{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}} \tag{215}$$

$$LA_{v\delta_a} = \lambda_1 - \lambda_2 \quad (216)$$

$$LA_{\psi\delta_a} = \lambda_3 - \lambda_2 \quad (217)$$

$$LA_{\phi\delta_a} = \lambda_4 - \lambda_2 \quad (218)$$

$$\frac{v_a}{\phi_a} = \frac{\dot{v}_a}{p_a} = \frac{\mu_{v\delta_a}}{\mu_{\phi\delta_a}} = \sqrt{\frac{(\omega_a G_1)^2 + (-G_2)^2}{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}} \quad (225)$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical lateral stability coefficients used in this analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec.

μ_b = 28.074

l_v = -7.19205

l_r = 3.7174

l_p = -27.693

n_v = 2.5782

n_r = -3.1307

n_p = -.63842

l_{δ_a} = 306.52

y_v = -.65

y_ϕ = .32687

From equations 193, 194, 141, 142, 143, 144, 201, 202, 203, 207 and 208:

$G_1 = .96529$	$J_1 = 23.659$
$G_2 = 1.0233$	$J_2 = .63842$
$C_3 = 2257.2$	$J_3 = .41497$
$C_4 = 31.474$	$J_4 = 3.7807$
$C_5 = 118.67$	$J_5 = 74.415$
$C_6 = 181.49$	

Using equations 195, 198, 204, 209, 216, 217 and 218, the lead angles, $LA_{v\delta_a}$, $LA_{\psi\delta_a}$ and $LA_{\phi\delta_a}$, were computed for chosen values of Ω_A. These computations are shown in Table L-12, and the results are plotted in Figure L-17.

Using equations 213, 214 and 215, the amplitude response ratios, $\mu_{v\delta_a}$, $\mu_{\psi\delta_a}$ and $\mu_{\phi\delta_a}$, were computed for chosen values of Ω_A. These computations are shown in Table L-12, and the results are plotted in Figure L-18. Equation 225 was used to compute the ratio $\frac{v_\alpha}{\phi_\alpha} = \frac{\dot{v}_\alpha}{p_\alpha} = \frac{\mu_{v\delta_a}}{\mu_{\phi\delta_a}}$ for chosen values of Ω_A. These computations are shown in Table L-12, and the results are plotted in Figure L-19.

Equations 176-a, 210, 211 and 212 were used to compute the time history of $\frac{\delta_a}{\delta_{a_\alpha}}$, $\frac{v}{\delta_{a_\alpha}}$, $\frac{\psi}{\delta_{a_\alpha}}$ and $\frac{\phi}{\delta_{a_\alpha}}$, respectively. These computations are shown in Tables L-13 and L-14, and the results are plotted in Figures L-20, L-21 and L-22.

-3-a: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW.

With reference to section B-3-a in Appendix H, a summary of the basic equations used in this analysis follows:

$$\phi_3 = \tan^{-1}\left(\frac{-\omega_{ar}^2 - l_r y_\phi}{l_p \omega_{ar}}\right) \tag{239}$$

$$\phi_4 = \tan^{-1}\left(\frac{-n_r y_\phi}{(n_p - y_\phi)\omega_{ar}}\right) \tag{242}$$

$$\delta_r = \delta_{r_a} \sin \omega_{ar}\gamma \tag{227}$$

$$\delta_a = \mu_{\delta_a \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\delta_a \delta_r}) \tag{248}$$

$$\phi = \mu_{\phi \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\phi \delta_r}) \tag{249}$$

$$\psi = \mu_{\psi \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\psi \delta_r}) \tag{250}$$

$$\mu_{\delta_a \delta_r} = \left(\frac{n_{\delta_r}}{l_{\delta_a}}\right) \sqrt{\frac{(l_p \omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}} \tag{251}$$

$$\mu_{\phi \delta_r} = \frac{n_{\delta_r}}{\sqrt{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}} \tag{252}$$

$$\mu_{\psi \delta_r} = \frac{n_{\delta_r} y_\phi}{\omega_{ar}\sqrt{(n_p - y_\phi)^2 \omega_{ar}^2 + (-n_r y_\phi)^2}} \tag{253}$$

$$LA_{\delta_a \delta_r} = \phi_3 - \phi_4 \tag{254}$$

$$LA_{\phi\delta_r} = \frac{\pi}{2} - \phi_4 \qquad (255)$$

$$LA_{\psi\delta_r} = - \phi_4 \qquad (256)$$

$$\frac{\phi_a}{\psi_a} = \frac{\dot{\phi}_a}{\dot{\psi}_a} = \frac{\mu_{\phi\delta_r}}{\mu_{\psi\delta_r}} = \frac{\omega_{ar}}{y_\phi} \qquad (263)$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical lateral dynamic stability coefficients used in this analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec.

μ_b = 28.074

y_ϕ = .32687

l_r = 3.7174

l_p = -27.693

n_r = -3.1307

n_p = -.63842

l_{δ_a} = 306.52

n_{δ_r} = 105.35

From equations 239, 242, 254, 255 and 256, the lead angles, $LA_{\delta_a\delta_r}$, $LA_{\phi\delta_r}$ and $LA_{\psi\delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in table L-15, and the results are plotted in Figure L-23.

Using equations 251, 252 and 253, the amplitude response ratios, $\mu_{\delta_a \delta_r}$, $\mu_{\phi \delta_r}$ and $\mu_{\psi \delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in Table L-15, and the results are plotted in Figure L-24. Equation 263 was used to compute the ratio $\frac{\phi_a}{\psi_a} = \frac{\dot{\phi}_a}{r_a} = \frac{\mu_{\phi \delta_r}}{\mu_{\psi \delta_r}}$ for chosen values of Ω_{AR}. These computations are shown in Table L-15, and the results are plotted in Figure L-25.

Equations 227, 248, 249 and 250 were used to compute the time history of $\frac{\delta_r}{\delta_{r_a}}$, $\frac{\delta_a}{\delta_{r_a}}$, $\frac{\phi}{\delta_{r_a}}$ and $\frac{\psi}{\delta_{r_a}}$, respectively.

These computations are shown in Tables L-16, L-17 and L-18, and the results are plotted in Figures L-26, L-27 and L-28.

B-3-b: ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW.

With reference to Section B-3-b in Appendix H, a summary of the basic equations used in this analysis follows:

$$P_1 = -l_v y_\phi \mu_b \tag{273}$$

$$P_2 = -(y_v + l_p) \tag{274}$$

$$P_3 = l_p y_v \tag{275}$$

$$\psi_2 = \tan^{-1} \left[\frac{\omega_{ar}(P_3 - \omega_{ar}^2)}{P_1 - \omega_{ar}^2 P_2}\right] \tag{276}$$

$$P_4 = n_v y_\phi \mu_b \tag{279}$$

$$P_5 = -n_p \tag{280}$$

$$P_6 = n_p y_v \tag{281}$$

$$\psi_3 = \tan^{-1}\left(\frac{\omega_{ar}P_6}{-P_4 - P_5\omega_{ar}^2}\right) \tag{282}$$

$$\psi_4 = \tan^{-1}\left(\frac{\omega_{ar}}{-y_v}\right) \tag{285}$$

$$\delta_r = \delta_{r_a} \sin \omega_{ar}\gamma \tag{227}$$

$$\delta_a = \mu_{\delta_a \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\delta_a \delta_r}) \tag{288}$$

$$\phi = \mu_{\phi \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{\phi \delta_r}) \tag{289}$$

$$v = \mu_{v \delta_r} \delta_{r_a} \sin(\omega_{ar}\gamma + LA_{v \delta_r}) \tag{290}$$

$$\mu_{\delta_a \delta_r} = \left(\frac{n_{\delta_r}}{l_{\delta_a}}\right) \sqrt{\frac{(P_1 - \omega_{ar}^2 P_2)^2 + \omega_{ar}^2 (P_3 - \omega_{ar}^2)^2}{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}} \tag{291}$$

$$\mu_{\phi \delta_r} = n_{\delta_r} \sqrt{\frac{(-y_v)^2 + (\omega_{ar})^2}{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}} \tag{292}$$

$$\mu_{v \delta_r} = \frac{n_{\delta_r} y_\phi}{\sqrt{(-P_4 - P_5\omega_{ar}^2)^2 + (\omega_{ar}P_6)^2}} \tag{293}$$

$$LA_{\delta_a \delta_r} = \psi_2 - \psi_3 \tag{294}$$

$$LA_{\phi\delta_r} = \psi_4 - \psi_3 \tag{295}$$

$$LA_{v\delta_r} = -\psi_3 \tag{296}$$

$$\frac{v_a}{\phi_a} = \frac{\dot{v}_a}{\dot{\phi}_a} = \frac{\mu_{v\delta_r}}{\mu_{\phi\delta_r}} = \frac{y_\phi}{\sqrt{(-y_v)^2 + (\omega_{ar})^2}} \tag{303}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical lateral stability coefficients used in this analysis, for the A-26 airplane, were (see Appendix D) as follows:

$T_A = 4.4716$ sec.

$\mu_b = 28.074$

$l_v = -7.19205$

$l_p = -27.693$

$n_v = 2.5782$

$n_p = -.63842$

$l_{\delta_a} = 306.52$

$n_{\delta_r} = 105.35$

$y_\phi = .32687$

$y_v = -.65$

From equations 273, 274, 275, 279, 280, and 281:

$P_1 = 65.998$	$P_4 = 23.659$
$P_2 = 28.343$	$P_5 = .63842$
$P_3 = 18.000$	$P_6 = .41497$

Using equations 276, 282, 285, 294, 295 and 296, the lead angles, $LA_{\delta_a \delta_r}$, $LA_{\phi\delta_r}$ and $LA_{v\delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in Table L-19, and the results are plotted in Figure L-29.

From equations 291, 292 and 293 the amplitude ratios, $\mu_{\delta_a \delta_r}$, $\mu_{\phi\delta_r}$ and $\mu_{v\delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in Table L-19, and the results are plotted in Figure L-30. Equation 303 was used to compute the ratio $\frac{v_\alpha}{\phi_\alpha} = \frac{\dot{v}_\alpha}{p_\alpha} = \frac{\mu_{v\delta_r}}{\mu_{\phi\delta_r}}$ for chosen values of Ω_{AR}. These computations are shown in Table L-19, and the results are plotted in Figure L-31.

Equations 227, 288, 289 and 290 were used to compute the time history of $\frac{\delta_r}{\delta_{r_\alpha}}$, $\frac{\delta_a}{\delta_{r_\alpha}}$, $\frac{\phi}{\delta_{r_\alpha}}$ and $\frac{v}{\delta_{r_\alpha}}$, respectively. These computations are shown in L-20, L-21 and L-22, and the results are plotted in Figures L-32, L-33 and L-34.

B-3-c:- ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK.

With reference to section B-3-c in Appendix H, a summary of the basic equations used in this analysis follows:

$$\sigma_1 = \tan^{-1}\left(\frac{l_r y_v + \mu_b l_v}{l_r \omega_{ar}}\right) \tag{312}$$

$$Q_1 = -y_v - n_r \tag{315}$$

$$Q_2 = n_v \mu_b + n_r y_v \tag{316}$$

$$\sigma_2 = \tan^{-1}\left(\frac{Q_2 - \omega_{ar}^2}{-Q_1\omega_{ar}}\right) \quad (317)$$

$$\sigma_3 = \tan^{-1}\left(\frac{\omega_{ar}}{-y_v}\right) \quad (320)$$

$$\delta_r = \delta_{r_\alpha} \sin \omega_{ar}\gamma \quad (227)$$

$$\delta_a = \mu_{\delta_a\delta_r}\delta_{r_\alpha} \sin\left(\omega_{ar}\gamma + LA_{\delta_a\delta_r}\right) \quad (323)$$

$$\psi = \mu_{\psi\delta_r}\delta_{r_\alpha} \sin\left(\omega_{ar}\gamma + LA_{\psi\delta_r}\right) \quad (324)$$

$$v = \mu_{v\delta_r}\delta_{r_\alpha} \sin\left(\omega_{ar}\gamma + LA_{v\delta_r}\right) \quad (325)$$

$$\mu_{\delta_a\delta_r} = \frac{n_{\delta_r}}{l_{\delta_a}} \sqrt{\frac{(l_r\omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}} \quad (326)$$

$$\mu_{\psi\delta_r} = \frac{n_{\delta_r}}{\omega_{ar}} \sqrt{\frac{(-y_v)^2 + (\omega_{ar})^2}{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}} \quad (327)$$

$$\mu_{v\delta_r} = \frac{n_{\delta_r}}{\sqrt{(-Q_1\omega_{ar})^2 + (Q_2 - \omega_{ar}^2)^2}} \quad (328)$$

$$LA_{\delta_a\delta_r} = \sigma_1 - \sigma_2 \quad (329)$$

$$LA_{\psi\delta_r} = \sigma_3 - \sigma_2 \quad (330)$$

$$LA_{v\delta_r} = \frac{3\pi}{2} - \sigma_2 \quad (331)$$

$$\frac{v_\alpha}{\psi_\alpha} = \frac{\dot{v}_\alpha}{\dot{\psi}_\alpha} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}} = \frac{\omega_{ar}}{\sqrt{(-y_v)^2 + (\omega_{ar})^2}} \quad (338)$$

An initial airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical lateral stability coefficients used in this analysis, for the A-26 airplane, were (see Appendix D) as follows:

$T_A = 4.4716$ sec.

$\mu_b = 28.074$

$l_v = -7.19205$

$l_r = 3.7174$

$n_v = 2.5782$

$n_r = -3.1307$

$l_{\delta_a} = 306.52$

$n_{\delta_r} = 105.35$

$y_v = -.65$

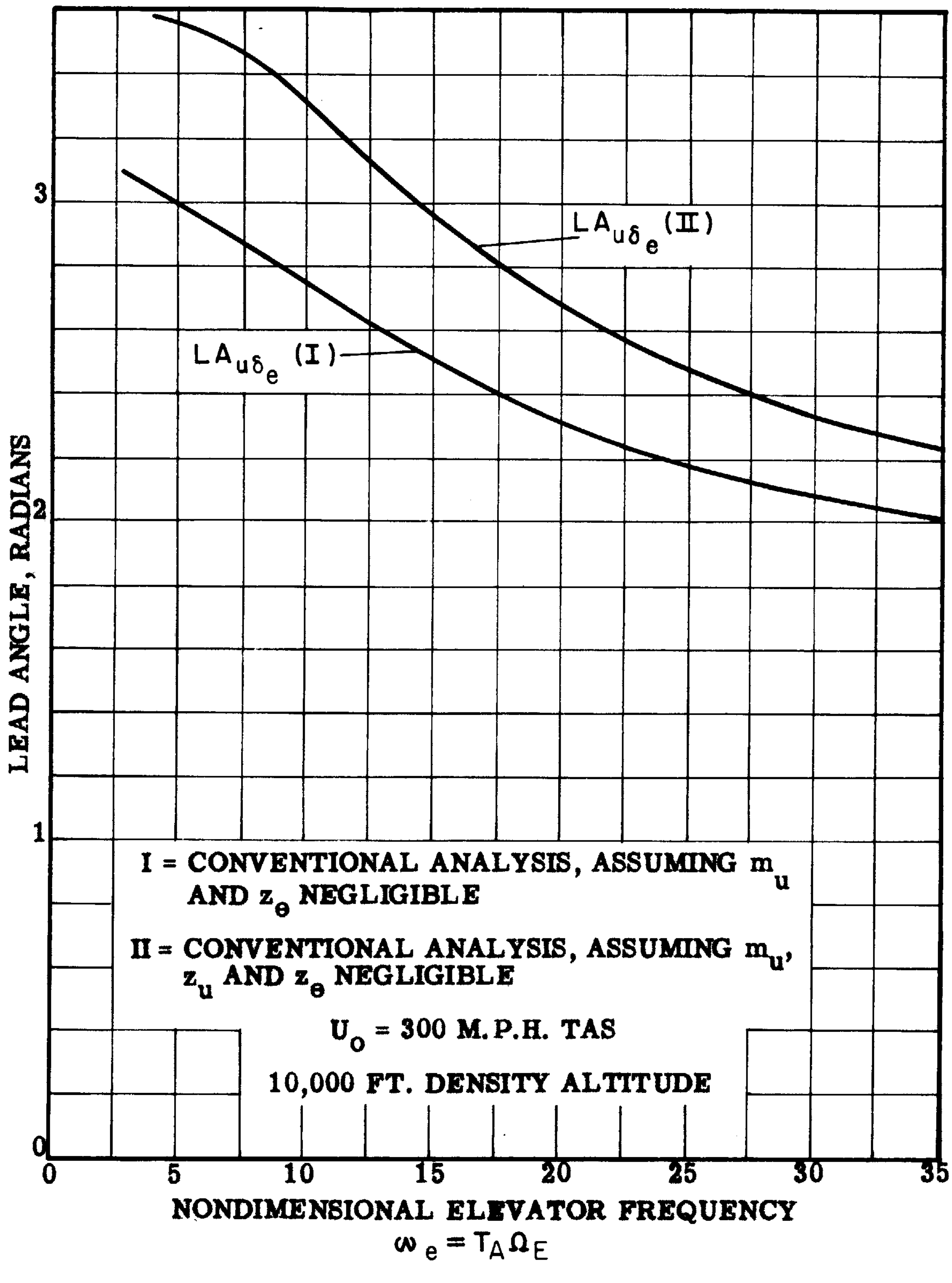
From equations 315 and 316,

$Q_1 \doteq 3.7807$

$Q_2 = 74.415$

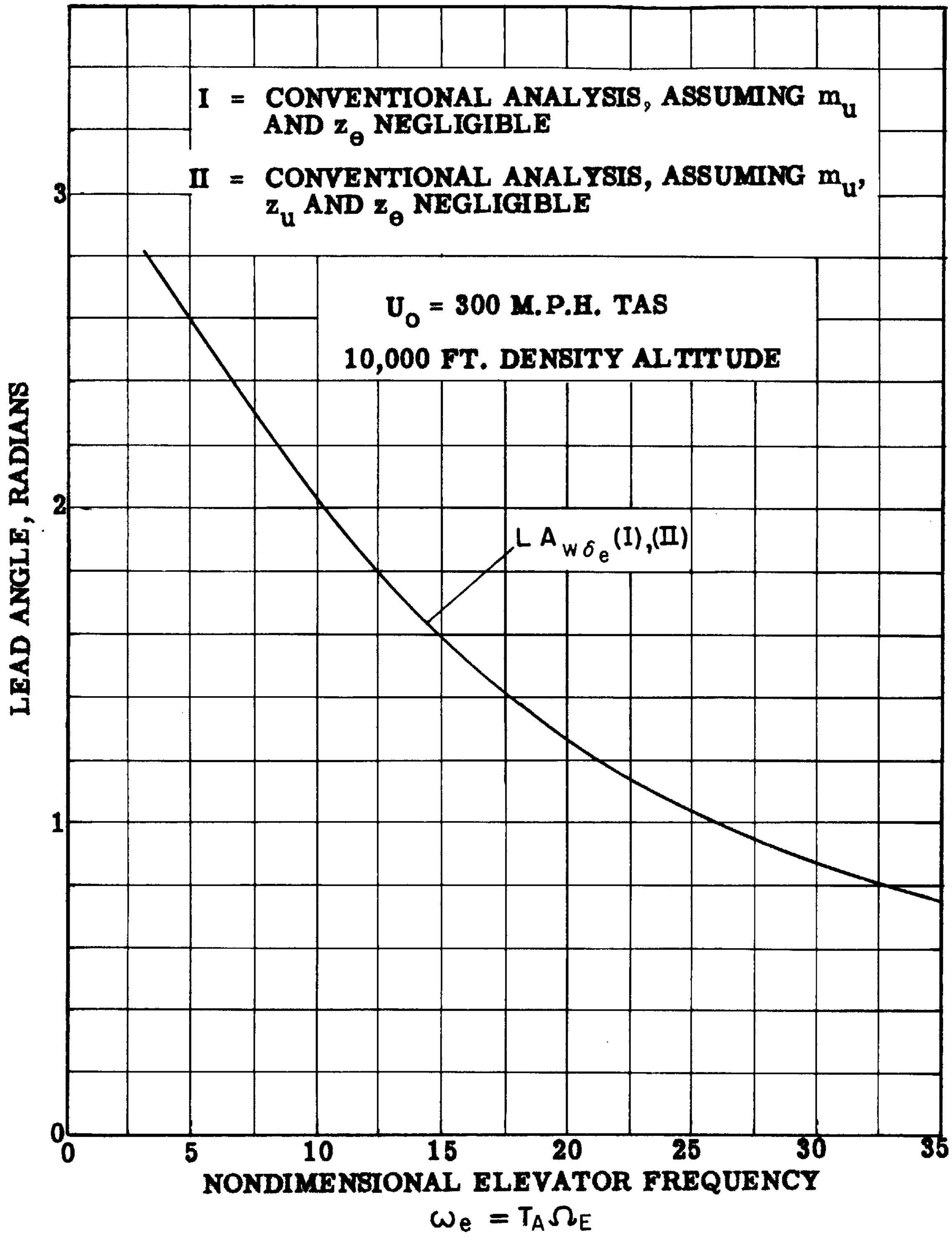
Using equations 312, 317, 320, 329, 330 and 331, the lead angles $LA_{\delta_a \delta_r}$, $LA_{\psi\delta_r}$ and $LA_{v\delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in Table L-23, and the results are plotted in Figure L-35. From equations 326, 327 and 328 the amplitude ratios, $\mu_{\delta_a \delta_r}$, $\mu_{\psi\delta_r}$ and $\mu_{v\delta_r}$, were computed for chosen values of Ω_{AR}. These computations are shown in Table L-23, and the results are plotted in Figure L-36. Equation 338 was used to compute the ratio $\frac{v_\alpha}{\psi_\alpha} = \frac{\dot{v}_\alpha}{r_\alpha} = \frac{\mu_{v\delta_r}}{\mu_{\psi\delta_r}}$ for chosen values of Ω_{AR}. These computations are shown in Table L-23, and the results are plotted in Figure L-37.

Equations 227, 323, 324 and 325 were used to compute the time history of $\frac{\delta_r}{\delta_{r_\alpha}}$, $\frac{\delta_a}{\delta_{r_\alpha}}$, $\frac{\psi}{\delta_{r_\alpha}}$, and $\frac{v}{\delta_{r_\alpha}}$, respectively. These computations are shown in Tables L-24, L-25 and L-26, and the results are plotted in Figures L-38, L-39 and L-40.



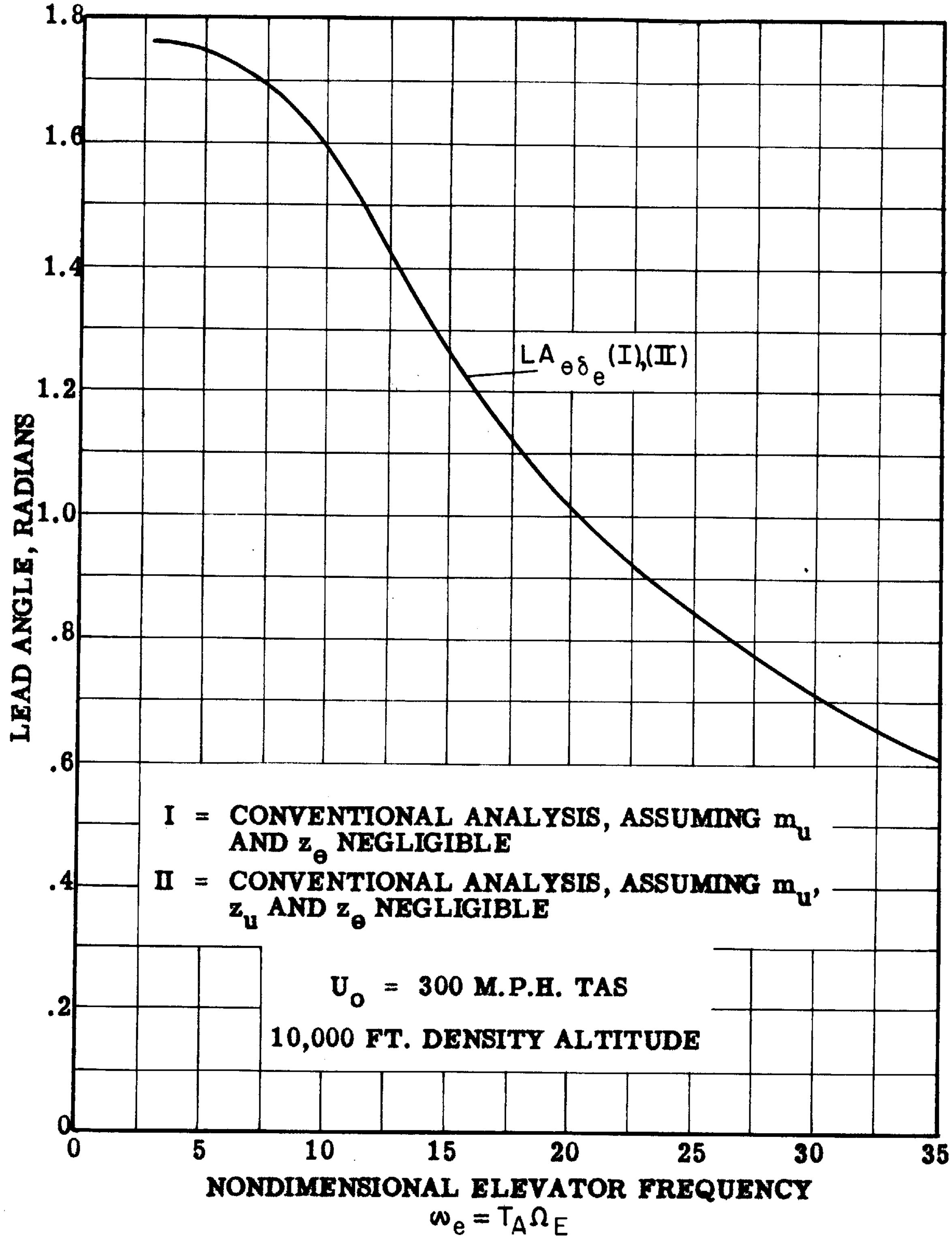
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL VELOCITY PER AIRPLANE TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE

FIGURE L-1



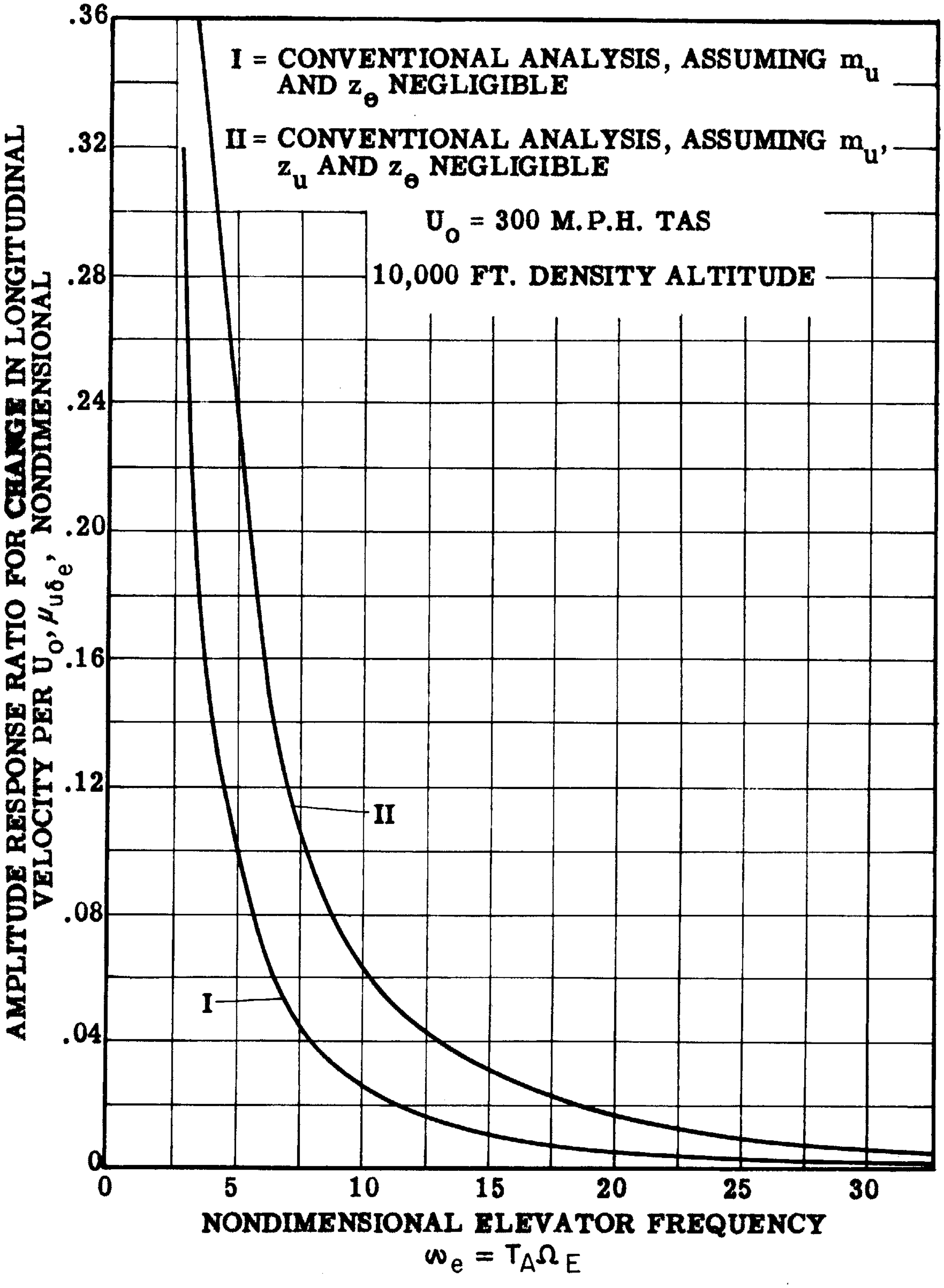
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN NORMAL VELOCITY PER AIRPLANE TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE

FIGURE L-2



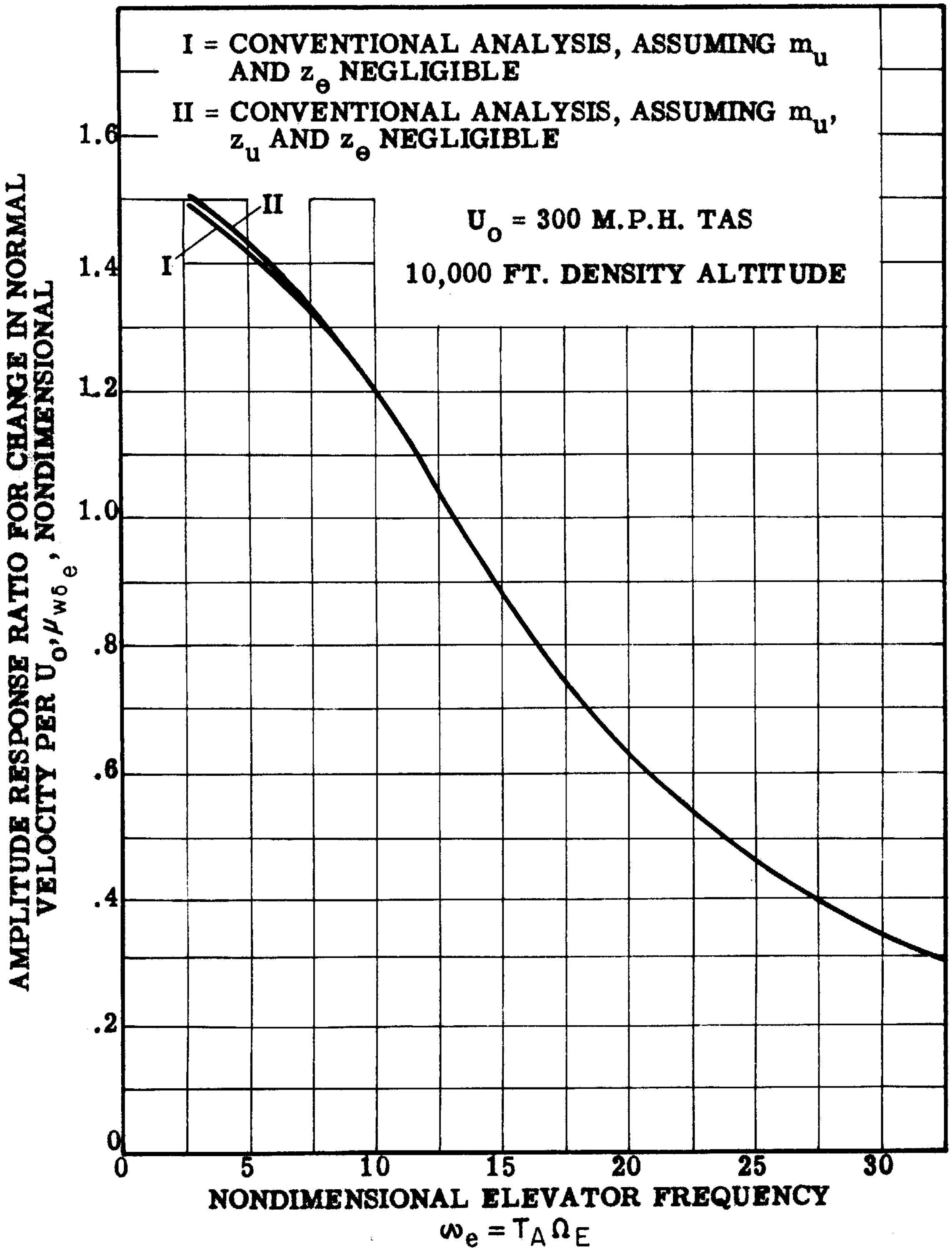
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN PITCH ANGLE AND CHANGE IN ELEVATOR ANGLE COMPUTED FOR A-26 AIRPLANE

FIGURE L-3



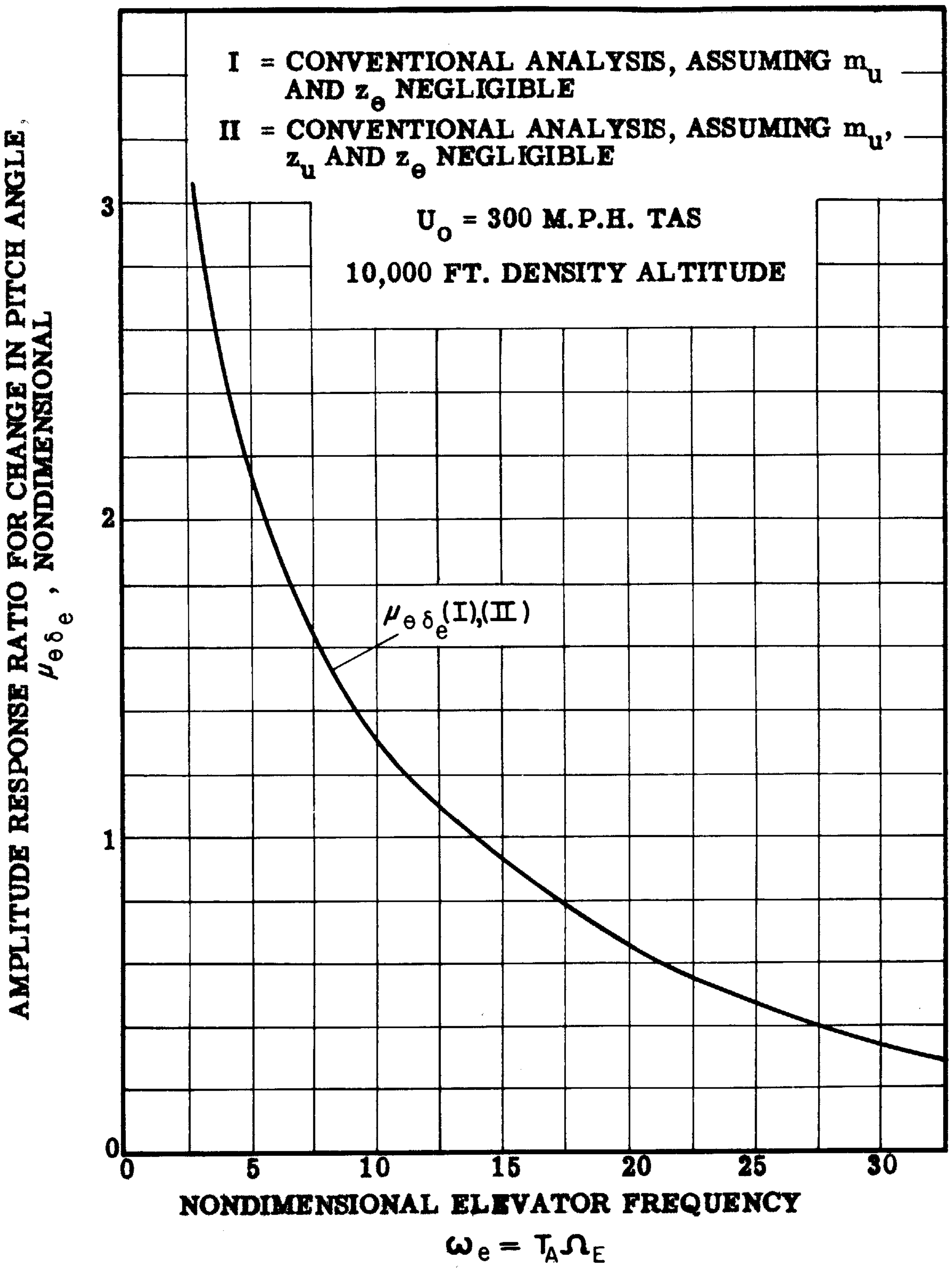
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN LONGITUDINAL VELOCITY PER AIRPLANE TRIM SPEED, COMPUTED FOR A-26 AIRPLANE

FIGURE L-4



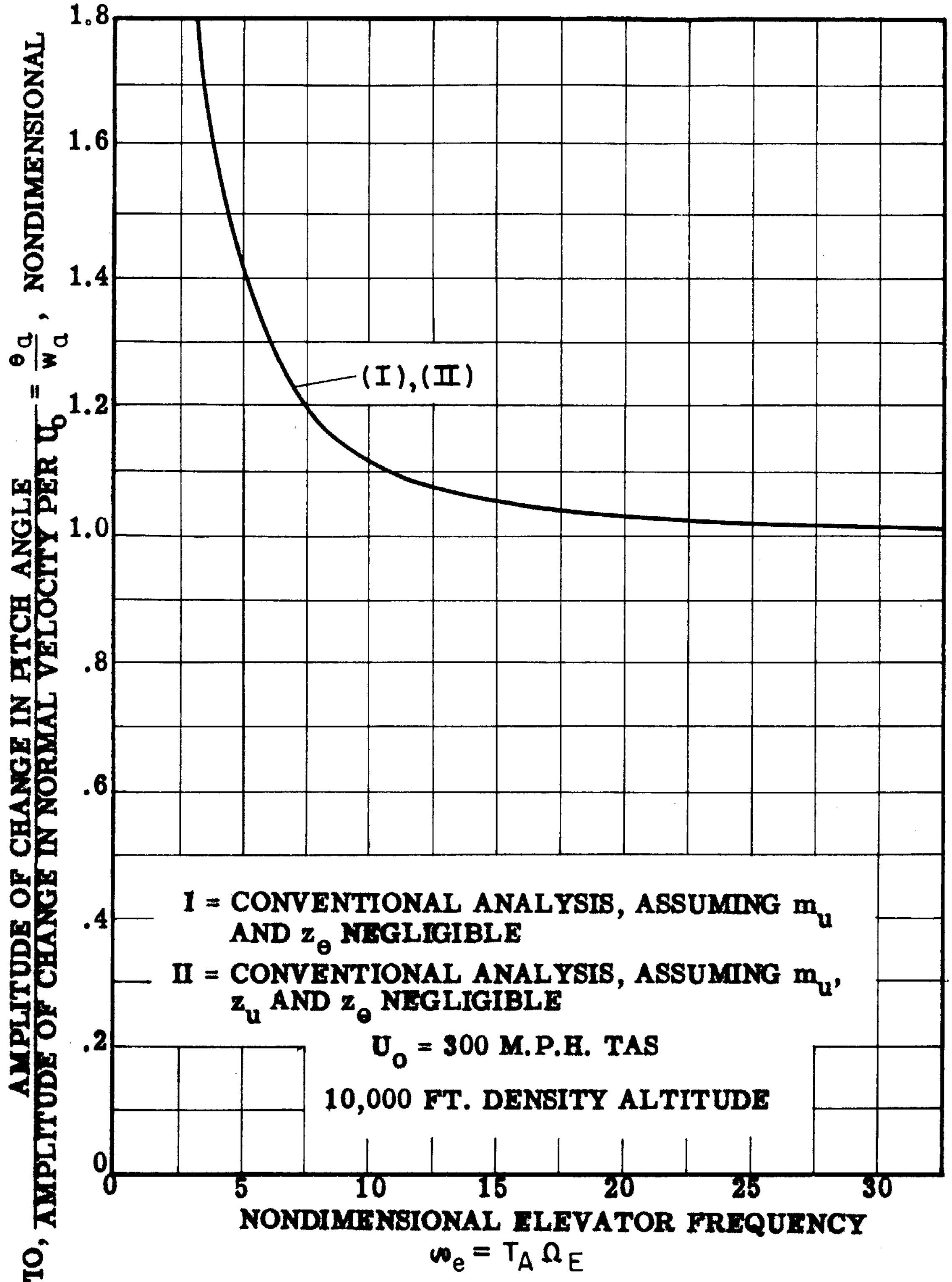
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN NORMAL VELOCITY PER AIRPLANE TRIM SPEED, COMPUTED FOR A-26 AIRPLANE

FIGURE L-5



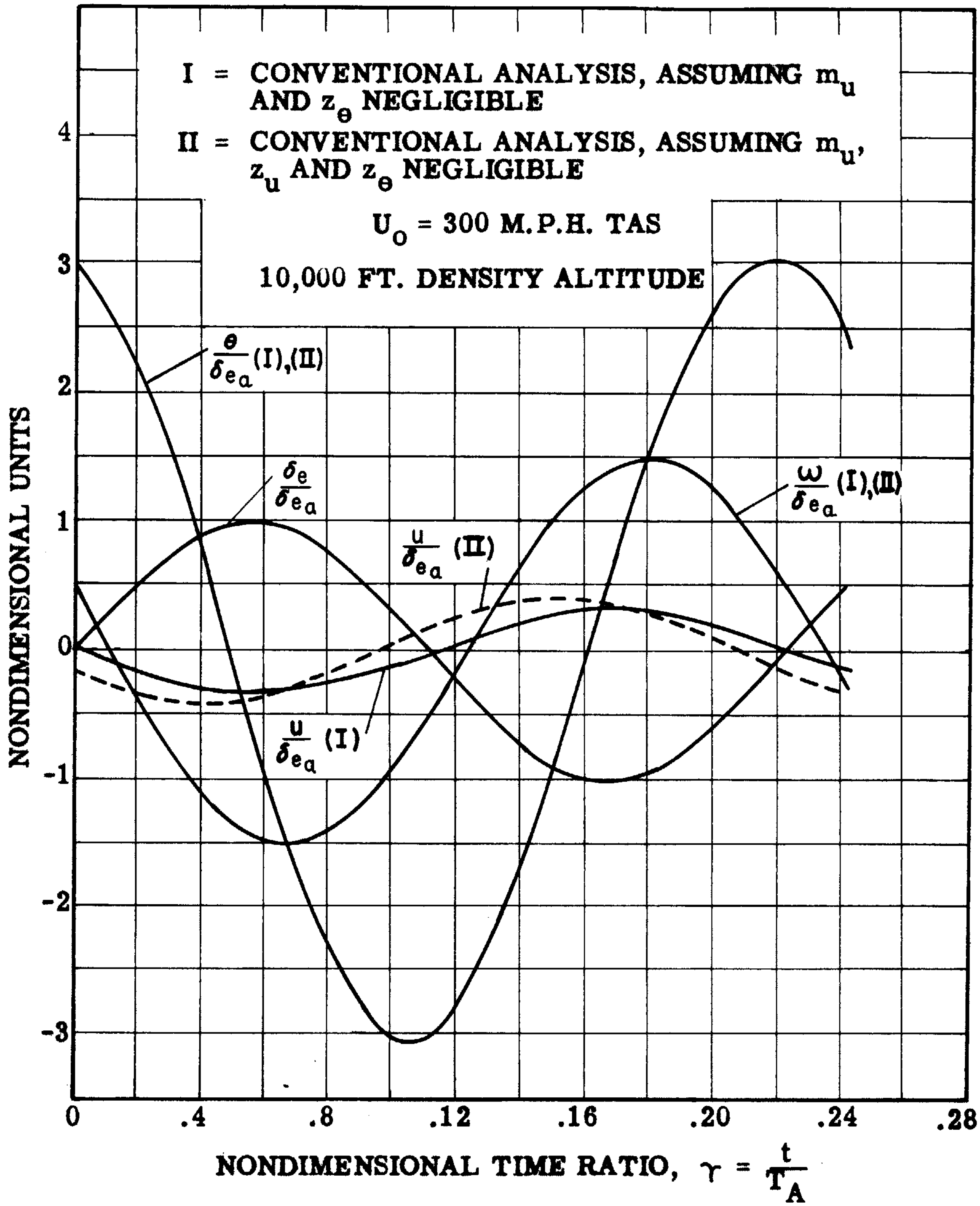
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN PITCH ANGLE, COMPUTED FOR A-26 AIRPLANE

FIGURE L-6



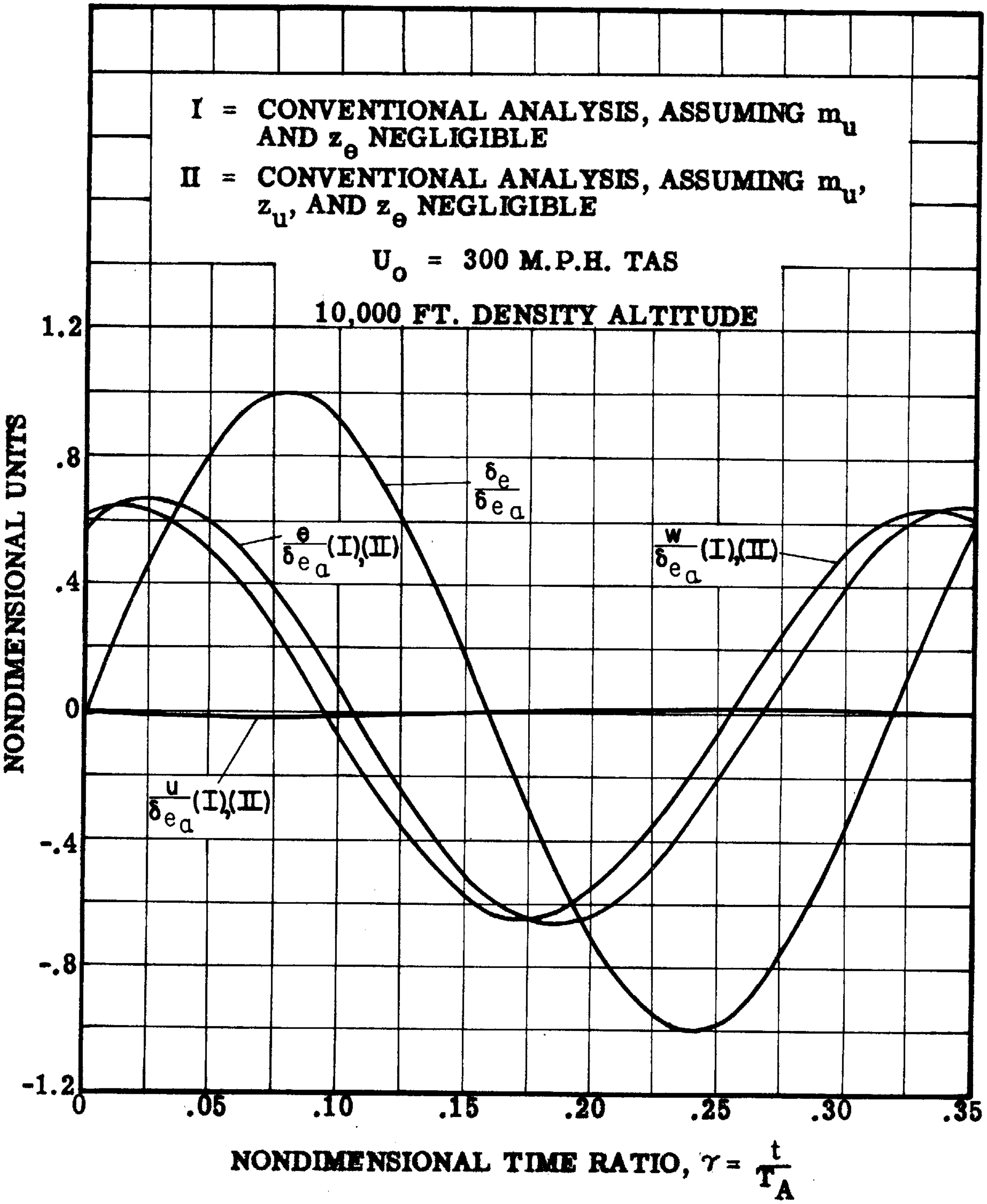
EFFECT OF CHANGE IN ELEVATOR FREQUENCY ON RATIO OF AMPLITUDE OF CHANGE IN PITCH ANGLE TO AMPLITUDE OF CHANGE IN NORMAL VELOCITY PER AIRPLANE TRIM SPEED, COMPUTED FOR A-26 AIRPLANE

FIGURE L-7



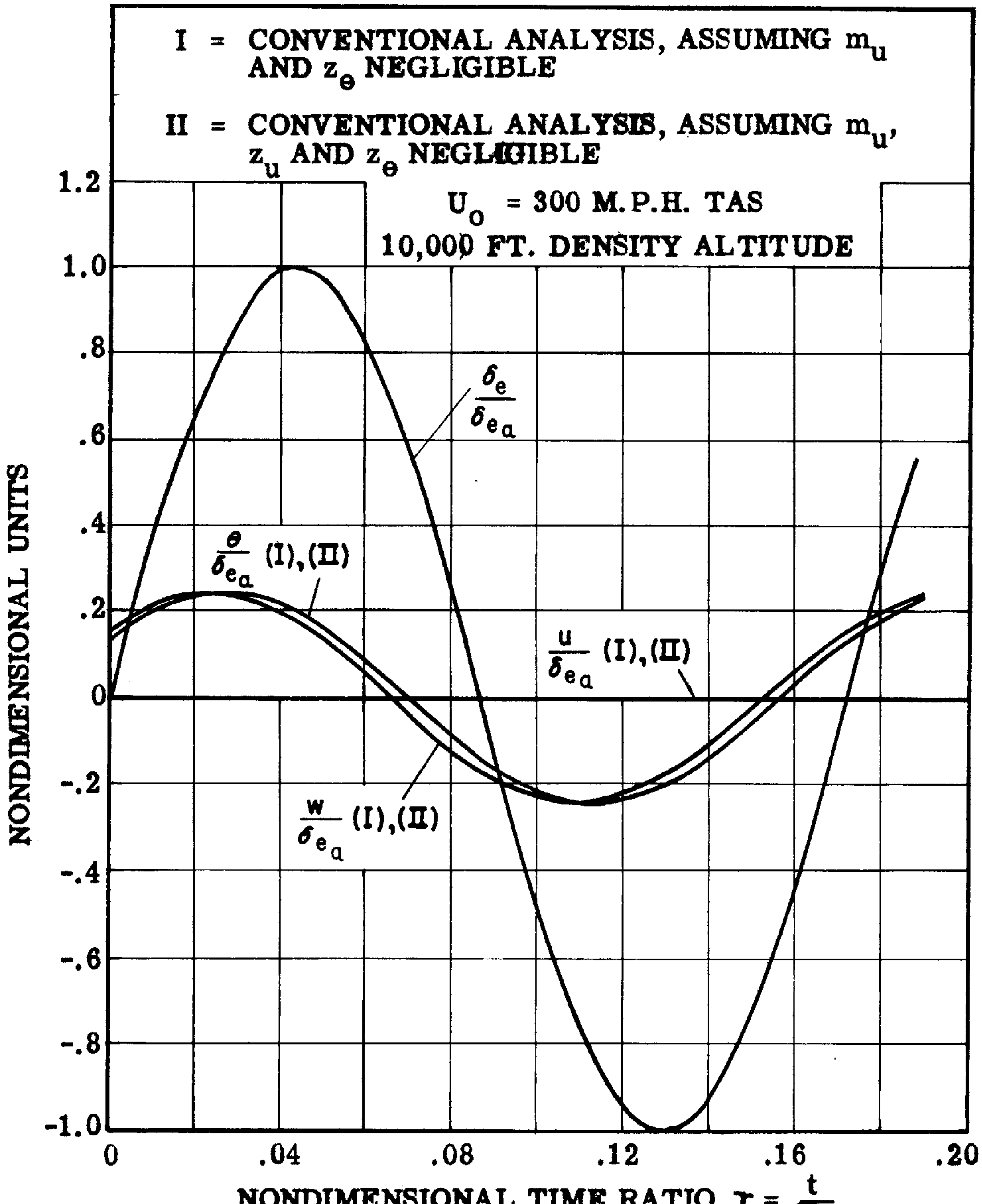
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, ELEVATOR FREQUENCY = .1 CYCLE PER SECOND

FIGURE L-8



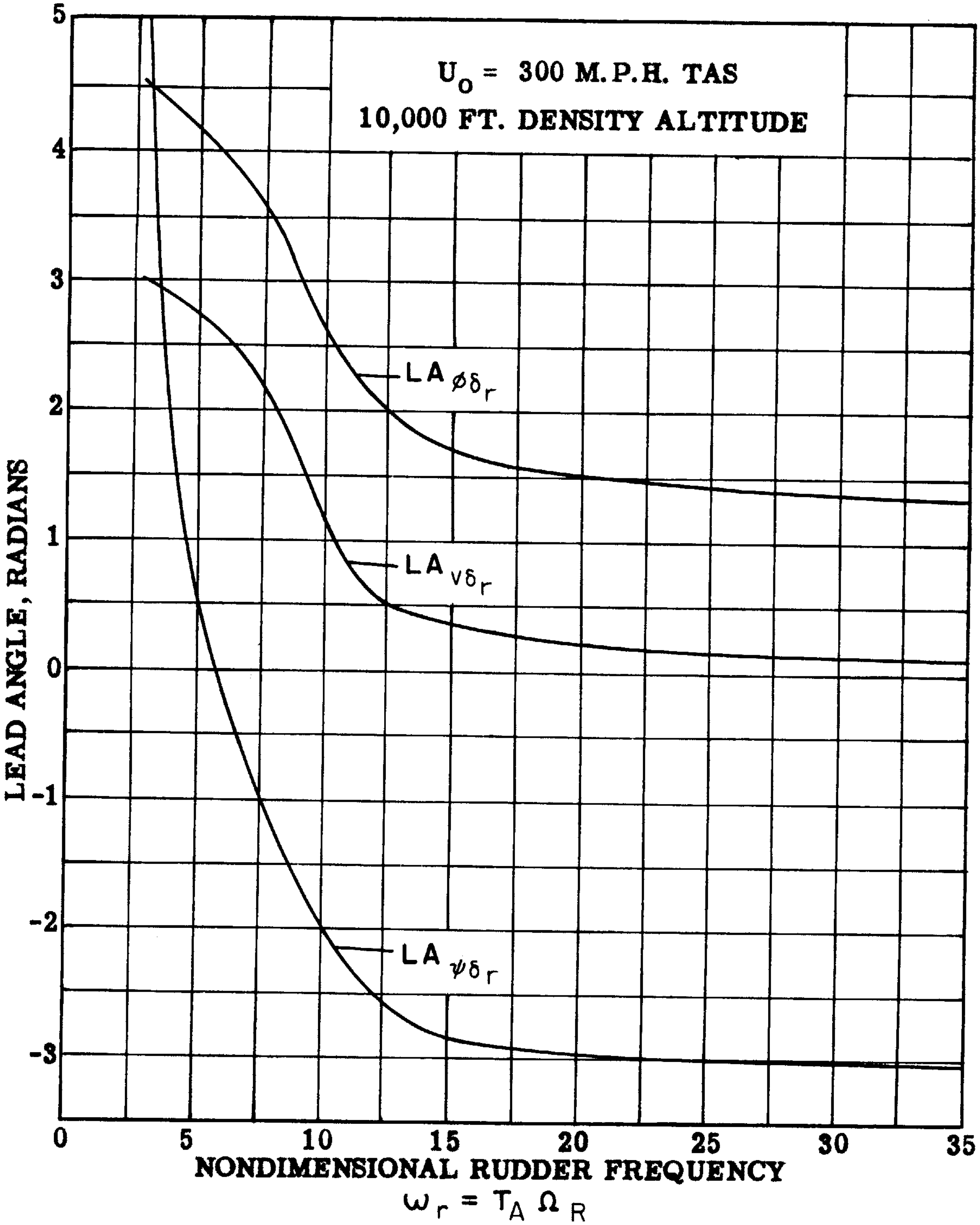
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, ELEVATOR FREQUENCY EQUAL .7 CYCLE PER SECOND

FIGURE L-9



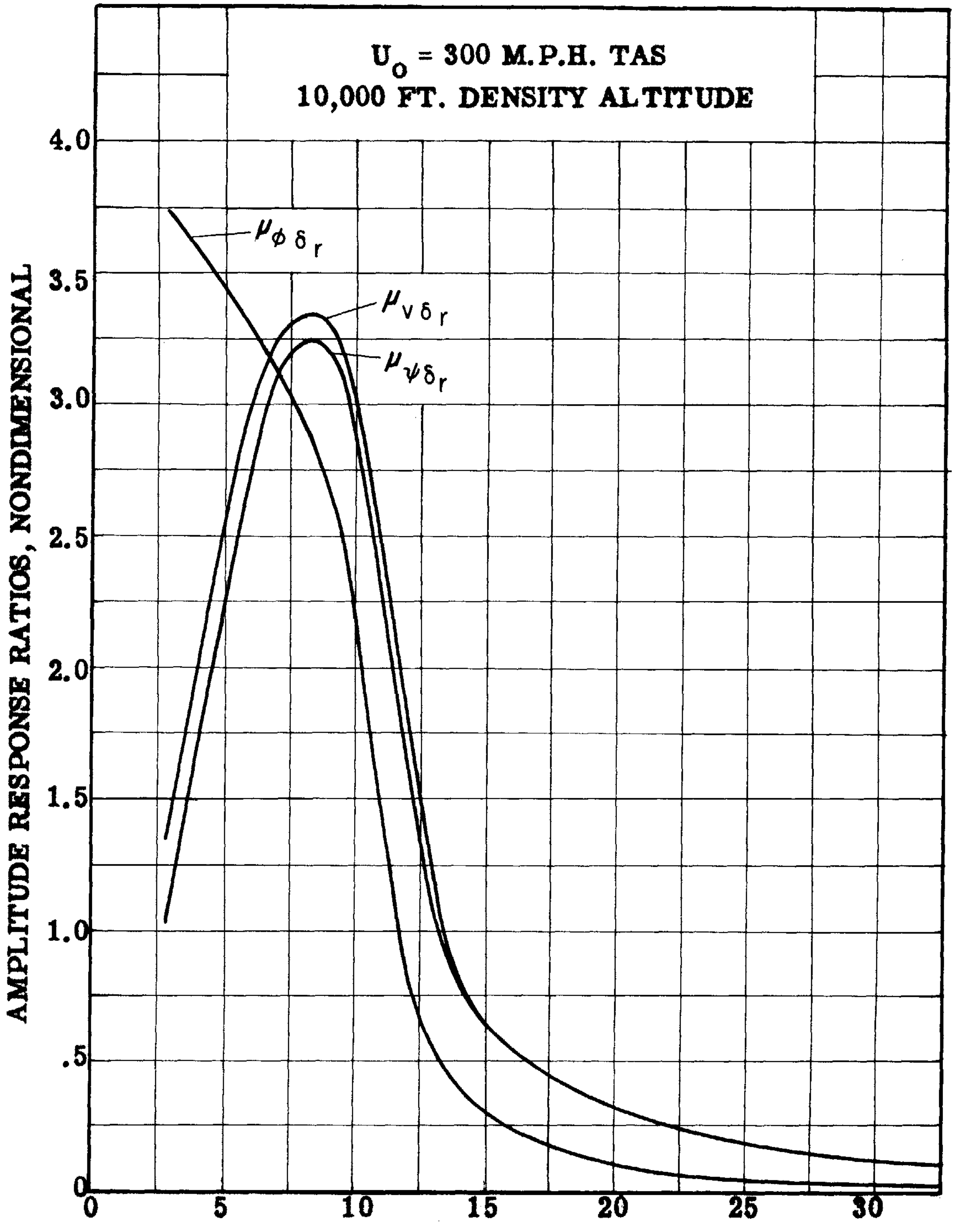
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, ELEVATOR FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-10



EFFECT OF CHANGE IN RUDDER FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN ANGLE OF BANK AND CHANGE IN RUDDER ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN AERODYNAMIC YAW AND CHANGE IN RUDDER ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN GEOMETRIC YAW AND CHANGE IN RUDDER ANGLE, COMPUTED FOR A-26 AIRPLANE, WITH FIXED AILERONS

FIGURE L-11

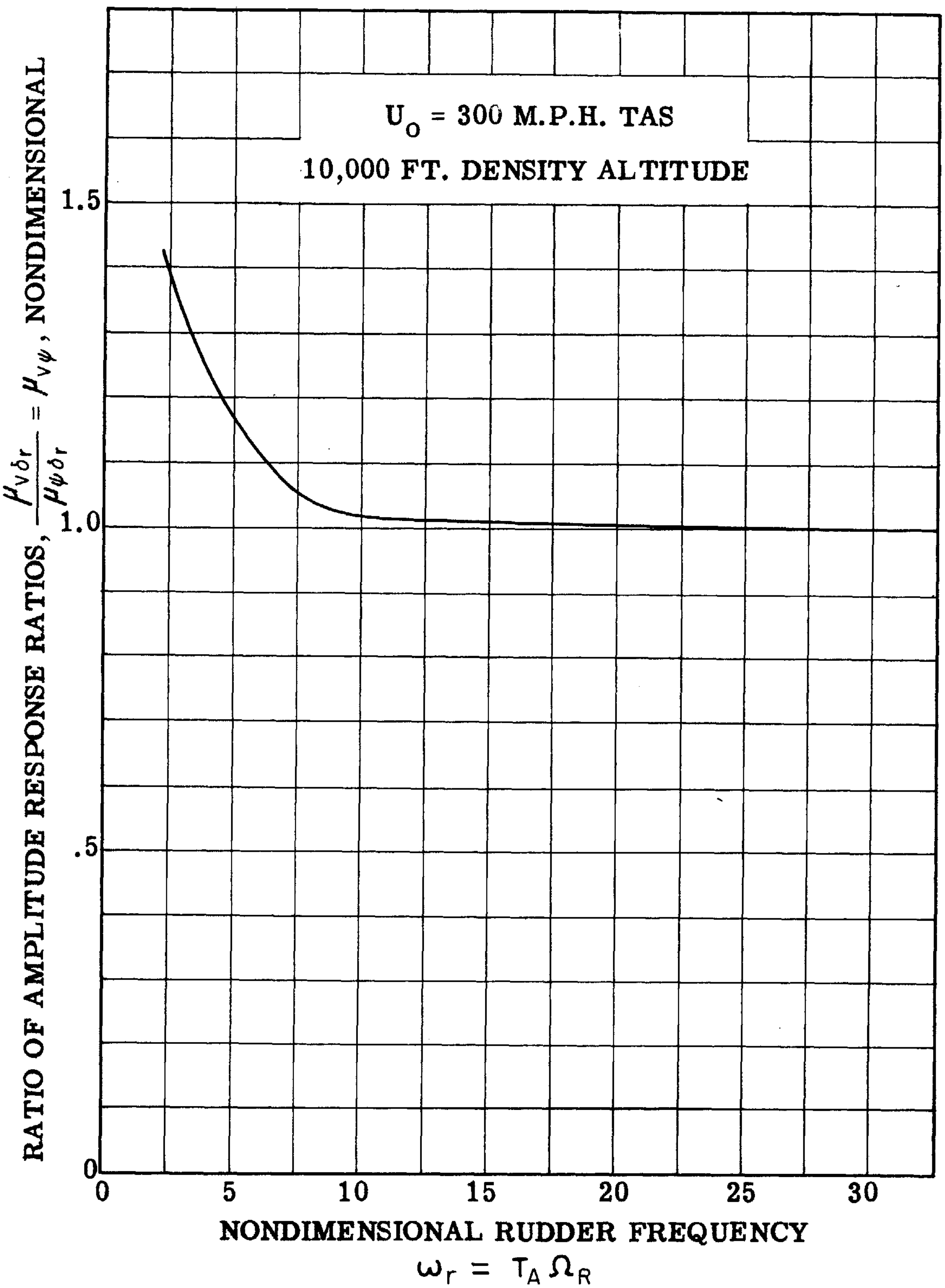


NONDIMENSIONAL RUDDER FREQUENCY

$\omega_r = T_A \ \Omega_R$

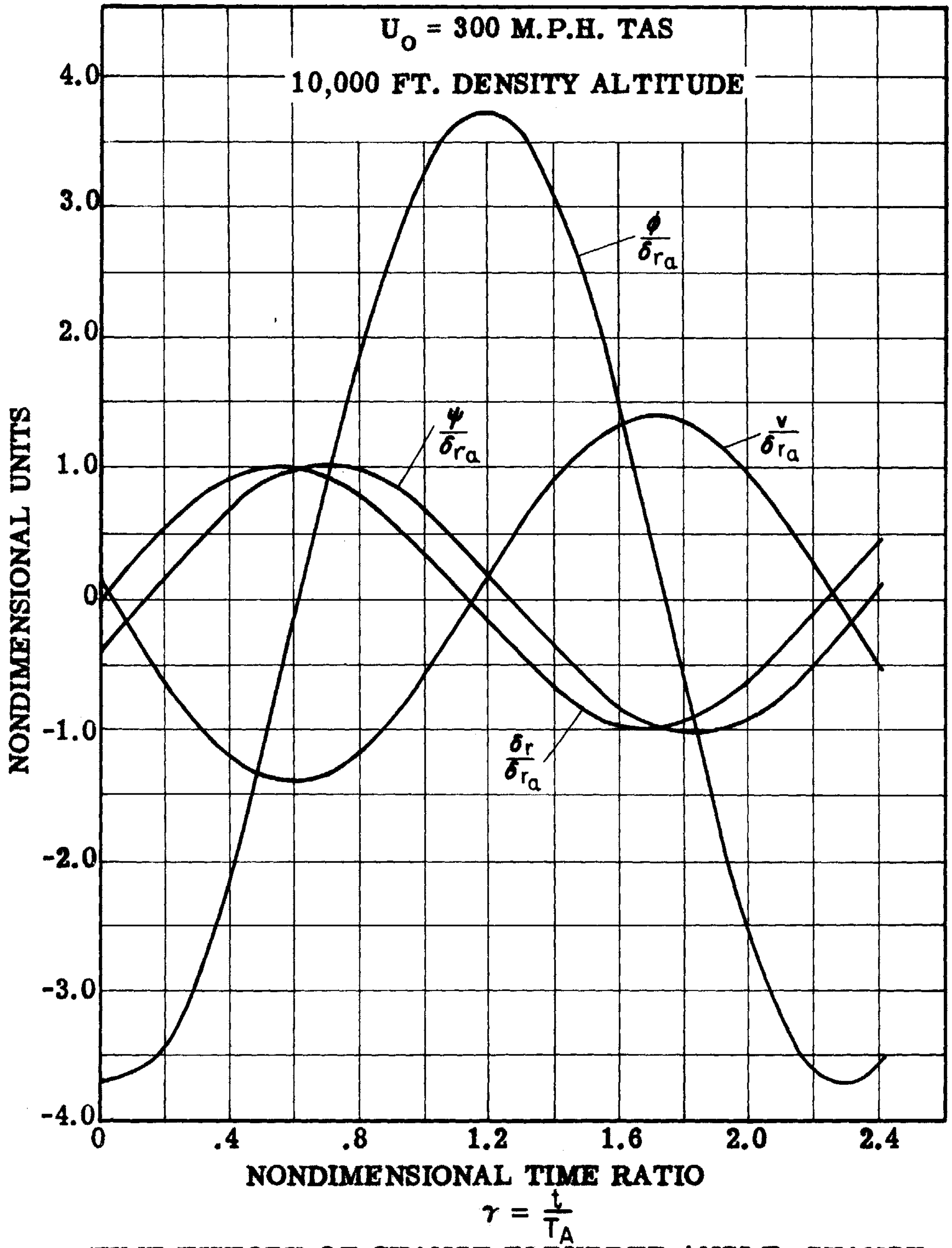
EFFECT OF CHANGE IN RUDDER FREQUENCY ON AMPLITUDE RESPONSE RATIOS COMPUTED FOR A-26 AIRPLANE, WITH FIXED AILERONS

FIGURE L-12



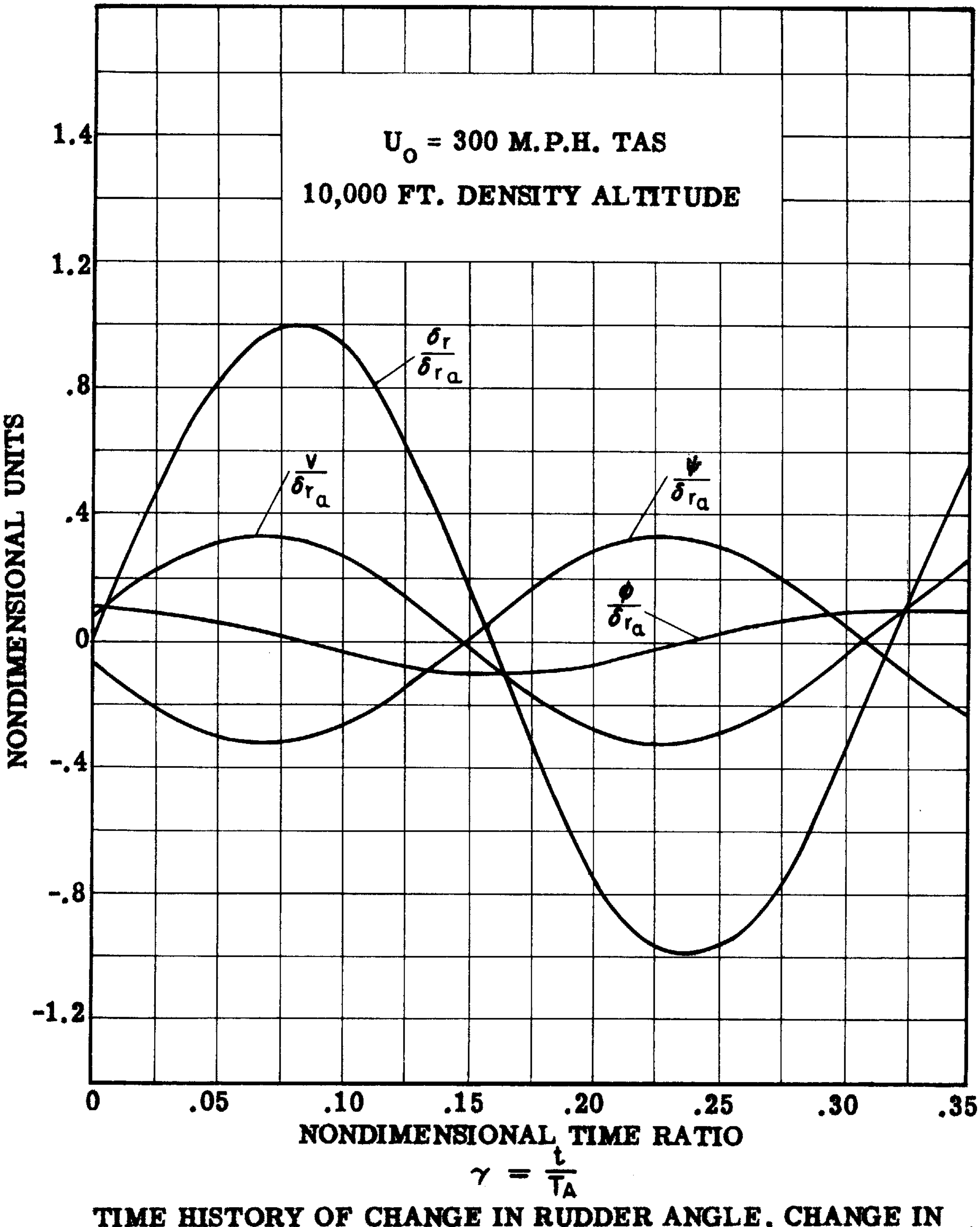
EFFECT OF CHANGE IN RUDDER FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR ANGLE OF AERODYNAMIC YAW TO AMPLITUDE RESPONSE RATIO FOR ANGLE OF GEOMETRIC YAW, COMPUTED FOR A-26 AIRPLANE WITH FIXED AILERONS

FIGURE L-13



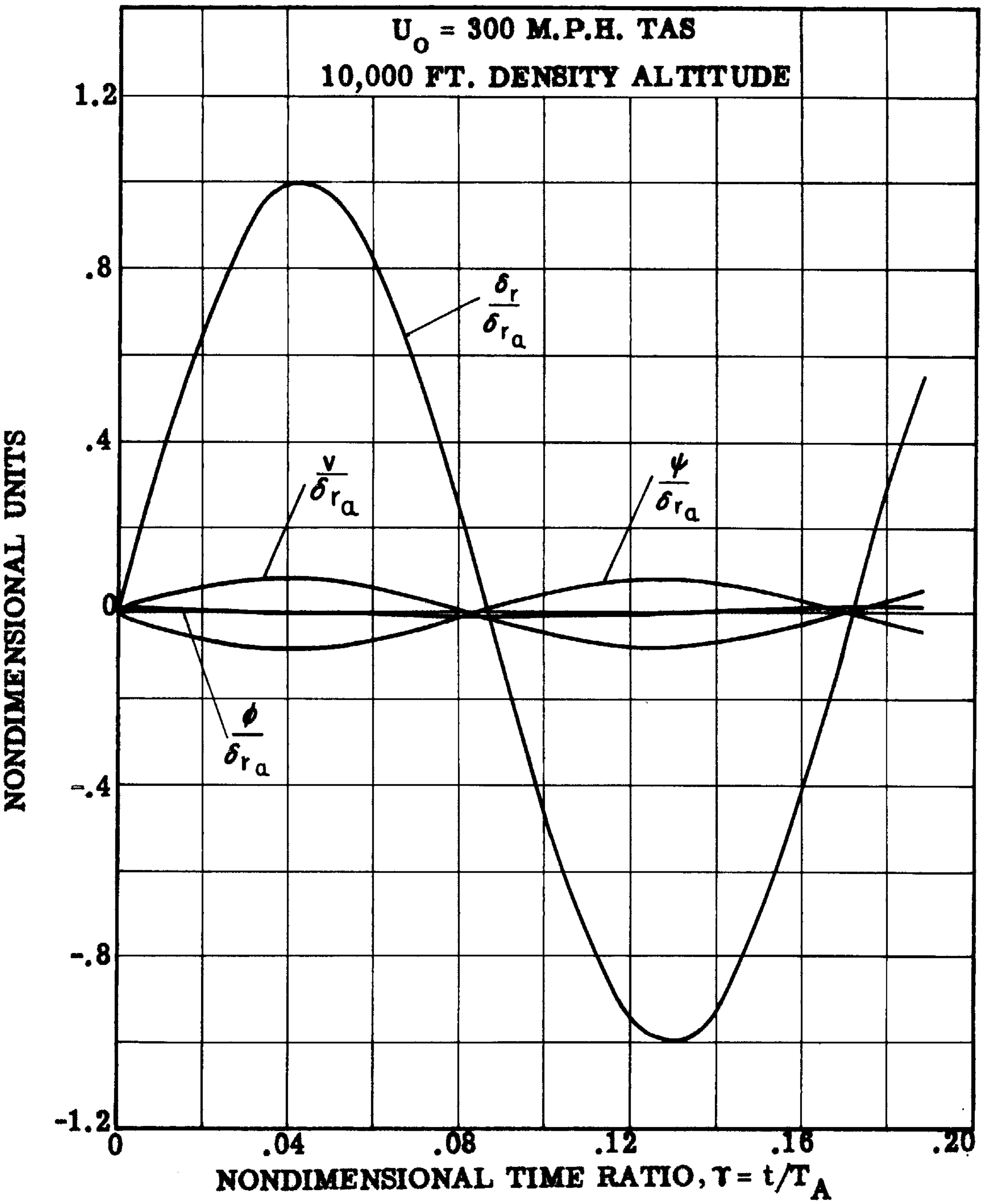
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN ANGLE OF BANK, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF AERODYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS, RUDDER FREQUENCY = .1 CYCLE PER SECOND

FIGURE L-14



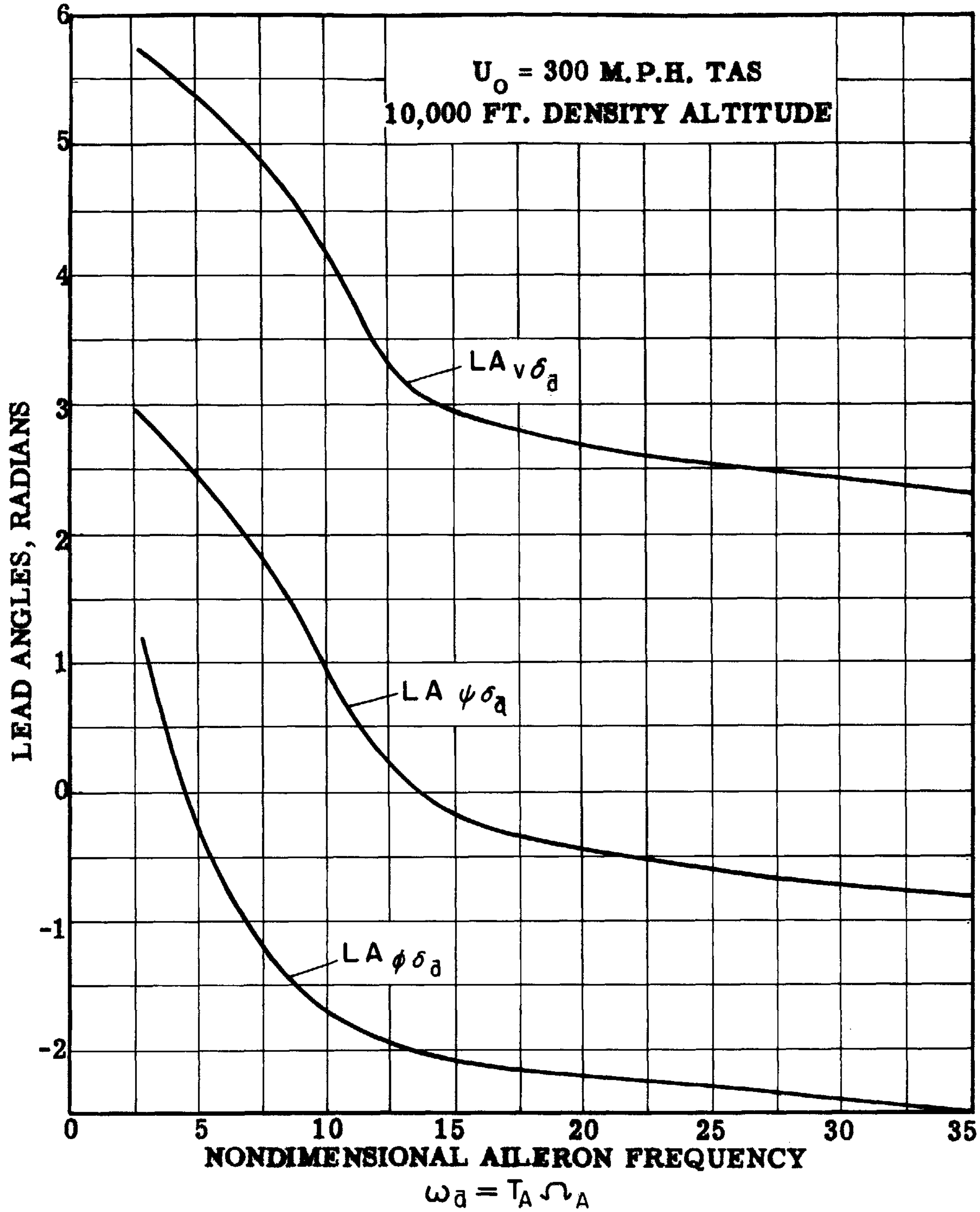
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN ANGLE OF BANK, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF AERODYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS, RUDDER FREQUENCY = .7 CYCLE PER SECOND

FIGURE L-15



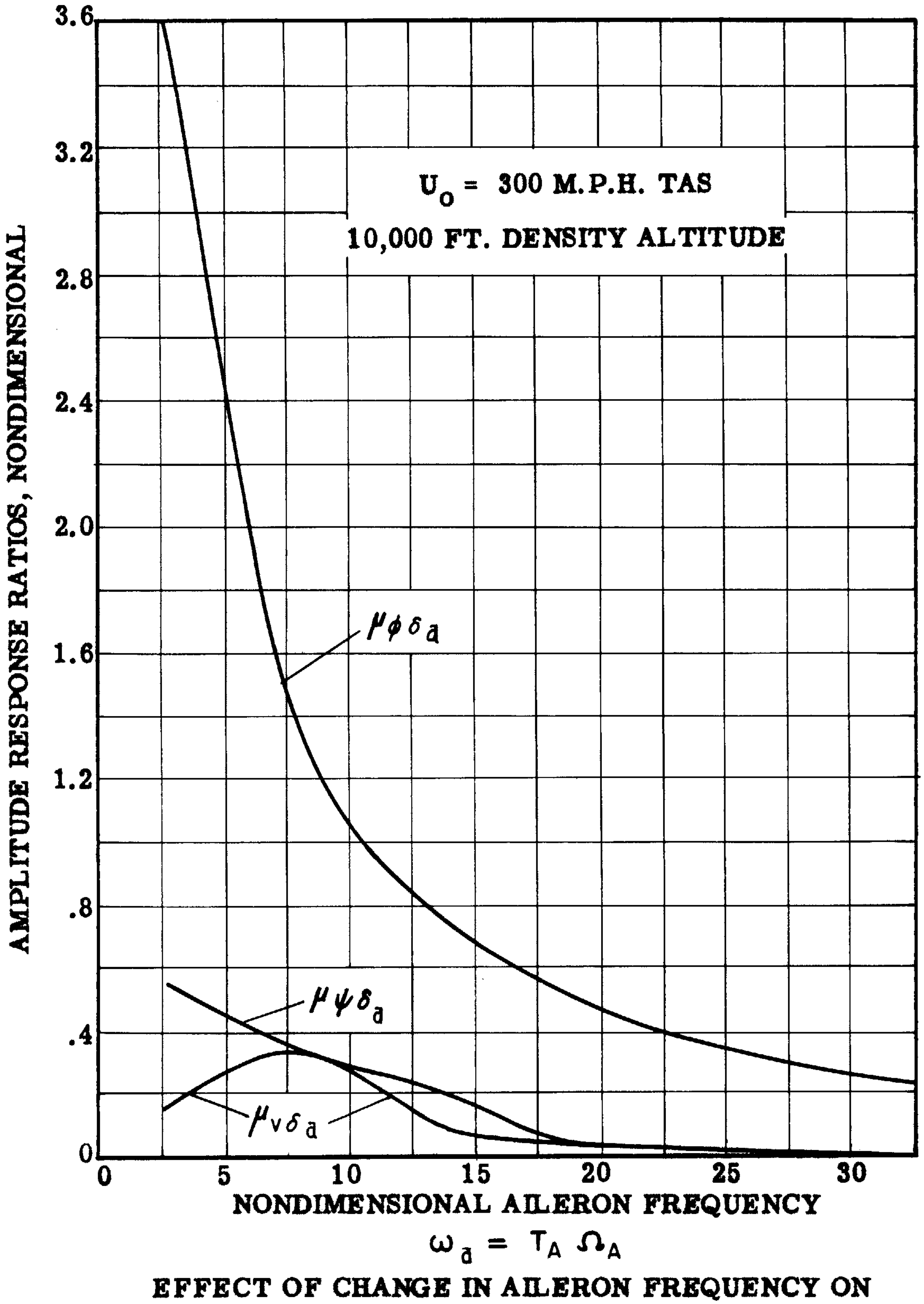
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN ANGLE OF BANK, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF AERODYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS, RUDDER FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-16



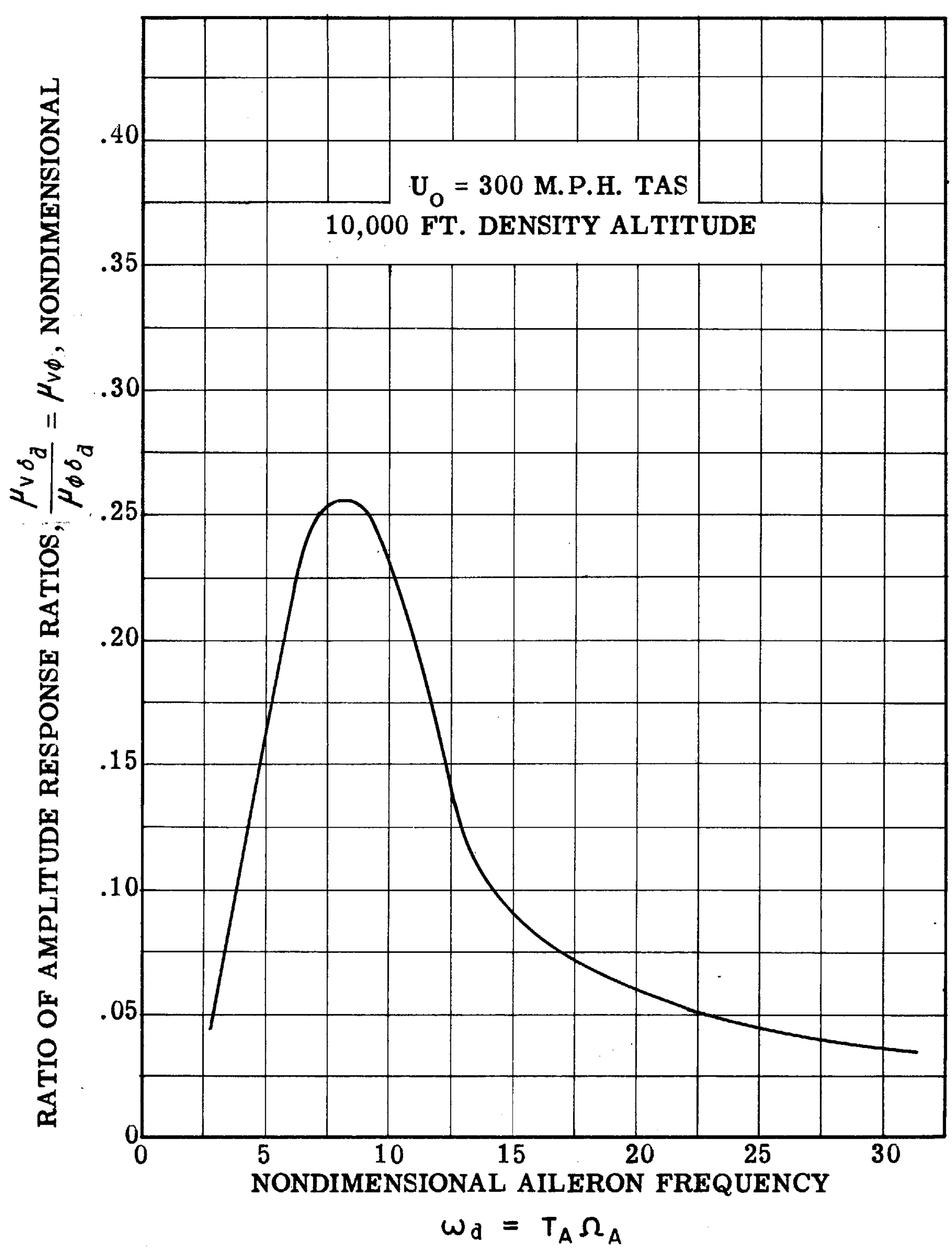
EFFECT OF CHANGE IN AILERON FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN AERODYNAMIC YAW AND CHANGE IN AILERON ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN GEOMETRIC YAW AND CHANGE IN AILERON ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN ANGLE OF BANK AND CHANGE IN AILERON ANGLE, COMPUTED FOR A-26 AIRPLANE, WITH FIXED RUDDER

