Three Essays on International Trade

by

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Abstract

This thesis consists of three essays about international trade and wage inequality.

Essay I characterizes optimal trade and FDI policies in a model with monopolistic competition and firm-level heterogeneity similar to Helpman et al. (2004). I find that both the optimal import tariffs and the optimal FDI subsidies discriminate against the more profitable foreign firms. This is because of the existence of a wedge between the private incentives of exporting and FDI firms, and the incentive of the representative agent.

Essay II develops an elementary theory of global supply chains. It considers a world economy with an arbitrary number of countries, one factor of production, a continuum of intermediate goods, and one final good. Production of the final good is sequential and subject to mistakes. In the unique free trade equilibrium, countries with lower probabilities of making mistakes at all stages specialize in later stages of production. Using this simple theoretical framework, it offers a first look at how vertical specialization shapes the interdependence of nations.

Essay III proposes a model that has as ingredients heterogeneity of workers and firms, complementarity between occupations within each firm and complementarity between workers and firms/occupations. The competitive equilibrium features positive assortative matching and leads to both within- and between- firm wage variations. Comparative static results are then derived to generate new insights about changes in these components of wage inequality.

Thesis Supervisor: Arnaud Costinot
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Contents

Chapter 1

Optimal Trade and FDI Policies with Firm Heterogeneity

1.1 Introduction

Multinationals and their subsidiaries have been playing a more and more important role in world economy and international trade. For instance, McKinsey (2010) documented that despite employing only 11 percent of the private sector laborforce in the United States, US multinationals accounted for 23 percent of US private sector GDP in 2007 and contributed 31 percent of growth in real GDP since 1990. In addition, collectively, they accounted for almost half of exports and more than a third of imports of the United States in 2007.

This trend in the increasing importance of multinationals echoes a recent trend in cross-border trade deals and trade talks that have increasingly shifted their objectives from building a conventional free trade agreement, which focuses on cutting tariffs, to a more comprehensive trade and investment pact that tries to coordinate across countries a broader array of policy instruments that could affect or distort trade and investment flows across borders. The US attempt to build the Trans-Pacific Partnership (TPP) under the Obama administration is a stark example of this. Despite its likely fate, the proposed agreement that was very close to being signed covered a series of subjects including intellectual property, competition policy, dispute settlement, trade-related environmental matters and labor standards (Schott (2016)).

However, the increasing enthusiasm in policymakers of discussing these "unconventional" trade policies in general, and foreign direct investment (FDI) policies in particular, has not been very well matched by the same level of enthusiasm among trade economists.[1] As argued in Costinot et al. (2016), firm heterogeneity, which has changed the way economists think about international trade, has not been thoroughly examined for their policy implications regarding multinationals and FDIs. The purpose of this paper, therefore, is to take the state-of-the-art trade model featuring firm heterogeneity, as developed by Melitz (2003a), as well as export and FDI choices, extend it into a set-up in which a government tries to find its optimal trade and FDI policies, and solve for the unilateral optimal policies.

This paper builds on and extends the firm heterogeneity with export/FDI choice model from Helpman et al. (2004). As argued in Helpman et al. (2004), "firms can serve foreign buyers through a variety of channels: they can export their products to foreign customers, serve them through foreign subsidiaries, or license foreign firms to produce their products". This paper extends this reasoning to the policy scope: given this variety of channels of firms to serve foreign buyers, a government's policy instruments across these channels will necessarily have mutual influences and must also be construed as a whole. I find that optimal trade and FDI policy requires firm-level taxes that discriminate against the most profitable foreign firms, whether they enter Home market via exporting or via FDI. In contrast, the optimal export taxes should still be uniform. Although optimal FDI policy favors the least productive foreign firms, and may go all the way to subsidize them to attract them into producing at Home, this does not follow policymakers' usual argument that doing so would boost domestic labor demand or increases Home GDP. Rather, it stems from the fact that regarding the entry of a single foreign

[1] In fact, in the Handbook of International Economics 3, Grossman and Rogoff (1995), there is no mention of FDI policies at all.

firm, Home's welfare is not completely aligned with the firm's profitability. From Home's perspective, under any given uniform FDI policy rate, it is always beneficial to promote the entry of the marginal foreign firm. Since Home's welfare is concave whereas a firm's profitability is linear in quantity, at the margin, the cost to attract the foreign firm into entry is always outweighed by the welfare gain associated with its entry.

Although, as argued above, there has not been a lot of research on the topic of FDI policies from trade economists, some of the insights that this paper finds can be related to earlier works. The terms-of-trade externality, as well exposed in Bagwell and Staiger (2002), is one of the motives behind Home's optimal tariff and FDI decisions in this model. The firm-delocation externality, as first identified in Venable (1985), also arises in my model since markets are imperfectly competitive and there is free entry, as in Venable's original set-up. Within the international trade literature, this paper is also related to the literature of optimal tariff, specifically when governments are benevolent (i.e. without political economic considerations, as in Grossman and Helpman (1995)). Notable precedents includes Dixit (1985) and Bagwell and Staiger (1999). Jones (1967) also uses optimal tariff arguments to study the taxation of capital movements in a static model with two goods. Finally, another strand of literature that this paper can be related to is the public finance literature on tax competition for FDI. Razin and Sadka (2008) uses a set-up where capital is the only factor of production to study the impact of taxes on both the intensive and the extensive margins of FDI. They emphasize the "fiscal externality" through which tax base could be shifted from the FDI source country to the host country. Haufler and Wooton (1999) consider tax competition between two countries of different sizes to attract a monopolist. They allow both a lump-sum tax and a tariff as policy instrument, and conclude that the larger market would always win the competition for international mobile capital. More recently, Demidova and Rodríguez-Clare (2009), Felbermayr et al. (2013) and Costinot et al. (2016) try to answers questions of optimal tariff and subsidies under firm heterogeneity. All of them do not consider the possibility for firms to do FDI, which is what this paper is focused on. Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013) are also restricted to environments in which only uniform tax rates are considered, while this paper considers the optimal trade and FDI policies that could vary at the firm-level.

In terms of methodology, this paper builds on the frameworks of Helpman et al. (2004) and Costinot et al. (2016). Specifically, I extend the framework of Costinot et al. (2016) to include FDI options for firms and use a primal approach to characterize the optimality conditions for Home, before drawing the implications on trade and FDI taxes. The novelty of this paper is to consider together the two potential choices of a firm to enter the foreign market, and compute the Lagrangians related to each of the options. This method is illustrative in the sense that it could also be useful when firms face other choices of production, be they about technologies or different factors of productions.

The remainder of this paper is composed of four sections. In Section 2, I describe the basic model. In Section 3, I set up and solve the micro and macro optimization problem for a Home social planner. In Section 4, I characterize the optimal trade and FDI policies that implement the solution to the problem in Section 3. Finally, I provide some concluding comments in Section 5.

1.2 The Basic Model

1.2.1 Set-up

Preferences. Consider a world of two countries, Home and Foreign, both of which use labor to produce goods in 2 sectors. One sector produces a homogeneous product that will be used as a numeraire; the other sector is composed of a continuum of varieties with a constant elasticity of substitution. The preferences of the representative consumer at Home and in Foreign are given by a standard utility function:

$$U = U(C, C_n),$$

where

$$C^{1/\mu} = \int_{\omega \in \Omega} c(\omega)^{1/\mu} \, d\omega,$$

$\sigma > 1$ is the elasticity of substitution for the continuum of varieties (with $\mu \equiv \frac{\sigma}{\sigma-1}$ being the mark-up), and C_n is the consumption of the homogeneous good. Ω is the set of all varieties of consumed goods, with $c(\omega)$ being the level of consumption for each $\omega \in \Omega$.

As shown in Dixit and Stiglitz (1977), if we define the aggregate price index as

$$P = \left(\int_{\omega \in \Omega} p(\omega)^{1-\sigma} \, d\omega \right)^{\frac{1}{1-\sigma}},$$

the optimal consumption follows

$$c(\omega) = C \left[\frac{p(\omega)}{P} \right]^{-\sigma}.$$

In what follows, all foreign variables are denoted with asterisks to be separated from their Home counterparts.

Production. I assume that labor is the only factor of production and is inelastically supplied at L and L^* in both countries. In addition, labor markets in both countries are assumed to be perfectly competitive.

As in Helpman et al. (2004), I assume that the production of the numeraire good exhibits constant returns to scale in both countries. In addition, it is assumed that in order to produce one unit of it, it always takes one unit of labor in Foreign and w^{-1} units of labor at Home, with $w > 0$. This good is assumed to be freely traded between both countries (with no trade costs) and its international price is normalized to unity. It then follows that the wage in Foreign is always equal to 1 and wage at Home equal to w. L_n and L_n^* denote the amount of Home and Foreign labor used to produce this good. As in Helpman et al. (2004), I assume that the parameters of the model are such that at all equilibria, there is positive production of the numeraire good and the composite good in both countries, $0 < L_n, L_n^* < L$.

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In the other sector, as in Melitz (2003a), there are a continuum of monopolistically competitive firms both at home and abroad. The set-up for fixed and varying cost of production is similar to Helpman et al. (2004). In particular, in order to enter the market, a firm has to pay a fixed cost of entry, f^E, measured in labor units of the country where the firm is located. The fixed cost is assumed to be constant across firms. After paying the fixed cost, an entrant draws a productivity φ from a distribution $G(\varphi)$ with a support Φ. [3] A firm that draws φ has its varying cost of producing any quantity q is equal to $\frac{q}{\varphi}$. Thus $\frac{1}{\varphi}$ is the unit labor requirement. Upon observing this draw, a firm may decide to exit and not produce, or to produce after paying an additional fixed overhead cost, f_d, measured again in labor units in the country where the firm is located. After paying f_d, the firm may enter the market in the country it is located.

There are two ways for the firm to enter the foreign market: exporting or FDI If it wants to export, it bears an additional fixed cost f_m (measured in labor units in the firm's home country) to enter the foreign market. Serving the foreign market via exporting also incurs an ice-berg trade cost, $\tau \geq 1$. Alternatively, the firm can choose to do FDI, which induces an additional fixed costs f_i, this time measured in labor units of the country the firm does FDI in. [4] [5]Finally, firms are owned by consumers in the country they are located and we assume that firms' profits are paid back to consumers in a uniform manner.

Since I allow firms to export or invest in the other country, in equilibrium, the

[2]A priori, with policies being endogenous, one could worry that with the optimal policy, Home may want to be at a corner. However, when the trade costs are very high such that both export and FDI are relatively small compared to the other modes of production, for instance, the two sectors are de facto a tradable sector (numeraire) and a non-tradable sector (composite good). Here, as long as the country sizes are proper, there will be active production of both the numeraire good and the composite good in both countries, which justifies the original assumption.

[3]For ease of notation, I assume that the distributions are identical for both countries. Having them different does not change the results in a qualitative way, see Costinot et al. (2016).

[4]Alternatively, one could assume f_i is measured in labor units of the firm's home country, or it could be a combination of labor in both countries. These specifications do not change the results in a qualitative way.

[5]It is also assumed that it is never profitable for firms to "offshore", i.e. serving its home market via production in its subsidiaries in the foreign country. One possible sufficient condition for this to happen is $\tau w, \frac{\tau}{w} > 1$.

consumption goods at Home can be produced by domestic firms, imported from foreign firms or produced by foreign subsidiaries at Home. Formally, I can write the domestic consumption of the composite good as

$$C^{1/\mu} = \int_{\Omega} c_d(\omega)^{1/\mu} d\omega + \int_{\Omega^*} [c_m(\omega) + c_i(\omega)]^{1/\mu} d\omega,$$

where Ω denotes the set of varieties produced at Home and Ω^* the set of goods produced abroad. For a foreign variety ω, $c_m(\omega)$ denotes the amount of Home consumption via imports and $c_i(\omega)$ the amount of Home consumption via FDI Also, define $c_{m,i}(\omega) = c_m(\omega) + c_i(\omega)$. Since the equilibrium is symmetric for all firms with the same productivity, I finally re-write the previous equation as

$$\begin{aligned} C^{1/\mu} &= C_d^{1/\mu} + C_{m,i}^{1/\mu} \\ &= N \int_{\Phi_d} c_d(\varphi)^{1/\mu} dG(\varphi) + N^* \int_{\Phi_{m,i}} c_{m,i}(\varphi)^{1/\mu} dG(\varphi), \end{aligned} \tag{1.1}$$

where N and $N^*$are respectively the measures of domestic and foreign entrants; Φ_d and $\Phi_{m,i}$ are respectively the set of productivities that correspond to varieties being produced at Home for domestic consumption and those being produced by foreign firms for Home consumption (via imports or FDI). The notations for foreign consumption goods are analogous with asterisks over each of the variables.

To simplify notations, for a variety φ associated with the production type $j = d, d^*, m, m^*, i, i^*$, I define the labor cost of producing $c_j(\varphi)$ units as

$$l_j(c_j(\varphi), \varphi) \equiv \frac{\tau_j c_j(\varphi)}{\varphi} + \mathbb{I}_{c_j(\varphi)>0} f_j,$$

with $\mathbb{I}$ being the characteristic function such that $\mathbb{I}_{c>0} = 1$ if $c > 0$ and $\mathbb{I}_{c>0} = 0$ if $c = 0$, and τ following the convention that $\tau_j = \tau$ if $j = m, m^*$ and $\tau_j = 1$ otherwise.

Taxes. In the baseline model, I focus on an environment in which the home government has access to a full set of ad-valorem consumption and trade taxes that can vary across firms. For a firm with blueprint φ that sells in the home market, $t_j(\varphi)$ denotes the tax charged by the Home government, with $j = d, m, i$ corresponding to the three different types of consumption goods: products of home firms (d), home imports of foreign varieties (m) and home FDI products of foreign varieties (i); for a firm with blueprint φ that is producing at Home, $s_j(\varphi)$ denotes the subsidy paid by home government, with $j = d, m^*, i$ corresponding to the three different destination markets of the firm: home domestic consumption of its own varieties (d), home's exporting varieties (m^*) and foreign varieties that are being produced at Home via FDI (i). All taxes and subsidies could be positive or negative. For instance, $t_m(\varphi) > 0$ signifies an import tariff whereas $t_m(\varphi) < 0$ signifies an import subsidy.

Notice that by defining $s_d(\varphi)$ and $s_{m^*}(\varphi)$ separately, we allow Home government to charge different taxes on the same good if it is produced for different destination markets. Similarly, $t_m(\varphi)$ and $t_i(\varphi)$ being potentially different gives Home government the possibility to tax differently a foreign variety, depending on if it's being imported or produced locally. I will come back to this assumption later.

Since Home cannot determine consumption tax rates on its exporting varieties, define $t^*_m(\varphi) \equiv 0$; similarly, Home cannot choose the production subsidy for its imported foreign varieties, so I define $s_m(\varphi) \equiv 0$. With this convention, Home's full set of policy choices is given by $\{t_j(\varphi), s_j(\varphi) \,|\, j = d, m, m^*, i\}$.

1.2.2 Decentralized Equilibrium

For all producers who sell at Home, their profit maximization problem is:

$$\pi_j(\varphi) \equiv \max_{c,p} \left(1 + s_j(\varphi)\right) cp - w_j l_j(c, \varphi),$$

where $j = d, m, i$; $c(\varphi)$ and $p(\varphi)$, the pre-sales-tax price, satisfy the domestic demand curve condition

$$\frac{c}{C} = \left[\frac{\left(1 + t_j(\varphi)\right) p}{P}\right]^{-\sigma}; \tag{1.2}$$

w_j is the labor cost with $w_m = 1$ and $w_d = w_i = w$.

Profit maximization implies that domestic prices satisfy

$$\begin{aligned} p_j(\varphi) &= \frac{1}{1+s_j(\varphi)} \frac{\mu \tau_j w_j}{\varphi}, \text{ if } \frac{(\mu-1)\, \tau_j c_j(\varphi | p_j(\varphi))}{\varphi} \geq f_j \\ p_j(\varphi) &= \infty, \text{otherwise} \end{aligned} \tag{1.3}$$

where μ is the constant mark-up and $c_j(\varphi | p_j(\varphi))$ given by (1.2).

Aggregate price index satisfy

$$P^{1-\sigma} = \sum_{j=d,i,m} N_j \int_{\Phi} \left[(1+t_j(\varphi))\, p_j(\varphi) \right]^{1-\sigma} dG(\varphi), \tag{1.4}$$

where N_j is either the measure of domestic or foreign entrants, depending on the origins of the related producers.[6]

Free entry condition for Home is

$$\sum_{j=d,i^*,m^*} \int_{\Phi} \pi_j(\varphi)\, dG(\varphi) = w f^E. \tag{1.5}$$

Labor market clearing condition is

$$\sum_{j=d,i,x} N_j \int_{\Phi} l_j(c_j(\varphi), \varphi)\, d\omega + N f^E = L - L_n. \tag{1.6}$$

Market clearing for the numeraire good is

$$C_n + C_n^* = w L_n + L_n. \tag{1.7}$$

Optimality condition between the numeraire good and the aggregate good for Home

$$\frac{U_C(C, C_n)}{U_{C_n}(C, C_n)} = P. \tag{1.8}$$

[6] In particular, $N_d = N_x = N$ and $N_i = N_m = N^*$.

Finally, Home's budget balance condition is

$$\sum_{j=d,m,i,m^*} N_j \int_\Phi \left(t_j(\varphi) - s_j(\varphi)\right) c_j(\varphi)\, p_j(\varphi)\, dG(\varphi) = T. \tag{1.9}$$

The same conditions in Foreign are analogous and omitted.

1.2.3 Home's Optimization Problem

I assume that Home is a strategic country that sets ad-valorem trade taxes $t(\omega)$ for each importing goods and subsidies $s(\omega)$ for each domestically produced variety (either by a domestic producer or a foreign FDI subsidiary). Foreign is assumed to be passive. This leads to the following definition of Home's problem.

Definition 1. Home's problem is to

$$\max_{T,\left\{t_j(\varphi),s_j(\varphi)\right\}_{j=d,m,i,m^*},\left\{c_j(\varphi),c_j^*(\varphi),p_j(\varphi),p_j^*(\varphi)\right\}_{j=d.m.i},C,C^*,P,P^*,C_n,C_n^*,L_n,L_n^*,N,N^*} U,$$

subject to conditions (1.1) through (1.9) both for Home and for the foreign country.

In the next section, i try to characterize the unilateral optimal trade and FDI taxation for Home by using the primal approach.

1.3 Optimal Allocation

In this section, I follow a similar strategy that is used in Costinot et al. (2016), by ignoring home prices and taxes and try to solve for the first-best allocation for Home. Assume that a social planner at Home chooses domestic consumptions, C_n, $c_d(\varphi)$, $c_m(\varphi)$ and $c_i(\varphi)$, foreign consumptions, C_n^*, $c_d^*(\varphi)$, $c_m^*(\varphi)$ and $c_i^*(\varphi)$, the measures of domestic and foreign entrants, N and N^*, domestic prices of foreign products $p_m(\varphi)$, $p_i(\varphi)$, as well as all foreign prices, $p_d^*(\varphi)$, $p_m^*(\varphi)$, $p_i^*(\varphi)$, subject to conditions (1.1) through (1.6) for Foreign and conditions (1.1), (1.6) and (1.7) for Home.

By Walras' Law, Homes' current account balance constraint is automatically satisfied so it can be omitted from the equilibrium conditions. But the constraint is quite interesting in itself in that it consists of two parts. First, Home's exports and imports generate a trade balance. Second, the difference between profits earned by Home firms' subsidiaries in the foreign country and those earned by foreign firms' subsidiaries at Home constitutes the capital account balance. The sum of the two should be equal to zero:

$$
\begin{aligned}
0 \leq & \underbrace{\int_{\Phi_m^*} N c_m^* (\varphi)\, p_m^* (\varphi)\, dG (\varphi) - \int_{\Phi_m} N^* p_m (\varphi)\, c_m (\varphi)\, dG (\varphi) + X_n}_{\text{trade balance}} \\
& + \underbrace{\int_{\Phi_i^*} N \left[c_i^* (\varphi)\, p_i^* (\varphi) - l_i^* \left(c_i^* (\varphi), \varphi\right)\right] dG (\varphi)}_{\text{net international income}} \\
& - \underbrace{\int_{\Phi_i} N^* [(1 + s_i (\varphi))\, c_i (\varphi)\, p_i (\varphi) - w l_i \left(c_i (\varphi), \varphi\right)] dG (\varphi)}_{\text{net international income}}, \qquad (1.10)
\end{aligned}
$$

with $X_n = wL_n - C_n$ Home's export of the numeraire good. I will solve this problem in 2 steps: first, I take the aggregate production of domestic varieties, C_d, $C_{m,i}^*$ and X_n, as given and solve for productions of domestic micro varieties and the measure of domestic entrants, N; second, still taking the aggregate production of domestic varieties as given, I solve for productions of the numeraire good and foreign micro varieties, as well as the measure of foreign entrants, N^*. Lastly, I use the aggregate constraints to solve for the macro variables C_d, $C_{m,i}^*$ and X_n.

Before solving the micro quantities, however, I start by looking at the optimality conditions for the numeraire good to make some simplification on the Lagrange multipliers. Taking the first order conditions with respect to L_n and L_n^*, I get:

$$
\begin{aligned}
\lambda_n w &= \lambda_L, \\
\lambda_n &= \lambda_L^*,
\end{aligned}
$$

where λ_n, λ_L and $\lambda_L^*$are, respectively, the Lagrange multipliers for the numeraire good market clearing condition (1.7) and Home and Foreign's labor market clearing constraints (1.6). These implies

$$\lambda_L = w\lambda_L^*. \tag{1.11}$$

will be shown, when trying to solve the optimization problem, the Home government always faces a trade-off between using domestic labor and using foreign labor. From (1.11), the presence of the numeraire good guarantees a fixed relative price between the two, such that a unit of domestic labor is always worth of w units of foreign labor.

1.3.1 Production of domestic varieties

In what follows, I will solve for the optimality conditions for domestic and foreign varieties. As in Helpman et al. (2004), the following assumption would turn out to be very helpful to guarantee that in a free-trade equilibrium, firms in both countries engage in both exporting and FDI.

$$w^{\frac{1}{\sigma-1}} \leq \tau \leq \left(\frac{f_i}{f_m}\right)^{\frac{1}{\sigma-1}}, \frac{\tau}{w} \leq \left(\frac{f_i}{f_m}\right)^{\frac{1}{\sigma-1}}. \tag{1.12}$$

Consider home's problem of minimizing the (Home and Foreign) labor cost of producing C_d units of aggregate consumption of home varieties and $C_{m,i}^*$ units of aggregate consumption for foreign consumers. This can be expressed as:

$$\mathfrak{L}\left(C_d, C_{m,i}^*\right) = \min_{c_d, c_{m^*}, c_{i^*}, N} N\left[\int_\Phi \sum_{j=d,m^*} l_j\left(c_j\left(\varphi\right), \varphi\right) + \frac{1}{w} l_i^*\left(c_i^*\left(\varphi\right), \varphi\right) dG\left(\varphi\right) + f^E\right], \tag{1.13}$$

$$N\int_\Phi \left(c_d\left(\varphi\right)\right)^{1/\mu} dG\left(\varphi\right) \geq C_d^{1/\mu}, \tag{1.14}$$

$$N\int_\Phi \left(c_{m^*}\left(\varphi\right) + c_{i^*}\left(\varphi\right)\right)^{1/\mu} dG\left(\varphi\right) \geq C_{m,i}^{*1/\mu}, \tag{1.15}$$

where $\frac{1}{w}$, as explained above, is the relative weight Home puts on the usage of domestic and foreign labor. This problem is a priori not convex at the points

$c_j(\varphi) = 0$. See Costinot et al. (2016) for a thorough discussion of the issue. One can show that any solution to it must minimize the associated Lagrangian, given by $\mathfrak{L} = N\mathfrak{L}'$ where

$$\begin{aligned}\mathfrak{L}' \equiv & \int_\Phi \left[\sum_{j=d,m^*} l_j(c_j(\varphi), \varphi) + \tfrac{1}{w} l_i^*(c_i^*(\varphi), \varphi)\right] dG(\varphi) \\ & - \int_\Phi \left[\lambda_d c_d(\varphi)^{1/\mu} + \lambda_{m,i}(c_m^*(\varphi) + c_i^*(\varphi))^{1/\mu}\right] dG(\varphi) + f^E\end{aligned}$$

for some Lagrangian multipliers, $\lambda_d, \lambda_{m,i} > 0$.

Thanks to the additive separability of $\mathfrak{L}$ and l in $c_j(\varphi)$, I could use a variety-by-variety approach to solve this problem. Start with the domestic consumptions. For any given variety φ, the optimal allocation $c_d(\varphi)$ must be the solution to:

$$c_d(\varphi) = \arg\min_c l_d(c, \varphi) - \lambda_d c^{1/\mu}. \tag{1.16}$$

When not at the corner of $c = 0$, the first order condition to this problem is

$$\frac{\lambda_d}{\mu} c^{-\frac{1}{\sigma}} = \frac{1}{\varphi}.$$

It remains to compare if this interior solution generates a higher Lagrangian than the corner solution at $c(\varphi) = 0$. Combining both cases, the solution of (1.16) follows a simple cut-off rule:

$$\begin{aligned} c_d(\varphi|\lambda_d) &= \left(\frac{\mu}{\lambda_d \varphi}\right)^{-\sigma}, \text{if } \varphi \geq \varphi_d^0 \\ &= 0, \text{if } \varphi < \varphi_d^0 \end{aligned} \tag{1.17}$$

with the cut-off productivity given by

$$\varphi_d^0(\lambda_d) = \left(\frac{\mu}{\lambda_d}\right)^\mu \left(\frac{f_d}{\mu - 1}\right)^{\frac{1}{\sigma-1}}.$$

For notational simplicity, denote $\Phi_d = \{\varphi \in \Phi, \varphi \geq \varphi_d^0\}$ the set of home blueprints being produced for home consumption.

Unlike in Costinot et al. (2016), the solution to the foreign consumption of domestic varieties is more complicated. This is because there are two ways for Home to enter the foreign market: either through producing at Home and exporting or through producing in Foreign via FDI. For each blueprint φ, the question that Home faces is the following: should the foreign market be served, and, if so, by which way of production? Formally, for any given variety φ, the optimal allocation $\left(c_m^*(\varphi), c_i^*(\varphi)\right)$ must be the solution to:

$$\begin{aligned} c_m^*(\varphi), c_i^*(\varphi) &= \arg\min_{c,c'} \mathfrak{L}_{m,i}^*(\varphi) \\ &= \arg\min_{c,c'} l_m^*(c,\varphi) + \frac{1}{w} l_i^*\left(c',\varphi\right) - \lambda_{m,i}\left(c+c'\right)^{1/\mu}. \end{aligned}$$

A priori, both exporting and FDI could be active for the same variety. In practice, however, thanks to the increasing returns to scale assumptions, it is never optimal for home to serve the foreign market in both ways.[7]As a result, for any given variety φ, the previous problem is equivalent to the following:

$$\mathfrak{L}_{m,i}^*(\varphi) = \min\left\{\mathfrak{L}_m^*(\varphi), \mathfrak{L}_i^*(\varphi)\right\},$$

where

$$\mathfrak{L}_m^*(\varphi) = \min_c l_m^*(c,\varphi) - \lambda_{m,i} c^{1/\mu}$$

and

$$\mathfrak{L}_i^*(\varphi) = \min_c \frac{1}{w}\left(l_i^*(c,\varphi) - \lambda_{m,i} c^{1/\mu}\right)$$

are respectively the optimal micro problems for exporting and FDI.

Each of the two problems is analogous to the problem for domestic consumption. The solution to $\mathfrak{L}_{m,i}^*(\varphi)$ is given by

[7] See Appendix A.1.2 for a proof. Intuitively, if both modes of serving the foreign market are active, it is always strictly better for home to either serve the combined quantity through exporting only, or serve it through FDI only. In a dynamic model with uncertainty, one can break this increasing-returns-to-scale argument so that an individual firm may choose to serve a foreign market through both exports and FDI, see Rafael and Nikolaos (2003)for a rigorous treatment of this case.

$$c_m^* (\varphi|\lambda_{m,i}) = \left(\frac{\tau\mu}{\lambda_{m,i}\varphi}\right)^{-\sigma}, \text{if } \varphi \geq \varphi_m^{0*}$$
$$= 0, \text{if } \varphi \leq \varphi_m^{0*}, \tag{1.18}$$

where $\varphi_m^{0*} (\lambda_{m,i}) = \tau \left(\frac{\mu}{\lambda_{m,i}}\right)^{\mu} \left(\frac{f_m}{\mu-1}\right)^{\frac{1}{\sigma-1}}$.

Similarly, the solution of $\mathbb{L}_i^* (\varphi)$ is

$$c_i^* (\varphi|\lambda_{m,i}) = \left(\frac{\mu}{\lambda_{m,i}\varphi}\right)^{-\sigma}, \text{if } \varphi \geq \varphi_i^{0*}, \tag{1.19}$$
$$= 0, \text{if } \varphi \leq \varphi_i^{0*},$$

where $\varphi_i^{0*} (\lambda_{m,i}) = \left(\frac{\mu}{\lambda_{m,i}}\right)^{\mu} \left(\frac{f_i}{\mu-1}\right)^{\frac{1}{\sigma-1}}$.

Figure 1-1 shows the relationship between the two Lagrangians. See Appendix A.1.1 for the expressions of the Lagrangians. The solid line represents the Lagrangian for exports, $\mathfrak{L}_m^*$, and the dashed line the Lagrangian for FDI, $\mathfrak{L}_i^*$. One could see that when productivity φ is sufficiently high, it is always more efficient for Home to do FDI relative to exporting. For notational simplicity, denote $\Phi_m^* = \left[\varphi_m^{0*}, \varphi'^*\right]$ the set of home blueprints that are exported and $\Phi_i^* = [\varphi'^*, \infty)$ the set of home blueprints that are produced in Foreign via FDI . Under condition (1.12), both sets are non-empty.

This should come with no surprise. When considering how to serve the foreign market, home government is facing a similar proximity-concentration trade-off that a typical firm faces in a competitive equilibrium: under condition (1.12), the fixed cost is higher for FDI than for exporting, where as for the variable cost, the opposite is true. As a result, it is only when productivity is sufficiently high and enough quantity is being produced, FDI dominates exporting, so that the optimal solution is such that home varieties associated with the highest productivities will enter foreign market via FDI, those associated with less high productivities will export, and those associated with the lowest productivities will not enter the

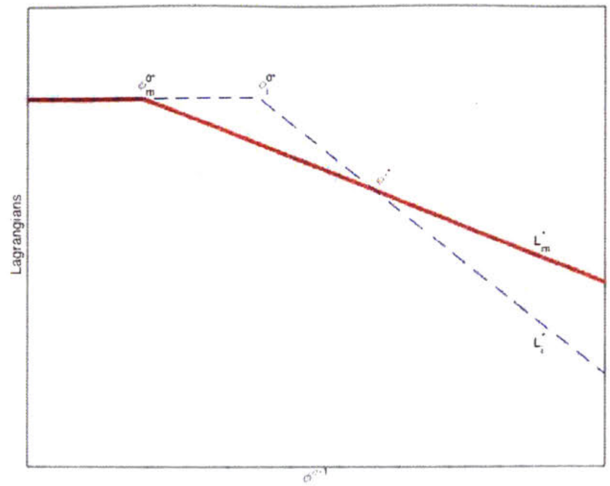

Figure 1-1: Lagrangians $\mathfrak{L}^*_{m,i}(\varphi) = \min\left\{\mathfrak{L}^*_m(\varphi), \mathfrak{L}^*_i(\varphi)\right\}$.

foreign market.

Since I have expressed micro quantities as functions of the two Lagrange multipliers λ_d and $\lambda_{m,i}$, it is straight-forward to plug them back into Equations (1.14) and (1.15) to solve for λ_d and $\lambda_{m,i}$ as functions of N, C_d and $C^*_{m,i}$. Finally, to determine the the optimal domestic measure of entrants, I go back to problem (1.13). By the Envelope Theorem, as long as $N > 0$ at the optimum, it should satisfy:

$$\begin{aligned}
&\int_\Phi \left[\sum_{j=d,m^*} l_j\left(c_j(\varphi), \varphi\right) + \tfrac{1}{w} l^*_i\left(c^*_i(\varphi), \varphi\right)\right] dG(\varphi) \\
-&\int_\Phi \left[\lambda_d c_d(\varphi)^{1/\mu} + \lambda_{m,i}\left(c^*_m(\varphi) + c^*_i(\varphi)\right)^{1/\mu}\right] dG(\varphi) + f^E = 0.
\end{aligned}$$

This determines the optimal measure of domestic entrants, $N\left(C_d, C^*_{m,i}\right)$. As a result, the optimal quantities for each home variety can also be expressed as functions of C_d and $C^*_{m,i}$. This would also allow us to express the three usages of labor for domestic varieties as functions of C_d and $C^*{}_{m,i}$:

$$\begin{aligned}
L_d\left(C_d, C^*_{m,i}\right) &= \int_{\Phi_d} l_d(\varphi)\, dG(\varphi), \\
L^*_m\left(C_d, C^*_{m,i}\right) &= \int_{\Phi^*_m} l^*_m(\varphi)\, dG(\varphi), \\
L^*_i\left(C_d, C^*_{m,i}\right) &= \int_{\Phi^*_i} l^*_m(\varphi)\, dG(\varphi).
\end{aligned}$$

At this point, one can check that among blueprints $\varphi, \varphi' \in \Phi_d$ for domestic consumption, the relative ratio of their optimal quantities is the same under the planning problem and in the decentralized equilibrium. The same result holds for the exporting and FDI varieties, too. This is a result of the efficiency of firm-level decisions under Melitz type of models with monopolistic competition and CES preferences, see Dixit and Stiglitz (1977) and Dhingra and Murrow (2012) for closed-economy version of this result. It is also similar to the related results in Costinot et al. (2016), while extends it into an environment with endogenous FDI choices. The result implies that the home government may want to impose a uniform domestic consumption tax or production subsidy, in order to manipulate the allocation of labor among the four possible uses (domestic production, exporting, FDI at home or numeraire); but it never wants to impose domestic consumption taxes or production subsidies that vary across domestic firms.

1.3.2 Foreign's offer curve

Next, I consider the problem of maximizing domestic consumption of foreign varieties, subject to a budget constraint and Foreign's free entry condition and labor market clearing condition. Denote $X_n = wL_n - C_n$ Home's export of the numeraire good. Formally, this can be written as[8]

$$C_{m,i}^{1/\mu}\left(C_d, C_{m,i}^*, X_n\right) = \max_{c_m, c_i, C_d^*, N^*, C_n^*, L_n, L_n^*} N^* \int_{\Phi} \left(c_m(\varphi) + c_i(\varphi)\right)^{1/\mu} dG(\varphi) \tag{1.20}$$

[8] A careful reader might think that, since N appears both in Home's maximization problem in the previous sub-section and here, there could be an issue with the two-step strategy that tries to solve Home's labor minimization problem and Foreign's offer curve one by one. However, one should note that here N only appears in equations (1.24) and (1.25), and, thanks to equation (1.11), the ratio of the Lagrange multipliers of these two equations is exactly w. As a result, the Lagrangian of the current problem contains exactly $\mathfrak{L}\left(C_d, C_{m,i}^*\right)$, the Lagrangian obtained in the previous sub-section; and N would appear in the current Lagrangian only inside the term $\mathfrak{L}\left(C_d, C_{m,i}^*\right)$. Thus, the two-step strategy employed here is justified.

subject to

$$L^* = P_d^* \left(C_d^*, N^*\right) \left[C_d^* + C_{m,i}^* \left(\frac{C_{m,i}^*}{C_d^*}\right)^{-1/\sigma}\right] + C_n^*, \tag{1.21}$$

$$\frac{U_C}{U_{C_n}} \left(C_d^*, C_{m,i}^*, C_n^*\right) = P_d^* \left(C_d^*, N^*\right) \left(1 + \left(\frac{C_{m,i}^*}{C_d^*}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{1-\sigma}}. \tag{1.22}$$

$$N^* f^E = \Pi_d^* \left(C_d^*, N^*\right) + N^* \int_\Phi \sum_{j=m,i} \pi_j \left(c_j\left(\varphi\right), \varphi, w\right) dG\left(\varphi\right), \tag{1.23}$$

$$\begin{aligned} L^* &= N^* f^E + L_d^* \left(C_d^*, N^*\right) + N^* \int_\Phi l_m \left(c_j\left(\varphi\right), \varphi\right) dG\left(\varphi\right), \\ &+ N\left(C_d, C_{m,i}^*\right) \int_\Phi l_i^* \left(c_i^*\left(\varphi\right), \varphi\right) dG\left(\varphi\right) + L_n^* \end{aligned} \tag{1.24}$$

$$\begin{aligned} L &= N\left(C_d, C_{m,i}^*\right) \left[f^E + \int_\Phi \sum_{j=d,m^*} l_j \left(c_j\left(\varphi\right), \varphi\right) dG(\varphi)\right], \\ &+ N^* \int_\Phi l_i \left(c_i\left(\varphi\right), \varphi\right) dG\left(\varphi\right) + L_n \end{aligned} \tag{1.25}$$

$$C_n^* - L_n^* = C_n - L_n = X_n, \tag{1.26}$$

$$\begin{aligned} \pi_m &= \frac{\mu}{\varphi} c_m\left(\varphi\right) - l_m\left(c_m\left(\varphi\right), \varphi\right) \geq 0, \\ \pi_i &= w\left[\frac{\mu}{\varphi} c_i\left(\varphi\right) - l_i\left(c_i\left(\varphi\right), \varphi\right)\right] \geq 0. \end{aligned} \tag{1.27}$$

As established in Appendix A.1.3, constraint (1.21) summarizes Foreign's utility maximization problem, whereas constraints (1.23) through (1.26) summarize its free entry condition, labor market clearing conditions for Home and Foreign, and numeraire market clearing condition, after taking into account the equilibrium values of local prices and quantities in Foreign, $p_d^*\left(\varphi\right)$, $P_d^*\left(C_d^*, N^*\right)$ and $c_d^*\left(\varphi\right)$.

