DESIGN OF AUTOMATIC RUDDER COORDINATION SYSTEMS FOR AIRCRAFT
by
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(1944)

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ABSTRACT

In the design of automatic control systems for the lateral control of aircraft, there is usually a requirement that sideslip be minimized during both transient and steady-state maneuvers. As the demand for higher performance flight control systems evolved, it has been increasingly difficult to use a direct measurement of sideslip to control transient sideslip since the high gain and lead compensation required amplify the noise existing at the data source.

Consequently rudder coordination systems have been built utilizing for a rudder command signal some quantity which indicates that a maneuver is being initiated so that the rudder can be deflected in a manner that prevents the occurrence of sideslip. These systems have evolved from a more or less "cut and try" procedure and have operated in an open-loop manner depending upon a calibrated variation of gains with flight condition.

This thesis shows how signal flow diagrams can be used to provide a theoretical design procedure for such systems. In addition an error quantity has been derived that measures the performance of the rudder coordination system. This error quantity can be computed from the information supplied by roll and yaw rate gyros only. It has then been shown that the error quantity can be used to change the gains of the system in a closed loop manner as flight conditions or loading conditions may require. One of the secondary outputs of the system is a measure of true airspeed independent of air data inputs.
ACKNOWLEDGMENT

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Mr. Robert Bairnsfather and his group of the laboratory's analog computer department made possible the special switching set-up for the G. P. S. simulator necessary to simulate this system and were very helpful throughout the simulation program.
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<tr>
<td>( W_{(1,2)}(\text{axis}) )</td>
<td>Component of angular velocity of coordinate system 2 with respect to coordinate system 1 taken along the axis of the second subscript. For example, ( W_{(1A)}X_A ) is the ( X_A ) component of the angular velocity of the airplane with respect to inertial space</td>
</tr>
<tr>
<td>( \alpha_{(1-2)} )</td>
<td>Angle measured from direction 1 to direction 2</td>
</tr>
<tr>
<td>( X, Y, Z )</td>
<td>Orthogonal coordinate axes</td>
</tr>
<tr>
<td>( g )</td>
<td>Acceleration of gravity</td>
</tr>
<tr>
<td>( U_0 )</td>
<td>( X_A ) component of the trimmed airspeed of the aircraft</td>
</tr>
<tr>
<td>( V_A )</td>
<td>Airplane resultant airspeed</td>
</tr>
<tr>
<td>( S_{(1)}[q_{(in)}/q_{(out)}] )</td>
<td>Static sensitivity of component 1 for ( q_{(in)} ) input and ( q_{(out)} ) output</td>
</tr>
<tr>
<td>( (RF)<em>{(1)}[q</em>{(in)}]/q_{(out)} )</td>
<td>Relating function of component 1 relating the ( q_{(in)} ) input to the ( q_{(out)} ) output</td>
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<tr>
<td>( I_A(\ ) )</td>
<td>Aircraft moment of inertia about the axes listed in subscripts within the parentheses</td>
</tr>
<tr>
<td>( q )</td>
<td>Dynamic pressure</td>
</tr>
<tr>
<td>( b )</td>
<td>Wing span</td>
</tr>
<tr>
<td>( S )</td>
<td>Wing area</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass</td>
</tr>
<tr>
<td>( C_\cdot(\ ) )</td>
<td>Standard N. A. C. A. stability derivative notation</td>
</tr>
<tr>
<td>( E_0 )</td>
<td>Trimmed elevation angle of the aircraft X-axis from the horizontal plane</td>
</tr>
<tr>
<td>( \delta_a )</td>
<td>Aileron angle measured from trim position</td>
</tr>
<tr>
<td>( \delta_r )</td>
<td>Rudder angle measured from trim position</td>
</tr>
<tr>
<td>( p )</td>
<td>Laplace operator</td>
</tr>
<tr>
<td>( T_{n-m} )</td>
<td>Transmittance of signal path between points ( n ) and ( m )</td>
</tr>
<tr>
<td>( r )</td>
<td>Filter time constant</td>
</tr>
<tr>
<td>( \sigma_A )</td>
<td>Airplane angle of attack</td>
</tr>
<tr>
<td>( \beta_A )</td>
<td>Airplane angle of sideslip</td>
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<tr>
<td>( C )</td>
<td>General constant</td>
</tr>
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<td>Description</td>
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<tr>
<td>B</td>
<td>Bank</td>
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<tr>
<td>g</td>
<td>Gyro</td>
</tr>
<tr>
<td>rcc</td>
<td>Rudder coordination computer</td>
</tr>
<tr>
<td>ac</td>
<td>Aileron coordination path</td>
</tr>
<tr>
<td>rrc</td>
<td>Roll rate coordination path</td>
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<tr>
<td>I</td>
<td>Inertial space</td>
</tr>
<tr>
<td>A</td>
<td>Airplane</td>
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<tr>
<td>E</td>
<td>Earth</td>
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OBJECT

The objects of this thesis are to present a method for specifying the relating functions involved in providing automatic control of transient sideslip and to design a self-adjusting rudder coordination system for aircraft.
CHAPTER 1
INTRODUCTION

A conventional winged aircraft changes its heading by establishing an angle of roll (or bank) while keeping sideslip nearly zero. This maneuver rotates the lift vector about the roll axis of the aircraft producing a component of lift along a horizontal axis perpendicular to the earth’s local vertical. If negligible aerodynamic side-force exists, there will be no additional forces applied to the airplane along this horizontal axis, and the aircraft accordingly develops a component of acceleration in the horizontal plane resulting in a turning rate. A similar acceleration could be developed by yawing the aircraft so as to produce aerodynamic sideslip, but this is usually undesirable since only very small accelerations can be achieved in this manner, and since the lateral acceleration in this case is applied to occupants of the airplane in a direction transverse rather than parallel to their spinal axes and is hence more uncomfortable. In military aircraft the launching of projectiles or missiles is greatly simplified if the launching axis is aligned with the relative airstream at least in yaw. With the addition to the aircraft of high performance automatic flight control systems, it was also discovered that the response time necessary to establish a desired yaw angular velocity depended greatly upon the degree to which transient sideslip was minimized. For these reasons a requirement is usually placed upon the design of automatic flight control systems to minimize both transient and steady-state sideslip.

Since the aileron control surface generates predominately a rolling moment and the rudder control surface predominately a yawing moment, the aileron is the primary control for the lateral control of the airplane while the rudder is coordinated with the aileron to control sideslip. The requirement of minimizing sideslip therefore dictates the design of the rudder control system, and there have been three basic approaches to this problem in the past.
A. Closed-Loop Sideslip Feedback

The most straightforward method is to measure sideslip directly or a quantity proportional to sideslip such as lateral acceleration and feed a corresponding signal to the rudder servo to form a closed-loop control system.
Unfortunately due to the difficulty in predicting the local airflow conditions near the surface of an airplane which change with Mach number and angle of attack, it is not easy to obtain an accurate, direct measure of sideslip. In addition, with an aerodynamically clean aircraft at the smaller angles of sideslip that the system must control, the lateral acceleration may be of the order of the noise level appearing at the output of an accelerometer due to extraneous inputs such as structural vibrations.

These feedbacks produce the same effect as increasing $C_{n\beta}$ which in turn decreases the damping ratio of the lateral oscillation. This is shown in the root loci of Fig. 1-1. Part (a) of the figure presents the root locus for the yaw rate damping loop, and part (b) is the root locus for adding sideslip feedback to the damped airplane. As the sideslip loop gain is increased, the roots rise almost vertically, decreasing the closed-loop damping ratio. Increasing the yaw damping gain to counteract this trend is not too effective since it also increases the transient sideslip, which requires a still higher sideslip loop gain. Thus one is forced to try lead compensation networks if the damping is too low.

Figure 1-2 shows a simulated response of the sideslip angle of a jet interceptor to a 3-second aileron pulse. An open-loop gain requiring a ratio of rudder deflection to sideslip angle equal to 5 will still produce a peak sideslip which is 23\% of the value with no sideslip feedback. From the standpoint of rudder noise in turbulence, such a gain is already much too high. For these reasons this method of sideslip control is not effective in control of transient sideslip for high performance aircraft.

B. Computed Rudder Command

In the closed-loop control of the first method the rudder is controlled after the sideslip develops. Since the sideslip is related to the control surface deflection by the aircraft equations of motion, an alternative method is to compute the rudder required to minimize sideslip as a function of the aileron maneuvering inputs and to feed the rudder a corresponding command signal at the same time that maneuvers are initiated. In this manner better control of transient sideslip can be obtained.