FIGURE L-17



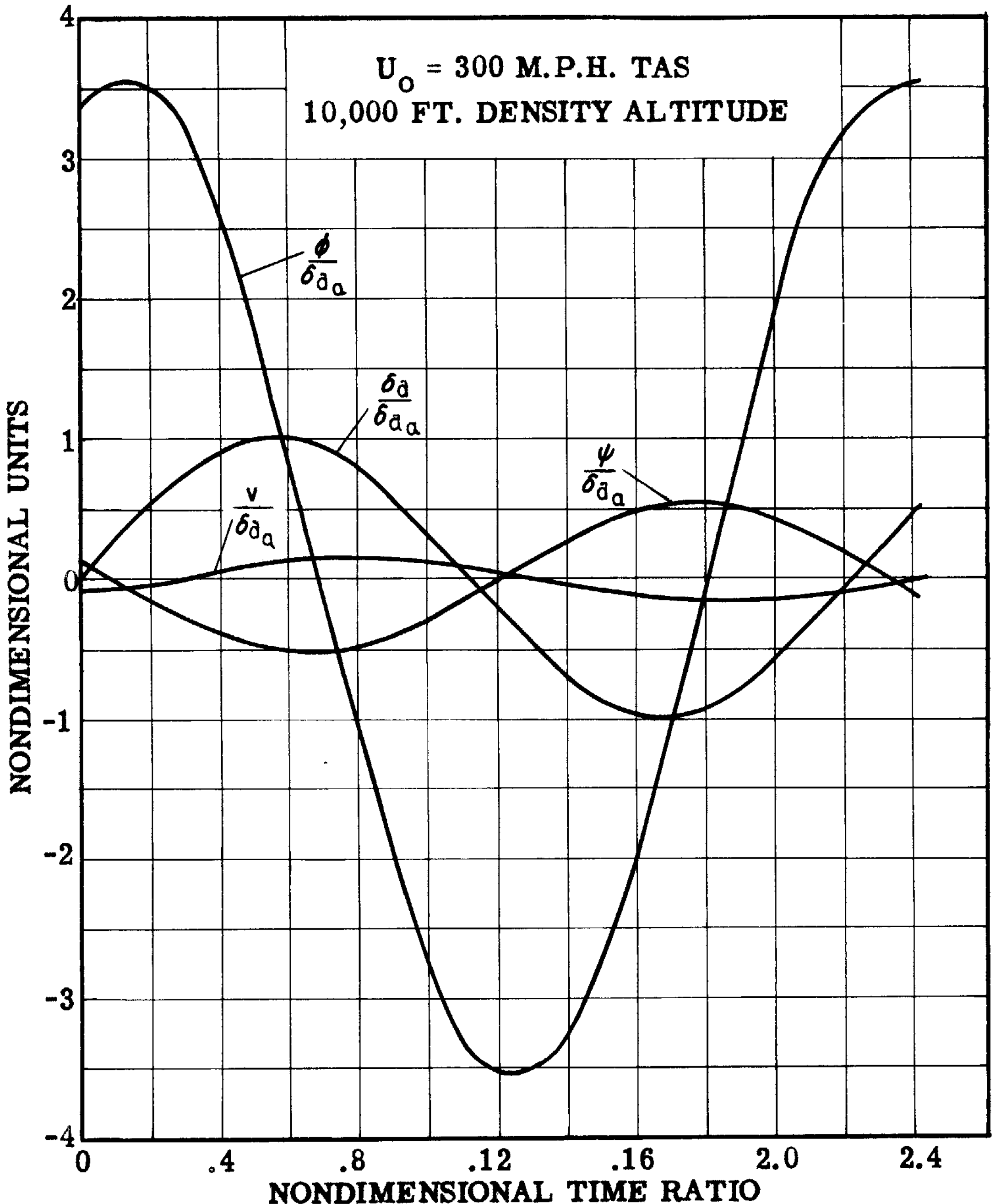
EFFECT OF CHANGE IN AILERON FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, WITH FIXED RUDDER

FIGURE L-18



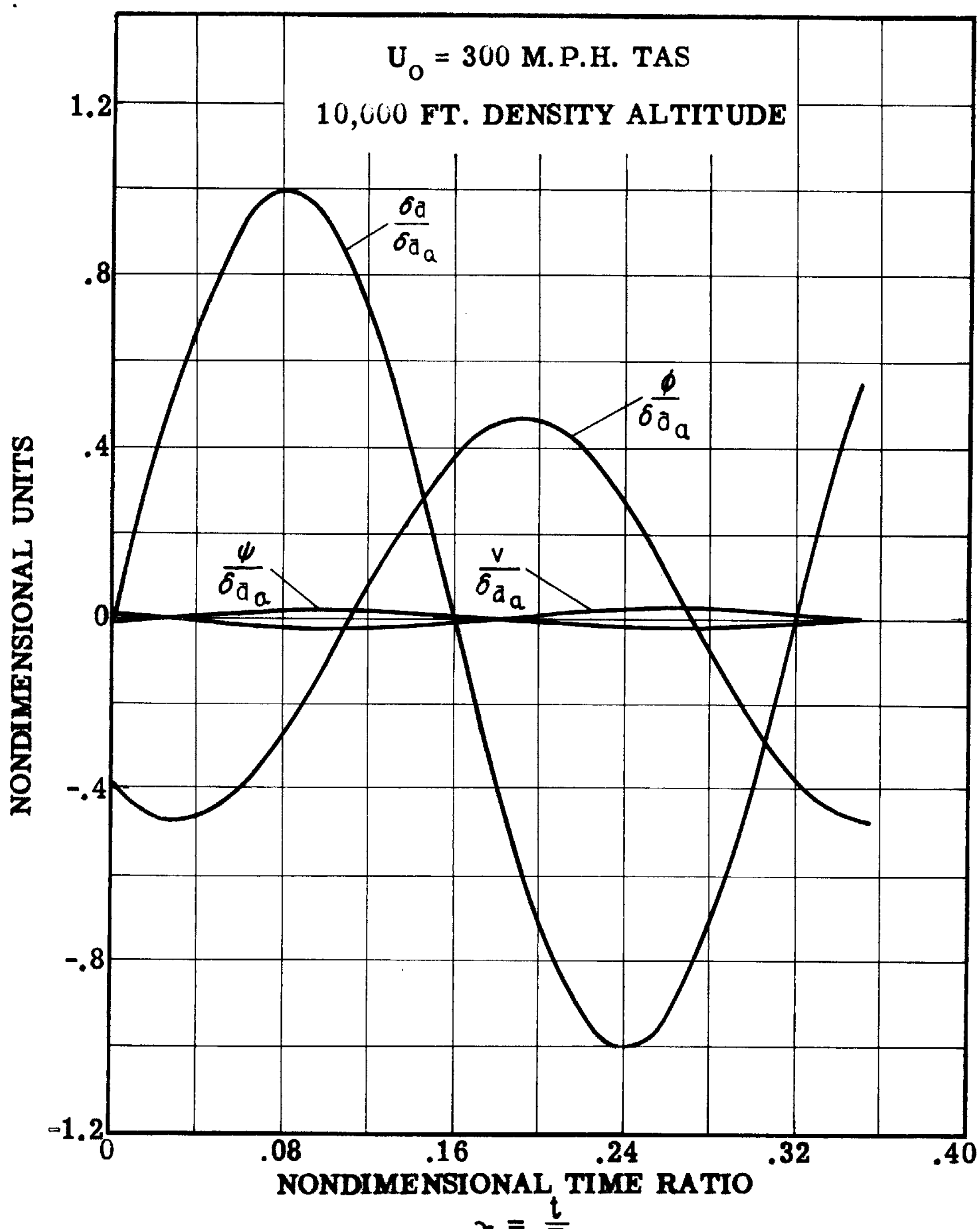
EFFECT OF CHANGE IN AILERON FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR ANGLE OF AERODYNAMIC YAW TO AMPLITUDE RESPONSE RATIO FOR ANGLE OF BANK, COMPUTED FOR A-26 AIRPLANE, WITH FIXED RUDDER

FIGURE L-19



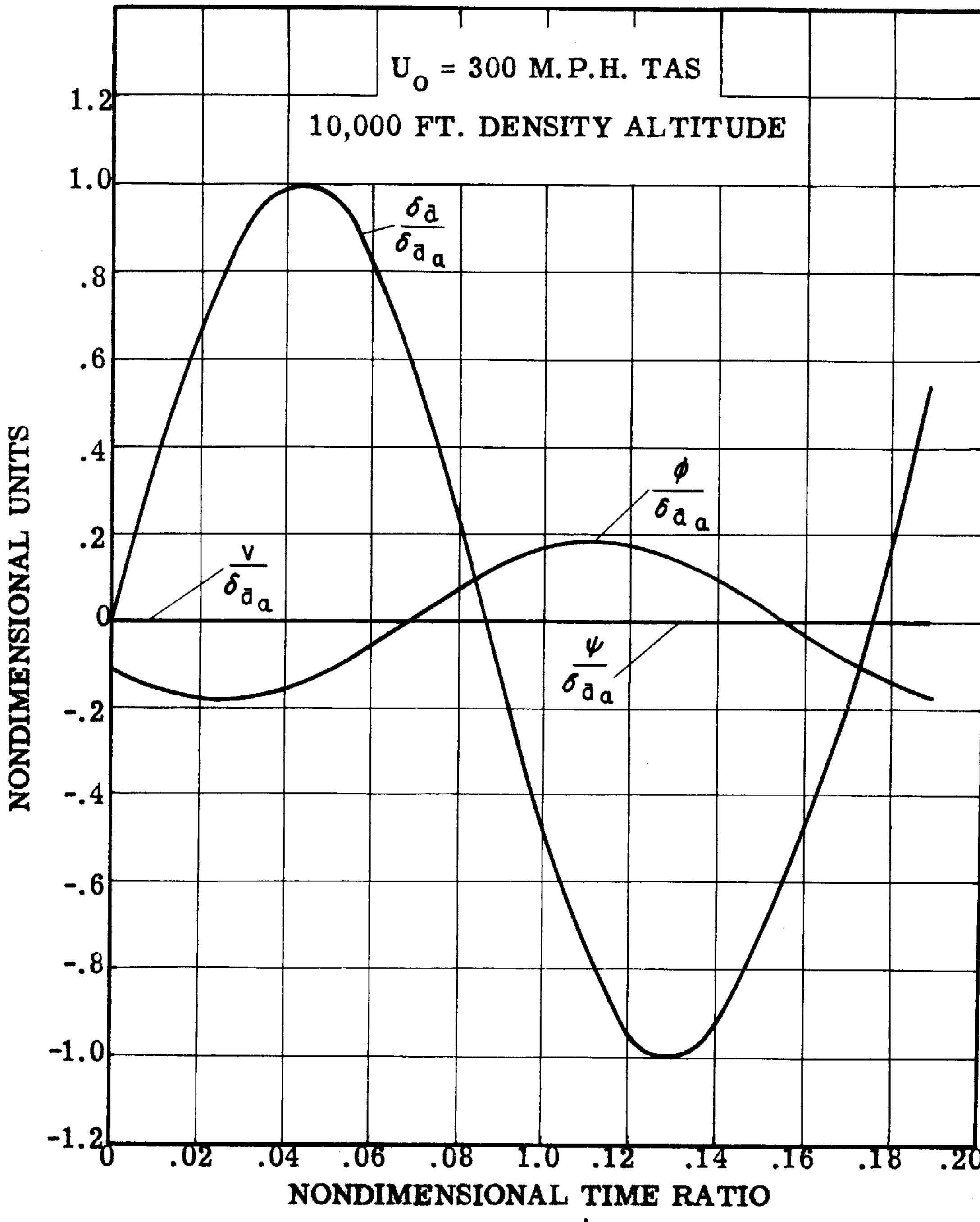
TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF BANK, PER AMPLITUDE OF CHANGE IN AILERON ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER, AILERON FREQUENCY = .1 CYCLE PER SECOND

FIGURE L-20



TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF BANK, PER AMPLITUDE OF CHANGE IN AILERON ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER, AILERON FREQUENCY = .7 CYCLE PER SECOND

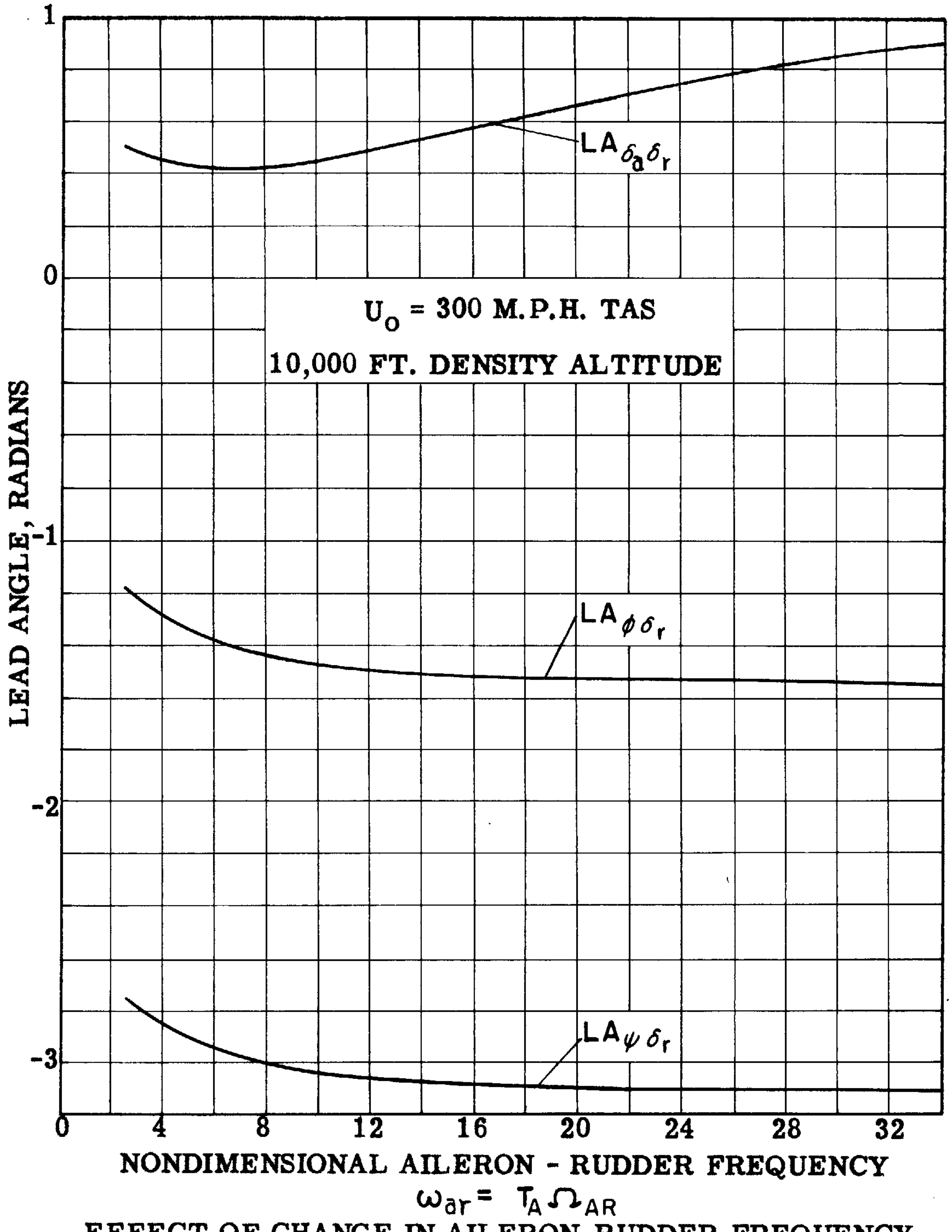
FIGURE L-21



$$\gamma = \frac{t}{T_A}$$

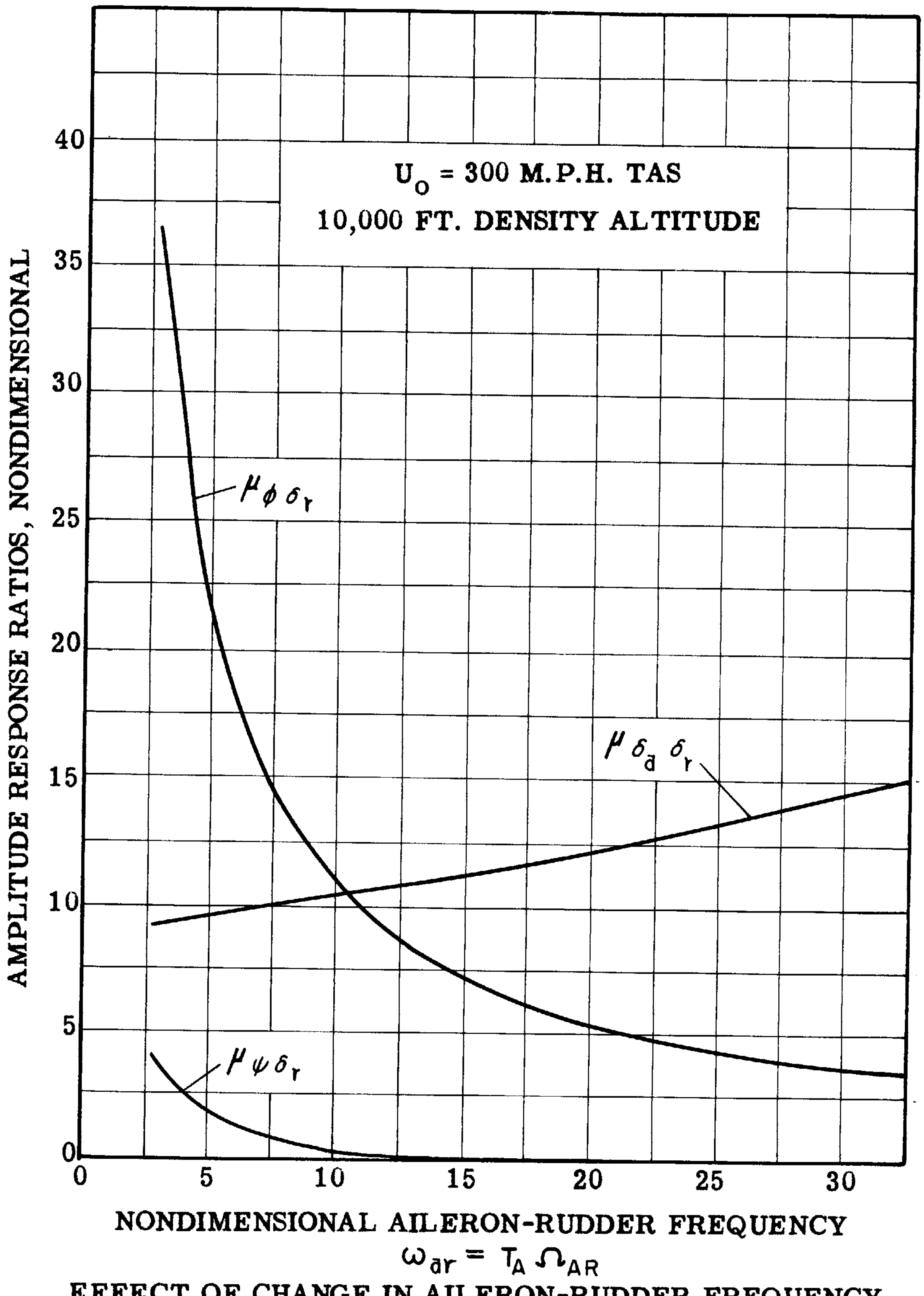
TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN ANGLE OF BANK, PER AMPLITUDE OF CHANGE IN AILERON ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER, AILERON FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-22



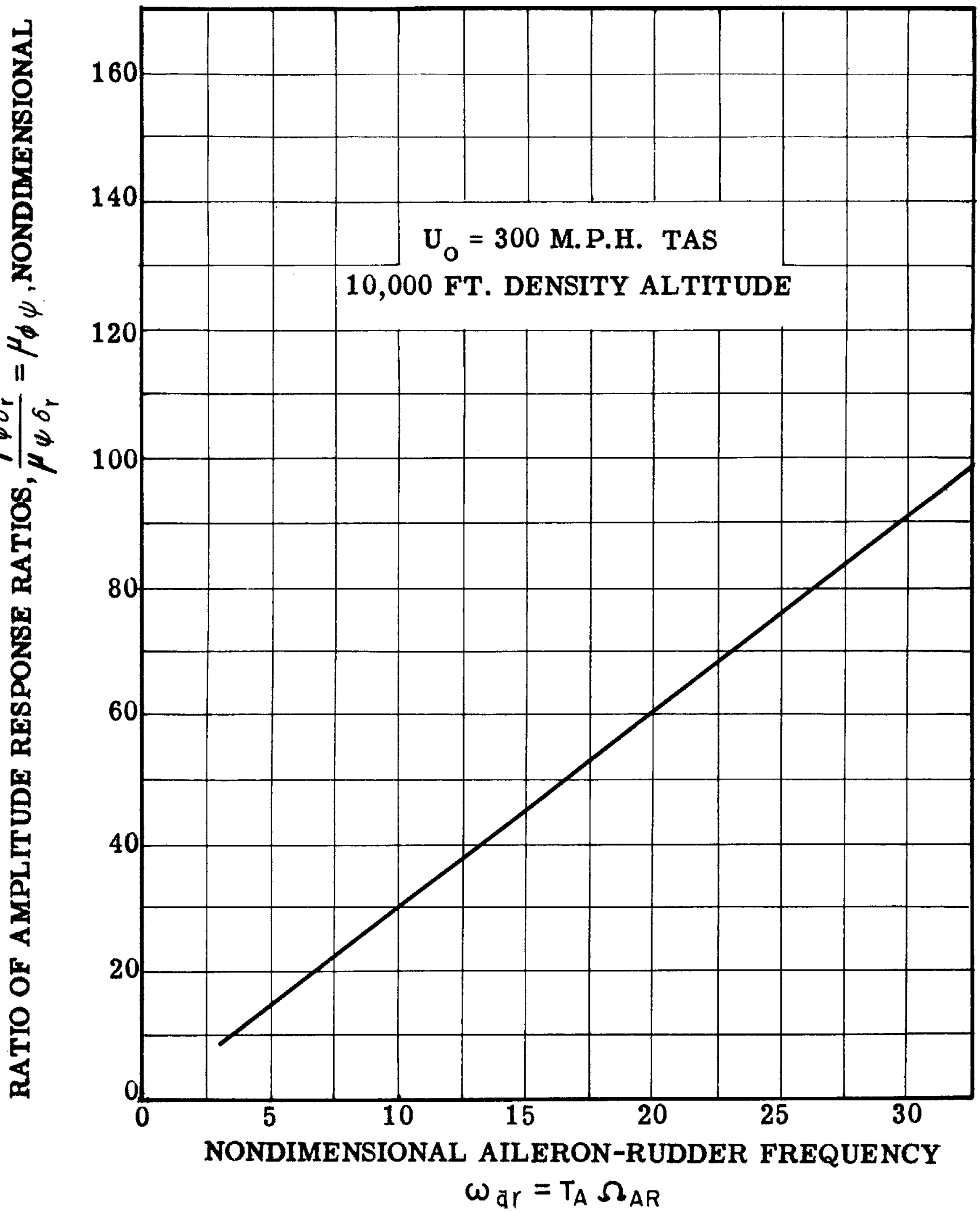
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN AILERON ANGLE AND CHANGE IN RUDDER ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN ANGLE OF BANK AND CHANGE IN RUDDER ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN GEOMETRIC YAW AND CHANGE IN RUDDER ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO AERODYNAMIC YAW

FIGURE L-23



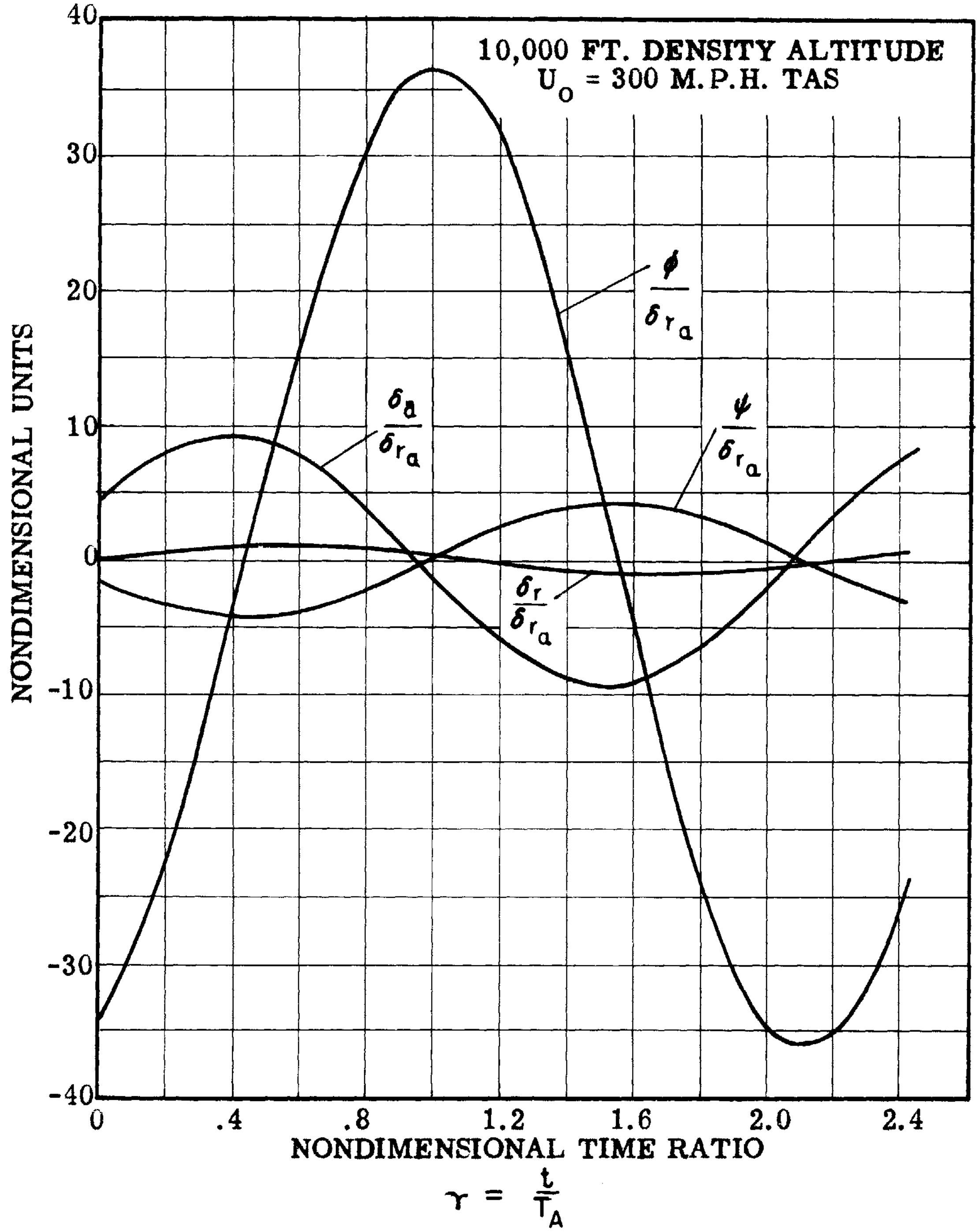
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO AERODYNAMIC YAW

FIGURE L-24



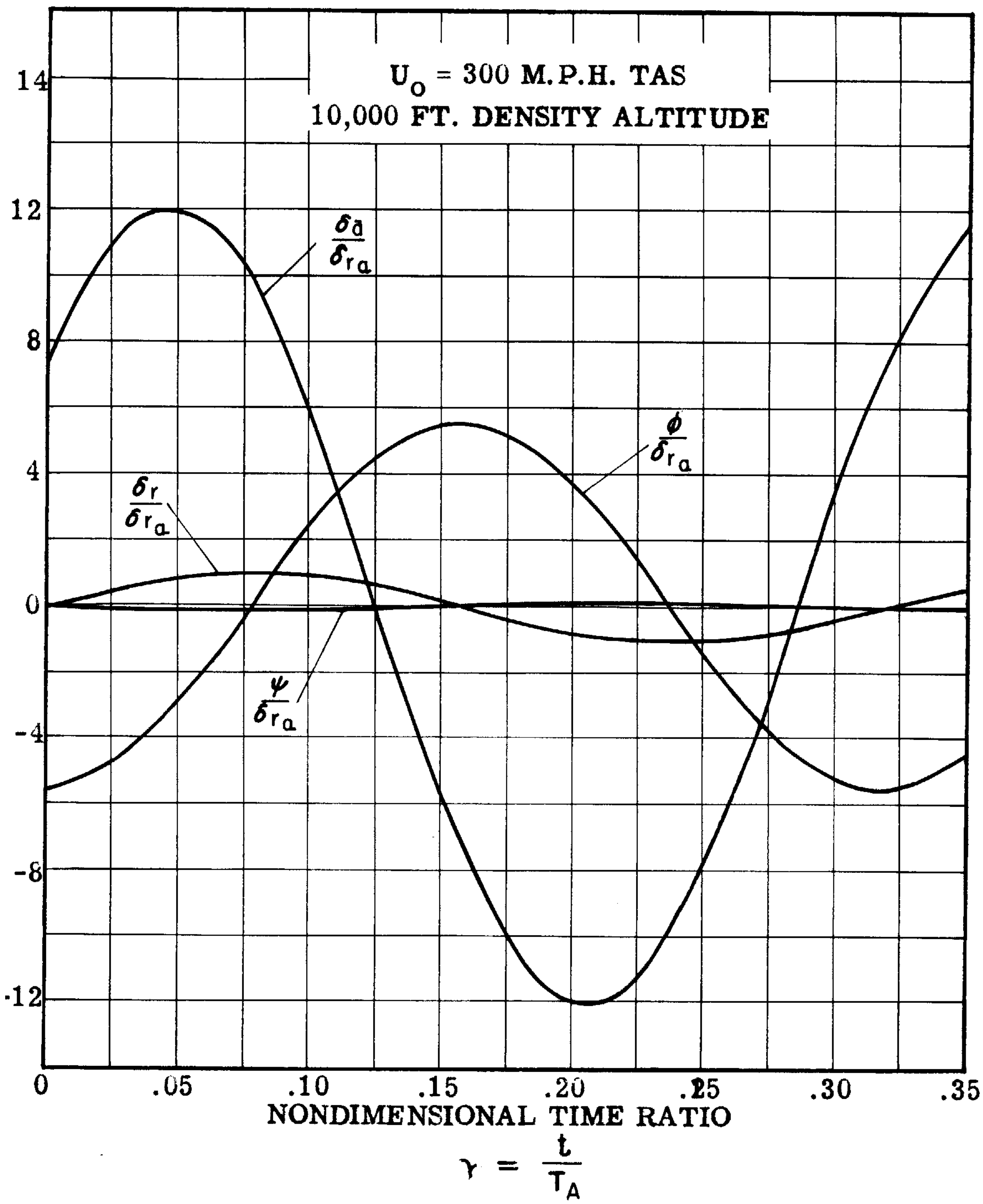
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR ANGLE OF BANK TO AMPLITUDE RESPONSE RATIO FOR ANGLE OF GEOMETRIC YAW, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO AERODYNAMIC YAW

FIGURE L-25



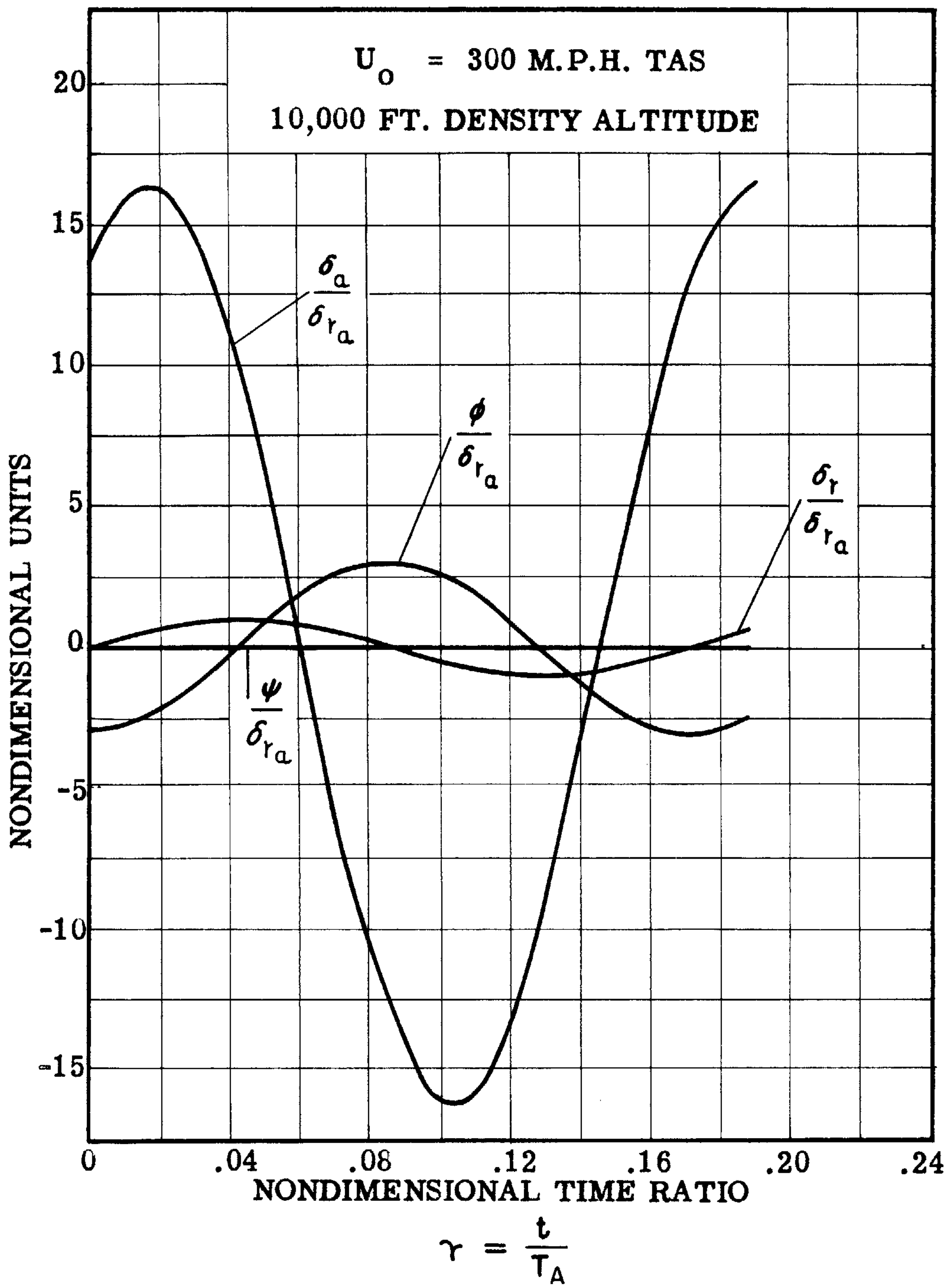
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN AILERON ANGLE, CHANGE IN BANK ANGLE, AND CHANGE IN ANGLE OF GEOMETRIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO AERODYNAMIC YAW, FREQUENCY OF AILERONS AND RUDDER = .1 CYCLE PER SECOND

FIGURE L-26



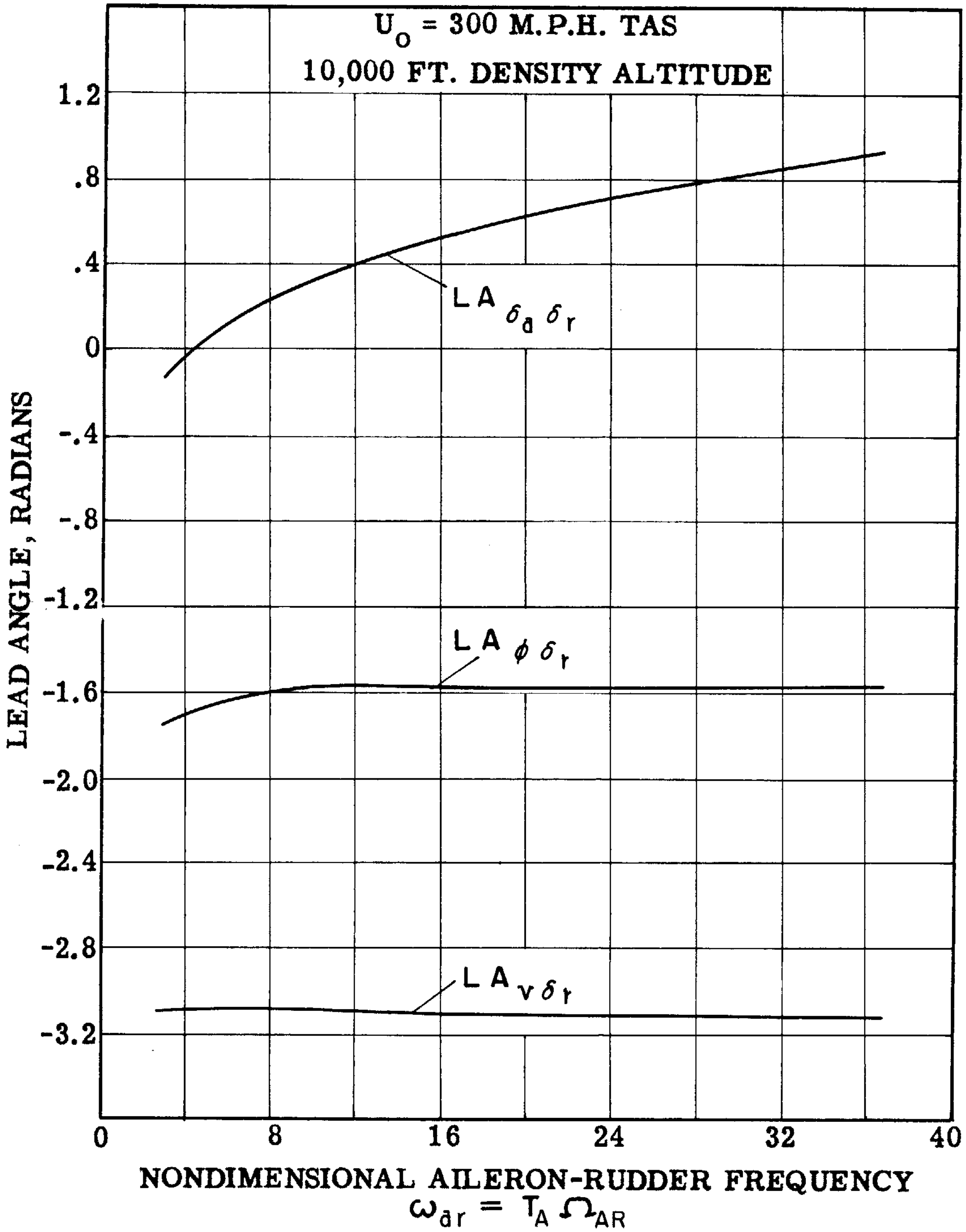
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN AILERON ANGLE, CHANGE IN BANK ANGLE, AND CHANGE IN ANGLE OF GEOMETRIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO AERODYNAMIC YAW, FREQUENCY OF AILERONS AND RUDDER = .7 CYCLE PER SECOND

FIGURE L-27



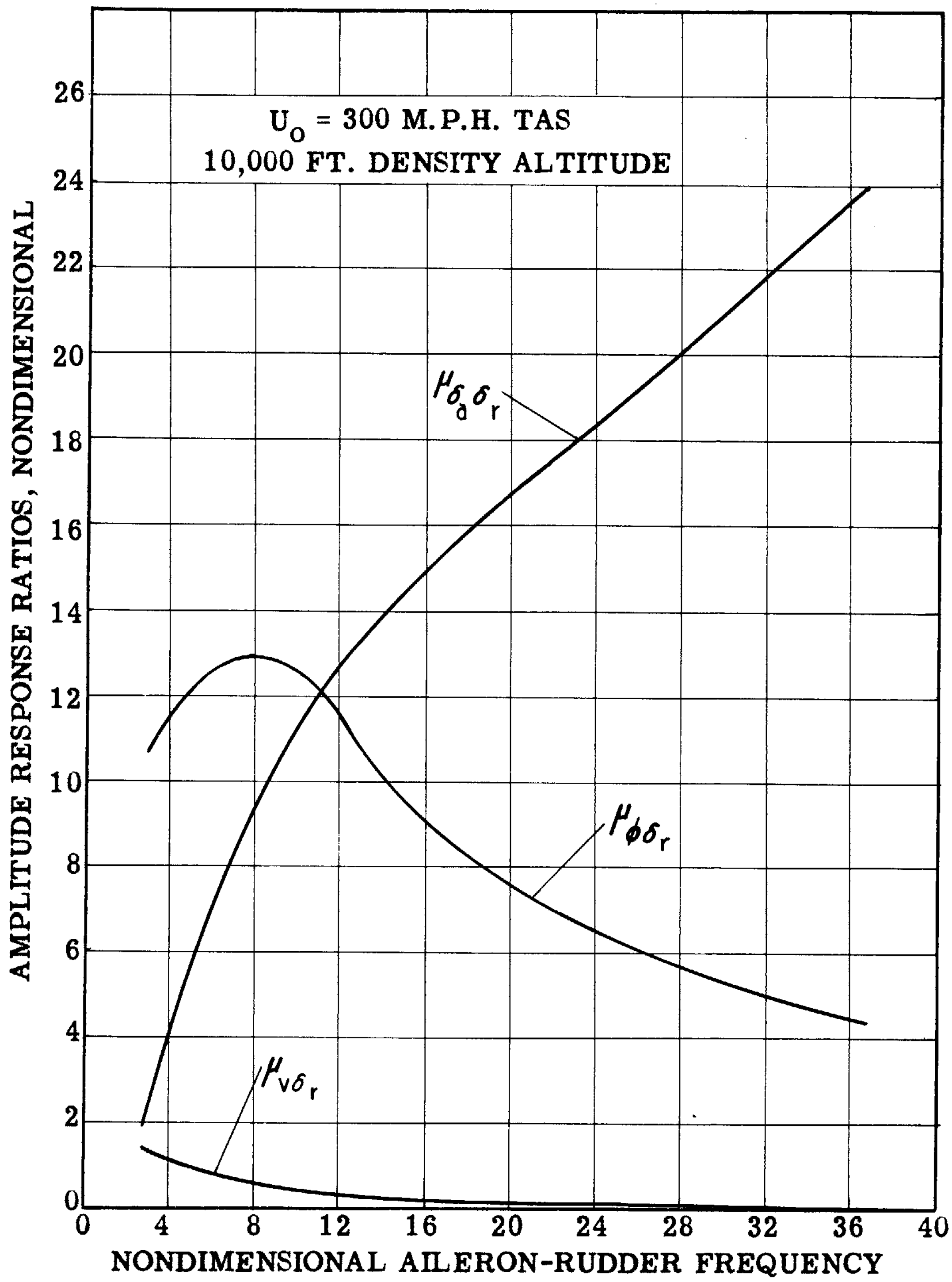
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN AILERON ANGLE, CHANGE IN BANK ANGLE, AND CHANGE IN ANGLE OF GEOMETRIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO AERODYNAMIC YAW, AILERON-RUDDER FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-28



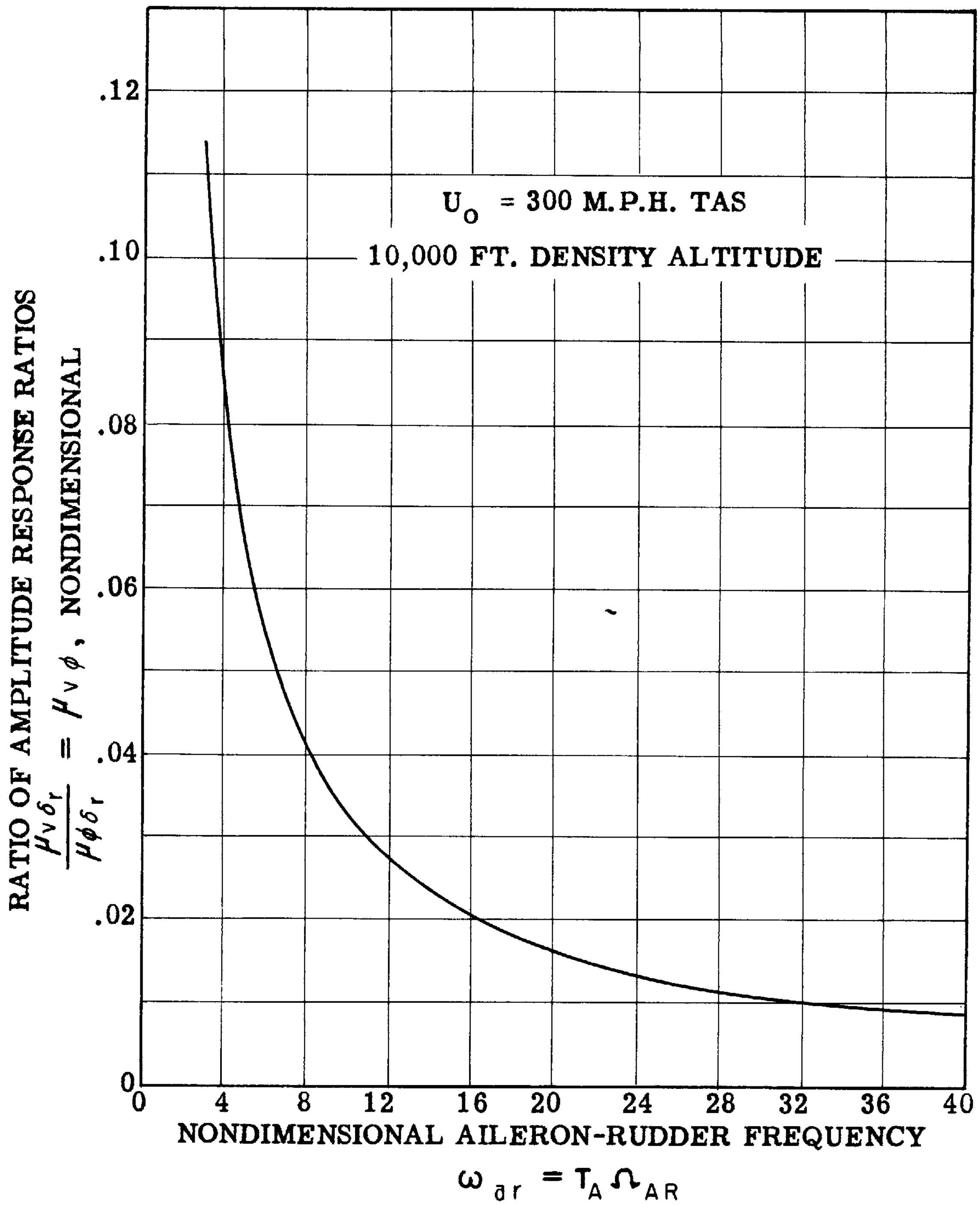
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN AILERON ANGLE AND CHANGE IN RUDDER ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN ANGLE OF BANK AND CHANGE IN RUDDER ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN LATERAL VELOCITY PER AIRPLANE TRIM SPEED AND CHANGE IN RUDDER ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO GEOMETRIC YAW

FIGURE L-29



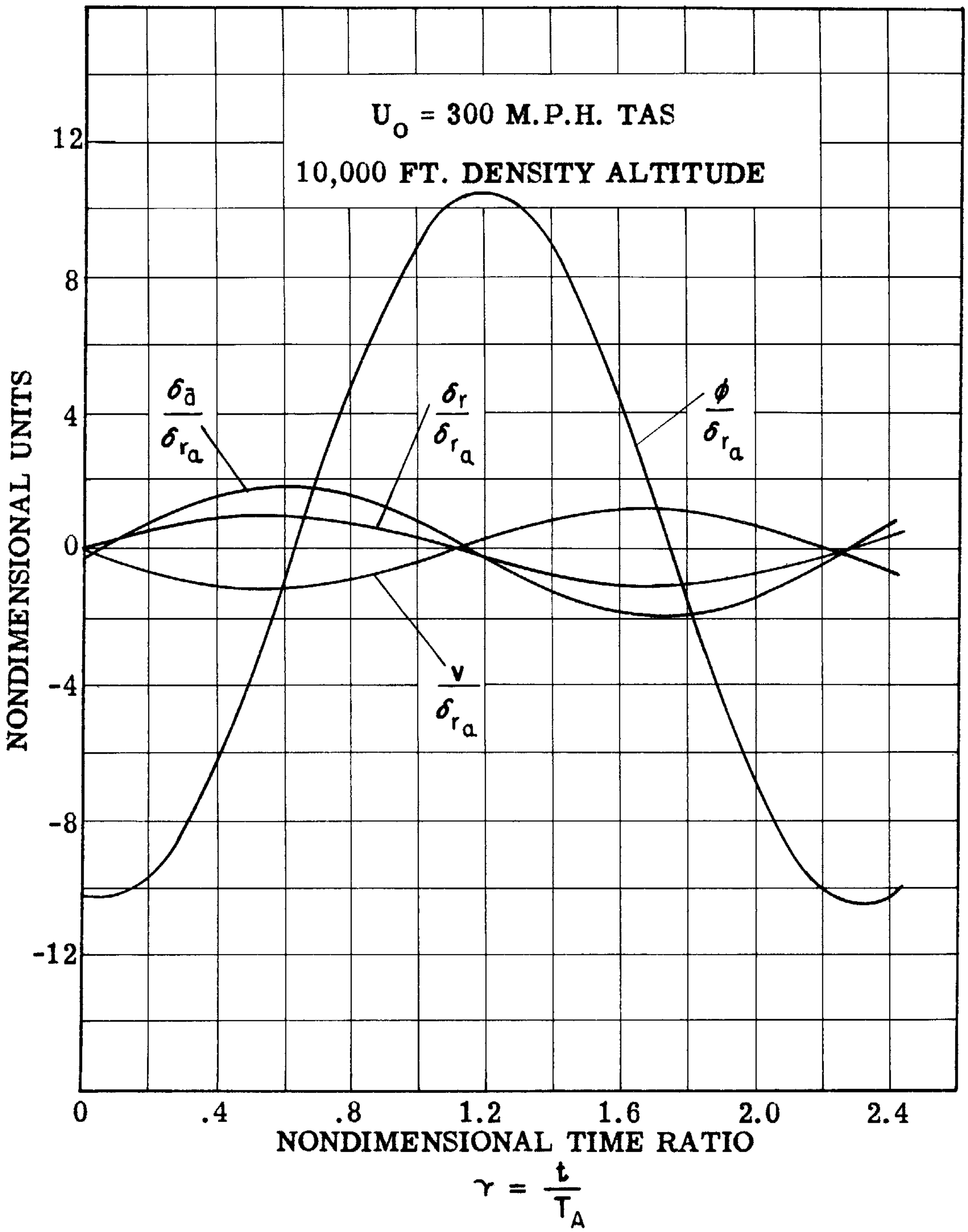
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO GEOMETRIC YAW

FIGURE L-30



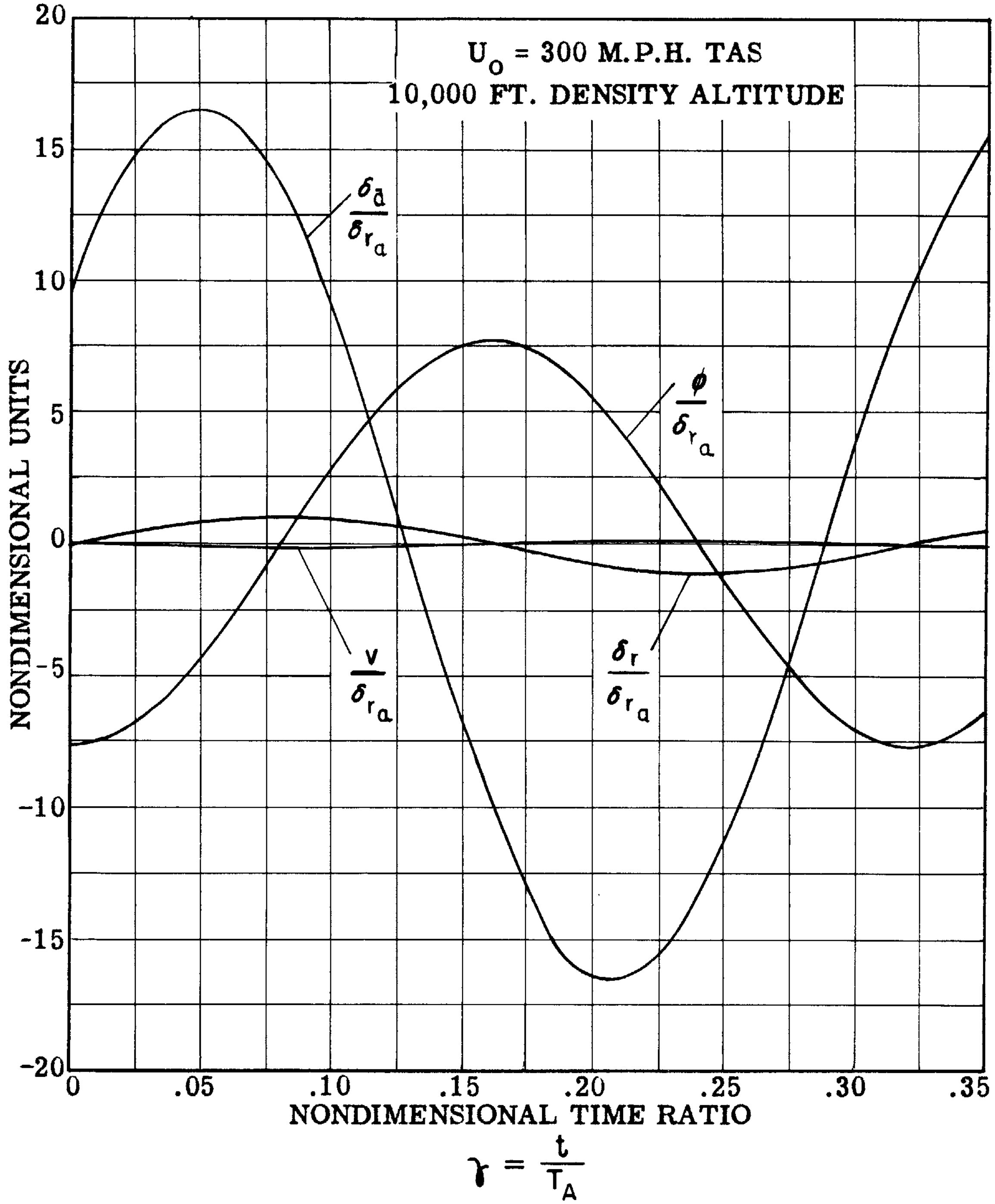
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR CHANGE IN LATERAL VELOCITY PER AIRPLANE TRIM SPEED TO AMPLITUDE RESPONSE RATIO FOR CHANGE IN ANGLE OF BANK, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO GEOMETRIC YAW

FIGURE L-31



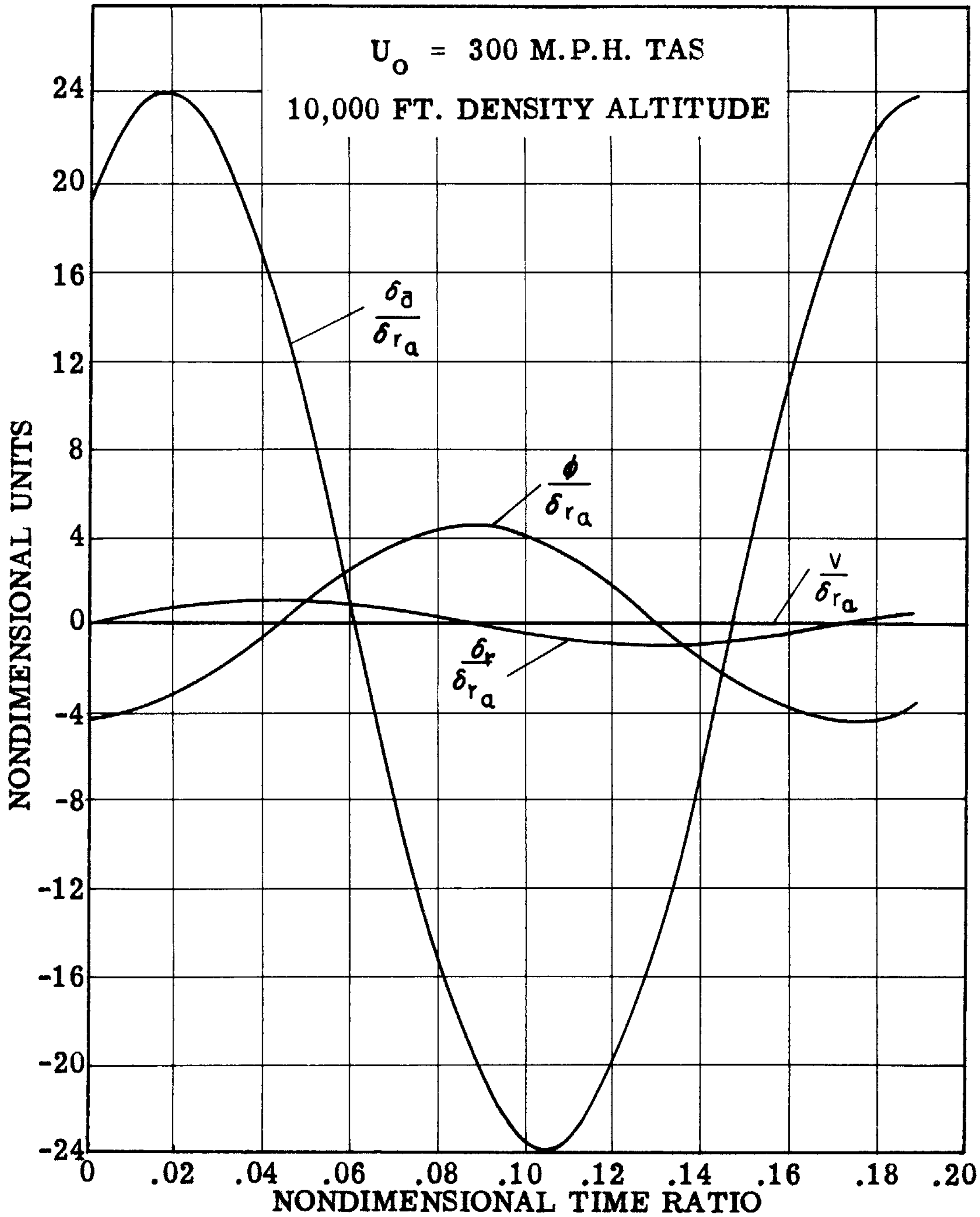
TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN AILERON ANGLE, CHANGE IN BANK ANGLE, AND CHANGE IN ANGLE OF AERODYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FRO ZERO GEOMETRIC YAW, AILERON-RUDDER FREQUENCY = .1 CYCLE PER SECOND

FIGURE L-32



TIME HISTORY OF CHANGE IN RUDDER ANGLE, CHANGE IN AILERON ANGLE, CHANGE IN BANK ANGLE, AND CHANGE IN ANGLE OF AERODYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO GEOMETRIC YAW, AILERON-RUDDER FREQUENCY = .7 CYCLE PER SECOND

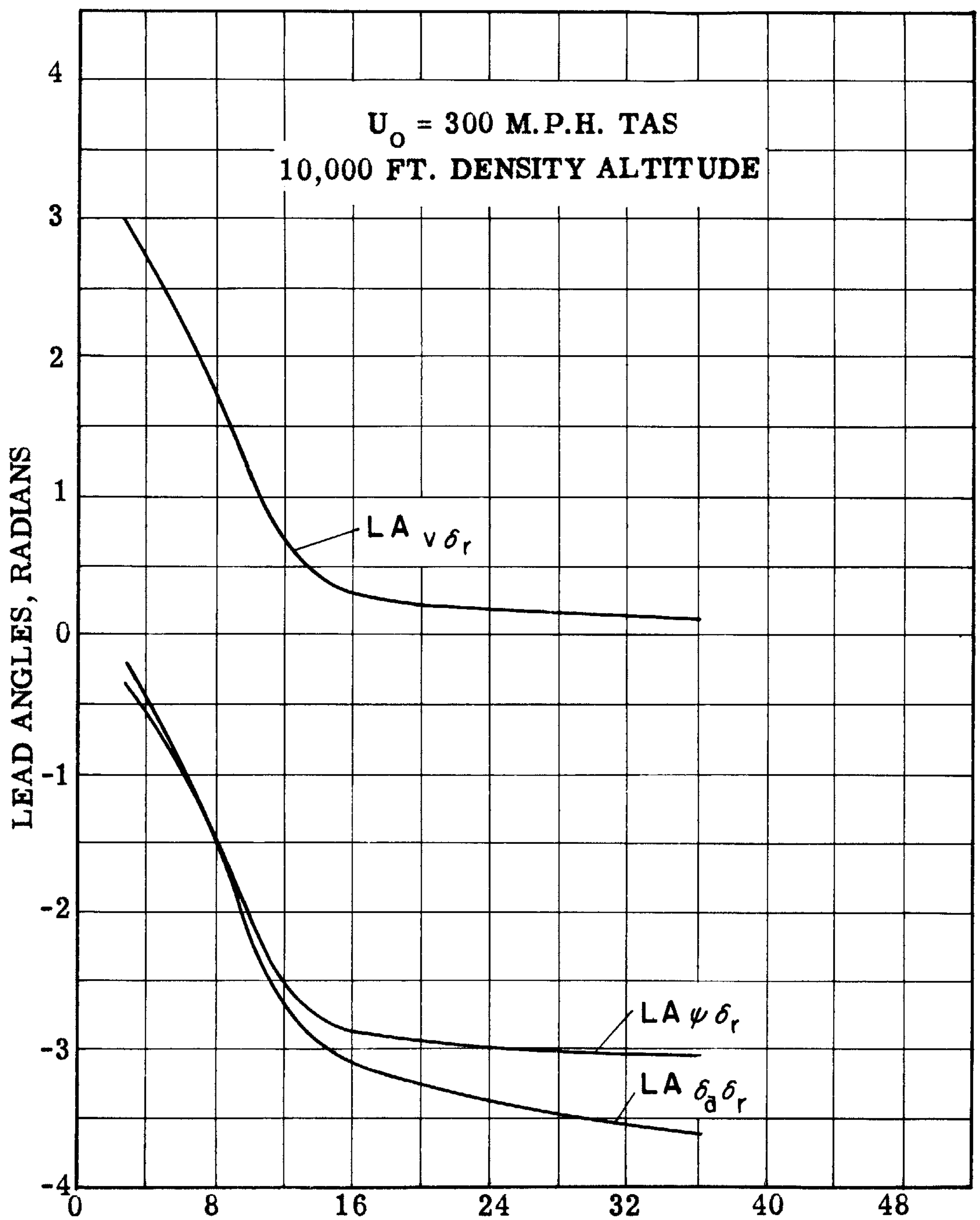
FIGURE L-33



$$\gamma = \frac{t}{T_A}$$

TIME HISTORY OF CHANGE IN RUDDER ANGLE CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF BANK, AND CHANGE IN ANGLE OF AERO DYNAMIC YAW, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO GEOMETRIC YAW, AILERON-RUDDER FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-34

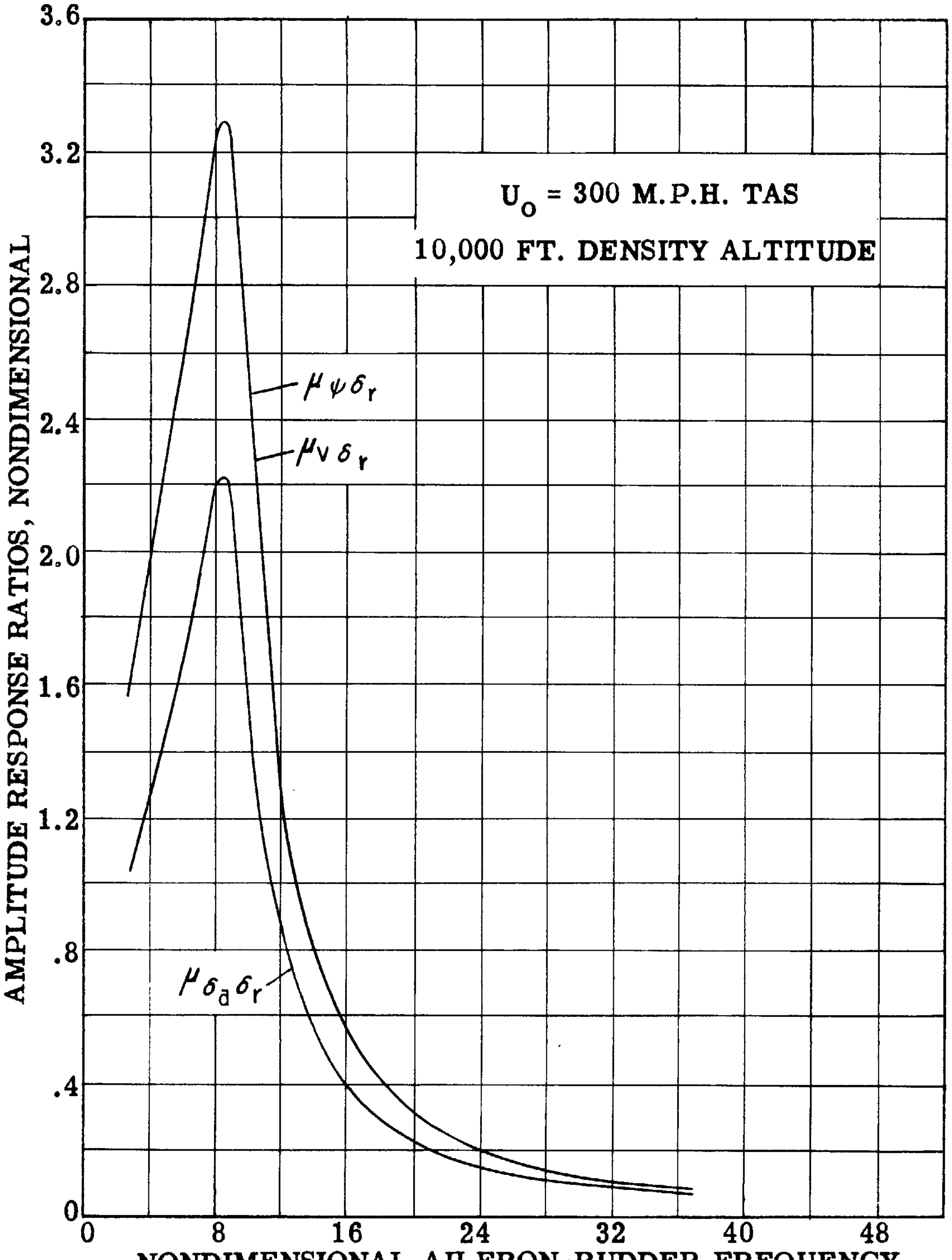


NONDIMENSIONAL AILERON-RUDDER FREQUENCY

$\omega_{ar} = T_A \Omega_{AR}$

EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN AERODYNAMIC YAW AND CHANGE IN RUDDER ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN GEOMETRIC YAW AND CHANGE IN RUDDER ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN AILERON ANGLE AND CHANGE IN RUDDER ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO ANGLE OF BANK

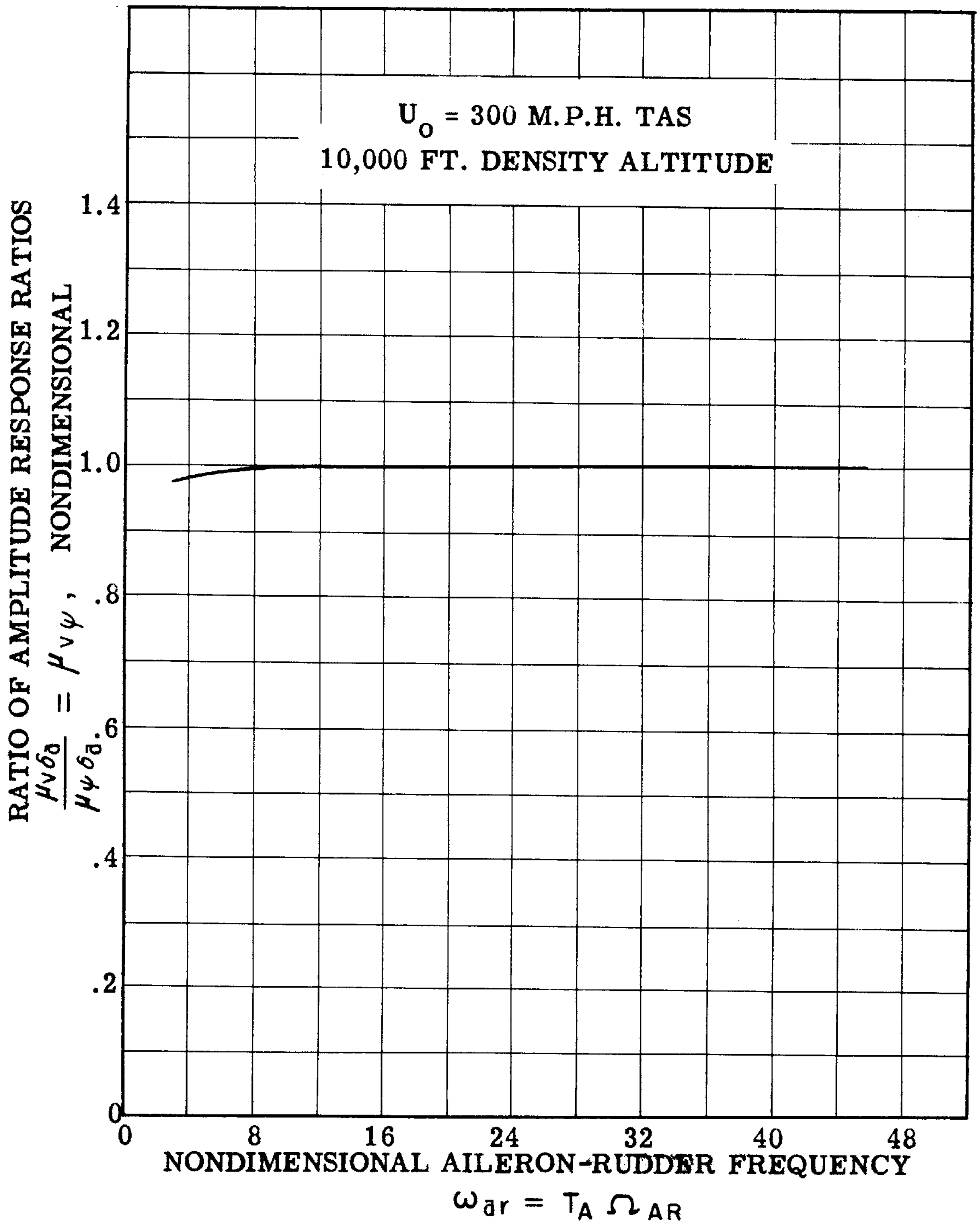
FIGURE L-35



$$\omega_{ar} = T_A \Omega_{AR}$$

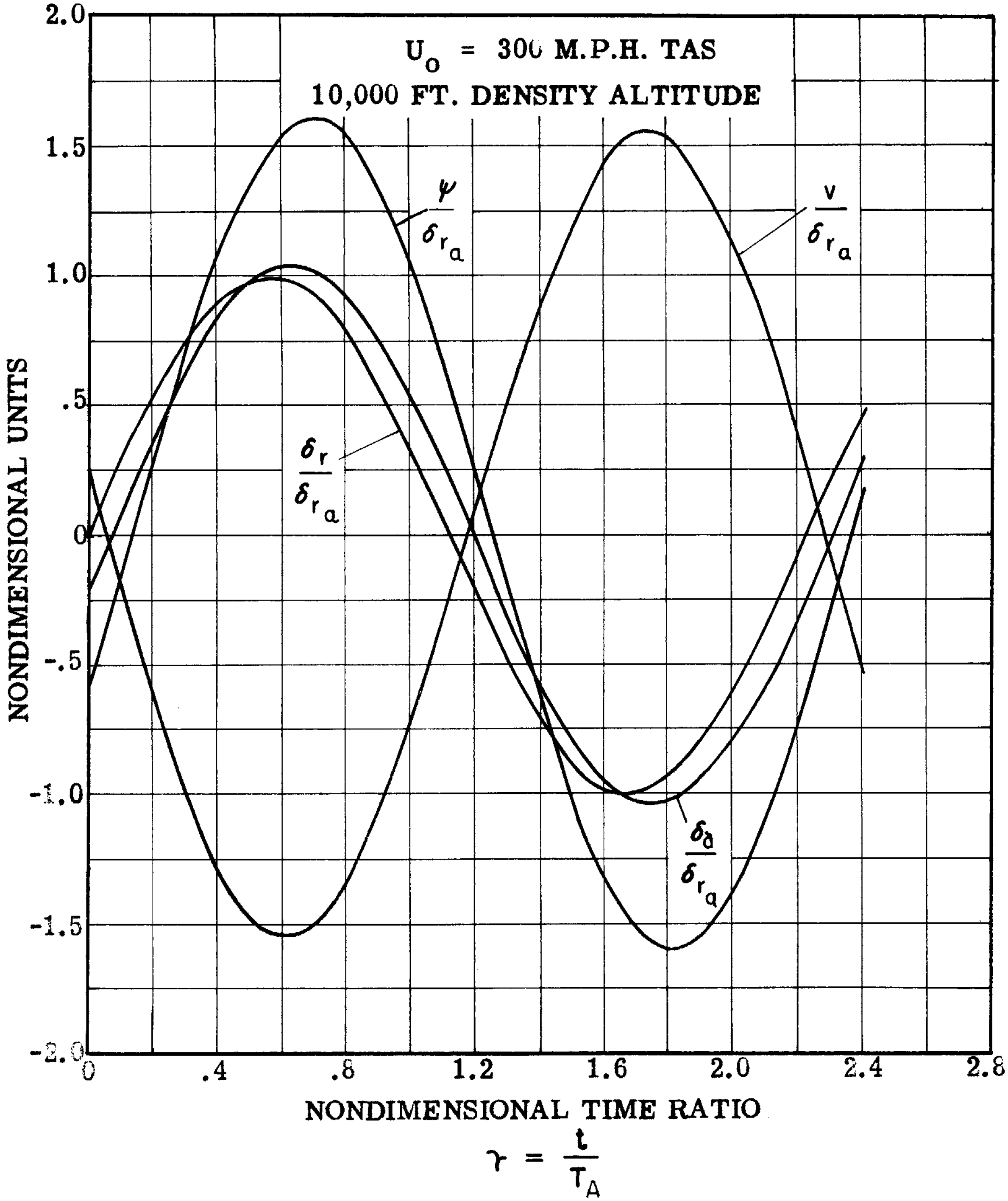
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO ANGLE OF BANK

FIGURE L-36



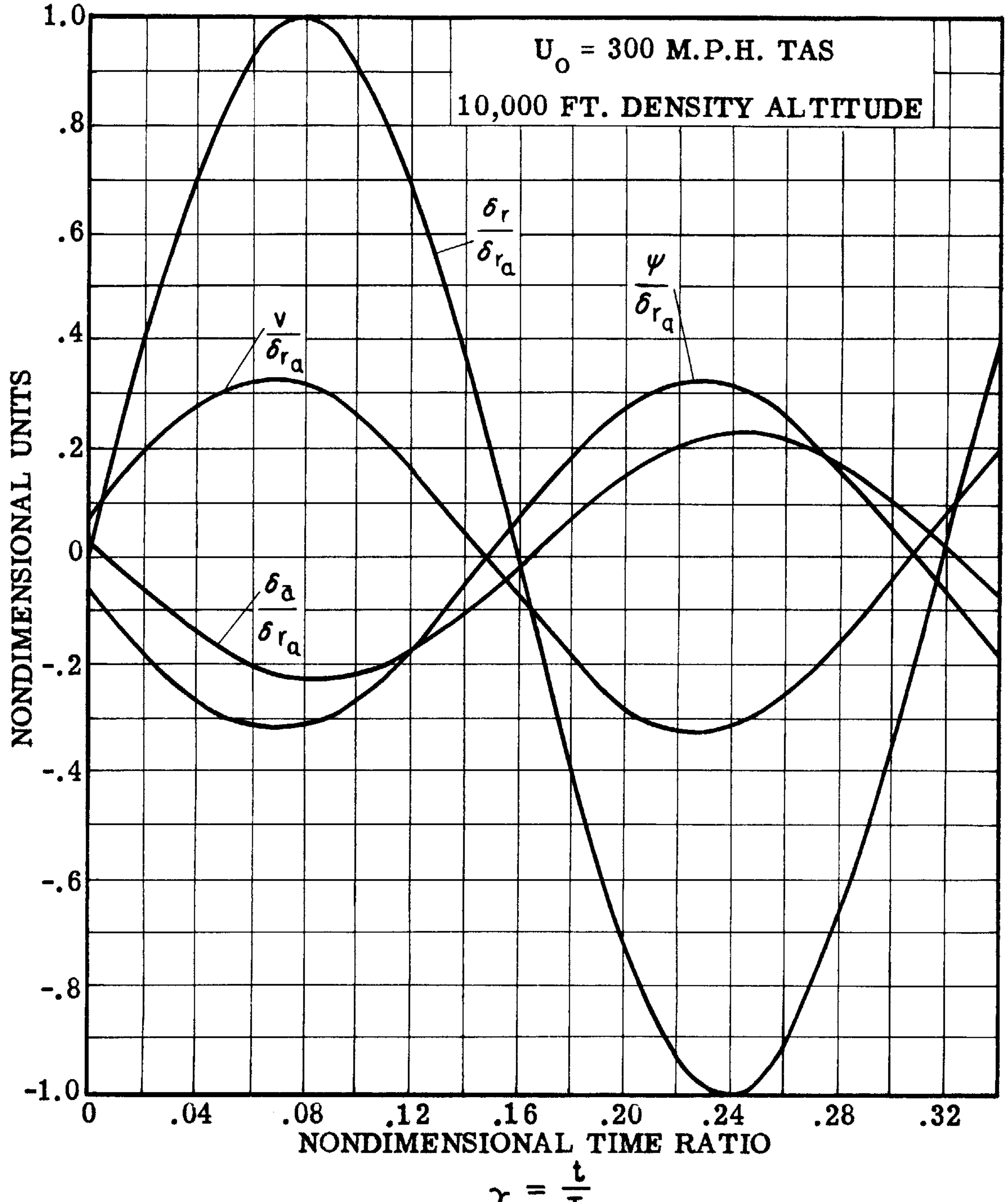
EFFECT OF CHANGE IN AILERON-RUDDER FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR ANGLE OF AERODYNAMIC YAW TO AMPLITUDE RESPONSE RATIO FOR ANGLE OF GEOMETRIC YAW, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO ANGLE OF BANK

FIGURE L-37



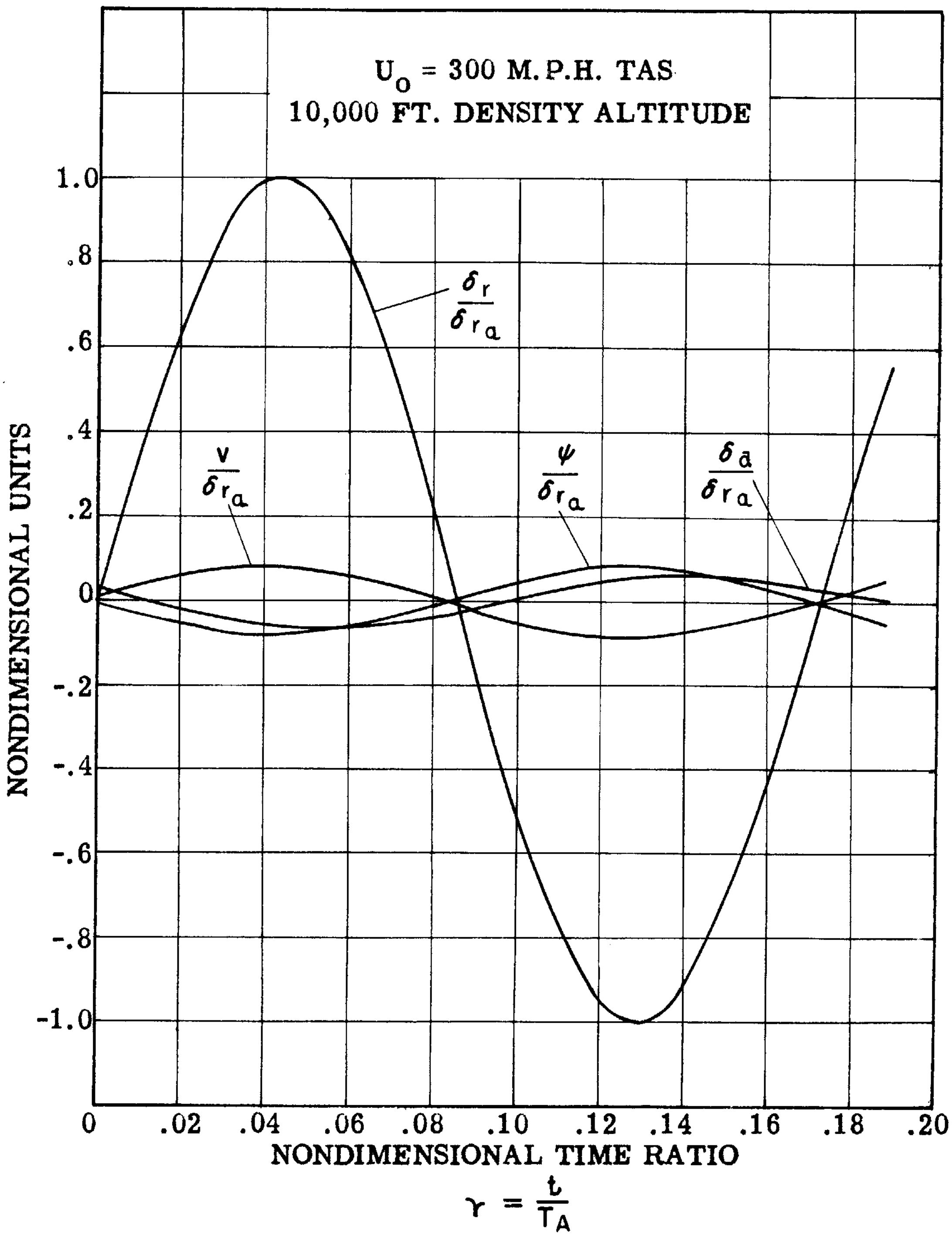
TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN RUDDER ANGLE, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO ANGLE OF BANK, AILERON-RUDDER FREQUENCY = .1 CYCLE PER SECOND

FIGURE L-38



TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN RUDDER ANGLE, PER AMPLITUDE OF CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO ANGLE OF BANK, AILERON-RUDDER FREQUENCY = .7 CYCLE PER SECOND

FIGURE L-39



TIME HISTORY OF CHANGE IN AILERON ANGLE, CHANGE IN ANGLE OF AERODYNAMIC YAW, CHANGE IN ANGLE OF GEOMETRIC YAW, AND CHANGE IN RUDDER ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS AND RUDDER, ADJUSTED FOR ZERO ANGLE OF BANK, AILERON-RUDDER FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE L-40

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION

TABLE L-1

①	Ω_E, cycles per second.	0.1	0.3	0.5	0.7	0.9	1.1	1.3
②	$\omega_e = T_A \cdot 2\pi \cdot \Omega_E$ = ①28.096	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
㊱	−② H_2	-.39242	-1.1773	-1.9621	-2.7469	-3.5317	-4.3166	-5.1014
㊲	㊱/($-H_1$) = tan β_1	+.24651	+.73956	+1.2326	+1.7255	+2.2185	+2.7116	+3.2046
㊳	⑨②	526.94	15,789.	73,678	202,610.	430,970.	787,300.	.12999×10^+7
㊴	㊳/⑥ = tan φ_2	.30375	1.3871	10.917	-3.3758	-1.6532	-1.1392	-.88386
㊵	$\omega_e C_2$ = ② C_2	.24517	.73551	1.2258	1.7162	2.2065	2.6969	3.1872
㊶	−③ + C_1	-7.6802	-70.831	-197.14	-386.58	-639.17	-954.97	-1333.9
㊷	㊵/㊶ = tan φ_1	-.031922	-.010384	-.0062179	-.0044394	-.0034621	-.0028241	-.0023894
㊸	② K_2	13.928	41.785	69.642	97.497	125.35	153.21	181.07
㊹	−③ + K_1	-7.3466	-70.498	-196.80	-386.24	-638.83	-954.63	-1333.6
㊺	㊸/㊹ = tan ψ_1	-1.8958	-.59271	-.35387	-.25243	-.19622	-.16049	-.13578
㊻	$\beta_1 = \tan^{-1}$ ㊲, degrees.	193.85	216.48	230.95	239.91	245.74	249.76	252.67
㊼	$\varphi_2 = \tan^{-1}$ ㊴, "	16.896	54.211	84.766	106.50	121.17	131.28	138.53
㊽	$\varphi_1 = \tan^{-1}$ ㊷, "	178.17	179.40	179.64	179.75	179.80	179.84	179.86
㊾	$\psi_1 = \tan^{-1}$ ㊺, "	117.81	149.34	160.51	165.83	168.90	170.88	172.27
㊿	.0174533 ㊻, radians, β_1	3.3833	3.7783	4.0308	4.1872	4.2890	4.3591	4.4099
51	" ㊼, " , φ_2	.29489	.94616	1.4794	1.8588	2.1148	2.2913	2.4178
52	" ㊽, " , φ_1	3.1097	3.1311	3.1353	3.1372	3.1381	3.1388	3.1392
53	" ㊾, " , ψ_1	2.0562	2.6065	2.8014	2.8943	2.9479	2.9824	3.0067
54	$LA_{u\delta_e} = \beta_1 - \varphi_2$, radians, ㊿−51	3.0884	2.8321	2.5514	2.3284	2.1742	2.0678	1.9921
55	$LA_{w\delta_e} = \varphi_1 - \varphi_2$, " , 52−51	2.8148	2.1849	1.6559	1.2784	1.0233	.8475	.7214
56	$LA_{\theta\delta_e} = \psi_1 - \varphi_2$, " , 53−51	1.7613	1.6603	1.3220	1.0365	.8331	.6911	.5889

TRUE AIRSPEED = 300 MPH
10,000 FEET DENSITY ALTITUDE
COMPUTATIONS FOR LEAD ANGLES
$LA_{u\delta_e}$, $LA_{w\delta_e}$, $LA_{\theta\delta_e}$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION

TABLE L-2

1	Ω_E, cycles per second	0.1	0.3	0.5	0.7	0.9	1.1	1.3
2	$\omega_e = T_A \cdot 2\pi \cdot \Omega_E$ = (1) 28.096	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
3	ω_e^2 = (2)^2	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
4	ω_e^4 = (3)^2	62.314	5047.4	38947.	149,610.	408,810.	912,370.	1,779800.
5	$\omega_e^2 C_3$ = (3)C_3	1829.1	16,462	45,728	89,623.	148,150.	221,320.	309,120.
6	−(4) + (5) − C_4	1734.8	11,383.	6,749.0	−60,019.	−260,690.	−691,080.	−1,470,700.
7	(6)^2	$.30095 \times 10^{+7}$	$.12957 \times 10^{+9}$	$.45549 \times 10^{+8}$	$.36023 \times 10^{+10}$	$.67959 \times 10^{+11}$	$.47759 \times 10^{+12}$	$.21630 \times 10^{+13}$
8	$\omega_e^2 C_5 = C_5$(3)	210.71	1896.4	5267.9	10,325	17,067.	25,497	35,611.
9	(8) − C_6	187.55	1873.2	5244.7	10,302.	17,044.	25,474	35,588.
10	(9)^2(3)	$.27767 \times 10^{+6}$	$.24929 \times 10^{+9}$	$.54285 \times 10^{+10}$	$.41050 \times 10^{+11}$	$.18574 \times 10^{+12}$	$.61984 \times 10^{+12}$	$.16896 \times 10^{+13}$
11	(7) + (10) = denominator under $\sqrt{\ }$	$.32872 \times 10^{+7}$	$.37886 \times 10^{+9}$	$.54740 \times 10^{+10}$	$.44652 \times 10^{+11}$	$.25370 \times 10^{+12}$	$.10974 \times 10^{+13}$	$.38526 \times 10^{+13}$
12	H_1^2	2.5341	2.5341	2.5341	2.5341	2.5341	2.5341	2.5341
13	$(\omega_e H_2)^2 = [-(2) H_2]^2$ = (36)^2	.15399	1.3860	3.8498	7.5455	12.473	18.633	26.024
14	(12) + (13)	2.6881	3.9201	6.3839	10.080	15.007	21.167	28.558
15	(14)/(11)	$.81776 \times 10^{-6}$	$.10347 \times 10^{-7}$	$.11662 \times 10^{-8}$	$.22575 \times 10^{-9}$	$.59153 \times 10^{-10}$	$.19288 \times 10^{-10}$	$.74127 \times 10^{-11}$
16	$\sqrt{(15)}$	.00090430	.00010172	.000034150	.000015025	$.76911 \times 10^{-5}$	$.43918 \times 10^{-5}$	$.27226 \times 10^{-5}$
17	$-m_{\delta_e}$ (16) = $\mu_{u\delta_e}$	.31827	.035800	.012019	.0052880	.0027069	.0015457	.00095822
18	$(\omega_e^2 - C_1)^2 = [-(3) + C_1]^2$ = (41)^2	58.985	5017.0	38,864.	$.14944 \times 10^{+6}$	$.40854 \times 10^{+6}$	$.91197 \times 10^{+6}$	$.17793 \times 10^{+7}$
19	$(\omega_e^2 C_2)^2 = [(2) C_2]^2$ = (40)^2	.060108	.54097	1.5026	2.9453	4.8686	7.2733	10.158
20	(18) + (19)	59.045	5017.5	38,866	$.14944 \times 10^{+6}$	$.40854 \times 10^{+6}$	$.91198 \times 10^{+6}$	$.17793 \times 10^{+7}$
21	(20)/(11)	.000017962	.000013244	$.71001 \times 10^{-5}$	$.33468 \times 10^{-5}$	$.16103 \times 10^{-5}$	$.83104 \times 10^{-6}$	$.46184 \times 10^{-6}$
22	$\sqrt{(21)}$	.0042382	.0036392	.0026646	.0018294	.0012690	.00091161	.00067959
23	$-m_{\delta_e}$ (22) = $\mu_{w\delta_e}$	1.4916	1.2808	.93781	.64384	.44662	.32084	.23918
24	$[-(3) + K_1]^2$ = (44)^2	53.973	4970.0	38,730.	$.14918 \times 10^{+6}$	$.40810 \times 10^{+6}$	$.91132 \times 10^{+6}$	$.17785 \times 10^{+7}$
25	$[(2) K_2]^2$ = (43)^2	193.99	1746.0	4850.0	9505.7	15,713.	23,473.	32,786
26	(24) + (25)	247.96	6716.0	43,580.	$.15869 \times 10^{+6}$	$.42381 \times 10^{+6}$	$.93479 \times 10^{+6}$	$.18113 \times 10^{+7}$
27	(26)/(11)	.000075432	.000017727	$.79613 \times 10^{-5}$	$.35539 \times 10^{-5}$	$.16705 \times 10^{-5}$	$.85182 \times 10^{-6}$	$.47015 \times 10^{-6}$
28	$\sqrt{(27)}$	.0086852	.0042104	.0028216	.0018852	.0012925	.00092294	.00068568
29	$-m_{\delta_e}$ (28) = $\mu_{\theta\delta_e}$	3.0568	1.4819	.99306	.66350	.45490	.32483	.24133
30	(29)/(23) = $\mu_{\theta\delta_e} / \mu_{w\delta_e} = \mu_{q\dot{w}}$	2.0493	1.1570	1.0589	1.0305	1.0185	1.0124	1.0090

COMPUTATIONS FOR AMPLITUDE RESPONSE RATIOS

$\mu_{u\delta_e}$, $\mu_{w\delta_e}$, $\mu_{\theta\delta_e}$ and $\mu_{q\dot{w}}$

TRUE AIRSPEED 300 MPH

10,000 FEET DENSITY ALTITUDE

$C_1 = .21369$	$K_1 = .54735$
$C_2 = .087261$	$K_2 = 4.9574$
$C_3 = 231.71$	$H_1 = 1.5919$
$C_4 = 31.960$	$H_2 = .13967$
$C_5 = 26.693$	$m_{\delta_e} = -351.95$
$C_6 = 23.157$	$T_A = 4.4716$ sec.

