A STUDY OF RADIO INTERFERENCE FROM THE
ELECTRIC IGNITION OF INTERNAL COMBUSTION MOTORS

by

ALBERT CARRUTHERS HALL
B.S., Agricultural and Mechanical College of Texas
1936

SUBMITTED IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

From The

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
1938

Signature of Author

Signature redacted

Department of Electrical Engineering, Jan. 17, 1938

Signature of Professor in Charge of Research

Signature redacted

Signature of Chairman of Department Committee on Graduate Students

Signature redacted
ACKNOWLEDGMENT

The author wishes to express his sincere thanks to Professor E.L. Bowles, who suggested the problem, and especially to Professor R.D. Bennett who gave his assistance so freely whenever it was needed.
TABLE OF CONTENTS

I  Introduction:                       1
II  History:                           3
III Description of the Apparatus:     6
IV  Theory and Experimental Results:  15
    Spark phenomena with no suppressor
    Spark phenomena with suppressor
    Effect of the distributor
V   Effect of Suppressor on Ignition:  57
VI   Conclusions:                     61

Bibliography:                        63

Appendix:                           64
    Measurement of coil parameters
    Description of oscillograph
INTRODUCTION

While this paper is devoted to a study of the radio interference caused by electric ignition, it should be pointed out that the principles involved are equally applicable to a wider problem of interference from any man-made device. This type of radio interference is known as man-made static.

A little thought will show that in practically every case of man-made static, the interference can be traced to the occurrence of some sort of electrical discharge, generally a spark. For instance Curtis\(^1\) gives the following sources of interference arising from the operation of an automobile:

(a) Interference from the action of the spark plugs.
(b) Interference from the action of the high tension distributor or from faulty connections in the leads in its circuit.
(c) Interference from the action of the low tension interrupter.
(d) Interference from the action of the generator brushes.

In every one of the above-listed sources, a spark takes place. We can then assume that the presence of the spark is intimately connected with the presence of radio disturbances.
This investigation was primarily devoted to the study of the action of the spark plug, and secondarily to the effect of the distributor. But because of the close relationship between the particular and the more general problem, the principles involved should be applicable to the whole problem of man-made static.
II
HISTORY

The problem of ignition interference became of primary importance when the value of radio communication to aeronautics was realized. It has long been known that communication channels between an airplane and a ground station or between two airplanes was highly desirable, but since the rapid increase in the use of aircraft as passenger carriers during the last decade, it has been imperative to have such facilities.

Then a few years ago automobile radios were popularized, and the problem again came to the foreground. While the problem is not as vital in this case as in the former, there was a greater demand on the part of the public for a solution.

Again, it is easy to predict that the problem will gain prominence within a few years. For, as shall be discussed later, the interference is the most troublesome in the ultra high frequency band (thirty to three hundred megacycles). And that region will undoubtedly be the region used by television facilities when commercialization takes place.

Since the phase of the problem as it affects aircraft is most important, it is here that the greatest care has gone into a solution. It is important to prevent interference, but it is even more important not to
endanger engine performance in doing so. If shielding is used to decrease the interference, care must be taken not only that the shielding is not such that will adversely affect the engine performance, but that every portion of the ignition system is completely shielded, as otherwise the interference may be increased.

In automobile engines, however, the engine performance standard is not so high as it is in aircraft. Therefore some solution less expensive and more convenient as to installation and maintenance was sought. It was found that the use of resistors having between 15,000 and 20,000 ohms and placed in the ignition leads immediately adjacent to the spark plugs was effective.

This artifice does not remove all the interference and what remains after their installation is most noticeable in the ultra high frequency range.

In order to get a better explanation of the problem, it was decided to study the phenomenon primarily from the standpoint of the current and voltage characteristics that exist in the ignition system. This required an oscillographic study, so thought was turned toward selecting a suitable instrument. After a process of elimination a variation of the Dufour cathode ray oscillograph was chosen.

In the investigation over 100 oscillograms were recorded and analyzed, typical ones of which are repro-
duced in this paper.

Certain other investigators have studied the voltage and current relations in the spark by means of the oscillograph. See, for example, references 2, 3, and 7 in the bibliography. Most of this work, however, was done in studying ignition efficiency. The investigation of the writer was carried out with the primary purpose of studying the causes of radio interference.

In the study, special attention was paid to the effect of placing resistances in the ignition leads to suppress the interference, as no oscillograms were found in the literature to record the effect of this artifice.
III
DESCRIPTION OF THE APPARATUS

The whole spark phenomenon lasts only a few milliseconds, and that portion of the phenomenon that is most important from the standpoint of radio interference occurs in time intervals of a fraction of a microsecond. Therefore the principal difficulty lay in recording such rapid transients.

A bifilar oscillograph is incapable of recording transients containing frequencies much above 5,000 cycles per second. The solution was to use some sort of cathode ray oscillograph, the moving element of which can for almost all purposes be considered massless.

The next problem was to find a method of recording the path of the cathode ray beam. At the time the investigation was begun there were no fluorescent screens of sufficiently great sensitivity to permit recording the transient by photographing the trace on the screen. The logical solution was to allow the beam to impinge directly upon the film.

The Dufour cathode ray oscillograph makes use of this means of recording. While this instrument was not available, a modification of it was. This instrument is completely described in the appendix.

The Dufour type of oscillograph is unavailable in most laboratories, and is not used very frequently. A highly developed technique is required to use the in-
Spark Set Up and Synchronizing Circuit

Figure 1

\[ C_1 = \frac{1}{4} \mu \text{fd} \]
\[ C_2 = 0-20 \mu \text{fd} \]
\[ C_3 = 0-200 \mu \text{fd} \]
\[ C_4 = 0.02 \mu \text{fd} \]
\[ C_5 = 18.75 \mu \text{fd} \]
\[ R_1 = 0-25,000 \text{ ohms} \]
\[ R_2 = 0-3,000 \text{ ohms} \]
\[ R_3 = 0-200,000 \text{ ohms} \]
\[ R_4 = 2.0 \text{ megohms} \]
\[ R_5 = 50,000 \text{ ohms} \]
instrument at all satisfactorily. Therefore much time had to be devoted to the mastering of the technique before any results could be obtained.

Another difficulty lay in synchronizing the spark and the operation of the oscillograph. The apparatus for accomplishing this is shown in figures 1, 2, and 3. Figure 2 and the upper section of figure 1 show the ignition system and figure 3 and the lower section of fig. 1 show the apparatus for starting the oscillograph. Diagrams and photographs of the oscillograph itself will be found in the appendix.

Apparatus was chosen to simulate as nearly as possible the actual apparatus used in an automobile engine. The induction coil is a Delco-Remy coil. The distributor D, fig. 1, is a Delco-Remy six point distributor. The condenser C<sub>1</sub> across the primary of the induction coil is a Wizard of one-fourth microfarad capacity. The spark plug is a Champion spark plug with a 0.022 inch gap. The ignition cable running from the coil to the distributor and from the distributor to the spark plug is standard automotive ignition cable with an overall length of 47 inches. Thirty-one inches of this was between the distributor and the spark gap. The apparatus, as shown, was set up to simulate at least approximately the engine conditions as far as length of
Fig. 2

Fig. 3
lead and lead-ground capacity are concerned.

The motor in fig. 2 could be used to rotate the distributor. In the automobile this function is performed by a shaft geared to the engine.

$C_a$ and $C_b$ comprise a capacity voltage divider, and $P$ is one pair of plates on the oscillograph. During part of the investigation a resistance potential divider was used instead of the capacity divider. In this case the resistance to ground introduced by the divider was never less than fifteen megohms, so that its effect on the circuit was negligible. In case of the capacity divider, the capacity to ground through the divider was never more than 20 micro-microfarads.