Geometrically the process of controlling the airplane so that sideslip is zero is the same as giving the airplane a yaw component of angular velocity equal to the yaw component of the angular velocity of the aircraft's linear velocity vector. (Both angular velocity components are with respect to inertial space.) Thus as the airplane rolls during maneuvers, the rudder adjusts the yaw angular velocity to minimize sideslip.
F-94A, 22,000 ft, M = 0.6

No rudder servo dynamics
No rate gyro dynamics
High pass filter $r = 2.5$ sec

For

$S_{yd}(w_z, \delta_z) = 0.72$

Closed loop poles are

$p = -0.6$
$p = -0.95 \pm 1.63 j$

For

$S_{yd}(w_z, \delta_z) = 0.72$

Closed loop poles are

$p = -0.6$
$p = -0.95 \pm 1.63 j$

a) Yaw damping loop

b) Sideslip control loop

Fig. 1-1. Root loci for the yaw damping and sideslip control loops.
Sideslip vs Time

F-94A
$M = 0.6$
22,000 ft

Yaw Damping Loop
$S_{y_1}(w_2, \delta_2) = 0.72$
$r = 2.5 \text{ sec}$

Input: 3 second aileron pulse
No lead compensation

Fig. 1-2. Simulated sideslip response of an airplane with a sideslip feedback control loop.
For coordinated turn:

\[ W_{(1A)ZA} = \left( \frac{1}{D_0} \right) \sin A_B \]

Fig. 1-3. Control of sideslip through control of yaw angular velocity; system 1 — command yaw rate coordinated with commanded bank angle, no sideslip feedback.
In the past the computation of either the rudder or the yaw rate required has been a calibrated procedure which is open-loop with respect to sideslip. The gains of the coordination control path were programmed with flight condition in accordance with a preset calibration. Several approaches have been used. In one a signal proportional to aileron was modified by a filter network and sent to the rudder. The filter characteristics and gains were set by a cut-and-try procedure utilizing an analogue computer. A similar system using roll rate as an input signal has also been used. Employing control system analysis techniques, such as the root locus, for these systems is a laborious procedure since the yaw rate damping loop necessary to damp the lateral oscillation has a large effect upon transient sideslip, and the roll and yaw rates are each functions of both the aileron and rudder deflections. These analysis methods also do not provide a straightforward approach to the specification of system requirements. To provide a more rational design procedure for these systems is one of the objects of this thesis.

A similar approach to the rudder coordination problem involves a yaw rate command loop around the rudder. Examples of these are given in Figs. 1-3, 1-4, and 1-5. In the system of Fig. 1-3, the aileron is used to control bank angle by feeding back a bank angle signal obtained from a two-degree-of-freedom vertical gyro. The rudder is used to control yaw rate by feeding back a yaw rate signal obtained from a rate gyro. For a constant altitude turn the yaw rate and bank angle are related in the steady state by Eq. (1-1)

\[ W_{(IA)} \frac{z_A}{z} = \frac{g}{U_0} \sin A_B \]  

Thus the yaw rate command should be \( \frac{g}{U_0} \) times the sine of the bank angle command. This functional relation depends upon true airspeed and a trigonometric function of bank angle. In practice the system is mechanized so that the yaw rate command is a constant times the bank angle with the constant chosen for one desired bank angle at a specified flight condition such as a cruise condition. At the chosen flight conditions steady-state sideslip will be minimized and transient sideslip is greatly reduced. At other flight conditions of bank angle and airspeed, greater sideslip is accepted.

The system of Fig. 1-4 differs in the mechanization used to satisfy Eq. (1-1). Since a roll rate loop is closed around the aileron, a bank angle is generated as the integral of a roll rate command pulse. This pulse is obtained by feeding the yaw rate command signal through a derivative filter to the roll rate loop. In general the resulting bank angle will be incorrect for the commanded yaw rate. In this system the bank angle is corrected by applying a torque to the roll rate gyro proportional to the output signal of a Y-axis pendulum which in turn is proportional to aerodynamic side force. This generates a roll rate until the bank angle is such as to satisfy Eq. (1-1).
Fig. 1-4. Control of sideslip through control of yaw angular velocity; system 2 - command yaw rate coordinated with commanded roll rate, correction of roll rate with steady-state lateral acceleration.
Fig. 1-5. Control of sideslip through control of yaw angular velocity; system 3 – command turn rate coordinated with commanded bank angle, correction of turn rate with steady-state rudder deflection.
In the system of Fig. 1-5 the same general approach is used. In this case, however, a bank angle loop is used in conjunction with a yaw rate command loop as in Fig. 1-3, but differing in mechanization details. Corrections for sideslip are made in this case by correcting the yaw rate command so as to satisfy Eq. (1-1), rather than by correcting the bank angle. The stator of a signal generator mounted on a two-degree-of-freedom directional gyro is driven relative to the airplane at the desired turning rate. As the airplane turns, the rotor of the gyro remains fixed in inertial space, and the airplane carries the stator back into its null position relative to the rotor. If there is an error in rate, there will be an output of the signal generator, and this signal is fed to the rudder. The rate at which the servo motor drives the stator determines the turning rate of the airplane. If the yawing moment due to yaw rate is neglected, the rudder in trim would be at zero deflection. Therefore the rudder position signal is used to correct the turn motor rate to reduce steady-state sideslip.

These methods do not provide particularly close control of transient sideslip since they depend again upon a calibrated relationship during the turn entry.

C. Combined Systems

The third approach is merely a combination of the other two in which transient control commands are fed directly to the rudder and some form of integration of sideslip is used to control the steady-state condition.

The rudder coordination system developed in this thesis provides a method of eliminating the calibration requirements of these systems by automatic variation of system parameters on a closed-loop basis without a direct measure of sideslip.
CHAPTER 2

APPLICATION OF THE SIGNAL-FLOW DIAGRAM TO DETERMINE Rudder Coordination Requirements

The signal-flow diagram was originated by S. J. Mason (Reference 1). This diagram is a means of visualizing the flow of signals (or forces, torques, etc.) through a complex interconnected control system. It provides a valuable addition to the block diagram in analysis of system response to multiple inputs. In the signal-flow diagram each of the variables of the system is represented by a small circle, or node. Lines are drawn connecting each node between which signals are flowing. An arrow on each line indicates the direction of the flow. The transmittance, or the function relating the two nodes through the signal path, is written beside each line. The diagram is interpreted as indicating that the signal flowing in any branch is the variable from which the branch emanates multiplied by the transmittance of the branch, and the value of the variable represented by a node is equal to sum of all signals entering the node. For example the expression

\[ W_x = \left[ \frac{C_L \delta_a}{\left( \frac{I_x}{\eta S_b} \right) \rho + \left( \frac{b}{2U_o} \right) C_L \rho} \right] \delta_a = T[\delta_a - W_x] \delta_a \]  

is represented by

Feedback paths are apparent as closed loops on the diagram. Rules exist (Refs. 1, 2) for reduction of the complete system diagram to one involving a desired input and the desired outputs, which enable one to determine the essential relationship between these quantities. This is often difficult to do by other methods with complicated multiple input and output systems.

Signal-Flow Diagram for the Aircraft Lateral Response

To construct a signal-flow diagram the equations representing the system are first written in a form such that each of the dependent variables in turn is
expressed in terms of the other variables. In other words this is the expression which would be written upon inspection of a node of the signal-flow diagram.

If there are \( n \) equations involving \( n \) dependent variables, there is an arbitrary choice of which equation to use to express any one variable as a function of the others. In the case of the airplane, the following choice was made: the rolling moment equation was used to obtain an expression for roll angular velocity \( W_{(IA)X_A} \); the yawing moment equation for yaw angular velocity, \( W_{(IA)Z_A} \); and the side force equation for sideslip angle, \( \beta_A \). For the system under design here the rudder is being controlled to minimize sideslip considering the aileron to be the primary input to the airplane. Since the main contribution of the rudder is a yawing moment rather than a side force, obtaining the sideslip expression from the side-force equation rather than from the yawing moment equation means that the primary signal path between the rudder and the quantity it is controlling involves an intermediate node, \( W_{(IA)Z_A} \). However this choice does result in the advantage of keeping higher orders of the Laplace operator \( p \) in the denominators of the expressions for the transmittances. In this case the rudder is looked upon as providing the yawing moment required to generate a yaw angular velocity equal to the yaw angular velocity of the velocity vector so that sideslip will be zero. When the yawing moment equation is used to obtain sideslip, the rudder is looked upon as providing the yawing moment required to align the aircraft with the relative wind vector. Either choice will of course lead to the same end result.

Although the signal-flow diagram can give insight into the flow of forces and torques for such effects as inertial cross-coupling or nonlinear aerodynamics, the rules for reducing the diagram do not apply to a nonlinear system. Hence the linearized lateral aircraft equations are used here, and the nonlinear effects were investigated on the analog simulator.

**Coordinate Systems**

Figure 2-1 defines the coordinate systems that are used to orient the aircraft and its velocity vector with respect to the earth. Five orthogonal right-hand coordinate systems are defined:

1. the aircraft coordinates, \( A \); these are established by axes fixed to the aircraft with \( X_A \) an arbitrarily chosen longitudinal axis in the plane of symmetry positive forward; \( Y_A \) perpendicular to the plane of symmetry, positive along the right wing; \( Z_A \) perpendicular to \( X_A \) and \( Y_A \), positive down.
Fig. 2-1. Aircraft coordinate systems.
(2) the gyro-coordinates, \( \mathbf{g} \); these are established by axes fixed to the aircraft with \( X_g \) in the aircraft plane of symmetry and parallel to the roll gyro input axis, positive forward; \( Y_g \) perpendicular to the plane of symmetry, and \( Z_g \) perpendicular to the other two axes and also parallel to the yaw gyro input axis, positive down.