First, as argued previously, the first order conditions on L_n and L_n^* imply that

$$\lambda_L = w\lambda_L^*.$$

where λ_L and λ_L^* are, respectively, the Lagrange multiplier for constraints (1.25) and (1.24).

Next, like in the previous sub-section, I first take the aggregate quantities C_d^*, C_n^*, L_n, L_n^* and N^* as given and try to solve for the micro quantities, using the additive separability of the objective function. Here again, because of the increasing-returns-to-scale technology firms have, it is never optimal for any foreign firm to engage in both exporting and FDI. As a result, for any given firm with blueprint φ, the problem is equivalent to

$$\mathfrak{L}_{m,i}(\varphi) = \max\left\{\mathfrak{L}_m(\varphi), \mathfrak{L}_i(\varphi)\right\},$$

where

$$\mathfrak{L}_m(\varphi) \equiv \max_c c^{1/\mu} - \kappa\frac{\tau c}{\varphi},$$

$$\mathfrak{L}_i(\varphi) \equiv \max_c c^{1/\mu} - \kappa'\frac{wc}{\varphi}$$

subject to

$$(\mu - 1)\frac{\tau c_m}{\varphi} \geq f_m,$$

$$(\mu - 1)\frac{wc_i}{\varphi} \geq f_i.$$

where $\kappa = (\mu - 1)\lambda_E + \lambda_L^*$ and $\kappa' = (\mu - 1)\lambda_E + \lambda_L$, with λ_E being the Lagrange multiplier for the foreign free entry condition (1.23).[9]The solution of $\mathfrak{L}_m(\varphi)$ is

$$\begin{aligned}c_m(\varphi|\kappa) &= (\kappa\mu\tau)^{-\sigma}\varphi^{\sigma}, \text{ if } \varphi \geq \varphi_m^u \\ &= \frac{f_m}{\tau(\mu-1)}\varphi, \text{ if } \varphi \in [\varphi_m^c, \varphi_m^u] \\ &= 0, \text{ if } \varphi \leq \varphi_m^c\end{aligned} \tag{1.28}$$

with

$$\varphi_m^u(\kappa) = \tau(\kappa\mu)^{\mu}\left(\frac{f_m}{\mu-1}\right)^{\frac{1}{\sigma-1}},$$
$$\varphi_m^c(\kappa) = \tau\kappa^{\mu}\left(\frac{f_m}{\mu-1}\right)^{\frac{1}{\sigma-1}}$$

being the cut-off productivities for firms that are constrained or unconstrained by the positive profit conditions (1.27).

Now let's turn to the problem for foreign producers doing FDI at Home. The solution of $\mathfrak{L}_i(\varphi)$ is completely analogous to that of $\mathfrak{L}_m(\varphi)$:

$$\begin{aligned}c_i(\varphi|\kappa', w) &= (\kappa'\mu w)^{-\sigma}\varphi^{\sigma}, \text{ if } \varphi \geq \varphi_i^u \\ &= \frac{f_i}{w(\mu-1)}\varphi, \text{ if } \varphi \in [\varphi_i^c, \varphi_i^u] \\ &= 0, \text{ if } \varphi \leq \varphi_i^c\end{aligned} \tag{1.29}$$

with

$$\varphi_i^u = w(\kappa'\mu)^{\mu}\left(\frac{f_i}{\mu-1}\right)^{\frac{1}{\sigma-1}},$$
$$\varphi_i^c = w\kappa'^{\mu}\left(\frac{f_i}{\mu-1}\right)^{\frac{1}{\sigma-1}}.$$

[9]Both κ and κ' must be positive, see Appendix A.4 for a rigorous proof. Intuitively, social planner at Home internalizes the fact that he can manipulate the measure of entrants in Foreign. At the optimum, such considerations lead Home to select a lower measure of foreign entrants than under laissez faire. Yet doing so tend to increase foreign firms' profits, thus the right side of the free entry condition, which means at the optimum, the Lagrange multiplier associated with it will be strictly positive.

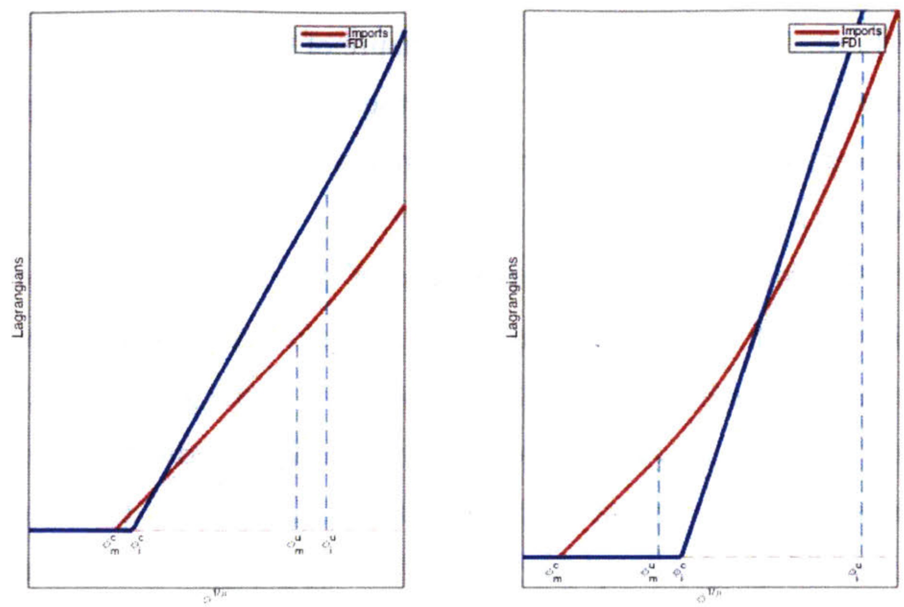

Figure 1-2: Lagrangians in the two scenarios of foreign varieties.

See Appendix A.1.1 for the expressions of the Lagrangians $\mathfrak{L}_m$, $\mathfrak{L}_i$ and $\mathfrak{L}_{m,i}$. One can check at the equilibrium, if $\frac{\kappa}{\kappa'} \in \left[\frac{w}{\tau}, \frac{w}{\tau}\left(\frac{f_i}{f_m}\right)^{\frac{1}{\sigma-1}}\right]$, under condition (1.12), the thresholds are always such that $\varphi_m^c < \varphi_i^c$ and $\varphi_m^u < \varphi_i^u$, and, when productivity φ is sufficiently high, the Lagrangian $\mathfrak{L}_i$ always dominates $\mathfrak{L}_m$.[10] Denote $\varphi' > \varphi_m^c$ the productivity at which $\mathfrak{L}_i(\varphi') = \mathfrak{L}_m(\varphi')$, i.e., it is equally beneficial for Home to consume the foreign variety via imports or via FDI. For notational simplicity, denote $\Phi_m = [\varphi_m^c, \varphi']$ the set of foreign blueprints that are exported and $\Phi_i = [\varphi', \infty)$ the set of foreign blueprints that are produced in Home via FDI .Depending on whether $\varphi_i^c < \varphi_m^u$, Figure 1-2 shows the two possible scenarios for the Lagrangians.

Next, since I have expressed the micro quantities, $c_m(\varphi)$ and $c_i(\varphi)$, as functions of the Lagrange multipliers λ_L, λ_L^* and λ_E, the optimization problem (1.20) is reduced to a standard maximization over five macro variables C_d^*, C_n, L_n, L_n^* and N^*, and can be solved using standard techniques. Since L_n only shows up in

[10]In fact, when $\frac{\kappa}{\kappa'} < \frac{w}{\tau}$ or $\frac{\kappa}{\kappa'} > \frac{w}{\tau}\left(\frac{f_i}{f_m}\right)^{\frac{1}{\sigma-1}}$, either FDI (in the former case) or exporting (in the latter case) is not active in the equilibrium.

equation (1.25), it is possible to ignore it for now (which will be dealt with in the next section). As a result, the optimal measure of foreign entrants, N^* and foreign consumption of its own varieties C_d^* and the numeraire C_n^*, and its production of numeraire goods, L_n^*, as well as the Lagrange multipliers, λ_E, λ_L and λ_L^*, can all be expressed as functions of C_d, $C_{m,i}^*$ and X_n.

Note that for foreign producers with $\varphi, \varphi' \in \Phi_m^u \equiv \Phi_m \cup [\varphi_m^u, \infty)$[11], $\frac{c_m(\varphi)}{c_m(\varphi')} = \left(\frac{\varphi}{\varphi'}\right)^\sigma$ is satisfied. This is exactly the same as the relative consumption ratio under the decentralized equilibrium, which means that Home government does not impose any distortion for foreign producers within this productivity range - as I will show in Section 4, the optimal import tariff is flat for $\varphi \in \Phi_m^u$. However, for foreign producers with productivity $\varphi \in \Phi_m^c \equiv \Phi_m \cup [\varphi_m^c, \varphi_m^u]$, the fact that the optimal import quantity is linearly w.r.t. productivity φ (instead of being proportional to φ^σ as is the case for $\varphi \in \Phi_m^u$) suggests that the import tariff increases as φ increases within this range. Intuitively, had Home imposed the same level of import tariff on these firms as they do for the more productive ones with $\varphi \geq \varphi_m^u$, it would not be profitable for the foreign firms to export to Home. So Home has to lower its tariff because it wants to raise its imports in order to make sure that the least profitable firms in Foreign are willing to produce and export strictly positive amounts.

The same situation applies to foreign firms over the FDI decision. At the optimum, Home sets a flat sales tax (or, as shown in Section 4., production subsidy) for $\varphi \in \Phi_i^u \equiv \Phi_i \cup [\varphi_i^u, \infty)$, yet the optimal sales tax is increasing for $\varphi \in \Phi_i^c \equiv \Phi_i \cup [\varphi_i^c, \varphi_i^u]$.[12]

1.3.3 Macro Problem

Until now, I have expressed all micro quantities as functions of C_d, $C_{m,i}^*$ and C_n. It is now the time to go for the choices of the macro quantities, subject to Home's resource constraint and Foreign's offer curve. Formally, the problem can be written as

[11] Notice that Φ_m^u may well be empty, as shown in Figure 1-2(on the left).

[12] Similarly, Φ_i^c may be empty, as shown in Figure 1-2(on the right).

$$\max_{C_d, C^*_{m,i}, C_{m,i}, C_n, L_n} U\left[\left((C_d)^{1/\mu} + (C_{m,i})^{1/\mu}\right)^{\mu}, C_n\right]$$

subject to

$$C_{m,i} \leq C_{m,i}\left(C_d, C^*_{m,i}, wL_n - C_n\right) \tag{1.30}$$

$$L = L\left(C_d, C^*_{m,i}, wL_n - C_n\right) + L_n, \tag{1.31}$$

where $L\left(C_d, C^*_{m,i}, X_n\right)$ is the usage of home labor for production of all varieties from previous sections. This problem is similar to a terms-of-trade manipulation problem with a notable difference. The possibility of FDI introduces an additional channel of international transfer so that the two-good Lerner Symmetry, as showcased in Costinot et al. (2016), will not hold. Thus, it is no longer possible to use the concept of terms-of-trade in a meaningful way. See Blanchard (2009) for a similar discussion. One can also see this from the fact that Foreign's offer curve, $C_{m,i}\left(C_d, C^*_{m,i}, X_n\right)$, depends not only on the traded goods, $C^*_{m,i}$ and X_n, but also on the non-traded home varieties C_d.

At any interior solution to this problem, the necessary first order conditions are

$$\begin{aligned}
U_C \frac{\partial C}{\partial C_d} + \Lambda_F \frac{\partial C_{m,i}}{\partial C_d} &= \Lambda_L \frac{\partial L}{\partial C_d}, \\
\Lambda_F \frac{\partial C_{m,i}}{\partial C^*_{m,i}} &= \Lambda_L \frac{\partial L}{\partial C^*_{m,i}}, \\
U_C \frac{\partial C}{\partial C_{m,i}} &= \Lambda_F, \\
U_{C_n} - \Lambda_F \frac{\partial C_{m,i}}{\partial X_n} &= - \Lambda_L \frac{\partial L}{\partial X_n}, \\
w\Lambda_F \frac{\partial C_{m,i}}{\partial X_n} &= w\Lambda_L \frac{\partial L}{\partial X_n} + 1,
\end{aligned}$$

where Λ_F and Λ_L are the Lagrange multipliers for (1.30) and (1.31), respectively.

Re-arranging the equations, we have:

$$\frac{MRS_{d/m,i} + \frac{\partial C_{m,i}}{\partial C_d}}{\frac{\partial C_{m,i}}{\partial C^*_{m,i}}} = \frac{\frac{\partial L}{\partial C_d}}{\frac{\partial L}{\partial C^*_{m,i}}}, \tag{1.32}$$

$$\frac{MRS_{n/m,i} - \frac{\partial C_{m,i}}{\partial X_n}}{\frac{\partial C_{m,i}}{\partial C^*_{m,i}}} = -\frac{\frac{\partial L}{\partial X_n}}{\frac{\partial L}{\partial C^*_{m,i}}}. \tag{1.33}$$

$$U_{c_n} = \frac{1}{w}. \tag{1.34}$$

where $MRS_{d/m,i} = (\partial U/\partial C_d) \,/\, (\partial U/\partial C_{m,i})$ is the marginal rate of substitution between consumptions of domestic and foreign varieties at home and $MRS_{n/m,i} = U_{C_n} / (U_C \partial C/\partial C_{m,i})$ the marginal rate of substitution between consumptions of foreign varieties and the numeraire good at home.

For both of the equations (1.32) and (1.33), the left hand side represents the relative changes in utility if domestic production sees a marginal increase in one good and a marginal decrease in another good. The right hand side represents the implicit marginal rate of transformation between the two. At the optimum, both sides have to be the same. If $\frac{\partial C_{m,i}}{\partial C_d} = 0$, which happens, for example, when the FDI channel is completely shut down, Equation (1.32) reduces to the optimal tariff formula in Costinot et al. (2016).

1.4 Optimal Taxation

In this section, I use the solutions in the previous section to derive necessary properties that ad-valorem taxes and subsidies implementing the first-best allocation must satisfy. Since they replicate the solution to Home's relaxed planning problem, they a fortiori solve the home government's problem described in Definition 1.

1.4.1 Taxes on Domestic Varieties

Consider a schedule of taxes that implements the first-best allocation. Denote $\{(s_d(\varphi), t_d(\varphi))\}$ its domestic component. Fix a benchmark domestic variety φ_d that is produced and sold in the home market in the first-best allocation and denote $(s_d, t_d) = (s_d(\varphi), t_d(\varphi))$ the domestic taxes imposed on it.

Lemma 1. *In a first-best allocation, domestic taxes should be such that all domestically produced home varieties $\varphi \in \Phi_d$ satisfy*

$$\frac{1+s_d(\varphi)}{1+t_d(\varphi)} = \frac{1+s_d}{1+t_d}.$$

Proof. By equations (1.2), (1.3) and (1.17), for any domestic variety $\varphi \in \Phi_d$,

$$\left(\frac{1+t_d(\varphi)}{\varphi\left[1+s_d(\varphi)\right]}\frac{\varphi_d(1+s_d)}{1+t_d}\right)^{-\sigma} = \frac{c_d(\varphi)}{c_d(\varphi_d)} = \left(\frac{\varphi}{\varphi_d}\right)^{\sigma},$$

which leads to the result. □

Similarly, fix a benchmark domestic variety φ_m^* that is produced and exported to foreign and denote $s_m^* = s_m^*(\varphi_m^*)$, we have the following result:

Lemma 2. *In a first-best allocation, domestic subsidy on any exporting home variety $\varphi \in \Phi_m^*$ satisfies*

$$s_m^*(\varphi) = s_m^*.$$

1.4.2 Taxes on Foreign Varieties

Foreign goods may enter home market via imports or FDI. Consider first a home import tax schedule, $\{t_m(\varphi)\}$, that implements the first-best allocation. Fix a benchmark foreign variety φ_m that has positive imports and such that $\varphi_m \in \Phi_m^u$[13], i.e., the corresponding foreign firms' profitability constraint is not binding. Denote

[13] As argued in Section 3, Φ_m^u may be empty, but this does not affect our calculation below.

$t_m = t_m(\varphi_m)$. By equations (1.2), (1.3) and (1.28), for any foreign variety φ that is imported in the equilibrium with $\varphi \geq \varphi_m^u$,

$$t_m(\varphi) = t_m,$$

whereas for any foreign variety φ that is imported with $\varphi < \varphi_m^u$,

$$t_m(\varphi) = \alpha_m (1 + t_m) \varphi^{1/\mu} - 1,$$

with $\alpha_m = \frac{1}{\kappa\mu} \left[\frac{f_m \tau^{\sigma-1}}{\mu-1} \right]^{-\frac{1}{\sigma}}$. It is straightforward to show that $\alpha_m \varphi^{1/\mu} \leq 1$ as long as $\varphi \leq \varphi_m^u$, with equality achieved when $\varphi = \varphi_m^u$. Therefore, the previous results can be summarized as

Lemma 3. *In a first-best allocation, domestic tax on any importing foreign variety $\varphi \in \Phi_m$ satisfies*

$$t_m(\varphi) = (1 + t_m) \min\left\{1, \alpha_m \varphi^{1/\mu}\right\} - 1.$$

Next, consider a home FDI tax and subsidy schedule, $\{s_i(\varphi), t_i(\varphi)\}$ that implements the first-best allocation. Fix a benchmark foreign variety φ_i that is being produced via FDI in equilibrium and such that $\varphi_i \in \Phi_i^u$. The results in section 3.2 guarantees that for sufficiently productive foreign firms, it is more profitable to enter the home market via FDI than exporting, so such a φ_i always exists. Let $(s_i, t_i) = (s_i(\varphi_i), t_i(\varphi_i))$ be the subsidy and tax for this benchmark variety. By equations (1.2), (1.3) and (1.29), for any foreign variety φ that is produced at home via FDI with $\varphi \geq \varphi_m^u$,

$$\frac{1 + s_i(\varphi)}{1 + t_i(\varphi)} = \frac{1 + s_i}{1 + t_i},$$

whereas for any foreign variety that is produced at home via FDI with $\varphi < \varphi_m^u$,

$$\frac{1 + s_i(\varphi)}{1 + t_i(\varphi)} = \frac{1 + s_i}{1 + t_i} \frac{1}{\alpha_i \varphi^{\frac{1}{\mu}}},$$

with $\alpha_i = \frac{1}{\kappa'\mu} \left[\frac{f_i w^{\sigma \ 1}}{\mu-1} \right]^{-\frac{1}{\sigma}}$. It is again straightforward to show that $\alpha_m \varphi^{1/\mu} \leq 1$ as long as $\varphi \leq \varphi_i^u$, with equality achieved when $\varphi = \varphi_i^u$. Therefore, the previous results can be summarized as

Lemma 4. *In a first-best allocation, domestic tax on any importing foreign variety $\varphi \in \Phi_i$ satisfies*

$$\frac{1+s_i(\varphi)}{1+t_i(\varphi)} = \frac{1+s_i}{1+t_i}\frac{1}{min\left\{1, \alpha_i \varphi^{1/\mu}\right\}}.$$

Similar to Costinot et al. (2016), in the context of a model of intra-industry trade and FDI where heterogeneous firms select into exporting or FDI, optimal import taxes are higher for more profitable exporters; optimal sales taxes are higher for more profitable foreign firms doing FDI. However, such heterogeneous taxes do not reflect the home government's desire to prevent the more productive foreign firms from entering home market. Instead, they reflect the desire to let less productive foreign firms also enter the home market. This motive leads to both taxes being constant among the most productive foreign firms, but vary among the least productive ones.

1.4.3 Overall Level of Taxes

Next, I characterize the overall level of taxes that is necessary for a decentralized equilibrium to implement the first-best allocation. Thanks to the two previous sections, this boils down to characterizing the six benchmark tax rates, s_d, t_d, s_m^*, t_m, s_i, t_i.

At the decentralized equilibrium, marginal rates of substitution must be equal to the relative prices. Using equations (1.3) (1.4) and previous results in this section, we have

$$MRS^*_{d/m,i} = \frac{P_d}{\left(P^*_m\left(s^*_m\right)^{1-\sigma} + P_i^{*1-\sigma}\right)^{\frac{1}{1-\sigma}}},$$

$$MRS_{d/m,i} = \frac{P_d\left(\frac{1+t_d}{1+s_d}\right)^{\frac{1}{1-\sigma}}}{\left(P_m\left(t_m\right)^{1-\sigma} + P_i\left(\frac{1+t_i}{1+s_i}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}},$$

$$MRS_{n/m,i} = \left(P_m\left(t_m\right)^{1-\sigma} + P_i\left(\frac{1+t_i}{1+s_i}\right)^{1-\sigma}\right)^{-\frac{1}{1-\sigma}},$$

where

$$P_d\left(\frac{1+t_d}{1+s_d}\right)^{1-\sigma} = \int_{\Phi_d} N\left(\frac{1+t_d}{1+s_d}\frac{\mu w}{\varphi}\right)^{1-\sigma} dG(\varphi),$$

$$P_m\left(t_m\right)^{1-\sigma} = \int_{\Phi_m} N^*\left((1+t_m)\min\left\{1, \alpha_m\varphi^{1/\mu}\right\}\frac{\mu}{\varphi}\right)^{1-\sigma} dG(\varphi),$$

$$P_i\left(\frac{1+t_i}{1+s_i}\right)^{1-\sigma} = \int_{\Phi_i} N^*\left(\frac{1+t_i}{1+s_i} min\left\{1, \alpha_i\varphi^{1/\mu}\right\}\frac{\mu w}{\varphi}\right)^{1-\sigma} dG(\varphi),$$

$$P_d^{1-\sigma} = \left(\int_{\Phi_d^*} N^*\left(\frac{\mu}{\varphi}\right)^{1-\sigma} dG(\varphi)\right)$$

$$P^*_m\left(s^*_m\right)^{1-\sigma} = \int_{\Phi^*_m} N\left(\frac{1}{1+s^*_m}\left(\frac{\tau\mu w}{\varphi}\right)\right)^{1-\sigma} dG(\varphi),$$

$$P_i^{*1-\sigma} = \int_{\Phi_i^*} N\left(\frac{\mu}{\varphi}\right)^{1-\sigma} dG(\varphi).$$

Combining this with equations (1.32) and (1.33), one can obtain the following result.

Lemma 5. *In a first-best allocation, the overall level of optimal taxes,* s_d, t_d, s^*_m, t_m, s_i, t_i, *should be such that the following conditions are all satisfied.*

$$MRS^*_{d/m,i} = \frac{P_d}{\left(P^*_m \left(s^*_m\right)^{1-\sigma} + P_i^{*1-\sigma}\right)^{\frac{1}{1-\sigma}}},$$

$$\frac{\partial C_{m,i}}{\partial C^*_{m,i}} \frac{\frac{\partial L}{\partial C_d}}{\frac{\partial L}{\partial C^*_{m,i}}} - \frac{\partial C_{m,i}}{\partial C_d} = \frac{P_d \left(\frac{1+t_d}{1+s_d}\right)^{\frac{1}{1-\sigma}}}{\left(P_m \left(t_m\right)^{1-\sigma} + P_i \left(\frac{1+t_i}{1+s_i}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}},$$

$$\frac{\partial C_{m,i}}{\partial C^*_{m,i}} \frac{\frac{\partial L}{\partial X_n}}{\frac{\partial L}{\partial C^*_{m,i}}} + \frac{\partial C_{m,i}}{\partial X_n} = \left(P_m \left(t_m\right)^{1-\sigma} + P_i \left(\frac{1+t_i}{1+s_i}\right)^{1-\sigma}\right)^{-\frac{1}{1-\sigma}},$$

where $MRS^*_{d/m,i}$, $\frac{\partial C_{m,i}}{\partial C^*_{m,i}}$, $\frac{\partial C_{m,i}}{\partial C_d}$, $\frac{\partial L}{\partial C_d}$, $\frac{\partial L}{\partial C^*_{m,i}}$ *and* $\frac{\partial L}{\partial X_n}$ *are their corresponding values in the planning problem.*

1.4.4 Implementation

There are, a priori, many pairs of taxes/subsidies that satisfy Lemma (1) through (5). In particular, one can set $t_m = s_i = s_d = 0$ and derive t_d, t_i and s^*_m from Lemma 5. This leads to the following result.

Lemma 6. *There exists a decentralized equilibrium with taxes that implements any allocation that solves the relaxed planning problem.*

To prove this lemma, one first constructs a first-best allocation using quantities and prices from Section 3 and taxes from this section, then shows that equilibrium conditions (1.1) through (1.9) are satisfied for both for Home and for the foreign country. The proof is very similar to Costinot et al. (2016) and omitted.

Proposition 1. *At the micro-level, unilaterally optimal taxes should be such that: (i) domestic taxes are uniform across all domestic producers (condition); (ii) export taxes are uniform across all exporters (condition 22); (iii) import taxes are uniform across Foreign's most profitable exporters and strictly increasing with profitability across its least profitable ones (condition 24); (iv) subsidies/taxes on Foreign firms doing FDI are uniform across the* *most profitable ones, and strictly increasing with profitability across least profitable ones. At the macro-level,*

1.4.5 How Does FDI affect Optimal Trade Policy?

The possibility of FDI adds quite a lot of complexity to the model. Here are a couple of key insights this delivers.

First, firm heterogeneity affects optimal trade policy, as it leads to heterogeneous taxes across foreign exporters and foreign FDI firms. Because fixed export or FDI costs do not affect Home welfare at the margin yet they play an important role in firms' exporting or FDI decision via the profitability condition, Home will find it optimal to discriminate among the foreign exporting and FDI firms by lowering the related taxes for the least productive ones. In line with the similar result in Costinot et al. (2016), this typically implies that the import tariffs or sales taxes imposed on the most profitable firms from abroad are higher, relative to other taxes, than they would be in the absence of selection.

Second, international transfers, such as profits from foreign FDI subsidiaries, breaks the Lerner symmetry between import tariffs and export taxes, as seen in Lemma 5. As in Blanchard (2009), when there is cross-border ownership because of FDI, trade balance is no longer the correct concept to consider. In fact, one country could run a permanent trade deficit, while the other runs a permanent trade surplus; and as long as the latter pays remittances to the former equal to its trade surplus, the current account balance condition will hold. In particular, this means that at the macro level, Home's problem is no longer a standard terms-of-trade manipulation problem as it is the case in Costinot et al. (2016).

Third, it is essential that there is wages at Home are fixed by a numeraire sector thus are not part of the policy variables. In fact, in an environment without the numeraire sector, Home has every incentive to set domestic wage as low as possible. The reason for that is wage paid by domestic firms do not affect Home welfare, as it only constitutes a transfer from domestic firm-owners to domestic wage-earners, which are, by assumption, the same representative agent. On the contrary, wage affects international transfers to foreign investors in Home, because it affects both the fixed and the variable costs of production for foreign FDI firms. Since foreign firms' profits are proportional to home wage, by setting wage equal to zero, all foreign firms will be making zero profits and are also completely indifferent between investing in Home or not. Thus, home can minimize its remittances to Foreign and increase its welfare.

1.5 Closing Remarks

In this paper, I characterize optimal trade and FDI policies in a model with monopolistic competition and firm-level heterogeneity similar to Helpman et al. (2004). I find that optimal trade and FDI policies requires firm-level taxes that discriminate against the most profitable foreign firms, whether they enter Home market via exporting or via FDI. In contrast, the optimal export taxes should still be uniform. The reason for the discrimination within foreign firms that enter the home market stems from the fact that the marginal foreign firm chooses not to do FDI because the profitability constraint is binding. From Home's perspective, it is beneficial to promote the entry of the marginal foreign firm because the fixed cost that the marginal foreign firms face are typically more costly than the related Home welfare cost; by lowering their tax, those firms would start producing at Home.

The results of this paper imply that it might be optimal for policymakers to use preferential taxes to attract foreign firms into FDI. However, they should only do so to the extent that the foreign firm breaks even after entering the home market. While in this paper I assume that the only policy instrument available for Home (for FDI firms) is sales taxes and production subsidies, had a lump-sum tax been available, the optimal FDI policy would involve the usage of the lump-sum tax, because doing so would allow extracting more profits from the most productive foreign firms as well (although in this case, there would still be discrimination against the most productive foreign firms because their higher profits imply that

Home would want to impose them higher taxes).

A related limitation of the findings in this paper is that I assumed the policymakers have access to a full set of tax instrument. In reality, it might not be always possible for countries to set firm-specific taxes and subsidies. A uniform intra industry tax might be more realistic. Similarly, international trade agreements may also limit the scope of potential policy instruments that countries could apply.

Finally, the model in this paper assumes that only horizontal FDI is possible. With the increase in fragmentation of world production (Baldwin (2006)), it would be very interesting to study the optimal trade and FDI policies in a set-up that allows both firm heterogeneity and vertical FDI. This would, in particular, open up the possibilities for firms to offshore their productions and for policymakers to "onshore" them back, which were assumed away in this paper. Much remains to be done on the normative side of the literature to close the gap between theory and practice.

Chapter 2

An Elementary Theory of Global Supply Chains[1]

> "One man draws out the wire, another straights it, a third cuts it, a fourth points it, a fifth grinds it at the top for receiving the head; to make the head requires two or three distinct operations; to put it on, is a peculiar business, to whiten the pins is another; it is even a trade by itself to put them into the paper; and the important business of making a pin is, in this manner, divided into about eighteen distinct operations, which, in some manufactories, are all performed by distinct hands, though in others the same man will sometimes perform two or three of them." Adam Smith (1776)

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2.1 Introduction

Most production processes consist of a large number of sequential stages. In this regard the production of pins in late eighteenth century England is no different from today's production of tee-shirts, cars, computers, or semiconductors. Today, however, production processes increasingly involve global supply chains spanning multiple countries, with each country specializing in particular stages of a good's production sequence, a phenomenon which Hummels, Ishii, and Yi (2001) refer to as vertical specialization.

This worldwide phenomenon has attracted a lot of attention among policy makers, business leaders, and trade economists alike. On the academic side of this debate, a large literature has emerged to investigate how the possibility to fragment production processes across borders may affect the volume, pattern, and consequences of international trade; see e.g. Feenstra and Hanson (1996), Yi (2003), and Grossman and Rossi-Hansberg (2008). In this paper, we propose to take a first look at a distinct, but equally important question: Conditional on production processes being fragmented across borders, how does technological change, either global or local, affect different countries participating in the same supply chain? In other words, how does vertical specialization shape the interdependence of nations?

From a theoretical standpoint, this is not an easy question. General equilibrium models with an arbitrary number of goods and countries—with or without sequential production—rarely provide sharp and intuitive comparative static predictions.[2] In order to make progress, we therefore start by proposing a simple theory of trade with sequential production. In Section 2 we consider a world economy with multiple countries, one factor of production (labor), and one final good. Production is sequential and subject to mistakes, as in Sobel (1992) and Kremer (1993). Production of the final good requires a continuum of intermediate stages. At each of these stages, production of one unit of an intermediate good requires

[2] Ethier (1984) offers a review of theoretical results in high-dimensional trade models.

one unit of labor and one unit of the intermediate good produced in the previous stage. Mistakes occur along the supply chain at a constant Poisson rate, which is an exogenous technological characteristic of a country. When a mistake occurs at some stage, the intermediate good is entirely lost. By these stark assumptions, we aim to capture the more general idea that because of less skilled workers, worse infrastructure, or inferior contractual enforcement, both costly defects and delays in production are more likely in some countries than in others.

Section 3 describes the properties of the free trade equilibrium in our basic environment. Although our model allows for any finite number of countries and a continuum of stages, the unique free trade equilibrium is fully characterized by a simple system of first-order non-linear difference equations. This system can be solved recursively by first determining the assignment of countries to different stages of production and then computing the wages and export prices sustaining that allocation as an equilibrium outcome. In our model, the free trade equilibrium always exhibits vertical specialization: countries with a lower probability of making mistakes, at *all* stages, specialize in later stages of production, where mistakes are more costly. Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage among nations.

Using this simple model, the rest of our paper offers a comprehensive exploration of how technological change, either global or local, affects different countries participating in the same global supply chain. Section 4 analyzes the consequences of global technological change. We investigate how an increase in the length of production processes, which we refer to as an increase in "complexity," and a uniform decrease in failure rates worldwide, which we refer to as "standardization," may affect the pattern of vertical specialization and the world income distribution. Building solely on the idea that labor markets must clear both before and after a given technological change, we demonstrate that although both an increase in complexity and standardization lead all countries to "move up" the supply chain, they have opposite effects on inequality between nations. While an increase in complexity increases inequality around the world, standardization benefits poor countries, i.e., countries with higher failure rates, disproportionately more. According to our model, standardization may even lead to a welfare loss in the most technologically advanced country, a form of immiserizing growth.

Section 5 focuses on how local technological change may spill over, through terms-of-trade effects, to other countries participating in the same supply chain. We consider two forms of local technological change: (i) labor-augmenting technical progress; and (ii) a decrease in a country's failure rate, which we refer to as "routinization." In a world with sequential production, we show that local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom, depending on the nature of technological changes, all countries either move up or down, but whatever they do, movements along the chain fully determine changes in inequality between nations. At the top of the chain, by contrast, local technological progress always leads all countries to move up, but even conditioning on the nature of technological change, inequality between nations may either fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

Section 6 demonstrates how more realistic features of global supply chains may easily be incorporated into our simple theoretical framework. Our first extension introduces trading frictions, which we refer to as "coordination costs." Among other things, we demonstrate that a decrease in coordination costs may lead to "overshooting:" more stages of production may be offshored to a small country at intermediate levels of coordination costs than under perfectly free trade. Our second extension allows for the existence of multiple parts, each produced sequentially and then assembled, with equal productivity in each country, into a unique final good using labor. In this environment, the poorest countries tend to specialize in assembly, while the richest countries tend to specialize in the later stages of the most complex parts. Our third extension allows for imperfect observability of mistakes. In this situation, we show how differences in failure rates and "quality control" across countries jointly determine the pattern of vertical specialization.

We conclude by providing sufficient conditions such that our cross-sectional predictions remain unchanged for more general production functions.