TIME HISTORY FOR $\frac{\delta_e}{\delta_{e_a}}$, $\frac{u}{\delta_{e_a}}$, $\frac{w}{\delta_{e_a}}$ and $\frac{\theta}{\delta_{e_a}}$
FOR Ω_E = .1 CYCLES PER SECOND
(ω_e = 2.8096)

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR

CASE I: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION.

CASE II: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT.

TABLE L-3

								CASE I						
①	t, seconds	0	.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
②	$\gamma = t/T_a$ = ①/4.4716	0	.20127	.40254	.60381	.80508	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
③	$\omega_e\gamma$ = 2.8096 × ②	0	.56549	1.1310	1.6965	2.2620	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
④	$\omega_e\gamma + \beta_1 - \varphi_2$ = ③ + 3.0884	3.0884	3.6639	4.2194	4.7849	5.3504	5.9160	6.4813	7.0468	7.6124	8.1777	8.7433	9.3089	9.8741
⑤	$\omega_e\gamma + \varphi_1 - \varphi_2$ = ③ + 2.8148	2.8148	3.3803	3.9468	4.5113	5.0768	5.6424	6.2077	6.7732	7.3388	7.9041	8.4697	9.0353	9.6005
⑥	$\omega_e\gamma + \psi_1 - \varphi_2$ = ③ + 1.7613	1.7613	2.3268	2.8923	3.4578	4.0233	4.5889	5.1542	5.7197	6.2853	6.8506	7.4162	7.9818	8.5470
⑦	δ_e/δ_{e_a} = sin ③	0	+.53583	+.90483	+.99211	+.77051	+.30885	-.24869	-.72897	-.98232	-.92978	-.58779	-.062616	+.48160
⑧	sin ④	+.053207	-.49014	-.88089	-.99738	-.80323	-.35902	+.19680	+.69151	+.97097	+.94805	+.63000	+.11563	-.43429
⑨	sin ⑤	+.32094	-.23650	-.72031	-.97986	-.93433	-.59777	-.075501	+.47070	+.87018	+.99875	+.81634	+.37978	-.17485
⑩	sin ⑥	+.98189	+.72753	+.24666	-.31101	-.77185	-.99238	-.90401	-.53420	+.0020944	+.53745	+.90572	+.99185	+.76929
⑪	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑧ = .31827 × ⑧	+.016934	-.15600	-.28036	-.31744	-.25564	-.11427	+.062636	+.22009	+.30903	+.30174	+.20061	+.036802	-.13822
⑫	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑨ = 1.4916 × ⑨	+.47871	-.35276	-1.0744	-1.4616	-1.3936	-.89163	-.11262	+.70210	+1.2980	+1.4897	+1.2177	+.56648	-.26081
⑬	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑩ = 3.0568 × ⑩	+3.0014	+2.2239	+.75399	-.95070	-2.3594	-3.0335	-2.7634	-1.6329	+.0064022	+1.6429	+2.7686	+3.0319	+2.3516
								CASE II						
⑭	$\omega_e\gamma + \frac{3\pi}{2} - \varphi_2$ = ③ + 2.8159	2.8159	3.3814	3.9469	4.5124	5.0779	5.6435	6.2088	6.7743	7.3399	7.9052	8.4708	9.0364	9.6016
⑮	$\omega_e\gamma + \psi_1 - \varphi_2$ = ③ + 1.7683	1.7683	2.3338	2.8993	3.4648	4.0303	4.5959	5.1612	5.7267	6.2923	6.8576	7.4232	7.9888	8.5540
⑯	$\omega_e\gamma + \angle A_{u\delta_e}$ = ③ + 3.5839	3.5839	4.1494	4.7149	5.2804	5.8459	6.4115	6.9768	7.5423	8.1079	8.6732	9.2388	9.8044	10.370
⑰	sin ⑭	+.31995	-.23752	-.72104	-.98006	-.93396	-.59693	-.074282	+.47163	+.87079	+.99869	+.81573	+.37865	-.17588
⑱	sin ⑮	+.98055	+.72273	+.23989	-.31764	-.77627	-.99323	-.90100	-.52814	+.0090756	+.54332	+.90865	+.99094	+.76481
⑲	sin ⑯	-.42799	-.84563	-1.0000	-.84302	-.42341	+.12793	+.63930	+.95181	+.96793	+.68276	+.18498	-.37056	-.81066
⑳	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑰ = 1.5062 × ⑰	+.48191	-.35775	-1.0860	-1.4762	-1.4067	-.89910	-.11188	+.71037	+1.3116	+1.5042	+1.2287	+.57032	-.26491
㉑	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑱ = 3.0143 × ⑱	+2.9557	+2.1785	+.72310	-.95746	-2.3399	-2.9939	-2.7159	-1.5920	+.027357	+1.6377	+2.7389	+2.9870	+2.3054
㉒	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑲ = .40493 × ⑲	-.17545	-.34665	-.40993	-.34658	-.17357	+.052442	+.26207	+.39018	+.39678	+.27988	+.075829	-.15190	-.33231

TIME HISTORY FOR $\frac{\delta_e}{\delta_{e_a}}$, $\frac{u}{\delta_{e_a}}$, $\frac{w}{\delta_{e_a}}$ and $\frac{\theta}{\delta_{e_a}}$
FOR $\Omega_E = .7$ CYCLES PER SECOND
($\omega_e = 19.667$)

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR
CASE I: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION.
CASE II: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT

TABLE L-4

	CASE I													
①	t, seconds	0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
②	$\gamma = t/T_A$ = ①/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③	$\omega_e \gamma$ = 19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④	$\omega_e \gamma + \beta_1 - \varphi_2$ = ③ + 2.3284	2.3284	2.9002	3.4719	4.0437	4.6155	5.1872	5.7589	6.3308	6.9026	7.4743	8.0460	8.6179	9.1896
⑤	$\omega_e \gamma + \varphi_1 - \varphi_2$ = ③ + 1.2784	1.2784	1.8502	2.4219	2.9937	3.5655	4.1372	4.7089	5.2808	5.8526	6.4243	6.9960	7.5679	8.1396
⑥	$\omega_e \gamma + \psi_1 - \varphi_2$ = ③ + 1.0355	1.0355	1.6073	2.1790	2.7508	3.3226	3.8943	4.4660	5.0379	5.6097	6.1814	6.7531	7.3250	7.8967
⑦	δ_e/δ_{e_a} = sin ③	0	.54111	.91009	.98958	.75425	.27899	-.28486	-.75836	-.99046	-.90748	-.53597	.006318	.54639
⑧	sin ④	.72645	.23904	-.32441	.78467	-.99531	-.88942	-.50060	.047630	.58056	.92881	.98163	.72212	.23294
⑨	sin ⑤	.95757	.96121	.65921	.14729	-.41136	-.83905	-.99999	-.84273	-.41739	+.14073	+.65396	+.95936	+.95951
⑩	sin ⑥	.86012	.99933	.82065	.38091	-.18000	-.68366	-.96979	-.94749	-.62374	-.10158	+.45275	+.86331	+.99909
⑪	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑧ = .0052880 × ⑧	.0038415	.0012640	-.0017155	-.0041493	-.0052632	-.0047033	-.0026472	.00025187	.0030700	.0049115	.0051909	.0038186	.0012318
⑫	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑨ = .64386 × ⑨	.61654	.61888	.42444	.094834	-.26486	-.54023	-.64385	-.54260	-.26874	+.090610	+.42105	+.61769	+.61779
⑬	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑩ = .66350 × ⑩	.57069	.66306	.54450	.25273	-.11943	-.45361	-.64346	-.62866	-.41385	-.067398	+.30040	+.57281	+.66290
	CASE II													
⑭	$\omega_e \gamma + \frac{3\pi}{2} - \varphi_2$ = ③ + 1.2785	1.2785	1.8503	2.4220	2.9938	3.5656	4.1373	4.7090	5.2809	5.8527	6.4244	6.9961	7.5680	8.1397
⑮	$\omega_e \gamma + \psi_1 - \varphi_2$ = ③ + 1.0357	1.0357	1.6075	2.1792	2.7510	3.3228	3.8945	4.4662	5.0381	5.6099	6.1816	6.7533	7.3252	7.8969
⑯	$\omega_e \gamma + \angle A_{u\delta_e}$ = ③ + 2.6974	2.6974	3.2692	3.8409	4.4127	4.9845	5.5562	6.1279	6.6998	7.2716	7.8433	8.4150	8.9869	9.5586
⑰	sin ⑭	.95759	.96121	.65908	.14729	-.41136	-.83915	-.99999	-.84273	-.41739	+.14073	.65408	.95936	.95946
⑱	sin ⑮	.86022	.99933	.82055	.38075	-.18018	-.68378	-.96983	-.94743	-.62361	-.10140	+.45306	+.86340	+.99908
⑲	sin ⑯	+.42972	-.12724	-.64372	-.95543	-.96321	-.66458	-.15471	+.40466	+.83514	+.99994	+.84675	+.42404	-.13347
⑳	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑰ = .64410 × ⑰	.61678	.61912	.42451	.094869	-.26496	-.54050	-.64409	-.54280	-.26884	.090644	.42129	.61792	.61799
㉑	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑱ = .66355 × ⑱	.57080	.66311	.54448	.25265	-.11956	-.45372	-.64353	-.62867	-.41380	-.067284	.30063	.57291	.66294
㉒	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑲ = .017043 × ⑲	.0073237	-.0021686	-.010971	-.016283	-.016416	-.011326	-.0026367	.0068966	.014233	.017042	.014431	.0072269	-.0022747

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR

TIME HISTORY FOR $\frac{\delta_e}{\delta_{e_a}}, \frac{u}{\delta_e}, \frac{w}{\delta_e}$ and $\frac{\theta}{\delta_{e_a}}$
FOR $\Omega_E = 1.3$ (cycles per second)
($\omega_e = 36.525$)

CASE I: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS INCLUDED IN ALL THREE EQUATIONS OF MOTION.
CASE II: WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT.

TABLE L-5

		CASE I												
①	t, seconds	0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
②	$\tau = t/T_A$ = ①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③	$\omega_e \tau$ = 36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④	$\omega_e \tau + \beta_1 - \varphi_2$ = ③ + 1.9921	1.9921	2.5639	3.1357	3.7074	4.2792	4.8510	5.4227	5.9945	6.5661	7.1381	7.7097	8.2817	8.8533
⑤	$\omega_e \tau + \varphi_1 - \varphi_2$ = ③ + .7214	.7214	1.2932	1.8650	2.4367	3.0085	3.5803	4.1520	4.7238	5.2954	5.8674	6.4390	7.0110	7.5826
⑥	$\omega_e \tau + \psi_1 - \varphi_2$ = ③ + .5889	.5889	1.1607	1.7325	2.3042	2.8760	3.4478	4.0195	4.5913	5.1629	5.7349	6.3065	6.8785	7.4501
⑦	δ_e/δ_{e_a} = sin ③	0	.54111	.91013	.98958	.75425	.27849	-.28502	-.75836	-.99044	-.90748	-.53597	.006458	.54639
⑧	sin ④	.91255	.54610	.0059341	-.53612	-.90763	-.99041	-.75813	-.28468	.27916	.75448	.98960	.90989	.54083
⑨	sin ⑤	.66043	.96172	.95702	.64799	.13278	-.42483	-.84703	-.99994	-.83485	-.40386	.15523	.66523	.96340
⑩	sin ⑥	.55544	.91708	.98695	.74291	.26253	-.30137	-.76940	-.99267	-.90024	-.52116	+.023385	.56078	.91955
⑪	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑧ = .00095822 × ⑧	.00087442	.00052328	.0000056862	-.00051372	-.00086971	-.00094903	-.00072646	-.00027279	.00026750	.00072296	.00094825	.00087187	.00051823
⑫	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑨ = .23918 × ⑨	.15796	.23002	.22890	.15499	.031758	-.10161	-.20259	-.23917	-.19968	-.096595	.037128	.15911	.23043
⑬	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑩ = .24133 × ⑩	.13404	.22132	.23818	.17929	.063356	-.072730	-.18568	-.23956	-.21725	-.12577	.0056435	.13533	.22192
		CASE II												
⑭	$\omega_e \tau + \frac{3\pi}{2} - \varphi_2$ = ③ + .7214	.7214	1.2932	1.8649	2.4367	3.0085	3.5803	4.1520	4.7238	5.2954	5.8674	6.4390	7.0110	7.5826
⑮	$\omega_e \tau + \psi_1 - \varphi_2$ = ③ + .5887	.5887	1.1605	1.7322	2.3040	2.8758	3.4476	4.0193	4.5911	5.1627	5.7347	6.3063	6.8783	7.4499
⑯	$\omega_e \tau + \angle A_{u\delta_e}$ = ③ + 2.2099	2.2099	2.7817	3.3534	3.9252	4.4970	5.0688	5.6405	6.2123	6.7839	7.3559	7.9275	8.4995	9.0711
⑰	sin ⑭	.66043	.96172	.95707	.64799	.13278	-.42483	-.84703	-.99994	-.83485	-.40386	+.15523	+.66523	+.96340
⑱	sin ⑮	.55528	.91700	.98700	.74303	.26269	-.30121	-.76929	-.99265	-.90032	-.52146	+.023036	+.56064	+.91948
⑲	sin ⑯	+.80261	+.35217	-.21030	-.70587	-.97690	-.93716	-.59930	-.070801	+.48007	+.87848	+.99730	+.79874	+.34628
⑳	$w/\delta_{e_a} = \mu_{w\delta_e}$ ⑰ = .2390 × ⑰	.15797	.23004	.22893	.15500	.031761	-.10162	-.20261	-.23919	-.19970	-.096603	+.037131	+.15912	+.23045
㉑	$\theta/\delta_{e_a} = \mu_{\theta\delta_e}$ ⑱ = .24132 × ⑱	.13400	.22129	.23818	.17931	.063392	-.072688	-.18565	-.23955	-.21727	-.12584	+.0055590	+.13529	+.22189
㉒	$u/\delta_{e_a} = \mu_{u\delta_e}$ ⑲ = .0033786 × ⑲	.0027117	.0011898	-.00071052	-.0023849	-.0033006	-.0031663	-.0020248	-.00023921	+.0016220	+.0029680	+.0033695	+.0026986	+.0011699

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT

TABLE L-6

(1)	Ω_E, cycles per second	0.1	0.3	0.5	0.7	0.9	1.1	1.3
(2)	ω_e	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
(37)	(15)(29) = $\mu_{\theta\delta_e} \sin\beta_1$	-.68307	-.14889	+.23817	+.33585	+.30479	+.24971	+.20036
(38)	$-x_\theta$ (37)	-.22328	-.048668	+.077851	+.10978	+.099627	+.081623	+.065492
(39)	(10)(31) = $\mu_{w\delta_e} \sin\beta_2$	+1.4413	+.75001	+.085731	-.18289	-.23119	-.21170	-.17922
(40)	$-x_w$ (39)	-.26981	-.14040	-.016049	+.034237	+.043279	+.039630	+.033550
(41)	(38) + (40) = B	-.49309	-.18907	+.061802	+.14402	+.14291	+.12125	+.099042
(42)	(41)/(36) = $\frac{B}{A}$ = tan $LA_{u\delta_e}$	+.47344	+.27952	-.12612	-.47583	-.78548	-1.0726	-1.3453
(43)	$LA_{u\delta_e} = \tan^{-1}$ (42), degrees	205.34	195.62	172.81	154.55	141.85	132.99	126.62
(44)	.0174533 × (43) = $LA_{u\delta_e}$, radians	3.5839	3.4142	3.0161	2.6974	2.4758	2.3211	2.2099
(45)	$(36)^2 = A^2$	1.0847	.45753	.24012	.091609	.033102	.012778	.0054199
(46)	$(41)^2 = B^2$	.24314	.035747	.0038195	.020742	.020423	.014702	.0098093
(47)	(45) + (46) = $A^2 + B^2$	1.3278	.49328	.24394	.11235	.053525	.027480	.015229
(48)	$(x_u)^2$ + (3)	7.9015	71.053	197.36	386.80	639.39	955.19	1334.1
(49)	(47)/(48)	.16804	.0069424	.0012360	.00029046	.000083713	.000028769	.000011416
(50)	$\sqrt{(49)} = \mu_{u\delta_e}$	.40993	.083321	.035157	.017043	.0091495	.0053637	.0033786
(51)	$LA_{w\delta_e} = \frac{3\pi}{2} - \varphi_2$, radians, $= 4.7124 - \varphi_2$	2.8159	2.1852	1.6560	1.2786	1.0233	.8475	.7214
(52)	$LA_{\theta\delta_e} = \psi_1 - \varphi_2$, "	1.7683	1.6612	1.3223	1.0357	.8331	.6911	.5887

COMPUTATIONS FOR LEAD ANGLES

$LA_{u\delta_e}$, $LA_{w\delta_e}$, $LA_{\theta\delta_e}$

TRUE AIRSPEED 300 MPH

10,000 FEET DENSITY ALTITUDE

$x_\theta = -.32687$ $x_u = -.087261$

$x_w = .18720$ $x_u^2 = .0076145$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR NORMAL FORCE AND PITCHING MOMENT

TABLE L-7

(1)	Ω_E, Cycles per second	0.1	0.3	0.5	0.7	0.9	1.1	1.3
(2)	$\omega_e = T_A \cdot 2\pi \cdot \Omega_E = 28.096$ (1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
(3)	$\omega_e^2 =$ (2)^2	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
(4)	$C_7^2 \omega_e^2 = 707.83$ (3)	5587.5	50,288.	139,690	273,780.	452,570.	676,110.	944,320.
(5)	$+C_8 - \omega_e^2 = C_8 -$ (3)	221.38	158.22	31.92	-157.52	-410.11	-725.91	-1104.8
(6)	(5)^2	49,009	25,034	1018.9	24,813.	168,190	526,950.	1,220,600.
(7)	(4) + (6) = denominator under $\sqrt{\ }$	54,596.	75,322	140,710.	298,590.	620,760.	1,203,100.	2,164,900.
(8)	1/(7)	$.18316 \times 10^{-4}$	$.13276 \times 10^{-4}$	$.71068 \times 10^{-5}$	$.33491 \times 10^{-5}$	$.16109 \times 10^{-5}$	$.83119 \times 10^{-6}$	$.46192 \times 10^{-6}$
(9)	$\sqrt{(8)}$	.0042797	.0036436	.0026659	.0018301	.0012692	.0009117	.00067965
(10)	$-m_{\delta_e}$ (9) $= \mu_{w\delta_e}$	1.5062	1.2824	.93826	.64410	.44669	.32087	.23920
(11)	$(z_w)^2 +$ (3) $= 23.718 +$ (3)	31.612	94.763	221.07	410.51	663.10	978.90	1357.8
(12)	(11)/(7)	.00057902	.0012581	.0015711	.0013748	.0010682	.00081365	.00062719
(13)	$\sqrt{(12)}$	.024063	.035470	.039637	.037078	.032684	.028525	.025044
(14)	$-m_{\delta_e}$ (13)	8.4690	12.484	13.950	13.050	11.503	10.039	8.8142
(15)	(14)/(2) $= \mu_{\theta\delta_e}$	3.0143	1.4811	.99302	.66355	.45492	.32482	.24132
(16)	$-$(2)/(z_w) $= \tan \psi_1$	.57691	1.7307	2.8846	4.0383	5.1921	6.3461	7.4998
(17)	(5)/[$-$(2)C_7] $= \tan \varphi_2$	-2.9616	-.70556	-.085406	+.30105	+.60962	+.88283	+1.1369
(18)	$\tan^{-1}$ (16) $= \psi_1$, degrees.	209.98	239.98	250.88	256.09	259.10	261.04	262.40
(19)	$\tan^{-1}$ (17) $= \varphi_2$, " .	108.66	144.80	175.12	196.75	211.37	221.44	228.67
(20)	.0174533 (18) $= \psi_1$, radians.	3.6648	4.1884	4.3787	4.4696	4.5222	4.5560	4.5797
(21)	" (19) $= \varphi_2$, " .	1.8965	2.5272	3.0564	3.4339	3.6891	3.8649	3.9910
(22)	$\mu_{\theta\delta_e} / \mu_{w\delta_e} =$ (15)/(10)	2.0013	1.1549	1.0584	1.0302	1.0184	1.0123	1.0089
(23)	$-$(2)/$x_u = \tan \alpha_1 = -$(2)/$-.087261$	32.198	96.593	160.99	225.38	289.77	354.18	418.57
(24)	$\alpha_1 = \tan^{-1}$ (23), degrees.	268.22	269.41	269.64	269.75	269.80	269.84	269.86
(25)	α_1, radians = .0174533 x (24)	4.6813	4.7021	4.7061	4.7080	4.7089	4.7096	4.7099
(26)	(20) $-$ (21) $-$ (25) $= \beta_1$, radians.	-2.9130	-3.0409	-3.3838	-3.6723	-3.8758	-4.0185	-4.1212
(27)	$\pi/2 -$ (21) $-$ (25) $= \beta_2$, radians.	-5.0070	-5.6586	-6.1917	-6.5711	-6.8272	-7.0037	-7.1301
(28)	$\cos \beta_1 = \cos$ (26)	-.97398	-.99493	-.97081	-.86245	-.74237	-.63954	-.55735
(29)	$\sin \beta_1 = \sin$ (26)	-.22661	-.10053	+.23984	+.50614	+.66999	+.76876	+.83027
(30)	$\cos \beta_2 = \cos$ (27)	+.29035	+.81114	+.99582	+.95884	+.85565	+.75148	+.66231
(31)	$\sin \beta_2 = \sin$ (27)	+.95692	+.58485	+.091372	-.28394	-.51756	-.65976	-.74923
(32)	(15) (28) $= \mu_{\theta\delta_e} \cos \beta_1$	-2.9359	-1.4736	-.96403	-.57228	-.33772	-.20774	-.13450
(33)	$-x_\theta$ (32)	-.95966	-.48168	-.31511	-.18706	-.11039	-.067904	-.043964
(34)	(10) (30) $= \mu_{w\delta_e} \cos \beta_2$	+.43733	+1.0402	+.93434	+.61759	+.38221	+.24113	+.15842
(35)	$-x_w$ (34)	-.081868	-.19473	-.17491	-.11561	-.071550	-.045140	-.029656
(36)	(33) + (35) = A	-1.0415	-.67641	-.49002	-.30267	-.18194	-.11304	-.073620

COMPUTATIONS FOR AMPLITUDE RESPONSE RATIOS

$\mu_{u\delta_e}$, $\mu_{w\delta_e}$, $\mu_{\theta\delta_e}$ and $\mu_{q\dot{w}}$

TRUE AIRSPEED 300 MPH
10,000 FEET DENSITY ALTITUDE

$C_7 = 26.605$ $\quad m_{\delta_e} = -351.95$
$C_8 = 229.27$ $\quad x_u = -.087261$
$z_w = -4.8701$ $\quad x_w = .18720$
$z_w^2 = 23.718$ $\quad x_\theta = -.32687$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS

TABLE L-8.

COMPUTATIONS FOR $\mu_{v\delta_r}$, $\mu_{\psi\delta_r}$, $\mu_{\varphi\delta_r}$ and $\mu_{v\delta_r}/\mu_{\psi\delta_r} = \mu_{v\psi}$

$V_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

①	Ω_R	0.1	0.3	0.5	0.7	0.9	1.1	1.3
②	$\omega_r = 2\pi T_A \Omega_R = 4.4716(2\pi)$①	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
③	②$^2 = \omega_r^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
④	$C_4$③ $= C_4\omega_r^2$	248.45	2236.1	6211.4	12174.	20124.	30063.	41989.
⑤	C_3 − ④	2008.8	21.1	−3954.2	−9916.8	−17867.	−27806.	−39732.
⑥	⑤^2	$40353.\times10^{+2}$	445.21	$15636.\times10^{+3}$	$98343.\times10^{+3}$	$31923.\times10^{+4}$	$77317.\times10^{+4}$	$15786.\times10^{+5}$
⑦	⑥③	$31854.\times10^{+3}$	31630.	$30858.\times10^{+5}$	$38038.\times10^{+6}$	$20411.\times10^{+7}$	$73852.\times10^{+7}$	$21060.\times10^{+8}$
⑧	③$C_6 = \omega_r^2 C_6$	1432.7	12894.	35817.	70194.	$11604.\times10^{+1}$	$17336.\times10^{+1}$	$24213.\times10^{+1}$
⑨	③$^2 = \omega_r^4$	62.314	5047.4	38947.	$14961.\times10^{+1}$	$40881.\times10^{+1}$	$91237.\times10^{+1}$	$17798.\times10^{+2}$
⑩	$-C_5$ + ⑧ − ⑨	12517.	7727.9	−3248.7	−79530.	$-29289.\times10^{+1}$	$-73913.\times10^{+1}$	$-15378.\times10^{+2}$
⑪	⑩^2	$15668.\times10^{+2}$	$59720.\times10^{+3}$	$10554.\times10^{+4}$	$63250.\times10^{+5}$	$85785.\times10^{+6}$	$54631.\times10^{+7}$	$23648.\times10^{+8}$
⑫	⑦ + ⑪ = denominator under $\sqrt{\ }$	$33421.\times10^{+3}$	$59752.\times10^{+3}$	$31913.\times10^{+5}$	$44363.\times10^{+6}$	$28990.\times10^{+7}$	$12848.\times10^{+8}$	$44708.\times10^{+8}$
⑬	$-C_1$② $= -C_1\Omega_R$	−77.806	−233.42	−389.03	−544.64	−700.25	−855.88	−1011.5
⑭	⑬^2	6053.8	54485.	$15134.\times10^{+1}$	$29663.\times10^{+1}$	$49035.\times10^{+1}$	$73253.\times10^{+1}$	$10231.\times10^{+2}$
⑮	−③ − C_2	−9.1090	−72.260	−198.57	−388.01	−640.60	−956.40	−1335.3
⑯	⑮^2	82.974	5221.5	39430.	$15055.\times10^{+1}$	$41037.\times10^{+1}$	$91470.\times10^{+1}$	$17830.\times10^{+2}$
⑰	⑭ + ⑯	6136.8	59706.	$19077.\times10^{+1}$	$44718.\times10^{+1}$	$90072.\times10^{+1}$	$16472.\times10^{+2}$	$28061.\times10^{+2}$
⑱	⑰/⑫	$.18362\times10^{-3}$	$.99923\times10^{-3}$	$.59778\times10^{-4}$	$.10080\times10^{-4}$	$.31070\times10^{-5}$	$.12821\times10^{-5}$	$.62775\times10^{-6}$
⑲	$\sqrt{⑱}$	$.13551\times10^{-1}$	$.31611\times10^{-1}$	$.77316\times10^{-2}$	$.31749\times10^{-2}$	$.17627\times10^{-2}$	$.11323\times10^{-2}$	$.79231\times10^{-3}$
⑳	M_{δ_r}⑲ $= \mu_{v\delta_r}$	1.4065	3.3302	.81452	.33448	.18570	.11929	$.83470\times10^{-1}$
㉑								
㉒	(K_2+C_1)③	223.74	2013.6	5593.5	10963.	18122.	27073.	37812.
㉓	$-K_1$ + ㉒	157.74	1947.6	5527.5	10897.	18056.	27007.	37746.
㉔	㉓^2	24882.	$37931.\times10^{+2}$	$30553.\times10^{+3}$	$11874.\times10^{+4}$	$32602.\times10^{+4}$	$72938.\times10^{+4}$	$14248.\times10^{+5}$
㉕	$-C_1K_2$ + ③ $= -C_1K_2 + \omega_r^2$	−10.106	+53.045	179.35	368.79	621.38	937.18	1316.1
㉖	㉕^{2}③	806.20	$19991.\times10^{+1}$	$68480.\times10^{+2}$	$52607.\times10^{+3}$	$24687.\times10^{+4}$	$83894.\times10^{+4}$	$23108.\times10^{+5}$
㉗	㉔ + ㉖	25688.	$39930.\times10^{+2}$	$36901.\times10^{+3}$	$17135.\times10^{+4}$	$57289.\times10^{+4}$	$15683.\times10^{+5}$	$37356.\times10^{+5}$
㉘	㉗/⑫	$.76862\times10^{-3}$	$.66826\times10^{-1}$	$.11563\times10^{-1}$	$.38625\times10^{-2}$	$.19762\times10^{-2}$	$.12207\times10^{-2}$	$.83556\times10^{-3}$
㉙	$\sqrt{㉘}$	$.27724\times10^{-1}$	.25851	.10753	$.62149\times10^{-1}$	$.44455\times10^{-1}$	$.34938\times10^{-1}$	$.28906\times10^{-1}$
㉚	M_{δ_r} ㉙/② $= \mu_{\psi\delta_r}$	1.0395	3.2311	.80638	.33291	.18521	.11909	$.83373\times10^{-1}$
㉛	$\mu_{v\delta_r}/\mu_{\psi\delta_r}$ = ⑳/㉚	1.3531	1.0307	1.0101	1.0047	1.0026	1.0017	1.0012
㉜	②H_1	10.444	31.333	52.222	73.110	93.998	114.89	135.78
㉝	㉜^2 + H_2^2	41860.	42733.	44478.	47096.	50587.	54951.	60187.
㉞	㉝/⑫	$.12525\times10^{-2}$	$.71517\times10^{-3}$	$.13937\times10^{-4}$	$.10616\times10^{-5}$	$.17450\times10^{-6}$	$.42770\times10^{-7}$	$.13462\times10^{-7}$
㉟	$\sqrt{㉞}$	$.35391\times10^{-1}$	$.26743\times10^{-1}$	$.37332\times10^{-2}$	$.10303\times10^{-2}$	$.41775\times10^{-3}$	$.20681\times10^{-3}$	$.11603\times10^{-3}$
㊱	M_{δ_r}㉟ $= \mu_{\varphi\delta_r}$	3.7284	2.8174	.39329	.10854	$.44008\times10^{-1}$	$.21787\times10^{-1}$	$.12224\times10^{-1}$

$C_1 = 27.693$
$C_2 = 1.21510$
$C_3 = 2257.2$
$C_4 = 31.474$
$C_5 = 118.67$
$C_6 = 181.49$
$K_1 = 65.998$
$K_2 = .65$
$H_1 = 3.7174$
$H_2 = 204.33$
$M_{\delta_r} = 105.35$
$K_2 + C_1 = 28.343$
$H_2^2 = 41751.$
$C_1K_2 = 18.000$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS

TABLE L-9

COMPUTATIONS FOR LEAD ANGLES, $LA_{v\delta_r}$, $LA_{\psi\delta_r}$, $LA_{\phi\delta_r}$, U_0 = 300 mph TAS at 10,000 FT. DENSITY ALTITUDE

(1)	Ω_R	0.1	0.3	0.5	0.7	0.9	1.1	1.3
(2)	$\omega_r = 2\pi T_A \Omega_R = 4.4716(2\pi)$(1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
(37)	(15)/13 = $\tan\alpha_1$	.11707	.30957	.51042	.71242	.91482	1.1174	1.3201
(38)	(10)/(5)(2) = $\tan\alpha_2$	.22178	43.452	.58484×10'	.40778	.64830	.86008	1.0597
(39)	(25)(2)/(23) = $\tan\alpha_3$	-.18000	.22957	.45581	.66560	.87018	1.0725	1.2735
(40)	$-H_2$/(32) = $\tan\alpha_4$	-19.564	-6.5212	-3.9127	-2.7948	-2.1738	-1.7785	-1.5049
(41)	$\tan^{-1}$(37) = α_1, degrees	186.68	197.20	207.04	215.47	222.45	228.17	232.86
(42)	" (38) = α_2, "	12.505	88.682	183.35	202.18	212.96	220.71	226.66
(43)	" (39) = α_3, "	349.80	12.929	24.504	33.648	41.029	47.004	51.860
(44)	" (40) = α_4, "	272.93	278.72	284.34	289.70	294.70	299.35	303.60
(45)	(41)-(42) = $\alpha_1-\alpha_2$ = $LA_{v\delta_r}$, degrees	174.18	108.52	23.69	13.29	9.49	7.46	6.20
(46)	(43)-(42) = $\alpha_3-\alpha_2$ = $LA_{\psi\delta_r}$ "	337.30	-75.753	-158.85	-168.53	-171.93	-173.71	-174.80
(47)	(44)-(42) = $\alpha_4-\alpha_2$ = $LA_{\phi\delta_r}$ "	260.42	190.04	100.99	87.52	81.74	78.64	76.94
(48)	.0174533 (45) = $LA_{v\delta_r}$, radians	3.0400	1.8940	.41347	.23195	.16563	.13020	.10821
(49)	" (46) = $LA_{\psi\delta_r}$ "	5.8853	-1.3221	-2.7725	-2.9414	-3.0007	-3.0318	-3.0508
(50)	" (47) = $LA_{\phi\delta_r}$ "	4.5452	3.3168	1.7626	1.5276	1.4266	1.3725	1.3429

$C_1 = 27.693$ $K_1 = 65.998$
$C_2 = 1.21510$ $K_2 = .65$
$C_3 = 2257.2$ $H_1 = 3.7174$
$C_4 = 31.474$ $H_2 = 204.33$
$C_5 = 118.7$
$C_6 = 181.49$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS

TABLE L-10

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{r_a}, v/δ_{r_a}, ψ/δ_{r_a} and φ/δ_{r_a} — $U_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

$\omega_r = 2.8096$, $\Omega_R = .1 \frac{\text{CYCLES}}{\text{SEC}}$

(1)	t, seconds	0	.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
(2)	$\tau = t/T_A$, (1)/4.4716	0	.20127	.40254	.60381	.80608	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
(3)	$\omega_r\tau$ (radians), 2.8096 × (2)	0	.56549	1.1310	1.6965	2.2620	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
(4)	$\omega_r\tau + \angle A_{\psi\delta_r}$, (3) + 5.8853	5.8853	6.4508	7.0163	7.5818	8.1473	8.7129	9.2782	9.8437	10.409	10.975	11.540	12.106	12.671
(5)	$\sin(\omega_r\tau + \angle A_{\psi\delta_r})$, sin (4)	−.38752	+.16682	.66913	.96316	.95727	.65329	.14608	−.40674	−.83282	−.99979	−.85545	−.44432	.10435
(6)	ψ/δ_{r_a} = 1.0395 × (5)	−.40283	.17341	.69556	1.0012	.99508	.67909	.15185	−.42281	−.86572	−1.0393	−.88924	−.46187	.10847
(7)	$\omega_r\tau + \angle A_{v\delta_r}$, (3) + 3.0400	3.0400	3.6055	4.1710	4.7365	5.3020	5.8676	6.4329	6.9984	7.5640	8.1293	8.6949	9.2605	9.8257
(8)	$\sin(\omega_r\tau + \angle A_{v\delta_r})$, sin (7)	.10140	−.44745	−.85699	−.99971	−.83118	−.40370	+.14919	.65580	.95822	.96236	.66679	.16350	−.39025
(9)	v/δ_{r_a} = 1.4065 × (8)	.14262	−.62934	−1.2054	−1.4061	−1.1691	−.56780	+.20984	.92238	1.3477	1.3536	.93784	.22996	−.54889
(10)	$\omega_r\tau + \angle A_{\varphi\delta_r}$, (3) + 4.5452	4.5452	5.1107	5.6762	6.2417	6.8072	7.3728	7.9381	8.5036	9.0692	9.6345	10.200	10.766	11.331
(11)	$\sin(\omega_r\tau + \angle A_{\varphi\delta_r})$, sin (10)	−.98605	−.92173	−.57043	−.41004 $\times 10^{-1}$	.50030	.88645	.99646	.79632	.34808	−.20825	−.69991	−.97378	−.94426
(12)	φ/δ_{r_a} = 3.7284 × (11)	−3.6764	−3.4366	−2.1268	−.15288	1.8653	3.3050	3.7152	2.9690	1.2978	−.77644	−2.6095	−3.6306	−3.5206
(13)	$\delta_r/\delta_{r_a} = \sin\omega_r\tau$ = sin (3)	0	.53583	.90483	.99211	.77051	.30885	−.24869	−.72897	−.98232	−.92978	−.58779	−.62616 $\times 10^{-1}$	+.48160

$\omega_r = 19.667$, $\Omega_R = .7 \frac{\text{CYCLES}}{\text{SEC}}$

(1)	t, seconds	0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
(2)	$\tau = t/T_A$, (1)/4.4716	0	.29072 $\times 10^{-1}$	.58145 $\times 10^{-1}$	.87217 $\times 10^{-1}$	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
(3)	$\omega_r\tau$ (radians), 19.667 × (2)	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
(4)	$\omega_r\tau + \angle A_{\psi\delta_r}$, (3) − 2.9414	−2.9414	−2.3696	−1.7979	−1.2261	−.6543	−.8260 $\times 10^{-1}$	+.4891	1.0610	1.6328	2.2045	2.7762	3.3481	3.9198
(5)	$\sin(\omega_r\tau + \angle A_{\psi\delta_r})$, sin (4)	−.19885	−.69754	−.97433	−.94118	−.60860	−.82495 $\times 10^{-1}$	.46983	.87284	.99808	.80582	.35739	−.20501	−.70203
(6)	ψ/δ_{r_a} = .33291 × (5)	−.66199 $\times 10^{-1}$	−.23222	−.32436	−.31333	−.20261	−.27463 $\times 10^{-1}$	.15641	.29058	.33227	.26827	.11898	−.68250 $\times 10^{-1}$	−.23371
(7)	$\omega_r\tau + \angle A_{v\delta_r}$, (3) + .23195	.23195	.80371	1.3754	1.9472	2.5190	3.0908	3.6624	4.2344	4.8062	5.3778	5.9496	6.5214	7.0932
(8)	$\sin(\omega_r\tau + \angle A_{v\delta_r})$, sin (7)	.22986	.71993	.98097	.92997	.58312	.50767 $\times 10^{-1}$	−.49758	−.88790	−.99561	−.78661	−.32738	+.23599	.72429
(9)	v/δ_{r_a} = .33448 × (8)	.76884 $\times 10^{-1}$	.24080	.32811	.31106	.19504	.16981 $\times 10^{-1}$	−.16643	−.29698	−.33301	−.26311	−.10950	+.78934 $\times 10^{-1}$	.24226
(10)	$\omega_r\tau + \angle A_{\varphi\delta_r}$, (3) + 1.5275	1.5275	2.0993	2.6710	3.2428	3.8146	4.3863	4.9580	5.5299	6.1017	6.6734	7.2451	7.8170	8.3887
(11)	$\sin(\omega_r\tau + \angle A_{\varphi\delta_r})$, sin (10)	.99906	.86357	.45337	−.10106	−.62333	−.94732	−.97000	−.68404	−.18052	+.38038	.82025	.99932	.86039
(12)	φ/δ_{r_a} = .10854 × (11)	.10844	.93732 $\times 10^{-1}$	.49209 $\times 10^{-1}$	−.10969 $\times 10^{-1}$	−.67656 $\times 10^{-1}$	−.10282	−.10528	−.74246 $\times 10^{-1}$	−.19594 $\times 10^{-1}$	+.41286 $\times 10^{-1}$	.89030 $\times 10^{-1}$	.10847	.93387 $\times 10^{-1}$
(13)	$\delta_r/\delta_{r_a} = \sin\omega_r\tau$, sin (3)	0	.54111	.91008	.98958	.75425	.27899	−.28485	−.75836	−.99046	−.90748	−.53597	+.63006 $\times 10^{-2}$	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS

TABLE L-11

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{r_a}, v/δ_{r_a}, ψ/δ_{r_a} and φ/δ_{r_a} V_0=300 mph TAS, 15000 FT DENSITY ALT.

$\omega_r = 36.525$, $\Omega_R = 1.3 \frac{\text{CYCLES}}{\text{SEC}}$

①	t, seconds		0	.07	.14	.21	.28	.35
②	$\gamma = t/T_r$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272
③	$\omega_r\gamma$ (radians)	36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589
④	$\omega_r\gamma + \angle A_{\psi\delta_r}$	③ − 3.0508	−3.0508	−2.4790	−1.9072	−1.3355	−.7637	−.1919
⑤	$\sin(\omega_r\gamma + \angle A_{\psi\delta_r})$	sin ④	$-.90633\times10^{-1}$	−.61511	−.94397	−.97244	−.69159	−.19072
⑥	ψ/δ_{r_a} =	.083373 × ⑤	$-.75563\times10^{-2}$	$-.51284\times10^{-1}$	$-.78702\times10^{-1}$	$-.81075\times10^{-1}$	$-.57660\times10^{-1}$	$-.15901\times10^{-1}$
⑦	$\omega_r\gamma + \angle A_{v\delta_r}$	③ + .10821	.10821	.67997	1.2518	1.8235	2.3953	2.9671
⑧	$\sin(\omega_r\gamma + \angle A_{v\delta_r})$	sin ⑦	.10798	.62876	.94955	.96824	.67893	.17365
⑨	v/δ_{r_a} =	.083470 × ⑧	$.90131\times10^{-2}$	$.52483\times10^{-1}$	$.79259\times10^{-1}$	$.80819\times10^{-1}$	$.56670\times10^{-1}$	$.14495\times10^{-1}$
⑩	$\omega_r\gamma + \angle A_{\varphi\delta_r}$	③ + 1.3429	1.3429	1.9147	2.4865	3.0582	3.6300	4.2018
⑪	$\sin(\omega_r\gamma + \angle A_{\varphi\delta_r})$	sin ⑩	.97414	.94147	.60918	.083330	−.46916	−.87250
⑫	φ/δ_{r_a} =	.012224 × ⑪	$.11908\times10^{-1}$	$.11509\times10^{-1}$	$.74466\times10^{-2}$	$.10186\times10^{-2}$	$-.57350\times10^{-2}$	$-.10665\times10^{-1}$
⑬	$\delta_r/\delta_{r_a} = \sin\omega_r\gamma$ =	sin ③	0	.54111	.91013	.98958	.75425	.27899

①	.42	.49	.56	.63	.70	.77	.84
②	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④	+.3798	.9516	1.5232	2.0952	2.6668	3.2388	3.8104
⑤	+.37072	.81434	.99886	.86559	.45710	$-.97062\times10^{-1}$	−.62005
⑥	$+.30908\times10^{-1}$	$.67894\times10^{-1}$	$.83278\times10^{-1}$	$.72167\times10^{-1}$	$.38110\times10^{-1}$	$-.80924\times10^{-2}$	$-.51695\times10^{-1}$
⑦	3.5388	4.1106	4.6822	5.2542	5.8258	6.3978	6.9694
⑧	−.38687	−.82432	−.99954	−.85681	−.44166	+.11442	.63365
⑨	$-.32292\times10^{-1}$	$-.68806\times10^{-1}$	$-.83432\times10^{-1}$	$-.71518\times10^{-1}$	$-.36865\times10^{-1}$	$+.95506\times10^{-2}$	$.52891\times10^{-1}$
⑩	4.7735	5.3453	5.9169	6.4889	7.0605	7.6325	8.2041
⑪	−.99813	−.80634	−.35835	+.20433	.70141	.97557	.93933
⑫	$-.12201\times10^{-1}$	$-.98567\times10^{-2}$	$-.43805\times10^{-2}$	$.24977\times10^{-2}$	$.85740\times10^{-2}$	$.11925\times10^{-1}$	$.11482\times10^{-1}$
⑬	−.28502	−.75836	−.99044	−.90748	−.53597	$+.64053\times10^{-2}$	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER

TABLE L-12

COMPUTATIONS FOR $LA_{v\delta_a}$, $LA_{\psi\delta_a}$, $LA_{\phi\delta_a}$, $\mu_{v\delta_a}$, $\mu_{\psi\delta_a}$, $\mu_{\phi\delta_a}$ and $\mu_{v\phi}$

(1)	Ω_A	0.1	0.3	0.5	0.7	0.9	1.1	1.3
(2)	$\omega_a = 2\pi T_A \Omega_A = 28.096$ (1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
(3)	$(2)^2 = \omega_a^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
(4)	G_1 (2)	2.7121	8.1362	13.560	18.984	24.408	29.833	35.257
(5)	$(4)^2 = (G_1 \omega_a)^2$	7.3555	66.198	183.87	360.39	595.75	890.01	1243.1
(6)	(5) $+ G_2^2$	8.4026	67.245	184.92	361.44	596.80	891.06	1244.1
(7)	(12) from Rudder Analysis = denom. under $\sqrt{\ }$	$33421. \times 10^{+3}$	$59752. \times 10^{+3}$	$31913. \times 10^{+5}$	$44363. \times 10^{+6}$	$28990. \times 10^{+7}$	$12848. \times 10^{+8}$	$44768. \times 10^{+8}$
(8)	(6)/(7)	$.25142 \times 10^{-6}$	$.11254 \times 10^{-5}$	$.57945 \times 10^{-7}$	$.81473 \times 10^{-8}$	$.20586 \times 10^{-8}$	$.69354 \times 10^{-9}$	$.27827 \times 10^{-9}$
(9)	$\sqrt{(8)}$	$.50142 \times 10^{-3}$	$.10609 \times 10^{-2}$	$.24072 \times 10^{-3}$	$.90263 \times 10^{-4}$	$.45372 \times 10^{-4}$	$.26335 \times 10^{-4}$	$.16681 \times 10^{-4}$
(10)	l_{δ_a} (9) $= \mu_{v\delta_a}$	.15370	.32519	$.73785 \times 10^{-1}$	$.27667 \times 10^{-1}$	$.13907 \times 10^{-1}$	$.80722 \times 10^{-2}$	$.51131 \times 10^{-2}$
(11)	(3) J_2	5.0396	45.357	125.99	246.93	408.19	609.81	851.72
(12)	$-J_1 -$ (11)	-28.699	-69.016	-149.65	-270.59	-431.85	-633.47	-875.38
(13)	$(12)^2$	823.63	4763.2	22395.	73219.	$18649. \times 10^{+1}$	$40128. \times 10^{+1}$	$76629. \times 10^{+1}$
(14)	(2) J_3	1.1659	3.4977	5.8295	8.1612	10.493	12.825	15.757
(15)	$(14)^2$	1.3593	12.234	33.983	66.605	110.10	164.48	229.73
(16)	(13) + (15)	824.99	4775.4	22429.	73286.	$18660. \times 10^{+1}$	$40144. \times 10^{+1}$	$76652. \times 10^{+1}$
(17)	(16)/(7)	$.24685 \times 10^{-4}$	$.79920 \times 10^{-4}$	$.70282 \times 10^{-5}$	$.16520 \times 10^{-5}$	$.64367 \times 10^{-6}$	$.31245 \times 10^{-6}$	$.17145 \times 10^{-6}$
(18)	$\sqrt{(17)}$	$.49684 \times 10^{-2}$	$.89398 \times 10^{-2}$	$.83834 \times 10^{-2}$	$.12853 \times 10^{-2}$	$.80229 \times 10^{-3}$	$.55897 \times 10^{-3}$	$.41407 \times 10^{-3}$
(19)	$(l_{\delta_a}$ (18))/(2) $= \mu_{\psi\delta_a}$	.54203	.32510	.18292	$.20032 \times 10^{-1}$	$.97255 \times 10^{-2}$	$.55439 \times 10^{-2}$	$.34749 \times 10^{-2}$
(20)	(2) J_4	10.622	31.867	53.111	74.355	95.599	116.85	138.09
(21)	$(20)^2$	112.83	1015.5	2820.8	5528.7	9139.2	13654.	19069.
(22)	$-J_5 +$ (3)	-66.521	-3.370	+122.94	312.38	564.96	880.76	1259.7
(23)	$(22)^2$	4425.	11.357	15114.	97581.	$31918. \times 10^{+1}$	$77574. \times 10^{+1}$	$15868. \times 10^{+2}$
(24)	(21) + (23)	4537.8	1026.9	17935.	$10311. \times 10^{+1}$	$32832. \times 10^{+1}$	$78939. \times 10^{+1}$	$16059. \times 10^{+2}$
(25)	(24)/(7)	$.13578 \times 10^{-3}$	$.17186 \times 10^{-4}$	$.56200 \times 10^{-5}$	$.23242 \times 10^{-5}$	$.11325 \times 10^{-5}$	$.61441 \times 10^{-6}$	$.35920 \times 10^{-6}$
(26)	$\sqrt{(25)}$	$.11652 \times 10^{-1}$	$.41456 \times 10^{-2}$	$.23707 \times 10^{-2}$	$.15245 \times 10^{-2}$	$.10642 \times 10^{-2}$	$.78385 \times 10^{-3}$	$.59933 \times 10^{-3}$
(27)	l_{δ_a} (26) $= \mu_{\phi\delta_a}$	3.5716	1.2707	.72667	.46729	.32620	.24027	.18371
(28)	$-G_2$/(4) $= \tan \lambda_1$	-.37731	-.12577	$-.75465 \times 10^{-1}$	$-.53903 \times 10^{-1}$	$-.41925 \times 10^{-1}$	$-.34301 \times 10^{-1}$	$-.29024 \times 10^{-1}$
(29)	(14)/(12) $= \tan \lambda_3$	$-.40625 \times 10^{-1}$	$-.50680 \times 10^{-1}$	$-.38954 \times 10^{-1}$	$-.30161 \times 10^{-1}$	$-.24298 \times 10^{-1}$	$-.20246 \times 10^{-1}$	$-.17315 \times 10^{-1}$
(30)	(22)/(20) $= \tan \lambda_4$	-6.2626	-.10575	+2.3148	4.2012	5.9097	7.5375	9.1223
(31)	$\tan^{-1}$ (28) $= \lambda_1$, degrees.	339.33	352.83	355.68	356.91	357.60	358.04	358.34
(32)	" (29) $= \lambda_3$, "	177.67	177.10	177.77	178.27	178.61	178.84	179.01
(33)	" (30) $= \lambda_4$, "	-80.93	-6.04	+66.636	76.611	80.396	82.443	83.744
(34)	.0174533 (31) $= \lambda_1$, radians.	5.9224	6.1580	6.2078	6.2293	6.2413	6.2490	6.2542
(35)	" (32) $= \lambda_3$, "	3.1009	3.0910	3.1027	3.1114	3.1173	3.1213	3.1243
(36)	" (33) $= \lambda_4$, "	1.4125	.10542	1.1630	1.3371	1.4032	1.4389	1.4616
(37)	" (42) from Rudder Analysis, λ_2, radians	.21825	1.5478	3.2001	3.5287	3.7169	3.8521	3.9560
(38)	(34) − (37) $= \lambda_1 - \lambda_2 = LA_{v\delta_a}$, radians.	5.7042	4.6102	3.0077	2.7006	2.5244	2.3969	2.2982
(39)	(35) − (37) $= \lambda_3 - \lambda_2 = LA_{\psi\delta_a}$, "	2.8826	1.5432	-.0974	-.4173	-.5996	-.7308	-.8317
(40)	(36) − (37) $= \lambda_4 - \lambda_2 = LA_{\phi\delta_a}$, "	1.1942	-1.4424	-.20371	-2.1916	-2.3137	-2.4132	-2.4944
(41)	$\mu_{v\delta_a} / \mu_{\phi\delta_a}$ = (10)/(27) $= \mu_{v\phi}$	$.43047 \times 10^{-1}$	.25591	.10154	$.59207 \times 10^{-1}$	$.42633 \times 10^{-1}$	$.33596 \times 10^{-1}$	$.27832 \times 10^{-1}$

V_0 = 300 mph TAS

10,000 FT. DENSITY ALTITUDE

$G_1 = 9.6529$ $J_5 = 74.415$

$G_2 = 1.0233$ $G_2^2 = 1.0471$

$J_1 = 23.659$ $l_{\delta_a} = 306.52$

$J_2 = .63842$

$J_3 = .41497$

$J_4 = 2.7807$

U_0 = 300 mph TAS
10,000 FT. DENSITY ALTITUDE

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER

TABLE L-13

COMPUTATIONS FOR TIME HISTORY OF δ_a/δ_{a_a}, v/δ_{a_a}, ψ/δ_{a_a} and φ/δ_{a_a}

Ω_A = .1 CYCLES/SEC
ω_a = 2.8096

① t, seconds	0	.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
② $\Upsilon = t/T_A$ ①/4.4716	0	.20127	.40254	.60381	.80508	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
③ $\omega_a\Upsilon$, (radians), 2.8096 × ②	0	.56549	1.1310	1.6965	2.2620	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
④ $\omega_a\Upsilon + \lambda_1 - \lambda_2$, ③ + 5.7042	5.7042	6.2697	6.8352	7.4007	7.9662	8.5318	9.0971	9.6626	10.228	10.794	11.359	11.925	12.490
⑤ $\sin(\omega_a\Upsilon + \lambda_1 - \lambda_2)$ sin ④	-.54712	$-.13439 \times 10^{-1}$	+.52443	.89902	.99371	.77890	.32177	-.23565	-.71958	-.97975	-.93470	-.59832	$-.76371 \times 10^{-1}$
⑥ v/δ_{a_a} .15370 × ⑤	$-.84092 \times 10^{-1}$	$-.20656 \times 10^{-2}$	$+.80605 \times 10^{-1}$	.13818	.15273	.11972	$.49456 \times 10^{-1}$	$-.36219 \times 10^{-1}$	-.11060	-.15059	-.14366	$-.91962 \times 10^{-1}$	$-.11738 \times 10^{-1}$
⑦ $\omega_a\Upsilon + \lambda_3 - \lambda_2$, ③ + 2.8826	2.8826	3.4481	4.0136	4.5791	5.1446	5.7102	6.2755	6.8410	7.4066	7.9719	8.5375	9.1031	9.6683
⑧ $\sin(\omega_a\Upsilon + \lambda_3 - \lambda_2)$, sin ⑦	.25612	-.30170	-.76560	-.99112	-.90807	-.54215	$-.76794 \times 10^{-2}$	+.52933	.90161	.99305	.77539	.31615	-.24108
⑨ ψ/δ_{a_a} .54203 × ⑧	.13882	-.16353	-.41498	-.53721	-.49220	-.29386	$-.41625 \times 10^{-2}$	.28691	.48870	.53826	.42028	.17136	-.13067
⑩ $\omega_a\Upsilon + \lambda_4 - \lambda_2$, ③ + 1.1942	1.1942	1.7596	2.3252	2.8907	3.4562	4.0218	4.5871	5.1526	5.7182	6.2835	6.8491	7.4147	7.9799
⑪ $\sin(\omega_a\Upsilon + \lambda_4 - \lambda_2)$ sin ⑪	.92992	.98222	.72873	.24835	-.30951	-.77085	-.99216	-.90468	-.53538	$+.2967 \times 10^{-3}$	.53612	.90505	.99209
⑫ φ/δ_{a_a} = 3.5716 × ⑪	3.3213	3.5081	2.6027	.88701	-1.1054	-2.7532	-3.5436	-3.2312	-1.9122	$.10597 \times 10^{-2}$	1.9148	3.2341	3.5433
⑬ $\delta_a/\delta_{a_a} = \sin\omega_a\Upsilon$ = sin ③	0	.53583	.90483	.99211	.77051	.30885	-.24869	-.72897	-.98232	-.92978	-.58779	$-.62616 \times 10^{-1}$	+.48160

L-77

Ω_A = .7 CYCLES/SEC.
ω_a = 19.667

① t, seconds	0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
② $\Upsilon = t/T_A$ ①/4.4716	0	$.29072 \times 10^{-1}$	$.58145 \times 10^{-1}$	$.87217 \times 10^{-1}$	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③ $\omega_a\Upsilon$, (radians), 19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④ $\omega_a\Upsilon + \lambda_1 - \lambda_2$, ③ + 2.7006	2.7006	3.2724	3.8441	4.4159	4.9877	5.5594	6.1311	6.7030	7.2748	7.8465	8.4182	8.9901	9.5618
⑤ $\sin(\omega_a\Upsilon + \lambda_1 - \lambda_2)$ sin ④	.42688	-.13035	-.64612	-.95636	-.96236	-.66223	-.15143	.40753	.83696	.99997	.84498	.42119	-.13658
⑥ v/δ_{a_a} $.27667 \times 10^{-1}$ × ⑤	$.11810 \times 10^{-1}$	$-.36064 \times 10^{-2}$	$-.17876 \times 10^{-1}$	$-.26460 \times 10^{-1}$	$-.26626 \times 10^{-1}$	$-.18322 \times 10^{-1}$	$-.41896 \times 10^{-2}$	$+.11275 \times 10^{-1}$	$+.23156 \times 10^{-1}$	$.27666 \times 10^{-1}$	$.23378 \times 10^{-1}$	$.11653 \times 10^{-1}$	$-.37788 \times 10^{-2}$
⑦ $\omega_a\Upsilon + \lambda_3 - \lambda_2$, ③ − .4173	-.4173	.15446	.7262	1.2980	1.8698	2.4415	3.0132	3.5851	4.1569	4.7286	5.3003	5.8722	6.3899
⑧ $\sin(\omega_a\Upsilon + \lambda_3 - \lambda_2)$, sin ⑦	-.40529	+.15383	.66403	.96302	.95564	.64426	.12810	-.42909	-.84962	-.99986	-.83215	-.39955	.10644
⑨ ψ/δ_{a_a} $.20032 \times 10^{-1}$ × ⑧	$-.81188 \times 10^{-2}$	$.30815 \times 10^{-2}$	$.13302 \times 10^{-1}$	$.19291 \times 10^{-1}$	$.19143 \times 10^{-1}$	$.12906 \times 10^{-1}$	$.25661 \times 10^{-2}$	$-.85955 \times 10^{-2}$	$-.17020 \times 10^{-1}$	$-.20029 \times 10^{-1}$	$-.16670 \times 10^{-1}$	$-.80038 \times 10^{-2}$	$+.21322 \times 10^{-2}$
⑩ $\omega_a\Upsilon + \lambda_4 - \lambda_2$, ③ − 2.1916	-2.1916	-1.6198	-1.0481	-.4763	+.0955	.6672	1.2389	1.8108	2.3826	2.9543	3.6260	4.0979	4.6696
⑪ $\sin(\omega_a\Upsilon + \lambda_4 - \lambda_2)$, sin ⑩	-.81341	-.99880	-.86646	-.45848	$.95342 \times 10^{-1}$	.61878	.94542	.97134	.68823	.18618	-.37493	-.81704	-.99909
⑫ φ/δ_{a_a} = .46729 × ⑪	-.38010	-.46673	-.40489	-.21424	$.44552 \times 10^{-1}$	.28915	.44179	.45390	.32160	$.87000 \times 10^{-1}$	-.17520	-.38179	-.46686
⑬ $\delta_a/\delta_{a_a} = \sin\omega_a\Upsilon$ = sin ③	0	.54111	.91008	.98958	.75425	.27899	-.28485	-.75836	-.99046	-.90748	-.53597	$.63006 \times 10^{-2}$	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER

TABLE L-14

$\Omega_A = 1.3$ CYCLES/SEC.
$\omega_a = 36.525$

COMPUTATIONS FOR TIME HISTORY OF δ_a/δ_{a_a}, v/δ_{a_a}, ψ/δ_{a_a} and φ/δ_{a_a}

① t, seconds	0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
② $\gamma = t/T_A$ ①/4.4716	0	$.15654 \times 10^{-1}$	$.31309 \times 10^{-1}$	$.46963 \times 10^{-1}$	$.62617 \times 10^{-1}$	$.78272 \times 10^{-1}$	$.93926 \times 10^{-1}$	.10958	.12523	.14089	.15654	.17220	.18785
③ $\omega_a \gamma$ (radians), 36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④ $\omega_a \gamma + \lambda_1 - \lambda_2$ ③ + 2.2982	2.2982	2.8700	3.4418	4.0135	4.5853	5.1571	5.7288	6.3006	6.8722	7.4442	8.0158	8.5878	9.1594
⑤ $\sin(\omega_a \gamma + \lambda_1 - \lambda_2)$ sin ④	.74687	.26825	-.29571	-.76560	-.99194	-.90274	-.52636	$.17400 \times 10^{-1}$	.55557	.91720	.98694	.74268	.26253
⑥ v/δ_{a_a} $.51131 \times 10^{-2}$ × ⑤	$.38188 \times 10^{-2}$	$.13716 \times 10^{-2}$	$-.15120 \times 10^{-2}$	$-.39146 \times 10^{-2}$	$-.50719 \times 10^{-2}$	$-.46158 \times 10^{-2}$	$-.26913 \times 10^{-2}$	$.88968 \times 10^{-4}$	$.28407 \times 10^{-2}$	$.46897 \times 10^{-2}$	$.50463 \times 10^{-2}$	$.37974 \times 10^{-2}$	$.13423 \times 10^{-2}$
⑦ $\omega_a \gamma + \lambda_3 - \lambda_2$, ③ − .8317	-.8317	-.25994	+.3119	.8836	1.4554	2.0272	2.5989	3.1707	3.7423	4.3143	4.8859	5.4579	6.0295
⑧ $\sin(\omega_a \gamma + \lambda_3 - \lambda_2)$ sin ⑦	-.73907	-.25701	.30686	.77302	.99335	.89764	.51638	$-.29143 \times 10^{-1}$	-.56526	-.92180	-.98499	-.73480	-.25106
⑨ ψ/δ_{a_a} $.34749 \times 10^{-2}$ × ⑧	$-.25682 \times 10^{-2}$	$-.89308 \times 10^{-3}$	$.10663 \times 10^{-2}$	$.26862 \times 10^{-2}$	$.34518 \times 10^{-2}$	$.31192 \times 10^{-2}$	$.17944 \times 10^{-2}$	$-.10127 \times 10^{-3}$	$-.19642 \times 10^{-2}$	$-.32032 \times 10^{-2}$	$-.34227 \times 10^{-2}$	$-.25534 \times 10^{-2}$	$-.87241 \times 10^{-3}$
⑩ $\omega_a \gamma + \lambda_4 - \lambda_2$, ③ − 2.4944	-2.4944	-1.9226	-1.3508	-.7791	-.2073	+.3645	.9362	1.5080	2.0796	2.6516	3.2232	3.7952	4.3668
⑪ $\sin(\omega_a \gamma + \lambda_4 - \lambda_2)$ sin ⑩	-.60293	-.93873	-.97590	-.70264	-.20581	.35648	.80531	.99803	.87335	.47055	$-.81591 \times 10^{-1}$	-.60807	-.94088
⑫ φ/δ_{a_a} = .18371 × ⑪	-.11076	-.17245	-.17928	-.12908	$-.37809 \times 10^{-1}$	$+.65489 \times 10^{-1}$	.14794	.18335	.16044	$.86445 \times 10^{-1}$	$-.14989 \times 10^{-1}$	-.11171	-.17285
⑬ $\delta_a/\delta_{a_a} = \sin \omega_a \gamma$, sin ③	0	.54111	.91013	.98958	.75425	.27899	-.28502	-.75836	-.99044	-.90748	-.53597	$.64577 \times 10^{-2}$	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS,
ADJUSTED FOR ZERO AERODYNAMIC YAW

TABLE L-15

COMPUTATIONS FOR $LA_{\delta_a \delta_r}$, $LA_{\varphi \delta_r}$, $LA_{\psi \delta_r}$, $\mu_{\delta_a \delta_r}$, $\mu_{\varphi \delta_r}$, $\mu_{\psi \delta_r}$ and $\mu_{\varphi \delta_r}/\mu_{\psi \delta_r} = \mu_{\varphi \psi}$

①	Ω_{AR}	0.1	0.3	0.5	0.7	0.9	1.1	1.3
②	$\omega_{ar} = 2\pi T_A \Omega_{AR} = 28.096$ ①	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
③	②$^2 = \omega_{ar}^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
④	$-$③ $- l_r y_\varphi$, $-$③ $- 1.2151$	-9.1090	-72.260	-198.57	-388.01	-640.60	-956.40	-1335.3
⑤	$l_p \omega_{ar} = l_p$ ②	-77.806	-233.42	-389.03	-544.64	-700.26	-855.88	-1011.5
⑥	④/⑤ $= \tan \varphi_3$	.11707	.30957	.51042	.71242	.91482	1.1174	1.3201
⑦	$(m_p - y_\varphi)$ ②	-2.7121	-8.1362	-13.560	-18.984	-24.408	-29.833	-35.257
⑧	$-m_r y_\varphi$/⑦ $= \tan \varphi_4$	-.37731	-.12577	-.075465	-.053903	-.041925	-.034301	-.029024
⑨	$\tan^{-1}$ ⑥ $= \varphi_3$, degrees	186.68	197.20	207.04	215.47	222.45	228.17	232.86
⑩	" ⑧ $= \varphi_4$, "	159.33	172.83	175.68	176.92	177.60	178.04	178.34
⑪	.0174533 ⑨ $= \varphi_3$, radians	3.2582	3.4418	3.6135	3.7607	3.8825	3.9823	4.0642
⑫	" ⑩ $= \varphi_4$, "	2.7808	3.0165	3.0662	3.0878	3.0997	3.1074	3.1126
⑬	⑪ $-$ ⑫ $= \varphi_3 - \varphi_4 = LA_{\delta_a \delta_r}$, radians	.4774	.4253	.5473	.6729	.7828	.8749	.9516
⑭	1.57080 $-$ ⑫ $= \pi/2 - \varphi_4 = LA_{\varphi \delta_r}$ "	-1.2100	-1.4457	-1.4954	-1.5170	-1.5289	-1.5366	-1.5418
⑮	$-$⑫ $= -\varphi_4 = LA_{\psi \delta_r}$, radians	-2.7808	-3.0165	-3.0662	-3.0878	-3.0997	-3.1074	-3.1126
⑯	③^2	6053.8	54,485.	$15134. \times 10^{+1}$	$29662. \times 10^{+1}$	$49035. \times 10^{+1}$	$73253. \times 10^{+1}$	$10231. \times 10^{+2}$
⑰	④^2	82.974	5221.5	39430.	$15056. \times 10^{+1}$	$41037. \times 10^{+1}$	$91470. \times 10^{+1}$	$17830. \times 10^{+2}$
⑱	⑯ + ⑰	6136.8	59,706.	$19077. \times 10^{+1}$	$44718. \times 10^{+1}$	$90072. \times 10^{+1}$	$16472. \times 10^{+2}$	$28061. \times 10^{+2}$
⑲	⑦^2	7.3555	66.198	183.87	360.39	595.75	890.01	1243.1
⑳	⑲ $+ (-m_r y_\varphi)^2$ = denominator under $\sqrt{\ }$	8.4026	67.245	184.92	361.44	596.80	891.06	1244.1
㉑	⑱/⑳	730.35	887.89	1031.6	1237.2	1509.2	1848.6	2265.5
㉒	$\sqrt{㉑}$	27.025	29.797	32.119	35.174	38.848	42.995	47.492
㉓	$(m_{\delta_r}/l_{\delta_a})$ ㉒ $= \mu_{\delta_a \delta_r}$	9.2885	10.241	11.039	12.089	13.352	14.777	16.323
㉔	$\sqrt{⑳}$	2.8987	8.2003	13.599	19.012	24.429	29.851	35.272
㉕	m_{δ_r}/㉔ $= \frac{105.35}{㉔} = \mu_{\varphi \delta_r}$	36.344	12.847	7.7469	5.5412	4.3125	3.5292	2.9868
㉖	$m_{\delta_r} y_\varphi$/② $= 34.436$/②	12.257	4.0855	2.4513	1.7510	1.3619	1.1142	.94281
㉗	㉖/㉔ $= \mu_{\psi \delta_r}$	4.2284	.49821	.18026	.092100	.055749	.037325	.026730
㉘	$\mu_{\varphi \delta_r}/\mu_{\psi \delta_r}$ = ㉕/㉗	8.5952	25.786	42.976	60.165	77.356	94.553	111.74

$U_0 = 300$ mph TAS
10,000 FT. DENSITY ALTITUDE

$l_p = -27.693$
$m_{\delta_r} = 105.35$
$l_{\delta_a} = 306.52$
$l_r y_\varphi = 1.2151$

$m_p - y_\varphi = -.96529$
$m_r y_\varphi = -1.0233$
$y_\varphi = .32687$
$(-m_r y_\varphi)^2 = 1.0471$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW

TABLE L-16

COMPUTATIONS FOR TIME HISTORY OF $\frac{\delta_r}{\delta_{r_a}}$, $\frac{\delta_a}{\delta_{r_a}}$, $\frac{\varphi}{\delta_{r_a}}$ and $\frac{\psi}{\delta_{r_a}}$

$\omega_{ar} = 2.8096$ (U_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE)

(Ω_{AR} = .1 CYCLES/SEC)

①	t, seconds.	0	.9	1.8	2.7	3.6
②	$\tau = t/T_A$, ①/4.4716	0	.20127	.40254	.60381	.80508
③	$\omega_{ar}\tau$, 2.8096 × ②	0	.56549	1.1310	1.6965	2.2620
④	δ_r/δ_{r_a}, sin ③	0	+.53583	+.90483	+.99211	+.77051
⑤	$\omega_{ar} + \angle A_{\delta_a \delta_r}$, ③ + .4774	.4774	1.0429	1.6084	2.1739	2.7394
⑥	$\sin(\omega_{ar}\tau + \angle A_{\delta_a \delta_r})$, sin ⑤	+.45947	+.86387	+.99929	+.82353	+.39137
⑦	δ_a/δ_{r_a} = 9.2885 × ⑥	4.2678	8.0241	9.2819	7.6494	3.6352
⑧	$\omega_{ar}\tau + \angle A_{\varphi\delta_r}$ = ③ − 1.2100	−1.2100	−.64451	−.07900	+.4865	1.0520
⑨	$\sin(\omega_{ar}\tau + \angle A_{\varphi\delta_r})$ = sin ⑧	−.93562	−.60081	−.078918	+.46753	+.86842
⑩	φ/δ_{r_a} = 36.344 × ⑨	−34.004	−21.836	−2.8682	+16.992	+31.562
⑪	$\omega_{ar}\tau + \angle A_{\psi\delta_r}$ = ③ − 2.7808	−2.7808	−2.2153	−1.6498	−1.0843	−.5188
⑫	$\sin(\omega_{ar}\tau + \angle A_{\psi\delta_r})$ = sin ⑪	−.35298	−.79937	−.99688	−.88398	−.49584
⑬	ψ/δ_{r_a} = 4.2284 × ⑫	−1.4925	−3.3801	−4.2152	−3.7378	−2.0966

①	t, seconds.	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
②	$\tau = t/T_A$, ①/4.4716	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
③	$\omega_{ar}\tau$, 2.8096 × ②	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
④	δ_r/δ_{r_a}, sin ③	+.30885	−.24869	−.72897	−.98232	−.92978	−.58779	−.062616	+.48160
⑤	$\omega_{ar} + \angle A_{\delta_a \delta_r}$, ③ + .4774	3.3050	3.8703	4.4358	5.0014	5.5667	6.1323	6.6979	7.2631
⑥	$\sin(\omega_{ar}\tau + \angle A_{\delta_a \delta_r})$, sin ⑤	−.16264	−.66588	−.96198	−.95852	−.65672	−.15040	+.40291	+.83040
⑦	δ_a/δ_{r_a} = 9.2885 × ⑥	−1.5107	−6.1850	−8.9354	−8.9032	−6.0999	−1.3970	3.7424	7.7132
⑧	$\omega_{ar}\tau + \angle A_{\varphi\delta_r}$ = ③ − 1.2100	1.6176	2.1829	2.7484	3.3140	3.8793	4.4449	5.0105	5.5757
⑨	$\sin(\omega_{ar}\tau + \angle A_{\varphi\delta_r})$ = sin ⑧	+.99890	+.81845	+.38317	−.17159	−.67263	−.96442	−.95590	−.64998
⑩	φ/δ_{r_a} = 36.344 × ⑨	+36.304	+29.746	+13.926	−6.2363	−24.446	−35.051	−34.741	−23.623
⑪	$\omega_{ar}\tau + \angle A_{\psi\delta_r}$ = ③ − 2.7808	.0468	.6121	1.1776	1.7432	2.3085	2.8741	3.4397	4.0049
⑫	$\sin(\omega_{ar}\tau + \angle A_{\psi\delta_r})$ = sin ⑪	.046775	.57459	.92369	.98518	.73998	.26438	−.29371	−.75995
⑬	ψ/δ_{r_a} = 4.2284 × ⑫	.19778	2.4296	3.9057	4.1657	3.1289	1.1179	−1.2419	−3.2134

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW

TABLE L-17

$\omega_{ar} = 19.667$
(V_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE) COMPUTATIONS FOR TIME HISTORY OF $\frac{\delta_r}{\delta_{r_a}}$, $\frac{\delta_a}{\delta_{r_a}}$, $\frac{\varphi}{\delta_{r_a}}$ and $\frac{\psi}{\delta_{r_a}}$ (Ω_{AR} = .7 CYCLES/SEC)

①	t, seconds.	0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
②	$\gamma = t/T_A$, ①/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③	$\omega_{ar}\gamma$, 19.667 x ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④	δ_r/δ_{r_a}, sin ③	0	.54111	.91009	.98958	.75425	.27899	-.28485	-.75836	-.99046	-.90748	-.53597	+.0063180	+.54639
⑤	$\omega_{ar}\gamma + LA_{\delta_a\delta_r}$, ③+.6729	.6729	1.2447	1.8164	2.3882	2.9600	3.5317	4.1034	4.6753	5.2471	5.8188	6.3905	6.9624	7.5341
⑥	$\sin(\omega_{ar}\gamma + LA_{\delta_a\delta_r})$, sin ⑤	.62325	.94730	.97000	.68417	.18052	-.38026	-.82025	-.99931	-.86039	-.44792	+.10711	+.62823	+.94926
⑦	δ_a/δ_{r_a} = 12.089 x ⑥	7.5346	11.452	11.726	8.2709	2.1823	-4.5970	-9.9160	-12.081	-10.401	-5.4149	1.2949	7.5947	11.476
⑧	$\omega_{ar}\gamma + LA_{\varphi\delta_r}$ = ③-1.5170	-1.5170	-.94524	-.3735	.19830	.7701	1.3418	1.9135	2.4854	3.0572	3.6289	4.2006	4.7725	5.3442
⑨	$\sin(\omega_{ar}\gamma + LA_{\varphi\delta_r})$, sin ⑧	-.99855	-.81063	-.36488	.19701	.69620	.97389	.94182	.61015	.084374	-.46824	-.87190	-.99820	-.80696
⑩	φ/δ_{r_a} = 5.542 x ⑨	-5.5332	-4.4919	-2.0219	1.0917	3.8578	5.3965	5.2188	3.3810	.46753	-2.5946	-4.8314	-5.5312	-4.4715
⑪	$(\omega_{ar}\gamma + LA_{\psi\delta_r})$ = ③-3.0878	-3.0878	-2.5160	-1.9443	-1.3725	-.8007	-.2290	+.3427	.9146	1.4864	2.0581	2.6298	3.2017	3.7734
⑫	$\sin(\omega_{ar}\gamma + LA_{\psi\delta_r})$ = sin ⑪	-.053730	-.58552	-.93106	-.98040	-.71785	-.22701	.33603	.79232	.99644	.88360	.48969	-.060003	-.59061
⑬	ψ/δ_{r_a} = .092100 x ⑫	-.0049486	-.053926	-.085751	-.090295	-.066114	-.020908	.030948	.072973	.091772	.081380	.045100	-.0055263	-.054395

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW

COMPUTATIONS FOR TIME HISTORY OF $\frac{\delta_r}{\delta_{r_a}}$, $\frac{\delta_a}{\delta_{r_a}}$, $\frac{\varphi}{\delta_{r_a}}$ and $\frac{\psi}{\delta_{r_a}}$

TABLE L-18

$\omega_{ar} = 36.525$

(U_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE)

(Ω_{AR} = 1.3 CYCLES/SEC)

① t, seconds	0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
② $\gamma = t/T_A$ ①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③ $\omega_{ar}\gamma$ = 36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④ δ_r/δ_{r_a} = sin ③	0	.54111	.91013	.98958	.75425	.27899	-.28502	-.75836	-.99044	-.90748	-.53597	+.0064577	+.54639
⑤ $\omega_{ar}\gamma + \angle A_{\delta_a\delta_r}$, ③+.9516	.9516	1.5234	2.0952	2.6669	3.2387	3.8105	4.3822	4.9540	5.5256	6.0976	6.6692	7.2412	7.8128
⑥ $\sin(\omega_{ar}\gamma + \angle A_{\delta_a\delta_r})$, sin ⑤	.81435	.99888	.86559	.45710	-.096888	-.62019	-.94597	-.97097	-.68721	-.18447	+.37655	+.81805	+.99915
⑦ δ_a/δ_{r_a} = 16.323 × ⑥	13.293	16.305	14.129	7.4612	-1.5815	-10.123	-15.441	-15.849	-11.217	-3.0111	6.1464	13.353	16.309
⑧ $\omega_{ar}\gamma + \angle A_{\varphi\delta_r}$, ③-1.5418	-1.5418	-.97004	-.39820	.17350	.74530	1.3171	1.8888	2.4606	3.0322	3.6042	4.1758	4.7478	5.3194
⑨ $\sin(\omega_{ar}\gamma + \angle A_{\varphi\delta_r})$, sin ⑧	-.99958	-.82491	-.38776	+.17263	+.67819	+.96799	+.94986	+.62956	+.10918	-.44628	-.85946	-.99937	-.82136
⑩ φ/δ_{r_a} = 2.9868 × ⑨	-2.9855	-2.4638	-1.1582	.51561	2.0256	2.8912	2.8370	1.8804	.32610	-1.3329	-2.5670	-2.9849	-2.4532
⑪ $(\omega_{ar}\gamma + \angle A_{\psi\delta_r})$, ③-3.1126	-3.1126	-2.5408	-1.9690	-1.3973	-.82550	-.25370	.31800	.88980	1.4614	1.9334	2.6050	3.1770	3.7486
⑫ $\sin(\omega_{ar}\gamma + \angle A_{\psi\delta_r})$ = sin ⑪	-.028999	-.56530	-.92176	-.98499	-.73489	-.25099	.31267	.77695	.99402	.93498	.51121	-.035398	-.57041
⑬ ψ/δ_{r_a} = .026730 × ⑫	-.00077514	-.015110	-.024639	-.026329	-.019644	-.0067090	.0083577	.020768	.026570	.024992	.013665	-.00094619	-.015247

$P_1 = 65.998$ $P_4 = 23.659$ $y_v = -.65$
$P_2 = 28.343$ $P_5 = .63842$ $m_{\delta_r} = 105.35$
$P_3 = 18.000$ $P_6 = .41497$ $l_{\delta_3} = 306.52$
$(-y_v)^2 = .42250$ $y_a = .32687$ $\mu_b = 28.074$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW

TABLE L-19

COMPUTATIONS FOR $LA_{\delta_a\delta_r}$, $LA_{\varphi\delta_r}$, $LA_{v\delta_r}$, $\mu_{\delta_a\delta_r}$, $\mu_{\varphi\delta_r}$, $\mu_{v\delta_r}$, and $\mu_{v\varphi}$

(V_0 = 300 mph TAS, 10,000 FT DENSITY ALTITUDE)

(1)	Ω_{ar}	0.1	0.3	0.5	0.7	0.9	1.1	1.3
(2)	$\omega_{ar} = 2\pi/T_a$ (1) = 29.096 (1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
(3)	(2)^2 = ω_{ar}^2	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
(4)	P_3 − (3) 18.000 − (3)	10.106	−53.045	−179.35	−368.79	−621.38	−937.18	−1316.1
(5)	(2) (4) = $\omega_{ar}(P_3 - \omega_{ar}^2)$	28.394	−447.11	−2519.5	−7253.0	−15,712.	−28964.	−48,071.
(6)	(3) P_2 28.343 × (3)	223.74	2013.6	5693.5	10963.	18,122.	27073.	37,812.
(7)	P_1 − (6) = $P_1 - \omega_{ar}^2 P_2$	−157.74	−1947.6	−5527.5	−10,897.	−18,056.	−27,007.	−37,746.
(8)	(5)/(7) = tan ψ_2	−.18001	+.22957	+.45581	+.66560	+.87018	+1.0725	+1.2735
(9)	(2) P_6 .41497 × (2)	1.1659	3.4977	5.8295	8.1612	10.493	12.826	15.157
(10)	P_5 (3) .63842 × (3)	5.0396	45.357	125.99	246.93	408.19	609.81	851.72
(11)	$-P_4$ − (10) −23.659 − (10)	−28.699	−69.016	−149.65	−270.59	−431.85	−633.47	−875.38
(12)	(9)/(11) = tan ψ_3	−.040625	−.050680	−.038954	−.030161	−.024298	−.020246	−.017315
(13)	(2)/[$-y_v$] = tan ψ_4	+4.3225	12.967	21.612	30.257	38.902	47.548	56.192
(14)	tan^{-1} (8) = ψ_2 degrees	169.80	192.93	204.50	213.65	221.03	227.00	231.86
(15)	" (12) = ψ_3 "	177.67	177.10	177.77	178.27	178.61	178.84	179.01
(16)	(13) = ψ_4 "	76.974	85.590	87.351	88.107	88.528	88.795	88.980
(17)	.0174533 (14) = ψ_2 radians	2.9636	3.3673	3.5692	3.7289	3.8577	3.9619	4.0467
(18)	" (15) = ψ_3 "	3.1009	3.0910	3.1027	3.1114	3.1173	3.1213	3.1243
(19)	" (16) = ψ_4 "	1.3435	1.4938	1.5246	1.5378	1.5451	1.5498	1.5530
(20)	(17) − (18) = $\psi_2 - \psi_3 = LA_{\delta_a\delta_r}$ "	−.1373	.2763	.4665	.6175	.7404	.8406	.9224
(21)	(19) − (18) = $\psi_4 - \psi_3 = LA_{\varphi\delta_r}$ "	−1.7574	−1.5972	−1.5781	−1.5736	−1.5722	−1.5715	−1.5713
(22)	−(18) = $-\psi_3 = LA_{v\delta_r}$ "	−3.1009	−3.0910	−3.1027	−3.1114	−3.1173	−3.1213	−3.1243
(23)	(7)^2	24,882.	$37931.\times10^{+2}$	$30553.\times10^{+3}$	$11874.\times10^{+4}$	$32602.\times10^{+4}$	$72938\times10^{+4}$	$14248\times10^{+5}$
(24)	(5)^2	806.22	$19991.\times10^{+1}$	$63479.\times10^{+2}$	$52606\times10^{+3}$	$24687.\times10^{+4}$	$83891.\times10^{+4}$	$23108.\times10^{+5}$
(25)	(23) + (24)	25,688.	$39930.\times10^{+2}$	$36901.\times10^{+3}$	$17135.\times10^{+4}$	$57289.\times10^{+4}$	$15683.\times10^{+5}$	$37356.\times10^{+5}$
(26)	(11)^2	823.63	4763.2	22,395.	73,219.	$18649.\times10^{+1}$	$40128.\times10^{+1}$	$76629.\times10^{+1}$
(27)	(9)^2	1.3593	12.234	33.983	66.605	110.10	164.48	229.73
(28)	(26) + (27) = denominator under $\sqrt{\ }$	824.99	4775.4	22,429.	73,286.	186,600.	$40,144.\times10^{+1}$	$76652.\times10^{+1}$
(29)	(25)/(28)	31.137	836.16	1645.2	2338.1	3070.2	3906.7	4873.5
(30)	[redacted]	5.5801	28.916	40.561	48.354	55.409	62.504	69.810
(31)	[redacted] = $\mu_{\delta_a\delta_r}$ .34370 × (30)	1.9179	9.9384	13.941	16.619	19.044	21.483	23.994
(32)	$(-y_v)^2$ + (3) .42250 + (3)	8.3164	71.468	197.77	387.21	639.80	955.60	1334.5
(33)	(32)/(28)	.010081	.014966	.0088176	.0052835	.0034287	.0023804	.0017410
(34)	[redacted]	.10040	.12234	.093902	.072688	.058555	.048789	.041725
(35)	m_{δ_r} (33) = $\mu_{\varphi\delta_r}$	10.577	12.889	9.8926	7.6577	6.1688	5.1399	4.3957
(36)	$1/\sqrt{(28)}$	.034815	.014471	.0066774	.0036940	.0023150	.0015783	.0011422
(37)	$m_{\delta_r} y_a$ (36) = 34.436 (36) = $\mu_{v\delta_r}$	1.1989	.49832	.22994	.12721	.079719	.054350	.039333
(38)	(37)/(35) = $\frac{v_a}{\varphi_a} = \frac{iv_a}{\varphi_a} = \frac{\mu_{v\delta_r}}{\mu_{\varphi\delta_r}} = \mu_{v\varphi}$	.11335	.038662	.023244	.016612	.012923	.010574	.0089481

