Macroeconomics and Financial Fragility

by

Nicolás Eduardo Caramp

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Abstract

This thesis consists of three chapters.

Chapter 1 studies the interaction between the ex-ante production of assets and ex-post adverse selection in financial markets. Positive shocks that increase market liquidity and prices exacerbate the production of low-quality assets and can increase the likelihood of a financial market collapse. An increase in government bonds increases total liquidity and reduces the incentives to produce bad assets, but can exacerbate adverse selection in private asset markets. Optimal policy balances these two effects, requiring more issuances when the liquidity premium is high. I also study transaction taxes and asset purchases, showing that policy should lean against the wind of market liquidity.

Chapter 2, joint work with David Colino and Pascual Restrepo, studies how consumer durables amplify business cycle fluctuations. We show that employment in durable manufacturing industries is more cyclical than in other industries, and that this cyclicality is amplified in general equilibrium. We provide evidence of three mechanisms that generate amplification. First, employment changes propagate through input-output linkages. Second, the reduction of employment in durables negatively affects employment in non-tradable sectors. Third, workers do not completely reallocate to other less cyclical tradable industries.

Chapter 3, joint work with Dejanir Silva, studies how the level, maturity structure and characteristics of government debt affects the severity of crises and the effectiveness of stabilization policies. We find that both fiscal and monetary policies become less powerful in high debt economies, and that in response to a preference shock that pushes the economy into a liquidity trap, high debt economies experience larger and more prolonged recessions. Long-term bonds and indexed debt improve the effectiveness of stabilization policies.

Thesis Supervisor: Iván Werning
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Chapter 1

Sowing the Seeds of Financial Crises: Endogenous Asset Creation and Adverse Selection

1.1 Introduction

It is widely believed that the recession that hit the US economy in 2008 originated in the financial sector. The years previous to the crisis were characterized by a rapid increase in the private production of assets that were considered safe, mostly through securitization. Many of the markets for these assets then collapsed, marking the starting point of the deepest recession in the post-war era. The extent to which this boom sowed the seeds of the posterior crisis is an important open question. Although many scholars have pointed to adverse selection to explain the observed collapse in these markets (e.g., Kurlat (2013), Chari et al. (2014), Guerrieri and Shimer (2014a), Bigio (2015)), many important questions remain unanswered: where does the asset heterogeneity come from, how does it relate to the underlying state of the economy, and how does it interact with other sources of liquidity? These are the questions I seek to explore in this paper.

Safety refers to a characteristic of assets that are perceived as high quality, have an active (*liquid*) market, and facilitate financial transactions (as collateral or media of exchange more generally).[1] While traditionally this characteristic was mostly

[1] This has been recently emphasized, for instance, by Calvo (2013), Gorton et al. (2012) and Gorton (2016).

limited to government bonds and bank deposits, in the last 30 years there has been a large increase in the use of other privately produced assets, such as asset- and mortgage-backed securities.[2] Securitization was the instrument used by the private sector to provide the market with the safe assets it was demanding. This expanded the type of loans that were made, and riskier and more opaque borrowers were accepted. This process was particularly stark in the mortgage market, which saw an explosion of non-standard, low-documentation mortgages and low credit score borrowers.[3] In fact, Bank for International Settlements (2001) articulated an early warning about the deterioration of the quality of assets used as collateral.

This paper presents a theory of asset quality determination in which the ex-ante production of assets interacts with ex-post adverse selection in financial markets. Assets in the economy derive their value from the dividends they pay and the liquidity services they provide. Better quality assets pay higher dividends, but because of adverse selection in markets, they sell at a pooling price with lower-quality assets. This cross-subsidization between high- and low-quality assets introduces a motive for agents to produce relatively more lemons when they expect prices to be high, since they expect to sell the assets rather than keep them until maturity. As a consequence, the theory predicts that the production of low-quality assets is more responsive to market conditions than that of high-quality assets. Therefore, shocks that improve the functioning of financial markets exacerbate the production of lemons and may even increase the exposure of the economy to a financial market collapse–a process that disrupts liquidity.

Moreover, the supplies of privately produced tradable assets and government bonds (private and public liquidity) interact through the liquidity premium. When the supply of public liquidity is low, the private sector's incentives to produce close substitutes increase.[4] But because low-quality assets are more sensitive to changes in the value of liquidity services, their production increases proportionally more, reducing the average quality composition in the economy. Indeed, my model predicts that the reductions in US government bonds in the late 90s due to sustained

[2]See Gorton et al. (2012).

[3]See Ashcraft and Schuermann (2008). While origination of non-agency mortgages (subprime, Alt-A and Jumbo) were \$680 billion in 2001, they increased in 2006 to \$1,480 billion, a 118% growth. On the other hand, origination of agency (prime) mortgages decreased by 27%, from \$1443 billion in 2001 to \$1040 billion in 2006. Moreover, while only 35% of non-agency mortgages were securitized in 2001, that figure grew to 77% in 2006.

[4]This channel has been found empirically, for example, by Greenwood et al. (2015) and Krishnamurthy and Vissing-Jorgensen (2015).

fiscal surpluses, as well as the increased foreign demand for US-produced safe assets in the early 2000s (a consequence of the so called "savings glut"), both generated perverse effects on the quality composition of privately produced assets.[5]

While the theory presented is silent about the specifics of the asset production process, I believe that the economic forces that it highlights are typical of the full process of transforming illiquid assets into liquid ones. In my interpretation, the production process constitutes both the origination process of loans (e.g., mortgages) and the posterior securitization process (e.g., AAA-rated private-label mortgage-backed securities).[6] In both cases the "producers" know more than other market participants about the underlying quality of these products, either because they have collected information that cannot be credibly transmitted, or because they know how much effort they put into the process. Hence, the problem of quality production and adverse selection can be present in the whole intermediation chain.

The mechanics of the model hinge upon the behavior of the shadow valuation of different qualities. Suppose there are only two qualities. Agents with high-quality assets sell them only if their liquidity needs are high relative to the price discount they suffer in the market due to the adverse selection problem. In contrast, agents with low-quality assets always sell their holdings. Anticipating that this will be their strategy in the market, agents adjust their quality production decisions to the expected market conditions. If the market's expectations are high–in the sense that volume traded is high–agents anticipate that the probability they will sell their assets is relatively high, independent of the quality of those assets. In this case, more low-quality assets are produced because it is less attractive to exert effort to produce high-quality assets. That is, low-quality assets are produced for *speculative motives*: not for their fundamental value but for the profit the agent can make just from selling in the market. In this sense, good times can sow the seeds of a future crisis by providing incentives that lead to asset quality deterioration.

I consider two comparative statics that improve the functioning of financial

[5] See Caballero (2006) and Caballero (2010) for a discussion on safe asset shortages.

[6] An important issue is the role of tranching in avoiding adverse selection. In my opinion there are two reasons why tranching can have a limited effect. First, if the balance sheets of financial intermediaries are difficult to monitor, then intermediaries can always go back to the market to sell any remaining fraction of assets. Second, certification by third parties (e.g., rating agencies) can have limited success if players learn how to game the rating models or if the incentives of the third party are compromised.

markets: an improvement in the expected payoff of bad assets (or a reduction in their default probability) and an increase in liquidity needs (which can derive from increased productivity in the real economy or changes in the supply of public liquidity). I show that in both cases there is an increase in the production of assets and a deterioration of the asset quality composition, which can even lead to an increased exposure of the economy to a financial market collapse. While the direct effect of the exogenous shocks tends to *increase financial stability*, the endogenous response of the economy through a worsening of the asset quality composition tends to *increase financial fragility*. To understand the importance of this result, consider what would happen if the asset quality distribution were exogenously given. A positive shock would improve market conditions, which would increase the volume traded and equilibrium prices. Since this quality composition would be fixed, the result would be an unambiguous reduction in the probability that the market would collapse. Hence, when asset quality is exogenous, positive shocks increase financial stability. However, when agents can react to the improved conditions of the market, the quality distribution deteriorates, which is a force that increases financial fragility. Which effect dominates depends on the relative size of each force. Moreover, I show that if the shock is transitory, financial fragility increases as the shock dies out, whatever its effect on impact. Hence, a boom can set the stage for a financial crisis.

I also consider the effects of reducing transaction costs. Financial innovation can reduce the cost of trading financial assets by facilitating the transformation of illiquid assets (e.g., mortgages) into liquid ones (e.g., MBS, ABS, CDOs). I show that if transaction costs are high, then the market for these assets remains inactive and agents produce only high-quality assets. As transaction costs decrease, agents who have sufficiently high liquidity needs find it optimal to sell their assets. Interestingly, while transaction costs remain relatively high, the presence of a secondary market is not enough to attract the production of low-quality assets. Therefore, while transaction costs remain at middle-range levels, the economy features a market for assets in which a low volume is traded and only high-quality assets are produced. Lastly, when transaction costs are sufficiently low, the production of low-quality assets becomes profitable and the economy can enter into a state in which high volumes are traded but with significant financial risk. These dynamics are consistent with developments of the last 30 years in the US economy, wherein the early stages of financial innovation could have improved the efficiency of the economy with no increased exposure to risk, but further innovation could have created perverse effects during the early 2000s, that culminated in a complete financial collapse in 2008.

On a more technical note, I show that a large amplification mechanism is present in the model. Due to the interaction between asset quality production and markets that suffer from adverse selection, prices might not be able to perform their role of clearing markets and guiding incentives. Suppose that the payoff of low-quality assets is distributed uniformly with bounds given by ϵ of distance around a mean. I show that there is a positive measure of parameter values such that as ϵ goes to zero (that is, the exogenous risk goes to zero), the endogenous risk of the economy remains positive and bounded away from zero. This is so because of the discontinuity of market prices to state variables in the presence of adverse selection. As the exogenous risk vanishes, the fundamentals of the economy in all states of nature become very similar. However, it can happen that similar prices in all states do not give the right incentives to agents during the production stage, when they make their investment decisions. If prices are low in all states, then agents have low incentives to produce low-quality assets, which is inconsistent with prices being low. On the other hand, if prices are high, the incentives to produce low-quality assets can be too high, which is inconsistent with prices being high. A fixed-point type of logic would argue for middle-range prices. However, these prices can be inconsistent with market clearing, because of the discontinuity of equilibrium market prices. Endogenous risk *convexifies* the expected prices, so that while prices clear the markets, risk adjusts incentives during the production stage. As I demonstrate, the limit of an economy that has vanishing exogenous aggregate risk is the unique equilibrium of an economy that has no exogenous aggregate risk but does have sunspots.

Another important determinant of the dynamics of privately produced safe assets is the supply of public liquidity. A significant number of recent papers document that private production of safe assets increases when the supply of government bonds is low (and vice-versa). Gorton et al. (2012) and Krishnamurthy and Vissing-Jorgensen (2015) show that the supply of government bonds and the production of private substitutes in general are negatively correlated. Greenwood et al. (2015) find a negative correlation between the supply of US Treasuries and the supply of unsecured financial commercial papers, while Sunderam (2015) finds a similar result with respect to asset-backed commercial papers. Krishnamurthy and Vissing-Jorgensen (2012) shows that an increase in the supply of government bonds reduces the liquidity premium.

In my model, a higher volume of bonds increases the liquidity in the economy, which decreases the liquidity premium. As a consequence, government bonds crowd out private liquidity, which disproportionally reduces the incentives to produce low-quality assets. Hence, a shortage of safe assets induces a deterioration of private asset quality. This result seems to suggest that the government should provide all the liquidity the financial sector requires (a type of *Friedman Rule* applied to this setting). This appealing solution separates the liquidity value of assets from their dividend value so that assets are produced only for fundamental reasons. However, this policy might not be feasible for two reasons. First, the fiscal costs associated with it are likely to be large. Second, even if costs were low, there is no guarantee that the government bonds would end up in the hands of those who needed them the most, since agents with good investment opportunities would not purchase bonds. These two factors indicate why securitization can have social value: it allows investors to mitigate the trade-off they face between undertaking investment opportunities and keeping enough liquidity to satisfy future needs. Hence, any feasible intervention would tend to complement the private markets rather than replace them. In such a case, the government faces a subtle trade-off. On the one hand, it wants to provide the agents with the liquidity they need and reduce the production of bad assets. On the other hand, by crowding out the private markets, the government could exacerbate the adverse selection problem, since agents are less willing to sell their good assets at a discount to satisfy their liquidity needs, which are partly satisfied by government bonds. In the extreme case in which the quality distribution is exogenous, the presence of government bonds unambiguously increases the adverse selection problem and, consequently, fragility. That said, if the production elasticity of bad trees is high, government bonds can increase stability. Nonetheless, I find that the government should issue more bonds when the liquidity premium is high and less when the liquidity premium is low.

Ex-post policies could also be used. Tirole (2012) and Philippon and Skreta (2012) study how to restart a market that has collapsed because of adverse selection. In the optimal policy, the government buys from some agents assets that could be of the worst quality. From an ex-ante perpective, the anticipation of such policies exacerbates the production of lemons in the economy. To compensate for this, the government could tax financial transactions (and hence, lower market liquidity) in high-liquidity states.[7]

Literature Review. This paper is most closely related to the literature that incorporates adverse selection in financial markets into macroeconomic models. Recently, adverse selection in financial markets has been invoked to explain certain phenomena experienced during the Great Recession, including the sudden collapse of the market of assets collateralized by mortgage related products. The work of Eisfeldt (2004), Kurlat (2013) and Bigio (2015) exemplifies this literature. They build dynamic general equilibrium models in which agents trade assets under asymmetric information in order to obtain the resources to satisfy some liquidity needs (fund investment projects in the case of Eisfeldt (2004) and Kurlat (2013), and obtaining working capital in Bigio (2015)). They show that adverse selection in financial markets can be an important source of amplification of exogenous shocks. In particular, Bigio (2015) demonstrates that adverse selection quantitatively explains the dynamics of the economy during the Great Recession. However, all of these papers take the distribution of asset quality as exogenously given. This paper builds on these insights but, taking a step back, it focuses on the endogenous determination of asset quality distribution. This extension is key to understanding the build-ups of risks emphasized in these papers. Also in this literature, Guerrieri and Shimer (2014a) and Chari et al. (2014) study similar economies but under the assumption that markets are exclusive. However, they also assume that the quality distribution is exogenous.

Also relevant is Gorton and Ordoñez (2014), who study a dynamic model of credit booms and busts that emphasizes the information-insensitivity of assets that serve as collateral and, second, how changes in the incentives to produce information about the quality of the underlying assets can trigger a crisis. In contrast, I demonstrate that positive shocks play a role in reducing the incentives to produce good quality assets. Gorton and Ordonez (2013) also study the interaction between public and private liquidity, but their focus is in the production of infor-

[7] This *leaning against the wind* logic for policy is similar to Diamond and Rajan (2012) with respect to monetary policy.

mation, whereas my model highlights the liquidity premium and the production of quality. In contrast to Gorton and Ordonez (2013), I find that government bonds have an ambiguous effect on the economy, and they can even increase financial fragility because they increase the adverse selection problem in private markets.

On the normative side, the focus has been on the problem of how to deal with markets that collapsed. Tirole (2012) and Philippon and Skreta (2012), who take an ex-post point of view,[8] ask how markets that have suffered from adverse selection can be efficiently restored. My paper, which adopts an ex-ante perspective, studies two sets of policy instruments: government bonds and transaction taxes and subsidies (or asset purchase programs).

In addition, there is an empirical literature that tries to measure the extent of adverse selection in financial markets. Keys et al. (2010) use a regression discontinuity approach to ask whether the quality of loans that had a lower probability of being securitized was higher than those that had a higher probability, and they find that it was. Loans with a low probability of being securitized were about 10–25% less likely to default than similar loans that had a higher probability of being securitized. This suggests that originators most carefully screened the loans they were most likely to keep. Other papers that show that asymmetric information could have been relevant in financial markets before the crisis include Demiroglu and James (2012), Downing et al. (2009), Krainer and Laderman (2014), and Piskorski et al. (2015).

Closest in theme and content to this paper is Neuhann (2016). In his independently developed model, bankers produce loans that are subject to aggregate risk. Because their funding ability is constrained by their net worth and their risk exposure, a secondary market for loans allows them to reduce their risk exposure and ultimately increase lending. The price in the market depends on the wealth in the hands of the buyers, so that when buyers' net worth is high, the market price is high enough to prompt to some bankers to begin originating low-quality assets. Therefore, investment efficiency falls. When a negative shock hits the economy, low-quality assets default and buyers' wealth contracts, which makes the secondary market collapse. My paper takes a different approach. In my setup, the buyers' wealth channel is absent. I highlight the importance of the economy's fundamentals and the liquidity premium. I show that asset quality deteriorates

[8]Tirole (2012) presents an ex-ante analysis but does not study the possibility of manipulating incentives through a combination of taxes and subsidies in different states of the economy.

after positive shocks, such as an increase in the fundamental value of low-quality assets, a reduction in trading costs, or an increase in the productivity in the real economy, and after a reduction in the supply of government bonds. This difference is important for our normative analysis. In contrast to Neuhann (2016), who argues that the growth of the buyer's net worth should be controlled, I study the optimal supply of government bond and transaction taxes (and subsidies).

This paper also contributes to the literature that emphasizes the role played by public liquidity in the facilitation of financial transactions. Woodford (1990) shows that when agents face binding borrowing constraints, a higher supply of government bonds can increase welfare. Government bonds supply the agents with the instruments necessary to respond to variations in income and spending opportunities through trade in secondary markets, which improves the allocation of resources. Holmström and Tirole (1998) also highlight the role of tradable instruments when agents cannot fully pledge their future income. They demonstrate that government bonds can complement private liquidity when the latter is not sufficient to satisfy all of the demand.

Gorton et al. (2012), Greenwood et al. (2015), Sunderam (2015), and Krishnamurthy and Vissing-Jorgensen (2015) document the negative relation between the private and public supply of money-like assets. Finally, Moreira and Savov (2016) emphasize the role of "shadow-banking" in supplying "money-like" assets. They show that "shadow-money" allows for higher growth but exposes the economy to aggregate risk. In this case, however, there is no asymmetric information problem in the economy.

Outline. The rest of the paper is organized as follows. In section 2, I present a simple three-period model that features linear demand for liquidity to show the main forces of the model and study its positive implications. Section 3 extends the basic model to incorporate decreasing returns to liquidity, and it analyzes the interaction between the real economy and financial markets. Section 4 studies the effects of government bonds on the production of private assets. It also explores the role of transaction taxes and subsidies. In section 5, I extend the model to an infinite horizon setting. Section 6 concludes. All the proofs are presented in the appendix.

1.2 Basic Model

In this section I present a simple three-period model that highlights the main forces of the economy. In the first period, agents choose the quality of the assets they produce anticipating that in the future they will face a "liquidity shock" that affects their intertemporal preferences for consumption, and a market for assets that suffers from adverse selection.

1.2.1 The Environment

Agents. There are three dates, 0, 1, and 2, and two types of goods: non-storable final consumption good, and Lucas (1978) trees. The economy is populated by a measure one of agents, $i \in [0,1]$. Agents receive an endowment of final consumption good of W_0 in period 0, and W_1 in period 1.[9] In period 0 they operate a technology that transforms final consumption goods into trees, which pay a dividend in period 2.

Agents' preferences are given by

$$U = d_0 + E_0 \left[\mu_1 d_1 + d_2\right],$$

where d_t is consumption in $t = 0, 1, 2$, μ_1 is a random idiosyncratic "liquidity shock" (uncorrelated across agents), which is private information of the agents, and the expectation is taken with respect to μ_1 and an aggregate state of the economy, described below. The liquidity shock affects the agent's marginal utility of consumption in period 1. From period 0 point of view, μ_1 is distributed according to the cumulative distribution function $G(\mu_1)$ in $[1, \mu^{\max}]$. I assume that G is such that with probability π, $\mu_1 = 1$, and with probability $1-\pi$, μ_1 has a continuous cumulative distribution G_μ in $[1, \mu^{\max}]$. The mass of probability in $\mu_1 = 1$ simplifies the analysis of equilibrium prices below. In the extension of the model presented in the next section, π arises endogenously in equilibrium.

Technology. Agents have access to a technology to produce trees in period 0. This technology is idiosyncratic to each agent. There are two types of trees. An agent of type ξ can transform $q_G(\xi)$ units of the consumption good into 1 unit of

[9] I assume that all agents receive the same endowment. As I show later, this assumption is without loss of generality.

high quality, "good", tree (denoted by G), and $q_B(\xi)$ units of the consumption good into 1 unit of low quality, "bad", tree (denoted by B), and ξ is distributed in the population uniformly in $[0,1]$.[10] The distribution of liquidity shocks in the population is independent of the types in period 0, ξ. I make the following assumptions.

Assumption 1. *The functions $q_G(\xi)$ and $q_B(\xi)$ are such that*

1. *$q_G(\xi)$ and $q_B(\xi)$ are continuous and increasing in ξ, with $q_G(0) = q_B(0) = 1$,*
2. *$q_G(\xi) \geq q_B(\xi)$ for all ξ,*
3. *$\frac{q_G(\xi)}{q_B(\xi)}$ is increasing in ξ.*

The first assumption implies that the cost of producing each type of tree is perfectly positively correlated, and that the agent with the lowest cost faces the same cost of producing good and bad trees (normalized to 1). The second assumption implies that producing bad trees is cheaper than producing good trees for every agent, which is needed so that bad trees have a chance of being produced. Finally, the third assumption implies that the cost of producing good trees grows faster than the cost of producing bad trees. That is, high (low) cost agents have a comparative advantage in producing bad (good) trees. Thus, one can interpret $q_B(\xi)$ as the *efficiency type* of the agent, and the difference $q_G(\xi) - q_B(\xi)$ as the *effort cost* required to obtain a good tree. Thus, for less efficient agents, the cost of increasing the quality of the tree produced is higher. Below, I discuss the robustness of my results to these assumptions.

Trees deliver fruits in final consumption good in period 2. A unit of good tree pays Z with certainty at maturity. On the other hand, only a fraction α of bad trees deliver fruit in period 2, so that one unit of bad tree in period 0 pays αZ in period 2.

[10] This assumption is WLOG since ξ affects the economy only through q_G and q_B. In particular, for any continuous cumulative distribution function $\Omega(\tilde{\xi})$ with support in $[0,1]$ and associated density $\omega(\tilde{\xi})$, and differentiable functions $\tilde{q}_G$ and $\tilde{q}_B$ satisfying Assumption 1, it is possible to find differentiable functions q_G and q_B such that the distributions of $\tilde{q}_G$ and $\tilde{q}_B$ under $\tilde{\xi}$ coincide with the distribution of q_G and q_B under $\xi \sim U[0,1]$:

$$Prob(\tilde{q}_j(\tilde{\xi}) \leq \overline{q}) = \int_1^{\tilde{q}_j^{-1}(\overline{q})} \omega(\tilde{\xi}) d\tilde{\xi} = \int_1^{q_j^{-1}(\overline{q})} d\xi = Prob(q_j(\xi) \leq \overline{q})$$

if and only if q_j satisfies

$$\frac{\omega(\tilde{q}_j^{-1}(\overline{q}))}{\tilde{q}_j'(\tilde{q}_j^{-1}(\overline{q}))} = \frac{1}{q_j'(q_j^{-1}(\overline{q}))},$$

for $j \in \{G, B\}$.

The fraction of bad trees that deliver fruit is known one period in advance. Thus, in period 1 the fraction α is common knowledge. However, in period 0 agents believe that α is a random variable distributed according to the cumulative distribution function F in the interval $[\underline{\alpha}, \overline{\alpha}] \subseteq [0,1]$. One can interpret α as an aggregate shock to the productivity of bad trees, so that higher α implies higher quality of bad trees, or $1 - \alpha$ as a default rate of bad trees in period 2. Initially I assume that F is continuous and non-degenerate. I analyze what happens if this assumption is violated later in this section.

Finally, I assume that the investment opportunities are private information of the agents. Moreover, only the owner of a tree can determine its quality. These elements will be important when I describe the financial markets below.

Denote by H_t^G and H_t^B the total amount of good and bad trees in the economy in period t, respectively. Let λ_t^E denote the fraction of good trees in the economy in t, that is $\lambda_t^E \equiv \frac{H_t^G}{H_t^G + H_t^B}$.

Financial Markets. Due to the idiosyncratic liquidity shocks in period 1, there are gains from trade among agents. I assume that financial markets are incomplete. In particular, I limit the financial markets to trade of existing trees. This market is meant to be a metaphor of collateralized debt markets, like "repos" or short-term commercial paper.[11]

I follow Kurlat (2013) and Bigio (2015) and assume that there is one market in which trees are traded, that buyers cannot distinguish the quality of a specific unit of tree but can predict what fraction of each type there is in this market, and that the market is anonymous, non-exclusive and competitive. These assumptions imply that the market features a pooling price, P_1^M.[12] Buyers get a diversified pool of trees from the market, where λ_1^M is the fraction of good trees in the pool. Note that since agents don't hold any trees initially, there is no trade in period 0.

In order to make the distinction between good and bad trees stark, I make the following assumptions.

Assumption 2. *The expected payoff of each type of tree satisfies*

[11] Bigio (2015) presents an equivalence result between a market for trading assets and a repo contract when there is no cost of defaulting besides delivering the collateral to the creditor. This is a standard assumption in papers of collateralized debt. See for example Geanakoplos (2010) and Simsek (2013).

[12] There is a literature that assumes exclusive markets and assets of different qualities can trade at different prices. See for example Chari et al. (2014) and Guerrieri and Shimer (2014a).

1. $Z > 1 = q_G(0)$,

2. $E[\mu_1 \alpha Z] < 1 = q_B(0)$,

3. $E[\mu_1 Z] < q_B(1) < q_G(1)$.

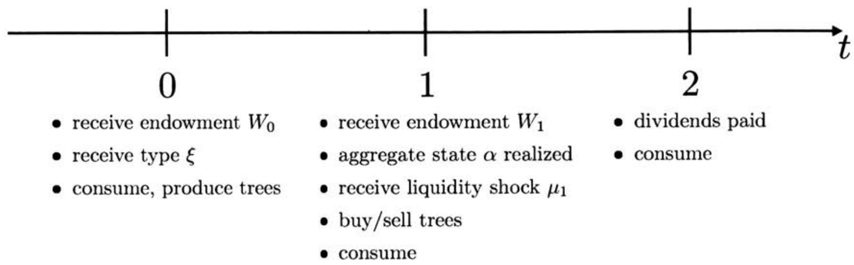
The first assumption states that at least some agents will always find it profitable to produce good trees, even if there were no market for trees in period 1. The second assumption states that if the quality of trees was observable in the market, the net present value of bad trees would be lower than the production cost of the most efficient agent. This implies that in an economy with perfect information bad trees would never be produced. The third assumptions implies that the agents with the highest costs do not produce trees.

Aggregate State and Timing. In period 1, the exogenous state of the economy is given by the distribution of liquidity shocks in the population and the realized quality of bad trees, α. The endogenous state is given by the cross-section distribution of trees and shocks across agents. Hence, the aggregate state of the economy in period 1 is $X_1 \equiv \{\alpha, \Gamma_1\} \in \mathbb{X}_1$, where $\Gamma_1(h_G, h_B, \mu_1)$ is the cumulative distribution of agents over holdings of each type of tree and liquidity shocks. In period 2, the state of the economy is given by the quality of bad trees in the current period, and the cross-section distribution of trees across agents, $X_2 \equiv \{\alpha, \Gamma_2\} \in \mathbb{X}_2$, where $\Gamma_2(h_G, h_B)$ is the cumulative distribution of agents over holdings of each type of tree.

To summarize, the timing of the economy is as follows. Agents start period 0 with an endowment of final consumption good W_0. At the beginning of the period they are assigned a type, indexed by ξ, which determines their cost of producing trees of different qualities. Given the production costs they face, agents decide whether to produce trees, and in case they do, of what quality, or consume.

In period 1, agents receive an endowment of final consumption good W_1. The aggregate shock α is realized, and agents receive an idiosyncratic liquidity shock. Since some agents may hold trees that they produced in period 0, the secondary market in period 1 may be active. Agents choose among two possible uses of the consumption goods they hold, which I call *liquid wealth*: consume or buy trees in the secondary market.

Figure 1-1: Timing



Finally, in period 2 all good trees pay Z, a fraction α of bad trees pays Z, and agents consume. Figure 1-1 summarizes the timing.

I find the equilibrium of this economy in steps. First, I solve the agents' problem. I show that the policy functions are linear in both the quantity of good and bad trees. This implies an aggregation result by which equilibrium prices and aggregate quantities are independent of the portfolio distribution of the agents in period 1. Second, I study the market for trees in period 1 and define a partial equilibrium for this market, which is an intermediate step for solving the full equilibrium of the economy. I show that finding an equilibrium of the economy simplifies to solving a fixed point problem in the fraction of good trees in the economy in period 1, λ_1^E. Finally, I study the equilibrium properties of the model and some comparative statics.

1.2.2 Agents' Problem

The problem the agents face in period 2 is very simple. They just collect the dividends from the trees they own and consume. Their value function is given by

$$V_2(h_G, h_B; X_2) = Zh_G + \alpha Z h_B, \tag{P2}$$

where h_G and h_B are their holdings of good and bad trees, respectively.

Let's turn to period 1. Denote purchases of trees in the secondary market by m. If an agent buys m units of trees, a fraction λ_1^M of them is good, while a fraction $1 - \lambda_1^M$ is bad. Let s_B denote sales of bad trees and s_G sales of good trees. The agents' problem in state X_1 is given by:

$$V_1(h_G, h_B; \mu_1, X_1) = \max_{\substack{d, m, s_G, s_B, \\ h'_G, h'_B}} \mu_1 d + V_2(h'_G, h'_B; X_2), \tag{P1}$$

subject to

$$d + P_1^M(X_1)(m - s_G - s_B) \leq W_1, \tag{1.1}$$

$$h'_G = h_G + \lambda_1^M(X_1)m - s_G, \tag{1.2}$$

$$h'_B = h_B + (1 - \lambda_1^M(X_1))m - s_B, \tag{1.3}$$

$$d \geq 0, \quad m \geq 0, \quad s_G \in [0, h_G], \quad s_B \in [0, h_B].$$

Constraint (1.1) is the agent's budget constraint, which states that consumption plus net purchases of trees cannot be larger than the endowment W_1. Constraints (1.2) and (1.3) are the laws of motion of good and bad trees respectively, which are given by the agents' initial holdings of trees plus a fraction of the purchases they make (where the fraction is given by the market composition of each type) minus the sales they make.

Given the linearity of the budget constraint and the utility function, both in current consumption, d, and the holdings of each type of trees for period 2, h'_G and h'_B, the agents' decisions are characterized by two thresholds on μ_1: μ_1^B, that determines when to consume or buy trees, and μ_1^S, that determines when to sell good trees.

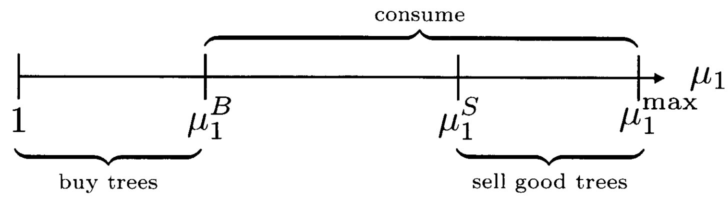
Lemma 1 (Agents' Choice in Period 1). *Consider an agent with liquidity shock μ_1. There exist thresholds μ_1^B and μ_1^S that may depend on the state of the economy, X_1, such that*

- *if $\mu_1 \leq \mu_1^B$, then the agent buys trees ($m > 0$), keeps all his good trees ($s_G = 0$), and if $\mu_1 < \mu_1^B$ his consumption in period 1 is zero ($d = 0$);*
- *if $\mu_1 > \mu_1^B$ and $\mu_1 \leq \mu_1^S$, then the agent's consumption in period 1 is positive ($d > 0$), and he does not buy trees nor sell good trees ($m = s_G = 0$);*
- *if $\mu_1 > \mu_1^S$, then the agent's consumption is positive ($d > 0$), his purchases of trees are zero ($m = 0$), and he sells all his good trees in order to consume the proceeds ($s_G = h_G$).*

All agents always sell their holdings of bad trees, i.e. $s_B = h_B$. If π is sufficiently big, then $\mu_1^B = 1$.

The result in Lemma 1 is fairly straightforward. First, all agents sell their holdings of bad trees because there is an arbitrage opportunity. By selling one unit of

Figure 1-2: Agents' Choice in period 1



bad tree they get P_1^M units of final good, which they can use to buy trees in the secondary market to obtain λ_1^M units of good trees and $1 - \lambda_1^M$ units of bad trees. Since $\lambda_1^M \in [0, 1]$, this strategy is always weakly optimal.[13] Second, the return from buying trees in the market is given by $\mu_1^B \equiv \frac{\lambda_1^M Z + (1-\lambda_1^M)\alpha Z}{P_1^M}$, which is the same for all agents. Because the utility from consuming in period 1 and the return from the market are both linear, agents just compare μ_1 and μ_1^B to decide whether to use their liquid wealth to consume or to buy trees. If $\mu_1 > \mu_1^B$ agents strictly prefer to consume, while they prefer to buy trees if $\mu_1 < \mu_1^B$. If π is sufficiently big, there are enough agents with $\mu_1 = 1$ so that they have enough wealth to purchase all the trees in the market, pushing the market price up until the return is equal to 1. In what follows, I will proceed under the assumption that $\mu_1^B = 1$. Note that in this case, μ_1 is also the marginal utility of *liquid wealth*, that is, the marginal utility of holding an extra unit of final consumption good, in contrast to just holding wealth in illiquid form, like trees.

The decision to sell good trees involves similar calculations. The market price of trees is always below the fundamental value of good trees, Z. This implies that the market price is lower than the payoff the agent would obtain if he kept the good tree until maturity. Hence, the only reason the agent would sell his good trees is if the utility derived from consuming in period 1 instead of period 2 compensates for the loss. This happens if $\mu_1 \geq \mu_1^S$, where $\mu_1^S \equiv \frac{Z}{P_1^M} \geq \mu_1^B$. Figure 1-2 summarizes these choices.

An important result that will greatly simplify the analysis that follows is the linearity of the agents' value function with respect to their holdings of each type of tree.