Our paper is related to several strands of the literature. First, we draw some ideas from the literature on hierarchies in closed-economy (and mostly partial-equilibrium) models. Important contributions include Lucas (1978), Rosen (1982), Sobel (1992), Kremer (1993), Garicano (2000) and Garicano and Rossi-Hansberg (2006). As in Sobel (1992) and Kremer (1993), we focus on an environment in which production is sequential and subject to mistakes, though we do so in a general equilibrium, open-economy setup. Models of hierarchies have been applied to the study of international trade issues before, but with very different goals in mind. For instance, Antràs, Garicano, and Rossi-Hansberg (2006) use the knowledge economy model developed by Garicano (2000) to study the matching of agents with heterogeneous abilities across borders and its consequences for within-country inequality. Instead, countries are populated by homogeneous workers in our model.[3]

In terms of techniques, our paper is also related to a growing literature using assignment or matching models in an international context; see, for example, Grossman and Maggi (2000), Grossman (2004), Yeaple (2005), Ohnsorge and Trefler (2007), Nocke and Yeaple (2008), Costinot (2009), Blanchard and Willmann (2010), and Costinot and Vogel (2010). Here, like in some of our earlier work, we exploit the fact that the assignment of countries to stages of production exhibits positive assortative matching, i.e., more productive countries are assigned to later stages of production, in order to generate strong and intuitive comparative static predictions in an environment with a large number of goods and countries.

In terms of focus, our paper is motivated by the recent literature documenting the importance of vertical specialization in world trade. On the empirical side, this literature builds on the influential work of Hummels, Rappoport, and Yi (1998), Hummels, Ishii, and Yi (2001), and Hanson, Mataloni, and Slaughter (2005). Our

[3]Other examples of trade papers using hierearchy models to study within-country inequality include Kremer and Maskin (2006), Sly (2010), Monte (2010), and Sampson (2010).

focus on how vertical specialization shapes the interdependence of nations is also related to the work of Kose and Yi (2001, 2006), Burstein, Kurz, and Tesar (2008), and Bergin, Feenstra, and Hanson (2009) who study how production sharing affects the transmission of shocks at business cycle frequency.

On the theoretical side, the literature on fragmentation is large and diverse; see Antràs and Rossi-Hansberg (2009) for a recent overview. Among existing papers, our theoretical framework is most closely related to Dixit and Grossman (1982), Sanyal (1983), Yi (2003, 2010), Harms, Lorz, and Urban (2009), Baldwin and Venables (2010), and Antràs and Chor (2011) who also develop trade models with sequential production. None of these papers, however, investigate how technological change, either global or local, may differentially impact countries located at different stages of the same supply chain. This is the main focus of our analysis.

2.2 Basic Environment

We consider a world economy with multiple countries, indexed by $c \in \mathcal{C} \equiv \{1, ..., C\}$, one factor of production, labor, and one final good. Labor is inelastically supplied and immobile across countries. L_c and w_c denote the endowment of labor and wage in country c, respectively. Production of the final good is sequential and subject to mistakes. To produce the final good, a continuum of stages $s \in \mathcal{S} \equiv (0, S]$ must be performed. At each stage, producing one unit of intermediate good requires one unit of the intermediate good produced in the previous stage and one unit of labor.

Mistakes occur along the supply chain at a constant Poisson rate, $\lambda_c > 0$, which is an exogenous technological characteristic of a country. Countries are ordered so that λ_c is strictly decreasing in c. When a mistake occurs on a unit of intermediate good at some stage, that intermediate good is entirely lost. Formally, consider two consecutive stages, s and $s + ds$, with ds infinitesimal. If a firm from country c combines $q(s)$ units of intermediate good s with $q(s)ds$ units of labor, its output of intermediate good $s + ds$ is given by

$$q\,(s + ds) = (1 - \lambda_c ds)\, q\,(s)\,. \tag{2.1}$$

Note that letting $q'\,(s) \equiv [q\,(s + ds) - q\,(s)]\,/ds$, Equation (2.1) can be written as $q'\,(s)\,/q\,(s) = -\lambda_c$. In other words, moving along the supply chain in country c, potential units of the final good get destroyed at a constant rate, λ_c. In the rest of our analysis, we often refer to λ_c as a measure of total factor productivity.[4] Since λ_c is strictly decreasing in c, countries with a higher index c are more productive.

All markets are perfectly competitive and all goods are freely traded. $p(s)$ denotes the world price of intermediate good s. For expositional purposes, we assume that "intermediate good 0" is in infinite supply and has zero price, $p(0) = 0$. "Intermediate good S" corresponds to the unique final good mentioned before, which we use as our numeraire, $p\,(S) = 1$. For technical reasons, we further assume that if a firm produces intermediate good $s + ds$, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Formally, for any intermediate good $s + ds$, we assume the existence of $s_\Delta < s + ds \leq s_\Delta + \Delta$ such that if $q\,(s + ds) > 0$, then $q\,(s') > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. This implies that each unit of the final good is produced by a finite, though possibly arbitrarily large number of firms.

[4] Although labor is the only primary factor of production, λ_c is *not* a measure of labor productivity. Instead it measures how much output at each stage can be produced by one unit of labor and one unit of intermediate good from the previous stage. In an environment with a discrete number of stages, the production function corresponding to equation (2.1) would be simply given by a Leontief production function $q(s+1) = e^{-\lambda_c} \min\{q(s), l(s+1)\}$, where $q\,(s)$ and $l(s+1)$ are the inputs used in stage $s + 1$. We come back to this issue in Section 2.6.4.

2.3 Free Trade Equilibrium

2.3.1 Definition

In a free trade equilibrium, all firms maximize their profits taking world prices as given and all markets clear. Profit maximization requires that for all $c \in \mathcal{C}$,

$$\begin{aligned} &p(s+ds) \leq (1+\lambda_c ds)\, p(s) + w_c ds, \\ &p(s+ds) = (1+\lambda_c ds)\, p(s) + w_c ds, \text{ if } Q_c(s') > 0 \text{ for all } s' \in (s, s+ds], \end{aligned} \tag{2.2}$$

where $Q_c(s')$ denotes total output at stage s' in country c. Condition (2.2) states that the price of intermediate good $s + ds$ must be weakly less than its unit cost of production, with equality if intermediate good $s + ds$ is actually produced by a firm from country c. To see this, note that the production of one unit of intermediate good $s + ds$ requires $1/(1 - \lambda_c ds)$ units of intermediate good s as well as labor for all intermediate stages in $(s, s + ds]$. Thus the unit cost of production of intermediate good $s + ds$ is given by $[p(s) + w_c ds] / (1 - \lambda_c ds)$. Since ds is infinitesimal, this is equal to $(1 + \lambda_c ds) p(s) + w_c ds$.

Good and labor market clearing further require that

$$\sum_{c=1}^{C} Q_c(s_2) - \sum_{c=1}^{C} Q_c(s_1) = -\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c Q_c(s)\, ds, \text{ for all } s_1 \leq s_2, \tag{2.3}$$

$$\int_0^S Q_c(s)\, ds = L_c, \text{ for all } c \in \mathcal{C}, \tag{2.4}$$

Equation (2.3) states that the change in the world supply of intermediate goods between stages s_1 and s_2 must be equal to the amount of intermediate goods lost due to mistakes in all countries between these two stages. Equation (2.4) states that the total amount of labor used across all stages must be equal to the total supply of labor in country c. In the rest of this paper, we formally define a free trade equilibrium as follows.

Definition 1. *A free trade equilibrium corresponds to output levels $Q_c(\cdot) : \mathcal{S} \longrightarrow \mathbb{R}^+$ for all $c \in \mathcal{C}$, wages $w_c \in \mathbb{R}^+$ for all $c \in \mathcal{C}$, and intermediate good prices $p(\cdot) : \mathcal{S} \longrightarrow \mathbb{R}^+$*

such that conditions (2.2)-(2.4) *hold.*

2.3.2 Existence and Uniqueness

We first characterize the pattern of international specialization in any free trade equilibrium.

Proposition 1. *In any free trade equilibrium, there exists a sequence of stages* $S_0 \equiv 0 < S_1 < ... < S_C = S$ *such that for all* $s \in \mathcal{S}$ *and* $c \in \mathcal{C}$, $Q_c(s) > 0$ *if and only if* $s \in (S_{c-1}, S_c]$. .

According to Proposition 1, there is vertical specialization in any free trade equilibrium with more productive countries producing and exporting at later stages of production. The formal proof as well as all subsequent proofs can be found in the Appendix.[5] The intuition behind Proposition 1 can be understood in two ways. One possibility is to look at Proposition 1 through the lens of the hierarchy literature; see e.g. Lucas (1978), Rosen (1982), and Garicano (2000). Since countries that are producing at later stages can leverage their productivity on larger amounts of inputs, efficiency requires countries to be more productive at the top. Another possibility is to note that since new intermediate goods require both intermediate goods produced in previous stages and labor, prices must be increasing along the supply chain. Thus the labor cost share is relatively lower in the production of intermediate goods produced at later stages, which makes them relatively cheaper to produce in countries with higher wages. In our model these are the countries with higher productivity in all stages. Because of the sequential nature of production, absolute productivity differences—here in the form of uniformly lower failure rates at *all* stages of production—are a source of comparative advantage among nations.[6]

[5]A result similar to Lemma 1 in an environment with a discrete number of stages can also be found in Sobel (1992) and Kremer (1993). In Section 2.6 we extend it to more general production functions.

[6]In his early work on fragmentation, Jones (1980) pointed out that if some factors of production are internationally mobile, then absolute advantage may affect the pattern of international specialization. The basic idea is that if physical capital is perfectly mobile and one country has an absolute

We refer to the vector $(S_1, ..., S_C)$ as the *"pattern of vertical specialization"* and denote by $Q_c \equiv Q_c(S_c)$ the total amount of intermediate good S_c produced and exported by country c. Using the previous notation, the pattern of vertical specialization and export levels can be jointly characterized as follows.

Lemma 1. *In any free trade equilibrium, the pattern of vertical specialization and export levels satisfy the following system of first-order non-linear difference equations:*

$$S_c = S_{c-1} - \left(\frac{1}{\lambda_c}\right) \ln\left(1 - \frac{\lambda_c L_c}{Q_{c-1}}\right), \text{for all } c \in \mathcal{C}, \tag{2.5}$$

$$Q_c = e^{-\lambda_c(S_c - S_{c-1})} Q_{c-1}, \text{for all } c \in \mathcal{C}, \tag{2.6}$$

with boundary conditions $S_0 = 0$ and $S_C = S$.

Lemma 1 derives from the goods and labor market clearing conditions (2.3) and (2.4). Equation (2.5) reflects the fact that the exogenous supply of labor in country c must equal the amount of labor demanded to perform all stages from S_{c-1} to S_c. This amount of labor depends both on the rate of mistakes λ_c as well as the total amount Q_{c-1} of intermediate good S_{c-1} imported from country $c-1$. Equation (2.6) reflects the fact that intermediate goods get lost at a constant rate at each stage when produced in country c.

In the rest of this paper, we refer to the vector of wages $(w_1, ..., w_C)$ as the *"world income distribution"* and to $p_c \equiv p(S_c)$ as the price of country c's exports (which is also the price of country $c+1$'s imports under free trade). Let $N_c \equiv S_c - S_{c-1}$ denote the measure of stages performed by country c within the supply chain. In the next lemma, we show how the measures of stages being performed in all countries $(N_1, ..., N_C)$ shape the world income distribution.

advantage in producing capital services, then it will specialize in capital intensive goods. The logic of our results is very different and intimately related to the sequential nature of production. Mathematically, a simple way to understand why sequential production processes make abolute productivity differences a source of comparative advantage is to consider the cumulative amount of labor necessary to produce all stages from 0 to $s \leq S$ in country c for a potential unit of the final good. By equation (2.1), this is equal to $e^{\lambda_c s}$ which is log-supermodular in (λ_c, s). This is the exact same form of complementarity that determines the pattern of international specialization in standard Ricardian models; see Costinot (2009).

Lemma 2. *In any free trade equilibrium, the world income distribution and export prices satisfy the following system of first-order linear difference equations:*

$$w_{c+1} = w_c + (\lambda_c - \lambda_{c+1})\, p_c, \textit{for all } c < C, \tag{2.7}$$

$$p_c = e^{\lambda_c N_c} p_{c-1} + \left(e^{\lambda_c N_c} - 1\right)(w_c/\lambda_c), \textit{for all } c \in \mathcal{C}, \tag{2.8}$$

with boundary conditions $p_0 = 0$ and $p_C = 1$.

Lemma 2 derives from the zero-profit condition (2.2). Equation (2.7) reflects the fact that for the "cutoff" good, S_c, the unit cost of production in country c, $(1 + \lambda_c ds)\, p_c + w_c ds$, must be equal to the unit cost of production in country $c + 1$, $(1 + \lambda_{c+1} ds)\, p_c + w_{c+1} ds$. Equation (2.8) directly derives from the zero-profit condition (2.2) and the definitions of N_c and p_c. It illustrates the fact that the price of the last intermediate good produced by country c depends on the price of the intermediate good imported from country $c - 1$ as well as the total labor cost in country c.

Combining Proposition 1 with Lemmas 1 and 2, we can establish the existence of a unique free trade equilibrium and characterize its main properties.

Proposition 2. *There exists a unique free trade equilibrium. In this equilibrium, the pattern of vertical specialization and export levels are given by equations* (2.5) *and* (2.6), *and the world income distribution and export prices are given by equations* (2.7) *and* (2.8).

The proof of Proposition 2 formally proceeds in two steps. First, we use Lemma 1 to construct the unique pattern of vertical specialization and vector of export levels. In equations (2.5) and (2.6), we have one degree of freedom, Q_0, which corresponds to total input used at the initial stage of production. Since S_C is decreasing in Q_0, it can be set to satisfy the final boundary condition $S_C = S$. Once $(S_1, ..., S_C)$ and $(Q_0, ..., Q_{C-1})$ have been determined, all other output levels can be computed using equation (2.1) and Proposition 1. Second, we use Lemma 2 together with the equilibrium measure of stages computed before, $(N_1, ..., N_C)$, to characterize the unique world income distribution and vector of export prices. In equations

(2.7) and (2.8), we still have one degree of freedom, w_1. Given the monotonicity of p_C in w_1, it can be used to satisfy the other final boundary condition, $p_C = 1$. Finally, once $(w_1, ..., w_C)$ and $(p_1, ..., p_C)$ have been determined, all other prices can be computed using the zero-profit condition (2.2) and Proposition 1.

2.3.3 Discussion

As a first step towards analyzing how vertical specialization shapes the interdependence of nations, we have provided a full characterization of free trade equilibria in a simple trade model with sequential production. Before turning to our comparative static exercises, we briefly discuss the cross-sectional implications that have emerged from this characterization.

First, since rich countries specialize in later stages of production while poor countries specialize in earlier stages, our model implies that rich countries tend to trade relatively more with other rich countries (from whom they import their intermediates and to whom they export their output) while poor countries tend to trade relatively more with other poor countries, as documented by Hallak (2010). Second, since intermediate goods produced in later stages have higher prices and countries producing in these stages have higher wages, our model implies that rich countries both tend to import goods with higher unit values, as documented by Hallak (2006), and to export goods with higher unit values, as documented by Schott (2004), Hummels and Klenow (2005), and Hallak and Schott (2010).

Following Linder (1961), the two previous stylized facts have traditionally been rationalized using non-homothetic preferences; see e.g. Markusen (1986), Flam and Helpman (1987), Bergstrand (1990), Stokey (1991), Murphy and Shleifer (1997) Matsuyama (2000), Fieler (2010), and Fajgelbaum, Grossman, and Helpman (2009). The common starting point of the previous papers is that rich countries' preferences are skewed towards high quality goods, so they tend to import goods with higher unit values. Under the assumption that rich countries are also relatively better at producing high quality goods, these models can further explain why rich countries tend to export goods with higher unit values and why countries with similar levels of GDP per capita tend to trade more with each other.[7]

The complementary explanation offered by our elementary theory of global supply chains is based purely on supply considerations. According to our model, countries with similar per-capita incomes are more likely to trade with one another because they specialize in nearby regions of the same supply chain. Similarly, countries with higher levels of GDP per capita tend to have higher unit values of imports and exports because they specialize in higher stages in the supply chain, for which inputs and outputs are more costly. Note that our supply-side explanation also suggests new testable implications. Since our model only applies to sectors characterized by sequential production and vertical specialization, if our theoretical explanation is empirically relevant, one would therefore expect "Linder effects"—i.e. the extent of trade between countries with similar levels of GDP per capita—to be higher, all else equal, in sectors in which production processes are vertically fragmented across borders in practice.

The previous cross-sectional predictions, of course, should be interpreted with caution. Our theory is admittedly stylized. In Section 2.6, we will discuss how the previous results may be affected (or not) by the introduction of more realistic features of global supply chains.

2.4 Global Technological Change

Many technological innovations, from the discovery of electricity to the internet, have impacted production processes worldwide. Our first series of comparative static exercises focuses on the impact of global technological changes on different countries participating in the same supply chain. Our goal is to investigate how an increase in the length of production processes, perhaps associated with the de-

[7] In Fajgelbaum, Grossman, and Helpman (2009), such predictions are obtained in the absence of any exogenous relative productivity differences. In their model, a higher relative demand for high-quality goods translates into a higher relative supply of these goods through a "home-market" effect.

velopment of higher quality goods, as well as a uniform decrease in failure rates worldwide, perhaps due to the standardization of production processes, may affect the pattern of vertical specialization and the world income distribution.

2.4.1 Definitions

It is useful to introduce first some formal definitions describing the changes in the pattern of vertical specialization and the world income distribution in which we will be interested.

Definition 2. *Let* $(S'_1, ..., S'_C)$ *denote the pattern of vertical specialization in a counterfactual free trade equilibrium. A country* $c \in \mathcal{C}$ *is moving up (resp. down) the supply chain relative to the initial free trade equilibrium if* $S'_c \geq S_c$ *and* $S'_{c-1} \geq S_{c-1}$ *(resp.* $S'_c \leq S_c$ *and* $S'_{c-1} \leq S_{c-1}$*).*

According to Definition 2, a country is moving up or down the supply chain if we can rank the set of stages that it performs in the initial and counterfactual free trade equilibria in terms of the strong set order. Among other things, this simple mathematical notion will allow us to formalize a major concern of policy makers and business leaders in developed countries, namely the fact that China and other developing countries are "moving up the value chain;" see e.g. OECD (2007).

Definition 3. *Let* $(w'_1, ..., w'_C)$ *denote the world income distribution in a counterfactual free trade equilibrium. Inequality is increasing (resp. decreasing) among a given group* $\{c_1, ...c_n\}$ *of adjacent countries if* $w'_{c+1}/w'_c \geq w_{c+1}/w_c$ *(resp.* $w'_{c+1}/w'_c \leq w_{c+1}/w_c$*) for all* $c_1 \leq c \leq c_{n-1}$*.*

According to Definition 3, inequality is increasing (resp. decreasing) within a given group of adjacent countries, if for any pair of countries within that group, the relative wage of the richer country is increasing (resp. decreasing). This property offers a simple way to conceptualize changes in the world income distribution in our model.[8]

[8]For expositional purposes, we have chosen to state all definitions and propositions in Sections 2.4 and 2.5 using weak inequalities. Most of our comparative static results also hold as strict inequalities; see Appendix.

2.4.2 Increase in complexity

At the end of the eighteenth century, Adam Smith famously noted that making a pin was divided into about 18 distinct operations. Today, as mentioned by Levine (2010), making a Boeing 747 requires more than 6,000,000 parts, each of them requiring many more operations. In this section we analyze the consequences of an increase in the measure of stages S necessary to produce a final good, which we simply refer to as an "increase in complexity."[9]

Our approach, like in subsequent sections, proceeds in two steps. We characterize first the changes in the pattern of vertical specialization and second the associated changes in the world income distribution. Our first comparative static results can be stated as follows.

Proposition 3. *An increase in complexity leads all countries to move up the supply chain and increases inequality between countries around the world.*

The changes in the pattern of vertical specialization and the world income distribution associated with an increase in complexity are illustrated in Figure 1. The broad intuition behind changes in the pattern of vertical specialization is simple. An increase in complexity tends to decrease total output at all stages of production. Since labor supply must remain equal to labor demand, this decrease in output levels must be accompanied by an increase in the measure N_c of stages performed in all countries. Proceeding by iteration from the bottom of the supply chain, we can then show that this change in N_c can only occur if all countries move up.

The logic behind the changes in the world income distribution is more subtle. From equation (2.7) in Lemma 2, we know that relative wages of countries $c + 1$

[9]For simplicity, we abstract from any utility gains that may be associated with the production of more complex goods in practice. Our analytical results on the pattern of vertical specialization and the inequality between nations do not depend on this simplification. But it should be clear that changes in real wages depend on it. Here an increase in complexity necessarily lowers totat output, and in turn, real wages. If the utility level associated with the consumption of one unit of the final good were allowed to increase with the complexity of the production process, then an increase in complexity may very well raise real wages.

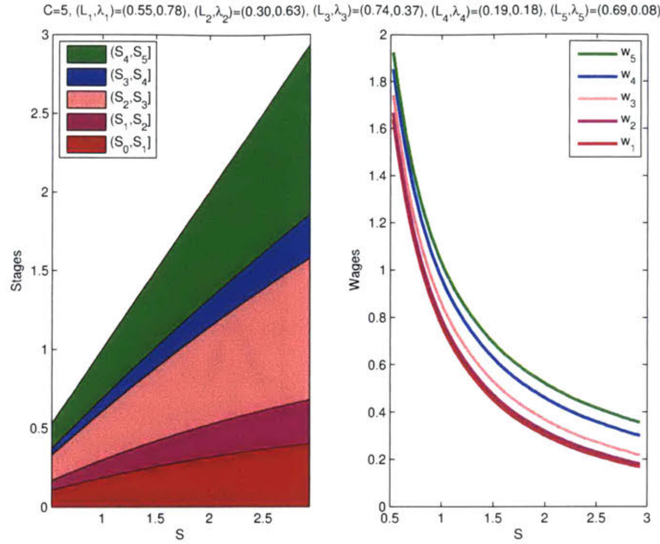

Figure 2-1: Consequences of an increase in complexity.

and c must equate the unit cost of production of the cutoff good S_c,

$$\frac{w_{c+1}}{w_c} = 1 + \frac{\lambda_c - \lambda_{c+1}}{(w_c/p_c)}, \text{ for all } c < C. \tag{2.9}$$

Thus, w_{c+1}/w_c is decreasing in w_c/p_c, which we refer to as the labor cost share of country c's exports.[10] This is reminiscent of the mechanism underlying terms-of-trade effects in a Ricardian model; see e.g. Dornbusch, Fischer, and Samuelson (1977) and Krugman (1986). From an economic standpoint, equation (2.9) captures the basic idea that the wage of country $c+1$ should increase relative to the wage of country c if and only if $c+1$ moves into sectors in which it has a comparative advantage. In our model, since country $c+1$ has a higher wage, these are the sectors with lower labor cost shares. In a standard Ricardian model, these would be the sectors in which country $c+1$ is relatively more productive instead.

There is, however, one important difference between a standard Ricardian model and our model with sequential production. In a standard Ricardian model, the pattern of comparative advantage only depends on *exogenous* productivity differences. In our model, the same pattern depends on *endogenous* differences in labor

[10] We slightly abuse terminology. Strictly speaking, the share of wages in the unit cost of production incurred at stage S_c, that is the stage at which country c exports, is equal to $(w_c/p_s)\,ds$.

cost shares across stages. According to equation (2.8) in Lemma 2, we have

$$\frac{w_c}{p_c} = \left[\left(e^{\lambda_c N_c} \right) \left(\frac{p_{c-1}}{w_c} \right) + \left(\frac{e^{\lambda_c N_c} - 1}{\lambda_c} \right) \right]^{-1}. \tag{2.10}$$

Hence the labor cost share of country c's exports depends on: (i) the price of country c's imports, p_{c-1}; (ii) the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$; and (iii) the associated amount of labor necessary to transform imports into exports, $\left(e^{\lambda_c N_c} - 1\right) / \lambda_c$. Having characterized how an increase in complexity affects the pattern of vertical specialization, it is fairly easy to evaluate how it affects each of these three components. From the first part of Proposition 3, we know that countries are both moving up into higher stages and performing more stages. Moving up into higher stages tends to increase import prices and hence, p_{c-1}/w_c. Performing more stages also increases the amount of labor necessary to transform imports into exports, $\left(e^{\lambda_c N_c} - 1\right) / \lambda_c$. In addition, it raises the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$. In this situation, all three effects tend to lower the labor cost share of intermediate goods that are being traded, which explains why inequality between nations increases.

2.4.3 Standardization

In most industries, production processes become more standardized as goods mature over time. In order to study the potential implications of this particular type of technological change within our theoretical framework, we now consider a uniform decrease in failure rates from λ_c to $\lambda'_c \equiv \beta \lambda_c$ for all $c \in \mathcal{C}$, with $\beta < 1$, which we simply refer to as "standardization." The consequences of standardization on the pattern of vertical specialization and the world income distribution can be described as follows.

Proposition 4. *Standardization leads all countries to move up the supply chain and decreases inequality between countries around the world.*

The consequences of standardization are illustrated in Figure 2. For a given

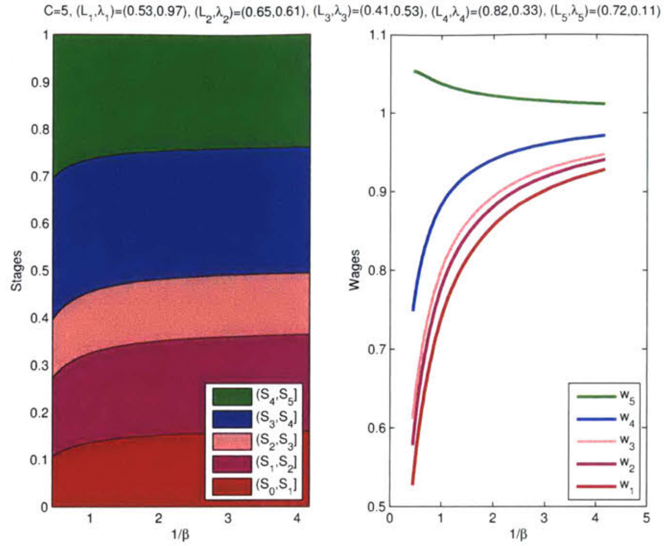

Figure 2-2: Consequences of standardization.

pattern of vertical specialization, standardization tends to raise total output—and, therefore, the demand for labor—at all stages of production. Since labor supply must remain equal to labor demand, this increase in output levels must be partially offset by a reduction of output at earlier stages of production. Hence, poor countries must increase the measure of stages that they perform, pushing all countries up the supply chain.

Like in our first comparative static exercise, the logic behind the changes in the world income distribution is more subtle. The direct effect of standardization on relative wages is to decrease inequality. By construction, for any pair countries c_2 and c_1 such that $c_2 > c_1$, we have $\lambda'_{c_1} - \lambda'_{c_2} = \beta(\lambda_{c_1} - \lambda_{c_2}) > 0$. Thus the productivity gap between poor and rich countries is lower for any $\beta \in (0,1)$. In the extreme case in which $\beta = 0$, having a lower rate of mistakes λ_c does not provide any benefit. There is, however, an indirect, general equilibrium effect associated with changes in the pattern of vertical specialization. To establish that the direct effect necessarily dominates the indirect one, the basic idea behind our proof is to normalize the measures of stages performed and export prices by β. Under this normalization, standardization is equivalent to a reduction in complexity: because both tend to reduce output lost to mistakes, they both require countries to move down the (normalized) supply chain, which leads to a fall in inequality between countries.[11] It is interesting to note that while standardization and an increase in complexity both cause all countries to move up the supply chain, they have opposite effects on inequality between nations.

The previous comparative static results are reminiscent of Vernon's (1966) "product cycle hypothesis;" see also Grossman and Helpman (1991) and Antràs (2005). In our model, as a particular production process becomes more standardized, less productive countries start performing a broader set of stages. As this happens, our analysis demonstrates that inequality between nations decreases around the world. Figure 2 also illustrates that although the direct effect of standardization is to increase output in all countries, welfare may fall in the most technologically advanced countries through a terms-of-trade deterioration. This is reminiscent of Bhagwati's (1958) "immiserizing growth." Two key differences, however, need to be highlighted. First, standardization increases productivity in all countries in the global supply chain, whereas Bhagwati's (1958) immiserizing growth occurs in response to an outward shift in the production possibility frontier in a single country. Second, and more importantly, standardization proportionately increases productivity at all stages of production, whereas Bhagwati's (1958) immiserizing growth occurs in response to an outward shift in the export sector. In our model, it is the sequential nature of production that makes uniform productivity growth endogenously act as export-biased technological change in the more technologically advanced countries.[12]

[11]Formally, while N_c rises for poor countries and falls for rich countries, βN_c falls for all countries. Hence, whereas all countries move up the chain, they move down the normalized chain. Under this normalization, countries: (i) are performing fewer stages, and (ii) are moving down into lower stages. Both effects tend to lower the normalized price βp_c of intermediate goods that are being traded, and in turn, to increase their labor cost share. This explains why inequality between nations decreases.

[12]Like in Bhagwati's original paper, however, it should be clear that immiserizing growth arises in this environment because of strong complementarities between goods. In our model, producing one unit of intermediate good always requires one unit of the intermediate good produced in the previous stage and one unit of labor. This explains why technological changes may have large (and adverse) terms-of-trade effects.

2.5 Local Technological Change

Two of the major changes in today's world economy are: (i) the increased fragmentation of the production process, which Baldwin (2006) refers to as the "Great Unbundling;" and (ii) the rise of China and other developing countries, such as India and Brazil. While both phenomena have been studied separately, we know very little about their interaction, either theoretically or empirically. The goal of this section is to use our elementary theory of global supply chains to take a first stab at this issue. To do so, our second series of comparative static exercises focuses on the impact of labor-augmenting technical progress and routinization in one country and describes how they spill over to other countries in the same supply chain through terms-of-trade effects.

2.5.1 Labor-augmenting technical progress

We first study the impact of labor-augmenting technical progress, which increases the total efficiency units of labor in a given country c_0 from L_{c_0} to $L'_{c_0} > L_{c_0}$. Following the same two-step logic as in Section 2.4, the consequences of labor-augmenting technical progress can be described as follows.[13]

Proposition 5. *Labor-augmenting technical progress in country* c_0 *leads all countries* $c < c_0$ *to move down the supply chain and all countries* $c > c_0$ *to move up. This decreases inequality among countries* $c \in \{1, ..., c_0\}$*, increases inequality among countries* $c \in \{c_0, ..., c_1\}$*, and decreases inequality among countries* $c \in \{c_1, ..., C\}$*, with* $c_1 \in \{c_0 + 1, ..., C\}$*.*

The spillover effects associated with labor-augmenting technical progress are illustrated in Figure 3. The broad intuition behind the changing patterns of specialization is simple. An increase in the supply of labor (in efficiency units) in one

[13] In line with our previous comparative static exercises, when discussing changes in the world income distribution, we focus on changes in wages *per efficiency units*. For all countries $c \neq c_0$, this is equivalent to changes in wages per worker. For country c_0, however, one should keep in mind that the two types of changes are distinct. If M_{c_0} denotes population size in country c_0, then the wage per worker is $w_{c_0} L_{c_0} / M_{c_0}$. Thus a decrease in w_{c_0} does not necessarily imply a decrease in wages per worker in that country.

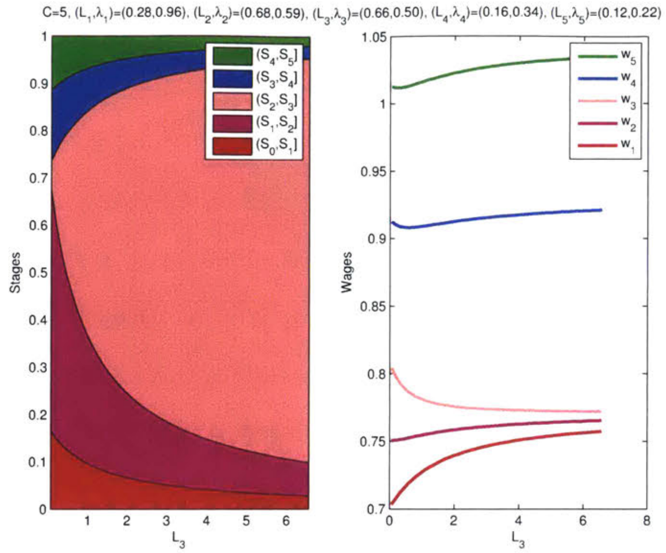

Figure 2-3: Consequences of labor-augmenting technical progress in country 3.

country tends to raise total output at all stages of production. Since labor supply must remain equal to labor demand, this increase in output levels must be accompanied by a decrease in the measure of stages N_c performed in each country $c \neq c_0$. Proceeding by iteration from the bottom and the top of the supply chain, we can then show that this change in N_c can only occur if all countries below c_0 move down and all countries above c_0 move up. Finally, since the total measure of stages must remain constant, the measure of stages N_{c_0} performed in country c_0 must increase.

Changes in the pattern of vertical specialization naturally translate into changes in the world income distribution. Countries at the bottom of the chain are moving down into lower stages and performing fewer stages. Both changes tend to increase the labor cost share of intermediate goods that are being traded and decrease inequality between nations at the bottom of the chain. The non-monotonic effects on inequality at the top of the chain reflect two conflicting forces. On the one hand, countries at the top of the chain are moving up. This tends to increase their import prices, reduce the labor cost share of their exports, and in turn, increase inequality. On the other hand, countries at the top of the chain are performing fewer stages. This decreases both the volume of imports and amount of labor necessary to produce one unit of their exports, which tends to raise the labor cost share of their exports, and in turn, decrease inequality.[14]

The previous non-monotonic effects stand in sharp contrast to the predictions of standard Ricardian models and illustrate nicely the importance of modeling the sequential nature of production for understanding the consequences of technological changes in developing and developed countries on their trading partners worldwide. To see this, consider a Ricardian model without sequential production in which there is a ladder of countries with poor countries at the bottom and rich countries at the top. Krugman (1986) is a well-known two-country example. In such an environment, if foreign labor-augmenting technical progress leads the richest countries to move up, inequality among these countries necessarily increases. The reason is simple. On the one hand, the relative wage of two adjacent countries is equal to their relative productivity in the "cutoff" sector. On the other hand, richer countries are relatively more productive in sectors higher up the ladder (otherwise they would not be specializing in these sectors in equilibrium). By contrast, Proposition 5 predicts that as the richest countries move up, inequality may decrease at the very top of the chain. This counterintuitive result derives from the fact that the pattern of comparative advantage in a model with sequential production is not exogenously given, but depends instead on endogenous labor cost shares along the supply chain. This subtle distinction breaks the monotonic relationship between the pattern of international specialization and inequality between nations. Although later stages necessarily have lower labor cost shares *in a given equilibrium*, the labor cost shares of later stages *in the new equilibrium* may be higher than the labor cost shares of earlier stages *in the initial equilibrium*. At the top of the chain, poorer countries may therefore benefit disproportionately more from being pushed into later stages of production.

[14]Note that since $c_1 \in \{c_0 + 1, ..., C\}$, the third group of countries, $\{c_1, ..., C-1\}$, is non-empty if $c_1 < C$, but empty if $c_1 = C$. We have encountered both cases in our simulations.

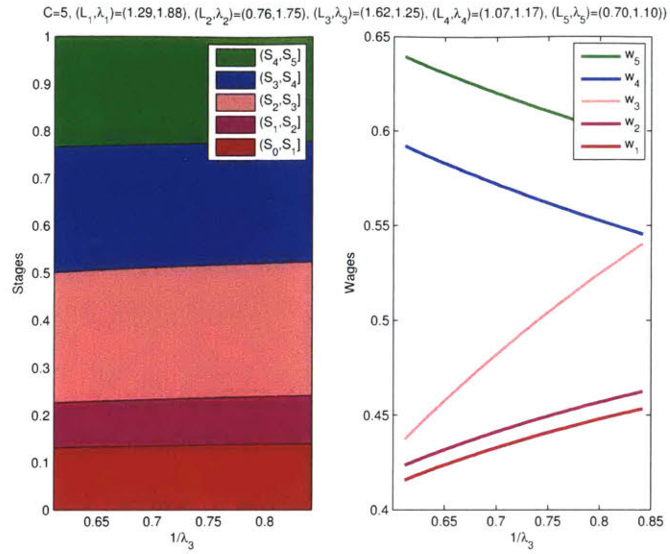

Figure 2-4: Consequences of routinization in country 3.