(V_0 = 300 mph TAS, AT 10,000 FT. DENSITY ALTITUDE)
Ω_{AR} = .1 CYCLES/SEC.
ω_{ar} = 2.8096

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW

TABLE L-20

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{ra}, δ_a/δ_{ra}, φ/δ_{ra} and v/δ_{ra}

①	t		0	.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
②	$\gamma = t/T_a$	①/4.4716	0	.20127	.40254	.60381	.80508	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
③	$\omega_{ar}\gamma$	2.8096 × ②	0	.56549	1.1310	1.6965	2.2620	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
④	$\delta_r/\delta_{ra} = \sin \omega_{ar}\gamma$ =	sin ③	0	.63583	.90483	.99211	.77051	.30885	-.24869	-.72897	-.98232	-.92978	-.58779	-.062616	+.48160
⑤	$\omega_{ar}\gamma + \psi_2 - \psi_3$	③ -.1373	-.1373	.42819	.9937	1.5592	2.1247	2.6903	3.2556	3.8211	4.3867	4.9520	5.5176	6.0832	6.6484
⑥	$\sin(\omega_{ar}\gamma + \psi_2 - \psi_3)$	sin ⑤	-.13687	.41522	.83805	.99993	.85044	.43617	-.11372	-.62837	-.94743	-.97142	-.69290	-.19868	+.35723
⑦	$\delta_a/\delta_{ra} = \mu_{\delta_a\delta_r} \sin$ ⑤ =	1.9179 × ⑥	-.26250	.79635	1.6073	1.9178	1.6311	.83653	-.21810	-1.2052	-1.8171	-1.8631	-1.3289	-.38105	+.68513
⑧	$\omega_{ar}\gamma + \psi_4 - \psi_3$	③ -1.7574	-1.7574	-1.1919	-.6264	-.0609	+.5046	1.0702	1.6355	2.2010	2.7666	3.3319	3.8975	4.4631	5.0283
⑨	$\sin(\omega_{ar}\gamma + \psi_4 - \psi_3)$	sin ⑧	-.98265	-.92907	-.58623	-.060886	.48345	.87730	.99791	.80789	.36634	-.18910	-.68595	-.96910	-.95052
⑩	$\varphi/\delta_{ra} = \mu_{\varphi\delta_r} \sin$ ⑧ =	10.577 × ⑨	-10.393	-9.8268	-6.2006	-.64399	5.1135	9.2792	10.555	8.5451	3.8748	-2.0001	-7.2553	-10.250	-10.054
⑪	$\omega_{ar}\gamma - \psi_3$	③ -3.1009	-3.1009	-2.5354	-1.9699	-1.4044	-.8389	-.2733	+.2920	.8575	1.4231	1.9884	2.5540	3.1196	3.6848
⑫	$\sin(\omega_{ar}\gamma - \psi_3)$	sin ⑪	-.040655	-.56971	-.92139	-.98619	-.74390	-.26991	.28786	.75621	.98911	.91404	.55441	.021989	-.51683
⑬	$v/\delta_{ra} = \mu_{v\delta_r} \sin$ ⑪ =	1.1989 × ⑫	-.048741	-.68303	-1.1047	-1.1823	-.89186	-.32360	.34512	.90662	1.1858	1.0958	.66468	.026363	-.61963

(U_0 = 300 mph TAS, AT 10,000 FT. DENSITY ALTITUDE)
Ω_{AR} = .7 CYCLES/SEC.
ω_{ar} = 19.667

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW

TABLE L-21

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{ra}, δ_a/δ_{ra}, φ/δ_{ra} and v/δ_{ra}

① t		0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
② $\gamma = t/T_A$	①/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③ $\omega_{ar}\gamma$	19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④ $\delta_r/\delta_{ra} = \sin \omega_{ar}\gamma$ =	sin ③	0	.54111	.91009	.98958	.75425	.27899	−.28485	−.75836	−.99046	−.90748	−.53597	+.0063006	+.54639
⑤ $\omega_{ar}\gamma + \psi_2 - \varphi_3$	③ + .6175	.6175	1.1893	1.7610	2.3328	2.9046	3.4763	4.0480	4.6199	5.1917	5.7634	6.3351	6.9070	7.4787
⑥ $\sin(\omega_{ar}\gamma + \psi_2 - \varphi_3)$	sin ⑤	.57900	.92811	.98196	.72345	.23480	−.32854	−.78726	−.99572	−.88733	−.49667	+.051813	+.58411	+.93042
⑦ $\delta_a/\delta_{ra} = \mu_{\delta a \delta r} \sin$ ⑤ =	16.619 × ⑥	9.6224	15.424	16.319	12.023	3.9021	−5.4600	−13.083	−16.548	−14.747	−8.2542	+.86108	+9.7073	+15.463
⑧ $\omega_{ar}\gamma + \psi_4 - \psi_3$	③ −1.5736	−1.5736	−1.0018	−.4301	+.1417	.7135	1.2852	1.8569	2.4288	3.0006	3.5723	4.1440	4.7159	5.2876
⑨ $\sin(\omega_{ar}\gamma + \psi_4 - \psi_3)$	sin ⑧	−1.0000	−.84244	−.41696	.14122	.65449	.95949	.95936	.65395	.14056	−.41755	−.84273	−.99999	−.83905
⑩ $\varphi/\delta_{ra} = \mu_{\varphi\delta r} \sin$ ⑧ =	7.6577 × ⑨	−7.6577	−6.4512	−3.1930	1.0814	5.0119	7.3475	7.3465	5.0078	1.0764	−3.1975	−6.4534	−7.6576	−6.4252
⑪ $\omega_{ar}\gamma - \psi_3$	③ −3.1114	−3.1114	−2.5396	−1.9679	−1.3961	−.8243	−.2526	+.3191	.8910	1.4628	2.0345	2.6062	3.1781	3.7498
⑫ $\sin(\omega_{ar}\gamma - \psi_3)$	sin ⑪	−.030190	−.56626	−.92220	−.98478	−.73407	−.24992	+.31371	.77771	.99417	.89439	.51024	−.036469	−.57143
⑬ $v/\delta_{ra} = \mu_{v\delta r} \sin$ ⑪ =	.12721 × ⑫	−.0038405	−.072034	−.11731	−.12527	−.093381	−.031792	.039907	.098932	.12647	.11378	.064908	−.0046392	−.072692

(V_0 = 300 mph TAS, AT 10,000 FT. DENSITY ALTITUDE)
Ω_{AR} = 1.3 CYCLES/SEC
ω_{ar} = 36.525

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW

TABLE L-22

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{ra}, δ_a/δ_{ra}, φ/δ_{ra} and v/δ_{ra}

① t		0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
② $\gamma = t/T_A$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③ $\omega_{ar}\gamma$	36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④ $\delta_r/\delta_{ra} = \sin \omega_{ar}\gamma =$	sin ③	0	.54111	.91013	.98958	.75425	.27899	-.28502	-.75836	-.99044	-.90748	-.53597	+.0064577	+.54639
⑤ $\omega_{ar}\gamma + \psi_2 - \varphi_3$	③ + .9224	.9224	1.4942	2.0660	2.6377	3.2095	3.7813	4.3530	4.9248	5.4964	6.0684	6.6400	7.2120	7.7836
⑥ $\sin(\omega_{ar}\gamma + \psi_2 - \varphi_3)$	sin ⑤	.79706	.99707	.87990	.48282	-.067841	-.59693	-.93612	-.97753	-.70809	-.21320	+.34923	+.80094	+.99753
⑦ $\delta_a/\delta_{ra} = \mu_{\delta_a\delta_r} \sin ⑥$	= 23.994 × ⑥	19.125	23.924	21.112	11.585	-1.6278	-14.323	-22.461	-23.455	-16.990	-5.1155	8.3794	19.218	23.935
⑧ $\omega_{ar}\gamma + \psi_4 - \psi_3$	③ - 1.5713	-1.5713	-.99954	-.4277	+.1440	.7158	1.2876	1.8593	2.4311	3.0027	3.5747	4.1463	4.7183	5.2899
⑨ $\sin(\omega_{ar}\gamma + \psi_4 - \psi_3)$	sin ⑧	-1.0000	-.84122	-.41477	+.14351	.65622	.96017	.95867	.65223	.13848	-.41977	-.84405	-.99998	-.83781
⑩ $\varphi/\delta_{ra} = \mu_{\varphi\delta_r} \sin ⑧ =$	4.3957 × ⑨	-4.3957	-3.6978	-1.8232	+.63083	2.8845	4.2206	4.2140	2.8670	.60872	-1.8452	-3.7102	-4.3956	-3.6828
⑪ $\omega_{ar}\gamma - \psi_3$	③ - 3.1243	-3.1243	-2.5525	-1.9807	-1.4090	-.8372	-.2654	+.3063	.8781	1.4497	2.0217	2.5933	3.1653	3.7369
⑫ $\sin(\omega_{ar}\gamma - \psi_3)$	sin ⑪	-.017278	-.55557	-.91713	-.98694	-.74277	-.26229	.30154	.76952	.99268	.90009	.52116	-.023734	-.56078
⑬ $v/\delta_{ra} = \mu_{v\delta_r} \sin ⑪$	= .039333 × ⑫	-.00067960	-.021852	-.036073	-.038819	-.029215	-.010317	.011860	.030268	.039045	.035403	.020499	-.00093353	-.022057

$Q_1 = 3.7807$, $l_r = 3.7174$, $(-y_v)^2 = .4222$
$Q_2 = 74.415$, $n_{\delta_r} = 105.35$
$y_v = -.65$, $l_{\delta a} = 306.52$
$l_r y_v + \mu_b l_v = -204.33$, $(-204.33)^2 = 41,751.$

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK

TABLE L-23

COMPUTATIONS FOR $\angle A_{\delta a \delta r}$, $\angle A_{\psi \delta r}$, $\angle A_{v \delta r}$, $\mu_{\delta a \delta r}$, $\mu_{\psi \delta r}$, $\mu_{v \delta r}$ AND $\mu_{v \psi}$

$U_0 = 300$ mph TAS
10,000 FT DENSITY ALTITUDE

No.														
(1)	Ω_{dr}	0.1	0.3	0.5	0.7	0.9	1.1	1.3						
(2)	$\omega_{dr} = 2\pi \Omega_{dr}$ 28.096 (1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525	6	7	9.2			
(3)	$(2)^2 = \omega_{dr}^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1	39	49	84.64			
(4)	$Q_2 - (3)$	66.521	3.3700	-122.94	-312.38	-564.96	-880.76	-1259.7	35.415	25.415	-10.225			
(5)	$-Q_1 (2)$	-10.622	-31.867	-53.111	-74.355	-96.599	-116.85	-138.09	-22.684	-26.465	-34.782			
(6)	$(4)/(5) = \tan \sigma_2$	-6.2626	-.10575	+2.3148	+4.2012	+5.9097	+7.5375	+9.1223						
(7)	$(3)/[-y_v] = \tan \sigma_3$	4.3225	12.967	21.612	30.257	38.902	47.548	56.192						
(8)	l_r (2)	10.444	31.333	52.222	73.110	93.998	114.89	135.78	22.304	26.022	34.200			
(9)	$-Q_2/(8) = \tan \sigma_1$	19.564	6.5212	3.9127	2.7948	2.1738	1.7785	1.5049						
(10)	$\tan^{-1}$ (9) $= \sigma_1$ degrees	87.074	81.282	75.664	70.312	65.296	60.652	56.396						
(11)	" (6) $= \sigma_2$ "	99.072	173.96	246.64	256.61	260.40	262.44	263.74						
(12)	" (7) $= \sigma_3$ "	76.974	85.590	87.351	88.107	88.528	88.795	88.980						
(13)	.0174533 (10) $= \sigma_1$ radians	1.5197	1.4186	1.3206	1.2272	1.1396	1.0586	.98430						
(14)	" (11) $= \sigma_2$ "	1.7291	3.0362	4.3047	4.4787	4.5448	4.5804	4.6031						
(15)	" (12) $= \sigma_3$ "	1.3435	1.4938	1.5246	1.5378	1.5451	1.5498	1.5530						
(16)	$(13) - (14) = \sigma_1 - \sigma_2 = \angle A_{\delta a \delta r}$ "	-.2094	-1.6176	-2.9841	-3.2515	-3.4052	-3.5218	-3.6188						
(17)	$(15) - (14) = \sigma_3 - \sigma_2 = \angle A_{\psi \delta r}$ "	-.3856	-1.5424	-2.7801	-2.9409	-2.9997	-3.0306	-3.0501						
(18)	$4.71240 - (14) = \frac{3\pi}{2} - \sigma_2 = \angle A_{v \delta r}$ "	2.9833	1.6762	.4077	-.2337	.1676	.1320	.1093						
(19)	$(5)^2$	112.83	1015.5	2820.8	5528.7	9139.2	13,654.	19,069.	514.56	700.40	1209.8			
(20)	$(4)^2$	4425.0	11.357	15,114.	97,581.	$31918. \times 10^{+1}$	$77574. \times 10^{+1}$	$15868. \times 10^{+2}$	1254.2	645.92	104.55			
(21)	(19) + (20) = denominator under $\sqrt{\ }$	4537.8	1026.9	17,935.	$10311. \times 10^{+1}$	$32832. \times 10^{+1}$	$78939. \times 10^{+1}$	$16057. \times 10^{+2}$	1768.8	1346.3	1314.4			
(22)	$(8)^2$	109.08	981.76	2727.1	5345.1	8835.6	13200.	18436.	497.47	677.14	1169.6			
(23)	$(22) + (-Q_2)^2 =$	41,860.	42,733.	44,478.	47,096.	50,587	54,951	60,187.	42,248.	42,428.	42,921.			
(24)	(23)/(21)	9.2247	41.614	2.4800	.45675	.15408	.069612	.037479	23.885	31.515	32.654			
(25)	$\sqrt{(24)}$	3.0372	6.4509	1.5748	.67583	.39253	.26384	.19359	4.8872	5.6138	5.7144			
(26)	$n_{\delta r}$ (25)$/l_{\delta a} = \mu_{\delta a \delta r} = .34370$ (25)	1.0439	2.2172	.54126	.23228	.13491	.090682	.066537	1.6797	1.9295	1.9640			
(27)	$(-y_v)^2 + (3) = .42250 + (3)$	8.3164	71.468	197.77	387.21	639.80	955.60	1334.5	39.422	49.422	85.062			
(28)	(27)/(21)	.0018327	.069596	.011027	.0037553	.0019487	.0012106	.00083100	.022287	.036710	.064715			
(29)	$\sqrt{(28)}$	.042810	.26381	.10501	.061280	.044144	.034794	.028827	.14929	.19160	.25439			
(30)	$n_{\delta r}$ (29)/(2) $= \mu_{\psi \delta r}$ 105.35 (29)/(2)	1.6052	3.2973	.78751	.32826	.18392	.11860	.083146	2.6213	2.8836	2.9130			
(31)	$\sqrt{(21)}$	67.363	32.045	133.92	321.11	572.99	888.48	1267.2	42.057	36.692	36.255			
(32)	$n_{\delta r}/(31) = \mu_{v \delta r}$ 105.35/(31)	1.5639	3.2876	.78666	.32808	.18386	.11857	.083136	2.5049	2.8712	2.9058			
(33)	$(32)/(30) = \frac{\mu_{v \delta r}}{\mu_{\psi \delta r}} = \frac{v_a}{\psi_a} \cdot \frac{v_a}{\psi_a} = \mu_{v \psi}$	.97427	.99706	.99892	.99945	.99967	.99975	.99988						
(34)														

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK

TABLE L-24

Ω_{AR} = .1 CYCLES/SECOND
ω_{ar} = 2.8096

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{ra}, δ_a/δ_{ra}, ψ/δ_{ra} and v/δ_{ra}

U_0 = 300 mph TAS, 10,000 FT DENSITY ALT.

①	t, seconds		0	.9	1.8	2.7	3.6	4.5	5.4	6.3	7.2	8.1	9.0	9.9	10.8
②	$\gamma = t/\tau_A$	= ①/4.4716	0	.20127	.40254	.60381	.80508	1.0064	1.2076	1.4089	1.6102	1.8114	2.0127	2.2140	2.4152
③	$\omega_{ar}\gamma$	2.8096 × ②	0	.56549	1.1310	1.6965	2.2620	2.8276	3.3929	3.9584	4.5240	5.0893	5.6549	6.2205	6.7857
④	$\sin \omega_{ar}\gamma = \delta_r/\delta_{ra}$ =	sin ③	0	.53583	.90483	.99211	.77051	.30885	-.24869	-.72897	-.98232	-.92978	-.58779	-.062616	+.48160
⑤	$\omega_{ar}\gamma + \sigma_1 - \sigma_2$	③ -.2094	-.2094	.35609	.92160	1.4871	2.0526	2.6182	3.1835	3.7490	4.3146	4.8799	5.4455	6.0111	6.5763
⑥	$\sin(\omega_{ar}\gamma + \sigma_1 - \sigma_2)$	sin ⑤	-.20788	.34860	.79657	.99650	.88612	.49985	-.041876	-.57071	-.92193	-.98600	-.74314	-.26875	+.28886
⑦	δ_a/δ_{ra} =	1.0439 × ⑥	-.21701	.36390	.83154	1.0402	.92502	.52179	-.043714	-.59576	-.96240	-1.0293	-.77576	-.28055	+.30154
⑧	$\omega_{ar}\gamma + \sigma_3 - \sigma_2$	③ -.3856	-.3856	.17989	.7454	1.3109	1.8764	2.4420	3.0073	3.5728	4.1384	4.7037	5.2693	5.8349	6.4001
⑨	$\sin(\omega_{ar}\gamma + \sigma_3 - \sigma_2)$	sin ⑧	-.37611	.17892	.67826	.96642	.95366	.64386	.13381	-.41803	-.83971	-.99996	-.84888	-.43334	+.11667
⑩	ψ/δ_{ra}	1.6052 × ⑨	-.60373	.28720	1.0887	1.5513	1.5308	1.0335	.21479	-.67102	-1.3479	-1.6051	-1.3626	-.69560	+.18728
⑪	$\omega_{ar}\gamma + \frac{3\pi}{2} - \sigma_2$	③ + 2.9833	2.9833	3.5488	4.1143	4.6798	5.2453	5.8109	6.3762	6.9417	7.5073	8.0726	8.6382	9.2038	9.7690
⑫	$\sin(\omega_{ar}\gamma + \frac{3\pi}{2} - \sigma_2)$	sin ⑪	.15764	-.39603	-.82639	-.99947	-.86136	-.45492	+.092892	+.61194	+.94053	+.97618	+.70797	+.21917	-.33742
⑬	v/δ_{ra}	1.5639 × ⑫	.24653	-.61935	-1.2924	-1.5631	-1.3471	-.71145	.14527	.95701	1.4709	1.5266	1.1072	.34276	-.52769

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK

TABLE L-25

Ω_{AR} = .7 CYCLES/SECOND
ω_{ar} = 19.667

COMPUTATIONS FOR TIME HISTORY OF $\delta r/\delta r_a$, $\delta a/\delta r_a$, $\psi/\delta r_a$ and $v/\delta r_a$

U_0 = 300 mph TAS
10,000 FT. DENSITY ALT.

(1)	t, seconds		0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
(2)	$\gamma = t/T_A$	(1)/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
(3)	$\omega_{ar}\gamma$	19.667 x (2)	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
(4)	$\sin\omega_{ar}\gamma = \delta r/\delta r_a$ =	sin (3)	0	.54111	.91009	.98958	.75425	.27899	-.28485	-.75836	-.99046	-.90748	-.53597	+.0062831	+.54639
(5)	$\omega_{ar}\gamma + \tau_1 - \tau_2$	(3) - 3.2515	-3.2515	-2.6797	-2.1080	-1.5362	-.9644	-.3927	+.1790	.7509	1.3227	1.8944	2.4661	3.0380	3.6097
(6)	$\sin(\omega_{ar}\gamma + \tau_1 - \tau_2)$	sin (5)	+.10973	-.44557	-.85914	-.99940	-.82171	-.38268	+.17805	.68229	.96938	.94810	.62524	.10349	-.45119
(7)	$\delta a/\delta r_a$ =	.23228 x (6)	+.025488	-.10350	-.19956	-.23214	-.19087	-.088889	.041357	.15848	.22517	.22022	.14523	.024039	-.10480
(8)	$\omega_{ar}\gamma + \tau_3 - \tau_2$	(3) -2.9409	-2.9409	-2.3691	-1.7974	-1.2256	-.6538	-.0821	+.4896	1.0615	1.6333	2.2050	2.7767	3.3486	3.9203
(9)	$\sin(\omega_{ar}\gamma + \tau_3 - \tau_2)$	sin (8)	-.19937	-.69792	-.97445	-.94101	-.60821	-.082008	+.47027	.87308	.99805	.80551	.35690	-.20552	-.70240
(10)	$\psi/\delta r_a$	.32826 x (9)	-.065445	-.22910	-.31987	-.30890	-.19965	-.026920	+.15437	.28660	.32762	.26442	.11716	-.067464	-.23057
(11)	$\omega_{ar}\gamma + \frac{3\pi}{2} - \tau_2$	(3) +.2337	.2337	.80546	1.3772	1.9490	2.5208	3.0925	3.6642	4.2361	4.8079	5.3796	5.9513	6.5232	7.0949
(12)	$\sin(\omega_{ar}\gamma + \frac{3\pi}{2} - \tau_2)$	sin (11)	.23158	.72114	.98132	.92933	.58170	.049024	-.49909	-.88870	-.99545	-.78553	-.32590	+.23769	+.72549
(13)	$v/\delta r_a$	.32808 x (12)	.075977	.23659	.32196	.30489	.19084	.016084	-.16374	-.29156	-.32659	-.25772	-.10692	.077981	.23802

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK

TABLE L-26

Ω_{AR} = 1.3 CYCLES/SECOND
ω_{ar} = 36.525

COMPUTATIONS FOR TIME HISTORY OF δ_r/δ_{r_a}, δ_a/δ_{r_a}, ψ/δ_{r_a} and v/δ_{r_a}

V_0 = 300 mph TAS
10,000 FT. DENSITY ALT.

①	t		0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
②	$\gamma = t/T$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③	$\omega_{ar}\gamma$	36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④	$\sin \omega_{ar}\gamma = \delta_r/\delta_{r_a}$	sin ③	0	.54111	.91013	.98958	.75425	.27899	-.28502	-.75836	-.99044	-.90748	-.53597	+.0064577	+.54639
⑤	$\omega_{ar}\gamma + \sigma_1 - \sigma_2$	③ − 3.6188	-3.6188	-3.0470	-2.4752	-1.9035	-1.3317	-.7599	-.1882	+.3836	+.9552	+1.5272	+2.0988	+2.6708	+3.2424
⑥	$\sin(\omega_{ar}\gamma + \sigma_1 - \sigma_2)$	sin ⑤	+.45927	-.094456	-.61813	-.94518	-.97155	-.68885	-.18709	+.37427	+.81643	+.99905	+.86384	+.45352	+.10071
⑦	δ_a/δ_{r_a}	.066537 × ⑥	+.030558	-.0062848	-.041129	-.062889	-.064644	-.045834	-.012448	+.024903	+.054323	+.066474	+.057477	+.030176	+.0067009
⑧	$\omega_{ar}\gamma + \sigma_3 - \sigma_2$	③ − 3.0501	-3.0501	-2.4783	-1.9065	-1.3348	-.7630	-.1912	+.3805	+.9523	1.5239	2.0959	2.6675	3.2395	3.8111
⑨	$\sin(\omega_{ar}\gamma + \sigma_3 - \sigma_2)$	sin ⑧	-.091328	-.61566	-.94420	-.97228	-.69110	-.19004	+.37138	+.81475	+.99890	+.86524	+.45648	-.097757	-.62060
⑩	ψ/δ_{r_a}	.083146 × ⑨	-.0075936	-.051190	-.078506	-.080841	-.057462	-.015801	+.030879	+.067743	+.083055	+.071941	+.037954	-.0081281	-.051600
⑪	$\omega_{ar}\gamma + \frac{2\pi}{2} - \sigma_2$	③ + .1093	.1093	.68106	1.2529	1.8246	2.3964	2.9682	3.5399	4.1117	4.6833	5.2553	5.8269	6.3989	6.9705
⑫	$\sin(\omega_{ar}\gamma + \frac{3\pi}{2} - \sigma_2)$	sin ⑪	.10908	.62962	.94990	.96797	.67816	.17244	-.38784	-.82492	-.99958	-.85618	-.44057	+.11546	+.63446
⑬	v/δ_{r_a}	.083136 × ⑫	.0090685	.052344	.078971	.080473	.056380	.014336	-.032243	-.068581	-.083101	-.071179	-.036627	+.0095989	+.052746

APPENDIX M

ANALYSES FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO STEP FUNCTIONS

APPENDIX M

ALYSES FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE STEP FUNCTIONS

A. ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP FUNCTION.

With reference to Section A in Appendix I, a summary of the basic equations used in this analysis follows:

$$a_1 d^4 + a_2 d^3 + a_3 d^2 + a_4 d + a_5 = 0 \quad (1)$$

$$a_1 = 1 \quad (2)$$

$$a_2 = -(z_w + m_q + \mu_c m_{\dot{w}} + x_u) \quad (3)$$

$$a_3 = x_u(z_w + m_q) + \mu_c m_{\dot{w}}(x_u - z_\theta) + z_w m_q - \mu_c m_w - x_w z_u \quad (4)$$

$$a_4 = \mu_c m_{\dot{w}}(z_\theta x_u - z_u x_\theta) + \mu_c m_w (x_u - z_\theta)$$

$$+ \mu_c m_u (x_\theta + x_w) - m_q (x_u z_w - x_w z_u) \quad (5)$$

$$a_5 = \mu_c m_w (x_u z_\theta - x_\theta z_u) + \mu_c m_u (x_\theta z_w - x_w z_\theta)$$

$$-\mu_c m_w x_\theta z_u \quad (6)$$

$$u = e^{\alpha_1 \gamma} u_1 \sin(\beta_1 \gamma + LA_{u1}) + e^{\alpha_2 \gamma} u_2 \sin(\beta_2 \gamma + LA_{u2}) \quad (7)$$

$$w = e^{\alpha_1 \gamma} w_1 \sin(\beta_1 \gamma + LA_{w1}) + e^{\alpha_2 \gamma} w_2 \sin(\beta_2 \gamma + LA_{w2}) \quad (8)$$

$$\theta = e^{\alpha_1 \gamma} \theta_1 \sin(\beta_1 \gamma + LA_{\theta 1}) + e^{\alpha_2 \gamma} \theta_2 \sin(\beta_2 \gamma + LA_{\theta 2}) \quad (9)$$

$$u_1 = \sqrt{A_u^2 + B_u^2} \quad (10)$$

$$u_2 = \sqrt{C_u^2 + D_u^2} \quad (11)$$

$$w_1 = \sqrt{A_w^2 + B_w^2} \quad (12)$$

$$w_2 = \sqrt{C_w^2 + D_w^2} \quad (13)$$

$$\theta_1 = \sqrt{A_\theta^2 + B_\theta^2} \quad (14)$$

$$\theta_2 = \sqrt{C_\theta^2 + D_\theta^2} \quad (15)$$

$$LA_{u1} = \tan^{-1} \frac{A_u}{B_u} \quad (16)$$

$$LA_{u2} = \tan^{-1} \frac{C_u}{D_u} \quad (17)$$

$$LA_{w1} = \tan^{-1} \frac{A_w}{B_w} \quad (18)$$

$$LA_{w2} = \tan^{-1} \frac{C_w}{D_w} \quad (19)$$

$$LA_{\theta 1} = \tan^{-1} \frac{A_\theta}{B_\theta} \quad (20)$$

$$LA_{\theta 2} = \tan^{-1} \frac{C_\theta}{D_\theta} \quad (21)$$

$$\dot{u}_o = x_u u_o + x_w w_o + x_\theta \theta_o \quad (22)$$

$$\dot{w}_o = z_u u_o + z_w w_o + \dot{\theta}_o + z_\theta \theta_o \quad (23)$$

$$\ddot{\theta}_o = \mu_c m_u u_o + \mu_c m_{\dot{w}} \dot{w}_o + \mu_c m_w w_o + m_q \dot{\theta}_o \quad (24)$$

$$\ddot{u}_o = x_u \dot{u}_o + x_w \dot{w}_o + x_\theta \dot{\theta}_o \quad (25)$$

$$\ddot{w}_o = z_u \dot{u}_o + z_w \dot{w}_o + \ddot{\theta}_o + z_\theta \dot{\theta}_o \quad (26)$$

$$\dddot{\theta}_o = \mu_c m_u \dot{u}_o + \mu_c m_{\dot{w}} \ddot{w}_o + \mu_c m_w \dot{w}_o + m_q \ddot{\theta}_o \quad (27)$$

$$\dddot{u}_o = x_u \ddot{u}_o + x_w \ddot{w}_o + x_\theta \ddot{\theta}_o \quad (28)$$

$$\dddot{w}_o = z_u \ddot{u}_o + z_w \ddot{w}_o + \dddot{\theta}_o + z_\theta \ddot{\theta}_o \quad (29)$$

A_u, B_u, C_u, D_u, A_w, B_w, C_w, D_w, A_θ, B_θ, C_θ and D_θ were found by determinants, as described in Section A of Appendix I.

Assumed Boundary Conditions:

$u_o = 0$, $w_o = .05$, $\theta_o = .05$, $\dot{\theta}_o = 0$, $U_o = 300$ mph TAS at 10,000 feet density altitude (30)

Computed Boundary Conditions:

Using the values for the stability coefficients which were computed in Section A-1 of Appendix D, the computed boundary conditions were found from equations 22 to 29 inclusive.

Results were as follows:

$$\begin{aligned} \dot{u}_o &= -.0069835 & \dddot{\theta}_o &= 164.15 \\ \dot{w}_o &= -.24350 & \dddot{u}_o &= 1.0886 \\ \ddot{\theta}_o &= -6.1708 & \dddot{w}_o &= 188.43 \\ \ddot{u}_o &= -.044974 & & \\ \ddot{w}_o &= -4.9803 & & \end{aligned} \quad (31)$$

Characteristic Equation

Using the values for the stability coefficients which were computed in Section A-1 of Appendix D, the coefficients for the characteristic equation were computed, using equations 2 to 5 inclusive. Then, from equation 1, the characteristic equation was found to be:

$$d^4 + 26.6926\, d^3 + 231.712\, d^2 + 23.156\, d + 31.959 = 0 \quad (32)$$

Using the Graeffe method, the roots to equation 32 were as follows:

$$d_1 = \alpha_1 + i\beta_1 = -13.303 + 7.2348\, i \quad (33)$$

$$d_2 = \alpha_1 - i\beta_1 = -13.303 - 7.2348\, i \quad (34)$$

$$d_3 = \alpha_2 + i\beta_2 = -.042404 + .37090\, i \quad (35)$$

$$d_4 = \alpha_2 - i\beta_2 = -.042404 - .37090\, i \quad (36)$$

The roots and the boundary conditions were then used to solve for A_u, B_u, C_u, D_u, A_w, B_w, C_w, D_w, A_θ, B_θ, C_θ, and D_θ, by determinants, as described between equations 54 and 57 in Section A of Appendix I. These calculations are shown in Tables M-1 and M-2.

Then u_1, u_2, w_1, w_2, θ_1 and θ_2 were computed from equations 10 to 15, inclusive. These calculations are shown in Table M-2.

Using equations 7, 8 and 9, the short-period and the long-period components of the transient response for each variable were computed. Calculations for the short-period component of the change in longitudinal velocity per U_o are shown in Table M-3, and the results are plotted in Figure M-1. Calculations for the long-period component of the change in longitudinal velocity per U_o are shown in Tables M-4 and M-5, and the results are plotted in Figure M-2. Calculations for the short-period component of the change in normal velocity per U_o are shown in Table M-6, and the results are plotted in Figure M-3. Calculations for the long-period component of the change in normal velocity per U_o are shown in Table M-7, and the results are plotted in Figure M-4. Calculations for the short-period component of the change in pitch angle are shown in Table M-8, and the results are plotted in Figure M-5. Calculations for the long-period component of the change in pitch angle are shown in Table M-9, and the results are plotted in Figure M-6.

The total transient responses, for change in longitudinal velocity per U_o, change in normal velocity per U_o, and change in pitch angle are plotted in Figures O-1, O-2 and O-3, respectively.

B. ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

With reference to Section B in Appendix I, a summary of the basic equations used in this analysis follow:

$$d(b_1 d^4 + b_2 d^3 + b_3 d^2 + b_4 d + b_5) = 0 \qquad (37)$$

$$b_1 = 1 \qquad (38)$$

$$b_2 = -(n_r + l_p + y_v) \qquad (39)$$

$$b_3 = (l_p n_r - l_r n_p) + y_v(n_r + l_p) + \mu_b n_v \qquad (40)$$

$$b_4 = y_v(l_r n_p - l_p n_r) + \mu_b(l_v n_p - l_p n_v - l_v y_\phi) \qquad (41)$$

$$b_5 = \mu_b y_\phi(l_v n_r - l_r n_v) \qquad (42)$$

$$v = v_1 + v_2 e^{d_2\gamma} + v_3 e^{d_3\gamma} + e^{\sigma\gamma} v_4 \sin(\epsilon\gamma + LA_v) \qquad (43)$$

$$\psi = \psi_1 + \psi_2 e^{d_2\gamma} + \psi_3 e^{d_3\gamma} + e^{\sigma\gamma} \psi_4 \sin(\epsilon\gamma + LA_\psi) \qquad (44)$$

$$\phi = \phi_1 + \phi_2 e^{d_2\gamma} + \phi_3 e^{d_3\gamma} + e^{\sigma\gamma} \phi_4 \sin(\epsilon\gamma + LA_\phi) \qquad (45)$$

$$v_4 = \sqrt{A_v^2 + B_v^2} \qquad (46)$$

$$\psi_4 = \sqrt{A_\psi^2 + B_\psi^2} \qquad (47)$$

$$\phi_4 = \sqrt{A_\phi^2 + B_\phi^2} \qquad (48)$$

$$LA_v = \tan^{-1}\frac{A_v}{B_v} \qquad (49)$$

$$LA_\psi = \tan^{-1}\frac{A_\psi}{B_\psi} \qquad (50)$$

$$LA_\phi = \tan^{-1}\frac{A_\phi}{B_\phi} \quad (51)$$

$$\ddot{\phi}_o = \mu_b l_v v_o + l_r\dot{\psi}_o + l_p\dot{\phi}_o \quad (52)$$

$$\ddot{\psi}_o = \mu_b n_v v_o + n_r\dot{\psi}_o + n_p\dot{\phi}_o \quad (53)$$

$$\dot{v}_o = y_v v_o - \dot{\psi}_o + y_\phi \phi_o \quad (54)$$

$$\dddot{\phi}_o = \mu_b l_v \dot{v}_o + l_r\ddot{\psi}_o + l_p\ddot{\phi}_o \quad (55)$$

$$\dddot{\psi}_o = \mu_b n_v \dot{v}_o + l_r\ddot{\psi}_o + l_p\ddot{\phi}_o \quad (56)$$

$$\ddot{v}_o = y_v \dot{v}_o - \ddot{\psi}_o + y_\phi \dot{\phi}_o \quad (57)$$

$$\ddddot{\phi}_o = \mu_b l_v \ddot{v}_o + l_r\dddot{\psi}_o + l_p\dddot{\phi}_o \quad (58)$$

$$\ddddot{\psi} = \mu_b n_v \ddot{v}_o + n_r\dddot{\psi}_o + n_p\dddot{\phi}_o \quad (59)$$

$$\dddot{v}_o = y_v \ddot{v}_o - \dddot{\psi}_o + y_\phi \ddot{\phi}_o \quad (60)$$

$$\overset{.....}{\phi}_o = \mu_b l_v \dddot{v}_o + l_r\ddddot{\psi}_o + l_p\ddddot{\phi}_o \quad (61)$$

$$\overset{.....}{\psi}_o = \mu_b n_v \dddot{v}_o + n_r\ddddot{\psi}_o + n_p\ddddot{\phi}_o \quad (62)$$

$$\ddddot{v}_o = y_v \dddot{v}_o - \ddddot{\psi}_o + y_\phi \dddot{\phi}_o \quad (63)$$

v_1, v_2, v_3, ψ_1, ψ_2, ψ_3, ϕ_1, ϕ_2, ϕ_3, A_v, B_v, A_ψ, B_ψ, A_ϕ and B_ϕ were found by determinants, as described in Section A of Appendix I. v_1, v_2, v_3, A_v and B_v were found by using equations

117, 118, 119, 120 and 121 in Section B of Appendix I, and the rest were found by similar equations for ψ and ϕ.

Assumed Boundary Conditions:

U_o = 300 mph TAS at 10,000 ft. density altitude,

$\psi_o = 0$, $\dot{\psi}_o = 0$, $v_o = 0$, $\phi_o = .05$ and $\dot{\phi}_o = 0$.

Computed Boundary Conditions:

Using the values for the stability coefficients which were computed in Section B-1 of Appendix D, the computed boundary conditions were found by equations 52 to 63, inclusive.

Results were as follows:

$\dot{v}_o = .016344$	$\ddot{\phi}_o = 0$	$\ddot{\psi}_o = 0$
$\ddot{v}_o = -.010624$	$\dddot{\phi}_o = -3.3000$	$\dddot{\psi}_o = 1.1830$
$\dddot{v}_o = -1.1761$	$\ddddot{\phi}_o = 97.930$	$\ddddot{\psi}_o = -2.3658$
$\ddddot{v}_o = 2.0516$		

Characteristic Equation:

Using the values for the stability coefficients which were computed in Section B-1 of Appendix D, the coefficients for the characteristic equation were computed, using equations 38 to 42, inclusive. Then, using equation 37, the characteristic equation was found to be:

$$d(d^4 + 31.4737d^3 + 181.49d^2 + 2257.22d + 118.67 = 0. \quad (64)$$

Using the Graeffe method, the roots to equation 64 were as follows:

$d_1 = 0$ $\quad$ $\bar{d}_4 = -1.7795 + 8.8037\ i$

$d_2 = -27.862$ $\quad$ $\bar{d}_5 = -1.7795 - 8.8037\ i$

$d_3 = -.052796$

The roots and the boundary conditions were then used to solve for v_1, v_2, v_3, ψ_1, ψ_2, ψ_3, ϕ_1, ϕ_2, ϕ_3, A_v, B_v, A_ψ, B_ψ, A_ϕ and B_ϕ, by the method of determinants. This method is described between equations 54 and 57 in Section A of Appendix I. The determinants and the results are shown in Table M-10.

Then v_4, ψ_4, ϕ_4, LA_v, LA_ψ and LA_ϕ were computed from equations 46 to 51 respectively. These calculations are also shown in Table M-10.

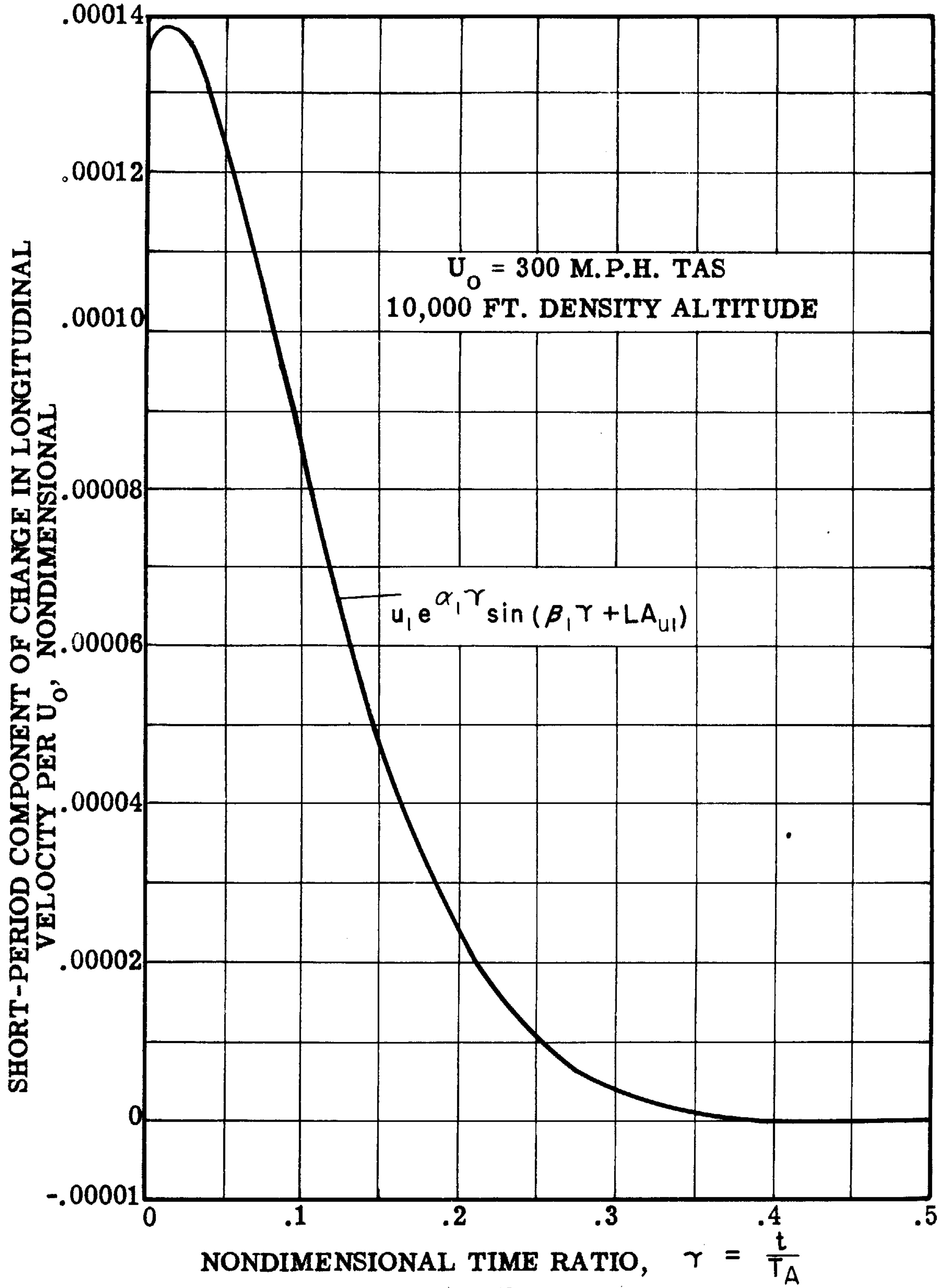
The root $d_2 = -27.862$, controls the "roll subsidence" which is a highly damped component. The root $d_3 = -.052796$ controls the "spiral stability". If d_3 were positive the airplane would be spirally unstable, but since it is negative it indicates spiral stability. The complex conjugate roots $\bar{d}_4 = -1.7795 + 8.8037\ i$, and $\bar{d}_5 = -1.7795 - 8.8037$, control the "Dutch roll". Since $\sigma = -1.7795$, is negative, this component also damps out.

The "subsidence" components of v, ψ and ϕ were computed in Tables 11, 12 and 13, and the results are plotted in Figure M-7.

The "spiral stability" components of v, ψ and ϕ were computed in Tables 11, 12 and 13, and the results are plotted in Figure M-8.

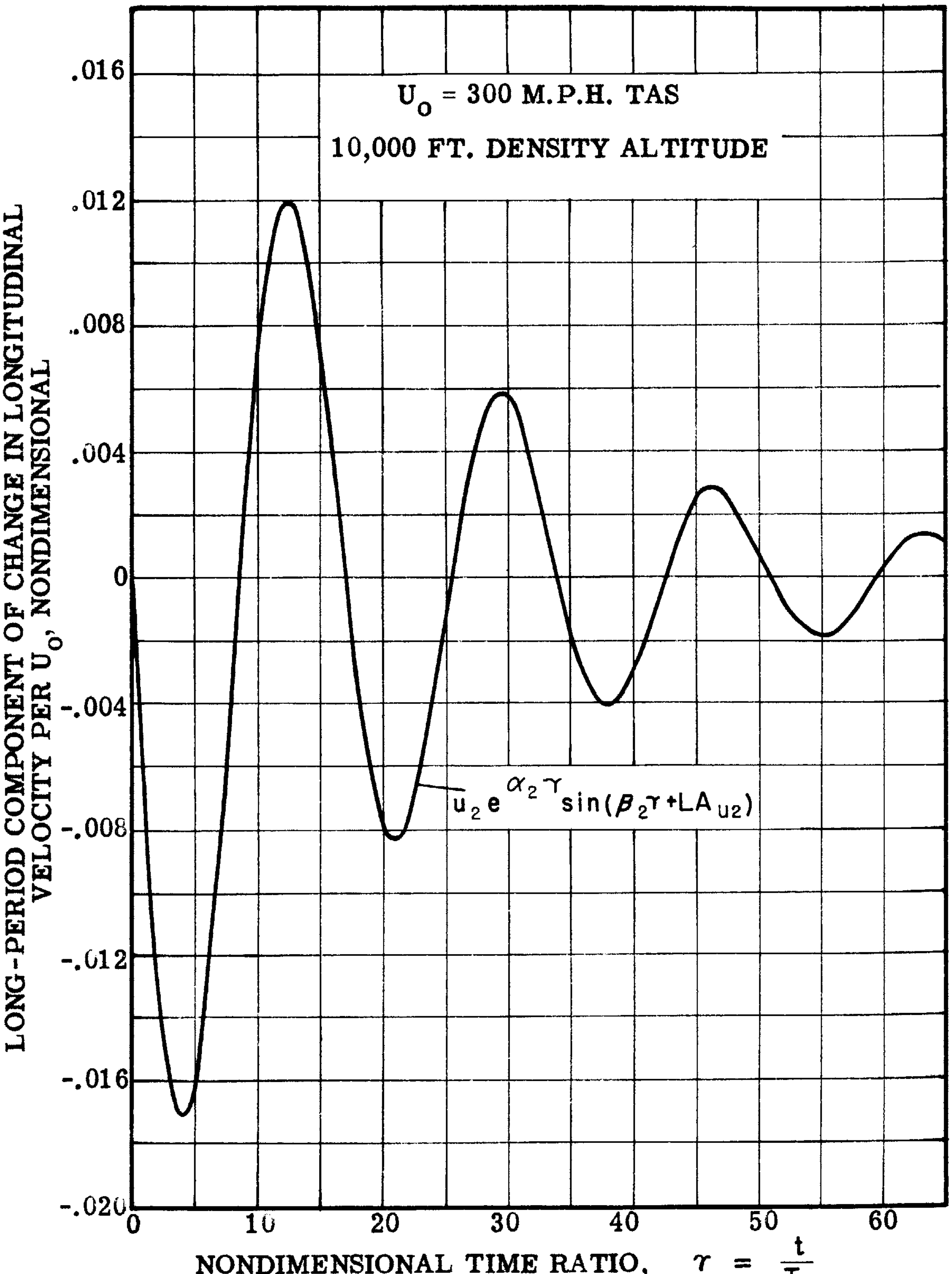
The "Dutch Roll" components of v, ψ and ϕ were computed in Tables 11, 12 and 13, and the results are plotted in Figure M-9.

The total motion for v, ψ and ϕ was obtained by adding all the components, using equations 43, 44 and 45. These computations are shown in Tables 11, 12 and 13, and the results are plotted in Figure M-10.



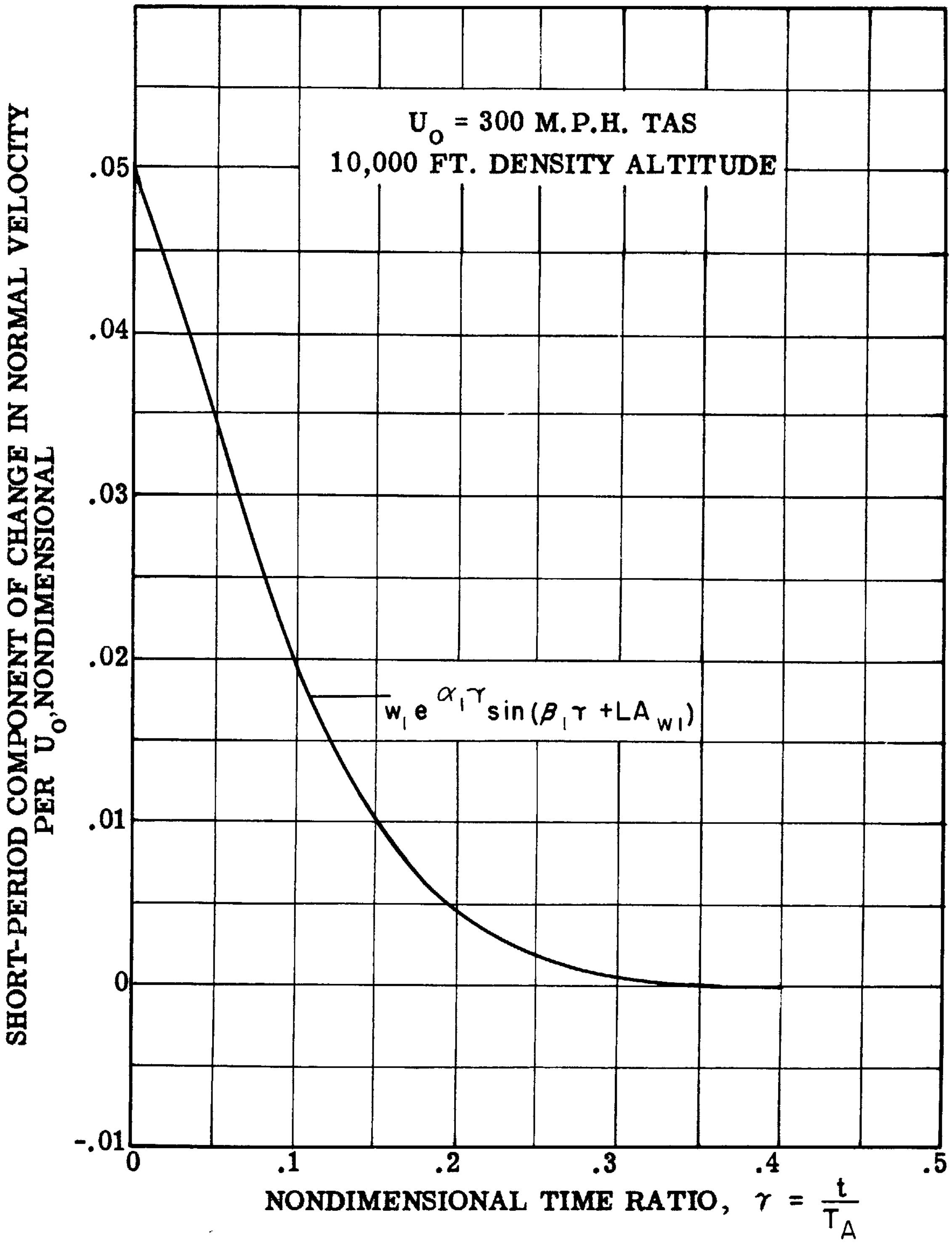
SHORT-PERIOD COMPONENT OF CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-1



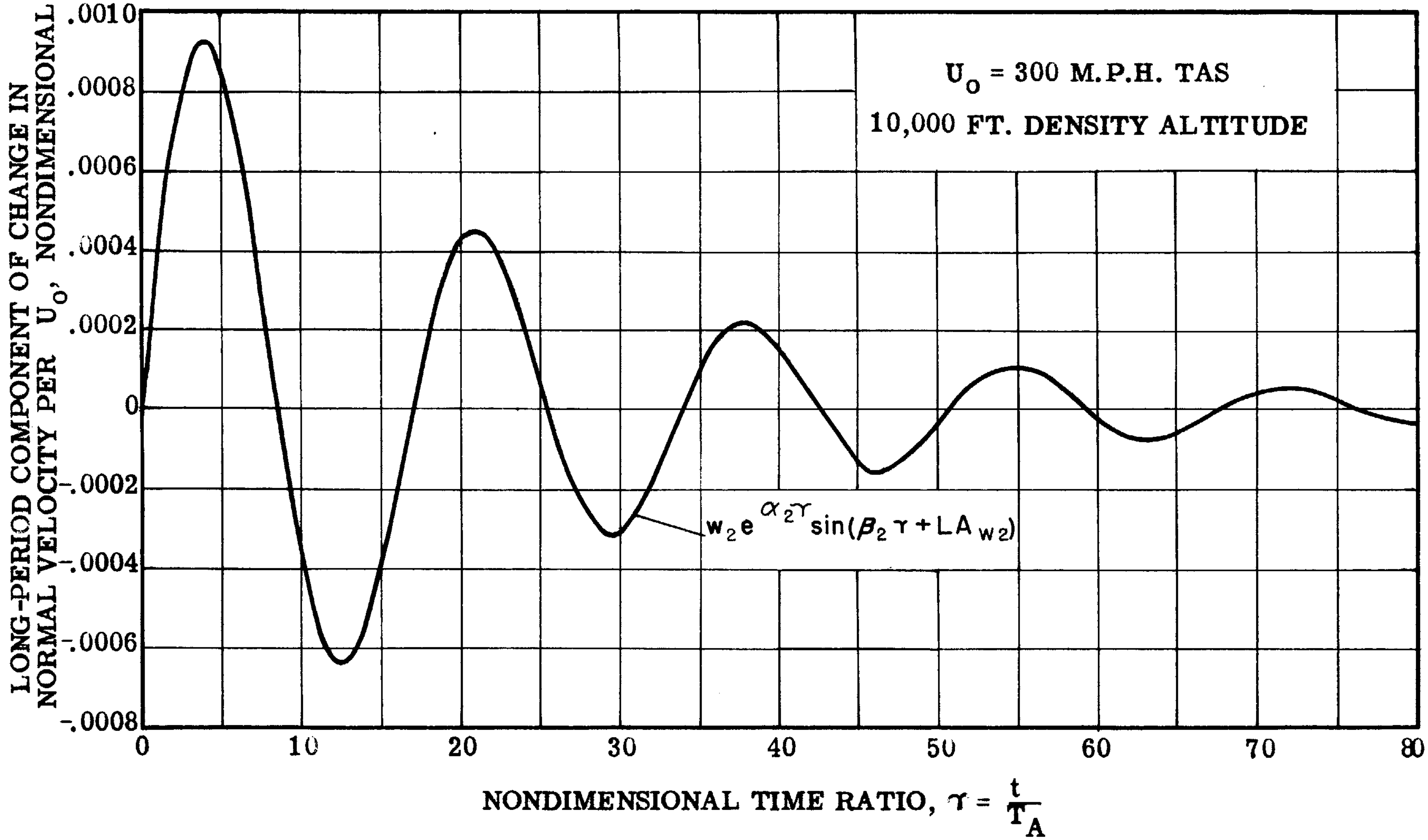
LONG-PERIOD COMPONENT OF CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-2



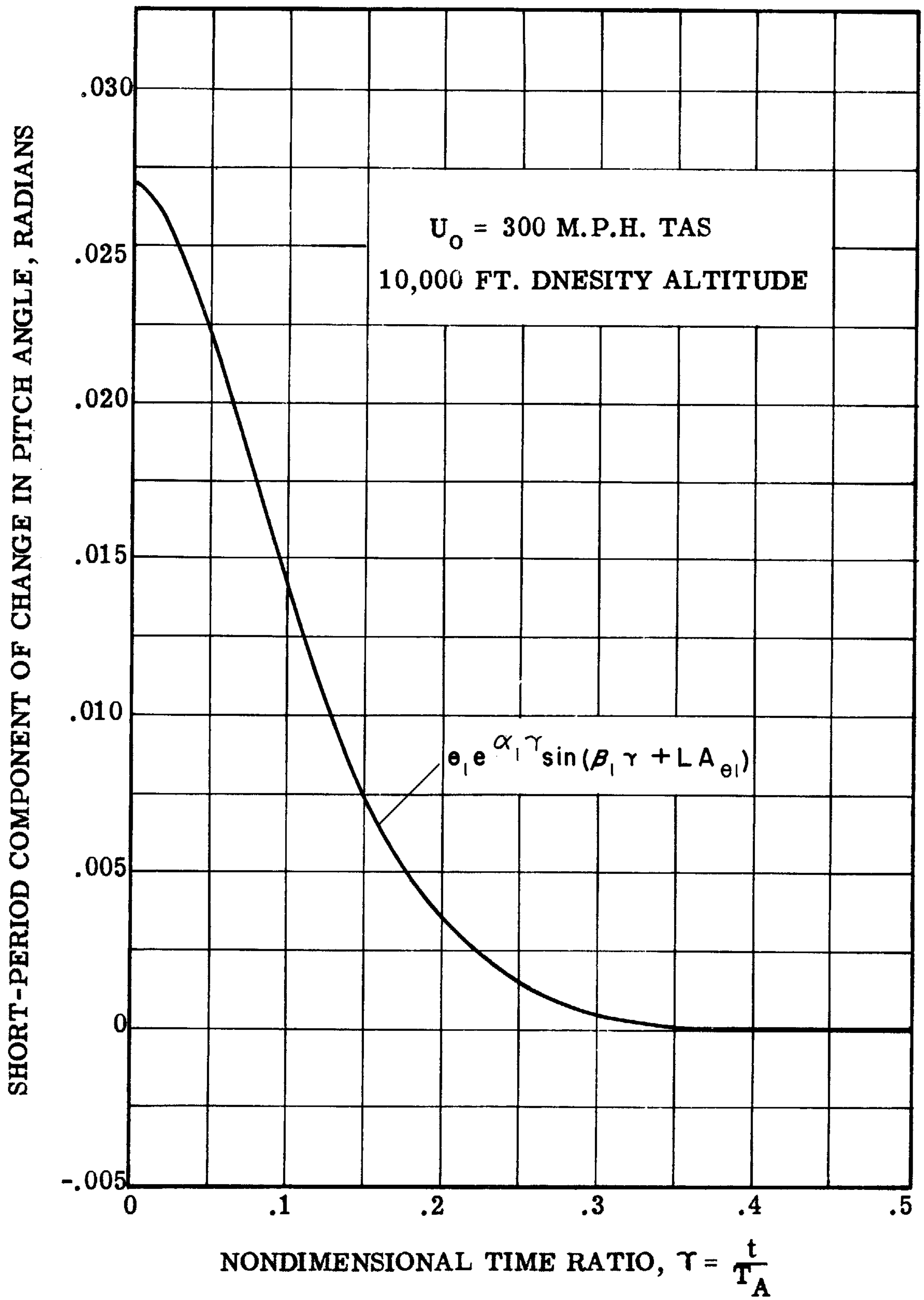
SHORT-PERIOD COMPONENT OF CHANGE IN NORMAL VELOCITY PER TRIM SPEED, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-3



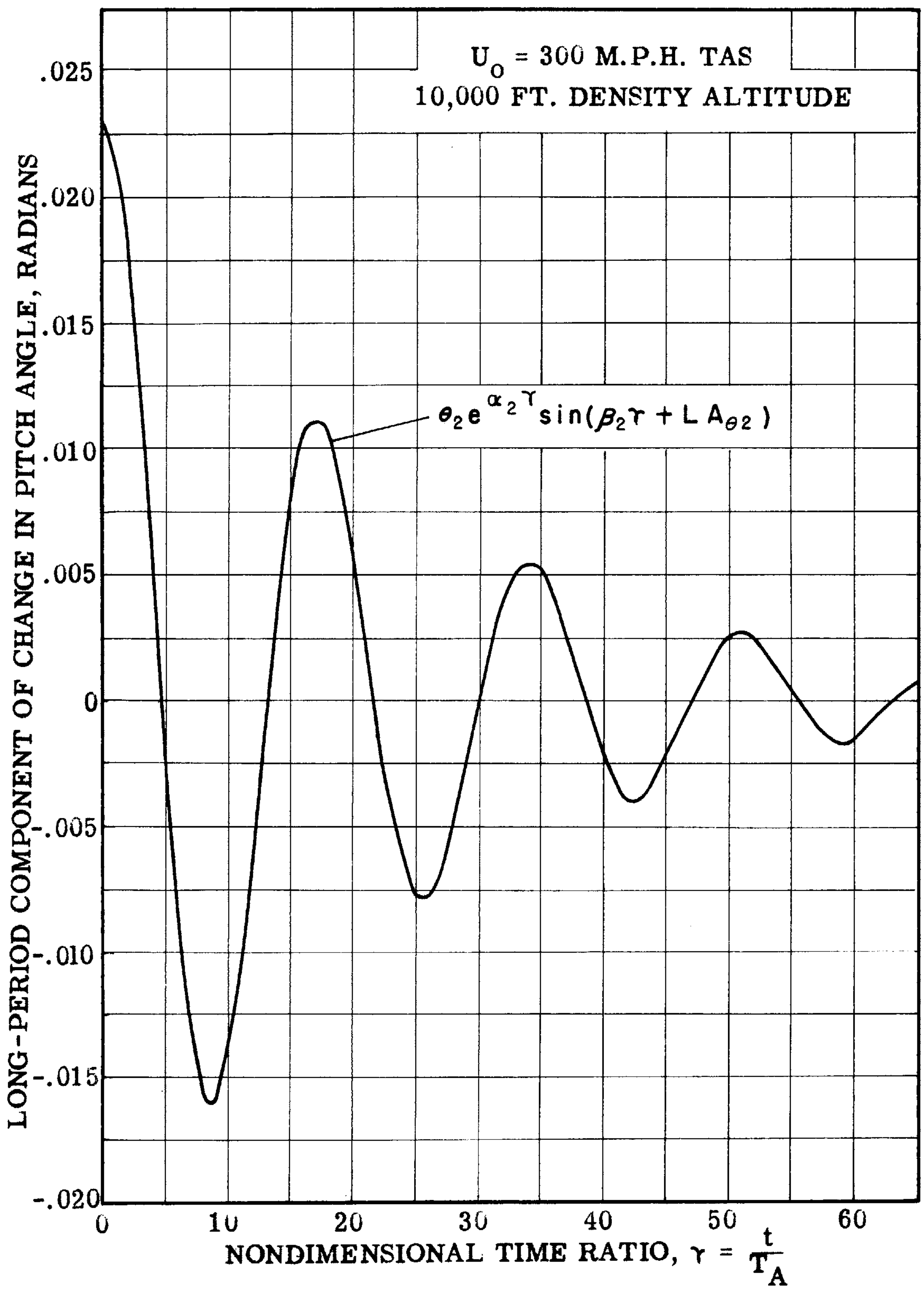
LONG-PERIOD COMPONENT OF CHANGE IN NORMAL VELOCITY PER TRIM SPEED, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-4



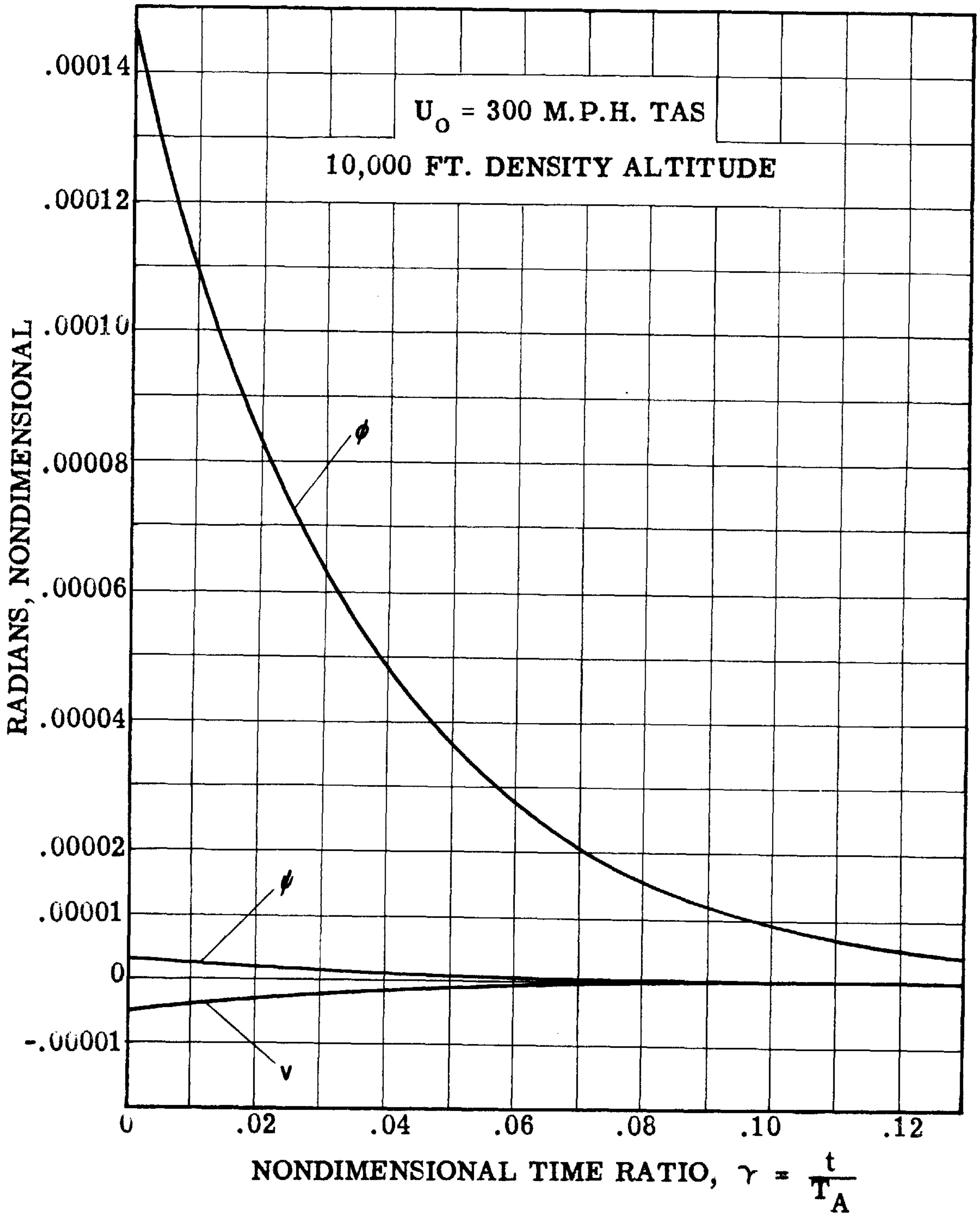
SHORT-PERIOD COMPONENT OF CHANGE IN PITCH ANGLE, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-5



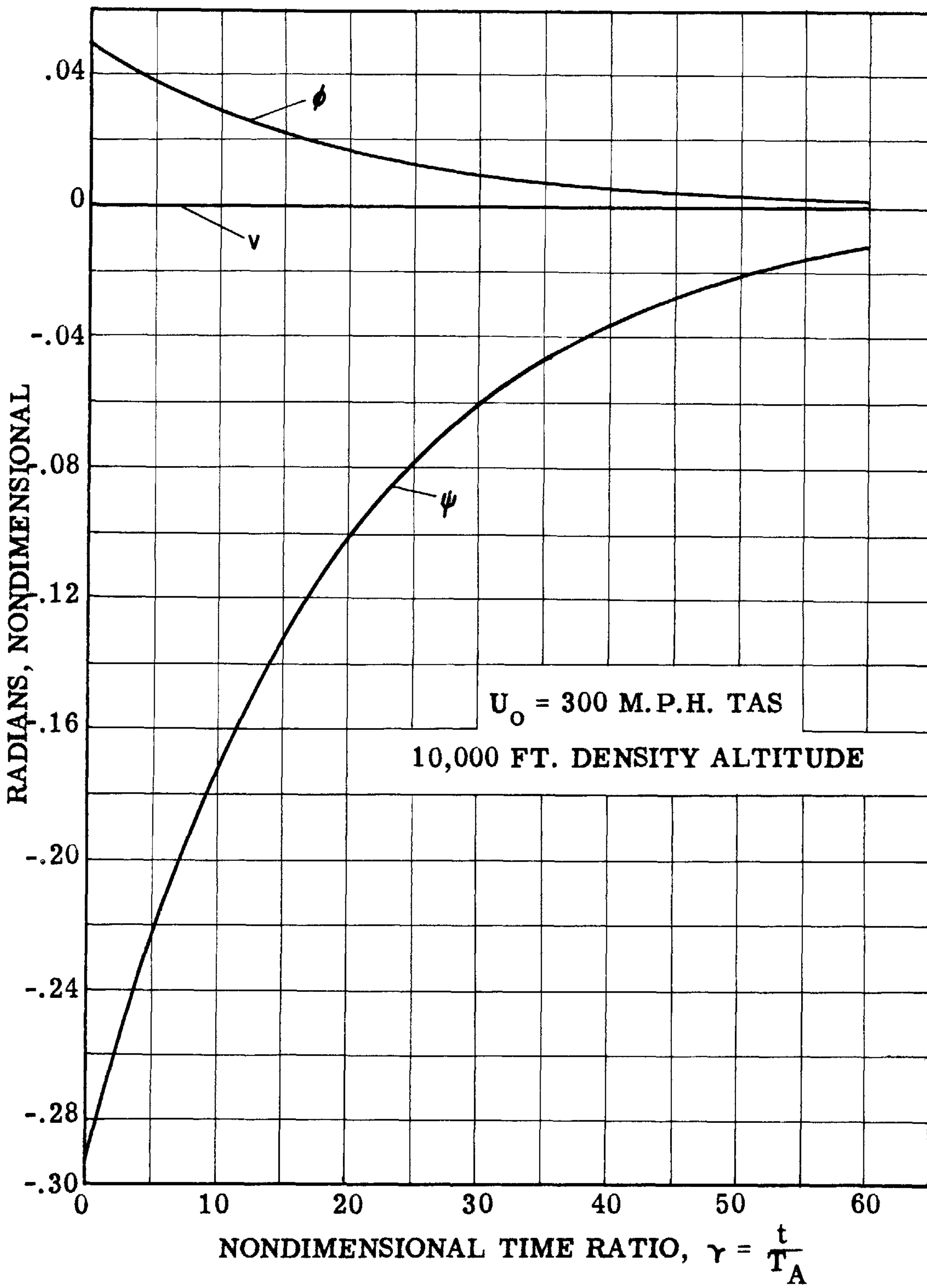
LONG-PERIOD COMPONENT OF CHANGE IN PITCH ANGLE FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

FIGURE M-6



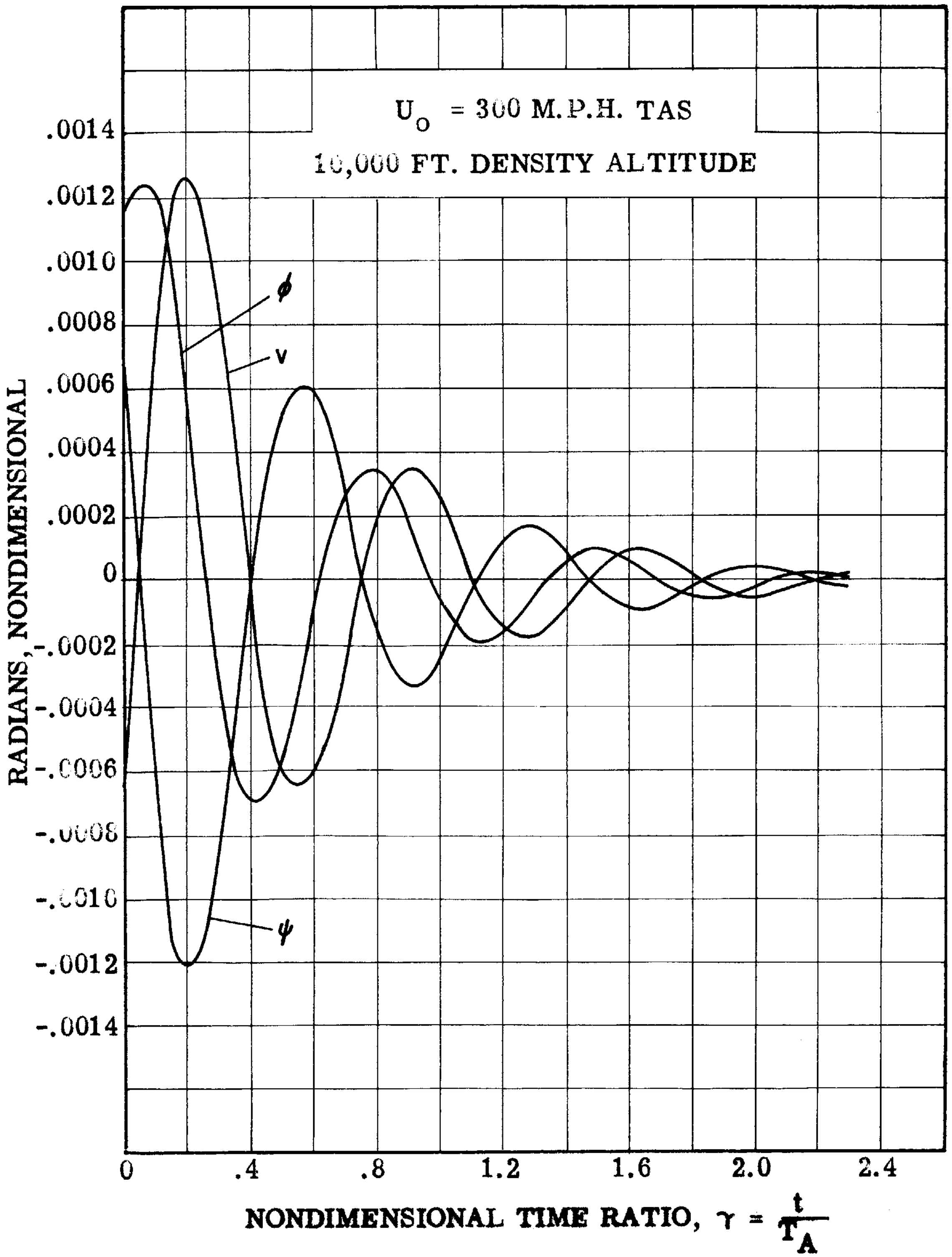
TIME HISTORY OF SUBSIDENCE COMPONENTS OF ANGLE OF BANK, ANGLE OF GEOMETRIC YAW, AND ANGLE OF AERODYNAMIC YAW, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

FIGURE M-7



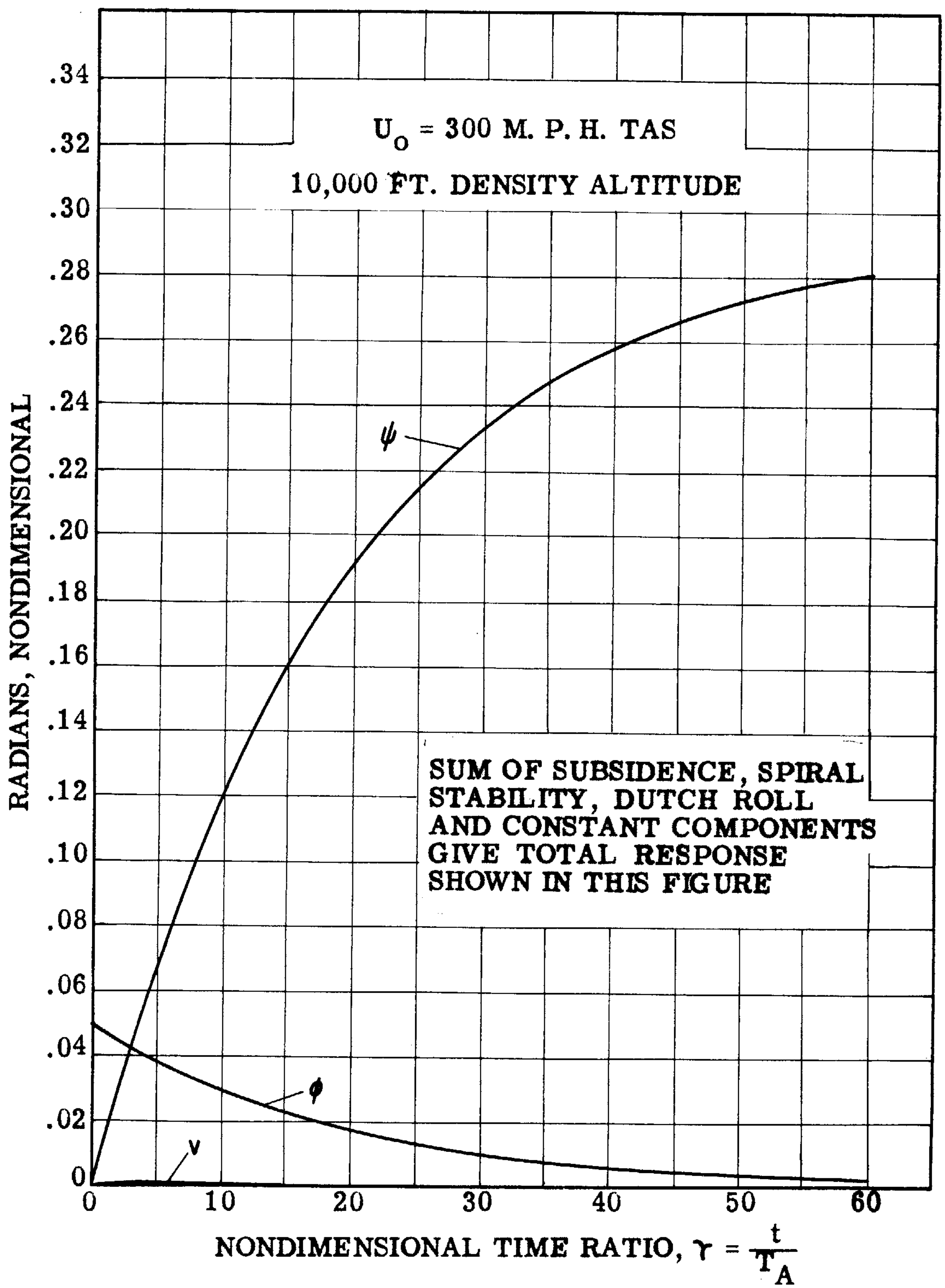
TIME HISTORY OF SPIRAL STABILITY COMPONENTS OF ANGLE OF BANK, ANGLE OF GEOMETRIC YAW, AND ANGLE OF AERODYNAMIC YAW, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

FIGURE M-8



TIME HISTORY OF DUTCH ROLL COMPONENTS OF ANGLE OF BANK, ANGLE OF GEOMETRIC YAW, AND ANGLE OF AERODYNAMIC YAW, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

FIGURE M-9



TIME HISTORY OF TOTAL RESPONSE OF ANGLE OF BANK, ANGLE OF GEOMETRIC YAW, AND ANGLE OF AERODYNAMIC YAW, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

FIGURE M-10

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-1

$U_0 = 300$ mph TAS; 10,000 FT. D.A.

DETERMINANTS
FOR A_u, B_u, C_u, D_u, A_w, B_w, C_w, D_w, A_θ, B_θ, C_θ, AND D_θ

					u						w		
(1)	α_1	-13.303	Δ_1 =	1	0	1	0						
(2)	α_2	-.042404		-13.303	7.2348	-.042404	.37090						
(3)	β_1	7.2348	(denominator)	124.63	-192.49	-.13577	-.031456				same as u	Δ_1	
(4)	β_2	.37090		-265.30	3462.2	+.017424	-.049024						
(5)	$(1)^2 = \alpha_1^2$	176.97											
(6)	$(3)^2 = \beta_1^2$	52.342	Δ_2 =	0	0	1	0			.05	0	1	0
(7)	$(5) - (6) = \alpha_1^2 - \beta_1^2$	124.63		-.0069835	7.2348	-.042404	.37090			-.24350	7.2348	-.042404	.37090
(8)	$(1)(3) = \alpha_1 \beta_1$	-96.245	for (A_u)	-.044974	-192.49	-.13577	-.031456		for (A_w)	-4.9803	-192.49	-.13577	-.031456
(9)	$(2)^2 = \alpha_2^2$	.0017981		1.0886	3462.2	+.017424	-.049024			188.43	3462.2	+.017424	-.049024
(10)	$(4)^2 = \beta_2^2$	.13757											
(11)	$(9) - (10) = \alpha_2^2 - \beta_2^2$	-.13577	Δ_3 =	1	0	1	0			1	.05	1	0
(12)	$(2)(4) = \alpha_2 \beta_2$	-.015728		-13.303	-.0069835	-.042404	.37090			-13.303	-.24350	-.042404	.37090
(13)	$(5)(1) = \alpha_1^3$	-2354.2	for (B_u)	124.63	-.044974	-.13577	-.031456		for (B_w)	124.63	-4.9803	-.13577	-.031456
(14)	$-3(8)(3) = -3\alpha_1\beta_1^2$	+2088.9		-265.30	1.0886	+.017424	-.049024			-265.30	188.43	+.017424	-.049024
(15)	$(13) + (14) = \alpha_1^3 - 3\alpha_1\beta_1^2$	-265.30											
(16)	$(6)(3) = \beta_1^3$	378.68	Δ_4 =	1	0	0	0			1	0	.05	0
(17)	$3(5)(1) = 3\alpha_1^2\beta_1$	3840.9		-13.303	7.2348	-.0069835	.37090			-13.303	7.2348	-.24350	.37090
(18)	$-(16) + (17) = -\beta_1^3 + 3\alpha_1^2\beta_1$	3462.2	for (C_u)	124.63	-192.49	-.044974	-.031456		for (C_w)	124.63	-192.49	-4.9803	-.031456
(19)	$(2)(9) = \alpha_2^3$	$-.76247 \times 10^{-4}$		-265.30	3462.2	1.0886	-.049024			-265.30	3462.2	188.43	-.049024
(20)	$-3(12)(4) = -3\alpha_2\beta_2^2$	+.017500											
(21)	$(19) + (20) = \alpha_2^3 - 3\alpha_2\beta_2^2$	+.017424	Δ_5 =	1	0	1	0			1	0	1	.05
(22)	$(10)(4) = \beta_2^3$	.051025		-13.303	7.2348	-.042404	-.0069835			-13.303	7.2348	-.042404	-.24350
(23)	$3(12)(2) = 3\alpha_2^2\beta_2$	.0020008		124.63	-192.49	-.13577	-.044974		for (D_w)	124.63	-192.49	-.13577	-4.9803
(24)	$-(22) + (23) = -\beta_2^3 + 3\alpha_2^2\beta_2$	-.049024	for (D_u)	-265.30	3462.2	+.017424	1.0886			-265.30	3462.2	+.017424	188.43

U_0 = 300 mph TAS, 10,000 FT. D.A.

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-2

DETERMINANTS FOR A_u, B_u, C_u, D_u, A_w, B_w, C_w, D_w, A_θ, B_θ, C_θ AND D_θ

		θ		
Δ_1 = (denominator)		same as Δ_1		
Δ_2 =	.05	0	1	0
	0	7.2348	-.042404	.37090
for (A_θ)	-6.1708	-192.49	-.13577	-.031456
	164.15	3462.2	+.017424	-.049024
Δ_3 =	1	.05	1	0
	-13.303	0	-.042404	.37090
for (B_θ)	124.63	-6.1708	-.13577	-.031456
	-265.30	164.15	+.017424	-.049024
Δ_4 =	1	0	.05	0
	-13.303	7.2348	0	.37090
for (C_θ)	124.63	-192.49	-6.1708	-.031456
	-265.30	3462.2	164.15	-.049024
Δ_5 =	1	0	1	.05
	-13.303	7.2348	-.042404	0
for (D_θ)	124.63	-192.49	-.13577	-6.1708
	-265.30	3462.2	+.017424	164.15

		u	w	θ
(1)	Δ_2/Δ_1 = A	.00013502	.050017	.026926
(2)	Δ_3/Δ_1 = B	.00032448	.058258	.049474
(3)	Δ_4/Δ_1 = C	-.00013500	-.000017166	.023073
(4)	Δ_5/Δ_1 = D	-.020330	+.0011001	.0033938
(5)	$(1)^2 = A^2$	$.18230 \times 10^{-7}$	.0025017	.00072501
(6)	$(2)^2 = B^2$	$.10529 \times 10^{-6}$	.0033940	.0024477
(7)	(5) + (6)	$.12352 \times 10^{-6}$	.0058957	.0031727
(8)	$\sqrt{(7)} = \sqrt{A^2+B^2} = u_1, w_1, \theta_1$	.00035145	.076784	.056327
(9)	$(3)^2 = C^2$	$.18225 \times 10^{-7}$	$.29467 \times 10^{-9}$	.00053236
(10)	$(4)^2 = D^2$	.00041331	$.12102 \times 10^{-5}$	$.11518 \times 10^{-4}$
(11)	(9) + (10)	.00041333	$.12105 \times 10^{-5}$	.00054388
(12)	$\sqrt{(11)} = \sqrt{C^2+D^2} = u_2, w_2, \theta_2$	.020331	.0011002	.023321
(13)	(1)/(2) = A/B = tan ∠A_1	.41611	.85854	.54425
(14)	(3)/(4) = C/D = tan ∠A_2	+.0066404	-.015604	+6.7986
(15)	$\tan^{-1}$ (13) = ∠A_1 degrees	22.593	40.647	28.557
(16)	$\tan^{-1}$ (14) = ∠A_2 "	180.38	359.11	81.633
(17)	.0174533 (15) = ∠A_1 radians	.39432	.70942	.49841
(18)	" (16) = ∠A_2 "	3.1482	6.2677	1.4248

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-3

CALCULATIONS FOR SHORT-PERIOD TRANSIENT RESPONSE FOR CHANGE IN LONGITUDINAL VELOCITY

$U_0 = 300$ mph TAS; 10,000 FT. D.A.