Lemma 2. *The value function in period* 1, $V_1(h_G, h_B; \mu_1, X_1)$, *is linear in each type of*

[13] Note that the arbitrage opportunity is independent of the price level. It does not rely on the market price being higher than the bad trees fundamental value αZ, but on the fact that the market composition cannot be worse than getting only bad trees.

tree:

$$V_1(h_G, h_b; \mu_1, X_1) = \mu_1 W_1 + \tilde{\gamma}_1^G(\mu_1, X_1)h_G + \tilde{\gamma}_1^B(\mu_1, X_1)h_B,$$

where

$$\tilde{\gamma}_1^G(\mu_1, X_1) = \max\{\mu_1 P_1^M(X_1), Z\}, \tag{1.4}$$

$$\tilde{\gamma}_1^B(\mu_1, X_1) = \mu_1 P_1^M(X_1). \tag{1.5}$$

This result follows directly from the linearity of the objective function and the budget constraint, and it assumes that π is high enough so that $\mu_1^B(X_1) = 1$. For the agents, the marginal utility of an extra unit of consumption good is given by μ_1. If $\mu_1 > 1$, this utility comes from consuming in period 1. If $\mu_1 = 1$, the agent is indifferent between consuming in period 1 and buying trees in order to consume in period 2, which generates marginal utility of 1. Since agents always sell their holdings of bad trees, their liquid wealth is no less than $W_1 + P_1^M(X_1)h_B$. As described above, agents might not be willing to sell their good trees unless their preference for consumption in period 1 is high enough. By selling a unit of good tree and consuming the proceeds, the agent gets $\mu_1 P_1^M(X_1)$ in period 1. On the other hand, by keeping the tree until maturity, the agents gets Z in period 2. Since there is no extra discounting between periods 1 and 2, the value of an extra unit of good tree is given by $\max\{\mu_1 P_1^M(X_1), Z\}$. Note that the coefficient on bad trees does not directly depend on its payoff in period 2. This is because no agent that *starts* the period owning bad trees holds them until maturity.

As a consequence of the linearity of the value function, prices and aggregate quantities do not depend on the distribution of portfolios in the population. Therefore, the relevant state in periods 1 and 2 is $X = \{\lambda_1^E, H_1, \alpha\} \in \mathbb{X}$, where $H_1 \equiv H_1^G + H_1^B$.

Finally, the problem of an agent in period 0 is given by

$$V_0(\xi) = \max_{\substack{d, i_G, i_B, \\ h'_G, h'_B}} d + E_0[V_1(h'_G, h'_B; \mu_1, X)], \tag{P0}$$

subject to

$$d + q_G(\xi)i_G + q_B(\xi)i_B \leq W_0, \tag{1.6}$$

$$h'_G = i_G, \quad h'_B = i_B. \tag{1.7}$$

$$d \geq 0, \quad i_G \geq 0, \quad i_B \geq 0.$$

Constraint (1.6) is the agent's budget constraint, which states that consumption plus expenditures in the production of trees cannot be larger than the endowment W_0, and constraint (1.7) are the laws of motion of good and bad trees respectively, which are simply given by the investment agents make.

In order to decide whether to invest or not, agents compare their cost of production and their shadow valuation of holding trees in period 1, with the utility they get from period 0 consumption, which is equal to 1. Next, I define the shadow values of trees in this economy.

Definition 1 (Shadow Values of Trees). *The shadow values of trees are given by*

$$\gamma_0^G \equiv E_0 \left[\tilde{\gamma}_1^G(\mu_1, X) \right] = E_0 \left[\max \left\{ \mu_1 P_1^M(X), Z \right\} \right],$$
$$\gamma_0^B \equiv E_0 \left[\tilde{\gamma}_1^B(\mu_1, X) \right] = E_0 \left[\mu_1 P_1^M(X) \right].$$

The shadow values of trees are just the expected values of the marginal utility of each type of tree in period 1, given by (1.4) and (1.5). They can be decomposed into three different elements: a fundamental value, a liquidity premium, and an adverse selection tax/subsidy. That is:

$$\gamma_0^G = E[\underbrace{Z}_{\text{fund. value}} + \underbrace{(\mu_1 - 1)Z}_{\text{liq. premium}} - \min\{\mu_1 \underbrace{(Z - P_1^M(X))}_{\text{adv. sel. tax}}, (\mu_1 - 1)Z\}], \tag{1.8}$$

$$\gamma_0^B = E[\underbrace{\alpha Z}_{\text{fund. value}} + \underbrace{(\mu_1 - 1)\alpha Z}_{\text{liq. premium}} + \mu_1 \underbrace{(P_1^M(X) - \alpha Z)}_{\text{adv. sel. subs.}}]. \tag{1.9}$$

First, the fundamental value is given by the dividend each type of tree pays in period 2, given by Z for good trees, and αZ for bad trees. Second, trees in this economy derive value from the fact that they can be traded in period 1, transforming their payoff in period 2 into resources in period 1, when they are potentially more valuable. The liquidity premium is a consequence of the *liquidity services* tradable assets provide in economies with incomplete markets, as emphasized by Holmström and Tirole (2001). Finally, the asymmetric information problem in the market for trees introduces a wedge in the market price that is negative for good trees and positive for bad trees. Let's focus on the shadow value of good trees first, given by (1.8). As I show below, the market price of trees is always between the fundamental value of good and bad trees, that is, $P_1^M(X) \in [\alpha Z, Z]$. Therefore, the adverse selection tax is always weakly positive. This tax is charged only if the tree is sold. Hence, the agents have a choice: sell the tree and pay the tax, generating a utility loss of $\mu_1(Z - P_1^M(X))$, or keep the tree and give up the liquidity services associated to it, generating a utility loss of $(\mu_1 - 1)Z$. The agent optimally chooses the option that generates the smallest loss. On the other hand, the pooling price implies an implicit subsidy for bad trees, as the last term in (1.9) shows. It is the size of this cross-subsidization between good and bad trees that shapes the incentives to produce different qualities. Moreover, note that all agents have the same ex-ante valuation for an extra unit of tree (good or bad) in the following period. This result relies mainly on the linearity of the agents' problem and greatly simplifies the analysis.[14]

A consequence of these expressions is that the shadow values have heterogeneous elasticities to market prices. Let $\gamma_0^i(P_1^M)$ be the shadow value of type $i \in \{G, B\}$ as a function of future prices $\{P_1^M(X)\}_{X\in\mathbb{X}}$, and let $D_\kappa\gamma_0^i(P_1^M)$ be the associated directional derivative with respect to future prices in the direction $\kappa(X)$.

Proposition 1 (Sensitivity of Shadow Values to Prices). *The shadow value of bad trees is more sensitive to future prices than the shadow value of good trees, that is*

$$\frac{D_\kappa\gamma_0^B(P_1^M + \kappa)}{\gamma_0^B(P_1^M)} > \frac{D_\kappa\gamma_0^G(P_1^M + \kappa)}{\gamma_0^G(P_1^M)} > 0,$$

for $\kappa(X) > 0\ \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to $\mathbb{X}$.

This is the key result of the model. It says that the private valuation of bad trees is more sensitive to changes in expected market prices than that of good trees. Or put differently, the private valuation of good trees is more insulated from shocks to the market than that of bad trees. As explained above, and explicit in equation (1.8), good trees have the option value of being kept until maturity if market conditions are not sufficiently good, or if liquidity needs are low, while this strategy

[14]It also depends on the fact that liquidity shocks in period 1 are independent of the types in period 0. However, allowing for correlation would not complicate the analysis, since at the individual level the shadow values would still be independent of the individual portfolio, which is the key property for tractability.

is always dominated for bad trees. Bad trees are produced only to be sold in the future, that is, for *speculative motives*. Since the fundamental value of bad trees is lower than its cost, it is never profitable to produce bad trees in order to keep them until maturity. The only reason to produce bad trees is the expectation of high prices in the secondary market, that can produce high returns when bad trees are sold as good ones. On the other hand, good trees have a high fundamental value. Since their market price is always below the discounted value of its dividends, agents only sell their good trees if their liquidity shock is high enough, that is, if the benefits of current consumption are sufficiently attractive so as to compensate for the loss from selling good trees below their private valuation. Thus, there are states of nature in which agents strictly prefer not to sell their good trees, isolating its value from price changes. This channel is at the core of the positive and normative analysis that follows. Moreover, it is important to note that this result is independent of Assumption 1. It only relies on the cross-subsidization between good and bad trees due to the pooling price, independently of their costs.

Now, I'm ready to characterize the agents' choice in period 0. As in period 1, the linearity of the agents' problem implies that their decisions are characterized by cutoffs. Given the shadow valuation of trees, γ_0^G and γ_0^B, agents decide whether to produce trees or not by comparing the return per unit invested of each option (good or bad) and the utility of consumption (which is 1). Since agents with low ξ have a comparative advantage in producing good trees, there always exists a threshold ξ_G such that agents with $\xi \leq \xi_G$ produce good trees. Agents with $\xi > \xi_G$ have a comparative advantage in producing bad trees. However, the cost of production might not be low enough to compensate for the opportunity cost of consuming immediately. If $\frac{\gamma_0^B}{q_B(\xi_G)} \leq 1$, then the shadow value of bad trees is too low compared to the cost of production. In this case, the marginal investor equalizes the return from producing good trees with the utility of consuming immediately, that is, $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$. Agents with $\xi \in (\xi_G, 1]$ consume all their endowment.

On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} > 1$, then there are agents with $\xi \in (\xi_G, \xi_G + \varepsilon)$, for some $\varepsilon > 0$, that face a cost of producing good trees that is too high, but have a positive return if they produce bad trees. Hence, there exists $\xi_B > \xi_G$ such that if $\xi \in (\xi_G, \xi_B]$ the agent produces bad trees. The marginal investors of each type are determined as follows. The marginal investor of good trees is indifferent between producing good trees and bad trees, so ξ_G satisfies $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$. The marginal investor of bad trees is indifferent between producing bad trees and consuming in period 0, thus ξ_B satisfies $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$. Finally, all agents for which $\frac{\gamma_0^B}{q_B(\xi)} < 1$ do not produce trees but consume. In order to simplify notation, I set $\xi_B = \xi_G$ whenever $\frac{\gamma_0^B}{q_B(\xi_G)} < 1$ (that is, there is no production of bad trees). The next lemma summarizes this result.

Lemma 3. *There exists $\xi_G \in (0,1)$ such that $i_G(\xi) = \frac{W_0}{q_G(\xi)}$ if and only if $\xi \leq \xi_G$. If $\frac{\gamma_0^B}{q_B(\xi_G)} \leq 1$, then ξ_G satisfies $\frac{\gamma_0^G}{q_G(\xi_G)} = 1$, and $i_B(\xi) = 0$ for all ξ. On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} > 1$, then ξ_G is such that $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$. In this case, there exists $\xi_B \in (\xi_G, 1]$ such that $i_B(\xi) = \frac{W_0}{q_B(\xi)}$ if and only if $\xi \in (\xi_G, \xi_B]$, where ξ_B satisfies $\frac{\gamma_0^B}{q_B(\xi_B)} = 1$.*

Define aggregate investment in good and bad trees as $I_0^G = \int_0^1 i_G(\xi)d\xi$ and $I_0^B = \int_0^1 i_B(\xi)d\xi$, respectively. Then

$$I_0^G = \int_0^{\xi_G} \frac{W_0}{q_G(\xi)} d\xi,$$
$$I_0^B = \int_{\xi_G}^{\xi_B} \frac{W_0}{q_B(\xi)} d\xi.$$

In Proposition 1 I showed that the shadow value of bad trees is more sensitive to changes in the market conditions than the shadow value of good trees. Now, I extend the result to the behavior of aggregate investment.

As future prices increase, the shadow value of both good and bad trees increases. However, the shadow value of bad trees increases proportionally more. If $I_0^B > 0$, then ξ_G is defined such that $\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)}$, or $\frac{\gamma_0^G}{\gamma_0^B} = \frac{q_G(\xi_G)}{q_B(\xi_G)}$. When expected prices increase, the left hand side of the expression *decreases*, since the shadow value of bad trees increases by more than the shadow value of good trees by Proposition 1, hence ξ_G *decreases* and the production of bad trees partially crowds out the production of good trees. The intuition is simple. Before the change in prices, the marginal agent was indifferent between producing good and bad trees. Now that prices increased, the production of bad trees is more profitable, hence the production of bad trees partially crowds out the production of good trees. Moreover, ξ_B, the type of the marginal investor of bad trees, increases, reinforcing the increase in the production of bad trees. Thus, an increase in expected prices reduces the production of good trees while it increases the production of bad trees.

On the other hand, if $\frac{\gamma_0^B}{q_B(\xi_G)} < 1$, so there is no production of bad trees, then

ξ_G is defined so that $\gamma_0^G = q_G(\xi_G)$. Therefore, a small increase in expected prices *increases* ξ_G. Therefore, when $I_0^B = 0$, an increase in expected prices increases the production of good trees. The next proposition summarizes this result.

Proposition 2. *Let $I_0^G(P_1^M)$ and $I_0^B(P_1^M)$ be the aggregate investment functions of good and bad capital, respectively, as a function of future prices $\{P_1^M(X)\}_{X\in\mathbb{X}}$. If $I_0^B(P_1^M) = 0$, then $D_\kappa I_0^G(P_1^M + \kappa) > 0$. If $I_0^B(P_1^M) > 0$, then*

$$D_\kappa I_0^B(P_1^M + \kappa) > 0 > D_\kappa I_0^G(P_1^M + \kappa),$$

for $\kappa(X) > 0 \; \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to $\mathbb{X}$.

While the result on the sensitivity of shadow values in Proposition 1 does not depend on Assumption 1, the result in Proposition 2 does. For the result in shadow valuations to translate into a result on quantities produced, some structure is necessary on the mass of agents that change their behavior after expected prices change. In particular, for Proposition 2 to hold, it is necessary that when the shadow value of bad trees moves more than that of good trees, a bigger mass of agents decide to produce bad trees than good trees. The positive correlation of the cost functions and the comparative advantage assumptions are sufficient conditions for this to be true. Moreover, the result that the production of good trees decreases because of the crowding-out effect is a partial equilibrium one. In general equilibrium, shocks that increase market prices can generate an increase in the production of both types of trees. I will analyze general equilibrium effects later in this section.

Proposition 2 implies that the production of lemons is more elastic to *future* prices than the production of non-lemons. It is related to the result in Akerlof (1970), who shows that the decision to *sell* non-lemons is more sensitive to prices than the decision to sell lemons. In my model, this result still holds in the secondary market for trees. But the lower exposure of the private valuation of good trees to market shocks reverses the result when considering production.

An immediate corollary of Proposition 2 is that the fraction of good trees in the economy in period 1, λ_1^E, decreases when agents expect higher market prices in the future. Moreover, the total amount of trees in the economy, $H_1 = H_1^G + H_1^B$, increases.

Corollary 2.1. *Let $\lambda_1^E(P_1^M)$ be the fraction of good trees in the economy in period 1, and $H_1(P_1^M)$ the total amount of trees in period 1, as a function of future prices $\{P_1^M(X)\}_{X \in \mathbb{X}}$. Then,*

$$D_\kappa \lambda_1^E(P_1^M + \kappa) \leq 0, \quad \textit{with strict inequality if } I_0^B > 0,$$

and

$$D_\kappa H_1(P_1^M + \kappa) > 0,$$

for $\kappa(X) > 0 \, \forall X \in A$ with $\nu(A) > 0$ for some $A \subseteq \mathbb{X}$, where ν is the measure associated to $\mathbb{X}$.

Next, I turn to the equilibrium in the secondary market for trees.

1.2.3 Market for Trees

The economy features a unique market in which all trees for sale are traded, as in Kurlat (2013) and Bigio (2015). By assuming that π is sufficiently large, the market for trees becomes a market with a demand and supply of quality, rather than quantity, in which agents with $\mu_1 = 1$ are willing and able to buy all the trees in the market as long as the price is fair. The inverse demand of tree quality is then given by

$$P_1^M = \lambda_1^M Z + (1 - \lambda_1^M)\alpha Z,$$

and hence the demand is

$$\lambda_1^M = \frac{P_1^M - \alpha Z}{(1-\alpha)Z}. \tag{1.10}$$

Meanwhile, Lemma 1 states that there exists μ_1^S such that agents with $\mu_1 = \mu_1^S$ are indifferent between selling their good trees and keeping them. All agents with $\mu_1 > \mu_1^S$ sell their holdings of good trees (recall that all agents sell their bad trees). Therefore, the supply of trees is given by

$$S = \int_{\mu_1^S}^{\overline{\mu}} H_1^G dG(\mu_1) + H_1^B = \left[1 - G\left(\mu_1^S\right)\right] H_1^G + H_1^B.$$

Using the result that $\mu_1^S = \frac{Z}{P_1^M}$, the implied fraction of good trees supplied is given by

$$\lambda_1^M = \frac{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] H_1^G}{S},$$
$$= \frac{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] \lambda_1^E}{\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] \lambda_1^E + (1 - \lambda_1^E)}. \tag{1.11}$$

In order to organize the analysis of the equilibrium of the economy, it is useful to define the partial equilibrium of this market for each state, taking λ_1^E as given.

Definition 2 (Partial Equilibrium in the Market for Trees). *A partial equilibrium in the market for trees in state X is a price P_1^M and a fraction of good trees in the market λ_1^M such that, given λ_1^E and α, the demand for tree quality* (1.10) *equals the supply of tree quality* (1.11).

There are two well-known characteristics of the set of partial equilibria in markets that suffer from adverse selection. The first one is what I call a *market collapse,* also known as *market unraveling*. If at every price greater than αZ the fraction of good trees supplied by sellers is too low compared to the *break-even* condition of buyers given by (1.10), then the only possible partial equilibrium has $P_1^M = \alpha Z$ and $\lambda_1^M = 0$. Because bad trees are inefficient (Assumption 2), if agents expected the price to be αZ in all states of the economy, no one would have incentives to produce bad trees. Since this paper studies how the incentives to produce different qualities varies with the underlying conditions of the economy, I will restrict the analysis to parameter values and functional forms such that for any realization of the exogenous state $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, there exists a threshold $\overline{\lambda}_1^E(\alpha) \in [0, 1)$ such that if the fraction of good trees in the economy is greater than $\overline{\lambda}_1^E(\alpha)$, then (1.10) and (1.11) intersect at an interior point with $\lambda_1^M > 0$. A necessary condition for this is that G_μ is convex at least over some interval of its support $[1, \mu_1^{\max}]$. In order to simplify exposition, I make the following assumption.

Assumption 3. *The distribution function G_μ is (weakly) convex in all its support* $[1, \mu_1^{\max}]$.

The second characteristic of markets that suffer from adverse selection is the multiplicity of partial equilibria. Consider figure 1-3. The panel (a) shows a market in which the quality of bad trees is high and there are multiple partial equilibria.

The literature has adopted the convention of selecting the partial equilibrium that features the highest volume of trade (see Kurlat (2013), Bigio (2015), Chari et al. (2014)). Later in this section I discuss the microfoundations of this selection criterion and how it affects the equilibrium of the economy. For now, I make the same selection.

As the quality of bad trees, α, decreases, the demand function (1.10) moves down. When α is low enough, the economy transitions to the market depicted in figure 1-3(b). In this case, the highest volume of trade equilibrium disappears, generating a market collapse. This has two implications. First, there exists a threshold $\alpha^*(\lambda_1^E)$ such that if $\alpha < \alpha^*$, then the market collapses and only bad trees are traded. On the other hand, if $\alpha \geq \alpha^*$, then both good and bad trees are traded in the market. Second, as λ_1^E increases, the threshold α^* decreases, meaning that the set of states such that there is a market collapse shrinks. This leads to the following definition of *market fragility*.

Definition 3. *Define market fragility as*

$$MF_1(\lambda_1^E) \equiv Prob(\alpha < \alpha^*(\lambda_1^E)).$$

Market fragility is the probability of a market collapse, that is, the probability that the economy features a market in which only bad trees are traded. It is straightforward to see that market fragility, $MF_1(\lambda_1^E)$, is decreasing in λ_1^E.

Even though market fragility is not a direct measure of welfare, it is a property that is tightly connected to the efficiency of the economy. The collapse of a market is the extreme case in which the flow of resources is severely impaired.

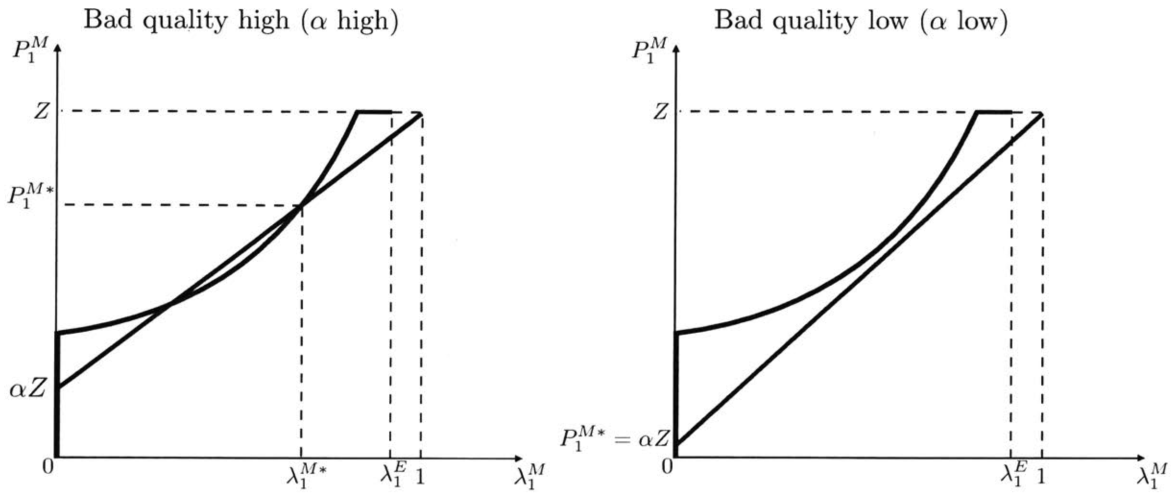
1.2.4 Equilibrium

Let's define an equilibrium for this economy.

Definition 4 (Equilibrium). *An equilibrium in this economy consists of prices* $\{P_1^M(X)\}$*; fraction of good trees in the market* $\{\lambda_1^M(X)\}$*; decision rules* $\{d_0(\xi), d_1(h_G, h_B; \mu_1, X), d_2(h_G, h_B; X)\}$, $\{i_G(\xi), i_B(\xi)\}$, $\{h'_G(h_G, h_B; \mu_1, X), h'_B(h_G, h_B; \mu_1, X)\}$, $\{m(h_G, h_B; \mu_1, X),\ s_G(h_G, h_B; \mu_1, X),\ s_B(h_G, h_B; \mu_1, X)\}$*; a fraction of good trees in the economy,* λ_1^E*, and a total amount of trees* H_1*, such that*

1. $\{d_0(\xi), d_1(h_G, h_B; \mu_1, X), d_2(h_G, h_B; X)\}$, $\{i_G(\xi), i_B(\xi)\}$, $\{h'_G(h_G, h_B; \mu_1, X),$

Figure 1-3: Market Equilibrium in period 1. (a) Multiple Equilibria: Maximal Volume of Trade Selected. (b) Unique Equilibrium: Market Collapse.



$h'_B(h_G, h_B; \mu_1, X)\}$, $\{m(h_G, h_B; \mu_1, X), s_G(h_G, h_B; \mu_1, X), s_B(h_G, h_B; \mu_1, X)\}$ *solve the agents' problems* (P0), (P1) *and* (P2), *taking* $\{P_1^M(X)\}$, $\{\lambda_1^M(X)\}$, λ_1^E *and* H_1 *as given;*

2. $\{P_1^M(X)\}$ *and* $\{\lambda_1^M(X)\}$ *are the maximum volume of trade in partial equilibrium state by state;*

3. λ_1^E *and* H_1 *are consistent with individual decisions.*

Because of the linearity of the agents' problem in period 1, prices are independent of the total amount of trees, H_1, while aggregate variables are linear in H_1. Hence, in order to complete the characterization of the equilibrium, I just need to determine the fraction of good trees in period 1, λ_1^E, which is given by

$$\lambda_1^E = \frac{I_0^G}{I_0^G + I_0^B}.$$

Note that the decision to produce trees in period 0, and of what quality, depends on market prices in period 1 in each state. But prices in period 1 in each state depend on the fraction of good trees in the economy, which in turn are determined by aggregate investment in period 0. It is useful to define the following mapping

$$T(\lambda_1^E) = \frac{I_0^G(\lambda_1^E)}{I_0^G(\lambda_1^E) + I_0^B(\lambda_1^E)}. \tag{1.12}$$

An equilibrium of this economy requires that $T(\lambda_1^E) = \lambda_1^E$. The mapping T is decreasing in λ_1^E, since higher λ_1^E implies higher expected prices, and the result follows from Proposition 2. When the distribution of α, F, is continuous, then $I_0^G(\lambda_1^E)$ and $I_0^B(\lambda_1^E)$ are continuous, and hence T is continuous. Therefore, the equilibrium of the economy exists and is unique. The following proposition summarizes these results.

Proposition 3. *An equilibrium of the economy always exists and is unique.*

Next, I study some properties and comparative statics of the economy. Propositions 4 and 5 formalize the idea that positive shocks to fundamentals distort the quality production decisions, since they increase the production of bad trees relative to that of good trees so that the average tree quality in the economy decreases. Next, I show that a reduction of transaction costs has a similar effect, and I lay out a plausible story for the development of the US financial sector in the last 30 years that could have led to the financial crisis of 2008. Finally, I show that the endogenous production of asset quality can interact with markets that suffer from adverse selection in such a way that the amplification of risk in the economy can be very large, to the extreme that endogenous risk remains positive and bounded away from zero even as exogenous risk vanishes away.

The Quality of Bad Trees

Consider the effect of an anticipated (from period 0 point of view) increase in the expected quality of bad trees (or an expected reduction of *default rates*).[15] In particular, suppose that the distribution of α is indexed by a parameter $\theta : F(\alpha|\theta)$, where a higher θ means a better distribution in the sense of first-order stochastic dominance. It can be shown that an increase in θ is equivalent to an increase in prices for all states under the initial distribution. From Proposition 2, we know that the partial equilibrium effect is an increase in the production of bad trees, a reduction in the production of good trees, and a reduction in the fraction of good trees in the economy, λ_1^E. This reduction in λ_1^E feeds back to the prices, through a general equilibrium effect. This partially offsets the increase in production of bad trees and the reduction in production of good trees. However, the overall effect is an increase in the investment in bad trees, a reduction in the fraction of good

[15] Or equivalently, consider two economies with different distributions of bad tree quality.

trees in the economy, an ambiguous effect on the investment in good trees, but an increase in the total production of trees, $H_1 = I_0^G + I_0^B$.

Since λ_1^E decreases, the market price *for each realization* of α decreases, so the threshold α^* increases. This endogenous adjustment of the economy is a force towards more fragility. However, the direct effect of the shock is an improvement in the distribution of shocks, which is a force towards less fragility. In general, the result is ambiguous and depends on the nature of the shock and the elasticities of production of trees. Recall that market fragility is the probability that the quality of bad trees, α, is below the threshold, α^*, that is $MF = F(\alpha^*(\lambda_1^E)|\theta)$. Differentiating this expression with respect to θ we get

$$\frac{d\text{MF}}{d\theta} = \underbrace{\frac{\partial F(\alpha^*|\theta)}{\partial\theta}}_{\leq 0} + f(\alpha^*;\theta)\underbrace{\frac{\partial\alpha^*(\lambda_1^E)}{\partial\lambda_1^E}}_{<0}\underbrace{\frac{\partial\lambda_1^E}{\partial\theta}}_{<0}.$$

For example, suppose the change in F is concentrated in very high values of α, so that $\frac{\partial F(\alpha^*|\theta)}{\partial\theta} = 0$. Then, the effect of the endogenous adjustment mechanism of the economy dominates, and market fragility *increases*. On the other hand, consider what would happen if the fraction of good trees in the economy was exogenously given, as in Eisfeldt (2004) and Kurlat (2013). In that case, $\frac{\partial\lambda_1^E}{\partial\theta} = 0$, so that market fragility would *decrease* after the shock. The next proposition summarizes these results.

Proposition 4 (Increase in Bad Trees' Expected Quality). *Consider an anticipated increase in θ, so that $F(\alpha|\theta)$ increases in FOSD sense. Then,*

1. *total investment in trees, $I_0^G + I_0^B$, increases;*

2. *the fraction of good trees in the economy, λ_1^E, decreases;*

3. *market prices in period 1, P_1^M, decrease in every state;*

4. *the threshold α^* increases;*

5. *the effect on market fragility is ambiguous.*

This is an important result since it states that a "positive" shock to the economy can endogenously increase the fragility of its financial markets, in the sense that the probability of a market collapse is higher. Thus, it formalizes the idea that positive shocks can set the stage for a financial crisis. Next, I show that changes to the agents' liquidity needs have similar effects.

Liquidity Shocks

An increase in the distribution of liquidity shocks increases the value of trees coming from their medium of exchange role. This is a positive shock in the sense that it improves the functioning of the market for trees.[16] Since liquidity shocks and market prices enter symmetrically in the expressions for the shadow value of trees, an increase in liquidity shocks triggers a qualitatively similar response from period 0.

Proposition 5 (Increase in Liquidity Shocks). *Consider an anticipated change in the distribution of μ_1 from $G(\mu_1)$ to $\tilde{G}(\mu_1)$ such that $\tilde{G} > G$ in FOSD sense. Then,*

1. *total investment in trees, $I_0^G + I_0^B$, increases;*
2. *the fraction of good trees in the economy, λ_1^E, decreases;*
3. *the effect on market prices in period 1, P_1^M, is ambiguous;*
4. *the effect on the threshold α^* is ambiguous;*
5. *the effect on market fragility is ambiguous.*

The incentives to produce lemons increase with the value of liquidity services. However, the effect on market fragility is, again, ambiguous. On the one hand, as G increases, more agents sell their good trees so market fragility decreases. On the other hand, the endogenous response of the economy reduces the average quality of trees, increasing fragility. The overall effect depends on the interaction between these two forces.

Note that if the change in expectations does not reflect a change in the actual distributions (in the sense that it is just unfounded *optimism*) then fragility always increases for both types of shocks. Moreover, even though the effect of shocks to the economy's fundamentals on market fragility is ambiguous on impact, in the infinite horizon extension I show that if the shock is transitory, then market

[16] In this model liquidity shocks are "good" shocks in the sense that they increase the agents' valuation for consumption. Similarly one could assume that the shocks are "bad" and they reduce the utility of consumption in period 2. In both cases, an increase in the distribution of liquidity shocks is good news for the functioning of the market.

fragility increases as the shock dies out. This is another way in which good times sow the seeds of the next crisis.

Finally, in the next section I extend the model of this section and microfound these shocks so that changes in the distribution of G arise from shocks to the "real economy", or shocks to the supply of government bonds. This introduces a new set of comparative statics and sources of risk build-up in the economy.

Transaction Costs

Financial innovation can reduce the cost of trading financial assets. Many scholars argue that in the last 30 years the financial sector underwent a process that facilitated the transformation of illiquid assets (e.g. mortgages) into liquid ones (e.g. MBS, ABS, CDOs).[17] Securitization and repo contracts seem to have been some of the stars of this process. Here, I show that a reduction in transactions costs naturally leads to a deterioration of the quality of assets in the economy.

Consider a variant of the economy described before in which sellers receive $P_1^S = P_1^M - c$ per tree sold, where P_1^M is the price payed by buyers, an c is a pecuniary cost that summarizes all the costs the seller has to incur in order to be able to transfer property of the tree to another agent. The main characteristics of the equilibrium with trading costs follow from the previous discussion, in particular existence and uniqueness. An important difference is that the market for trees can be inactive for some values of c, or have only good trees being traded. Obviously, if $c = 0$ the equilibrium is exactly the one described above.

Suppose $c \geq Z$. Since prices cannot be higher than Z, agents get no net resources from the sale of trees. Therefore, there will be no active market for trees in this economy, and producers of trees keep them until maturity. Since $E[\mu_1 \alpha Z] < 1$ by assumption, no agent produces bad trees, and the economy has $\lambda_1^E = 1$. Since the maximum utility agents can get from consumption is $\mu_1^{\max}$, this result holds for all $c \in (c_1, \infty)$, where $c_1 \equiv \frac{\mu_1^{\max}-1}{\mu_1^{\max}} Z$.

For a cost c slightly lower than c_1, one of two things can happen, depending on parameter values. If $\mu_1^{\max}$ is relatively high, then the cost c can be high and still incentivize some agents with high μ_1 to sell their good trees. But if c is high, then the price the sellers receive is low, so the returns from selling trees are not sufficiently high to incentivize speculative production of bad trees. In that case,

[17]See for instance Adrian and Shin (2010).

there exists a $c_2 < c_1$ such that if $c \in (c_2, c_1)$ there is an active market of trees in period 1, $\lambda_1^E = 1$, and $P_1^M = Z$ in all states of the economy. Also note that I_0^G increases as c decreases in this region. The reason is that the liquidity premium increases as the cost of trading trees decreases, and while the production of bad trees is inefficient, the incentives of producing good trees increases. On the other hand, if $c < c_2$, the transaction cost is sufficiently low to attract the production of bad trees, so $\lambda_1^E \in (0, 1)$.

If $\mu_1^{\max}$ is relatively low, then the cost c has to be low in order to incentivize agents with good trees to sell in the market. In this case, the price sellers get from selling trees, $P_1^S = Z - c$ is relatively high when there are no bad trees. Thus, if c is low enough, some agents will have incentives to produce bad trees. Therefore, when $\mu_1^{\max}$ is low, if $c < c_1$ there is an active market in period 1 and $\lambda_1^E \in (0, 1)$. For notational convenience I set $c_2 = c_1$ when this happens.

Finally, the fraction of good trees in the economy decreases as c decreases in the region $c \in [0, c_2)$. The next proposition summarizes these results.

Proposition 6. *Suppose sellers receives $P_1^S = P_1^M - c$ per tree sold, where c is a transaction cost. There exists c_1 and c_2 with $c_1 \geq c_2$ such that*

- *if $c > c_1$, there is no market for trees and $I_B = 0$,*
- *if $c \in (c_2, c_1)$, there is an active market for trees in period 1, $I_B = 0$, and $\frac{\partial I_G}{\partial c} < 0$,*
- *if $c < c_2$, there is an active market for trees in period 1, $I_B > 0$, and $\frac{\partial \lambda_1^E}{\partial c} > 0$.*

This result introduces a plausible story for the development of the US financial sector in the last 30 years. When the main financial innovations were introduced, the cost of trading certain assets (e.g., ABS, MBS, CDOs) decreased. However, if the reduction in costs was gradual, then the economy could have spent some time in the range at which there was an active market but no production of low quality assets, since the market return did not make their production profitable. Hence, the economy completely benefited from further innovation and cost reductions, increasing the high-quality asset production and volume traded, and improving the allocation of resources. However, at some point the transaction costs could have decreased so much that some agents found it profitable to produce low quality assets to take advantage of the market. Further reductions of the transaction tax further reduced the average quality of the assets, which exposed the economy to financial risk, as experienced in 2008.

Financial Risk

The previous exercises were meant to convey the idea that "positive" shocks give bad incentives in terms of asset quality production, since they improve the functioning of markets and increase prices, which in turn reduces the incentives to produce high quality assets. Here I make a digression in order to show that the interaction between the production of asset quality and the presence of markets that suffer from adverse selection can generate a large amplification of exogenous shocks, to the point that endogenous risk can remain positive and bounded away from zero even as exogenous risk vanishes away.

Consider an economy in which the distribution of bad tree quality is given by

$$\alpha = \tilde{\alpha} + u, \quad u \sim \mathcal{U}[-\epsilon, \epsilon], \tag{1.13}$$

for some $\epsilon > 0$, and where $\mathcal{U}$ denotes the uniform distribution. Let $P_1^M(\alpha|\epsilon)$ denote the equilibrium price in period 1 when the exogenous state is α and the spread of the uniform distribution is given by ϵ. I want to determine what happens to the variance of the price as the exogenous risk vanishes, that is, as $\epsilon \to 0$.

