AN AUTOMATIC CURVE FOLLOWER
FOR
THE ROCKEFELLER DIFFERENTIAL ANALYZER
by
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Finally, the author wishes to thank his wife who typed this thesis report. Her many sacrifices have been a continual source of inspiration.
TO MY

MOTHER AND FATHER
ABSTRACT

The Rockefeller Differential Analyzer at the Massachusetts Institute of Technology is an analogue type computer capable of evaluating the solutions of ordinary differential equations. Rotating shafts, whose positions represent the values of the variables involved, are interconnected by means of mechanical computing units in a manner dictated by the equations to be solved. One type of mechanical unit is an input function unit which relates by any arbitrary function the rotational motion of one shaft to the given rotational motion of a second. The relationship is established by allowing the given shaft motion to rotate a drum on which the arbitrary function is plotted in cartesian coordinates while an operator keeps an index on the curve by rotating the output shaft. This thesis has developed a means of following the plotted curve automatically.

A photoelectric pickup unit receives pulses of light from a small segment of the line. The voltage pulses across the phototube plate resistor are electronically limited and gated such that a train of voltage pulses results whose widths are a linear function of the pickup position. The d.c. component of this pulse train is applied to the field circuit of a d.c. motor so as to control the position of the pickup and thus
cause it to follow the line.

Tests have shown that the unit will follow a pencil line of 56 degrees slope to within ± 0.005 inches. The system requires no special preparation of the function paper and is relatively insensitive to variations in contrast between the line and the paper.
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CHAPTER I
INTRODUCTION

The development of mechanized computation has gained considerable impetus over the past two decades. More and more man has learned of the worth of these machines in performing routine calculations which heretofore had hindered his progress by retarding the effective use of his mind. Having undergone a slowly developed infancy, this science experienced a period of very rapid growth necessitated by the needs of the recent war. Now it enjoys a period of intellectual maturity characterized by such studies as the stability, reliability, and economics of large scale computing machines.

These latter problems are most interesting. Their study is essential to the science because the answers to them are the criteria by which such computers are evaluated. The ability of mathematical machines to perform certain computations accurately and automatically and to thus allow a saving of human time and labor is the mark of their worth.

An excellent example of such a computer is the Rockefeller Differential Analyzer*2 at the Massachusetts Institute of Technology. Completed in 1942 under the direction of Bush and

* Superscript numerals refer to the bibliography on page 67.
Caldwell, this machine is capable of giving the numerical solutions of ordinary differential equations quickly and with good accuracy (from about .05% to about 1% depending upon the equations). An essential component of the differential analyzer is an "input function unit", which relates by any arbitrary function the rotational motion of one shaft of the machine to the given rotational motion of a second shaft by allowing the given shaft motion to rotate a drum on which the arbitrary function is plotted while an operator keeps an index on the curve by rotating the output shaft. This thesis develops a device that performs this function; i.e., "follows" the plotted curve, automatically.

In order to establish the problem more clearly for the reader, a brief outline of the use of differential analyzers in analogue computation followed by a review of past automatic curve followers will be given here.

**Analogue Computation by Means of Differential Analyzers**

Generally speaking, computational devices may be subdivided into two types: "analogue computers", and "digital computers". They are defined subsequently.

If two or more physical systems are described by equations of the same mathematical form, the solution of the equations may be interpreted to define the action of any of the systems.
Such systems are said to be analogous; one is the analogue of the other. Now, since analogous physical systems are governed by the same equations, it is theoretically possible to study a problem by means of any of its analogues. Therefore, the most gainful procedure would be to set up the analogue in that system whose parameters and variables are most readily measured and controlled. This is the principle of analogue computation. More specifically, an analogue computer may be defined as a physical system characterized by algebraic, differential, and integral equations of the same mathematical form as the physical system (or mathematical system) to be studied, and possessing the advantage that its own variables and parameters are more easily measured and controlled. A digital computer, on the other hand, may be defined as a device that adds, subtracts, multiplies, and divides numbers directly according to certain input directions. An analogue computer is a "measuring" device; a digital computer is a "counting" device.

The differential analyzer is an analogue type computer in which the values of the variables involved are represented by the positions of rotating shafts. These shafts are interconnected by means of mathematical units in a manner dictated by the equations to be solved such that the
completed interconnection is an analogue of the given mathematical system. Thus to obtain the desired numerical results it is merely necessary to measure the angular rotations of the shafts whose motions represent the desired functions and record each as a function of the motion of the one shaft that represents the independent variable.

The mathematical units referred to above may be considered as "function generating" equipment; that is, in interconnecting shafts to form the mechanical analogue each unit "generates" an output shaft motion that bears a definite mathematical relationship to the motion of its input shaft or shafts. Table I, for example, lists certain components of a mechanical differential analyzer and the function each generates.

<table>
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<th>Computing Element</th>
<th>Input Variable(s)</th>
<th>Function Generated</th>
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<tr>
<td>Integrator</td>
<td>$x, y$</td>
<td>$\int y , x , dy$</td>
</tr>
<tr>
<td>Adder</td>
<td>$x, y$</td>
<td>$x + y$</td>
</tr>
<tr>
<td>Gear box</td>
<td>$x$</td>
<td>$(\text{constant}) \cdot x$</td>
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Table I. Functions Generated by Certain Components of a Mechanical Differential Analyzer.