①	γ		0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2
②	α, γ	-13.303 × ①	0	-1.3303	-2.6606	-3.9909	-5.3212	-6.6515	-7.9818	-9.3121	-10.642	-11.973	-13.303	-14.633	-15.964
③	$e^{\alpha,\gamma}$	$e^{②}$	1.0000	.26440	.069909	.018483	.0048869	.0012921	.00034163	.000090325	.000023891	$.63125 \times 10^{-5}$	$.16695 \times 10^{-5}$	$.44154 \times 10^{-6}$	$.11666 \times 10^{-6}$
④	$u, e^{\alpha,\gamma}$	.00035145 × ③	.00035145	.000092923	.000024570	$.64959 \times 10^{-5}$	$.17175 \times 10^{-5}$	$.45411 \times 10^{-6}$	$.12007 \times 10^{-6}$	$.31745 \times 10^{-7}$	$.83965 \times 10^{-8}$	$.22185 \times 10^{-8}$	$.58675 \times 10^{-9}$	$.15518 \times 10^{-9}$	$.41000 \times 10^{-10}$
⑤	β, γ	7.2348 × ①	0	.72348	1.4470	2.1704	2.8939	3.6174	4.3409	5.0644	5.7878	6.5113	7.2348	7.9583	8.6818
⑥	$\beta, \gamma + \angle A_{u}$	⑤ + .39432	.39432	1.1178	1.8413	2.5647	3.2882	4.0117	4.7352	5.4587	6.1821	6.9056	7.6291	8.3526	9.0761
⑦	$\sin(\beta, \gamma + \angle A_{u})$	sin ⑥	.38418	.89914	.96363	.54537	-.14608	-.76436	-.99974	-.73420	-.10088	+.58297	.97984	.87823	.34169
⑧	$u, e^{\alpha,\gamma} \sin(\beta, \gamma + \angle A_{u})$	④ × ⑦	.00013502	.000083551	.000023676	$.35427 \times 10^{-6}$	$-.25089 \times 10^{-6}$	$-.34710 \times 10^{-6}$	$-.12004 \times 10^{-6}$	$-.23307 \times 10^{-7}$	$-.84704 \times 10^{-9}$	$+.12933 \times 10^{-8}$	$.57199 \times 10^{-9}$	$.13628 \times 10^{-9}$	$.14009 \times 10^{-10}$
⑨															
⑩															
⑪															
⑫	γ		.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	.33	.36	.39
⑬	α, γ	-13.303 × ⑫	-.39909	-.79818	-1.1973	-1.5964	-1.9954	-2.3945	-2.7936	-3.1927	-3.5918	-3.9909	-4.3900	-4.7891	-5.1882
⑭	$e^{\alpha,\gamma}$	$e^{⑬}$	.67093	.45014	.30201	.20262	.13596	.091218	.061202	.041061	.027549	.018483	.012401	.0083200	.0055820
⑮	$u, e^{\alpha,\gamma}$	.00035145 × ⑭	.00023580	.00015820	.00010614	.000071211	.000047783	.000032059	.000021509	.000014431	$.96821 \times 10^{-5}$	$.64959 \times 10^{-5}$	$.43583 \times 10^{-5}$	$.29241 \times 10^{-5}$	$.19618 \times 10^{-5}$
⑯	β, γ	7.2348 × ⑫	.21704	.43409	.65113	.86818	1.0852	1.3023	1.5193	1.7364	1.9534	2.1704	2.3875	2.6045	2.8216
⑰	$\beta, \gamma + \angle A_{u}$	⑯ + .39432	.61136	.82841	1.0454	1.2625	1.4795	1.6966	1.9136	2.1307	2.3477	2.5647	2.7818	2.9988	3.2159
⑱	$\sin(\beta, \gamma + \angle A_{u})$	sin ⑰	.57398	.73685	.86513	.95285	.99584	.99210	.94182	.84731	.71313	.54537	.35201	.14228	-.074282
⑲	$u, e^{\alpha,\gamma} \sin(\beta, \gamma + \angle A_{u})$	⑮ × ⑱	.00013534	.00011657	.000091825	.000067853	.000047584	.000031806	.000020258	.000012228	$.69046 \times 10^{-5}$	$.35427 \times 10^{-5}$	$.15342 \times 10^{-5}$	$.41604 \times 10^{-6}$	$-.14573 \times 10^{-6}$
⑳															
(21)															
(22)															
(23)	γ		.01	.02											
(24)	α, γ	-13.303 × (23)	-.13303	-.26606											
(25)	$e^{\alpha,\gamma}$	$e^{(24)}$	.87544	.76639											
(26)	$u, e^{\alpha,\gamma}$	.00035145 × (25)	.00030767	.00026935											
(27)	β, γ	7.2348 × (23)	.072348	.14470											
(28)	$\beta, \gamma + \angle A_{u}$	(27) + .39432	.46667	.53902											
(29)	$\sin(\beta, \gamma + \angle A_{u})$	sin (28)	.44991	.51330											
(30)	$u, e^{\alpha,\gamma} \sin(\beta, \gamma + \angle A_{u})$	(26) × (29)	.00013842	.00013826											

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-4

CALCULATIONS FOR LONG-PERIOD TRANSIENT RESPONSE FOR CHANGE IN LONGITUDINAL VELOCITY

U_0 = 300 mph TAS; 10,000 FT. D.A.

① γ		0.	5	10	15	20	25	30	35	40	45	50	55	60
② $\alpha_2\gamma$	$-.042404\times$①	0	-.21202	-.42404	-.63606	-.84808	-1.0601	-1.2721	-1.4841	-1.6962	-1.9082	-2.1202	-2.3322	-2.5442
③ $e^{\alpha_2\gamma}$	$e^{②}$	1.0000	.80899	.65440	.52937	.42824	.34642	.28024	.22671	.18338	.14835	.12001	.097082	.078539
④ $u_2 e^{\alpha_2\gamma}$	$.020331\times$③	.020331	.016447	.013305	.010763	.0087065	.0070431	.0056976	.0046092	.0037283	.0030161	.0024399	.0019738	.0015968
⑤ $\beta_2\gamma$	$.37090\times$①	0	1.8545	3.7090	5.5635	7.4180	9.2725	11.127	12.982	14.836	16.690	18.545	20.400	22.254
⑥ $\beta_2\gamma+\angle A_{u2}$,	⑤ + 3.1482	3.1482	5.0027	6.8572	8.7117	10.566	12.421	14.275	16.130	17.984	19.838	21.693	23.188	25.402
⑦ $\sin(\beta_2\gamma+\angle A_{u2})$,	sin ⑥	-.0066000	-.95817	+.54303	+.65421	-.90916	-.14487	+.99051	-.40960	-.76154	+.83485	+.29404	-.93106	+.26556
⑧ $u_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{u2})$,	④ × ⑦	-.00013418	-.015759	+.0072250	+.0070413	-.0079156	-.0010203	+.0056435	-.0018879	-.0028392	+.0025180	0.0071743	-.0018377	+.00042405

① γ		1	2	3	4	6	7	8	9	11	12	13	14	16
② $\alpha_2\gamma$	$-.042404\times$①	-.042404	-.084808	-.12721	-.16962	-.25442	-.29683	-.33923	-.38164	-.46644	-.50885	-.55125	-.59366	-.67846
③ $e^{\alpha_2\gamma}$	$e^{②}$	.95849	.91869	.88055	.84398	.77536	.74317	.71232	.68274	.62723	.60119	.57623	.55230	.50740
④ $u_2 e^{\alpha_2\gamma}$	$.020331\times$③	.019487	.018678	.017902	.017159	.015764	.015109	.014482	.013881	.012752	.012223	.011715	.011229	.010316
⑤ $\beta_2\gamma$	$.37090\times$①	.37090	.74180	1.1127	1.4836	2.2254	2.5963	2.9672	3.3381	4.0799	4.4508	4.8217	5.1926	5.9344
⑥ $\beta_2\gamma+\angle A_{u2}$,	⑤ + 3.1482	3.5191	3.8900	4.2609	4.6318	5.3736	5.7445	6.1154	6.4863	7.2281	7.5990	7.9699	8.3408	9.0826
⑦ $\sin(\beta_2\gamma+\angle A_{u2})$,	sin ⑥	-.36861	-.68047	-.89979	-.99675	-.78930	-.51294	-.16694	+.20176	+.81045	+.96767	+.99329	+.88385	+.33562
⑧ $u_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{u2})$	④ × ⑦	-.0071831	-.012710	-.016108	-.017103	-.012443	-.0077500	-.0024176	+.0028006	+.010335	+.011828	+.011636	+.0099248	+.0034623

① γ		17	18	19	21	22	23	24	27	29	31	33	36	38
② $\alpha_2\gamma$	$-.042404\times$①	-.72087	-.76327	-.80568	-.89048	-.93289	-.97529	-1.0177	-1.1449	-1.2297	-1.3145	-1.3993	-1.5265	-1.6114
③ $e^{\alpha_2\gamma}$	$e^{②}$	.48632	.46614	.44679	.41046	.39341	.37708	.36143	.31826	.29238	.26861	.24677	.21729	.19961
④ $u_2 e^{\alpha_2\gamma}$	$.020331\times$③	.0098874	.0094771	.0090837	.0083451	.0079984	.0076664	.0073482	.0064705	.0059444	.0054611	.0050171	.0044177	.0040583
⑤ $\beta_2\gamma$	$.37090\times$①	6.3053	6.6762	7.0471	7.7889	8.1598	8.5307	8.9016	10.014	10.756	11.498	12.240	13.352	14.094
⑥ $\beta_2\gamma+\angle A_{u2}$,	⑤ + 3.1482	9.4535	9.8244	10.195	10.937	11.308	11.679	12.050	13.162	13.904	14.646	15.388	16.500	17.242
⑦ $\sin(\beta_2\gamma+\angle A_{u2})$,	sin ⑥	-.028794	-.38912	-.69629	-.99828	-.95159	-.77539	-.49379	+.56107	+.97294	+.87335	+.31449	-.71178	-.99932
⑧ $u_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{u2})$,	④ × ⑦	-.00028470	-.0036877	-.0063249	-.0083307	-.0076112	-.0059444	-.0036285	+.0036304	+.0057835	+.0047695	+.0015778	-.0031444	-.0040565

① γ		42	47	52	57	62	65	67	70	72	75	77	80	85
② $\alpha_2\gamma$	$-.042404\times$①	-1.7810	-1.9930	-2.2050	-2.4170	-2.6290	-2.7563	-2.8411	-2.9683	-3.0531	-3.1803	-3.2651	-3.3923	-3.6043
③ $e^{\alpha_2\gamma}$	$e^{②}$	.16847	.13629	.11025	.089189	.072152	.063529	.058364	.051393	.047213	.041573	.038193	.033631	.027207
④ $u_2 e^{\alpha_2\gamma}$	$.020331\times$③	.0034252	.0027709	.0022415	.0018133	.0014669	.0012916	.0011866	.0010449	.00095989	.00084522	.00077650	.00068375	.00055315
⑤ $\beta_2\gamma$	$.37090\times$①	15.578	17.432	19.287	21.141	22.996	24.108	24.850	25.963	26.705	27.818	28.559	29.672	31.526
⑥ $\beta_2\gamma+\angle A_{u2}$,	⑤ + 3.1482	18.726	20.580	22.435	24.289	26.144	27.256	27.998	29.111	29.853	30.966	31.707	32.820	34.674
⑦ $\sin(\beta_2\gamma+\angle A_{u2})$,	sin ⑥	-.12360	+.98741	-.42894	-.74664	+.84712	+.85081	+.27228	-.74198	-.99996	-.43523	+.28736	+.98600	-.11667
⑧ $u_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{u2})$	④ × ⑦	-.00042335	+.0027360	-.00096147	-.0013539	+.0012426	+.0010989	+.00032309	-.00077529	-.00095985	-.00036787	+.00022314	+.00067418	-.00006454

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE
TO LONGITUDINAL STEP-FUNCTION

TABLE M-5

CALCULATIONS FOR LONG-PERIOD TRANSIENT RESPONSE
FOR CHANGE IN LONGITUDINAL VELOCITY

U_0 = 300 mph TAS; 10,000 FT. D.A.

(1)	γ		0	.03	.06	.09	.12	.15	.18	.21	.24	.27	.30	.33	.36
(2)	$\alpha_2\gamma$	$-.042404 \times$ (1)	0	-.0012721	-.0025442	-.0038164	-.0050885	-.0063606	-.0076327	-.0089048	-.010177	-.011449	-.012721	-.013993	-.015265
(3)	$e^{\alpha_2\gamma}$	$e^{(2)}$	1.0000	.99873	.99746	.99619	.99492	.99366	.99240	.99114	.98987	.98861	.98736	.98611	.98485
(4)	$u_2 e^{\alpha_2\gamma}$	$.020331 \times$ (3)	.020331	.020305	.020279	.020254	.020228	.020202	.020176	.020151	.020125	.020099	.020074	.020049	.020023
(5)	$\beta_2\gamma$	$.37090 \times$ (1)	0	.011127	.022254	.033381	.044508	.055635	.066762	.077889	.089016	.10014	.11127	.12240	.13352
(6)	$\beta_2\gamma + \angle A_{u2}$,	(5) + 3.1482	3.1482	3.1593	3.1705	3.1816	3.1927	3.2038	3.2150	3.2261	3.2372	3.2483	3.2595	3.2706	3.2817
(7)	$\sin(\beta_2\gamma + \angle A_{u2})$,	sin (6)	-.0066000	-.017699	-.028896	-.039989	-.051078	-.062160	-.073334	-.084399	-.095454	-.10650	-.11763	-.12864	-.13964
(8)	$u_2 e^{\alpha_2\gamma} \sin(\beta_2\gamma + \angle A_{u2})$,	(4) × (7)	-.00013418	-.00035938	-.00058598	-.00080994	-.0010332	-.0012558	-.0014796	-.0017007	-.0019210	-.0021405	-.0023613	-.0025791	-.0027960
(9)															
(10)															
(11)															
(12)	γ		.01	.02	.1	.2	.39	.4	.5	.6	.7	.8	.9	1.0	1.1
(13)	$\alpha_2\gamma$	$-.042404 \times$ (12)	-.00042404	-.00084808	-.0042404	-.0084808	-.016538	-.016962	-.021202	-.025442	-.029683	-.033923	-.038164	-.042404	-.046644
(14)	$e^{\alpha_2\gamma}$	$e^{(13)}$	.99958	.99915	.99577	.99156	.98360	.98318	.97902	.97488	.97075	.96665	.96256	.95849	.95443
(15)	$u_2 e^{\alpha_2\gamma}$	$.020331 \times$ (14)	.020322	.020314	.020245	.020159	.020000	.019989	.019904	.019820	.019736	.019653	.019570	.019487	.019405
(16)	$\beta_2\gamma$	$.37090 \times$ (12)	.0037090	.0074180	.037090	.074180	.14465	.14836	.18545	.22254	.25963	.29672	.33381	.37090	.40799
(17)	$(\beta_2\gamma + \angle A_{u2})$,	(16) + 3.1482	3.1519	3.1556	3.1853	3.2224	3.2928	3.2966	3.3336	3.3707	3.4078	3.4449	3.4820	3.5191	3.5562
(18)	$\sin(\beta_2\gamma + \angle A_{u2})$,	sin (17)	-.010300	-.014000	-.043686	-.080712	-.15062	-.15438	-.19082	-.22710	-.26307	-.29867	-.33386	-.36860	-.40282
(19)	$u_2 e^{\alpha_2\gamma} \sin(\beta_2\gamma + \angle A_{u2})$,	(15) × (18)	-.00020932	-.00028440	-.00088442	-.0016271	-.0030124	-.0030859	-.0037981	-.0045011	-.0051919	-.0058698	-.0065336	-.0071829	-.0078167

$$u = u_1 e^{\alpha_1\gamma} \sin(\beta_1\gamma + \angle A_{u1}) + u_2 e^{\alpha_2\gamma} \sin(\beta_2\gamma + \angle A_{u2})$$

γ	0	.01	.02	.03	.06	.09	.1	.12	.15	.18	.2	.21	.24
u	$+.84 \times 10^{-6}$	-.000070900	-.00014614	-.00022404	-.00046941	-.00071812	-.00080087	-.00096535	-.0012082	-.0014478	-.0016034	-.0016804	-.0019088
γ	.27	.30	.33	.36	.39	.40	.5	.6	.7	.8	.9	1.0	1.1
u	-.0021336	-.0023578	-.0025776	-.0027956	-.0030125	-.0030862	-.0037984	-.0045012	-.0051919	-.0058698	-.0065336	-.0071829	-.0078167

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE
TO LONGITUDINAL STEP-FUNCTION

TABLE M-6

CALCULATIONS FOR SHORT-PERIOD TRANSIENT RESPONSE
FOR CHANGE IN NORMAL VELOCITY

U_0 = 300 mph TAS; 10,000 FT. D.A.

	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2
(1) γ	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2
(2) $e^{\alpha_1 \gamma}$ copy line (3) of 1-A	1.0000	.26440	.069909	.018483	.0048869	.0012921							
(3) $\omega_1 e^{\alpha_1 \gamma}$ .076784 × (2)	.076784	.020302	.0053679	.0014192	.00037524	.000099213							
(4) $\beta_1 \gamma$ copy lines (5) 1-A	0	.72348	1.4470	2.1704	2.8939	3.6174							
(5) $\beta_1 \gamma + \angle A_{w1}$, (4) + .70942	.70942	1.4329	2.1564	2.8798	3.6033	4.3268							
(6) $\sin(\beta_1 \gamma + \angle A_{w1})$, sin (5)	.65140	.99051	.83340	.25882	-.44542	-.92659							
(7) $\omega_1 e^{\alpha_1 \gamma} \sin(\beta_1 \gamma + \angle A_{w1})$, (3) × (6)	.050017	.020109	.0044736	.00036732	-.00016714	-.000091930							

	.01	.02	.03	.06	.09	.12	.15	.18	.21	.24	.27	.33	.36
(1) γ	.01	.02	.03	.06	.09	.12	.15	.18	.21	.24	.27	.33	.36
(2) $e^{\alpha_1 \gamma}$ copy ($e^{\alpha_1 \gamma}$) of 1-A	.87544	.76639	.67093	.45014	.30201	.20262	.13596	.091218	.061202	.041061	.027549	.012401	.0083200
(3) $\omega_1 e^{\alpha_1 \gamma}$ .076784 × (2)	.067220	.058846	.051517	.034564	.023190	.015558	.010440	.0070041	.0046993	.0031528	.0021153	.00095220	.00063884
(4) $\beta_1 \gamma$ copy ($\beta_1 \gamma$) of 1-A	.072348	.14470	.21704	.43409	.65113	.86818	1.0852	1.3023	1.5193	1.7364	1.9534	2.3875	2.6045
(5) $\beta_1 \gamma + \angle A_{w1}$, (4) + .70942	.78177	.85412	.92646	1.1435	1.3606	1.5776	1.7946	2.0117	2.2287	2.4458	2.6628	3.0969	3.3139
(6) $\sin(\beta_1 \gamma + \angle A_{w1})$, sin (5)	.70454	.75399	.79950	.91009	.97799	.99998	.97507	.90438	.79122	.64105	.46066	.044666	-.17144
(7) $\omega_1 e^{\alpha_1 \gamma} \sin(\beta_1 \gamma + \angle A_{w1})$, (3) × (6)	.047359	.044369	.041188	.031456	.022680	.015558	.010180	.0063344	.0037182	.0020211	.00097443	.000042531	-.00010952

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-7

CALCULATIONS FOR LONG-PERIOD TRANSIENT RESPONSE FOR CHANGE IN NORMAL VELOCITY

U_0 = 300 mph TAS; 10,000 FT. D.A.

① γ	0	5	10	15	20	25	30	35	40	45	50	55	60
② $e^{\alpha_2\gamma}$ copy	1.0000	.80894	.65440	.52937	.42824	.34642	.28024	.22671	.18338	.14835	.12001	.097082	.078539
③ $\omega_2 e^{\alpha_2\gamma}$ .0011002 × ②	.0011002	.00089000	.00071997	.00058241	.00047115	.00038113	.00030832	.00024943	.00020175	.00016321	.00013204	.00010681	.000086409
④ $\beta_2\gamma$ copy	0	1.8545	3.7090	5.5635	7.4180	9.2725	11.127	12.982	14.836	16.690	18.545	20.400	22.254
⑤ $\beta_2\gamma+\angle A_{w2}$, ④ + 6.2677	6.2677	8.1222	9.9767	11.831	13.686	15.540	17.395	19.250	21.104	22.958	24.813	26.668	28.522
⑥ $\sin(\beta_2\gamma+\angle A_{w2})$, sin ⑤	-.015533	+.96423	-.52428	-.67082	+.89994	+.16711	-.99325	+.38912	+.77494	-.82314	-.31399	+.99939	-.24531
⑦ $\omega_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{w2})$, ③ × ⑥	-.000017089	+.00085816	-.00037747	-.00039069	+.00042401	+.000063691	-.00030624	+.000097058	+.00015634	-.00013434	-.000041459	+.00010674	-.000021197

① γ	1	2	3	4	6	7	8	9	11	12	13	14	16
② $e^{\alpha_2\gamma}$ copy	.95849	.91869	.88055	.84398	.77536	.74317	.71232	.68274	.62723	.60119	.57623	.55230	.50740
③ $\omega_2 e^{\alpha_2\gamma}$ .0011002 × ②	.0010545	.0010107	.00096878	.00092855	.00085305	.00081764	.00078369	.00075115	.00069008	.00066143	.00063397	.00060764	.00055824
④ $\beta_2\gamma$ copy	.37090	.74180	1.1127	1.4836	2.2254	2.5963	2.9672	3.3381	4.0799	4.4508	4.8217	5.1926	5.9344
⑤ $\beta_2\gamma+\angle A_{w2}$, ④ + 6.2677	6.6386	7.0095	7.3804	7.7513	8.4931	8.8640	9.2349	9.6058	10.348	10.718	11.089	11.460	12.202
⑥ $\sin(\beta_2\gamma+\angle A_{w2})$, sin ⑤	.34792	.66406	.88997	.99474	.80261	.53184	.18875	-.18000	-.79758	-.96174	-.99564	-.89408	-.35641
⑦ $\omega_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{w2})$, ③ × ⑥	.00036688	.00067117	.00086219	.00092367	.00068467	.00043485	.00014792	-.00013521	-.00055039	-.00063612	-.00063121	-.00054328	-.00019896

① γ	17	18	19	21	22	23	24	27	29	31	33	36	38
② $e^{\alpha_2\gamma}$ copy	.48632	.46614	.44679	.41046	.39341	.37708	.36143	.31826	.29238	.26861	.24677	.21729	.19961
③ $\omega_2 e^{\alpha_2\gamma}$ .0011002 × ②	.00053506	.00051285	.00049156	.00045159	.00043283	.00041486	.00039765	.00035015	.00032168	.00029552	.00027150	.00023906	.00021961
④ $\beta_2\gamma$ copy	6.3053	6.6762	7.0471	7.7889	8.1598	8.5307	8.9016	10.014	10.756	11.498	12.240	13.352	14.094
⑤ $\beta_2\gamma+\angle A_{w2}$, ④ + 6.2677	12.573	12.944	13.315	14.057	14.428	14.798	15.169	16.282	17.024	17.766	18.508	19.620	20.362
⑥ $\sin(\beta_2\gamma+\angle A_{w2})$, sin ⑤	+.0066322	.36877	.68059	.99679	.95802	.78951	.51324	-.54303	-.96771	-.88377	-.33545	+.69591	+.99834
⑦ $\omega_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{w2})$, ③ × ⑥	$.35486\times10^{-5}$	.00018912	.00033455	.00045014	.00041466	.00032754	.00020409	-.00019014	-.00031129	-.00026117	-.000091075	+.00016636	+.00021925

① γ	42	47	52	57	62	65	67	70	72	75	77	80	85	82
② $e^{\alpha_2\gamma}$ copy	.16847	.13629	.11025	.089189	.072152	.063529	.058364	.051393	.047213	.041573	.038193	.033631	.027207	.030897
③ $\omega_2 e^{\alpha_2\gamma}$ .0011002 × ②	.00018535	.00014995	.00012130	.000098126	.000079382	.000069895	.000064212	.000056543	.000051944	.000045739	.000042020	.000037001	.000029933	.000033993
④ $\beta_2\gamma$ copy	15.578	17.432	19.287	21.141	22.996	24.108	24.850	25.963	26.705	27.818	28.559	29.672	31.526	30.414
⑤ $\beta_2\gamma+\angle A_{w2}$, ④ + 6.2677	21.846	23.700	25.555	27.409	29.264	30.376	31.118	32.231	32.973	34.086	34.827	35.940	37.794	36.682
⑥ $\sin(\beta_2\gamma+\angle A_{w2})$, sin ⑤	+.14436	-.99051	+.40992	+.76154	-.83581	-.86251	-.29404	+.72777	+.99990	+.45399	-.26556	-.98229	+.094108	-.85081
⑦ $\omega_2 e^{\alpha_2\gamma}\sin(\beta_2\gamma+\angle A_{w2})$, ③ × ⑥	+.000026757	-.00014853	+.000049723	+.000074727	-.000066348	-.000060285	-.000018881	+.000041150	+.000051939	+.000020765	-.000011159	-.000036346	+.0000028169	-.000028922

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION

TABLE M-8

CALCULATIONS FOR SHORT-PERIOD TRANSIENT RESPONSE FOR CHANGE IN PITCH ANGLE

V_o = 300 mph TAS; 10,000 FT. D.A.

① γ	0	.1	.2	.3	.4	.5							
② $e^{\alpha,\gamma}$ copy	1.0000	.36440	.069909	.018483	.0048869	.0012921							
③ $\theta, e^{\alpha,\gamma}$ .056327 × ②	.056327	.014893	.0039378	.0010411	.00027526	.000072780							
④ β,γ copy	0	.72348	1.4470	2.1704	2.8939	3.6174							
⑤ $\beta,\gamma + \angle A_{\theta 1}$ ④ + .49841	.49841	1.2219	1.9454	2.6688	3.3923	4.1158							
⑥ $\sin(\beta,\gamma + \angle A_{\theta 1})$, sin ⑤	.47803	.93975	.93067	.45539	-.24801	-.82728							
⑦ $\theta, e^{\alpha,\gamma} \sin(\beta,\gamma + \angle A_{\theta 1})$, ③ × ⑥	.026926	.013996	.0036648	.00047411	-.000068267	-.000060209							
① γ	.01	.02	.03	.06	.09	.12	.15	.18	.21	.24	.27	.33	.36
② $e^{\alpha,\gamma}$ copy	.87544	.76639	.67093	.45014	.30201	.20262	.13596	.091218	.061202	.041061	.027549	.012401	.0083200
③ $\theta, e^{\alpha,\gamma}$ .056327 × ②	.049311	.043168	.037791	.025355	.017011	.011413	.0076582	.0051380	.0034473	.0023128	.0015518	.00069851	.00046864
④ β,γ copy	.072348	.14470	.21704	.43409	.65113	.86818	1.0852	1.3023	1.5193	1.7364	1.9534	2.3875	2.6045
⑤ $\beta,\gamma + \angle A_{\theta}$ ④ + .49841	.57076	.64311	.71546	.93250	1.1495	1.3666	1.5836	1.8007	2.0177	2.2348	2.4518	2.8859	3.1029
⑥ $\sin(\beta\gamma + \angle A_{\theta 1})$, sin ⑤	.54027	.59968	.65595	.80311	.91256	.97922	.99992	.97370	.90176	.78758	.63635	.25291	.038737
⑦ $\theta, e^{\alpha,\gamma} \sin(\beta,\gamma + \angle A_{\theta 1})$, ③ × ⑥	.026641	.025887	.024789	.020363	.015524	.011176	.0076576	.0050029	.0031086	.0018215	.00098749	.00017666	.000018157

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE
TO LONGITUDINAL STEP-FUNCTION

TABLE M-9

CALCULATIONS FOR LONG-PERIOD TRANSIENT RESPONSE
FOR CHANGE IN PITCH ANGLE

U_0 = 300 mph TAS; 10,000 FT. D.A.

① γ	0	5	10	15	20	25	30	35	40	45	50	55	60
② $e^{\alpha_2 \gamma}$ copy	1.0000	.80894	.65440	.52937	.42824	.34642	.28024	.22671	.18338	.14835	.12001	.097082	.078539
③ $\theta_2 e^{\alpha_2 \gamma}$ .023321 × ②	.023321	.018865	.015261	.012345	.0099870	.0080789	.0065355	.0052871	.0042766	.0034597	.0027988	.0022640	.0018316
④ $\beta_2 \gamma$ copy	0	1.8545	3.7090	5.5635	7.4180	9.2725	11.127	12.982	14.836	16.690	18.545	20.400	22.254
⑤ $\beta_2 \gamma + \angle A_{\theta 2}$, ④ + 1.4248	1.4248	3.2793	5.1338	6.9883	8.8428	10.697	12.552	14.407	16.261	18.115	19.970	21.825	23.679
⑥ $\sin(\beta_2 \gamma + \angle A_{\theta 2})$, sin ⑤	+.98936	-.13727	-.91255	+.64812	+.54975	-.95574	-.014311	+.96382	-.52532	-.67043	+.90032	+.16505	-.99317
⑦ $\theta_2 e^{\alpha_2 \gamma} \sin(\beta_2 \gamma + \angle A_{\theta 2})$, ③ × ⑥	+.023073	-.0025896	-.013926	+.0080010	+.0054904	-.0077213	-.000093530	+.0050958	-.0022466	-.0023195	+.0025198	+.00037367	-.0018191

① γ	1	2	3	4	6	7	8	9	11	12	13	14	16
② $e^{\alpha_2 \gamma}$ copy	.95849	.91869	.88055	.84398	.77536	.74317	.71232	.68274	.62723	.60119	.57623	.55230	.50740
③ $\theta_2 e^{\alpha_2 \gamma}$ .023321 × ②	.022353	.021425	.020535	.019682	.018082	.017331	.016612	.015922	.014628	.014020	.013438	.012880	.011833
④ $\beta_2 \gamma$ copy	.37090	.74180	1.1127	1.4836	2.2254	2.5963	2.9672	3.3381	4.0799	4.4508	4.8217	5.1926	5.9344
⑤ $\beta_2 \gamma + \angle A_{\theta 2}$, ④ + 1.4248	1.7957	2.1666	2.5375	2.9084	3.6502	4.0211	4.3920	4.7629	5.5047	5.8756	6.2465	6.6174	7.3592
⑥ $\sin(\beta_2 \gamma + \angle A_{\theta 2})$, sin ⑤	.97480	.82767	.56799	.23107	-.48695	-.77040	-.94910	-.99873	-.70215	-.39635	-.036644	+.32804	+.88006
⑦ $\theta_2 e^{\alpha_2 \gamma} \sin(\beta_2 \gamma + \angle A_{\theta 2})$, ③ × ⑥	.021790	.017733	.011664	.0045479	-.0088050	-.013352	-.015766	-.015902	-.010271	-.0055568	-.00049242	+.0042252	+.010414

① γ	17	18	19	21	22	23	24	27	29	31	33	36	38
② $e^{\alpha_2 \gamma}$ copy	.48632	.46614	.44679	.41046	.39341	.37708	.36143	.31826	.29238	.26861	.24677	.21729	.19961
③ $\theta_2 e^{\alpha_2 \gamma}$ .023321 × ②	.011341	.010871	.010420	.0095723	.0091747	.0087939	.0084289	.0074221	.0068186	.0062643	.0057549	.0050674	.0046551
④ $\beta_2 \gamma$ copy	6.3053	6.6762	7.0471	7.7889	8.1598	8.5307	8.9016	10.014	10.756	11.498	12.240	13.352	14.094
⑤ $\beta_2 \gamma + \angle A_{\theta 2}$, ④ + 1.4248	7.7301	8.1010	8.4719	9.2137	9.5846	9.9555	10.326	11.439	12.181	12.923	13.665	14.777	15.519
⑥ $\sin(\beta_2 \gamma + \angle A_{\theta 2})$, sin ⑤	+.99233	+.96966	+.81513	+.20945	-.15919	-.50618	-.78413	-.90326	-.37590	+.34906	+.89061	+.80219	+.18790
⑦ $\theta_2 e^{\alpha_2 \gamma} \sin(\beta_2 \gamma + \angle A_{\theta 2})$, ③ × ⑥	.011254	.010541	.0084937	.0020049	-.0014605	-.0044513	-.0066094	-.0067041	-.0025631	.0021866	.0051254	.0040650	.00087469

① γ	42	47	52	57	62	65	67	70	72	75	77	80	85
② $e^{\alpha_2 \gamma}$ copy	.16847	.13629	.11025	.089189	.072152	.063529	.058364	.051393	.047213	.041573	.038193	.033631	.027207
③ $\theta_2 e^{\alpha_2 \gamma}$ .023321 × ②	.0039289	.0031784	.0025711	.0020800	.0016827	.0014816	.0013611	.0011985	.0011011	.00096952	.00089070	.00078431	.00063449
④ $\beta_2 \gamma$ copy	15.578	17.432	19.287	21.141	22.996	24.108	24.850	25.963	26.705	27.818	28.559	29.672	31.526
⑤ $\beta_2 \gamma + \angle A_{\theta 2}$, ④ + 1.4248	17.003	18.857	20.712	22.566	24.421	25.533	26.275	27.388	28.130	29.243	29.984	31.097	32.951
⑥ $\sin(\beta_2 \gamma + \angle A_{\theta 2})$, sin ⑤	-.96222	+.0069813	+.95782	-.54317	-.65342	+.38912	+.90924	+.77494	+.14436	-.82413	-.99027	-.31399	+.99939
⑦ $\theta_2 e^{\alpha_2 \gamma} \sin(\beta_2 \gamma + \angle A_{\theta 2})$, ③ × ⑥	-.0037805	+.000022189	+.0024627	-.0011298	-.0010995	+.00057652	+.0012376	+.00092877	+.00015895	-.00079901	-.00088203	-.00024627	+.00063410

M-29

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

DETERMINANTS

TABLE M-10

FOR $v_1, v_2, v_3, A_v, B_v, \psi_1, \psi_2, \psi_3, A_\psi, B_\psi, \varphi_1, \varphi_2, \varphi_3, A_\varphi, B_\varphi$

$U_0 = 300$ mph

10,000 FT DENSITY ALT.

①	σ	-1.7795		
②	ϵ	8.9037		
③	①$^2 = \sigma^2$	3.1666		
④	②$^2 = \epsilon^2$	77.605		
⑤	③ − ④ $= \sigma^2 - \epsilon^2$	-74.338		
⑥	①② $= \sigma\epsilon$	-15.666		
⑦	2⑥ $= 2\sigma\epsilon$	-31.332		
⑧	③① $= \sigma^3$	-5.6360		
⑨	3⑥ $= 3\sigma\epsilon$	-46.998		
⑩	⑨② $= 3\sigma\epsilon^2$	-413.76		
⑪	⑧ − ⑩ $= \sigma^3 - 3\sigma\epsilon^2$	$+408.12$		
⑫	④② $= \epsilon^3$	682.33		
⑬	⑨① $= 3\sigma^2\epsilon$	83.633		
⑭	−⑫ + ⑬ $= -\epsilon^3 + 3\sigma^2\epsilon$	-598.70		
⑮	⑤$^2 = (\sigma^2 - \epsilon^2)^2$	5526.1		
⑯	⑦$^2 = 4\sigma^2\epsilon^2$	981.69		
⑰	⑮ − ⑯ $= (\sigma^2 - \epsilon^2)^2 - 4\sigma^2\epsilon^2$	4544.4		
⑱	2⑦⑤ $= 4\sigma\epsilon(\sigma^2 - \epsilon^2)$	$+4658.3$		
⑲	d_2	-27.862		
⑳	⑲$^2 = d_2^2$	776.29		
㉑	⑲⑳ $= d_2^3$	$-21,629.$		
㉒	⑳$^2 = d_2^4$	$60263. \times 10^{+1}$		
㉓	d_3	$-.052796$		
㉔	㉓$^2 = d_3^2$	$.0027874$		
㉕	㉓㉔ $= d_3^3$	$-.00014716$		
㉖	㉔$^2 = d_3^4$	$.77696 \times 10^{-5}$		
㉗	$\Delta_2/\Delta_1 = v_1 \quad \psi_1 \quad \varphi_1$	0	$+.29383$	$+.000056433$
㉘	$\Delta_3/\Delta_1 = v_2 \quad \psi_2 \quad \varphi_2$	$-.50847 \times 10^{-5}$	$+.32479 \times 10^{-5}$	$+.00014648$
㉙	$\Delta_4/\Delta_1 = v_3 \quad \psi_3 \quad \varphi_3$	$+.00065181$	$-.29453$	$+.048758$
㉚	$\Delta_5/\Delta_1 = A_v \quad A_\psi \quad A_\varphi$	$-.00063358$	$.00070310$	$+.0011176$
㉛	$\Delta_6/\Delta_1 = B_v \quad B_\psi \quad B_\varphi$	$+.0017163$	$-.0016139$	$+.00098200$
㉜	㉚^2	$.40142 \times 10^{-6}$	$.494350 \times 10^{-6}$	$.12490 \times 10^{-5}$
㉝	㉛^2	$.29457 \times 10^{-5}$	$.26047 \times 10^{-5}$	$.96432 \times 10^{-6}$
㉞	㉜ + ㉝	$.33471 \times 10^{-5}$	$.30990 \times 10^{-5}$	$.22133 \times 10^{-5}$
㉟	$\sqrt{㉞} = v_4 \quad \psi_4 \quad \varphi_4$	$.0018295$	$.0017604$	$.0014877$
㊱	㉚/㉛ = tan $LA_v \quad LA_\psi \quad LA_\varphi$	$-.36915$	$-.43565$	$+1.1381$
㊲	$\tan^{-1}$ ㊱ = $LA_v \quad LA_\psi \quad LA_\varphi$ deg	339.74	156.46	48.696
㊳	.0174533 ㊲ = " " " rad	5.9296	2.7307	$.84991$
㊴				
		v	ψ	φ

BASIC DETERMINANT SET-UPS:

(A)

	v_1	v_2	v_3	A_v	B_v
v_0	1	1	1	1	0
$\dot{v}_0$	0	d_2	d_3	σ	ϵ
$\ddot{v}_0$	0	d_2^2	d_3^2	$\sigma^2 - \epsilon^2$	$2\sigma\epsilon$
$\dddot{v}_0$	0	d_2^3	d_3^3	$\sigma^3 - 3\sigma\epsilon^2$	$-\epsilon^3 + 3\sigma^2\epsilon$
$\ddddot{v}_0$	0	d_2^4	d_3^4	$(\sigma^2-\epsilon^2)^2 - 4\sigma^2\epsilon^2$	$4\sigma\epsilon(\sigma^2-\epsilon^2)$

(B)

	ψ_1	ψ_2	ψ_3	A_ψ	B_ψ
ψ_0	1	1	1	1	0
$\dot{\psi}_0$	0	d_2	d_3	σ	ϵ
$\ddot{\psi}_0$	0	d_2^2	d_3^2	$\sigma^2 - \epsilon^2$	$2\sigma\epsilon$
$\dddot{\psi}_0$	0	d_2^3	d_3^3	$\sigma^3 - 3\sigma\epsilon^2$	$-\epsilon^3 + 3\sigma^2\epsilon$
$\ddddot{\psi}_0$	0	d_2^4	d_3^4	$(\sigma^2-\epsilon^2)^2 - 4\sigma^2\epsilon^2$	$4\sigma\epsilon(\sigma^2-\epsilon^2)$

(C)

	φ_1	φ_2	φ_3	A_φ	B_φ
φ_0	1	1	1	1	0
$\dot{\varphi}_0$	0	d_2	d_3	σ	ϵ
$\ddot{\varphi}_0$	0	d_2^2	d_3^2	$\sigma^2 - \epsilon^2$	$2\sigma\epsilon$
$\dddot{\varphi}_0$	0	d_2^3	d_3^3	$\sigma^3 - 3\sigma\epsilon^2$	$-\epsilon^3 + 3\sigma^2\epsilon$
$\ddddot{\varphi}_0$	0	d_2^4	d_3^4	$(\sigma^2-\epsilon^2)^2 - 4\sigma^2\epsilon^2$	$4\sigma\epsilon(\sigma^2-\epsilon^2)$

$v_1 = 0$ ψ_1 = not needed φ_1 = not needed
$v_2 = -.50847 \times 10^{-5}$ ψ_2 = " φ_2 = "
$v_3 = +.00065181$ ψ_3 = " φ_3 = "
$v_4 = .0018295$ ψ_4 = " φ_4 = "
$LA_v = 5.9296$ LA_ψ = " LA_φ = "

$d_2 = -27.862$
$d_3 = -.052796$
$\sigma = -1.7795$
$\epsilon = 8.8037$

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

TABLE M-11

COMPUTATIONS FOR TIME HISTORY OF v

(U_0 = 300 mph TAS at 10,000 FT. DENSITY ALT.)

(1) t													
(2) γ	0	5	10	15	20	25	30	35	40	45	50	55	60
(3) $d_2\gamma$ = −27.862 × (2)	0	−139.31	−278.62	−417.93	−557.24	−696.55	−835.86	−975.17	−1114.5	−1253.8	−1393.1	−1532.4	−1671.7
(4) $d_3\gamma$ = −.052796 × (2)	0	−.26398	−.52796	−.79194	−1.0559	−1.3199	−1.5839	−1.8479	−2.1118	−2.3758	−2.6398	−2.9038	−3.1678
(5) $\sigma\gamma$ = −1.7795 × (2)	0	−8.8975	−17.795	−26.692	−35.590	−44.488	−53.385	−62.282	−71.180	−80.078	−88.975	−97.872	−106.77
(6) $\epsilon\gamma$ = 8.8037 × (2)	0	44.018	88.037	132.06	176.07	220.09	264.11	308.13	352.15	396.17	440.18	484.20	528.22
(7) $e^{d_2\gamma}$ = $e^{(3)}$	1	—	—										
(8) $e^{d_3\gamma}$ = $e^{(4)}$	1	.76799	.58980	.45296	.34788	.26716	.20517	.15757	.12102	.092940	.071378	.054817	.042098
(9) $e^{\sigma\gamma}$ = $e^{(5)}$	1	.00013673	—										
(10) $\epsilon\gamma + LA_v$ = (rad.) (6) + 5.9296	5.9296	49.948	—										
(11) $\sin(\epsilon\gamma + LA_v)$ = sin (10)	−.34628	−.31233	—										
(12) $v_2 e^{d_2\gamma}$ = $-.50847 \times 10^{-5}$ × (7)	$-.50847 \times 10^{-5}$	—	—										
(13) $v_3 e^{d_3\gamma}$ = +.00065181 × (8)	+.00065181	.00050058	.00038444	.00029524	.00022675	.00017414	.00013373	.00010271	.000078882	.000060579	.000046525	.000035730	.000027440
(14) $v_4 e^{\sigma\gamma}$ = .0018295 × (9)	.0018295	$.25015 \times 10^{-6}$	—										
(15) $v_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_v)$ = (14) × (11)	−.00063352	$-.78129 \times 10^{-7}$	—										
(16) $v = v_1 + (12) + (13) + (15)$	.000013205	.00050050	.00038444	.00029524	.00022675	.00017414	.00013373	.00010271	.000078882	.000060579	.000046525	.000035730	.000027440

(1) t													
(2) γ	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13
(3) $d_2\gamma$ = −27.862 × (2)	−.27862	−.55724	−.83586	−1.1145	−1.3931	−1.6717	−1.9503	−2.2290	−2.5076	−2.7862	−3.0648	−3.3434	−3.6221
(4) $e^{d_2\gamma}$ = $e^{(3)}$	.75683	.57279	.43350	.32808	.24830	.18793	.14223	.10764	.081467	.061657	.046665	.035318	.026727
(5) $v_2 e^{d_2\gamma}$ = $-.50847 \times 10^{-5}$ × (4)	$-.38483 \times 10^{-5}$	$-.29125 \times 10^{-5}$	$-.22042 \times 10^{-5}$	$-.16682 \times 10^{-5}$	$-.12625 \times 10^{-5}$	$-.95557 \times 10^{-6}$	$-.72320 \times 10^{-6}$	$-.54732 \times 10^{-6}$	$-.41424 \times 10^{-6}$	$-.31351 \times 10^{-6}$	$-.23728 \times 10^{-6}$	$-.17958 \times 10^{-6}$	$-.13590 \times 10^{-6}$

(1) t													
(2) γ	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
(3) $\sigma\gamma$ −1.7795 × (2)	−.17795	−.35590	−.53385	−.71180	−.88975	−1.0677	−1.2456	−1.4236	−1.6016	−1.7795	−1.9574	−2.1354	−2.3134
(4) $\epsilon\gamma$ 8.8037 × (2)	.88037	1.7607	2.6411	3.5215	4.4018	5.2822	6.1626	7.0430	7.9233	8.8037	9.6841	10.564	11.445
(5) $e^{\sigma\gamma}$ $e^{(3)}$	.83698	.70054	.58634	.49076	.41076	.34380	.28777	.24086	.20157	.16872	.14123	.11820	.098924
(6) $\epsilon\gamma + LA_v$ (rad.) (4) + 5.9296	6.8100	7.6903	8.5707	9.4511	10.331	11.212	12.092	12.973	13.853	14.733	15.614	16.494	17.375
(7) $\sin(\epsilon\gamma + LA_v)$ = sin (6)	+.50272	+.98663	+.75402	−.026351	−.78715	−.97667	−.45679	+.39555	+.95990	+.82767	+.093761	−.70760	−.99538
(8) $v_4 e^{\sigma\gamma}$ = .0018295 × (5)	.0015313	.0012816	.0010727	.00089785	.00075149	.00062898	.00052648	.00044064	.00036877	.00030867	.00025838	.00021625	.00018098
(9) $v_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_v)$ = (8) × (7)	+.00076982	+.0012646	+.00080884	−.000023659	−.00059154	−.00061431	−.00024049	+.00017430	+.00035398	+.00025548	+.000024226	−.00015302	−.00018014
(2) γ	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3			
(3) $\sigma\gamma$ −1.7795 × (2)	−2.4913	−2.6692	−2.8472	−3.0252	−3.2031	−3.3810	−3.5590	−3.7370	−3.9149	−4.0928			
(4) $\epsilon\gamma$ 8.8037 × (2)	12.325	13.206	14.086	14.966	15.847	16.727	17.607	18.488	19.368	20.249			
(5) $e^{\sigma\gamma}$ $e^{(3)}$	.082805	.069310	.058008	.048548	.040636	.034013	.028467	.023826	.019943	.016693			
(6) $\epsilon\gamma + LA_v$ (rad) (4) + 5.9296	18.265	19.136	20.016	20.896	21.777	22.657	23.537	24.418	25.298	26.179			
(7) $\sin(\epsilon\gamma + LA_v)$ = sin (6)	−.56064	+.28234	+.91914	+.88862	+.21303	−.61704	−.99970	−.65606	+.16505	+.86515			
(8) $v_4 e^{\sigma\gamma}$ = .0018296 × (5)	.00015149	.00012680	.00010613	.000088819	.000074344	.000062227	.000052080	.000043590	.000036486	.000030540			
(9) $v_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_v)$ = (8) × (7)	−.000084931	+.000035801	+.000097548	+.000078926	+.000015838	−.000038397	−.000052064	−.000028578	+.0000060220	+.000026422			

v_1 = not needed $\psi_1 = +.29383$ φ_1 = not needed
v_2 = " $\psi_2 = +.32479 \times 10^{-5}$ φ_2 = "
v_3 = " $\psi_3 = -.29453$ φ_3 = "
v_4 = " $\psi_4 = .0017604$ φ_4 = "
LA_v = $LA_\psi = 2.7307$ LA_φ = "

$d_2 = -27.862$
$d_3 = -.052796$
$\sigma = -1.7795$
$\epsilon = 8.8037$

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

TABLE M-12

COMPUTATIONS FOR TIME HISTORY OF ψ (U_0 = 300 mph TAS, at 10,000 FT. DENSITY ALT.)

① t													
② γ	0	5	10	15	20	25	30	35	40	45	50	55	60
③ $e^{d_2\gamma}$ copy	1	—	—										
④ $e^{d_3\gamma}$ copy	1	.76799	.58980	.45296	.34788	.26716	.20517	.15757	.12102	.092940	.071378	.054817	.042098
⑤ $e^{\sigma\gamma}$ copy	1	.00013673	$.18695 \times 10^{-7}$										
⑥ $\epsilon\gamma + LA_\psi$ Line 6 (Table M-11) + 2.7307	2.7307	46.749	90.768	134.79	178.80	222.82	266.84	310.86	354.88	398.9	442.91	486.93	530.95
⑦ $\sin(\epsilon\gamma + LA_\psi)$ = sin ⑥	+.39939	+.36650	+.33216										
⑧ $\psi_2 e^{d_2\gamma} = +.32479 \times 10^{-5}$ × ③	$+.32479 \times 10^{-5}$	—	—										
⑨ $\psi_3 e^{d_3\gamma} = -.29453$ × ④	-.29453	-.22620	-.17371	-.13341	-.10246	-.078687	-.060429	-.046409	-.035644	-.027374	-.021023	-.016145	-.012399
⑩ $\psi_4 e^{\sigma\gamma}$ = .0017604 × ⑤	.0017604	$.24070 \times 10^{-6}$	$.32911 \times 10^{-10}$										
⑪ $\psi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_\psi)$ ⑩ × ⑦	+.00070309	$+.88217 \times 10^{-7}$	$+.10932 \times 10^{-10}$										
⑫ $\psi = \psi_1$ + ⑧ + ⑨ + ⑪	$+.63379 \times 10^{-5}$	+.067630	+.12012	+.16042	+.19137	+.21514	+.23340	+.24742	+.25819	+.26646	+.27281	+.27768	+.28143
① t													
② γ	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13
③ $e^{d_2\gamma}$ copy	.75683	.57279	.43350	.32808	.24830	.18793	.14223	.10764	.081467	.061657	.046665	.035318	.026727
④ $\psi_2 e^{d_2\gamma}$ $+.32479 \times 10^{-5}$ × ③	$.24581 \times 10^{-5}$	$.18604 \times 10^{-5}$	$.14080 \times 10^{-5}$	$.10656 \times 10^{-5}$	$.80645 \times 10^{-6}$	$.61038 \times 10^{-6}$	$.46195 \times 10^{-6}$	$.34960 \times 10^{-6}$	$.26460 \times 10^{-6}$	$.20026 \times 10^{-6}$	$.15156 \times 10^{-6}$	$.11471 \times 10^{-6}$	$.86807 \times 10^{-7}$
① t													
② γ	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
③ $\epsilon\gamma$ copy 8.8037 × ②	.88037	1.7607	2.6411	3.5215	4.4018	5.2822	6.1626	7.0430	7.9233	8.8037	9.6841	10.564	11.445
④ $e^{\sigma\gamma}$ copy (TABLE M-11)	.83698	.70054	.58634	.49076	.41076	.34380	.28777	.24085	.20157	.16872	.14123	.11820	.098924
⑤ $\epsilon\gamma + LA_\psi$ ③ + 2.7307	3.6111	4.4914	5.3718	6.2522	7.1325	8.0129	8.8933	9.7737	10.654	11.534	12.415	13.295	14.176
⑥ $\sin(\epsilon\gamma + LA_\psi)$ sin ⑤	-.45243	-.97569	-.79037	-.031062	+.75080	+.98739	+.50679	-.34186	-.94223	-.85851	-.15074	+.66588	+.99925
⑦ $\psi_4 e^{\sigma\gamma}$ = .0017604 × ④	.0014734	.0012332	.0010322	.00086393	.00072310	.00060523	.00050659	.00042399	.00035484	.00029701	.00024862	.00020808	.00017415
⑧ $\psi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_\psi)$ = ⑥ × ⑦	-.00066661	-.0012032	-.00081582	-.000026835	+.00054290	+.00059760	+.00025673	-.00014495	-.00033434	-.00025499	-.000037477	+.00013856	+.00017402
①													
② γ	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3			
③ $\epsilon\gamma$ copy 8.8037 × ②	12.325	13.206	14.086	14.966	15.847	16.727	17.607	18.488	19.368	20.249			
④ $e^{\sigma\gamma}$ copy (TABLE M-11)	.082805	.069310	.058008	.048548	.040636	.034013	.028467	.023826	.019943	.016693			
⑤ $\epsilon\gamma + LA_\psi$ ③ + 2.7307	15.056	15.937	16.817	17.697	18.578	19.458	20.338	21.219	22.099	22.980			
⑥ $\sin(\epsilon\gamma + LA_\psi)$ sin ⑤	+.60682	-.22699	-.89525	-.91355	-.26892	+.57215	+.99664	+.69717	-.10800	-.83581			
⑦ $\psi_4 e^{\sigma\gamma}$ .0017604 × ④	.00014577	.00012201	.00010212	.000085464	.000071536	.000059876	.000050113	.000041943	.000035108	.000029386			
⑧ $\psi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA_\psi)$ = ⑥ × ⑦	+.000088456	-.000027696	-.000091423	-.000078076	-.000019237	+.000034258	+.000049945	+.000029241	-.0000037917	-.000024561			

v_1 = not needed, v_2 = ", v_3 = ", v_4 = ", LA_v = "

ψ_1 = not needed, ψ_2 = ", ψ_3 = ", ψ_4 = ", LA_ψ = "

$\varphi_1 = +.000056433$, $\varphi_2 = +.00014648$, $\varphi_3 = +.048758$, $\varphi_4 = +.0014877$, $LA_\varphi = .84991$

$d_2 = -27.862$, $d_3 = -.052796$, $\sigma = -1.7795$, $\epsilon = 8.8037$

ANALYSIS FOR FIXED-CONTROL TRANSIENT RESPONSE OF A-26 AIRPLANE TO LATERAL STEP-FUNCTION

TABLE M-13

COMPUTATIONS FOR TIME HISTORY OF φ

(U_0 = 300mph TAS at 10,000 FT. DENSITY ALT.)

① t													
② γ	0	5	10	15	20	25	30	35	40	45	50	55	60
③ $e^{d_2\gamma}$ copy	1	—	—										
④ $e^{d_3\gamma}$ "	1	.76799	.58980	.45296	.34788	.26716	.20517	.15757	.12102	.092940	.071378	.054817	.042098
⑤ $e^{\sigma\gamma}$ "	1	.00013673	$.18696 \times 10^{-7}$										
⑥ $\epsilon\gamma$ "	0	44.018	88.037	132.06	176.07	220.09	264.11	308.13	352.15	396.17	440.18	484.20	528.22
⑦ $\epsilon\gamma + LA\varphi$ = ⑥ + .84991	.84991	44.868	88.887	132.91	176.92	220.94	264.96	308.98	353.00	397.02	441.03	485.05	529.07
⑧ $\sin(\epsilon\gamma + LA\varphi)$ = sin ⑦	+.75122	+.77384	+.79653										
⑨ $\varphi_2 e^{d_2\gamma}$ = +.00014648 × ③	+.00014648	—	—										
⑩ $\varphi_3 e^{d_3\gamma}$ = +.048758 × ④	+.048758	+.037446	.028767	.022086	.016962	.013026	.010004	.0076828	.0059007	.0045316	.0034802	.0026728	.0020526
⑪ $\varphi_4 e^{\sigma\gamma}$ = +.0014877 × ⑤	+.0014877	$.20341 \times 10^{-6}$	$.27813 \times 10^{-10}$										
⑫ $\varphi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA\varphi)$ = ⑪ × ⑧	.0011176	$.15741 \times 10^{-6}$	$.22164 \times 10^{-10}$										
⑬ $\varphi = \varphi_1$ + ⑨ + ⑩ + ⑫	+.050079	.037503	.028813	.022141	.017018	.013082	.010050	.0077392	.0059571	.0045880	.0035366	.0027292	.0021090

① t													
② γ	.01	.02	.03	.04	.05	.06	.07	.08	.09	.10	.11	.12	.13
③ $e^{d_2\gamma}$ copy	.75683	.57279	.43350	.32808	.24830	.18793	.14223	.10764	.081467	.061657	.046665	.035318	.026727
④ $\varphi_2 e^{d_2\gamma}$ +.00014648 × ③	.00011086	.000083902	.000063499	.000048057	.000036371	.000027528	.000020884	.000015767	.000011933	$.90315 \times 10^{-5}$	$.68355 \times 10^{-5}$	$.51734 \times 10^{-5}$	$.39150 \times 10^{-5}$

① t													
② γ	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
③ $\epsilon\gamma$ copy 8.8037 × ②	.88037	1.7607	2.6411	3.5215	4.4018	5.2822	6.1626	7.0430	7.9233	8.8037	9.6841	10.564	11.445
④ $e^{\sigma\gamma}$ copy (Table M-11)	.83698	.70054	.58634	.49076	.41076	.34380	.28777	.24085	.20157	.16872	.14123	.11820	.098924
⑤ $\epsilon\gamma + LA\varphi$ ③ + .84991	1.7303	2.6106	3.4910	4.3714	5.2517	6.1321	7.0125	7.8929	8.7732	9.6536	10.534	11.414	12.295
⑥ $\sin(\epsilon\gamma + LA\varphi)$ sin ⑤	.98731	.50633	-.34235	-.94241	-.85806	-.15067	+.66640	+.99924	+.60640	-.22682	-.89532	-.91376	-.26808
⑦ $\varphi_4 e^{\sigma\gamma}$ = +.0014877 × ④	.0012462	.0010422	.00087230	.00073010	.00061109	.00051147	.00042812	.00035831	.00029988	.00025100	.00021011	.00017585	.00014717
⑧ $\varphi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA\varphi)$ ⑥ × ⑦	+.0012294	+.00052770	-.00029863	-.00068806	-.00052435	-.000077012	+.00028530	+.00035804	+.00018185	-.000056932	-.00018812	-.00016068	-.000039453

① t													
② γ	1.4	1.5	1.6	1.7	1.8	1.9	2.0	2.1	2.2	2.3			
③ $\epsilon\gamma$ copy 8.8037 × ②	12.325	13.206	14.086	14.966	15.847	16.727	17.607	18.488	19.368	20.249			
④ $e^{\sigma\gamma}$ copy (Table M-11)	.082805	.069310	.058008	.048548	.040636	.034013	.028467	.023826	.019943	.016693			
⑤ $\epsilon\gamma + LA\varphi$ ③ + .84991	13.175	14.056	14.936	15.816	16.697	17.577	18.457	19.338	20.218	21.099			
⑥ $\sin(\epsilon\gamma + LA\varphi)$ sin ⑤	+.57172	+.99671	+.69754	-.10783	-.83552	-.95579	-.38268	+.46947	+.97958	+.77824			
⑦ $\varphi_4 e^{\sigma\gamma}$ = +.0014877 × ④	.00012319	.00010311	.000086299	.000072225	.000060454	.000050601	.000042350	.000035446	.000029669	.000024834			
⑧ $\varphi_4 e^{\sigma\gamma} \sin(\epsilon\gamma + LA\varphi)$ ⑥ × ⑦	+.000070430	+.00010277	+.000060197	$-.77880 \times 10^{-5}$	-.000050511	-.000048364	-.000016206	+.000016641	+.000029063	+.000019327			

APPENDIX N

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL VARIATION IN OUTPUT SIGNAL FROM OSCILLATOR WHICH DRIVES ELEVATOR SERVO, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

APPENDIX N

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL VARIATION IN OUTPUT SIGNAL FROM OSCILLATOR WHICH DRIVES ELEVATOR SERVO, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

With reference to Appendices J and H, a summary of the basic equations used in this analysis follows:

$$C_1 = x_\theta z_u - x_u z_\theta. \tag{1}$$

$$C_2 = z_\theta - x_u. \tag{2}$$

$$\phi_1 = \tan^{-1}\left(\frac{\omega_e C_2}{C_1 - \omega_e^2}\right). \tag{3}$$

$$C_3 = (x_u z_w - x_w z_u) + (x_u + z_w)m_q + \mu_c m_{\dot{w}}(x_u - z_\theta)$$
$$-m_{\delta_e} k_\theta - \mu_c m_w. \tag{4}$$

$$C_4 = m_{\delta_e} k_\theta (x_w z_u - x_u z_w) + \mu_c m_w (x_u z_\theta - x_\theta z_u)$$
$$+ \mu_c m_u (x_\theta z_w - x_w z_\theta). \tag{5}$$

$$C_5 = -x_u - z_w - m_q - \mu_c m_{\dot{w}}. \tag{6}$$

$$C_6 = (x_w z_u - x_u z_w)m_q + (x_u + z_w)m_{\delta_e} k_\theta + (x_u - z_\theta)\mu_c m_w$$
$$+ (x_u z_\theta - x_\theta z_u)\mu_c m_{\dot{w}} - (x_w + x_\theta)\mu_c m_u. \tag{7}$$

$$\phi_2 = \tan^{-1}\left[\frac{\omega_e(\omega_e^2 C_5 - C_6)}{-\omega_e^4 + \omega_e^2 C_3 - C_4}\right]. \tag{8}$$

$$K_1 = x_u z_w - x_w z_u. \tag{9}$$

$$K_2 = -x_u - z_w. \tag{10}$$

$$\psi_1 = \tan^{-1}\left(\frac{\omega_e K_2}{K_1 - \omega_e^2}\right). \tag{11}$$

$$H_1 = x_\theta z_w - x_w z_\theta. \tag{12}$$

$$H2 = -x_w - x_\theta. \tag{13}$$

$$\beta_1 = \tan^{-1}\left(\frac{-\omega_e H_2}{-H_1}\right). \tag{14}$$

$$\delta_e = \delta_{e_a} \sin \omega_e \gamma. \tag{15}$$

$$u = \mu_{u\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{u\delta_e}). \tag{16}$$

$$w = \mu_{w\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{w\delta_e}). \tag{17}$$

$$\theta = \mu_{\theta\delta_e} \delta_{e_a} \sin(\omega_e \gamma + LA_{\theta\delta_e}). \tag{18}$$

$$\mu_{u\delta_e} = -m_{\delta_e} \sqrt{\frac{(-H_1)^2 + (-\omega_e H_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2 (\omega_e^2 C_5 - C_6)^2}} \tag{19}$$

$$\mu_{w\delta_e} = -m_{\delta_e} \sqrt{\frac{(C_1 - \omega_e^2)^2 + (\omega_e C_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2 (\omega_e^2 C_5 - C_6)^2}} \tag{20}$$

$$\mu_{\theta\delta_e} = -m_{\delta_e} \sqrt{\frac{(K_1 - \omega_e^2)^2 + (\omega_e K_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2 (\omega_e^2 C_5 - C_6)^2}} \tag{21}$$

$$LA_{u\delta_e} = \beta_1 - \phi_2. \tag{22}$$

$$LA_{w\delta_e} = \phi_1 - \phi_2. \tag{23}$$

$$LA_{\theta\delta_e} = \psi_1 - \phi_2. \tag{24}$$

$$\frac{\theta_a}{w_a} = \frac{\dot{\theta}_a}{\dot{w}_a} \doteq \sqrt{\frac{(K_1 - \omega_e^2)^2 + (\omega_e K_2)^2}{(C_1 - \omega_e^2)^2 + (\omega_e C_2)^2}} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}} . \tag{25}$$

$$k_\theta = \frac{\partial \delta_e}{\partial \theta} = \frac{\partial \Delta_E}{\partial \theta} . \tag{26}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis, and a value of $k_\theta = .5$ was arbitrarily chosen. The theoretical longitudinal dynamic stability coefficients used in this analysis, for the A-26 Airplane, were (see Appendix D) as follows:

$T_A = 4.4716$ sec.

$\mu_c = 241.43$

$x_u = -.087261$

$x_w = .18720$

$x_\theta = -.32687$

$z_u = -.65374$

$z_w = -4.8701$

z_θ negligible for $\theta_o = 0$

m_u assumed negligible for A-26 Airplane.

$m_w = -.61949$

$m_{\dot{w}} = -.022236$

$m_q = -16.3668$

$m_{\delta_e} = -351.95$

From equations 1, 2, 4, 5, 6, 7, 9, 10, 12 and 13:

$C_1 = .21369$ $\quad$ $K_1 = .54735$

$C_2 = .087261$ $\quad$ $K_2 = 4.9574$

$C_3 = 407.69$ $\quad$ $H_1 = 1.5919$

$C_4 = 128.28$ $\quad$ $H_2 = .13967$

$C_5 = 26.693$

$C_6 = 895.53$

Note that in this case, only C_3, C_4 and C_6 depend on k_θ.

Using equations 4, 8, 11, 14, 22, 23 and 24, the lead angles, $LA_{u\delta_e}$, $LA_{w\delta_e}$ and $LA_{\theta\delta_e}$, were computed for chosen values of Ω_E. These computations are shown in Table N-1, and the results are plotted in Figures N-1, N-2 and N-3. The results of the analysis for lead angles for $k_\theta = 0$, (see Section A-1 of Appendix L), are also included in Figures N-1, N-2 and N-3 for comparison. Using equations 19, 20 and 21, the amplitude response ratios, $\mu_{u\delta_e}$, $\mu_{w\delta_e}$ and $\mu_{\theta\delta_e}$, were computed for chosen values of Ω_E. These computations are also shown in Table N-1, and the results are plotted in Figures N-4, N-5 and N-6. The results of the analysis for amplitude response ratios for $k_\theta = 0$, (see Section

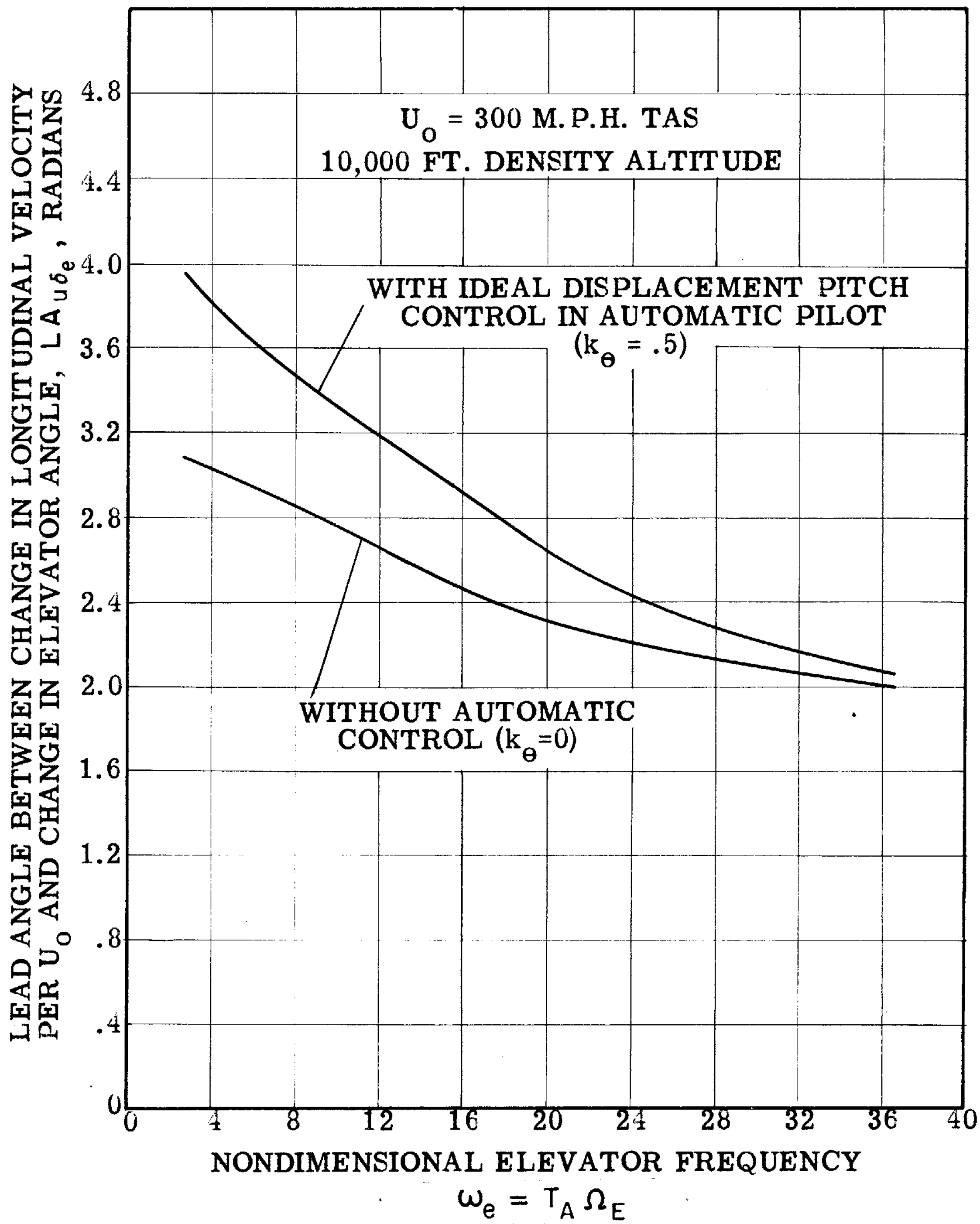
A-1 of Appendix L), are also included in Figures N-4, N-5 and N-6 for comparison. Equation 25 was used to compute the ratio

$$\frac{\theta_\alpha}{w_\alpha} = \frac{q_\alpha}{\dot{w}_\alpha} = \frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}}$$

for chosen values of Ω_E. These computations are shown in Table N-1, and the results are plotted in Figure N-7. Note that the ratio $\frac{\mu_{\theta\delta_e}}{\mu_{w\delta_e}}$ is independent of k_θ.

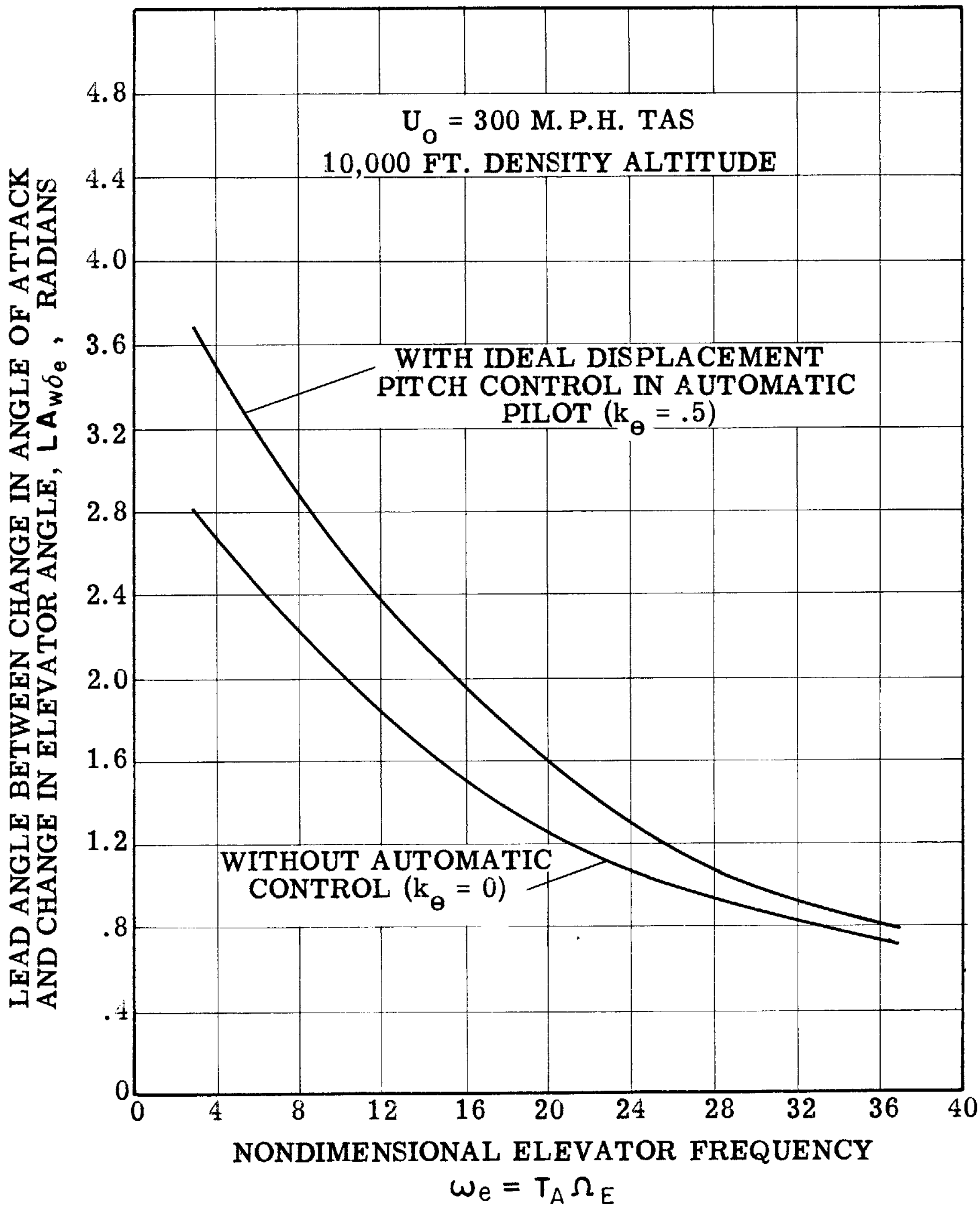
Equations 16, 17 and 18 could be used to compute the time history of $\frac{\delta_e}{\delta_{e_\alpha}}$, $\frac{u}{\delta_{e_\alpha}}$, $\frac{w}{\delta_{e_\alpha}}$ and $\frac{\theta}{\delta_{e_\alpha}}$, respectively, (analogous to the time histories shown in Figures L-8, L-9 and L-10), but this analysis will not be included.

It should be noted that δ_e used in this analysis is the elevator angle which would exist if k_θ were equal to zero. It is NOT equal to the resultant δ_{e_r} which could be computed from equation 21 in Appendix J (see discussion on page J-2). An extension of this analysis could be made by computing the amplitude of δ_{e_r} and then putting the response computed in this appendix on the basis of δ_{e_r} instead of δ_e. The resulting curves should then be independent of k_θ.



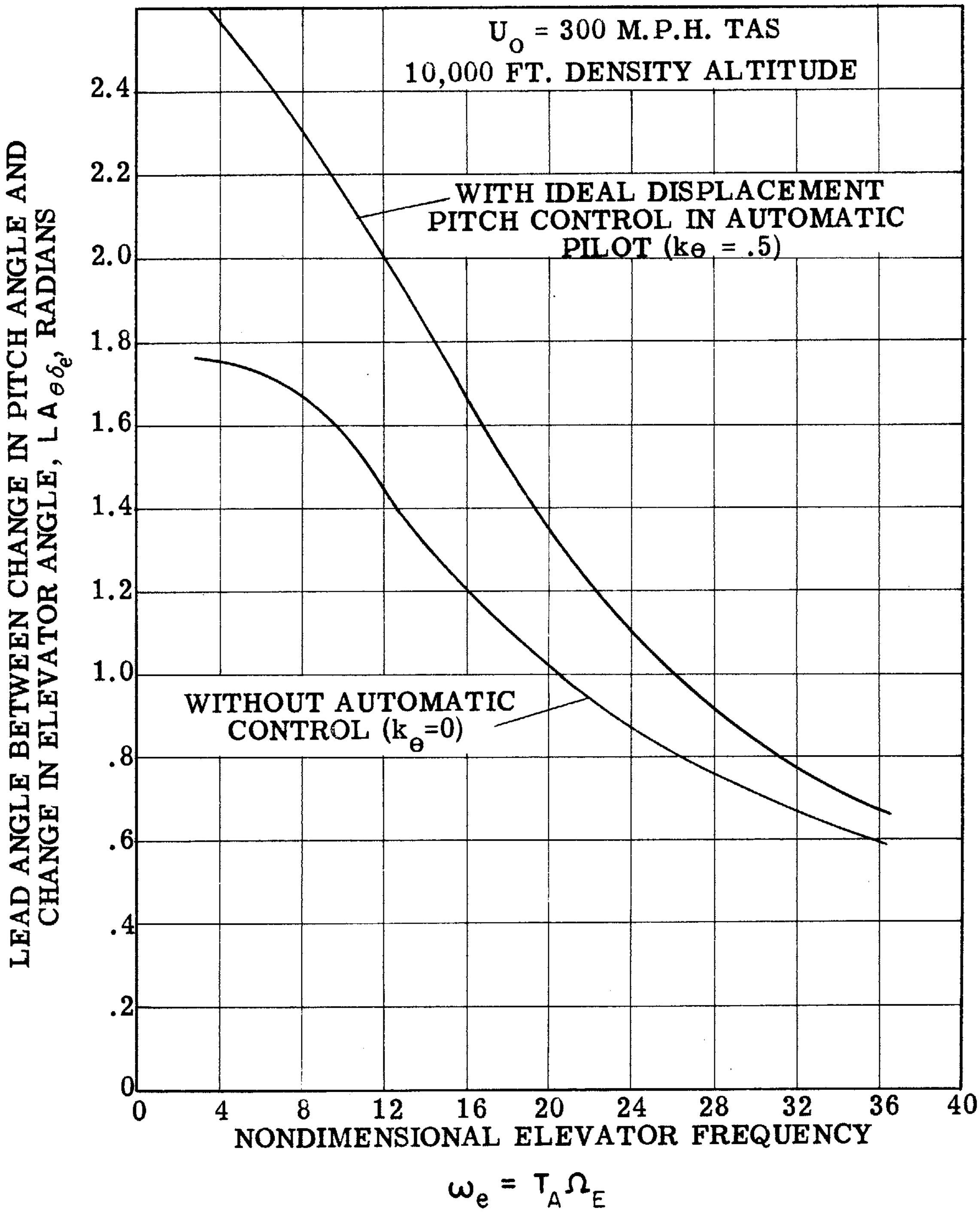
EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON LEAD ANGLE BETWEEN CHANGE IN LONGITUDINAL VELOCITY PER AIRPLANE TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT = 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

FIGURE N-1



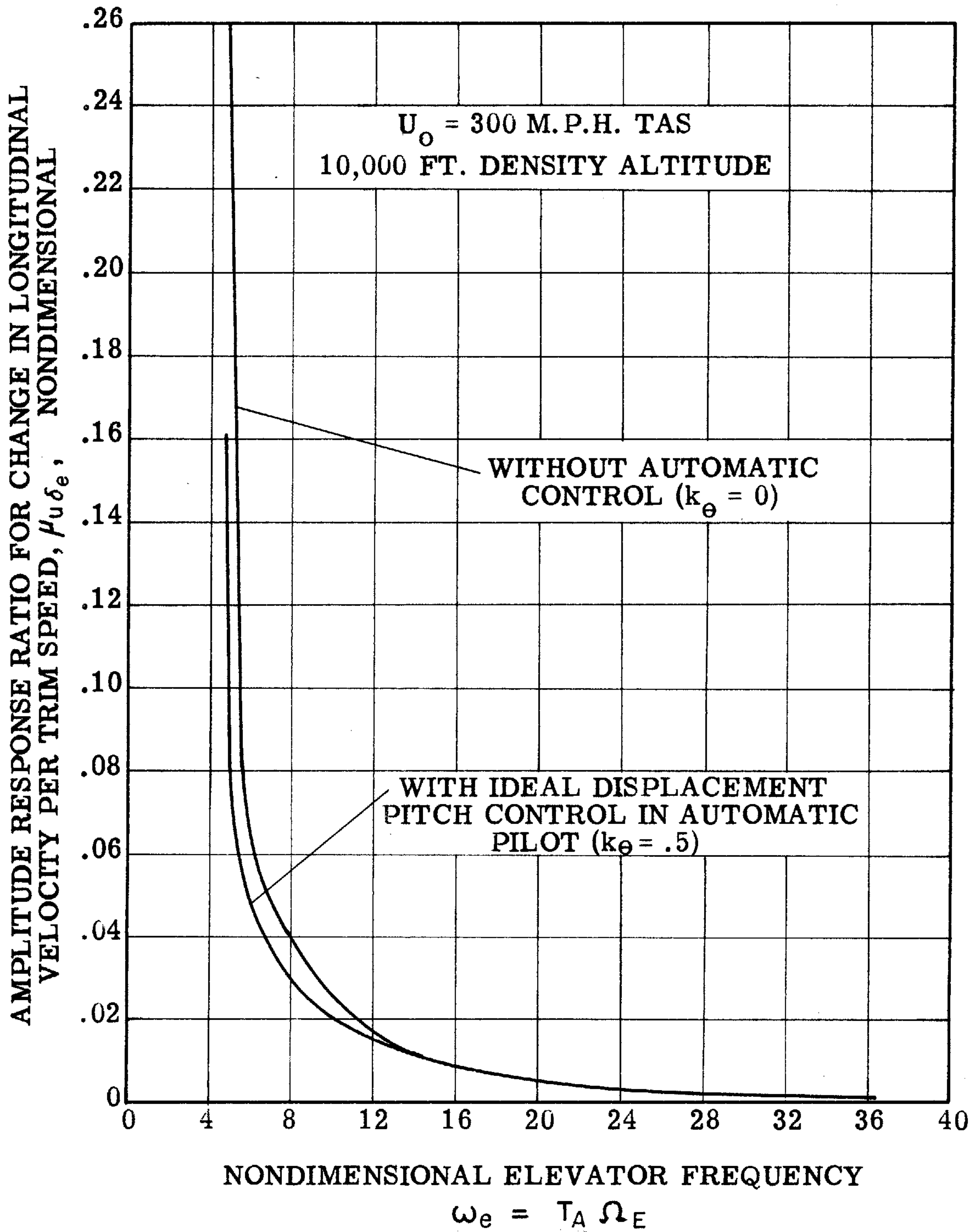
EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON LEAD ANGLE BETWEEN CHANGE IN ANGLE OF ATTACK AND CHANGE IN ELEVATOR ANGLE, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT EQUAL 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

FIGURE N-2



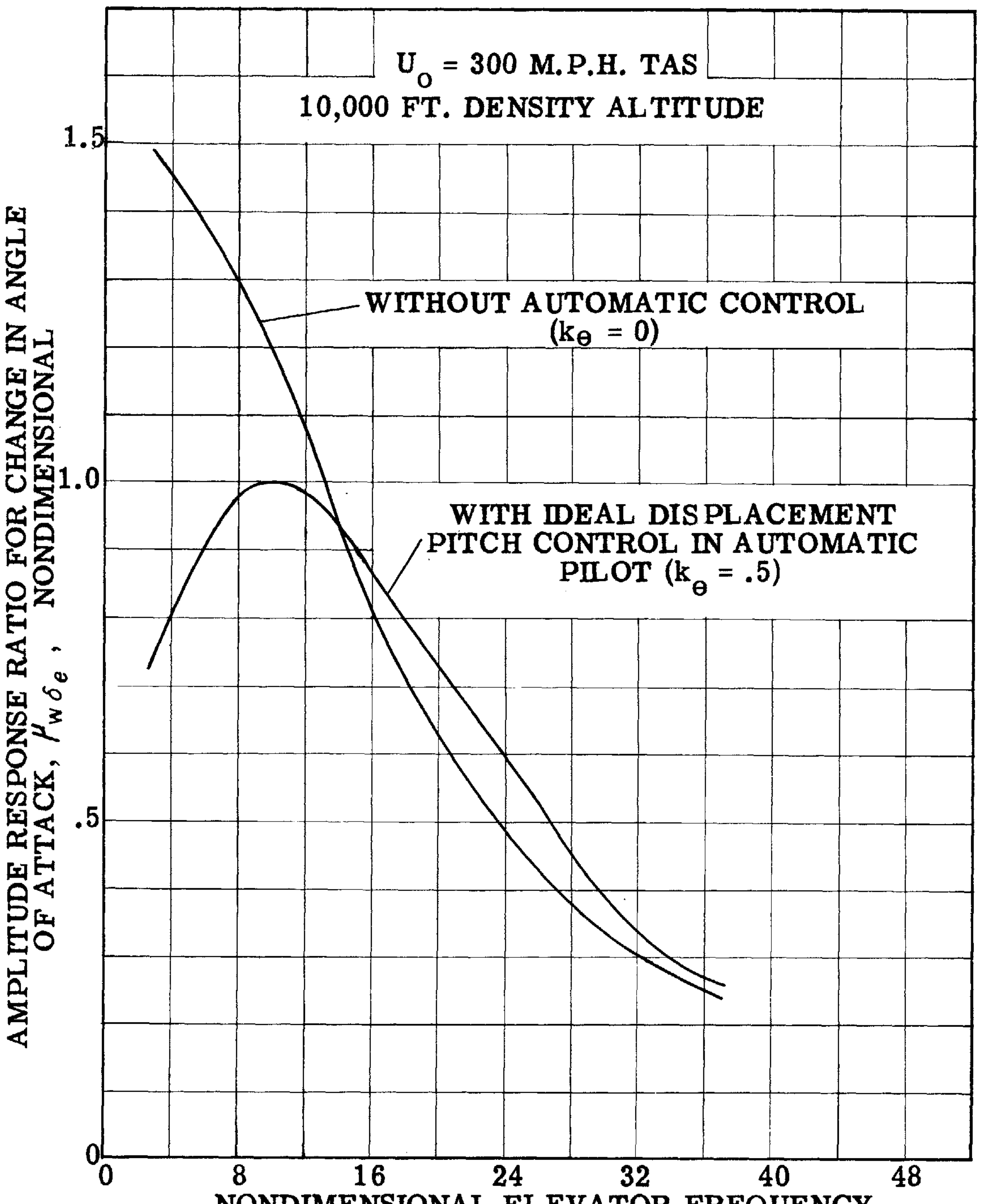
EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON LEAD ANGLE BETWEEN CHANGE IN PITCH ANGLE AND CHANGE IN ELEVATOR ANGLE, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT EQUAL 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

FIGURE N-3



EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT = 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

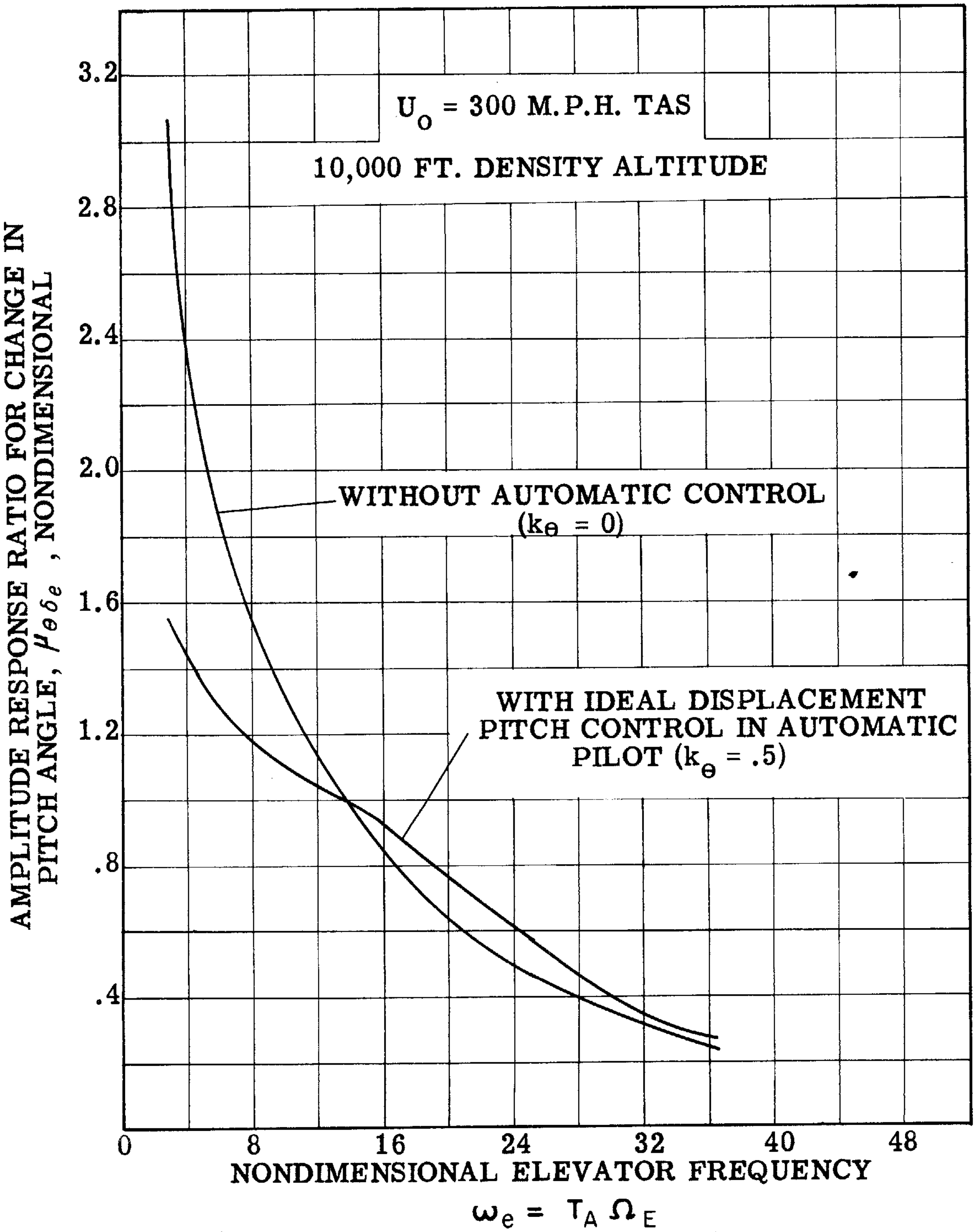
FIGURE N-4



$\omega_e = T_A \Omega_E$

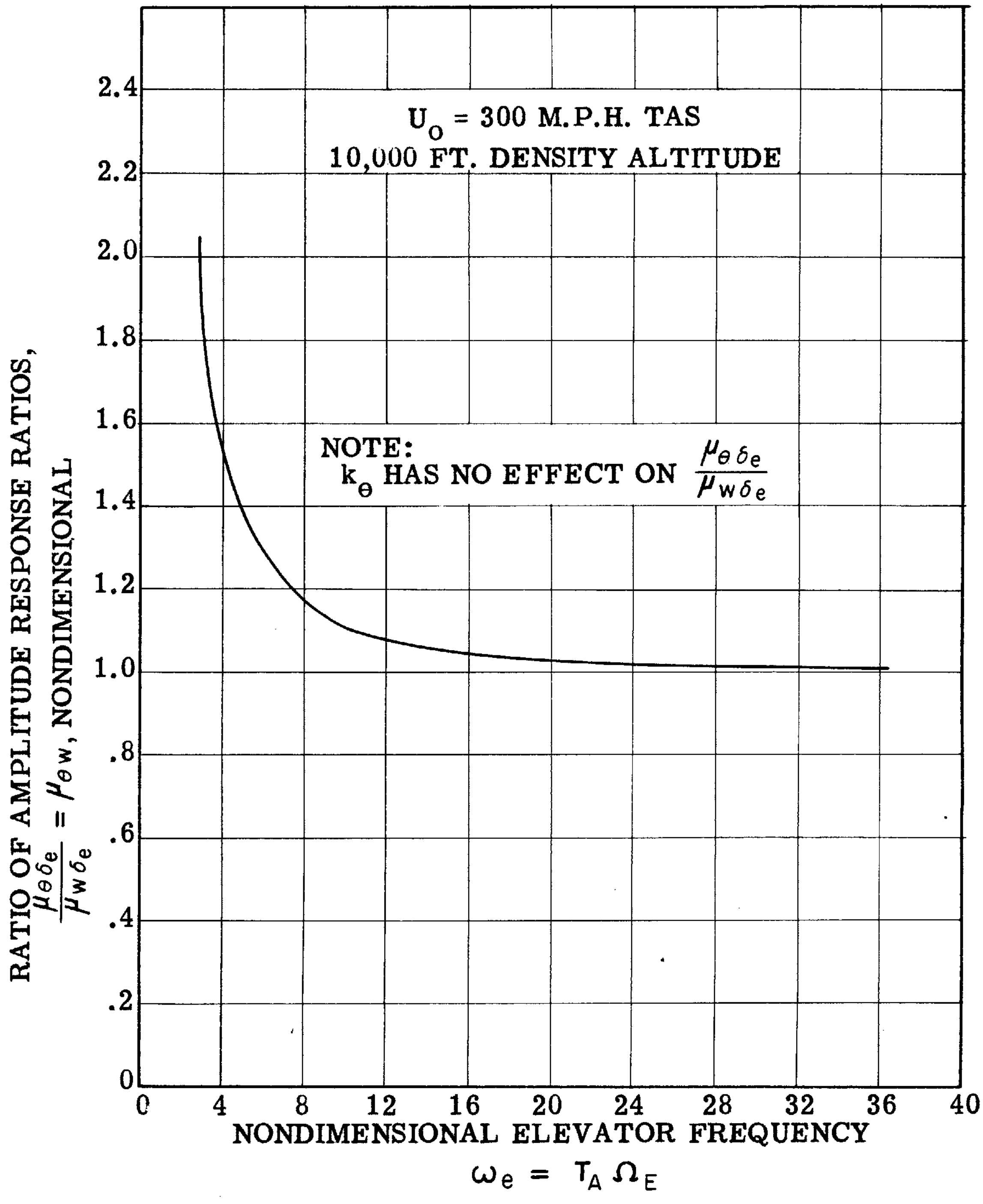
EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN ANGLE OF ATTACK, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT = 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