In order to understand how the economy behaves as exogenous risk vanishes it is useful to note that prices perform two roles in this economy. First, they clear markets, which in this case means that the quality supplied has to be consistent with the quality demanded. Second, prices send signals to the agents and shape investment decisions in period 0. Note that this dual role of prices is not special to this economy but appears every time agents have investment opportunities and there is a market for that investment (think of physical capital in a standard neoclassical model, in which the rental rate clears the market for available capital but also gave incentives to produce capital in the past). What is special about markets that suffer from adverse selection is that prices can be discontinuous in state variables. In particular, the market price in a given state α is discontinuous in the fraction of good trees in the economy, λ_1^E. This discontinuity will be key to understanding the role of risk in the economy.

As $\epsilon \to 0$, the fundamentals of the economy in every state get very similar to each other. If prices were continuous, the prices in different states would also get closer to each other. At what level should they be? If prices were low in every state, such that markets collapse for every realization of α, then no agent would produce bad trees, contradicting that the prices in period 1 are low. On the other hand, if all prices are high, then it might be that too many bad trees are produced, which is inconsistent with prices being high. Hence, in order for prices to give the right incentives to invest, they should be in a "middle" range. However, those prices can be inconsistent with market clearing. Does this mean that there is no equilibrium for some pair $\tilde{\alpha}$ and ϵ, with ϵ small but positive? We already know the answer is no, because Proposition 3 guarantees existence for any continuous distribution function of the aggregate shock α. Hence, the result is that the equilibrium cannot feature prices that are *continuous* in the aggregate state. Hence, even though the difference between the lowest state $\tilde{\alpha} - \epsilon$ and the highest state $\tilde{\alpha} + \epsilon$ can be made arbitrarily small, the economy might need discontinuous prices to give the right incentives to the agents in period 0. The risk introduced by market fragility allows the economy to obtain a "middle range" price on average, when that price is not consistent with market clearing in any state in period 1. The next proposition summarizes this result.

Proposition 7. *Consider an economy in which α is distributed according to* (1.13). *There exists an open set $B \subset [0,1]$ such that if $\tilde{\alpha} \in B$ then*

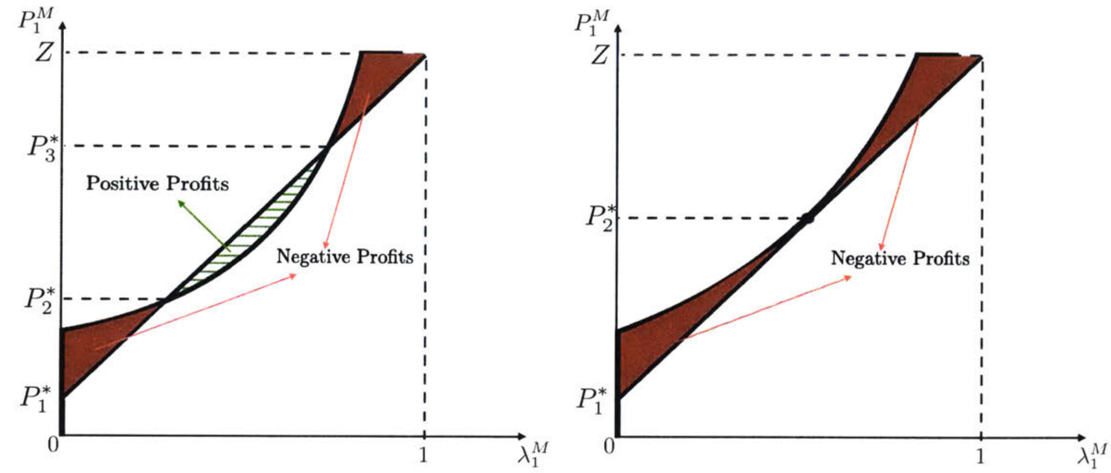
$$\lim_{\epsilon \to 0} \mathbb{V}ar[P_1^M(\alpha|\epsilon)] = \sigma^2(\tilde{\alpha}),$$

for some $\sigma^2(\tilde{\alpha}) > 0$.

Finally, the result in Proposition 7 is related to what happens to the economy if the distribution $F(\alpha)$ has atoms. As noted above, the proof of existence of equilibrium uses the fact that F is continuous so that the mapping T is continuous, which guarantees that a fixed point exists. I now show that the limit $\sigma^2(\tilde{\alpha})$ is the variance of the price in an economy with no exogenous aggregate risk, that is, F is degenerate at $\alpha = \tilde{\alpha}$, and with an equilibrium definition that allows for sunspots.

In order to explain the role of sunspots in the perfect foresight economy, it is useful to take a step back and study the theoretical justifications for the selection of the maximal volume of trade partial equilibrium I made before. The choice of the maximal volume of trade equilibrium can be justified as being the generic outcome of a game in which buyers can make different offers but choose not to in equilibrium (see, for instance, Mas-Colell et al. (1995) and Attar et al. (2011)). Consider the cases depicted in figure 1-4. Figure 1-4(a) shows the case in which

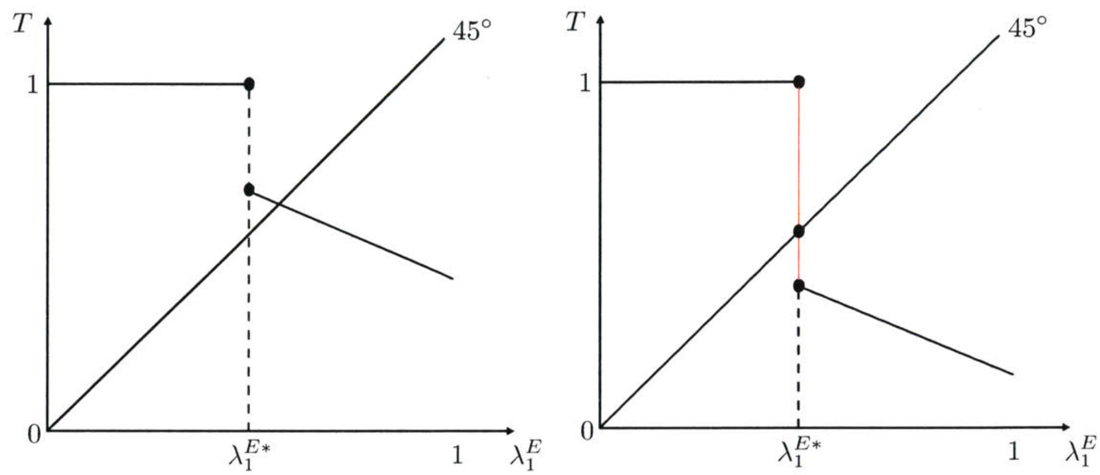
Figure 1-4: Market Equilibrium in period 1. (a) Unique Equilibrium. (b) Multiple Equilibria.



the game-theoretic approach selects the highest volume of trade equilibrium. The intuition is fairly simple: if the equilibrium featured prices P_1^* or P_2^*, some buyer could offer a price slightly higher than P_2^*, and attract a relatively large number of sellers of good trees, and make a profit. P_3^* is the only price at which there is no profitable deviation. On the other hand, figure 1-4(b) shows a case in which both P_1^* and P_2^* are consistent with equilibrium. Suppose the equilibrium has P_1^*. There is no deviation for buyers that can get them positive profits. The same happens with P_2^*. Hence, both prices are consistent with agents' optimization. This case is not relevant when the distribution of exogenous aggregate risk F is continuous and non-degenerate, since given λ_1^E there is only one state α in which the multiplicity can arise. Since that state has probability zero from the point of view of period 0, selecting the maximal volume of trade had no impact on agents choices in period 0. However, this logic doesn't hold when F has atoms.

Consider the case in which F is degenerate in some $\tilde{\alpha}$, so the economy does not face any exogenous aggregate risk (agents still face idiosyncratic liquidity shocks). As before, an equilibrium of the economy requires that $T(\lambda_1^E) = \lambda_1^E$, with the mapping T defined in (1.12). However, the mapping T can be discontinuous in λ_1^E. Let $\lambda_1^{E*} \equiv \sup\{\lambda_1^E \in [0,1] : P_1^M(\lambda_1^E; \tilde{\alpha}) = \tilde{\alpha}Z\}$, that is, the threshold fraction of good trees in the economy such that if $\lambda_1^E < \lambda_1^{E*}$ the market in period 1 collapses. Note that λ_1^{E*} corresponds to figure 1-4(b), so that both prices can be part of an equilibrium. The key to finding an equilibrium in this economy is to determine what happens when $\lambda_1^E = \lambda_1^{E*}$. Since bad trees are inefficient, I already know that if the

Figure 1-5: Equilibrium in period 0 when F is degenerate. (a) No Aggregate Risk. (b) Positive Aggregate Risk (Sunspot Equilibrium).



low price equilibrium is selected, $T(\lambda_1^{E*}) = 1$. If the high price is selected, then existence depends on whether $T(\lambda_1^{E*})$ is greater or smaller than λ_1^{E*}. If $T(\lambda_1^{E*}) \geq \lambda_1^{E*}$, the discontinuity in T does not prevent a fixed point from existing, so the equilibrium of the economy has the same properties as the economies with continuous F. This case is depicted in figure 1-5(a).

On the other hand, if $T(\lambda_1^{E*}) < \lambda_1^{E*}$, then a fixed point does not exist. In order to obtain existence of equilibrium in this case as well, I need to modify the definition of equilibrium. Motivated by the fact that the economy in the limit to perfect foresight featured positive endogenous risk, I define a Sunspot Equilibrium (SE) in which there is a random variable that selects a partial equilibrium in period 1. Note that when the fixed point of T exists (that is, cases like figure 1-5(a)), then the SE coincides with the previous equilibrium definition. But when the mapping T does not have a fixed point, the sunspot convexifies the mapping T so that it crosses the 45° line, as shown in figure 1-5(b). Moreover, the SE is unique.

When the sunspot is not trivial, the economy faces strictly positive endogenous aggregate risk even though the exogenous aggregate risk is zero. The reason for this result is the tension between the discontinuity of prices with respect to λ_1^E and the endogenous production decisions/*portfolio choices* of the agents, as in the limit above. When prices cannot align agents' incentives, risk helps, and that is what the sunspot is doing. In this sense, I view the financial markets as not just amplifying exogenous risk but as *creating* endogenous risk. Moreover, it turns out that

$$\mathbb{V}ar[P_1^M(\tilde{\alpha})] = \sigma^2(\tilde{\alpha}).$$

That is, the limit of the variance of an economy with vanishing exogenous risk coincides with the variance introduced by the sunspot in a perfect foresight equilibrium.

The next proposition summarizes these results.

Proposition 8 (Fundamental Endogenous Financial Risk). *A Sunspot Equilibrium (SE) always exists and is unique. It coincides with the maximal volume of trade equilibrium whenever the latter exists. If it doesn't, the SE features strictly positive randomization. Moreover, the SE is the limit of the maximal volume of trade equilibrium with uniform exogenous aggregate risk and vanishing volatility, in the sense that*

$$\lim_{\epsilon \to 0} \mathbb{V}ar[P_1^M(\alpha|\epsilon)] = \sigma^2(\tilde{\alpha}) = \mathbb{V}ar[P_1^M(\tilde{\alpha})].$$

1.3 Extended Model and Positive Implications

The analysis in the previous section shows that it is the dual role that trees play that exposes the economy to financial risk. On the one hand, they are a form of real investment, in the sense of being a technology that transforms units of goods in one period into units of goods in others. On the other hand, they facilitate transactions in period 1, so that resources can flow among agents even if the tree did not produce any dividend. In reality, the government is an important provider of instruments that perform the second role, through government bonds. There are both theoretical (see for example Woodford (1990), Holmström and Tirole (1998)) and empirical works (see for example Krishnamurthy and Vissing-Jorgensen (2012), Greenwood et al. (2015), Sunderam (2015), Krishnamurthy and Vissing-Jorgensen (2015)) that study the interaction between private and public liquidity. On the theoretical side they show that government bonds can be welfare enhancing when the economy cannot produce enough financial instruments to optimally transfer resources among agents (for example, because markets are incomplete or there is limited pledgeability of future income). On the empirical side, they show that the production of private liquid instruments increases when the supply of government bonds decreases, which seems to be driven by changes in the liquidity premium.

In this section, I extend the basic model and incorporate decreasing returns to liquidity in order to obtain a more stable demand for liquid assets and to be able to study the interaction between private and public liquidity in a meaningful way. To do so, I change the source of the liquidity risk that agents face. In particular, I now assume that in period 1 instead of receiving a shock to preferences, agents are endowed with a technology that transforms final consumption good into physical capital (denoted by k), and the marginal rate of transformation is random and idiosyncratic. The agents' preferences are now given by

$$d_0 + E[d_1 + d_2].$$

Moreover, agents operate a linear technology that transforms final consumption good into capital at a rate A, where A is distributed independently across agents according to the cumulative distribution function G in $[0, A^{\max}]$.

In period 2, agents then rent the capital they own to a representative firm that operates the following technology

$$Y = Z^Y f(K),$$

where $f'(K) > 0$, $f''(K) < 0$, and $f(K)$ satisfies the Inada conditions, where K is the amount of capital operated by the firm, and Z^Y is the TFP level. I assume that the market for renting capital is competitive, hence the rental rate is $r(K) = Z^Y f'(K)$. Moreover, I assume that the profits of the firm, $\Pi = f(K) - r(K)K$, are transferred to the agents uniformly in period 2. The state of the economy in period 1 is given by $X_1 = \{\lambda_1^E, H_1, \alpha\} \in \mathbb{X}_1$, and the state of the economy in period 2 is given by $X_2 = \{\lambda_1^E, H_1, K, \alpha\} \in \mathbb{X}_2$.

This extension allows me to study the interaction between private and public liquidity in a model that is only a small departure from the one in the previous section. The specific modeling choices have two main features. First, they incorporate decreasing returns to liquidity in a way that keeps the linearity of the agents' problem, so that cross-section distributions of agents' portfolios are not necessary to determine aggregate allocations.[18] Second, they incorporate a different sector

[18] A different approach would have been to incorporate decreasing marginal utility of consumption at the individual level in the basic model. However, this would have implied that the agents' problem is not linear, hence losing some tractability of the problem.

of the economy in a parsimonious way, and formally establish the connection between financial markets and the "real economy". Better functioning markets imply a better allocation of resources and hence a higher efficiency of investment, but also higher productivity of the real economy implies a higher demand for liquidity and hence affects the quality production decisions of the agents.

Because of the intertemporal linkages between periods 1 and 2 that capital introduces, the model requires a modification of the definition of equilibrium that allows for a richer set of markets for trees. Instead of forcing that all transactions take place in the same market, I allow for the existence of many markets that operate simultaneously.[19] Each market ω is defined by a positive price $P_1^M(\omega) \in \mathbb{R}_+$. Without loss of generality, I assume that if $\omega' > \omega$ then $P_1^M(\omega') > P_1^M(\omega)$. The set of all markets is denoted by Ω. As in the previous section, only sellers know the quality of the tree they hold. Buyers do not observe the quality of a tree being offered, and they can only form some expectation about the quality distribution in each market. Moreover, markets need not clear. A fraction of the trees offered in a specific market may remain unsold.

Importantly, I keep the assumption that markets are non-exclusive. Sellers can offer the same unit of tree for sale in any subset of markets simultaneously. They are only restricted not to sell more trees than they own. From the seller's point of view, markets are characterized both by their prices, $P_1(\omega)$, as well as an amount of rationing, $\eta(\omega)$. The amount of rationing specifies the fraction of supplied trees a seller will be able to sell in market ω. I assume that trees are perfectly divisible, so $\eta(\omega)$ is the fraction of trees the seller actually sells rather than being the probability of selling an indivisible unit. The amount of rationing η is an equilibrium object that results from the equilibrium supply and demand decisions of the agents in each market and state of the economy. Finally, let Ω^B be the set of markets with positive supply.

Next, I state the agents' problem for this economy. I show that the main features of the equilibrium are isomorphic to the basic economy of the previous section, so the main insights still hold. The main difference is that the marginal utility of liquid wealth is now decreasing in total liquidity, creating a two-way feedback effect between the financial markets and the real economy. In the next section, I use this result to study the effect of the supply of public liquidity on the incentives

[19]See, for example, Guerrieri and Shimer (2014a), Guerrieri and Shimer (2014b), and Kurlat (2016) for models with adverse selection and many markets.

to produce tree quality.

1.3.1 Agents' Problem

The problem agents' face in period 2 now is

$$V_2(h_G, h_B, k; X_2) = Zh_G + \alpha Z h_B + r(X_2)k + \Pi(X_2). \tag{P2'}$$

The only difference between (P2') and (P2) is that in (P2'), besides the dividend from the trees, agents receive the rental rate $r(X_2)$ for their holdings of capital k, and the profits of the representative firm, $\Pi(X_2)$.

The problem that agents face in period 1 is slightly more complicated. The program they solve has to accommodate the new investment opportunities and the availability of many markets. Therefore, the agents solve the following program:

$$V(h_G, h_B; A, X_1) = \max_{\substack{d, i_K, m, s_G, \\ s_B, h'_G, h'_B, k'}} d + V_2(h'_G, h'_B, k'; X_2), \tag{P1'}$$

subject to

$$d + i_K + \sum_{\omega \in \Omega^B} P_1^M(\omega) m(\omega) \leq W_1 + \sum_{\omega \in \Omega} P_1^M(\omega)(s_G(\omega) + s_B(\omega))\eta(\omega; X_1), \tag{1.14}$$

$$h'_G = h_G + \sum_{\omega \in \Omega^B} \lambda_1^M(\omega; X_1) m(\omega) - \sum_{\omega \in \Omega} s_G(\omega)\eta(\omega; X_1), \tag{1.15}$$

$$h'_B = h_B + \sum_{\omega \in \Omega^B} (1 - \lambda_1^M(\omega; X_1)) m(\omega) - \sum_{\omega \in \Omega} s_B(\omega)\eta(\omega; X_1), \tag{1.16}$$

$$k' = A i_K, \tag{1.17}$$

$$\sum_{\omega \in \Omega} s_G(\omega)\eta(\omega; X_1) \leq h_G, \quad \text{and} \quad \sum_{\omega \in \Omega} s_B(\omega)\eta(\omega; X_1) \leq h_B, \tag{1.18}$$

$$d \geq 0, \quad i_K \geq 0,$$

$$m(\omega) \geq 0, \quad s_G(\omega) \in [0, h_G], \quad s_B(\omega) \in [0, h_B], \quad \forall \omega \in \Omega.$$

Constraint (1.14) is the agent's budget constraint, which states that consumption plus investment in capital and purchases in all markets cannot be larger than the endowment W_1 plus the sale of trees in different markets. Constraints (1.15) and (1.16) are the laws of motion of good and bad trees respectively, while constraint (1.17) is the law of motion of capital. Finally, constraint (1.18) establishes that agents cannot sell more trees than they hold. Note that the measure used is $\eta(m; X_1)$, implying that the restriction is over the actual sales, not on the number of trees the agents send to the market. This is the non-exclusivity assumption.

Following Kurlat (2016), I focus on solutions to this problem that are robust to small perturbations of η, in order to rule out self-fulfilling equilibria in which sellers do not supply in certain markets because there are no buyers, and buyers do not demand in some markets because there are no sellers, even though a small amount of trade would trigger a response from them. See Appendix 1.7.2 for the details.

It is useful to define $\tilde{\omega}(X)$ as the market with the lowest price such that if an agent sends his trees to all markets with prices at least as high, they would be able to sell all their holdings in equilibrium. Formally,

$$\tilde{\omega}(X) \equiv \max \left\{ \omega' \in \Omega : \sum_{\omega \geq \omega'} \eta(\omega; X) \geq 1 \right\}. \tag{1.19}$$

The interpretation of $\tilde{\omega}(X)$ is that it is the market with the lowest price that can have active trading, given the rationing in the other markets.

The solution to (P1′) is the analogue of Lemma 1 in the previous section.

Lemma 4 (Agents' Choice). *Consider an agent with investment opportunity A. There exists thresholds A_1^B and $A_1^S(\omega)$ (with $A_1^S(\omega)$ decreasing in ω) that may depend on the state of the economy, X_1, such that*

- *if $A \leq A_1^B$, then the agent does not produce capital ($i_K = 0$), consumes or buys trees in some markets ($d \geq 0$, $m(\omega) \geq 0$) and does not sell his good trees ($s_G(\omega) = 0$ for all $\omega \in \Omega$);*
- *if $A > A_1^B$, then the agent produces capital ($i_k > 0$), does not consume ($d = 0$), does not buy trees ($m(\omega) = 0$ for all $\omega \in \Omega$), and might sell his good trees in some markets ($s_G(\omega) = h_G$ for all $\omega \in \Omega$ such that $A > A_1^S(\omega)$ and $s_G(\omega) = 0$ for all $\omega \in \Omega$ such that $A < A_1^S(\omega)$).*

All agents always sell their holding of bad trees in the markets with the highest prices, i.e. $s_B(\omega) = h_B$ for all $\omega > \tilde{\omega}$ and $s_B(\omega) = 0$ for all $\omega < \tilde{\omega}$.

The decisions of the agents are very similar to those in the basic model. First, agents compare their productivity A with the return from buying trees in the market and the utility from consumption (which is equal to 1). Since all agents face the same alternatives to investing, there is a threshold A_1^B common to all agents such that only those with productivity higher than A_1^B produce capital. Those with productivity below A_1^B use their liquid wealth to buy trees in the market and consume, whichever provides the highest utility. Moreover, while all agents sell all of their bad trees, only agents with high enough productivity sell their good trees. Since there are many markets, they choose in which market to sell given their productivity, which determines the thresholds $\{A_1^S(\omega)\}_{\omega\in\Omega}$.

In the previous section, the marginal utility of liquid wealth coincided with the liquidity shocks μ_1. Now, it is given by the Lagrange multiplier associated to the agents' budget constraint:

$$\mu_1(A, X_1) = \max\left\{1, Ar(X_2), \left\{\frac{\lambda_1^M(\omega; X_1)Z + (1-\lambda_1^M(\omega; X_1))\alpha Z}{P_1(\omega)}\right\}_{\omega\in\Omega^B}\right\}.$$

μ_1 represents the utility derived from the use of resources that provides the maximum return on the margin. Agents have three possible uses. They can consume and obtain 1 unit of utility; they can produce capital at a rate A and obtain a payoff $Ar(X_2)$ in period 2 per unit invested; or they can buy trees in some markets and obtain a return of $\frac{\lambda_1^M(\omega;X_1)Z+(1-\lambda_1^M(\omega;X_1))\alpha Z}{P_1(\omega)}$.

Even though the model is richer, the mapping between the liquidity shocks of the previous section and the liquidity services of this section is very direct. Let $\mu_1^B(X_1)$ be the return from buying trees in the secondary market.[20] Agents with low A have a marginal utility of liquid wealth of $\mu_1(A, X_1) = \max\{1, \mu_1^B(X_1)\}$. Following the same logic as in the previous section, if W_1 is high enough, then there are enough agents willing to buy trees rather than invest in capital, so that $\mu_1^B(X_1) = 1$. Thus, liquidity services simplify to

$$\mu_1(A, X_1) = \max\{1, Ar(X_1)\}. \tag{1.20}$$

[20] That is

$$\mu_1^B(X_1) \equiv \max_{\omega\in\Omega^B} \frac{\lambda_1^M(\omega; X_1)Z + (1-\lambda_1^M(\omega; X_1))\alpha Z}{P_1(\omega)}.$$

Therefore, there is a mass of agents with low enough A such that $\mu_1(A, X) = 1$, analogous to the mass π of agents with $\mu_1 = 1$ in the previous section. Note that this case implies that aggregate consumption is positive in period 1, hence restricting parameter values so that there is positive aggregate consumption every period leads to the same result. Moreover, as A increases, $\mu_1(A, X_1)$ increases, and the cross-section distribution of $\mu_1(A, X_1)$ is ultimately governed by the distribution of A, and the value of $r(X_2)$.

In order to maintain the assumption that bad trees are an inefficient investment, I assume that the amount of investment in period 1 that would prevail in an economy with no markets for trees would be such that the liquidity services are not too large. Let $\tilde{\mu}_1(A, X_1)$ denote the liquidity services that would prevail in such an economy.

Assumption 4. *The payoff of bad trees is such that*

$$E[\tilde{\mu}_1(A, X_1)\alpha Z] < 1.$$

This assumption holds if W_1 is large enough.

Finally, the program the agents solve in period 0 does not change except that V_1 is now given by (P1'). Next, I briefly describe the determination of partial equilibria in the markets for trees, define the equilibrium of the full economy and characterize it.

1.3.2 Equilibrium

Most of the analysis in the previous section follows through after these modifications. The main difference is that more than one market may be active in equilibrium. It turns out that under my assumptions, at most two markets can be active: a high price market in which good and bad trees are traded, and a low price market in which only bad trees are traded. The low price market can have positive volume of trade only if there is rationing in the high price market (market collapse is an extreme case in which there is 100% rationing in the high price market). Moreover, the structure of active markets follows very closely the discussion on the multiplicity of partial equilibria of the previous section when the economy was forced to have only one active market, as depicted in figure 1-4.

Consider once again the demand and supply of trees studied in the previous section, modified to the specifics of the economy in this section. The demand in each market is given by

$$\lambda_1^M(P_1) = \frac{P_1 - \alpha Z}{(1-\alpha)Z}, \tag{1.21}$$

while the supply is given by

$$\lambda_1^M(P_1) = \frac{\left[1 - G\left(\frac{Z}{r(K)P_1}\right)\right]\lambda_1^E}{\left[1 - G\left(\frac{Z}{r(K)P_1}\right)\right]\lambda_1^E + (1-\lambda_1^E)}. \tag{1.22}$$

Given λ_1^E and K, the partial equilibrium of the markets for trees can take 3 different forms:

1. if one of the intersections between (1.21) and (1.22) happens at a point in which $\lambda_1^M > 0$ and the game-theoretic foundation developed in the previous section selects the maximal volume of trade partial equilibrium, then the economy with many potential markets has only one active market in equilibrium which corresponds to the maximal volume of trade equilibrium and there is no rationing in the market;

2. if there is only one intersection between (1.21) and (1.22) which happens at $\lambda_1^M = 0$, then there is also only one active market in equilibrium, which corresponds to a market collapse;

3. if one of the intersections between (1.21) and (1.22) happens at a point in which $\lambda_1^M > 0$ and the game-theoretic foundation developed in the previous section *does not* select the maximal volume of trade partial equilibrium, then there can be *two* active markets in equilibrium, which correspond to the two intersections of (1.21) and (1.22), as in figure 1-4(b). Sellers of bad trees send their trees to both markets. They sell all they can in the high price market and then sell the rest in the low price market. Sellers of good trees only send their trees to the high price market. If there is rationing, they keep the units they were not able to sell.

See Appendix 1.7.2 for details.

Therefore, the equilibrium is pooling when there is only one active market, and semi-separating when there is more than one active market. In both cases, there is some degree of cross-subsidization among types of trees. Since the low price market is active only when there is rationing in the high price market, and all agents try to sell their trees in the high price market before trying to sell in the low price market, I denote by $\eta(X_1)$ the rationing in the high price market and $1-\eta(X_1)$ the fraction of trees sold in the low price market. Note that $\eta(X_1)$ indexes all the possibilities described above. Moreover, I denote by $P_1^M(\omega_H;X_1)$ and $P_1^M(\omega_L;X_1)$ the high price and the low price, respectively.

Let's define an equilibrium for this economy.

Definition 5 (Equilibrium). *An equilibrium in this economy consists of prices $\{P_1^M(\omega_H;X_1),\ P_1^M(\omega_L;X_1), r(X_2)\}$; fraction of good trees in the market ω_H, $\{\lambda_1^M(\omega_H;X_1)\}$; a rationing function $\eta(X_1)$; decision rules $\{d_0(\xi),\ d_1(h_G,h_B;A,X_1),$ $d_2(h_G,h_B,k;X_2)\}$, $\{i_G(\xi), i_B(\xi), i_K(h_G,h_B;A,X_1\}$, $\{h'_G(h_G,h_B;A,X_1),$ $h'_B(h_G,h_B;A,X_1)\}$, $\{m(h_G,h_B;\omega_H,A,X_1),\ m(h_G,h_B;\omega_L,A,X_1),\ s_G(h_G,h_B;A,X_1),$ $s_B(h_G,h_B;A,X_1)\}$; a fraction of good trees in the economy, λ_1^E, a total amount of trees H_1, and aggregate capital $\{K(X_1)\}$, such that*

1. *$\{d_0(\xi), d_1(h_G,h_B;A,X_1), d_2(h_G,h_B,k;X_2)\}$, $\{i_G(\xi), i_B(\xi), i_K(h_G,h_B;A,X_1\}$, $\{h'_G(h_G,h_B;A,X_1), h'_B(h_G,h_B;A,X_1)\}$, $\{m(h_G,h_B;\omega_H,A,X_1),$ $m(h_G,h_B;\omega_L,A,X_1), s_G(h_G,h_B;A,X_1), s_B(h_G,h_B;A,X_1)\}$ solve the agents' problems* (P0), (P1′) *and* (P2′), *taking $\{P_1^M(\omega_H;X_1),\ P_1^M(\omega_L;X_1)\}$, $\{\lambda_1^M(\omega_H;X_1)\}$, $\eta(X_1)$, λ_1^E, H_1, and $\{K(X_1)\}$ as given;*

2. *$\{P_1^M(\omega_H;X_1), P_1^M(\omega_L;X_1)\}$, $\{\lambda_1^M(\omega_H;X_1)\}$ and $\eta(X_1)$ are the partial equilibrium of the markets for trees state by state;*

3. *the rental rate $r(X_2)$ equals the marginal product of capital, $r(X_2)=f'(K(X_1))$;*

4. *λ_1^E, H_1 and $\{K(X_1)\}$ are consistent with individual decisions.*

It is important to note that the change in the definition of equilibrium does not imply a fundamental change in the functioning of the economy. In particular, if I used this definition of equilibrium in the previous section, all the results when the distribution F is continuous would hold. This is reassuring in the sense that the main forces of the economy do not change by allowing for a richer set of markets.

Finding an equilibrium involves similar steps than in the previous section. In particular, shadow prices are defined following the same logic. There are two differences. First, the investment in physical capital in period 1 connects the outcomes of period 1 and period 2, so finding an equilibrium of the economy starting in period 1 is a little more involved than before. Second, the economy does not scale linearly in H_1, so the fixed point I will need to solve is two dimensional in λ_1^E and H_1.

I solve for the equilibrium by backward induction. First, I find an equilibrium of the economy starting in period 1. Then, I move to period 0 and solve for the equilibrium of the full economy.

Define aggregate investment in physical capital as

$$I_1^K(X_1) = \int_0^{A^{\max}} A i_K(h_G, h_B; A, X_1) d\Gamma_1(h_G, h_B, A),$$

where $\Gamma_1(h_G, h_B, A)$ is the cross section distribution of portfolio holdings and investment opportunities. Then

$$I_1^K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} A[W_1 + [\eta(X_1)P_1^M(\omega_H; X_1) + (1-\eta(X_1))P_1^M(\omega_L; X_1)]H_1^B]dG(A) + \int_{A_1^S(X_1)}^{A^{\max}} A\eta(X_1)P_1^M(\omega_H; X_1)H_1^G dG(A). \quad (1.23)$$

Since the return on capital that the agents get depends on the aggregate capital of the economy, $A_1^B(X_1)$ and $A_1^S(X_1)$ depend on K, and in turn affect the market prices and rationing functions. Hence, it is useful to define the mapping $T_K(K; X_1) = I_1^K(K; X_1)$. An equilibrium of the economy in period 1 requires that $T_K(K; X_1) = K$. If I didn't allow for multiple markets and rationing, the mapping T_K could be discontinuous in K. Hence, the extension in the market for trees guarantees that there is a fixed point for any value of X_1.

Let's turn to period 0. In the previous section, finding an equilibrium involved finding a fixed point of a mapping that depended on λ_1^E, but not on H_1. The reason for this was that the economy starting in period 1 was linear in H_1 since there were constant returns to liquidity. Now, because f has decreasing returns in capital (i.e., r is decreasing in K), this is not true anymore. Therefore, I define a vector mapping $T(\lambda_1^E, H_1)$ given by

$$T(\lambda_1^E, H_1) = \begin{bmatrix} \frac{I_0^G(\lambda_1^E, H_1)}{I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)} \\ I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1) \end{bmatrix} \quad (1.24)$$

An equilibrium requires that

$$T(\lambda_1^E, H_1) = \begin{bmatrix} \lambda_1^E \\ H_1 \end{bmatrix}$$

The next proposition establishes existence of the equilibrium of the full economy.

Proposition 9. *An equilibrium of the economy always exists.*

While the equilibrium may not be unique, I will focus on the equilibrium with the highest fraction of good trees. This equilibrium is stable.

Next, I use the model to characterize the interaction between the financial markets and the real economy. First, better functioning markets increase the flow of resources to those with the best investment opportunities, hence aggregate capital in the economy increases. Second, higher productivity in the real economy, both through higher TFP of the representative firm, Z^Y, and investment opportunities, A, increases market prices and hence *worsens* the tree quality production in period 0.

Interaction Between Financial Markets and Real Investment

Real investment and the financial markets relate to each other through two channels. First, if there is more liquidity in the market then agents with good investment opportunities can invest more and aggregate capital in the economy goes up. Second, if TFP of the firm in period 2 goes up, investment opportunities are more profitable so more agents sell their good trees, which improves liquidity of the market. For future reference, define investment efficiency as

$$\frac{K(X_1)}{\int_0^{A^{\max}} i_K(h_G, h_B; A, X_1) d\Gamma_1(h_G, h_B, A)}$$

where the denominator is the total amount of resources used in the production of K.

Consider first how the functioning of the secondary markets affects the real economy.

Lemma 5 (Contagion). *Aggregate capital and investment efficiency are increasing in α and in H_1^G.*

Even though the two sectors of the economy are not directly related, the efficiency of the economy's investment depends on how well the secondary markets function. If the liquidity in the market is high, agents with good investment opportunities will be able to invest their endowment and obtain funds from the market. This implies that the total amount of capital they produce is relatively high, which crowds out low productivity agents. Hence, the efficiency of investment increases. Interestingly, if the crowding-out effect is strong enough, the aggregate consumption in period 1 may also increase. This is because the low productivity agents that switch from investing to not investing will now consume and buy trees. Hence, if the flow of new consumers is larger than the increased expenditures due to the price increase, total consumption in the economy goes up.

Next, I study the interaction between the real sector and the incentives to produce asset qualities. In particular, I consider the effects of an increase in the TFP level of the representative firm, Z^Y, and an increase in agents' investment opportunities, from A to ϕA, for some $\phi > 1$.