(3) the velocity or flight path coordinates, \( V_A \); these are established by axes fixed to the velocity vector with \( X_{V_A} \) parallel to the aircraft's velocity vector, positive forward; \( Y_{V_A} \) perpendicular to \( X_{V_A} \) in the projection plane passed through \( V_A \) perpendicular to the aircraft plane of symmetry, and \( Z_{V_A} \) perpendicular to the other two axes, positive down.

(4) the earth-airplane coordinates, \( \mathbf{E}_A \); these are derived from the aircraft axes and the horizontal plane such that \( X_{E_A} \) is the intersection of the earth horizontal plane and the vertical plane passed through \( X_A \), positive forward; \( Y_{E_A} \) is perpendicular to \( X_{E_A} \) in the horizontal plane, and \( Z_{E_A} \) is perpendicular to the horizontal plane, positive down.

(5) the velocity-plane of symmetry coordinates, \( V_A \) (PS); these are established by axes which are the same as aircraft coordinate axes when the X-axis is taken along the projection of the velocity vector into the plane of symmetry.

Equations of Motion

The aircraft equations of motion are the linearized relations commonly used to express the aircraft's behavior during small deviations from trimmed flight. The aircraft axes are assumed to be coincident with the velocity coordinate system in trimmed steady-state flight. If both the trimmed angular velocity of the aircraft with respect to inertial space and the sideslip angle are zero, then \( W(IA)X_A \) becomes the deviation in the roll angular velocity of the airplane with respect to inertial space from trimmed flight; \( W(IA)Z_A \) becomes the deviation in the yaw angular velocity of the airplane with respect to inertial space from trimmed flight; \( \beta_A \) becomes the airplane's sideslip angular deviation from trimmed flight. Also if the rotation of the earth is neglected, let

\[
W(IA)X_A = W(EA)X_A
\]

\[
W(IA)Z_A = W(EA)Z_A
\]

Finally since all components of angular velocity will be measured with respect to inertial space (or the earth by Eq. (2-2), let

25
\[ W_{(IA)} = W_{(EA)}X_A = W_X \]
\[ W_{(IA)}Z_A = W_{(EA)}Z_A = W_Z \]

Since the gyro input axes do not in general coincide with the aircraft axes, the corresponding definitions for the angular velocity inputs to the roll and yaw gyro are
\[ W_{(IG)}X = W_{(EG)}X = W_X \]
\[ W_{(IG)}Z_q = W_{(EG)}Z_q = W_Z \]

The equations of motion for the aircraft are then,

**Rolling Moment:**
\[ \left[ \frac{I_A(x)}{q^{1/2}} \right] p \left[ \frac{C_1(x)}{q^{1/2}} \right] W_X + \left[ \frac{I_A(x)}{q^{1/2}} \right] p - \left[ \frac{b}{2U_o} \right] C_{1r} W_Z A - C_{\beta A} + C_{\delta A} \]

**Yawing Moment:**
\[ - \left[ \frac{I_A(x)}{q^{1/2}} \right] p + \left[ \frac{b}{2U_o} \right] C_{n_p} W_X + \left[ \frac{I_A(x)}{q^{1/2}} \right] p - \left[ \frac{b}{2U_o} \right] C_{n_r} W_Z A - C_{n A} + C_{n A} \]

**Side Force:**
\[ - \left[ \frac{C_{L_o}}{p} \right] W_X + \left[ \frac{mU_o}{q^{1/2}} \right] W_Z A + \left[ \frac{C_{L_o} \sin E_o}{p} \right] W_Z A \]

In addition there are the equations relating the inputs to the rate gyro to \( W_X \) and \( W_Z \) and that of the rudder to its input command.

**Rate Gyros:**

The angle between the X-gyro input axis and the X-aircraft axis is \( \theta_{X} \) which is written as \( A \) when no ambiguity results. Thus
\[ W_{(IA)}X_q = W_{(IG)}X_q = W_X - W_X A \cos A - W_Z A \sin A \]
\[ W_{(IA)}Z_q = W_{(IG)}Z_q = W_Z = W_X A \sin A + W_Z A \cos A \]

**Rudder Control:**
\[ \delta_r = (RF)_{ac}[\delta_a,\delta_z] \delta_a + (RF)_{rc}[\delta_r,\delta_z] W_X + S_{\delta}[\delta_x,\delta_z] \left( \frac{r_p}{1 + r_p} \right) W_Z \]
(the dynamic effects of the rudder servo and the rate gyros have been neglected).

The Signal-Flow Diagram for the Aircraft

Equations (2-5) through (2-10) can be rewritten in the form suitable for the construction of the signal-flow diagram as follows:

Rolling Velocity: (from rolling moment equation)

\[
W_{X_A} = \frac{1}{\left[\left(\frac{1}{qSb}\right)qSb - \left(\frac{b}{2U_o}\right)C_{\phi}\right]} \left\{ C_{\phi}\delta_a + \left[\left(\frac{1}{qSb}\right)qSb \right] p + \left(\frac{b}{2U_o}\right) C_{\phi}\right\} W_{Z_A} + C_{\phi}\beta_A
\]

(2-11)

Yawing Velocity: (from yawing moment equation)

\[
W_{Z_A} = \frac{1}{\left[\left(\frac{1}{qSb}\right)qSb - \left(\frac{b}{2U_o}\right)C_{\phi}\right]} \left\{ C_{n}\delta_a + \left[\left(\frac{1}{qSb}\right)qSb \right] p + \left(\frac{b}{2U_o}\right) C_{n}\right\} W_{X_A} + C_{n}\beta + C_{n}\delta_A
\]

(2-12)

Sideslip Angle: (from side force equation)

\[
\beta_A = \frac{1}{\left(\frac{mU_o}{qS}\right)qS - C_{r}\beta}\left\{ C_{L_0}\cos E_o \right\} W_{X_A} - \left[\left(\frac{mU_o}{qS}\right)qS - C_{L_0}\sin E_o \right\} W_{Z_A} + C_{r}\delta_A
\]

(2-13)

Roll Gyro:

\[
W_{X_q} = W_{X_A}\cos \alpha - W_{Z_A}\sin \alpha
\]

(2-14)

Yaw Gyro:

\[
W_{Z_q} = W_{X_A}\sin \alpha + W_{Z_A}\cos \alpha
\]

(2-15)

Rudder Angle:

\[
\delta_r = (RF)_{ao}(\delta_a, \delta_q) \delta_a + (RF)_{tc}(W_{X_A}, \delta_i) W_{X_q} + \gamma_{ao}(W_{Z_A}, \delta_i) \left(\frac{r_p}{1 + r_p}\right) W_{Z_q}
\]

(2-16)

The coefficients of the terms in Eqs. (2-11) through (2-16) are functions of \(p\), and they define the transmittances of the signal paths. Thus,

Aileron Angle: Node 1

\[
\delta_a = q_{(in)}
\]

Rolling Velocity: Node 2

\[
W_{X_A} = T_{1-2}\delta_a + T_{3-2}W_{Z_A} + T_{4-2}\beta_A
\]

(2-17)

Yawing Velocity: Node 3

\[
W_{Z_A} = T_{1-3}\delta_a + T_{2-3}W_{X_A} + T_{4-3}\beta_A + T_{5-3}\delta_r
\]

(2-18)
Fig. 2-2. Signal-flow diagram for the aircraft lateral equations of motion.

Fig. 2-3. Signal-flow diagram for aircraft rudder coordination system.
Table 2-1. Definition of signal-flow path transmittances.

<table>
<thead>
<tr>
<th>Aileron Signal Paths</th>
<th>Yaw Rate Signal Paths</th>
<th>Rudder Signal Paths</th>
<th>Derived Transmittances</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_{1-2} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{5-4} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{1-4} = T_{1-4} )</td>
<td>( T_{3-4} = T_{3-4} + T_{3-4}T_{3-4} )</td>
</tr>
<tr>
<td>( T_{1-3} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{5-4} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-5} = T_{2-5} + T_{2-5}T_{2-5} )</td>
<td>( T_{3-5} = T_{3-5} + T_{3-5}T_{3-5} )</td>
</tr>
<tr>
<td>( T_{1-5} = (RF)<em>{\alpha}[\delta</em>{\alpha} \partial_{\alpha}] )</td>
<td>( T_{5-4} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-5} = T_{2-5} + T_{2-5}T_{2-5} )</td>
<td>( T_{3-5} = T_{3-5} + T_{3-5}T_{3-5} )</td>
</tr>
<tr>
<td>( T_{2-1} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{6-5} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-3} = T_{2-3} + T_{2-3}T_{2-3} )</td>
<td>( T_{3-4} = T_{3-4} + T_{3-4}T_{3-4} )</td>
</tr>
<tr>
<td>( T_{2-4} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{6-5} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-3} = T_{2-3} + T_{2-3}T_{2-3} )</td>
<td>( T_{3-4} = T_{3-4} + T_{3-4}T_{3-4} )</td>
</tr>
<tr>
<td>( T_{2-7} = \cos A_\gamma )</td>
<td>( T_{6-5} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-3} = T_{2-3} + T_{2-3}T_{2-3} )</td>
<td>( T_{3-4} = T_{3-4} + T_{3-4}T_{3-4} )</td>
</tr>
<tr>
<td>( T_{2-8} = \sin A_\gamma )</td>
<td>( T_{6-5} = \left[ \frac{a_{\alpha}(\dot{\gamma})}{\phi_0} \right] + (\beta) )</td>
<td>( T_{2-3} = T_{2-3} + T_{2-3}T_{2-3} )</td>
<td>( T_{3-4} = T_{3-4} + T_{3-4}T_{3-4} )</td>
</tr>
</tbody>
</table>