In order to facilitate the taking of oscillograms of the voltage- and current-time relationships, during most of the investigation the distributor was not rotated. Instead it was set in position for the spark to occur, and the primary circuit broken whenever it was desired that a spark should occur. In this way, certain timing and shielding difficulties were overcome, without changing any of the fundamental conditions.

$S_a$ and $S_b$ are switches for controlling the time that the spark occurs and the time the electron beam in the oscillograph begins its sweep. When $S_a$ is opened, the spark plug "fires". When $S_b$ is closed, the oscillograph sweep begins, as is explained below.
The apparatus controlling the opening of $S_2$ and the closing of $S_3$ is shown in fig. 4. $S_2$ and $S_3$ are toggle switches clamped on vertical meter sticks in such a manner that when the large weight is dropped it throws each toggle successively, thus opening one circuit and closing another. Thus by varying the relative heights of the switches, the interval of time between their operation can be controlled. The two rods running the length of the apparatus are guides for the falling weight. The weight rests on a finger arrangement in such a manner
that when it is desired to drop the weight, it is only necessary to pull forward the handle projecting above the top guide rod support. The box at the bottom of the runners contains lead shot for breaking the fall of the weight. The whole apparatus rests on a cork mat for further damping the vibrations from the fall of the weight.

When $S_3$ is closed a voltage of 135 volts is applied to the grid of the thyratron, through the resistance-capacity delay network. The thyratron, an FG 27, has in its plate circuit the coils for tripping the oscillograph, and an eighteen microfarad condenser charged to a potential of 345 volts. Thus when the grid rises in potential to the point where the tube becomes conducting, the condenser discharges through the tube and starts the sweep, as explained in the appendix.

The thyratron is biased 22.5 volts negatively. The delay circuit consists of a 100,000 ohm adjustable resistor in series with a 0.01 microfarad condenser placed between the grid and cathode, as shown. It was found that by using large values of bias and impulse voltages, more consistent results could be obtained. The adjustment of the resistance shown in the diagram as $R_a$ gives a fine control for the starting of the sweep, and was used in conjunction with the adjustment of the relative heights of $S_2$ and $S_3$ which was a coarser adjustment.
With the apparatus set up in this manner, a high degree of consistency could be obtained. It has been found that the time interval between the operation of the two switches $S_2$ and $S_3$ will be constant within a few microseconds.

Oscillograms were taken of the voltage across the gap and of the current in the ignition lead, for various values of resistance $R_1$ (fig. 1) in the lead. The switch $S_1$ in the left position would impress voltage transients on the oscillograph, and in the right position, current transients. $R_2$ was always zero when voltage transients were being recorded. The current was recorded by obtaining the voltage across $R_g$ which was between 1,000 and 3,000 ohms. Oscillograms were taken with the distributor both in the circuit and out. The effect of the distributor will be discussed later.

The time calibration wave was obtained from a beat frequency oscillator. Frequencies used were 1,000, 2,000, 5,000, 10,000, 20,000 and 40,000 cycles. It was found not to be necessary to impress timing waves on each film, as the sweep of the oscillograph was very consistent. Thus the time axis could be determined from a preceding or following oscillogram of the same sweep speed and on which a timing wave had been impressed.

The magnitude of the transient was determined
by calculation, knowing the sensitivity of the oscillograph and the potentiometer constant, and by direct comparison with a known impressed voltage.
IV

THEORY AND EXPERIMENTAL RESULTS

Physically, the apparatus is very simple. As described in the previous section, it consists only of an induction coil, battery, breaker, condenser to prevent sparking at the terminals, spark gap, and interconnecting leads.

Electrically, however, the apparatus gives rise to very complex phenomena which it is impossible to describe completely by mathematical means. Nevertheless it is possible to predict, to a close approximation, the behavior of the phenomenon. The circuit shown in fig. 5 is, for most purposes, the electrical equivalent of the physical set up.

\[ C_1 \text{ is a capacity for increasing the voltage obtainable with the coil and for preventing sparking at the breaker points. } R_1, L_1, M, L_2, R_2, \text{ and } C_2 \text{ are constants of the induction coil. } R_1 \text{ is the primary resistance,} \]
L₁, the primary inductance, M, the mutual inductance, L₂, the secondary inductance, R₂, the secondary resistance, and C₂ is the secondary distributed capacity. For the Delco coil used these parameters had the following values.

\[ \begin{align*}
R₁ & = 1.37 \text{ ohms} \\
R₂ & = 3,175 \text{ ohms} \\
L₁ & = 7.3 \text{ millihenries} \\
M & = 0.374 \text{ henry} \\
C₁ & = 0.25 \text{ microfarad} \\
C₂ & = 262 \text{ micro-microfarads}
\end{align*} \]

The procedure used in measuring these constants will be found in the appendix.

C₂ is the distributed capacity of the secondary winding. There is also, of course, a distributed capacity on the primary side, but this is very small compared with C₁, the capacity across the breaker. G is the spark gap with electrical characteristics difficult to express exactly. The portion of the circuit within the broken line section is the ignition lead connecting the secondary of the coil with the spark plug. For low frequencies, its parameters are negligible, but for high frequencies, it must be considered not as a lumped constant network but as a distributed constant network. In effect it acts as a transmission line between the secondary of the coil and the spark gap. It is this line that radiates most of the energy causing interference.
The gap due to the distributor has not been included in the equivalent circuit. For the present it will be assumed that the breaker makes perfect contact, and that this contact is made just prior to the instant the breaker switch $S_1$ opens, and is not broken until all the energy of the coil has been dissipated. This is not true as will be explained later, but for the present discussion the assumption is accurate enough.

Prior to the opening of the breaker $S_1$, a constant current flows in the primary circuit, the value of which is determined by the primary resistance $R_1$, and the impressed electromotive force of the battery. Now when the breaker $S_1$ opens, this current will drop to zero in some manner, and the energy that has been stored in the electromagnetic field must be dissipated. Since a coupling exists between the primary and secondary of the induction coil, this fall of current will induce a rise of potential across the secondary. This potential will rise in some manner until it reaches a value at which the gap will break down and the spark will occur. Then after a certain amount of the coil energy has been dissipated, the potential across the secondary will fall to such a value that the spark will be extinguished and the remaining energy will dissipate itself internally.

The above description is included to give a qualitative idea of the spark mechanism. A more exact
Due to the complex nature of the spark, it is not possible to write an exact mathematical expression representing the complete phenomenon. However, it is possible to separate the phenomenon into certain major parts and to write the mathematical expression for each of these parts to a good degree of accuracy. The phenomenon can be divided chronologically as follows:

Primary circuit interrupted
   Voltage build up. (No discharge at gap)
Gap breakdown
   Capacitive discharge
   Inductive discharge
   Reignition
   Cathode Failure
Spark extinguished
   Dissipation of remaining energy

First these will be discussed in brief to establish a connected picture of the phenomenon. Then each division will be more fully developed.

Voltage build up

Until the spark gap breaks down the circuit consists only of the coupled resonance circuit shown in fig. 6. The equations for the secondary voltage of this circuit are developed later in the paper showing that there will be two frequencies present. The lower frequency will have the larger amplitude and the smaller damping. The first cycle of this voltage has been calculated and
plotted in fig. 7.

Gap breakdown

When the voltage across the coil reaches the breakdown value, the spark occurs. During the first microsecond or so, the spark acts like a resistance. During this time a very large current flows due to the discharge of the secondary capacity of the coil through the gap. This portion of the spark is called the capacitive discharge, and it is this discharge that causes the interference observed in the ultra high frequency band.