For generating functions not listed in Table I, two methods are available. If the desired function is the solution of a known differential equation it is possible to interconnect the basic elements listed above to solve this defining equation
and so to obtain the desired function. However, often times there is need to generate empirical, experimental, or other functions that are not conveniently defined analytically. To accomplish this an "input function unit" is used. Such a unit is capable of producing an arbitrary output shaft motion \( f(x) \), given the function \( f(x) \) and the input shaft motion \( x \).

One of the three input units used by the Rockefeller Differential Analyzer is shown in Fig. 1. The shaft motion representing the argument \( x \) is allowed to rotate a drum on which a curve of the desired function is plotted in cartesian coordinates while an operator, using a handwheel to drive an index parallel to the axis of the drum, keeps the index on the curve. The motion of the handwheel, then, represents the function \( f(x) \).

Now, it is quite possible to follow such a plotted curve very accurately by the manual means described -- in fact with the same accuracy as the automatic curve follower to be discussed in the following pages. However, and this is the main argument for installing an automatic device to perform this function, the human operator will not constantly maintain this accuracy. To follow a curve for any length of time is an unquestionably wearying task and the intentness of the operator quickly diminishes with time. Moreover, other
**Fig. 1. General View of the Input Function Unit**
uncontrollable factors such as distractions or even the particular mood the operator happens to be in considerably affect the following accuracy. On the other hand, an automatic follower would not experience these fatigue effects, nor would its ability be as unpredictable. Besides all this, an obvious argument in favor of using automatic curve followers on the differential analyzer is, of course, that a reduction in the number of required operators would be effected. All of these considerations, therefore, lead to the desirability of converting the present input function units of the Rockefeller Differential Analyzer from manual to automatic operation.

**Development of Automatic Curve Followers**

The need for an automatic curve follower was evidently realized quite soon after the differential analyzer was developed because the first curve follower, described in 1936 by Hazen, Jaeger, and Brown, was used on the first differential analyzer. The curve $f(x)$ is plotted on a horizontal table and the lower half blackened in. A screw mounted in a horizontal position perpendicular to the motion of the table supports a lamp and a slit through which the photocell views the table. As the table moves proportional to the argument $x$ the output of the photocell controls the motion of the screw so that the slit follows the boundary between the highly reflecting and blackened light-absorbing surfaces, the motion of the screw being taken as
representative of the function $f(x)$. The device is reliable and successful - in fact, two present day differential analyzers (one at the University of Pennsylvania, the other at the Aberdeen Proving Ground, Maryland) employ essentially the same scheme. However, this method suffers a decided disadvantage in that a considerable amount of time is required to prepare the curve for the follower. Furthermore, it is impossible to plot more than one curve at a time on the function table, thus necessitating a replacement of the function paper after each curve is used.

Another scheme tried on the original differential analyzer is to cause a shaft whose angular rotation is proportional to the argument $x$ to rotate a metal drum onto which the function $f(x)$ is engraved. A needle point to which a light coil in a magnetic field is attached controls its own axial position by sensing any deviation from the groove. Obviously, such a technique is economically justifiable only if the required number of analogue solutions using the same input function is of the order of hundreds or more.

In 1949, R. A. Arrison, Jr. completed a master's thesis research project at M.I.T. wherein he designed and constructed an automatic curve follower that did successfully follow a curve plotted on a function unit of the type shown in Fig. 1. A lens focuses the image of the line onto the plane of a half-disc rotating in front of a photocell. The transmitted light,
and therefore the photocell current, varies in an approximately sinusoidal manner. Sufficient information is gained from its amplitude and phase to control the position of the pickup and thus to cause it to follow the line. Although Arrison's work produced significant results, the unit was not perfected for the following reasons. First, since the amplitude of the modulated light depends not only on the magnitude of the error between the curve and the optical center of the pickup but also on the slope of the line, relatively complex electronic circuits are required to interpret the photocell signal. Secondly, the slightest deviation from axial symmetry in the optical system generates an unwanted light modulation at the signal frequency. Finally, the magnitude of the signal varies with the reflectivity of the paper and the absorptivity of the plotted black line.

A photoelectric technique whereby mechanically produced light pulses are modulated in width by the line position seems most promising. Considerable variations in light intensity have no effect on the system since over a wide range the circuits are insensitive to the pulse amplitudes. The development and testing of a curve follower utilizing these ideas is now described.
CHAPTER II
AN AUTOMATIC CURVE FOLLOWER

If linear methods of synthesis are to be used in the servomechanism design, the sensing unit of an automatic curve follower must have an output that is: (a) continuous, and (b) linearly related to the following error. However, pulsed data may be used if the pulse repetition frequency is many times the highest expected signal frequency so that for all practical purposes the information flow may be considered as continuous. In the system developed here a phototube receives pulses of light that are modulated in width proportional to the distance between the right edge of the line and a fixed reference point on the pickup. The repetition frequency is 408 pulses per second. Error in one direction causes the pulse widths to decrease; error in the other direction causes them to increase. Therefore, if the voltage pulses appearing across the phototube plate load resistor are coupled to an electronic limiter so that the resulting pulse amplitudes are held constant, the d.c. component of the pulse-train will be a linear function of the following error. Finally, if the d.c. component of the pulses at a pulse width midway between the maximum and minimum widths is taken as a reference, and if the distance between the
right edge of the line and the reference point on the pickup corresponding to this pulse width is defined as zero following error, then at all times the difference between this reference voltage and the d.c. component of the pulses will be in magnitude and sign proportional to the following error. This differential voltage may then be amplified and applied to the field circuit of a d.c. motor so as to control the position of the pickup and thus cause it to follow the line.