FIGURE N-5



EFFECT OF IDEAL DISPLACEMENT PITCH CONTROL ON AMPLITUDE RESPONSE RATIO FOR CHANGE IN PITCH ANGLE, AT VARIOUS VALUES OF ELEVATOR FREQUENCY, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, FOR k_θ IN AUTOMATIC PILOT = 0 AND .5, ELEVATOR MOTION FOR k_θ = 0 BASIS FOR COMPARISON

FIGURE N-6



EFFECT OF ELEVATOR FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR CHANGE IN PITCH ANGLE TO AMPLITUDE RESPONSE RATIO FOR CHANGE IN NORMAL VELOCITY PER AIRPLANE TRIM SPEED, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR

FIGURE N-7

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL VARIATION IN OUTPUT SIGNAL FROM OSCILLATOR WHICH DRIVES ELEVATOR SERVO, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

$k_\theta = \frac{\partial \delta_e}{\partial \theta} = .5$

TABLE N-1

(BASED UPON δ_e FOR $k_\theta = 0$)

0	Ω_E, cycles per second	0.1	0.3	0.5	0.7	0.9	1.1	1.3
2	$\omega_e = T_A \cdot 2\pi \cdot \Omega_E =$ (1) 28.096	2.8046	8.4288	14.048	19.667	25.286	30.906	36.525
3	$\omega_e^2 =$ (2)^2	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
4	$\omega_e^4 =$ (3)^2	62.314	5047.4	38947.	$14961.\times10^{+1}$	$40881\times10^{+1}$	$91237\times10^{+1}$	$17798\times10^{+2}$
5	$\omega_e^2 C_3 =$ (3) C_3	3218.3	28964.	80458.	$15769.\times10^{+1}$	$26067.\times10^{+1}$	$38942.\times10^{+1}$	$54390.\times10^{+1}$
6	$-$(4) + (5) $- C_4$	3027.7	23788.	41383.	7951.7	$-14827.\times10^{+1}$	$-32308.\times10^{+1}$	$-12360.\times10^{+2}$
7	(6)^2	$91670.\times10^{+2}$	$56587.\times10^{+4}$	$17126.\times10^{+5}$	$63230.\times10^{+3}$	$21984.\times10^{+4}$	$27361.\times10^{+7}$	$15277.\times10^{+8}$
8	$\omega_e^2 C_5 =$ (8) TABLE L-2	210.71	1896.4	5267.9	10325.	17067.	25497.	35611.
9	(8) $- C_6$	-684.82	+1000.9	4372.4	9429.5	16171.	24601.	34715.
10	(9)^2(3)	$37021\times10^{+2}$	$7173.\times10^{+3}$	$37729.\times10^{+5}$	$34391.\times10^{+6}$	$16080.\times10^{+7}$	$57808.\times10^{+7}$	$1607.\times10^{+8}$
11	(7) + (10) = denominator under $\sqrt{\ }$	$12869.\times10^{+3}$	$63704.\times10^{+4}$	$54855.\times10^{+5}$	$34454.\times10^{+6}$	$16300.\times10^{+7}$	$85169\times10^{+7}$	$31354.\times10^{+8}$
12	$H_1^2 + (\omega_e H_2)^2 =$ (14) TABLE L-2	2.6881	3.9201	6.3839	10.080	15.007	21.167	28.558
13	(12)/(11)	20888×10^{-6}	$.61536\times10^{-8}$	$.11638\times10^{-8}$	$.29256\times10^{-9}$	$.92067\times10^{-10}$	24853×10^{-10}	$.91082\times10^{-11}$
14	$\sqrt{(13)}$	$.45703\times10^{-3}$	$.78445\times10^{-4}$	$.34115\times10^{-4}$	$.17104\times10^{-4}$	$.95952\times10^{-5}$	$.49853\times10^{-5}$	$.30180\times10^{-5}$
15	$-m_{\delta_e}$ (14) $= \mu_{u\delta_e}$	.16085	.027609	.012007	.0060198	.0033770	.0017546	.0010622
16	(20) TABLE L-2	59.045	5017.5	38866.	$14944.\times10^{+1}$	$40864.\times10^{+1}$	$91198.\times10^{+1}$	$17793.\times10^{+2}$
17	(16)/(11)	$.45882\times10^{-5}$	$.78763\times10^{-5}$	$.70852\times10^{-5}$	$.43374\times10^{-5}$	$.25064\times10^{-5}$	$.10708\times10^{-5}$	$.56749\times10^{-6}$
18	$\sqrt{(17)}$	$.21420\times10^{-2}$	$.28065\times10^{-2}$	$.26618\times10^{-2}$	$.20826\times10^{-2}$	$.15832\times10^{-2}$	$.10348\times10^{-2}$	$.75332\times10^{-3}$
19	$-m_{\delta_e}$ (18) $= \mu_{w\delta_e}$	.75388	.98775	.93682	.73297	.55721	.36420	.26513
20	(26) TABLE L-2	247.96	6716.0	43,580.	$15869.\times10^{+1}$	$42381.\times10^{+1}$	$93479.\times10^{+1}$	$18113.\times10^{+2}$
21	(20)/(11)	$.19268\times10^{-4}$	$.10543\times10^{-4}$	$.79446\times10^{-5}$	$.46058\times10^{-5}$	$.26001\times10^{-5}$	$.10976\times10^{-5}$	$.57769\times10^{-6}$
22	$\sqrt{(21)}$	$.43895\times10^{-2}$	$.32470\times10^{-2}$	$.28186\times10^{-2}$	$.21461\times10^{-2}$	$.16125\times10^{-2}$	$.10477\times10^{-2}$	$.76006\times10^{-3}$
23	$-m_{\delta_e}$ (22) $= \mu_{\theta\delta_e}$	1.5449	1.1428	.99201	.75532	.56752	.36874	.26750
24	(23)/(19) $= \mu_{\theta\delta_e}/\mu_{w\delta_e} = \mu_{\gamma/\alpha}$	2.0493	1.1570	1.0589	1.0305	1.0185	1.0125	1.0089
25	(9)(2) $= \omega_e[\omega_e^2 C_5 - C_6]$	-1924.1	8436.4	61423.	$18545.\times10^{+1}$	$40890.\times10^{+1}$	$76032.\times10^{+1}$	$12680.\times10^{+2}$
26	(25)/(6) $= \tan\varphi_2$	-.63550	.35465	1.4843	23.322	-2.7578	-1.4535	-1.0259
27	$\tan^{-1}$ (26) $= \varphi_2$ degrees	-32.436	19.527	56.032	87.545	109.93	124.53	134.27
28	.0174533 (27) $= \varphi_2$ radians	-.56612	.34081	.97794	1.5279	1.9186	2.1735	2.3435
29	(52) TABLE L-1 $= \varphi_1$	3.1097	3.1311	3.1353	3.1372	3.1381	3.1388	3.1392
30	(50) " $= \beta_1$	3.3833	3.7783	4.0308	4.1872	4.2890	4.3591	4.4099
31	(53) " $= \psi_1$	2.0562	2.6065	2.8014	2.8943	2.9479	2.9824	3.0067
32	$LA_{u\delta_e} = \beta_1 - \varphi_2 =$ (30) $-$ (28) radians	3.9494	3.4375	3.0529	2.6593	2.3704	2.1856	2.0664
33	$LA_{w\delta_e} = \varphi_1 - \varphi_2 =$ (29) $-$ (28) "	3.6758	2.7903	2.1574	1.6093	1.2195	.9653	.7957
34	$LA_{\theta\delta_e} = \psi_1 - \varphi_2 =$ (31) $-$ (28) "	2.6223	2.2657	1.8235	1.3664	1.0293	.8089	.6632
35								

COMPUTATIONS FOR AMPLITUDE RESPONSE RATIOS AND LEAD ANGLES

$\mu_{u\delta_e}$, $\mu_{w\delta_e}$, $\mu_{\theta\delta_e}$ and $\mu_{\gamma/\alpha}$ and $LA_{u\delta_e}$, $LA_{w\delta_e}$, $LA_{\theta\delta_e}$

TRUE AIRSPEED = 300 MPH
10,000 FEET DENSITY ALTITUDE

$C_1 = .21369$	$K_1 = .54735$
$C_2 = .087261$	$K_2 = 4.9574$
$C_3 = 407.69$	$H_1 = 1.5919$
$C_4 = 128.28$	$H_2 = .13967$
$C_5 = 26.693$	$m_{\delta_e} = -351.95$
$C_6 = 895.53$	$T_A = 4.4716$ sec

APPENDIX O

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

APPENDIX O

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT

The equations for transient response of aircraft to longitudinal step-function, with ideal displacement, integral and derivative pitch-control in automatic pilot, were derived in section A of Appendix K. When k_q and $\frac{k_\theta}{d}$ are both equal to zero, these equations reduce to:

$$a_1 d^4 + a_2 d^3 + a_3 d^2 + a_4 d + a_5 = 0, \tag{1}$$

$$a_1 = 1, \tag{2}$$

$$a_2 = -(z_w + m_q + \mu_c m_{\dot{w}} + x_u), \tag{3}$$

$$a_3 = x_u(z_w + m_q + \mu_c m_{\dot{w}}) + z_w m_q$$

$$- m_{\delta_e} k_\theta - z_\theta \mu_c m_{\dot{w}} - \mu_c m_w - x_w z_u, \tag{4}$$

$$a_4 = x_u(-z_w m_q + m_{\delta_e} k_\theta + z_\theta \mu_c m_{\dot{w}} + \mu_c m_w)$$

$$+ x_w z_u m_q - x_w \mu_c m_u + m_{\delta_e} z_w k_\theta - z_\theta \mu_c m_w$$

$$- x_\theta(z_u \mu_c m_{\dot{w}} + \mu_c m_u), \quad \text{and} \tag{5}$$

$$a_5 = x_u(-m_{\delta_e} z_w k_\theta + z_\theta \mu_c m_w)$$

$$+ x_w(z_u m_{\delta_e} k_\theta - z_\theta \mu_c m_u) - x_\theta(z_u \mu_c m_w - z_w \mu_c m_u). \tag{6}$$

By comparison with the equations presented in Section A of Appendix M, we see that a_1 and a_2 are not affected by k_θ, while a_3, a_4 and a_5 are affected.

The additional equations used in this analysis were as follows:

$$u = e^{\alpha_1 \gamma} u_1 \sin(\beta_1 \gamma + LA_{u1}) + u_2 e^{d_3 \gamma} + u_3 e^{d_4 \gamma}. \quad (7)$$

$$w = e^{\alpha_1 \gamma} w_1 \sin(\beta_1 \gamma + LA_{w1}) + w_2 e^{d_3 \gamma} + w_3 e^{d_4 \gamma}. \quad (8)$$

$$\theta = e^{\alpha_1 \gamma} \theta_1 \sin(\beta_1 \gamma + LA_{\theta 1}) + \theta_2 e^{d_3 \gamma} + \theta_3 e^{d_4 \gamma}. \quad (9)$$

$$u_1 = \sqrt{A_u^2 + B_u^2}. \quad (10)$$

$$w_1 = \sqrt{A_w^2 + B_w^2}. \quad (11)$$

$$\theta_1 = \sqrt{A_\theta^2 + B_\theta^2}. \quad (12)$$

$$LA_{u1} = \tan^{-1} \left(\frac{A_u}{B_u}\right). \quad (13)$$

$$LA_{w1} = \tan^{-1} \left(\frac{A_w}{B_w}\right). \quad (14)$$

$$LA_{\theta 1} = \tan^{-1} \left(\frac{A_\theta}{B_\theta}\right). \quad (15)$$

$$\dot{u}_o = x_u u_o + x_w w_o + x_\theta \theta_o. \quad (16)$$

$$\dot{w}_o = z_u u_o + z_w w_o + \dot{\theta}_o + z_\theta \theta_o. \quad (17)$$

$$\ddot{\theta}_o = \mu_c m_u u_o + \mu_c m_{\dot{w}} \dot{w}_o + \mu_c m_w w_o + m_q \dot{\theta}_o + m_{\delta_e} k_\theta \theta_o. \quad (18)$$

$$\ddot{u}_o = x_u \dot{u}_o + x_w \dot{w}_o + x_\theta \dot{\theta}_o. \qquad (19)$$

$$\ddot{w}_o = z_u \dot{u}_o + z_w \dot{w}_o + \ddot{\theta}_o + z_\theta \dot{\theta}_o. \qquad (20)$$

$$\dddot{\theta}_o = \mu_c m_u \dot{u}_o + \mu_c m_{\dot{w}} \ddot{w}_o + \mu_c m_w \dot{w}_o + m_q \ddot{\theta}_o + m_{\delta_e} k_\theta \dot{\theta}_o. \qquad (21)$$

$$\dddot{u}_o = x_u \ddot{u}_o + x_w \ddot{w}_o + x_\theta \ddot{\theta}_o. \qquad (22)$$

$$\dddot{w}_o = z_u \ddot{u}_o + z_w \ddot{w}_o + \dddot{\theta}_o + z_\theta \ddot{\theta}_o. \qquad (23)$$

A_u, B_u, u_2, u_3, A_w, B_w, w_2, w_3, A_θ, B_θ, θ_2 and θ_3 were found by determinants, as described in Section A of Appendix I.

Assumed Boundary Conditions:

$u_o = 0$, $w_o = .05$, $\theta_o = .05$, $U_o = 300$ mph TAS,

at 10,000 feet density altitude (24)

Computed Boundary Conditions:

Using the values for the stability coefficients which were computed in Section A-1 of Appendix D, the computed boundary conditions were found from equations 16 to 23, inclusive. Results were as follows:

$\dot{u}_o = -.0069835$	$\dddot{\theta}_o = 355.41$	
$\dot{w}_o = -.24351$	$\dddot{u}_o = 2.3176$	
$\ddot{\theta}_o = -14.970$	$\dddot{w}_o = 422.55$	(25)
$\ddot{u}_o = -.044976$		
$\ddot{w}_o = -13.780$		

Characteristic Equation:

Using the values of the stability coefficients which were computed in Section A-1 of Appendix D, the coefficients for the characteristic equation (equation 1) were computed, using equations 2 to 5, inclusive. The characteristic equation was found to be:

$$d^4 + 26.693\ d^3 + 407.69\ d^2 + 895.53d + 128.28 = 0. \qquad (26)$$

Using the Graeffe method, the roots to equation 26 were found to be:

$$d_1 = \alpha_1 + i\beta_1 = -12.063 + 14.138\ i \qquad (27)$$

$$d_2 = \alpha_1 - i\beta_1 = -12.063 - 14.138\ i \qquad (28)$$

$$d_3 = -2.4130 \qquad (29)$$

$$d_4 = -.15392 \qquad (30)$$

The roots and the boundary conditions were then used to solve for A_u, B_u, u_2, u_3, A_w, B_w, w_2, w_3, A_θ, B_θ, θ_2 and θ_3, by determinants. For example, the basic determinant for the determination of A_u, B_u, u_2 and u_3 is:

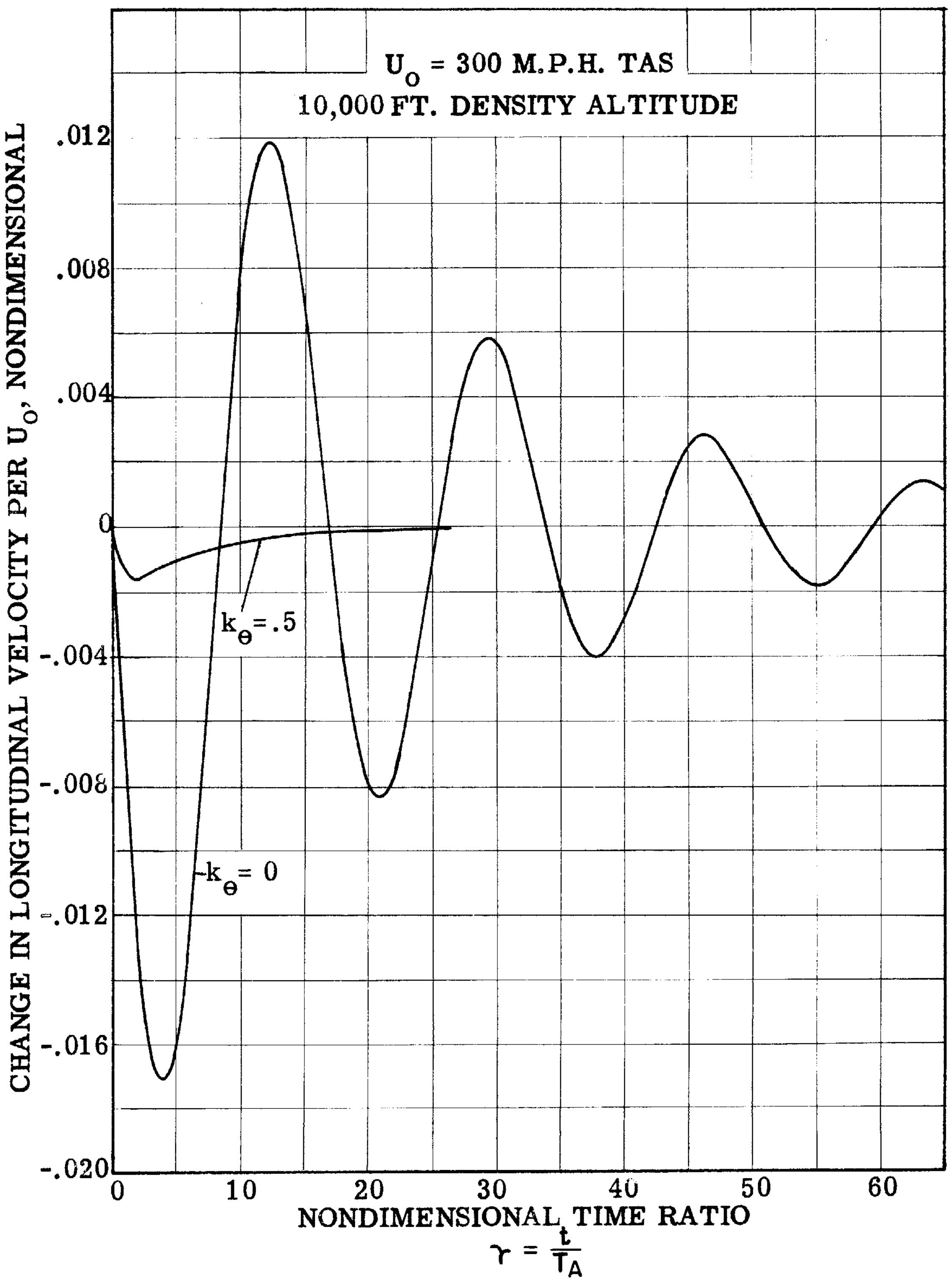
	A_u	B_u	u_2	u_3
u_o	1	0	1	1
$\dot{u}_o$	α_1	β_1	d_3	d_4
$\ddot{u}_o$	$\alpha_1^2 - \beta_1^2$	$2\alpha_1\beta_1$	d_3^2	d_4^2
$\dddot{u}_o$	$\alpha_1^3 - 3\alpha_1\beta_1^2$	$-\beta_1^3 + 3\alpha_1^2\beta_1$	d_3^3	d_4^3

(31)

These calculations are shown in Table O-1. Then u_1, w_1 and

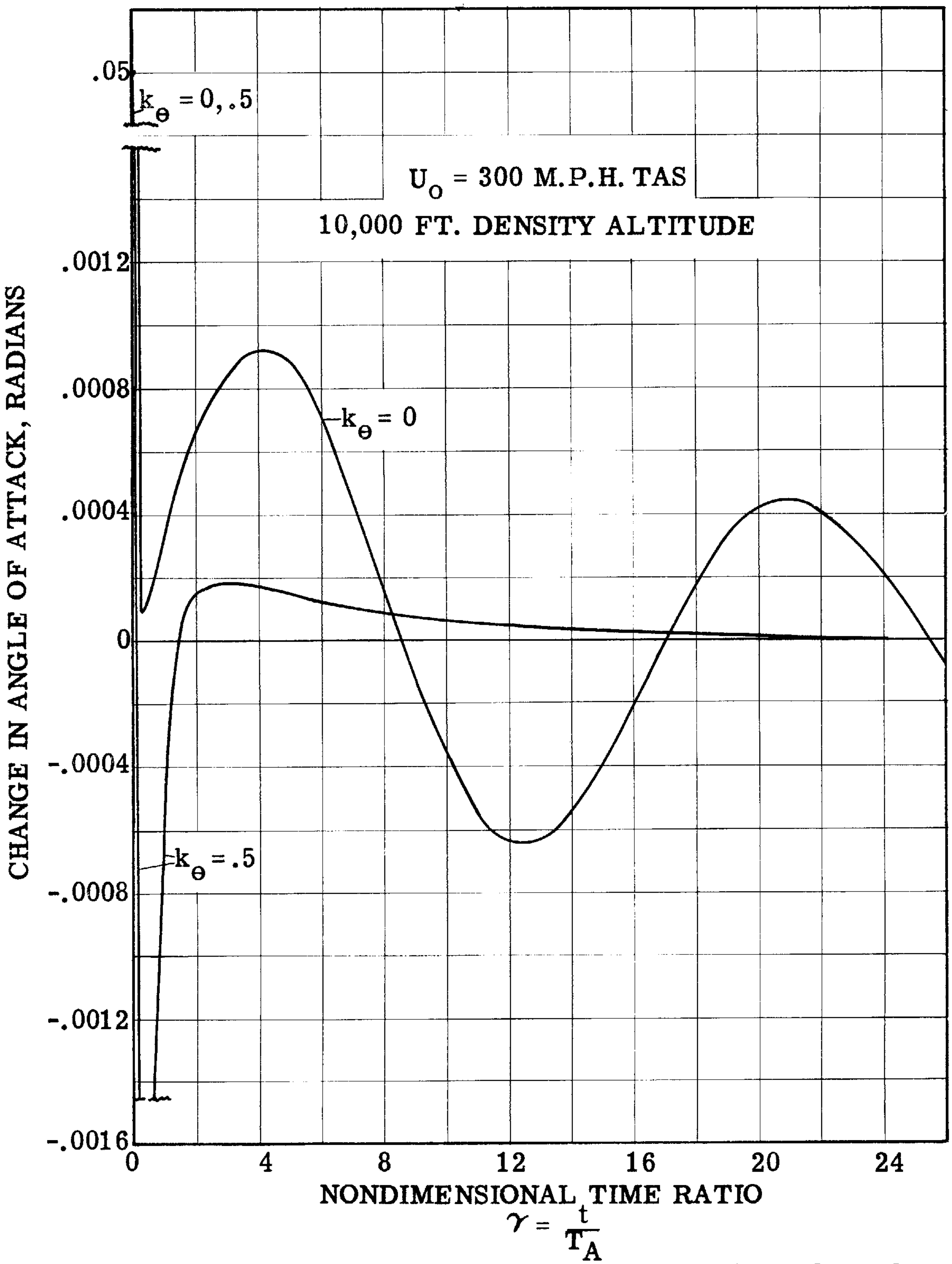
θ_1 were computed from equations 10, 11 and 12, respectively, and LA_{u1}, LA_{w1} and $LA_{\theta 1}$ were computed from equations 13, 14 and 15, respectively. These calculations are also shown in Table O-1. Equations 7, 8 and 9 were then used to compute the time history for u, w and θ, respectively. The calculations for the time history of u are shown in Table O-2, and the results are plotted in Figure O-1. Also included in Figure O-1 is a plot of the time history of u for the case when $k_\theta = 0$, (see section A, Appendix M). The calculations for the time history of w are shown in Table O-3, and the results are plotted in Figure O-2. Also included in Figure O-2 is a plot of the time history of w for the case when $k_\theta = 0$, (see Section A, Appendix M). The calculations for the time history of θ are shown in Table O-4, and the results are plotted in Figure O-3. Also included in Figure O-3 is a plot of the time history of θ for the case when $k_\theta = 0$, (see Section A, Appendix M).

The same type of analysis could be made for any combination of ideal displacement, integral and derivative pitch control, using the basic equations developed in Section A of Appendix K.



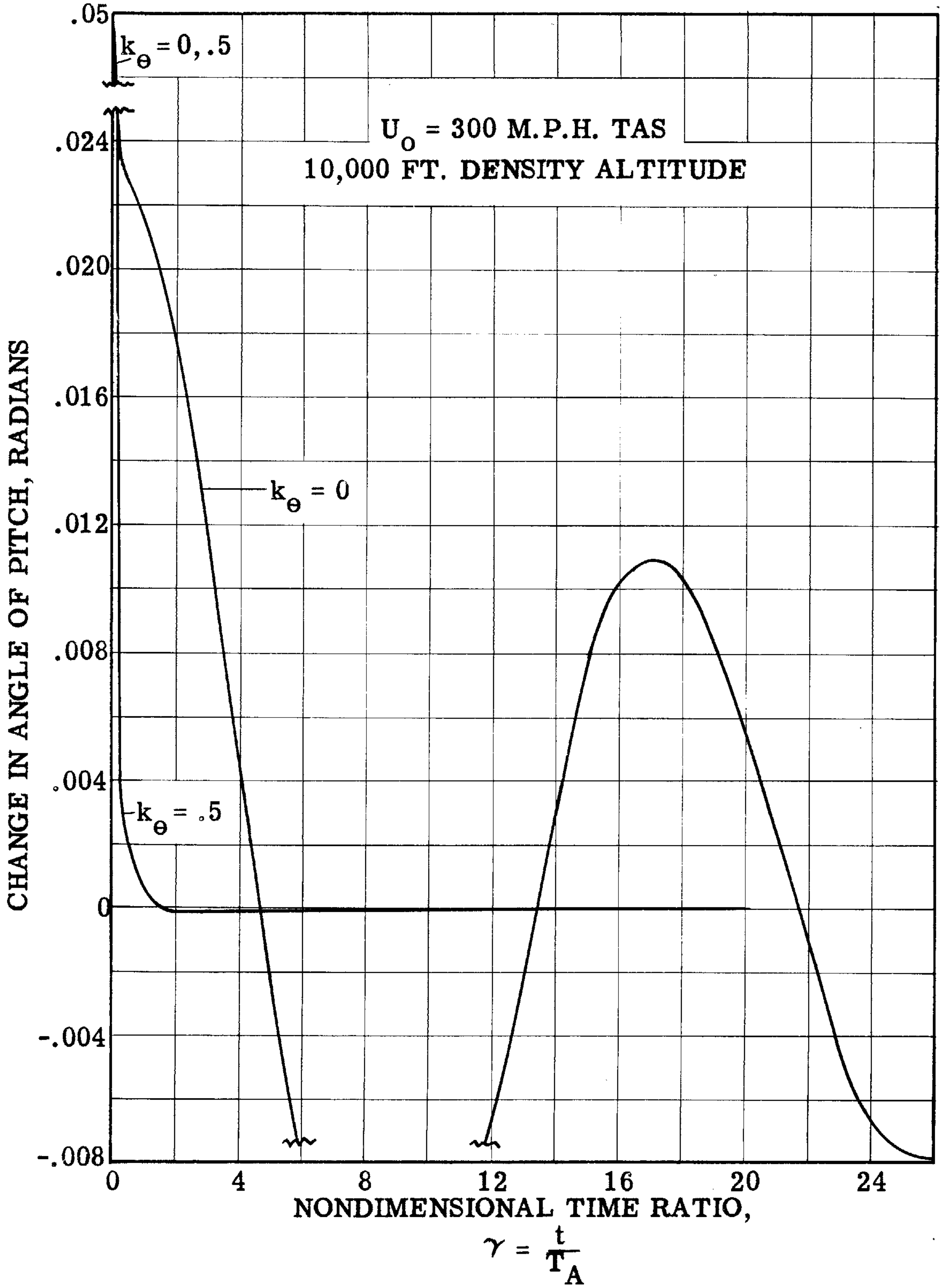
TIME HISTORY OF CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, k_Θ = .5 COMPARED TO k_Θ = 0

FIGURE O-1



TIME HISTORY OF CHANGE IN ANGLE OF ATTACK, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, k_θ = .5 COMPARED TO k_θ = 0

FIGURE O-2



TIME HISTORY OF CHANGE IN ANGLE OF PITCH, FOR COMPUTED TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, k_θ = .5 COMPARED TO k_θ = 0

FIGURE O-3

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, $k_\theta = .5$

TABLE O-1

(DETERMINANTS FOR A_u, A_w, A_θ, B_u, B_w, B_θ, u_2, w_2, θ_2, u_3, w_3 and θ_3) (U_0 = 300 mph TAS, at 10,000 FT. D.A.)

(1)	α_1	−12.063		
(2)	β_1	14.138		
(3)	$(1)^2 = \alpha_1^2$	145.52		
(4)	$(2)^2 = \beta_1^2$	199.88		
(5)	$(3) - (4) = \alpha_1^2 - \beta_1^2$	−54.36		
(6)	$(1)(2) = \alpha_1 \beta_1$	−170.55		
(7)	$2(6) = 2\alpha_1 \beta_1$	−341.10		
(8)	$(3)(1) = \alpha_1^3$	−1755.4		
(9)	$3(6) = 3\alpha_1 \beta_1$	−511.65		
(10)	$9(1) = 3\alpha_1 \beta_1^2$	−7233.7		
(11)	$(8) - (10) = \alpha_1^3 - 3\alpha_1 \beta_1^2$	+5478.3		
(12)	$(4)(2) = \beta_1^3$	2825.9		
(13)	$(9)(1) = 3\alpha_1^2 \beta_1$	+6172.0		
(14)	$-(12) + (13) = -\beta_1^3 + 3\alpha_1^2 \beta_1$	+3346.1		
(15)	d_3	−2.4130		
(16)	$(15)^2 = d_3^2$	5.8226		
(17)	$(15)(16) = d_3^3$	−14.050		
(18)	d_4	−0.15392		
(19)	$(18)^2 = d_4^2$	.023691		
(20)	$(18)(19) = d_4^3$	−.0036465		
(21)				
(22)				
(23)		u	w	θ
(24)	$\Delta_2/\Delta_1 = A_u, A_w, A_\theta$	.00036360	+.058186	.042101
(25)	$\Delta_3/\Delta_1 = B_u, B_w, B_\theta$	.00010503	+.030984	.037336
(26)	$\Delta_4/\Delta_1 = u_2, w_2, \theta_2$	.0018318	−.0084909	.0081895
(27)	$\Delta_5/\Delta_1 = u_3, w_3, \theta_3$	−.0021953	+.00030957	−.00029393
(28)	$(24)^2$	$.13220 \times 10^{-6}$	.0033856	.0017725
(29)	$(25)^2$	$.11031 \times 10^{-7}$	.00096001	.0013940
(30)	$(28) + (29)$	$.14323 \times 10^{-6}$	.0043456	.0031665
(31)	$\sqrt{(30)} = u_1, w_1, \theta_1$	.00037846	.065921	.056272
(32)	$\tan \angle A = (24)/(25)$	3.4619	1.8779	1.1276
(33)	$\tan^{-1}(32) = \angle A_1$, degrees.	73.888	61.964	48.432
(34)	$.0174533\ (33) = \angle A_1$, radians.	1.2896	1.0815	.84530
(35)				
(36)				
(37)				

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, $k_\theta = .5$

TABLE O-2

TIME HISTORY FOR μ

(U_0 = 300 mph TAS, at 10,000 FT D.A.)

(1) t													
(2) γ	0	2	4	6	8	10	12	14	16	18	20	22	
(3) $\alpha_1\gamma$ = −12.063 × (2)	0	−24.126	−48.252	−72.378									
(4) $e^{\alpha_1\gamma}$ = $e^{(3)}$	1.0000	$.33282 \times 10^{-10}$											
(5) $e^{\alpha_1\gamma}\mu_1$ = (4) × .00037846	.00037846	$.12596 \times 10^{-13}$											
(6) $\beta_1\gamma$ = 14.138 × (2)	0	28.276											
(7) $\beta_1\gamma + \angle A_{\mu_1}$ = (6) + 1.2896 radian	1.2896	29.566											
(8) $\sin(\beta_1\gamma + \angle A_{\mu_1})$ = sin (7)	.96073	−.96126											
(9) $e^{\alpha_1\gamma}\mu_1 \sin(\beta_1\gamma + \angle A_{\mu_1})$ = (5) × (8)	.00036360	$-.12108 \times 10^{-13}$											
(10) $d_3\gamma$ = −2.4130 × (2)	−0	−4.8260	−9.6520	−14.478	−19.304	−24.130	−	−	−	−	−	−	−
(11) $e^{d_3\gamma}$ = $e^{(10)}$	1.0000	.0080187	.000064297	$.51557 \times 10^{-6}$									
(12) $\mu_2 e^{d_3\gamma}$ = .0018318 × (11)	.0018318	.000014689	$.11778 \times 10^{-6}$	$.94442 \times 10^{-9}$									
(13) $d_4\gamma$ = −.15392 × (2)	0	−.30784	−.61568	−.92352	−1.2314	−1.5392	−1.8470	−2.1549	−2.4627	−2.7706	−3.0784	−3.3862	−
(14) $e^{d_4\gamma}$ = $e^{(13)}$	1.0000	.73503	.54027	.39712	.29188	.21455	.15771	.11591	.085205	.062627	.046033	.033837	
(15) $\mu_3 e^{d_4\gamma}$ = −.0021953 × (14)	−.0021953	−.0016136	−.0011861	−.00087180	−.00064076	−.00047100	−.00034622	−.00025446	−.00018705	−.00013749	−.00010106	−.000074282	
(16) μ = (9) + (12) + (15)	+.0000001	−.0015989	−.0011860	−.00087180	−.00064076	−.00047100	−.00034622	−.00025446	−.00018705	−.00013749	−.00010106	−.000074282	
(1) t													
(2) γ	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
(3) $d_3\gamma$ = −2.4130 × (2)	−.24130	−.48260	−.72390	−.96520	−1.2065	−1.4478	−1.6891	−1.9304	−2.1717	−2.4130	−2.6543	−2.8956	−3.1369
(4) $e^{d_3\gamma}$ = $e^{(3)}$	.78561	.61718	.48486	.38091	.29924	.23509	.18469	.14509	.11398	.089546	.070351	.055267	.043418
(5) $\mu_2 e^{d_3\gamma}$ = .0018318 × (4)	.0014391	.0011306	.00088817	.00069775	.00054815	.00043064	.00033832	.00026578	.00020879	.00016403	.00012887	.00010124	.000079533
(1) t													
(2) γ	.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26
(3) $\alpha_1\gamma$ = −12.063 × (2)	−.24126	−.48252	−.72378	−.96504	−1.2063	−1.4476	−1.6888	−1.9301	−2.1713	−2.4126	−2.6539	−2.8951	−3.1364
(4) $e^{\alpha_1\gamma}$ = $e^{(3)}$	.78564	.61723	.48492	.38097	.29930	.23513	.18474	.14513	.11403	.089582	.070378	.055296	.043440
(5) $e^{\alpha_1\gamma}\mu_1$ = (4) × .00037846	.00029733	.00023360	.00018352	.00014418	.00011327	.000088987	.000069917	.000054926	.000043156	.000033903	.000026635	.000020927	.000016440
(6) $\beta_1\gamma$ = 14.138 × (2)	.28276	.56552	.84828	1.1310	1.4138	1.6966	1.9793	2.2621	2.5448	2.8276	3.1104	3.3931	3.6759
(7) $\beta_1\gamma + \angle A_{\mu_1}$ = (6) + 1.2896	1.5724	1.8551	2.1379	2.4206	2.7034	2.9862	3.2689	3.5517	3.8344	4.1172	4.4000	4.6827	4.9655
(8) $\sin(\beta_1\gamma + \angle A_{\mu_1})$ = sin (7)	1.0000	.95985	.84349	.66013	.42436	.15471	−.12689	−.39875	−.63863	−.82806	−.95159	−.99956	−.96816
(9) $e^{\alpha_1\gamma}\mu_1 \sin(\beta_1\gamma + \angle A_{\mu_1})$ = (5) × (8)	.00029733	.00022422	.00015480	.000095178	.000048067	.000013767	$-.88718 \times 10^{-5}$	$-.21902 \times 10^{-4}$	$-.27561 \times 10^{-4}$	$-.28074 \times 10^{-4}$	$-.25346 \times 10^{-4}$	$-.20918 \times 10^{-4}$	$-.15916 \times 10^{-4}$
(1)													
(2) γ	.28	.30	.32	.34	.36	.38	.40	.42	.44	.46	.48	.50	
(3) $\alpha_1\gamma$ = −12.063 × (2)	−3.3776	−3.6189	−3.8602	−4.1014	−4.3427	−4.5839	−4.8252	−5.0665	−5.3077	−5.5490	−5.7902	−6.0315	
(4) $e^{\alpha_1\gamma}$ = $e^{(3)}$	.034129	.026812	.021064	.016550	.013001	.010215	.0080252	.0063046	.0049534	.0038914	.0030575	.0024019	
(5) $e^{\alpha_1\gamma}\mu_1$ = (4) × .00037846	.000012916	.000010147	.0000079719	.0000062635	.0000049204	$.38660 \times 10^{-5}$	$.30372 \times 10^{-5}$	$.23860 \times 10^{-5}$	$.18747 \times 10^{-5}$	$.14727 \times 10^{-5}$	$.11571 \times 10^{-5}$	$.90902 \times 10^{-6}$	
(6) $\beta_1\gamma$ = 14.138 × (2)	3.9586	4.2414	4.5242	4.8069	5.0897	5.3724	5.6552	5.9380	6.2207	6.5035	6.7862	7.0690	
(7) $\beta_1\gamma + \angle A_{\mu_1}$ = (6) + 1.2896	5.2482	5.5310	5.8138	6.0965	6.3793	6.6620	6.9448	7.2276	7.5103	7.7931	8.0758	8.3586	
(8) $\sin(\beta_1\gamma + \angle A_{\mu_1})$ = sin (7)	−.85985	−.68327	−.45228	−.18567	+.096820	+.36975	+.61442	+.81014	+.94153	+.99815	+.97550	+.87538	
(9) $e^{\alpha_1\gamma}\mu_1 \sin(\beta_1\gamma + \angle A_{\mu_1})$ = (5) × (8)	−.000011106	$-.69331 \times 10^{-5}$	$-.36055 \times 10^{-5}$	$-.11629 \times 10^{-5}$	$+.47246 \times 10^{-6}$	$+.14296 \times 10^{-5}$	$+.18661 \times 10^{-5}$	$+.19330 \times 10^{-5}$	$+.17651 \times 10^{-5}$	$+.14700 \times 10^{-5}$	$+.11289 \times 10^{-5}$	$+.79574 \times 10^{-6}$	

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, $k_\theta = .5$

TABLE O-3

TIME HISTORY FOR w (U_0 = 300 mph TAS at 10,000 FT. D.A.)

① t														
② γ		0	2	4	6	8	10	12	14	16	18	20	22	24
③ $\alpha_1\gamma$	$-12.063 \times$ ②	0	-24.126	-48.252	-72.378	–	–	–	–	–	–	–	–	–
④ $e^{\alpha_1\gamma}$	$e^{③}$	1.0000	$.33282\times10^{-10}$											
⑤ $e^{\alpha_1\gamma}w_1$	④ $\times$ .065921	.065921	$.21940\times10^{-11}$											
⑥ $\beta_1\gamma$	$14.138 \times$ ②	0	28.276											
⑦ $\beta_1\gamma + \angle A_{w_1}$	⑥ + 1.0815	1.0815	29.358											
⑧ $\sin(\beta_1\gamma + \angle A_{w_1})$	sin ⑦	.88266	-.88377											
⑨ $e^{\alpha_1\gamma}w_1 \sin(\beta_1\gamma+\angle A_{w_1})$	⑤ $\times$ ⑧	.058186	$-.19390\times10^{-11}$											
⑩ $d_3\gamma$	$-2.4130 \times$ ②	0	-4.8260	-9.6520	-14.478									
⑪ $e^{d_3\gamma}$	$e^{⑩}$	1.0000	.0080187	.000064297	$.51557\times10^{-6}$									
⑫ $w_2 e^{d_3\gamma}$	$-.0084909 \times$ ⑪	-.0084909	-.000068086	$-.54594\times10^{-6}$	$-.43777\times10^{-8}$									
⑬ $d_4\gamma$	$-.15392 \times$ ②	0	-.30784	-.61568	-.92352	-1.2314	-1.5392	-1.8470	-2.1549	-2.4627	-2.7706	-3.0784	-3.3862	
⑭ $e^{d_4\gamma}$	$e^{⑬}$	1.0000	.73503	.54027	.39712	.29188	.21455	.15771	.11591	.085205	.062627	.046033	.033837	
⑮ $w_3 e^{d_4\gamma}$	$+.00030957 \times$ ⑭	.00030957	.00022754	.00016726	.00012294	.000090357	.000066418	.000048822	.000035882	.000026377	.000019387	.000014250	.000010475	
⑯ w =	⑨ + ⑫ + ⑮	+.050006	+.00015945	.00016670	.00012294	.000090357	.000066418	.000048822	.000035882	.000026377	.000019387	.000014250	.000010475	

① t														
② γ		.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
③ $d_3\gamma$	$-2.4130 \times$ ②	-.24130	-.48260	-.72390	-.96520	-1.2065	-1.4478	-1.6891	-1.9304	-2.1717	-2.4130	-2.6543	-2.8956	-3.1369
④ $e^{d_3\gamma}$	$e^{③}$	.78561	.61718	.48486	.38091	.29924	.23509	.18469	.14509	.11398	.089546	.070351	.056267	.043418
⑤ $w_2 e^{d_3\gamma}$	$-.0084909 \times$ ④	-.0066705	-.0052404	-.0041169	-.0032343	-.0025408	-.0019961	-.0015682	-.0012319	-.00096779	-.00076033	-.00059734	-.00046927	-.00036866

① t														
② γ		.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26
③ $\alpha_1\gamma$	$-12.063 \times$ ②	-.24126	-.48252	-.72378	-.96504	-1.2063	-1.4476	-1.6888	-1.9301	-2.1713	-2.4126	-2.6539	-2.8951	-3.1364
④ $e^{\alpha_1\gamma}$	$e^{③}$	.78564	.61723	.48492	.38097	.29930	.23513	.18474	.14613	.11403	.089582	.070378	.055296	.043440
⑤ $e^{\alpha_1\gamma}w_1$	④ $\times$ .065921	.051790	.040688	.031966	.025114	.019730	.015500	.012178	.0095671	.0075170	.0059053	.0046394	.0036452	.0028636
⑥ $\beta_1\gamma$	$14.138 \times$ ②	.28276	.56552	.84828	1.1310	1.4138	1.6966	1.9793	2.2621	2.5448	2.8276	3.1104	3.3931	3.6759
⑦ $\beta_1\gamma + \angle A_{w_1}$	⑥ + 1.0815	1.3643	1.6470	1.9298	2.2125	2.4953	2.7781	3.0608	3.3436	3.6263	3.9091	4.1919	4.4746	4.7574
⑧ $\sin(\beta_1\gamma + \angle A_{w_1})$	sin ⑦	.97876	.99710	.93624	.80104	.60223	.35560	.080721	-.20056	-.46592	-.69428	-.86759	-.97188	-.99899
⑨ $e^{\alpha_1\gamma}w_1 \sin(\beta_1\gamma+\angle A_{w_1})$	⑤ $\times$ ⑧	.050690	.040570	.029928	.020117	.011882	.0055118	.00098302	-.0019188	-.0035023	-.0040999	-.0040251	-.0035427	-.0028607

① t														
② γ		.28	.30	.32	.34	.36	.38	.40	.42	.44	.46	.48	.50	
③ $\alpha_1\gamma$	$-12.063 \times$ ②	-3.3776	-3.6189	-3.8602	-4.1014	-4.3427	-4.5839	-4.8252	-5.0665	-5.3077	-5.5490	-5.7902	-6.0315	
④ $e^{\alpha_1\gamma}$	$e^{③}$	.034129	.026812	.021064	.016550	.013001	.010215	.0080252	.0063046	.0049534	.0038914	.0030575	.0024019	
⑤ $e^{\alpha_1\gamma}w_1$	④ $\times$ .065921	.0022498	.0017675	.0013886	.0010910	.00085704	.00067338	.00052903	.00041561	.00032653	.00025652	.00020165	.00015834	
⑥ $\beta_1\gamma$	$14.138 \times$ ②	3.9586	4.2414	4.5242	4.8069	5.0897	5.3724	5.6552	5.9380	6.2207	6.5035	6.7862	7.0690	
⑦ $\beta_1\gamma + \angle A_{w_1}$	⑥ + 1.0815	5.0401	5.3229	5.6057	5.8884	6.1712	6.4539	6.7367	7.0195	7.3022	7.5850	7.8677	8.1505	
⑧ $\sin(\beta_1\gamma + \angle A_{w_1})$	sin ⑦	-.94676	-.81935	-.62688	-.38462	-.11182	+.16987	+.43806	+.67159	+.85454	+.96405	+.99990	+.95636	
⑨ $e^{\alpha_1\gamma}w_1 \sin(\beta_1\gamma+\angle A_{w_1})$	⑤ $\times$ ⑧	-.0021300	-.0014482	-.00087049	-.00041962	-.000096839	+.00011439	+.00023175	+.00027912	+.00027886	+.00024730	+.00020153	+.00015143	

ANALYSIS FOR TRANSIENT RESPONSE OF A-26 AIRPLANE TO LONGITUDINAL STEP-FUNCTION, WITH IDEAL DISPLACEMENT PITCH CONTROL IN AUTOMATIC PILOT, $k_\theta = .5$

TABLE O-4

TIME HISTORY FOR θ

(U_0 = 300 mph TAS at 10,000 FT. D.A.)

(1) t														
(2) γ		0	2	4	6	8	10	12	14	16	18	20	22	
(3) $\alpha_1\gamma$	$-12.063 \times$ (2)	0	-24.126	-48.252	-72.378									
(4) $e^{\alpha_1\gamma}$	$e^{(3)}$	1.0000	$.33282 \times 10^{-10}$											
(5) $e^{\alpha_1\gamma}\theta_1$	(4) $\times .056272$	.056272	$.18728 \times 10^{-11}$											
(6) $\beta_1\gamma$	$14.138 \times$ (2)	0	28.276											
(7) $\beta_1\gamma + \angle A_{\theta_1}$	(6) + .84530	.84530	29.121											
(8) $\sin(\beta_1\gamma + \angle A_{\theta_1})$	sin (7)	.74817	-.74896											
(9) $e^{\alpha_1\gamma}\theta_1 \sin(\beta_1\gamma + \angle A_{\theta_1})$	= (5) × (8)	.042101	$-.14027 \times 10^{-11}$											
(10) $d_3\gamma$	$-2.4130 \times$ (2)	0	-4.8260	-9.6520	-14.478									
(11) $e^{d_3\gamma}$	$e^{(10)}$	1.0000	.0080187	.000064297	$.51557 \times 10^{-6}$									
(12) $\theta_2 e^{d_3\gamma}$	.0081895 × (11)	.0081895	.000065669	$.52656 \times 10^{-6}$	$.42223 \times 10^{-8}$									
(13) $d_4\gamma$	$-.15392 \times$ (2)	0	-.30784	-.61568	-.92352	-1.2314	-1.5392	-1.8470	-2.1549	-2.4627	-2.7706	-3.0784	-3.3862	
(14) $e^{d_4\gamma}$	$e^{(13)}$	1.0000	.73503	.54027	.39712	.29188	.21455	.15771	.11591	.085205	.062627	.046033	.033837	
(15) $\theta_3 e^{d_4\gamma}$	$-.00029393 \times$ (14)	-.00029393	-.00021605	-.00015880	-.00011673	-.000085792	-.000063063	-.000046354	-.000034069	-.000025044	-.000018408	-.000013530	-.0000099457	
(16) θ =	(9) + (12) + (15)	+.049997	-.00015038	-.00015827	-.00011673	-.000085792	-.000063063	-.000046356	-.000034069	-.000025044	-.000018408	-.000013530	-.0000099457	
(1) t														
(2) γ		.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0	1.1	1.2	1.3
(3) $d_3\gamma$	$-2.4130 \times$ (2)	-.24130	-.48260	-.72390	-.96620	-1.2065	-1.4478	-1.6891	-1.9304	-2.1717	-2.4130	-2.6543	-2.8956	-3.1369
(4) $e^{d_3\gamma}$	$e^{(3)}$	.78561	.61718	.48486	.38091	.29924	.23509	.18469	.14509	.11398	.089546	.070351	.055267	.043418
(5) $\theta_2 e^{d_3\gamma}$	.0081895 × (4)	.0064338	.0050544	.0039708	.0031195	.0024506	.0019253	.0015125	.0011882	.00093344	.00073334	.00057614	.00045261	.00035557
(1) t														
(2) γ		.02	.04	.06	.08	.10	.12	.14	.16	.18	.20	.22	.24	.26
(3) $\alpha_1\gamma$ =	$-12.063 \times$ (2)	-.24126	-.48252	-.72378	-.96504	-1.2063	-1.4476	-1.6888	-1.9301	-2.1713	-2.4126	-2.6539	-2.8951	-3.1364
(4) $e^{\alpha_1\gamma}$ =	$e^{(3)}$	.78564	.61723	.48492	.38097	.29930	.23513	.18474	.14513	.11403	.089582	.070378	.055296	.043440
(5) $e^{\alpha_1\gamma}\theta_1$ =	(4) × .056272	.044210	.034733	.027287	.021438	.016842	.013231	.010396	.0081668	.0064167	.0050410	.0039603	.0031116	.0024445
(6) $\beta_1\gamma$ =	$14.138 \times$ (2)	.28276	.56552	.84828	1.1310	1.4138	1.6966	1.9793	2.2621	2.5448	2.8276	3.1104	3.3931	3.6759
(7) $\beta_1\gamma + \angle A_{\theta_1}$ =	(6) + .84530	1.1281	1.4108	1.6936	1.9763	2.2591	2.5419	2.8246	3.1074	3.3901	3.6729	3.9557	4.2384	4.5212
(8) $\sin(\beta_1\gamma + \angle A_{\theta_1})$	sin (7)	.90360	.98723	.99247	.91893	.77229	.56439	.31167	.034202	-.24598	-.50664	-.72705	-.88974	-.98179
(9) $e^{\alpha_1\gamma}\theta_1 \sin(\beta_1\gamma + \angle A_{\theta_1})$ =	(5) × (8)	.039948	.034289	.027082	.019700	.013007	.0074674	.0032401	.00027932	-.0015784	-.0025540	-.0028793	-.0027685	-.0024000
(1) t														
(2) γ		.28	.30	.32	.34	.36	.38	.40	.42	.44	.46	.48	.50	
(3) $\alpha_1\gamma$ =	$-12.063 \times$ (2)	-3.3776	-3.6189	-3.8602	-4.1014	-4.3427	-4.5839	-4.8252	-5.0665	-5.3077	-5.5490	-5.7902	-6.0315	
(4) $e^{\alpha_1\gamma}$ =	$e^{(3)}$	.034129	.026812	.021064	.016550	.013001	.010215	.0080252	.0063046	.0049534	.0038914	.0030575	.0024019	
(5) $e^{\alpha_1\gamma}\theta_1$ =	(4) × .056272	.0019205	.0015088	.0011853	.00093130	.00073159	.00057482	.00045159	.00035477	.00027874	.00021898	.00017205	.00013516	
(6) $\beta_1\gamma$ =	$14.138 \times$ (2)	3.9586	4.2414	4.5242	4.8069	5.0897	5.3724	5.6552	5.9380	6.2207	6.5035	6.7862	7.0690	
(7) $\beta_1\gamma + \angle A_{\theta_1}$ =	(6) + .84530	4.8039	5.0867	5.3695	5.6522	5.9350	6.2177	6.5005	6.7833	7.0660	7.3488	7.6315	7.9143	
(8) $\sin(\beta_1\gamma + \angle A_{\theta_1})$	sin (7)	-.99582	-.93074	-.79176	-.58990	-.34120	-.065403	+.21559	+.47946	+.70525	+.87513	+.97534	+.99814	
(9) $e^{\alpha_1\gamma}\theta_1 \sin(\beta_1\gamma + \angle A_{\theta_1})$ =	(5) × (8)	-.0019125	-.0014043	-.00093847	-.00054937	-.00024962	-.000037595	+.000097358	+.00017010	+.00019658	+.00019164	+.00016781	+.00013491	

APPENDIX P

METHODS FOR DETERMINING DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

APPENDIX P

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APPENDIX P

METHODS FOR DETERMINING DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF CONTROL SURFACES

A-1: METHOD FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS CONSIDERED IN ALL THREE EQUATIONS OF MOTION:

The equations for steady-state response of aircraft to forced sinusoidal motion of elevator, when the effect of change in longitudinal velocity is included in all three equations of motion, were derived in Section A-1 of Appendix H. From equation 73 of Appendix H, we have:

$$\frac{q_\alpha}{\dot{w}_\alpha} = \frac{\theta_\alpha}{w_\alpha} = \sqrt{\frac{(K_1-\omega_e^2)^2 + (\omega_e K_2)^2}{(C_1-\omega_e^2)^2 + (\omega_e C_2)^2}} \quad . \tag{1}$$

Q_a can be measured directly in flight, for various values of Ω_E, but $\dot{W}_a$ can only be obtained indirectly by measuring the total normal acceleration:

$$n_z g = \dot{W} - U_o Q - g \cos \theta. \tag{2}$$

Thus, since $\Delta\dot{W} = \dot{W}$,

$$\Delta\dot{W} = n_z g + U_o Q + g \cos \theta. \tag{3}$$

For small changes in θ, $\cos \theta \cong 1$. Thus,

$$\dot{w}_\alpha = \left(\frac{\Delta\dot{W}_\alpha}{U_o}\right)T_A = \left(\frac{T_A}{U_o}\right)(n_z g + U_o Q + g)_\alpha, \qquad (4)$$

or

$$\dot{w}_\alpha = [C_{L_o}(n_z + 1) + q]_\alpha . \qquad (5)$$

Also,

$$q_\alpha = T_A Q_\alpha = \omega_e \theta_\alpha . \qquad (6)$$

Dividing equation 6 by equation 4, we have:

$$\frac{q_\alpha}{\dot{w}_\alpha} = \frac{Q_\alpha}{\left[Q + \frac{(n_z + 1)g}{U_o}\right]_\alpha} \qquad (7)$$

Q and n_z can be measured directly in flight, for any value of Ω_E. Thus the ratio $\frac{q_\alpha}{\dot{w}_\alpha}$ can be determined easily from the flight-test records, for values of ω_e.

For convenience, we may let $\xi = \frac{q_\alpha}{\dot{w}_\alpha}$. Then, from equation 1,

$$\xi^2 = \left[\frac{\omega_e^4 + \omega_e^2(K_2^2 - 2K_1) + K_1^2}{\omega_e^4 + \omega_e^2(C_2^2 - 2C_1) + C_1^2}\right]. \qquad (8)$$

We may also let

$$A = K_2^2 - 2K_1, \qquad (9)$$

$$B = K_1^2, \qquad (10)$$

$$L = C_2^2 - 2C_1 \text{ and} \qquad (11)$$

$$D = C_1^2 . \qquad (12)$$

Then equation 8 becomes:

$$\omega_e^2 A + B - (\xi\omega_e)^2 L - \xi^2 D = (\xi^2 - 1)\omega_e^4 \qquad (13)$$

With any four different values of ω_e and the corresponding values of ξ, there are four different equations like equation 13. These could be solved simultaneously for A, B, L and D, and then K_1, K_2, C_1 and C_2 could be found from equations 9, 10, 11 and 12. From equations 38, 39, 50 and 51 of Appendix H we have, respectively:

$$C_1 = x_\theta z_u - x_u z_\theta, \qquad (14)$$

$$C_2 = z_\theta - x_u, \qquad (15)$$

$$K_1 = x_u z_w - x_w z_u \text{ and} \qquad (16)$$

$$K_2 = -x_u - z_w. \qquad (17)$$

For small changes in θ from the horizontal, $z_\theta \cong 0$, and $x_\theta = -C_{L_o}$. Thus,

$$z_u = -\left(\frac{C_1}{C_{L_o}}\right). \qquad (18)$$

$$x_u = -C_2, \qquad (19)$$

$$z_w = C_2 - K_2 \text{ and} \qquad (20)$$

$$x_w = \frac{C_{L_o}}{C_1}\left[K_1 + C_2(C_2 - K_2)\right]. \qquad (21)$$

In this manner, z_u, x_u, z_w and x_w may be computed.

From equations 63 and 72 in Appendix H, we have:

$$\mu_{q\delta_e} = -m_{\delta_e}\omega_e \sqrt{\frac{(K_1 - \omega_e^2)^2 + (\omega_e K_2)^2}{(-\omega_e^4 + \omega_e^2 C_3 - C_4)^2 + \omega_e^2(\omega_e^2 C_5 - C_6)^2}} \qquad (22)$$

m_{δ_e} can be determined from static flight-tests, by shifting weight along the longitudinal axis of the aircraft and measuring the resulting change in elevator angle required for level flight. Thus, m_{δ_e} may be considered as a known quantity in equation 22. Also,

$$\mu_{q\delta_e} = \frac{T_A Q_\alpha}{\Delta_{E_\alpha}}, \qquad (23)$$

where Q_α and Δ_{E_α} can be measured in flight. Thus, equation 22 may be rearranged to give:

$$\omega_e^6 N - \omega_e^4 Q + \omega_e^2 T - W = \omega_e^8 - \left(\frac{m_{\delta_e}\omega_e}{\mu_{q\delta_e}}\right)^2 [(K_1 - \omega_e^2)^2 + (\omega_e K_2)^2], \qquad (24)$$

where, by definition,

$$N = 2C_3 - C_5^2 , \qquad (25)$$

$$Q = C_3^2 + 2C_4 - 2C_5C_6, \qquad (26)$$

$$T = 2C_3C_4 - C_6^2 \text{ and} \qquad (27)$$

$$W = C_4^2 . \qquad (28)$$

For any four different values of ω_e there are four corresponding values of $\mu_{q\delta_e}$, as found by using flight-test data in equation 23. Thus, there are four corresponding values for the right-hand side of equation 24, because K_1 and K_2 are already known from the fore-going analysis. Then N, Q, T and W can be determined from four simultaneous equations similar to equation 24, with each equation corresponding to a different value of ω_e. Once N, Q, T and W have been determined, C_3, C_4, C_5 and C_6 may be found from equations 25, 26, 27 and 28.

From equations 43, 44, 45 and 46 in Appendix H, we have, respectively:

$$C_3 = (x_u z_w - x_w z_u) + (x_u + z_w)m_q + \mu_c m_{\dot{w}}(x_u - z_\theta) - \mu_c m_w, \tag{29}$$

$$C_4 = \mu_c m_w (x_u z_\theta - x_\theta z_u) + \mu_c m_u (x_\theta z_w - x_w z_\theta), \tag{30}$$

$$C_5 = -x_u - z_w - m_q - \mu_c m_{\dot{w}} \quad \text{and} \tag{31}$$

$$C_6 = (x_w z_u - x_u z_w)m_q + (x_u - z_\theta)\mu_c m_w + (x_u z_\theta - x_\theta z_u)\mu_c m_{\dot{w}} - (x_w + x_\theta)\mu_c m_u. \tag{32}$$

For $z_\theta = 0$, and for $x_\theta = -C_{L_o}$, equations 29, 30, 31 and 32 may be rearranged respectively, as follows:

$$(x_u + z_w)m_q + (\mu_c x_u)m_{\dot{w}} - \mu_c m_w = (C_3 - x_u z_w + x_w z_u), \tag{33}$$

$$(\mu_c C_{L_o} z_u)m_w - (\mu_c C_{L_o} z_w)m_u = C_4, \tag{34}$$

$$-m_q - \mu_c m_{\dot{w}} = (C_5 + x_u + z_w) \quad \text{and} \tag{35}$$

$$(x_w z_u - x_u z_w)m_q + (C_{L_o} z_u \mu_c)m_{\dot{w}} + (\mu_c x_u)m_w + \mu_c(C_{L_o} - x_w)m_u = C_6. \tag{36}$$

Determinants may now be used to solve equations 33, 34, 35 and 36 for the only unknowns, m_q, $m_{\dot{w}}$, m_w, and m_u.

Thus, it has been shown that the important longitudinal dynamic stability coefficients for aircraft can be determined, at least <u>theoretically</u>, from frequency response curves for $\frac{q_\alpha}{\dot{w}_\alpha}$ and $\mu_{q\delta_e}$. Whether or not this method of analysis will actually work remains to be seen, but it is the author's belief that, with present-day techniques of dynamic flight-testing, the inaccuracy-spread of the flight-test data would be too large to justify such an elaborate analysis.

Similar methods can be shown, based upon phase-angle response, or a combination of phase-angle response and amplitude-ratio response, but they are equally as complex as the amplitude-ratio method just discussed. But one of the advantages of the phase-angle method is that m_{δ_e} need not be known for the determination of C_3, C_4, C_5 and C_6.

A-2: METHOD FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF ELEVATOR, WHEN THE EFFECT OF CHANGE IN LONGITUDINAL VELOCITY IS NEGLECTED IN THE EQUATIONS OF MOTION FOR CHANGE IN NORMAL FORCE AND CHANGE IN PITCHING MOMENT

The equations for steady-state response of aircraft to forced sinusoidal motion of elevator, when the effect of change in longitudinal velocity is neglected in the equations of motion for change in normal force and change in pitching moment, were derived in section A-2 of Appendix H.

From equations 82, 83, 87, 99 and 100, in section A-2 of Appendix H, we have:

$$LA_{\dot{w}q} = LA_{w\delta_e} - LA_{\theta\delta_e} = \frac{3\pi}{2} - \psi_1. \quad (37)$$

Thus,

$$LA_{\dot{w}q} = \frac{3\pi}{2} - \tan^{-1}\left(\frac{-\omega_e}{z_w}\right), \quad (38)$$

and

$$z_w = -\frac{\omega_e}{\tan\left(\frac{3\pi}{2} - LA_{\dot{w}q}\right)}. \quad (39)$$

For any ω_e and the corresponding $LA_{\dot{w}q}$, z_w may be computed directly from equation 39. $LA_{\dot{w}q}$ can be obtained from the flight records of n_z and Q, since

$$\dot{w} = C_{L_o}(n_z + 1) + T_A Q. \quad (40)$$

From equation 102 of Appendix H,

$$\frac{q_a}{\dot{w}_a} = \frac{\theta_a}{w_a} = \sqrt{\left(\frac{z_w}{\omega_e}\right)^2 + 1}. \quad (41)$$

Thus,

$$z_w = -\omega_e \times \sqrt{\xi^2 - 1}, \quad (42)$$

where $\xi = \frac{q_a}{\dot{w}_a}$, as defined in Section A-1 of this Appendix.

Equations 39 and 42 show that z_w can be obtained by either the phase-angle or the amplitude-ratio method.

The other stability coefficients, such as m_q, $m_{\dot{w}}$, m_w, m_u, z_u, x_u, etc., can not be obtained by this method of analysis, although the effective damping coefficient, b, and the equivalent spring coefficient, k, for the short period oscillation only, can be obtained. The coefficients b and k are defined by equations 84 and 85 in Section A-2 of Appendix H. From equations 99, 82 and 86, in Section A-2 of Appendix H,

$$\tan LA_{\dot{w}\delta_e} = -\left(\frac{\omega_e^2 - k}{\omega_e b}\right). \qquad (43)$$

From equation 43, the coefficients b and k may be obtained by using any two different values of ω_e and the corresponding values of $LA_{\dot{w}\delta_e}$. Values of $LA_{\dot{w}\delta_e}$ may be obtained from the flight records of n_z, Q and Δ_E, using equation 40 as a basis. The coefficients b and k can also be obtained from equations 80 and 99 in Section A-2 of Appendix H. These equations combine to give:

$$\mu_{\dot{w}\delta_e} = \frac{-m_{\delta_e}\omega_e}{\sqrt{(-\omega_e b)^2 + (k - \omega_e^2)^2}}. \qquad (44)$$

Providing m_{δ_e} is known, b and k may be obtained from equation 44 with any two different values of ω_e and their corresponding values of $\mu_{\dot{w}\delta_e}$. Values of $\mu_{\dot{w}\delta_e}$ can be obtained from the flight records of n_z, Q and Δ_E, If we use equation 40 as a basis, we have:

$$\mu_{\dot{w}\delta_e} = \frac{\dot{w}_a}{\delta_{e_a}} = \frac{[C_{L_0}(n_z + 1) + T_A Q]_a}{\Delta_{E_a}} \qquad (45)$$

The coefficients b and k may also be obtained by using the circle diagram method presented in reference 1, (Also see Appendix T).

B-1: METHOD FOR DETERMINING LATERAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER, WITH FIXED AILERONS:

The equations for steady-state response of aircraft to forced sinusoidal motion of rudder, with fixed ailerons, were derived in Section B-1 of Appendix H. From equation 171 in Section B-1 of Appendix H, we have:

$$\frac{\dot{v}_a}{\dot{\psi}_a} = \frac{v_a}{\psi_a} = \omega_r \sqrt{\frac{(-\omega_r C_1)^2 + (-\omega_r^2 - C_2)^2}{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1K_2 + \omega_r^2)^2}} \qquad (46)$$

Also,

$$n_y g = \dot{V} + U_o R - g \sin\phi, \qquad (47)$$

$$\dot{\psi} \equiv r \equiv T_A R, \qquad (48)$$

$$\Delta\dot{V} = \dot{V}, \qquad (49)$$

and $$\dot{v} \equiv \frac{T_A}{U_o}\Delta\dot{V}. \qquad (50)$$

Thus, $$\Delta\dot{V} = (n_y + \sin\phi)g - U_o R, \qquad (51)$$

or $$\dot{v} = C_{L_o}(n_y + \sin\phi) - r. \qquad (52)$$

Dividing the amplitude of equation 52 by the amplitude of equation 48, and using dimensional notation on the right hand side, we have:

$$\frac{\dot{v}_a}{\dot{\psi}_a} = \frac{\left[\frac{(n_y + \sin\phi)g}{U_o} - R\right]_a}{R_a}. \qquad (53)$$

The factors on the right-hand side of equation 53 can be taken directly from flight-test records, so the ratio $\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha}$ can be plotted against ω_r. From equation 46, and using the general method applied in Section A-1 of this Appendix, C_1, C_2, K_1 and K_2 may be computed for any four different values of ω_r. Then, from equations 136, 137, 148 and 149 in Section B-1 of Appendix H, we have:

$$l_p = -C_1, \tag{54}$$

$$y_v = -K_2, \tag{55}$$

$$l_v = -\frac{K_1}{\mu_b y_\phi} \doteq -\frac{K_1}{\mu_b C_{L_o}}, \tag{56}$$

and

$$l_r = \frac{C_2}{y_\phi} = \frac{C_2}{C_{L_o}}. \tag{57}$$

Combining equations 160 and 169 in Section B-1 of Appendix H, we have:

$$\mu_{\dot{\psi}\delta_r} = \frac{T_A R_\alpha}{\Delta_{R_\alpha}} = n_{\delta_r}\sqrt{\frac{[-K_1 + \omega_r^2(K_2 + C_1)]^2 + \omega_r^2(-C_1K_2 + \omega_r^2)^2}{(\omega_r C_3 - \omega_r^3 C_4)^2 + (-C_5 + \omega_r^2 C_6 - \omega_r^4)^2}}. \tag{60}$$

Now, using a method analogous to the one outlined between equations 22 and 36, we can <u>theoretically</u> solve for the rest of the lateral coefficients. But again it is the author's belief that, with present-day techniques of dynamic flight-testing, the inaccuracy-spread of the flight-test data would be too large to justify such an elaborate analysis.

B-2: METHOD FOR DETERMINING LATERAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF AILERONS, WITH FIXED RUDDER:

The equations for steady-state response of aircraft to forced sinusoidal motion of ailerons, with fixed rudder, were derived in Section B-2 of Appendix H. From equation 225 in Section B-2 of Appendix H, we have:

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{v_\alpha}{\phi_\alpha} = \sqrt{\frac{(\omega_a G_1)^2 + (-G_2)^2}{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}} \; . \qquad (61)$$

Also, $\dot{\phi} \equiv p \equiv T_A P$. (62)

From equations 52 and 62, we have:

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{[\frac{g}{U_o}(n_y + \sin\phi) - R]_\alpha}{P_\alpha} \qquad (63)$$

The factors on the right-hand side of equation 63 can be taken directly from flight-test records, so the ratio $\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha}$ can be plotted against ω_a. From equation 61, and using the general method applied in section A-1 of this Appendix, G_1, G_2, J_4 and J_5 may be computed for any four different values of ω_a. Then, from equations 193, 194, 207 and 208 in section B-2 of Appendix H, we have:

$$n_p = y_\phi - G_1, = C_{L_o} - G_1, \qquad (64)$$

$$n_r = -\frac{G_2}{y_\phi} = -\frac{G_2}{C_{L_o}} \; , \qquad (65)$$

$$y_v = \frac{G_2}{y_\phi} - J_4 = \frac{G_2}{C_{L_o}} - J_4, \tag{66}$$

and $$n_v = \frac{1}{\mu_b}[J_5 + \frac{G_2}{C_{L_o}}(\frac{G_2}{C_{L_o}} - J_4)]. \tag{67}$$

Combining equations 215 and 224 in Section B-2 of Appendix H, we have:

$$\mu_{\dot{\phi}\delta_a} = \frac{T_A P_\alpha}{\Delta_{A_\alpha}} = \omega_a l_{\delta_a} \sqrt{\frac{(\omega_a J_4)^2 + (-J_5 + \omega_a^2)^2}{(\omega_a C_3 - \omega_a^3 C_4)^2 + (-C_5 + \omega_a^2 C_6 - \omega_a^4)^2}}. \tag{68}$$

Now, using a method analogous to the one outlined between equations 22 and 36, we can theoretically solve for the rest of the lateral coefficients. But it is again the author's belief that, with present-day techniques of dynamic flight-testing, the inaccuracy-spread of the flight-test data would be too large to justify such an elaborate analysis.

B-3-a: METHOD FOR DETERMINING LATERAL DYNAMIC STABILITY COEFFICIENTS FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO AERODYNAMIC YAW

The equations for steady-state response of aircraft to forced sinusoidal motion of rudder and ailerons, adjusted for zero aerodynamic yaw, were derived in Section B-3-a of Appendix H. From equation 263 in Section B-3-a of Appendix H, we have:

$$\frac{\dot{\phi}_\alpha}{\dot{\psi}_\alpha} = \frac{\phi_\alpha}{\psi_\alpha} = \frac{\omega_{ar}}{y_\phi} . \tag{69}$$

Also, $$\frac{\dot{\phi}_\alpha}{\dot{\psi}_\alpha} = \frac{P_\alpha}{R_\alpha}, \tag{70}$$

where P_α and R_α can be taken directly from flight records

Thus, $$y_\phi = \omega_{ar}\left(\frac{R_\alpha}{P_\alpha}\right) = \omega_{ar}\left(\frac{\psi_\alpha}{\phi_\alpha}\right) \tag{71}$$

For a given ω_{ar}, the corresponding value of the ratio $\frac{R_\alpha}{P_\alpha}$, or the ratio $\frac{\psi_\alpha}{\phi_\alpha}$, can be determined from the flight records, and then equation 71 can be used to compute y_ϕ. This result can then be checked with the value of C_{L_o}, which is the theoretical value for y_ϕ.

From equations 239, 242 and 256 in section B-3-a of Appendix H, we have:

$$LA_{\psi\delta_r} = -\tan^{-1}\left[\frac{-n_r y_\phi}{(n_p - y_\phi)\omega_{ar}}\right], \tag{72}$$

and $$LA_{\psi\delta_a} = -\tan^{-1}\left[\frac{-\omega_{ar}^2 - l_r y_\phi}{l_p \omega_{ar}}\right]. \tag{73}$$

Both $LA_{\psi\delta_r}$ and $LA_{\psi\delta_a}$ may be obtained from flight records. Thus, for any two different values of ω_{ar} and the corresponding values of $LA_{\psi\delta_r}$, equation 72 can be used to compute n_r and n_p. And, for any two different values of ω_{ar} and the corresponding values of $LA_{\psi\delta_a}$, equation 73 can be used to compute l_r and l_p.

From equations 252 and 264 in Section B-3-a of Appendix H,

$$n_{\delta_r} = \mu_{\phi\delta_r} \sqrt{(n_p - y_\phi)^2\omega_{ar}^2 + (-n_r y_\phi)^2} , \tag{74}$$

and $l_{\delta_a} = \mu_{\phi\delta_a} \sqrt{(l_p\omega_{ar})^2 + (-\omega_{ar}^2 - l_r y_\phi)^2}$. (75)

Now, $\mu_{\phi\delta_r}$ is equal to $\frac{\phi_\alpha}{\delta_{r_\alpha}}$, which can be obtained from flight records for any value of ω_{ar}, so n_{δ_r} can be computed from equation 74. Analogously, $\mu_{\phi\delta_a}$ can be obtained from flight records for any value of ω_{ar}, so l_{δ_a} can be computed from equation 75.

Thus, all of the important lateral stability coefficients, except l_v, n_v and y_v, can be obtained by this method.

B-3-b: METHOD FOR DETERMINING LATERAL DYNAMIC STABILITY COEFFICIENTS FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO GEOMETRIC YAW.

The equations for steady-state response of aircraft to forced sinusoidal motion of rudder and ailerons, adjusted for zero geometric yaw, were derived in section B-3-b of Appendix H. From equation 303 in Section B-3-b of Appendix H, we have:

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{v_\alpha}{\phi_\alpha} = \frac{y_\phi}{\sqrt{(-y_v)^2 + (\omega_{ar})^2}} \qquad (76)$$

Also, from equation 63,

$$\frac{\dot{v}_\alpha}{\dot{\phi}_\alpha} = \frac{\frac{g}{U_0}(n_y + \sin\phi)_\alpha}{P_\alpha} \qquad (77)$$

because $R \equiv 0$.

The factors on the right-hand side of equation 77 can be be taken directly from flight records, so a value of $\frac{\dot{v}_a}{\dot{\phi}_a}$ may be obtained for any ω_{ar}. Then, solving for y_v from equation 76, we have:

$$y_v = -\sqrt{\frac{y_\phi^2}{\left(\frac{\dot{v}_a}{\dot{\phi}_a}\right)^2} - \omega_{ar}^2} \,. \tag{78}$$

Thus, equation 78 may be used to compute y_v, using any value of ω_{ar} and the corresponding value of $\frac{\dot{v}_a}{\dot{\phi}_a}$. This assumes that y_ϕ is known. It can be computed by the method explained in section B-3-a of this Appendix. From equations 273, 274, 275, 276, 279, 280, 281, 282, 296 and 302 in Section B-3-b of Appendix H, we have:

$$LA_{\dot{v}\delta_r} = \frac{\pi}{2} - \tan^{-1}\left[\frac{\omega_{ar} n_p y_v}{-n_v y_\phi \mu_b + n_p \omega_{ar}^2}\right], \tag{79}$$

and

$$LA_{\dot{v}\delta_a} = \frac{\pi}{2} - \tan^{-1}\left[\frac{\omega_{ar}(l_p y_v - \omega_{ar}^2)}{-l_v y_\phi \mu_b + \omega_{ar}^2 (y_v + l_p)}\right]. \tag{80}$$

From equation 52,

$$\dot{v} = C_{L_o}(n_y + \sin\phi) \tag{81}$$

because $r = 0$. Thus, a plot of $\dot{v}$ against time may be made from the flight records of n_y and ϕ. And so $LA_{\dot{v}\delta_r}$ and $LA_{\dot{v}\delta_a}$ can be obtained from the flight records, for various values of ω_{ar}. Then, for any two different values of ω_{ar}, equation 79 can be used to solve for n_p and n_v, since y_ϕ can be established by the method of Section B-3-a of this Appendix, and y_v can be determined from equation 78. Analogously, for any two different values of ω_{ar}, equation 80 can be used to solve for l_p and l_v.

From equations 273, 274, 275, 279, 280, 281, 293 and 305 in Section B-3-b of Appendix H, we have:

$$n_{\delta_r} = \left(\frac{\mu_{v\delta_r}}{y_\phi}\right) \sqrt{(-n_v y_\phi \mu_b + n_p \omega_{ar}^2)^2 + (\omega_{ar} n_p y_v)^2}, \tag{82}$$

and

$$l_{\delta_a} = \left(\frac{\mu_{v\delta_a}}{y_\phi}\right) \sqrt{[-l_v y_\phi \mu_b + \omega_{ar}^2 (y_v + l_p)]^2 + \omega_{ar}^2 (l_p y_v - \omega_{ar}^2)^2}. \tag{83}$$

Now, $\mu_{v\delta_r}$ and $\mu_{v\delta_a}$ can be obtained from flight records, so n_{δ_r} can be computed from equation 82, and l_{δ_a} can be computed from equation 83.

Thus, all the important lateral stability coefficients, except l_r, n_r and y_ϕ, can be obtained by this method.

B-3-c: METHOD FOR DETERMINING LATERAL DYNAMIC STABILITY COEFFICIENTS FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF RUDDER AND AILERONS, ADJUSTED FOR ZERO ANGLE OF BANK.

The equations for steady-state response of aircraft to forced sinusoidal motion of rudder and ailerons, adjusted for zero angle of bank, were derived in Section B-3-c of Appendix H. From equation 338 in Section B-3-c of Appendix H, we have:

$$\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha} = \frac{v_\alpha}{\psi_\alpha} = \frac{\omega_{ar}}{\sqrt{(-y_v)^2 + (\omega_{ar})^2}} \tag{84}$$

Also, from equation 53,

$$\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha} = \frac{(n_y \frac{g}{U_0} - R)_\alpha}{R_\alpha} \tag{85}$$

because $\phi \equiv 0$.

The factors on the right-hand side of equation 85 can be taken directly from flight records, so the ratio $\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha}$ can be plotted against ω_{ar}. Then, solving for y_v from equation 84, we have:

$$y_v = -\omega_{ar} \sqrt{\frac{1}{(\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha})^2} - 1.} \tag{86}$$

Thus, equation 86 may be used to compute y_v, using any value of ω_{ar} and the corresponding value of $\frac{\dot{v}_\alpha}{\dot{\psi}_\alpha}$. Note that in equation 86, y_ϕ need not be known to compute y_v, which is not the case for equation 78.

From equations 312, 315, 316, 317, 331 and 337 in Section B-3-c of Appendix H, we have:

$$LA_{\dot{v}\delta_r} = -\tan^{-1}[\frac{n_v\mu_b + n_r y_v - \omega_{ar}^2}{\omega_{ar}(y_v + n_r)}], \tag{87}$$

and $$LA_{\dot{v}\delta_a} = -\tan^{-1}[\frac{l_r y_v + \mu_b l_v}{l_r\omega_{ar}}]. \tag{88}$$

From equation 52,

$$\dot{v} = C_{L_o} n_y - T_A R \tag{89}$$

because $\phi = 0$. Thus, a plot of $\dot{v}$ against time may be made from the flight records of n_y and R. And so $LA_{\dot{v}\delta_r}$ and $LA_{\dot{v}\delta_a}$ can be obtained from the flight records, for various values of ω_{ar}. Then, for any two different values of ω_{ar}, equation 87 can be used to solve for n_v and n_r, since y_v can be found from equation 86. Analagously, l_v and l_r can be found from equation 88 for any two different values of ω_{ar}. From equations 315, 316, 328 and 340 in Section B-3-c of Appendix H, we have:

$$n_{\delta_r} = \mu_{v\delta_r} \sqrt{\omega_{ar}^2 (y_v + n_r)^2 + (n_v \mu_b + n_r y_v - \omega_{ar}^2)^2}, \tag{90}$$

and

$$l_{\delta_a} = \mu_{v\delta_a} \sqrt{(l_r \omega_{ar})^2 + (l_r y_v + \mu_b l_v)^2}\ . \tag{91}$$

Now, $\mu_{v\delta_r}$ and $\mu_{v\delta_a}$ can be obtained from flight records, so n_{δ_r} can be computed from equation 90, and l_{δ_a} can be computed from equation 91.

Thus, all the important lateral stability coefficients, except l_p, n_p and y_ϕ, can be obtained by this method.

APPENDIX Q

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR

APPENDIX Q

DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR

A. DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY

When both throttle and elevator are forced to oscillate sinusoidally at the same frequency,

$$\omega_e = \omega_{th} = \omega_{e-th} = \omega_{t_x}, \tag{1}$$

and equations 29, 30 and 31 of Section A-1 in Appendix H become:

$$(-d + x_u)u + (x_w)w + (x_\theta)\theta = t_x, \tag{2}^*$$

$$(z_u)u + (-d + z_w)w + (d + z_\theta)\theta = 0 \tag{3}$$

and

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d^2 + dm_q)\theta = -m_{\delta_e}\delta_e. \tag{4}$$

where

$$t_x = \left(\frac{T_A}{U_o M_A}\right)T_X = t_{x_a}\mathcal{R}e^{i\omega_{e-th}\gamma},\ \theta = \bar{\theta}_a\mathcal{R}e^{i\omega_{e-th}\gamma},$$

$$u = \bar{u}_a\mathcal{R}e^{i\omega_{e-th}\gamma}, \tag{5}$$

$$\delta_e = \bar{\delta}_{e_a}\mathcal{R}e^{i\omega_{e-th}\gamma} \text{ and } w = \bar{w}_a\mathcal{R}e^{i\omega_{e-th}\gamma}. \tag{6}$$

Here, we arbitrarily consider t_{x_a} as real and known and $\bar{\delta}_{e_a}$ as complex and unknown. The problem is to adjust $\bar{\delta}_{e_a}$

*Note: The coefficient $x_{t_x} = 1$, so it is not needed in the term on the right hand side of equation 2.

mathematically so that there will be zero change in longitudinal velocity. For convenience in derivation, we will drop the "e-th" subscript.