The capacitive portion of the discharge lasts less than a microsecond. After this is over, a glow discharge is established, characterized by a current of much smaller value than existed during the capacitive discharge, and by a voltage across the gap that is almost constant. This is called the inductive discharge.

Two other phenomena occur during this portion of the spark: reignition and cathode failure. These both will be discussed later as both are sources of interference.

Spark extinguished

After the coil is no longer able to sustain the inductive discharge, the gap is extinguished, and the coupled resonant circuit again controls the frequencies. By this time the oscillation of higher frequency has damped out, and only the lower frequency remains, which
also damps out in a short period.

Now that the whole phenomenon has been described briefly, each portion will be discussed in more detail.

**Voltage Build Up**

Until the spark gap breaks down, the system consists effectively of a coupled resonant circuit as shown in fig. 6. The differential equations controlling the behavior of this circuit are:

\[
L_1 \frac{d^2 q_1}{dt^2} + M \frac{d^2 q_2}{dt^2} + R_1 \frac{dq_1}{dt} + \frac{q_1}{C_1} = 0 \tag{1}
\]

\[
L_2 \frac{d^2 q_2}{dt^2} + M \frac{d^2 q_1}{dt^2} + R_2 \frac{dq_2}{dt} + \frac{q_2}{C_2} = 0 \tag{2}
\]

$L_1$, $L_2$, $M$, $R_1$, $R_2$, $C_1$, and $C_2$ are the parameters given on page 16, and $q_1$ and $q_2$ are the instantaneous charges on the capacitances $C_1$ and $C_2$, respectively. While $C_2$ is actually the distributed capacity of the secondary winding, it can be considered to be a lumped capacity to a close approximation.

If "p" is substituted for the differential operator and $q_1$ eliminated from the equations above, the
following differential equation is obtained containing $q_a$ alone:

$$
\left(p^4(L_{12} - \omega^2) + p^3(L_{12} + L_{22}) + p^2(R_{12} + \frac{L_2}{C_1} + \frac{L_1}{C_2})
+ p\left(\frac{R_1}{C_2} + \frac{R_2}{C_1} + \frac{1}{C_1C_2}\right)\right)q_a = 0 \quad (3)
$$

The expression for $q_1$ may be written down on inspection by simply changing $q_a$ to $q_1$ in equation (3).

To obtain the solution, it is necessary to solve the quartic equation for $p$. While a general solution of a fourth degree equation is possible, it is not of sufficient importance to repeat here.

The secondary voltage is equal to $q_2/C_2$ and is of the form

$$
V_a = A_1 e^{-k_1t} \sin(2\pi n_1 t - \delta_1)
- A_2 e^{-k_2t} \sin(2\pi n_2 t - \delta_2),
$$

where $A_1$ and $A_2$ are the initial amplitudes of the two frequency components $n_1$ and $n_2; \delta_1$ and $\delta_2$ are the initial phases; and $k_1$ and $k_2$ are the damping factors. Taylor Jones gives the following approximate expressions for these parameters that are sufficiently accurate for the purposes here.

The frequencies $n_1$ and $n_2$ may be calculated from the expression $\omega = 2\pi n$, where $\omega$ is given by

$$
\omega = \sqrt{\frac{\omega_1^2 + \omega_2^2 \pm \sqrt{(\omega_1^2 + \omega_2^2)^2 - 4}\omega_1^2\omega_2^2(1-K^2)}}{2(1-K^2)}.
$$
The larger value of $\omega$ (obtained by using the positive sign for the inner radical) corresponds to $n_2$, and the smaller value to $n_1$. $K$ is the coefficient of coupling of the coil, and $\omega_1$ and $\omega_2$ are the uncoupled natural angular frequencies of the primary and secondary respectively. They are given by $\omega_1 = 1/\sqrt{L_1C_1}$ and $\omega_2 = 1/\sqrt{L_2C_2}$.

These expressions for the frequencies $n_1$ and $n_2$ are based on the assumption of small damping. Using the same assumption, $A_1$ and $A_2$ can be written as

$$A_1 = 2\pi M_0 n_1 n_2/(n_2^2 - n_1^2)$$
$$A_2 = 2\pi M_0 n_1^2 n_2/(n_2^2 - n_1^2) ,$$

where $i_0$ is the current flowing before the breaker is opened.

The damping factors $k_1$ and $k_2$ are given by

$$k_1 = 4\pi^2 n_1^2 (\frac{\theta_1 + \theta_2}{2} - \beta)$$
$$k_2 = 4\pi^2 n_1^2 (\frac{\theta_1 + \theta_2}{2} + \beta) ,$$

where $\theta_1 = R_1 C_1/2$, $\theta_2 = R_2 C_2/2$ and $\beta = 2\pi^2 M_0 n_2^2/(n_2^2 - n_1^2)(\theta_1 - \theta_2)(L_1 C_1 - L_2 C_2)$

The phases $\delta_1$ and $\delta_2$ are given by

$$\delta_1 = \arctan \frac{2\pi n_1 \left[2\beta n_2^2 + n_2^2 \right] - (\theta_1 - 2\theta_2)}{n_2^2 - n_1^2} ,$$

and $\delta_2 = \arctan \frac{n_2^2}{n_1^2} \tan \delta_1$

in which $\theta_0 = R_0 \sigma_1/2$, and $R_0 = E/i_0$.

When these constants are calculated using the coil parameters as given on page 16 the following values are obtained:
\[ n_1 = 1,760 \text{ cycles per second} \]
\[ n_2 = 8,220 \text{ cycles per second} \]
\[ A_1 = 18,900 \text{ volts} \]
\[ A_2 = 4,040 \text{ volts} \]
\[ k_1 = 36.8 \]
\[ k_2 = 761.0 \]
\[ \delta_1 = \arctan 0.63 \times 10^{-4} \]
\[ \delta_2 = \arctan 2.94 \times 10^{-4} \]

The phase angles are small enough to be neglected, so the expression for the secondary voltage can be written
\[ V_2 = 18,900 e^{-36.8t} \sin 2\pi(1760t) - 4,040 e^{-761t} \sin 2\pi(8,220t) \]

This expression has been plotted for the first cycle in fig. 7.

The solution assumes that the inductances \( L_1 \) and \( L_2 \) are constant and that the mutual inductance from the primary to the secondary will be the same as that from the secondary to the primary. The first assumption will not be true because of the iron core in the coil and the second will not be true because of a non-uniform voltage distribution on the secondary. For further discussion of the problem, the reader is referred to "Induction Coils" by Taylor Jones\(^5\). For the purposes of this investigation this expression is satisfactory.

A study of the rise of secondary voltage plotted in fig. 7 will show that electric ignition can be expected to occur in a very short length of time after the primary circuit of the induction coil is broken. It only
Figure 7
Voltage across Secondary
of Induction Coil if
No Spark Occurs

Time in Microseconds
requires ninety microseconds for the voltage to rise to its peak value which is almost 20,000 volts. The spark plug used had a gap of 0.022 inches, and required a voltage of 3600 volts to cause breakdown at atmospheric pressure. Thus there should be a lapse of only thirty microseconds between the opening of the breaker and the attainment of breakdown voltage.

The form of the voltage across the spark gap is shown in fig. 8. The voltage build up begins at about 600 microseconds on the time axis, and the capacitive discharge begins at the peak that is seen at about 1400 microseconds. The voltage immediately drops to a value at which it remains almost constant, until the spark extinguishes at 2600 microseconds. The voltage immediately builds up again, but is not able to reignite the spark, so oscillates until the damping of the coil brings it to zero in the neighborhood of 4,000 microseconds. Fig. 9 shows the same characteristic. The timing wave on this
film has a frequency of 1000 cycles per second.