2.5.2 Routinization

We now turn our attention to the consequences of a decrease in the failure rate λ_{c_0} of a given country c_0, which we refer to as "routinization." For simplicity we restrict ourselves to a small change in λ_{c_0}, in the sense that it does not affect the ranking of countries in terms of failure rates. The consequences of routinization can be described as follows.

Proposition 6. *Routinization in country c_0 leads all countries to move up the supply chain, increases inequality among countries $c \in \{1, ..., c_0\}$, decreases inequality among countries $c \in \{c_0, c_0 + 1\}$, increases inequality among countries $c \in \{c_0 + 1, ..., c_1\}$, and decreases inequality among countries $c \in \{c_1, ..., C\}$, with $c_1 \in \{c_0 + 1, ..., C\}$.*

The spillover effects associated with routinization are illustrated in Figure 4. According to Proposition 6, all countries move up the supply chain. In this respect, the consequences of routinization are the same as the consequences of labor-augmenting technical progress at the top of the chain, but the exact opposite at the bottom.

To understand this result, consider first countries located at the top of the chain. Since total output of the final good must rise in response to a lower failure rate in country c_0, countries at the top of the chain must perform fewer stages for labor markets to clear. By a simple iterative argument, these countries must therefore move further up the supply chain, just like in Proposition 5. At the top of the chain, the consequences of routinization for inequality are the same as the consequences of labor-augmenting technical progress. The non-monotonicity—with inequality rising among countries $c \in \{c_0+1, ..., c_1\}$ and decreasing among countries $c \in \{c_1, ..., C\}$—arises from the same conflicting forces: countries move up the chain but produce fewer stages.

At the bottom of the chain, the broad intuition behind the opposite effects of labor-augmenting technical progress and routinization for changes in the pattern of specialization can be understood as follows. Holding the pattern of vertical specialization fixed, labor-augmenting technical progress in country c_0 increases the total labor supply of countries $c \geq c_0$, but leaves their labor demand unchanged. Thus labor market clearing requires countries at the bottom of the chain to reduce the number of stages they perform, to move down the chain, and to increase their output, thereby offsetting the excess labor supply at the top. By contrast, routinization in country c_0 increases the total labor demand of countries $c \geq c_0$ (since country c_0 now produces more output at each stage), but leaves their labor supply unchanged. As a result, countries at the bottom of the chain now need to increase the number of stages they perform, to move up the chain, and to reduce their output in order to offset the excess labor demand at the top. The consequences for inequality follow from the same logic as in the previous section.[15]

Our goal in this section was to take a first stab at exploring theoretically the relationship between vertical specialization and the recent emergence of developing countries like China. Much remains to be done to assess whether the effects identified in this paper are empirically important. Nevertheless, we view our theoretical analysis as a useful first step towards understanding how vertical specialization

[15]The only difference is that in the middle of the chain, inequality decreases among countries $c \in \{c_0, c_0+1\}$ because of the direct effect of a reduction in λ_{c_0}, which tends to decrease inequality between c_0 and c_0+1, as seen in equation (2.9). This force was absent from our previous comparative static exercise since labor endowments (in efficiency units) did not directly affect zero profit conditions.

shapes the interdependence of nations. A key insight that has emerged is that because of sequential production, local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom of the chain, depending on the nature of technological changes, countries may move up or down, but regardless of the nature of technological changes, movements along the chain fully determine changes in the world income distribution within that region. At the top of the chain, by contrast, local technological progress always leads countries to move up, but even conditioning on the nature of technological change, inequality between nations within that region may fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

2.6 Extensions

Our elementary theory of global supply chains is special along several dimensions. First, all intermediate goods are freely traded. Second, production is purely sequential. Third, mistakes are perfectly observable. Fourth, labor and intermediate goods are assumed to be perfect complements and all stages of production are subject to the same failure rates. In this section we demonstrate how more realistic features of global supply chains may be incorporated into our theoretical framework. To save on space, we focus on sketching alternative environments and summarizing their main implications. A detailed analysis can be found in our online Addendum.

2.6.1 Coordination Costs

An important insight of the recent trade literature is that changes in trade costs affect the pattern and consequences of international trade not only by affecting final goods trade, but also by affecting the extent of production fragmentation across borders; see e.g. Feenstra and Hanson (1996), Yi (2003), and Grossman and Rossi-Hansberg (2008). We now discuss how the introduction of trading frictions in our simple environment would affect the geographic structure of global supply chains, and in turn, the interdependence of nations.

A natural way to introduce trading frictions in our model is to assume that the likelihood of a defect in the final good is increasing in the number of times the intermediate goods used in its production have crossed a border. We refer to such costs, which are distinct from standard iceberg trade costs, as "coordination costs." Formally, if the production of a given unit u of the final good involves n international transactions—i.e. export and import at stages $0 < S_1^u \leq S_2^u \leq ... \leq S_n^u < S$—then the final good is defect free with probability $(1-\tau)^n$. The parameter $\tau \in (0,1)$ measures the extent of coordination costs. Section 2.2 corresponds to the limit case when coordination costs go to zero. Upon completion of each unit of the final good, we assume that consumers perfectly observe whether the unit is defect free or not. A unit with a defect has zero price. Like in Section 2.2, we assume that the (defect-free) final good is freely traded and we use it as our numeraire. Finally, we assume that all international transactions are perfectly observable by all firms so that two units of the same intermediate good s may, in principle, command two different prices if their production requires a different number of international transactions. Accordingly, competitive equilibria remain Pareto optimal in the presence of coordination costs.

The analysis of this generalized version of our model is considerably simplified by the fact that, in spite of coordination costs, a weaker version of vertical specialization must still hold in any competitive equilibrium. Let $c^u(s)$ denote the country in which stage s has been performed for the production of a given unit u. Using the previous notation, the pattern of international specialization can be characterized as follows.

Proposition 1 [Coordination Costs] *In any competitive equilibrium, the allocation of stages to countries,* $c^u : \mathcal{S} \rightarrow \mathcal{C}$, *is increasing in* s *for all* $u \in [0, \sum_{c \in \mathcal{C}} Q_c(S)]$.

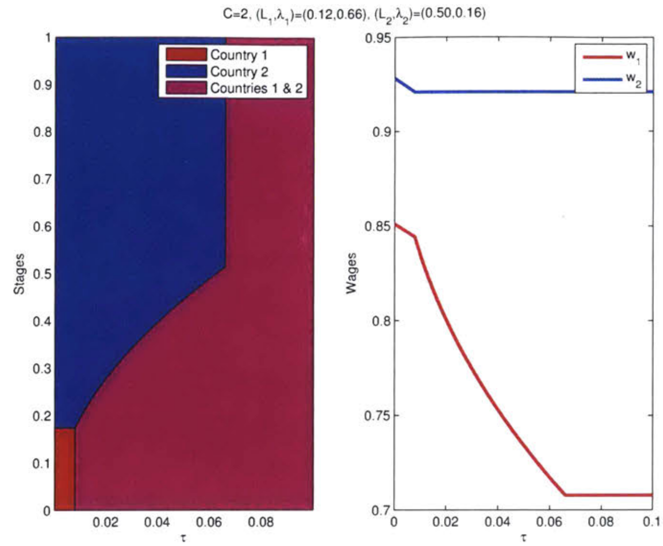

Figure 2-5: Consequences of a change in coordination costs.

According to this variation of Proposition 1, for any unit of the final good, production must still involve vertical specialization, with less productive countries specializing in earlier stages of production. This result is weaker, however, than the one derived in Section 2.2 in that it does not require $c^u(\cdot)$ to be the same for all units. This should be intuitive. Consider the extreme case in which τ is arbitrarily close to one. In this situation all countries will remain under autarky in a competitive equilibrium. Thus the same stages of production will be performed in different countries. In the presence of coordination costs, one can therefore only expect vertical specialization to hold within each supply chain, whether or not all chains are identical, which is what our new proposition establishes. Armed with this proposition, we can characterize competitive equilibria using the same approach as in Section 2.2. The only difference is that we now need to guess first the structure of the equilibrium (e.g. some units are produced entirely in country 1, whereas all other units are produced jointly in all countries) and then verify ex post that our guess is correct.

Figure 5 illustrates how the structure of competitive equilibria varies with the magnitude of coordination costs in the two-country case.[16] There are three distinct

[16]Details about the construction of these competitive equilibria are available upon request.

regions. For sufficiently high coordination costs, all stages are being performed in both countries and there is no trade. Conversely, for low enough coordination costs, the pattern of vertical specialization is the same as under free trade. In this region, reductions in coordination costs have no effect on the pattern of specialization, but raise wages in all countries. The most interesting case arises when coordination costs are in an intermediate range. In this region, the large country (country 2) is incompletely specialized, whereas the small country (country 1) is completely specialized in a subset of stages. As can easily be shown analytically, the set of stages that are being offshored to the small country is necessarily *increasing* in the level of coordination costs over that range. Hence starting from autarky and decreasing coordination costs, there will be "overshooting:" a broader set of stages will be performed in the poor country at intermediate levels of coordination costs than under perfectly free trade. This pattern of overshooting does not arise from coordination failures, heterogeneity in trade costs, or the imperfect tradability of the final good, as discussed in Baldwin and Venables (2010). It simply reflects the fact that in a perfectly competitive model with sequential production and trading frictions, a sufficiently large set of stages must be performed in the small country for firms to find it profitable to fragment production across borders. Accordingly, the larger the coordination costs, the larger the set of stages being performed in the small country!

Figure 5 also illustrates that sequential production does not hinder the ability of smaller countries to benefit from international trade. On the contrary, smaller countries tend to benefit more from freer trade. In the above example, a decrease in coordination costs either only benefits the small country (for intermediate levels of coordination costs) or affects real wages in both countries in the same proportional manner (for low enough coordination costs). Finally, Figure 5 highlights that how many stages of the production process are being offshored to a poor country may be a very poor indicator of the interdependence of nations. Here, when the measure of stages being offshored is the largest, the rich country is completely insulated from (small) technological shocks in the poor country.

2.6.2 Simultaneous versus Sequential Production

Most production processes are neither purely sequential, as assumed in Section 2.2, nor purely simultaneous, as assumed in most of the existing literature. Producing an aircraft, for example, requires multiple parts, e.g. fuselage, stabilizer, landing gears, entry doors, seats, and windows. Many of these parts are produced simultaneously before being assembled, but each of these parts requires a large number of sequential stages. For instance, the construction of a mid-fuselage section for the Boeing 787 Dreamliner in the United States involves the fabrication of wing-to-body fairing in Canada using panels made in China; see Gates (2005). More generally, extraction of raw materials comes before refining, which itself comes before manufacturing.

With this is mind, we turn to a generalization of our original model in which there are multiple supply chains, indexed by $n \in \mathcal{N} \equiv \{1, ..., N\}$, each associated with the production of a part. We allow supply chains to differ in terms of their complexity, S^n, but for simplicity, we require failure rates to be constant across chains and given by λ_c, as in Section 2.2. Hence countries do not have a comparative advantage in particular parts. Parts are ordered such that S^n is weakly increasing in n, so that parts with a higher index n are more complex.

Parts are assembled into a unique final good using labor. Formally, the output Y_c of the final good in country c is given by

$$Y_c = F\left(X_c^1, ..., X_c^N, A_c\right),$$

where $F(\cdot)$ is a production function with constant returns to scale, X_c^n is the amount of part n used in the production of the final good in country c, and $A_c \leq L_c$ corresponds to the amount of labor used for assembly in country c. Note that the production function $F(\cdot)$ is assumed to be identical across countries, thereby capturing the idea that assembly is sufficiently standardized for mistakes in this activity to be equally unlikely in all countries. Note also that by relabelling each part n as a distinct final good and the production function $F(\cdot)$ as a utility function, with $F(\cdot)$

independent of A_c, this section can also be interpreted as a multi-sector extension of our baseline framework.[17]

In this generalized version of our model, the pattern of international specialization still takes a very simple form, as the next proposition demonstrates.

Proposition 1 [Simultaneous Production] *In any free trade equilibrium, there exists a sequence of stages* $S_0 \equiv 0 \leq S_1 \leq ... \leq S_C = S^N$ *such that for all* $n \in \mathcal{N}$, $s \in (0, S^n]$, *and* $c \in \mathcal{C}$, $Q_c^n(s) > 0$ *if and only if* $s \in (S_{c-1}, S_c]$. *Furthermore, if country c is engaged in parts production,* $A_c < L_c$, *then all countries* $c' > c$ *are only involved in parts production,* $A_{c'} = 0$.

This strict generalization of Proposition 1 imposes three restrictions on the pattern of international specialization. First, the poorest countries tend to specialize in assembly, while the richest countries tend to specialize in parts production. This directly derives from the higher relative productivity of the poorest countries in assembly. Second, amongst the countries that produce parts, richer countries produce and export at later stages of production. This result also held in Section 2.3, and the intuition is unchanged. Third, whereas middle-income countries tend to produce all parts, the richest countries tend to specialize in only the most complex ones. Intuitively, even the final stage S^n of a simple part is sufficiently labor intensive that high-wage, high-productivity countries are less competitive at that stage. Viewed through the lens of the hierarchy literature, the final output of a simple chain does not embody a large enough amount of inputs to merit, from an efficiency standpoint, leveraging the productivity of the most productive countries.

Compared to the simple model analyzed in Section 2.3, the present model suggests additional cross-sectional predictions. Here, trade is more likely to be concentrated among countries with similar levels of GDP per capita if exports and imports tend to occur along the supply chain associated with particular parts rather than at the top between "part producers" and "assemblers." Accordingly, one should

[17] More generally, one could interpret the present model as a multi-sector economy with one "outside" good, that can be produced one-to-one from labor in all countries, and multiple "sequential" goods, whose production is as described in Section 2.2. Under this interpretation, A_c would be the amount of labor allocated to the outside good in country c.

expect trade to be more concentrated among countries with similar levels of GDP per capita in industries in which the production process consists of very complex parts.

2.6.3 Imperfect Observability of Mistakes

In our benchmark model, all mistakes are perfectly observable so that each country's exports are equally and fully reliable. In practice, quality concerns are a major determinant of the organization of global supply chains; see e.g. Manuj and Mentzer (2008). To capture such considerations within our framework, we now generalize our model to allow countries to differ not only in terms of the rate at which they make mistakes, but also the probability with which mistakes are observed, i.e. their "quality control."

As before, when a mistake occurs on a given unit u at stage s, any intermediate good produced after stage s using unit u is also defective and the associated final good is worthless. Our only point of departure from our benchmark model is that mistakes are imperfectly observed with a country-specific probability $\beta_c \in [0, 1]$. The location in which different stages associated with a given unit have been performed is public information. All markets are perfectly competitive and all goods are freely traded. Thus different units of a given intermediate good s produced at different locations may command different price depending on their "quality," i.e., the commonly known probability that they are defect free.

In this environment, if a firm from country c combines $q\left[s, \theta\left(s\right)\right]$ units of intermediate good s with quality $\theta\left(s\right)$ with $q\left[s, \theta\left(s\right)\right] ds$ units of labor, its output at stage $s + ds$ is given by

$$q\left[s + ds, \theta\left(s + ds\right)\right] = \left(1 - \beta_c \lambda_c ds\right) q\left[s, \theta\left(s\right)\right]. \tag{2.11}$$

Using Bayes' rule and a first-order Taylor expansion, the quality at stage $s + ds$ can be computed as

$$\theta\left(s + ds\right) = \left[1 - \left(1 - \beta_c\right) \lambda_c ds\right] \theta\left(s\right). \tag{2.12}$$

For simplicity, we restrict ourselves to "symmetric" free trade equilibria in which all units of the final good are produced in the same manner. The next proposition demonstrates how both β_c and λ_c shape the pattern of international specialization in this situation.

Proposition 1 [Imperfect Observability of Mistakes] *Suppose that $\beta_c \lambda_c$ is strictly decreasing in c and λ_c is weakly decreasing in c. Then in any symmetric free trade equilibrium, there exists a sequence of stages $S_0 \equiv 0 < S_1 < ... < S_C = S$ such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$.*

This strict generalization of Proposition 1 states that countries with higher failures rates and better quality controls tend to specialize in the earlier stages of global supply chains. The tendency for less productive countries to produce and export at earlier stages of production is driven by the same forces as in our benchmark model. But there is now also a tendency for countries with better quality controls to specialize in the same stages. Intuitively, if a country makes an unobserved mistake in producing a unit of output u in stage s, which occurs at rate $\lambda_c (1 - \beta_c)$, then firms will continue to add labor to this defective unit as it moves up the chain. If this unobserved mistake occurs lower down the chain, then more labor gets wasted. Hence, it is efficient to have countries with better quality control, all else equal, at the bottom of the chain. This again leads to more nuanced cross-sectional predictions than our benchmark model. Since richer countries are likely to have both lower failure rates and better quality control, the relative importance of these two considerations in different industries may determine whether or not they tend to operate at the top of global supply chains in practice.

2.6.4 General Production Functions

In order to focus attention in the simplest possible way on the novel aspects of an environment with sequential production, we have focused on a very stylized production process. Stages only differ in the order in which they are performed, and countries only differ in the rates at which mistakes occur along the supply chain. Formally, the production process underlying equation (2.1) corresponds to the limit, when δs goes to zero, of the following Leontief production function:

$$q(s+\delta s) = e^{-\lambda_c \delta s} \min \{q(s), l(s)/\delta s\}.$$

In this final subsection, we relax the three main features of the above production function: perfect complementarity between labor and intermediate goods at all stages of production, symmetry between all stages of production, and production that is subject to mistakes ($\lambda_c > 0$). Instead we assume that our production process with a continuum of stages corresponds to the limit, when δs goes to zero, of the following CES production function:

$$q(s+\delta s) = e^{-\lambda_c(s)\delta s} \left\{(1-\delta s)q(s)^{\frac{\sigma-1}{\sigma}} + \delta s \left[l(s)/\delta s\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}},$$

where $\sigma \geq 0$ denotes the elasticity of substitution at all stages of production and $\lambda_c(s) \in \mathbb{R}$ is a measure of country c's total factor productivity at stage s. As demonstrated in our online Addendum, starting from the previous production function and substituting in the optimal labor demand, we obtain the following generalization of equation (2.1):

$$q(s+ds) = \left\{1 - \left[\lambda_c(s) - \frac{\sigma}{1-\sigma}\left(1 - (w_c/p(s))^{1-\sigma}\right)\right] ds\right\} q(s).$$

Equation (2.1) corresponds to the special case: $\sigma = 0$ and $\lambda_c(s) \equiv \lambda_c > 0$. For technical reasons, we further assume that in order to produce one unit of intermediate good 0, firms need to hire a small number $\varepsilon > 0$ of workers. Thus perfect competition guarantees that $p(0) > 0$.[18] In the next proposition we provide sufficient conditions under which the pattern of international specialization can still be described as in Proposition 1 in this more general environment.

Proposition 1 [General Production Function] *Suppose that $\sigma < 1$ and that $\lambda_c(s)$ is*

[18]If $p(0) = 0$ and $\sigma > 0$, firms producing intermediate good ds have zero labor demand, zero costs, and in turn, $p(ds) = 0$. Iterating, this implies zero prices at all stages.

strictly decreasing in c, differentiable in s with either $\lambda_c'(s) > 0$ or $\lambda_c'(s) = 0$ for all s, and weakly submodular in (s, c). Then in any free trade equilibrium, there exists a sequence of stages $S_0 \equiv 0 < S_1 < ... < S_C = S$ such that for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$.

According to this strict generalization of Proposition 1, our cross-sectional predictions are unchanged if: (i) the elasticity of substitution between labor and intermediate goods is not too high, $\sigma < 1$; (ii) later stages of production tend to be more costly to produce, $\lambda_c'(s) > 0$ or $= 0$; and (iii) more productive countries (at all stages) are relatively more productive in later stages of production, $\lambda_c(s)$ is weakly submodular. In order to understand this result, it is useful to go back to the intuition behind Proposition 1. In Section 2.3 prices were increasing along the supply chain. Thus the cost share of labor was relatively lower in the production of intermediate goods produced at later stages, which made them relatively cheaper to produce in countries with higher wages. Conditions (i)-(iii) guarantee that the same logic applies. By condition (ii), prices are still increasing along the supply chain. And by condition (i), this implies that the cost share of labor remains relatively lower in the production of intermediate goods produced at later stages. Thus absent any comparative advantage across stages, more productive countries should tend to specialize in later stages of production. By condition (iii), the previous pattern of international specialization is reinforced by the comparative advantage of more productive countries in later stages.

While the basic forces emphasized in Sections 2.4 and 2.5 still shape the interdependence of nations, deriving comparative static predictions in this more general environment is more involved. Consider the smallest departure from our baseline model: assuming that production is no longer subject to mistakes, $\lambda_c < 0$ for all c. In this situation, all of the comparative static results on vertical specialization derived in Sections 2.4 and 2.5 continue to hold. Nevertheless, the implications for the world income distribution are more subtle as changes in the measures of stages performed in each country are no longer sufficient to predict how changes in the pattern of vertical specialization affect inequality between nations.[19] A similar issue arises in environments in which failure rates are no longer constant across stages of production. Finally, without perfect complementarity between labor and intermediate goods, our model is no longer block-recursive: prices affect labor demand, and thus, the assignment of countries to stages of production.

These issues notwithstanding, the pattern of international specialization in this extension, as well as the extensions presented in Sections 2.6.1-2.6.3, always exhibits vertical specialization. This implies that the free trade equilibrium remains characterized by a simple system of non-linear difference equations, akin to the ones presented in Lemmas 1 and 2. Accordingly, it is still easy to use simulations in order to investigate how global supply chains shape the interdependence of nations. We hope that this appealing feature of our theoretical framework will make it a useful guide for future work on global supply chains in richer, more realistic environments.

2.7 Concluding Remarks

In this paper, we have developed an elementary theory of global supply chains. The key feature of our theory is that production is sequential and subject to mistakes. In the unique free trade equilibrium, countries with lower probabilities of making mistakes at all stages specialize in later stages of production. Because of the sequential nature of production, absolute productivity differences are a source of comparative advantage among nations.

Using this simple theoretical framework, we have taken a first step towards analyzing how vertical specialization shapes the interdependence of nations. Among other things, we have shown that local technological changes tend to spillover very differently at the bottom and the top of the chain. At the bottom of the chain, de-

[19]Specifically, if $\lambda_c < 0$, then while increasing the measure of stages performed in a country c still increases the amount of labor necessary to transform imports into one unit of export, $\left(e^{\lambda_c N_c} - 1\right) / \lambda_c$, it now decreases the volume of imports necessary to produce one unit of export, $e^{\lambda_c N_c}$, thereby pushing labor cost shares, and in turn, relative wages in the opposite direction.

pending on the nature of technological changes, countries may move up or down, but whatever they do, movements along the chain fully determine changes in the world income distribution within that region. At the top of the chain, by contrast, local technological progress always leads countries to move up, but even conditioning on the nature of technological change, inequality between nations within that region may fall or rise. Perhaps surprisingly, while richer countries at the bottom of the chain benefit disproportionately more from being pushed into later stages of production, this is not always true at the top.

Our model is admittedly stylized, but we believe that its tractability lends itself to a variety of extensions and applications. The previous section has explored some of them. There are many others. For instance, we have focused on a perfectly competitive environment. It would be interesting to extend our framework to allow for monopolistic competition and endogenous innovation. We have also ignored any policy-related issues. It would be interesting to investigate how global supply chains may affect countries' commercial policies, and in turn, the optimal design of international trade agreements. One could also analyze competition between countries—perhaps through infrastructure spending or education policies—to capture higher positions in supply chains. Last but not least, it would be interesting to incorporate our simple model with sequential production into a standard quantitative trade model, such as Eaton and Kortum (2002), to explore the quantitative implications of global supply chains for the interdependence of nations. Such a model would be ideally suited to organize recent empirical evidence on trade in value added (e.g. Johnson and Noguera 2011).

Although we have emphasized the consequences of vertical specialization for the interdependence of nations, we believe that our general results also have useful applications outside of international trade. Sequential production processes are pervasive in practice. They may involve workers of different skills, as emphasized in the labor and organizations literature. They may also involve firms of different productivities, as in the industrial organization literature. Whatever the particular context may be, our theoretical analysis may help shed a new light on how vertical specialization shapes the interdependence between different actors of a given supply chain.

Chapter 3

Assortative Matching and Wage Inequality within and across Firms

3.1 Introduction

Striking changes have taken place in the production structure and distribution of earnings over the last few decades. As documented by many studies, the return to skills, for example measured by the college premium, has had an increasing trend for some decades (see Katz and Autor (1999) and Acemoglu and Autor (2011)). On the other hand, it has been recently made available data sets with matched employers and employees. New questions concerning the variation of wages across individuals within and across firms have been raised and there have been some empirical, as well as theoretical attempts made to reveal these new facts (See, for instance, Lazear and Shaw (2008) and Helpman et al. (2012)). The general view here is that heterogeneities among workers, firms/plants and the assortative matching between them have all contributed to the increase in inequality (Card et al. (2012)).

The classical and most straightforward way to model wage differences across individuals is, of course, to assume that individuals are endowed with different numbers of "efficiency units" of labor. This would imply that there is perfect substitutability across individuals. A very unpleasant feature of this assumption is, as posited by Kremer and Maskin (1996), that it generates no prediction at all with regard neither to the skill-levels, nor to the within- and across-firm levels of inequality.

A natural step moving forward is then to consider the difference between skilled and unskilled labor and analyze shifts in supply and demand between them (Goldin and Katz (2008)). However, it is increasingly difficult to explain variations in inequality among workers with only two sub-groups (Acemoglu and Autor (2011)) and the recent abundance of firm- and plant-level income data also cries out for a more nuanced model to explain its richer granularity and dynamics.

Following this availability of high-granularity data, more recent literature applies assignment models to labor market or international trade to model directly the assortative matching between workers and firms (Costinot and Vogel (2010)). However, in most of the matching models seen so far, the focus has been on making predictions on factor prices and there is typically perfect correlation between workers' skill and firms' characteristic, which leaves no space to account for any within-firm variation of worker skills and wages. Yet, in the empirical labor literature, economists debate on whether within-firm pay inequality is a large component in explaining recent increase in wage inequality(Lazear and Shaw (2008), Song et al. (2016)). Taken these considerations together, it only appears natural to consider a more general framework that would allow for work and firm heterogeneity and use assortative matching models to explain both within- and across-firm levels of income inequality.

This paper proposes exactly such a model. It features (i) heterogeneity of workers and firms, (ii) complementarity between occupations within each firm and (iii) complementarity between workers and firms/occupations. There are competitive good and factor markets. In equilibrium. workers sort into firms *and* occupations on the basis of their comparative advantage. However, the correlation between worker skill and firm characteristic is not perfect, which generates within-firm wage variation.

The model allows for a certain degree of freedom between movements in overall wage inequality and its within- and across-firm components. In particular, the comparative statics show that it is possible to have a pervasive fall in inequality, within a decreasing within-firm inequality at the lower-end of the firm technology distribution and an increasing within-firm inequality at the higher-end of the distribution.

-Related Literature. This paper is first and foremost related to the literature that uses assignment models in the labor market or in the international context, including Grossman and Maggi (2000), Antràs et al. (2006), Ohnsorge and Trefler (2007), Costinot (2009), Costinot and Vogel (2010), Helpman et al. (2010), Sampson (2012). Among them, this paper is most closely related to Costinot and Vogel (2010), who develop a model of labor assignment in the context of international trade that allows them to given predictions on comparative statics on factor allocation and factor prices. Compared to the present paper, however, their analyses, as well as many other models in this literature, typically do not have anything to say about the within-firm component of wage inequality, which stems from the fact that the assignment result in their model makes worker ability perfectly correlated with firm characteristic. In terms of allowing two degrees of matching - matching heterogeneous workers to firms and different jobs within a firm, the model has some of the flavor of Grossman et al. (2017), which uses assortative matching to discuss impacts of world prices of traded goods on wage inequality. Finally, the present paper is also related to the literature in labor economics on wage inequality, including, for instance, Acemoglu and Autor (2011) and Kremer and Maskin (1996).

The rest of our paper is organized as follows. Section 3.2 presents the theoretical framework. Section 3.3 derives comparative static results in the closed economy. Section 3.4 concludes.

3.2 The Closed Economy Model

3.2.1 Environment

-Preference

I consider an economy in which there is a representative consumer, whose preference is given by a Cobb-Douglas utility function over a continuum of goods indexed by their skill intensity or technology $\varphi \in \Phi \subset [0, +\infty)$:

$$\ln U = \int_{\Phi} \beta(\varphi) \ln y(\varphi)\, d\varphi,$$

with the normalization $\int_{\Phi} \beta(\varphi)\, d\varphi = 1$. Good markets are competitive. Denote by $p(\varphi)$ the price of a good produced with technology φ. An aggregate price index can then be computed by:

$$\ln P = \int_{\varphi \in \Phi} \beta(\varphi) \ln p(\varphi)\, d\varphi, \tag{3.1}$$

which will be used as the numeraire. The assumption of the Cobb-Douglas preferences imply that the demand functions for each of the differentiated goods are given by:

$$y(\varphi)\, p(\varphi) = \beta(\varphi) R, \tag{3.2}$$

where R is the aggregate income or expenditure.

-Production

It is assumed that there are a continuum of perfectly competitive firms in this economy, each of which is characterized by its skill intensity, φ. The production of a good φ requires the participation of a continuum of occupations or tasks, indexed by the skill intensity, $\theta \in \Theta \subset [0, +\infty)$. Although the assumption of competitive market and constant returns to scale would make the boundary of a firm irrelevant, I assume, as in Melitz (2003b), that a firm is characterized by its technology of production φ.[1] As will be shown, this will allow me to compare wages both within and across the firms in the economy. In addition, I assume that the set of occupations are the same across firms and does not vary with different the skill intensity of the product, φ.

The production function of good φ is assumed to take the following CES form:

$$y(\varphi)^{\epsilon-1/e} = \int_{\Theta} y(\theta,\varphi)^{\epsilon-1/e}\, d\theta, \tag{3.3}$$

where $y(\theta,\varphi)$ is the (endogenous) output of task θ in producing φ. Thus, a worker is hired at an occupation θ in a firm that has technology or skill intensity φ. In what follows, I shall refer to the firm-occupation pair (θ,φ) as a job whenever this does not create confusion.

Labor is the only factor used in the production process. There is a continuum of workers in this economy, indexed by their skill $\sigma \in \mathbb{R}$. Let $L(\sigma)$ be the fixed, inelastic supply of workers with skill σ. I impose the restriction that L has a compact support, so $L(\sigma) > 0$ if and only if $\sigma \in \Sigma \subset [0,+\infty)$. For technical reasons, I assume that L is continuously differentiable.

The output from the job (θ,φ) takes the following form

$$y(\theta,\varphi) = \int_{\Sigma} A(\sigma,\theta,\varphi)\, L(\sigma,\theta,\varphi)\, d\sigma, \tag{3.4}$$

where $L(\sigma,\theta,\varphi)$ is the measure of workers of skill σ hired at the job (θ,φ). A is the labor productivity function, representing the amount of output that a worker with skill σcould produce if he is hired by a firm of technology φ at the occupation θ.

Crucially, I make the following assumption on the structure of comparative advantage.

Assumption 1. *Assume that A is twice-differentiable and strictly increasing in σ. Assume that there is a continuous, strictly increasing and differentiable function $J : \Theta \times \Phi \to \mathbb{R}^{+}$ such that A is strictly log-supermodular with respect to σ and (θ,φ) where the*

[1] One could, of course, reinterpret a firm as a plant. I prefer the firm interpretation mainly for the reason that it will allow me to talk about "within-firm wage inequality" rather than "within-plant wage inequality" later in the text.

latter is equipped by the order induced by J:

$$\frac{A(\sigma',\theta',\varphi')}{A(\sigma,\theta',\varphi')} > \frac{A(\sigma',\theta,\varphi)}{A(\sigma,\theta,\varphi)}, \textit{ for all } \sigma' > \sigma, J(\theta',\varphi') > J(\theta,\varphi). \tag{3.5}$$

This assumption specifies that the jobs, which have two dimensions of heterogeneity across products and occupations, can be ordered in such a way that there is no ambiguity about the relative skill intensity between two jobs. In what follows, I shall refer to the mapping J as the sophistication of a job. This assumption then says that the production function, A, exhibits complementarity between job sophistication and worker skill. In other words, workers with higher skill have a comparative advantage in more "sophisticated" jobs. The assumption on the monotonicity of the sophistication function implies that, holding occupation fixed, producing a good with higher skill intensity is more sophisticated, and therefore high-skill workers have a comparative advantage there. Holding the product or the firm fixed, high skill workers also have a comparative advantage in more senior occupations.[2]

Since J is continuous, define $J(\Theta \times \Phi) = J_0$. Assumption 3.5 has the following implication.

Lemma 7. *There exists a function $\widetilde{A} : \sigma \times J_0 \to \mathbb{R}^+$ that is twice-differentiable, strictly increasing, and supermodular in (σ, j), such that*

$$A(\sigma,\theta,\varphi) = \widetilde{A}(\sigma, J(\theta,\varphi)).$$

For notational simplicity, in what follows, I shall still use A for the function $\widetilde{A}$ when this does not generate confusion. Also, the log of a variable X shall be denoted by x.

Firms hire workers in a perfectly competitive labor market. Let $W(\sigma)$ denote the equilibrium wage paid to workers of skill σ. Since there is constant returns to scale in production, all φ firms take good and factor prices as given, and solve the

[2] Ohnsorge and Trefler (2007) develop a model of labor assignment which features two dimension of worker heterogeneity and one dimension of firm heterogeneity.

following standard cost-minimization problem:

$$L^1(\sigma,\theta,\varphi) \in \arg\min_{L'(\sigma,\theta,\varphi)} \int_\Theta \int_\Sigma W(\sigma)\, L'(\sigma,\theta,\varphi)\, d\sigma d\theta \tag{3.6}$$

subject to the constraint

$$1 \leq \int_\Theta \left[\int_\Sigma A(\sigma,\theta,\varphi)\, L(\sigma,\theta,\varphi)\, d\sigma\right]^{\epsilon-1/e} d\theta.$$

Given constant returns to scales, the total labor hired at a job (θ,φ) is then given by $L(\sigma,\theta,\varphi) = L^1(\sigma,\theta,\varphi)\, y(\varphi)$.

-Market Clearing

The competitive labor market clearing condition is such that for any skill type σ, demand equals supply:

$$L(\sigma) = \int_\Phi \int_\Theta L(\sigma,\theta,\varphi)\, F(\varphi)\, d\theta d\varphi. \tag{3.7}$$

Good market clearing requires:

$$y(\varphi) = p(\varphi)^{-1} \beta(\varphi) R, \tag{3.8}$$

where, as above, firms' output is indexed by technology. Finally, the competitiveness of the good market requires that

$$p(\varphi)^{1-\epsilon} = \int_\Theta \left[\int_\Sigma W(\sigma)\, L^1(\sigma,\theta,\varphi)\, d\sigma\right]^{1-\epsilon} d\theta, \tag{3.9}$$

or price equals marginal cost. As I take the aggregate price index as the numeraire, this requires

$$0 = \int_\Phi \beta(\varphi) \ln p(\varphi)\, d\varphi. \tag{3.10}$$

In the rest of this paper, I formally define an equilibrium as follows.

Definition 2. A competitive equilibrium corresponds to labor inputs, $L : \sigma \times \Theta \times \Phi \to \mathbb{R}^+$, output levels, $y : \Phi \to \mathbb{R}^+$, outputs by occupations, $y : \Theta \times \Phi \to \mathbb{R}^+$, prices, $p : \Phi \rightarrow \mathbb{R}^+$, and equilibrium wages, $W : \sigma \rightarrow \mathbb{R}^+$ such that conditions (3.3)-(3.10) hold.

3.2.2 Matching workers with jobs

This section mostly revisits some of the results that have been established in the matching literature (such as in Costinot and Vogel (2010) or Sampson (2012)) and adapt them to the current model so that they could be used for the rest of the paper.