Lemma 6 (Shocks to the Real Economy). *An increase in the TFP level, Z^Y, or in the investment opportunities, from A to ϕA, increases $\mu_1(A, X_1)$ for every state (A, X_1). As a consequence, the production of trees, H_1, increases, and the fraction of good trees in the economy, λ_1^E, decreases.*

This lemma is an extension of Proposition 5 in the previous section. A higher demand for intermediation driven by a stronger real sector increases the liquidity premium and the incentives to produce low quality assets. Conditional on λ_1^E and H_1, P_1^M and λ_1^M are increasing in Z^Y and ϕ for every realization of α. Hence, λ_1^E decreases in equilibrium. This result can be quantitatively important to understand the build-up to the crisis. Bigio (2015) finds that, in the years previous to the crisis, the measured TFP for the US economy was above trend, and the crisis was triggered by a substantial drop in TFP *followed* by an increase in the adverse selection problem in financial markets. An interpretation of the data through the lens of my model is that the abnormally high TFP worsened the asset quality distribution in the years previous to the crisis, which was latent while TFP remained high but generated a collapse in financial markets when TFP declined.

1.4 Normative Implications

In this section, I use the model to analyze how the economy behave under government intervention. First, I study how the economy reacts to changes in the public supply of liquidity, with particular interest in how it affects the incentives to produce tree quality. Then, I analyze the role that transaction taxes and purchase programs play in shaping incentives and improving liquidity.

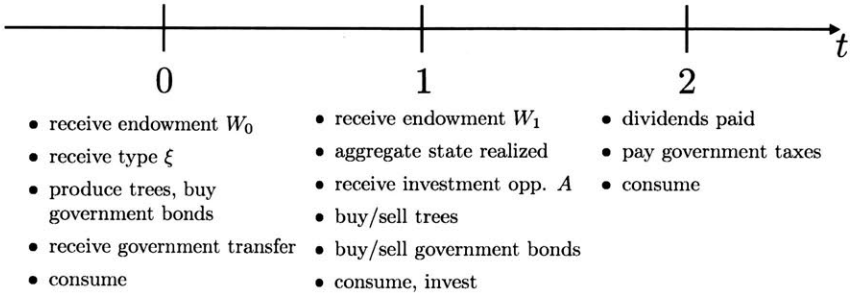
1.4.1 Government Bonds

Government bonds contribute to the total amount of liquidity in the economy. Intuitively, a higher volume of bonds allows for a greater volume of transactions, which increases investment. However, because of the decreasing returns to capital, this in turn reduces the marginal return to liquidity, reducing the liquidity premium on *all* tradable assets. Therefore, the incentives to produce trees decreases. Because bad trees are more sensitive to changes in the value of liquidity services, an increase in the supply of government bonds reduces the shadow value of bad trees disproportionally more than that of good trees, so that the fraction of good trees in the economy increases. However, government bonds can also have negative effects. For a given fraction of good trees in the economy, a larger supply of government bonds *increases* the adverse selection problem in the market. The reason is that government bonds crowd out private markets. Since bad trees are always sold, it is some of the good trees that leave the market, increasing the adverse selection wedge. Which effect dominates depends on the relative strength of each channel.

Consider the following timing. As before, agents start with an endowment W_0 of final goods. Agents receive a type ξ and decide whether to produce trees or not. But now, agents have a different alternative to consumption. They can buy government bonds at price P_0^{GB}. Government bonds pay one unit of final good in period 2. I still want to focus on economies that have positive consumption in all periods and states, so I assume that the supply of government bonds, B_0, is not too large compared to W_0 and W_1. In that case,

$$P_0^{GB} = \gamma_0^{GB} = E[\mu_1(A, X_1)].$$

Figure 1-6: Timing with government bonds.



That is, the price is equal to the liquidity services the bonds provide in period 1. Note that I am already imposing that the market price in period 1 is equal to one. The reason for this is that as long as aggregate consumption is positive, the return of bonds between periods 1 and 2 has to be equal to the intertemporal marginal rate of substitution of the buyers (who are the non-investors), which is equal to one.

For simplicity, I assume that the government rebates the proceeds of selling bonds in period 0 to the agents lump-sum, and then taxes agents lump-sum in period 2. In order to keep the mechanics of the model as close as possible to the previous sections, I assume that the government's transfers in period 0 occur after investment takes place, so that they cannot be used for investment. I make this assumption to isolate the market incompleteness in period 0 from the market incompleteness in period 1. Allowing the alternative would not change the main message, but would incorporate a distributive role of government bonds that is unlikely to be relevant in reality.[21] Figure 1-6 summarizes the new timing.

The quantity of government bonds affects the liquidity services and hence the *risk free* interest rate of the economy, which is given by

$$i_0 \equiv \frac{1}{E[\mu_1(A, X_1)]} - 1.$$

The first result shows that incomplete reallocation pushes interest rates down.

Lemma 7 (Laissez-faire Interest Rates). *Consider an equilibrium with positive consumption in every period and state. The interest rate in the laissez-faire equilibrium is*

[21] I could alternatively assume there is a different set of agents with linear preferences and no liquidity needs that receive the transfers. The result would be the same. This is the assumption in Holmström and Tirole (1998).

lower than in first best.

In first best, $\mu(A, X_1) = 1$ in all states if aggregate consumption is positive, since there is no limitation to the reallocation of resources among agents. As long as there is incomplete reallocation, $\mu_1(A, X_1) > 1$ for some A, hence the interest rate is lower.

Government bonds affect the economy through the quantity of liquid instruments, which in turn affects the liquidity premium of assets. This has three effects: the direct effect is to increase the flow of resources in the economy, since more government bonds implies more instruments to trade for goods, that is, more liquidity; second, it reduces the incentives to sell good trees, reducing both the quantity of assets traded as well as their price in the secondary market, which increases the adverse selection wedge for a given λ_1^E; last, in period 0, anticipating the effect government bonds have in period 1, it reduces the incentives to produce bad trees, hence increasing the equilibrium fraction of good trees in the economy. Next, I formally study these effects.

Consider the economy in period 1. Suppose that agents hold a total of $B_0 > 0$ of government bonds, distributed uniformly among all agents.[22] These bonds pay one unit of consumption good in period 2. How does the equilibrium in period 1 change with an increase in B_0? Assuming that B_0 is not too large, the price of government bonds between period 1 and 2 is equal to one. Keeping everything else fixed, an increase in B_0 increases investment:

$$I_1^K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} A[W_1 + [\eta(X_1)P_1^M(\omega_H; X_1) + (1-\eta(X_1))P_1^M(\omega_L; X_1)]H_1^B + B_0]dG(A) + \int_{A_1^S(X_1)}^{A^{\max}} A\eta(X_1)P_1^M(\omega_H; X_1)H_1^G dG(A).$$

However, as K increases, $r(K)$ decreases. This has two separate effects. On the one hand, $A_1^B(X_1)$ increases. As the return on capital decreases, investment becomes less attractive, so the agents that have the marginal productivity, $A_1^B(X_1)$, decide not to invest under the new conditions. That is, the presence of government bonds improves the flow of resources so that high productivity agents are able to invest more, while low productivity agents choose not to invest. Hence, a higher supply of government bonds increases investment efficiency.

[22] Because of the linearity of the value function and the iid assumption on investment opportunities, this is without loss of generality.

On the other hand, $A_1^S(X_1)$ also increases. Since the return of investing in capital decreases, fewer agents are willing to sell their good trees to produce capital. Note that this can have perverse effects in the secondary market for trees. While the demand for trees is not affected, the supply and quality of trees decrease. Hence, the market price or the rationing η decrease.

It is useful to define the total amount of liquidity in the economy. Total liquidity is the value of all the assets available for trade. In this economy it is given by

$$TL(X_1) \equiv \underbrace{B_0}_{\text{public liquidity}} + \underbrace{\eta(X_1)P_1^M(\omega_H;X_1)\left[[1-G(A_1^S(X_1))]H_1^G + H_1^B\right] + (1-\eta(X_1))P_1^M(\omega_L;X_1)H_1^B}_{\text{private liquidity}}.$$

Hence, a higher volume of government bonds in period 1 increases the investment in the economy and its efficiency, but it partially crowds out the market for trees, increasing the adverse selection wedge for a fixed λ_1^E. The next proposition summarizes these results.

Proposition 10. *Consider an economy in period 1 with some fraction of good trees, λ_1^E, and total amount of trees, H_1. Suppose the total amount of government bonds in the hands of agents increases. Then*

1. *the total amount of liquidity in the economy increases;*
2. *aggregate capital and investment efficiency increase;*
3. *the volume traded in the market for trees decreases;*
4. *liquidity services, $\mu_1(A, X_1)$, decrease for every state (A, X_1).*

Now, let's consider period 0. The government sells government bonds to agents. Agents anticipate that more public liquidity in period 1 reduces the liquidity premium and hence the shadow value of trees in period 0. This has a bigger impact on the shadow value of bad trees, so production of bad trees, I_0^B, decreases, and the fraction of good trees in the economy, λ_1^E, increases. Moreover, because the liquidity premium decreases, the risk-free interest rate of the economy increases. The next proposition summarizes the results.

Proposition 11. *An increase in the supply of government bonds reduces the production of bad trees, I_0^B, and increases the fraction of good trees in the economy, λ_1^E. The equilibrium interest rate in period 0 increases.*

The overall effect of B_0 on market fragility is ambiguous. On the one hand, the incentive to sell good trees decreases, but on the other hand the fraction of good trees in the economy increases. If the asset quality composition of the economy is sufficiently inelastic (exogenously given quality distribution is an extreme case), then a higher supply of government bonds increases fragility. However, below I describe an example that shows the forces at play, and why a reduction in market fragility is a plausible outcome. In the next section, I show that in an infinite horizon context, the dynamics of the economy also shape the effects of government bonds.

But first consider an extension of the model in which an external agent demands domestic government bonds. Even though the model is of a closed economy, one could easily extend it to incorporate international financial transactions. Suppose there is a foreign agent that buys government bonds. This reduces the local supply of government bonds (while increasing current consumption). The effect on the production of trees is analogous to a reduction in the supply of government bonds.

Corollary 11.1. *If a foreign agent buys government bonds, λ_1^E falls. The risk free interest rate also falls.*

This result also connects to stories of safe asset shortages due to the world's savings glut in the early 2000s, which put excessive pressure on the US financial sector to produce safe assets.[23]

Next I consider a particular technology for the representative firm that shows how government bonds can reduce financial fragility.

Example: Government Bonds Reduce Financial Fragility

Suppose the production function in period 2 is given by

$$f(K) = Z^Y \max\{K, K^*\}, \tag{1.25}$$

[23]See, for instance, Caballero (2006).

where K^* is a technological parameter. This production function has the special feature of being linear in the region $[0, K^*)$, and the marginal product of capital drops to zero when $K \geq K^*$. In order for the concave part of the production function to affect the economy, I assume that K^* is such that if α is high, $K(\alpha) = K^*$. That is, I assume that when the liquidity in the market is high, the economy achieves the first best quantity of capital (note that this does not imply that the allocation is first best for two reasons: first, bad trees were produced, which is socially inefficient; second, the composition of the investment in capital is also inefficient, since some investment is undertaken by low productivity agents). If α is low, then $K(\alpha) < K^*$. Moreover, K^* is high enough so that the financial fragility threshold α^* is such that $K(\alpha^*) < K^*$.

Consider an increase in B_0. In low α states, K increases but there is no impact on $r(K)$, hence the market for trees is not affected. In high α states, government bonds partially crowd out the private market, in this extreme case by increasing rationing. Hence, the direct effect of the increase in the supply of public liquidity is a reduction in the shadow value of trees, with the shadow value of bad trees decreasing more than that of good trees. Therefore, the production of bad trees decreases and the fraction of good trees in the economy, λ_1^E, increases. Since the state α^* features $K < K^*$, the overall effect of an increase in government bonds is a drop in the probability of a market collapse, that is, market fragility decreases when the supply of public liquidity increases. The reason why market fragility unambiguously decreases here is that, with this production function, government bonds *crowd out* private liquidity in high liquidity states, but *complement* private liquidity in low liquidity states. While this sounds like a reasonable result, it does not immediately hold for more general production functions. In those cases, the overall effect also depends on the elasticity of production of tree quality. This will be particularly interesting in the infinite horizon version of the model, where the elasticity of the fraction of good trees in the economy does not only depend on the elasticity of production but also on the stock and composition of trees in the economy from previous periods' production.

Optimal Policy

It is well known that allocations in economies with markets that suffer from adverse selection are usually interim-constrained Pareto Optimal, since it is not possible to improve efficiency in the economy without lowering the well-being of those benefiting from the asymmetric information.[24] However, here I am interested in aggregate allocations rather than distributional concerns. Therefore, in order to study optimal policy, I assume that the planner maximizes a utilitarian welfare function that puts equal weight on all agents. Hence, the planner maximizes:

$$\mathbb{W} = D_0 + E_0[D_1(X_1) + D_2(X_2)], \tag{1.26}$$

subject to the equilibrium conditions

$$\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)},$$
$$\frac{\gamma_0^B}{q_B(\xi_B)} = \frac{\gamma_0^{GB}}{P_0^{GB}} = 1,$$

where D_0, D_1 and D_2 are the aggregate consumption functions. This program is isomorphic to one in which the planner maximizes the expected utility of the representative agent *before* its type ξ is realized in period 0.

From the previous analysis one could conclude that the optimal policy should involve issuing enough government bonds so as to completely crowd out the private market. That is, the government could use its taxing power to become the monopolist producer of liquid instruments in the economy. This is an appealing solution since it separates the liquidity value of assets from their dividend value, so that assets are produced only for fundamental reasons. This logic resembles the Friedman Rule for monetary policy, that is, the government should completely satiate the liquidity needs of the agents.

However, there are at least two problems with this solution. First, the amount of bonds needed can be very large, so that the fiscal cost of the intervention could be very high. In order for the liquidity premium to be equal to zero, the agents with $A = A^{\max}$ need to hold enough liquidity to invest the optimal amount in period 1. Since investment opportunities are random, all agents that have a chance of getting the best investment opportunity need to be holding enough government bonds in advance. Moreover, the smaller the set of agents that can get the best shock, the larger the reallocation that is needed in period 1, so the larger the supply of bonds needed. In the limit in which the measure of agents with $A = A^{\max}$ is zero, the

[24]See Bigelow (1990).

amount of bonds the government has to issue in period 1 is infinite.

Second, even if the fiscal cost was zero, the dynamics of the economy might cause the bonds to end up in the wrong hands. Suppose the government is willing and able to issue all the bonds needed to completely satiate agents' liquidity needs in period 1. The problem is that some agents will prefer to invest in trees in period 0 (for fundamental reasons) instead of buying government bonds. And it is exactly because of this that *securitization* has a valuable social role. It allows investors to mitigate the trade-off they face between undertaking investment opportunities and keeping enough liquidity available to satisfy future needs. Hence, even if it wanted to, it is unlikely the planner can satisfy the full demand for liquidity with government bonds.

Given this discussion, I will continue my analysis under the assumption that if the government issues bonds in period 0, in period 2 it has to pay a cost q_{GB} per unit of bond issued (the shadow cost of taxation). This is a similar strategy to the one adopted by Holmström and Tirole (1998) and Tirole (2012). The benefit of this assumption, instead of using the model to determine the costs of taxation, is that the model was not built to take a stand on the cheapest way of collecting revenue. However, the exercise is still insightful to understand what optimal policy should look like. With this positive cost, the government will choose to complement the market rather than fully substitute for it.

In an interior solution, it must be that[25]

$$\begin{aligned}\gamma_0^{GB} + E\left[\frac{\partial\tilde{\gamma}_1^G(X_1)}{\partial P_1^M(\omega_H;X_1)}\frac{\partial P_1^M(\omega_H;X_1)}{\partial B_0} + \frac{\partial\tilde{\gamma}_1^G(X_1)}{\partial\eta(X_1)}\frac{\partial\eta(X_1)}{\partial B_0}\right] H_G + \\ E\left[\frac{\partial\tilde{\gamma}_1^B(X_1)}{\partial P_1^M(\omega_H;X_1)}\frac{\partial P_1^M(\omega_H;X_1)}{\partial B_0} + \frac{\partial\tilde{\gamma}_1^B(X_1)}{\partial\eta(X_1)}\frac{\partial\eta(X_1)}{\partial B_0}\right] H_B = 1 + q_{GB}. \quad (1.27)\end{aligned}$$

The LHS is the sum of the liquidity value of an extra unit of government bond, γ_0^{GB}, and the change in the value of private liquidity for a fixed level of K. The first term is analogous to the force that justifies government intervention in Woodford (1990) and Holmström and Tirole (1998). This effect would still be there even if there was perfect information in private markets.

But government bonds also affect the functioning of private markets. The change in the value of private liquidity depends on how prices and rationing react

[25]See the details of the derivation in Appendix 1.7.3.

to government bonds. For a fixed fraction of good trees in the economy, λ_1^E, both prices and rationing decrease because of the crowding-out effect of government bonds. Moreover, the drop in the liquidity premium disproportionally reduces the incentives to produce bad trees, so the average asset quality in the economy increases. Hence, while previous work suggested that optimal policy should equalize the liquidity premium to the shadow cost of taxation, when private markets are fragile it should also take into account potentially negative crowding-out effects.

Still, the government should try to smooth the changes of the liquidity premium in response to shocks. To see this, note that, in an optimum, the second order condition (SOC) has to be negative. But the effect of B_0 over the variables in the SOC works indirectly through the liquidity premium. The quantity of bonds affects the amount of investment and hence the amount of capital for period 2, which determines the rate of return of capital, $r(K)$, and hence the liquidity services $\mu_1(A, X_1)$. And it is the change in $\mu_1(A, X_1)$ that affects γ_0^B, $A_1^S(X_1)$, H_1^G, H_1^B, and $P_1^M(\omega_h; X_1)$. This is important because then I can sign the effect of any shock that affects the FOC only through the liquidity premium by determining if its effect has the same or opposite sign to the SOC. For example, an increase in Z^Y has the opposite effect of an increase in government bonds, hence the FOC increases with Z^Y, and optimal B_0 increases with Z^Y.

The next proposition summarizes this result.

Proposition 12. *Optimal policy takes the form of increasing the supply government bonds when shocks occur that increase the liquidity premium, and lowering it when shocks occur that reduce the liquidity premium.*

1.4.2 Transaction Tax

An alternative policy tool the government could use are transaction taxes and subsidies (or purchase programs). In fact, the government used purchase programs to improve liquidity in financial markets after the crisis hit. Tirole (2012) and Philippon and Skreta (2012) study how to optimally intervene in markets that collapse due to adverse selection from an ex-post point of view. Here, I analyze the problem from an ex-ante perspective. Since subsidies and purchase programs are equivalent in this setting, I assume that the government uses subsidies for notational convenience, even though purchase programs are better from a practical point of view.[26]

Suppose the price the sellers receive is $P_1^S(X) = P_1^M(X) + c(X)$, where $P_1^M(X)$ is the price paid by the buyers and $c(X)$ is the government's subsidy (or tax if negative).[27] By manipulating the price, the government is effectively doing two things. First, given H_1 and λ_1^E, it is deciding how much liquidity there is in the market. Second, it shapes the incentives to invest in period 0. While the government wants the highest possible liquidity in the markets and the highest possible quality production in period 0, transaction taxes and subsidies trade-off one for the other. So the question is what is the optimal way to balance these forces. In particular, what states should be taxed and what states should be subsidized? Below I show that the answer depends on whether the marginal value of liquidity in low liquidity states is high enough compared to the marginal value in high liquidity states. In the likely case that the value of liquidity in low liquidity states is sufficiently higher than in high liquidity states, then optimal policy requires that taxes are pro-cyclical (and potentially subsidize low liquidity states), in a *leaning against the liquidity* type of policy.

For simplicity, suppose that the quality of bad trees can only take two values: α_H and α_L, with $\alpha_H > \alpha_L$. The probability that $\alpha = \alpha_H$ is denoted by ζ_H. Moreover, assume that the production function is given by

$$f(K) = \begin{cases} Z^Y K & \text{if } K \leq K^* \\ Z^Y(K^* + \delta K) & \text{if } K > K^* \end{cases} \tag{1.28}$$

with $\delta \in [0, 1]$. Note that (1.25) is a particular case of (1.28) with $\delta = 0$. For this exercise it is more convenient to work with this form. I choose K^* such that if the state is α_H, in the laissez-faire equilibrium $K > K^*$. On the other hand, if the state is α_L, then $K < K^*$.This implies that $r(K(\alpha_H)) = Z^Y\delta < Z^Y = r(K(\alpha_L))$.

[26]With subsidies to transactions, agents could just buy and sell the same asset from one another repetedely only to receive the subsidy. By buying the asset, the government avoids this type of behavior. However, the two policies differ with respect to the timing of payments, even though they have the same net present value.

[27]I use this notation instead of the more standard ad-valorem subsidy/tax for analytical convenience. It is always possible to define the implicit ad-valorem subsidy/tax as $\tau(X) \equiv \frac{P_1^M(X)+c(X)}{P_1^M(X)} - 1$.

Maximizing (1.26) by choosing the values for $\{c(\alpha)\}$ gives the following FOC

$$\frac{\partial W}{\partial c(\alpha')} = E\left[\left[\int_{A_1^B(\alpha)}^{A^{\max}} Ar(K(\alpha))dG(A) + G(A_1^B(\alpha))\right]\left[\frac{\partial P_1^M(\alpha)}{\partial c(\alpha')} + \frac{\partial c(\alpha)}{\partial c(\alpha')}\right]\right] H_1^B +$$
$$E\left[\int_{A_1^S(\alpha)}^{A^{\max}} Ar(K(\alpha))dG(A)\left[\frac{\partial P_1^M(\alpha)}{\partial c(\alpha')} + \frac{\partial c(\alpha)}{\partial c(\alpha')}\right]\right] H_1^G = E\left[\frac{\partial T(\alpha)}{\partial c(\alpha')}\right],$$

where $T(\alpha) = c(\alpha)\left[[1 - G(A_1^S(\alpha))]H_1^G + H_1^B\right]$, is the fiscal cost (revenues if negative) of the policy $\{c(\alpha)\}$.

So, should taxes be pro-cyclical or counter-cyclical? The answer depends on the value of δ and ζ_H. If $\delta = 1$, it could be optimal to subsidize the high α state and tax the low α state. The reason is that market liquidity is convex in the selling price. When the production function is linear, $A_1^B(\alpha_H) = A_1^B(\alpha_L)$, since $r(K(\alpha_H)) = r(K(\alpha_L))$, but $A_1^S(\alpha_H) < A_1^S(\alpha_L)$, since $P_1^M(\alpha_H) > P_1^M(\alpha_L)$. Therefore, the direct benefits from the subsidy $E\left[\left[\int_{A_1^B(\alpha)}^{A^{\max}} Ar(K(\alpha))dG(A) + G(A_1^B(\alpha))\right]\frac{\partial c(\alpha)}{\partial c(\alpha')}\right] H_1^B +$ $E\left[\int_{A_1^S(\alpha)}^{A^{\max}} Ar(K(\alpha))dG(A)\frac{\partial c(\alpha)}{\partial c(\alpha')}\right] H_1^G$ are higher for the high liquidity state. If ζ_H is not too high, $\frac{\partial P_1^M(\alpha)}{\partial c(\alpha_L)} < \frac{\partial P_1^M(\alpha)}{\partial c(\alpha_H)} < 0$, since a one unit increase in the price of the low α state induces a higher production of lemons that a unit increase in the price of the high α state. Therefore, the optimal policy would require to increase liquidity in high liquidity states and lower it in low liquidity states.

This result is counter-intuitive and an artifact of the fact that agents are risk neutral, so that the elasticity of substitution across states of nature is infinite. Moreover, it goes in the opposite direction than the result in Tirole (2012), who finds that subsidies should be higher for low liquidity states. Even though Tirole (2012) also has agents with linear preferences, the production function has an extreme form of concavity at the individual level. I can achieve a similar result by choosing a δ that is low enough. In that case, extra liquidity in the high liquidity state is less valuable than in the low liquidity state because in the former it gives a return of $AZ^Y\delta$ while in the latter the return is AZ^Y. It is straightforward to see that as δ goes to zero, the benefits from extra liquidity in the high liquidity state vanish away. In that case, the optimal policy prescribes a pro-cyclical transaction tax (whether it implies subsidizing the low state depends on parameter values). The next proposition summarizes these results.

Proposition 13. *There exists $\delta^* \in (0, 1]$ such that if $\delta < \delta^*$, the optimal transaction tax* *is pro-cyclical.*

1.5 Infinite Horizon

In the previous sections I presented a three-period model which allowed me to study the interaction between the incentives to produce assets of different qualities and changes in the economy's fundamentals and government policy. This section builds a tractable extension to an infinite horizon model in order to get some insights about the dynamic behavior of these mechanisms.

1.5.1 The Environment

The model is analogous to the three-period version with two main differences. First, agents operate the technology to produce trees and physical capital every period. Second, there are markets for trees every period.

There is a continuum of infinitely lived agents. There are three types of goods: a final consumption good, Lucas (1978) trees (which can be good or bad), and physical capital. Agents maximize utility:

$$U_t = E_t \left[\sum_{s=t}^{\infty} \beta^s d_s \right],$$

where d_s is consumption in $s = \{t, t+1, t+2, ...\}$, β is the agents' discount factor, and the expectation is taken with respect to their idiosyncratic investment opportunities and an aggregate state of the economy, both described below. For convenience, I assume that agents receive an endowment W of final goods every period. This will give me a flexible way of guaranteeing that there is positive aggregate consumption in all states around the stochastic steady state of the economy (described below), so that pricing is risk neutral like in the previous sections.

Agents have access to two technologies every period: one that produces trees and one that produces physical capital. The technologies and payoff of trees and capital are a natural extension of the ones described in the three-period models, with some simplifying assumptions.

I assume that trees are long lived and depreciate at a rate δ_H. While good trees pay a dividend Z every period, bad trees pay αZ, where $\alpha \sim F(\alpha)$ with support in $[0,1]$, for some non-degenerate continuous cumulative distribution function F. For simplicity, I assume that α is iid over time. That is, trees of different qualities die at the same rate, but while good trees always pay Z, bad trees pay a fraction of that. Moreover, I assume that all agents face the same cost of producing trees. I normalize the cost of producing bad trees to one, so that $q_B = 1$. On the other hand, the cost of producing good trees has two components: producing one unit of good tree costs $q_G \phi(I_G)$, where I_G is the aggregate production of good trees. The term q_G could be interpreted as the unit cost of production, with $q_G > q_B$. The term $\phi(I_G)$ is an investment adjustment cost, with $\phi(0) = 1$, $\phi' > 0$, and $\phi'' > 0$. This avoids that the stock of good trees grows without bound in this linear environment.[28]

I keep the production technology of physical capital from the previous section. That is, agents receive a productivity A drawn from a convex and continuous cumulative distribution function G with support $[0, A^{\max}]$. In order to simplify the dynamic interactions of the economy, I assume that physical capital fully depreciates after use. There is a representative firm that operates a concave production function $f(K)$ and it rents capital from the agents in competitive markets, so that the rental rate of capital is given by $r(K) = f'(K)$. Firm's profits are distributed uniformly across all agents.

The structure of the market for capital is analogous to the extended three-period model from the previous section. Every period there could be up to two active markets. In one market only bad trees are traded, and there is no rationing. In the other market, good and bad trees are traded at a pooling price and there can be rationing in equilibrium.

Finally, the government supplies an amount B of one-period bonds every period, that pay 1 unit of final good at maturity. The per-period budget constraint of the government is given by

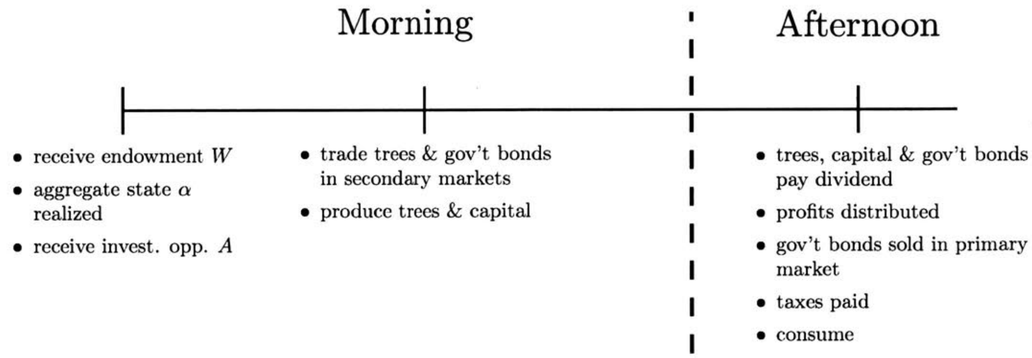
$$B - P_{GB} B = T,$$

where P_{GB} is the price at which the government sells the bonds in the primary market, and T is a lump-sum tax to the agents.

The timing within a period is as follows. For tractability, I assume that each

[28]There are different assumptions that would bound the amount of trees in equilibrium, and convex investment adjustment costs is a very tractable one. Note that, by making the adjustment cost depend on aggregate investment, the problem of the agents remains linear, so there is no need to keep track of cross-section distribution of agents' portfolio holdings to determine aggregate allocations.

Figure 1-7: Within Period Timing



period is divided into two sub-periods, which I denote by "morning" and "afternoon". In the morning, agents receive the endowment W, and both the aggregate state, α, and the idiosyncratic investment opportunity, A, are realized. Moreover, the secondary markets for trees and government bonds open and production of trees and physical capital takes place. Note that this implies that only the own endowment W and the proceeds from trading assets can be used in production. In the afternoon, trees pay their dividend, production in the representative firm takes place, the rental rate of capital is paid and profits distributed, the government pays the outstanding bonds and sells new bonds in the primary market, and agents pay taxes and consume. This timing makes each period in the infinite horizon model as close as possible to the timing in the three-period economy from the previous sections and greatly simplifies the dynamics of the economy, as descried below. Figure 1-7 depicts the timing within a period.

I will look for a recursive competitive equilibrium of the economy with $X \equiv \{\lambda_E, H, K, B; \alpha\}$ as a state variable, where λ_E is the fraction of good trees, H is the total amount of trees, K is the amount of physical capital, B is the supply of government bonds, and α is the exogenous quality of bad trees.

1.5.2 Agents' Problem and Equilibrium

Agents start the period with a portfolio of trees (good and bad), capital, and government bonds. An agent's investment opportunity is given only by A, since all agents face the same cost of producing trees. Every period there can be two markets for trees active. The first features a price $P_M(\omega_H, X)$ and both good and bad trees are traded, and $\lambda_M(X)$ denotes the fraction of good trees in the market. However, only a fraction $\eta(X)$ of the trees supplied are actually sold. In the second market, the price is $P_M(\omega_L, X)$ and only bad trees are sold. Since a fraction $\eta(X)$ of the bad trees are sold in the high-price market, only the remainder $1 - \eta(X)$ is sold in this low-price market. Moreover, buyers can decide in which market to trade. Let $m(\omega_H)$ denote the purchases in the high price market, and $m(\omega_L)$ denote the purchases in the low price market (where they know they are getting bad trees with probability one). Finally, let $Q(X)$ denote the price of the government bonds traded in the secondary markets and $\tilde{b}$ the holdings of government bonds at the end of the morning.

Thus, an agent's budget constraint in the morning is given by

$$\begin{aligned}\phi(I_G)q_G i_G + i_B + i_K + P_M(\omega_H, X)m(\omega_H) + P_M(\omega_L, X)m(\omega_L) + Q(X)\tilde{b} \leq W+ \\ P_M(\omega_H, X)\eta(X)(s_G + s_B) + P_M(\omega_L, X)(1 - \eta(X))s_B + Q(X)b, \quad (1.29)\end{aligned}$$

which states that expenditures in investment and purchases of trees and government bonds cannot exceed the sum of endowment, trees sold and government bonds sold.

I assume that the final good cannot be stored between periods, but it can be stored between morning and afternoon. Let Δ denote the surplus in the morning (note that $\Delta \geq 0$). Then, the agent's budget constraint in the afternoon is given by

$$\begin{aligned}&d + P_{GB}(X)b' \leq \Delta + [h_G + \lambda_M(X)m(\omega_H) - \eta(X)s_G]Z+ \\ &[h_B + (1 - \lambda_M(X))m(\omega_H) + m(\omega_L) - s_B]\alpha Z + r(X)k + \tilde{b} + \Pi(X) - T(X), \quad (1.30)\end{aligned}$$

which states that expenditures in consumption and purchases of government bonds in the primary market cannot exceed the sum of the surplus from the morning, the dividends received (from trees, capital, firms and government bonds), and government transfers.

Finally, agents face the following laws of motion of their portfolio holdings

$$h'_G = (1 - \delta_H)[h_G + \lambda_M(X)m(\omega_H) - \eta(X)s_G] + i_G, \quad (1.31)$$

$$h'_B = (1 - \delta_H)[h_B + (1 - \lambda_M(X))m(\omega_H) + m(\omega_L) - s_B] + i_B, \quad (1.32)$$

$$k' = Ai_K. \quad (1.33)$$

Therefore, the problem of an agent with investment opportunity A is given by

$$V(h_G, h_B, k, b; A, X) = \max_{\substack{d, i_G, i_B, i_K, m, s_G, s_B \\ h'_G, h'_B, k', \tilde{b}, b', \Delta}} d + \beta E[V(h'_G, h'_B, k', b'; A', X')|X], \quad \text{(P)}$$

subject to (1.29), (1.30), (1.31), (1.32) and (1.33), and

$$d \geq 0, \quad i_G \geq 0, \quad i_B \geq 0, \quad i_K \geq 0, \quad m \geq 0, \quad b' \geq 0,$$

$$s_G \in [0, h_G], \quad s_B \in [0, h_B].$$

As in the three period models, the value function V is linear in *each* element of the agents' portfolio:

$$V(h_G, h_B, k, b; A, X) = \tilde{\gamma}_G(A, X)h_G + \tilde{\gamma}_B(A, X)h_B + \tilde{\gamma}_K(A, X)k + \tilde{\gamma}_{GB}(A, X)b,$$

where

$$\begin{aligned}
\tilde{\gamma}_G(A, X) =& \max\{\mu(A, X)\eta(X)P_M(\omega_H, X) + (1 - \eta(X))[Z + (1 - \delta_H)\gamma_G(X)], \\
& Z + (1 - \delta_H)\gamma_G(X)\}, \\
\tilde{\gamma}_B(A, X) =& \mu(A, X)[\eta(X)P_M(\omega_H, X) + (1 - \eta(X))P_M(\omega_L, X)], \\
\tilde{\gamma}_K(A, X) =& r(X), \\
\tilde{\gamma}_{GB}(A, X) =& \mu(A, X),
\end{aligned}$$

and liquidity services are given by

$$\mu(A, X) = \max\left\{1, \frac{\gamma_G(X)}{\phi(I_G)q_G}, \gamma_B(X), A\gamma_K(X), \mu_B(X), \frac{1}{Q(X)}, \frac{\gamma_{GB}(X)}{P_{GB}(X)}\right\}, \quad (1.34)$$

where $\mu_B(X) \equiv \max\left\{\frac{\lambda_M(X)[Z+(1-\delta_H)\gamma_G(X)]+(1-\lambda_M(X))[\alpha Z+(1-\delta_H)\gamma_B(X)]}{P_M(\omega_H,X)}, \frac{\gamma_B(X)}{P_1^M(\omega_L,X)}\right\}$ is the return from buying trees in the secondary market.[29] Finally, the shadow prices are given by

$$\gamma_j(X) = \beta E[\tilde{\gamma}_j(A', X')|X], \quad j \in \{G, B, K, GB\}.$$

The agents' choices follow the same logic than in the simple three-period model.

