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Tipping Points: The Gender Segregating and Desegregating Effects of Network Recruitment

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ABSTRACT

Current scholarship commonly posits that network recruitment contributes to job sex segregation, and that the segregated nature of personal contact networks explains this effect. A variety of empirical findings inconsistent with this explanation demonstrate its inadequacy. Building upon Kanter's observation that recruitment processes often resemble "homosocial reproduction," we develop a population dynamics model of network recruitment. The resulting formal model builds a parsimonious theory regarding the segregating effects of network recruitment resolving the puzzles and inconsistencies revealed by recent empirical findings. This revised theory also challenges conventional understandings of how network recruitment segregates: in isolation, network recruitment – even with segregated networks – is more likely to *desegregate* rather than segregate. Network recruitment segregates primarily through its interactions with other supply-side (e.g., gendered self-sorting) or demand-side (e.g., gendered referring rates) biasing mechanisms. Our model reveals whether and to what extent network recruitment segregates or desegregates, and reveals opportunities for organizational intervention. There is an easily-calculable tipping point where demand-side factors such as gender differences in referring can counteract and neutralize other segregating effects from referring. Independent of other personnel practices, organizational policies affecting employees' referring behaviors can tip the balance to determine whether network recruitment serves as a segregating or desegregating force. We ground our model empirically using 3 organizational cases.

Keywords: social networks, labor market, job segregation, recruitment

Job sex segregation is a proximate cause of gender wage inequality (Bielby and Baron 1986). Men's wage advantages over women are significantly diminished once men and women doing the same job are compared (e.g., Bayard et al. 2003; England et al. 1994; Petersen & Morgan 1995). The most common mode of organizational hiring – network recruitment (DeVaro 2005; Marsden & Gorman 2001) – is commonly understood to be an important contributor to job segregation (Bielby 2000; Marsden & Gorman 2001; Reskin, McBrier, & Kmec 1999; Trimble & Kmec 2011). This understanding holds that network recruitment preserves or exacerbates job segregation because of the segregated nature of personal contact networks (Marsden & Gorman 2001; Reskin & Padavic 2002; Roos & Reskin 1984). However, the mechanisms by which network recruitment segregates, and specifically the role of segregated contact networks in this process, remain under-theorized and under-examined. Several surprising empirical findings highlight the inadequacy of current understandings of how network recruitment contributes to job segregation.

Recent evidence challenges the tenet that network recruitment tends to perpetuate or even exacerbate job segregation – preserving or extending the advantages of the majority. In her study of the Registered Nurse labor market across sixty Pacific Northwest hospitals, Kmec (2008) found that hospitals with *higher* levels of network recruitment had *less* gender segregation than those with lower levels of network recruitment. In the context of racial/ethnic job segregation via network recruitment, Waldinger and Lichter's (2003) study of immigrant worker niches provides additional contrary evidence. They document instances where network recruitment was instrumental in reversing the composition of a job – turning the minority group into a majority, and the erstwhile majority into the minority. Waldinger and Lichter quote a hiring manager describing these effects from employee referrals: "'Once one has one Hispanic, you have two or three more and this brings about a chain reaction. The same thing happened with our accounting department—once we hired an Asian, they seemed to be all Asian'" (Waldinger & Lichter 2003:108-109).

Current understandings also entail the expectation that "persons hired through networks tend to be socially similar to those referring them" (Marsden & Gorman 2001:471). Challenging this expectation, Fernandez and Sosa (2005) document a setting where the referral hires generated by male referrers were mostly (85%) *female*. Even the referral *applicants* generated by male referrers were mostly (56%) female.

Clearly, theory relating network recruitment to job segregation merits an update. The revised theory must be able to specify the conditions under which network recruitment segregates or desegregates and when referrers generate majority in-group or majority out-group referrals in a manner aligning with the empirical record. This more consistent revised theory is the goal of the current study. One important step in developing this revised theory is embracing a more balanced view of network recruitment. Decades after Granovetter's (1973) watershed study of the role of networks in job-changers' job searches, the network recruitment literature has continued to privilege the job-seeker's perspective (Rubineau & Fernandez 2015). As a result, the segregating effects of network recruitment commonly have been viewed as a primarily supply-side mechanism (e.g., Calvo-Armengol & Ioannides 2008; Fernandez & Sosa 2005; Kmec 2005). But network recruitment has both supply-side and demand-side components (Lin et al. 2009; Reskin & Roos 1990; Rubineau & Fernandez 2015). The behavior of referrers – the employees who initiate network recruitment – is an important but under-studied demand-side component to network recruitment and its segregating effects (Kmec, McDonald, & Trimble 2010; Rubineau & Fernandez 2013). In particular, the analysis of the demand-side behaviors of referrers in Rubineau and Fernandez's (2013) simulation study suggest that network recruitment could contribute to desegregation. Specifically, they point to under-represented groups referring at higher relative rates as a potential way to achieve this outcome. However, that analysis leaves unanswered some important questions we address in this paper. When would network recruitment be expected to be segregating and when desegregating? What factors determine the direction of these effects? How does current theory need to be revised to explain these varying effects?

There are two key reasons why the Rubineau and Fernandez (2013) study of referrer behaviors was not able to answer these questions. First, that paper explicitly adopted an exclusively demand-side perspective (2013:2473). Second, the key theoretical explanatory mechanism for the segregating effects of network recruitment – homophily in contact networks – was treated as constant (2013:2474). A theory of network recruitment that embraces both the supply-side and demand-side components of network recruitment and their dynamic interactions with segregated contact networks is needed to provide a more complete and

accurate understanding of its segregating effects. We use an insight from management scholar Rosabeth Moss Kanter paired with analytical tools from population dynamics to develop this more encompassing theory.

In discussing managers' tendencies to replace themselves with demographically similar others, Kanter coined the term "homosocial reproduction" (1977:48). While developed in the context of white collar corporate managers, Kanter's conceptualization of the recruitment process yields two general insights applicable to network recruitment and job segregation. First, network recruitment involves demand-side dynamics, as organizational members are the ones initiating reproduction. Second, network recruitment can be viewed as a form of social reproduction, and can thus be modeled and analyzed using tools adapted from demography. With this perspective, and a focus on job sex segregation, we use tools used to study population dynamics to elucidate network recruitment dynamics and their segregating effects.

The result is a single parsimonious model that is wholly consistent with the empirical evidence and that defines the conditions under which network recruitment contributes to job segregation. This formal model serves as the tool needed to build a revised theory of network recruitment and job segregation aligned with the empirical record. Using several detailed quantitative case studies of gendered referring dynamics in distinct organizational settings, we ground our model with empirical data.

This study complements previous research on network recruitment and job segregation by integrating the demand-side aspects of network recruitment from the perspective of referrers – organizational members who connect job-seekers to the organization by providing job opportunity information to their contacts. This article extends theoretical understandings of the segregating effects of network recruitment by integrating the net effects of supply-side biases (e.g., gendered self-sorting), demand-side biases (e.g., gendered referring rates), and their interactions in the context of network recruitment.

We find that segregated contact networks are a necessary but not a sufficient condition for network recruitment to contribute to job segregation. Surprisingly, we find that in isolation, network recruitment – even with highly segregated networks – is more likely to *desegregate* rather than segregate. Network recruitment can readily segregate when interacting with other supply-side or demand-side biases. Moreover, there is an easily-calculable tipping point where network recruitment's interactions with demand-side biases can

counteract its interactions with supply-side biases. When demand-side biases (i.e., gendered referring rates) are below this point, network recruitment segregates. On the other side of this tipping point, however, network recruitment desegregates. Further, we show that referrer behaviors can tip the balance to determine whether network recruitment serves as a segregating or desegregating force. We conclude by suggesting how organizational policies based on these insights can influence segregation outcomes.

REVISING NETWORK RECRUITMENT THEORY THROUGH FORMAL MODELING

We operationalize network recruitment as a population process, where the current population generates its new members. Here, the population is job holders within a firm, and reproduction is referring. Our initial model uses a Markov process (Keyfitz and Caswell 2005), a common analytical tool for studying population dynamics and sometimes labor market dynamics (e.g., Montgomery 1994; Tassier 2005).

In building our Markov model, we create two population states or stocks: one representing the men in the firm, and one representing the women in the firm. We model how referring affects the relative composition of these stocks. Although our analysis invokes the model's equilibrium state, our claims explicitly require out-of-equilibrium empirical contexts. One of our goals is to reveal the conditions determining the *direction* of the segregating effects of network recruitment. When does network recruitment act as a pushing force preserving or exacerbating job segregation, and when does is act as a pulling force bringing a job towards greater desegregation? The model's equilibrium identifies the eventual composition towards which network recruitment pushes or pulls the current job composition. The movement of the job composition from its current out-of-equilibrium state towards the equilibrium state entailed by network recruitment represents its segregating or desegregating effects.

Consider a hypothetical organization composed of men and women all with the same job title who generate the next generation of organizational members through referring. The proportion male among new hires generated by male referrers is m, and $1 - m$ of the men-generated hires are female. The proportion female among new hires generated by female referrers is w, and $1 - w$ of the women-generated hires are male. When m and w are not representative of the labor market, the result is biased network recruitment. Because of homophily, the general tendency for people to be tied to socially similar others (McPherson, Smith-Lovin,

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and Cook 2001), m and w are both expected to be greater than 0.5, and biased network recruitment is expected to be homophilous. These same-sex referring rates define a transition matrix M representing the reproduction process of referring. This transition matrix M is specified in equation (1), and its corresponding Markov model diagram is presented in Figure 1A.[1]

$$M = \begin{vmatrix} m & 1-w \\ 1-m & w \end{vmatrix} \tag{1}$$

This model entails a hypothetical job[2] for which referring is the only recruitment mode, every job holder refers a single replacement for herself or himself, that each of these referrals are hired, and each new generation of hires completely replaces the previous generation. We first explore this highly restricted model to cultivate an intuition for the dynamics of homophilous network recruitment. Relaxing these unrealistic assumptions neither contradicts nor alters the inferences from our model. (Proofs appear in the Appendix.)

We define j_t to be a 2-element column vector consisting of the count of men and women in the job at time t. The matrix product Mj_t gives the count of men and women in the job in the next generation, j_{t+1}, after a time-step of network recruitment has occurred. Each application of the transition matrix to the job's latest generation returns the next generation. If there is some firm composition j^* such that $Mj^* = j^*$, then the firm is at an equilibrium defined by the transition matrix. This equilibrium composition, j^* is a *fixed column vector* of M, and is the eigenvector of M when M has an eigenvalue equal to one (Grinstead and Snell 1997). Our transition matrix necessarily has an eigenvalue of one, for all possible values of m and w (proof in Appendix). Thus, there will always be an equilibrium job composition completely determined by the parameters m and w. Equation (2) below gives the formula for f^*, the equilibrium proportion female of the job.

$$f^* = \frac{1-m}{2-m-w} \tag{2}$$

Figure 2 illustrates the effects of varying levels of homophilous recruiting for m and w, ranging from 0.5 (gender parity in referring) to 0.99 (almost exclusively same-sex referring). As is shown in the wireframe

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[1] We note that we are modeling this population process as a first-order Markov process where the sex of the referral is not influenced by the sex of the referrer's referrer. In this choice, we err on the side of simplicity. We could find neither theoretical nor empirical statements addressing whether or higher-order influences might exist.
[2] Because we are modeling a firm with only a single job title, we will use "job" and "firm" interchangeably.

of Figure 2A, each specific value of *m* and *w* determines the equilibrium job composition towards which network recruitment alone pushes a job. When *m* ≠ *w*, the equilibrium composition depends upon the difference between the two rates, with larger differences pushing the equilibrium composition further from parity in favor of the group with more homophilous referrring. If men have a higher same-sex referring rate than women (i.e., *m* > *w*), then network recruitment will push jobs towards a male-dominated equilibrium composition, even if the job was initially female-dominated, and vice-versa.