Since firms' cost-minimization problem is independent across occupations, a firm producing good φ solves the following cost-minimization problem for occupation θ:

$$\min_{L'(\sigma,\theta,\varphi)} \int_{\Sigma} W(\sigma)\, L'(\sigma,\theta,\varphi)\, d\sigma, \tag{3.11}$$

subject to the quantity constraint

$$\int_{\Sigma} A(\sigma, J(\theta,\varphi))\, L(\sigma,\theta,\varphi)\, d\sigma \geq 1.$$

It then follows that $L(\sigma,\theta,\varphi) > 0$ only if $\sigma \in \arg\min_{\sigma'} \frac{W(\sigma')}{A(\sigma', J(\theta,\varphi))}$. Given the assumptions on the labor production function, A, this condition imposes strong restrictions on the competitive equilibrium.

Lemma 8. *In a competitive equilibrium, there exists a continuous, strictly increasing matching function $M : J_0 \rightarrow \sigma$ such that (i) $L(\sigma,\theta,\varphi) > 0$ if and only if $\sigma = M(J(\theta,\varphi))$; (ii) $M(\inf J_0) = \inf \Sigma$ and $M(\sup J_0) = \sup \Sigma$.*

The intuition for Lemma 8 is as follows. First, because markets are perfectly competitive, the equilibrium features economic efficiency. Second, because of the complementarity of the labor productivity function, A, high-skill workers have a comparative advantage in more sophisticated jobs. This implies the monotonicity of the matching function.

It follows from Lemma 8 that in a firm φ would only hire workers with skill $M(J(\theta,\varphi))$ for an occupation θ such that

$$M\left(J\left(\theta,\varphi\right)\right)=\arg\min_{\sigma}\frac{W\left(\sigma\right)}{A\left(\sigma,J\left(\theta,\varphi\right)\right)}.$$

Cost minimization by a firm φ then implies:

$$p\left(\varphi\right)^{1-\epsilon}=\int_{\Theta}\left(\frac{W\left(M\left(J\left(\theta,\varphi\right)\right)\right)}{A\left(M\left(J\left(\theta,\varphi\right)\right),J\left(\theta,\varphi\right)\right)}\right)^{1-\epsilon}d\theta.$$

Since the cost of producing one unit of output at occupation θ in firm φ is equal to $\frac{W(M(J(\theta,\varphi)))}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}$, the demand function (3.8) then implies

$$\frac{y\left(\theta,\varphi\right)}{y\left(\varphi\right)}=\left[\frac{W\left(M\left(J\left(\theta,\varphi\right)\right)\right)}{p\left(\varphi\right)A\left(M\left(J\left(\theta,\varphi\right)\right),J\left(\theta,\varphi\right)\right)}\right]^{-\epsilon}. \tag{3.12}$$

And finally, the labor demand of labor by firm φ at occupation θ can be written as

$$L\left(\sigma,\theta,\varphi\right)=y\left(\varphi\right)\left[\frac{W\left(M\left(J\left(\theta,\varphi\right)\right)\right)}{p\left(\varphi\right)A\left(M\left(J\left(\theta,\varphi\right)\right),J\left(\theta,\varphi\right)\right)}\right]^{-\epsilon}\delta\left(\sigma-M\left(J\left(\theta,\varphi\right)\right)\right), \tag{3.13}$$

where δ is the Dirac function. The rest of our analysis depends crucially on the following lemma.

Lemma 9. *In a competitive equilibrium, the wage schedule and the matching function satisfy the following differential equations*

$$M'\left(j\right)=\frac{W\left(M\left(j\right)\right)^{-\epsilon}A\left(M\left(j\right),j\right)^{\epsilon-1}}{L\left(M\left(j\right)\right)}RI\left(j\right), \tag{3.14}$$

$$\frac{d\ln W}{d\sigma}\left(M\left(j\right)\right)=\frac{\partial\ln A}{\partial\sigma}\left(M\left(j\right),j\right), \tag{3.15}$$

where $I\left(j\right)=\int_{\Phi}\mathbf{1}_{j\in J(\Theta,\varphi)}p\left(\varphi\right)^{\epsilon-1}\beta\left(\varphi\right)J_{\varphi}^{-1\prime}\left(j\right)d\varphi$. *The boundary conditions are* $M\left(\inf J_0\right)=\inf\Sigma$ *and* $M\left(\sup J_0\right)=\sup\Sigma$.

According to this lemma, the two key endogenous variables of the model, the matching function, M, and the wage schedule, W, are given by the solution of a system of ordinary differential equations. Equation (3.14) captures the effect of labor market clearing on the shape of the matching function. Formally, the slope of the matching function, $M'(j)$, equates the supply of workers of any skill, $L(M(j))$, with its demand,

$$W(M(j))^{-\epsilon} A(M(j), j)^{\epsilon-1} I(j),$$

where the integral is over all firms φ that actually have jobs with sophistication j (and therefore actually hires workers of skill $M(j)$). Equation (3.15) captures how profit maximization determines the wage schedule. Intuitively, differences in relative productivity, $\frac{\partial \ln A}{\partial \sigma}(M(j), j)$, must be reflected in differences in relative wages, $\frac{d \ln W}{d\sigma}(M(j))$.

3.2.3 Wages and Within-firm Inequality

In order to make predictions on within- and across- firm inequality, I make the following assumptions on the job sophistication function and the labor productivity function.

Assumption 2. *Assume the labor productivity function is log-convex in j and σ, i.e.,*

$$\frac{d^2 \ln A}{dj^2}(\sigma, j), \frac{d^2 \ln A}{d\sigma^2}(\sigma, j) \geq 0.$$

Assume that the Job sophistication function J is linear in θ: $J(\theta, \varphi) = J_1(\varphi)\theta + J_2(\varphi)$, with $\frac{dJ_1}{d\varphi} \geq 0$.

Assumption 3. *Assume that tasks within a firm are gross substitutes: $\epsilon > 1$.*

The assumption of different tasks within a firm being gross substitutes is broadly consistent with recent development in both theoretical and empirical literature (Acemoglu and Autor (2011), Heckman et al. (1998)).

Proposition 2. *The average wage in a firm producing φ is given by*

$$W^m(\varphi) = \frac{\int_\Theta \frac{W(M(J(\theta,\phi)))}{A(M(J(\theta,\phi)),J(\theta,\phi))} \left[\frac{W(M(J(\theta,\phi)))}{A(M(J(\theta,\phi)),J(\theta,\phi))}\right]^{-\epsilon} d\theta}{\int_\Theta \frac{1}{A(M(J(\theta,\phi)),J(\theta,\phi))} \left[\frac{W(M(J(\theta,\phi)))}{A(M(J(\theta,\phi)),J(\theta,\phi))}\right]^{-\epsilon} d\theta}.$$

In particular, the average wage is a strictly increasing function of the skill intensity, φ.

The intuition of this proposition is as follows. By assumption, a fixed occupation θ in a more technologically advanced firm always exhibits a higher degree of job sophistication than in a less technologically advanced firm. Since both the matching function and the wage schedule are strictly increasing, this implies that a firm with better technology always pays more for any given occupation. On the other hand, the fact that the production function is Cobb-Douglas implies that this direct effect is not reversed by a even larger reduction of the employment of workers at higher occupations. As a result, average wage is higher in a more technologically advanced firm.

In order to study the within and across firm wage inequality, I need to propose a measure that will help characterize the effect of matching on the distribution of wages, and in particular wage inequality. One possible way to represent the wage distribution within a firm φ is to draw the corresponding Lorenz curve. The following proposition characterizes the variation of within-firm wage inequality across firms. Formally, consider a wage distribution with the associate C.D.F. $F(w)$, the Lorenz curve is defined as

$$LC(t) = \frac{\int_{-\infty}^{w_t} w dF(w)}{\int_{-\infty}^{+\infty} w dF(w)}, \text{ where } w_t = \inf\{w, F(w) \geq t\}.$$

The Lorenz curve captures the share of total income possessed by the the population who are within the t-th quantile of the wage distribution. It is also useful to have the following definition.

Definition 3. Let $F^1(w)$ and $F^2(w)$ be two C.D.Fs of wage and $LC^1(t)$ and $LC^2(t)$ the associated Lorenz curves. Following Atkinson (1970), we say that $F^1(w)$ weakly (resp. strongly) Lorenz dominates $F^2(w)$ if and only if $LC^1(t) \geq LC^2(t)$ for any $t \in [0,1]$ ($LC^1(t) > LC^2(t)$ for any $t \in (0,1)$).

Lorenz dominance is a partial order in the sense that there is no guarantee that given two wage distributions one would Lorenz dominate. However, when this does happen, then the Lorenz dominating distribution exhibits less inequality by all standard measures. It is also worth pointing out that although there is no direct relationship between Lorenz dominance and the more generally used concept of stochastic dominance, a variance of Lorenz dominance called Generalized Lorenz dominance, which is obtained simply by multiplying $LC(t)$ by the mean of the underlying variable, is strictly equivalent to second order stochastic dominance.

The equilibrium within-firm wage distribution satisfies the following property.

Proposition 3. *Consider two firms of skill intensities φ and φ'. The wage distribution of the former firm strictly Lorenz dominates that of the latter if and only if $\varphi < \varphi'$.*

This proposition says that within-firm inequality is strictly smaller in a firm that produces more skill-intensive good. Intuitively, because of because of the complementarity between worker skill and job sophistication, more technologically advanced firms tend to hire relatively more people in more senior occupations. This implies that for any quantile τ, the τ-th quantile worker in the wage distribution is always hired in a more senior occupation at a more technologically advanced firm. Finally, similar to the intuition described in Acemoglu and Autor (2011), the fact that occupations across the firm are gross substitutes would then assure that increasing wages do not overturn this result.

This intuition of why firms at the higher end of the technology distribution have smaller wage inequality echoes with Kremer and Maskin (1996). Kremer and Maskin argue that with two types of workers, the equilibrium will exhibit what they call "cross-matching", that is, high skilled workers are matched with low skilled workers in firms, if and only if the two levels of skill are not too far from each other. When that difference becomes too large, the complementarity between jobs will imply that it is better to have "self-matching", that is, workers work with the same skill types in firms. In my model, since more technologically advanced firm hires higher skilled workers, the previous result says that in some sense the degree of "self-matching" increases, and therefore wage inequality falls.

3.3 Comparative Statics in a Closed Economy

In this section, I assume that the labor skill follows a Pareto distribution, which has been widely documented and modeled in the related literature (Jones and Kim (2014)).

Assumption 4. *Assume that skill is distributed according to a truncated Pareto distribution: for $\sigma \leq \bar{\sigma}$, $L(\sigma) = \frac{C}{\sigma^{\mu+1}}$ with $\mu \geq 1$ and $C > 0$.*

1. Changes in Skill Abundance

I first consider a change in labor supply. Specifically, assume that of the distribution of skills changes from $L(\sigma)$ to $\widehat{L}(\sigma)$ such that

$$\widehat{L}(\sigma')\, L(\sigma) \geq \widehat{L}(\sigma)\, L(\sigma') \text{ for all } \sigma' \geq \sigma. \tag{3.16}$$

This assumption corresponds to the monotone likelihood ratio property (see Milgrom (1981)). Intuitively, when this is true, there are relatively more high skilled workers with $\widehat{L}$ than with L. Note that this condition would hold trivially if $L(\sigma') = 0$ or $\hat{L}(\sigma) = 0$. Under assumption (4), this is equivalent to $\hat{\mu} > \mu$.

I start by analyzing the impact of a change in skill abundance on the matching function.

Lemma 10. *Under assumption (4), suppose that $L(\sigma)$ and $\widehat{L}(\sigma)$ are such that condition (3.16) holds, then $\widehat{M}(j) \geq M(j)$ for all $j \in J(\Theta, \Phi)$.*

The result is similar to Costinot and Vogel (2010). This lemma says that there is a skill upgrading from the standpoint of the jobs. For a job with a given level of sophistication, it is now performed by a worker of higher skill. On the other hand, from the workers' standpoint, there is job downgrading. A worker with a given skill type is now performing a less sophisticated job. Intuitively, since the relative supply of highs skilled workers increases, one would expect that more jobs are performed by the high-skill workers.

The upward shift of the matching function implies that there is smaller wage inequality under $\widehat{L}$ than under L for two workers of given skill types. Specifically, from equation (3.15), we know

$$\frac{d \ln W}{d\sigma}(\sigma) = \frac{\partial \ln A}{\partial \sigma}\left(\sigma, M^{-1}(\sigma)\right).$$

Now, since there is job downgrading, log-supermodularity of the labor productivity function implies that the R.H.S. of this equation goes down. Integrating it over two skill levels $\sigma' > \sigma$, we obtain

$$\frac{W(\sigma')}{W(\sigma)} \geq \frac{\widehat{W}(\sigma')}{\widehat{W}(\sigma)}.$$

Therefore, moving from L to $\widehat{L}$ results in a pervasive fall in inequality, meaning that for any two workers, the difference in their log-wage goes down when high-skill workers become more abundant. This result is not surprising: in this model, the increase in the relative supply of high-skill workers leads to a reallocation of high-skilled towards less sophisticated jobs. As the high-skilled have a comparative advantage in more sophisticated jobs, this translates into a decrease in relative wage.

If there is a pervasive fall in wage inequality, does this mean that the within and between firm inequalities also go down? The following results show that this might not be the case.

Proposition 4. *There exists $j_0 \in J(\Theta, \Phi)$, such that $\widehat{M'}(j) \geq M'(j)$ for all $j \leq j_0$ and $\widehat{M'}(j) \leq M'(j)$ for all $j \geq j_0$. For any pair of jobs, $j \leq j' \leq j_0$, we have $\frac{\widehat{W}(\widehat{M}(j_2))}{\widehat{W}(\widehat{M}(j_1))} \geq \frac{W(M(j_2))}{W(M(j_1))}$.*

Intuitively, the differential equation on the matching function, (3.14), implies that there are three forces that govern the slope of the matching function at a job j. First, the higher the wage of the workers working at j is, the smaller the slope is. This is because of labor substitution within firms: the more costly workers at job j are, the more firms will substitute workers at other jobs for workers at j, so the smaller the total number of workers hired at j is. Second, the more workers of skill $M(j)$ are, the smaller the slope is. This is because, *ceteris paribus*, the higher

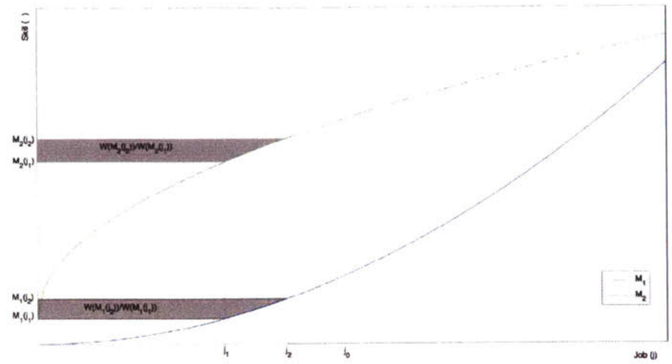

Figure 3-1: Changes in matching and wage inequality

$L(M(j))$ is, the more jobs these workers will occupy. Lastly, as these workers are hired at different firms, and the slope of the matching function at job j depends on the total labor demand of these firms for $M(j)$ workers, the more firms there are that have occupations of sophistication j, the steeper the slope is. The assumptions on the signs of second order derivatives then guarantee that all three forces would work in the same direction such that moving from L to $\widehat{L}$ leads to a matching function whose slope is relatively less steep at the higher end.

Once we know the changes in the slope of the matching function, it is straightforward to predict changes in inequality across jobs. From equation (3.15), we know that

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln A}{\partial \sigma}(M(j), j) M'(j)\, dj.$$

Since there is skill upgrading from L to $\widehat{L}$, assumptions on the log-convexity of A implies that the relative wage difference between two jobs would increase whenever the matching function between the two jobs is steeper.

For an illustrative purpose, consider the following labor productivity function: $\ln A(\sigma, j) = \sigma j$. Then the previous equation becomes

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} jM'(j)\, dj = \int_{M(j_1)}^{M(j_2)} M^{-1}(\sigma)\, d\sigma.$$

Graphically, this implies that the relative wage difference between can be represented by the shaded areas in Figure 3-1. Moreover, this also implies that the relative wage difference is uniquely determined by the slope of the matching functions. Since moving from L to $\widehat{L}$ leads to a steeper matching function at the lower end and a flatter at the high end, this further implies that wage inequality between two jobs becomes smaller at the lower end and larger at the higher end.

Changes in relative wage inequality between jobs lead directly to the following result in within-firm wage inequality.

Corollary 1. *For any firm φ such that $J\left(\overline{\theta}, \varphi\right) \leq j_0$, the within-firm wage distribution under $\widehat{L}$ is Lorenz dominated by that under L. Moreover, if $\frac{\partial^2 \ln A}{\partial \sigma^2} = 0$, then for any firm φ such that $J\left(\underline{\theta}, \varphi\right) \geq j_0$, the within-firm wage distribution under $\widehat{L}$ Lorenz dominates that under L.*

Hence, within-firm wage distribution becomes more unequal in firms with low skill intensity and becomes smaller in firms with high skill intensity. The intuition is that the higher relative supply in high-skill workers imply that within-firm skill dispersion goes up at the lower end and goes down at the higher end, which leads to this result.

2. Changes in Technology

I next consider a change in technology. Specifically, consider an increase in the complementarity between worker skills and job sophistication, which amounts to a high-skill biased technological change. Formally, I assume that the labor productivity function has changed in such a way that

$$\widehat{A}\left(\sigma, j\right) = A\left(\sigma, j\right) Z\left(\sigma, j\right), \tag{3.17}$$

where Z is assumed to be strictly increasing and strictly log-supermodular in (σ, j).

Lemma 11. *Suppose that A and $\widehat{A}$ are such that condition (3.17) holds, then $\widehat{M}\left(j\right) \geq M\left(j\right)$ for all $j \in J\left(\Theta \times \Phi\right)$.*

The result is similar as the previous comparative statics, in the sense that we obtain a skill upgrading from the standpoint of the jobs and a job downgrading from the standpoint of the workers. The result is also reminiscent of the comparative static result derived by Acemoglu and Autor (2011) with one dimension of jobs. Intuitively, the high-skill workers are becoming relatively more productive on their jobs and therefore expand the measure of jobs that they perform at, which leads to the upward shift of the matching function.

Note that here an upward shift of the matching function does not lead to a pervasive fall in inequality: although there is job downgrading, which tends to make relative wage difference smaller as before, the increased complementarity between skills and jobs work in the other direction.

The following proposition helps us predict movements in within and between firm wage inequalities.

Proposition 5. *There exists $j_0 \in \left[\underline{j}, \bar{j}\right]$, such that $\widehat{M}'(j) \geq M'(j)$ for all $j \leq j_0$ and $\widehat{M}'(j) \leq M'(j)$ (resp. $\widehat{M}'(j) \leq M'(j)$) for all $j \geq j_0$. For any pair of jobs, $j \leq j' \leq j_0$, we have $\frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} \geq \frac{W(M(j_2))}{W(M(j_1))}$.*

Notice that here even if A is log-linear in skill, we will not obtain the result that wage inequality between two jobs is smaller at the high end. The reason is that there is now this additional factor, Z, which makes skill and jobs more complementary, thus increasing the wage inequalities. This lemma has the following implications on within-firm wage inequalities.

Corollary 2. *For any firm φ such that $J\left(\overline{\theta}, \varphi\right) \leq j_0$, the within-firm wage distribution under $\widehat{A}$ is Lorenz dominated by that under A.*

Corollary 3. *In the limiting case in which Θ is a singleton, the Lorenz curve of the firms' average wage distribution under $\widehat{A}$ is Lorenz dominated by that under A.*

In the limiting case in which Θ is a singleton, there is a one-to-one mapping between firm skill intensity φ and worker skill type σ at the equilibrium. The Lorenz curve of firms' average wage (weighted by the number of workers) therefore coincides with the Lorenz curve of wages in the economy. Intuitively, since the distribution of worker skills remain the same, the τ-th quantile in the skill distribution remains at the same skill level. It then follows from the job downgrading that the poorest τ percent of workers would now work in a more limited number of firms, which then implies that collectively, they receive a lower wage payment.

3.4 Concluding Remarks

This paper proposes a matching model that features heterogeneity of workers and firms, complementarity between occupations within each firm and complementarity between workers and firms/occupations. There are competitive good and factor markets. In equilibrium. workers sort into firms and occupations on the basis of their comparative advantage. The correlation between worker skill and firm characteristic (skill intensity, in this model) is not perfect, which generates within-firm wage variation.

The model predicts that both a shift in labor supply biased towards the skilled workers and an increase in the degree of complementarity between workers and jobs would lead to a job downgrading. Moreover, following these two movements, the within-firm wage inequality always rises for firms with low skill intensity, yet they differ in the response of the between-firm wage inequality. In the former case, the movement in between-firm wage inequality is ambiguous, whereas in the latter case, it tends to go up.

While this framework opens up the possibility of modeling a higher level of granularity of matching of heterogeneous workers across firms and occupations within firms, much is yet to be done especially in terms of applying these models to data and test the fit and the results. With the proliferation of modern employer-employee datasets, I am hopeful that more granular models will be built and tested not only at the level of stylized facts, but also at establishment and occupation levels.

Appendix A

Appendix for Optimal Trade and FDI Policies

A.1 Proofs

A.1.1 Production of domestic varieties (Section 1.3.1)

The corresponding Lagrangians are

$$\begin{aligned}\mathfrak{L}_m^*(\varphi) &= f_m - (\mu - 1)\left(\frac{\varphi}{\tau}\right)^{\sigma-1}\left(\frac{\mu}{\lambda_{m,i}}\right)^{-\sigma}, \text{if } \varphi \geq \varphi_m^{0*} \\ &= 0, \text{if } \varphi \leq \varphi_m^{0*}\end{aligned}$$

and

$$\begin{aligned}\mathfrak{L}_i^*(\varphi) &= \frac{1}{w}\left[f_i - (\mu - 1)\,\varphi^{\sigma-1}\left(\frac{\mu}{\lambda_{m,i}}\right)^{-\sigma}\right], \text{if } \varphi \geq \varphi_i^{0*}, \\ &= 0, \text{if } \varphi \leq \varphi_i^{0*}.\end{aligned}$$

Under condition (1.12), one can check that that $\varphi_i^{0*} \geq \varphi_m^{0*}$, so that there will always be both exporting and FDI for domestic varieties. In addition, when productivity φ is sufficiently high, the Lagrangian $\mathfrak{L}_i^*$ is always smaller than $\mathfrak{L}_m^*$. Denote

$\varphi'^{*} > \varphi_i^{0*}$ the productivity at which $\mathfrak{L}_i^*(\varphi'^*) = \mathfrak{L}_m^*(\varphi'^*)$, i.e., it is equally beneficial for Home to export or to do FDI The solution to $\mathfrak{L}_{m,i}^*(\varphi)$ is finally given by

$$\begin{aligned}
\mathfrak{L}_{m,i}^*(\varphi) &= 0, \text{ if } \varphi \leq \varphi_m^{0*}, \\
&= \mathfrak{L}_m^*(\varphi), \text{ if } \varphi \in \Phi_m^* \equiv \left[\varphi_m^{0*}, \varphi'^*\right], \\
&= \mathfrak{L}_i^*(\varphi), \text{ if } \varphi \in \Phi_i^* \equiv [\varphi'^*, \infty).
\end{aligned}$$

A.1.2 Footnote 7

Here I try to establish the result in footnote 6, namely, that the solution to the following minimization problem

$$\begin{aligned}
c_m^*(\varphi), c_i^*(\varphi) &= \arg\min_{c,c'} \mathfrak{L}_{m,i}^*(c, c', \varphi) \\
&= \arg\min_{c,c'} l_m^*(c, \varphi) + \frac{1}{w} l_i^*(c', \varphi) - \lambda_{m,i}(c + c')^{1/\mu} \qquad \text{(A.1)}
\end{aligned}$$

is such that $c_m^*(\varphi)$ and $c_i^*(\varphi)$ cannot be positive at the same time.

Assume by contradiction that it is the case. Since both quantities are positive, the fixed costs are paid in for both means of serving the foreign market:

$$\begin{aligned}
l_m^*(c_m^*(\varphi), \varphi) &= \frac{\tau c_m^*(\varphi)}{\varphi} + f_m, \\
l_i^*(c_i^*(\varphi), \varphi) &= \frac{c_i^*(\varphi)}{\varphi} + f_i.
\end{aligned}$$

Two scenarios are possible here, either the variable cost of exporting is cheaper than that of FDI, or vice versa. Assume it is the case that $\frac{\tau}{\varphi} \leq \frac{1}{w\varphi}$, or $\tau w \leq 1$.

Define $c = c_m^* (\varphi) + c_i^* (\varphi)$,

$$\begin{aligned}
\mathfrak{L}_{m,i}^* (c, 0, \varphi) &= \frac{\tau \left[c_m^* (\varphi) + c_i^* (\varphi)\right]}{\varphi} + f_m - \lambda_{m,i} \left(c_m^* (\varphi) + c_i^* (\varphi)\right)^{1/\mu} \\
&\leq \frac{\tau c_m^* (\varphi)}{\varphi} + f_m + \frac{c_i^* (\varphi)}{w\varphi} - \lambda_{m,i} \left(c_m^* (\varphi) + c_i^* (\varphi)\right)^{1/\mu} \\
&< \frac{\tau c_m^* (\varphi)}{\varphi} + f_m + \frac{c_i^* (\varphi)}{w\varphi} + \frac{f_i}{w} - \lambda_{m,i} \left(c_m^* (\varphi) + c_i^* (\varphi)\right)^{1/\mu} \\
&= \mathfrak{L}_{m,i}^* \left(c_m^* (\varphi), c_i^* (\varphi), \varphi\right),
\end{aligned}$$

which contradicts the assumption in (A.1). The proof in the other scenario is analogous and omitted here.

A.1.3 Foreign's Offer Curve (Section 1.3.2)

The full problem of maximizing Home's consumption of foreign goods, $C_{m,i}$, conditional on its own aggregate quantities C_d, $C_{m,i}^*$ and C_n, is

$$C_{m,i}^{1/\mu} \left(C_d, C_{m,i}^*, C_n\right) \equiv \max_{c_{d^*}, c_m, c_i, p_d^*, p_m, p_i, P_{m,i}^*, P_d^*, C_d^*, C_n^*, N^*} N^* \int_\Phi \left(c_m (\varphi) + c_i (\varphi)\right)^{1/\mu} dG (\varphi),$$

subject to

$$\begin{aligned}
p_d^* (\varphi) &= \frac{\mu}{\varphi}, \text{ if } \frac{(\mu - 1)\, c_d^* (\varphi)}{\varphi} \geq f_d, \\
&= \infty, \text{ otherwise,}
\end{aligned} \tag{A.2}$$

$$P_d^{*1-\sigma} = N^* \int_\Phi p_d^* (\varphi)^{1-\sigma} dG (\varphi), \tag{A.3}$$

$$\begin{aligned}
c_d^* (\varphi) &= \left[p_d^* (\varphi) / P_d^*\right]^{-\sigma} C_d^*, \text{ if } \frac{(\mu - 1)\, c_d^* (\varphi)}{\varphi} \geq f_d, \\
&= 0, \text{ otherwise,}
\end{aligned} \tag{A.4}$$

$$C_{m,i}^*, C_d^* \in \arg\max_{C_{m,i}^*, C_d^*} \left\{U \left(C^*, C_n\right) | C_{m,i}^* P_{m,i}^* + C_d^* P_d^* + C_n^* = L^*\right\}, \tag{A.5}$$

where

$$C^{*1/\mu} = C_d^{*1/\mu} + C_{m,i}^{*1/\mu},$$

$$f^E = \int_\Phi \sum_{j=d^*,m,i} \pi_j^* \left(c_j(\varphi), \varphi, w\right) dG(\varphi), \tag{A.6}$$

$$L^* = N^* \int_\Phi \sum_{j=d^*,m} l_j \left(c_j(\varphi), \varphi\right) dG(\varphi) + N\left(C_d, C_{m,i}, \lambda\right) \int_\Phi l_i^* \left(c_i^*(\varphi), \varphi\right) dG(\varphi) + L_n^* \tag{A.7}$$

The first three constraints (A.2)-(A.4) allow us to express foreign micro quantities and prices, $c_d^*(\varphi)$ and $p_d^*(\varphi)$, as well as the aggregate price, P_d^*, as functions of C_d^* and N^*. It is then possible to express total profits and total employment associated with local sales of foreign firms, Π_d^* and L_d^*, as functions of C_d^* and N^*:

$$\Pi_d^*\left(C_d^*, N^*\right) = N^* \int_\Phi \pi_d^*\left(c_d^*\left(\varphi | C_d^*, N^*\right), \varphi\right),$$

$$L_d^*\left(C_d^*, N^*\right) = N^* \int_\Phi l_d^*\left(c_d^*\left(\varphi | C_d^*, N^*\right), \varphi\right) dG(\varphi).$$

Next, foreign utility maximization (A.5) implies the following first order conditions:

$$P_{m,i}^* = P_d^*\left(C_d^*, N^*\right)\left(\frac{C_{m,i}^*}{C_d^*}\right)^{-1/\sigma},$$

$$\frac{U_C}{U_{C_n}} = P^* = P_d^*\left(C_d^*, N^*\right)\left(1 + \left(\frac{C_{m,i}^*}{C_d^*}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{1-\sigma}}.$$

Since $P^{*1-\sigma} = P_d^{*1-\sigma} + P_{m,i}^{*1-\sigma}$, this can be finally expressed as:

$$\begin{aligned} L^* &= P_d^*\left(C_d^*, C_{m,i}^*\right)\left[C_d^* + C_{m,i}^*\left(\frac{C_{m,i}^*}{C_d^*}\right)^{-1/\sigma}\right] + C_n^*, \\ \frac{U_C}{U_{C_n}}\left(C_d^*, C_{m,i}^*, C_n^*\right) &= P_d^*\left(C_d^*, N^*\right)\left(1 + \left(\frac{C_{m,i}^*}{C_d^*}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{1-\sigma}}. \end{aligned}$$

The corresponding Lagrangians are

$$\begin{aligned}
\mathfrak{L}_m(\varphi) &= 0, \text{ if } \varphi \leq \varphi_m^c \\
&= \left[\frac{f_m}{(\mu-1)\tau}\right]^{\frac{\sigma-1}{\sigma}} \varphi^{\frac{\sigma-1}{\sigma}} - \kappa\frac{f_m}{(\mu-1)}, \text{ if } \varphi \in [\varphi_m^c, \varphi_m^u] \\
&= \frac{1}{\sigma}(\kappa\mu\tau)^{1-\sigma}\varphi^{\sigma-1}, \text{ if } \varphi \geq \varphi_m^u,
\end{aligned}$$

$$\begin{aligned}
\mathfrak{L}_i(\varphi) &= 0, \text{ if } \varphi \leq \varphi_i^c \\
&= \left[\frac{f_i}{(\mu-1)w}\right]^{\frac{\sigma-1}{\sigma}} \varphi^{\frac{\sigma-1}{\sigma}} - \kappa'\frac{f_i}{\mu-1}, \text{ if } \varphi \in [\varphi_i^c, \varphi_i^u] \\
&= \frac{1}{\sigma}(\kappa'\mu w)^{1-\sigma}\varphi^{\sigma-1}, \text{ if } \varphi \geq \varphi_i^u,
\end{aligned}$$

and

$$\begin{aligned}
\mathfrak{L}_{m,i}(\varphi) &= 0, \text{ if } \varphi \leq \varphi_m^0, \\
&= \mathfrak{L}_m(\varphi), \text{ if } \varphi \in \Phi_m \equiv \left[\varphi_m^c, \varphi'\right], \\
&= \mathfrak{L}_i(\varphi), \text{ if } \varphi \in \Phi_i \equiv [\varphi', \infty).
\end{aligned}$$

A.1.4 Positive Discrimination

In this section, I try to establish that $\lambda_E > 0$. Let me first establish $\lambda_E \geq 0$, which is equivalent to showing that one can relax constraint (1.23) into

$$N^* f^E \geq \Pi_d^*(C_d^*, N^*) + N^* \int_\Phi \sum_{j=m,i} \pi_j(c_j(\varphi), \varphi, w)\, dG(\varphi). \qquad \text{(A.8)}$$

To do so, it is convenient to again separate (1.20) into an inner problem that takes C_d^*, $C_{m,i}$, L_n and L_n^* as given and maximizes over c_m and c_i and an outer problem that maximizes over C_d^*, $C_{m,i}$, L_n and L_n^*. If (A.8) is slack, the inner problem becomes

$$C_{m,i}^{1/\mu}\left(C_d^*, C_{m,i}, L_n, L\right) = \max_{c_m . c_i} N^* \int_\Phi \left(c_m\left(\varphi\right) + c_i\left(\varphi\right)\right)^{1/\mu} dG\left(\varphi\right)$$

$$L^* = N^* f^E + L_d^*\left(C_d^*, N^*\right) + N^* \int_\Phi l_m\left(c_j\left(\varphi\right), \varphi\right) dG\left(\varphi\right),$$
$$+ N\left(C_d, C_{m,i}^*\right) \int_\Phi l_i^*\left(c_i^*\left(\varphi\right), \varphi\right) dG\left(\varphi\right) + L_n^*$$

$$L = N\left(C_d, C_{m,i}^*\right) f^E + L_d\left(C_d, C_{m,i}^*\right) + L_m^*\left(C_d, C_{m,i}^*\right),$$
$$+ N^* \int l_i\left(c_i\left(\varphi\right), \varphi\right) dG\left(\varphi\right) + L_n$$

$$\begin{aligned} \pi_m &= \frac{\mu}{\varphi} c_m\left(\varphi\right) - l_m\left(c_m\left(\varphi\right), \varphi\right) \geq 0, \\ \pi_i &= w\left[\frac{\mu}{\varphi} c_i\left(\varphi\right) - l_i\left(c_i\left(\varphi\right), \varphi\right)\right] \geq 0. \end{aligned}$$

which is just the dual of minimizing the foreign cost of producing for Home consumers. Hence, the optimal quantities c_m and c_i must satisfy the same conditions as in Section 3.1. Differentiating the previous expression, by the Envelope Theorem,

$$\frac{\partial C_{m,i}^{1/\mu}}{\partial N^*} = -\lambda_L^*\left[f^E - \Pi_d^*\left(C_d^*, N^*\right)/N^* + \int_\Phi \sum_{j=m,i} \pi_j\left(c_j\left(\varphi\right), \varphi, w\right) dG\left(\varphi\right)\right] < 0.$$

Now consider the outer problem of maximizing the previous value function with respect to C_d^*, N^*, $C_{m,i}$, L_n and L_n^* subject to constraints (1.21) through (1.26). Checking the monotonicity of the functions, at a solution of Home's relaxed problem, constraint (1.21) and can be relaxed as:

$$L^* \leq P_d^* \left(C_d^*, N^*\right) \left[C_d^* + C_{m,i}^* \left(\frac{C_{m,i}^*}{C_d^*}\right)^{-1/\sigma}\right] + C_n^*,$$

$$\left(1 + \left(\frac{C_{m,i}^*}{C_d^*}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{1}{\sigma-1}} \frac{U_C}{U_{C_n}} \left(C_d^*, C_{m,i}^*, C_n^*\right) \leq P_d^* \left(C_d^*, N^*\right).$$

If this constraint was slack, Home could reduce $C_{m,i}^*$ and increase C_d, while still satisfying (1.31), and hence increase the utility function in Home's macro problem. Since that price of foreign varieties, $P_d^* \left(C_d^*, N^*\right)$, must be decreasing in N^* , we can then rearrange these constraint as an upper-bound on the measure of foreign entrants,

$$N^* \leq N^* \left(C_d^*, C_{m,i}^*, C_n^*\right).$$

The rest of the proof is identical to Costinot et al. (2016) and omitted.

Appendix B

Appendix for An Elementary Theory of Global Supply Chains

B.1 Proofs (I): Free Trade Equilibrium

Proof of Proposition 1. As mentioned in the main text, if a firm in country c produces intermediate good s, then it necessarily produces a measure $\Delta > 0$ of intermediate goods around that stage. Specifically, there exists an $s_\Delta < s \leq s_\Delta + \Delta$ such that $Q_c(s') > 0$ for all $s' \in (s_\Delta, s_\Delta + \Delta]$. Throughout this proof we define $\Delta(s) \equiv (s_\Delta, s_\Delta + \Delta)$, for some s_Δ satisfying the previous conditions. The local properties that follow do not depend on which exact s_Δ we choose. We proceed in four steps.