System Design

A single line block diagram of the automatic curve follower is shown in Fig. 2. An outline of the design will be followed by a detailed discussion of the electrical and mechanical components.

Reflected light from a small segment of the curve is magnified and focused onto a thin plate which has a circular aperture 0.205 inches in diameter centered along the optical axis of the lens. (Refer to Fig. 3.) Rotating at constant speed between the aperture plate and an electron-multiplier type phototube is a disc containing 12 small holes near its periphery. Each hole is 0.013 inches in diameter. The light transmitted to the phototube, then, has the negative
Fig. 2. Single Line Block Diagram of the Automatic Curve Follower.
Fig. 3. Sketch of Scanning Mechanism
of the waveshape shown in Fig. 4a. The intensity of the light is zero when none of the scanning holes are coincident with the aperture area, increases suddenly when a scanning hole enters the area, decreases momentarily as the hole passes over the image of the line, and returns to zero again as the hole leaves the area. Voltage pulses are therefore developed across the phototube plate load resistor at a repetition frequency of 408 pulses per second and are coupled by means of a cathode follower to an electronic limiter circuit. The action of the limiter is twofold; first, it causes the system to be relatively insensitive to changes in the amplitude of the light pulses or to variations in the absorptivity of the black line, and secondly, it acts as a pulse-shaper. The limiter output pulses (Fig. 4b) are coupled to the control grid of a gate tube whose suppressor grid is biased at cathode potential. Hence, the gate is normally opened. However, the limiter pulses are also coupled to a differentiating and triggering circuit which produces positive triggers (Fig. 4c) that are coincident in time with those two portions of the limiter pulses characterized by highly negative slopes. The first trigger of each pair actuates a cathode-coupled monostable multivibrator that is adjusted to produce negative pulses (Fig. 4d) slightly longer than the limiter pulses. These are
Fig. 4. Composite Sketch of Pulse Waveshapes.
coupled to the suppressor grid of the gate tube. Thus, the gate is opened for the first portions of the limiter pulses, acting like a pulse amplifier during these times, and is closed by the action of the multivibrator pulses. Note, therefore, that the width of the resulting gate tube pulses (Fig. 4e) is proportional to the distance between the right edge of the line image and the right edge of the circular aperture.

A diode clamping circuit coupled to the output of the gate tube clamps the base of the resulting pulse-train to ground potential so that the d.c. component is directly proportional to the pulse widths. Finally, these pulses are passed thru a low-pass filter which attenuates the a.c. components and couples the d.c. component to the input of a d.c. amplifier. The d.c. error voltage to the servo loop, therefore, is a push-pull voltage proportional to the difference between the d.c. component of the pulse-train and a reference voltage which is the d.c. component of the pulses at zero following error.

The actions of the remaining system components are quite conventional. A field controlled d.c. motor is connected thru a gear-train to the pickup unit thus completing the servo loop. Stabilization is accomplished by the simultaneous action of a phase lead network and a magnetic damper
attached to the motor shaft.

The following chapter discusses the system components in greater detail.
CHAPTER III
SYSTEM COMPONENTS

The function unit of the Rockefeller Differential Analyzer shown in Fig. 1 is designed for use either as an input unit or as an output unit. If the index employed when the unit is used as an input device is replaced by a pen and if the shaft motion $x$ is allowed to rotate the drum while the shaft motion $y$ rotates the lead-screw which imparts linear motion to the pen, a plot of $y$ versus $x$ will be recorded on the drum. As in the other units of the Rockefeller Differential Analyzer these motions are transmitted by means of servomechanisms. This point is brought forward here because for reasons of economy and convenience some components of the pen-drive servomechanism are utilized in the automatic curve follower. Referring to Fig. 2, therefore, all of the components are newly designed elements of the automatic curve follower with the exception of the d.c. servo amplifier, the d.c. motor, and the mechanical gearing. These three components are also part of the pen-drive servomechanism referred to above, and as such were designed previously.

With the exception of the phototube, miniature tubes have been used throughout the system design. These tubes afford a saving of space, yet have voltage and current ratings similar to the larger types.
The power supply problem has been minimized by designing the circuits to utilize the voltages already available at the function unit panel. A special power supply is required, however, to furnish one milliampere at -1000 volts to the dynode resistors of the electron-multiplier phototube.

**Pickup Unit**

The pickup unit is given in block form in Fig. 5. A photograph of the unit mounted on the lead-screw of the input function table is shown in Fig. 6, and a front-view close-up is shown in Fig. 7.

![Diagram of Pickup Unit](image)

*Fig. 5. The Pickup Unit*
Fig. 6. General View of the Pickup Unit
Fig. 7. Front-View Close-Up of the Pickup
Mechanical System

Since the d.c. component of the gate tube output pulses is to be proportional to the pulse widths only, the pulse repetition frequency must be held constant. This is accomplished by using a synchronous motor to drive the scanning mechanism. A 3-phase, 400 cycle Kollsman synchronous motor was available for experimental purposes and is adapted here for 60 cycle, single-phase operation by applying reduced voltage to one pair of stator terminals and connecting a 10μfd capacitor from one of these to the third terminal. The dimensions of the scanning gears are such that the light pulses occur 408 times each second when the motor is thus running synchronously at 3600 rpm.

The details of the scanning mechanism have already been given in Fig. 3. A close-up photograph of the scanning gears is shown in Fig. 8.