While these oscillograms are valuable for getting a qualitative picture of the spark mechanism, they do not give an entirely accurate representation of the phenomenon because they were taken with a resistance potential divider to decrease the voltage on the plates of the oscillograph. In order to prevent the resistance to ground from affecting the phenomenon, it is made large, generally being about 15 megohms from ignition lead to ground. In the case of oscillogram 14 in Fig. 9, the total resistance was 15.5 megohms, with 6.5 megohms across the plates of the oscillograph, thus giving a multiplying factor of 0.42. Now if the capacity of the deflecting plates of the oscillograph and its leads is 25 micro-microfarads, then the time constant for charging the plates is approximately 225 microseconds, or about one-fourth millisecond. Similarly the time constant for discharging
the plates is about 150 microseconds.

The effect of this long time constant is especially evident on the oscillograms in the trace made by the voltage dropping from the gap breakdown value to the inductive discharge portion of the spark. Actually, the voltage drops so rapidly that no trace is visible. The traces present in oscillograms 1 and 14 (figs. 8 and 9) are due to the discharge of the deflecting plate capacity through the voltage divider.

After it was discovered that the resistance potential divider was causing this distortion, the remaining oscillograms were taken using a capacity potential divider. The capacity that is added to the ignition circuit is very small, as explained on page 10.

Figs. 10, 11, and 12 are examples of the true speed with which the voltage drops from breakdown to the inductive discharge. The voltage drops so rapidly that no trace is left on the film. The periodic phenomenon superimposed on the inductive discharge is reignition. This will be discussed later as it is very important factor in the interference problem.

The voltage at which the trace disappears is 3600 volts plus or minus 100 volts. This agrees with the value found for the breakdown of the gap, using a slowly rising unidirectional potential.

The way the voltage rises to the breakdown value
does not agree with the theory previously developed. Instead of rising at once to the breakdown value, the voltage rises to a potential of about 1000 volts, oscillates about this voltage for a period of almost 0.4 millisecond, then rises rapidly to the breakdown potential. This is shown in fig. 10 very well. The frequency of oscillation is approximately 10,000 cycles. These values of time and frequency have been measured on all the oscillograms taken under similar conditions and the values obtained averaged. They vary little between individual oscillograms, and what variation that is present is chiefly due to the impossibility of measuring the oscillograms precisely.

The explanation for this type of voltage build up is probably that arcing is taking place at the breaker in the primary. If it is assumed that complete interruption is not immediately made, such a build up is possible. The following sequence of events is thought to take place:

The breaker opens at time \( t = 0 \). It does not completely interrupt the circuit, however, and some sort of discharge takes place. Although complete break does not take place the current is reduced enough to cause the secondary of the coil to rise to a potential of about 1000 volts. In from 0.3 to 0.4 millisecond, the arc at the breaker is extinguished and the voltage across the
secondary again rises in potential until it reaches the breakdown value of the gap.

Since the time lag between the first and second rises of secondary voltage is so nearly constant for all the oscillograms it is probable that the arc at the primary breaker is extinguishing at the same part of the cycle each time. This is most likely to occur when the voltage across the primary reaches a zero value, which occurs one-half cycle after the breaker is opened. One half cycle corresponds to 0.3 millisecond. This is not strictly true because a discharge will change the frequency of oscillation somewhat, but as a first approximation it tends to show a correlation between the primary frequency of oscillation and the length of time before breakdown occurs.

**Capacitive discharge**

It has been shown how the voltage across the coil secondary rises to the breakdown value of the gap. The next thing that occurs is the spark. The first portion of the discharge is characterized by its large current and short duration. For a particular case, Finch calculated the maximum current to be over 100 amperes. As stated before this is the portion of the spark that causes the interference observed in the ultra high frequency band, so a discussion of its characteristics should be valuable.
For a short length of time after breakdown of the gap, and for pulses of very high frequency, the spark can be replaced by a resistance. When the gap initially breaks down, the effect is the same as discharging a condenser through a transmission line terminated in a resistance. The circuit corresponding to this period is shown in fig. 13. The condenser corresponds to

![Fig. 13](image)

Equivalent Circuit for Capacitive Discharge

the distributed capacity of the coil. The transmission line is the lead from the coil to the spark plug. This line is assumed to consist of inductance and capacity only; the resistance and leakage being neglected. This is a common assumption when dealing with radio frequencies. The resistance $R$ is considered to be the effective resistance of the spark (for the duration of the capacitive discharge) plus any resistance that may be placed next to the gap. The inductance of the coil is assumed to have infinite impedance for the frequencies present, and so can be neglected. When the gap breaks down the effect is the same as if a switch had been
thrown across the gap connecting the resistance $R$ to the
ground return.

Professor M.F. Gardner, of this department, has
solved this problem from the standpoint of wave functions.
The expression follows:

$$\frac{V(0,t)}{V_1} = 1 - \frac{Z}{Z + R} - \frac{2ZR}{(Z + R)^2} \left[ 1 - 2e^{-\alpha(t - \tau)} \right] \mu(t - \tau)$$

$$+ \frac{2ZR(Z - R)}{(Z + R)^3} \left[ 1 - 4\alpha(t - 2\tau) e^{-\alpha(t - 2\tau)} \right] \mu(t - 2\tau)$$

$$- \frac{2ZR(Z - R)^2}{(Z + R)^4} \left\{ 1 - 2 \left[ 1 - 2\alpha(t - 3\tau) + 2\alpha^2(t - 3\tau)^2 \right] e^{-\alpha(t - 3\tau)} \right\} \mu(t - 3\tau) + \cdots$$

where

$V_1$ = breakdown voltage of gap
$V(0,t)$ = voltage at end of line (see fig. 13)
$Z$ = surge impedance of line; $= \sqrt{L/C}$
$L$ = line inductance
$C$ = line capacity
$\tau$ = time for wave to travel twice the length of the
line; $= \sqrt{LC}$
$\alpha = 1/ZC$
$C_1$ = distributed capacity of coil
$u(t - n\tau)$ = unit function that is zero for all neg-
ative values of the parenthesis
$(t - n\tau)$ and is unity for all posi-
tive values.

The values of line inductance and capacity have
been calculated from the physical dimensions of the cir-
cuit using the following expressions given by Morecroft:

$$\mu(t - n\tau) = \left\{ \begin{array}{ll}
1 & t > n\tau \\
0 & t < n\tau
\end{array} \right.$$
\[
L = 2 \left\{ \ln \frac{2h}{r} + \frac{1}{4} \right\} \text{ centimeters}
\]
\[
C = \frac{L}{2 \ln \left( \frac{2h}{r} \right)} \text{ centimeters}
\]

where

\[
\begin{align*}
1 &= \text{length of line in centimeters}, \\
h &= \text{distance of wire above ground in centimeters}, \\
r &= \text{radius of wire in centimeters}.
\end{align*}
\]

For the set up used in this investigation \( L \) was found to be \( 1.005 \times 10^{-6} \) henry, and \( C \) was found to be 9.72 micro-microfarads.

The plot of this expression is given in fig. 14, for the first three reflections. Actually an infinite number occur, but three are sufficient to give a physical picture of what is occurring.