Further, as the values of *m* and *w* approach each other, the equilibrium composition *f** approaches 0.5. When *m=w*, *f**=0.5, even when *m* and *w* are both close to 1. For example, if both men and women generate referral applicants that are 95 percent same-sex, then the 5 percent of women generated by male referrers go on to generate mostly women, and vice versa. As this process continues, the resulting equilibrium composition will necessarily be 50:50.[3] So even in the presence of very high levels of same-sex referring by both men and women, network recruitment can still pull a firm towards gender parity, and it can do so *regardless of either the initial composition of the firm or the degree of same-sex bias in referring.*

One implication of equation (2) is that any particular equilibrium composition can be expressed as a linear combination of same-sex referring parameters *m* and *w*. For the gender parity equilibrium composition (50% female), that line is *m=w*. Figure 2B presents a "top" view of Figure 2A, indicating the decile composition lines and regions relating *m* and *w* for equilibrium compositions from 0% to 100%, in 10% increments. This depiction, resembling an overhead view of a circular stairway, highlights several useful model implications. First, Figure 2B shows the singularity in our model when *m=w*=1. In only and exactly this extreme case, network referring can yield any composition. When *m=w*=1, network recruitment simply preserves the current composition of the firm, whatever that is. If same-sex referring rates are anything less than 100% for both men and women, however, the equilibrium composition from network recruitment alone is a single percentage determined by equation (2) regardless of the initial composition of the job. This independence from initial conditions for all cases except a singular extreme scenario is a second key

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[3] Jackson (1989) showed a similar convergence to parity in the reproduction of gendered nurturing norms. Similarly, Heckathorn (1997) showed that equal in-group nomination rates yields representative equilibria.

implication of our model. The third implication derives from the empirical evidence of same-sex referring rates discussed in greater detail below. Across a variety of empirical estimates of same-sex referring rates, none reached as large as 80%, suggesting that the likely set of equilibria from network recruiting alone would be located within the lower-left quadrant of Figure 2B, or between 30% female and 70% female. Given the context of high and enduring levels of job sex segregation in the U.S. (Weeden & Sørensen 2004; Blau, Brummand, & Liu 2013), network recruitment alone is likely to act more commonly as a desegregating mechanism than as a segregating one.

These results obtain even when we relax our simplifying assumptions, i.e., when there is non-referral recruitment, when a subset of employees refer, and when current employees exit gradually with time. Rather than affecting the final equilibrium proportion female and male in the job, the more complex version of the model only changes how quickly the job reaches the equilibrium defined by m and w (proofs in Appendix). Because our concern is the direction of the push or pull towards or away from segregation being exerted on a job by network recruitment, factors that alter the speed of composition changes are not of primary interest. So although more complex definitions of $\boldsymbol{M}$ may provide greater verisimilitude, because these additional factors do not affect the direction of the segregating or desegregating "force" of network recruitment, we proceed using the simpler version of the transition matrix defined in equation (1).

This simple version of our referring model already presents several fundamental challenges to the conventional wisdom of the segregating effects of network recruitment (e.g., Bielby 2000:123; Marsden & Gorman 2001:471). First, realistic homophilous network recruitment by itself *cannot* simultaneously preserve or exacerbate segregation for both male-dominated *and* female-dominated jobs. Other job-specific or firm-specific mechanisms must be involved for network recruitment not to push all jobs towards the same equilibrium. Second, the segregating effects of network recruitment are likely to be much less sensitive to the current demographic composition of the job, and be much more sensitive to differences in the within-group referring rates of referrers, than previously appreciated. These implications motivate our first proposition:

Proposition 1: *Regardless of the current composition of the job, the composition of referral applicants is expected to move towards the equilibrium composition determined by the same-sex referring rates of job holders.*

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Proposition 1 relates the effects of network recruitment to the composition of referral applicants. Unless there is a strong bias against hiring referral applicants – and empirical evidence shows biases often favor referral applicants (Fernandez & Galperin 2013; Petersen, Saporta, & Seidel 2000) – then over time, the composition of the job will also shift in the direction of the composition of referral applicants.

If homophilous network recruitment alone, operating similarly across all jobs, moves all jobs towards the same equilibrium composition, what types of job-specific or firm-specific mechanisms could change this outcome? One possibility is that the network segregation levels vary widely by job. For example, network recruitment could preserve or exacerbate segregation in male dominated jobs if the men in those jobs had highly segregated networks while the women in those jobs had relatively low levels of segregation in their contact networks. The reverse would need to be true for female-dominated jobs. Whether this variation in contact network segregation exists is an empirical question. Research examining this question does *not* support such a correlation between segregation among contacts and job composition. Belliveau (2005) examined homophily in the networks of female college graduates as they transition to the labor market. There were no significant correlations between women's networks' gender homophily and either women's preference for working in a male-dominated occupation or the general gender composition (by job title) of their job offers. An alternative explanation was offered by a pair of scholars documenting apparent heterophilous recruitment (Fernandez & Sosa 2005). Their explanation relied upon the interaction of network recruitment with existing supply-side biasing mechanisms.

Interactions with Supply-Side Effects: Biases in Who Applies

Fernandez and Sosa (2005) documented a case where male referrers generated majority female referral applicants (and hires). Rather than conclude that men's referring was more heterophilous than homophilous, Fernandez and Sosa invoked the interaction of network recruitment with supply-side mechanisms (gendered applicant preferences for a Customer Service Representative job). Arguing that the composition of non-referral external job applicants can be used to represent the net effects of supply-side

biases for that particular job,[4] Fernandez and Sosa compare the gender composition of referral applicants from male referrers (44% male) to the gender composition of non-referral external job applicants (35% male). They conclude that male referrers still generate a set of applicants that are more male than expected.

Supply-side mechanisms serve to bias the composition of a firm or job even before job-seekers have any formal interactions with a particular hiring organization. When considering the segregating effects of network recruitment, an important question is whether the people a referrer contacts about a job opportunity are similarly affected by the collection of supply-side mechanisms in a way that influences their decision to apply for the job. These effects could be exerted via the choices of the referrer in deciding whom to inform about an opportunity (e.g., Fernandez & Castilla 2001; Kmec, McDonald, & Trimble 2010; Marin 2012; Smith 2005), or the job-seeker in deciding whether to apply (e.g., Fernandez & Friedrich 2011), or both. Recent research finds that job-seekers feel more pressure to pursue job opportunities identified through referrals from friends than non-referred jobs (Sterling 2014). While it is hard to imagine a job-seeker being wholly immune from these mechanisms simply because a contact told them about a particular opportunity, the act of encouragement from a person in the firm, along with possible additional information the referrer could provide, could attenuate the effects of some of those supply-side mechanisms. Acknowledging the need for additional study, we expand our model to include interactions with supply-side biases. Potential referral applicants are subject to the same supply-side biasing mechanisms as those affecting non-referral applicants.

We add a single parameter, s, defined as a positive real number, to our model to represent the net effects of supply-side mechanisms. If the proportion female in the labor market is l, and the observed proportion female of the non-referral external job applicants surviving the supply-side mechanisms is f, we use the following formula relating f, l and s: $f=ls/(ls+1-l)$.[5] If $s=1$, indicating no net supply-side biases, then the

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[4] We admit, an over-simplification. Firms influence the composition of applicants through advertisements (e.g., Gorman 2005) and location (Fernandez & Su 2004) among other mechanisms. The composition of non-referral external applicants represents an upper bound for the net effects of supply-side biases (Fernandez & Sosa 2005).

[5] To explain this formula, consider a population with 2 groups: Ps and Qs, with P and Q representing the counts of their members. The proportion of Ps in the population is $P/(P+Q)$. If we wish to scale (either increasing or decreasing) the proportion of Ps in the population by a scaling parameter x, we can call the new count of Ps Px, and the new proportion of Ps is $Px/(Px + Q)$. In our specific example, Ps are the proportion of women in the labor market, represented here as l. Even though l is a proportion rather than a count, the math is the same. The other group, men, are then represented as $1-l$. Before scaling, we can re-write the proportion of women in the labor market as we did with Ps and Qs as: $l/(l+ 1-l)$. The scaling parameter is s, so the new proportion female in the labor market after scaling becomes: $ls/(ls + 1-l)$.

proportion female among non-referral external job applicants will be equal the proportion female in the labor

market ($f=l$), and no biasing effects are manifest. If $s > 1$, meaning the net supply-side biases favor women,

then non-referral applicants will be more female than the labor market ($f > l$, but f will never exceed one even

as l approaches one, regardless of the value of s). If $s < 1$, meaning the net supply-side biases favor men, then

non-referral applicants will be less female than the labor market ($f < l$, but f will never be less than 0, even as l

approaches 0, regardless of the value of s). In a labor market with gender parity (i.e., that l=0.5), we can re-

write the relation between f and s as follows: $f = s/(s+1)$.[6] Solving for s, we get $s=f/(1-f)$. By using the

proportion female among non-referral external job applicants as an indicator for f, firms can estimate

(arguably an upper-bound of) the net effects of supply-side biases quite easily, as we demonstrate below.

Supply-side biases affect distinctly the behavior of male and female potential job applicants. The

female potential referral applicants, whether generated by male or female referrers, will become actual

applicants with a likelihood scaled by $s/(1+s)$. The male potential referral applicants, whether generated by

male or female referrers, will become actual applicants with a likelihood scaled by $1/(1+s)$. We update the

transition matrix defining our modeled population process as shown in equation (3) below:

$$\boldsymbol{M} = \begin{vmatrix} \dfrac{m}{1+s} & \dfrac{1-w}{1+s} \\ \dfrac{(1-m)s}{1+s} & \dfrac{ws}{1+s} \end{vmatrix} = \frac{1}{1+s} \begin{vmatrix} m & 1-w \\ (1-m)s & ws \end{vmatrix} \tag{3}$$

The corresponding population process model diagram is shown in Figure 1B. Population process

modeling uses both Markov and non-Markov models (Humphries & Baron 2012). This change to the

structure of the transition matrix, as well as subsequent changes to the matrix, makes it a non-Markov

population process. (Matrix entries are now transition rates whose column sums may be values other than

one rather than probabilities that necessarily sum to one). The population outcomes of non-Markov

transition matrices are still analyzed in terms of their dominant eigenvector,[7] which represents the stable

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[6] Parity isn't a requirement. Non-parity markets have different scalar coefficients in the equilibria equations (2, 4, and 6).
[7] In non-Markov models, the dominant eigenvector has an associated dominant eigenvalue that may not be one. Eigenvalues greater than one reflect growing populations, while those less than one reflect shrinking populations. Growth or decline doesn't affect relative composition. We are interested in the direction of network recruiting's effects in terms of segregation or desegregation. The direction is best understood based on the stable composition represented

distribution of the population across the states modeled resulting from those transition rates (Pastor 2009) –

in our case, the stable distribution of men and women in the job.

How do changes in *s* affect the equilibrium composition of a job? The revised formula for the

equilibrium proportion female in the job, *f**, based on *m*, *w*, and *s* is given in equation (4).

$$f^* = \frac{ws - m + \sqrt{w^2 s^2 + 2mws + m^2 + 4s - 4ws - 4ms}}{2 - 2w + ws - m + \sqrt{w^2 s^2 + 2mws + m^2 + 4s - 4ws - 4ms}} \tag{4}$$

When *s*=1, equation (4) reduces to equation (2). To illustrate how homophilous network recruitment interacts

with supply-side mechanisms, we graph these segregating effects using three scenarios of biased network

recruiting. The first scenario is the case where men and women have equally segregated contact networks (i.e.,

m=*w*). As discussed above, this yields an equilibrium of 50% female whenever *m*=*w* <1. The second scenario

is the linear relation between *m* and *w* yielding an equilibrium composition of 67% female (i.e., *m*=2.03*w –

1.03). The third scenario is the linear relation between *m* and *w* yielding an equilibrium composition of 83%

female (i.e., *m*=4.88*w – 3.88)[8]. These equilibria for these scenarios are plotted in Figure 3 as a set of surfaces

where one axis is the same-sex referring rate among women, *w* (which also defines the same-sex referring rate

among men, *m* in a scenario-specific manner), and the other is the supply-side mechanism parameter, *s*.