Step 1: $p(\cdot)$ *is continuous.*

Consider a stage $s_0 \in (0, S]$. By the goods market clearing condition, we know that there must be at least one country, call it c_0, producing intermediate good s_0, which requires $Q_{c_0}(s) > 0$ for all $s \in \Delta(s_0)$. By condition (2.2), we therefore have

$$p(s) = (1 + \lambda_{c_0} ds)\, p(s - ds) + w_{c_0} ds, \text{ for all } s \in \Delta(s_0),$$

which implies

$$\frac{dp(s)}{ds} = \lambda_{c_0} p(s) + w_{c_0}, \text{ for all } s \in \Delta(s_0). \tag{B.1}$$

Thus $p(\cdot)$ is piecewise differentiable over $(0, S]$, and in turn, continuous almost everywhere. To conclude let us show that p cannot have any jump. We proceed by contradiction. Suppose that there exists $s_0 \in (0, S)$ such that $p(s_0^+) \neq p(s_0^-)$. Then there must exist $c_0 \neq c_1$ such that firms in country c_0 produce intermediate good s_0 and sell it to firms in country c_1. If $p(s_0^+) > p(s_0^-)$, then $p(s_0^+) > (1 + \lambda_{c_0} ds) p(s_0 - ds) + w_{c_0} ds$, which violates condition (2.2). If instead $p(s_0^+) < p(s_0^-)$, then $p(s_0 + ds) > (1 + \lambda_{c_1} ds) p(s_0^-) + w_{c_1} ds$, which also violates condition (2.2).

Step 2: *If $s_2 > s_1$, then $p(s_2) > p(s_1)$.*

By Step 1 we know that $p(\cdot)$ is continuous. By Step 1 we also know that for any stage $s \in (0, S]$, there is a small neighborhood $\Delta(s)$ of s such that $p(\cdot)$ is a solution of Equation $(B.1)$. Thus $p'(s) > 0$ for almost all $s \in (0, S]$. The fact that $p(\cdot)$ is strictly increasing directly derives from these two observations.

Step 3: *If $c_2 > c_1$, then $w_{c_2} > w_{c_1}$.*

Consider two countries c_2 and c_1 with $c_2 > c_1$. Factor market clearing requires country c_1 to produce at least one intermediate good in $(0, S]$, call it s_1. By assumption, this requires $Q_{c_1}(s) > 0$ for all $s \in \Delta(s_1)$. Thus condition (2.2) implies

$$p(s_1) = (1 + \lambda_{c_1} ds) p(s_1 - ds) + w_{c_1} ds, \tag{B.2}$$

$$p(s_1) \leq (1 + \lambda_{c_2} ds) p(s_1 - ds) + w_{c_2} ds. \tag{B.3}$$

Since $\lambda_{c_2} < \lambda_{c_1}$, equation $(B.2)$ and inequality $(B.3)$ imply $w_{c_2} > w_{c_1}$.

Step 4: *If $c_2 > c_1$ and $Q_{c_1}(s_1) > 0$, then $Q_{c_2}(s) = 0$ for all $s < s_1$.*

We proceed by contradiction. Suppose that there exist two countries, $c_2 > c_1$, and two intermediate goods, $s_1 > s_2 > 0$, such that c_1 produces s_1 and c_2 produces s_2. By assumption, this requires $Q_{c_1}(s) > 0$ for all $s \in \Delta(s_1)$ and $Q_{c_2}(s) > 0$ for all

$s \in \Delta(s_2)$. Thus condition (2.2) implies

$$\begin{aligned} p(s_1) &= (1+\lambda_{c_1} ds)\, p(s_1 - ds) + w_{c_1} ds, \\ p(s_2) &= (1+\lambda_{c_2} ds)\, p(s_2 - ds) + w_{c_2} ds, \\ p(s_1) &\leq (1+\lambda_{c_2} ds)\, p(s_1 - ds) + w_{c_2} ds, \\ p(s_2) &\leq (1+\lambda_{c_1} ds)\, p(s_2 - ds) + w_{c_1} ds. \end{aligned}$$

Combining the four previous expressions, we get

$$\begin{aligned} &[(1+\lambda_{c_2} ds)\, p(s_1 - ds) + w_{c_2} ds]\, [(1+\lambda_{c_1} ds)\, p(s_2 - ds) + w_{c_1} ds] \\ &\qquad \geq [(1+\lambda_{c_1} ds)\, p(s_1 - ds) + w_{c_1} ds]\, [(1+\lambda_{c_2} ds)\, p(s_2 - ds) + w_{c_2} ds], \end{aligned}$$

which can be rearranged as

$$(1+\lambda_{c_2} ds)\, [p(s_1 - ds) - p(s_2 - ds)]\, w_{c_1} \geq (1+\lambda_{c_1} ds)\, [p(s_1 - ds) - p(s_2 - ds)]\, w_{c_2}.$$

By Step 2, we know that $p(s_1 - ds) - p(s_2 - ds) > 0$. Thus the previous inequality implies

$$(1+\lambda_{c_2} ds)\, w_{c_1} \geq (1+\lambda_{c_1} ds)\, w_{c_2}. \tag{B.4}$$

Since $\lambda_{c_2} < \lambda_{c_1}$, inequality $(B.4)$ implies $w_{c_1} > w_{c_2}$, which contradicts Step 3.

To conclude the proof of Proposition 1, let us define $S_c \equiv \sup \{s \in \mathcal{S} | Q_c(s) > 0\}$ for all $c \in \mathcal{C}$. By Step 4, we must have $S_0 \equiv 0 < S_1 < ... < S_C = S$, and for all $s \in \mathcal{S}$ and $c \in \mathcal{C}$, $Q_c(s) > 0$ if $S_{c-1} < s < S_c$ and $Q_c(s) = 0$ if $s < S_{c-1}$ or $s > S_c$. Since $Q_c(s) > 0$ requires $Q_c(s') > 0$ for all $s' \in (s - ds, s]$, we must also have $Q_c(S_c) > 0$ and $Q_c(S_{c-1}) = 0$ for all $c \in \mathcal{C}$. Thus $Q_c(s) > 0$ if and only if $s \in (S_{c-1}, S_c]$. Finally, by the goods market clearing condition, country C must produce stage S, so that $S_C = S$. $\square$

Proof of Lemma 1. We first consider equation (2.6). Proposition 1 and equation (2.3) imply

$$Q_c(s_2) - Q_c(s_1) = -\lambda_c \int_{s_1}^{s_2} Q_c(s)\, ds, \text{ for all } s_1, s_2 \in (S_{c-1}, S_c]. \tag{B.5}$$

Taking the derivative of the previous expression with respect to s_2, we get

$$\frac{dQ_c(s)}{ds} = -\lambda_c Q_c(s), \text{ for all } s \in (S_{c-1}, S_c).$$

The solution of the previous differential equation must satisfy

$$Q_c(S_c) = e^{-\lambda_c(S_c - S_{c-1})} \lim_{s \to S_{c-1}^+} Q_c(s). \tag{B.6}$$

Proposition 1 and equation (2.3) also imply

$$Q_c(S_{c-1} + ds) - Q_{c-1}(S_{c-1} - ds) = -\left[\lambda_c \lim_{s \to S_{c-1}^+} Q_c(s) + \lambda_{c-1} Q_{c-1}(S_{c-1} - ds)\right] ds.$$

Since ds is infinitesimal, this further implies

$$\lim_{s \to S_{c-1}^+} Q_c(s) = \lim_{s \to S_{c-1}^-} Q_{c-1}(s) = Q_{c-1}(S_{c-1}). \tag{B.7}$$

Equation (2.6) derives from equations $(B.6)$ and $(B.7)$ and the definition of $Q_c \equiv Q_c(S_c)$.

Let us now turn to equation (2.5). By Proposition 1 and equation (2.4), we know that

$$\int_{S_{c-1}}^{S_c} Q_c(s)\, ds = L_c, \text{ for all } c \in \mathcal{C}. \tag{B.8}$$

By equations $(B.5)$ and $(B.7)$, we also know that

$$\int_{S_{c-1}}^{S_c} Q_c(s)\, ds = \frac{1}{\lambda_c}\left[Q_{c-1}(S_{c-1}) - Q_c(S_c)\right]. \tag{B.9}$$

Equations $(B.8)$ and $(B.9)$ imply

$$L_c = \frac{1}{\lambda_c}\left[Q_{c-1}\left(S_{c-1}\right) - Q_c\left(S_c\right)\right], \text{ for all } c \in \mathcal{C}. \tag{B.10}$$

Equation (2.5) derives from equations (2.6) and $(B.10)$ and the definition of $Q_c \equiv Q_c\left(S_c\right)$. The boundary conditions $S_0 = 0$ and $S_C = S$ have already been established in the proof of Proposition 1. □

Proof of Lemma 2. We first consider equation (2.7). Proposition 1 and condition (2.2) imply

$$\begin{aligned} p\left(S_c + ds\right) - \left(1 + \lambda_{c+1} ds\right) p\left(S_c\right) - w_{c+1} ds &\geq p\left(S_c + ds\right) - \left(1 + \lambda_c ds\right) p\left(S_c\right) - w_c ds, \\ p\left(S_c\right) - \left(1 + \lambda_c ds\right) p\left(S_c - ds\right) - w_c ds &\geq p\left(S_c\right) - \left(1 + \lambda_{c+1} ds\right) p\left(S_c - ds\right) - w_{c+1} ds, \end{aligned}$$

for any $c < C$. After simplifications, the two previous inequalities can be rearranged as

$$\left(\lambda_c - \lambda_{c+1}\right) p\left(S_c\right) \geq w_{c+1} - w_c \geq \left(\lambda_c - \lambda_{c+1}\right) p\left(S_c - ds\right).$$

Since p is continuous and ds is infinitesimal, we get

$$w_{c+1} - w_c = \left(\lambda_c - \lambda_{c+1}\right) p\left(S_c\right), \text{ for all } c < C.$$

which is equivalent to equation (2.7) by the definition of $p_c \equiv p\left(S_c\right)$.

Let us now turn to equation (2.8). Proposition 1 and condition (2.2) imply

$$p\left(s + ds\right) = \left(1 + \lambda_c ds\right) p\left(s\right) + w_c ds, \text{ for all } s \in \left(S_{c-1}, S_c\right],$$

which further implies

$$\frac{dp\left(s\right)}{ds} = \lambda_c p\left(s\right) + w_c, \text{ for all } s \in \left(S_{c-1}, S_c\right).$$

The solution of the previous differential equation must satisfy

$$p\left(S_c\right) = e^{\lambda_c\left(S_c - S_{c-1}\right)} \lim_{s \to S_{c-1}^{+}} p\left(S_{c-1}\right) + \left[e^{\lambda_c\left(S_c - S_{c-1}\right)} - 1\right]\left(w_c / \lambda_c\right),$$

which is equivalent to equation (2.8) by the continuity of $p\left(\cdot\right)$ and the definitions of $N_c \equiv S_c - S_{c-1}$ and $p_c \equiv p\left(S_c\right)$. The boundary conditions derive from the fact that $p_0 = p\left(S_0\right) = p(0) = 0$ and $p_C = p\left(S_C\right) = p\left(S\right) = 1$. □

Proof of Proposition 2. We proceed in four steps.

Step 1: $(S_0, ..., S_C)$ *and* $(Q_0, ..., Q_C)$ *satisfy equations* (2.5) *and* (2.6) *if and only if*

$$\begin{aligned} S_c &= S_0 + \textstyle\sum_{c'=1}^{c} \left(\frac{1}{\lambda_{c'}}\right) \ln \left[\frac{Q_0 - \sum_{c''=1}^{c'-1} \lambda_{c''} L_{c''}}{Q_0 - \sum_{c''=1}^{c'} \lambda_{c''} L_{c''}}\right], \textit{for all } c \in \mathcal{C}, \qquad &(\text{B.11}) \\ Q_c &= Q_0 - \textstyle\sum_{c'=1}^{c} \lambda_{c'} L_{c'}, \textit{for all } c \in \mathcal{C}. &(\text{B.12}) \end{aligned}$$

Let us first show that if $(S_0, ..., S_C)$ and $(Q_0, ..., Q_C)$ satisfy equations (2.5) and (2.6), then they satisfy equations $(B.11)$ and $(B.12)$. Consider equation $(B.12)$. Equations (2.5) and (2.6) imply

$$Q_c = Q_{c-1} - \lambda_c L_c, \text{ for all } c \in \mathcal{C},$$

By iteration we therefore have

$$Q_c = Q_0 - \textstyle\sum_{c'=1}^{c} \lambda_{c'} L_{c'}, \text{ for all } c \in \mathcal{C}.$$

Now consider equation $(B.11)$. Starting from equation (2.5) and iterating we get

$$S_c = S_0 - \textstyle\sum_{c'=1}^{c} \left(\frac{1}{\lambda_{c'}}\right) \ln\left(1 - \frac{\lambda_{c'} L_{c'}}{Q_{c'-1}}\right), \text{ for all } c \in \mathcal{C}.$$

Equation $(B.11)$ directly derives from the previous expression and equation $(B.12)$. It is a matter of simple algebra to check that if $(S_0, ..., S_C)$ and $(Q_0, ..., Q_C)$ satisfy equations $(B.11)$ and $(B.12)$, then they satisfy equations (2.5) and (2.6).

Step 2: *There exists a unique pair of vectors* $(S_0, ..., S_C)$ *and* $(Q_0, ..., Q_C)$ *satisfying equations* (2.5) *and* (2.6) *and the boundary conditions:* $S_0 = 0$ *and* $S_C = S$.

Let $\underline{Q}_0 \equiv \sum_{c=1}^{C} \lambda_c L_c$. By Step 1, if $Q_0 \leq \underline{Q}_0$, then there does not exist a pair of vectors $(S_0, ..., S_C)$ and $(Q_0, ..., Q_C)$ that satisfy equations (2.5) and (2.6). Otherwise $(Q_0, ..., Q_C)$ and $(S_0, ..., S_C)$ would also satisfy equations $(B.11)$ and $(B.12)$, which cannot be the case if $Q_0 \leq \underline{Q}_0$. Now consider $Q_0 > \underline{Q}_0$. From equation $(B.11)$, it is easy to check that $\partial S_C / \partial Q_0 < 0$ for all $Q_0 > \underline{Q}_0$; $\lim_{Q_0 \to \underline{Q}_0^+} S_C = +\infty$; and $\lim_{Q_0 \to +\infty} S_C = S_0$. Thus conditional on having set $S_0 = 0$, there exists a unique $Q_0 > \underline{Q}_0$ such that $(S_0, ..., S_C)$ and $(Q_0, ..., Q_C)$ satisfy equations $(B.11)$ and $(B.12)$ and $S_C = S$. Step 2 derives from Step 1 and the previous observation.

Step 3: *For any* $(N_1, ..., N_C)$*, there exists a unique pair of vectors* $(w_1, ..., w_C)$ *and* $(p_0, ..., p_C)$ *satisfying equations* (2.7) *and* (2.8) *and the boundary conditions:* $p_0 = 0$ *and* $p_C = 1$.

For any $(N_1, ..., N_C)$, w_1, and p_0, there trivially exists a unique pair of vectors $(w_2, ..., w_C)$ and $(p_1, ..., p_C)$ that satisfy equations (2.7) and (2.8). Thus taking $(N_1, ..., N_C)$ as given and having set $p_0 = 0$, we only need to check that there exists a unique w_1 such that $p_C = 1$. To do so, we first establish that p_C is strictly increasing in w_1. We proceed by iteration. By equation (2.8), we know that p_1 is strictly increasing in w_1. Thus by equation (2.7), w_2 must be strictly increasing in w_1 as well. Now suppose that p_{c-1} and w_c are strictly increasing in w_1 for $c < C$. Then p_c must be strictly increasing in w_1, by equation (2.8), and w_{c+1} must be strictly increasing in w_1, by equation (2.7). At this point we have established, by iteration, that p_{C-1} and w_C are strictly increasing in w_1. Combining this observation with equation (2.8), we obtain that p_C is strictly increasing in w_1. To conclude, let us note that, by equations (2.7) and (2.8), we also have $\lim_{w_1 \to 0} p_C = 0$ and $\lim_{w_1 \to +\infty} p_C = +\infty$. Since p_C is strictly increasing in w_1, there therefore exists a unique w_1 such that $p_C = 1$.

Steps 1-3 imply the existence and uniqueness of $(S_0, ..., S_C)$, $(Q_0, ..., Q_C)$, $(w_1, ..., w_C)$, and $(p_0, ..., p_C)$ that satisfy equations (2.5)-(2.8) with boundary conditions $S_0 = 0$,

$S_C = S$, $p_0 = 0$, and $p_C = 1$. Now consider the following output levels and intermediate good prices

$$
\begin{aligned}
Q_c(s) &= e^{-\lambda_c(s-S_{c-1})} Q_{c-1}, \text{ for all } s \in (S_{c-1}, S_c], \\
p(s) &= e^{\lambda_c(s-S_{c-1})} p_{c-1} + \left[e^{\lambda_c(s-S_{c-1})} - 1\right](w_c/\lambda_c) \text{ for all } s \in (S_{c-1}, S_c].
\end{aligned}
$$

By construction, $[Q_1(\cdot), ..., Q_C(\cdot)]$, $(w_1, ..., w_C)$, and $p(\cdot)$ satisfy conditions (2.2)-(2.4). Thus a free trade equilibrium exists. Since $(S_0, ..., S_C)$, $(Q_0, ..., Q_C)$, $(w_1, ..., w_C)$, and $(p_0, ..., p_C)$ are unique, the free trade equilibrium is unique as well by Proposition 1 and Lemmas 1 and 2. □

B.2 Proofs (II): Global Technological Change

Proof of Proposition 3. We decompose the proof of Proposition 3 into three parts. First, we show that an increase in S increases the measure of stages N_c performed in all countries. Second, we show that an increase in S leads all countries to move up the supply chain. Third, we show that an increase in S increases inequality between countries around the world.

Part I: *If $S' > S$, then $N'_c > N_c$ for all $c \in \mathcal{C}$.*

We first show that $N'_1 > N_1$ by contradiction. Suppose that $N'_1 \leq N_1$. By equation (2.5), equation (2.6) and the definition of $N_c \equiv S_c - S_{c-1}$, we know that

$$
N_c = -\left(\frac{1}{\lambda_c}\right) \ln\left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right)\left(e^{\lambda_{c-1} N_{c-1}} - 1\right)\right], \text{ for all } c > 1. \tag{B.13}
$$

According to equation $(B.13)$, we have $\partial N_c / \partial N_{c-1} > 0$. Thus by iteration, $N'_1 \leq N_1$ implies $N'_c \leq N_c$ for all countries $c \in \mathcal{C}$. This further implies $\sum_{c=1}^{C} N'_c = S'_C - S'_0 \leq S_C - S_0 = \sum_{c=1}^{C} N_c$, which contradicts $S'_C - S'_0 > S_C - S_0$ by Lemma 1. Starting from $N'_1 > N_1$, we can then use equation $(B.13)$ again to show by iteration that $N'_c > N_c$ for all $c \in \mathcal{C}$.

Part II: *If $S' > S$, then $S'_c > S_c$ for all $c \in \mathcal{C}$.*

We proceed by iteration. By Lemma 1 and Part I, we know that $S_1' = N_1' > S_1 = N_1$. Thus $S_c' > S_c$ is satisfied for $c = 1$. Let us now show that if $S_c' > S_c$ for $1 \leq c < C$, then $S_{c+1}' > S_{c+1}$. By definition, we know that $S_{c+1}' = S_c' + N_{c+1}'$ and $S_{c+1} = S_c + N_{c+1}$. By Part I, we also know that $N_{c+1}' > N_{c+1}$. Thus $S_c' > S_c$ implies $S_{c+1}' > S_{c+1}$.

Part III: *If $S' > S$, then $(w_{c+1}/w_c)' > (w_{c+1}/w_c)$ for all $c < C$.*

The proof of Part III proceeds in two steps.

Step 1: *If $N_1' > N_1$, then $(w_2/w_1)' > (w_2/w_1)$.*

Since $p_0 = 0$, we know from equations (2.7) and (2.8) that

$$\frac{w_2}{w_1} = 1 + \frac{1}{\lambda_1} (\lambda_1 - \lambda_2) \left(e^{\lambda_1 N_1} - 1 \right). \tag{B.14}$$

Combining equation (B.14) and $N_1' > N_1$, we obtain $(w_2/w_1)' > (w_2/w_1)$. This completes the proof of Step 1.

Step 2: *For any country $1 < c < C$, if $N_c' > N_c$ and $(w_c/w_{c-1})' \geq (w_c/w_{c-1})$, then $(w_{c+1}/w_c)' > (w_{c+1}/w_c)$.*

Consider a country $1 < c < C$. Equations (2.7) and (2.8) imply

$$\frac{w_{c+1}}{w_c} = 1 + (\lambda_c - \lambda_{c+1}) \left[\left(\frac{e^{\lambda_c N_c} - 1}{\lambda_c} \right) + e^{\lambda_c N_c} \left(\frac{w_{c-1}}{w_c} \right) \left(\frac{p_{c-1}}{w_{c-1}} \right) \right]. \tag{B.15}$$

By equation (2.7), we also know that

$$\frac{w_c}{w_{c-1}} = 1 + (\lambda_{c-1} - \lambda_c) \left(\frac{p_{c-1}}{w_{c-1}} \right), \tag{B.16}$$

which further implies

$$\left(\frac{w_{c-1}}{w_c} \right) \left(\frac{p_{c-1}}{w_{c-1}} \right) = \frac{(p_{c-1}/w_{c-1})}{1 + (\lambda_{c-1} - \lambda_c) (p_{c-1}/w_{c-1})}. \tag{B.17}$$

Since $(w_c/w_{c-1})' \geq (w_c/w_{c-1})$ and $\lambda_{c-1} > \lambda_c$, equation ($B$.16) immediately implies

$$\left(\frac{p_{c-1}}{w_{c-1}}\right)' \geq \left(\frac{p_{c-1}}{w_{c-1}}\right).$$

Combining this observation with equation $(B.17)$—the right-hand side of which is increasing in (p_{c-1}/w_{c-1})—we obtain

$$\left(\frac{w_{c-1}}{w_c}\right)' \left(\frac{p_{c-1}}{w_{c-1}}\right)' \geq \left(\frac{w_{c-1}}{w_c}\right) \left(\frac{p_{c-1}}{w_{c-1}}\right). \tag{B.18}$$

To conclude, note that $N_c' > N_c$ implies $e^{\lambda_c N_c'} > e^{\lambda_c N_c}$. Thus equation $(B.15)$ and inequality $(B.18)$ imply $(w_{c+1}/w_c)' > (w_{c+1}/w_c)$. This completes the proof of Step 2. Combining Part I with Steps 1 and 2, it is then easy to establish Part III by iteration. □

Proof of Proposition 4. We decompose the proof of Proposition 4 into three parts. First, we show that a decrease in β increases the measure of stages N_c performed by all countries $c < c_1$ and decreases the measure of stages N_c performed by all countries $c \geq c_1$, with $1 < c_1 \leq C$. Second, we show that a decrease in β leads all countries to move up. Third, we show that a decrease in β decreases inequality between countries around the world.

Part I: *If $\beta' < \beta$, then there exists $1 < c_1 \leq C$ such that $N_c' > N_c$ if $c < c_1$, $N_{c_1}' \leq N_{c_1}$, and $N_c' < N_c$ if $c > c_1$.*

Equation (2.5), equation (2.6), and the definition of N_c imply

$$N_c = -\left(\frac{1}{\beta\lambda_c}\right) \ln\left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right)\left(e^{\beta\lambda_{c-1}N_{c-1}} - 1\right)\right], \text{ for all } c > 1. \tag{B.19}$$

After some algebra, one can check that $\partial N_c/\partial N_{c-1} > 0$ and $\partial N_c/\partial\beta > 0$. Since $\beta' < \beta$, equation $(B.19)$ implies that if $N_{c-1}' \leq N_{c-1}$ for $c > 1$, then $N_c' < N_c$. This further implies the existence of $1 \leq c_1 \leq C+1$ such that $N_c' > N_c$ if $c < c_1$, $N_{c_1}' \leq N_{c_1}$, and $N_c' < N_c$ if $c > c_1$. To conclude the proof of Part I, note that if $c_1 = 1$, then $\sum_{c=1}^{C} N_c' = S_C' - S_0' < S_C - S_0 = \sum_{c=1}^{C} N_c$, which contradicts $S_C' - S_0' = S_C - S_0$ by Lemma 1. Similarly, if $c_1 = C+1$, then $\sum_{c=1}^{C} N_c' = S_C' - S_0' > S_C - S_0 = \sum_{c=1}^{C} N_c$, which also contradicts $S'_C - S'_0 = S_C - S_0$ by Lemma 1.

Part II: *If $\beta' < \beta$, then $S'_c > S_c$ for all $c \in \{1, ..., C-1\}$.*

We first show by iteration that $S'_c > S_c$ if $c < c_1$. By Lemma 1 and Part I, we know that $S'_1 = N'_1 > S_1 = N_1$. Thus $S'_c > S_c$ is satisfied for $c = 1$. Let us now show that if $S'_c > S_c$ for $1 \leq c < c_1 - 1$, then $S'_{c+1} > S_{c+1}$. By definition, we know that $S'_{c+1} = S'_c + N'_{c+1}$ and $S_{c+1} = S_c + N_{c+1}$. By Part I, we also know that $N'_{c+1} > N_{c+1}$ if $c < c_1 - 1$. Thus $S'_c > S_c$ for $1 \leq c < c_1 - 1$ implies $S'_{c+1} > S_{c+1}$. This establishes that $S'_c > S_c$ if $c < c_1$. If $c_1 = C$, our proof is complete. If instead $c_1 < C$, we still need to show that $S'_c > S_c$ if $C - 1 \geq c \geq c_1$. We again proceed by iteration. For country $C - 1$, we know from Part I that $S'_{C-1} = S - N'_C > S - N_C = S_{C-1}$. Thus $S'_c > S_c$ is satisfied for $c = C - 1$. Now suppose that $S'_c > S_c$ for $c > c_1 + 1$. Then $S'_{c-1} = S'_c - N'_c > S_c - N_c = S_{c-1}$, since $S'_c > S_c$ by assumption and $N'_c < N_c$ by Part I. This establishes that $S'_c > S_c$ for all $C - 1 \geq c \geq c_1$, which completes the proof of Part II.

Part III: *If $\beta' < \beta$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < C$.*

Throughout this part of the proof, we let $\widetilde{N}_c \equiv \beta N_c$ and $\widetilde{p}_c \equiv \beta p_c$. The proof of Part III proceeds in two steps.

Step 1: *If $\beta' < \beta$, then $\widetilde{N}'_c < \widetilde{N}_c$ for all countries $c \in \mathcal{C}$.*

The proof is similar to Part I of the proof of Proposition 3. We first show that $\widetilde{N}'_1 < \widetilde{N}_1$ by contradiction. Suppose that $\widetilde{N}'_1 \geq \widetilde{N}_1$. By equation (2.5), equation (2.6) and the definition of $\widetilde{N}_c$, we know that

$$\widetilde{N}_c = -\left(\frac{1}{\lambda_c}\right) \ln \left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right) \left(e^{\lambda_{c-1} \widetilde{N}_{c-1}} - 1\right)\right], \text{ for all } c > 1, \tag{B.20}$$

where $\partial \widetilde{N}_c / \partial \widetilde{N}_{c-1} > 0$. Thus by iteration, $\widetilde{N}'_1 \geq \widetilde{N}_1$ implies $N'_c \geq N_c$ for all countries $c \in \mathcal{C}$. This implies $\beta' \left(\sum_{c=1}^{C} N'_c\right) = \sum_{c=1}^{C} \widetilde{N}'_c \geq \sum_{c=1}^{C} \widetilde{N}_c = \beta \left(\sum_{c=1}^{C} N_c\right)$. Since $\beta' < \beta$, this further implies $\sum_{c=1}^{C} N'_c = S'_C - S'_0 > S_C - S_0 = \sum_{c=1}^{C} N_c$, which contradicts $S'_C - S'_0 = S_C - S_0$ by Lemma 1. Starting from $\widetilde{N}'_1 < \widetilde{N}_1$, we can now use equation $(B.20)$ to show by iteration that $\widetilde{N}'_c < \widetilde{N}_c$ for all $c \in \mathcal{C}$.

Step 2: *If $\widetilde{N}_c' < \widetilde{N}_c$ for all countries $c \in \mathcal{C}$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < C$.*

Equations (2.7) and (2.8) can be rearranged as

$$\begin{aligned} w_{c+1} &= w_c + (\lambda_c - \lambda_{c+1})\, \widetilde{p}_c, \text{ for all } c < C, & \text{(B.21)} \\ \widetilde{p}_c &= e^{\lambda_c \widetilde{N}_c} \widetilde{p}_{c-1} + \left(e^{\lambda_c \widetilde{N}_c} - 1\right)(w_c/\lambda_c), \text{ for all } c \in \mathcal{C}. & \text{(B.22)} \end{aligned}$$

Following the exact same strategy as in Part III of the proof of Proposition 3, it is then easy to show by iteration that $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < C$. Part II directly follows from Steps 1 and 2. □

B.3 Proofs (III): Local Technological Change

Proof of Proposition 5. We decompose the proof of Proposition 5 into three parts. First, we show that an increase in L_{c_0} increases the measure of stages performed in country c_0 and decreases the measure of stages performed in any other country. Second, we show that an increase in L_{c_0} leads all countries all countries $c < c_0$ to move down and all countries $c > c_0$ move up. Third, we show that an increase in L_{c_0} decreases inequality among countries $c \in \{1, ..., c_0\}$, increases inequality among countries $c \in \{c_0, ..., c_1\}$, and decreases inequality among countries $c \in \{c_1, ..., C\}$, with $c_1 \in \{c_0 + 1, ..., C\}$.

Part I: *If $L_{c_0}' > L_{c_0}$, then $N_{c_0}' > N_{c_0}$ and $N_c' < N_c$ for all $c \neq c_0$.*

Like in our previous proofs, we will repeatedly use the following relationship

$$N_c = -\left(\frac{1}{\lambda_c}\right) \ln\left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right)\left(e^{\lambda_{c-1} N_{c-1}} - 1\right)\right], \text{ for all } c > 1, \quad (23)$$

where $\partial N_c / \partial N_{c-1} > 0$ and $\partial N_c / \partial L_c > 0$. The proof of Part I proceeds in two steps.

Step 1: *If $L_{c_0}' > L_{c_0}$, then $N_c' < N_c$ for all $c > c_0$.*

Let us first establish that $Q_C' > Q_C$. By Proposition 1, we know that $Q_C' \equiv Q_C'(S) = \sum_{c=1}^{C} Q_c'(S)$. By the First Welfare Theorem, we also know that the allocation in a free trade equilibrium is Pareto optimal. Thus Q_C' must be the maximum output level of the final good attainable given the new resource and technological constraints, i.e.,

$$Q'_C = \underset{\widetilde{Q}_1(\cdot),...,\widetilde{Q}_C(\cdot)}{\arg\max} \; \textstyle\sum_{c=1}^{C} \widetilde{Q}_c\left(S\right),$$

subject to

$$\begin{aligned} \textstyle\sum_{c=1}^{C} \widetilde{Q}_c\left(s_2\right) - \sum_{c=1}^{C} \widetilde{Q}_c\left(s_1\right) &\leq -\textstyle\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda_c \widetilde{Q}_c\left(s\right) ds, \text{ for all } s_1 \leq s_2, && \text{(B.23)} \\ \int_0^S \widetilde{Q}_c\left(s\right) ds &\leq L'_c, \text{ for all } c \in \mathcal{C}, && \text{(B.24)} \end{aligned}$$

where $L'_{c_0} > L_{c_0}$ and $L'_c = L_c$ for all $c \neq c_0$. Now consider $\widetilde{Q}_1\left(\cdot\right), ..., \widetilde{Q}_C\left(\cdot\right)$ such that

$$\widetilde{Q}_{c_0}\left(s\right) \equiv Q_{c_0}\left(s\right) + \left(\frac{\lambda_{c_0} e^{-\lambda_{c_0} s}}{1 - e^{-\lambda_{c_0} S}}\right)\left(L'_{c_0} - L_{c_0}\right), \text{ for all } s \in \mathcal{S},$$

and

$$\widetilde{Q}_c\left(s\right) \equiv Q_c\left(s\right), \text{ for all } s \in \mathcal{S} \text{ and } c \neq c_0.$$

Since $Q_1\left(\cdot\right), ..., Q_C\left(\cdot\right)$ satisfies the initial resource and technological constraints, as described by conditions (2.3) and (2.4), $\widetilde{Q}_1\left(\cdot\right), ..., \widetilde{Q}_C\left(\cdot\right)$ must satisfy, by construction, the new resource and technological constraints, as described by conditions $(B.23)$ and $(B.24)$. Since $L'_{c_0} > L_{c_0}$, we must also have

$$\widetilde{Q}_{c_0}(S) + \widetilde{Q}_C\left(S\right) = \left(\frac{\lambda_{c_0} e^{-\lambda_{c_0} S}}{1 - e^{-\lambda_{c_0} S}}\right)\left(L'_{c_0} - L_{c_0}\right) + Q_C > Q_C.$$

Since $Q'_C \geq \widetilde{Q}_{c_0}\left(S\right) + \widetilde{Q}_C\left(S\right)$, the previous inequality implies $Q'_C > Q_C$. By equation (2.5), equation (2.6), and the definition of N_c, we also know that

$$N_C = \left(\frac{1}{\lambda_c}\right) \ln\left(1 + \frac{\lambda_C L_C}{Q_C}\right).$$

Thus if $C > c_0$, $Q'_C > Q_C$ and $L'_C = L_C$ imply $N'_C < N_C$. To conclude the proof of Step 1, note that if $N'_c < N_c$ for $c > c_0 + 1$, then $L'_{c-1} = L_{c-1}$ and equation $(B.13)$ imply $N'_{c-1} < N_{c-1}$. Thus by iteration, $N'_c < N_c$ for all $c > c_0$.

Step 2: *If $L'_{c_0} > L_{c_0}$, then $N'_c < N_c$ for all $c < c_0$.*

We first show by contradiction that if $L'_{c_0} > L_{c_0}$ and $c_0 > 1$, then $N'_1 < N_1$. Suppose that $N'_1 \geq N_1$. Since $L'_c = L_c$ for all $c < c_0$, we can use equation $(B.13)$—the fact that $\partial N_c / \partial N_{c-1} > 0$—to establish by iteration that $N'_c \geq N_c$ for all $c < c_0$. Since $L'_{c_0} > L_{c_0}$ and $L'_{c_0-1} = L_{c_0-1}$, we can further use equation $(B.13)$—the facts that $\partial N_c / \partial N_{c-1} > 0$ and that $\partial N_c / \partial L_c > 0$—to establish that $N'_{c_0} > N_{c_0}$. To show that $N'_{c_0+1} > N_{c_0+1}$, we use the two following relationships:

$$Q_{c-1} = \frac{\lambda_c L_c}{1 - e^{-\lambda_c N_c}}, \text{ for all } c \in \mathcal{C}, \tag{B.25}$$

$$Q_c = \frac{\lambda_c L_c e^{-\lambda_c N_c}}{1 - e^{-\lambda_c N_c}}, \text{ for all } c \in \mathcal{C}. \tag{B.26}$$

Equation $(B.25)$ derives from equation (2.5) and the definition of $N_c \equiv S_c - S_{c-1}$. Equation $(B.26)$ further uses equation (2.6). Since $N'_{c_0-1} > N_{c_{0-1}}$ and $L'_{c_0-1} = L_{c_0-1}$, equation $(B.26)$—in particular, the fact that $\partial Q_c / \partial N_c < 0$—implies $Q'_{c_0-1} < Q_{c_0-1}$. Since $Q'_{c_0-1} < Q_{c_0-1}$ and $N'_{c_0} > N_{c_0}$, equation (2.6) implies $Q'_{c_0} < Q_{c_0}$. Finally, since $Q'_{c_0} < Q_{c_0}$ and $L'_{c_0+1} = L_{c_0+1}$, equation $(B.25)$—in particular, the fact that $\partial Q_{c-1} / \partial N_c < 0$—implies $N'_{c_0+1} > N_{c_0+1}$, which contradicts Step 1. At this point, we have established that if $L'_{c_0} > L_{c_0}$ and $c_0 > 1$, then $N'_1 < N_1$. To conclude the proof of Step 2, note that if $N'_c < N_c$ for $c < c_0 - 1$, then $L'_{c+1} = L_{c+1}$ and equation $(B.13)$ imply $N'_{c+1} < N_{c+1}$. Thus by iteration, $N'_c < N_c$ for all $c < c_0$. Part I directly derives from Step 1, Step 2, and the fact that $\sum_{c=1}^{C} N'_c = \sum_{c=1}^{C} N_c = S$, by Lemma 1.