[29]There is a slight abuse of notation since there are states in which only one market is active. In that case $\mu_B(X)$ is the return on the active market.

Consider the problem of an agent with investment opportunity given by A. There exists $A_B(X)$ such that the agent chooses to produce physical capital if and only if $A \geq A_B(X)$. Moreover, there exists $A_S(X) \geq A_B(X)$ such that if $A \geq A_S(X)$, the agent sells his good trees in order to invest in capital. Agents with $A < A_B(X)$ use their liquid wealth to consume, produce trees or buy trees and government bonds in the market. Since at least one market for trees is always active in equilibrium, optimality requires that

$$\mu_B(X) \geq 1, \tag{1.35}$$

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} \leq \mu_B(X), \tag{1.36}$$

$$\gamma_B(X) \leq \mu_B(X), \tag{1.37}$$

$$\frac{1}{Q(X)} \leq \mu_B(X), \tag{1.38}$$

$$\frac{\gamma_{GB}(X)}{P_{GB}(X)} \leq \mu_B(X). \tag{1.39}$$

Therefore, $A_B(X) \equiv \frac{\mu_B(X)}{\gamma_K(X)}$ and $A_S(X) \equiv \frac{\gamma_G(X)}{P_M(\omega_H,X)\gamma_K(X)}$.

Consider now the market for trees. Let $M(\omega_H, X)$ and $M(\omega_L, X)$ be the total demand in markets ω_H and ω_L respectively. They must satisfy

$$P_M(\omega_H, X)M(\omega_H, X) + P_M(\omega_L, X)M(\omega_L, X) \leq \int_0^{A_B(X)} W dG(A). \tag{1.40}$$

Moreover, (1.35) imposes the following restrictions on prices

$$P_M(\omega_H, X) \leq \lambda_M(X)[Z + (1-\delta_H)\gamma_G(X)] + (1-\lambda_M(X))[\alpha Z + (1-\delta_H)\gamma_B(X)], \tag{1.41}$$

$$P_M(\omega_L, X) \leq \alpha Z + (1-\delta_H)\gamma_B(X), \tag{1.42}$$

where

$$\lambda_M(X) = \frac{[1-G(A_S(X))]\lambda_E}{[1-G(A_S(X))]\lambda_E + (1-\lambda_E)}. \tag{1.43}$$

On the other hand, let $S(\omega_H, X)$ and $S(\omega_L, X)$ be the supply of trees in market

ω_H and ω_L respectively. Then

$$S(\omega_H, X) = \int_{A_S(X)}^{A^{\max}} H_G dG(A) + H_B, \tag{1.44}$$

$$S(\omega_L, X) = H_B. \tag{1.45}$$

A partial equilibrium in the markets for trees requires that $M(\omega_H, X) = \eta(X)S(\omega_H, X)$ and $M(\omega_L, X) = (1 - \eta(X))S(\omega_L, X)$, and that prices are the highest consistent with (1.41) and (1.42) (maximal volume of trade equilibrium). If (1.40) is satisfied with strict inequality, then (1.41) and (1.42) hold with strict equality. In the previous sections I simplified the problem by assuming that the analogous to (1.40) was always satisfied with strict inequality. Here, I will assume that the same holds around the stochastic steady state I define below.

Finally, the laws of motion of λ_E, H and K are given by

$$\lambda_E'(X) = \lambda_E \theta(X) + \frac{I_G(X)}{I_G(X) + I_B(X)}(1 - \theta(X)), \tag{1.46}$$

where $\theta(X) \equiv \frac{(1-\delta_H)H}{(1-\delta_H)H + I_G(X) + I_B(X)}$,

$$H'(X) = (1 - \delta_H)H + I_G(X) + I_B(X), \tag{1.47}$$

and

$$K'(X) = \int_{\frac{1}{\gamma_K(X)}}^{A^{\max}} A[W + [\eta(X)P_M(\omega_H, X) + (1 - \eta(X))P_M(\omega_L, X)]H_B + Q(X)B]dG(A) + \int_{\frac{\gamma_G(X)}{\gamma_K(X)P_M(\omega_H, X)}}^{A^{\max}} A\eta(X)P_M(\omega_H, X)H_G dG(A). \tag{1.48}$$

where $\lambda_E'(X)$, $H'(X)$ and $K'(X)$ are the fraction of good trees, total amount of trees, and aggregate physical capital, respectively, one period ahead. Note that (1.48) is the analogous to its three-period counterpart (1.23).

I define an equilibrium for this economy.

Definition 6 (Equilibrium). *An equilibrium consists of prices* $\{P_M(\omega_H, X), P(\omega_L, X), Q(X), P_{GB}(X), r(X)\}$*; market fraction of good trees* $\lambda_M(X)$ *and rationing* $\eta(X)$ *of market* ω_H*; a value function* $V(h_G, h_B, k, b; A, X)$*, shadow values* $\{\gamma_G(X), \gamma_B(X), \gamma_K(X), \gamma_{GB}(X)\}$ *and decision rules* $\{d, i_G, i_B, i_K, m(\omega_H), m(\omega_L), s_G, s_B, h_G', h_B', k', \tilde{b}, b', \Delta\}$ *that* *depend on* $(h_G, h_B, k, b; A, X)$, *such that*

1. $\{d, i_G, i_B, i_K, m(\omega_H), m(\omega_L), s_G, s_B, h'_G, h'_B, k', \tilde{b}, b', \Delta\}$, *and* $V(h_G, h_B, k, b; A, X)$ *solve program* (P) *taking* $P_M(\omega_H, X)$, $P_M(\omega_L, X)$, $Q(X)$, $P_{GB}(X)$, $r(X)$, $\lambda_M(X)$, *and* $\eta(X)$ *as given;*

2. *the markets for trees clear:* $M(\omega_H, X) = S(\omega_H, X)$ *and* $M(\omega_L, X) = S(\omega_L, X)$*;*

3. $P_M(\omega_H, X)$, $P_M(\omega_L, X)$ *and* $\lambda_M(X)$ *satisfy* (1.40), (1.41), (1.42) *and* (1.43)*;*

4. *the primary and secondary markets for government bonds clear;*

5. *the rental rate* $r(X)$ *equals the marginal product of capital,* $r(X) = f'(K)$*;*

6. *the laws of motion of* λ_E*,* H *and* K*, given by* (1.46), (1.47) *and* (1.48)*, are consistent with individual decisions and rationing* $\eta(X)$*.*

1.5.3 Stochastic Steady State

I study the economy around a stochastic steady state. This is a natural starting point, and as I will show, provides a tractable laboratory to study the dynamics of the economy. I do this in stages. First, I analyze the characteristics of the steady state. I show that there exists an equilibrium in which λ_E and H are constant over time, and K fluctuates with the aggregate shock α. Then, I characterize the dynamic properties of the economy around this equilibrium. Finally, I perform some comparative statics exercises.

I guess and verify that the equilibrium with λ_E and H constant over time exists. The laws of motion of λ_E and H (equations (1.46) and (1.47)) imply that

$$\Delta\lambda_E = 0 \Leftrightarrow \lambda_E = \frac{I_G}{I_G + I_B}, \tag{1.49}$$

$$\Delta H = 0 \Leftrightarrow H = \frac{I_G + I_B}{\delta_H}. \tag{1.50}$$

Thus, (λ_E, H) are constant over time if and only if I_G and I_B are constant over time, independently of K and α. Since $\lambda_E \in (0,1)$, both good and bad trees have to be produced in equilibrium. Moreover, I assume that W is large enough so that $\mu_B = 1$ in all states in the steady state (below I put a lower bound on W so that this is satisfied), (1.41) and (1.42) are satisfied with equality, and $Q(X) = 1$ and

$P_{GB}(X) = \gamma_{GB}(X)$. Therefore, agents that produce trees have to be indifferent between this and consuming, thus

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} = \gamma_B(X) = 1,$$

which implies that γ_G and γ_B have to be constant over time. Two things remain to be shown. First, that constant γ_G and γ_B are consistent with the definitions of the shadow values. Second, that there exists λ_E and H consistent with this equilibrium. The shadow values of trees are given by

$$\begin{aligned}\gamma_G(X) =&\beta E[\max\{\mu(A', X')\eta(X')P_M(\omega_H, X') + (1-\eta(X'))[Z + (1-\delta_H)\gamma_G(X')],\\ &Z + (1-\delta_H)\gamma_G(X')\}|X], \qquad (1.51)\\ \gamma_B(X) =&\beta E[\mu(A', X')[\eta(X')P_M(\omega_H, X') + (1-\eta(X'))P_M(\omega_L, X')]|X],, \qquad (1.52)\end{aligned}$$

where $\mu(A, X) = \max\{1, A\gamma_K(X)\}$. Then, (1.51) and (1.52) are constant over time conditional on (λ_E, H) being constant, if and only if they are independent of α and K. First, since α is iid, the shadow values do not directly depend on α. Second, the shadow values do not depend explicitly on K and $K'(X)$. Hence, it is sufficient to show that $K'(X)$ does not depend on K. But it is immediate from (1.48) that $K'(X)$ does not depend on K, since prices P_M, fraction λ_M, and rationing η only depend on shadow values and current realization of α, proving that the shadow values of trees are constant over time if (λ_E, H) are constant over time. This result relies on three assumptions: first, aggregate shocks are iid; second, capital fully depreciate after use; third, trees and capital pay their dividend after trade and investment (in trees and capital) takes place. Below I discuss how changing these assumptions would change the results.

Finally, I need to show that there exists a pair (λ_E, H) consistent with the equilibrium. Recall that a constant path of λ_E and H solves

$$\lambda_E = \frac{I_G}{I_G + I_B}, \qquad (1.53)$$

and

$$H = \frac{I_G + I_B}{\delta_H}, \qquad (1.54)$$

where I_G and I_B have to be consistent with individual optimality conditions. These two equations determine two curves in the space (λ_E, H). A steady state is characterized by an intersection of these curves. To see that at least one intersection exists consider the following. On the one hand, (1.53) is strictly greater than zero when $H = 0$ because $I_G > 0$, and has a limit $\tilde{\lambda}_E < 1$ when $H \to \frac{W}{\delta_H}$, since $I_B > 0$. On the other hand, (1.54) is greater than zero when $\lambda_E = 0$ since $I_G > 0$, and less than $\frac{W}{\delta_H}$ when $\lambda_E = \tilde{\lambda}_E$, since at least some endowment is used to produce physical capital. Since they are both continuous functions, an intersection exists. However, there could be multiple intersections. Although this could potentially be an interesting phenomenon to study, it is beyond the scope of this paper. Therefore, I select the one that features the highest λ_E.

Finally, a sufficient restriction on W is that

$$G(A_M(X))W > P_M(\omega_H, X)\eta(X)[[1 - G(A_S(X))]H_G + H_B] + \\ P_M(\omega_L, X)(1 - \eta(X))H_B + [1 - G(A_M(X))]B,$$

for all states α. Since the left hand side is increasing in W (because $A_M(X)$ is increasing in W) and unbounded, while the right hand side is decreasing in W and bounded, there exists an open set in $\mathbb{R}_+$ such that the condition is satisfied.

In the stochastic steady state the realization of α only affects the liquidity in the market and therefore the production of capital. Higher α implies higher volume traded and therefore more reallocation towards the agents with the highest productivities. Thus, this infinite horizon extension keeps the main insights from the previous sections while maintaining tractability.

Finally, it is easy to see that in the intersection with the highest level of λ_E, (1.53) crosses (1.54) from below in the space (λ_E, H). This property is key to show that the steady state is stable and the economy converges monotonically to the steady state from any initial conditions (λ_E, H) that are sufficiently close to it. I show this in Appendix 1.7.4.

Next, I study the dynamic response of the economy to a transitory shock.

1.5.4 Transitory Shock

When studying the effect of shocks on market fragility, a general result stated that the overall effect was ambiguous. While the fundamental shock reduces the fragility of the system, the endogenous response generates an opposite effect. The overall result depends on functional forms and parameter values. A natural question is what happens if the economy goes back to its initial value or trend, in a impulse response type of exercise.

Here I study a transitory increase in the distribution of the quality of bad trees, α. Suppose the economy is in its stochastic steady state in period T and the distribution of α in $T+1$ increases from F to $\tilde{F}$ such that $\tilde{F} > F$ in first order stochastic dominance sense, and then it goes back to F in $T+2$. On impact, this generates an increase in the shadow value of trees, with the shadow value of bad trees increasing proportionally more than that of good trees. Therefore, production of bad trees increases and the fraction of good trees in the economy decreases. This is the same effect I found in the three-period model. In $T+1$, the agents anticipate that fundamentals go back to their initial level, so their incentives to produce bad trees decreases. Moreover, the incentives to produce good trees is also lower than in the steady state. To see this note that

$$\lambda_E' = \lambda_E \theta(\lambda_E, H) + \frac{I_G}{I_G + I_B}(1 - \theta(\lambda_E, H)).$$

Therefore, if λ_E is lower, $\frac{I_G}{I_G+I_B}$ has to be higher. But for each level of investment, the shadow value of good trees is lower than in the stochastic steady state, since it results in a lower fraction of good trees and hence in lower expected prices. Therefore, market fragility in $T+2$ is higher than in the stochastic steady state. The next proposition states the main result of this section.

Proposition 14. *Consider an economy that starts in its stochastic steady state. Suppose in period T the economy is hit by a shock that increases the distribution of the quality of bad trees α in FOSD sense for one period. Then, the fraction of good trees in $T+1$ decreases and market fragility in period $T+2$ is higher than in the stochastic steady state.*

This result is important because it shows that a transitory shock sows the seeds of the a crisis by generating perverse incentives in the boom that exposes the economy to a bust when conditions go back to "normal". The result can also be extended to the other sources of risk studied before, like a transitory increase in the TFP of the representative firm or a transitory reduction in the supply of government bonds. Note that the intuition is a natural extension from the insight gained in the three period models: a positive shock worsens the composition of assets in the economy, and since assets are long lived, when the shock vanishes away the fraction of good assets in the economy is smaller than before the shock, while the exogenous state goes back to its initial level, so fragility increases.

Moreover, this setting allows me to study the effects of the timing of unexpected government intervention. The government could issue bonds as soon as the shock hits or wait until the economy goes back to its trend. If the issuance occurs when the shock hits, higher public liquidity reduces the liquidity premium, which reduces the incentives to produce bad trees. Therefore, a small increase in government bonds reduces market fragility with respect to a situation of no government intervention. On the other hand, if the intervention occurs when the shock dies out, the increase in public liquidity crowds out the production of *good* trees, increasing market fragility even more.

Proposition 15. *A small increase in government bonds in T reduces market fragility. A small increase in government bonds in $T+1$ increases market fragility.*

1.5.5 Discussion

In order to keep the tractability of the infinite horizon model I made several assumptions about the fundamentals and the timing of the economy. Without them, the steady state I analyzed would not exist, and both λ_E and H would fluctuate with the realization of the aggregate shock α. Here I briefly discuss what would change in the more general setting.

If shocks were not iid and capital did not fully depreciate after use, shocks in one period would carry information about the economy in future periods. I conjecture that under different assumptions, as long as the aggregate shock has positive auto-correlation, the main results should not change. That is, a positive transitory shock would disproportionally increase the production of bad trees and, thus, reduce the fraction of good trees in the economy. As the shock vanishes away, the economy faces the same conditions than in my simplified economy: the same fundamentals than before the shock (for a *given* path of realizations of the aggregate state) but a worse asset quality composition in the economy. Therefore, market fragility would increase.

Moreover, I assumed that trees and capital pay after the market for trees close and investment is undertaken. Each assumption performs a different role. Trees need to pay after the market closes so that with iid aggregate shocks there is some risk in the market. On the other hand, if the dividends from capital were used for investment, it would introduce a term that connects past shocks with current control variables, so that a steady state with constant λ_E and H would not exist. However, the main forces of the economy do not change, so the results are likely to survive. But more research is needed to fully understand the implications of the infinite horizon economy.

1.6 Conclusion

I have developed a model in which ex-ante production of assets interacts with ex-post adverse selection in financial markets. The production of low-quality assets is more sensitive to changes in markets conditions and the value of liquidity services than that of high-quality assets. Therefore, shocks that improve market functioning, such as reductions in the "default rate" of low-quality assets, increases in the productivity of the real economy, or reductions in transaction costs, deteriorate the asset quality composition of the economy and can even increase the probability of a financial crisis, defined as an event in which the financial markets collapse. Moreover, the supply of public liquidity also affects the private incentives to produce asset quality. I show that an increase in government bonds increases the total liquidity available in the economy and reduces the incentives to produce low-quality assets, but it can also exacerbate the adverse selection problem in private markets. If the production of trees is sufficiently elastic, then a reduction in government bonds can increase market fragility.

All these comparative statics point to plausible sources of risk build-up in the US before the Great Recession: perceived low risk on subprime mortgages that ended when house prices started to decrease; strong growth rates at the wake of the "dot-com" crisis; financial innovation that reduced the costs of trading illiquid assets; safe asset shortage due to fiscal surpluses in the late 90s as well as foreign demand in the early 2000s (the "savings glut").

Moreover, I study optimal policy in this setting. I find that the government should take into account the crowding-out effect on private markets when choosing the supply of bonds. Still, supply should increase when the liquidity premium increases (and viceversa). Moreover, I show that if the liquidity in low-liquidity states is sufficiently more valuable than in high-liquidity states, transaction taxes (or subsidies) that "lean against liquidity" are optimal.

Finally, I extend the insights from the basic models to an infinite horizon setting. I show that financial fragility is a natural outcome after transitory shocks, and that government intervention through the issuance of bonds should take place when the shock hits rather than when it dies out, since in the latter case it can exacerbate the negative effects of the lower asset quality distribution. In this analysis, I had to make strong assumptions in order to keep the model tractable. A natural next step would be to study whether these results survive in more realistic models (which would probably need to be solved numerically).

1.7 Appendix

1.7.1 Basic Model

Proof of Lemma 1. Let $\kappa_1(h_G, h_B; \mu_1, X_1)$ be the Lagrange multiplier associated to the budget constraint of program (P1). The FOC with respect to d is

$$\mu_1 - \kappa_1(h_G, h_B; \mu_1, X_1) \leq 0.$$

Moreover, the FOC with respect to m is

$$-\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) + \lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z \leq 0.$$

Therefore,

$$\kappa_1(h_G, h_B; \mu_1, X_1) = \max\left\{\mu_1, \frac{\lambda_1^M(X_1)Z + (1 - \lambda_1^M(X_1))\alpha Z}{P_1^M(X_1)}\right\}.$$

Define $\mu_1^B(X_1) \equiv \frac{\lambda_1^M(X_1)Z+(1-\lambda_1^M(X_1))\alpha Z}{P_1^M(X_1)}$. This is the return from the market, which is the same for all agents. Therefore, if $\mu_1 < \mu_1^B(X_1)$, then $m > 0$ and $d = 0$. If $\mu_1 = \mu_1^B(X_1)$, the agent is indifferent between consuming and buying trees in the market. On the other hand, if $\mu_1 > \mu_1^B(X_1)$, then $m = 0$ and $d > 0$.

Moreover, the FOC with respect to s_G is

$$\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) - Z.$$

Therefore, if $\mu_1 \leq \mu_1^B(X_1)$, $\kappa_1(h_G, h_B; \mu_1, X_1) = \mu_1^B(X_1)$, and hence $\kappa_1(h_G, h_B; \mu_1, X_1)P_1^M(X_1) - Z < 0$ as long as $\lambda_1^M(X_1) < 1$, and $s_G = 0$. Let $\mu_1^S(X_1) \equiv \frac{Z}{P_1^M(X_1)}$. If $\mu_1 > \mu_1^S(X_1)$, then $s_G = h_G$, and zero otherwise. It is straightforward to see that $s_B = h_B$ for all μ_1.

Finally, note that the demand for trees is

$$M(X_1) = \int_1^{\mu_1^B(X_1)} \frac{W_1}{P_1^M(X_1)} dG(\mu_1),$$

while the supply is

$$S(X_1) = \int_{\mu_1^S(X_1)}^{\mu_1^{\max}} H_G dG(\mu_1) + H_B.$$

As π increases, M increases for all prices, while S decreases. If π is high enough, agents with μ_1 have enough wealth to buy all the trees in the market and also consume. Therefore, they have to be indifferent between the return from the market, $\mu_1^B(X_1)$ and the utility from consumption, 1. Hence, $\mu_1^B(X_1) = 1$ and $P_1^M(X_1) = \lambda_1^M(X_1)Z + (1-\lambda_1^M(X_1))\alpha Z$. ■

Proof of Lemma 2. Since agents always sell their bad trees, and agents with $\mu_1 > 1$ consume, while agents with $\mu_1 = 1$ consume and buy trees in the market (with a return of one), the utility from a unit of good tree is given by $\mu_1 P_1^M(X_1)$. Similarly, the utility from the endowment W_1 is given by μ_1. On the other hand, only agents with $\mu_1 > \mu_1^S(X_1)$ sell their good trees, in which case the get a utility of $\mu_1 P_1^M(X_1)$. If they don't sell, they a utility of Z in period 2. Note that $\mu_1 > \mu_1^S(X_1) \Leftrightarrow \mu_1 P_1^M(X_1) > Z$. Hence, the utility from holding one unit of good tree un period 1 is given by $\max\{\mu_1 P_1^M(X_1), Z\}$. Therefore, the value function in period 1 is

$$V(h_G, h_B; \mu_1, X_1) = \mu_1 W_1 + \max\{\mu_1 P_1^M(X_1), Z\}h_G + \mu_1 P_1^M(X_1)h_B.$$

■

Proof of Proposition 1. First, let's calculate $D_\kappa \gamma_0^B(P_1^M + \kappa)$:

$$D_\kappa \gamma_0^B(P_1^M + \kappa) = \lim_{\varepsilon \to 0} \frac{\gamma_0^B(P_1^M + \varepsilon\kappa) - \gamma_0^B(P_1^M)}{\varepsilon} = E_0[\mu_1 \kappa(X)] > 0.$$

On the other hand, $D_\kappa \gamma_0^G(P_1^M + \kappa)$ is given by:

$$D_\kappa \gamma_0^G(P_1^M + \kappa) = E_{0,\alpha}\left[E_{0,\mu_1}\left[\mu_1 \kappa(X) | \mu_1 \geq \frac{Z}{P_1^M(X)}\right]\left[1 - G\left(\frac{Z}{P_1^M(X)}\right)\right]\right] > 0.$$

Hence, $D_\kappa \gamma_0^B(P_1^M + \kappa) > D_\kappa \gamma_0^G(P_1^M + \kappa) > 0$ as long as $P_1^M(X) < Z$ in some states with positive measure. Moreover, since $\gamma_0^B < \gamma_0^G$, then

$$\frac{D_\kappa \gamma_0^B(P_1^M + \kappa)}{\gamma_0^B(P_1^M)} > \frac{D_\kappa \gamma_0^G(P_1^M + \kappa)}{\gamma_0^G(P_1^M)} > 0.$$

■

Proof of Lemma 3. Let $\kappa_0(\xi)$ be the Lagrange multiplier associated to the budget constraint of program (P0). The FOC with respect to d is

$$1-\kappa_0(\xi)\leq 0.$$

Moreover, the FOCs with respect to i_G and i_B are

$$\kappa_0(\xi)q_G(\xi)+\gamma_0^G\leq 0,$$
$$\kappa_0(\xi)q_B(\xi)+\gamma_0^B\leq 0.$$

Therefore

$$\kappa_0(\xi)=\max\left\{1,\frac{\gamma_0^G}{q_G(\xi)},\frac{\gamma_0^B}{q_B(\xi)}\right\}.$$

First, note that by Assumption 2, $Z>1$ and so $\gamma_0^G\geq Z>1=q_G(0)$. Second, since $\gamma_0^B>\gamma_0^B$, then $\frac{\gamma_0^G}{q_G(0)}>\frac{\gamma_0^B}{q_B(0)}$. By continuity of q_G and q_B, there exists $\xi_G(0,1)$ such that $\max\left\{1,\frac{\gamma_0^G}{q_G(\xi)},\frac{\gamma_0^B}{q_B(\xi)}\right\}=\frac{\gamma_0^G}{q_G(\xi)}$, and hence $i_G(\xi)=\frac{W_0}{q_G(\xi)}$ if and only if $\xi\leq\xi_G$. Agents with $\xi>\xi_G$ will choose to consume or produce bad trees. If $\frac{\gamma_0^B}{q_B(\xi_G)}\leq 1$ then $\max\left\{1,\frac{\gamma_0^G}{q_G(\xi)},\frac{\gamma_0^B}{q_B(\xi)}\right\}=1$ for all $\xi>\xi_G$, and ξ_G is defined such that the marginal investors is indifferent between producing trees and consuming, $\frac{\gamma_0^G}{q_G(\xi_G)}=1$. If $\frac{\gamma_0^B}{q_B(\xi_G)}>1$, then there exists $\xi_B\in(\xi_G,1)$ such that $\max\left\{1,\frac{\gamma_0^G}{q_G(\xi)},\frac{\gamma_0^B}{q_B(\xi)}\right\}$, and $i_B(\xi)=\frac{W_0}{q_B(\xi)}$ if and only if $\xi\in(\xi_G,\xi_B]$. In this case, ξ_G is defined so that the marginal investor of good trees is indifferent between producing good and bad trees, $\frac{\gamma_0^G}{q_G(\xi_G)}=\frac{\gamma_0^B}{q_B(\xi_G)}$, and the marginal investor of bad trees is indifferent between producing bad trees and consuming, $\frac{\gamma_0^B}{q_B(\xi_B)}=1$. ■

Proof of Proposition 2. From Proposition 1 we know that shadow values increase, but γ_0^B increases by more than γ_0^G when prices increase. If $I_0^B=0$, then optimality implies that $\frac{\gamma_0^G}{q_G(\xi_G)}=1$. Since γ_0^G increases, ξ_G has to increase, so I_G increases.

On the other hand, if $I_0^B>0$, the optimality conditions are $\frac{\gamma_0^G}{q_G(\xi_G)}=\frac{\gamma_0^B}{q_B(\xi_G)}$ and $\frac{\gamma_0^B}{q_B(\xi_B)}=1$. Hence, as prices increase, ξ_G decreases and ξ_B increases, giving the desired result. ■

Proof of Corollary 2.1. That λ_1^E decreases with prices is immediate from Proposition 2. To see that H_1 increases note two things. First, the mass of agents that invests increases since it is given by ξ_B. Second, since $q_B(\xi) < q_G(\xi)\ \forall \xi \in [0,1]$, when ξ_G decreases to $\xi_G - \Delta$ for some $\delta > 0$, investment of the agents in $\xi \in (\xi_G, \xi_G - \Delta]$ goes up, since $\frac{W_0}{q_G(\xi)} < \frac{W_0}{q_B(\xi)}$. ■

Proof of Proposition 3. Since the cdf of α, F is continuous, the shadow values of good and bad trees are continuous in λ_1^E even if market prices are discontinuous in the state of the economy. Because I_0^G and I_0^B are continuous functions of the shadow values, the mapping T is continuous in λ_1^E. Moreover, since prices are increasing in λ_1^E, Proposition 2 implies that I_0^G is decreasing in λ_1^E while I_0^B is increasing, so the mapping T is decreasing in λ_1^E. Therefore, a fixed point of T exists and is unique. ■

Proof of Proposition 4. Note that the change in the distribution of F has no effect on the equilibrium in period 1 as long as λ_1^E doesn't change. So the key is to see how λ_1^E changes, which reduces to determining how the mapping T defined in (1.12) changes.

First note that since P_1^M is increasing in α for any value of λ_1^E and H_1, the increase in F to $\tilde{F}$ is mathematically equivalent to an increase in prices in each state α by $\phi(\alpha) \geq 0$. To see this note that

$$Prob_{\tilde{F}}(P_1^M(X) \leq P) \leq Prob_F(P_1^M(X) \leq P) \Rightarrow Prob_{\tilde{F}}(P_1^M(X) \leq P) = Prob_F(P_1^M(X) + \phi(X) \leq P).$$

By Proposition 2, an increase in prices reduces I_0^G and increases I_0^B as functions of λ_1^E, so that the mapping T decreases for all λ_1^E. Hence, the fixed point $\lambda_1^{E*} = T(\lambda_1^{E*}$ decreases. Note that the sign of the change in I_0^G is ambiguous since the partial equilibrium effect of prices reduces it but the endogenous change in λ_1^E increases it.

Because ξ_B increases and ξ_G decreases, total investment increases (recall that those who switch from producing good trees to bad trees face a lower cost, so they produce more trees). Moreover, because λ_1^E decreases, equilibrium prices decrease in all states, so the threshold α^* increases.

Finally, market fragility is ambiguous since the change in F reduces it but the endogenous change in λ_1^E increases it. The overall effect depends on parameters and functional forms. ■

Proof of Proposition 5. The change in the distribution G is equivalent to an increase in μ_1 to $\mu_1 + \phi(\mu_1)$ with $\phi(\mu_1) \geq 0$. Keeping the prices fixed, the change in the shadow values are given by

$$\begin{aligned}
\Delta\gamma_0^G =& E[\max\{(\mu_1 + \phi(\mu_1))P_1^M, Z\}] - E[\max\{\mu_1 P_1^M, Z\}], \\
=& E_{0,\alpha}\left[E_{0,\mu_1}\left[\phi(\mu_1)P_1^M | \mu_1 \geq \frac{Z}{P_1^M}\right]\left[1 - G\left(\frac{Z}{P_1^M}\right)\right] + \right. \\
& \left. E_{0,\mu_1}\left[(\mu_1 + \phi(\mu_1))P_1^M - Z | \mu_1 < \frac{Z}{P_1^M} \ \&\ \mu_1 + \phi(\mu_1) \geq \frac{Z}{P_1^M}\right]\left[G\left(\frac{Z}{P_1^M}\right) - \tilde{G}\left(\frac{Z}{P_1^M}\right)\right]\right], \\
\Delta\gamma_0^B =& E[(\mu_1 + \phi(\mu_1))P_1^M] - E[\mu_1 P_1^M] = E[\phi(\mu_1)P_1^M].
\end{aligned}$$

Since $(\mu_1 + \phi(\mu_1))P_1^M - Z < \phi(\mu_1)P_1^M$ when $\mu_1 < \frac{Z}{P_1^M}$, then $\Delta\gamma_)^B > \Delta\gamma_0^G$ and hence $\frac{\Delta\gamma_0^B}{\gamma_0^B} > \frac{\Delta\gamma_0^G}{\gamma_0^G}$. Therefore, $I_)^G$ decreases and I_0^B increases as functions of λ_1^E, so the mapping T decreases for all λ_1^E. Hence, $\lambda_1^{E*} = T(\lambda_1^{E*}$ decreases.

Note that now the effect on P_1^M is ambiguous since more agents want to sell good trees, but there is a smaller fraction of good trees in the economy. Hence the effect on α^* and MF are ambiguous.

■

Proof of Proposition 6. Let $c_1 \equiv \frac{\mu_1^{\max} - 1}{\mu_1^{\max}} Z$. Therefore, if $c > c_1$, the price sellers get is $P_1^S < Z - \frac{\mu_1^{\max} - 1}{\mu_1^{\max}} Z = \frac{Z}{\mu_1^{\max}}$. Hence, no agent sells their good trees.

Consider now $c \leq c_1$. If $c = c_1$, and only good trees were produced, the shadow value of bad trees would be

$$\gamma_0^B = E\left[\mu_1 \frac{Z}{\mu_1^{\max}}\right] = E[\mu_1]\frac{Z}{\mu_1^{\max}}.$$

Fixing $E[\mu_1]$, note that if $\mu_1^{\max} \geq ZE[\mu_1]$, then $\gamma_0^B \leq 1$ even when $\lambda_1^E = 1$, so no agent will produce bad trees when the cost is in the neighborhood of c_1. On the other hand, note that as $mu_1^{\max} \to E[\mu_1]$, then $\gamma_0^B \to Z > 1$ when $\lambda_1^E = 1$. Hence, there exists $\overline{\mu}_1^{\max}$ such that if $\mu_1^{\max} \geq \overline{\mu}_1^{\max}$, there exists $c_2 \leq c_1$ such that if $c \in (c_2, c_1)$, only good trees are produced and there is some trade in the secondary market. On the other hand, if $\mu_1^{\max} < \overline{\mu}_1^{\max}$, $\gamma_0^B > 1$ and there is some production of bad trees.

Moreover, since an increase in c is equivalent to a reduction in prices, it is straightforward to see that if $c \in (c_2, c_1)$, then $\frac{\partial I_0^G}{\partial c} < 0$. If $c < c_2$, note that a reduction of c moves the mapping T down, so $\frac{\partial \lambda_1^E}{\partial c} < 0$.

■

Sketch of Proof of Proposition 7. Consider an economy with F degenerated at $\tilde{\alpha}$. I choose $\tilde{\alpha}$ such that $T(\lambda_1^{E*}) < \lambda_1^{E*}$, where λ_1^{E*} is the fraction of good trees such that if $\lambda_1^E < \lambda_1^{E*}$ the market collapses. Denote the associated price at λ_1^{E*} as P_1^{M*}.

Now consider another economy in which α is distributed according to

$$\alpha = \tilde{\alpha} + u, \quad u \sim \mathcal{U}[-\epsilon, \epsilon],$$

where $\mathcal{U}$ denotes the uniform distribution. The objective is to show that as $\epsilon \to 0$, $\mathbb{V}ar(P_1^M(\alpha|\epsilon)) \nrightarrow 0$.

Suppose $\mathbb{V}ar(P_1^M(\alpha|\epsilon)) \to 0$ as $\epsilon \to 0$. This means that $P_1^M(\alpha|\epsilon) \to \tilde{P}_1^M$, for some $\tilde{P}_1^M$. Note that $P_1^M(\alpha|\epsilon)$ needs to be higher than αZ for all α as $\epsilon \to 0$, since otherwise agents would not produce bad trees, which cannot be an equilibrium. Moreover, the mapping $T(\lambda_1^E|\epsilon)$ is continuous in ϵ and converges pointwise to $T(\lambda_1^E)$ (the mapping of an economy with F degenerate at $\tilde{\alpha}$), except at λ_1^{E*}. Therefore, $\tilde{P}_1^M = P_1^{M*}$. But then $T(\lambda_1^{E*}(\epsilon)|\epsilon) < \lambda_1^{E*}(\epsilon)$, so this cannot be an equilibrium. Therefore, $P_1^M(\alpha|\epsilon) \nrightarrow \tilde{P}_1^M$ and $\mathbb{V}ar(P_1^M(\alpha|\epsilon)) \nrightarrow 0$. In particular, there exists $\alpha^*(\epsilon)$ such that $P_1^M(\alpha|\epsilon) = \alpha Z$ for all $\alpha < \alpha^*(\epsilon)$, and $P_1^M(\alpha|\epsilon) > \alpha Z$ for all $\alpha \geq \alpha^*(\epsilon)$.