Figure 3A though Figure 3C plots the equilibrium values as wire frame surfaces along with the solid gray

surface showing the segregating effects of supply-side mechanisms alone (i.e., *s*/(1+*s*)). As suggested in Figure

2B, the latter two scenarios using values for *m* and *w* representing more segregated equilibrium compositions

require a smaller range of the higher values of *w* to yield these outcomes[9]. This smaller range is reflected in

Figures 3B and 3C.

Figure 3 shows that homophilous network recruitment interacts strongly with supply-side biases.

Recall that in the absence of supply-side biases (i.e., *s*=1), the three scenarios each define their own single

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by the eigenvector. The Appendix shows that adding non-network recruitment, tenure, and turnover rates only alters the time required to reach the stable distribution, but does not change the distribution itself. Understanding the direction of effects of network recruitment thus depends upon the dominant eigenvector but not upon the dominant eigenvalue.

[8] Because our model is symmetric to gender, the identical dynamics in terms of percent male would obtain from an examination of increasingly male equilibria.

[9] Mathematically, the range could be extended, but this entails men disproportionately referring women. We model network recruitment biased via homophily, requiring same-sex referring rates *m* and *w* both to be within [0.5,1].

equilibrium composition (50%, 67%, and 83% female, respectively). In Figures 3A-3C, the surfaces intersect the axis representing s=1 at a flat line at those levels. At the axis, increasing w (and m commensurately) does not alter the equilibrium composition. This relationship immediately and rapidly changes away from the axis in the presence of supply-side biases. In the presence of supply-side biases even slightly greater than one, increases in same-sex referring rates have an acceleratingly larger effect on the equilibrium composition of the job. This accelerating effect is strongest when s is close to 1, and lessens to a linear effect as s increases.

By subtracting the segregating effects of the two processes alone, i.e., the maximum of homophilous network recruiting alone (the planes at 50%, 67%, and 83% for the three scenarios) and supply-side biases alone (gray $s/(1+s)$ surfaces in Figures 3A-3C), from their interaction (wireframes in Figures 3A-3C), we show the increase in segregation attributable to the interaction between homophilous network recruitment and supply-side biases. These magnitudes are plotted as a distinct surfaces in Figures 3D-3E. The greatest effects occur when network recruitment is more homophilous (i.e., higher values of m and w), and for supply-side biases in the range of s=[1,2]. The diminishment in the segregating effects from increasing s is attributable to the fact that as s increases, the range for possible increases in segregation diminishes.

Allowing s to take on values less than one, representing a male-favoring supply-side bias, reveals a similarly rapid reversing acceleration towards male-dominated equilibria. This reversal is illustrated in Figures 3G-3I, which plots the equilibrium composition resulting from homophilous network recruitment and supply-side biases where the supply-side bias ranges from female-favoring bias of s=2 to the symmetric male-favoring bias of s=0.5. In Figure 3G, where the gendered same-sex referring rates are equal (m=w), the graph is perfectly symmetric about s=1. Figures 3H and 3I show the effects of male-favoring supply-side biases in the presence of biased referring that would otherwise result in a female-dominated applicant pool (67% female or 83% female, respectively). Even in these scenarios, interactions with male-favoring supply-side biases can yield *male-favoring* segregating effects of referring above-and-beyond (or as depicted in Figures 3H and 3I, below) the segregating effects of male-favoring supply-side biases alone.

The interactions between homophilous network recruitment and supply-side biases can yield the kind of effects commonly described in the literature: male-typed jobs can become even more male, and female-

typed jobs can become even more female. Importantly, homophily in referrers' contact networks alone *does not* drive this outcome. Contact network homophily is a necessary, but not a sufficient condition for network recruitment to contribute to job segregation.

In addition to showing how network recruitment can segregate both male-typed and female-typed jobs, the interactions with supply-side effects can help explain the empirical findings where male referrers generate majority female referral applicants (e.g., Fernandez & Sosa 2005). Analysis of our model shows that whenever $m/(1-m)<s$, where m is men's same-sex referring rate, and s is the scaling factor for the net effect of supply-side mechanisms, male referrers will generate *majority female referral applicants*. This outcome is the result of strong supply-side biases altering the composition of an initially majority male set of potential referral applicants from male referrers' contact networks. Similarly, whenever $(1-w)/w>s$, female referrers will generate majority-male referral applicants, despite sharing the job opportunity information with a group of majority-female personal contacts.

Biasing supply-side biases distinctly alter men's and women's probabilities of applying for a job opportunity after learning about the opportunity from their contacts. By definition, supply-side biases that affect whether a male (female) potential job applicant applies for a job come in to play whether the man (woman) learned of the job opportunity from a male or a female referrer. Demand-side biases act differently.

Interactions with Demand-Side Effects: Biases in Who Referrers

"Demand-side" biases that can interact with network recruitment in ways that depend on the gender of the referrer. Although our model allows men's and women's contact networks to have independently varying levels of segregation, it has assumed that men and women are equally likely to engage in referring. Can we relax this assumption? If gender differences in referring rates bias the composition of referral applicants, this would be a demand-side biasing mechanism by definition, as the organization, through the behaviors of its members, would be the source of the bias. Gendered referring rates are one among many possible referrer behaviors that could contribute to job segregation (Rubineau & Fernandez 2013:2472-3).

We represent a gender bias in referring rates using the demand-side bias parameter, d, defined as the ratio of the referring rate among female employees to the referring rate among male employees. The

construction of this parameter is analogous to the supply-side parameter, s. Women's probability of referring is scaled by $d/(d+1)$, while men's probability of referring is scaled by $1/(d+1)$. The construction of this scaling parameter ensures referring rates remain between 0 and 1 for men and women for all non-negative values of d. If both groups refer at the same rate, then $d=1$. In this case, referrers are perfectly representative of all employees (even in the case where not all employees refer). If d is greater than 1, then women are over-represented among referrers, and if d is less than 1, then men are over-represented among referrers.

Adding demand-side biases to our model modifies our transition matrix $\boldsymbol{M}$ as represented in equation (5) below. As before, the common denominator means that the equilibrium composition of the job defined by the transition matrix is determined by the numerators of the matrix. The model diagram for this population process is provided in Figure 1C. We first examine the effects of biased network recruitment with the demand-side bias in isolation, and then when interacting with both demand-side and supply-side biases.

$$\boldsymbol{M} = \begin{vmatrix} \dfrac{m}{(1+s)(1+d)} & \dfrac{(1-w)d}{(1+s)(1+d)} \\ \dfrac{(1-m)s}{(1+s)(1+d)} & \dfrac{wsd}{(1+s)(1+d)} \end{vmatrix} = \frac{1}{(1+s)(1+d)} \begin{vmatrix} m & (1-w)d \\ (1-m)s & wsd \end{vmatrix} \tag{5}$$

The full formula for the equilibrium proportion female in the job in the presence of homophilous network recruitment interacting with both supply-side and demand-side biases is given in equation (6).

$$f^* = \frac{wsd - m + \sqrt{w^2s^2d^2 + 2mwsd + m^2 + 4sd - 4wsd - 4msd}}{2d - 2wd + wsd - m + \sqrt{w^2s^2d^2 + 2mwsd + m^2 + 4sd - 4wsd - 4msd}} \tag{6}$$

Setting s, the parameter for the net effect of supply-side biases, equal to one allows us first to examine the effects of demand-side mechanisms' interactions with biased network referring in isolation.

Unlike the effects of supply-side mechanisms, demand-side gender biases in referring rates alone do not affect the composition of job applicants absent segregated networks. By definition, in the absence of other supply-side biases, if personal contact networks are not segregated, then the gender composition of men's referral applicants would be the same as the composition produced by women. In this case, the over-representation or under-representation of men or women in referring is immaterial to the gender composition of referral applicants. The demand-side biases of gendered referring only alter the gender composition of

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referral applicants in the presence of segregated contact networks. Figure 4 illustrates the segregating effects of homophilous network recruitment when interacting with demand-side biases, using the same three scenarios from Figure 3. Because the demand-side bias of gendered referring does not bias the composition of job applicants in isolation, we subtract the scenario-specific equilibrium composition from referring alone (50%, 67%, and 83% female) from the equilibrium composition to represent the effects attributable to the interaction between demand-side biases and homophilous network recruitment. Comparing Figures 4A-4C with the corresponding Figures 3D-3F, it is clear that the interactions with demand-side mechanisms generate a similar pattern of effects as did the interactions with supply-side effects. One similarity is the rapid acceleration of segregation with increases in same-sex referring rates when the bias parameter (here, *d*) is close to, but not equal to 1. A second similarity is the diminishing returns to increases of the bias parameter above 2. One difference is that because demand-side biases in referring have no segregating effects in isolation (affecting only the representativeness of referrers, not the composition of applicants), their interaction with homophilous network recruitment can yield ostensibly larger segregating effects.

Another important similarity in the interaction effects is the rapid reversal in composition outcomes when the demand-side biasing parameter *d* takes on values representing a male-favoring bias (men being more likely to refer than women). Figures 4D-4F plot the results as demand-side bias parameter *d* ranges from 2 to 0.5 in a manner analogous to the plots in Figures 3G-3I. Despite relative differences in same-sex referring rates that would otherwise yield female-dominated referral applicants, interactions with male-favoring demand-side biases can quickly reverse these effects to yield increasingly male applicants. Homophilous network recruitment's interactions with job-specific demand-side biases can yield the kind of outcomes usually attributed to the operation of network recruitment alone: that some jobs can become more male while other jobs can become more female through network recruitment. The magnitudes of the effects of interactions with demand-side biases can be as large as or even larger than the effects of interactions with comparable supply-side biases. Despite having been overlooked by much of the literature on referral recruitment (for a few recent exceptions, see Kmec 2010; Rubineau & Fernandez 2013), demand-side processes likely play a large and important role in the segregating effects of network recruitment.

Demand-Side Effects, Supply-Side Effects, and Tipping Points

Examining the joint interactions of homophilous network recruitment with supply-side and demand-side biases requires a mathematical analysis of equation (6). Graphing the findings from this analysis requires an unavailable additional spatial dimension. Our x- and y- axes need to be the supply-side and demand-side biases, and the z-axis needs to be their compositional effects. To include variations in same-sex referring rates m and w, we again use the previous 3 scenarios, and select 3 pairs of values for m and w within each scenario. The results are plotted in Figures 5A-5C. Figure 5 allows both biasing parameters s and d to range from 0.5 to 2, and plots them both on log scales to illustrate the dynamics on both sides of the neutral-bias boundary. When $s=1$ and $d=1$, all three surfaces in each scenario necessarily obtain the equilibrium composition from network recruitment acting in isolation, given in equation (2): 50%, 67% and 83%, respectively.

Figures 5A-5C also reveal the competing effects of these two types of biases. When both biasing parameters work in the same direction, either both towards more female ($s, d > 1$) or both towards more male ($s, d < 1$) outcomes, the equilibria comport accordingly. When $s>1$ but $d <1$, that is, supply-side biases indicate female-favoring outcomes while demand-side biases indicate male-favoring outcomes, all three scenarios show equilibrium compositions more female than the $s=1$, $d=1$ outcome. The supply-side biases dominate. When $s<1$ but $d >1$, supply-side biases indicate male-favoring outcomes while demand-side indicates more female outcomes. Again, all three scenarios show equilibria more male than the $s=1$, $d=1$ outcome. Again, the supply-side biases dominate. The obvious implication is that despite the relative magnitudes plotted in Figures 3 and 4, when both biases are co-present, supply-side biases play a stronger role in determining outcomes than equivalent demand-side biases. The instructive implication is the potential for compensating or counteracting effects one type of bias may have on the other.