While the solution of this problem in terms of normal functions would be valuable, as this method of attack would give the frequencies present and their amplitudes as a function of the terminating resistance \( R \), such a solution is very laborious and beyond the scope of this paper. It is possible to give the fundamental frequency that will be present, however. It is the frequency associated with the period \( 2\pi \) for terminating resistances greater than the surge impedance. Thus for all values of \( R \) greater than 322 ohms the fundamental frequency will be \( 8 \times 10^7 \) cycles per second for this particular ignition lead. The wavelength corresponding to this frequency is 3.77 meters.
Fig. 14
Voltage across Gap and Resistor during Capacitive Discharge

\[ V(0,t) \text{ in Percent of Breakdown Voltage} \]

\[ R = 10,000 \text{ ohms} \]
\[ R = 1,000 \text{ ohms} \]
\[ R = 321.5 \text{ ohms} \]
\[ R = 100 \text{ ohms} \]

\[ \text{Time in } 10^{-3} \text{ Microseconds} \]
This will not be the only frequency present in the practical case. In addition to the harmonics of the fundamental, there may be a variation in the fundamental by a factor of ten. The frequency calculated here is the frequency caused by a specific ignition lead, with a certain configuration with relation to the ground plane. This frequency is dependent upon the distributed inductance and capacitance of the lead. In the actual automobile, there may be from four to sixteen leads, all of different lengths and having different distances to ground. For the particular case here, a lead was chosen that was of average length and spacing. Since there are actually a number of different transmission lines present, there will be a spectrum of frequencies covering the ultra high frequency band.

**Inductive discharge**

After the capacitive discharge has been completed, the inductive discharge is established. If reignition does not occur, and the voltage during the inductive discharge is assumed constant as is shown in figs. 8 and 9, then we can write another set of differential equations for the coil, in which the voltage across the secondary is held constant. Fig. 15 shows the circuit.

The differential equations governing the circuit are the following:
\[
\frac{L_1 \, di_1}{dt} + M \frac{di_2}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt = 0 \quad \text{and}
\]
\[
\frac{L_2 \, di_2}{dt} + M \frac{di_1}{dt} + R_2 i_2 = E
\]
in which \(i_1\) and \(i_2\) are the primary and secondary currents respectively, \(E\) is the voltage across the secondary maintained by the discharge, and \(L_1, L_2, R_1, R_2, \) and \(C_1\) are the parameters shown in the figure.

When \(i_1\) is eliminated from the above equations, and \(p\) is substituted for the differential operator \(\frac{d}{dt}\), the following third order differential equation is obtained:

\[
[p^3(L_1L_2 - M^2) + p^2(R_1L_2 + R_2L_1) + p(R_1R_2 + L_2/C_1) + R_2\sqrt{C_1}] i_2 = 0
\]

The expression for \(i_1\) is obtained by simply interchanging \(i_1\) with \(i_2\) in the above equation.

The solution for this equation, which requires solving a cubic equation in \(p\), has been solved by R. Ruedy. The current is of the form

\[
i_2 = A_1 e^{a_1 t} + A_2 e^{a_2 t} + A_3 e^{a_3 t} - \frac{E}{R_2}
\]
in which \(a_1, a_2,\) and \(a_3\) are the roots of the preceding
A cubic equation. One of the roots is real or all three are real; and if there are any complex solutions, they must occur in conjugate pairs. The latter case is the case here. The current consists of an oscillatory discharge (corresponding to the two complex roots) of a single frequency, superimposed upon an aperiodic term (corresponding to the real root). Ruedy gives a general solution, making certain approximations. For the problem here, the particular frequency has been solved for exactly. The frequency present is 7,470 cycles per second. The formula given by Ruedy for the frequency is

$$n = \frac{1}{2\pi \sqrt{L_1 C_1 (1 - k^2)}}$$

in which $k$ is the coefficient of coupling of the coil. Calculated from this expression $n$ is 7,440 cycles per second.

This phenomenon is shown in figure 16. The oscillatory portion of the characteristic dies away in about 0.8 millisecond, while the aperiodic portion has not completely vanished at the end of the film, which is 1.8 milliseconds from the time of the beginning of

![Fig. 16](image-url)
the discharge.

The indistinctness in the first cycle and half is due to what is known as "cathode failure". The beam is travelling so rapidly and with such an amplitude as not to leave any trace on the film. This will be discussed more fully later.

The frequency of oscillation of the current in the inductive discharge disagrees rather decidedly with that predicted by the preceding theory. The frequency of the oscillation in figure 16 is 13,750 cycles per second, while the averaged frequency of all the oscillograms taken for this case is 14,150 cycles. The frequency predicted by the above developed theory is 7,470 cycles per second.

Reignition

In the above discussion of the characteristics of the inductive discharge, the assumption has been made that the voltage across the spark gap is constant for this period. Actually, however, it may not be constant, as fig. 10 shows very well. This oscillogram shows that the discharge is being extinguished and reignited. Each time the arc reignites, another capacitive discharge takes place. It is this type of discharge that contains the most interference energy.

This reignition phenomenon is due to the inability of the coil to sustain the discharge. If the cur-
rent falls to such a value that the discharge is extin-
guished, the coil will recharge the condenser until
breakdown again occurs.

Shallreuter\textsuperscript{9} has given an equation for the period
at which reignition will take place. The expression
follows:

\[ t = R_s C_a \ln \left( \frac{E - E_0}{E - E_1} \right) + C_a R_a \ln \frac{E_1}{E_0} \]

in which

- \( t \) = the period of reignition
- \( R_s \) = the secondary resistance of the coil
- \( C_a \) = the secondary distributed capacity
- \( E \) = the voltage the coil can sustain
- \( E_0 \) = the extinguishing voltage for the gap
- \( E_1 \) = the igniting voltage for the gap.

The period has been plotted in fig. 17 as a func-
tion of \( E_1 \) the voltage of the coil. The values for \( R_s \)
and \( C_a \) used are those given on page 16. \( E_0 \) was taken to
be 320 volts and \( E_1 \), the breakdown voltage was taken to
be 3600 volts, the first breakdown value, and 1000 volts,
the value at which reignition will break down the gap
at 15,000 cycles per second.

It was not possible to verify the presence of
these frequencies by analysis of the oscillograms. The
average frequency of reignition found during the induct-
tive portion of the discharge in the voltage oscillo-
grams is about 15,000 cycles per second. Examples of
this are figs. 11 and 12.
Fig. 17
Theoretical Reignition Period

\[ E = \text{Secondary voltage} \]
\[ E_i = \text{Breakdown voltage} \]
\[ E_0 = \text{Extinguishing voltage} \]

Period in Microseconds

\[ E_i = 3600 \text{ Volts} \]
\[ E_i = 1000 \text{ Volts} \]

Secondary Potential in Kilovolts
This frequency is almost that of the current during the inductive discharge. The average current frequency, as has been stated before, is 14,200 cycles. Furthermore, a study of the current oscillograms shows that the current actually comes to zero in the trough of the first few cycles of the inductive discharge current transient. This is shown very clearly in figures 32, 33, 34, and 35, which were taken with the distributor in the circuit, as will be discussed later. The current also falls to zero for the case with the distributor out of the circuit, but is zero for shorter periods of time so does not show so clearly on the oscillograms.

Evidently there is a major period of reignition controlled by the frequency of the current. When the current goes to zero, the voltage rises, until reignition occurs. The current then goes through another cycle. After the current has passed through a few cycles, it no longer goes to zero and this reignition does not recur. The total number of reignitions that occur during a discharge does not seem to be consistent but varies from oscillogram to oscillogram.

There is a possibility that this reignition is not a single occurrence as has been explained above, but a multiple phenomenon. In other words each reignition shown in fig. 12 may consist of a large number of reignitions, whose frequency is given by fig. 17. While this
may be what is occurring, there is not sufficient data from the oscillograms to prove or disprove it.