For u = 0, equations 2, 3, 4, 5 and 6 give:

$$(x_w)\bar{w}_\alpha + (x_\theta)\bar{\theta}_\alpha = t_{x_\alpha}. \tag{7}$$

$$(-i\omega + z_w)\bar{w}_\alpha + (i\omega + z_\theta)\bar{\theta}_\alpha = 0 \tag{8}$$

$$(\mu_c m_{\dot{w}} i\omega + \mu_c m_w)\bar{w}_\alpha + (\omega^2 + i\omega m_q)\bar{\theta}_\alpha + m_{\delta_e}\bar{\delta}_{e_\alpha} = 0 \tag{9}$$

Solving equations 7, 8 and 9 for $\bar{w}_\alpha$, $\bar{\theta}_\alpha$ and $\bar{\delta}_{e_\alpha}$, we have:

$\bar{w}_\alpha = \Delta_2/\Delta_1$, $\bar{\theta}_\alpha = \Delta_3/\Delta_1$ and $\bar{\delta}_{e_\alpha} = \Delta_4/\Delta_1$,

where

$$\Delta_1 = \begin{vmatrix} (x_w) & (x_\theta) & (0) \\ (-i\omega + z_w) & (i\omega + z_\theta) & (0) \\ (\mu_c m_{\dot{w}} i\omega + \mu_c m_w) & (\omega^2 + i\omega m_q) & (m_{\delta_e}) \end{vmatrix}, \tag{10}$$

$$= -m_{\delta_e} \sqrt{(x_\theta z_w - x_w z_\theta)^2 + \omega^2(-x_w - x_\theta)^2}\; e^{i\psi_1}, \tag{11}$$

and

$$\psi_1 = \tan^{-1}\left[\frac{\omega(-x_w - x_\theta)}{x_\theta z_w - x_w z_\theta}\right]. \tag{12}$$

Also,

$$\Delta_2 = \begin{vmatrix} (t_{x_\alpha}) & (x_\theta) & (0) \\ (0) & (i\omega + z_\theta) & (0) \\ (0) & (\omega^2 + i\omega m_q) & (m_{\delta_e}) \end{vmatrix}, \tag{13}$$

$$= -m_{\delta_e} t_{x_\alpha} \sqrt{(-z_\theta)^2 + (-\omega)^2}\; e^{i\psi_2}, \tag{14}$$

and

$$\psi_2 = \tan^{-1} \left(\frac{-\omega}{-z_\theta}\right), \; = \frac{3\pi}{2} \text{ for } z_\theta = 0. \tag{15}$$

Further,

$$\Delta_3 = \begin{vmatrix} (x_w) & (t_{x_\alpha}) & (0) \\ (-i\omega + z_w) & (0) & (0) \\ (\mu_c m_{\dot{w}} i\omega + \mu_c m_w) & (0) & (m_{\delta_e}) \end{vmatrix}, \tag{16}$$

$$= -m_{\delta_e} t_{x_\alpha} \sqrt{(z_w)^2 + (-\omega)^2}\; e^{i\psi_3}, \tag{17}$$

and $\psi_3 = \tan^{-1} \left(\frac{-\omega}{+z_w}\right).$ (18)

Also,

$$\Delta_4 = \begin{vmatrix} (x_w) & (x_\theta) & (t_{x_\alpha}) \\ (-i\omega + z_w) & (i\omega + z_\theta) & (0) \\ (\mu_c m_{\dot{w}} i\omega + \mu_c m_w) & (\omega^2 + i\omega m_q) & (0) \end{vmatrix}, \tag{19}$$

$$= t_{x_\alpha} \sqrt{(-\omega^2 L_1 - L_3)^2 + \omega^2(-\omega^2 + L_2)^2}\; e^{i\psi_4}, \tag{20}$$

and

$$\psi_4 = \tan^{-1} \left[\frac{(-\omega^2 + L_2)\omega}{-\omega^2 L_1 - L_3}\right], \tag{21}$$

where

$$L_1 = -m_q - z_w - \mu_c m_{\dot{w}}, \tag{22}$$

$$L_2 = m_q z_w - \mu_c m_w - z_\theta \mu_c m_{\dot{w}}, \tag{23}$$

and $L_3 = z_\theta \mu_c m_w$ (24)

Now, using the subscript "e-th", we have:

$$w = \mu_{wt_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{wt_x}), \tag{25}$$

$$\theta = \mu_{\theta t_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{\theta t_x}) \tag{26}$$

and

$$\delta_e = \mu_{\delta_e t_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{\delta_e t_x}). \tag{27}$$

where, for $z_\theta = 0$, we have:

$$\mu_{wt_x} = \frac{w_\alpha}{t_{x_\alpha}} = \frac{\omega_{e-th}}{\sqrt{(x_\theta z_w)^2 + \omega_{e-th}^2(-x_w - x_\theta)^2}} , \tag{28}$$

$$\mu_{\theta t_x} = \frac{\theta_\alpha}{t_{x_\alpha}} = \sqrt{\frac{(z_w)^2 + (-\omega_{e-th})^2}{(x_\theta z_w)^2 + \omega_{e-th}^2(-x_w - x_\theta)^2}} \tag{29}$$

and

$$\mu_{\delta_e t_x} = \frac{\delta_{e_\alpha}}{t_{x_\alpha}} = \frac{\omega_{e-th}}{-m_{\delta_e}} \sqrt{\frac{(-\omega_{e-th}L_1)^2 + (-\omega_{e-th}^2 + L_2)^2}{(x_\theta z_w)^2 + \omega_{e-th}^2(-x_w - x_\theta)^2}} . \tag{30}$$

Also, for $z_\theta = 0$, we have:

$$LA_{wt_x} = \frac{3\pi}{2} - \psi_1, \tag{31}$$

$$LA_{\theta t_x} = \psi_3 - \psi_1 \tag{32}$$

and

$$LA_{\delta_e t_x} = \psi_4 - \psi_1. \tag{33}$$

Analogously,

$$\mu_{w\delta_e} = \frac{w_\alpha}{\delta_{e_\alpha}} = - \frac{m_{\delta_e}}{\sqrt{(-\omega_{e-th}L_1)^2 + (-\omega_{e-th}^2 + L_2)^2}} , \tag{34}$$

$$\mu_{\theta\delta_e} = \frac{\theta_a}{\delta_{e_a}} = \frac{-m_{\delta_e} \sqrt{(z_w)^2 + (-\omega_{e-th})^2}}{\omega_{e-th} \sqrt{(-\omega_{e-th}L_1)^2 + (-\omega_{e-th}^2 + L_2)^2}} \; , \tag{35}$$

$$\text{and} \quad \mu_{t_x\delta_e} = \frac{t_{x_a}}{\delta_{e_a}} = \frac{1}{\mu_{\delta_e t_x}} \; . \tag{36}$$

Also, for $z_\theta = 0$:

$$LA_{w\delta_e} = \frac{3\pi}{2} - \psi_4, \tag{37}$$

$$LA_{\theta\delta_e} = \psi_3 - \psi_4, \tag{38}$$

$$\text{and} \quad LA_{t_x\delta_e} = \psi_1 - \psi_4. \tag{39}$$

After differentiating equations 25, 26 and 27, we see that:

$$\mu_{\dot{w}t_x} = \omega_{e-th}\mu_{wt_x} \; , \qquad \mu_{\dot{w}\delta_e} = \omega_{e-th}\mu_{w\delta_e} \; , \tag{40}$$

$$\mu_{qt_x} = \omega_{e-th}\mu_{\theta t_x} \; , \qquad \mu_{q\delta_e} = \omega_{e-th}\mu_{\theta\delta_e} \; , \tag{41}$$

$$\mu_{\dot{\delta}_e t_x} = \omega_{e-th}\mu_{\delta_e t_x}, \qquad \mu_{\dot{t}_x\delta_e} = \omega_{e-th}\mu_{t_x\delta_e}, \text{ etc.} \tag{42}$$

and,

$$LA_{\dot{w}t_x} = LA_{wt_x} + \pi/2, \qquad LA_{\dot{w}\delta_e} = LA_{w\delta_e} + \pi/2 \tag{43}$$

$$LA_{qt_x} = LA_{\theta t_x} + \pi/2, \qquad LA_{q\delta_e} = LA_{\theta\delta_e} + \pi/2 \tag{44}$$

$$LA_{\dot{\delta}_e t_x} = LA_{\delta_e t_x} + \pi/2, \qquad LA_{\dot{t}_x\delta_e} = LA_{t_x\delta_e} + \pi/2, \text{etc.} \tag{45}$$

If we divide equation 34 by equation 35, we have:

$$\frac{\mu_{w\delta_e}}{\mu_{\theta\delta_e}} = \mu_{w\theta} = \mu_{\dot{w}q} = \frac{1}{\sqrt{\left(\frac{z_w}{\omega_{e-th}}\right)^2 + 1}} \tag{46}$$

B. DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK)

From equations 2, 3 and 4, for w = 0, we have:

$$(-d + x_u)u + (x_\theta)\theta = t_x, \tag{47}$$

$$(z_u)u + (d + z_\theta)\theta = 0 \tag{48}$$

and

$$(\mu_c m_u)u + (-d^2 + dm_q)\theta + m_{\delta_e}\delta_e = 0. \tag{49}$$

If we combine equations 47, 48, 49,5 and 6, and then solve for u, θ and δ_e, using the method shown in section A, we have:

$$u = \mu_{ut_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{ut_x}), \tag{50}$$

$$\theta = \mu_{\theta t_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{\theta t_x}), \tag{51}$$

and

$$\delta_e = \mu_{\delta_e t_x} t_{x_\alpha} \sin(\omega_{e-th}\gamma + LA_{\delta_e t_x}), \tag{52}$$

where

$$\mu_{ut_x} = \sqrt{\frac{(-z_\theta)^2 + (-\omega_{e-th})^2}{(L_4 - \omega_{e-th}^2)^2 + \omega_{e-th}^2 L_5^2}}, \tag{53}$$

$$\mu_{\theta t_x} = -z_u \sqrt{\frac{1}{(L_4 - \omega_{e-th}^2)^2 + \omega_{e-th}^2 L_5^2}}, \tag{54}$$

and

$$\mu_{\delta_e t_x} = -\left(\frac{1}{m_{\delta_e}}\right) \sqrt{\frac{(z_u\omega_{e-th}^2 - \mu_c m_u z_\theta)^2 + \omega_{e-th}^2(z_u m_q - \mu_c m_u)^2}{(L_4 - \omega_{e-th}^2)^2 + \omega_{e-th}^2 L_5^2}}$$

And,

$$LA_{ut_x} = \phi_2 - \phi_1, \quad (56)$$

$$LA_{\theta t_x} = \pi - \phi_1, \quad (57)$$

$$LA_{\delta_e t_x} = \phi_3 - \phi_1. \quad (58)$$

Further,

$$L_4 = -x_u z_\theta + x_\theta z_u, \quad (59)$$

$$L_5 = -x_u + z_\theta, \quad (60)$$

and,

$$\phi_1 = \tan^{-1}\left(\frac{L_5 \omega_{e-th}}{L_4 - \omega_{e-th}^2}\right), \quad (61)$$

$$\phi_2 = \tan^{-1}\left(\frac{-\omega_{e-th}}{-z_\theta}\right), \; = \frac{3\pi}{2} \text{ for } z_\theta = 0, \quad (62)$$

and

$$\phi_3 = \tan^{-1}\left[\frac{\omega_{e-th}(z_u m_q - \mu_c m_u)}{z_u \omega_{e-th}^2 - \mu_c m_u z_\theta}\right] = \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right),$$

for z_θ and $m_u = 0$. (63)

Also, for z_θ and $m_u = 0$:

$$\mu_{u\delta_e} = \frac{m_{\delta_e}}{z_u \sqrt{\omega_{e-th}^2 + m_q^2}} \quad (64)$$

$$\mu_{q\delta_e} = -\left(\frac{m_{\delta_e}}{\sqrt{\omega_{e-th}^2 + m_q^2}}\right) \quad (65)$$

Thus,

$$\mu_{uq} = \frac{\mu_{u\delta_e}}{\mu_{q\delta_e}} = -\frac{1}{z_u}. \tag{66}$$

Further, for z_θ and $m_u = 0$

$$LA_{\theta\delta_e} = \pi - \phi_3 = \pi - \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right), \tag{67}$$

and

$$LA_{q\delta_e} = \frac{3\pi}{2} - \phi_3 = \frac{3\pi}{2} - \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right). \tag{68}$$

C. DERIVATION OF EQUATIONS FOR STEADY-STATE RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANCE IN PITCH ANGLE

From equations 2, 3 and 4, for $\theta = 0$, we have:

$$(-d + x_u)u + (x_w)w = t_x, \tag{69}$$

$$(z_u)u + (-d + z_w)w = 0, \tag{70}$$

and

$$(\mu_c m_u)u + (d\mu_c m_{\dot{w}} + \mu_c m_w)w + m_{\delta_e}\delta_e = 0. \tag{71}$$

If we combine equations 69, 70, 71, 5 and 6, and then solve for u, w and δ_e, using the method shown in Section A, we have:

$$u = \mu_{ut_x} t_{x_a} \sin(\omega_{e-th}\gamma + LA_{ut_x}), \tag{72}$$

$$w = \mu_{wt_x} t_{x_a} \sin(\omega_{e-th}\gamma + LA_{wt_x}), \tag{73}$$

and

$$\delta_e = \mu_{\delta_e t_x} t_{x_a} \sin(\omega_{e-th}\gamma + LA_{\delta_e t_x}), \tag{74}$$

where

$$\mu_{ut_x} = \sqrt{\frac{(-z_w)^2 + \omega_{e-th}^2}{(-L_6 + \omega_{e-th}^2)^2 + (-L_7\omega_{e-th})^2}}, \tag{75}$$

$$\mu_{wt_x} = \frac{-z_u}{\sqrt{(-L_6 + \omega_{e-th}^2)^2 + (-L_7\omega_{e-th})^2}}, \tag{76}$$

and

$$\mu_{\delta_e t_x} = -\left(\frac{1}{m_{\delta_e}}\right)\sqrt{\frac{L_8^2 + \omega_{e-th}^2 L_9^2}{(-L_6 + \omega_{e-th}^2)^2 + (-L_7\omega_{e-th})^2}}. \tag{77}$$

Further,

$$LA_{ut_x} = \lambda_2 - \lambda_1, \tag{78}$$

$$LA_{wt_x} = \pi - \lambda_1 \tag{79}$$

and

$$LA_{\delta_e t_x} = \lambda_3 - \lambda_1. \tag{80}$$

Also,

$$L_6 = x_u z_w - x_w z_u, \tag{81}$$

$$L_7 = -z_w - x_u, \tag{82}$$

$$L_8 = z_u \mu_c m_w - \mu_c m_u z_w, \tag{83}$$

and

$$L_9 = z_u \mu_c m_{\dot{w}} + \mu_c m_u. \tag{84}$$

And,

$$\lambda_1 = \tan^{-1}\left(\frac{-\omega_{e-th}L_7}{-L_6 + \omega_{e-th}^2}\right), \tag{85}$$

$$\lambda_2 = \tan^{-1} \left(\frac{+\ \omega_{e-th}}{-\ z_w}\right), \tag{86}$$

and

$$\lambda_3 = \tan^{-1} \left(\frac{\omega_{e-th} L_9}{L_8}\right). \tag{87}$$

Now, for $m_u = 0$:

$$\mu_{wu} = \frac{\mu_{w\delta_e}}{\mu_{u\delta_e}} = \frac{\mu_{wt_x}}{\mu_{ut_x}} = \frac{\mu_{\dot{w}\delta_e}}{\mu_{\dot{u}\delta_e}} = \frac{-z_u}{\sqrt{(-z_w)^2 + \omega_{e-th}^2}}\ , \tag{88}$$

and

$$LA_{w\delta_e} = \pi - \lambda_3 = -\tan^{-1} \left(\frac{-\omega_{e-th} m_{\dot{w}}}{-m_w}\right) \tag{89}$$

etc.

APPENDIX R

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR

APPENDIX R

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR.

A. ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in longitudinal velocity, were derived in Section A of Appendix Q. As would be expected, the equations for w and θ, when based upon elevator motion, are identical with the equations for w and θ derived in Section A-2 of Appendix H, where the effect of change in longitudinal velocity was neglected in the differential equations of motion for normal force and pitching moment, and where the throttle was assumed to be fixed.

With reference to Section A of Appendix Q, a summary of the basic equations used in this analysis is as follows:

$$L_1 = -m_q - z_w - \mu_c m_{\dot{w}} . \tag{1}$$

$$L_2 = m_q z_w - \mu_c m_w - z_\theta \mu_c m_{\dot{w}}. \tag{2}$$

$$L_3 = z_\theta \mu_c m_w. \tag{3}$$

$$\psi_1 = \tan^{-1} \left[\frac{\omega_{e-th}(-x_w - x_\theta)}{x_\theta z_w - x_w z_\theta}\right]. \tag{4}$$

$$\psi_2 = \tan^{-1} \left(\frac{-\omega_{e-th}}{-z_\theta}\right), = \frac{3\pi}{2} \text{ for } z_\theta = 0. \tag{5}$$

$$\psi_3 = \tan^{-1}\left(\frac{-\omega_{e-th}}{+z_w}\right). \tag{6}$$

$$\psi_4 = \tan^{-1}\left[\frac{(-\omega^2_{e-th} + L_2)\omega_{e-th}}{-\omega^2_{e-th}L_1 - L_3}\right]. \tag{7}$$

$$LA_{w\delta_e} = \psi_2 - \psi_4. \tag{8}$$

$$LA_{\theta\delta_e} = \psi_3 - \psi_4. \tag{9}$$

$$LA_{t_x\delta_e} = \psi_1 - \psi_4. \tag{10}$$

$$\mu_{w\delta_e} = \frac{w_a}{\delta_{e_a}} = \frac{-m_{\delta_e}}{\sqrt{(-\omega_{e-th}L_1)^2 + (-\omega^2_{e-th} + L_2)^2}} . \tag{11}$$

$$\mu_{\theta\delta_e} = \frac{\theta_a}{\delta_{e_a}} = \frac{-m_{\delta_e} \sqrt{(z_w)^2 + (-\omega_{e-th})^2}}{\omega_{e-th} \sqrt{(-\omega_{e-th}L_1)^2 + (-\omega^2_{e-th} + L_2)^2}} \tag{12}$$

$$\mu_{t_x\delta_e} = \frac{t_{x_a}}{\delta_{e_a}} = \left(\frac{-m_{\delta_e}}{\omega_{e-th}}\right) \sqrt{\frac{(x_\theta z_w)^2 + \omega^2_{e-th}(-x_w - x_\theta)^2}{(-\omega_{e-th}L_1)^2 + (-\omega^2_{e-th} + L_2)^2}} . \tag{13}$$

$$w = \mu_{w\delta_e}\delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{w\delta_e}) \tag{14}$$

$$\theta = \mu_{\theta\delta_e}\delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{\theta\delta_e}) \tag{15}$$

$$t_x = \mu_{t_x\delta_e}\delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{t_x\delta_e}) \tag{16}$$

$$\delta_e = \delta_{e_a} \sin \omega_{e-th} \gamma \tag{17}$$

$$\frac{\mu_{\theta \delta_e}}{\mu_{w \delta_e}} = \mu_{\theta w} = \mu_{q \dot{w}} = \sqrt{\left(\frac{z_w}{\omega_{e-th}}\right)^2 + 1} \tag{18}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical longitudinal dynamic stability coefficients used in this Analysis, for the A-26 Airplane, were (see Appendix D) as follows:

T_A = 4.4716 sec.

μ_c = 241.43

x_w = .18720

x_θ = -.32687

z_w = -4.8701

z_θ negligible

m_w = -.61949

$m_{\dot{w}}$ = -.022236

m_q = -16.3668

m_{δ_e} = -351.95

From equations 1, 2 and 3:

L_1 = 26.605,

L_2 = 229.27,

and L_3 = 0.

Using equations 4, 5, 6, 7, 8, 9 and 10, the lead angles, $LA_{w\delta_e}$, $LA_{\theta\delta_e}$, $LA_{t_x\delta_e}$, were computed for chosen values of

Ω_{E-TH}. These computations are shown in Table R-1, and the results are plotted in Figure R-1.

From equations 11, 12 and 13, the amplitude response ratios, $\mu_{w\delta_e}$, $\mu_{\theta\delta_e}$ and $\mu_{t_x\delta_e}$, were computed for chosen values of Ω_{E-TH}. These computations are shown in table R-1, and the results are plotted in Figure R-2. Equation 18 was used to compute the ratio $\mu_{\theta w} = \mu_{q\dot{w}}$, for chosen values of Ω_{E-TH}. These computations are shown in Table R-1, and the results are plotted in Figure R-3.

Equations 14, 15, 16 and 17 were used to compute the time history of w/δ_{e_a}, θ/δ_{e_a}, t_x/δ_{e_a} and δ_e/δ_{e_a}, respectively. These computations are shown in Tables R-2, R-3 and R-4, and the results are plotted in Figures R-4, R-5 and R-6.

B. ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK)

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in normal velocity, were derived in Section B of Appendix Q. With reference to this section a summary of the basic equations used in this analysis is as follows:

$$L_4 = -x_u z_\theta + x_\theta z_u. \tag{1}$$

$$L_5 = -x_u + z_\theta. \tag{2}$$

$$\phi_1 = \tan^{-1} \left(\frac{L_5\omega_{e-th}}{L_4 - \omega^2_{e-th}}\right). \quad (3)$$

$$\phi_2 = \tan^{-1}\left(\frac{-\omega_{e-th}}{-z_\theta}\right), \quad = \frac{3\pi}{2} \text{ for } z_\theta = 0. \quad (4)$$

$$\phi_3 = \tan^{-1}\left[\frac{\omega_{e-th}(z_u m_q - \mu_c m_u)}{z_u\omega^2_{e-th} - \mu_c m_u z_\theta}\right], \quad (5)$$

$$= \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right) \text{ for } z_\theta \text{ and } m_u = 0. \quad (5\text{-a})$$

$$LA_{u\delta_e} = \phi_2 - \phi_3. \quad (6)$$

$$LA_{\theta\delta_e} = \pi - \phi_3 \quad (7)$$

$$LA_{t_x\delta_e} = \phi_1 - \phi_3 \quad (8)$$

$$\mu_{u\delta_e} = \frac{m_{\delta_e}}{z_u \times \sqrt{\omega^2_{e-th} + m^2_q}}, \text{ for } z_\theta \text{ and } m_u = 0. \quad (9)$$

$$\mu_{q\delta_e} = \frac{-m_{\delta_e}}{\sqrt{\omega^2_{e-th} + m^2_q}}, \text{ for } z_\theta \text{ and } m_u = 0. \quad (10)$$

$$\mu_{t_x\delta_e} = \left(\frac{m_{\delta_e}}{z_u\omega_{e-th}}\right)\sqrt{\frac{(L_4 - \omega^2_{e-th})^2 + \omega^2_{e-th}L_5^2}{\omega^2_{e-th} + m^2_q}},$$

$$\text{for } z_\theta \text{ and } m_u = 0. \quad (11)$$

$$\mu_{qu} = -z_u \quad (12)$$

$$u = \mu_{u\delta_e}\delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{u\delta_e}) \quad (13)$$

$$\theta = \mu_{\theta\delta_e}\delta_{e_\alpha}\sin(\omega_{e-th}\gamma + LA_{\theta\delta_e}) \tag{14}$$

$$t_x = \mu_{t_x\delta_e}\delta_{e_\alpha}\sin(\omega_{e-th}\gamma + LA_{t_x\delta_e}) \tag{15}$$

$$\delta_e = \delta_{e_\alpha}\sin\omega_{e-th}\gamma \tag{16}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical longitudinal dynamic stability coefficients used in this analysis (see Appendix D) were as follows:

T_A = 4.4716 sec.	m_q = -16.3668
μ_c = 241.43	m_{δ_e} = -351.95
x_u = -.087261	z_θ negligible
x_θ = -.32687	m_u negligible
z_u = -.65374	

From equations 1 and 2,

L_4 = .21369

and L_5 = .087261.

Using equations 3, 4, 5-a, 6, 7 and 8, the lead angles, $LA_{u\delta_e}$, $LA_{\theta\delta_e}$, $LA_{t_x\delta_e}$, were computed for chosen values of Ω_{E-TH}. These computations are shown in Table R-5, and results are plotted in Figure R-7.

From equations 9, 10 and 11, the amplitude response ratios, $\mu_{u\delta_e}$, $\mu_{q\delta_e}$ and $\mu_{t_x\delta_e}$, were computed for chosen values of Ω_{E-TH}. These computations are shown in Table R-5, and the results are plotted in Figure R-8. Note that $\mu_{qu} = -z_u$ is independent of Ω_{E-TH}.

Equations 13, 14, 15 and 16 were used to compute the time history of u/δ_{e_α}, θ/δ_{e_α}, t_x/δ_{e_α} and $\delta_e/\delta_{e_\alpha}$, respectively. These computations are shown in tables R-6, R-7 and R-8, and the results are plotted in Figures R-9, R-10 and R-11.

C. ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE.

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in pitch angle, were derived in Section C of Appendix Q. With reference to this section a summary of the basic equations used in this analysis is as follows:

$$L_6 = x_u z_w - x_w z_u. \qquad (1)$$

$$L_7 = -z_w - x_u. \qquad (2)$$

$$L_8 = z_u \mu_c m_w - \mu_c m_u z_w. \qquad (3)$$

$$L_9 = z_u \mu_c m_{\dot{w}} + \mu_c m_u. \qquad (4)$$

$$\lambda_1 = \tan^{-1}\left(\frac{-\omega_{e-th} L_7}{-L_6 + \omega_{e-th}^2}\right). \qquad (5)$$

$$\lambda_2 = \tan^{-1}\left(\frac{+\omega_{e-th}}{-z_w}\right). \qquad (6)$$

$$\lambda_3 = \tan^{-1}\left(\frac{\omega_{e-th} L_9}{L_8}\right). \qquad (7)$$

$$LA_{u\delta_e} = \lambda_2 - \lambda_3. \qquad (8)$$

$$LA_{w\delta_e} = \pi - \lambda_3 \qquad (9)$$

$$LA_{t_x \delta_e} = \lambda_1 - \lambda_3. \tag{10}$$

$$\mu_{u\delta_e} = -m_{\delta_e} \sqrt{\frac{(-z_w)^2 + \omega_{e-th}^2}{L_8^2 + \omega_{e-th}^2 L_9^2}} . \tag{11}$$

$$\mu_{w\delta_e} = \frac{m_{\delta_e} z_u}{\sqrt{L_8^2 + \omega_{e-th}^2 L_9^2}} \tag{12}$$

$$\mu_{t_x \delta_e} = -m_{\delta_e} \sqrt{\frac{(-L_6 + \omega_{e-th}^2)^2 + (-L_7 \omega_{e-th})^2}{L_8^2 + \omega_{e-th}^2 L_9^2}} . \tag{13}$$

$$\mu_{uw} = -\left(\frac{1}{z_u}\right) \sqrt{(-z_w)^2 + \omega_{e-th}^2} \tag{14}$$

$$u = \mu_{u\delta_e} \delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{u\delta_e}) \tag{15}$$

$$w = \mu_{w\delta_e} \delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{w\delta_e}) \tag{16}$$

$$t_x = \mu_{t_x \delta_e} \delta_{e_a} \sin(\omega_{e-th}\gamma + LA_{t_x \delta_e}) \tag{17}$$

$$\delta_e = \delta_{e_a} \sin \omega_{e-th}\gamma \tag{18}$$

An initial true airspeed of 300 mph, at 10,000 feet density altitude, was chosen for the basis of this analysis. Thus, the theoretical longitudinal dynamic stability coefficients used in this analysis (see Appendix D) were as follows:

T_A = 4.4716 sec.

μ_c = 241.43

x_u = -.087261

$x_w = .18720$

$z_u = -.65374$

$z_w = -4.8701$

m_u negligible

$m_w = -.61949$

$m_{\dot{w}} = -.022236$

$m_{\delta_e} = -351.95$

From equations 1, 2, 3 and 4,

$L_6 = .54735$

$L_7 = 4.9574$

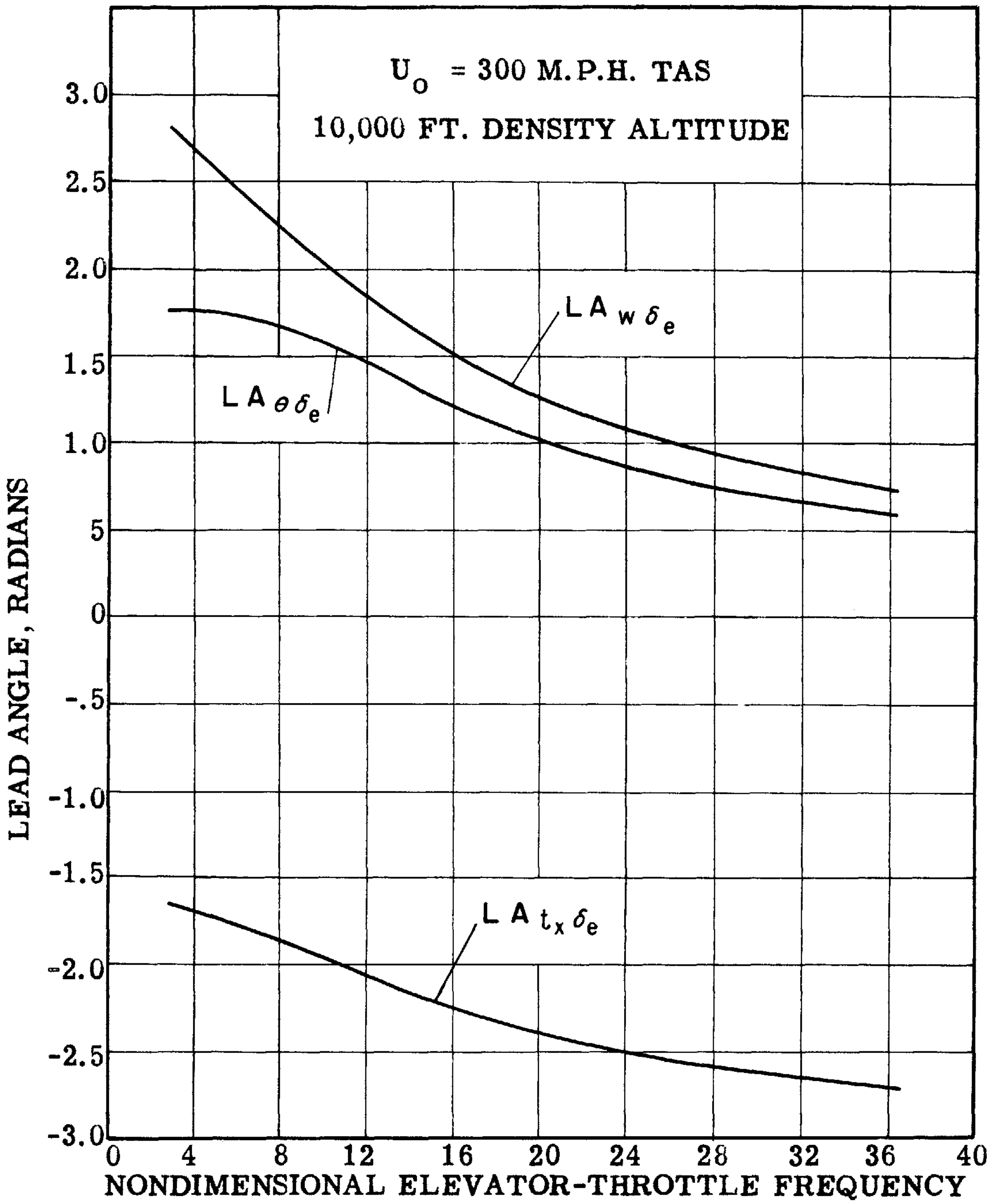
$L_8 = 97.773$

$L_9 = 3.5095$

Using equations 5, 6, 7, 8, 9 and 10, the lead angles, $LA_{u\delta_e}$, $LA_{w\delta_e}$ and $LA_{t_x\delta_e}$, were computed for chosen values of Ω_{E-TH}. These computations are shown in Table R-9, and the results are plotted in Figure R-12.

From equations 11, 12 and 13, the amplitude response ratios, $\mu_{u\delta_e}$, $\mu_{w\delta_e}$ and $\mu_{t_x\delta_e}$, were computed for various values of Ω_{E-TH}. The computations are shown in Table R-9, and the results are plotted in Figure R-13. From equation 14, the ratio $\mu_{u\delta_e}/\mu_{w\delta_e}$ was computed for chosen values of Ω_{E-TH}. These computations are shown in Table R-9, and the results are plotted in Figure R-14.

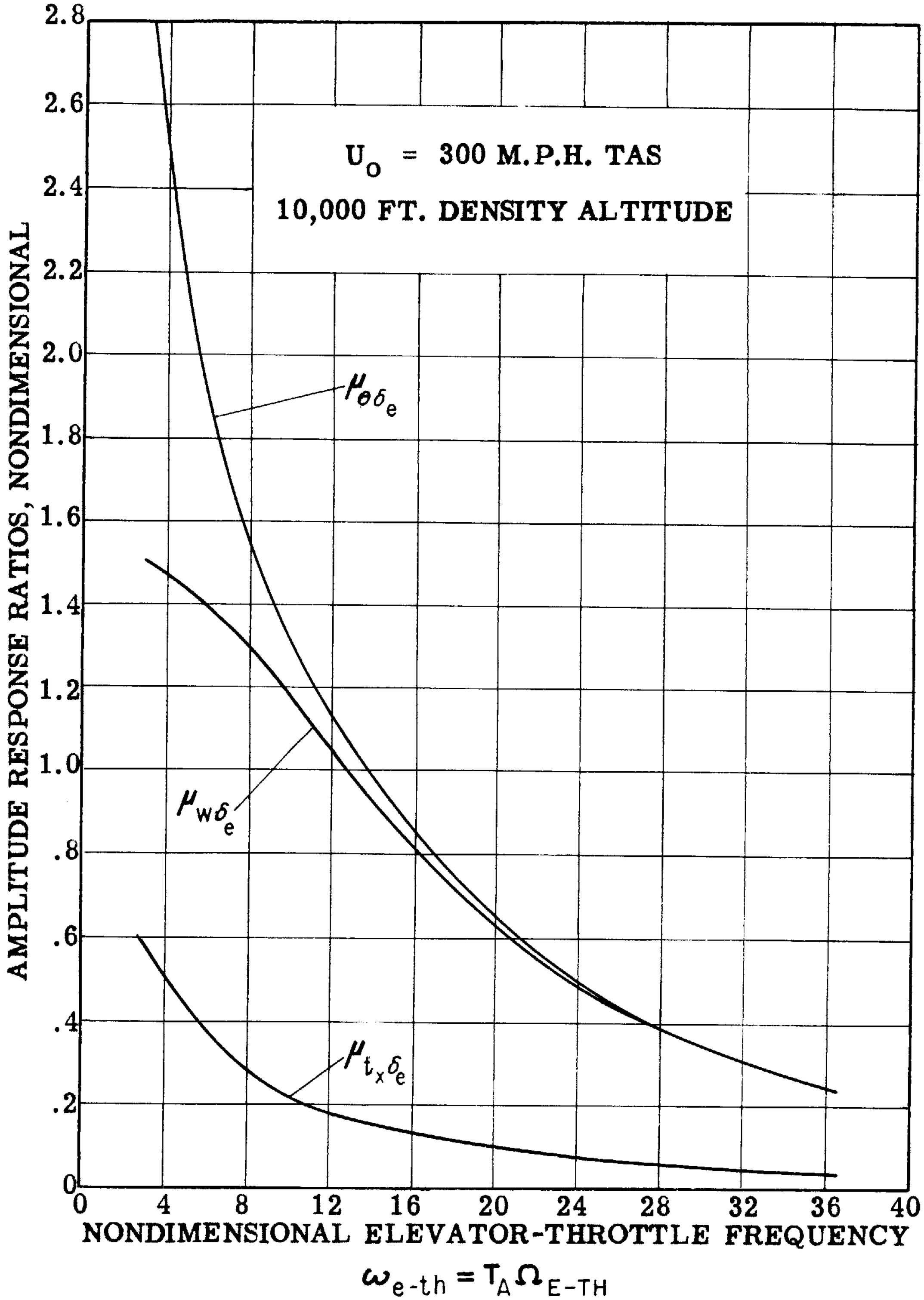
Equations 15, 16, 17 and 18 were used to compute the time history of u/δ_{e_α}, w/δ_{e_α}, t_x/δ_{e_α} and $\delta_e/\delta_{e_\alpha}$, respectively. These computations are shown in Tables R-10, R-11 and R-12, and the results are plotted in Figures R-15, R-16 and R-17.



$\omega_{e-th} = T_A \Omega_{E-TH}$

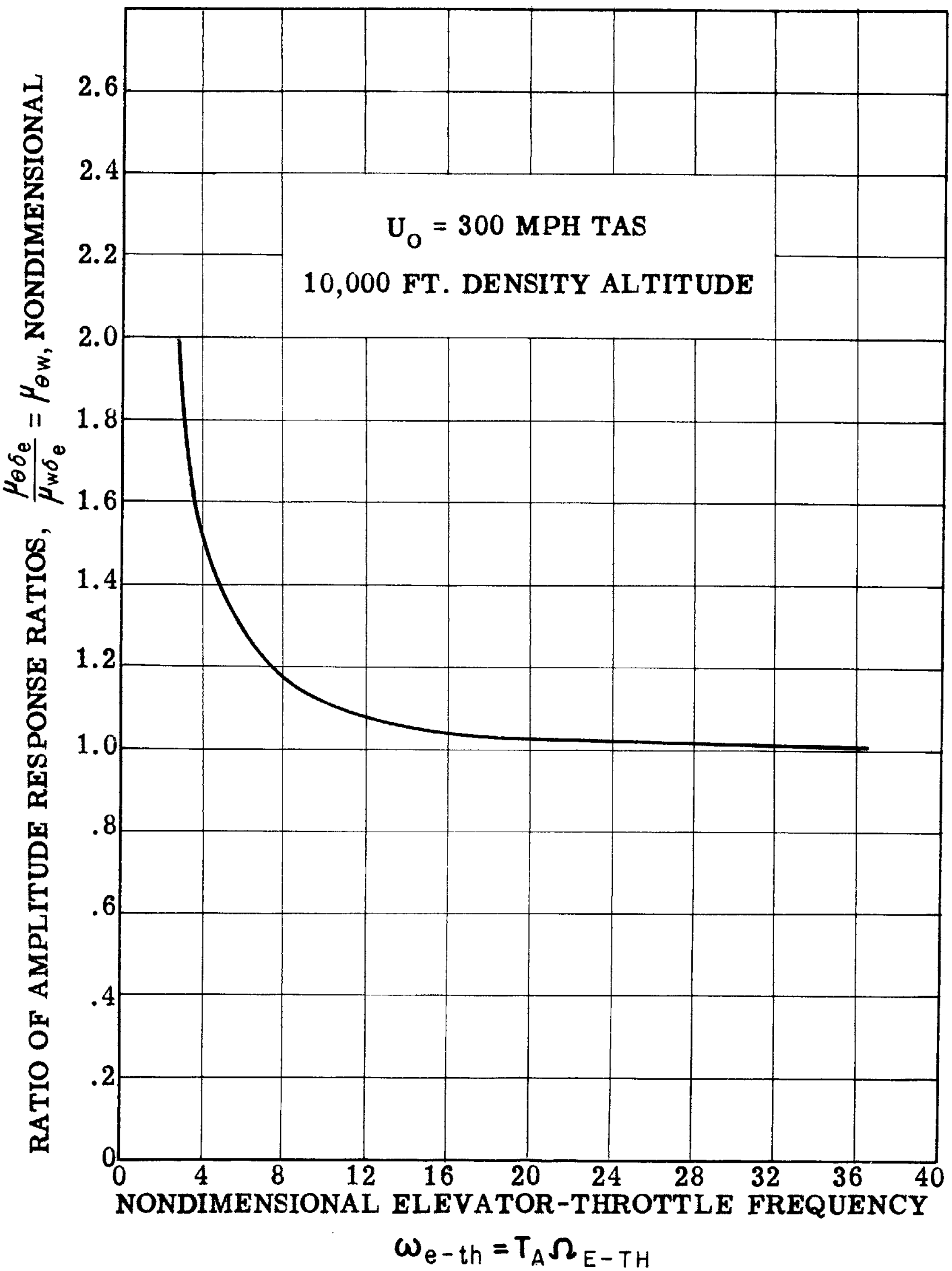
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN NORMAL VELOCITY PER TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN PITCH ANGLE AND CHANGE IN ELEVATOR ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL THRUST AND CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY

FIGURE R-1



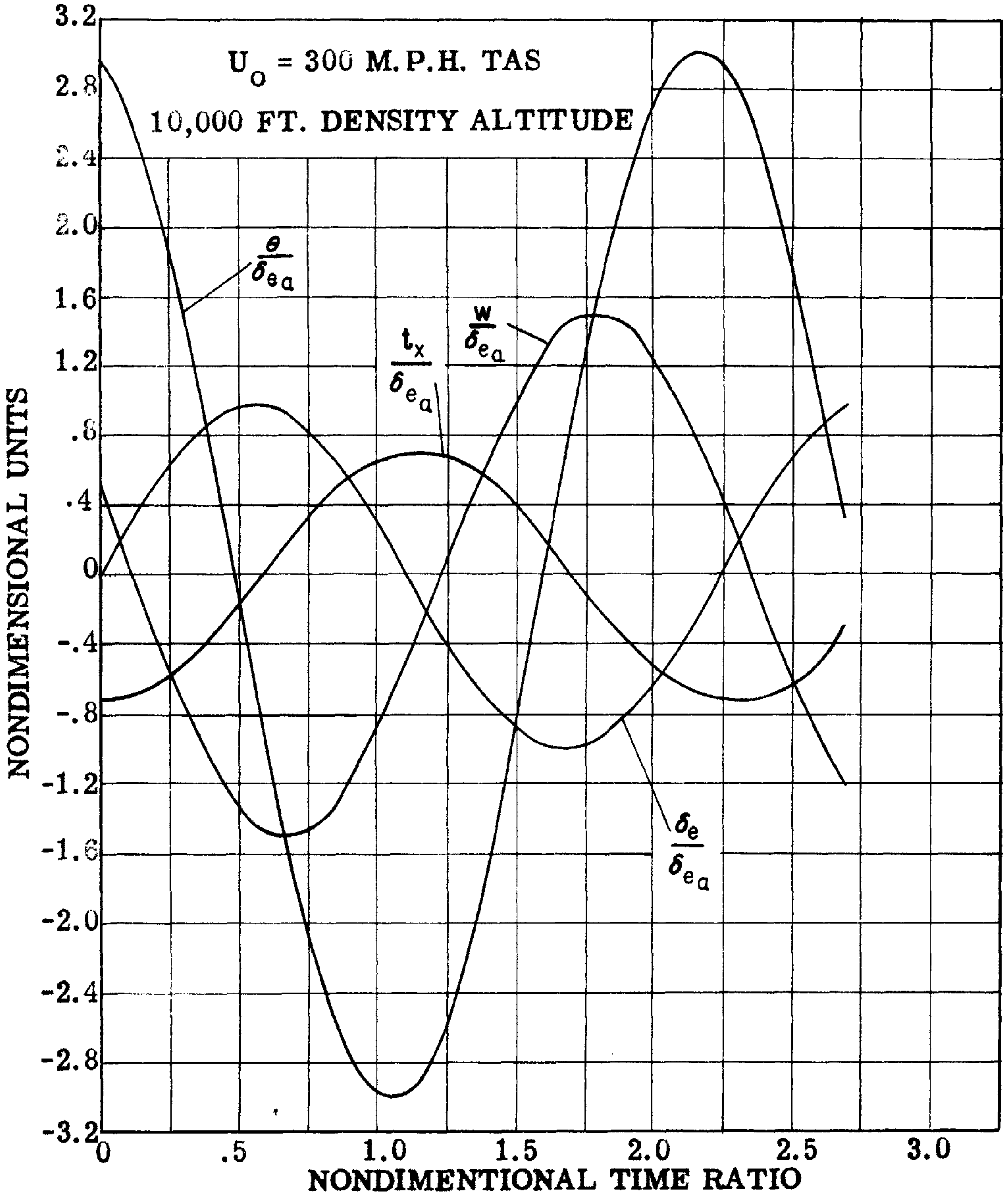
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY

FIGURE R-2



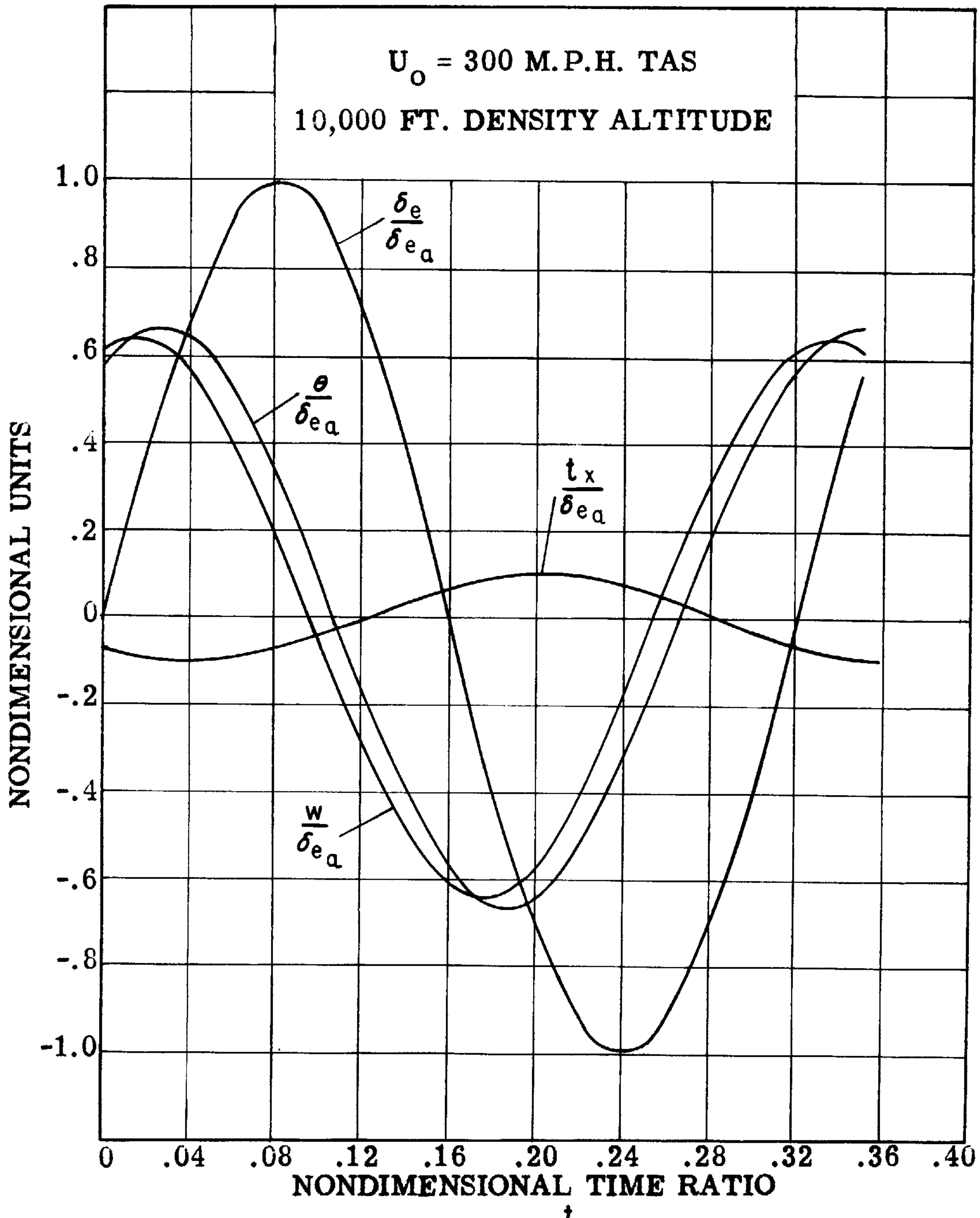
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR ANGLE OF PITCH TO AMPLITUDE RESPONSE RATIO FOR NORMAL VELOCITY PER TRIM SPEED, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY

FIGURE R-3



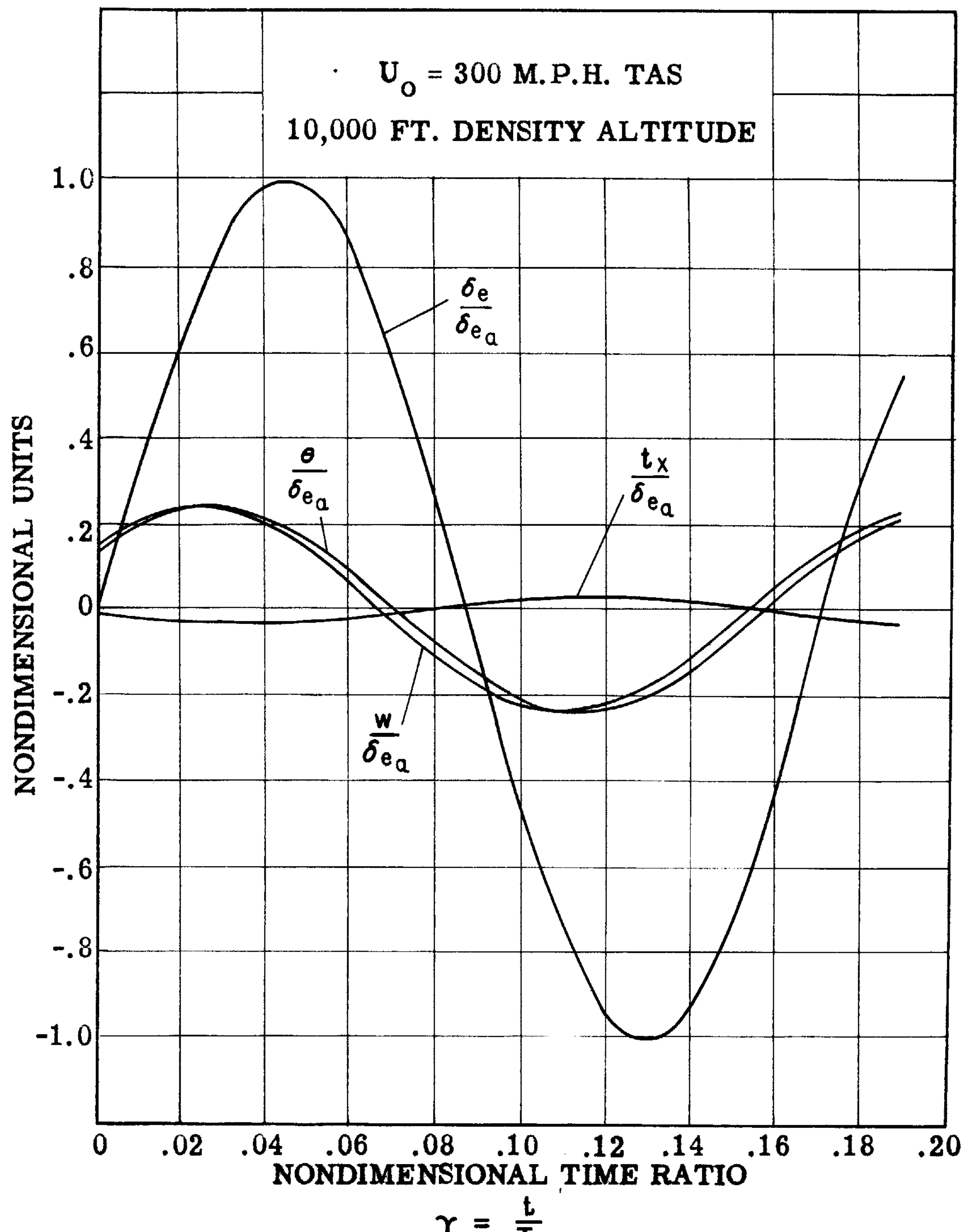
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY, ELEVATOR-THROTTLE FREQUENCY = .1 CYCLE PER SECOND

FIGURE R-4



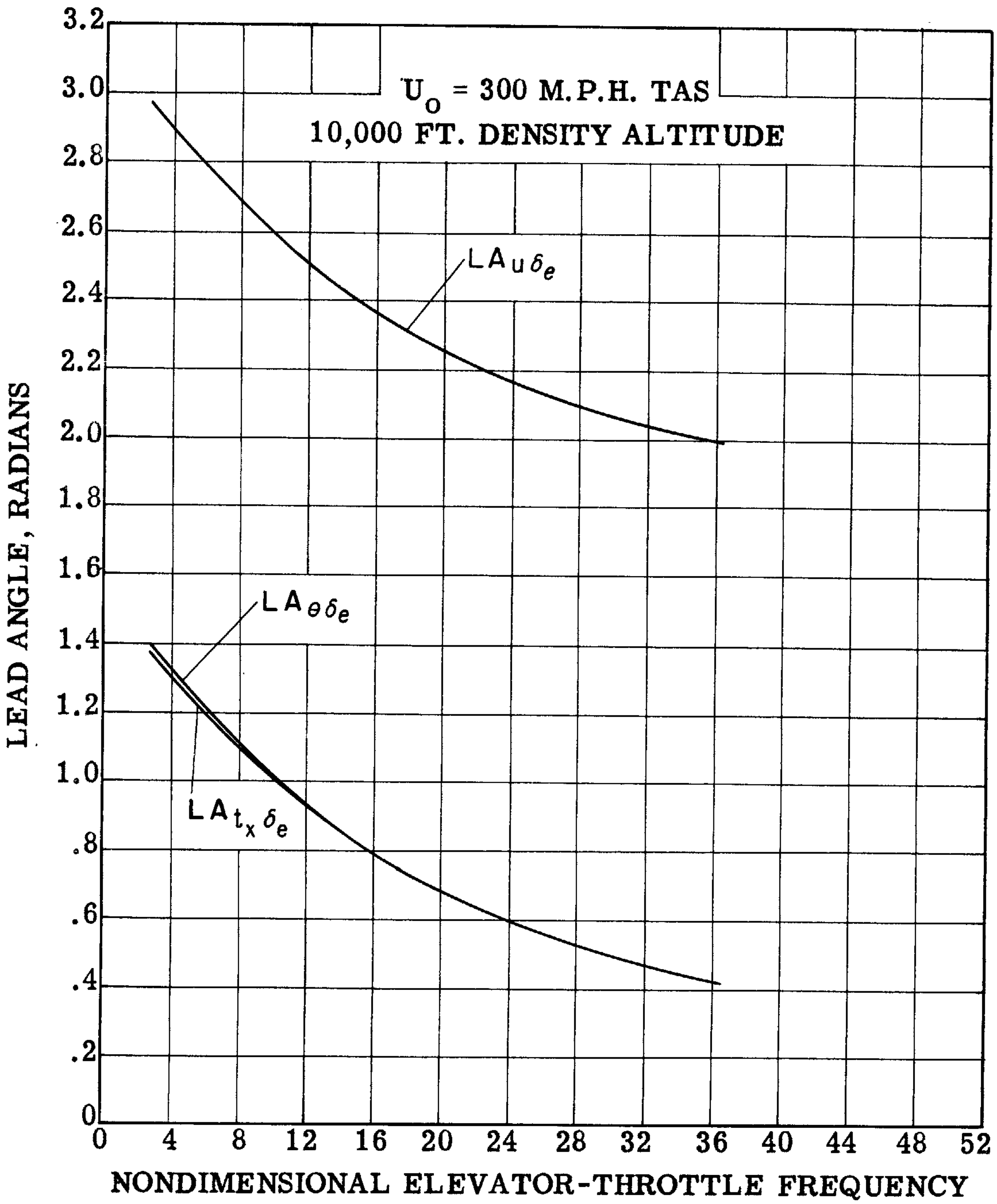
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY, ELEVATOR-THROTTLE FREQUENCY = .7 CYCLE PER SECOND

FIGURE R-5



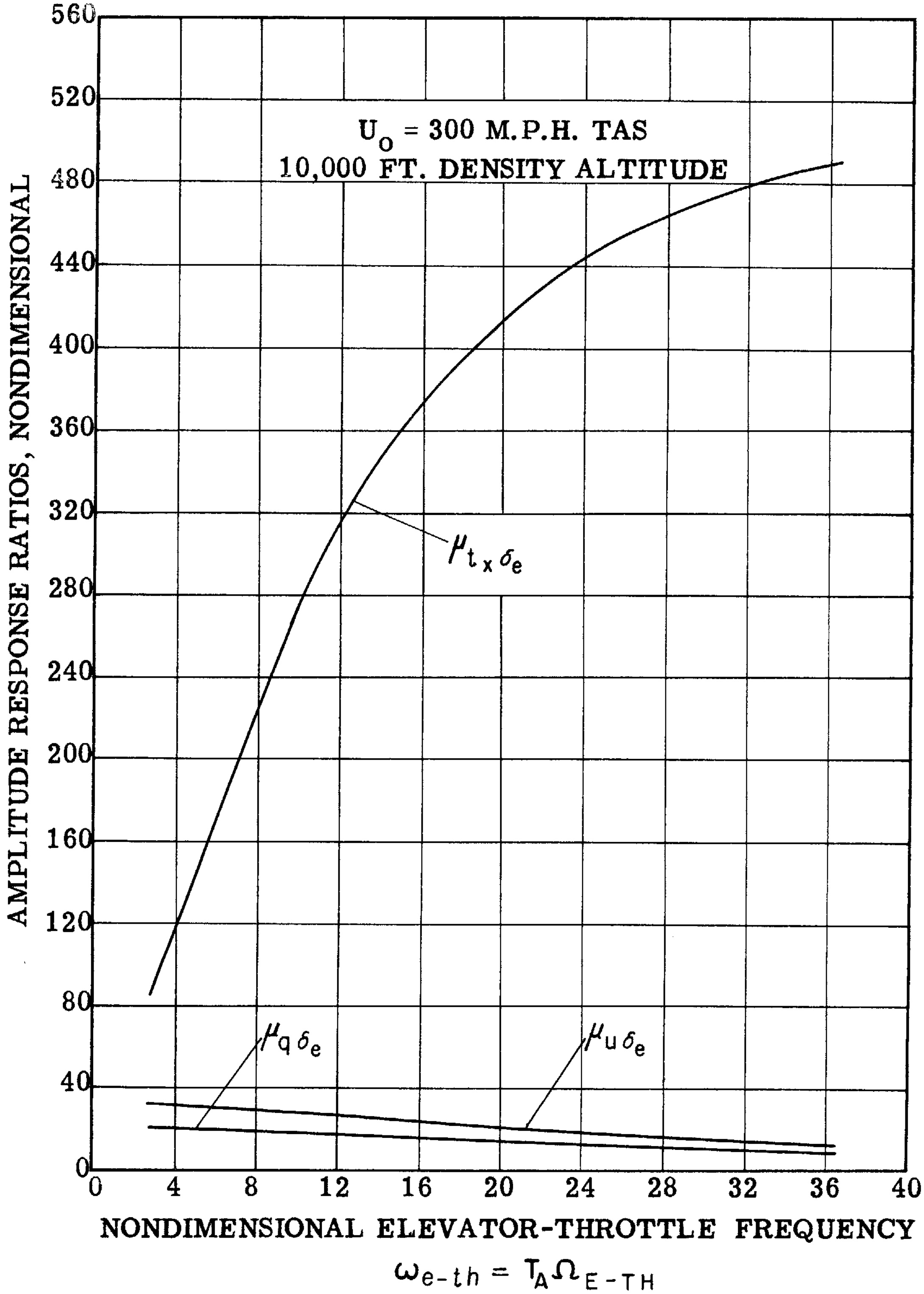
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN NORMAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR, AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY, ELEVATOR-THROTTLE FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE R-6



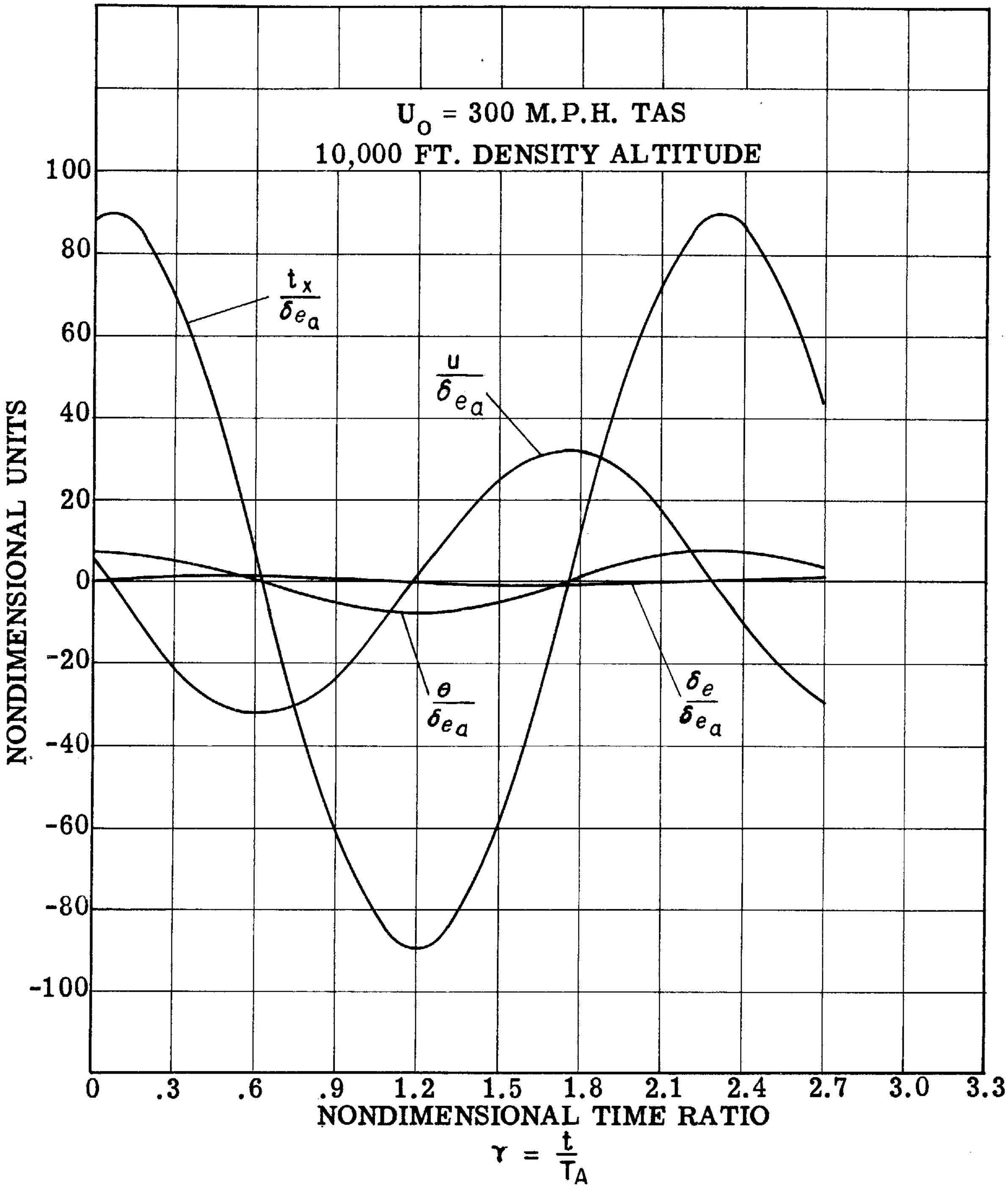
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN PITCH ANGLE AND CHANGE IN ELEVATOR ANGLE, AND THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL THRUST AND CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN ANGLE OF ATTACK

FIGURE R-7



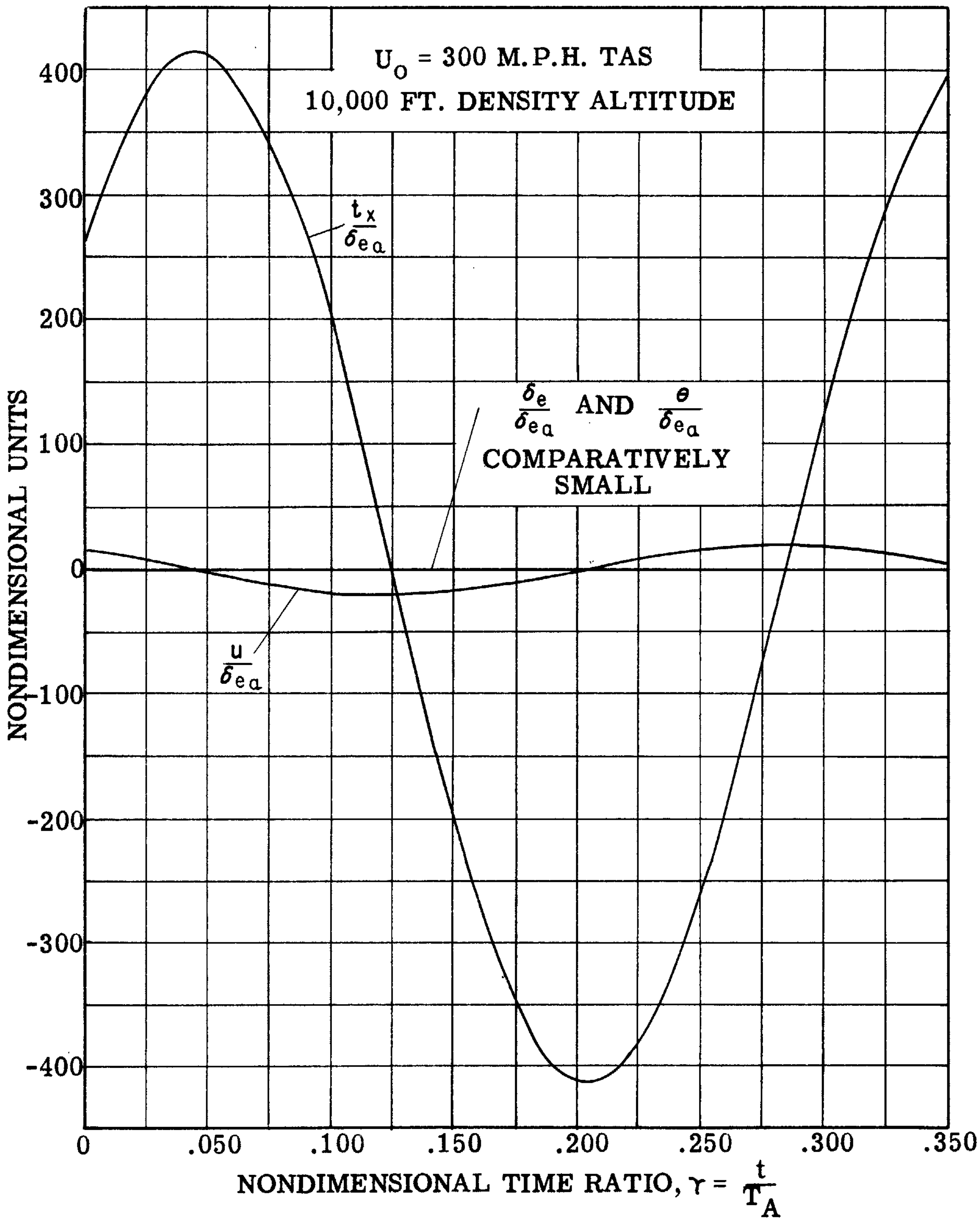
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN ANGLE OF ATTACK

FIGURE R-8



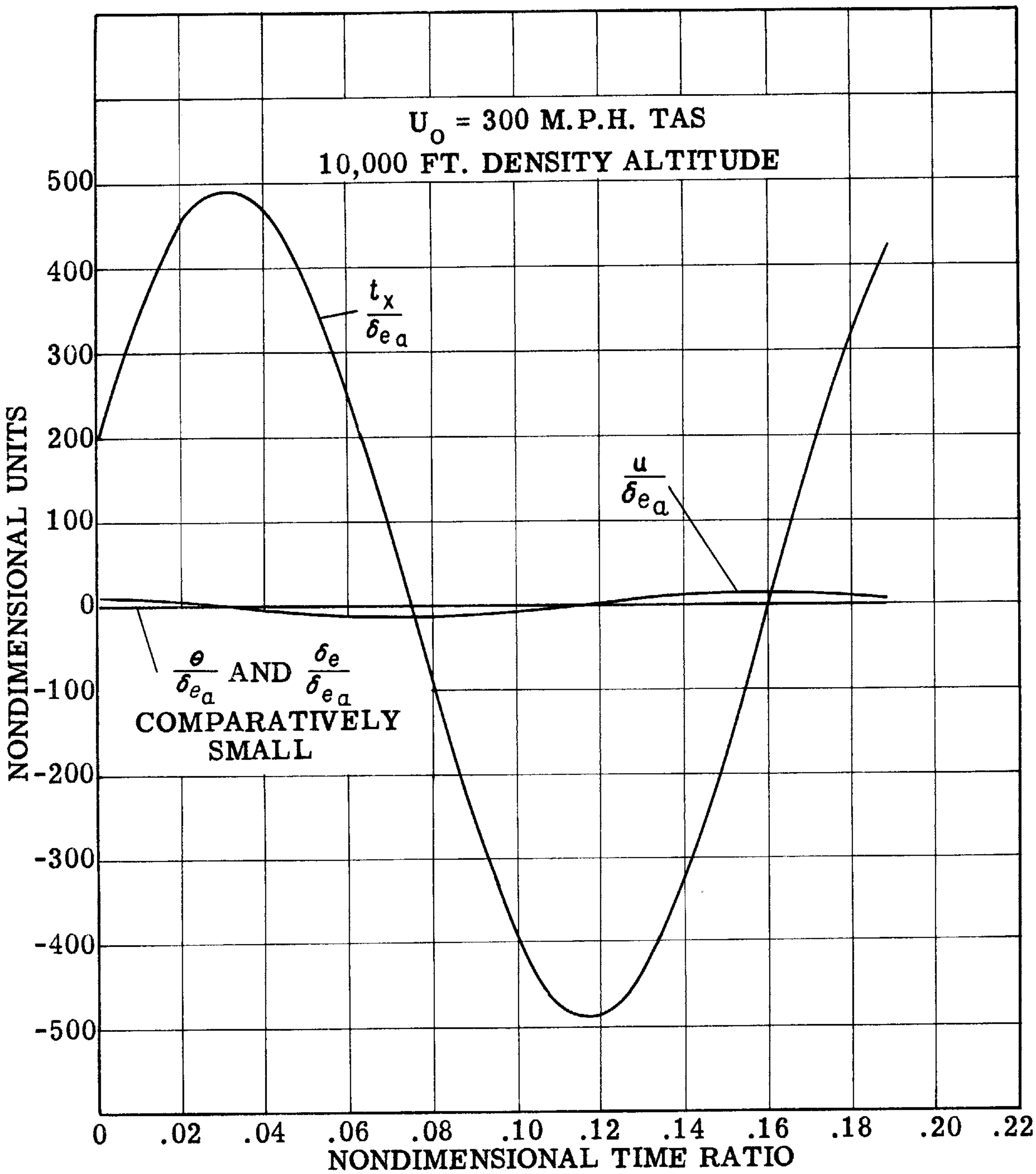
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN ANGLE OF ATTACK, ELEVATOR-THROTTLE FREQUENCY = .1 CYCLE PER SECOND

FIGURE R-9



TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN ANGLE OF ATTACK, ELEVATOR-THROTTLE FREQUENCY = .7 CYCLE PER SECOND

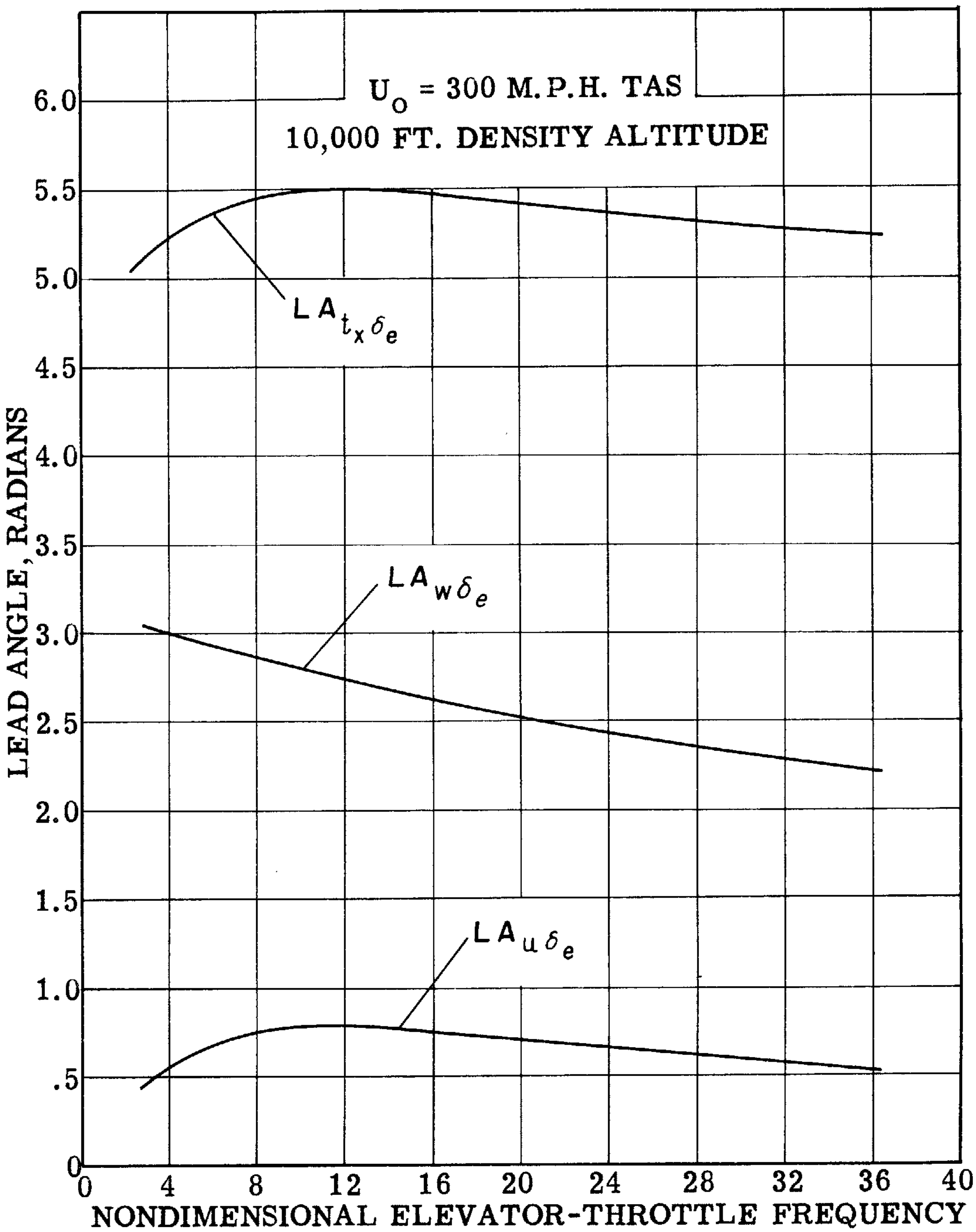
FIGURE R-10



$$\tau = \frac{t}{T_A}$$

TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN PITCH ANGLE, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN ANGLE OF ATTACK, ELEVATOR-THROTTLE FREQUENCY = 1.3 CYCLES PER SECOND

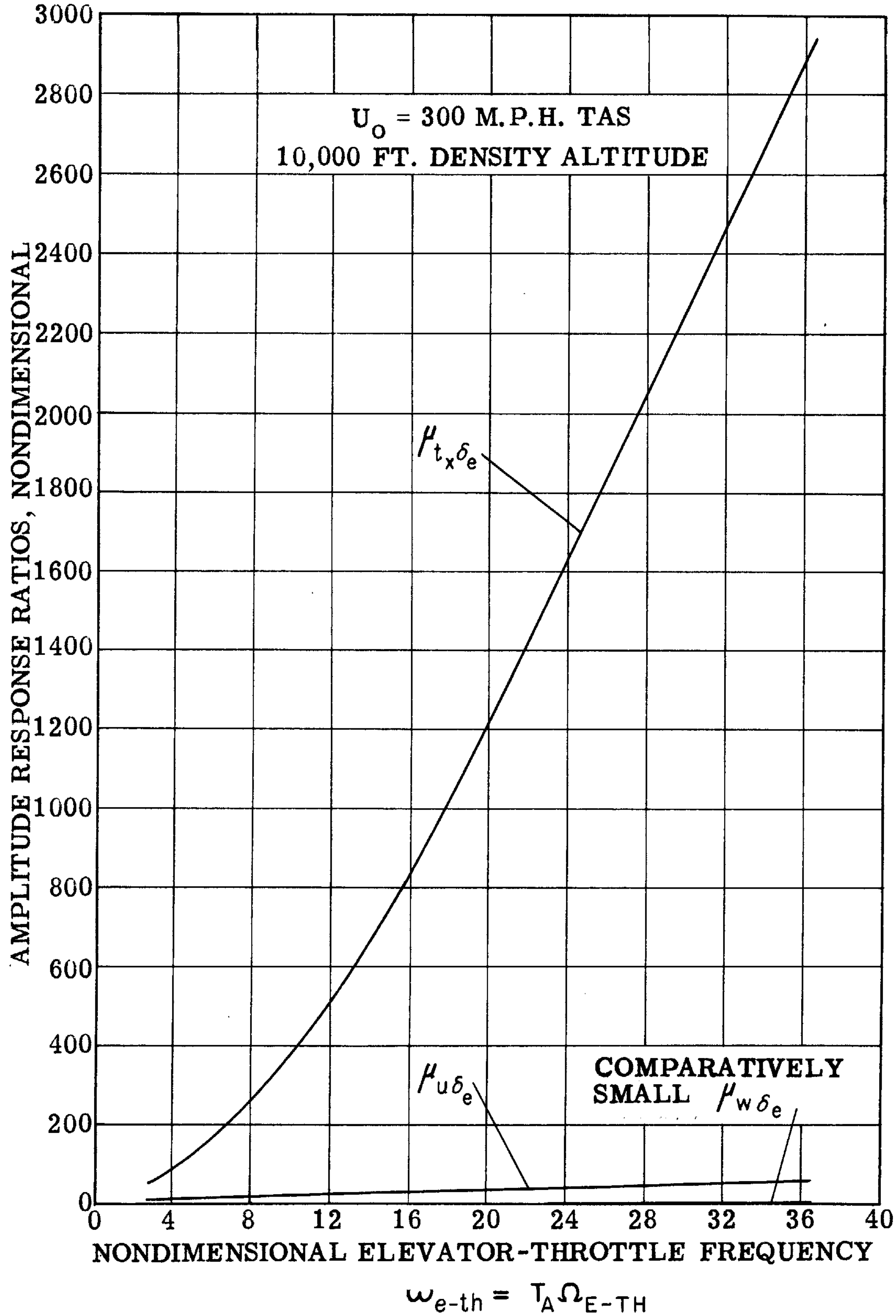
FIGURE R-11



$\omega_{e\text{-}th} = T_A \Omega_{E\text{-}TH}$

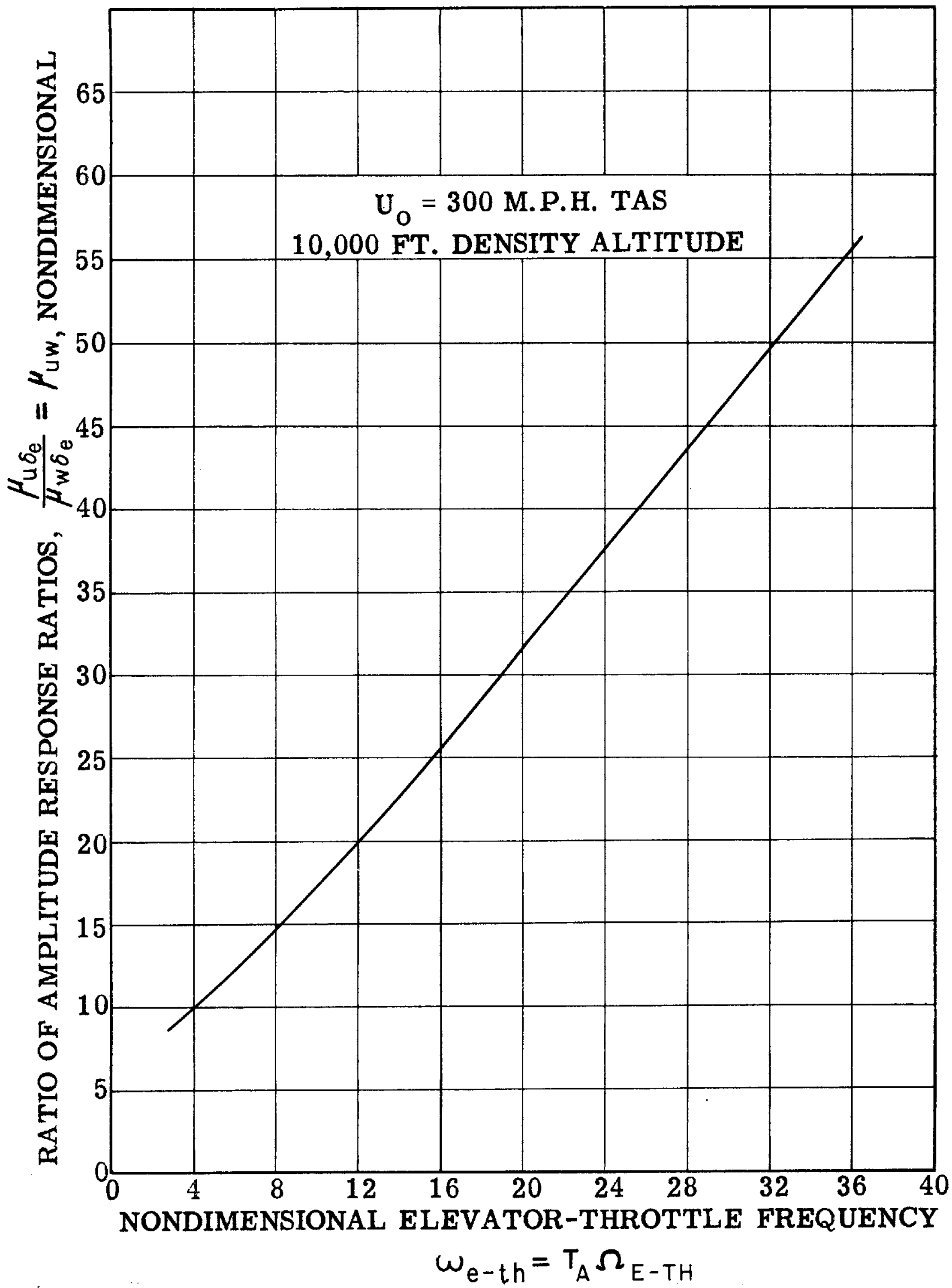
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED AND CHANGE IN ELEVATOR ANGLE, THE PHASE ANGLE BETWEEN CHANGE IN ANGLE OF ATTACK AND CHANGE IN ELEVATOR ANGLE , AND THE PHASE ANGLE BETWEEN CHANGE IN LONGITUDINAL THRUST AND CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE

FIGURE R-12



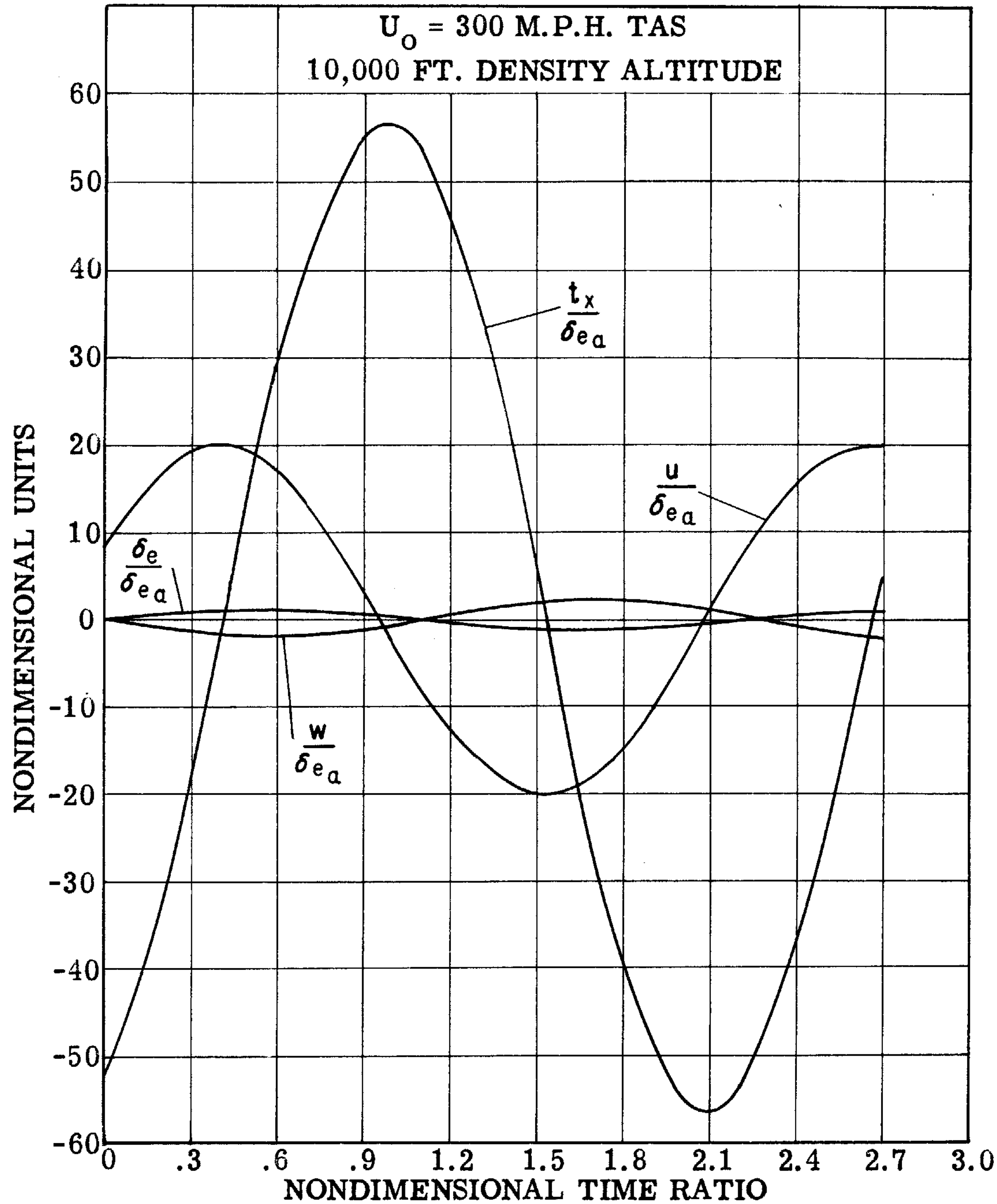
EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON AMPLITUDE RESPONSE RATIOS, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE

FIGURE R-13



EFFECT OF CHANGE IN ELEVATOR-THROTTLE FREQUENCY ON RATIO OF AMPLITUDE RESPONSE RATIO FOR CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED TO AMPLITUDE RESPONSE RATIO FOR CHANGE IN ANGLE OF ATTACK, COMPUTED FOR A-26 AIRPLANE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE

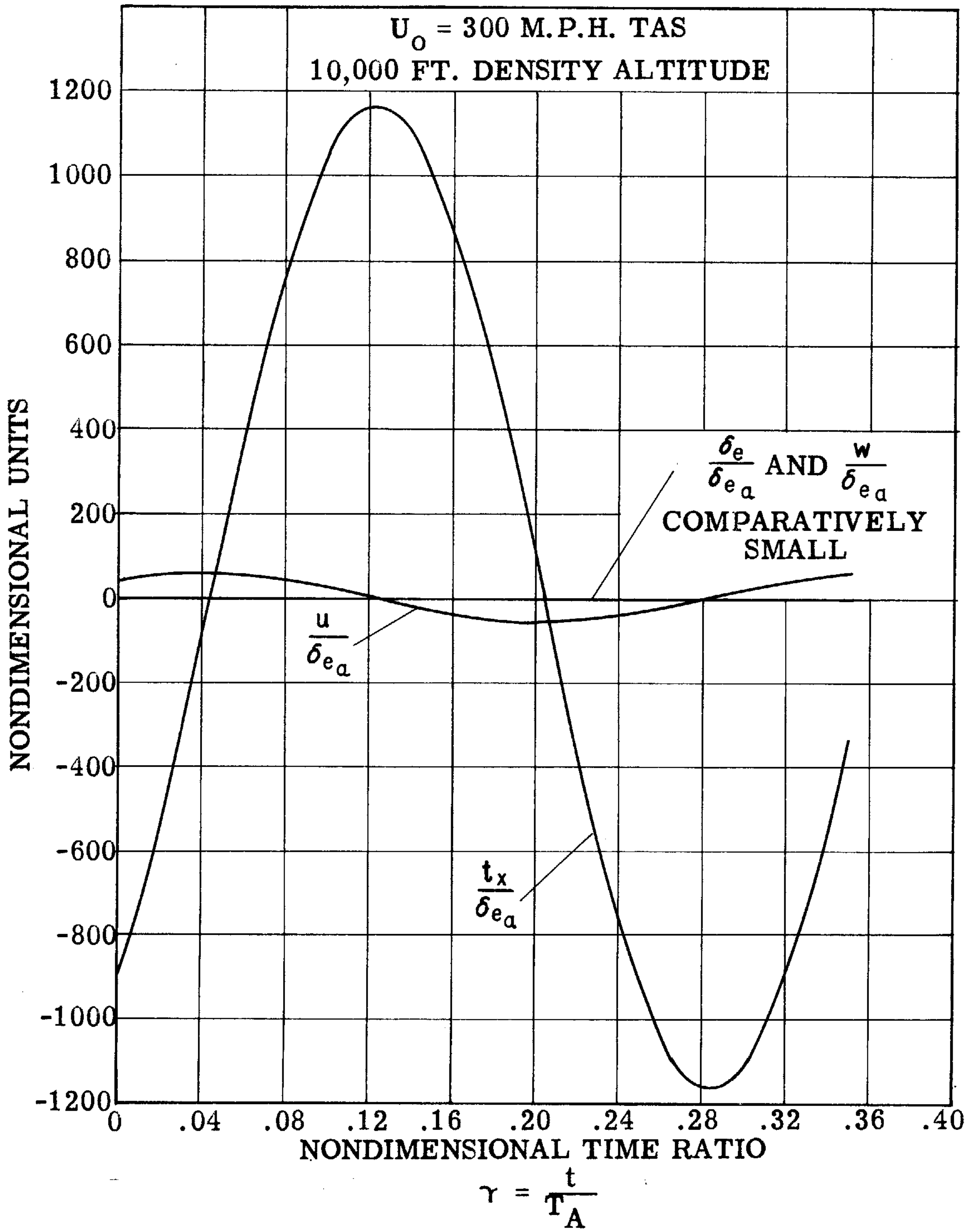
FIGURE R-14



$$\tau = \frac{t}{T_A}$$

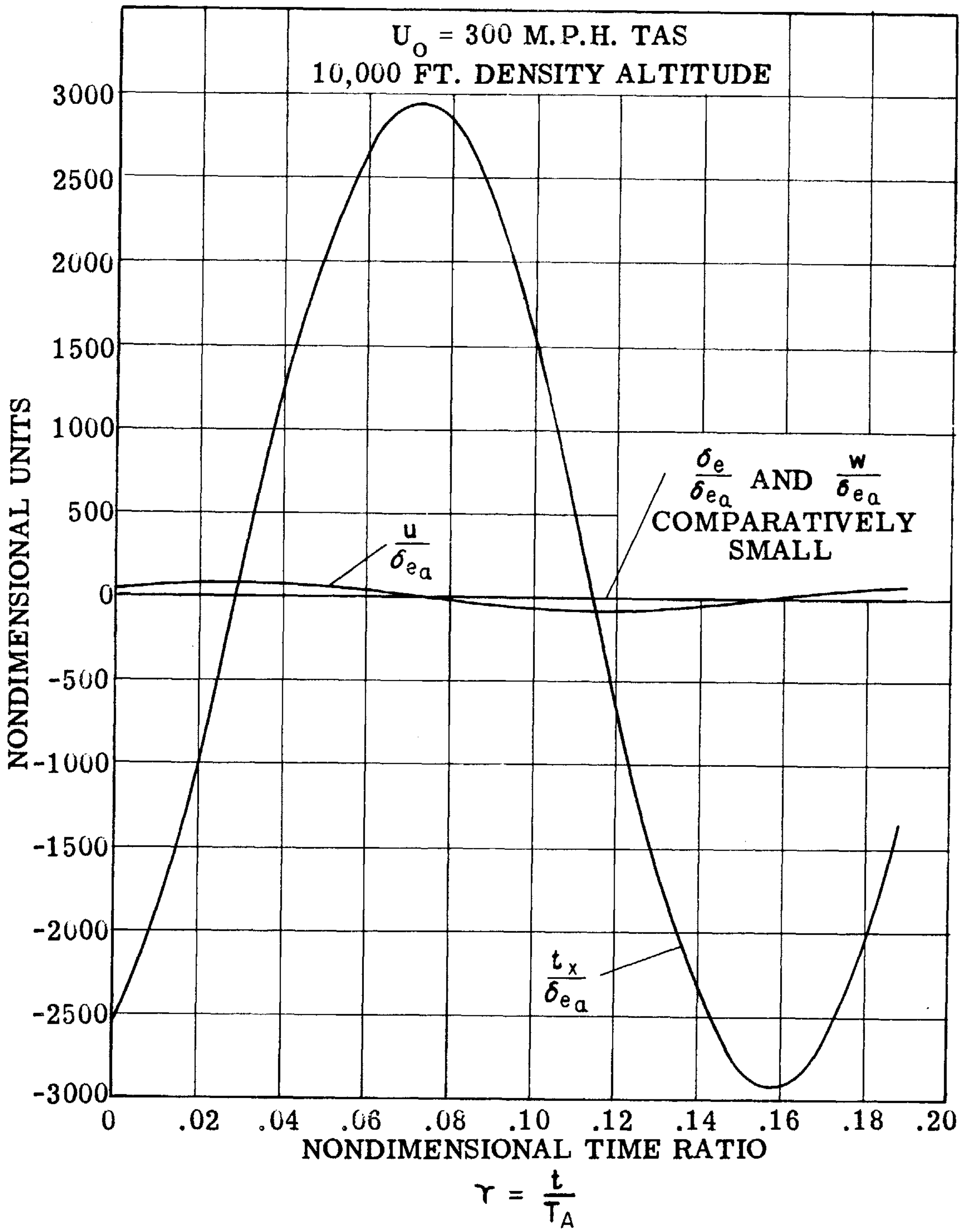
TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN ANGLE OF ATTACK, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE, ELEVATOR-THROTTLE FREQUENCY = .1 CYCLE PER SECOND

FIGURE R-15



TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN ANGLE OF ATTACK, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE, ELEVATOR-THROTTLE FREQUENCY = .7 CYCLE PER SECOND

FIGURE R-16



TIME HISTORY OF CHANGE IN ELEVATOR ANGLE, CHANGE IN ANGLE OF ATTACK, CHANGE IN LONGITUDINAL VELOCITY PER TRIM SPEED, AND CHANGE IN NONDIMENSIONAL LONGITUDINAL THRUST, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FOR COMPUTED RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF ELEVATOR AND THROTTLE, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE, ELEVATOR-THROTTLE FREQUENCY = 1.3 CYCLES PER SECOND

FIGURE R-17

TABLE R-1

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY.