Of particular organizational interest are the potential compensating effects of parameters over which an organization may have putative influence. The segregated nature of contact networks is "law-like" (McPherson, Smith-Lovin, & Cook 2001:438), and scholars argue there is little justification to attempt external interventions to alter this tendency (Dabady, Blank, & Citro 2004:43, in the context of race). This exempts same-sex referring rates m and w from organizational influence. Similarly, supply-side biases by

definition are outside the realm of organizational control. This leaves demand-side biases, definitionally located within the realm of organizational influence, as the sole potential organizational lever. We model demand-side biases using model parameter d: the ratio of the likelihood that female organizational members engage in referring to that likelihood for male members. Is there a value of demand-side bias parameter, d, that can counteract the collected segregating effects of the system given parameters m, w, and s?

The answer to this question depends upon the answer to a second question: what gender composition represents the elimination of the segregating effects from referring? This simple question has more than one answer. From the dynamic modeling perspective, the answer is the composition obtained in the absence of homophilous network recruitment. That composition is $s/(s+1)$, or the result of supply-side biases alone. Dynamically, in the presence of some supply-side bias and in the absence of any homophilous network recruitment (or other biases), a job will obtain this composition. Practically, an organization may have a particular desired composition. Examples of plausible compositional goals include the gender composition of labor market generally (with the goal of representativeness) or the current composition of the job (with the goal of no additional segregating effects). We can represent a particular gender composition goal as *f(goal)*. With these answers, we can return to our model to identify the value of parameter d that can counteract or compensate for the segregating effects of the system.

We call this critical value of d, d^*, or the **tipping point** for the segregating effects of homophilous network recruitment in a job with a given level of supply-side bias, s. When $d > d^*$, homophilous network recruitment has an equilibrium composition more female than the compositional goal, *f(goal)*. When $d < d^*$, the equilibrium is more male than *f(goal)*, and when $d = d^*$, the equilibrium composition is *f(goal)* exactly.

This definition of the tipping point guides the process by which we calculate d^* from our model. We find the tipping point for a system and a particular *f(goal)* by setting the equilibrium composition determined by parameters m, w, s and d (i.e., equation 6) equal to *f(goal)*, and solving for d. When we use the dynamical definition *f(goal)*, that is the equilibrium resulting from the operation of supply-side mechanisms as determined by s alone, or $s/(s+1)$, the resulting equation for tipping point d^* is given in equation (7) below.

$$\textbf{\textit{Tipping point:}}\ d^* = (2m\text{-}1)/(2ws\text{-}s) \tag{7}$$

When *f(goal)* is some other value, the tipping point may be determined via calculation, with parameters *m*, *w*, and *s* available from organizational data, as we discuss in more detail below.

Recalling that the demand-side parameter *d* is the ratio of the percent referring among female employees to the percent referring among male employees, reaching the tipping point, *d** most reasonably involves increasing the referring rates of the under-represented group, although theoretically it could include discouraging referring by the over-represented group. Holding the proportion referring among women in the firm constant (to match the labeling in Table 1, we'll call this proportion D), the proportion referring among men would need to be D/*d**. If the original proportion referring among men in the firm is C (again, matching the labels in Table 1), then the increase in referring among men would need to be: (D/*d**) – C. This implication from our model yields our second proposition:

Proposition 2: *In the presence of homophilous network recruitment and its interactions with supply-side biases, increasing the relative referring rate of the under-represented group to its tipping point will counteract the segregating effects of referring.*

Our formula for the tipping point in equation (7) reveals several intuitive boundary conditions. First are the cases when either men or women refer in a completely unbiased manner (i.e., *m*=0.5 or *w*=0.5). When one group engages in biased network recruitment (either homophilous or heterophilous) but the other group's network recruitment is unbiased, then the job composition necessarily moves in the direction of the biased network recruitment. Balancing this bias is only possible if the biased group does not refer. In terms of the formula for the tipping point, *d** is either zero or infinite depending on which group manifests bias in its network recruitment. Second, if for a particular job, either *m* or *w* is above 0.5 while the other is below 0.5, then both groups are preferentially referring the same group. In this case, network recruitment can only increase the composition of that preferentially referred group, and no practical tipping point exists. Beyond these scope conditions, the relative composition of referring groups would not be a successful lever for managing the segregating effects of network recruitment. In short, our tipping point analysis holds when the network recruiting behavior of referrers exhibits some level of homosocial reproduction. We now turn to several empirical cases to examine the reasonableness of this scope condition.

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EMPIRICAL CASES

Our formal analytical model has made numerous simplifying assumptions to allow us to scrutinize the dynamics of homophilous network recruitment. We have defined and operationalized parameters that distill large sets of social processes into single terms. How well do these model parameters translate into observable empirical contexts, and what values do they obtain? To answer these questions, we use data from several detailed empirical case studies to calculate empirical realizations of the variables in our analytical model. The results from these case calculations are presented in Table 1.

The first empirical case (Fernandez and Sosa 2005) is drawn from a call-center located in the mid-western United States serving a large bank. The dominant job title at this call center, and the job to which the case study data pertains, is that of Customer Service Representative (CSR). All CSR job applicants, including referral applicants and information about their matched referrers, applying to the firm between January 1995 and December 1996 provide the data for this case study. A total of 4,114 people (70% female) were employed at the call center during that observation window. Over one-third of all job applicants were referral applicants. These 1,539 referral applicants (70% female) were generated by 1,223 referrers (74% female).

The second empirical case (Fernandez and Fernandez-Mateo, 2006) uses data from the set of entry-level factory production jobs (machine operators and tenders) at the single factory headquarters and main production site of a mid-size multi-site company in the western United States. During the 3+ year observation period (from September 1, 1997 to November 30, 2000), there were 2,605 applications to this firm's entry-level jobs at this site. This firm site employed 557 people (63% female) during the observation window. Among these, 216 (61% female) employees generated 602 (59% female) referral applicants.

The third empirical case uses job application and firm data over a one-year window from the U.S. operations of a large multinational firm employing approximately 9,500 workers. During that one-year window, the firm was 47.6% female, there were over 60,000 distinct external job applicants, and more than 95% of these applicants could be assigned both referral/non-referral status and gender categories. External non-referral job applicants were 40.2% female. Although referrals were the third most commonly-used mode for new external job applicants, they represented only 6% of all external job applicants. Most of the referral

applicants could be matched to a referrer allowing the gender of both referrer and referral to be coded and compared. Of the firm's employees during the one-year window, 21.0% of male employees and 24.9% of female employees generated at least one referral applicant. Male employees' referral applicants were 59.8% male, and female employees' referral applicants were 58.5% female.

These empirical case studies vary in terms of industry, region, firm-size, and the prevalence of referring. The first two are similar in that they are low-wage female-dominated jobs with high turnover rates. Our model should be most applicable to jobs with high levels of referral recruitment and turnover rates, as this turnover allows incumbent workers to be replaced quickly by the next generation of referred workers. The third case is an exception, and potentially provides a more challenging case for our model.

Table 1 shows our calculations of our model parameters from the case details. The first section of Table 1 shows measurable case variables needed to calculate our model parameters. The first model parameter we calculate is s, representing supply-side biases. We find s by taking the ratio of the percent female among non-referral external job applicants to the percent male. We find s to be 1.86, 1.35, and 0.67, indicating (upper bounds for) two cases with female-favoring supply-side biases and one with male-favoring biases.

These values for the supply-side parameter s, along with the same-sex referring *outcomes* based on the matched referrer-referral data (A and B in Table 1), allow us to estimate the same-sex referring *rates* for male and female referrers (m and w). Men's same-sex referring rates (parameter m) prior to supply-side biasing mechanisms are 0.591, 0.629, and 0.500, respectively. Women's same-sex referring rates (parameter w) are 0.618, 0.614, and 0.677. The observed range of same-sex referring rates for both women and men fall in the lower half of our modeled range, [0.5,0.99]. We do not observe a tendency towards heterophilous referring even in the case (Case 1) where men's referrals are mostly female. Another data source provides additional support for the idea that parameters m and w tend to reflect homophilous referring, but not at extreme levels. In 2012, a GSS module asked a nationally-representative set of respondents about their referring behavior. The most recent referrals reported by male referrers were 71% male. The most recent referrals reported by female referrers were 69% female. Restricting to referrals for jobs at the respondents' own employer, men report generating 78% male referrals, and women report generating 76% female referrals.

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The demand-side bias parameter d is the ratio of the percent referring among women at the firm to the percent referring among men. We find d takes on the values of 1.2, 1.06, and 1.19. These values reflect a slightly higher likelihood for female employees to engage in referring relative to male employees. Data from the 2012 GSS referring module do not show a significant gender difference in referring rates.

Having calculated all 4 model parameters for the three cases, we can calculate the case-specific equilibria entailed by homophilous network recruitment and its interactions with supply-side and demand-side biases. The calculated equilibria are 70.6% female, 59.2% female, and 49.8% female, respectively. These equilibria allow us to examine and answer the question: does network recruitment act as a segregating or desegregating force? We have different answers to this question for each of the three cases. In Case 1, the firm is 70.1% female, non-referral external job applicants (our indicator for supply-side biases alone, or $s/(s+1)$) are 65.0% female, and the labor market generally is close to 50% female. For each of these plausible reference or target compositions, the equilibrium from referring is more female, suggesting that homophilous network recruitment plus its interactions with existing supply-side and demand-side biases are pushing the job towards greater segregation. Skipping Case 2 for the moment, Case 3 displays the opposite pattern. Here, the firm is 47.6% female, and non-referral external job applicants are 40.2% female. The calculated equilibrium of 49.8% female is more male than the other two compositions and represents almost perfect gender parity. In this case, network recruitment is clearly pulling the firm towards desegregation. The segregating effect of network recruitment in Case 2 depends upon the reference or target composition. The firm in Case 2 is 63.2% female, and non-referral external job applicants are 57.4% female. The model-identified equilibrium of 59.2% female is more female (more segregating) than the composition entailed by supply-side biases alone. This equilibrium is also more male (more desegregating) than the current composition of the firm and job. Theoretically, the equilibrium composition in the presence of homophilous network is slightly (1.8 percentage points) more segregated than the composition would be in its absence. Practically, the current effects of network recruitment is to desegregate the job relative to its current composition.

For each of these cases, we can calculate the specific tipping point, d^*, needed to eliminate the segregating effects of referring relative to various compositional goals. Practically, this need only be done for

Case 1, where current network recruitment is contributing to greater job segregation. Here, the tipping point relative to the firm's current composition is 1.05, and relative to the equilibrium for the job in the absence of homophilous network recruiting, the tipping point is 0.41. These two tipping points represent the needed value of parameter d, the ratio of women's referring to men's referring, to achieve the compositional goals representing neutral segregating effects. The tipping point relative to the firm's current composition is not very different from the current parameter d, 1.05 versus 1.20. Practically, this means reducing women's higher referring likelihoods relative to men from a 20% advantage to a 5% advantage would be sufficient to render neutral the segregating effects of network recruitment relative to the current gender composition of the firm. Why would preserving a higher rate of referring among women counteract the segregating effects of recruiting, when in this case women have a higher same-sex referring rate than men (w=0.618 vs. m=0.591), and there is a strong female-favoring supply-side bias (s=1.86)? It does so relative to the current composition of the job. The Case 1 parameters at the d^*=1.05 tipping point yields a 70.1% female equilibrium: precisely the current composition of the firm. The dynamics still favor women, but do not increase the percent of women beyond the current composition. To fully counteract the segregating effects of homophilous network recruitment such that the equilibrium would be the composition of the firm or job in the absence of homophilous network recruitment (i.e., the composition under supply-side biases alone, or 65% female) requires a tipping point of d^*=0.41. This means going from a gendered referring situation where women's likelihood of referring is 20% greater than that of men, to a situation where men's likelihood of referring is 144% greater than that of women. This large change is needed precisely because the other parameters, including the difference between same-sex referring rates and supply-side biases all still favor women. Further, as we found in our model analysis, supply-side biases tend to dominate demand-side biases such that larger changes in the latter are needed to counteract the effects of the former.