**Cathode Failure**

There is one other frequency present in the current characteristic that does not fall in any of the classes mentioned up to the present. It can be explained only by a form of discharge instability, or what is sometimes spoken of as "cathode failure". As the current increases after one of the capacitive discharges, it reaches a value at which very rapid oscillations begin. The oscillograph does not have sufficient resolving power to enable the frequency to be determined accurately, but it shows its presence, and allows estimates of the frequency to be made. It is not a reignition phenomenon, because the amplitude of the oscillation is only 35 to 45 milliamperes, and because the oscillations only exist while the current is in the upper part of its swing. The oscillations are of almost constant amplitude and form an envelope about the top of each current wave. Fig. 18 shows the form of the characteristic.

![Fig. 18](image-url)
The frequency of the oscillations varies between about 150,000 and 500,000 or more cycles per second. The oscillations occur at the highest frequency on the first peaks of the inductive discharge current characteristic, and occur less and less frequently on each following wave. This is shown in fig. 19.

![Fig. 19](image)

An explanation of this oscillation has been advanced by Dr. G.S. Timoshenko. During the glow discharge a cathode spot is formed. This spot does not remain at one point but will reform at other positions. This phenomenon, known as cathode failure, cause variations in the discharge current, which show as oscillations on the oscillograms.

**Spark extinguished**

After the last reignition has occurred, and the coil has no longer sufficient energy to establish another discharge, the remaining energy is dissipated within the coupled resonant circuit.
This "tail" is shown very well in figs. 8 and 9. The average frequency of the tail oscillation has been found to be 1790 cycles per second. The frequency predicted from the constants of the induction coil is 1760 cycles per second. This is a very good agreement.

There is only one frequency observable in the tail oscillation which is what would be expected, as the larger frequency has a much larger damping factor.

**Effect of resistors as interference suppressors**

It has been found that a resistance of 16,000 to 25,000 ohms placed in the spark plug lead immediately adjacent to the spark plug will greatly decrease the interference from ignition. In order to investigate the reasons for this observation, a series of oscillograms were taken of current and voltage characteristics for various size resistors in the lead. Oscillograms were taken with values of resistance in the lead of 1,000, 2,000, 3,000, 10,000, 15,000 and 25,000 ohms.

Oscillograms of current transients with 1,000, 2,000, and 3,000 ohms in the line showed little differences between them. Figs. 16, 18, and 19 are examples. With 10,000 ohms in the line, however marked differences are present. Figs. 20 and 21 are examples. Fig. 20 has 10,000 ohms in the line, while Fig. 21 has 9,500 ohms.

There are three very important points that are
evident from these oscillograms. First, the last portion of the capacitive discharge trace is visible; second, there are no reignitions during the cycles shown in the oscillograms; and third, the number of variations due to cathode failure has been greatly decreased.

Fig. 21 has two separate sparks recorded on the film. While the last portion of the capacitive discharge is not clear, still there are traces of it present. The first spark on fig. 21 has only one cathode fail variation, while the second spark has three. In fig. 20 there are more variations of this sort, but still far less than on the recordings with no resistance in the lead. In this film the last part of the trace of the capacitive discharge is much clearer.

Figs. 22 and 23 are recordings of current transients taken under the same conditions as figs. 20 and 21, except the suppressor resistance has been increased to 15,000 ohms. The capacitive discharge trace is clearer yet, and the number of cathode fail variations has been again reduced. One reignition is observed to take place in the second spark on fig. 22.

Figs. 24 and 25 have had the suppressor resistance still further increased until it is now 25,350 ohms. The trace made by the current in the capacitive discharge is very clear, showing that the transient is now much slower. Also there is no evidence of either
reignition or cathode fail interference during the cycles recorded.

Except for reduction in the number of reignitions, the voltage trace has a very similar form with and without suppressors in the ignition lead. Fig. 26 has 10,000 ohms in the line. Before the initial breakdown, the voltage trace is substantially the same. After breakdown, the only noticeable difference is in the number of reignitions. Only one is recorded in this oscillogram.

Referring again to fig. 14, it is seen that increasing the resistance in the lead does two things to the capacitive discharge: It decreases the initial amplitude of each reflection and decreases the rate at which it is absorbed. The result is that while the time required for the current or voltage to die away is increased, the energy radiated will be less, because the frequency amplitude will be roughly proportional to the amplitude of the front of the wave. In fig. 14, only three reflections have been plotted, which is sufficient,
however, to give a physical interpretation.

It can be concluded, therefore, that using resistor suppressors does three things:

(1) It reduces the high frequency energy in the capacitive discharge.

(2) It greatly reduces, or entirely suppresses, the energy in the cathode failure type of oscillation, the fundamental frequency of which is between 150,000 and 500,000 cycles.

(3) It reduces the number of reignitions occurring, or may suppress them entirely.

These conclusions were confirmed with a radio receiver. The interference dropped off noticeably with 10,000 ohms resistance in the line, and with 25,000 ohms resistance in the line the interference was almost entirely suppressed. There was, however, some interference still present, which would be predicted from the shape of figs. 24 and 25. For, as these oscillograms show, the capacitive discharge was still present. As would be expected, this interference was most noticeable in the high frequency band.

Effect of the distributor

So far in the discussion the assumption has been made that the distributor makes perfect contact. In the oscillograms taken, this assumption was made to be true by shorting the distributor, so there was no possibility
of poor contact.

In actual practice, perfect contact is seldom attained. Only in the ideal case will contact be made just prior to the opening of the breaker points, and not be broken until the complete phenomenon has taken place. In the first place the rotating arm and the contact points in the distributor are likely to become worn, so that there is a small clearance between them at all times. The other possibility is that the breaker opens before the contact arm quite reaches the point, in which case there is also a clearance, for at least part of the spark phenomenon.

In either case, a second gap is present in series with the gap at the spark plug.

A series of oscillograms were taken to investigate this effect. The distributor used was one that had been in actual service. There was a small clearance between the contact arm and each of the points, and in the investigation the contact arm was set adjacent to the desired point, and not rotated. The voltage build up was started in the same way as when the distributor was out of the circuit. Effectively, then a gap was placed in series with the spark plug.

The form of the voltage characteristic is shown in fig. 27. This oscillogram is open to the same criticism advanced before, as a resistance potential divider
was used. However, the form of the voltage is shown fairly well, although the trace does not follow the rapid variations perfectly.

Figs. 28, 29, and 30 were taken with no divider of any sort across the spark. Part of the trace is off the film, but the oscillograms are useful in showing the way the first portion of the voltage build up occurs.

Referring to fig. 27, it is believed the spark to be finally extinguished at a time of one millisecond on the oscillogram time scale. If one gap is extinguished the other will be extinguished also. This leaves the portion of the line from the distributor to the spark plug charged to a potential of several hundred volts and with no way to dissipate its charge and the charge on the oscillograph except through the potential divider and any line to ground leakage that may be present. This accounts for the sloping tails on the transients in figs. 27, 28, 29, and 30. The oscillations superimposed on the tail in figs. 29 and 30 are probably induced on the line by the oscillations in the induction coil. Their amplitude is only 90 to 100 volts.

In this case the complete discharge through the gap occurs in from 0.4 to 0.6 millisecond. This time agrees with the current oscillograms.

The current oscillograms are characterized by the large portion of the cycle during which the current is
zero. This is to be expected since in effect the gap separation has been increased, making larger voltages necessary to ignite and sustain the discharge. Typical oscillograms are figs. 31 to 35, inclusive.

Another characteristic is that the oscillation that has been called cathode failure covers a longer portion of each cycle than when the distributor was out of the circuit.

From these considerations, it can be predicted that more interference will be radiated when the distributor is in the line. This was confirmed with a radio receiver. While the difference was not great, it could easily be detected.
V

EFFECT OF SUPPRESSORS ON IGNITION

The chief objection to the use of suppressors in the ignition system is the possible detrimental effect that they may have on the engine performance.