Note: Circled numbers are node numbers.
Sideslip Angle: Node 4

$$\beta_A = T_{2.4} W_{XA} + T_{3.4} W_{ZA} + T_{5.4} \delta_r$$  \hfill (2-19)

Gyro Roll Rate: Node 7

$$W_{Xg} = T_{2.7} W_{XA} + T_{3.7} W_{ZA}$$  \hfill (2-20)

Gyro Yaw Rate: Node 8

$$W_{Zg} = T_{2.8} W_{XA} + T_{3.8} W_{ZA}$$  \hfill (2-21)

Rudder Angle: Node 5

$$\delta_r = T_{1.5} \delta_o + T_{7.5} W_{Xg} + T_{8.5} W_{Zg}$$  \hfill (2-22)

Output: Node 6

$$q_{(out)} = \beta_A$$

The transmittances are summarized in Table 2-1.

These equations then result in the signal-flow diagram of Figs. 2-2 and 2-3. Figure 2-2 presents the signal-flow diagram for the aircraft lateral equations, and Fig. 2-3 results when the rudder coordination signal paths are added. Figure 2-3 can be reduced through the steps presented in Appendix A to obtain that of Fig. 2-4. The diagram can be further simplified if the side force due to rudder is negligibly small, and the diagram of Fig. 2-5 results. In this diagram there are two signal-flow paths to the sideslip node. These are the sideslip generated by the aircraft’s rolling and yawing motions when the rudder is controlled in accordance with Eq. (2-16). In addition there is one self-generated closed loop at the sideslip node. This self-loop has the effect of dividing each of the transmittances of the signal paths entering the node by the quantity

$$\left[ 1 - \text{(transmittance of the self-loop)} \right]$$

However, the design objective of the rudder coordination system is to cause sideslip to be zero, and this then means that the sum of the signals entering the sideslip node is zero. Since the self-loop only affects the denominator of the sum of the signals, it can be eliminated as long as the system poles remain stable.

Thus Fig. 2-5 shows that the transmittances of the roll rate to rudder path and the aileron to rudder path must be chosen so that

$$T_{2.4} W_{X} + T_{3.4} W_{Z} = 0$$  \hfill (2-23)
Fig. 2-4. Reduced signal-flow diagram for the rudder coordination system.

If $T_{5-4} = 0$
then $T_{1-4} = 0$
$T'_{2-4} = T_{2-4}$
$T'_{3-4} = T_{3-4}$
$T''_{3-4} = T_{3-4} + T_{3-2}T_{2-4}$

Fig. 2-5. Reduced signal-flow diagram for the rudder coordination system neglecting side force due to rudder.
or
\[ T_{1-2} T_{2-4} \delta_a + (T'_{1-3} + T_{1-2} T''_{2-3}) T''_{3-4} \delta_a = 0 \]  
(2-24)

thus
\[ T_{1-2} T_{2-4} + (T'_{1-3} + T_{1-2} T''_{2-3}) T''_{3-4} = 0 \]  
(2-25)

Substituting the expressions of Table 2-1 for the primed quantities gives
\[ T_{1-2} T_{2-4} \left[ \frac{T_{1-3} + T_{1-5} T_{5-3} + T_{1-2} (T_{2-3} + T_{2-5} T_{5-3})}{1 - T_{3-3} - T_{3-2} (T_{2-3} + T_{2-5} T_{5-3})} \right] \times (T_{3-4} + T_{3-2} T_{2-4}) = 0 \]  
(2-26)

This equation can be further rewritten to obtain an expression relating the aileron to rudder coordination path, \( T_{1-5} \), and the roll rate to rudder coordination path, \( T_{7-5} \), to the aircraft parameters. The general expression including the effects of climbing or diving flight is presented in Appendix C. When the aircraft is in level flight so that the elevation angle, \( \theta \), is zero, Eq. (2-26) becomes
\[ T_{1-5} = \left\{ \left( \frac{g}{U_o} \right) \left( \frac{C_{n \delta_a}}{C_{n \delta_r}} \right) \left[ 1 + \frac{C_n \delta_a}{C_{\delta_a}} \frac{I_{A(zz)}}{I_{A(xz)}} \right] \right\} \right] \times \left[ \frac{C_n \delta_a}{C_{\delta_a}} \left( \frac{C_{\delta_a}}{C_{n \delta_a}} \right) \left( \frac{I_{A(xz)}}{I_{A(zz)}} \right) \right]
\]

In Eq. (2-27), \( T_{7-5} \) is the relating function of the roll rate to rudder path and \( T_{1-5} \) is the relating function of the aileron to rudder path. If one of these relating functions is specified, Eq. (2-27) can then be solved for the relating function the second path must have if sideslip is to remain zero. Three special cases are of interest:

(a) when the rudder coordination system input is roll rate only, i.e. \( T_{1-5} = 0 \)
(b) when input is aileron motion only, i.e. \( T_{7 \rightarrow 5} = 0 \)
(c) when both coordination paths are used, but the aileron to rudder path is restricted to have the form \( S \frac{\tau p}{1 + \tau p} \)

These results in the relating functions given by Eqs. (2-28), (2-29), and (2-30) respectively.

Case (a):

\[
(RF)_{rrc[W_X, \delta_t]} = T_{7 \rightarrow 5} = \left( \frac{g}{U_o} \right) \frac{I_{A(zz)}}{qS_b} \left[ 1 + \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{I_{A(zz)}} \right] \frac{1}{\cos A} \\
- \left( \frac{b}{2U_o} \right) \frac{C_{\delta r}}{C_{\delta r}} \left[ 1 - \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{C_{\delta a}} \left( \frac{1}{\cos A} \right) \right] \frac{1}{p} \\
- \left( \frac{1}{C_{\delta r}} \right) \frac{I_{A(zz)}}{qS_b} \left[ \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{C_{\delta a}} \left( \frac{1}{\cos A} \right) \right] \frac{p}{\cos A} \\
- \left( \frac{g}{U_o} \frac{b}{2U_o} \right) \frac{C_{\delta r}}{C_{\delta r}} \left[ 1 - \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{C_{\delta a}} \left( \frac{1}{\cos A} \right) \right] \frac{1}{p} \tag{2-28}
\]

Note: The term \( \frac{g}{U_o} \tan A \) occurs in Eq. (2-27) due to the airplane yaw rate component appearing as an input to the roll rate gyro. This term has been neglected.

Case (b):

\[
(RF)_{ac[\delta_r, \delta_t]} = T_{1 \rightarrow 5} \tag{2-29}
\]

where \( T_{1 \rightarrow 5} \) is given by Eq. (2-27) with \( T_{7 \rightarrow 5} = 0 \)

Case (c):

\[
(RF)_{rrc[W_X, \delta_t]} = T_{7 \rightarrow 5} = \left( \frac{g}{U_o} \right) \frac{I_{A(zz)}}{qS_b} \left[ 1 + \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{I_{A(zz)}} \right] \frac{1}{\cos A} \\
- \left( \frac{b}{2U_o} \right) \frac{C_{\delta r}}{C_{\delta r}} \left[ 1 - \frac{C_n \delta_a}{C_{\delta a}} \frac{I_{A(zz)}}{C_{\delta a}} \left( \frac{1}{\cos A} \right) \right] \frac{1}{p} \\
- \left( \frac{g}{U_o} \right) S_{y[A,W]} \left[ \frac{1 + \left( U_o / g \right) \tan A_c p}{\cos A} \right] \tag{2-30a}
\]
\[(RF)_{ac(w_{xx}, \delta_r)} = T_{1-S} = \frac{-C_{n\delta_a} \left( \frac{I_{A(xz)}}{qSb} \right) \left( 1 + \frac{C_{p\delta_a}}{C_{n\delta_a}} \frac{I_{A(xz)}}{I_{A(xz)}} \right) p}{\frac{I_{A(xz)}}{qSb} p + \left[ -\left( \frac{b}{2U_0} \right) C_{p} - \left( \frac{I_{A(xz)}}{qSb} \frac{g}{U_0} \right) \right]}
\]

(2-30b)

\[
\begin{align*}
\text{(neglecting the term } \left( \frac{b}{2U_0} \right) C_{p} \left( \frac{g}{U_0} \right) \text{)}
\end{align*}
\]

For steady-state sideslip: use slow integration of sideslip or lateral acceleration.