The application of our model to three empirical cases reveals a substantial policy challenge towards making network recruitment neutral to segregation for one of these cases. The concreteness of this outcome highlights the usefulness of this model. Across the cases, we could identify not only whether and the extent to

which homophilous network recruitment contributes to job segregation, but we could also precisely define policy goals sufficient to counteract the segregating effects when present.

DISCUSSION AND CONCLUSION

This paper develops a population process model to analytically formalize a key aspect of Kanter's insight that organizational recruitment can be seen as a reproductive process with segregating – or homosocial – consequences. Scholars typically argue that network recruitment preserves or exacerbates job segregation, that the segregated nature of personal networks explains this outcome, and view this as a supply-side bias. Our formal model counters each of these claims. In isolation, realistic homophilous network recruitment cannot simultaneously segregate both male-dominated and female-dominated jobs. Rather, the effects of homophilous network recruitment in isolation would be to push all jobs towards the same single equilibrium composition defined by same-sex referring rates it. In this way, network recruitment by itself more commonly *desegregates* rather than segregates. Network recruitment can preserve or exacerbate job segregation in both male-dominated and female-dominated jobs only through interactions with other supply-side or demand-side biases. From these interactions, there is an easily-calculable equilibrium composition towards which network recruitment would push a job. This equilibrium enables a firm to assess whether network recruitment is acting as a segregating or a desegregating force. In addition, there is a similarly easily-calculable tipping point where interactions with demand-side biases (model parameter d), such as gendered rates of employee referring, can counteract the segregating effects of network recruitment.

Is the model we present novel? Sociologists and economists have developed models of segregation in the labor market that incorporate network recruitment.[10] Three studies have models that come close to ours in terms of sharing the goal of understanding the segregating effects of network recruitment. Stovel and Fountain (2009) present an agent-based model examining the role of segregated networks on within-firm job segregation by sex or race. This model assumes that each firm has only one referrer – the hiring manager –

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[10] For the most part, however, these efforts focus on modeling an entire idealized labor market (e.g., Calvó-Armengol & Jackson 2004; Stovel & Fountain 2009; Tassier 2005). The outcomes investigated by the economists are rarely job segregation by sex or race and more commonly wage- or unemployment duration-inequality (e.g., Barr 2009; Gemkow & Neugart 2011). Reviews of the theoretical and empirical scholarship in economics on the topic include articles by Ioannides & Datcher Loury (2004) and Calvó-Armengol & Ioannides (2008).

for its cluster of jobs. These hiring managers are necessarily a member of only one of the two modeled demographic groups (e.g., men or women). This model choice creates a strong interaction with referrer behavior – a demand-side interaction, and hides potential opportunities for organizational action. For a particular organization, all referrers are in one sociodemographic group. In terms of our model, their modeling choice requires the demand-side parameter d to be either zero or infinity for all jobs. The only organizational lever from this model would be to change the category of the single referrer, effectively changing d to zero from infinity, or to infinity from zero. Tassier (2005) also uses a Markov model to investigate the association between referring and job segregation. That model makes the simplifying assumption that all contact networks are perfectly segregated, i.e., $m=w=1$. We show that this simplification yields a singularity in the dynamics of the system. Both of these models make extreme assumptions that unintentionally obscure the role of demand-side processes in network recruitment. A third study by Rubineau and Fernandez (2013) presents a combination of mathematical and agent-based models to examine the role of referrer behaviors in the segregating effects of network recruitment. Their model treats the level of within-group referring (in our model, both m and w) as a single constant. Their analysis leaves the role of segregated contact networks as a black box, focusing instead on the demand-side aspects of network recruitment. Although they do find that referrer behaviors can moderate the segregating effects of network recruitment, their analysis does not reveal the conditions under which network recruitment acts as a segregating or desegregating force. The current study opens the black box of how segregated networks interact with network recruitment, and makes evident the conditions under which network recruitment segregates or desegregates.

This paper yields several implications for theory. First, at the most general level, our model demonstrates that the segregating effects of network recruitment by themselves are not necessarily either a supply-side or a demand-side mechanism. Rather, network recruitment interacts with both supply- and demand-side processes to either segregate or desegregate. Second, per proposition 1, the effects of network recruitment are less dependent on the current composition of the job and more dependent on differences in homophilous referring rates (m & w). These rate dependencies better explain empirical experience than the prior conventional wisdom that referring alone preserves or exacerbates job segregation. In our model, it is

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clear that if one group generates substantially more similar referral applicants than the other, then absent unusually large supply-side or demand-side effects, then over time, that group will become the majority. This is precisely the observation by Waldinger & Lichter (2003) quoted above. They document the irrelevance of initial conditions for two jobs: one going from only one Hispanic to being majority Hispanic, and another going from minority Asian to majority Asian. The story of these two jobs is not merely one of segregation through network recruitment. Initially, the two jobs were majority non-Hispanic and non-Asian, and network recruitment neither segregated nor preserved these initial non-Hispanic and non-Asian majorities. It initially desegregated them. The changes in job composition did not stop at representativeness. The job compositions changed until they reached equilibria reflecting in-group referring rates – with majorities of the groups that refer the most. While Waldinger and Lichter's (2003) observations presented a challenge to the conventional understandings of the segregating effects of referring, they are wholly consistent with the presented model.

Our revised theory also provides insight into the puzzle of the apparent desegregating effects of network recruitment for nurses in hospitals across the Pacific Northwest identified by Kmec (2008). Based on our model, we would propose testable hypotheses about this empirical setting. First, we expect that non-referral external applicants would be more female than referral applicants, indicating strong supply-side gendering effects. Second, we expect that either the same-sex referring rate among male nurses was substantially higher than the same-sex referring rate among female nurses ($m > w$), or the rate at which men referred was greater than the rate for women ($d < 1$), or both. Our model also explains precisely why and how biased network referring can give rise to groups generating referral applicants that are majority out-group members. While previous theories of network recruitment would need to draw upon processes beyond referring to explain these observations, our theory is parsimonious: the groups that refer most and with the greatest within-group bias become dominant. A final theoretical implication of our model is that demand-side processes and other referrer behaviors can affect job segregation. More systematic research is needed to reveal other referrer behaviors that could affect the segregating effects of referring.

Our model also provides implications for managers. The supply-side perspective of biased network referring dominating the literature provides few opportunities for organizations to manage the segregating

effects of network recruitment beyond the choice of encouraging or discouraging the practice. In contrast, the demand-side biases (d: ratio of gendered referring likelihoods) highlighted here provide a powerful lever to organizations for managing the segregating effects of network recruitment. First, firms can collect data of the type shown in our cases to identify the supply-side, demand-side, and referring dynamics present for their specific jobs. Second, policies affecting employee referring behaviors are likely to affect the segregating effects of referring. Referral programs can be leveraged for this purpose. Job advertisements typically indicate that certain groups are particularly encouraged to apply. Similarly, internal communications about a referral program can particularly encourage the under-represented group to participate. Third, by recording referrer and applicant data, and by instituting policies to manage gendered referring rates for example, firms can calculate and work towards reaching their tipping point (d^*). At that point, network recruitment would pull the job towards a target composition, counteracting other segregating processes.

By integrating both supply-side and demand-side dynamics into a formal model of network recruitment, we have provided a more parsimonious and explanatory understanding of the nexus of two important labor market phenomena: network recruitment and job segregation. This new understanding also reveals powerful policy levers for firms in managing the segregating effects of referring. By managing referrer behaviors (e.g., d: gendered referring rates) to or beyond the identified tipping point, organizations can mitigate, eliminate, or even reverse the segregating effects of network recruitment. Whereas extant theories addressing job segregation focus on the elimination of biases in personnel decisions (e.g., Bielby 2000), network recruitment can exacerbate or reduce job segregation, *even in the absence of any biases in personnel hiring or exit decisions*. The theory developed here offers organizational theorists a complementary perspective on the processes perpetuating job segregation, and gives managers a new tool for reducing job segregation.

REFERENCES

Barr, T. 2009. With friends like these: Endogenous labor market segregation with homogeneous, nonprejudiced agents. *Amer. J. Econom. Sociol.* **68**(3) 703-746.

Bayard, K., J. Hellerstein, D. Neumark, K. Troske. 2003. New evidence on sex segregation and sex differences in wages from matched employee-employer data. *J. Labor Econom.* **21** 887-922.

Belliveau, M.A. 2005. Blind ambition? The effects of social networks and institutional sex composition on the job search outcomes of elite coeducational and women's college graduates. *Org. Sci.* **16**(2), 134-150.

Bielby, W.T. 2000. Minimizing workplace gender and racial bias. *Contemporary Sociol.* **29**(1) 120-129.

Bielby, W.T., J.N. Baron. 1986. Sex segregation within occupations. *Amer. Econom. Rev.* **76** 43-47.

Blau, F.D., P. Brummund, A.Y.H. Liu. 2013. Trends in occupational segregation by gender 1970–2009: Adjusting for the impact of changes in the occupational coding system. *Demography*, **50**(2) 471-492.

Calvó-Armengol, A., Y.M. Ioannides. 2008. Social networks in labour markets. S.N. Durlauf, L.E. Blume, eds. *The New Palgrave Dictionary of Economics. 2nd ed.* Palgrave Macmillan, Basingstoke, UK.

Calvó-Armengol, A., M. Jackson. 2004. The effects of social networks on employment and inequality. *Amer. Econom. Rev.* **94** 426–454.

Dabady, M., R.M. Blank, C.F. Citro, Eds.2004. *Measuring Racial Discrimination*. National Academies DC.

DeVaro, J. 2005. Employer recruitment strategies and the labor market outcomes of new hires. *Econom. Inquiry.* **43** 263-282.

England, P., M.S. Herbert, B. S. Kilbourne, L. L. Reid, and L. McCreary Megdal. 1994. The gendered valuation of occupations and skills: Earnings in 1980 census occupations. *Soc. Forces.* **73**,65-100.

Fernandez, R.M., E.J. Castilla. 2001. How much is that network worth? Social capital in employee referral networks. N. Lin, K. Cook, R. Burt, eds. *Social Capital: Theory and Research.* de Gruyter, NY 85-104.

Fernandez, R.M., E.J. Castilla, P. Moore. 2000. Social capital at work: Networks and employment at a phone center. *Amer. J. Sociology* **105**(5) 1288-1356.

Fernandez, R.M., I. Fernandez-Mateo. 2006. Networks, race and hiring. *Amer. Sociol. Rev.* **71** 42-71.

Fernandez, R.M., C. Friedrich. 2011. Gender sorting at the application interface. *Indust. Rel.* **50**(4) 591-609.

Fernandez, R.M., R.V. Galperin. 2014. The causal status of social capital in labor markets. *Contemporary Perspectives on Organizational Social Networks.* Emerald Group 445-462.

Fernandez, R.M., L. Sosa. 2005. Gendering the job: Networks and recruitment at a call center. *Amer. J. Sociology* **111** 859–904.

Fernandez, R.M., C. Su. 2004. Space in the study of labor markets. *Annual Rev. Sociol.* 545-569.

Gemkow, S., M. Neugart. 2011. Referral hiring, endogenous social networks, and inequality: An agent-based analysis. *J. Evolutionary Econom.* **21** 703–719.

Gorman, E. 2005. Gender stereotypes, same-gender preferences, and organizational variation in the hiring of women: Evidence from law firms. *Amer. Sociol.l Rev.* **70**(4) 702-728.

Granovetter, M.S. 1973. The strength of weak ties. *Amer. J. Sociol.* **78** 1360-1380.

Grinstead, C.M., J.L. Snell. 1997. *Introduction to Probability*. American Mathematical Society, Providence, RI.

Heckathorn, D.D. 1997. Respondent-driven sampling: a new approach to the study of hidden populations. *Soc. Problems* 174-199.