Part II: *If $L'_{c_0} > L_{c_0}$, then $S'_c < S_c$ for all $c < c_0$ and $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$.*

The proof of Part II proceeds in two steps.

Step 1: *If $L'_{c_0} > L_{c_0}$, then $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$.*

We proceed by iteration. Suppose that $c_0 \leq C - 1$. For country $C - 1$, we know from Part I that $S'_{C-1} = S - N'_C > S - N_C = S_{C-1}$. Thus $S'_c > S_c$ is satisfied for $c = C - 1$. Now suppose that $S'_c > S_c$ for $c > c_0 + 1$. Then $S'_{c-1} = S'_c - N'_c > S_c -$

$N_c = S_{c-1}$, since $S'_c > S_c$ by assumption and $N'_c < N_c$ by Part I. This establishes that $S'_c > S_c$ for all $C - 1 \geq c \geq c_0$, which completes the proof of Step 1.

Step 2: *If $L'_{c_0} > L_{c_0}$, then $S'_c < S_c$ for all $c < c_0$.*

We proceed by iteration. Suppose that $c_0 > 1$. For country $c = 1$, we know from Part I that $S'_1 = N'_1 < N_1 = S_1$. Thus, $S'_c < S_c$ is satisfied for $c = 1$. Now suppose that $S'_{c-1} < S_{c-1}$ for $c < c_0$. Then $S'_c = N'_c + S'_{c-1} < N_c + S_{c-1} = S_c$, since $S'_{c-1} < S_{c-1}$ by assumption and $N'_c < N_c$ by Part I. This establishes that $S'_c < S_c$ for all $c < c_0$, which completes the proof of Step 2. Part II directly follows from Steps 1 and 2.

Part III: *If $L'_{c_0} > L_{c_0}$, then there exists $c_0 + 1 \leq c_1 \leq C$ such that $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $1 \leq c < c_0$; $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 \leq c < c_1 - 1$; $(w_{c_1}/w_{c_1-1})' \geq w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.*

The proof of Part III proceeds in three steps.

Step 1: *If $L'_{c_0} > L_{c_0}$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < c_0$.*

By Part I, we know that $N'_c < N_c$ for all $c < c_0$. Thus we can use the same argument as in Part III of the proof of Proposition 3 to show that $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $c < c_0$.

Step 2: *If $L'_{c_0} > L_{c_0}$, then there exists $c_0 \leq c_1 \leq C$ such that $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 \leq c < c_1 - 1$; $(w_{c_1}/w_{c_1-1})' \geq w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.*

By Part I, we know that $N'_c < N_c$ for all $c > c_0$. Thus we can again use the same argument as in Part III of the proof of Proposition 3 to show that if there exists $\tilde{c} \geq c_0$ such that $(w_{\tilde{c}+1}/w_{\tilde{c}})' \leq (w_{\tilde{c}+1}/w_{\tilde{c}})$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $\tilde{c} \leq c < C$. To conclude the proof of Step 2, let us just define $c_1 \equiv \inf\{c \geq c_0 | (w_{c+1}/w_c)' < (w_{c+1}/w_c)\}$. By construction, w_{c+1}/w_c rises for all $c_0 \leq c < c_1$ and falls for all $c_1 \leq c < C$. In order to complete the proof of Part III, the only thing left to show is that $c_1 > c_0$, which is what we establish in our final step.

Step 3: *If $L'_{c_0} > L_{c_0}$ and $c_0 \neq C$, then $(w_{c_0+1}/w_{c_0})' > (w_{c_0+1}/w_{c_0})$.*

By Part II, we already know that $S'_c < S_c$ for all $c \in \{1, ..., c_0 - 1\}$ and $S'_c > S_c$ $\{c_0, ..., C-1\}$. Since the optimal allocation is the solution of a well-behaved planning problem, the maximum theorem implies the continuity of the pattern of vertical specialization in L_{c_0}. Hence if the change from L_{c_0} to $L'_{c_0} > L_{c_0}$ is small enough, the following chain of inequalities must hold:

$$S'_1 < S_1 < S'_2 < ... < S'_{c_0-1} < S_{c_0-1} < S_{c_0} < S'_{c_0} < S_{c_0+1} < ... < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

We first focus on this situation. For any $c \in \{1, ..., c_0 - 2\}$, since $S'_c < S_c < S'_{c+1} < S_{c+1}$, condition (2.2) implies

$$\begin{aligned}
\frac{p'\left(S'_{c+1}\right)}{w'_{c+1}} &= \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}p'\left(S_c\right)}{w'_{c+1}} + \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)} - 1}{\lambda_{c+1}}, \\
\frac{p\left(S'_{c+1}\right)}{w_{c+1}} &= \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)}p\left(S_c\right)}{w_{c+1}} + \frac{e^{\lambda_{c+1}\left(S'_{c+1}-S_c\right)} - 1}{\lambda_{c+1}}.
\end{aligned}$$

Since $S'_{c+1} > S_c$, the two previous equations further imply that for any $c \in \{1, ..., c_0 - 2\}$,

$$p'\left(S_c\right)/w'_{c+1} \geq p\left(S_c\right)/w_{c+1} \Rightarrow p'\left(S'_{c+1}\right)/w'_{c+1} \geq p\left(S'_{c+1}\right)/w_{c+1}. \tag{B.27}$$

In addition, for any $c \in \{1, ..., c_0 - 1\}$, since $S_{c-1} < S'_c < S_c$, condition (2.2) also implies that

$$\frac{p'\left(S_c\right)}{w'_{c+1}} = \frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}p'\left(S'_c\right)}{w'_{c+1}} + \frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)} - 1}{\lambda_{c+1}}, \tag{B.28}$$

$$\frac{p\left(S_c\right)}{w_{c+1}} = \frac{e^{\lambda_c\left(S_c-S'_c\right)}p\left(S'_c\right)}{w_{c+1}} + \frac{e^{\lambda_c\left(S_c-S'_c\right)} - 1}{\lambda_c}\left(\frac{w_c}{w_{c+1}}\right). \tag{B.29}$$

Let us now show that if $p'\left(S'_c\right)/w'_c \geq p\left(S'_c\right)/w_c$, then

$$\frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)}p'\left(S'_c\right)}{w'_{c+1}} \geq \frac{e^{\lambda_c\left(S_c-S'_c\right)}p\left(S'_c\right)}{w_{c+1}}, \tag{B.30}$$

$$\frac{e^{\lambda_{c+1}\left(S_c-S'_c\right)} - 1}{\lambda_{c+1}} \geq \frac{e^{\lambda_c\left(S_c-S'_c\right)} - 1}{\lambda_c}\left(\frac{w_c}{w_{c+1}}\right). \tag{B.31}$$

We start with inequality $(B.30)$, which can be rearranged as

$$e^{\lambda_{c+1}(S_c - S_c')} \frac{w_c'}{w_{c+1}'} \frac{p'(S_c')}{w_c'} \geq e^{\lambda_c(S_c - S_c')} \frac{w_c}{w_{c+1}} \frac{p(S_c')}{w_c}.$$

By equation (2.7), we know that

$$e^{\lambda_c(S_c - S_c')} \frac{w_c}{w_{c+1}} \frac{p(S_c')}{w_c} = \frac{\frac{p(S_c')}{w_c}}{1 + (\lambda_c - \lambda_{c+1}) \frac{p(S_c)}{w_c}}, \tag{B.32}$$

$$e^{\lambda_{c+1}(S_c - S_c')} \frac{w_c'}{w_{c+1}'} \frac{p'(S_c')}{w_c'} = \frac{\frac{p'(S_c')}{w_c'}}{1 + (\lambda_c - \lambda_{c+1}) \frac{p'(S_c')}{w_c'}}. \tag{B.33}$$

Under the assumption that $p'(S_c')/w_c' \geq p(S_c')/w_c$, equations $(B.32)$ and $(B.33)$ imply

$$e^{\lambda_{c+1}(S_c - S_c')} \frac{w_c'}{w_{c+1}'} \frac{p'(S_c')}{w_c'} \geq \frac{\frac{p(S_c')}{w_c}}{1 + (\lambda_c - \lambda_{c+1}) \frac{p(S_c')}{w_c}} > e^{\lambda_c(S_c - S_c')} \frac{w_c}{w_{c+1}} \frac{p(S_c')}{w_c},$$

where the second inequality also uses the fact that $S_c' < S_c$. Thus inequality $(B.30)$ holds. Let us now consider inequality $(B.31)$, which can be rearranged as

$$\frac{\lambda_c \left(e^{\lambda_{c+1}(S_c - S_c')} - 1\right)}{\lambda_{c+1} \left(e^{\lambda_c(S_c - S_c')} - 1\right)} \geq \frac{w_c}{w_{c+1}}. \tag{B.34}$$

Since $S_{c-1} < S_c'$ for any $c \in \{1, ..., c_0 - 1\}$, condition (2.2) implies $p(S_c)/w_c \geq \left[e^{\lambda_c(S_c - S_{c-1})} - 1\right]/\lambda_c > \left[e^{\lambda_c(S_c - S_c')} - 1\right]/\lambda_c$. Combining the previous inequality with equation (2.7), we obtain

$$\frac{w_c}{w_{c+1}} = \frac{1}{1 + (\lambda_c - \lambda_{c+1}) \frac{p(S_c)}{w_c}} < \frac{\lambda_c}{\lambda_c + (\lambda_c - \lambda_{c+1}) \left[e^{\lambda_c(S_c - S_c')} - 1\right]}. \tag{B.35}$$

By inequalities $(B.34)$ and $(B.35)$, a sufficient condition for inequality $(B.31)$ to hold is

$$\frac{\lambda_c \left[e^{\lambda_{c+1}(S_c - S_c')} - 1\right]}{\lambda_{c+1} \left[e^{\lambda_c(S_c - S_c')} - 1\right]} \geq \frac{\lambda_c}{\lambda_c + (\lambda_c - \lambda_{c+1}) \left[e^{\lambda_c(S_c - S_c')} - 1\right]},$$

which can be rearranged as $\lambda_c / \left[1 - e^{-\lambda_c(S_c - S_c')}\right] \geq \lambda_{c+1} / \left[1 - e^{-\lambda_{c+1}(S_c - S_c')}\right]$. The previous inequality necessarily holds since $f(x) \equiv \frac{x}{1-e^{-tx}}$ is increasing in x for $t > 0$. At this point, we have established that inequalities $(B.30)$ and $(B.31)$ hold if $p'\left(S_c'\right)/w_c' \geq p\left(S_c'\right)/w_c$. Combining this observation with equations $(B.28)$ and $(B.29)$, we further have that for any $c \in \{1, ..., c_0 - 1\}$,

$$p'\left(S_c'\right)/w_c' \geq p\left(S_c'\right)/w_c \Rightarrow p'\left(S_c\right)/w_{c+1}' \geq p\left(S_c\right)/w_{c+1}. \tag{B.36}$$

Since $p'(0) = p(0) = 0$, we know that $p'\left(S_0\right)/w_1' \geq p\left(S_0\right)/w_1$. Thus we can use implications $(B.27)$ and $(B.36)$ to establish, by iteration, that

$$\frac{p'\left(S_{c_0-1}\right)}{w_{c_0}'} \geq \frac{p\left(S_{c_0-1}\right)}{w_{c_0}}. \tag{B.37}$$

Since $S_{c_0} < S_{c_0}'$, we know from condition (2.2) that

$$\frac{p'\left(S_{c_0}\right)}{w_{c_0}'} = \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}p'\left(S_{c_0-1}\right)}{w_{c_0}'} + \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0\ 1}\right)}-1}{\lambda_{c_0}}, \tag{B.38}$$

$$\frac{p\left(S_{c_0}\right)}{w_{c_0}} = \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}p\left(S_{c_0-1}\right)}{w_{c_0}} + \frac{e^{\lambda_{c_0}\left(S_{c_0}-S_{c_0-1}\right)}-1}{\lambda_{c_0}}. \tag{B.39}$$

Inequality $(B.37)$ and equations $(B.38)$ and $(B.39)$ imply $p'\left(S_{c_0}\right)/w_{c_0}' \geq p\left(S_{c_0}\right)/w_{c_0}$. Finally, since $S_{c_0} < S_{c_0}'$, we also know that $p'\left(S_{c_0}'\right)/w_{c_0}' > p'\left(S_{c_0}\right)/w_{c_0}'$. Combining these two observations, we get $p'\left(S_{c_0}'\right)/w_{c_0}' > p\left(S_{c_0}\right)/w_{c_0}$. Together with equation (2.7), the previous inequality implies $\left(w_{c_0+1}/w_{c_0}\right)' > \left(w_{c_0+1}/w_{c_0}\right)$, which implies $c_1 \equiv \inf\left\{c \geq c_0 | \left(w_{c+1}/w_c\right)' < \left(w_{c+1}/w_c\right)\right\} > c_0$. This completes the proof of Step 3 for a small enough change from L_{c_0} to $L_{c_0}' > L_{c_0}$. Since the previous result holds for any initial value of L_{c_0}, it must hold for large changes as well. As mentioned above, Part III directly follows from Steps 1-3. □

Proof of Proposition 6. We decompose the proof of Proposition 6 into three parts. First, we show that a decrease in λ_{c_0} increases the measure of stages N_c performed in all countries $c < c_0$ and decreases the measure of stages N_c performed in all countries $c > c_0$. Second, we show that a decrease in λ_{c_0} leads all countries to move up. Third, we show that a decrease in λ_{c_0} increases inequality among countries $c \in \{1, ..., c_0\}$, decreases inequality among countries $c \in \{c_0, c_0 + 1\}$, increases inequality among countries $c \in \{c_0 + 1, ..., c_1\}$, and decreases inequality among countries $c \in \{c_1, ..., C\}$, with $c_1 \in \{c_0 + 1, ..., C\}$.

Part I: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c > N_c$ for all $c < c_0$ and $N'_c < N_c$ for all $c > c_0$.*

Like in our previous proofs, we will repeatedly use the following relationship

$$N_c = -\left(\frac{1}{\lambda_c}\right) \ln\left[1 - \left(\frac{\lambda_c L_c}{\lambda_{c-1} L_{c-1}}\right)\left(e^{\lambda_{c-1} N_{c-1}} - 1\right)\right], \text{ for all } c > 1, \tag{23}$$

where $\partial N_c / \partial N_{c-1} > 0$. The proof of Part I proceeds in two steps.

Step 1: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c < N_c$ for all $c > c_0$.*

Let us first establish that $Q'_C > Q_C$. By the same argument as in Step 1 of Proposition 5, Q'_C must be such that

$$Q'_C = \underset{\widetilde{Q}_1(\cdot),...,\widetilde{Q}_C(\cdot)}{\arg\max} \ \textstyle\sum_{c=1}^{C} \widetilde{Q}_c(S),$$

subject to

$$\begin{aligned} \textstyle\sum_{c=1}^{C} \widetilde{Q}_c(s_2) - \sum_{c=1}^{C} \widetilde{Q}_c(s_1) &\leq -\textstyle\int_{s_1}^{s_2} \sum_{c=1}^{C} \lambda'_c \widetilde{Q}_c(s)\, ds, \text{ for all } s_1 \leq s_2, && \text{(B.40)} \\ \int_0^S \widetilde{Q}_c(s)\, ds &\leq L_c, \text{ for all } c \in \mathcal{C}, && \text{(B.41)} \end{aligned}$$

where $\lambda'_{c_0} < \lambda_{c_0}$ and $\lambda'_c = \lambda_c$ for all $c \neq c_0$. Now consider $\widetilde{Q}_1(\cdot), ..., \widetilde{Q}_C(\cdot)$ such that

$$\begin{aligned} \widetilde{Q}_{c_0}(s) &\equiv e^{-\left(\lambda_{c_0} - \lambda'_{c_0}\right)\left(S_{c_0} - s\right)} Q_{c_0}(s) \\ &+ \left(\frac{\lambda'_{c_0} e^{-\lambda'_{c_0} s}}{1 - e^{-\lambda'_{c_0} S}}\right) \int_{S_{c_0 - 1}}^{S_{c_0}} \left[1 - e^{-\left(\lambda_{c_0} - \lambda'_{c_0}\right)\left(S_{c_0} - t\right)}\right] Q_{c_0}(t)\, dt, \text{ for all } s \in \mathcal{S}, \end{aligned}$$

and

$$\widetilde{Q}_c(s) \equiv Q_c(s), \text{ for all } s \in \mathcal{S} \text{ and } c \neq c_0.$$

Since $Q_1(\cdot), ..., Q_C(\cdot)$ satisfy the initial resource and technological constraints, as described by equations (2.3) and (2.4), $\widetilde{Q}_1(\cdot), ..., \widetilde{Q}_C(\cdot)$ must satisfy, by construction, the new resource and technological constraints, as described by equations $(B.40)$ and $(B.41)$. Since $\lambda'_{c_0} < \lambda_{c_0}$, we must also have

$$\widetilde{Q}_{c_0}(S) + \widetilde{Q}_C(S) = \left(\frac{\lambda_{c_0} e^{-\lambda_{c_0} S}}{1 - e^{-\lambda_{c_0} S}}\right) \int_{S_{c_0-1}}^{S_{c_0}} \left[1 - e^{-\left(\lambda_{c_0} - \lambda'_{c_0}\right)\left(S_{c_0} - t\right)}\right] Q_{c_0}(t)\, dt + Q_C > Q_C.$$

Since $Q'_C \geq \widetilde{Q}_{c_0}(S) + \widetilde{Q}_C(S)$, the previous inequality implies $Q'_C > Q_C$. Combining this observation with equation $(B.26)$ and the fact that $c_0 \neq C$, which implies $\lambda'_C = \lambda_C$, we get $N'_C < N_C$. To conclude the proof of Step 1, note that if $N'_c < N_c$ for $c > c_0 + 1$, then $\lambda'_{c-1} = \lambda_{c-1}$ and equation $(B.13)$—the fact that $\partial N_c / \partial N_{c-1} > 0$ —imply $N'_{c-1} < N_{c-1}$. Thus by iteration, $N'_c < N_c$ for all $c > c_0$.

Step 2: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $N'_c > N_c$ for all $c < c_0$.*

We first show by contradiction that if $\lambda'_{c_0} < \lambda_{c_0}$ and $c_0 > 1$, then $N'_1 > N_1$. Suppose that $N'_1 \leq N_1$. Since $\lambda'_c = \lambda_c$ for all $c < c_0$, we can use equation $(B.13)$—the fact that $\partial N_c / \partial N_{c-1} > 0$—to establish by iteration that $N'_c \leq N_c$ for all $c < c_0$. Since λ_c is strictly decreasing in c, equation (2.3) therefore implies

$$\textstyle\sum_{c=1}^{C} Q'_c(s) \geq \sum_{c=1}^{C} Q_c(s), \text{ for all } s \leq S'_{c_0-1}.$$

By Step 1, we also know that $N'_c < N_c$ for all $c > c_0$. Thus $N'_{c_0} = S - \sum_{c \neq c_0} N'_c > S - \sum_{c \neq c_0} N_c = N_{c_0}$. Since $\lambda'_{c_0} < \lambda_{c_0}$, equation (2.3) therefore also implies

$$\textstyle\sum_{c=1}^{C} Q'_c(s) > \sum_{c=1}^{C} Q_c(s), \text{ for all } S'_{c_0-1} \leq s \leq S_{c_0}.$$

Using the fact that $N'_c \leq N_c$ for all $c < c_0$ and $N'_c < N_c$ for all $c > c_0$, one can easily establish by iteration, as we do in Part II, that $S'_{c_0-1} \leq S_{c_0-1}$ and $S'_{c_0} > S_{c_0}$.

Combining these two observations with the previous inequality, we obtain

$$\int_{S'_{c_0-1}}^{S'_{c_0}} \sum_{c=1}^{C} Q'_c(s)ds > \int_{S_{c_0-1}}^{S_{c_0}} \sum_{c=1}^{C} Q'_c(s)ds > \int_{S_{c_0-1}}^{S_{c_0}} \sum_{c=1}^{C} Q_c(s)ds,$$

which contradicts the fact that $\int_{S'_{c_0-1}}^{S'_{c_0}} \sum_{c=1}^{C} Q'_c(s)ds = \int_{S_{c_0-1}}^{S_{c_0}} \sum_{c=1}^{C} Q_c(s)ds = L_{c_0}$, by equation (2.4). At this point, we have established that if $\lambda'_{c_0} < \lambda_{c_0}$ and $c_0 > 1$, then $N'_1 > N_1$. To conclude, note that if $N'_c > N_c$ for $c < c_0 - 1$, then $\lambda'_c = \lambda_c$, $\lambda'_{c+1} = \lambda_{c+1}$, and equation $(B.13)$ imply $N'_{c+1} > N_{c+1}$. Thus by iteration, $N'_c > N_c$ for all $c < c_0$. Part I directly derives from Steps 1 and 2.

Part II: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $S'_c > S_c$ for all $c \in \{1, ..., C-1\}$.*

The proof of Part II proceeds in two steps.

Step 1: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $S'_c > S_c$ for all $c \in \{c_0, ..., C-1\}$.*

The proof is identical to the proof of Step 1 Part II of Proposition 5 and omitted.

Step 2: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $S'_c > S_c$ for all $c \in \{1, ..., c_0 - 1\}$.*

We proceed by iteration. Suppose that $c_0 > 1$. For country $c = 1$, we know from Part I that $S'_1 = N'_1 > N_1 = S_1$. Thus, $S'_c > S_c$ is satisfied for $c = 1$. Now suppose that $S'_{c-1} > S_{c-1}$ for $c < c_0$. Then $S'_c = N'_c + S'_{c-1} > N_c + S_{c-1} = S_c$, since $S'_{c-1} > S_{c-1}$ by assumption and $N'_c > N_c$ by Part I. This establishes that $S'_c > S_c$ for all $c < c_0$, which completes the proof of Step 2. Part II directly follows from Steps 1 and 2.

Part III: *If $\lambda'_{c_0} < \lambda_{c_0}$, then there exist $c_0 < c_1 \leq C$ such that $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c < c_0$; $(w_{c_0+1}/w_{c_0})' < w_{c_0+1}/w_{c_0}$; $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 < c < c_1 - 1$; $(w_{c_1+1}/w_{c_1})' \geq w_{c_1+1}/w_{c_1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.*

The proof of Part III proceeds in four steps.

Step 1: *If $\lambda'_{c_0} < \lambda_{c_0}$, then $(w_{c+1}/w_c)' > (w_{c+1}/w_c)$ for all $c < c_0$.*

By Part I, we know that $N'_c > N_c$ for all $c < c_0$. Thus we can use the same argument as in Part III of the proof of Proposition 3 to show that $(w_{c+1}/w_c)' > (w_{c+1}/w_c)$ for all $c < c_0$.

Step 2: *If* $\lambda'_{c_0} < \lambda_{c_0}$*and* $c_0 > 1$, *then* $\lambda'_{c_0}\left(S'_{c_0} - S'_{c_0-1}\right) < \lambda_{c_0}(S_{c_0} - S'_{c_0-1})$.

By Part II, we already know that $S'_c > S_c$ for all $c \in \{1, ..., C-1\}$. Since the optimal allocation is the solution of a well-behaved planning problem, the maximum theorem implies the continuity of the pattern of vertical specialization in λ_{c_0}. Hence if the change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$ is small enough, the following chain of inequalities must hold:

$$S_1 < S'_1 < S_2 < ... < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

We first focus on this situation. In the proof of Step 2, we let $Q'(s)$ and $p'(s)$, denote the output at stage s and the price of stage s if the failure rate in country c_0 is equal to λ'_{c_0}. From Equation $(B.26)$, we have

$$Q_1 = \frac{\lambda_1 L_1 e^{-\lambda_1 N_1}}{1 - e^{-\lambda_1 N_1}}.$$

Similarly, we have

$$Q'(S_1) = \frac{\lambda_1 L'_1 e^{-\lambda_1 N'_1}}{1 - e^{-\lambda_1 N'_1}},$$

where L'_1 is the amount of labor from country 1 used to perform stages $(0, S_1)$ when the failure rate of country c_0 is equal to λ'_{c_0}. The two previous equations, together with $N'_1 > N_1$ and $L'_1 < L_1$, therefore, imply $Q'(S_1) < Q_1$. Assume that $Q'(S_c) < Q_c$ holds for some $1 \leq c \leq c_0 - 2$. Since $c + 1 < c_0$, we have

$$\begin{aligned} Q_{c+1} &= e^{-\lambda_{c+1} N_{c+1}} Q_c > e^{-\lambda_{c+1}(S_{c+1} - S'_c + S'_c - S_c)} Q'(S_c) \\ &\geq e^{-\lambda_c (S'_c - S_c) - \lambda_{c+1}(S_{c+1} - S'_c)} Q'(S_c) = e^{-\lambda_{c+1}(S_{c+1} - S'_c)} Q'(S'_c) = Q'(S_{c+1}). \end{aligned}$$

Therefore, by iteration, we obtain

$$Q'(S_c) < Q_c \text{ for all } 1 \leq c \leq c_0 - 1. \tag{B.42}$$

By equation $(B.42)$, which implies $Q'(S_{c_0-1}) < Q_{c_0-1}$, and equation (2.1), we have

$$\begin{aligned} Q'_{c_0} &= e^{-\lambda'_{c_0}(S'_{c_0}-S'_{c_0-1})-\lambda_{c_0-1}(S'_{c_0-1}-S_{c_0-1})}Q'(S_{c_0-1}) \\ &< e^{-\lambda'_{c_0}(S'_{c_0}-S'_{c_0-1})-\lambda_{c_0-1}(S'_{c_0-1}-S_{c_0-1})}Q_{c_0-1}. \end{aligned} \tag{B.43}$$

Equation (2.6) implies

$$Q_{c_0} = e^{-\lambda_{c_0}(S_{c_0}-S_{c_0-1})}Q_{c_0-1}. \tag{B.44}$$

By the proof of Part I of Proposition 6, we have $N'_{c_0+1} < N_{c_0+1}$. Equation $(B.25)$ and $N'_{c_0+1} < N_{c_0+1}$ imply $Q'_{c_0} > Q_{c_0}$. Equation $(B.43)$, equation $(B.44)$, and $Q'_{c_0} > Q_{c_0}$ imply

$$e^{-\lambda'_{c_0}(S'_{c_0}-S'_{c_0-1})-\lambda_{c_0-1}(S'_{c_0-1}-S_{c_0-1})} > e^{-\lambda_{c_0}(S_{c_0}-S_{c_0-1})},$$

which implies

$$S'_{c_0} - S'_{c_0-1} < \frac{\lambda_{c_0}}{\lambda'_{c_0}}(S_{c_0} - S'_{c_0-1}) - \frac{\lambda_{c_0-1} - \lambda_{c_0}}{\lambda'_{c_0}}(S'_{c_0-1} - S_{c_0-1}) \leq \frac{\lambda_{c_0}}{\lambda'_{c_0}}(S_{c_0} - S'_{c_0-1}),$$

concluding the proof of Step 2 for a small enough change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$. Since the previous result holds for any initial value of λ_{c_0}, it must hold for large changes as well.

Step 3: *If $\lambda'_{c_0} < \lambda_{c_0}$and $c_0 < C$, then $\left(w_{c_0+1}/w_{c_0}\right)' < w_{c_0+1}/w_{c_0}$.*

Like in the previous step, we first consider a change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$ small enough for the following chain of inequalities to hold:

$$S_1 < S'_1 < S_2 < ... < S'_{C-2} < S_{C-1} < S'_{C-1}.$$

In the same way as we have proceeded in Part III Step 3 of Proposition 5, one can show by iteration that

$$\frac{p'(S'_{c_0-1})}{w'_{c_0}} \leq \frac{p(S'_{c_0-1})}{w_{c_0}}.$$

By condition (2.2) and equation (2.7), we know that

$$\frac{w'_{c_0+1}}{w'_{c_0}} - 1 = (\lambda'_{c_0} - \lambda_{c_0+1}) \left[\frac{p'(S'_{c_0-1})}{w'_{c_0}} e^{\lambda'_{c_0}(S'_{c_0} - S'_{c_0-1})} + \frac{e^{\lambda'_{c_0}(S'_{c_0} - S'_{c_0-1})} - 1}{\lambda'_{c_0}} \right].$$

Combining the two previous expressions with Step 2, we therefore get

$$\frac{w'_{c_0+1}}{w'_{c_0}} - 1 < (\lambda'_{c_0} - \lambda_{c_0+1}) \left[\frac{p(S'_{c_0-1})}{w_{c_0}} e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} + \frac{e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} - 1}{\lambda'_{c_0}} \right]$$

Since $\lambda'_{c_0} < \lambda_{c_0}$, this implies

$$\frac{w'_{c_0+1}}{w'_{c_0}} - 1 < (\lambda_{c_0} - \lambda_{c_0+1}) \left[\frac{p(S'_{c_0-1})}{w_{c_0}} e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} + \frac{e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} - 1}{\lambda_{c_0}} \right]. \tag{B.45}$$

But by condition (2.2) and equation (2.7), we also know that

$$\frac{w_{c_0+1}}{w_{c_0}} - 1 = (\lambda_{c_0} - \lambda_{c_0+1}) \left[\frac{p(S'_{c_0-1})}{w_{c_0}} e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} + \frac{e^{\lambda_{c_0}(S_{c_0} - S'_{c_0-1})} - 1}{\lambda_{c_0}} \right]. \tag{B.46}$$

Equations $(B.45)$ and $(B.46)$ imply $w'_{c_0+1}/w'_{c_0} < w_{c_0+1}/w_{c_0}$. This completes the proof of Step 3 for a small enough change from λ_{c_0} to $\lambda'_{c_0} < \lambda_{c_0}$. Since the previous result holds for any initial value of λ_{c_0}, it must hold for large changes as well.

Step 4: *If $\lambda'_{c_0} < \lambda_{c_0}$, then there exists $c_0 < c_1 \leq C$ such that $(w_{c+1}/w_c)' > w_{c+1}/w_c$ for all $c_0 < c < c_1 - 1$; $(w_{c_1}/w_{c_1-1})' \geq w_{c_1}/w_{c_1-1}$; and $(w_{c+1}/w_c)' < w_{c+1}/w_c$ for all $c_1 \leq c < C$.*

By Part I, we also know that $N'_c < N_c$ for all $c > c_0$. Thus we can again use the same argument as in Part III of the proof of Proposition 3 to show that if there exists $\tilde{c} > c_0$ such that $(w_{\tilde{c}+1}/w_{\tilde{c}})' \leq (w_{\tilde{c}+1}/w_{\tilde{c}})$, then $(w_{c+1}/w_c)' < (w_{c+1}/w_c)$ for all $\tilde{c} \leq c < C$. To conclude the proof of Step 4, let us just define $c_1 \equiv \inf\{c > c_0 | \, (w_{c+1}/w_c)' < (w_{c+1}/w_c)\}$. By construction, w_{c+1}/w_c rises for all $c_0 < c < c_1$ and falls for all $c_1 \leq c < C$. This concludes the proof of Part III. $\square$

Appendix C

Appendix for Assortative Matching and Wage Inequality within and across Firms

C.1 Proofs (I): Competitive Equilibrium

Proof. [Proof of Lemma 8] I denote $\sigma(\theta, \varphi) = \left\{\sigma \in \sigma | \sigma \in \arg\min_{\sigma'} \frac{W(\sigma')}{A(\sigma', \theta, \varphi)}\right\}$ and $\Phi(\sigma) = \left\{(\theta, \varphi) \in \Theta \times \Phi | \sigma \in \arg\min_{\sigma'} \frac{W(\sigma')}{A(\sigma', \theta, \varphi)}\right\}$. Since occupations within a firm is complementary, $\sigma(\theta, \varphi) \neq \emptyset$ for all (θ, φ). On the other hand, labor market clearing condition (3.7) implies that $\Phi(\sigma) \neq \emptyset$ for all σ.

I next prove that in a competitive equilibrium, the wage schedule is necessarily a strictly increasing function of worker skill. I proceed by contradiction. Suppose that there exists $\sigma_1 > \sigma_0$ such that $W(\sigma_1) \leq W(\sigma_0)$. Since the production function, A, is strictly increasing in skill, this would imply that $\frac{W(\sigma_1)}{A(\sigma_1, \theta, \varphi)} < \frac{W(\sigma_0)}{A(\sigma_0, \theta, \varphi)}$ for any firm-occupation (θ, φ), so $\sigma_0 \notin \arg\min_{\sigma} \frac{W(\sigma)}{A(\sigma, \theta, \varphi)}$. Yet we have just established that $\Phi(\sigma_0) \neq \emptyset$, which is a contradiction.

Next, I show that $\sigma(\theta, \varphi)$ is weakly increasing in job (θ, φ), in the sense that if there are (θ_0, φ_0) and (θ_1, φ_1) such that $J(\theta_1, \varphi_1) > J(\theta_0, \varphi_0)$, then $\sigma(\theta_1, \varphi_1) \geq \sigma(\theta_0, \varphi_0)$ (meaning that for any $\sigma_1 \in \sigma(\theta_1, \varphi_1)$ and $\sigma_0 \in \sigma(\theta_0, \varphi_0)$, we have $\sigma_1 \geq$

σ_0). I proceed by contradiction. Suppose that this is not the case, so there exists (θ_0, φ_0) and (θ_1, φ_1) such that $J(\theta_1, \varphi_1) > J(\theta_0, \varphi_0)$ yet one can choose $\sigma_1 \in \sigma(\theta_1, \varphi_1)$ and $\sigma_0 \in \sigma(\theta_0, \varphi_0)$ with $\sigma_1 < \sigma_0$. Since $\sigma_0 \in \sigma(\theta_0, \varphi_0)$, we have, by definition

$$\frac{W(\sigma_0)}{A(\sigma_0, \theta_0, \varphi_0)} \leq \frac{W(\sigma_1)}{A(\sigma_1, \theta_0, \varphi_0)}.$$

On the other hand, thanks to the log-supermodularity of A, we can write

$$\begin{aligned}
\frac{W(\sigma_0)}{A(\sigma_0, \theta_0, \varphi_0)} &> \frac{W(\sigma_0)}{A(\sigma_0, \theta_1, \varphi_1)} \frac{A(\sigma_1, \theta_1, \varphi_1)}{A(\sigma_1, \theta_0, \varphi_0)} \\
&\geq \frac{W(\sigma_1)}{A(\sigma_1, \theta_1, \varphi_1)} \frac{A(\sigma_1, \theta_1, \varphi_1)}{A(\sigma_1, \theta_0, \varphi_0)} \\
&= \frac{W(\sigma_1)}{A(\sigma_1, \theta_0, \varphi_0)}.
\end{aligned}$$

The second inequality comes from the fact that $\sigma_1 \in \sigma(\theta_1, \varphi_1)$. This forms a contradiction.