Optical System

Some of the optical dimensions are illustrated in Fig. 9. In Fig. 9, the 16 mm, compound, microscopic objective lens used by the system to focus the image of the line onto the aperture plate is represented by an equivalent simple lens of the same focal length. The relationship between the dimensions d and y is:

\[ d = \frac{(5.4)y + (10.6)^2}{y - 26.6} \]
**Fig. B. Close-Up Photograph of the Scanning Gears**
Notes: Dimensions are in mm.

a — First focal point of the equivalent simple lens.

b — 16 mm., compound, microscopic objective lens.

c — Lens-support flange.

d = \frac{(5.4)y + (10.6)^2}{y - 26.6}

Fig. 9. Sketch Showing Some of the Dimensions of the Optical System.
Therefore, if the distance \( y \) is set at approximately 75 mm, then for the image to be in focus on the aperture plate the working distance \( d \) needs to be about 10.7 mm. The magnification of the line is about 3X when the dimensions are so set. The proper procedure for adjusting these optical quantities will be discussed in Chapter IV.

Two 2.2-volt pocket flashlight bulbs are mounted on the pickup unit to illuminate the drum surface in the immediate view of the lens.

Electrical System

A schematic of the electrical circuit of the pickup is given in Fig. 10. In Fig. 10, \( V_1 \) is a 10-stage electron-multiplier phototube operating at approximately 100 volts per dynode stage. The dynode potentials are established by means of dropping resistors connected to the -1000 volt supply. In order that these potentials be effectively independent of the signal currents, the quiescent current through the dropping resistors should be at least ten times the largest expected value of signal current. Here the total resistance is 0.9 megohms, or 0.1 megohms per stage. Potential between the anode and the 9th dynode (ground) is established by connecting the anode to the +250-volt supply through a 2 megohm resistor. With these voltages so set, therefore, the peak phototube current is approximately 100 microamperes. \( V_2 \) is a
Fig. 10. Circuit Diagram of Photoelectric Pickup
Fig. 11. Oscillographic Display of the Pulse Waveshape at the Cathode Follower Output. (The pulse amplitude is approx. 95 volts; the pulse width is approx. 1250 µsec.)
cathode follower coupled directly to the phototube $V_1$.

A photograph showing the pulse waveshape at the cathode follower output is given in Fig. 11.

**Limiter**

The limiter used to shape the pickup output pulses is shown in block form in Fig. 12, and the corresponding circuit diagram is given in Fig. 13.

![Diagram](image)

*Fig. 12. Limiter*

In Fig. 13, the output of the cathode follower $V_2$ is capacitor-coupled to the limiter in order to isolate the quiescent variables of each circuit. Point $a$ is biased to a positive potential with respect to ground. Thus, the grid-to-cathode voltage of the left half of $V_3$ is normally zero because of the large (1 megohm) grid resistor. The potential of the plate $P_1$ is about 20 volts and the limiter output voltage is also about 20 volts. Hence, when the potential of point $a$
Fig. 13. Circuit Diagram of the Limiter

From Cathode Follower of Fig. 10.

To Control Grid of V4 in Fig. 20.

To Differentiator in Fig. 16.
Fig. 14. Oscilloscopic Display of the Pulse Waveshape at the Limiter Output. (The pulse amplitude is approx. 100 volts; the pulse width is approx. 1200 μsec.)
is driven negatively by the action of a pulse from $V_2$ to about -2 volts or lower, the left half of $V_3$ cuts off, and the grid-to-cathode voltage of the right half of $V_3$ increases until grid current flowing through the 1 megohm plate resistor of the left half limits its value at essentially zero volts. For these conditions, the limiter output voltage is about 150 volts. Thus, if point $a$ is at zero volts or higher, the limiter output is about 20 volts; and if point $a$ is at about -2 volts or lower, the limiter output is approximately 150 volts. The 1500 ohm, common-cathode resistor introduces current feedback that aids in increasing the rate of change of the output voltage with respect to the input voltage in the transition region. The circuit is adjusted by means of $R_1$ so that the pickup output pulses are limited about a voltage level between their "crests" and "troughs". In this way the system is made relatively insensitive to variations in contrast between the black line and the white paper.

A photograph showing the waveshape of the limiter output pulses is given in Fig. 14.

Gate Trigger Circuits

The function of the gate trigger circuits is to cut-off the gate tube when the limiter pulse experiences the first point of negative slope and to hold the gate tube cut-off until the limiter
pulse is over. This is accomplished by means of a differentiating and triggering circuit which actuates a cathode-coupled, monostable multivibrator. The multivibrator couples negative pulses to the suppressor grid of the gate tube. By adjusting the length of time the multivibrator remains in its unstable state to be slightly longer than a limiter pulse, the gate tube is assured to be cut-off until the limiter pulse is over. These elements are shown schematically in Fig. 15, and the circuit diagram is given in Fig. 16.

**Fig. 15. Gate Trigger Circuits**

In Fig. 16, negative pulses corresponding to the rapidly decreasing portions of the limiter pulses are generated at point b by the action of an RC differentiator. These cut-off
From Limiter Circuit of Fig. 13.

To Suppressor Grid of \( V_4 \) in Fig. 20.

---

**Fig. 16.** Circuit Diagram of the Gate Trigger Circuits
Positive pulses at point \( b \) corresponding to the rapidly increasing portions of the limiter pulses are held to a small value by the low forward resistance of a germanium crystal diode shunted across the differentiator resistor. Also, the 1 megohm grid resistor of \( V_{5b} \) prevents these small positive pulses from driving the grid of that tube positive. Hence, only positive triggers appear at the plate \( P_2 \).