Humphries, H.C., J. Baron. 2012. Ecosystem Structure and Function Modeling. M.E. Jensen, P.S. Bourgeron, Eds. *A guidebook for integrated ecological assessments*. Springer 257-272.

Ioannides Y.M., L.D. Loury. 2004. Job Information networks, neighborhood effects, and inequality. *J. of Econo.Literature, Amer.Econo. Association* **42**(4) 1056-1093.

Jackson, R.M. 1989. The reproduction of parenting. *Amer. Sociol. Rev.* **54**(2) 215-232.

Kanter, R.M. 1977. *Men and Women of the Corporation*. Basic Books, New York, NY.

Keyfitz, N., H. Caswell. 2005. *Applied Mathematical Demography*. Springer, New York, NY.

Kmec, J.A. 2005. Setting occupational sex segregation in motion—Demand-side explanations of sex traditional employment. *Work and Occupations* **32**(3) 322-354.

Kmec, J.A. 2008. The process of sex segregation in a gender-typed field: The case of male nurses. *Sociological Perspectives* **51** 259-280.

Kmec, J.A., S. McDonald, L.B. Trimble. 2010. Making gender fit and "correcting" gender misfits sex segregated employment and the nonsearch process. *Gender & Soc.* **24**(2) 213-236.

Leicht, K.T., J. Marx. 1997. The consequences of informal job finding for men and women. *Acad. of Management J.* **40**(4) 967-987.

Lin, N., Y. Zhang, W. Chen , D. Ao , L. Song. 2009. Recruiting and Deploying Social Capital in Organizations: Theory and Evidence. *Res. Sociol. Work* **19** 225-251.

Marin, A. 2012. Don't mention it: Why people don't share job information, when they do, and why it matters. *Soc. Networks* **34** 181–192.

Marsden, P.V., E.H. Gorman. 2001. Social networks, job changes, and recruitment. I.E. Berg, A.L. Kalleberg, eds. *Sourcebook of Labor Markets: Evolving Structures and Processes.* Kluwer Academic, NY 503-530.

McPherson, M., L. Smith-Lovin, J.M. Cook. 2001. Birds of a feather: Homophily in social networks. *Annual Rev. Sociol.* **27** 415-444.

Montgomery, J.D. 1994. Weak ties, employment, and inequality-An equilibrium analysis. *Amer. J. Sociol.* **99** 1212-1236.

Pastor, J. 2011. *Mathematical ecology of populations and ecosystems.* John Wiley & Sons.

Petersen, T., L.A. Morgan. 1995. Separate and unequal: Occupation-establishment sex segregation and the gender wage gap. *Amer. J. Sociol.* **101**(2) 329-365.

Petersen, T., I. Saporta, M.D.L. Seidel. 2000. Offering a job: Meritocracy and social networks. *Amer. J. Sociol.* **106**(3) 763-816.

Reskin, B.F., I. Padavic. 2002. *Women and men at work.* Sage Publications, Thousand Oaks, CA.

Reskin, B.F., P.A. Roos. 1990. *Job Queues, Gender Queues: Explaining Women's Inroads into Male Occupations.* Temple University Press, Philadelphia, PA.

Reskin, B.F., D.B. McBrier, J.A. Kmec. 1999. The determinants and consequences of workplace sex and race composition *Annual Rev. Sociol.* **25** 335-361.

Roos, P. A., B. F. Reskin. 1984. Institutional Factors Contributing to Sex Segregation in the Workplace. B. F. Reskin, NRC (U.S.), Committee on Women's Employment and Related Social Issues, eds. *Sex segregation in the workplace: Trends, explanations, remedies.* National Academies, Washington DC, 235-260.

Rubineau, B., R.M. Fernandez. 2013. Missing links: Referrer behavior and job segregation. *Management Sci.* **59**(11), 2470-2489.

Rubineau, B., R.M. Fernandez. 2015. How do labor market networks work? R.A. Scott, S.M. Kosslyn, Eds. *Emerging Trends in the Social and Behavioral Sciences.* Wiley 1-15.

Smith, S.S. 2005, 'Don't put my name on it': Social capital activation and job-finding assistance among the black urban poor. *Amer. J. of Sociology* **111**(1) 1-57.

Stovel, K., C. Fountain. 2009. The social dynamics of matching processes. P. Hedstrom, P. Bearman, eds. *The Handbook of Analytical Sociology.* Oxford University, NY 365-390. Tassier, T. 2005. A Markov model of referral-based hiring and workplace segregation. *J. Math. Sociol.* **29**(3) 233-262.

Trimble, L.B., J.A. Kmec. 2011. The role of social networks in getting a job. *Sociol. Compass* **5** (2) 165-178.

Weeden, K. A., J.B. Sørensen. 2004. Two-dimensional models of sex segregation: industries and occupations. M. Charles, D.B. Gusky, eds. *Occupational Ghettos: The Worldwide Segregation of Women and Men.* Stanford University, Stanford 245-294.

Waldinger, R.D., M.I. Lichter. 2003. *How the Other Half Works Immigration and the Social Organization of Labor.* University of California, Berkeley, CA.

**Table 1: Deducing model parameters for 3 empirical cases.**

	Case 1[a]	Case 2[b]	Case 3[c]
Case Details			
Number of firm employees	4114	557	9500
Firm percent female	70.1%	63.2%	47.6%
(A) Percent male among applicants referred by men	43.7%	55.7%	59.8%
(B) Percent female among applicants referred by women	75.1%	68.2%	58.5%
(C) Percent referrers among men at firm	21.4%	34.6%	21.0%
(D) Percent referrers among women at firm	25.7%	36.6%	24.9%
(f) Percent female of non-referral applicants	65.0%	57.4%	40.2%
Supply-Side Biases Parameters			
(s) Supply-side effects parameter = f/(1-f)	1.86	1.35	0.67
Demand-Side Biases Parameters			
(d) Demand-side effects parameter = D/C	1.20	1.06	1.19
Same-Sex Referring Rates			
(m) Men's same-sex referring rate = As/(1-A+As)	0.591	0.629	0.500
(w) Women's same-sex referring rate = B/(s-Bs+B)	0.618	0.614	0.677
Equilibrium Compositions			
(f*) Predicted equilibrium percent female	70.6%	59.2%	49.8%
Distance between equilibrium and firm percent female	+0.5	-4.1	+2.2
Distance between equilibrium and supply-side effects alone	+5.6	+1.8	+9.6
Does referring **Push** (segregate) or **Pull** (desegregate)?	**Push**	**Depends**	**Pull**
Tipping Points			
Tipping Point relative to firm percent female	1.05	2.1	0.8
Tipping Point relative to supply-side effects	0.41	0.84	0.003

[a] Case 1 is the Customer Service Representatives (CSRs) reported in Fernandez & Sosa (2005).

[b] Case 2 is the entry-level jobs at one firm reported in Fernandez & Fernandez-Mateo (2006).

[c] Case 3 is from all applications within a one-year period to the U.S. operations of a large multinational firm. The identification of applicant gender and matched referrers was done probabilistically via first names using a database of over 200,000 names and the online https://genderize.io tool.

Figure 1: Population Process Model Diagrams
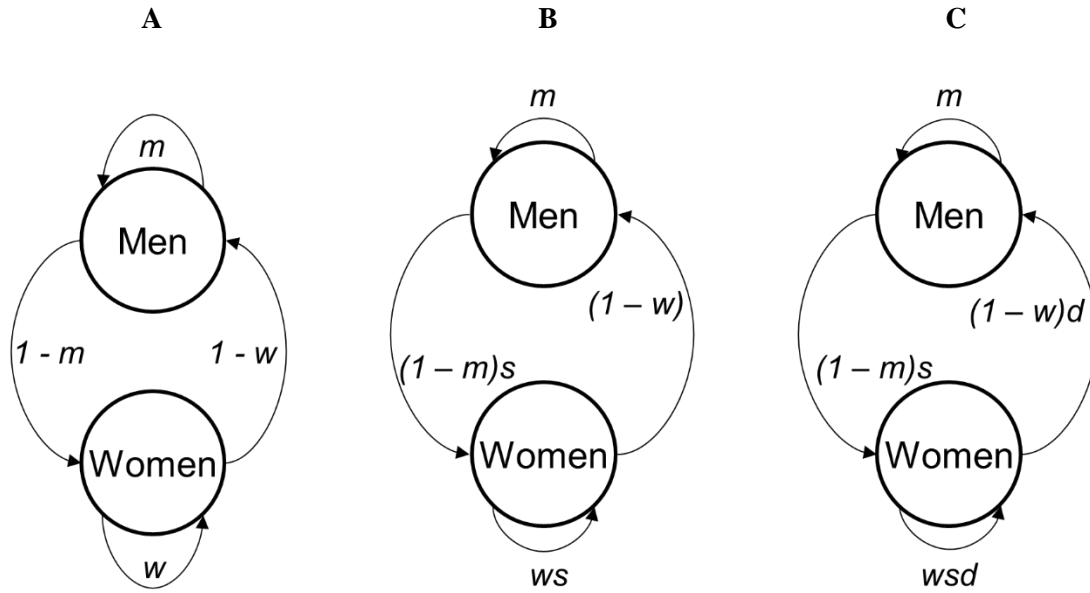
m = Men's same-sex referring rate. I.e., the proportion male among referral applicants generated by male referrers.

w = Women's same-sex referring rate. I.e., the proportion female among referral applicants generated by female referrers.

s = Supply-side parameter. A scaling term representing the net effects of supply-side mechanisms that affect the gender composition of applicants. When $s=1$, there are no net supply-side effects. When $s > 1$, supply-side mechanisms increase the proportion of female applicants. When $s < 1$, supply-side mechanisms decrease the proportion of female applicants.

d = Demand-side parameter. A scaling term representing gender differences in the likelihood of employees becoming referrers. When $d=1$, the gender composition of referrers is representative of the gender composition of employees. When $d > 1$, female employees are relatively more likely to refer. When $d < 1$, female employees are relatively less likely to refer.

FIGURE 2A

The wireframe surface shows the equilibrium job composition under varying values of same-sex referring for men and women (*m* and *w*) ranging from 0.5 (parity referring) to 0.99 (almost exclusively same-sex referring). The intersection of the wireframe surface with the gray plane highlights the values for *m* and *w* where the equilibrium composition is 50% female, that is, when *m*=*w*.
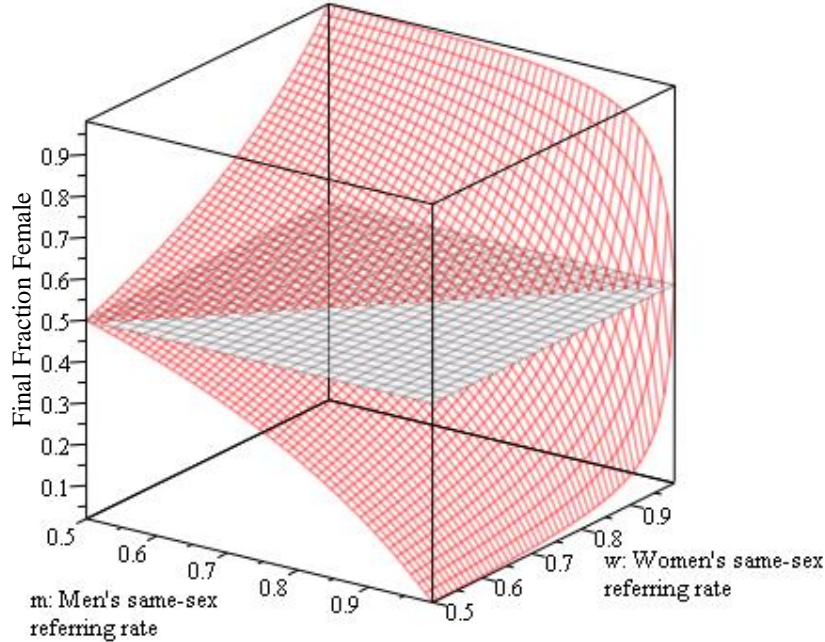