I then show that jobs with the same level of sophistication require the same skill type: $J(\theta_1, \varphi_1) = J(\theta_0, \varphi_0)$ implies $\sigma(\theta_1, \varphi_1) = \sigma(\theta_0, \varphi_0)$. Consider $\sigma_1 \in \sigma(\theta_1, \varphi_1)$ and $\sigma_0 \in \sigma(\theta_0, \varphi_0)$. From inequality (3.5), by continuity and strict monotonicity of J, one can show that for any σ', σ, (θ, φ) and (θ', φ') such that $J(\theta, \varphi) = J(\theta', \varphi')$, we have

$$\frac{A(\sigma', \theta', \varphi')}{A(\sigma, \theta', \varphi')} = \frac{A(\sigma', \theta, \varphi)}{A(\sigma, \theta, \varphi)}.$$

This implies that

$$\begin{aligned}
\frac{W(\sigma_0)}{A(\sigma_0, \theta_0, \varphi_0)} &= \frac{W(\sigma_0)}{A(\sigma_0, \theta_1, \varphi_1)} \frac{A(\sigma_1, \theta_1, \varphi_1)}{A(\sigma_1, \theta_0, \varphi_0)} \\
&\geq \frac{W(\sigma_1)}{A(\sigma_1, \theta_1, \varphi_1)} \frac{A(\sigma_1, \theta_1, \varphi_1)}{A(\sigma_1, \theta_0, \varphi_0)} \\
&= \frac{W(\sigma_1)}{A(\sigma_1, \theta_0, \varphi_0)},
\end{aligned}$$

where the second inequality comes from the fact that $\sigma_1 \in \sigma(\theta_1, \varphi_1)$. This implies that $\sigma_1 \in \sigma(\theta_0, \varphi_0)$. Since this is true for all $\sigma_1 \in \sigma(\theta_1, \varphi_1)$, we have $\sigma(\theta_1, \varphi_1) \subseteq \sigma(\theta_0, \varphi_0)$. Similarly, we must also have $\sigma(\theta_0, \varphi_0) \subseteq \sigma(\theta_1, \varphi_1)$, which completes the proof.

It follows that $\sigma(\theta,\varphi)$ is always a non-empty interval of σ. The fact that $\sigma(\theta,\varphi)$ is non-empty has been established. To show that it is an interval, consider two elements $\sigma_0, \sigma_1 \in \sigma(\theta,\varphi)$ with $\sigma_0 < \sigma_1$. Consider any skill type $\sigma_2 \in (\sigma_0, \sigma_1)$. It suffices to show that $\sigma_2 \in \sigma(\theta,\varphi)$. Suppose that this is not the case. Since $\Phi(\sigma_2) \neq \emptyset$, there exists (θ',φ') such that $\sigma_2 \in \sigma(\theta',\varphi')$. As $\sigma_0 < \sigma_2$ and $\sigma(.,.)$ is strictly increasing in job, it has to be the case that $J(\theta,\varphi) \leq J(\theta',\varphi')$. Similarly, $\sigma_2 < \sigma_1$ would imply that $J(\theta,\varphi) \geq J(\theta',\varphi')$ so $J(\theta,\varphi)$ must be equal to $J(\theta',\varphi')$. By the previous point, $\sigma(\theta,\varphi) = \sigma(\theta',\varphi')$, which forms a contradiction.

Next, denote by J_0 the set of jobs such whose required skills form a non-singleton interval: $J_0 = \{(\theta,\varphi) \,|\mu^B [\sigma(\theta,\varphi)] > 0\}$, where μ^B is the Borel measure. I shall show that $J(J_0)$ is countable. In other words, $\sigma(\theta,\varphi)$ is a singleton for all but a countable subset of degrees of sophistication. To do this, it suffices to construct an injection, f, from $J(J_0)$ to the set of rational numbers. For any $x \in J(J_0)$, choose any (θ,φ) such that $J(\theta,\varphi) = x$. By definition, $\sigma(\theta,\varphi)$ is an interval with strictly positive measure. By density of rational numbers in real numbers, one can choose y to be any rational number inside the interval $\sigma(\theta,\varphi)$ and define $f(x) = y$. I shall show that f is an injection. Consider x and x' two elements in $J(J_0)$. $f(x) = f(x')$ would imply that there exists (θ,φ) and (θ',φ') such that $J(\theta,\varphi) = x$ and $J(\theta',\varphi') = x'$. Moreover, we should have $f(x) \in \sigma(\theta,\varphi)$ and $f(x') \in \sigma(\theta',\varphi')$ by construction of f, so $\sigma(\theta,\varphi) \cap \sigma(\theta',\varphi') \neq \emptyset$. The only possibility for this to be true is that $J(\theta,\varphi) = J(\theta',\varphi')$, which means $x = x'$. This completes the proof.

Now, I show that $\sigma(\theta,\varphi)$ is a singleton for all degrees of sophistication. Consider any (θ,φ) such that $\sigma(\theta,\varphi)$ is not a singleton. We have $\mu^B[\sigma(\theta,\varphi)] > 0$. For any $\sigma \in \sigma(\theta,\varphi)$, by the previous points, we know that $\sigma \notin \sigma(\theta',\varphi')$ if $J(\theta,\varphi) \neq J(\theta',\varphi')$. This creates a contradiction since all the workers of skill type within $\sigma(\theta,\varphi)$ have to be hired in jobs of sophistication $J(\theta,\varphi)$, as the latter quantity is infinitesimal and the former is not.

Finally, notice that if I denote $\widehat{\sigma}(\theta,\varphi) = \{\sigma \in \sigma | L(\sigma,\theta,\varphi) > 0\}$, $\widehat{\sigma}(\theta,\varphi)$ is a non-empty subset of $\sigma(\theta,\varphi)$. The fact that $\sigma(\theta,\varphi)$ is always a singleton then implies that $\widehat{\sigma}(\theta,\varphi)$ is also a singleton. From the previous points, we know that $\widehat{\sigma}(\theta,\varphi)=\widehat{\sigma}(\theta',\varphi')$ if and only if $J(\theta,\varphi)=J(\theta',\varphi')$, so one could define a function $M: J(\Theta\times\Phi)\to\sigma$ such that $M(J(\theta,\varphi))=\widehat{\sigma}(\theta,\varphi)$. We have shown that the function M is strictly increasing. Since $M(J(\Theta\times\Phi))=\sigma$, M must be continuous and satisfy $M\left(J\left(\theta,\underline{\varphi}\right)\right)=\underline{\varphi}$ and $M\left(J\left(\overline{\theta},\overline{\varphi}\right)\right)=\overline{\sigma}$. □

Next, I prove Lemma 9.

Proof. [Proof of Lemma9] The first order condition of (3.11) is

$$\frac{d\ln W}{d\sigma}\left(M\left(J\left(\theta,\varphi\right)\right)\right)=\frac{\partial\ln A}{\partial\sigma}\left(M\left(J\left(\theta,\varphi\right)\right),J\left(\theta,\varphi\right)\right),$$

which completes the proof for (3.15). I now turn to equation (3.14). From the labor market clearing conditions,

$$L\left(\sigma\right)=\int_{\Theta\times\Phi}\frac{y\left(\theta,\varphi\right)}{A\left(M\left(J\left(\theta,\varphi\right)\right),J\left(\theta,\varphi\right)\right)}\delta\left(\sigma-M\left(J\left(\theta,\varphi\right)\right)\right)d\theta d\varphi.$$

Applying an integration by substitution,

$$L\left(\sigma\right)=\int_{\Phi}\int_{J(\Theta,\varphi)}\frac{y\left(J_{\varphi}^{-1}\left(j\right),\varphi\right)}{A\left(M\left(j\right),j\right)}\delta\left(\sigma-M\left(j\right)\right)J_{\varphi}^{-1\prime}\left(j\right)djd\varphi,$$

where $J_{\varphi}^{-1}(j)\in\Theta$ is such that $J\left(\varphi,J_{\varphi}^{-1}\left(j\right)\right)=j$. By Lemma 8, there exists j' such that $M(j')=\sigma$, this implies

$$L\left(M\left(j'\right)\right)=\int_{\Phi}\int_{J(\Theta,\varphi)}\frac{y\left(J_{\varphi}^{-1}\left(j\right),\varphi\right)}{A\left(M\left(j\right),j\right)}\delta\left(M\left(j'\right)-M\left(j\right)\right)J_{\varphi}^{-1\prime}\left(j\right)djd\varphi.$$

Applying another integration by substitution, we have

$$L\left(M\left(j'\right)\right)=\int_{\Phi}\int_{J(\Theta,\varphi)}\frac{y\left(J_{\varphi}^{-1}\left(j\right),\varphi\right)}{A\left(M\left(j\right),j\right)}\delta\left(j'-j\right)\frac{1}{M'\left(j'\right)}\left(\varphi\right)J_{\varphi}^{-1\prime}\left(j'\right)djd\varphi.$$

By definition of the Dirac function, this simplifies into

$$L\left(M\left(j'\right)\right)=\int_{\Phi}\mathbf{1}_{j'\in J(\Theta,\varphi)}\frac{y\left(J_{\varphi}^{-1}\left(j'\right),\varphi\right)}{A\left(M\left(j'\right),j'\right)}\frac{1}{M'\left(j'\right)}J_{\varphi}^{-1\prime}\left(j'\right)d\varphi,$$

where $\mathbf{1}$ is the characteristic function. Combining this with (3.12), we have

$$L\left(M\left(j'\right)\right)=\frac{W\left(M\left(j'\right)\right)^{-\epsilon}A\left(M\left(j'\right),j'\right)^{\epsilon-1}}{M'\left(j'\right)}R\int_{\Phi}\mathbf{1}_{j'\in J(\Theta,\varphi)}p\left(\varphi\right)^{\epsilon-1}\beta\left(\varphi\right)J_{\varphi}^{-1\prime}\left(j'\right)d\varphi.$$

Finally, re-arranging this, we have

$$M'\left(j\right)=\frac{W\left(M\left(j\right)\right)^{-\epsilon}A\left(M\left(j\right),j\right)^{\epsilon-1}}{L\left(M\left(j\right)\right)}R\int_{\Phi}\mathbf{1}_{j'\in J(\Theta,\varphi)}p\left(\varphi\right)^{\epsilon-1}\beta\left(\varphi\right)J_{\varphi}^{-1\prime}\left(j\right)d\varphi.$$

□

Next, I prove Proposition 2.

Proof. [Proof of Proposition 2] According to Lemma 8, the average wage in a firm φ is given by

$$W^{m}\left(\varphi\right)=\frac{\int_{\underline{\theta}}^{\overline{\theta}}\int_{\sigma}L\left(\sigma,\theta,\varphi\right)W\left(\sigma\right)d\sigma d\theta}{\int_{\theta}\int_{\sigma}L\left(\sigma,\theta,\varphi\right)d\sigma d\theta}.$$

Substituting labor demand function $L\left(\sigma,\theta,\varphi\right)$ using (3.13) and simplifying, we have

$$W^{m}\left(\varphi\right)=.\frac{\int_{0}^{1}\frac{W(M(J(\theta,\varphi)))}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}\left[\frac{W(M(J(\theta,\varphi)))}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}\right]^{-\epsilon}d\theta}{\int_{0}^{1}\frac{1}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}\left[\frac{W(M(J(\theta,\varphi)))}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}\right]^{-\epsilon}d\theta}\tag{C.1}$$

Next, I try to establish the fact that W^m is increasing in φ. First, to ease notions, define $f\left(\theta,\varphi\right)=\frac{W(M(J(\theta,\varphi)))}{A(M(J(\theta,\varphi)),J(\theta,\varphi))}$, $h\left(\theta,\varphi\right)=f\left(\theta,\varphi\right)^{1-\epsilon}$ and $g\left(\theta,\varphi\right)=\frac{h(\theta,\varphi)}{W(M(J(\theta,\varphi)))}$. Average wage in firm φ can be re-written as $W^{m}\left(\varphi\right)=\frac{\int_0^1 h}{\int_0^1 g}$.

Differentiating f while applying Equation (3.15):

$$\frac{d\ln f\left(\theta,\varphi\right)}{d\varphi}=w'M'J_{\varphi}-a_{\sigma}M'J_{\varphi}-a_{j}J_{\varphi}=-a_{j}J_{\varphi}<0,\tag{C.2}$$

where the lower letters signify the log of the respective functions. So f is decreasing in φ. Similarly, it is straightforward to show that f is decreasing in θ.

Under Assumption (2), differentiating (C.2) w.r.t. θ:

$$\frac{d^2 \ln f(\theta, \varphi)}{d\varphi d\theta} = -a_{\sigma j} M' J_\theta - a_{jj} J_\theta J_\varphi - a_j J_{\theta\varphi} < 0.$$

So f is log-submodular.

Differentiating Equation (C.1) w.r.t. φ, the sign of $\frac{dW^m}{d\varphi}$ is given by the sign of $\int_0^1 h_\varphi \int_0^1 g - \int_0^1 g_\varphi \int_0^1 h$. Since

$$\frac{d \ln h}{d\varphi} = (1-\epsilon) \frac{d \ln f}{d\varphi} > 0$$

when $\epsilon > 1$ and

$$\frac{g_\varphi}{g} = \frac{h_\varphi}{h} - \frac{W_\varphi}{W},$$

$\int_0^1 h_\varphi \int_0^1 g - \int_0^1 g_\varphi \int_0^1 h$ is positive if and only if $\frac{\int_0^\theta g_\varphi}{\int_0^\theta g} \frac{\int_0^\theta h}{\int_0^\theta h_\varphi} \leq 1$.

First

$$\frac{\int_0^\theta g_\varphi}{\int_0^\theta g} \frac{\int_0^\theta h}{\int_0^\theta h_\varphi} = \frac{\int_0^\theta \frac{h_\varphi}{W} - \frac{W_\varphi}{W} g}{\int_0^\theta g} \frac{\int_0^\theta h}{-\int h_\varphi} \leq \frac{\int_0^\theta \frac{h_\varphi}{W}}{\int_0^\theta \frac{h}{W}} \frac{\int_0^\theta h}{\int_0^\theta h_\varphi}.$$

So it suffices to show that $\frac{\int_0^\theta \frac{h_\varphi}{W}}{\int_0^\theta \frac{h}{W}} \leq \frac{\int_0^\theta h_\varphi}{\int_0^\theta h}$. Since

$$\frac{d \frac{h_\varphi}{h}}{d\theta} = \frac{d^2 \ln h}{d\theta d\varphi} = (1-\epsilon) \frac{d^2 \ln f(\theta, \varphi)}{d\varphi d\theta} > 0,$$

yet $\frac{1}{W(M(J(\theta,\varphi)))}$ is decreasing in θ, we have $\frac{\int_0^\theta \frac{h_\varphi}{W}}{\int_0^\theta \frac{h}{W}} \leq \frac{\int_0^\theta h_\varphi}{\int_0^\theta h}$, which concludes the proof. $\square$

I then prove Proposition 3.

Proof. [Proof of Proposition 3] Consider a firm with technology φ. Denote $\theta_\tau(\varphi)$ the τ-th quantile in the wage distribution of workers in this firm. θ_τ then satisfies the following identity:

$$\tau = \frac{\int_{\underline{\theta}}^{\theta_\tau} L(\theta, \varphi)\, d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta}, \tag{C.3}$$

where $L(\theta, \varphi)$ is the number of workers hired at occupation θ. Denote by $LC_\varphi(\tau)$ Lorenz curve of wages in firm φ. By definition of the Lorenz curve, we have

$$LC_\varphi(\tau) = \frac{\int_{\underline{\theta}}^{\theta_\tau} W(M(J(\theta, \varphi)))\, L(\theta, \varphi)\, d\theta}{\int_{\underline{\theta}}^{\overline{\theta}} W(M(J(\theta, \varphi)))\, L(\theta, \varphi)\, d\theta}.$$

Replacing $L(\theta, \varphi)$ using (3.13),

$$LC_\varphi(\tau) = \frac{\int_0^{\theta_\tau} h(\theta, \varphi)\, d\theta}{\int_0^1 h(\theta, \varphi)\, d\theta}. \tag{C.4}$$

Replacing $L(\theta, \varphi)$ in $(C.3)$ and differentiating w.r.t. φ, we have, after re-arrangement,

$$\frac{\partial \theta_\tau}{\partial \varphi} > 0.$$

To complete the proof, Differentiating $(C.4)$ w.r.t. φ, we have

$$\frac{\partial LC_\varphi(\tau)}{\partial \varphi} = \frac{\left[\frac{d\theta_\tau}{d\varphi} h + \int_0^{\theta_\tau} h_\varphi\right] \int_0^1 h - \int_0^{\theta_\tau} h \int_0^1 h_\varphi}{\left(\int_0^1 h\right)^2},$$

and noticing that $\int_0^{\theta_\tau} h_\varphi \int_0^1 h - \int_0^{\theta_\tau} h \int_0^1 h_\varphi \geq 0$ since

$$\frac{d^2 \ln h}{d\varphi d\theta} = (1 - \epsilon) \frac{d^2 \ln f}{d\varphi d\theta} > 0.$$

□

Next, I prove Lemma 10.

Proof. [Proof of Lemma 10]To show that the matching function moves up, I proceed by contradiction. Suppose that there exists $j \in J(\Theta \times \Phi)$ such that $\widehat{M}(j) < M(j)$. Thanks to Lemma 8, we know that both M and $\widehat{M}$ are continuous functions such that $M(\inf J(\Theta,\Phi)) = \widehat{M}(\inf J(\Theta,\Phi)) = \inf \Sigma$ and $M(\sup J(\Theta,\Phi)) = \widehat{M}(\sup J(\Theta,\Phi)) = \sup \Sigma$. Moreover, thanks to the first welfare theorem, the competitive equilibrium is efficient so it is not possible to have $\widehat{M}(j') < M(j')$ for all j'. As a result, there must exist $\inf J(\Theta,\Phi) \leq j_1 < j_2 \leq \sup J(\Theta,\Phi)$ such that (i) $M(j_1) = \widehat{M}(j_1)$ and $M(j_2) = \widehat{M}(j_2)$; (ii) $M'(j_1) \geq \widehat{M'}(j_1)$ and $M'(j_2) \leq \widehat{M'}(j_2)$; (iii) $M(j) > \widehat{M}(j)$ for all $j \in (j_1, j_2)$.

From (ii) and Equation (3.14),

$$\left(\frac{\hat{W}(\sigma_1)}{\hat{W}(\sigma_2)}\right)^{-\epsilon} \frac{\hat{L}(\sigma_2)}{\hat{L}(\sigma_1)} \leq \left(\frac{W(\sigma_1)}{W(\sigma_2)}\right)^{-\epsilon} \frac{L(\sigma_2)}{L(\sigma_1)}.$$

Because of assumption (3.16), this requires $\frac{\hat{W}(\sigma_1)}{\hat{W}(\sigma_2)} > \frac{W(\sigma_1)}{W(\sigma_2)}$, which cannot hold because of equation (3.15) and the log-supermodularity of A. $\square$

I then prove Lemma 4.

Proof. [Proof of Lemma4] Define $x(j) = \frac{\widehat{M'}(j)}{M'(j)}$. By Lemma 8, we know that the derivatives of matching functions are positive.

Differentiating $\ln x(j)$, at a point where $M'(j) = \hat{M}'(j)$,

$$\begin{aligned}
\frac{d\ln x}{dj} &= -\left(\frac{d\ln \hat{L}}{d\sigma}(\hat{M})\hat{M}' - \frac{d\ln L}{d\sigma}(M)M'\right) - \epsilon\left(\frac{d\ln \hat{W}}{d\sigma}(\hat{M})\hat{M}' - \frac{d\ln W}{d\sigma}(M)M'\right) \\
&\quad + (\epsilon-1)\left[\frac{d\ln \hat{A}}{d\sigma}(\hat{M},j)\hat{M}' - \frac{d\ln A}{d\sigma}(M,j)M' + \frac{d\ln A}{dj}(\hat{M},j) - \frac{d\ln A}{dj}(M)\right] \\
&= -\left(\frac{d\ln \hat{L}}{d\sigma}(\hat{M})\hat{M}' - \frac{d\ln L}{d\sigma}(M)M'\right) - \left(\frac{d\ln A}{d\sigma}(\hat{M},j)\hat{M}' - \frac{d\ln A}{d\sigma}(M)M'\right) \\
&\quad + (\epsilon-1)\left(\frac{d\ln A}{dj}(\hat{M},j) - \frac{d\ln A}{dj}(M,j)\right) \\
&= -\left(\frac{\mu+1}{M} - \frac{\hat{\mu}+1}{\hat{M}} + \frac{d\ln A}{d\sigma}(\hat{M},j) - \frac{d\ln A}{d\sigma}(M,j)\right)M' \\
&\quad + (\epsilon-1)\left(\frac{d\ln A}{dj}(\hat{M},j) - \frac{d\ln A}{dj}(M,j)\right)
\end{aligned}$$

Since $\epsilon > 1$, $\frac{d\ln x}{dj} < 1$ whenever $M'(j) = \hat{M}'(j)$. Denote j_0 the smallest $j \in J$

such that $M'(j) = \hat{M}'(j)$. The wage ratio between two jobs, j_1 and j_2, are given by

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln W}{\partial \sigma}(M(j))\, M'(j)\, dj.$$

Using equation (3.15), we have

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln A}{\partial \sigma}(M(j), j)\, M'(j)\, dj.$$

Therefore, we have

$$\frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} - \frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln A}{\partial \sigma}\left(\widehat{M}(j), j\right) \widehat{M'}(j) - \frac{\partial \ln A}{\partial \sigma}(M(j), j)\, M'(j)\, dj.$$

If $\frac{\partial^2 \ln A}{\partial \sigma^2} \geq 0$, since $\widehat{M}(j) \geq M(j)$ by Lemma 10, $0 \leq \frac{\partial \ln A}{\partial \sigma}\left(\widehat{M}(j), j\right) \leq \frac{\partial \ln A}{\partial \sigma}(M(j), j)$. Moreover, we have, from what have been previously established in this proof, that if $j_1 < j_2 \leq j_0$, $\widehat{M'}(j) \geq M'(j) \geq 0$, so $\frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} \geq \frac{W(M(j_2))}{W(M(j_1))}$. □

I then prove Corollary 1.

Proof. [Proof of Corollary 1] Choose any $\tau \in (0,1)$. From Equation (C.4), we have

$$LC_{\varphi}(\tau) = \int_{\underline{\theta}}^{\theta_\tau} B(\theta)\, d\theta.$$

Therefore, to show that the Lorenz curve goes down at τ, it suffices to show that $\theta_\tau > \widehat{\theta}_\tau$. By definition of $\theta\tau$ and $\widehat{\theta}_\tau$, we know

$$\begin{aligned} \tau \int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta &= \int_{\underline{\theta}}^{\theta_\tau} L(\theta, \varphi)\, d\theta, \\ \tau \int_{\underline{\theta}}^{\overline{\theta}} \widehat{L}(\theta, \varphi)\, d\theta &= \int_{\underline{\theta}}^{\widehat{\theta}_\tau} \widehat{L}(\theta, \varphi)\, d\theta. \end{aligned}$$

Replacing $L(\theta,\varphi)$ using (3.13), multiplying both equations by a constant, we have

$$\begin{aligned}
\tau\int_{\underline{\theta}}^{\overline{\theta}} B(\theta)\frac{W(M(J(\underline{\theta},\varphi)))}{W(M(J(\theta,\varphi)))}d\theta &= \int_{\underline{\theta}}^{\theta_\tau} B(\theta)\frac{W(M(J(\underline{\theta},\varphi)))}{W(M(J(\theta,\varphi)))}d\theta,\\
\tau\int_{\underline{\theta}}^{\overline{\theta}} B(\theta)\frac{\widehat{W}\left(\widehat{M}(J(\underline{\theta},\varphi))\right)}{\widehat{W}\left(\widehat{M}(J(\theta,\varphi))\right)}d\theta &= \int_{\underline{\theta}}^{\widehat{\theta}_\tau} B(\theta)\frac{\widehat{W}\left(\widehat{M}(J(\underline{\theta},\varphi))\right)}{\widehat{W}\left(\widehat{M}(J(\theta,\varphi))\right)}d\theta.
\end{aligned}$$

Taking the difference of the two equations,

$$\tau\int_{\underline{\theta}}^{\overline{\theta}} B(\theta)g(\theta)\,d\theta = \int_{\underline{\theta}}^{\theta_\tau} B(\theta)g(\theta)\,d\theta - \int_{\theta_\tau}^{\widehat{\theta}_\tau} B(\theta)\frac{\widehat{W}\left(\widehat{M}(J(\underline{\theta},\varphi))\right)}{\widehat{W}\left(\widehat{M}(J(\theta,\varphi))\right)}d\theta,$$

where $g(\theta) = \frac{W(M(J(\underline{\theta},\varphi)))}{W(M(J(\theta,\varphi)))} - \frac{\widehat{W}(\widehat{M}(J(\underline{\theta},\varphi)))}{\widehat{W}(\widehat{M}(J(\theta,\varphi)))}$. Therefore, to show $\theta_\tau > \widehat{\theta}_\tau$, we only need to establish

$$\tau\int_{\underline{\theta}}^{\overline{\theta}} B(\theta)g(\theta)\,d\theta > \int_{\underline{\theta}}^{\theta_\tau} B(\theta)g(\theta)\,d\theta.$$

But from Lemma 4, we know that if φ is such that $J\left(\overline{\theta},\varphi\right) \leq j_0$, we have

$$\frac{W(M(J(\underline{\theta},\varphi)))}{W(M(J(\theta,\varphi)))} > \frac{\widehat{W}\left(\widehat{M}(J(\underline{\theta},\varphi))\right)}{\widehat{W}\left(\widehat{M}(J(\theta,\varphi))\right)}.$$

Moreover, from the proof of Lemma 4, we know that g is increasing in θ. To conclude, define

$$s(t) = t\int_{\underline{\theta}}^{\overline{\theta}} B(\theta)g(\theta)\,d\theta - \int_{\underline{\theta}}^{\theta_t} B(\theta)g(\theta)\,d\theta.$$

Observe that $s(0) = s(1) = 0$. Differentiating s w.r.t. t,

$$s'(t) = \int_{\underline{\theta}}^{\overline{\theta}} B(\theta)g(\theta)\,d\theta - \frac{\partial\theta_t}{\partial t}B(\theta_t)g(\theta_t).$$

From the definition of θ_t, we have

$$\frac{\partial\theta_t}{\partial t}\frac{B(\theta_t)}{W(M(J(\theta_t,\varphi)))} = \int_{\underline{\theta}}^{\overline{\theta}}\frac{B(\theta)}{W(M(J(\theta,\varphi)))}d\theta.$$

Combining this with the previous equation,

$$s'(t) = \int_{\underline{\theta}}^{\overline{\theta}} B(\theta)\, g(\theta)\, d\theta - \left[\int_{\underline{\theta}}^{\overline{\theta}} \frac{B(\theta)}{W(M(J(\theta,\varphi)))} d\theta \right] W(M(J(\theta_t,\varphi)))\, g(\theta_t).$$

g, W, M and J are all increasing functions and θ_t is increasing in t. Therefore, $s'(t)$ is decreasing in t, so s is concave. Since $s(0) = s(1) = 0$, we conclude that $s(t) > 0$ for all $t \in (0,1)$. In particular, $s(\tau) > 0$, which completes the proof. The proof for firms such that $J(\underline{\theta},\varphi) \geq j_0$ is similar and omitted. □

I then prove Lemma 11.

Proof. [Proof of Lemma 11] To show that the matching function moves up, I proceed by contradiction. Suppose that there exists $j \in J(\Theta \times \Phi)$ such that $\widehat{M}(j) < M(j)$. Thanks to Lemma 8, we know that both M and $\widehat{M}$ are continuous functions such that $M\left(J\left(\underline{\theta},\underline{\varphi}\right)\right) = \underline{\sigma} = \widehat{M}\left(J\left(\underline{\theta},\underline{\varphi}\right)\right)$ and $M(J(\overline{\theta},\overline{\varphi})) = \overline{\sigma} = \widehat{M}(J(\overline{\theta},\overline{\varphi}))$. So there must exist $J\left(\underline{\theta},\underline{\varphi}\right) \leq j_1 < j_2 \leq J(\overline{\theta},\overline{\varphi})$ such that (i) $M(j_1) = \widehat{M}(j_1)$ and $M(j_2) = \widehat{M}(j_2)$; (ii) $M'(j_1) \geq \widehat{M'}(j_1)$ and $M'(j_2) \leq \widehat{M'}(j_2)$; (iii) $M(j) > \widehat{M}(j)$ for all $j \in (j_1, j_2)$.

Define $x(j) = \frac{\hat{M}'}{M'}(j)$. By the continuity, there exists $j \in (j_1, j_2)$ such that $x(j) = 1$. At any such point j, according to Equation (3.14), we have

$$\begin{aligned}
\frac{d\ln x}{dj} &= -\left(\frac{d\ln\hat{L}}{d\sigma}(\hat{M})\,\hat{M}' - \frac{d\ln L}{d\sigma}(M)\,M'\right) - \epsilon\left(\frac{d\ln\hat{W}}{d\sigma}(\hat{M})\,\hat{M}' - \frac{d\ln W}{d\sigma}(M)\,M'\right) \\
&\quad + (\epsilon-1)\left[\frac{d\ln\hat{A}}{d\sigma}(\hat{M},j)\,\hat{M}' - \frac{d\ln A}{d\sigma}(M,j)\,M' + \frac{d\ln A}{dj}(\hat{M},j) - \frac{d\ln A}{dj}(M,j)\right] \\
&= -\left(\hat{l}_\sigma(\hat{M})\,\hat{M}' - l_\sigma(M)\,M'\right) - \left(a_\sigma(\hat{M},j)\,\hat{M}' - a_\sigma(M)\,M'\right) + (\epsilon-1)\left(a_j(\hat{M},j) - a_j(M\right. \\
&= -\left(\hat{l}_\sigma(\hat{M}) - l_\sigma(M)\right)M' - \left(a_\sigma(\hat{M},j) - a_\sigma(M,j)\right)M' + (\epsilon-1)\left(a_j(\hat{M},j) - a_j(M,j)\right)
\end{aligned}$$

By (iii), $M(j) > \widehat{M}(j)$, so $a_j(\hat{M},j) - a_j(M,j) < 0$ under the log-supermodularity condition (3.5).

$$\hat{l}_\sigma(\hat{M}) > \hat{l}_\sigma(M) > l_\sigma(M)$$

, Since the matching functions are strictly increasing, condition (ii) implies

$$M'(j_1)/M'(j_2) \geq \widehat{M}'(j_1)/\widehat{M}'(j_2).$$

Replacing the derivatives of M and $\widehat{M}$ using equation (3.14), we obtain

$$\frac{W(M(j_2))}{W(M(j_1))} \geq \frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)}.$$

From equation (3.15), we have

$$\begin{aligned} \frac{W(M(j_2))}{W(M(j_1))} &= \int_{M(j_1)}^{M(j_2)} \frac{\partial \ln A}{\partial \sigma}\left(\sigma, M^{-1}(\sigma)\right) d\sigma, \\ \frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} &= \int_{\widehat{M}(j_1)}^{\widehat{M}(j_2)} \frac{\partial \ln \widehat{A}}{\partial \sigma}\left(\sigma, \widehat{M}^{-1}(\sigma)\right) d\sigma \end{aligned}$$

Combing this with condition (i) and (iii), together with the strict log-supermodularity of A and the fact that $\ln \widehat{A} - \ln A$ is increasing in σ, we obtain that $W(M(j_2))/W(M(j_1)) < \widehat{W}\left(\widehat{M}(j_2)\right)/\widehat{W}\left(\widehat{M}(j_1)\right)$, which forms a contradiction. □

Next, I prove Lemma 5.

Proof. [Proof of Lemma 5] Define $x(j) = \frac{\widehat{M}'(j)}{M'(j)}$. By Lemma 8, we know that both derivatives are positive. Moreover, we have

$$\ln x(j) = \ln \frac{\widehat{R}}{R} - \ln \frac{\widehat{W}\left(\widehat{M}(j)\right)}{W(M(j))} - \ln \frac{L\left(\widehat{M}(j)\right)}{L(M(j))}.$$

Differentiating w.r.t. j,

$$\begin{aligned} \frac{d \ln x}{dj}(j) &= -\left(\hat{l}_\sigma(\hat{M}) - l_\sigma(M)\right) M' - \left(a_\sigma(\hat{M}, j) - a_\sigma(M, j)\right) M' \\ &\quad + (\epsilon - 1)\left(a_j(\hat{M}, j) - a_j(M, j)\right). \end{aligned}$$

Two cases are possible. Either $\ln x(j)$ keeps a constant sign, in which case the claim is trivially true. Or, $\ln x(j)$ changes its sign. By continuity of h, there must be a non-empty set of jobs at which it is equal to zero. Define $j_0 = \inf(j | \ln x(j) = 0)$. From the previous argument, $\ln x(j) \leq 0$ for all $j \geq j_0$. But by definition of j_0, $\ln x$ keeps a constant sign over $j \leq j_0$, and the sign has to be positive, since $\ln x$ does not keep a constant sign over $j \geq j_0$. Therefore, $\ln x(j) \geq 0$ over $j \geq j_0$, which completes the proof.

Therefore, the wage ratio between two jobs, j_1 and j_2, are given by

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln W}{\partial \sigma}(M(j)) M'(j)\, dj.$$

Using equation (3.15), we have

$$\frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln A}{\partial \sigma}(M(j), j) M'(j)\, dj.$$

Therefore, we have

$$\frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} - \frac{W(M(j_2))}{W(M(j_1))} = \int_{j_1}^{j_2} \frac{\partial \ln \widehat{A}}{\partial \sigma}\left(\widehat{M}(j), j\right) \widehat{M'}(j) - \frac{\partial \ln A}{\partial \sigma}(M(j), j) M'(j)\, dj.$$

If $\frac{\partial^2 \ln A}{\partial \sigma^2} \geq 0$, since $\widehat{M}(j) \geq M(j)$ by Lemma 11, using condition 3.17, we have

$$\frac{\partial \ln \widehat{A}}{\partial \sigma}\left(\widehat{M}(j), j\right) > \frac{\partial \ln A}{\partial \sigma}\left(\widehat{M}(j), j\right) \geq \frac{\partial \ln A}{\partial \sigma}(M(j), j) \geq 0.$$

Moreover, we have, from what have been previously established in this proof, that if $j_1 < j_2 \leq j_0$, $\widehat{M'}(j) \geq M'(j) \geq 0$, so $\frac{\widehat{W}\left(\widehat{M}(j_2)\right)}{\widehat{W}\left(\widehat{M}(j_1)\right)} \geq \frac{W(M(j_2))}{W(M(j_1))}$. $\square$

Finally I prove Corollaries 2 and 3.

Proof. [Proof of Corollary2 and 3] The proof is analogous to that of Proposition 1 and is omitted.

Choose any $\tau \in (0, 1)$. Denote φ_τ the τ-th quantile in the wage distribution of all firms where the a firm is weighted by its number of workers. workers in this firm. φ_τ then satisfies the following identity:

$$\tau = \frac{\int_{\underline{\varphi}}^{\varphi_\tau} \int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta d\varphi}{\int_{\underline{\varphi}}^{\overline{\varphi}} \int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta d\varphi}. \tag{C.5}$$

By definition of the Lorenz curve of the between firm wages, we have

$$LC(\tau) = \frac{\int_{\underline{\varphi}}^{\varphi_\tau} W^m(\varphi) \int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta d\varphi}{\int_{\underline{\varphi}}^{\overline{\varphi}} W^m(\varphi) \int_{\underline{\theta}}^{\overline{\theta}} L(\theta, \varphi)\, d\theta d\varphi}.$$

Combining this with equation (3.13) and (C.1),

$$LC(\tau) = \int_{\underline{\varphi}}^{\varphi_\tau} \beta(\varphi)\, d\varphi.$$

Therefore, the $LC(\tau)$ becomes smaller if and only if φ_τ decreases.

In the limiting case in which $\overline{\theta} = \underline{\theta} = \theta$, equation $(C.5)$ can be re-written as

$$\tau = \frac{\int_{\underline{\varphi}}^{\varphi_\tau} L(\varphi, \theta)\, d\varphi}{\int_{\underline{\varphi}}^{\overline{\varphi}} L(\varphi, \theta)\, d\varphi}.$$

In this case, there is only one occupation in a firm and the positive assortative matching between workers and firms (instead of jobs) is complete. It then follows that, we can integrate the numerator by substitution, and the firm φ_τ always hires the worker whose skill type, $M(\varphi_\tau)$, that satisfies

$$\int_{\underline{\sigma}}^{M(\varphi_\tau)} L(\sigma)\, d\sigma = \tau \int_{\underline{\sigma}}^{\overline{\sigma}} L(\sigma)\, d\sigma.$$

Hence, $M(\varphi_\tau) = \widehat{M}(\widehat{\varphi}_\tau)$. By Lemma 11, we conclude that $\widehat{\varphi}_\tau \leq \varphi_\tau$ so the Lorenz curve shifts downward. □

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