The expressions for these relating functions contain constants, lead-lag terms, ideal derivative and integral terms. The integral term primarily controls the steady-state sideslip to provide the yawing moment required to balance the yawing moments due to yaw rate and due to the trim aileron deflection. In many high performance aircraft the steady-state sideslip is negligibly small. When this is not so, a slow acting integration of a direct measure of sideslip is better than an integration of roll rate.

For aircraft which do not exhibit a large yawing moment due to aileron, \(C_{n\delta_a}\), nor large products of inertia, the ideal derivative term can be neglected. Thus coordination systems based upon either of the equations of Case (a) or Case (b) give excellent performance. If \(C_{n\delta_a}\) is large however, the need for the derivative term may be difficult to fulfill in Case (a). Similarly with Case (b) there is difficulty in balancing out the aileron signal voltage corresponding to trimmed flight so that there will not be a corresponding rudder deflection. Thus Case (c) is a combination of the other two cases in which the roll rate path is providing the constant and the lead-lag terms, while the aileron path provides the lead term in the form of a high-pass filter network and thereby avoids any steady-state signal difficulties.
CHAPTER 3

DESIGN OF A CLOSED-LOOP SELF-ADJUSTING RUDDER COORDINATION SYSTEM

Proper coordination of the rudder with the aileron to minimize transient sideslip can be obtained by feeding the rudder a command signal which is a function of the roll angular velocity as shown in Chapter 2. In the past this has been done in a manner that was open-loop with respect to sideslip, and the performance depended upon a static calibration of the control path parameters. This chapter describes a method of providing closed-loop control to such a system so that the parameters will be automatically adjusted to minimize sideslip as flight conditions or the aircraft loading change.

The system under consideration is one in which only roll angular velocity is used for coordinating the rudder. The required relating function for the roll-rate to rudder control path is given by Eq. (2-28) of Chapter 2. This may be considered as the sum of four paths to the rudder: roll-rate direct, roll-rate through a lag, roll rate through a lead high-pass filter and roll rate through an integration. It was assumed that the integration requirement was met by integrating a measure of steady-state sideslip. Thus the system to be considered controls transient sideslip only.

Equation (2-28) shows that if the sensitivities of the direct, lag, and lead paths are properly adjusted, the transient sideslip is minimized. These sensitivities depend upon flight condition, the aircraft mass and aerodynamic parameters, and the characteristics of the yaw damper loop. Therefore a system is required that will automatically adjust these sensitivities on the basis of a measure of the transient sideslip occurring during maneuvering flight.

Due to the difficulty in measuring sideslip directly, an effort was made to find another quantity which was easily measurable and which not only gave a unique indication of whether or not sideslip was zero, but also gave an unambiguous indication of whether the sensitivities were set too high or too low. Such a quantity has been found in the relationship which exists between the roll and yaw angular velocities during a perfectly coordinated turn (zero sideslip).
Fig. 3-1. Orientation of the gyro input axes relative to the velocity vector.
Refer to Fig. 3-1:

If the components of the aircraft's angular velocity with respect to inertial space along the \(X_{VA}\) and \(Z_{VA}\) axes are \(W_{(IA)}X_{VA}\) and \(W_{(IA)}Z_{VA}\), then the components of angular velocity measured by rate gyros whose input axes are rotated through the angle \(A_{[X_{VA}, X_{A}]} = A_{q}\) from the velocity coordinate system are

\[
\begin{align*}
W_{(IA)}X_{A} &= W_{(IA)}X_{VA} \cos A_{q} - W_{(IA)}Z_{VA} \sin A_{q} \quad (3-1) \\
W_{(IA)}Z_{A} &= W_{(IA)}X_{VA} \sin A_{q} + W_{(IA)}Z_{VA} \cos A_{q} \quad (3-2)
\end{align*}
\]

For a constant altitude turn with zero sideslip,

\[
W_{(IA)}Z_{VA} = \left( \frac{g}{U_{o}} \right) \sin A_{A} \quad (3-3)
\]

which can be approximated as

\[
W_{(IA)}Z_{VA} = \left( \frac{g}{U_{o}} \right) \frac{W_{(IA)}X_{VA}}{p} \quad (3-4)
\]

Substitution of Eq. (3-4) into Eqs. (3-1) and (3-2) gives upon rearranging terms

\[
\begin{align*}
W_{(IA)}X_{A} &= W_{(IA)}X_{VA} \cos A_{q} \left[ 1 - \left( \frac{g}{U_{o}} \right) \tan A_{q} \right] \quad (3-5) \\
W_{(IA)}Z_{A} &= W_{(IA)}X_{VA} \cos A_{q} \left[ \left( \frac{g}{U_{o}} \right) \frac{1 + \tan A_{q}}{p} \right] \quad (3-6)
\end{align*}
\]

Eliminating \(W_{(IA)}X_{VA}\),

\[
\left[ 1 - \left( \frac{g}{U_{o}} \right) \tan A_{q} \right] W_{(IA)}Z_{A} = \left( \frac{g}{U_{o}} \right) \frac{1}{p} \left[ 1 + \left( \frac{U_{o}}{g} \right) (\tan A_{q}) p \right] W_{(IA)}X_{A} \quad (3-7)
\]

If Eq. (3-7) is multiplied by \((rp)/(1 + rp)\),

\[
\left[ 1 - \left( \frac{g}{U_{o}} \right) \tan A_{q} \right] \left( \frac{rp}{1 + rp} \right) W_{(IA)}Z_{A} = \left( \frac{g}{U_{o}} \right) \frac{1 + (U_{o}/g) (\tan A_{q}) p}{1 + rp} W_{(IA)}X_{A} \quad (3-8)
\]

Definition of an error quantity, \((\text{EQ})\)

\[
\begin{align*}
(\text{EQ}) &= \left( \frac{rp}{1 + rp} \right) W_{(IA)}Z_{A} - \left( \frac{g}{U_{o}} \right) \frac{1 + (U_{o}/g) (\tan A_{q}) p}{1 + rp} W_{(IA)}X_{A} \quad (3-9)
\end{align*}
\]

This expression neglects the term, \((g/U_{o})(\tan A_{q}/p)\), which is the yaw rate component sensed by the roll rate gyro which is small compared with the total roll rate.

By the condition of Eq. (3-3), \((\text{EQ})\) is zero when sideslip is zero.

Derivation Summary 3-1.
Referring to Fig. 3-1, Derivation Summary 3-1 shows that when sideslip is zero the components of the angular velocity of the aircraft with respect to inertial space measured by the roll and yaw rate gyros are related by Eq. (3-7). Since this relation involves integral terms, it is more practical to use the relationship which results when the gyro outputs are filtered by high-pass filters. Neglecting for the purpose of this discussion that voltage signals would be involved with the physical equipment, there results the relation of Eq. (3-9) which defines an error quantity, \((\text{EQ})\).

\[
\text{(EQ)} = \left( \frac{r_p}{1 + r_p} \right) W_{(IA)}z_q - \left( \frac{g}{U_o} \right) \left[ 1 + \left( \frac{U_o}{g} \tan A_q \right) \frac{1}{1 + r_p} \right] W_{(IA)}x_q \tag{3-9}
\]

Since Eq. (3-9) is true only when sideslip is zero, \((\text{EQ})\) will be zero when sidealp is zero and nonzero otherwise. \((\text{EQ})\) thus becomes a measure of how well the rudder coordination system is performing and can be used to change the parameters of the system so as to optimize that performance. The design of the self-adjusting system is based upon using the simple relationship defining \((\text{EQ})\) as a control quantity.

**Mechanization of the Error Quantity**

In Chapter 2 it was shown that the ideal rudder coordination signal should include terms proportional, after suitable filtering, to the roll rate, rolling acceleration and roll angle. In many cases the predominant terms arise from the presence of the yaw rate damper added to damp the aircraft’s natural lateral oscillation. Equation (2-28) of Chapter 2 presents the relating function of the rudder coordination system and shows that the term involving roll rate in the error quantity of Eq. (3-9) also appears as an important portion of the desired rudder coordination system’s relating function. Thus the mechanization of the error quantity also furnishes a control signal for the rudder.

Figure 3-2 is a functional block diagram for a possible mechanization of the error quantity to control the rudder coordination system. \((\text{EQ})\) can be written

\[
\text{(EQ)} = \left( \frac{r_p}{1 + r_p} \right) [W_{(IA)}z_q - \tan A_q W_{(IA)}x_q] - \left( \frac{g}{U_o} \right) r \left( \frac{1}{1 + r_p} \right) W_{(IA)}x_q \tag{3-10}
\]

Equation (3-10) shows that \((\text{EQ})\) can be computed by feeding the yaw rate gyro signal through a high-pass filter and portions of the roll rate gyro signal through the same filter as well as through a lag filter. The poles of the two filters are identical and thus the same \((\text{RC})\) network can be used for both paths with the inputs entering at different points.