Mole and Finch\(^3\) have studied the ignition phenomenon from an engine efficiency viewpoint, and tried to discover just what part of the current characteristic was responsible for ignition. They have concluded that the only part of the spark necessary to ignite the gas mixture is the initial portion of the inductive discharge, and that the capacitive discharge is very ineffectual compared to this.

They proved this to their satisfaction by eliminating first the capacitive discharge, and second, portions of the inductive discharge, and comparing the relative effects on the ignition.

To eliminate the capacitive discharge, Finch and Mole used a diode, acting as a current limiter. This prevents the high currents that characterize this type of discharge.

In order to eliminate portions of the inductive discharge they built a type of breaker for interrupting the primary current that could be reclosed at any desired time. When the primary of the induction coil is
reclosed while oscillations are taking place, the energy dissipates very rapidly, the voltage across the secondary drops, and if a discharge is in progress, it is extinguished in a very short time.

In their investigation, the breaker was reclosed in a shorter and shorter time after the initial interruption, thus removing more and more of the tail of the inductive discharge. They found in this way that the duration of the spark could be reduced to one-tenth that of its normal value, without reducing the effectiveness of the ignition.

If their conclusion is correct, then it is important to see what effect placing a resistor in the lead has on this portion of the discharge. To determine this, the maximum current flowing during the first cycle of the inductive discharge has been plotted against the value of the resistor. The plot is given in fig. 36.

The current values are the peaks of the inductive discharge characteristic and not the sum of the inductive discharge plus cathode fail variation. For instance, referring to fig. 21, note that after the capacitive discharge, the current oscillates fundamentally with the frequency of the inductive discharge. Superimposed upon this frequency is the cathode failure disturbance of small amplitude and much higher frequency. If the findings of Mole and Finch are correct, then these rapid
Fig. 36
Discharge Current vs Suppressor Resistance

Supressor Resistance in Thousand Ohms

Maximum Inductive Current-ma
fluctuations contribute little or nothing to the ignition phenomenon. Instead it is the slow pulse that causes the ignition. Therefore the cathode failure oscillations were disregarded in obtaining the above plot.

The curve shows that while individual sparks vary considerably, the average tendency is always in the same direction. In the change between 1000 and 25,000 ohms, the current drops only 12 milliamperes. Therefore the use of suppressors should have little effect on the ignition of gas mixtures.
VI

CONCLUSIONS

The following brief contains the salient points of the investigation:

(1) The existence of several audio and radio frequencies was confirmed.

(2) Radio interference was traced to one of three sources: (a) In the ultra high frequency band to the presence of a capacitive discharge, which causes the ignition lead to radiate at its natural period. (b) In the broadcast band, to the presence of a non-linear oscillation caused by cathode failure. (c) In the broadcast band, to a reignition phenomenon. In the normal spark all three sources are present.

(3) Resistors placed in the lead adjacent to the spark gap were found to decrease or suppress interference by (a) decreasing directly the energy radiated due to the capacitive discharge. (b) By decreasing or suppressing the oscillations from cathode failure. (c) By decreasing the number of reignitions that occur during a spark.

(4) Resistors in the ignition lead should have little effect upon ignition efficiency, if
they are not larger than 15,000 to 25,000 ohms.

(5) Distributors in the ignition leads increase interference by (a) increasing the number of reignitions. And (b) increasing the amplitude and duration of the cathode failure oscillation.
BIBLIOGRAPHY

1. L.F. Curtis
   Electrical Interference in Motor Car Receivers

2. G.I. Finch and R.W. Sutton
   Control of Ignition Coil Discharge Characteristics
   Proc. of Physical Society (London) Vol. 45, 1933,

3. G.I. Finch and G. Mole
   Mechanism of Electrical Ignition

4. R.H. George
   A New Type of Hot Cathode Oscillograph

5. Taylor Jones
   Induction Coil Theory and Applications (1932)

6. F.A. Laws
   Electrical Measurements (1917)

7. W. McFarlane
   Induction Coil Discharges
   Philosophical Mag., Vol. 16, 1933, pp. 865-896.

8. J.H. Morecroft
   Principles of Radio Communication (1927) pp. 193
   and 215

9. R. Ruedy
   Oscillations in the Spark from Induction Coils
   Canadian Journal of Research, Vol. 13, 1935,
   pp. 45-59.

10. J. Slepian
    The Electric Arc in Circuit Interrupters
    Journal of the Franklin Inst., Vol. 214, 1932,
    pp. 413-442.

11. F.E. Terman
MEASUREMENT OF INDUCTION COIL CONSTANTS

The following coil constants must be known in order to calculate the voltage and current relationships: the primary inductance, \( L_1 \); the primary resistance, \( R_1 \); the mutual inductance, \( M \); the secondary inductance, \( L_2 \); the secondary resistance, \( R_2 \); and the secondary distributed capacity, \( C_2 \).

Care must be exercised in controlling the core saturation when making these measurements, as the inductance of an iron-cored coil is a function of the flux density. After trying several circuits for obtain-
ing these constants, it was decided that the Carey Foster bridge \(^6\) was most suitable.

The circuit is shown in figure 37. The induction coil is placed in the lower right arm of the bridge, in the manner shown. The ammeter is in the circuit for determining the degree of saturation of the core. The measurements were made at sixty cycles so that a vibration galvanometer could be used as the balance indicator. The source consisted of a standard frequency oscillator, driving an amplifier, which is connected to the bridge by a shielded transformer. The frequency source is constant to within two parts in a million. The source is not free of harmonics, but the use of the vibration galvanometer as the null indicator removes any difficulty from this point. A Wagner ground is used, as shown in the figure.

This bridge was used because it lends itself to an easy control of the primary current and hence the core flux density, and because it compares mutual inductance directly against a standard capacity, thus making more accurate measurements possible.

The primary current is a function chiefly of the resistance \(R_1\) and the regulation of the source. Standard resistors of low values (ten ohms) and relatively large power dissipation ability can be used.
The equations for balance are:

\[ M = CR_1(R_a + r_a) \]
\[ L_a = \frac{M(R_1 + r_1)}{R_1} \]

where \( M \) = mutual inductance,
\( L_a \) = secondary inductance,
\( r_a \) = secondary resistance,
and \( C, R_1, R_a, \) and \( r_1 \) are standard parameters shown in fig. 37.

The secondary resistance, \( r_a \), must be measured by other means. This can be done easily and accurately.

To obtain the primary inductance the induction coil could be reversed in the bridge. The inductance is so small, however, that it is better to use some other method. The Maxwell bridge\(^6\) will measure this size inductance accurately, and the values of primary inductance used in this paper were found by the use of that bridge. Here also an ammeter was included to determine the core saturation.

Measurements were taken at several values of primary current for both bridges. The mutual and self inductances were plotted against current and values taken past the knees of the curves for the calculation of coil performance.

In the measurement of \( C_a \), the secondary distributed capacity, a beat frequency oscillator was connected to the primary, and a standard adjustable air condenser
was connected to the secondary, as shown in fig. 38.
The principle involved is that as the frequency is increased toward the resonant frequency on the input of a transformer, the secondary voltage will rise until resonance is reached, after which it will drop sharply. Therefore, using the vacuum tube voltmeter as a resonance indicator, it is possible to get a series of values of resonant frequency as a function of the added capacity $C_1$. If the reciprocal of the square of the resonant frequency is plotted against the value of the added capacity, $C_1$, the curve will be a straight line, as explained by Terman. If this line is extrapolated back to the capacity axis, it will have a negative intercept, the magnitude of which is equal to the magnitude of the fixed capacity across the secondary. This is the capacity that is taken as the equivalent of the distributed capacity of the winding.