Moreover, since λ_1^{E*} and market prices in period 1 are continuous in $\tilde{\alpha}$, there exists an open set $B \subset [0,1]$ such that if $\tilde{\alpha} \in B$, then $\mathbb{V}ar(P_1^M(\alpha|\epsilon)) \nrightarrow 0$. ■

Sketch of Proof of Proposition 8. I need to show that $MF(\varepsilon) \to \zeta$ as $\epsilon \to 0$, where ζ is the sunspot. Since $\lambda_1^E(\epsilon) \to \lambda_1^{E*}$, it holds that $E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \to E[P_1^M(\lambda_1^{E*}, \tilde{\alpha})]$. Note that

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] = \int_{\alpha^*(\epsilon)}^{\hat{\alpha}+\epsilon} \frac{P_1^M(\lambda_1^E(\epsilon), \alpha)}{2\epsilon} d\alpha + \int_{\hat{\alpha}-\epsilon}^{\alpha^*(\epsilon)} \frac{\alpha Z}{2\epsilon} d\alpha.$$

Hence,

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \geq Prob(\alpha \geq \alpha^*(\epsilon)) P_1^M(\lambda_1^E(\epsilon), \alpha^*(\epsilon)) + Prob(\alpha < \alpha^*(\epsilon))(\hat{\alpha} - \epsilon) Z,$$

and

$$E[P_1^M(\lambda_1^E(\epsilon), \alpha)] \leq Prob(\alpha \geq \alpha^*(\epsilon)) P_1^M(\lambda_1^E(\epsilon), \hat{\alpha} + \epsilon) + Prob(\alpha \geq \alpha^*(\epsilon)) \alpha^*(\epsilon) Z.$$

As ϵ goes to zero we get

$$\lim_{\varepsilon \to 0} E[P_1^M(\lambda_1^E(\varepsilon), \alpha)] = E[P_1^M(\lambda_1^{E*}, \alpha)] = (1-\tilde{\zeta})P_1^M(\lambda_1^{E*}, \tilde{\alpha}) + \tilde{\zeta}\tilde{\alpha}Z.$$

Hence, $\zeta = \tilde{\zeta}$. ■

1.7.2 Extended Model and Positive Implications

Proof of Lemma 4.

First I formally define the robustness of the solution to (P1′) to small perturbations of η, which is a modification of the analysis in Kurlat (2016) adjusted to the present setting.

Definition 7. *A solution to* (P1′) *is robust if there exists a sequence of strictly positive real numbers* $\{z_n\}_{n=1}^{\infty}$ *and a sequence of consumption, investment, buying and selling decisions*
$\{d^n, i_k^n, m^n, s_G^n, s_b^n, h_G'^n, h_B'^n, k'^n\}$ *such that, defining*

$$\eta^n(\omega; X_1) = \eta(\omega; X_1) + z_n, \quad \forall \omega \in \Omega$$

1. $\{d^n, i_k^n, m^n, s_G^n, s_b^n, h_G'^n, h_B'^n, k'^n\}$ solve the program

$$V(h_G, h_B; A, X_1) = \max_{\substack{d, i_K, m, s_G, \\ s_B, h'_G, h'_B, k'}} d + V_2(h'_G, h'_B, k'; X_2), \tag{P1′.A}$$

subject to

$$d + i_K + \sum_{\omega \in \Omega} P_1^M(\omega) m(\omega) \le W_1 + \sum_{\omega \in \Omega} P_1^M(\omega)(s_G(\omega) + s_B(\omega))\eta^n(\omega; X_1),$$

$$\begin{aligned} h'_G &= h_G + \sum_{\omega \in \Omega} \lambda_1^M(\omega; X_1) m(\omega) - \sum_{\omega \in \Omega} s_G(\omega)\eta^n(\omega; X_1), \\ h'_B &= h_B + \sum_{\omega \in \Omega} (1 - \lambda_1^M(\omega; X_1)) m(\omega) - \sum_{\omega \in \Omega} s_B(\omega)\eta^n(\omega; X_1), \\ k' &= A i_K, \end{aligned}$$

$$\sum_{\omega \in \Omega} s_G(\omega)\eta^n(\omega; X_1) \le h_G, \quad \text{and} \quad \sum_{\omega \in \Omega} s_B(\omega)\eta^n(\omega; X_1) \le h_B,$$

$$d \geq 0, \quad i_K \geq 0,$$

$$m(\omega) \geq 0, \quad s_G(\omega) \in [0, h_G], \quad s_B(\omega) \in [0, h_B], \quad \forall \omega \in \Omega.$$

2. $z_n \to 0$

3. $\{d^n, i_k^n, m^n, s_G^n, s_b^n, h_G'^n, h_B'^n, k'^n\} \to \{d, i_k, m, s_G, s_b, h_G', h_B', k'\}$.

Let $\mu_1(A, X_1)$ be the Lagrange multiplier associated to the budget constraint in program (P1'). The FOC with respect to d is

$$1 - \mu_1(A, X_1) \leq 0.$$

Moreover, the FOC with respect to $m(\omega)$ is

$$-\mu_1(A, X_1) P_1^M(\omega) + \lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z \leq 0,$$

while the FOC with respect to i_K and k' combined is

$$-\mu_1(A, X_1) + Ar(X_2) \leq 0.$$

Therefore

$$\mu_1(A, X_1) = \max \left\{ 1, Ar(X_2), \left\{ \frac{\lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z}{P_1^M(\omega)} \right\}_{\omega \in \Omega^B} \right\}.$$

Now, define $\mu_1^B(X_1) \equiv \max_{\omega \in \Omega} \frac{\lambda_1^M(\omega; X_1) Z + (1 - \lambda_1^M(\omega; X_1)) \alpha Z}{P_1^M(\omega)}$. Moreover, define $A_1^B(X_1) \equiv \frac{\max\{1, \mu_1^B(X_1)\}}{r(K(X_1))}$. If $A \leq A_1^B(X_1)$, the return from investing in capital, $Ar(K(X_1))$ is too low compared to the return of the best alternative between consuming and buying trees in some market. Hence, agents with $A \leq A_1^B(X_1)$ do not produce capital, and consume or buy trees. Moreover, since $P_1^M < Z$ in all markets in equilibrium, and ruling out arbitrage opportunities that are not consistent with equilibrium, agents do not sell good trees to consume or to buy trees in the markets, so that $s_G(\omega) = 0$ if $A \leq A_1^B(\omega)$.

On the other hand, if $A > A_1^B(X_1)$, agents produce capital and do not consume or buy trees.

Now let's switch to selling decisions. The decision to sell bad trees is straightforward: agents offer bad trees in all markets with the highest prices until they all the holdings are sold. On the other hand, the decision to sell is more involved and uses the definition of robust solution.

First, let's show that optimality requires that if $\omega' > \omega$, then if $s_G(\omega) > 0$, $s_G(\omega') = h_G$. Suppose this didn't hold. Then, the agent can increase its utility by reducing $s_G(\omega)$ by ϵ and increasing $s_G(\omega')$ by $\epsilon \frac{\eta(\omega;X_1)}{\eta(\omega';X_1)}$, for some $\epsilon > 0$. This policy is feasible and non-trivial unless $\eta(\omega;X_1) = 0$ or $\eta(\omega';X_1) = 0$. So consider a sequence $\eta^n(\omega;X_1) > 0$ and $\eta^n(\omega';X_1) > 0$. The solution to (P1'.A) must satisfy that if $s^n_G(\omega) > 0$ then $s^n_G(\omega') = h_G$ from the previous argument. But then $s^n_G(\omega') \to h_G$. Hence, agents sell there good trees only in markets that feature a high enough price.

Second, the FOC with respect to $s_G(\omega)$ is

$$\mu_1(A, X_1) P_1^M(\omega) \eta(\omega; X_1) - Z\eta(\omega; X_1).$$

Define $\tilde{A}(\omega; X_1) \equiv \frac{Z}{r(K(X_1))P_1^M(\omega)}$. It is straightforward to see that $\tilde{A}_1^S(\omega; X_1)$ is decreasing in ω. Then, an agent with productivity A sells in market ω if and only if $A > \tilde{A}_1^S(\omega; X_1)$ and $\omega \le \tilde{\omega}$, where $\tilde{\omega}$ is defined in (1.19). Therefore

$$A_1^S(\omega) \equiv \begin{cases} \tilde{A}_1^S(\omega; X_1) & \text{if } \omega \ge \tilde{\omega} \\ A^{\max} & \text{if } \omega < \tilde{\omega}. \end{cases}$$

■

Partial Equilibrium

If W_1 is sufficiently high, then buyers have enough wealth to drive down the return of the markets in which they participate to 1, so that they both buy trees and consume. In that case, active markets satisfy

$$P_1^M(\omega) = \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z, \tag{1.55}$$

with

$$\lambda_1^M(\omega; X_1) = \frac{[1 - G(A_1^S(\omega; X_1)]\lambda_E}{[1 - G(A_1^S(\omega; X_1)]\lambda_E + (1 - \lambda_E)},$$
$$= \frac{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E}{\left[1 - G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E + (1 - \lambda_E)}. \tag{1.56}$$

The intersection of these two curves defines the set of partial equilibria analogous to the previous section. I will show that a subset of these partial equilibria will determine active markets. First, recall that for each state X_1, the set of intersections between (1.55) and (1.56) could be one of three cases:

1. the two curves intersect only once, which can be at a price for which good and bad trees are traded, or a price at which only bad trees are traded, as in figure 1-3(b);

2. the two curves intersect three times, as depicted in figure 1-3(a);

3. the two curves intersect twice, as in figure 1-4(b).

First I show that active markets have to feature prices that belong to the set of partial equilibria. Then I establish what subset of these prices are actually active markets in equilibrium.

Suppose there was an active market in which $P_1^M(\omega) > \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. Then, buyers would get a higher utility from consuming than from buying in this market. Hence, $m(\omega) = 0$, which contradicts that the market was active. On the other hand, suppose that there are some markets in which $P_1^M(\omega) < \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. Define $\overline{\omega} \equiv \max_{\omega \in \Omega^B} \frac{\lambda_1^M(\omega;X_1)Z+(1-\lambda_1^M(\omega;X_1))\alpha Z}{P_1^M(\omega)}$. Then, buyers would want to spend all their liquid wealth in buying trees from market $\overline{\omega}$, but because the endowment W_1 is big, the demand for trees is greater than the supply, which is inconsistent with equilibrium. Hence, only markets that satisfy $P_1^M(\omega) = \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$ can have active trading.

This immediately implies that in case 1., there is only one active market in equilibrium. Can there be rationing? The answer is no. Suppose there were rationing. Then, there would be some agents with high investment opportunity that were not able to sell all they wanted. Then, from Lemma (4) we know that this agent will offer its trees in a market with a slightly lower price. Since the rationing in the markets is uniform, the fraction of good trees offered at that price is still given by $\lambda_1^M(\omega; X_1) = \frac{\left[1-G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E}{\left[1-G\left(\frac{Z}{r(K(X_1))P_1^M(\omega)}\right)\right]\lambda_E+(1-\lambda_E)}$. But then, in that market $P_1^M(\omega') < \frac{\left[1-G\left(\frac{Z}{r(K(X_1))P_1^M(\omega')}\right)\right]\lambda_E}{\left[1-G\left(\frac{Z}{r(K(X_1))P_1^M(\omega')}\right)\right]\lambda_E+(1-\lambda_E)}$, which we already know cannot be active in equilibrium. Hence, there cannot be rationing in this case.

In case 2., I now show that only the market with the highest price can be active. Suppose another market is active. Then, in a robust solution, seller are offering trees in markets with higher prices also, even though they are inactive. There are markets with price just below the highest partial equilibrium price such that $P_1^M(\omega) < \lambda_1^M(\omega; X_1)Z + (1 - \lambda_1^M(\omega; X_1))\alpha Z$. But then buyers would prefer to trade in this market instead, a contradiction. Hence, only the highest price market is active. Moreover, for the same reasons as in case 1., there is no rationing in equilibrium.

Finally, in case 3. both markets can be active in equilibrium, since there is no possible deviation of buyers and sellers that can rule out any of them. Moreover, the lower price market is active only if there is rationing in the higher price market. In fact, any rationing level in the high price market is consistent with partial equilibrium. However, no rationing can occur in the low price market, since some rationed agents, those with high A, would be willing to sell at a slightly lower price, a contradiction.

Sketch of Proof of Proposition 9.

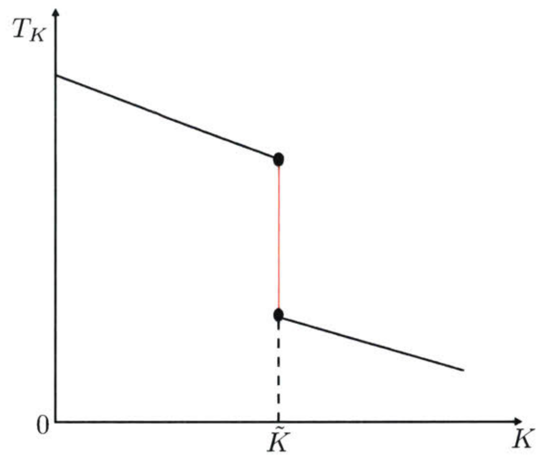
The proof has two parts. First, I show that there is an equilibrium of the economy starting in period 1 for any value of X_1. Then, I move to period 0 and show that an equilibrium of the full economy exists.

Given X_1, the mapping $T_K(L; X_1)$ has the following properties:

1. if K is "low", then the return on capital is "high" so many agents sell their trees in order to invest. Then there is a unique active market that features a "high" price, so T_K is high;

2. if K is "high", then the return on capital is "low" so few agents are willing to

Figure 1-8: Mapping $T_K(K; X_1)$



sell their trees in order to invest, hence only the market that trades bad trees is active and the price is "low", so T_K is low;

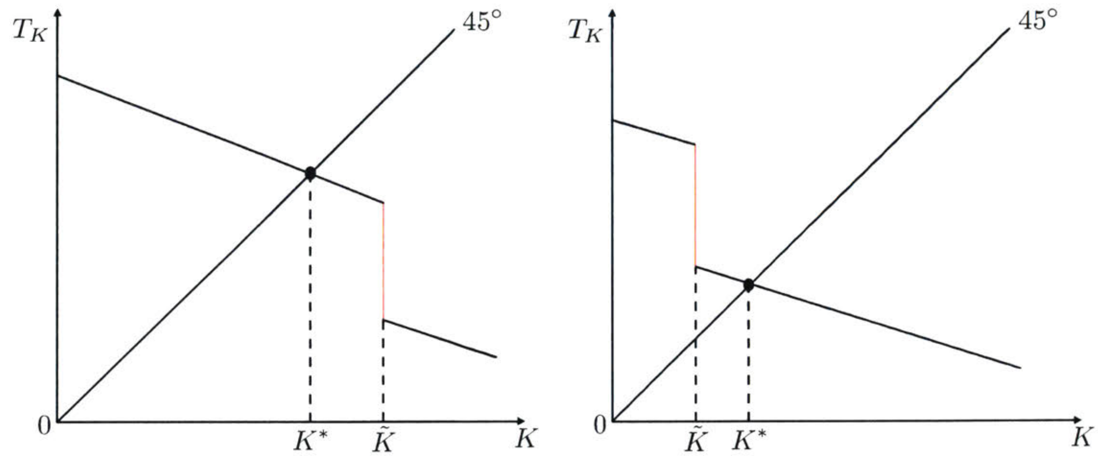
3. there exists a unique $\tilde{K}$ such the economy is in case 3., and two markets can be active.

Figure 1-8 shows the mapping $T_K(K; X_1)$. For $K < \tilde{K}$, T_K is high. For $K > \tilde{K}$, T_K is low. Note that T_K is always decreasing since the higher K is the lower the return on investment and hence total investment. Finally, if $K = \tilde{K}$, T_K can take a continuum of values indexed by η. This shows that extending the definition of equilibrium is necessary to guarantee that the mapping T_K is continuous in K and an equilibrium of the economy starting in period 1 always exists.

So the equilibrium can take one of three forms. If the state α is high, then the market for trees is liquid, so that the price is high and the equilibrium level of capital is high. This is the case depicted in figure 1-9(a). If the state α is low, then the market for trees collapses, only bad trees are traded at a low price, and the equilibrium capital is low. Figure 1-9(b) shows this case. Finally, if the state α is "middle-range", then the economy features two markets and the high price market is rationed. How is the amount of rationing determined? So that total investment is equal to $\tilde{K}$. Figure 1-10 shows this case.

So I established that an equilibrium of the economy in period 1 always exists. Let $K(\lambda_1^E, H_1; \alpha)$ denote the equilibrium capital as a function of the fraction of good trees in the economy, the total amount of trees, and the aggregate state α. Note that K is increasing in H_1. Now I switch to the determination of equilibrium period 0.

Figure 1-9: Equilibrium in period 1. (a) High state α: high price. (b) Low state α: low price.



Unlike in the basic model where the economy was linear in H_1, finding an equilibrium of this economy requires finding a fixed point of the two-dimensional mapping

$$T(\lambda_1^E, H_1) = \begin{bmatrix} \frac{I_0^G(\lambda_1^E, H_1)}{I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)} \\ I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1) \end{bmatrix}$$

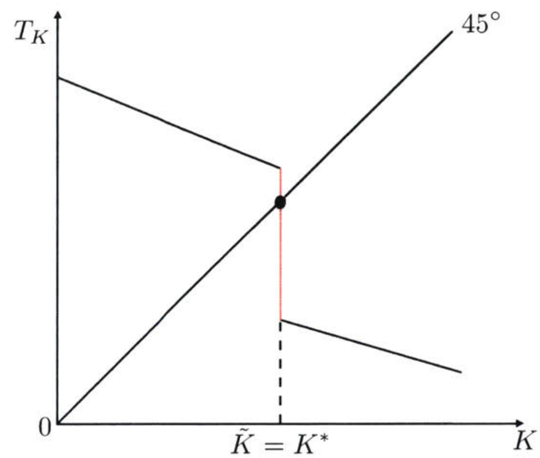
Consider first the mapping $T_H(H_1; \lambda_1^E) \equiv I_0^G(\lambda_1' H_1) + I_0^B(\lambda_1^E, H_1)$, which takes λ_1^E as given. Since higher H_1 increases K, which reduces the liquidity services of trees, T_H is decreasing in H_1, so equilibrium implies that $H_1(\lambda_1^E)$ is a continuous function of λ_1^E. Hence, finding an equilibrium of the economy reduces to finding a fixed point of the mapping $T_\lambda(\lambda_1^E) = \frac{I_0^G(\lambda_1^E, H_1(\lambda_1^E))}{I_0^G(\lambda_1^E, H_1(\lambda_1^E)) + I_0^B(\lambda_1^E, H_1(\lambda_1^E))}$. Since T_λ is continuous in λ_1^E and belongs to the compact space $[0, 1]$, a fixed point exists. However, there can be multiple fixed points. Though this could be a potentially interesting phenomena (generating a channel for self-fulfilling equilibria), it is beyond the scope of the paper. I will select the equilibrium that features the highest fraction of good trees. Note that this equilibrium is stable, since T_λ crosses the 45° from above.

■

Proof of Lemma 5. To prove this result it is enough to show that T_K increases for all K.

First note that for a fixed K (and hence a fixed return on capital), the equilibrium price in market ω_H is increasing in α and H_G. Consider two economies in period 1 with the same λ_1^E and H_1, but one has quality of bad trees α and the other α', with $\alpha' > \alpha$. For a fixed K, the demand of trees is higher in the economy with

Figure 1-10: Mapping $T_K(K; X_1)$



α', while the supplies are the same. Moreover, since the supply of trees is decreasing in K (since $r(K)$ is decreasing in K), $\tilde{K}(\alpha') > \tilde{K}(\alpha)$. Therefore, the equilibrium level of capital is higher in the economy with state α', strictly so if the market ω_H is active.

Similarly, consider two economies with the same H_B and α, but one has H_G of good trees and the other H'_G, with $H'_G > H_G$. For a fixed K, market liquidity increases for two reasons. First, there are more trees in the economy, so for the same price, volume traded is higher. Second, for a fixed H_B, the higher the fraction of good trees in the economy, and hence the higher the price in the market ω_H. Hence, the mapping T_K increases with H_G, so equilibrium capital increases.

■

Proof of Lemma 6.

First I need to show that increases in Z^Y and A increase $\mu_1(A, X_1)$. Then I show that this increase in $\mu_1(A, X_1)$ generates a reduction in λ_1^E and an increase in H_1.

For a fixed level of capital, an increase in Z^Y increases investment, both because more agents find it optimal to invest and because more agents sell their good trees to invest. Therefore, for each state X_1, T_K increases and hence the equilibrium level of capital increases. However, what matters for period 0 is what happens with the return on capital, since $\mu_1(A, X_1) = \max\{1, Ar(X_2)\}$. Since $r(K(X_1)) = Z^Y f'(K(X_1))$, r increases because of Z^Y but decreases because of $f'(K(X_1))$. Suppose $r(K(X_1))$ deçreases as a result, then less agents invest and less agents sell good trees to invest. But then total investment decreases, which contradicts that $K(X_1)$ increases.

Note that an increase of A to ϕA for some $\phi > 1$ has similar implications and enters the expression for $\mu_1(A, X_1)$ in an analogous ways as Z^Y, so $\mu_1(A, X_1)$ also increases when A increases to ϕA. Moreover, $P_1^M(X_1)$ increases because more agents sell their good trees (for fixed λ_1^E and H_1).

Now let's return to period 0. Recall that shadow values are given by

$$\begin{aligned}\gamma_0^G =& E[\max\{\mu_1(A, X_1)P_1^M(\omega_H; X_1)\eta(X_1) + (1-\eta(X_1))Z, Z\}],\\ \gamma_0^B =& E[\mu_1(A, X_1)[\eta(X_1)P_1^M(\omega_H; X_1) + (1-\eta(X_1))P_1^M(\omega_L; X_1)]].\end{aligned}$$

For fixed λ_1^E and H_1, γ_0^G and γ_0^B increase when μ_1 increases, but $|gamma_0^B$ increases by more. Therefore, $H(\lambda_1^E)$ defined as the fixed point of $T_H(H_1; \lambda_1^E) \equiv I_0^G(\lambda_1^E, H_1) + I_0^B(\lambda_1^E, H_1)$ taking λ_1^E as given, increases.

Finally, I need to show that $T_\lambda(\lambda_1^E) \equiv \frac{I_0^G(\lambda_1^E, H(\lambda_1^E))}{I_0^G(\lambda_1^E, H(\lambda_1^E)) + I_0^B(\lambda_1^E, H(\lambda_1^E))}$ decreases as a function of λ_1^E. From previous analysis we know that if μ_1 increases, then $T_\lambda(\lambda_1^E)$ decreases. Hence, $T_\lambda(\lambda_1^E)$ can increase only if the increase in $H(\lambda_1^E)$ provides so much liquidity in period 1 that μ_1 decreases. But a decrease in μ_1 contradicts that H increases in the first place, hence T_λ decreases.

Finally, since I am selecting the equilibrium with the highest λ_1^E, and this equilibrium happens in the intersection of T_λ with the 45° line from above, a decrease in T_λ reduces the equilibrium λ_1^E.

■

1.7.3 Normative Implications

Proof of Lemma 7. In first best, only agents with $A = A^{\max}$ invest and they do so until $A^{\max} r(K(X_1)) = 1$. Therefore, $\mu_1(A, X_1) = 1$ for all A, X_1, and $i_0 = 0$.

In laissez-faire, $\mu_1(A^{\max}, X_1) > 1$ for all X_1, hence $E[\mu_1(A, X_1)] > 1$ and $i_0 > 0$.

■

Proof of Proposition 10.

It is straightforward to see that for state X_1, the mapping $T_K(K; X_1)$ increases with B_0. Therefore, $K(X_1)$ increases with B_0 (this effect is strict except for states in which $K = \tilde{K}$). This immediately implies that $TL(X_1)$ increases. However, because the return on capital decreases, the supply of good trees decreases, so that

$P_1^M(\omega_H; X_1)$ decreases, strictly so except of states in which $K = \tilde{K}$, where $\eta(X_1)$ decreases. Therefore, $\mu_1(A, X_1)$ decreases for every state (A, X_1), strictly so except for states with $K = \tilde{K}$.

■

Proof of Proposition 11.

It follows directly from Lemma 6 since from the point of view of period 0 it only matters that $\mu_1(A, X_1)$ decreased as a function of λ_1^E and H_1.

■

Proof of Corollary 11.1.

Let $\tilde{B}_0 < B_0$ be the amount of government bonds bought by the foreign agent. The government collects revenues from selling to this agent of $P_0^{GB}\tilde{B}_0$, which are distributed after investment takes place. Hence, consumption in period 0 goes up and aggregate variables in period 1 are equivalent to those in an economy in which the government issues $B_0' \equiv B_0 - \tilde{B}_0$ bonds.

■

Optimal Policy: Government Bonds

The planner solves

$$\mathbb{W} = D_0 + E_0[D_1(X_1) + D_2(X_2)],$$

subject to

$$\begin{aligned}
D_0 =& (1 - \xi_B)W_0,\\
D_1(X_1) =& G(A_1^B(X_1))W_1 - P_1^M(\omega_H; X_1)\eta(X_1)\Big[[1 - G(A_1^S(X_1))]H_G + [1 - G(A_1^B(X_1))]H_B\Big] -\\
& P_1^M(\omega_L; X_1)(1 - \eta(X_1))[1 - G(A_1^B(X_1))]H_B,\\
D_2(X_2) =& [H_G + \alpha H_B]Z + f(K) - q_{GB}B_0,
\end{aligned}$$

$$\begin{aligned}
K(X_1) = \int_{A_1^B(X_1)}^{A^{\max}} & A[W_1 + [P_1^M(\omega_H; X_1)\eta(X_1) + P - 1^M(\omega_L; X_1)(1 - \eta(X_1))]]H_B + B_0]dG(A) + \\
& \int_{A_1^S(X_1)}^{A^{\max}} AP_1^M(\omega_H; X_1)\eta(X_1)H_G dG(A),
\end{aligned}$$

$$H_G = \int_0^{\xi_G} \frac{W_0}{q_G(\xi)} d\xi,$$
$$H_B = \int_{\xi_G}^{\xi_B} \frac{W_0}{q_B(\xi)} d\xi,$$

and the equilibrium conditions

$$\frac{\gamma_0^G}{q_G(\xi_G)} = \frac{\gamma_0^B}{q_B(\xi_G)},$$
$$\frac{\gamma_0^B}{q_B(\xi_B)} = \frac{\gamma_0^{GB}}{P_0^{GB}} = 1.$$

The first order condition is

$$\frac{\partial D_0}{\partial B_0} + E\left[\frac{\partial D_1(X_1)}{\partial B_0} + \frac{\partial D_2(X_2)}{\partial B_0}\right] = 0,$$

where

$$\frac{\partial D_0}{\partial B_0} = -\frac{\partial \xi_B}{\partial B_0} W_0,$$

$$\frac{\partial D_1(X_1)}{\partial B_0} = g(A_1^B(X_1))\frac{\partial A_1^B(X_1)}{\partial B_0}W_1 - \left[\frac{\partial P_1^M(\omega_H;X_1)}{\partial B_0}\eta(X_1) + P_1^M(\omega_H;X_1)\frac{\partial \eta(X_1)}{\partial B_0}\right]$$
$$[[1-G(A_1^S(X_1))]H_G + [1-G(A_1^B(X_1))]H_B] - P_1^M(\omega_H;X_1)\eta(X_1)\left[-g(A_1^S(X_1))\frac{\partial A_1^S(X_1)}{\partial B_0}H_G + \right.$$
$$\left.[1-G(A_1^S(X_1))]\frac{\partial H_G}{\partial B_0} - g(A_1^B(X_1))\frac{\partial A_1^B(X_1)}{\partial B_0}H_B + [1-G(A_1^B(X_1))]\frac{\partial H_B}{\partial B_0}\right] +$$
$$P_1^M(\omega_L;X_1)\frac{\partial \eta(X_1)}{\partial B_0}[1-G(A_1^B(X_1))]H_B + P_1^M(\omega_L;X_1)(1-\eta(X_1))g(A_1^B(X_1))\frac{\partial A_1^B(X_1)}{\partial B_0}H_B -$$
$$P_1^M(\omega_L;X_1)(1-\eta(X_1))[1-G(A_1^B(X_1))]\frac{\partial H_B}{\partial B_0} + g(A_1^B(X_1))\frac{\partial A_1^B(X_1)}{\partial B_0}B_0 - [1-G(A_1^B(X_1))],$$

$$\frac{\partial D_2(X_2)}{\partial B_0} = \left[\frac{\partial H_G}{\partial B_0} + (1-\delta)\frac{\partial H_B}{\partial B_0}\right] Z + r(K)\frac{\partial K}{\partial B_0} - q_{GB},$$

and

$$\frac{\partial K(X_1)}{\partial B_0} = -\frac{\partial A_1^B(X_1)}{\partial B_0} A_1^B(X_1)[W_1 + [P_1^M(\omega_H; X_1)\eta(X_1) + P_1^M(\omega_L; X_1)(1-\eta(X_1))]H_B + B_0]$$

$$g(A_1^B(X_1)) + \int_{A_1^B(X_1)}^{A^{\max}} A\left[\left[\frac{\partial P_1^M(X_1)}{\partial B_0}\eta(X_1) + [P_1^M(\omega_H; X_1) - P_1^M(\omega_L; X_1)]\frac{\partial \eta(X_1)}{\partial B_0}\right] H_B + \right.$$

$$\left.[P_1^M(\omega_H; X_1)\eta(X_1) + P_1^M(\omega_L; X_1)(1-\eta(X_1))]\frac{\partial H_B}{\partial B_0} + 1\right] G(dA) -$$

$$\frac{\partial A_1^S(X_1)}{\partial B_0} A_1^S(X_1) P_1^M(X_1)\eta(X_1) H_G g(A_1^S(X_1)) +$$

$$\int_{A_1^S(X_1)}^{A^{\max}} A\left[\left[\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0}\eta(X_1) + P_1^M(\omega_H; X_1)\frac{\partial \eta(X_1)}{\partial B_0}\right] H_G + P_1^M(\omega_H; X_1)\eta(X_1)\frac{\partial H_G}{\partial B_0}\right] G(dA)$$

After some algebra, it simplifies to

$$\frac{\partial \mathbb{W}}{\partial B_0} = E\left[\int_{A_1^B(X_1)}^{A^{\max}} Ar(K(X_1))dG(A) + G(A_1^B(X_1))\right] +$$

$$E\left[\left[\int_{A_1^B(X_1)}^{A^{\max}} Ar(K(X_1))dG(A) + G(A_1^B(X_1))\right]\right.$$

$$\left.\left[\frac{\partial P_1^M(X_1)}{\partial B_0}\eta(X_1) + [P_1^M(\omega_H; X_1) - P_1^M(\omega_L; X_1)]\frac{\partial \eta(X_1)}{\partial B_0}\right]\right] H_B +$$

$$E\left[\int_{A_1^S(X_1)}^{A^{\max}} Ar(K(X_1))dG(A)\left[\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0}\eta(X_1) + P_1^M(\omega_H; X_1)\frac{\partial \eta(X_1)}{\partial B_0}\right] - \right.$$

$$\left.\frac{\partial \eta(X_1)}{\partial B_0}[1 - G(A_1^S(X_1))]Z\right] H_G = 1 + q_{GB},$$

or

$$\frac{\partial \mathbb{W}}{\partial B_0} = \gamma_0^{GB} + E\left[\frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial P_1^M(\omega_H; X_1)}\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^G(X_1)}{\partial \eta(X_1)}\frac{\partial \eta(X_1)}{\partial B_0}\right] H_G +$$

$$E\left[\frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial P_1^M(\omega_H; X_1)}\frac{\partial P_1^M(\omega_H; X_1)}{\partial B_0} + \frac{\partial \tilde{\gamma}_1^B(X_1)}{\partial \eta(X_1)}\frac{\partial \eta(X_1)}{\partial B_0}\right] H_B = 1 + q_{GB}.$$

1.7.4 Infinite Horizon

It is possible to guess and verify that the value function of an agent with portfolio given by $\{h_G, h_B, k, b\}$ and investment opportunity A, when the aggregate state is

$X = \{\lambda_E, H, K, B; \alpha\}$, is given by

$$V(h_G, h_B, k, b; A, X) = \tilde{\gamma}_G(A, X)h_G + \tilde{\gamma}_B(A, X)h_B + \tilde{\gamma}_K(A, X)k + \tilde{\gamma}_{GB}b,$$

where

$$\begin{aligned}
\tilde{\gamma}_G(A, X) =& \max\{\mu(A, X)\eta(X)P_M(\omega_H; X) + (1-\eta(X))[Z + (1-\delta_H)\gamma_G(X)], \\
& Z + (1-\delta_H)\gamma_G(X)\}, \\
\tilde{\gamma}_B(A, X) =& \mu(A, X)[\eta(X)P_M(\omega_H; X) + (1-\eta(X))P_M(\omega_L; X), \\
\tilde{\gamma}_K(A, X) =& r(X), \\
\tilde{\gamma}_{GB}(A, X) =& \mu(A, X),
\end{aligned}$$

$$\gamma_j(X) = \beta E[\tilde{\gamma}_j(A', X')|X] \quad \text{for } j \in \{H, B, K, GB\}, \tag{1.57}$$

and

$$\begin{aligned}
\mu(A, X) = \max\Bigg\{&1, \frac{\gamma_G(X)}{\phi(I_G)q_G}, \gamma_B(X), A\gamma_K(X), \\
&\left\{\frac{\lambda_M(\omega; X)\gamma_G(X) + (1-\lambda_M(\omega; X))\gamma_B(X)}{P_M(\omega)}\right\}_{\omega\in\Omega^B}\Bigg\}.
\end{aligned}$$

Assuming that the endowment W is big, then the markets feature risk neutral pricing. Thus, $P_1^M(\omega_H; X)$ is determined by the intersection between

$$P_M = \lambda_M[Z + (1-\delta_H)\gamma_G(X)] + (1-\lambda_M)[\alpha Z + (1-\delta_H)\gamma_B(X)], \tag{1.58}$$

and

$$\lambda_M = \frac{\left[1 - G\left(\frac{\gamma_G(X)}{\gamma_K(X)P_M}\right)\right]\lambda_E}{\left[1 - G\left(\frac{\gamma_G(X)}{\gamma_K(X)P_M}\right)\right]\lambda_E + (1-\lambda_E)}, \tag{1.59}$$

and $P_M(\omega_L; X)$ is just the value of bad trees

$$P_M(\omega_L; X) = \alpha Z + (1-\delta_H)\gamma_B(X). \tag{1.60}$$

Since all agents have the same cost of producing trees, it means that buyers, consumers and producers of trees derive the same utility. This implies that it must hold

$$\frac{\gamma_G(X)}{\phi(I_G)q_G} = 1, \tag{1.61}$$

$$\gamma_B(X) \leq 1. \tag{1.62}$$

Moreover, capital is given by

$$K(X) = \int_{A_B(X)}^{A^{\max}} A[W + [\eta(X)P_M(\omega_H; X) + (1-\eta(X))P_M(\omega_L; X)]H_B + P_{GB}(X)B]dG(A) + \int_{A_S(X)}^{A^{\max}} A\eta(X)P_M(\omega_H; X)H_G dG(A), \tag{1.63}$$

where $A_B(X) \equiv \frac{1}{\gamma_K(X)}$ and $A_S(X) \equiv \frac{\gamma_G(X)}{\gamma_K(X)P_M(\omega_H;X)}$. Note that this expression is analogous to (1.23). Finally, the laws of motion of the fraction of good trees and the total amount of trees is given by

$$\lambda_E'(X) = \lambda_E\theta(X) + \frac{I_G(X)}{I_G(X) + I_B(X)}(1-\theta(X)), \tag{1.64}$$

$$H'(X) = (1-\delta_H)H + I_G(X) + I_B(X), \tag{1.65}$$

where $\theta(X) \equiv \frac{(1-\delta_H)H}{(1-\delta_H)H + I_G(X) + I_B(X)}$.