FIGURE 2B

A "top view" of Figure 2a showing decile ranges for the job composition equilibria under varying values of same-sex referring for men and women (*m* and *w*) ranging from 0.5 (parity referring) to 0.99 (almost exclusively same-sex referring).
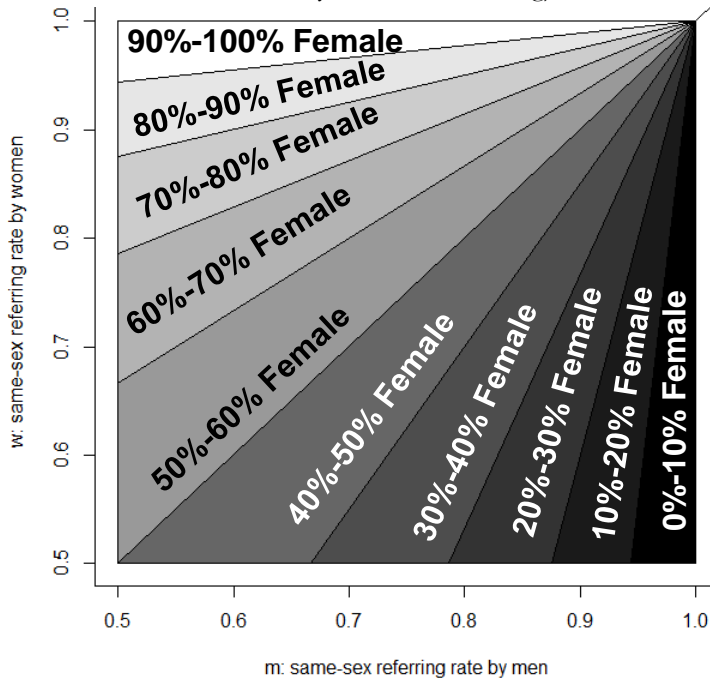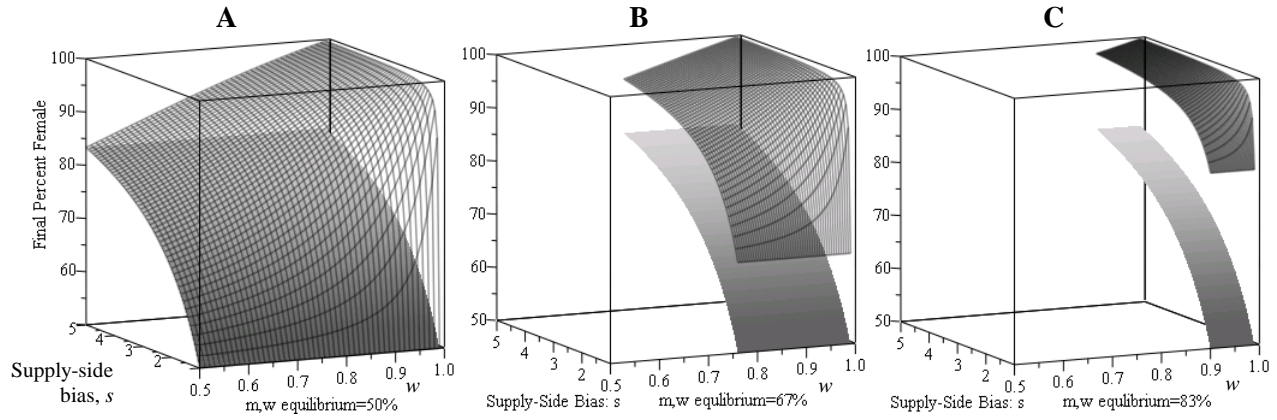

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FIGURE 3: Interactions with Supply-Side Bias

A-C: Wireframe surfaces show the equilibrium job composition of network referring for three scenarios (*m,w*→{50%, 67%, 83%} female) in the presence of supply-side bias, governed by parameter *s*. The gray curved surfaces show the expected job composition based on supply-side bias alone. *s*=[1,5], *w*=[0.5,0.99]

D-F: Surfaces represent the interaction effect magnitudes. They are the result of subtracting the maximum between the gray surfaces in the top panel and the network recruiting equilibria (50%, 67%, or 83%) from the corresponding wireframe surfaces in the top panel. *s*=[1,5], *w*=[0.5,0.99]
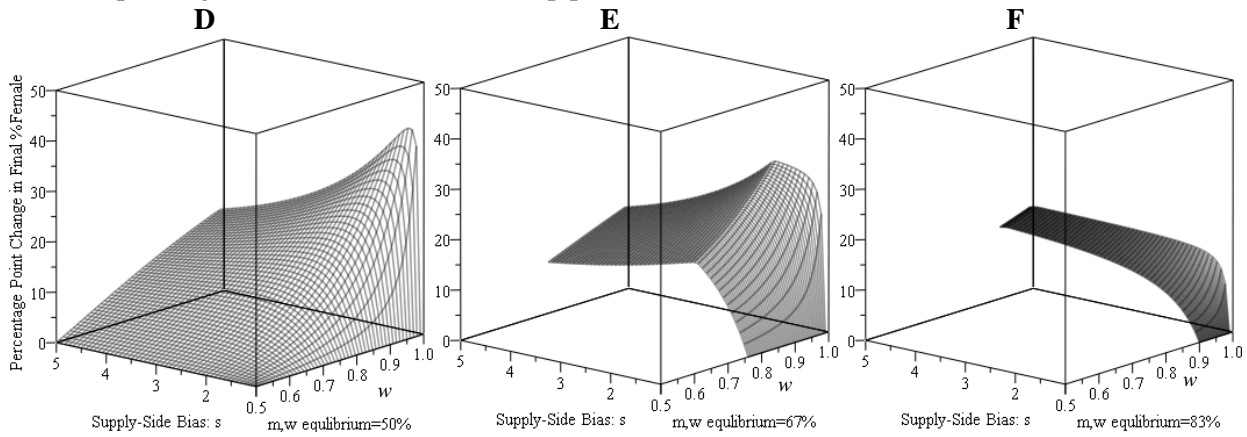
G-I: Equilibrium composition showing reversal of outcomes when *s*<1. These figures replicate Figures 3A-3C, but for a different range of values for *s*: [0.5,2], which are plotted on a log scale. *w*=[0.5,0.99]
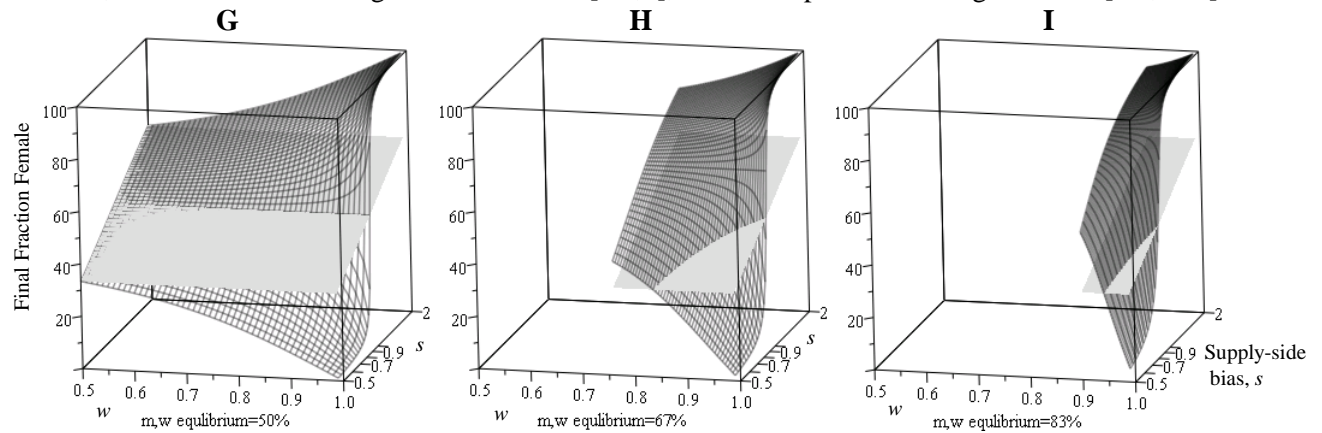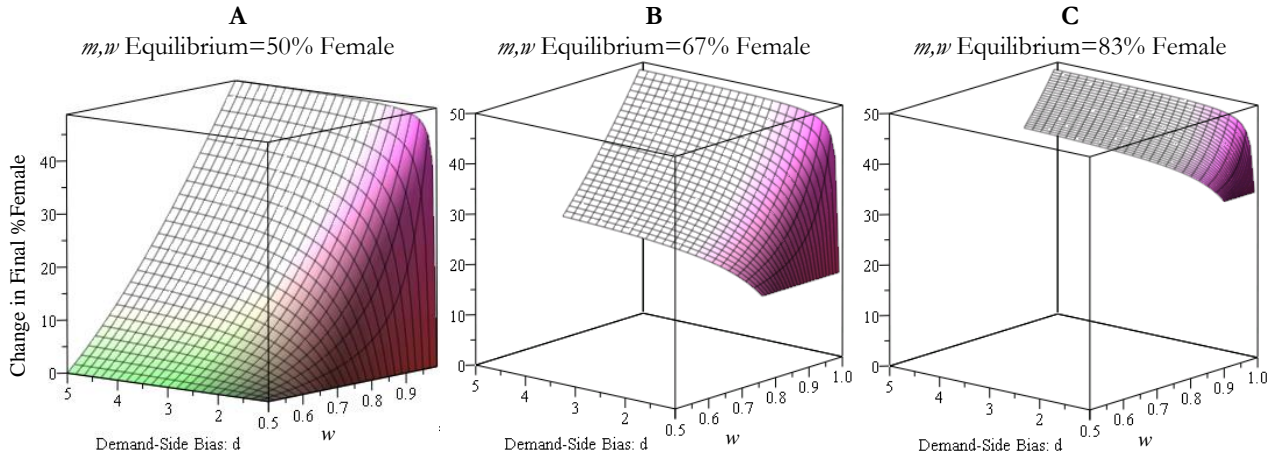

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FIGURE 4

A-C: Magnitude of the interaction effects between homophilous network recruitment and demand-side biases, represented using model parameter d, for three scenarios. w=[0.5,0.99], d=[1,5].