If the \(X\)-gyro axis is aligned with the aircraft’s zero lift line, the angle \(A_{g_2}\) is equal to the aircraft’s angle of attack, \(\alpha_A\).
Fig. 3-2. Functional schematic diagram of rudder coordination system.
Thus,
\[ \tan \Delta = \tan \alpha = a_A \quad (3-11) \]

But the angle of attack varies with flight condition as
\[ a_A = \frac{W/S}{(dC_L/da) \frac{p}{2} U_o^2} \quad (3-12) \]

or
\[ a_A = \frac{C}{U_o^2} \quad (3-13) \]

where C varies with loading, slope of the lift curve, and altitude. If C were assumed to be constant, the sensitivity of the roll rate high-pass filter path would have to be varied with flight condition as \((1/U_o)^2\) and therefore it would be proportional to the square of the sensitivity of the lag path. This relationship greatly simplifies the computing process since only one error quantity is then required rather than two to set the sensitivities of these two signal paths. The error quantity is therefore used to drive an instrument servo to position a shaft on which is mounted two potentiometers. By exciting one potentiometer with a roll rate signal its output determines the sensitivity of the lag path. By exciting the second pot by the output of the first pot, the output of the second pot determines the sensitivity of the lead-lag path.

The output of the second pot is combined with the yaw rate signal in a summing amplifier and fed to the filter network where all three signals are combined to form the error quantity. Since the terms forming the error quantity are also needed to control the rudder by Eq. (2-28) and to damp the lateral oscillation, the error quantity becomes one of the command paths to the rudder servo. To obtain the direct roll rate to rudder command path, the output of the first rudder coordination system pot is sent to the rudder servo. This approximates the variation with flight condition of the direct roll rate path of Eq. (2-28). The error quantity is then sent through the sign control switch and the sampling switch to the potentiometer control instrument servo. The need for the sign control switch can be seen from Eq. (3-9). If a turn is made to the right, both angular velocities will be positive. If the error quantity is positive, the value of \((gT/U_o)\) should be increased, i.e. the pot "turned up." If \((gT/U_o)\) is set too low and a turn to the left is made, the error quantity will be negative but the pot value should still be increased. Thus the sign of signal sent to the instrument servo must be controlled by an indication of which way the aircraft is turning. The sign of the aileron deflection or the sign of the command input to the flight control system can be used as the sign control quantity.
Since the pot settings should be constant for a given flight condition, it is desirable to sample the error only as often as necessary and to leave the pot alone if it is set correctly. The control quantity for the sampling switch should thus be a quantity which indicates a maneuver is to take place. The aileron command signal or the command input to the flight control system are acceptable quantities to use. The disadvantage of using the command signal to the flight control system is that it restricts the pot adjustment to those times during which a command signal exists. In maneuvers such as dives, it may not be desirable to upset the system by putting in lateral command inputs. Thus the aileron may be the better sampling quantity.
CHAPTER 4

RESULTS OF THE SIMULATION STUDY

The performance of the automatic self-adjusting rudder coordination system designed in Chapter 3 was studied on a G. P. S. high speed analogue simulator. The simulation set up is described in Appendix B. The ability of the system to coordinate the rudder with a simulated manual aileron input was examined first. This system is the same as that shown in Fig. 3-2. Later roll and yaw control loops were closed around this system so that the combined system became a completely automatic flight control system. The sensitivities of the outer control loops were assumed to be set in a manner independent of the rudder coordination system. In both cases it was assumed that yaw rate was fed to the rudder to damp the aircraft lateral oscillation.

Most of the study was done considering the system to be installed in an F-94A aircraft. The basic flight condition for comparison purposes was taken to be a Mach number of 0.6 at 22,000 foot altitude. The parameter variations studied are listed in Table 4-1. A brief examination of the performance of the system when installed in an F-100A aircraft was also made.

For the manual aileron inputs and for the first flight control system studies, the error quantity was sampled only when there was an aileron deflection from trim that produced a signal large enough to actuate the sampling switch. In the later flight control system studies, the sampling switch was in continuous operation. Since the sign of the error changes with the direction of roll, the sign of the instrument servo command signal must be made independent of the direction of rolling so that the sensitivity variation would always be in the proper direction. The algebraic sign of the aileron deflection from trim was used to control the "sign switch." For the case of the manual aileron inputs the aileron was deflected in only one direction, but in the case of the flight control system the sign of the aileron deflection changed whenever the system was oscillatory.

Transient time responses of the system are presented in Figs. 4-1 through 4-5, which are photographs of the oscilloscope on which the simulator outputs are displayed. Each curve represents approximately 10 seconds of the response following the initiation of the input. At the end of each sweep (or about 10 second
period) the solution with the exception of the variable sensitivity pot is clamped to zero until the input is repeated. Thus the family of curves shows the change in response as the rudder coordination sensitivity is varied. For the aileron pulse inputs of Fig. 4-1, the variable gain was only being adjusted while the aileron input existed which in this case occurred over the first three seconds of each transient. After that period of time the gain is constant for the rest of the sweep.

Thus on the plot of the sensitivity variation, the brightest dots shown should be interpreted as representing the end of successive sweeps. Figure 4-1(a) for example shows that the sensitivity will reach its final value after approximately 40 seconds of real time if successive 3 second pulses are applied to the aileron every 10 seconds. Alternatively it will take approximately 12 seconds of sampling time to set the gains.

The system and flight condition data are listed in Table 4-2. The time scales except for the F-100 simulation are 0.2 second per dot. The ordinate scales have been chosen to show the shape of the response, and cannot be read to determine quantitative data. This was necessary due to the voltage scaling used for the simulator which prevented an input high enough to permit constant oscilloscope gains without over loading parts of the simulator. The transient responses before and after the sensitivity adjustment are labelled "initial and "final" respectively.

Response to Aileron Pulse Inputs

Figure 4-1(a) through (g) presents the performance of the system installed in an F-94A aircraft at a Mach number of 0.6 at 22,000 foot altitude. Part (a) is a reference condition with which the other parts can be compared. A separate measurement indicated that the automatic coordination system has decreased the ratio of peak sideslip angle to maximum roll angle by a factor of five and the ratios are listed on the figure. The transient sideslip has been reduced to the level of the steady-state sideslip, and the turn entry is much smoother.

Figure 4-1(b) shows the effect of doubling the aileron input over that of part (a). This doubles the initial magnitude of the error quantity, and thus the pot instrument servo runs faster, and fewer transient pulses are needed to arrive at the steady-state sensitivity. Time did not permit an exhaustive study of how fast the sensitivity adjustment can be made, and there undoubtedly is an upper limit set by the system stability.

In this system \((\tan A_i)\) is computed as being proportional to \((1/U_0)^2\). The proportionality factor is a function of the slope of the lift curve, altitude, and weight. If a fixed constant of proportionality is used, it was found that acceptably
Table 4-1

Parameter Variations Studied                              Figure

Effect of input magnitude                                  4.1b
Effect of the sensitivity of the tan $A_q$ computing path   4.1c
Effect of the direct roll rate path to the rudder          4.1d
Effect of sensitivity setting which is initially too high  4.1e, 4.2c
Effect of gyro input axis alignment                         4.1f
Effect of $C_{nS}$                                         4.1g
Effect of flight condition                                 4.1h, 4.2d
Effect of dead-zone in the sampling switch                  4.2b
Aircraft comparison                                        4.5
Effect of aircraft loading condition                       4.3n
Table 4-2. System parameters for the simulator results.

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<tr>
<th>Figure No.</th>
<th>Airplane</th>
<th>Mach No.</th>
<th>Altitude (ft)</th>
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<th>Fuel</th>
<th>Bombs</th>
<th>Yaw Damper Sensitivity $S_y$, (sec)</th>
<th>Direct Rudder Coordination Sensitivity $S_{y,\theta}$, (sec)</th>
<th>$\tan A_\theta$, (g/(U_\infty))^3</th>
<th>$A_\theta$, (degree)</th>
<th>Initial Coordination Sensitivities</th>
<th>Flight Control System Yaw Rate Command</th>
<th>Manual 3-sec Aileron Pulse</th>
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Fig. 4-1. Response of F-94A airplane to pulse aileron input with the self-adjusting rudder coordination system, $M = 0.6$, 22,000 ft. (Page 1 of 4)
Fig. 4-1. Response of F-94A airplane to pulse aileron input with self-adjusting rudder coordination system, $M = 0.6$, 22,000 ft. (Page 2 of 4)
Fig. 4-1. Response of F-94A airplane to pulse aileron input with self-adjusting rudder coordination system, $M = 0.6$, 22,000 ft. (Page 3 of 4)
Fig. 4-1. Response of F-94A airplane to pulse aileron input with self-adjusting rudder coordination system, $M = 0.6$, 22,000 ft. (Page 4 of 4)
good sideslip response could be obtained at all flight conditions if the sensitivity of this path was adjusted to be approximately \(0.6 \left(\frac{g\tau}{U_0}\right)^2\). For the Mach-0.6, 22,000 foot flight condition this results in a computed value of \((\tan A_g)\) only 1/4 of the theoretical value. At the high speed, sea level condition, the computed \((\tan A_g)\) is approximately equal to the theoretical value. The gain of the roll-rate lead path, expressed as the ratio \((\tan A_g)/(g\tau/U_0)^2\), therefore has to be specified prior to flight although its variation with flight condition is automatic. Figure 4-1(c) shows that the accuracy of the value of \((\tan A_g)/(g\tau/U_0)^2\) need not be high.