To summarize, an equilibrium for this economy is characterized by:

1. Laws of motion of λ_E, H and K, which are given by (1.64), (1.65) and (1.63) respectively,

2. Shadow values of H_G, H_B, and K, given by (1.57),

3. Prices $P_M(\omega_H; X)$ and $P_M(\omega_L; X)$, and fraction of good trees $\lambda_M(X)$, that satisfy (1.58), (1.59) and (1.60), and rationing $\eta(X)$,

4. Equilibrium conditions (1.61) and (1.62).

To find an equilibrium of this economy I will follow similar steps than in the three period model. But first note that $X = (\lambda_E, H, K, B; \alpha)$ belongs to a compact set as long as B is bounded, because λ_E and α are bounded between 0 and 1, H is bounded between 0 and $\frac{W}{\delta_H}$, and K is bounded between 0 and $A^{\max}W$. Second, for a given non-exploding sequence of prices $P_M(\omega_H)$ and $P_M(\omega_L)$, rationing η, and rental rate r, γ_B and γ_K are well defined and unique. Moreover, the mapping defining γ_G is well defined and satisfies Blackwell's sufficient conditions hence γ_G exists and is unique.

Now, I proceed as in the previous section. First I characterize the partial equilibrium in the markets for trees, selecting the maximal volume of trade equilibrium whenever possible. Fixing γ_G, γ_B and γ_K, the partial equilibrium can take one of three cases:

1. only the market ω_H is active, hence $\eta = 1$,
2. only the market ω_L is active, hence $\eta = 0$,
3. both ω_H and ω_L are active, and $\eta \in [0,1]$.

Note that market prices are decreasing in α and increasing in λ_E. Importantly, they are independent of K.

First, I will find K' for fixed λ'_E and H'. First, note that K' does not directly depend on K. This is important for tractability, since past shocks do not affect future capital. Second, $\tilde{\gamma}_G$ and $\tilde{\gamma}_B$ are independent of K, so γ_G and γ_B are independent of K'. Define the mapping T_K as

$$T_K(K';X) = \int_{A_B(K';X)}^{A^{\max}} A[W + [\eta(K';X)P_M(\omega_H,K';X) + (1-\eta(K';X))P_M(\omega_L,K';X)]H_B + P_{GB}(K';X)B]dG(A) + \int_{A_S(K';X)}^{A^{\max}} A\eta(K';X)P_M(\omega_H,K';X)H_G dG(A).$$

It is straightforward to see that the mapping T_K is continuous in K' for fixed λ'_E and H' (the rationing η smooths out the discontinuity in the market). Moreover, because γ_K is decreasing in K', which makes volume traded (prices and rationing) decreasing in K', the mapping T_K is decreasing in K'. Therefore, there exists a unique fixed point $K' = T_K(K')$.

Second, I find H' for fixed λ'_E. Note that if H increases, K' increases, and hence μ and market volume (prices and rationing) decreases. Therefore, γ_G and γ_B are decreasing in H'. Define the mapping T_H as

$$T_H(H';X) = (1-\delta_H)H + I_G(H';X) + I_B(H;X).$$

It is straightforward to see that T_H is continuous in H'. Since in equilibrium $\gamma_G(X) = \phi(I_G)q_G$ and $\gamma_B(X) \leq 1$, I_G and I_B decrease with H'. Therefore, a fixed point $H' = T_H(H')$ for a given λ'_E exists and is unique.

Finally, define the mapping T_λ as

$$T_\lambda(\lambda'_E; X) = \lambda_E \theta(\lambda'_E; X) + \frac{I_G(\lambda'_E; X)}{I_G(\lambda'_E; X) + I_B(\lambda'_E; X)}(1 - \theta(\lambda'_E; X)).$$

This mapping is continuous in a compact space, so a fixed point $\lambda'_E = T_\lambda(\lambda'_E; X)$ exists. Let $L \equiv \{\tilde{\lambda}'_E \in [0,1] : \tilde{\lambda}'_E = T_\lambda(\tilde{\lambda}'_E)\}$. Since $T_\lambda(1; X) < 1$, the slope of T_λ at $\lambda'_E = \max L$ is less than one, that is, T_λ crosses the 45° line from above. If there are many intersections, I select the one with the highest λ'_E.

Stochastic Steady State

I look for a stochastic steady state of the economy. I guess and verify that there exists an equilibrium of this economy in which λ_E and H are constant over time.

The laws of motion of λ_E and H imply that

$$\Delta\lambda_E = 0 \Leftrightarrow \lambda_E = \frac{I_G}{I_G + I_B}, \tag{1.66}$$

$$\Delta H = 0 \Leftrightarrow H = \frac{I_G + I_B}{\delta_H}. \tag{1.67}$$

Hence, if λ_E and H are constant over time, I_G and I_B are constant over time. Moreover, the fact that capital fully depreciates, that α is iid, and the timing assumption on the payout of dividends, imply that $K(X)$ is connected to past periods only through λ_E and H. Therefore, if λ_E and H are constant over time, the distribution of K in the following period is constant over time. This means that $\gamma_K(X)$ is constant over time. But then, there is a solution to the recursive equations determining $\gamma_G(X)$ and $\gamma_B(X)$ that is constant over time.

So it only remains to be shown that there exists λ_E and H such that this equilibrium exists. First note that as H increases, $K(X)$ increases so that $\mu(A, X)$ decreases. Since γ_B is more sensitive to changes in the liquidity premium than γ_G, $\frac{I_G}{I_G+I_B}$ increases. Hence,

$$\lim_{H\to 0} \lambda_E = \lim_{H\to 0} \frac{I_G}{I_G + I_B} > 0.$$

Second, as $H \to \infty$, the liquidity premium goes to zero so $\mu(A, X) \to 1$. But as long as there is some trade in the market for trees, $\lim_{H\to\infty} I_B > 0$, hence

$$\lim_{H\to\infty} \lambda_E = \tilde{\lambda}_E < 1.$$

On the other hand, as $\lambda_E \to 0$, production of good trees remains positive, hence

$$\lim_{\lambda_E\to 0} H = \lim_{\lambda_E\to 0} \frac{I_G + I_B}{\delta_H} > 0.$$

And as $\lambda_E \to \tilde{\lambda}_E$ the production of trees remains finite, so

$$\lim_{\lambda_E\to\tilde{\lambda}_E} H < \infty.$$

Hence, (1.66) and (1.67) intersect at least once. Analyzing the possibility of multiple steady states is beyond the scope of this paper, so I choose the equilibrium that features the maximum fraction of good trees. It is possible to see that in that equilibrium (1.67) crosses (1.66) from above.

Moreover, this steady state is stable. Denote by $(\lambda_E^*, H*)$ the steady state levels of the fraction of good trees in the economy and the total amount of trees. If $\lambda_E < \lambda_E^*$, then γ_G and γ_B are lower (for all H'), but $\frac{\gamma_G}{\gamma_B}$ is higher, hence, $\frac{I_G}{I_G+I_B} > \lambda_E$. The opposite is true if $\lambda_E > \lambda_E^*$. Similarly, if $H < H^*$, the liquidity premium is high, so γ_G and γ_B are high, so $I_G + I_B > \delta_H H$. The opposite holds if $H > H^*$.

Chapter 2

Durable Crises (joint with David Colino and Pascual Restrepo)

2.1 Introduction

The consumption of durable goods is highly cyclical. Relative to other goods, it contracts sharply during recessions and expands in booms (Bils and Klenow, 1998; Bils et al., 2013). In this paper we explore the role that consumer durables play in amplifying business cycles. We ask if the cyclicality of demand for durable goods contributes to aggregate employment reductions during recessions and corresponding raises during booms, or if on the contrary, these sectoral demand shocks are mitigated by reallocation to other industries and do not affect the aggregate employment level.

We first document that employment in industries that produce consumer durables is more cyclical than employment in other industries.[1] Using a measure of life expectancy of consumer goods adapted from Bils and Klenow (1998) and U.S. data from the County Business Patterns (CBP) covering the period from 1988 to 2014, we show that, relative to other manufacturing industries, employment in durable industries declines sharply during downturns but also recovers faster subsequently.[2] Quantitatively, when the slack in the U.S. labor market (measured as

[1] In what follows, we use interchangeably the terms durable industries and industries that produce durable consumer goods. These do not include industries producing materials used mainly as intermediary goods, such as primary metals, concrete and cement, or lumber and wood products except furniture.

[2] This result, also shown recently by Bils et al. (2013), complements the literature showing that durable goods have a more cyclical demand (Bils and Klenow, 1998); affect the volatility of exports

the difference between the unemployment rate and the natural rate of unemployment) rises by 5 percentage points, as it did during the recent Great Recession, industry-level employment decreases by 2.265% more per additional year of expected life of the consumer good it produces. For the average durable industry, this translates into an additional 17% decline in employment with respect to industries that produce non-durables. These estimates imply that, when the slack in the U.S. labor market rises by 5 percentage points, employment in industries that produce consumer durables contracts by an additional 700 thousand jobs relative to other industries, or about half a percentage point of the labor force. Our results hold after we control for the secular decline in manufacturing and any potential trend that is specific to industries producing durable goods. The finding that employment in durable industries is particularly pro-cyclical is consistent with the view that business cycles may affect different industries heterogeneously (see Abraham and Katz (1986)).

We next explore the implications of this volatility on aggregate employment. Our estimates of the cyclicality of employment in durable industries capture the differential effect across industries of the volatility of consumer durable consumption. These effects do not, however, correspond to the equilibrium impact on aggregate employment, which also encompasses indirect channels that could mitigate or amplify the impact of this sectoral shock on employment levels. One could expect the reduction in durable employment during recessions to have a small or no aggregate impact if workers quickly reallocate across sectors, in which case our estimates could simply reflect the reallocation of workers to less cyclical industries during downturns (Loungani and Rogerson, 1989). If indeed workers reallocate to less cyclical industries, the volatility of durable good consumption would not affect the aggregate behavior of employment (Baxter, 1996). Even if workers need to spend some time in unemployment to reallocate, the large gross flows of workers across industries imply that this reallocation could be achieved without any significant impact on aggregate employment (Pilossoph, 2012).

To estimate the equilibrium impact of the volatility in durable employment on the labor market we exploit differences in the industry composition of U.S. commuting zones. Because the bulk of the adjustment to labor demand shocks, and especially the reallocation of workers, takes place locally, commuting zones pro-

(Engel and Wang, 2011); and affect the exposure to risk among firms that produce durables (Gomes et al., 2009).

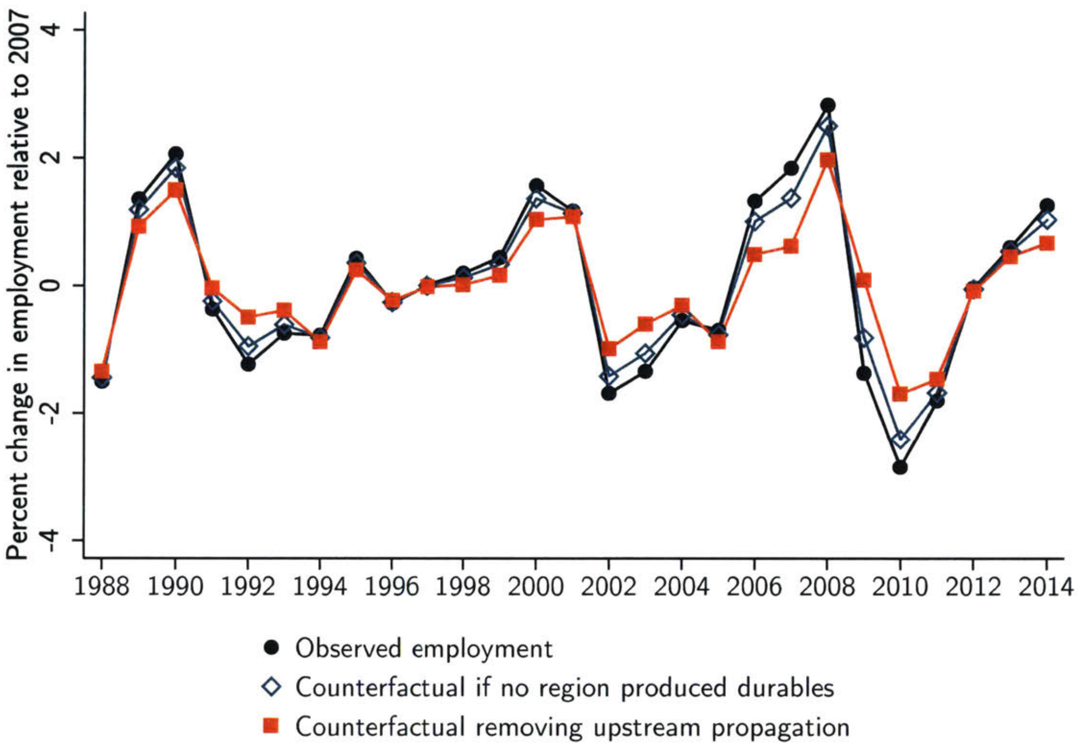
vide an ideal laboratory to investigate the aggregate effects from the decline in the demand for durables. Indeed, the evidence provided by Autor et al. (2013), Notowidigdo (2013), and Yagan (2014) suggests that the extent of workers' migration in response to labor market shocks is modest.[3] Using CBP data covering all commuting zones in the contiguous U.S., we document that employment is more cyclical in commuting zones that host more durable industries. This finding holds even after we control for the secular decline in manufacturing and any potential trend specific to commuting zones hosting durable industries.

Quantitatively, when the aggregate slack in the economy rises by 5 percentage points, employment in a commuting zone that produces consumer durables that last for one additional year declines by 3.25% more relative to a region that produces no durables. The estimated impact of durables on a commuting zone is larger than what a shift-share projection based on the initial industry estimates would predict, which suggests that rather than mitigating the shock to durables, the equilibrium forces that operate at the commuting zone level amplify the effect of the decline in the demand for durable goods. Although they affect a single sector, the substantial albeit temporary changes in the demand for durables during recessions and booms have aggregate effects at the local labor market level, and impact national employment cyclicality as a result. These novel results are quantitatively significant. A back of the envelope calculation suggests that if overall U.S. employment behaved as it does in areas that do not produce durables, national employment would be 20% less volatile. Figure 2-1 previews this result and plots the series for the cyclical component of the observed employment rate in the U.S. (in black circles) and a series of the counterfactual employment rate if no region produced durables (in blue hollow diamonds), resulting in decreased business cycle employment volatility.

We identify three mechanisms that explain why the volatility of demand for durables has a significant effect on aggregate employment. First, changes in the demand for consumer durables affect upstream industries that supply intermediate goods to durable goods producers. In line with this propagation through input-output linkages, we document that employment in upstream suppliers of durable

[3]Though this evidence is in the context of more persistent shocks, we find it reasonable to expect even less migration in response to temporary shocks as the ones we study in this paper. Indeed, we analyze migration patterns in section 2.4 and find evidence of only little reallocation between commuting zones.

Figure 2-1: Cyclical component of U.S. employment and its counterfactual behavior if no U.S. region produced durables, nor supplied durable industries.



Notes: This figure shows the cyclical component of U.S. non-farm private employment (in black circles), the cyclical component of the counterfactual employment absent industries producing consumer durables (in blue hollow diamonds), and the cyclical component of the counterfactual employment absent industries producing consumer durables and their upstream suppliers (in red squares) between 1988 and 2014. Series are expressed as log deviations from their trends, computed with the Holdrick-Prescott filter. More details of the calculations of the counterfactuals in sections 2.4 and 2.5.1.

industries is also highly cyclical, and so is employment in the commuting zones that host these suppliers. Quantitatively, when the aggregate slack in the economy rises by 5 percentage points, employment in a commuting zone with the average amount of linkages to durable goods declines by an additional 2.5% relative to a region with no linkages. A back of the envelope calculation suggests that if overall employment behaved as it does in areas that do not produce durables nor supply durable industries, U.S. employment would be 40% less volatile; input-output linkages double the contribution of consumer durables to the volatility of employment. The cyclical component for the counterfactual employment rate if the U.S.

produced no durables nor supplied durable industries is also shown in Figure 2-1 in red squares.

In addition, we document that industries locate close to their suppliers (Ellison et al., 2010). Thus, input-output linkages amplify the impact on employment in local labor markets that host durable industries, and contribute to explaining why other industries in affected areas do not expand to pick up the slack in the labor market. Quantitatively, the fact that upstream firms co-locate close to producers of consumer durables explains one third of the impact of durable goods on local labor markets. The importance of amplification through input-output linkages is in line with recent evidence showing that industry shocks affect upstream industries (Acemoglu et al., 2015, 2016; Pierce and Schott, 2016; Carvalho et al., 2014; Barrot and Sauvagnat, 2016), and with the theoretical literature emphasizing how sectoral shocks could have aggregate effects because of input-output linkages (Acemoglu et al., 2012).

Second, we show that employment in non-tradable services is more volatile in areas that host durable industries. This volatility cannot be explained by input-output linkages, and suggests that lower consumption by laid-off workers may affect employment in non-tradables through demand spillovers. This finding is in line with the empirical literature emphasizing how local declines in consumption affect employment in the non-tradable sector (Mian and Sufi, 2014), and with the literature emphasizing how demand externalities may amplify shocks when reallocation is imperfect (Beaudry et al., 2014). Quantitatively, the impact on non-tradable employment explains one fifth of the impact of durable industries on local labor market cyclicality. However, whether durable cyclicality results in negative spillovers on non-tradables at the national level as well will depend on the response of monetary and fiscal policy.[4]

Finally, we find little evidence of reallocation to non-durable tradable industries during crises. Abstracting from the impact of input-output linkages and the negative spillover on non-tradables, each additional year in the average expected life of consumer goods produced in a local labor market is associated with an extra decline of 1.5% in employment when the slack in the economy rises by 5 percentage points. This is smaller than our industry-level results predicts but still sug-

[4]Even if monetary or fiscal policy did fully offset the aggregate employment effects on non-tradables, our estimates still suggest major distributional impacts in terms of employment between commuting zones depending on their exposure to durables.

gests that workers do not fully reallocate to other sectors and industries that are less cyclical. In line with this observation, we find no evidence that non-durable tradable industries that are not affected by input-output linkages, expand more in regions that host durable industries compared to others. These results are at odds with models in which a frictionless or rapid reallocation of workers mitigates the aggregate impact of a sectoral shock to the durable industry (Baxter, 1996; Pilossoph, 2012). Instead, one interpretation that may be consistent with the data is that, due to reallocation costs and the expectation that sectoral conditions may revert, workers do not reallocate but remain "rest unemployed" (Jovanovic, 1987; Hamilton, 1988; Gouge and King, 1997; Alvarez and Shimer, 2011).

Besides the literature already mentioned, our paper relates to the debate on the role of sectoral shocks in generating employment fluctuations. Lilien and Hall (1986) emphasizes that sectoral shocks generate business cycles, while in our case business cycles are amplified because some sectors are more sensitive to the cycle as argued by Abraham and Katz (1986). A literature going back to Schumpeter (1942) emphasizes that firms in declining sectors may be permanently liquidated during recessions, which implies that permanent sectoral shifts may coincide with the onset of recessions.[5] Although manufacturing is on a secular decline in the U.S., we show that our results are robust to controlling in a number of ways for this decline and that our findings are specific to durable goods, rather than all manufacturing. Moreover, employment in durable industries and the commuting zones that host them rebounds in a pro-cyclical manner following a recession. Finally, our findings differ from Chodorow-Reich and Wieland (2016), who emphasize how secular reallocation, understood as the response of the economy to permanent sectoral shocks, may generate unemployment, especially during recessions.

The rest of the paper is structured as follows. Section 2.2 describes our data. Section 2.3 presents our evidence for industry employment and wages, which shows that there are large sectoral shocks that take place during recessions. Section 2.4 shows that these sectoral shocks have aggregate effects in U.S. commuting zones that host durable industries or their suppliers. Section 2.5 presents our investigation of mechanisms that generate amplification. Section 2.6 concludes by

[5]See also Davis and Haltiwanger (1990); Hall (1991); Caballero and Hammour (1994); Aghion and Saint-Paul (1998); Koenders and Rogerson (2005); Berger (2016); Jaimovich and Siu (2014); Restrepo (2015).

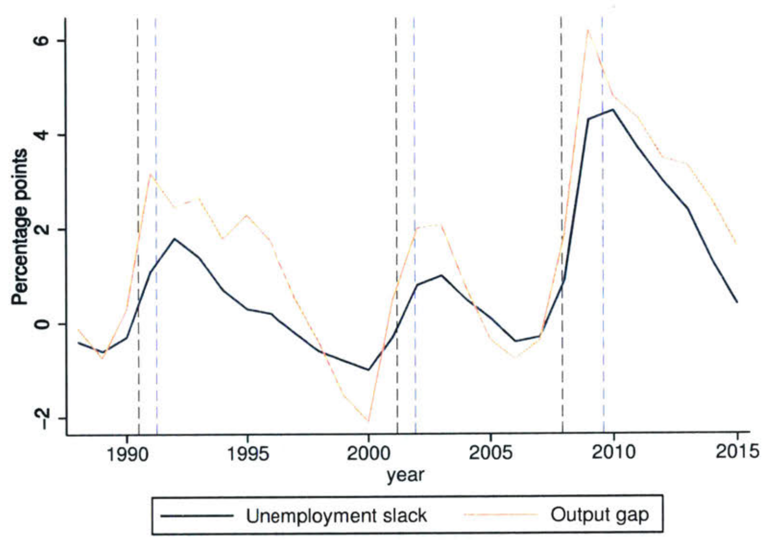
discussing the quantitative implications of our exercise and future avenues for research.

2.2 Data sources

We use yearly data from the County Business Patterns (CBP) between 1988 and 2014. CBP is an annual series covering U.S. employment during the week of March 12 and annual payroll data by county and industry. It covers all employment except self-employed individuals, employees of private households, railroad employees, agricultural production employees, and most government employees. In order to maintain a consistent panel of industries over our time period, we use the industry crosswalks in Autor et al. (2013) and aggregate our data to 479 industry codes. We restrict the analysis to the 48 states of the contiguous United States and aggregate the data to 722 commuting zones to study local labor markets. We supplement this data with information on within-U.S. net migration rates for each commuting zone from the Internal Revenue Service's Statistics of Income U.S. Population Migration Data, which records yearly migration flows between counties. In order to control for demographic covariates at the commuting zone-level, we use the 1990 Census. Finally, we use the long-term NAIRU, observed unemployment, and potential and realized GDP series from the Federal Reserve Bank of St. Louis Economic Data to define two measures of economic slack. The first measure is defined as the difference between the observed national unemployment rate and the long-term natural rate of unemployment, whereas the second one is defined as the difference in log points between the potential and the realized GDP. Both measures are plotted in Figure 2-2, with NBER recessions periods shaded in grey.

We explore different measures for consumer durable exposure, all of which yield qualitatively similar results. In the main text we focus on a measure adapted from Bils and Klenow (1998), which defines for every industry the durability of the consumer goods it produces. If an industry does not produce consumer durables, it is assigned a zero, which allow us to focus on how changes in consumers' demand for durables affect employment. The average durability of consumer goods is 0.35 years (which takes into account that some industries do not produce consumer durables). Among industries that produce consumer durables, the average durability is 7.5 years. These averages are weighted by the employed population

Figure 2-2: Unemployment slack and output gap in the U.S over time.



Notes: This figure plots yearly values between 1988 and 2015 for the unemployment slack in dark blue and output gap in light red. The unemployment slack is defined as the difference between the observed U.S. unemployment rate and the long-term NAIRU, and the output gap is defined as 100 $\times$ the difference between log of potential output and log of realized GDP. All data series are taken from the Federal Reserve Bank of St. Louis Economic Data. Also plotted are business cycle peaks (in dashed black) and troughs (in dashed blue) according to the NBER.

in each industry, to mimic our specifications in section 2.3.

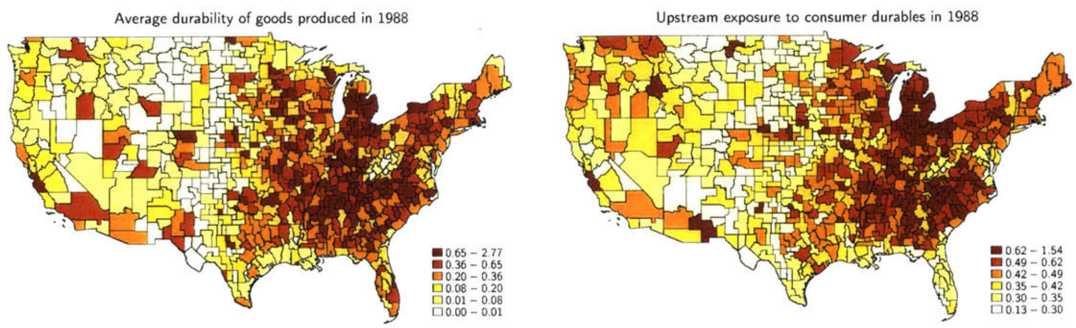
To measure the upstream and downstream exposure to industries that produce durables and investigate possible propagation through supply chain linkages, we use the 1992 input-output table for the U.S. economy from the Bureau of Economic Analysis.[6] We compute for each of the 497 industries a measure of the share of their sales that are directly or indirectly used in the production of consumer durables. In particular, we use the matrix of cross-industry sales $S = \{s_{ij}\}$ (in shares) from industry j to i to compute its Leontief inverse $L^U = (I - S)^{-1'} - I$. The row vector $L_i^U = (l_{i1}^U, l_{i2}^U, \ldots, l_{iI}^U)$ indicates the upstream exposure of industry i to shocks in all the industries it directly or indirectly sells its products to. We compute the *upstream propagation* for a non-durable industry as

$$\text{Upstream Propagation}_i = \sum_j l_{ij}^U \cdot \mathit{Durability}_j.$$

This measure captures the extent of upstream propagation on non-consumer

[6] The table is available at www.bea.gov/industry/io_benchmark.htm.

Figure 2-3: Average durability of goods and upstream exposure to consumer durables by commuting zone in 1988.



Notes: The map on the left shows the average durability of goods produced in each commuting zone in 1988, with the convention that industries that do not produce consumer durables have a durability of zero. The map on the right shows the upstream exposure of suppliers to consumer durables by commuting zone in 1988. This is calculated using the Leontief inverse of the 1992 BEA input-output table and the durability measure.

durable industries; it tells us the share of total production that is eventually used by industries to produce consumer durable goods, weighted by their respective durability. We also compute for each of the industries a measure of the share of their inputs that are consumer durables or are produced using consumer durables. In particular, we use the matrix of of cross-industry purchases $P = \{p_{ij}\}$ (in shares) from industry j to i to compute its Leontief inverse $L^D = (I - P)^{-1'} - I$. The row vector $L_i^D = (l_{i1}^D, l_{i2}^D, \ldots, l_{iI}^D)$ indicates the downstream exposure of industry i to shocks in all the industries it directly or indirectly purchases inputs from. We compute the *downstream propagation* for a non-durable industry as

$$\text{Downstream Propagation}_i = \sum_j l_{ij}^D \cdot Durability_j.$$

This measure captures the extent of downstream propagation on non-consumer durable industries; it tells us the share of consumer durable goods, weighted by their respective durability, that is needed to produce final goods in each industry. Figure 2-3 maps the geographic location of commuting zones that host durable industries and their upstream suppliers with extensive geographic variation across the US. The means of the 1990 Census covariates at the commuting zone level are shown in column (1) of Table 2.1.

Table 2.1: Descriptive statistics of covariates at the commuting zone level.

	Mean	Mean low durability	Mean high durability	Correlation durability	Partial correlation durability
	(1)	(2)	(3)	(4)	(5)
Share < 25	0.387	0.389	0.385	-0.005	-0.003
				(0.005)	(0.005)
Share 25-44	0.297	0.297	0.298	-0.002	0.004
				(0.003)	(0.003)
Share 45-64	0.189	0.186	0.191	0.008***	0.001
				(0.003)	(0.003)
Share college	0.151	0.154	0.147	-0.019***	0.011**
				(0.006)	(0.004)
Share high school	0.553	0.534	0.572	0.055***	-0.010
				(0.014)	(0.011)
Share hispanic	0.0583	0.0839	0.0327	-0.060**	-0.007
				(0.024)	(0.007)
Share black	0.0730	0.0405	0.105	0.047***	-0.020
				(0.018)	(0.020)
Log population	11.48	10.89	12.08	0.803***	0.410*
				(0.251)	(0.238)
Upstream exposure	0.464	0.368	0.560	0.257***	0.065**
				(0.018)	(0.027)
Construction share	0.0465	0.0461	0.0469	-0.005**	0.001
				(0.003)	(0.003)
Commuting zones	722	361	361	722	722

Notes: Column (1) shows the mean of 1990 Census covariates at the commuting zone level, while columns (2) and (3) split the sample between below- and above-median average durability. In column (4), each cell shows the coefficient, and standard error in parentheses clustered at the state level, of a regression involving the 1990 census covariate on the average durability of each commuting zone in 1988. Cells in column (5) are defined as in column (4), but with each specification including a control for the share of manufacturing industry employment in each commuting zone in 1988. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

2.3 Evidence from U.S. industries

We begin by exploring whether national-level employment in industries that produce consumer durables is more cyclical than in other industries over the period 1988-2014.[7] To that end, we estimate the industry-level model:

$$\begin{aligned}\ln E_{it} =& \alpha_i + \delta_t + \beta^I \cdot \text{Slack}_t \times \text{Durability industry}_i \\ &+ \gamma^I \cdot t \times \text{Durability industry}_i + \theta^I \cdot t \times \text{Manufacture}_i + \varepsilon_{it}, \end{aligned} \tag{2.1}$$

where $\ln E_{it}$ is the log of national employment in industry i in year t, Slack_t is our national-level measure of slack in the economy, $\text{Durability Industry}_i$ measures the durability of consumer goods produced by the industry, with the convention that industries that do not produce consumer goods are assigned a zero. Also, Manufacture_i is a dummy for manufacturing industries, and α_i and δ_t are a full set of industry and year fixed effects, respectively. ε_{it} is the error term, which we assume is independent across industries but may be serially correlated within each industry over time. When estimating equation (2.1) we weight observations by the employment in each industry in 1988 and report standard errors that are robust to heteroskedasticity and serial correlation within industries.[8]

The coefficient β^I that multiplies $\text{Slack}_t \times \text{Durability Industry}_i$ captures the additional cyclicality of durable industries compared to nondurable ones. Because recessions are measured through measures of positive slack, negative β^I will be associated with higher cyclicality. In the above model this effect is identified solely from cyclical fluctuations in employment, and does not confound the secular decline in manufacturing or any potential differential trend in durables. These two forces are accounted for by the trends $\gamma^I \cdot t \times \text{Durability Industry}_i$ and $\theta^I \cdot t \times \text{Manufacture}_i$.

Table 2.2 presents estimates of equation (2.1) using the industry-level data from 1988 to 2014 and covering the 479 industries defined in our data. We multiply our estimates by 100 so they can be interpreted in terms of log points. In panel A we

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[7] This period covers 3 recessions according to the NBER Business Cycle Dating Committee.

[8] In addition, in all of our models we control for a full set of year effects interacted with construction industry dummies, which leads to not considering construction in our industrial analysis. Though housing is an important durable good, we only focus on manufacturing durables as we want to abstract from the housing cycle and the impact of house prices on employment through net worth effects (Mian and Sufi, 2014). Nonetheless, our results are robust to including construction industry in the analysis.

use the unemployment measure of slack. In column (1) we present an estimate that excludes differential trends for durable and manufacturing industries. We estimate a statistically significant coefficient for β^I of -1.787, which suggests that durable industries are more cyclical. However, because our measure of slack rises sharply during the Great Recession, this estimate could confound cyclical movements in employment in durable industries with any secular trend affecting manufacturing or durables. To address this concern in column (2) we control for a time trend specific to durable industries. As expected, we find that employment in durable industries is on a statistically significant secular decline of 0.48% fewer jobs per year for every additional year of durability. Our estimate for the excess cyclicality of durable industries now falls to a still highly statistically significant -0.453, which shows the importance of accurately controlling for industry trends. This point estimate suggests that, when the slack in the U.S. labor market rises by one percentage point, employment declines by 0.453% more for every additional year of durability among consumer goods produced by a given industry. Compared to non-durable industries, employment in the average durable industry thus falls by 3.4% more for every percentage point increase in the aggregate unemployment slack. Our estimates for β^I remain essentially unchanged when we control for trends in the manufacturing sector in column (3) or industry-specific trends in column (5).[9] Finally, in column (4) we explore the volatility of other manufacturing industries, i.e. non-consumer durable manufacturing industries. The coefficients from column (3) change very little, and we find that non-durable manufacturing is much less pro-cyclical than durable manufacturing. If anything, the sign of the statistically insignificant point estimate suggests non-durable manufacturing industries could exhibit less cyclicality than the remaining industries.

In panel B we use the measure of slack defined by the output gap. We find similar results as above: when the slack in the U.S. labor market rises by one percentage point, employment declines by a statistically significant 0.278% more for every additional year of durability among consumer goods produced by a given industry. If we take into account Okun's law, which states that an increase in the unemployment rate of 1 percentage point is associated with an increase in the output gap of 2 percentage points, both sets of estimates yield similar quantitative implications. Using series of HP-filtered industry employment and real GDP be-

[9]The changes in the coefficients are smaller than the rounding level in the tables.

Table 2.2: Estimates at the industry level of the different response of durable industries to economic fluctuations.