A	B	C
m,w Equilibrium=50% Female	m,w Equilibrium=67% Female	m,w Equilibrium=83% Female

D-F: Equilibrium composition showing reversal of outcomes when d<1. These figures replicate Figures 4A-4C, but for a different range of values for d: [0.5,2], which are plotted on a log scale. w=[0.5,0.99].
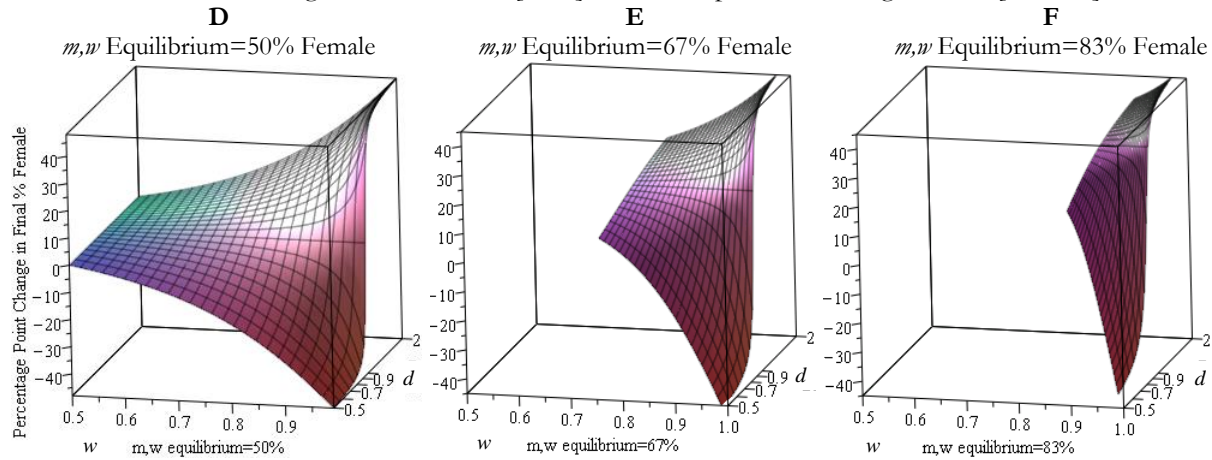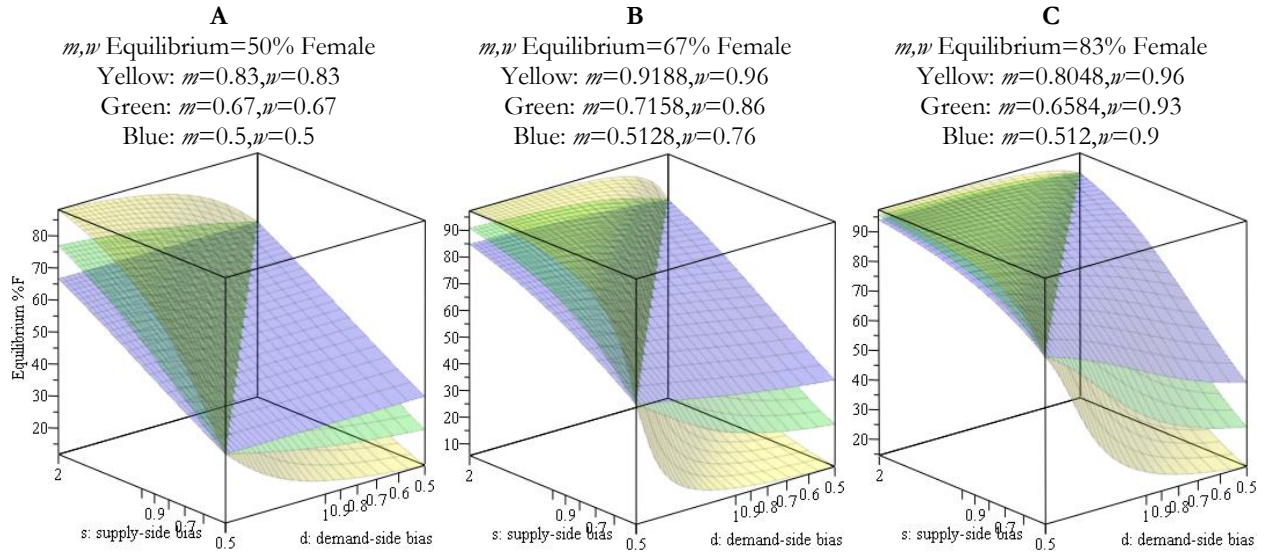

D	E	F
m,w Equilibrium=50% Female	m,w Equilibrium=67% Female	m,w Equilibrium=83% Female

FIGURE 5

Segregating effects of biased network referring when interacting simultaneously with supply-side effects, *s* and demand-side effects, *d*, for three scenarios (equilibrium from network recruitment alone=[50%, 67%, and 83% female]) and three distinct pairs of gender-specific same-sex referring rate paramaters *m*, *w* for each scenario.

A
m,w Equilibrium=50% Female
Yellow: *m*=0.83,*w*=0.83
Green: *m*=0.67,*w*=0.67
Blue: *m*=0.5,*w*=0.5

B
m,w Equilibrium=67% Female
Yellow: *m*=0.9188,*w*=0.96
Green: *m*=0.7158,*w*=0.86
Blue: *m*=0.5128,*w*=0.76

C
m,w Equilibrium=83% Female
Yellow: *m*=0.8048,*w*=0.96
Green: *m*=0.6584,*w*=0.93
Blue: *m*=0.512,*w*=0.9

Appendix: Model Proofs and Extensions

Claim from page 7: The referring transition matrix constructed only from the same-sex referring probability parameters, *m* and *w*, will always have an eigenvalue equal to 1.

Proof:
Let M be a 2 x 2 matrix constructed from only two parameters, *m* and *w* as defined in equation (1) above in the paper. Both *m* and *w* can take on any values in the range [0,1], inclusive.
M will have two eigenvalues: λ_1 and λ_2, each satisfying the definition of an eigenvalue:
det(M- λI) = 0. Given that M is a 2 x 2 matrix, the eigenvalues may be found by solving the quadratic formula below:

$$0 = (m - \lambda) * (w - \lambda) - (1 - m)*(1 - w)$$
$$= mw - \lambda w - \lambda m + \lambda^2 - (1 - m - w + mw)$$
$$= \lambda^2 - \lambda(m + w) + (m + w - 1)$$

The roots of this quadratic formula show that $\lambda_1 = 1$, and $\lambda_2 = (m + w - 1)$. For all possible values of *m* and *w* except $m = w = 1$, $\lambda_1 > \lambda_2$. When $m = w = 1$, $\lambda_1 = \lambda_2 = 1$. Thus 1 is always the largest eigenvalue of M as defined in equation (1) in the paper.

Claim from page 9: Model robustness to alternative specifications
First alternative: Relaxation of the Simplifying Assumptions: Our model of the referral process from the perspective of an organization makes a number of drastic assumptions. First, referring is the sole source of new organizational members. Second, all job holders engage in referring. All referral applicants are hired, and each new generation of job holders completely replaces the previous generation. Does the convenient analytical solution to our model hold when we relax these assumptions? We modify the transition matrix to relax all assumptions but the first. Because we are interested in elucidating referral processes, modeling a system where referrals are the sole source of applicants helps to highlight referring dynamics. We introduce three new parameters: *r* is the proportion of organizational members engaging in referring, *h* is the hiring rate of referral applicants, and *x* is the exit rate of current organizational members.[11] Each of these three new parameters may take values from 0 to 1, inclusive. With the addition of these parameters, the transition matrix can account for less-than-universal referring, less-than-universal hiring of applicants, and employee retention as well as turnover. The modified transition matrix is defined below, and the corresponding population process model is presented in the adjacent figure.
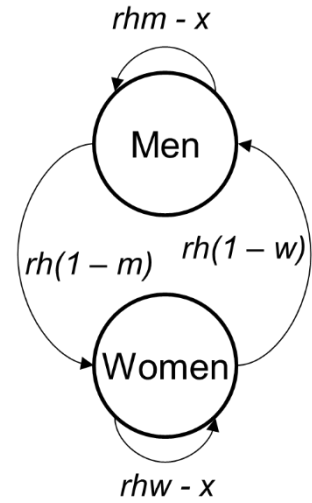
$$A = \begin{vmatrix} rhm - x & rh(1-w) \\ rh(1-m) & rhw - x \end{vmatrix} \tag{A1}$$

Although the introduction of these parameters can change A's eigenvalues relative to the simpler transition matrix (M) defined in equation (1) in the paper, it does not change the equilibrium eigenvector, f^* (proof follows). With the possibility of an eigenvalue other than 1, the equilibrium relation becomes $Af^* = \lambda f^*$, where λ is the largest eigenvalue of A. Consider the meaning of λ being other than one. When the composition of the job is f^*, the composition of the job at the next time step will be λf^*. When λ is greater than 1, the job will still have the same proportion of male and female employees, but the total population will increase by the scaling factor λ. Similarly, when $\lambda < 1$, the total population of the job decreases as determined by λ, but the proportion male and female remains the same.

Thus, we see that this new set of parameters does not change the equilibrium percentage of men and women of the job, only the job's size (which either grows to infinity or shrinks to zero depending on the

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[11] All applicants have equal probabilities of being hired, and all job holders have equal probabilities of exiting. Were either of these probabilities to differ by sex, these would represent screening and turnover biases, respectively. While such biases would certainly affect job composition, these effects derive from a host of mechanisms distinct from the referring process. Their inclusion is not needed to elucidate the segregating effects of referring.

value of λ). The new eigenvalue for transition matrix $\boldsymbol{A}$ in equation (A1) is rh-x, but the eigenvector giving the relative gender composition of the job at equilibrium is the same as for $\boldsymbol{M}$. So even with this more inclusive transition matrix, there is an equilibrium in the proportion female in the job, completely determined by referral homophily parameters m and w, and given by the same formula (2) from the paper. Rather than affecting the final equilibrium proportion female and male in the job, the three parameters r, h, and x simply affect how quickly the job reaches that equilibrium. Because the different definitions of $\boldsymbol{M}$ do not affect our outcome of interest, job segregation, we proceed using the simpler version of the transition matrix defined in equation (1).

   Claim: If v is an eigenvector of $\boldsymbol{M}$, where $\boldsymbol{M}$ is a function exclusively of m and w, defined as above, then v is also an eigenvector of $\boldsymbol{A}$, the transition matrix defined in equation (A1), with the additional r, h, and x parameters to allow for varying probabilities in referring, selection for hire, and exiting the job. As with m and w, the parameters r, h, and x are constrained to the range [0,1], inclusive.

Proof:

$\boldsymbol{M}v$	$= \lambda v$	by definition of eigenvector;
$\boldsymbol{A}$	$= (rh\boldsymbol{M} - xI)$	defining $\boldsymbol{A}$, the transition matrix in (A1) in terms of $\boldsymbol{M}$ in equation (1);
$\boldsymbol{A}v$	$= (rh\boldsymbol{M} - xI)v$	multiplying both sides by v, the eigenvector of $\boldsymbol{M}$;
	$= rh\boldsymbol{M}v - xv$	right distributive property of matrix multiplication;
	$= rh(\lambda v) - xv$	substitution from above;
	$= (rh\lambda - x)v$	distributive property of equality.

Thus, we see that if v is an eigenvector of $\boldsymbol{M}$ associated with eigenvalue λ, then v is also an eigenvector of $\boldsymbol{A}$ associated with the eigenvalue $(rh\lambda - x)$. As we showed above, the eigenvalue of interest of $\boldsymbol{M}$ is always equal to 1, therefore the corresponding eigenvalue for $\boldsymbol{A}$ is $(rh - x)$.

Second alternative: Dynamic Feedback: Although same-sex referring rates may vary by job across the labor market, they are modeled as static within the job. That is, we assume same-sex referring rates do not change with time or as the composition of the job changes. The composition of jobs may affect the use of referral recruitment (Taber and Hendricks 2003[12]). If so, would a model that relaxes this assumption and allows same-sex referring rates to be a function of the current composition of the job be preferable?

   We examined this possibility, although not using the same analytical approach from the paper. A different approach was necessary because a model parameter becomes a function of the current state of the model. Although the changes to the model's structure are somewhat complex, the behavior of this revised model is actually quite trivial. The results are the equivalent of a ball atop the peak of a hill. The ball can remain atop the peak, but that position is unstable. Any slight movement to one side or the other causes the ball to roll to the bottom of one of the sides of the hill. The dynamics are the same for the revised model. There is some value when the job remains at a constant composition (e.g., 50:50 male:female under equal same-sex referring rates), but this composition is unstable. If the composition ever deviates from this point, the deviation accelerates until the job is either 0% female or 100% female. The unstable critical point in this model with dynamic feedback is the same as the tipping point in our model, but because this point is unstable, deviations in either direction push the job to complete homogeneity. Because our empirical estimates of same-sex referring rates are surprisingly stable across distinct settings, because the behavior entailed by dynamic same-sex referring rates are both trivial and disconnected from any theoretical or empirical findings, and because our calculated tipping point is still an important critical value even in this model with feedback, we pursue our simpler model and neither present nor pursue further the model with dynamic feedback.

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[12] Taber, M.E., W. Hendricks. 2003. The effect of workplace gender and race demographic composition on hiring through employee referrals. *Human Resource Development Quart.* **14**(3) 303-319. This study bases its estimates of referring on post-hire results and the measured association is correlational rather than causal or even directional.