\( V_6 \) is a cathode-coupled multivibrator, the right half of which is normally conducting since its grid circuit is returned to the +250 volt supply. This current flowing through the 10,000 ohm cathode-resistor establishes the cathode at about +85 volts with respect to ground. The left half of \( V_6 \) is, therefore, cut-off since its grid \( G_1 \) is biased at +50 volts with respect to ground. When a positive trigger is generated at the plate of \( V_{5b} \), \( G_1 \) is driven positively with respect to its cathode, the left half of \( V_6 \) conducts, and the right half of \( V_6 \) is cut-off by the simultaneous action of the increased current through the common-cathode resistor and the negative pulse coupled to its grid from the plate of the left half.

The length of time the multivibrator remains in its unstable state is determined by the quiescent potential of \( G_1 \) and the time constant of the grid circuit of the right half of \( V_6 \).
Fig. 17. Oscilloscopic Display of the Pulse Waveshape at the Trigger-Tube Output. (The pulse amplitude is approx. 28 volts. For the case shown, the time between the two pulses is approx. 700 µsec.)
Fig. 13. Oscilloscopic Display of the Pulse Waveshape at the Multivibrator Output.
(The pulse amplitude is approx. 70 volts; the pulse width is approx. 1300 µsec.)
Here these values are such that the multivibrator pulses are approximately 1300 microseconds wide.

A photograph showing the waveshape of the positive triggers appearing at the plate of $V_{5b}$ is given in Fig. 17. Fig. 18 shows the waveshape of the multivibrator output pulses.

**Gate Circuit**

The block diagram of the gate circuit is shown in Fig. 19, and the corresponding circuit diagram is given in Fig. 20.

---

**Fig. 19. Gate Circuit**

Since the suppressor grid of $V_4$ in Fig. 19 is biased at cathode potential, the gate is normally opened and the limiter pulses coupled to the control grid appear inverted at the plate.
From Limiter Circuit of Fig. 13.

From Gate Trigger Circuits of Fig. 16.

To Low-Pass Filter of Fig. 24.

Fig. 20. Circuit Diagram of the Gate Circuit
Fig. 21. Oscilloscopic Display of the Pulse Waveshape at the Gate Tube Output. (The pulse amplitude is approx. 65 volts. For the case shown the pulse width is approx. 700 μsec.)
Fig. 22. Plot of Servo Error Voltage Against Pickup Position Error
of $V_4$. However, the occurrence of the first negative slope portion of the limiter pulse generates a multivibrator pulse which drives the suppressor grid negatively and cuts-off the tube. The waveshape of the gate tube output pulses is shown in Fig. 21.

The diode $V_{5a}$ clamps the bases of the gate tube pulses at ground potential. Therefore, since the pulse amplitudes are fixed, the d.c. component of the pulse-train is a linear function of the distance between the right edge of the line and the right edge of the aperture. (See Fig. 3.). Therefore, if this d.c. component is compared to a reference voltage equal to the d.c. component of the pulses at a pulse width midway between the minimum and maximum widths, the result is an error voltage that is proportional to the deviation of the pickup position from that position which corresponds to the reference pulse width. A plot of this error voltage against pickup position error is given in Fig. 22.

**Differential Amplifier**

The differential amplifier is shown in block form in Fig. 23. The corresponding circuit diagram is given in Fig. 24.

In Fig. 24, three stages of RC filtering are used to attenuate the a.c. components of the gate tube pulses and couple the d.c.
component to one grid of the differential amplifier tube V7. The other grid is set at a potential equal to the d.c. component of the pulses at the zero error position of the pickup.

Fig. 23. Differential Amplifier

Hence, the output is a push-pull voltage proportional to the pickup error, and although the input is single-ended, the output is essentially balanced because of the large cathode resistors. The amplification may be set to any value from about 5 to about 40 by means of the potentiometer R2.

Servo Amplifier and Motor

As was stated previously, the servo amplifier and servo motor are not newly designed elements of the automatic curve follower but are elements of the pen-drive servomechanism of
Fig. 24. Circuit Diagram of the Differential Amplifier
the function unit and are utilized here for reasons of convenience and economy. Their function is illustrated in schematic form in Fig. 25 and the circuit diagram is given in Fig. 26.

![Diagram of Servo Amplifier and Motor](image)

**Fig. 25. Servo Amplifier and Motor**

In Fig. 26, the output of the differential amplifier is coupled to the input of the servo amplifier by means of a phase lead network. The latter is used for system stability and will be discussed in greater detail in Chapter IV. $V_8$ and $V_9$ constitute a push-pull voltage amplifier that has an amplification of approximately 50.
Fig. 26. Circuit Diagram of the Servo Amplifier and Motor
Essentially constant current is supplied to the armature circuit of the servo motor from a high impedance source, and the currents through the opposing field circuits are controlled by the beam power pentodes $V_{10}$ and $V_{11}$. Hence, the torque developed by the motor is in magnitude and direction proportional to the difference between the two field currents.

**Mechanical Gearing**

A photograph of the gear train that is used to transmit motion from the servo motor to the pickup lead-screw is given in Fig. 27. In Fig. 27, the handwheel and pulley-belt drive used in the manual operation of the function unit is clearly visible. The smaller pulley is on the shaft from which the analogue output of the unit is taken, and the gearing from this shaft to the pickup lead-screw may be set to any one of 22 values by means of a selector switch which energizes the proper combination of magnetic clutches. To convert the unit to automatic operation, the belt is removed and the servo motor, shown in the right foreground of Fig. 27, is used to drive the gear train.