Equation (2-28) of Chapter 3 shows that the roll rate signal should be fed directly to the rudder in addition to the filtered signals. An approximation to the required variation of the sensitivity of the direct path with flight and loading condition was made by making this sensitivity proportional to \((g/U_0)W_X\), which is the output of the first instrument servo potentiometer. Figure 4-1(d) shows the response of the system with this direct path eliminated. The peak transient sideslip is reduced by a factor of 3 compared with the factor of 5 of Fig. 4-1(a).

In parts (a) through (d) of Fig. 4-1, the rudder coordination system sensitivities were zero initially, and the system increased them to the desired levels. The system must also be capable of reducing the sensitivities if they are initially set too high. Comparing part (e) with part (a) shows that the system works just as well in setting the pots from either direction, and the same steady-state solution is achieved.

Another installation setting that must be pre-selected is the alignment of the gyro input axes with the airplane axes. It has been shown that since the quantity \((1/U_0)^2\) is available, it is convenient to approximate \((\tan A_g)\) by setting it equal to the angle of attack. Thus the X-gyro axis should be aligned with the aircraft zero-lift line. Figure 4-1(f) shows that the gyros can be misaligned by at least 2.4° without causing an appreciable degradation in performance, so that accurate knowledge of the location of the zero-lift line does not seem to be required.

The sensitivity of the direct roll-rate to rudder path should vary with \(C_{n\delta a}\) and product of inertia. This system neglects these effects, and Fig. 4-1(g) shows that the performance is still good even if \(C_{n\delta a}\) is doubled. This cannot be taken as conclusive since the yawing moment due to aileron is not large for this airplane.

Figure 4-1(h) shows the response of the system at a Mach – 0.8, sea level condition. The yaw damping sensitivity was changed in the normal manner, but all other system parameters remained constant.
Response of a Complete Automatic Flight Control System

The performance of the self-adjusting rudder coordination system was next investigated considering it to form the inner control loops of a complete flight control system. Since the primary interest of this thesis is directed to the rudder coordination system, the details of the outer control loops of the flight control system are omitted here. The system is described merely as one that produces a yaw angular velocity proportional to an input command signal.

Figure 4-2 presents the response of the system assumed to be installed in the F-94A aircraft. The input to the system is a step function yaw angular velocity command signal. Parts (a) through (c) are for a Mach - 0.6, 22,000 foot flight condition, and part (d) presents the effect of changing flight condition. In part (a) it is seen that when the coordination gains are zero, the system exhibits a lightly damped, low frequency oscillation. As the coordination gains are increased by the automatic system the response becomes well damped emphasizing the importance of the proper adjustment of the rudder coordination sensitivities. Once the coordination system has been set to minimize sideslip, the outer control loop parameters of the flight control system can be adjusted to improve roll angle or yaw rate response characteristics.

It is important to recognize this necessity for separating the sideslip control problem from the roll angle or yaw rate control problem. This has been borne out by flight tests of such flight control systems. In flight it may be very confusing to a pilot or observer to try to determine which part of the system is producing instability. Thus making the sensitivity adjustment for the rudder coordination loop automatic will in itself greatly facilitate the setting of the sensitivities of the outer control loops.

Since the system of Fig. 4-2(a) is initially oscillatory, the sampling switch output and the sign inverting switch are activated on each departure of the aileron from its trim condition. It can be seen that the output of the sampling switch changes sign in accordance with the sign of the aileron, and in the particular case this causes the sensitivity adjustment to oscillate once before reaching its average steady-state value. Increasing the dead zone in the sampling switch causes the switch to be actuated on only the first half cycle of the oscillatory response as shown in part (b). This results in somewhat smoother adjustment of the sensitivity. It also shows that appreciable lag can be tolerated in the switch operation.

Figure 4-2(c) shows the response of the system when the coordination sensitivity is initially too high. Initially there exists a higher frequency oscillation which disappears as the proper sensitivity is reached.
Fig. 4-2. Response of an automatic flight control system to a step yaw angular velocity command signal with the self-adjusting rudder coordination system. F-94A, M = 0.6, 22,000 ft. (Page 1 of 3)
b) Effect of dead-zone in sampling switch

c) Effect of sensitivity setting which is initially too high

Fig. 4-2. Response of an automatic flight control system to a step yaw angular velocity command signal with the self-adjusting rudder coordination system. F-94A, M = 0.6, 22,000 ft. (Page 2 of 3)
Some rudder coordination system for all conditions; sensitivities of flight control system outer loops varied in the usual manner.

Fig. 4-2. Response of an automatic flight control system to a step yaw angular velocity command signal with the self-adjusting rudder coordination system, F-94A. (Page 3 of 3)
Figure 4-2(d) presents the results obtained for three additional flight conditions ranging from a high altitude, low dynamic pressure condition to a sea level, high dynamic pressure condition. The rudder coordination system was kept the same, and only the yaw damping and outer-loop control gains were changed. Performance is excellent at each condition.

In Figs. 4-1 and 4-2 the sampling switch was actuated by the aileron displacement from trim. It was then found that good performance was obtained when the sampling switch was actuated by the command input to the flight control system as shown in Fig. 4-3. Since the input was a step function, the sampling switch was thus always closed except for the slight delay at the initiation of each step. Somewhat faster adjustment of the coordination sensitivity resulted, and in fact there is an overshoot in the sensitivity variation as well as in each of the other system outputs. The responses of Fig. 4-3 are very similar to those of Fig. 4-2.

The system must also perform well when aircraft loading conditions change. With the F-94A, one of the largest changes is obtained by assuming the wing tip-tanks carry fuel and that two 500-pound bombs are carried at wing station 125. This triples the rolling moment of inertia and nearly doubles the yawing moment of inertia of the aircraft. Figure 4-3(n) presents the response of the system with this loading condition. Here again the rudder coordination system is the same as before, but some of the outer control loop gains were changed as was to be expected.

As shown in Chapter 3 the shaft position of the variable potentiometer in this self-adjusting system is proportional to \( \frac{g \gamma}{U_0} \). By measuring the average steady-state pot position at the various flight conditions, the ability of the system to predict this quantity is indicated in Fig. 4-4. Since the accuracy of the simulation is involved, the measured accuracy is not conclusive, but it is reasonable to expect that the pot could give a measure of true airspeed within approximately 5%. This proves to be a very useful quantity to have available for automatic adjustment of the outer control loop gains.

Most of the investigation of this thesis was made studying the response of the F-94A aircraft since the Instrumentation Laboratory could readily test the results in such an aircraft currently bailed to it by the Air Force. However with current aircraft design trends, it is important to investigate the system response in mass cross-coupled aircraft. This was briefly done using an F-100A in the simulation. This particular simulator set up was derived including the mass coupling terms and writing the airplane equations in terms of principal axes rather than stability axes, but otherwise the two studies were very similar.
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.

(Page 1 of 8)
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.

Note: Ordinate scales chosen to show shape of response only and cannot be used to obtain quantitative data. Time scale = 0.2 second/dot.

F-94A
M = 0.6
22,000 ft
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.

(Page 3 of 8)
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.

Ordinate scales chosen to show shape of response only and cannot be used to obtain quantitative data. Time scale = 0.2 second/dot.

F-94A
M = 0.6
22,000 ft

Initial

Final

Note: Initial

Final

g) Roll angular velocity of airplane along $X_{VA}$

h) Roll angular velocity measured by the roll gyro
i) Aileron angle

Note: Ordinate scales chosen to show shape of response only and cannot be used to obtain quantitative data. Time scale = 0.2 second/dot.

j) Rudder angle

Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists. (Page 5 of 8)
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists.

Note: Ordinate scales chosen to show shape of response only and cannot be used to obtain quantitative data. Time scale = 0.2 second/dot.

F-94A
M = 0.6
22,000 ft
Fig. 4-3. Response of an automatic flight control system to a step yaw angular velocity command signal input with the self-adjusting rudder coordination system. Error sampled when command input exists. (Page 8 of 8)
Figure 4-5 presents the performance of the flight control system with the F-100A airplane at Mach-0.9 at 35,000 foot altitude. It is seen that the rudder coordination system worked very well even though high cross-coupling effects are to be expected at this flight condition. The direct roll rate to rudder path was omitted. Further studies of this nature should be continued.

Fig. 4-4. Comparison of the value of \( \frac{\theta}{U_0} \) indicated by the coordination system pot versus the actual value.
Fig. 4-5. Response of an automatic flight control system to a step yaw angular velocity command signal with the self-adjusting rudder coordination system. F-100, M = 0.9, 35,000 ft.
CHAPTER 5

CONCLUSIONS

From the results of this study it can be concluded that:

(1) The signal flow diagram provides a useful tool for specifying the design requirements upon an aircraft automatic rudder coordination system for control of transient sideslip.

(2) A major source of transient sideslip is that caused by the rudder deflections introduced by the yaw rate feedback path used to damp the aircraft's lateral oscillation. If roll rate is used as the coordination system input, the filters in the roll rate path to the rudder must therefore have the same time constant as that of the high-pass filter used in the yaw damping loop.