![Circuit for Measuring Coil Distributed Capacity](image-url)
APPENDIX
DESCRIPTION OF THE OSCILLOGRAPH

Since most of the experimental results were obtained with the cathode ray oscillograph, a description of the instrument used should not be out of place.

The oscillograph is a modification of the Dufour cold cathode instrument. It was designed by R.H. George with the primary purpose of using it for recording lightning or switching surges. A diagram of the instrument is given in fig. 39, and photographs of the oscillograph and its associated apparatus are figs. 40 and 41. The oscillograph is made almost entirely of brass, except for an insulating section of bakelite.

The distinguishing features of the instrument is that it uses a hot cathode to obtain the electrons, and electrostatic focussing for bringing the beam to a point. The ability of the oscillograph of resolving high speed phenomena is obtained by allowing the electron beam to fall directly on the film. This feature requires that a good vacuum pump be an accessory as the vacuum must necessarily be broken to allow film to be placed in or removed from the special "camera" in the bell of the oscillograph.

Provision is also made for viewing slower transients by allowing the beam to fall upon a fluorescent screen placed over the exposed portion of the film. The
Diagram of the Oscillograph

Figure 39

- Cathode
- Cathode Shield
- Anode No.1
- Bakelite Insulating Section
- Focusing Coil
- Anode No.2 (Moveable)
- Deflecting Plates
- Observation Window
- Fluorescent Screen
- Door
- Tray for Phosphorous Pentoxide
- Camera
- Key for turning film
- Flip for raising screen

To Vacuum Pump
transient can then be viewed through the observation window, shown in fig. 39 without clouding the film. When photographic recordings are to be made, the window is covered and the fluorescent screen raised from over the film by means of a flip turning in a vacuum tight grease packing.

In order to reduce to a minimum the number of times the oscillograph must be opened, roll film of size $4\frac{1}{4}$ by $6\frac{1}{2}$ inches was used. The camera is so designed that daylight loading is possible. The film is rolled from one drum to another by means of a rod and key, also turning in a vacuum tight packing. To count the exposures one of the rolls over which the film moves is fastened to a brass disk with a bakelite segment. This disk makes an electrical contact which causes either a light to flash or a counter to operate. One revolution of the disk corresponds to another $3\frac{1}{2}$ inches of film turned into place under the fluorescent screen.

Two anodes are used as shown in fig. 39, and the focussing of the beam is obtained by varying the relative field intensities. The first anode is placed very close to the filament (about 0.05 inch). The cathode is protected from positive ion bombardment by a shield at cathode potential, and completely surrounding it except for a small hole for the electron beam.

The first accelerating anode was generally oper-
Beam Control and Power Supply Circuit

Figure 42
ated at a potential of about 2000 volts with respect to the cathode. This potential is adjustable by means of a multiplier shown to the left of the kenetron in fig. 42. The control is shown in fig. 40. It is the large dial in the left center of the control panel. This anode acts as a grid and controls the length of time the beam is on, as is explained below.

The second accelerating anode was operated at a potential of 10,000 volts with respect to the cathode. This is adjustable by means of the capacitor divider shown just below the first anode control in fig. 42. This control is the large dial in the right center of the control panel shown in fig. 40. The two anodes are insulated from each other by means of a bakelite section in the oscillograph.

The second anode consists of a bell shaped piece extending up into the bakelite section (fig. 39). It may be raised or lowered by means of a rack and pinion which turns in a vacuum tight grease packing. The electrostatic focussing is obtained by moving the anode up or down. The lower part of the oscillograph is at the same potential as the second accelerating anode, which is grounded for safety.

Arrangement is made for magnetic focussing to be used as an auxiliary means of bringing the beam to a small spot. This is done by a coil placed around the
bakelite section in such a manner that the direction of the magnetic field will be that of the electron beam.

The deflecting plates are held by rods insulated from the oscillograph by bakelite plugs. It is possible to adjust the separation, but it is easier to set them at a convenient point and use a potential divider to vary the sensitivity.

The high voltage for the oscillograph is obtained by means of a 10,000 volt transformer and a kenetron. Half wave rectification is used. The circuit is shown in fig. 42.

In the operation of the oscillograph, electrons are not emitted prior to a very short time before the occurrence of the transient that is to be recorded. Operation in this manner increases the life of the cathode and also gives greater contrast on the film. This is done by thyratron controlled sweep and trigger circuits. The trigger circuit controlling the cathode emission is shown in fig. 42, and the sweep circuit is shown in fig. 43.

A short interval before the spark occurs, a condenser is discharged through the coils marked "To the tripping circuit" in figs. 42 and 43. In the beam control circuit (fig. 42) this discharge causes the grid of the thyratron to become positive and the tube to become conducting. Current can now flow through the
Figure 43

110 V, a-c

\[ C_1 = 0.5 \mu F \, \text{fd.} \]
\[ C_2 = 0.5 \mu F \, \text{fd.} \]
\[ C_3 = 1 \mu F \, \text{fd.} \]
\[ C_4 = 1 \mu F \, \text{fd.} \]
\[ C_s = 1 \mu F \, \text{fd.} \]
\[ R_1 = 400 - 25,000 \, \text{ohms} \]
\[ R_2 = 0.5 \, \text{megohms} \]
\[ R_3 = 1 \, \text{megohms} \]
\[ R_4 = 16 \, \text{megohms} \]
\[ R_s = 10,000 \, \text{ohms} \]

To Tripping Circuit

To ungrounded oscillograph sweep plate
thyatron by two paths: through $R_1$, or through $C_1$ and $R_3$ in parallel and the 82 tube. $R_1$ and $R_3$ are large enough so that the current taken by them is not sufficient to keep the thyatron conducting. Therefore the length of time the thyatron conducts is determined by the time constant of $C_1$ and $R_4$. But the voltage on the first accelerating anode is equal to the voltage drop across $R_1$. Therefore there will be an accelerating voltage on this anode when the thyatron is conducting, and none when it is not conducting.

The sweep circuit which is shown in fig. 43, is tripped in the same way as the beam control circuit. When the thyatron becomes conducting due to the discharge of the condenser through the tripping coil, the condenser $C_a$ will charge at a rate determined by the product of $C_a$ and $R_1$.

The sweep plates have an initial bias on them due to the drop in $R_2$ and $R_3$. This keeps the beam off the screen until the sweep is tripped, thus removing one synchronization difficulty. The circuit parameters are so chosen that the beam control circuit will trip either previously to the sweep circuit, or very shortly afterwards. If the beam control is tripped first, no energy will be imparted to the screen or film until the sweep is tripped, due to the bias on the plates. And
the results will be equally satisfactory if the beam control is tripped shortly after the sweep, as long as the time interval is not so great that part of the film is not used, or part of the transient is unrecorded.

The sweep power supply uses a voltage doubler circuit which makes available a potential of about 1400 volts for sweeping the beam across the film. This potential is large enough to pull the beam off the film sufficiently long before the accumulating charge on \( C_3 \) has reached the saturation point, that the sweep is approximately linear.

When the charging current of \( C_3 \) reaches a low enough value, the sweep thyatron will again become non-conducting, and the beam will no longer be swept across the film. \( C_3 \) is one microfarad and \( R_4 \) is about 15 megohms so that \( C_3 \) discharges very slowly. Thus there is plenty of time for the beam control to cut off the cathode emission before the beam again falls on the film.

The instrument was unsatisfactory for investigation of transients of this short a duration. It was next to impossible to obtain a sharp focus of the beam, and the relatively low accelerating voltage rendered the oscillograph too insensitive to follow the rapid oscillations.