The variable gearing is necessary to the analogue computation. It properly relates the scale factor of the output shaft to the plotted scale of the function being generated. Hence, any one of 22 values from 10 to 1000 turns of the output shaft
Fig. 27. Showing the variable gear train of the input function unit.
per linear inch of the pickup movement may be had. The effect of this variable gearing in the feedback loop of the curve follower servomechanism will be discussed in Chapter IV.

The Automatic Curve Follower

A complete circuit diagram of the electrical components of the automatic curve follower is given in Fig. 28. The letter designations given the important elements of the circuits in the preceding paragraphs of this chapter are repeated in Fig. 28. Hence, the diagrams are consistent.

The operation and testing of the automatic curve follower are discussed in the following chapter.
FIG. 28 CIRCUIT DIAGRAM OF THE AUTOMATIC CURVE FOLLOWER
CHAPTER IV
OPERATION AND TESTING

After the tube filaments are allowed to reach their proper operating temperatures and the power supply connections indicated in Fig. 28 are made, the scanning motor is started by adjusting the variac output to about 90 volts. (In the following discussion, remarks concerning specific components may best be traced in the system diagram of Fig. 28 or in the photograph of Fig. 6.) After the motor starts, the variac output is reduced to about 30 volts. The motor will run synchronously at this value and will not overheat. Next, the lights are turned on by means of switch $S_1$, and the pickup is moved until the axis of the optical system is approximately centered on the curve. Now, several adjustments are required prior to operating the automatic curve follower to assure that optimum conditions exist. First, the optical system is adjusted.

Adjustment of the Optical System

As a rough guess of the correct position of the pickup and lens, the lens-support flange is set at about its mid-position and the base set screw of the pickup is fixed so that the tip of the objective lens is approximately 7/16 inches from the drum surface. Next, the vertical deflection plates of an oscilloscope
are connected to the cathode follower output, the lights are adjusted so that the area in the immediate view of the lens is properly illuminated, and the pickup is moved until the oscilloscope pattern is similar to that shown in Fig. 11. Finally, the lens-support flange is rotated until the "dip" in the center of the pulse, which is caused by a scanning hole crossing the line, is of maximum amplitude. The image of the line is now in focus on the aperture plate and the optical system is properly adjusted.

Adjustment of the Electrical Circuits

First, the vertical deflection plates of the oscilloscope are connected to the output of the limiter circuit, and the limiting action is set to occur between the "crests" and "troughs" of the pickup pulses by adjusting the potentiometer $R_1$ until the limiter output pulses have a waveshape similar to that shown in Fig. 14. It remains, now, to set the zero of the amplifier circuits. The oscilloscope is connected to the output of the gate circuit and the pickup is moved from right to left until the width of the output pulses viewed on the oscilloscope screen is about one-half the maximum width. The sensing scheme is now in the center of its linear range. Hence, the potentiometer $R_3$ is adjusted until a d.c. voltmeter connected to the output of the differential
amplifier reads zero. Finally, the potentiometers $R_4$ and $R_5$ are adjusted until the quiescent grid voltages on $V_{10}$ and $V_{11}$ are balanced and equal to -12 volts. The curve follower is now properly adjusted and will synchronize on the line when current is supplied to the armature circuit of the motor.

Although the above described adjustments may seem lengthy, it should be emphasized that once adjusted the unit should remain so for a reasonable length of time.

Proper adjustment of the amplifier gain is necessary for stability. In the experimental curve follower described here the gain is adjusted by means of the potentiometer $R_2$. However, as will be pointed out when the servomechanism problem of the curve follower is discussed in the concluding paragraphs of this chapter, the required gain value is a function of the variable mechanical gearing and may be set automatically by the same selector switch that is used to select the gearing ratio.

**Results of Following Tests**

The accuracy realized by an input function unit of the type being considered depends upon two factors. The first is the accuracy with which a given function can be plotted on the function unit drum. The second is the accuracy with which the curve can be followed. It is difficult to separate the two experi-
mentally. However, the overall system accuracy can be determined by plotting a known function on the unit and recording its generation as the pickup follows the curve. If this is done for several runs, and if the maximum error at each ordinate is observed, an approximate measure of the system accuracy is had. This was done for the function \( f(x) = x^2 \), and the results are given in Fig. 29. Note that the maximum overall error is 0.011 inches and occurs at a line slope of 56°. Since the maximum lead-screw travel is 18 inches, this represents a percentage error of 11 parts in 18,000, or 0.06%. Now, it seems reasonable to assume that the maximum plotting error is at least 0.006 inches, since this is only about one-half the thickness of the line. If this assumption is accepted, then, the maximum error due to the follower at a line slope of 56° may be set at 0.005 inches.

It is interesting to note from Fig. 29 that, generally speaking, the overall error increases with slope. At high slopes it is not only more difficult to plot the curve accurately, but since the velocities and accelerations required of the servomechanism are larger, it is also more difficult to follow it closely. This effect is not peculiar to automatic following, but is also observed in manual operation. In manual following, the human is the servomechanism and the same lags due to high velocities and rapid accelerations occur. Hence, whenever maximum accuracy is
Fig. 29. Plot of the Maximum Error Generated at Various Ordinates for 5 Runs of Following $f(x) = x^2$. 