$LA_{w\delta_e}$, $LA_{\theta\delta_e}$, $LA_{t_x\delta_e}$, COMPUTATIONS

U_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE

No.	Quantity							
1	Ω_{E-TH}	0.1	0.3	0.5	0.7	0.9	1.1	1.3
2	$\omega_{e-th} = 2\pi T_a$ ① $= 28.096$ ①	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
3	$\omega_{e-th}(-x_w - x_\theta)$ = ② [+.13967]	.39242	1.1773	1.9621	2.7469	3.5317	4.3166	5.1014
4	③/$[x_\theta z_w]$ = ③/+1.5919 = $\tan\psi_1$	.24651	.73956	1.2326	1.7255	2.2185	2.7116	3.2046
5	$-\omega_{e-th}/z_w$ = $-$②/[$-$4.8701] = $\tan\psi_3$	.57691	1.7307	2.8845	4.0383	5.1921	6.3461	7.4998
6	ω_{e-th}^2 = ②^2	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
7	$-$⑥ + L_2 = $-\omega_{e-th}^2 + L_2$	221.38	158.22	31.92	-157.52	-410.11	-725.91	-1104.8
8	⑦ ω_{e-th} = ⑦②	621.99	1333.6	448.41	-3097.9	-10370.	-22435	-40353.
9	$-$⑥ L_1 = $-\omega_{e-th}^2 L_1$	-210.02	-1890.2	-5250.5	-10291.	-17011.	-25413.	-35494.
10	⑨ $- L_3$ = $-\omega_{e-th}^2 L_1 - L_3$	-210.02	-1890.2	-5250.5	-10291.	-17011.	-25413.	-35494.
11	⑧/⑩ = $\tan\psi_4$	-2.9616	-.70553	-.085403	+.30103	.60961	.88282	1.1369
12	$\tan^{-1}$ ④ = ψ_1 degrees	13.848	36.485	50.948	59.906	65.737	69.757	72.669
13	" ⑤ = ψ_3 "	209.98	239.98	250.88	256.09	259.10	261.04	262.40
14	" ⑪ = ψ_4 "	108.66	144.80	175.12	196.75	211.37	221.44	228.67
15	.0174533 ⑫ = ψ_1 radians	.24169	.63678	.88921	1.0456	1.1473	1.2175	1.2683
16	" ⑬ = ψ_3 "	3.6648	4.1884	4.3787	4.4696	4.5222	4.5560	4.5797
17	" ⑭ = ψ_4 "	1.8965	2.5272	3.0564	3.4339	3.6891	3.8649	3.9910
18	$LA_{w\delta_e}$ = 4.71239 $-$ ⑰ = $\psi_2 - \psi_4$	2.8159	2.1852	1.6560	1.2785	1.0233	.84749	.72139
19	$LA_{\theta\delta_e}$ = ⑯ $-$ ⑰ = $\psi_3 - \psi_4$	1.7683	1.6612	1.3223	1.0357	.8331	.6911	.5887
20	$LA_{t_x\delta_e}$ = ⑮ $-$ ⑰ = $\psi_1 - \psi_4$	-1.6548	-1.8904	-2.1672	-2.3883	-2.5418	-2.6474	-2.7227
	$\mu_{w\delta_e}$, $\mu_{\theta\delta_e}$, $\mu_{t_x\delta_e}$, $\mu_{\theta w}$							
21	$(-\omega_{e-th} L_1)^2$ = $-$⑨ L_1	5587.6	50289.	13969 x10^{+1}	27379 x10^{+1}	45258. x10^{+1}	67611. x10^{+1}	94432. x10^{+1}
22	$(-\omega_{e-th}^2 + L_2)^2$ = ⑦^2	49009.	25034.	1012.5	24813.	16819. x10^{+1}	52695. x10^{+1}	12206. x10^{+2}
23	㉑ + ㉒	54597.	75323.	14070. x10^{+1}	29860. x10^{+1}	62077. x10^{+1}	12031. x10^{+2}	21649. x10^{+2}
24	$\sqrt{㉓}$	233.66	274.45	375.10	546.44	787.89	1096.9	1471.4
25	$-m_{\delta_e}$/㉔ = $\mu_{w\delta_e}$	1.5062	1.2824	.93828	.64408	.44670	.32086	.23919
26	z_w^2 + ⑥ = 23.718 + ⑥	31.612	94.763	221.07	410.51	663.10	978.90	1357.8
27	$\sqrt{㉖}$	5.6225	9.7346	14.868	20.261	25.757	31.287	36.848
28	㉕㉗/② = $\mu_{\theta\delta_e}$	3.0142	1.4811	.99302	.66355	.45503	.32482	.24131
29	③^2	.15399	1.3860	3.8498	7.5455	12.473	18.633	26.024
30	㉙ + $(x_\theta z_w)^2$ = ㉙ + 1.5919	1.7459	2.9779	5.4417	9.1374	14.065	20.225	27.616
31	$\sqrt{㉚}$	1.3213	1.7257	2.3327	3.0228	3.7503	4.4972	5.2551
32	㉛㉕/② = $\mu_{t_x\delta_e}$	.70832	.26255	.15580	.098993	.066254	.04669	.034415
33	㉘/㉕ = $\mu_{\theta\delta_e}/\mu_{w\delta_e}$ = $\mu_{\theta w}$	2.0012	1.1549	1.0583	1.0302	1.0186	1.0123	1.0089
34								
35								

m_{δ_e} = $-$351.95

x_w = .18720

x_θ = $-$.32687

z_w = $-$4.8701

z_θ = 0

L_1 = 26.605

L_2 = 229.27

L_3 = 0

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY.

TABLE R-2

$\omega_{e-th} = 2.8096$

$\Omega_{e-th} = .1$ CYCLE/SEC.

Time Histories of w/δ_{ea}, θ/δ_{ea}, $\tau x/\delta_{ea}$ and δ_e/δ_{ea} — $U_0 = 300$ mph TAS, 10,000 FT DENSITY ALTITUDE

①	t	0	1	2	3	4	5	6	7	8	9	10	11	12
②	$\gamma = t/T_A$ = ①/4.4716	0	.22363	.44727	.67090	.89453	1.1182	1.3418	1.5654	1.7891	2.0127	2.2363	2.4600	2.6836
③	$\omega_{e-th}\gamma$ = 2.8096 × ②	0	.62831	1.2566	1.8850	2.5133	3.1417	3.7699	4.3981	5.0267	5.6549	6.2831	6.9116	7.5398
④	$\omega_{e-th}\gamma + LA_{w\delta_e}$ = ③ + 2.8159	2.8159	3.4442	4.0725	4.7009	5.3292	5.9576	6.5858	7.2140	7.8426	8.4708	9.0990	9.7275	10.356
⑤	$\sin(\omega_{e-th}\gamma + LA_{w\delta_e})$ = sin ④	.31995	-.29804	-.80219	-.99993	-.81573	-.31979	+.29804	.80209	.99993	.81573	.32012	-.29804	-.80230
⑥	$w/\delta_{ea} = \mu_{w\delta_e}\sin$ ④ = 1.5062 × ⑤	.48191	-.44891	-1.2083	-1.5061	-1.2287	-.48167	+.44891	1.2081	1.5061	1.2287	.48216	-.44891	-1.2084
⑦	$\omega_{e-th}\gamma + LA_{\theta\delta_e}$ = ③ + 1.7683	1.7683	2.3966	3.0249	3.6533	4.2816	4.9100	5.5382	6.1664	6.7950	7.4232	8.0514	8.6799	9.3081
⑧	$\sin(\omega_{e-th}\gamma + LA_{\theta\delta_e})$ = sin ⑦	.98055	.67790	.11650	-.48969	-.90865	-.98055	-.67790	-.11650	+.48969	.90865	.98058	.67790	.11650
⑨	$\theta/\delta_{ea} = \mu_{\theta\delta_e}\sin$ ⑦ = 3.0142 × ⑧	2.9556	2.0433	.35115	-1.4760	-2.7389	-2.9556	-2.0433	-.35115	+1.4760	2.7389	2.9557	2.0433	.35115
⑩	$\omega_{e-th}\gamma + LA_{\tau x\delta_e}$ = ③ − 1.6548	-1.6548	-1.0265	-.3982	+.2302	.8585	1.4869	2.1151	2.7433	3.3719	4.0001	4.6283	5.2568	5.8850
⑪	$\sin(\omega_{e-th}\gamma + LA_{\tau x\delta_e})$ = sin ⑩	-.99648	-.85549	-.38776	.22816	.75686	.99648	.85545	.38784	-.22835	-.75688	-.99646	-.85545	-.38768
⑫	$\tau x/\delta_{ea} = \mu_{\tau x\delta_e}\sin$ ⑩ = .70832 × ⑪	-.70583	-.60596	-.27466	.16161	.53610	.70583	.60593	.27471	-.16174	-.53611	-.70581	-.60593	-.27460
⑬	$\delta_e/\delta_{ea} = \sin\omega_{e-th}\gamma$ = sin ③	0	.58777	.95105	.95106	.58779	-.0001745	-.58779	-.95100	-.95100	-.58779	-.0001745	+.58793	.95106

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY.

TABLE R-3

$\omega_{e-th} = 19.667$

$\Omega_{E-TH} = .7$ cycle/sec.

Time Histories of w/δ_{ea}, θ/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea}

V_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE

①	t	0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
②	$\gamma = t/T_1$ ①/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③	$\omega_{e-th}\gamma$ = 19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④	$\omega_{e-th}\gamma + \lambda_{Avge}$ = ③ + 1.2785	1.2785	1.8503	2.4220	2.9938	3.5656	4.1373	4.7090	5.2809	5.8527	6.4244	6.9961	7.5680	8.1397
⑤	$\sin(\omega_{e-th}\gamma + \lambda_{Avge})$ sin ④	.95759	.96121	.65908	.14729	−.41136	−.83915	−.99999	−.84273	−.41739	+.14073	.65408	.95936	.95946
⑥	$w/\delta_{ea} = \mu_{w\delta a} \sin$ ④ = .64408 × ⑤	.61676	.61910	.42450	.094867	−.26495	−.54048	−.64407	−.54279	−.26883	+.090641	.42128	.61790	.61797
⑦	$\omega_{e-th}\gamma + \lambda_{\theta avge}$ ③ + 1.0357	1.0357	1.6075	2.1792	2.7510	3.3228	3.8945	4.4662	5.0381	5.6099	6.1816	6.7533	7.3252	7.8969
⑧	$\sin(\omega_{e-th}\gamma + \lambda_{\theta avge})$ sin ⑦	.86022	.99933	.82055	.38075	−.18018	−.68378	−.96983	−.94743	−.62361	−.10140	+.45306	.86340	.99908
⑨	$\theta/\delta_{ea} = \mu_{\theta\delta a} \sin$ ⑦ = .66355 × ⑧	.57080	.66311	.54448	.25265	−.11956	−.45372	−.64353	−.62867	−.41380	−.067284	+.30063	.57291	.66294
⑩	$\omega_{e-th}\gamma + \lambda_{tx\,avge}$ = ③ − 2.3883	−2.3883	−1.8165	−1.2448	−.6730	−.1012	+.4705	1.0422	1.6141	2.1859	2.7576	3.3293	3.9012	4.4729
⑪	$\sin(\omega_{e-th}\gamma + \lambda_{tx\,avge})$ = sin ⑩	−.68404	−.96996	−.94733	−.62333	−.10102	+.45334	.86352	.99906	.81674	.37461	−.18652	−.68861	−.97147
⑫	$t_x/\delta_{ea} = \mu_{tx\delta a} \sin$ ⑩ .098993 × ⑪	−.067715	−.096019	−.093779	−.061705	−.010000	+.044877	.085482	.098900	.080852	.037084	−.018464	−.068168	−.096169
⑬	$\delta_e/\delta_{ea} = \sin \omega_{e-th}\gamma$ = sin ③	0	.54111	.91009	.98958	.75425	.27899	−.28485	−.75836	−.99046	−.90748	−.53597	+.0062831	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR,
ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY.

TABLE R-4

$\omega_{e-th} = 36.525$

$\Omega_{E-TH} = 1.3$ CYCLES/SEC.

Time Histories of w/δ_{ea}, θ/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea} $U_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

①	t		0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
②	$\gamma = t/T_1$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③	$\omega_{e-th}\gamma$	36.525 × ②	0	.67176	1.1436	1.7153	2.2871	2.8589	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④	$\omega_{e-th}\gamma + \lambda_{w\delta e}$	③ + .72139	.72139	1.2932	1.8650	2.4367	3.0085	3.5803	4.1520	4.7238	5.2954	5.8674	6.4390	7.0110	7.5826
⑤	$\sin(\omega_{e-th}\gamma + \lambda_{w\delta e})$	sin ④	.66043	.96172	.95702	.64799	.13278	-.42483	-.84703	-.99993	-.83485	-.40386	+.15523	.66523	.96340
⑥	$w/\delta_{ea} = \mu_{w\delta e} \sin$ ④ =	.23919 × ⑤	.15797	.23003	.22891	.15499	.031760	-.10162	-.20260	-.23917	-.19969	-.096599	+.037129	.15912	.23044
⑦	$\omega_{e-th}\gamma + \lambda_{\theta\delta e}$	③ + .5887	.5887	1.1605	1.7323	2.3040	2.8758	3.4476	4.0193	4.5911	5.1627	5.7347	6.3063	6.8783	7.4499
⑧	$\sin(\omega_{e-th}\gamma + \lambda_{\theta\delta e})$	sin ⑦	.55528	.91700	.98700	.74303	.26269	-.30121	-.76929	-.99265	-.90032	-.52146	+.023106	.56064	.91948
⑨	$\theta/\delta_{ea} = \mu_{\theta\delta e} \sin$ ⑦ =	.24131 × ⑧	.13399	.22128	.23817	.17930	.063390	-.072685	-.18564	-.23954	-.21726	-.12583	+.0055757	.13529	.22188
⑩	$\omega_{e-th}\gamma + \lambda_{t_x\delta e}$	③ - 2.7227	-2.7227	-2.1509	-1.5791	-1.0074	-.4356	+.1362	.7079	1.2797	1.8513	2.4233	2.9949	3.5669	4.1385
⑪	$\sin(\omega_{e-th}\gamma + \lambda_{t_x\delta e})$	sin ⑩	-.40674	-.83638	-.99996	-.84544	-.42195	+.13578	.65024	.95793	.96092	.65816	.14608	-.41263	-.83981
⑫	$t_x/\delta_{ea} = \mu_{t_x\delta e} \sin$ ⑩	.034415 × ⑪	-.013998	-.02878	-.034414	-.029096	-.014521	+.0046729	.022378	.032967	.033070	.022651	.0050273	-.014201	-.02890
⑬	$\delta_e/\delta_{ea} = \sin \omega_{e-th}\gamma$ =	sin ③	0	.54111	.91013	.98958	.75425	.27899	-.28502	-.75836	-.99044	-.90748	-.53597	+.10626	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK).

TABLE R-6.

V_0 = 300 mph TAS, 10,000 FT DENSITY ALTITUDE

$\angle A_{u\delta_e}$, $\angle A_{\theta\delta_e}$, $\angle A_{t_x\delta_e}$

①	Ω_{E-TH}	0.1	0.3	0.5	0.7	0.9	1.1	1.3
②	$\omega_{e-th} = 2\pi T_a$ ① = 28.096 ①	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525
③	②$^2 = \omega_{e-th}^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1
④	$L_\delta \omega_{e-th} = L_\delta$ ②	.24517	.73551	1.2258	1.7162	2.2065	2.6969	3.1872
⑤	L_y − ③	-7.6802	-70.831	-197.14	-386.58	-639.17	-954.97	-1333.9
⑥	④/⑤ = $\tan \varphi_1$	-.031922	-.010384	-.0062179	-.0044394	-.0034521	-.0028241	-.0023894
⑦	$z_u m_q \omega_{e-th}$ = ② [10.700]	30.063	90.188	150.31	210.44	270.56	330.69	390.82
⑧	$z_u \omega_{e-th}^2 = -.65374$ ③	-5.1606	-46.445	-129.02	-252.86	-417.99	-624.44	-872.15
⑨	⑦/⑧ = $\tan \varphi_3$	-5.8255	-1.9418	-1.1650	-.83224	-.64729	-.52958	-.44811
⑩	$\tan^{-1}$ ⑥ = φ_1 degrees	178.17	179.40	179.64	179.75	179.80	179.84	179.86
⑪	$\tan^{-1}$ ⑨ = φ_3 "	99.74	117.25	130.64	140.23	147.08	152.10	155.86
⑫	.0174533 ⑩ = φ_1 radians	3.1097	3.1311	3.1353	3.1372	3.1381	3.1388	3.1392
⑬	" ⑪ = φ_3 "	1.7408	2.0464	2.2801	2.4475	2.5670	2.6546	2.7203
⑭	$\angle A_{u\delta_e}$ = 4.71239 − ⑬ = $\varphi_2 - \varphi_3$	2.9716	2.6660	2.4323	2.2649	2.1454	2.0578	1.9921
⑮	$\angle A_{\theta\delta_e}$ = −⑬ + 3.1416 = $-\varphi_3 + \pi$	1.4008	1.0952	.8615	.6941	.5746	.4870	.4213
⑯	$\angle A_{t_x\delta_e}$ = ⑫ − ⑬ = $\varphi_1 - \varphi_3$	1.3689	1.0847	.8552	.6897	.5711	.4842	.4189

L_y = .21369
L_δ = .087261
z_u = −.65374
m_q = −16.3665
m_{δ_e} = −351.95

$\mu_{q\delta_e}$, $\mu_{u\delta_e}$, $\mu_{t_x\delta_e}$, μ_{qu}, $\mu_{\theta\delta_e}$

⑰	$\omega_e^2 z_u + m_q^2$ = ③ + 267.87	275.76	338.92	465.22	654.66	907.25	1223.0	1602.
⑱	$\sqrt{⑰}$	16.606	18.410	21.569	25.586	30.121	34.971	40.025
⑲	$-m_{\delta_e}$/⑱ = 351.95/⑱ = $\mu_{q\delta_e}$	21.194	19.117	16.317	13.756	11.685	10.064	8.7953
⑳	−⑲/z_u = ⑲/(.65374) = $\mu_{u\delta_e}$	32.420	29.243	24.959	21.042	17.874	15.394	13.451
㉑	⑤^2 = $(L_y - \omega_{e-th}^2)^2$	58.985	5017.0	38864.	14944. x10^{+1}	40854. x10^{+1}	91197. x10^{+1}	17793. x10^{+2}
㉒	$\omega_{e-th}^2 L_\delta^2$ = ③ [.0076145]	.060108	54097	1.5027	2.9452	4.8686	7.2732	10.159
㉓	㉑ + ㉒	59.045	5017.5	38866.	14944. x10^{+1}	40854. x10^{+1}	91198. x10^{+1}	17793. x10^{+2}
㉔	$\sqrt{㉓}$	7.6841	70.834	197.14	386.58	639.17	954.98	1333.9
㉕	㉔ ⑳/② = $\mu_{t_x\delta_e}$	88.667	245.75	350.26	413.61	451.83	475.67	491.23
㉖	$-z_u$ = μ_{qu} = $\mu_{q\delta_e}/\mu_{u\delta_e}$	65374	.65374	.65374	.65374	.65374	.65374	.65374
㉗	⑲/② = $\mu_{\theta\delta_e}$	7.5434	2.2681	1.1615	.69945	.46211	.32563	.24075

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK.)

TABLE R-6

$\omega_{E-TH} = 2.8096$

$\Omega_{E-TH} = .1$ cycle/sec.

Time Histories of u/δ_{Ea}, θ/δ_{Ea}, t_x/δ_{Ea}, δ_e/δ_{Ea}

$U_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

①	t	
②	$\gamma = t/T_R$	①/4.4716
③	$\omega_{E-TH}\gamma$ (radians)	2.8096 × ②
④	$\omega_{E-TH}\gamma + \lambda_{Au\delta_E}$	③ + 2.9716
⑤	$\sin(\omega_{E-TH}\gamma + \lambda_{Au\delta_E})$	sin ④
⑥	u/δ_{Ea} =	32.420 × ⑤
⑦	$\omega_{E-TH}\gamma + \lambda_{A\theta\delta_E}$	③ + 1.4008
⑧	$\sin(\omega_{E-TH}\gamma + \lambda_{A\theta\delta_E})$	sin ⑦
⑨	θ/δ_{Ea}	7.5434 × ⑧
⑩	$\omega_{E-TH}\gamma + \lambda_{Ax\delta_E}$	③ + 1.3689
⑪	$\sin(\omega_{E-TH}\gamma + \lambda_{Ax\delta_E})$	sin ⑩
⑫	t_x/δ_{Ea}	88.667 × ⑪
⑬	$\delta_e/\delta_{Ea} = \sin \omega_{E-TH}\gamma$ =	sin ③

Row	0	1	2	3	4	5	6	7	8	9	10	11	12
①	0	1	2	3	4	5	6	7	8	9	10	11	12
②	0	.22363	.44727	.67090	.89453	1.1182	1.3418	1.5654	1.7891	2.0127	2.2363	2.4600	2.6836
③	0	.62831	1.2566	1.8850	2.5133	3.1417	3.7699	4.3981	5.0267	5.6549	6.2831	6.9116	7.5398
④	2.9716	3.5999	4.2282	4.8566	5.4849	6.1133	6.7415	7.3697	7.9983	8.6265	9.2547	9.8832	10.511
⑤	.16918	-.44245	-.88507	-.98963	-.71618	-.16901	+.44245	.88499	.98960	.71618	.16918	-.44260	-.88491
⑥	5.4848	-14.344	-28.694	-32.084	-23.219	-5.4793	+14.344	28.691	32.083	23.219	5.4848	-14.349	-28.689
⑦	1.4008	2.0291	2.6574	3.2858	3.9141	4.5425	5.1707	5.7989	6.4275	7.0557	7.6839	8.3124	8.9406
⑧	.98558	.89680	.46546	-.14367	-.69792	-.98562	-.89680	-.46561	+.14384	.69792	.98556	.89672	.46546
⑨	7.4346	6.7649	3.5112	-1.0838	-5.2647	-7.4349	-6.7649	-3.5123	+1.0850	5.2647	7.4345	6.7643	3.5112
⑩	1.3689	1.9972	2.6255	3.2539	3.8822	4.5106	5.1388	5.7670	6.3956	7.0238	7.6520	8.2805	8.9087
⑪	.97969	.91047	.49349	-.11199	-.67469	-.97972	-.91047	-.49364	+.11216	.67469	.97968	.91040	.49349
⑫	86.866	80.729	43.756	-9.9298	-59.823	-86.869	-80.729	-43.770	+9.9449	59.823	86.865	80.722	43.756
⑬	0	.58777	.95105	.95106	.58779	-.0001745	-.58779	-.95100	-.95100	-.58779	-.0001745	+.58793	.95106

$\omega_{e-th} = 19.667$

$\Omega_{E-TH} = .7$ CYCLE/SEC.

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK)

TABLE R-7

Time Histories of u/δ_{ea}, θ/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea}

U_0 = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE

①	t		0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
②	$\gamma = t/T_a$	①/4.4716	0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③	$\omega_{e-th}\gamma$ (radians)	19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④	$\omega_{e-th}\gamma + hA_{u\delta e}$	③ + 2.2649	2.2649	2.8367	3.4084	3.9802	4.5520	5.1237	5.6954	6.2673	6.8391	7.4108	7.9825	8.5544	9.1261
⑤	$\sin(\omega_{e-th}\gamma + hA_{u\delta e})$	sin ④	.76862	.30021	-.26378	-.74373	-.98716	-.91657	-.55455	-.015882	+.52770	.90341	.99176	.76458	.29421
⑥	u/δ_{ea} =	21.042 × ⑤	16.173	6.3170	-5.5488	-15.650	-20.772	-19.286	-11.669	-.33419	+11.104	19.010	20.869	16.088	6.1908
⑦	$\omega_{e-th}\gamma + hA_{\theta\delta e}$	③ + .6941	.6941	1.2659	1.8376	2.4094	2.9812	3.5529	4.1246	4.6965	5.2683	5.8400	6.4117	6.9836	7.5553
⑧	$\sin(\omega_{e-th}\gamma + hA_{\theta\delta e})$	sin ⑦	63969	.95388	.96460	.66848	.15971	-.39987	-.83215	-.99987	-.84943	-.42878	+.12810	.64452	.95574
⑨	θ/δ_{ea}	.69945 × ⑧	.44743	.66719	.67469	.46757	.11171	-.27969	-.58205	-.69936	-.59414	-.29992	+.089600	.45081	.66849
⑩	$\omega_{e-th}\gamma + hA_{tx\delta e}$	③ + .6897	.6897	1.2615	1.8332	2.4050	2.9768	3.5485	4.1202	4.6921	5.2639	5.8356	6.4073	6.9792	7.5509
⑪	$\sin(\omega_{e-th}\gamma + hA_{tx\delta e})$	sin ⑩	.63631	.95255	.96579	.67172	.16401	-.39571	-.82970	-.99979	-.85173	-.43272	+.12377	.64118	.95440
⑫	t_x/δ_{ea} =	41361 × ⑪	263.18	393.98	399.46	277.83	67.836	-163.67	-343.18	-413.52	-352.28	-178.98	+51.193	265.20	394.75
⑬	$\delta_e/\delta_{ea} = \sin \omega_{e-th}\gamma$ =	sin ③	0	.54111	.91009	.98958	.75425	.27899	-.28485	-.75836	-.99846	-.90748	-.53597	+.0062831	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE AN ANGLE OF ATTACK)

TABLE R-8

$\omega_{e-th} = 36.525$

Ω_{E-TH}: 1.3 CYCLES/SEC.

Time Histories of u/δ_{ea}, θ/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea}

V_0 = 300 mph TAS, 10,000 FT, DENSITY ALTITUDE

①	t		0	.07	.14	.21	.28	.35	.42	.49	.56	.63	.70	.77	.84
②	$\gamma = t/T_A$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272	.095420	.10958	.12523	.14089	.15654	.17220	.18785
③	$\omega_{e-th}\gamma$ (radians)	36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589	3.4852	4.0024	4.5740	5.1464	5.7176	6.2896	6.8612
④	$\omega_{e-th}\gamma + hA_{uge}$	③ + 1.9921	1.9921	2.5639	3.1357	3.7074	4.2792	4.8510	5.4773	5.9945	6.5661	7.1385	7.7097	8.2817	8.8533
⑤	$\sin(\omega_{e-th}\gamma + hA_{uge})$	sin ④	.91255	.54610	.0059341	-.53612	-.90763	-.99041	-.72140	-.28468	+.27916	.75482	.98960	.90989	.54083
⑥	u/δ_{ea}	13.451 × ⑤	12.275	7.3456	.079820	-7.2114	-12.209	-13.322	-9.7036	-3.8292	+3.7550	10.153	13.311	12.239	7.2747
⑦	$\omega_{e-th}\gamma + hA_{\theta ge}$	③ + .4213	.4213	.99306	1.5649	2.1366	2.7084	3.2802	3.9065	4.4237	4.9953	5.5677	6.1389	6.7109	7.2825
⑧	$\sin(\omega_{e-th}\gamma + hA_{\theta ge})$	sin ⑦	.40894	.83770	.99998	84414	.41977	-.13814	-.69252	-.958602	-.96025	-.65593	-.14384	+.41485	.84113
⑨	θ/δ_{ea}	.24075 × ⑧	.098452	.20168	.24075	.20323	10106	-.033257	-.16672	-.23079	-.23118	-.15792	-.034629	+.099875	.20250
⑩	$\omega_{e-th}\gamma + hA_{txge}$	③ + .4189	.4189	.99066	1.5625	2.1342	2.7060	3.2778	3.9041	4.4213	4.9929	5.5653	6.1365	6.7085	7.2801
⑪	$\sin(\omega_{e-th}\gamma + hA_{txge})$	sin ⑩	.40675	.83607	.99996	.84545	.42199	-.13572	-.69076	-.95792	-.96092	-.65777	-.14608	+.41263	.83981
⑫	t_x/δ_{ea}	491.23 × ⑪	199.81	410.70	491.21	415.31	207.29	-66.670	-339.32	-470.56	-472.03	-323.12	-71.759	+202.70	412.54
⑬	$\delta_e/\delta_{ea} = \sin \omega_{e-th}\gamma$ =	sin ③	0	.54111	.91013	.98958	.75425	.27899	-.33693	-.75836	-.99044	-.90726	-.53597	+.0064053	.54639

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE.

TABLE R-9.

$\angle A_{u\delta_e}$, $\angle A_{w\delta_e}$, $\angle A_{t_x\delta_e}$, $\mu_{u\delta_e}$, $\mu_{w\delta_e}$, $\mu_{t_x\delta_e}$, μ_{uw}, COMPUTATIONS

U_o = 300 mph TAS, 10,000 FT. DENSITY ALTITUDE

No.	Computation								Constants
(1)	Ω_{E-TH}	0.1	0.3	0.5	0.7	0.9	1.1	1.3	
(2)	$\omega_{e-u} = 2\pi T_a \Omega_{ETH} = 28.096$ (1)	2.8096	8.4288	14.048	19.667	25.286	30.906	36.525	
(3)	$\omega_{e-u}^2 = (2)^2$	7.8939	71.045	197.35	386.79	639.38	955.18	1334.1	
(4)	$-\omega_{e-u} L_7 = -(2)[4.9574]$	-13.928	-41.785	-69.642	-97.497	-125.35	-153.21	-181.07	
(5)	$-L_6 + (3)$	7.3466	70.498	196.80	386.24	638.83	954.63	1333.6	
(6)	(4)/(5) = $\tan \lambda_1$	-1.8958	-.59271	-.35387	-.25243	-.19622	-.16049	-.13578	
(7)	(3)/$[-z_w]$ = (3)/4.8701 = $\tan \lambda_2$	.57691	1.7307	2.8845	4.0383	5.1921	6.3461	7.4998	
(8)	(2) L_9/L_8 = (2)[.035894] = $\tan \lambda_3$	.10085	.30254	.50424	.70593	.90762	1.1093	1.3110	L_6 = .54735
(9)	$\tan^{-1}$ (6) = λ_1 degrees	297.81	329.34	340.51	345.83	348.90	350.88	352.27	L_7 = 4.9574
(10)	" (7) = λ_2 "	29.981	59.981	70.880	76.092	79.098	81.045	82.405	L_8 = 97.773
(11)	" (8) = λ_3 "	5.759	16.833	26.759	35.220	42.228	47.967	52.665	L_9 = 3.5095
(12)	.0174533 (9) = λ_1 radians	5.1978	5.7481	5.9430	6.0359	6.0895	6.1240	6.1483	z_w = -4.8701
(13)	" (10) = λ_2 "	.52327	1.0469	1.2371	1.3281	1.3805	1.4145	1.4382	m_{δ_e} = -351.95
(14)	" (11) = λ_3 "	.10051	.29379	.46703	.61471	.73702	.83718	.91918	z_w = -.65374
(15)	$\angle A_{u\delta_e}$ = (13) − (14) = $\lambda_2 - \lambda_3$	.42276	.75311	.77007	.71339	.64330	.57732	.51902	
(16)	$\angle A_{w\delta_e}$ = −(14) + 3.1416 = $-\lambda_3 + \pi$	3.0411	2.8478	2.6746	2.5269	2.4046	2.3044	2.2224	
(17)	$\angle A_{t_x\delta_e}$ = (12) − (14) = $\lambda_1 - \lambda_3$	5.0973	5.4543	5.4760	5.4212	5.3525	5.2868	5.2291	
(18)	(3) + $(-z_w)^2$ = (3) + 23.718	31.612	94.763	221.07	410.51	663.10	978.90	1357.8	
(19)	(3) L_9^2 = (3)[12.317]	97.229	875.06	2430.8	4764.1	7875.2	11765.	16432.	
(20)	(19) + L_8^2 = (19) + 9559.6	9656.8	10435.	11990.	14324.	16935.	21325.	25992.	
(21)	(18)/(20)	$.32735 \times 10^{-2}$	$.90813 \times 10^{-2}$	$.18438 \times 10^{-1}$	$.28659 \times 10^{-1}$	$.39156 \times 10^{-1}$	$.45904 \times 10^{-1}$	$.52239 \times 10^{-1}$	
(22)	$\sqrt{(21)}$	$.57215 \times 10^{-1}$	$.95297 \times 10^{-1}$	.13579	.16929	.19788	.21425	.22856	
(23)	$-m_{\delta_e}$ (22) = $\mu_{u\delta_e}$	20.137	33.540	47.791	59.582	69.644	75.405	80.442	
(24)	$\sqrt{(20)}$	98.269	102.15	109.50	119.68	130.13	146.03	161.22	
(25)	$m_{\delta_e} z_w$/(24) = 230.08/(24) = $\mu_{w\delta_e}$	2.3413	2.2524	2.1012	1.9225	1.7681	1.5756	1.4271	
(26)	$-L_6$ + (3)	7.3466	70.498	196.80	386.24	638.83	954.63	1333.6	
(27)	$(26)^2$	53.973	4970.	38730.	$14918. \times 10^{+1}$	$40810. \times 10^{+1}$	$91132. \times 10^{+1}$	$17785. \times 10^{+2}$	
(28)	L_7^2 (3) = (3)[24.576]	194.00	1746.0	4850.1	9505.8	15713.	23475.	32787.	
(29)	(27) + (28)	247.97	6716.	43580.	$15869. \times 10^{+1}$	$42381. \times 10^{+1}$	$93480. \times 10^{+1}$	$18113. \times 10^{+2}$	
(30)	(29)/(20)	$.25678 \times 10^{-1}$	.64360	3.6347	11.079	25.026	43.836	69.687	
(31)	$\sqrt{(30)}$	.16024	.80225	1.9065	3.3285	5.0026	6.6209	8.3479	
(32)	$-m_{\delta_e}$ (31) = 351.95 (31) = $\mu_{t_x\delta_e}$	56.396	282.35	670.99	1171.5	1760.7	2330.2	2938.0	
(33)	(23)/(25) = $\mu_{u\delta_e}/\mu_{w\delta_e} = \mu_{uw}$	8.6008	14.891	22.745	30.992	39.389	47.858	56.367	

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR,
ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE.

TABLE R-10

$\omega_{e-th} = 2.8096$

$\Omega_{E-TH} =$ .1 CYCLE/SEC

Time Histories of u/δ_{ea}, w/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea}

V_0 = 300 mph TAS, 10000 FT. DENSITY ALTITUDE

①	t		0	1	2	3	4	5	6	7	8	9	10	11	12
②	$\gamma = t/T_a$	①/4.4716	0	.22363	.44727	.67090	.89453	1.1182	1.3418	1.5654	1.7891	2.0127	2.2363	2.4600	2.6836
③	$\omega_{e-th}\gamma$	2.8096 × ②	0	.62831	1.2566	1.8850	2.5133	3.1417	3.7699	4.3981	5.0267	5.6549	6.2831	6.9116	7.5398
④	$\omega_{e-th}\gamma + h_{Auge}$	③ +.42276	.42276	1.0511	1.6794	2.3078	2.9361	3.5645	4.1927	4.8209	5.4495	6.0777	6.7059	7.3344	7.9626
⑤	$\sin(\omega_{e-th}\gamma + h_{Auge})$	sin ④	.41027	.86797	.99411	.74045	.20398	-.41040	-.86794	-.99411	-.74045	-.20398	+.41024	.86803	.99411
⑥	u/δ_{ea} =	20.137 × ⑤	8.2616	17.478	20.018	14.910	4.1075	-8.2642	-17.478	-20.018	-14.910	-4.1075	+8.2610	17.480	20.018
⑦	$\omega_{e-th}\gamma + h_{Auge}$	③ + 3.0411	3.0411	3.6694	4.2977	4.9321	5.5544	6.1828	6.8110	7.4392	8.0678	8.6960	9.3242	9.9527	10.581
⑧	$\sin(\omega_{e-th}\gamma + h_{Auge})$	sin ⑦	+.10036	-.50362	-.91524	-.97595	-.66601	-.10019	+.50362	.91517	.97723	.66601	.10036	-.50377	-.91531
⑨	w/δ_{ea} =	2.2413 × ⑧	.23497	-1.1791	-2.1429	-2.2850	-1.5593	-.23457	+1.1791	2.1427	2.2880	1.5593	.23497	-1.1795	-2.1430
⑩	$\omega_{e-th}\gamma + h_{Auge}$	③ +5.0973	5.0973	5.7256	6.3539	6.9823	7.6106	8.2390	8.8672	9.4954	10.124	10.752	11.380	12.009	12.637
⑪	$\sin(\omega_{e-th} + h_{Auge})$	sin ⑩	-.92686	-.52918	+.070627	.64359	.97055	.92718	.52918	-.070627	-.64359	-.97046	-.92699	-.52903	+.070627
⑫	t_x/δ_{ea} =	56.396 × ⑪	-52.271	-29.844	+3.9831	36.296	54.735	52.289	29.844	-3.9831	-36.296	-54.730	-52.279	-29.835	+3.9831
⑬	δ_e/δ_{ea} =	sin ③	0	.58777	.95104	.95106	.58779	-.0001745	-.58779	-.95100	-.95100	-.58779	-.0001745	+.58793	.95100

$\omega_{e-th} = 19.667$

$\Omega_{E-TH} = .7$ CYCLE/SEC

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE
TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR,
ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE.

TABLE R-11

Time Histories of u/δ_{ea}, w/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea} $\quad U_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

①	t		0	.13	.26	.39	.52	.65	.78	.91	1.04	1.17	1.30	1.43	1.56
②	$\gamma = t/T_R$	①/4.4716	·0	.029072	.058145	.087217	.11629	.14536	.17443	.20351	.23258	.26165	.29072	.31980	.34887
③	$\omega_{e-th}\gamma$	19.667 × ②	0	.57176	1.1435	1.7153	2.2871	2.8588	3.4305	4.0024	4.5742	5.1459	5.7176	6.2895	6.8612
④	$\omega_{e-th}\gamma + h_{\Delta u \delta e}$	③ + .71339	.71339	1.2852	1.8569	2.4287	3.0005	3.5722	4.1439	4.7158	5.2876	5.8593	6.4310	7.0029	7.5746
⑤	$\sin(\omega_{e-th}\gamma + h_{\Delta u \delta e})$	sin ④	.65440	.95949	.95936	.65408	.14056	-.41739	-.84273	-.99999	-.83905	-.41136	+.14729	.65921	.96121
⑥	$u/\delta_{ea} =$	59.582 × ⑤	38.990	57.168	57.161	38.971	8.3748	-24.869	-50.212	-59.581	-49.992	-24.510	+8.7758	39.277	57.271
⑦	$\omega_{e-th}\gamma + h_{\Delta w \delta ea}$	③ + 2.5269	2.5269	3.0987	3.6704	4.2422	4.8140	5.3857	5.9574	6.5293	7.1011	7.6728	8.2445	8.8164	9.3881
⑧	$\sin(\omega_{e-th}\gamma + h_{\Delta w \delta e})$	sin ⑦	.57672	.042922	-.50453	-.89148	-.99485	-.78174	-.32012	+.24362	.72969	.98363	.92475	.57157	.036644
⑨	$w/\delta_{ea} =$	1.9225 × ⑧	1.1087	.082518	-.96996	-1.7139	-1.9126	-1.5029	-.61543	+.46836	1.4028	1.8910	1.7778	1.0988	.070448
⑩	$\omega_{e-th}\gamma + h_{\Delta u \delta e}$	③ + 5.4212	5.4212	5.9930	6.5647	7.1365	7.7083	8.2800	8.8517	9.4236	9.9954	10.567	11.139	11.711	12.282
⑪	$\sin(\omega_{e-th}\gamma + h_{\Delta x \delta ea})$	sin ⑩	-.75916	-.28619	+.27782	.75345	.98940	.91061	.54229	.0012217	-.54009	-.90953	-.98973	-.75482	-.28050
⑫	$t_x/\delta_{ea} =$	1171.5 × ⑪	-889.36	-335.27	+325.47	882.67	1159.1	1066.8	635.29	1.4312	-632.72	-1065.5	-1159.5	-884.27	-328.61
⑬	$\delta_e/\delta_{ea} =$	sin ③	0	.54111	.91009	.98958	.75425	.27899	-.28485	-.75836	-.99046	-.90748	-.53597	+.0062831	.54639

TABLE R-12

ANALYSIS FOR STEADY-STATE RESPONSE OF A-26 AIRPLANE TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE.

$\omega_{e-th} = 36.525$

$\Omega_{e-th} =$ 1.3 CYCLES/SEC.

Time Histories of u/δ_{ea}, w/δ_{ea}, t_x/δ_{ea} and δ_e/δ_{ea}

$U_0 = 300$ mph TAS, 10,000 FT. DENSITY ALTITUDE

No.	Quantity	Operation						
①	t		0	.07	.14	.21	.28	.35
②	$\gamma = t/T_a$	①/4.4716	0	.015654	.031309	.046963	.062617	.078272
③	$\omega_{e-th}\gamma$	36.525 × ②	0	.57176	1.1436	1.7153	2.2871	2.8589
④	$\omega_{e-th}\gamma + h_A u_{\delta e}$	③ + .51902	.51902	1.0908	1.6626	2.2343	2.8061	3.3779
⑤	$\sin(\omega_{e-th}\gamma + h_A u_{\delta e})$	sin ④	.49619	.88700	.99579	.78780	.32920	-.23412
⑥	$u/\delta_{ea} =$	80.442 × ⑤	39.915	71.352	80.103	63.372	26.482	-18.833
⑦	$\omega_{e-th}\gamma + h_A w_{\delta e}$	③ + 2.2224	2.2224	2.7942	3.3660	3.9377	4.5095	5.0813
⑧	$\sin(\omega_{e-th}\gamma + h_A w_{\delta e})$	sin ⑦	.79516	.34038	-.22257	-.71459	-.97950	-.93270
⑨	$w/\delta_{ea} =$	1.4271 × ⑧	1.1348	.48576	-.31763	-1.0198	-1.3978	-1.3311
⑩	$\omega_{e-th}\gamma + h_A t_{x\delta e}$	③ + 5.2291	5.2291	5.8009	6.3727	6.9444	7.5162	8.0880
⑪	$\sin(\omega_{e-th}\gamma + h_A t_{x\delta e})$	sin ⑩	-.86941	-.46376	+.089416	.61401	.94351	.97274
⑫	t_x/δ_{ea}	2938.0 × ⑪	-2554.3	-1362.5	+262.70	1804.0	2772.	2857.9
⑬	$\delta_e/\delta_{ea} =$	sin ③	0	.54111	.91013	.98958	.75425	.27899

No.								
①	.42	.49	.56	.63	.70	.77	.84
②	.093926	.10958	.12523	.14089	.15654	.17220	.18785
③	3.4306	4.0024	4.5740	5.1460	5.7176	6.2896	6.8612
④	3.9496	4.5214	5.0930	5.6650	6.2366	6.8086	7.3802
⑤	-.72297	-.98183	-.92842	.57957	-.046583	+.50151	.88981
⑥	-58.157	-78.980	-74684	-46.622	-3.7472	+40.342	71.578
⑦	5.6530	6.2248	6.7964	7.3684	7.9400	8.5120	9.0836
⑧	-.58934	-.058435	+.49090	.88442	.99630	.79122	.33463
⑨	-.84105	-.083393	+.70056	1.2622	1.4218	1.1292	.47755
⑩	8.6597	9.2315	9.8031	10.375	10.947	11.519	12.090
⑪	.69265	.19201	.36942	-.81351	-.99882	-.86611	-.45849
⑫	2035.0	564.13	-1085.4	-2390.1	-2934.5	-2544.6	-1347.0
⑬	.28502	-.75836	-.99044	-.90748	-.53597	+.0064053	.54639

APPENDIX S

METHODS FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR

APPENDIX S

METHODS FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR

A. METHOD FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN LONGITUDINAL VELOCITY.

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in longitudinal velocity, were derived in Section A of Appendix Q. From equation 46 of Appendix Q, we have:

$$\mu_{w\theta} = \mu_{\dot{w}q} = \frac{1}{\sqrt{\left(\frac{z_w}{\omega_{e-th}}\right)^2 + 1}} \tag{1}$$

As would be expected, this equation is exactly the same as equation 102 in Section A-2 of Appendix H, where the effect of change in longitudinal velocity was neglected in the equations of motion for normal force and pitching moment. Also, from equations 22, 23 and 24 in Section A of Appendix Q, we have:

$$L_1 = -m_q - z_w - \mu_c m_{\dot{w}}\ , \tag{2}$$

$$L_2 = m_q z_w - \mu_c m_w - z_\theta \mu_c m_{\dot{w}}\ , \tag{3}$$

and $$L_3 = z_\theta \mu_c m_w \tag{4}$$

For $z_\theta = 0$, $L_3 = 0$, and L_1 and L_2 become identical with "b" and "k", respectively, defined by equations 84 and 85 in Section A-2 of Appendix H.

Also, combining equations 18, 37, 38, 43 and 44, in Section A of Appendix Q, we have:

$$LA_{\dot{w}q} = \frac{3\pi}{2} - \tan^{-1}\left(\frac{-\omega_{e-th}}{z_w}\right), \tag{5}$$

which is identical with the equation obtained by combining equations 82, 83, 87, 99 and 100 in Section A-2 of Appendix H.

Thus it is seen that the method described in Section A-2 of Appendix P is generally applicable in this case. For example, equations 1 and 5 above, show that z_w can be obtained by either the amplitude-ratio or the phase-angle method. The other stability coefficients, such as m_q, $m_{\dot{w}}$, m_w, m_u, z_u, x_u, etc., can not be obtained by this method, although the effective damping coefficient, b, and the equivalent spring constant, k, for the short-period oscillation only, <u>can</u> be obtained. For $z_\theta = 0$, b and k may be obtained from

$$LA_{\dot{w}\delta_e} = -\tan^{-1}\left(\frac{\omega_{e-th}^2 - k}{\omega_{e-th} b}\right), \tag{6}$$

using any two different values of ω_{e-th} and the corresponding values of $LA_{\dot{w}\delta_e}$ obtained from the flight records. Equation 6 may be derived by combining equations 21, 22, 23, 37, and

43 in Section A of Appendix Q, and it is the same as equation 43 in Section A-2 of Appendix P. Also, b and k may be obtained by the circle diagram method described in reference 1.

B. METHOD FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN NORMAL VELOCITY (FOR SMALL OSCILLATIONS, SAME AS ZERO CHANGE IN ANGLE OF ATTACK)

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in normal velocity, were derived in Section B of Appendix Q. From equation 66 of Appendix Q, we have:

$$z_u = -\mu_{qu}. \qquad (7)$$

In this case, z_u is independent of ω_{e-th}, and can be easily obtained from the flight records.

From equation 65 in Appendix Q, we have:

$$\mu_{q\delta_e} = -\left(\frac{m_{\delta_e}}{\sqrt{\omega_{e-th}^2 + m_q^2}}\right). \qquad (8)$$

Since m_{δ_e} can be found from steady-state flight-tests, it can be considered as a known quantity in equation 8. Thus equation 8 may be used to solve for m_q, using any value of ω_{e-th} and the corresponding value of $\mu_{q\delta_e}$ obtained from the flight records. From equation 64 in Appendix Q, we have:

$$\mu_{u\delta_e} = \frac{m_{\delta_e}}{z_u \sqrt{\omega_{e-th}^2 + m_q^2}}\ , \qquad (9)$$

which can be used as a check on the values for z_u and m_q found from equations 7 and 8, respectively.

Also, m_q can be obtained from equations 67 or 68, of Appendix Q. These equations are, respectively, as follows:

$$LA_{\theta\delta_e} = \pi - \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right), \tag{10}$$

and $$LA_{q\delta_e} = 3\pi/2 - \tan^{-1}\left(\frac{m_q}{\omega_{e-th}}\right). \tag{11}$$

Thrust measurements need not be taken unless x_u and x_θ are required. Equations 57, 59, 60 and 61 of Appendix Q, combine to give:

$$LA_{\theta t_x} = \pi - \tan^{-1}\left(\frac{-x_u\omega_{e-th}}{x_\theta z_u - \omega^2_{e-th}}\right), \tag{12}$$

for $z_\theta = 0$. Knowing z_u, equation 12 may be used to solve for x_u and x_θ, using any two different values of ω_{e-th} and the corresponding values of $LA_{\theta t_x}$. But solving for x_θ is merely checking the effect of gravity, so the author recommends using the theoretical value of $x_\theta = -C_{L_o}$ (see Section A-1, Appendix B), and using the value of z_u obtained from equation 7, to solve for x_u from equation 12. In this way, the lead-angle response, $LA_{\theta t_x}$, for only one frequency is required. But this assumes that the thrust oscillation can be measured accurately. This is certainly possible, but rather difficult at the present state of development in flight-testing technique.

However, this technique should improve rapidly, since it is a basic requirement for the measurement of performance of jet-propelled aircraft.

C. METHOD FOR DETERMINING LONGITUDINAL DYNAMIC STABILITY COEFFICIENTS FOR AIRCRAFT FROM STEADY-STATE FLIGHT-TEST RESPONSE OF AIRCRAFT TO FORCED SINUSOIDAL MOTION OF THROTTLE AND ELEVATOR, ADJUSTED FOR ZERO CHANGE IN PITCH ANGLE

The equations for steady-state response of aircraft to forced sinusoidal motion of throttle and elevator, adjusted for zero change in pitch angle, were derived in Section B of Appendix Q. From equation 88 of Appendix Q, we have:

$$\mu_{wu} = \frac{-z_u}{\sqrt{(-z_w)^2 + \omega_{e-th}^2}} \,. \tag{13}$$

Here we can solve for z_u and z_w, using any two different values of ω_{e-th} and the corresponding values of μ_{wu} obtained from the flight records. However, z_w is easily obtained by the method discussed in section A, and z_u is easily obtained by the method discussed in Section B. Thus, equation 13 may be used as a check.

From equation 89 of Appendix Q, we have:

$$LA_{w\delta_e} = - \tan^{-1} \left(\frac{-\omega_{e-th} m_{\dot{w}}}{-m_w}\right), \tag{14}$$

for $m_u = 0$. Thus, we can solve for $m_{\dot{w}}$ and m_w, using any two different values of ω_{e-th} and the corresponding values of $LA_{w\delta_e}$ obtained from flight records.

Thrust measurements need not be taken unless x_u and x_w are required. Equations 79, 81, 82 and 85 of Appendix Q combine to give

$$LA_{wt_x} = -\tan^{-1}\left[\frac{-\omega_{e-th}(z_w + x_u)}{x_u z_w - x_w z_u - \omega_{e-th}^2}\right]. \tag{15}$$

z_w and z_u can be obtained easily by the methods discussed in Sections A and B, respectively, so they may be considered as known quantities in equation 15. Thus, x_u and x_w may be found, using any two different values of ω_{e-th} and the corresponding values of LA_{wt_x} obtained from flight-records.

If the characteristics of the long-period response are required, it is necessary to measure the thrust oscillation, because the long-period characteristics depend mostly upon x_u, x_w and x_θ. For example, the damping of the long-period is primarily dependent upon x_u, as shown by the equations developed in Section A of Appendix I.

APPENDIX T

DETERMINATION OF DAMPING COEFFICIENT AND SPRING CONSTANT FOR SHORT PERIOD COMPONENT OF LONGITUDINAL MOTION OF A-26 AIRPLANE, FROM FLIGHT-TEST DATA, USING CIRCLE DIAGRAM METHOD. COMPARISON OF COMPUTED RESPONSE WITH FLIGHT-TEST RESPONSE, FOR A-26 AIRPLANE.

APPENDIX T

DETERMINATION OF DAMPING COEFFICIENT AND SPRING CONSTANT FOR SHORT PERIOD COMPONENT OF LONGITUDINAL MOTION OF A-26 AIRPLANE, FROM FLIGHT-TEST DATA, USING CIRCLE DIAGRAM METHOD. COMPARISON OF COMPUTED RESPONSE WITH FLIGHT-TEST RESPONSE, FOR A-26 AIRPLANE.

A. DETERMINATION OF DAMPING COEFFICIENT AND SPRING CONSTANT FOR SHORT PERIOD COMPONENT OF LONGITUDINAL MOTION OF A-26 AIRPLANE, FROM FLIGHT-TEST DATA, USING CIRCLE DIAGRAM METHOD.*

1. Development of Theory:

From equations 74 and 75 in Section A-2 of Appendix H, for $z_\theta = 0$, we have:

$$(-d + z_w)w + q = 0, \text{ and} \tag{1}$$

$$(d\mu_c m_{\dot{w}} + \mu_c m_w)w + (-d + m_q)q = -m_{\delta_e}\delta_{e_a}\mathcal{J}e^{i\omega_e\gamma}. \tag{2}$$

From equation 5 in Section A-1 of Appendix P, we have:

$$\dot{w} = C_{L_o}(n_z + 1) + q, \text{ for } \cos\theta \cong 1. \tag{3}$$

Combining equations 1, 2 and 3 we have:

$$(\ddot{n_z + 1}) + (\dot{n_z + 1})b + (n_z + 1)k = m_{\delta_e}\left(\frac{z_w}{C_{L_o}}\right)\delta_{e_a}\mathcal{J}e^{i\omega_e\gamma}, \tag{4}$$

where

$$b = -z_w - m_q - \mu_c m_{\dot{w}}, \text{ and} \tag{5}$$

$$k = -\mu_c m_w + z_w m_q. \tag{6}$$

Note that these expressions for b and k are the same as those given by equations 84 and 85 in Section A-2 in Appendix H.

* Reference 1.

Since the reading of the normal accelerometer, n, is just 180° out of phase with n_z, equation 4 may be changed to:

$$\ddot{(n - 1)} + \dot{(n - 1)}b + (n - 1)k = -m_{\delta_e}\left(\frac{z_w}{C_{L_0}}\right)\delta_{e_\alpha}\mathcal{R}e\, e^{i\omega_e\gamma}. \tag{7}$$

Note that in Equation 7, a negative δ_e produces a positive $(n - 1)$, because m_{δ_e} and z_w are both negative. This agrees with physical concepts.

We may solve equation 7 by operational calculus as follows:

$$(-\omega_e^2 + i\omega_e b + k)\overline{(n - 1)}_\alpha = -m_{\delta_e}\left(\frac{z_w}{C_{L_0}}\right)\delta_{e_\alpha}. \tag{8}$$

Thus, since $(n - 1) = \overline{(n - 1)}_\alpha\, \mathcal{R}e\, e^{i\omega_e\gamma}$, we have:

$$(n - 1) = -m_{\delta_e}\left(\frac{z_w}{C_{L_0}}\right)\left(\frac{\delta_{e_\alpha}}{\sqrt{(k - \omega_e^2)^2 + (\omega_e b)^2}}\right) \sin\,(\omega_e\gamma + LA_{n\delta_e}), \tag{9}$$

where

$$LA_{n\delta_e} = LA_{(n - 1)\delta_e} = -\tan^{-1}\left(\frac{\omega_e b}{k - \omega_e^2}\right). \tag{10}$$

Differentiating equation 9 we have:

$$\dot{(n - 1)} = -\left(\frac{m_{\delta_e} z_w \delta_{e_\alpha}}{C_{L_0}\sqrt{\left(\frac{k}{\omega_e} - \omega_e\right)^2 + b^2}}\right) \sin\,(\omega_e\gamma + LA_{\dot{n}\delta_e}), \tag{11}$$

$$\text{where } LA_{\dot{n}\delta_e} = LA_{n\delta_e} + \pi/2, \tag{12}$$

$$= \pi/2 - \tan^{-1}\left(\frac{\omega_e b}{k - \omega_e^2}\right),$$

$$= \tan^{-1}\left[\frac{\left(\frac{k}{\omega_e}\right) - \omega_e}{b}\right]. \tag{13}$$

Now,

$$\cos LA_{\dot{n}\delta_e} = \frac{b}{\sqrt{(\frac{k}{\omega_e} - \omega_e)^2 + b^2}} \quad (14)$$

Thus, from equation 11 we have:

$$\frac{(\dot{n} - 1)_\alpha}{\delta_{e_\alpha}} = -(\frac{m_{\delta_e} z_w}{C_{L_o} b}) \cos LA_{\dot{n}\delta_e}$$

or,

$$\boxed{\frac{(\dot{n} - 1)_\alpha}{\delta_{e_\alpha}} = (\frac{m_{\delta_e} z_w}{C_{L_o} b}) \cos \text{Lag } A_{\dot{n}\delta_e}} \quad (15)$$

Equation 15 is the polar equation of a circle, with diameter equal to $(\frac{m_{\delta_e} z_w}{C_{L_o} b})$. Also,

$$\frac{(\dot{n} - 1)_\alpha}{\delta_{e_\alpha}} = \frac{\omega_e (n - 1)_\alpha}{\delta_{e_\alpha}} \quad (16)$$

Flight-test records may be taken for n, δ_e and $LA_{n\delta_e}$ at various values of $\omega_e = T_A \Omega_E$. Thus equations 12, 15 and 16 may be used to plot the circle diagram directly from the flight-test data. When $\text{Lag}A_{\dot{n}\delta_e} = 45^o$, $\cos \text{Lag}A_{\dot{n}\delta_e} = 1/\sqrt{2}$, and so

$$b = \frac{k}{(\omega_e)_{45^o}} - (\omega_e)_{45^o} \quad (17)$$

Analogously,

$$b = (\omega_e)_{-45^o} - \frac{k}{(\omega_e)_{-45^o}} . \quad (18)$$

When $\text{LagA}_{\dot{n}\delta_e} = 0$, $\cos \text{Lag}_{\dot{n}\delta_e} = 1$, and so

$$k = (\omega_e)^2_{0^\circ} \tag{19}$$

2. Reduction of Flight-test Data to Circle Diagram, Determination of b and k

Flight-test data* for the A-26C Airplane are presented in Figures T-1 and T-2. Figure T-1 presents values of $\frac{(n-1)_\alpha}{\delta_{e_\alpha}}$ and $LA_{n\delta_e}$, plotted against Ω_E, and Figure T-2 presents values of $\frac{Q_\alpha}{\delta_{e_\alpha}}$ and $LA_{q\delta_e}$, plotted against Ω_E. Figure T-3 shows the circle diagram which was computed from the flight-test data given in Figure T-1, using equations 12, 15 and 16. Computations for Figure T-3 are given in Table T-1.

Using equation 19, and the value of ω_e at the diameter of the circle in Figure T-3, k may be computed from the flight-test data as follows:

$$k = (\omega_e)^2_{0^\circ} = (18.6)^2 = 345.96 \tag{20}$$

Based upon equation 6 and the theoretical values for μ_c, m_w, z_w and m_q, (computed in the following section), the theoretical value of k is as follows:

$$\begin{aligned} k &= -(225.03)(-.58025) + (-4.8701)(-15.330) \\ &= 281.52 + 74.6586 \\ &= 356.18 \end{aligned} \tag{21}$$

Using equation 17 and the value of $(\omega_e)_{45^\circ}$ taken from the from the circle diagram in Figure T-3, and the value of k from equation 20, b may be computed from the flight-test

* Taken from Reference 10.

data as follows:

$$b = \frac{k}{(\omega_e)_{45^o}} - (\omega_e)_{45^o}$$

$$= (\frac{345.96}{9.8}) - 9.8 = 25.5 \qquad (22)$$

Based upon equation 5 and the theoretical values for z_w, m_q, μ_c and $m_{\dot{w}}$, (computed in the following section), the theoretical value of b is as follows:

$$b = -(-4.8701) - (-15.330) - (225.03)(-.022344)$$

$$= 4.8701 + 15.330 + 5.0281$$

$$= 25.23 \qquad (23)$$

The experimental lag angle shown in Figure T-3 is probably due to instrumentation lag in the normal accelerometer.

B. COMPARISON OF COMPUTED RESPONSE WITH FLIGHT-TEST RESPONSE, FOR A-26 AIRPLANE.

From equations 81, 100, and 100-a in Section A-2 of Appendix H we have:

$$\frac{q_\alpha}{\delta_{e_\alpha}} = - m_{\delta_e} \sqrt{\frac{(z_w)^2 + (-\omega_e)^2}{(-\omega_e b)^2 + (k - \omega_e^2)^2}}, \quad \text{and} \qquad (24)$$

$$LA_{q\delta_e} = \tan^{-1} [\frac{-\omega_e(k - \omega_e^2) - z_w\omega_e b}{z_w(k - \omega_e^2) - \omega_e^2 b}] \qquad (25)$$

Also,

$$\omega_e \equiv T_A\Omega_E, \text{ and } q_\alpha \equiv T_A Q_\alpha. \qquad (26)$$

Equations 24, 25 and 26 were used to determine the computed response for $\frac{Q_\alpha}{\delta_{e_\alpha}}$ and $LA_{q\delta_e}$ shown in Figure T-2. The corresponding computations are shown in Table T-2.

Computations for Theoretical Longitudinal Stability Coefficients for A-26C Airplane, EAS* = 180 mph, Pressure Altitude = 10,000 Ft., Free Air Temperature = -7° C, W_A = 28,100 lbs:

For pressure altitude = 10,000 ft, and free air temperature = -7°C, σ = .7420, and $\sigma^{\frac{1}{2}}$ = .8614.

$$\text{(a)}\quad T_A = \frac{17.83(\frac{W_A}{S})}{\sqrt{\sigma}\, U_{o_{mph,EAS}}} = \frac{(17.83)(28,100)}{(.8614)(180)(540)}$$

$$= 5.9839 \text{ seconds}$$

$$\text{(b)}\quad \mu_c = 26.12\,\frac{(\frac{W_A}{S})}{(\sigma)\ c_{ft.}} = \frac{(26.12)(28,100)}{(.7420)(8.14)(540)}$$

$$= 225.03$$

$$\text{(c)}\quad \frac{1}{2}\rho U^2_{o_{ft/sec}} = \frac{\sigma U^2_{o_{mph,TAS}}}{391} = \frac{U^2_{o_{mph,EAS}}}{391}$$

$$= \frac{(180)^2}{391} = 82.864 \text{ lbs/ft}^2$$

$$\text{(d)}\quad C_{L_o} = \frac{W_A/S}{\frac{1}{2}\rho U^2_{o_{ft/sec}}} = \frac{28,100}{(540)(82.864)}$$

$$= .62798$$

$$\text{(e)}\quad C_{D_o} = C_{D_{p_{min}}} + \frac{C^2_{L_o}}{e\pi R}$$

$$= .0244 + \frac{(.62798)^2}{(.8)(9.07)(3.1416)}$$

* Equivalent Airspeed, defined in NACA TN 1120, "Standard Symbols and Definitions".

$$= .0244 + .0173$$

$$= .0417$$

(f) $x_u = -3C_{D_o} = -3(.0417)$

$$= -.1251 \text{, power on.}$$

(g) $x_w = C_{L_o}[1 - (\frac{2}{e\pi R})(\frac{\partial C_L}{\partial \alpha})]$

$$= .62798 \left[1 - \frac{2(4.8701)}{(.8)(9.07)(3.1416)}\right]$$

$$= .35964$$

(h) $x_\theta = -C_{L_o} = -.62798$

(i) $z_u = -2C_{L_o} = -2(.62798) = -1.25596$

(j) $z_w = -(\frac{\partial C_L}{\partial \alpha}) = -4.8701$

(k) $z_\theta \cong 0$ for $\theta_o = 0$.

(l) m_u assumed negligible.

(m) $m_w = (\frac{M_A c^2}{I_Y})(\frac{\partial C_m}{\partial \alpha})$

$$= \frac{(28,100)(8.14)^2(-.48701)}{(32.2)(48,531)}$$

$$= -.58025$$

(n) $m_{\dot{w}} = (-\frac{\partial C_L}{\partial \alpha})'(\frac{\partial \epsilon}{\partial \alpha})(\frac{\frac{\rho}{2} s' l^2 c \eta'}{I_Y})$

$$= -(3.8904)(.41)\left[\frac{(.002378)(.7420)(116.1)(30.1)^2(8.14)(.9)}{(2)(48,531)}\right]$$

$$= -.022344$$

(o) $$m_q = -5/4\left(\frac{\partial C_L}{\partial \alpha}\right)'\left(\frac{S'}{S}\right)\left(\frac{\eta' M_A l^2}{I_Y}\right)$$

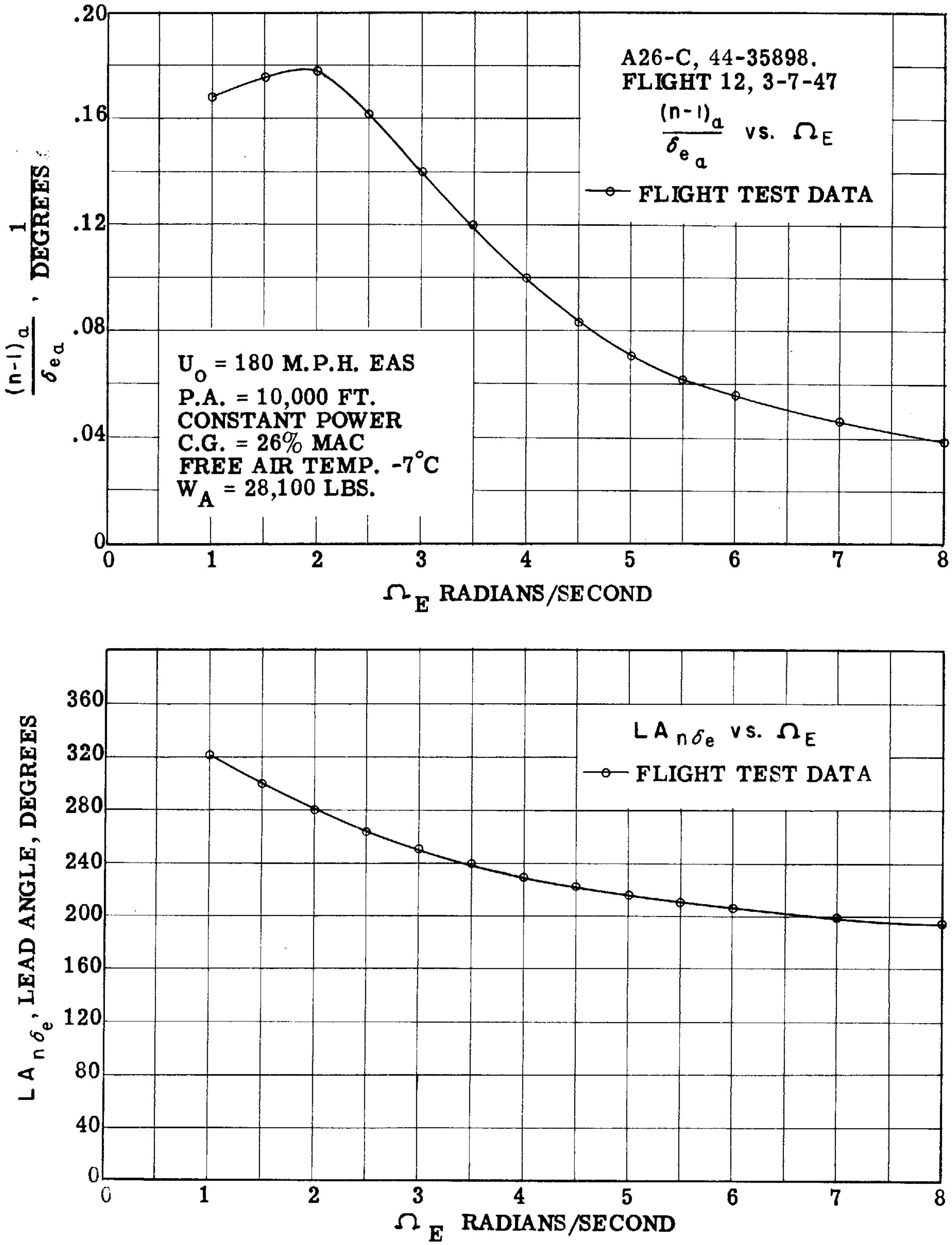
$$= -(5/4)(3.8904)\left(\frac{116.1}{540}\right)\left[\frac{(.9)(28,100)(30.1)^2}{(32.2)(48,531)}\right]$$

$$= -15.330$$

(p) $$m_{\delta_e} = \left(\frac{\partial C_m}{\partial \Delta_E}\right)\mu_c^2\left(\frac{\frac{\rho}{2} S c^3}{I_Y}\right)$$

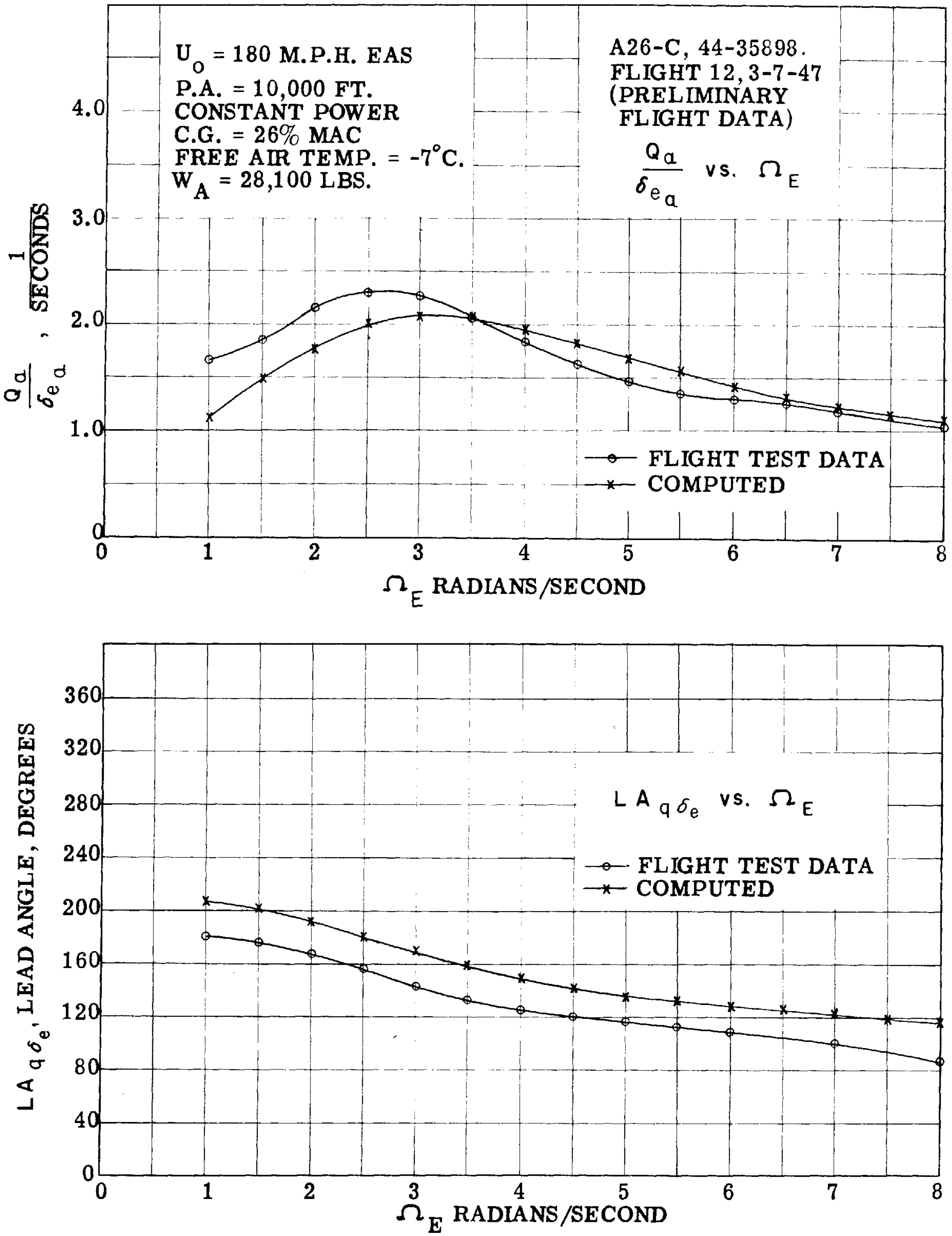
$$= (-1.146)(225.03)^2\left[\frac{(.7420)(.002378)(540)(8.14)^3}{(2)(48,531)}\right]$$

$$= -307.25$$



EFFECT OF ELEVATOR FREQUENCY ON PHASE AND AMPLITUDE OF CHANGE IN READING OF NORMAL ACCELEROMETER, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FLIGHT TEST DATA FOR A-26 AIRPLANE

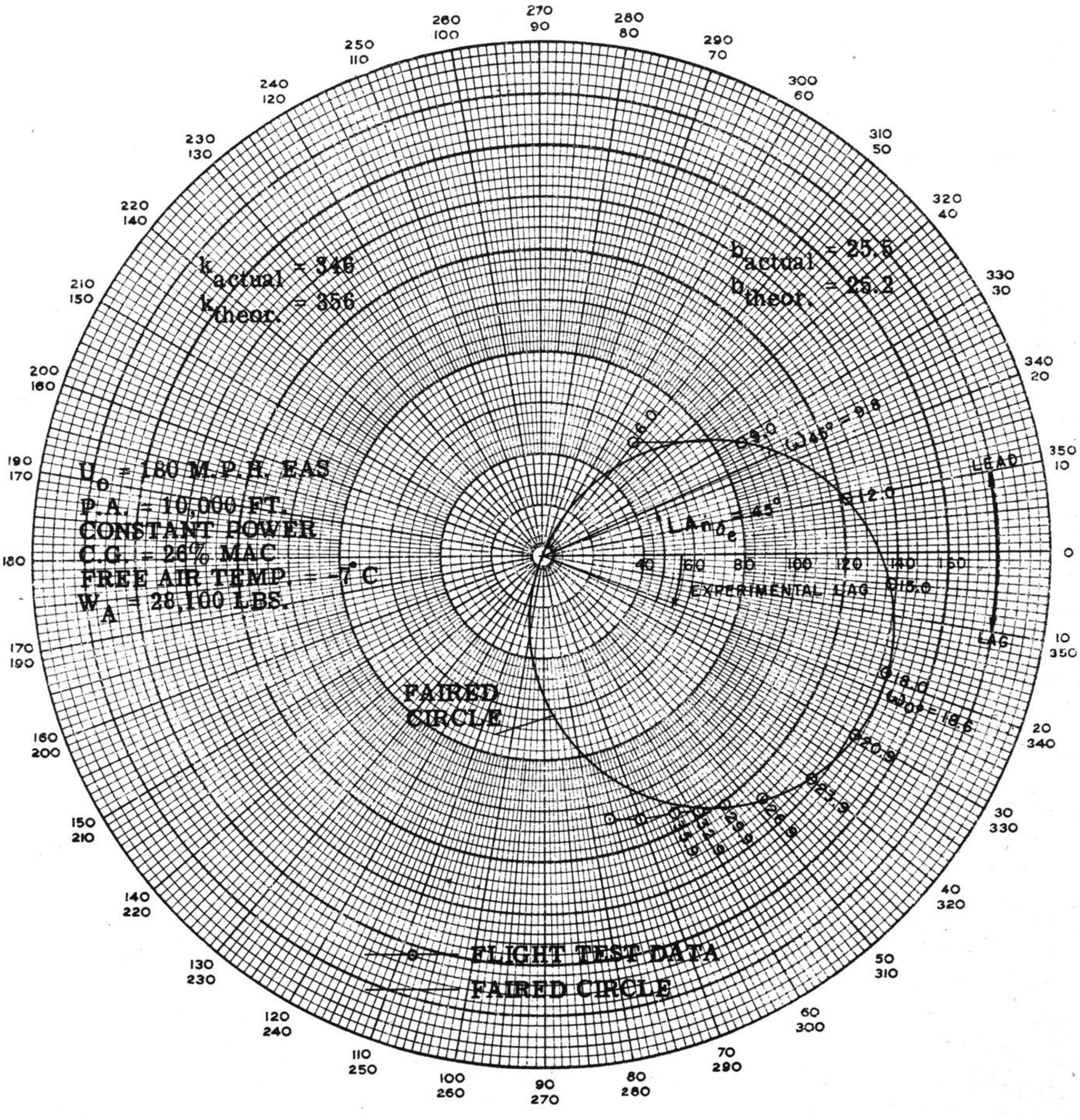
FIGURE T-1



EFFECT OF ELEVATOR FREQUENCY ON PHASE AND AMPLITUDE OF PITCHING VELOCITY, PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, FLIGHT TEST DATA COMPARED WITH COMPUTED DATA, FOR A-26 AIRPLANE

FIGURE T-2

A26-C, 44-35898.
BASED ON DATA FROM
FLIGHT 12, 3-7-47
(PRELIMINARY DATA)



CIRCLE DIAGRAM FOR DETERMINING DAMPING COEFFICIENT AND SPRING CONSTANT FOR SHORT PERIOD COMPONENT OF LONGITUDINAL MOTION OF A-26 AIRPLANE, BASED UPON FLIGHT TEST DATA

FIGURE T-3

CIRCLE DIAGRAM CALCULATIONS

BASED ON FLIGHT-TEST DATA FOR A-26 AIRPLANE

TABLE T-1

① Ω_E rad/sec	1	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	5.5	6.0	7.0	80
② $\frac{(n-1)_a}{\delta_{ea}}$ °/deg. from Figure T-1	.167	.175	.178	.161	.139	.118	.100	.083	.071	.062	.055	.046	.039
③ 57.29578 × ②	9.5684	10.0268	10.1986	9.2246	7.9641	6.7609	5.7296	4.7555	4.0680	3.5523	3.1513	2.6356	2.2345
④ $T_A \times \Omega_e$ = ① × 5.9839 = ω_e	5.984	8.976	11.968	14.960	17.952	20.944	23.936	26.928	29.920	32.912	35.904	41.888	47.872
⑤ $T_A \Omega$ = $(57.29578 \times \frac{(n-1)_a}{\delta_{ea}})$ ③ × ④	57.257	90.001	122.057	138.000	142.972	141.600	137.144	128.056	121.715	116.913	113.144	110.400	106.970
⑥ Lag Ang = 360 − L Ang from Figure T-1	39	61	80	95	109	120	130	138	144	149	154	160	166
⑦ L $A_{\delta e}$ (deg) 90° − ⑥	51	29	10	−5	−19	−30	−40	−48	−54	−59	−64	−70	−76

CALCULATIONS FOR EFFECT OF ELEVATOR FREQUENCY ON PHASE AND AMPLITUDE OF PITCHING VELOCITY PER AMPLITUDE OF CHANGE IN ELEVATOR ANGLE, COMPUTED FOR A-26 AIRPLANE

TABLE T-2

(1) Ω_E (plot)	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	7	8
(2) $TA\,\Omega_E = \omega_e = 5.9839 \times$ (1)	5.9839	8.9758	11.9678	14.9598	17.9517	20.9436	23.9356	26.9276	29.9195	32.9114	35.9034	41.8873	47.8712
(3) $(-\omega_e)^2 =$ (2)^2	35.8071	80.5650	143.2282	223.7956	322.2635	438.6344	572.9129	725.0956	895.1765	1083.1602	1289.0541	1754.5459	2291.6518
(4) $g\omega^2 = (-4.8701)^2$	23.7179												
(5) $g\omega^2 + (-\omega_e)^2 =$ (4) + (3)	59.5250	104.2829	166.9461	247.5135	345.9814	462.3523	596.6308	748.8135	918.8944	1106.8781	1312.7720	1778.2638	2315.3697
(6) $-\omega_e b = -$ (2) $\times$ 25.23	-150.9738	-226.4594	-301.9476	-377.4358	-452.9214	-528.4070	-603.8952	-679.3833	-754.8690	-830.3546	-905.8428	-1056.8166	-1207.7904
(7) $(-\omega_e b)^2 =$ (6)^2	22793.09	51283.86	91172.35	142,457.78	205,137.79	279,213.96	364,689.41	461,561.67	569,827.21	689,488.76	820,551.18	1,116,861.33	1,458,757.65
(8) $k - \omega_e^2 = 356.18 -$ (3)	320.373	275.615	212.952	132.384	33.916	-82.454	-216.733	-368.916	-538.996	-726.98	-932.874	-1398.360	-1935.472
(9) $(k - \omega_e^2)^2 =$ (8)^2	102638.86	75963.63	45348.55	17525.52	1150.30	6798.66	46973.19	136,099.02	290,516.69	528,499.92	870,253.90	1955410.69	3,746,051.86
(10) $(-\omega_e b)^2 + (k - \omega_e^2)^2 =$ (7) + (9)	125,431.95	127,247.49	136,520.90	159,983.30	206,288.09	286,012.62	411,662.60	597,660.69	860,343.90	1,217,988.68	1,690,805.08	3,072,272.02	5,204,809.51
(11) $\frac{(g\omega)^2 + (-\omega_e)^2}{(-\omega_e b)^2 + (k - \omega_e^2)^2} =$ (5)/(10)	.00047456	.00081953	.00122286	.00154712	.0016773	.0016165	.0014493	.0012529	.0010681	.00090878	.00077642	.00057881	.00044485
(12) $\sqrt{(11)}$	.02178	.02863	.03497	.03933	.04095	.04021	.03807	.03540	.03268	.03015	.02786	.02406	.02109
(13) $\dot{\theta}_a/\delta_{ea} = -m_{\delta_e}\sqrt{(11)} = -(-307.25) \times$ (12)	6.6919	8.7966	10.7445	12.0841	12.5819	12.3545	11.6970	10.8766	10.0409	9.2636	8.5600	7.3924	6.4799
(14) $\frac{q_a}{\delta_{ea}} = \dot{\theta}_a/\delta_{ea} \times \frac{1}{TA} =$ (13) $\times$ .16712	1.1184	1.4701	1.7956	2.0195	2.1027	2.0647	1.9548	1.8177	1.6780	1.5481	1.4305	1.2354	1.0829
(15) $\omega_e(k - \omega_e^2) =$ (2) $\times$ (8)	1917.08	2473.87	2548.67	1980.44	608.85	-1726.8836	-5187.6344	-9934.0225	-16126.49	-23925.93	-33493.36	-58573.52	-92453.37
(16) $g\omega(\omega_e b) = -4.8701 \times -$ (6)	-735.26	-1102.88	-1470.52	-1838.15	-2205.77	-2573.39	-2941.03	-3308.66	-3676.29	-4043.91	-4411.55	-5146.80	-5882.06
(17) $-[\omega_e(k - \omega_e^2) + g\omega\,\omega_e b] =$ (15) + (16)	+1181.82	+1370.99	+1078.05	-142.29	+1596.92	+4300.27	+8128.66	+13242.68	+19802.78	+27969.84	+37904.90	+63720.32	+98535.43
(18) $g\omega(k - \omega_e^2) = -4.8701 \times$ (8)	-1560.25	-1342.27	-1037.10	-644.72	-165.17	401.56	1065.51	1796.66	2624.96	3540.47	4543.19	6810.15	9425.94
(19) $\omega_e^2 b = 25.23 \times$ (3)	903.41	2032.65	3613.65	5646.36	8130.71	11066.15	14454.59	18294.16	22585.30	27328.13	32522.83	44267.19	57818.37
(20) $-[g\omega(k - \omega_e^2) + \omega_e^2 b] =$ (19) - (18)	+2463.66	+3374.92	+4650.75	+6290.08	+8295.88	+10665.19	+13399.08	+16497.50	+19960.34	+23787.66	+27979.64	+37457.03	+48392.43
(21) $\tan \angle A_{q\delta_e} =$ (17)/(20)	.47970	.40623	.23180	.02262	-.19250	-.40321	-.60666	-.80271	-.99211	-1.17581	-1.35473	-1.70116	-2.03617
(22) $\angle A_{q\delta_e} = \tan^{-1}$ (21) (deg)	205.627	202.108	193.051	181.296	169.104	158.040	148.756	141.246	135.227	130.380	126.283	120.448	116.156