(3) For the aircraft considered in this thesis good coordination can be obtained by using roll rate information only. In this case roll rate is fed to the rudder through three paths: a direct path, a lag filter path, and a high-pass filter path. If yawing moment due to aileron or the cross-product of inertia terms were much larger, either a path from aileron to rudder or a roll acceleration signal is also required if it is desired to provide control of the sideslip during the initial portion of the transient response.

(4) An error quantity has been derived based upon the relationship that must exist between the yaw and roll angular velocities that occur during a coordinated turn. This error quantity can be used to set the gains of the coordination system automatically as the flight and loading conditions change during flight.

(5) Since one of the outputs of the error quantity computer is a shaft position proportional to the reciprocal of true airspeed, the system provides a measure of true airspeed independent of air pressure measurements. The error in this measurement was indicated by the simulation studies to be less than 5%.

(6) The system provides good rudder coordination control of an F-94A interceptor aircraft. Results of simulation of F-100 airplane at one subsonic flight condition indicate that the error quantity can also provide control of sideslip for aircraft exhibiting strong mass cross-coupling effects.
(7) The proposed system design is a practical one and the necessary hardware could be easily built and tested.

(8) It is desirable to install the rate gyros for this system so that their input axes are parallel and perpendicular to the zero lift line of the aircraft. However, the location of the zero lift line need only be known to an accuracy of 2 or 3 degrees.

(9) The error quantity can either be sampled continuously or whenever there is an aileron deflection (or roll rate) from trim, although the former gave some improvement in response when an automatic flight control system was closed around the coordination system. Switching delays of up to 0.5 seconds in the sampling circuit do not degrade the performance.

(10) Steady state sideslip can best be reduced by a slow acting integration of a direct measure of sideslip or lateral acceleration.

(11) The error quantity can be mechanized neglecting the yaw rate term due to input axis alignment that appears in the output of the roll rate gyro.
Further investigations should be made to exploit the applications of the proposed system. It is recommended that the equipment be built and tested in the F-94 aircraft that has already been fully instrumented by the Instrumentation Laboratory and contains an automatic flight control system. Additional simulator studies would show the following:

(1) Performance for a high angle of attack conditions such as the landing condition for a delta wing aircraft.

(2) Effect of reducing the time constant of the high-pass filter in the yaw damping path.

(3) Effect of gust disturbances.

(4) Effects of the speed at which the gains are changed as a function of error. This would of necessity involve the effect of the magnitude of the input sent to the system.

(5) Effect of a larger product of inertia and greater $C_{n\delta a}$

(6) Effect of Mach number and variation of stability derivatives with angle of attack and angle of sideslip.

In addition it is recommended that a linearized root locus study be made of the gain control path to provide additional insight into the system's behavior. Further investigation should be made into the effect of complete mechanization of the error quantity as given by Eq. (3-8) and to the desirability of further separation of the error quantity signal from the rudder command signal so that the error quantity would be independent of how much of the computed rudder command signal was sent to the rudder to optimize the sideslip response.
APPENDIX A

REDUCTION OF THE AIRCRAFT SIGNAL-FLOW DIAGRAM

The step by step reduction of the signal-flow diagram for the lateral aircraft equations of motion are presented in Figs. A-1 through A-3. Each variable is represented by a node in accordance with Eqs. (2-17) through (2-22) of Chapter 2. The roll and yaw rate gyro input axes are displaced from the aircraft axes by the angle $A_g$ which gives rise to nodes 7 and 8. The signal path transmittances are given in Table 2-1.

Rules for reduction of signal-flow diagrams are given in references 1 and 2. Each step rearranges the diagram to simplify the picture of the flow of signals from input to output; but this is done in a manner which insures that the total information entering each of the retained nodes is unchanged. If the process were continued until only the input and output nodes remain, the final over-all transmittance is of course the same as that obtained from solving the simultaneous equations.
Fig. A-1. Signal-flow diagram for the aircraft lateral equations of motion.

Fig. A-2. Signal-flow diagram for aircraft rudder coordination system.
Fig. A-3. Reduction of signal-flow diagram of Fig. A-2. (Page 1 of 4)
c) Elimination of self-loop at node 3

\[ T_{1-3}'' = \frac{T_{1-3}}{1 - T_{3-3}} \]
\[ T_{2-3}'' = \frac{T_{2-3}}{1 - T_{3-3}} \]
\[ T_{4-3}' = \frac{T_{4-3}}{1 - T_{3-3}} \]

\[ T_{4-3}' = T_{4-3} + T_{4-2}T_{2-3} \]
\[ T_{4-4} = T_{4-2}T_{2-4} \]

d) Elimination of path 4-2

Fig. A-3. Reduction of signal-flow diagram of Fig. A-2. (Page 2 of 4)
Fig. A-3. Reduction of signal-flow diagram of Fig. A-2. (Page 3 of 4)
If $T_{5-4} = 0$ then $T_{1-4} = 0$, $T'_{2-4} = T_{2-4}$, $T'_{3-4} = T_{3-4}$, $T''_{3-4} = T_{3-4} + T_{3-3} T_{2-4}$

$g$) Neglecting side force due to rudder

Fig. A-3. Reduction of signal-flow diagram of Fig. A-2. (Page 4 of 4).
APPENDIX B

SIMULATION DIAGRAM

The simulation diagram for the rudder coordination system is presented in Fig. B-1. A G. P. S. high-speed simulator was used. The variable pots are represented by multipliers. The instrument servo for driving the pots is represented by the real time integrator which stores the value of the variable gain between sweeps. The drawing is for the F-94A simulation. The corresponding diagram for the F-100A involves changing the time scale by a factor of 2.
Fig. B-1. Simulation diagram for the rudder coordination system for the F-94A using a GPS high speed analog simulator.
APPENDIX C

GENERAL EXPRESSION FOR SPECIFYING THE RUDDER COORDINATION SIGNALS

The general expression for specifying the rudder coordination signals is given by Eq. (C-1). This expression includes the effects of the aircraft's being trimmed in a climb or dive with the aircraft's $X_A$-axis elevated through the angle $E_0$ from the earth horizontal plane.

$$T_{1.5} = \left\{ \left( \frac{g}{U_o} \right) \frac{I_{A(ZZ)}I_{qSb}}{C_{n}\delta_r} \cos E_o \left[ 1 + \left( \frac{C_{n}\delta_0}{C_{f}\delta_a} \right) I_{A(XZ)} \right] - \left( \frac{b}{2U_o} \right) \frac{C_{n p}}{C_{n}\delta_0} \left[ 1 - \left( \frac{C_{f p}}{C_{f}\delta_a} \right) C_{n}\delta_a \right] \right. \right.$$

$$- \left( \frac{g}{U_o} \right) S_{a}[w_{zz, \delta_0}] \cos A_0 \cos E_o \left[ 1 + \frac{\left( \frac{U_o}{q} \right) \tan A_q}{1 + r p} \right] - \frac{I_{A(XXY)}I_{qSb}}{C_{n}\delta_r} \left[ \frac{C_{n}\delta_0}{C_{f}\delta_a} + \frac{I_{A(XZ)}}{I_{A(XX)}} \right] p$$

$$- \left( \frac{g}{U_o} \right) \cos E_o \left[ \frac{b}{2U_o} \frac{C_{n p}}{C_{n}\delta_r} \left[ 1 - \frac{C_{f p}}{C_{f}\delta_r} \right] \right] 1 + T_{1.5} \cos A_0 \left[ 1 - \left( \frac{g}{U_o} \right) \cos E_o \tan A_q \right]$$

$$- \left( \frac{g}{U_o} \right) \cos E_o \left[ \frac{b}{2U_o} \frac{C_{n p}}{C_{n}\delta_r} \left[ 1 - \frac{C_{f p}}{C_{f}\delta_r} \right] \right] 1 + T_{1.5} \cos A_0 \left[ 1 - \left( \frac{g}{U_o} \right) \cos E_o \tan A_q \right]$$

$$- \left( \frac{g}{U_o} \right) \sin E_o \left[ \frac{b}{2U_o} \frac{C_{n p}}{C_{n}\delta_r} \left[ 1 - \frac{C_{f p}}{C_{f}\delta_r} \right] \right] 1 + T_{1.5} \cos A_0 \left[ 1 - \left( \frac{g}{U_o} \right) \cos E_o \tan A_q \right]$$

$$\left. \right\} \left( \frac{I_{A(XX)}}{qSb} \right)^2 + \left( \frac{b}{2U_o} \right) C_{p} \left( \frac{g}{U_o} \cos E_o \left( \frac{I_{A(XZ)}}{I_{A(XZ)}} \right) \left[ 1 + \frac{I_{A(XX)}}{I_{A(XZ)}} \tan E_o \right] \right)^p$$

$$\left. + \left( \frac{b}{2U_o} \right) \cos E_o \left( \frac{b}{2U_o} \right) C_{p} \left[ 1 - \frac{C_{f p}}{C_{f}\delta_a} \tan E_o \right] \right\} (C-1)$$

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APPENDIX D

BIBLIOGRAPHY