Conditions: Pencil line approx. 0.012" wide
Ordinate scale-- 0.1 unit/inch
Abscissa scale-- 0.1 unit/inch
required of the function unit, it would be advisable to scale the function plot to avoid slopes greater than, say, 60°.

Another criterion by which an automatic curve follower may be evaluated is the maximum following speed it can attain at a given line slope. Table II lists the results of three test-runs at slopes of 30, 60, and 75 degrees. Although the normal operating speed of the curve follower is much lower than the values listed, the maximum speed of the unit at a given slope is an indication of its reserve capabilities.

<table>
<thead>
<tr>
<th>Line Slope (degrees)</th>
<th>Maximum Drum Speed (in./min.)</th>
<th>Corresponding Maximum Pickup Speed (in./min.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>33.2</td>
<td>19.2</td>
</tr>
<tr>
<td>60</td>
<td>10.7</td>
<td>18.6</td>
</tr>
<tr>
<td>75</td>
<td>4.64</td>
<td>17.3</td>
</tr>
</tbody>
</table>

Table II. Maximum Speed of the Automatic Curve Follower at Three Values of Line Slope.

The Servomechanism Problem

The block diagram of the automatic curve follower considered as a servomechanism is given in Fig. 30. The servo loop consists of one integration due to the servo motor and three time lags due to the three stages of RC filtering. Hence, without the phase advances afforded by the phase lead network and the magnetic viscous damper, the system would be unstable.
Fig. 30. Block Diagram of the Automatic Curve Follower Servomechanism
Insufficient time was available during this thesis project to complete the study of the servomechanism problem. However, the system equations were formulated, the required parameters were measured, and a design of the compensating elements was carried out. Qualitative tests have shown this design to be sound. A marked improvement in system performance was noted. The results of this work are summarized below.

The variable quantities indicated in Fig. 30 are functions of frequency \( \omega \) and are defined by the following relationships.

(The time lag due to the motor field circuit is neglected).

\[
\begin{align*}
1) \quad x_1(j\omega) &= \frac{\theta(j\omega)}{2\pi T_d} \tan \lambda . \\
2) \quad \xi(j\omega) &= x_1(j\omega) - x_0(j\omega). \\
3) \quad e_1(j\omega) &= K_1 \xi(j\omega). \\
4) \quad e_2(j\omega) &= \frac{A e_1(j\omega)}{B(j\omega)^2 + C(j\omega)^2 + D(j\omega) + 1} . \\
5) \quad e_3(j\omega) &= K_2 e_2(j\omega) . \\
6) \quad e_4(j\omega) &= \frac{(1 + \alpha T_e j\omega) e_3(j\omega)}{\alpha(1 + T_e j\omega)} . \\
7) \quad Q_0(j\omega) &= \frac{(K_3/\tau_p + R_f)(1 + \frac{s}{\tau_a} j\omega) e_4(j\omega)}{J_m(1 + r)(j\omega)^2(1 + \frac{s}{(1+r)\tau_a} j\omega)} .
\end{align*}
\]
(8) \[ x_{\omega}(j\omega) = \frac{Q_{\omega}(j\omega)}{2\pi T_s} \]

where,

\[ x_{\omega} = \text{equivalent linear movement of the line} \]
\[ \phi = \text{input shaft rotation to the drum} \]
\[ T_d = \text{drum gearing factor} \]
\[ \lambda = \text{angle of line to the vertical} \]
\[ A = \text{filter constant} = 0.585 \]
\[ B = \text{"} = 9.03 \times 10^{-8} \]
\[ C = \text{"} = 7.55 \times 10^{-5} \]
\[ D = \text{"} = 1.76 \times 10^{-2} \]
\[ \alpha = \text{lead network parameter} \]
\[ T_e = \text{time constant} \]
\[ r_p = \text{plate resistance of beam power amplifier tube} \]
\[ = 10,000 \text{ ohms} \]
\[ R_f = \text{field resistance of d.c. motor} = 750 \text{ ohms} \]
\[ J_a = \text{damper inertia} = 0.922 \times 10^{-2} \text{ in.-oz.-sec}^2 \]
\[ f_a = \text{damper viscous friction coefficient} = 3.65 \times 10^{-2} \text{ in.-oz.-sec} \]
\[ J = \text{total inertia reflected to motor shaft} = J_{\text{motor}} + n^2 J_r. \]
\[ J_{\text{motor}} = \text{inertia of servo motor and gear train} = 1.09 \times 10^{-2} \text{ in.-oz.-sec}^2. \]
\[ J_r = \text{damper rotor inertia} = 4.56 \times 10^{-4} \text{ in.-oz.-sec}^2 \]
\[ r = \text{gearing up between motor and damper rotor} \]
\[ r = n^2 J_a / J_m \]
\[ T_d = \text{damper time constant} = \frac{J_a}{(1+r)f_a}. \]
$x_0 =$ linear output movement of the pickup

$T_s =$ screw gearing factor

$K_1, K_2, K_3 =$ sensitivity constants

Therefore if the loop transfer function $KG(j\omega)$ is defined as $x_0(j\omega)/E(j\omega)$, then,

$$KG(j\omega) = \frac{AK_1 K_2 K_3 /2\pi T_s (r_p + R_f) J_m (1 + r) \alpha}{(j\omega)^2 B(j\omega)^3 + C(j\omega)^2 + D(j\omega) + 1} \left(1 + \alpha T_e j\omega \right) \left(1 + \frac{J}{(1+r)f_a} j\omega \right)$$

It should be noted here that the effective motor inertia $J_{motor}$ is theoretically a function of the setting of the gearing ratio. Different values of load and gear train inertias are reflected to the motor shaft for different combinations of gears. However, complete computations of the total inertia reflected to the motor shaft were carried out for the maximum and minimum values of gearing. The results showed that in changing from the maximum to the minimum gear ratios, the total inertia varies only by about 10%. Similarly, measurements of the gearing friction and the torque-speed characteristics of the servo motor (Fig. 31.) have shown that the total damping of the system is negligibly small regardless of the gear setting. Hence, the inertia $J_{motor}$ is considered to be independent of the variable gearing and the motor friction coefficient is neglected.
Fig. 31. Torque-Speed Characteristics of the Servo Motor
Qualitative experimental results have demonstrated the advantages of a viscous damper in this application. On a linear basis, compensation by means of a lead network alone would be quite adequate. However, non-linear effects, such as stiction and backlash, are lessened by the mechanical filtering action of the damper. Moreover, since the addition of the damper inertia \( J_a \) increases the denominator of the \( KG(j\omega) \) function by a factor \( (1 + r) \), the required electronic gain is higher and a corresponding increase in the torque per unit error is had.

The system designed here has both a lead network and a viscous damper. In this way sufficient phase lead is attainable with a reasonable damper size. Also, a variable gearing ratio is included between the motor and the damper. This is useful in determining an experimental design since the time constant of the damper can be varied by merely changing the gearing ratio.

Since the servomechanisms of the Rockefeller Differential Analyzer have an approximate bandwidth of 3 cycles per second, this specification was chosen for the design of the servomechanism for the automatic curve follower. The desired maximum value of \( \left| x_0 / x_1(j\omega) \right| \) was set at 1.1 and the values 9 and 5 were chosen for \( \alpha \) and \( n \) respectively. The results of the de-
sign give a lead network time constant of 0.102 seconds and a damper time constant of 0.0218 seconds. The corresponding value of lead network capacitance for the resistance values chosen is 0.46µfd.

An analytical representation of the system response is given in Fig. 32. The system meets the design specifications.

Although the system behavior was markedly improved by the compensating elements described, much more thought needs to be given to this phase of the work. Specifically, it is noted from equation (9) that changes in the screw gearing factor $T_a$ affect the system sensitivity. In the experimental setup the gain was adjusted by means of the potentiometer $R_2$. However, in the final system design it will be necessary to have this gain be automatically set when a gearing factor $T_s$ is chosen. This can be accomplished by mechanically ganging the gearing selector and gain selector switches. Also, the non-linear effects, which are by no means negligible, need to be studied in greater detail.
Fig. 32. Magnitude and Phase of the System Function of the Automatic Curve Follower
CHAPTER V

CONCLUSIONS

The objective of this thesis research has been to construct and develop an automatic curve follower of sufficient simplicity and accuracy that would justify its use on the input function units of the Rockefeller Differential Analyzer. This objective has been realized. A sensing scheme has been developed that will allow the unit to follow a single pencil line of 56 degrees slope to within ± 0.005 inches. By virtue of the pulse techniques used, the system is relatively insensitive to variations in contrast between the black line and the white paper. An experimental setup consisting of a photoelectric pickup and a breadboard model of the electronic circuits has been constructed and has already been used in conjunction with a function unit of the Rockefeller Differential Analyzer during several problem solutions. To the author's knowledge the automatic curve follower developed here represents an improvement over past curve followers with respect to accuracy, yet requires no special preparation of the function paper.

Suggestions for Further Study

The present status of the automatic curve follower is such that many improvements can unquestionably be made.

With respect to the pickup, the 3-phase, 400-cycle, experimental scanning motor should be replaced by a single-phase, 60-cycle motor.
Secondly, a more elegant scheme for illuminating the line segment should be considered. The present method suffers the disadvantage that the illuminated spot moves as the optical system is adjusted. Finally, for ease of transfer from manual to automatic operation, and vice-versa, it would be convenient if the pickup were mounted on a two-position turret support. One position of the turret would set the manual index in position; the other would center the optical system of the pickup over the line. This scheme also possesses the advantage that the need for an oscilloscope in setting the initial position of the pickup would be eliminated.

The present method of obtaining a d.c. voltage proportional to the width of the gate tube pulses, namely the use of an RC filter, is inadequate. It is recalled that for system stability large electronic gains are required when a large gearing-down ratio is used between the motor and the pickup lead-screw. Sufficient a.c. voltage is present at the output of the filter to cause saturation of the motor field circuits at these high gains. Also, the RC filter introduces additional time lags into the system which must be compensated for. A study could be made of other means of obtaining a d.c. voltage proportional to the gate tube pulse-width. One method that might prove useful would be to allow the gate tube
pulses to trigger on a sawtooth generator at the beginning of each pulse and clamp the generator output at the end of each pulse. Hence, the peaks of the sawtooth pulses would be proportional to the widths of the gate tube pulses, and, peak detection of the sawtooth pulses would produce the required d.c. voltage.

More study needs to be given to the servomechanism problem. A final design of a viscous damper should be undertaken and the analytical results checked by experimental tests. A means of automatically setting the proper electronic gain when a gearing-ratio is chosen should be considered. One possibility already mentioned would be to mechanically gang the gain selector and gearing selector switches.

If these improvements are undertaken, the author feels that the automatic curve follower will be a valuable addition to the Rockefeller Differential Analyzer.
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