	Industry estimates from 1988 to 2014				
	(1)	(2)	(3)	(4)	(5)
	Panel A. Slack measured using unemployment.				
Industry that produces durables × Slack in year t	-1.787***	-0.453***	-0.453***	-0.436***	-0.453***
	(0.166)	(0.093)	(0.093)	(0.095)	(0.095)
Industry that produces durables × Yearly trend		-0.478***	-0.140**	-0.133**	
		(0.060)	(0.058)	(0.057)	
Industry in manufacture × Yearly trend			-3.976***	-4.060***	
			(0.289)	(0.318)	
Manufacturing industry that produces nondurables × Slack in year t				0.843	
				(0.752)	
Observations	12906	12906	12906	12906	12906
Number of industries	479	479	479	479	479
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
	Panel B. Slack measured using output gap.				
Industry that produces durables × Slack in year t	-1.048***	-0.278***	-0.278***	-0.274***	-0.278***
	(0.096)	(0.062)	(0.062)	(0.063)	(0.063)
Industry that produces durables × Yearly trend		-0.498***	-0.160***	-0.158***	
		(0.059)	(0.056)	(0.056)	
Industry in manufacture × Yearly trend			-3.976***	-3.995***	
			(0.289)	(0.302)	
Manufacturing industry that produces nondurables × Slack in year t				0.200	
				(0.430)	
Observations	12906	12906	12906	12906	12906
Number of industries	479	479	479	479	470
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
Unreported covariates:					
Industry and year effects	✓	✓	✓	✓	✓
Construction × year effects	✓	✓	✓	✓	✓
Industry trends					✓

Notes: Dependent variable is log employment at the industry and year level. All specifications include a full set of industry and year fixed effects, as well a set of contruction dummy-times-year fixed effects. Column (5) includes industry-specific time trends. In panel A slack is measured as the observed U.S. unemployment rate minus the natural unemployment rate, whereas it is defined as the U.S. output gap in panel B. Robust standard errors in brackets are clustered at the industry level. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

tween 1990 and 2011, Bils et al. (2013) find that the average durable industry is 1.79 times more volatile than GDP. Our estimates are of the same order or somewhat higher, and suggest that a one-point increase in GDP is associated with a 2.1% larger increase in the average durability.

Our results in this section support the view that the demand for durable goods is more cyclical and declines sharply during recessions. Our findings suggest that firms that produce consumer durables respond by reducing employment during downturns and expanding it during booms more than firms in other industries. However, the high cyclicality of employment in the durable sector need not affect aggregate employment. As explained in the introduction, our industry-level estimates could reflect reallocation of workers between industries, as other industries that are less cyclical expand (or decline less) during downturns to absorb workers displaced from durable industries. In the rest of the paper we explore whether the decline in the demand for durable goods and the vast employment losses in this industry contribute to the observed cyclicality of aggregate employment.

2.4 Evidence from U.S. local labor markets

We now analyze the impact of the excess cyclicality of employment in durable industries on the local labor markets that host them. We estimate the following model using data for 722 commuting zones in the contiguous U.S. covering the 1988-2014 period:

$$\begin{aligned}\ln E_{ct} =& \alpha_c + \delta_t + \beta^C \cdot \text{Slack}_t \times \text{Average durability}_{c1988} + \eta^C \cdot \text{Slack}_t \times \text{Manufacture}_{c1988} \\ &+ \gamma^C \cdot t \times \text{Average durability}_{c1988} + \theta^C \cdot t \times \text{Manufacture}_{c1988} + \varepsilon_{ct}, \quad (2.2)\end{aligned}$$

where $\ln E_{ct}$ is the log of the share of employment in commuting zone c in year t normalized by the the population in c at t,[10] Slack_t is again our measure of slack in the economy, and Average $\text{durability}_{c1988}$ is the average durability of consumer goods produced in the commuting zone in 1988, computed using the observed employment shares in that year and with the convention that industries that do

[10] We also estimate all the specifications using non-normalized log of employment in commuting zone c at year t as an outcome variable and find qualitatively the same results. Moreover, later on in this section we investigate migration responses to durable cyclicality and find only economically small impacts.

not produce consumer durables are assigned a zero. Also, $\text{Manufacture}_{c1988}$ is the share of employment in manufacturing industries measured in 1988 for commuting zone c, α_c and δ_t are a full set of commuting zone and year fixed effects, respectively, and ε_{ct} is the error term, which we assume may be serially correlated over time for all commuting zones in a given state. We use the durability and manufacturing shares at the beginning of our sample (1988) instead of the contemporaneous values in order to reduce endogeneity concerns of the local productive structure.[11] When estimating equation (2.2) we weight observations by the employment in each commuting zone in 1988 and report standard errors that are robust to heteroskedasticity and serial correlation within states.[12]

Just as in the previous section, the coefficient β^C that multiplies $\text{Slack}_t \times$ Average $\text{durability}_{c1988}$ captures the additional cyclicality of employment in areas that host durable industries. By including the two trends in the specification, we ensure that the secular decline in manufacturing or any potential trend in durables that could also affect employment in commuting zones is not confounded. The effect of interest is identified solely from cyclical fluctuations in employment.

Our approach exploits differences in the productive structure across commuting zones, in the extent to which they host consumer durable industries. Unlike our previous estimates, which compared relative changes in employment by industry, the impact of durables on the commuting zones that host them takes into account the possibility for reallocation, which could mitigate the aggregate effect of the shock to durables, or amplification mechanisms that could worsen the aggregate effects on employment. To the extent that most of the reallocation and adjustment to labor demand shocks takes place within a commuting zone, these estimates are informative about the equilibrium impact of the excess cyclicality of durables. To illustrate the value of contrasting our these two estimates consider the following scenarios.[13] Suppose that workers displaced from durable industries

[11]The average durability and the manufacturing share at the commuting zone level are highly persistent over time. The correlation between values in 1988 and 2007 are around 0.8.

[12]In addition, in all of our models we control for a full set of year effects interacted with the share of workers employed in construction in 1988. As with our industry analysis, this allows us to abstract from the housing cycle and the impact of house prices on employment through net worth effects (Mian and Sufi, 2014). Our results are robust to foregoing these controls.

[13]Notice that the average durability of consumer durables is computed using employment shares by industry in each commuting zone and $d \ln \left(\sum_i E_{ict}\right) = \sum_i s_{ict}\, d \ln E_{ict}$, where s_{ict} is the share of employment in industry i within commuting zone c and time t. As a result, the magnitudes of the coefficients β^C and β^I are directly comparable.

reallocate immediately to other manufacturing jobs in the same commuting zone. While we would still observe relative changes in employment by industry ($\beta^I < 0$), we would not observe any impact on the overall employment level of the commuting zone ($\beta^C = 0$). If instead, workers displaced from durable industries do not reallocate but remain unemployed or out of the labor force, we would observe relative changes in employment by industry that match the impact of the overall employment level of the commuting zone ($\beta^C \approx \beta^I < 0$). Finally, suppose that because of demand externalities or other possible amplification mechanisms, the decline in demand for durables spills over to other industries in the same commuting zone. In this case, we could have a larger impact on the overall employment level of the commuting zone than in the durable industries ($\beta^C < \beta^I < 0$). These examples illustrate that the difference between the industry and commuting-zone estimates, β^C and β^I, reflects the extent to which reallocation, demand externalities and other general equilibrium effects that operate in a commuting zone mitigate or amplify the sectoral shock to durables.

Table 2.3 presents estimates of equation (2.2). As before, we multiply our estimates by 100 so they can be interpreted in terms of log points. In panel A we use the measure of slack defined by the difference between the national unemployment rate and the natural unemployment rate (in percentage points). In column (1) we present an estimate that excludes the trends for commuting zones that host durable and manufacturing industries. We estimate a statistically significant coefficient for β^C of -1.977, which suggests that employment in commuting zones that host more durable industries behaves more cyclically than in other regions. However, because our measure of slack rises sharply during the Great Recession, this estimate could confound cyclical movements in employment in durable industries with any secular trend in manufacturing or durables. To address this concern, in column (2) we control for a time trend multiplied by the average durability of each commuting zone. As expected from the fact that employment in durable industries is on a secular decline, we find that employment in areas that host these industries is also on a decline over time. Our estimate for the excess cyclicality of durable industries now falls to a still significant -1.013, which shows the importance of accurately controlling for secular trends. This point estimate suggests that the average commuting zone experiences a decline in employment that is 0.46% larger than if it hosted no durable industries when the U.S. labor market slack increases

Table 2.3: Estimates at the commuting zone level of the different response of regions that host durable industries to economic fluctuations.

	Commuting zone estimates from 1988 to 2014						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A. Slack measured using unemployment.						
Baseline share of durables × Slack in year *t*	-1.977***	-1.013***	-1.013***	-0.870**	-0.747***	-0.649***	-0.649***
	(0.416)	(0.218)	(0.218)	(0.339)	(0.185)	(0.144)	(0.147)
Baseline share of durables × Yearly trend		-0.439***	-0.116	-0.131	-0.121	0.063	
		(0.092)	(0.121)	(0.118)	(0.108)	(0.081)	
Baseline share of manufacture × Slack in year *t*				-0.777			
				(1.154)			
Baseline share of manufacture × Yearly trend			-1.765***	-1.683***	-1.696**	-2.798***	
			(0.578)	(0.589)	(0.670)	(0.447)	
Observations	19494	19494	19494	19494	19494	19494	19494
Number of regions	722	722	722	722	722	722	722
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
	Panel B. Slack measured using output gap.						
Baseline share of durables × Slack in year *t*	-1.148***	-0.623***	-0.623***	-0.577***	-0.460***	-0.397***	-0.397***
	(0.245)	(0.147)	(0.147)	(0.214)	(0.121)	(0.093)	(0.094)
Baseline share of durables × Yearly trend		-0.484***	-0.160	-0.165	-0.154	0.034	
		(0.091)	(0.120)	(0.120)	(0.107)	(0.079)	
Baseline share of manufacture × Slack in year *t*				-0.250			
				(0.646)			
Baseline share of manufacture × Yearly trend			-1.765***	-1.740***	-1.696**	-2.798***	
			(0.578)	(0.585)	(0.670)	(0.447)	
Observations	19494	19494	19494	19494	19494	19494	19494
Number of regions	722	722	722	722	722	722	722
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
Unreported covariates:							
Commuting zone and year effects	✓	✓	✓	✓	✓	✓	✓
Share of construction × year effects		✓	✓	✓	✓	✓	✓
Census division × year effects					✓	✓	✓
Demographics × year effects						✓	✓
Commuting zone trends							✓

Notes: Dependent variable is log employment at the commuting zone and year level. All specifications include a full set of commuting zone and year fixed effects, and columns (2) to (7) include contruction share-times-year fixed effects. Columns (5) to (7) include fixed effects for the eight Census divisions interacted with year dummies, columns (6) and (7) include commuting zone-level demographic controls interacted with year fixed effects, and column (7) adds controls for commuting zone-specific time trends. In panel A slack is measured as the observed U.S. unemployment rate minus the natural unemployment rate, whereas it is defined as the U.S. output gap in panel B. Robust standard errors in brackets are clustered at the state level. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

by one percentage point.

Our estimates for β^C remain largely unchanged when we control for trends in the manufacturing sector in column (3), and remain largely unchanged when allowing areas that host manufacturing industries to have different cyclicality in column (4). Our estimates in columns (3) and (4) show that hosting non-durable manufacturing industries does not make a commuting zone more cyclical, and that once we control for the secular decline in manufacturing, commuting zones that host durables are not on a significant further secular decline. These findings suggest that our estimates for β^C do not confound the secular decline of employment in manufacturing or the possibility that this decline may concentrate during downturns (see Jaimovich and Siu (2014)). Both results reassure us that our estimates for β^C are capturing the specific impact of the excess cyclicality of durables on local labor markets, and not trends that are common to all manufacturing industries.

One concern with our previous estimates is that areas that host durable industries may differ in unobserved dimensions from the rest of the U.S., or from other areas that also specialize in manufacturing but mostly produce nondurable goods.

These differences could explain why these areas experience more pronounced recessions and booms. However, we find no significant geographic bunching of durable industries in Figure 2-3, instead documenting extensive dispersion. Moreover, in most of our empirical specifications we also control explicitly for the share of manufacturing employment in the commuting zone. That is, for a given share of local manufacturing, we are exploiting variation in the differential industrial specialization in consumer durables. In order to study whether commuting zones with a larger share of durables differ from others along observable characteristics, we estimate a set of regression specifications with 1990 Census covariates at the commuting zone-level as dependent variable, and the average durability in 1988 as well as, depending on the specification, the manufacturing share of employment in 1988 as independent variables. The coefficients of interest on the average durability for each regression are shown in Table 2.1, as well as sample means of the covariates split by whether the durability of the commuting zone is below or above median in columns (2) and (3). We find that, although Census covariates vary significantly across commuting zones depending on their average durability, commuting zones with a similar manufacturing share but different within-mix of durability only vary significantly in their population size and share of college graduates, with other demographic characteristics not statistically different. They also differ significantly in their exposure to upstream linkages, which we explore in more detail in subsection 2.5.1.

Nonetheless, we allay these concerns further in columns (5) to (7). Although the distribution of durable industries across the contiguous U.S. shown in Figure 2-3 seems not to be concentrated geographically, we control for eight Census division dummies interacted with a full set of year effects in column (5). These dummies guarantee that we identify β^C only by comparing areas that host durables with other areas in the same division, which ensures that our estimates do not confound broad and unobserved regional differences. Our estimates are somewhat reduced, but we still find an economically and statistically meaningful estimate for β^C of -0.747. Besides the division dummies, in column (6) we include a series of covariates measured for each commuting zone using the 1990 Census interacted with a full set of year effects. We control for the log of population, the log of the workforce, the share of people in different age bins, the shares of people with high school and college degrees, and the shares of Blacks and Hispanics. Though differences in these demographic characteristics could make some commuting zones more sensitive to business cycles, we do not find that their inclusion affects our estimates, as we find a coefficient for β^C of -0.649. Finally, in column (7) we include a full set of commuting zone trends, which control flexibly for unobserved heterogeneity and the possibility that areas that host durables are on a secular decline for reasons that are unrelated to the decline in manufacturing employment. Our estimates in column (7) suggest that, when the slack in the U.S. labor market rises by one percentage points, employment in the average commuting zone declines 0.23% more than if it hosted no durables.[14]

Panel B presents our findings when we measure the slack in the U.S. economy using the output gap. Our point estimates in column (7) show that when the output gap rises by one percentage point, employment in the average commuting zone declines 0.18% more than if it produced no consumer durables. This is again in line with the results of Panel A, taking Okun's law into account.

Another potential concern with our estimates is that workers may respond to the decline in the demand for durables by moving to other commuting zones. If this were the case, our cross-sectional estimates would confound the (potential) reallocation of workers across commuting zones with a decline in employment. Although the existing evidence suggests that changes in migration are not an important response to local shocks,[15] we can test directly if the decline in employment documented above is driven by migration. Table 2.4 has the same structure as Table 2.3 but explores whether the net migration rate (inflow minus outflow) is more cyclical in areas that host durables. Our point estimates are quite small and precisely estimated. Moreover, once we account for differences across commuting zones and trends in columns (6) and (7), we do not find a significant effect of durables on the cyclicality of net migration. Quantitatively, when the slack in the U.S. labor market rises by one percentage point, the yearly net migration rate in the average commuting zone declines by only a statistically insignificant 0.01% more than if it did not host durables.[16]

[14]The weighted average of the exposure to durables at the commuting zone is about 0.35.

[15]Bartik (2017) finds large geographic moving costs that inhibit labor market adjustment. Likewise, Autor et al. (2013) and Notowidigdo (2013) find large persistence in local labor market shocks, consistent with low geographic adjustment.

[16]Besides migration, there is an additional concern when interpreting our estimates of β^C as the equilibrium impact of the decline in the demand for durable goods. Because durables are traded across commuting zones, non-durable industries in other regions may benefit from the low price of durables and expand their employment. This reallocation of production through trade cannot be

Table 2.4: Estimates at the commuting zone level of the different response of net migration in regions that host durable industries to economic fluctuations.

	NET MIGRATION RATE AT THE COMMUTING ZONE FROM 1988 TO 2014						
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Panel A. Slack measured using unemployment.						
Baseline share of durables × Slack in year *t*	-0.020	-0.082*	-0.082*	-0.074*	-0.090***	-0.033	-0.033
	(0.040)	(0.041)	(0.041)	(0.040)	(0.032)	(0.029)	(0.029)
Baseline share of durables × Yearly trend		-0.009	-0.007	-0.008	0.002	-0.002	
		(0.007)	(0.008)	(0.010)	(0.007)	(0.006)	
Baseline share of manufacture × Slack in year *t*				-0.042			
				(0.166)			
Baseline share of manufacture × Yearly trend			-0.010	-0.006	-0.012	0.020	
			(0.045)	(0.055)	(0.043)	(0.039)	
Observations	18050	18050	18050	18050	18050	18050	18050
Number of regions	722	722	722	722	722	722	722
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
	Panel B. Slack measured using output gap.						
Baseline share of durables × Slack in year *t*	-0.013	-0.057*	-0.057*	-0.051*	-0.062***	-0.026	-0.026
	(0.028)	(0.029)	(0.029)	(0.027)	(0.022)	(0.021)	(0.022)
Baseline share of durables × Yearly trend		-0.013*	-0.011	-0.012	-0.002	-0.003	
		(0.007)	(0.009)	(0.009)	(0.007)	(0.006)	
Baseline share of manufacture × Slack in year *t*				-0.031			
				(0.117)			
Baseline share of manufacture × Yearly trend			-0.010	-0.007	-0.012	0.020	
			(0.045)	(0.050)	(0.043)	(0.039)	
Observations	18050	18050	18050	18050	18050	18050	18050
Number of regions	722	722	722	722	722	722	722
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
Unreported covariates:							
Commuting zone and year effects	✓	✓	✓	✓	✓	✓	✓
Share of construction × year effects		✓	✓	✓	✓	✓	✓
Census division × year effects					✓	✓	✓
Demographics × year effects						✓	✓
Commuting zone trends							✓

Notes: Dependent variable is log of net migration (population inflows minus outflows) at the commuting zone and year level. All specifications include a full set of commuting zone and year fixed effects, and columns (2) to (7) include contruction share-times-year fixed effects. Columns (5) to (7) include fixed effects for the eight Census divisions interacted with year dummies, columns (6) and (7) include commuting zone-level demographic controls interacted with year fixed effects, and column (7) adds controls for commuting zone-specific time trends. In panel A slack is measured as the observed U.S. unemployment rate minus the natural unemployment rate, whereas it is defined as the U.S. output gap in panel B. Robust standard errors in brackets are clustered at the state level. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

To gauge the economic significance of the estimates in this section we compute the counterfactual behavior of U.S. employment if it produced no durable goods. This counterfactual illustrates the behavior of employment in a scenario in which the demand for consumer durables were not more cyclical than the demand for other goods, or in which all durable consumer goods were imported from other countries. To compute our counterfactual, we multiply our estimate for β^C by the share of employment in durables in each commuting zone and subtract these employment losses or gains from the observed employment. This procedure gives us a series for employment in each commuting zone absent the effect of hosting durable industries:

$$E_{ct}^{nd} = \exp\left(\ln E_{ct} - \widehat{\beta}^C \cdot \text{Slack}_t \times \text{Average Durability}_{c1988}\right). \tag{2.3}$$

The observed and the counterfactual employment series coincide in areas that host no durables or when the aggregate slack in the economy is zero. We aggregate both series to compute their national average. For each average series we use the Holdrick-Prescott filter to compute the log deviations from its trend. Figure 2-1 plots the cyclical components of both series. As is evident from the figure, employment in the U.S. would be less cyclical if business cycles did not involve vast changes in the demand for durables. Quantitatively, the standard deviation of employment is 20% lower in the counterfactual scenario, which suggests that the high cyclicality of durable goods amplifies the impact of aggregate shocks by 20%.

2.5 Mechanisms that amplify the shock to durables

The evidence in the previous sections suggests that, when we look at local labor markets, the impact of the decline in the demand for durable goods is roughly of the same size as our industry estimates ($\beta^I \approx \beta^C < 0$), or even larger. We now explore three mechanisms that can explain why the sectoral shock to durables is not mitigated, and if anything is amplified, at the local labor market level. We first explore whether input-output linkages propagate the initial demand shock on durables across industries. We then analyze whether local demand spillovers

captured in our data and could lead to our estimates for β^C overstating the negative consequences of the decline in the demand for durables. However, in subsection 2.5.1 we find no evidence of benefits for downstream industries that use durables as inputs.

impact non-tradable employment at the commuting zone level. Last, we analyze patterns of reallocation of employment from durable industries to other tradable industries.

2.5.1 Input-output linkages

We explore the possibility that the cyclical changes in the demand for durables propagate through input-output linkages. In particular, we expect shocks to the demand for durables to negatively affect upstream industries that supply inputs to durable good producers –what we refer to as upstream propagation. On the other hand, changes in the demand for durables have an ambiguous effect on downstream firms that use durables –what we refer to as downstream propagation. Though downstream industries may benefit from having access to cheaper durable goods –the low demand by consumers implies there are more durables to be used by downstream industries– the shock to durables may also push some upstream firms out of business, thus affecting downstream firms.[17]

To assess the extent of upstream and downstream propagation at the industry level, we augment equation (2.1) as follows:

$$\begin{aligned} \ln E_{it} &= \alpha_i + \delta_t + \beta^I \cdot \text{Slack}_t \times \text{Durability Industry}_i \\ &\quad + \beta^I_U \cdot \text{Slack}_t \times \text{Upstream Propagation}_i + \beta^I_D \cdot \text{Slack}_t \times \text{Downstream Propagation}_i \\ &\quad + \gamma^I_i \cdot t + \varepsilon_{it}. \end{aligned} \tag{2.4}$$

Here, the terms $\beta^I_U \cdot \text{Slack}_t \times \text{Upstream Propagation}_i$ and $\beta^I_D \cdot \text{Slack}_t \times \text{Downstream Propagation}_i$ capture both potential sources of propagation. We also include industry trends $\gamma^I_i \cdot t$, specific to each of the groups of industries analyzed to isolate the effect of the secular decline in some industries from their cyclical responses.

Table 2.5 presents our industry-level estimates. Panel A uses the unemployment rate to measure slack while Panel B uses the output gap as a measure for slack. In column (1) we estimate the impact of upstream propagation controlling for industry trends. In panel A we estimate a statistically significant coefficient for upstream propagation of $\widehat{\beta}^U_I = -1.338$. This effect is large: our point estimate

[17]For example, it could be that existing customer-supplier relationships are more productive or involve customized inputs. Barrot and Sauvagnat (2016) find that idiosyncratic supplier production shocks impose large output losses on their customers, especially when suppliers produce specific inputs.

Table 2.5: Estimates at the industry level of the different response of durable industries and their suppliers to economic fluctuations.

	INDUSTRY ESTIMATES FROM 1988 TO 2014			
	(1)	(2)	(3)	(4)
	Panel A. Slack measured using unemployment.			
Industry that produces durables × Slack in year *t*	-0.521***	-0.524***	-0.521***	-0.490***
	(0.096)	(0.101)	(0.102)	(0.104)
Upstream propagation of durables × Slack in year *t*	-1.338***	-1.333***	-1.417***	-1.109***
	(0.177)	(0.186)	(0.262)	(0.193)
Downstream propagation of durables × Slack in year *t*		-0.249	-0.219	
		(1.823)	(1.820)	
Manufacturing industry that produces nondurables × Slack in year *t*			0.367	
			(0.632)	
Observations	12906	12906	12906	10584
Number of industries	479	479	479	392
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014
	Panel B. Slack measured using output gap.			
Industry that produces durables × Slack in year *t*	-0.320***	-0.324***	-0.323***	-0.302***
	(0.064)	(0.067)	(0.068)	(0.068)
Upstream propagation of durables × Slack in year *t*	-0.832***	-0.826***	-0.869***	-0.686***
	(0.121)	(0.127)	(0.168)	(0.122)
Downstream propagation of durables × Slack in year *t*		-0.307	-0.291	
		(1.148)	(1.151)	
Manufacturing industry that produces nondurables × Slack in year *t*			0.187	
			(0.402)	
Observations	12906	12906	12906	10584
Number of industries	479	479	479	392
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014
Unreported covariates and sample:				
Industry and year effects	✓	✓	✓	✓
Construction × year effects	✓	✓	✓	✓
Industry trends	✓	✓	✓	✓
Only manufacturing				✓

Notes: Dependent variable is log employment at the industry and year level. All specifications include a full set of industry and year fixed effects, a set of construction dummies-times-year fixed effects, and industry-specific time trends. Column(4) restricts the analysis to manufacturing industries. In panel A slack is measured as the observed U.S. unemployment rate minus the natural unemployment rate, whereas it is defined as the U.S. output gap in panel B. Robust standard errors in brackets are clustered at the industry level. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

suggests that, when slack in the U.S. labor market rises by one percentage points, employment in the average non-durable industry declines by an additional 0.7% as a consequence of upstream propagation.[18] Meanwhile, employment in the average durable industry declines by an additional 3.9%. In column (2) we also estimate the impact of downstream propagation but find no evidence of downstream spillovers. In panel A we estimate a coefficient for downstream propagation of $\widehat{\beta}_I^D = -0.249$. Though small and not significant, our imprecise estimates do not allow us to rule out large effects on downstream industries. However, since we are focusing on consumer durables, it is reasonable that there will not be significant downstream effects; few industries use consumer durables as intermediates while many act as suppliers to industries producing consumer durables.

In column (3) we explore if non-durable manufacturing industries are more cyclical once we take into account the upstream propagation of changes in the demand for durables. Our estimates show that, once we control for these sources of propagation, employment in non-durable manufacturing is not significantly less cyclical than employment in non-manufacturing industries. In contrast, employment in durable industries is considerably more cyclical. Column (4) goes one step further and restricts our analysis to manufacturing industries. It shows that once we account for upstream propagation, employment in durables is more cyclical than in other manufacturing industries.

We now explore how input-output linkages affect our commuting zone estimates. First, there is a high spatial correlation in the location of non-durable manufacturing firms and the durable industries they sell to in our data (see Ellison et al. (2010)). Because we cannot identify the individual consumer-supplier relationships between production plants, we use the BEA national input/output tables to obtain average supply relationships between industries. We find that commuting zones with larger average durabilities also host more industry that supply consumer durables. Because of agglomeration gains, it is likely that these supply industries are also more connected to the local durable manufacturing, and implies that, through input-output linkages, the decline in employment can be amplified in commuting zones hosting large shares of durable industries. In addition, the upstream propagation that we document implies that the high cyclicality of durables may also affect commuting zones that do not host durable industries but that do

[18]The average non-durable industry has an upstream exposure of 0.5.

host their suppliers.

To explore both mechanisms empirically at the commuting zone-level, we augment equation (2.2) as follows:

$$\begin{aligned}\ln E_{ct} =& \alpha_c + \delta_t + \beta^C \cdot \text{Slack}_t \times \text{Average durability}_{c1988} \\ &+ \beta_U^C \cdot \text{Slack}_t \times \text{Upstream Propagation}_{c1988} + \text{Trends} + \varepsilon_{ct},\end{aligned} \tag{2.5}$$

where Upstream $\text{Propagation}_{c1988}$ is the average upstream exposure to durables among industries in commuting zone c measured using employment shares in 1988 to mitigate endogeneity concerns.

We present the results from this exercise in columns (1) and (2) of Table 2.6. In column (1) we report our baseline estimates from column (7) in Table 2.3 for comparison purposes. In column (2) we augment this regression by estimating whether upstream propagation makes employment more cyclical in commuting zones that host upstream suppliers to consumer durables. We find that in a commuting zone with the average amount of upstream linkages to durables (0.5), employment declines by 0.55% more than in a region with no upstream linkages when labor market slack rises by one percentage point. Moreover, our estimate for the impact of hosting durable industries falls from -0.649 to -0.416. This is in line with the fact that part of the effect of durables estimated in column (1) reflects propagation to upstream firms that locate close to durable good producers. Quantitatively, this co-location of suppliers close to their customers explains about a third of the effect of consumer durables on local employment found in section 2.4.

To gauge the economic significance of the estimates in this subsection we compute the counterfactual behavior of overall U.S. employment if it produced no durable goods and absent the upstream propagation. To compute our counterfactuals, we multiply our estimate for β^C by the share of employment in durables in each commuting zone and subtract these employment losses or gains from the observed employment. This procedure gives us a series for employment in each commuting zone absent the effect of hosting durable industries, as in equation (3.18). The observed and the counterfactual employment series coincide in areas that host no durables or when the aggregate slack in the economy is zero. We then compute an additional counterfactual in which we also subtract the role of

Table 2.6: Estimates at the commuting zone level of the negative spillovers created by the decline in employment in the durable industry on other sectors.

	Commuting zone estimates from 1988 to 2014					
	Total employment		Non-tradable services		Non-durable manufacture	
	(1)	(2)	(3)	(4)	(5)	(6)
	Panel A. Slack measured using unemployment.					
Baseline share of durables × Slack in year t	-0.649***	-0.416***	-0.519*	-0.520*	0.365	0.676
	(0.147)	(0.131)	(0.265)	(0.266)	(0.773)	(0.766)
Upstream propagation for all industries × Slack in year t		-1.089***				
		(0.307)				
Upstream propagation for non-durables × Slack in year t						-1.679***
						(0.494)
Upstream propagation for retail and services × Slack in year t				-0.489		
				(2.761)		
Observations	19494	19494	19494	19494	19494	19467
Number of regions	722	722	722	722	720	720
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
	Panel B. Slack measured using output gap.					
Baseline share of durables × Slack in year t	-0.397***	-0.280***	-0.387**	-0.387**	0.427	0.619
	(0.094)	(0.088)	(0.172)	(0.173)	(0.533)	(0.533)
Upstream propagation for all industries × Slack in year t		-0.547***				
		(0.202)				
Upstream propagation for non-durables × Slack in year t						-1.033***
						(0.356)
Upstream propagation for retail and services × Slack in year t				0.555		
				(1.781)		
Observations	19494	19494	19494	19494	19494	19467
Number of regions	722	722	722	722	720	720
Years in panel	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014	1988-2014
Unreported covariates:						
Commuting zone and year effects	✓	✓	✓	✓	✓	✓
Share of construction × year effects	✓	✓	✓	✓	✓	✓
Census division × year effects	✓	✓	✓	✓	✓	✓
Demographics × year effects	✓	✓	✓	✓	✓	✓
Commuting zone trends	✓	✓	✓	✓	✓	✓

Notes: Dependent variable is log of total employment at the commuting zone and year level in columns (1) and (2), log of employment in non-tradable services at the commuting zone and year level in columns (3) and (4), and log of employment in non-durable manufacturing industries in columns (5) and (6). All specifications include a full set of commuting zone and year fixed effects, a set of construction share-times-year fixed effects, census division as well as commuting zone demographic controls interacted with year dummies, and commuting zone-specific time trends. In panel A slack is measured as the observed U.S. unemployment rate minus the natural unemployment rate, whereas it is defined as the U.S. output gap in panel B. Robust standard errors in brackets are clustered at the state level. ***, ** and * denote statistical significance at the 1%, 5% and 10% respectively.

upstream propagation:

$$E_{ct}^{ndu} = \exp(\ln E_{ct} - \widehat{\beta}^{C} \cdot \text{Slack}_t \times \text{Average durability}_{c1988} - \widehat{\beta}_{U}^{C} \cdot \text{Slack}_t \times \text{Upstream Propagation}_{c1988}). \tag{2.6}$$

We aggregate both counterfactual series to compute their national average. Figure 2-4 plots these counterfactual series (normalizing their level to 0 in 2007). As is evident from the figure, employment in the U.S. would be less cyclical if business cycles did not involve vast changes in the demand for durables. For each series we use the Holdrick-Prescott filter to compute log deviations from trend. Quantitatively, the cyclicality of industries that produce durable goods explains 13% of aggregate employment cyclicality. This is below our initial estimate because it does not take into account the propagation to suppliers that co-locate close to industries that produce consumer durables. Upstream propagation explains an additional 27% of the cyclicality of aggregate employment, of which 7% is due to propagation in areas that also host industries that produce consumer durables, and the rest is due to upstream propagation to other regions.

2.5.2 Demand spillovers affecting non-tradables

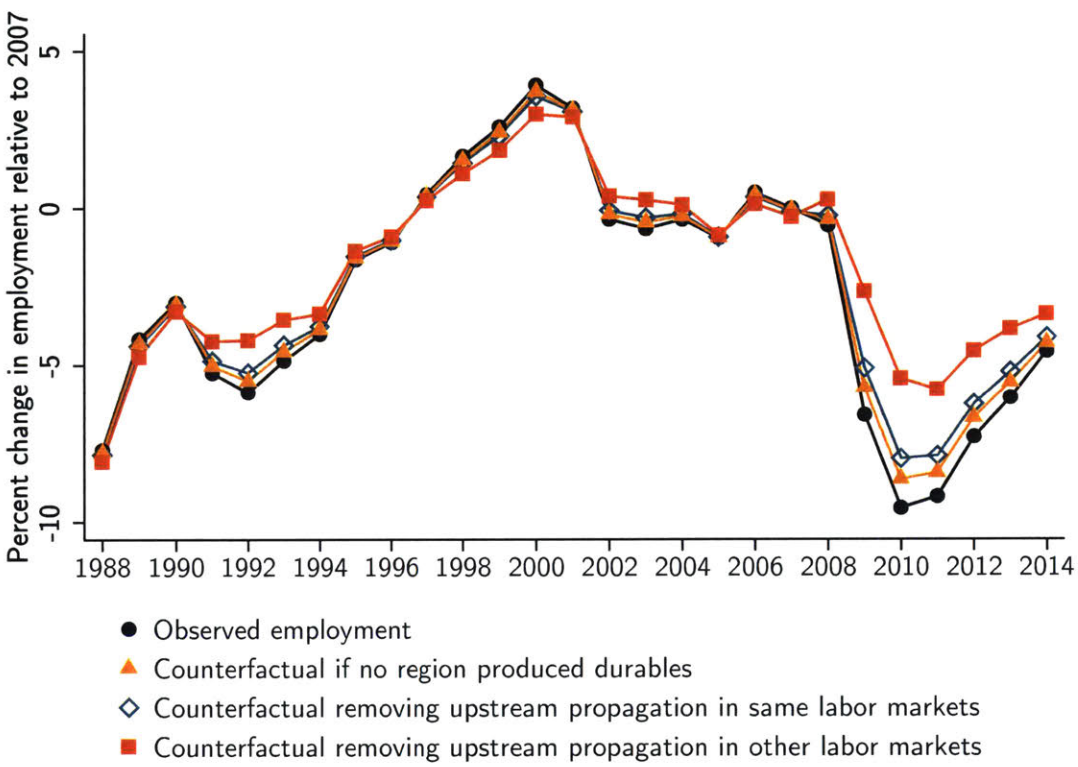
Another potential source of propagation is through demand spillovers. If unemployed workers consume less,[19] the demand for non-tradable goods produced and consumed locally in recessions may decline by more in areas more affected by the cyclicality of durable industries. If that is the case, non-tradables will not expand in relative terms to pick up the extra slack in the labor market caused by downsizing in durable industries.

To assess the extent of negative spillovers on non-tradables, we estimate equation (2.5) but use the log of commuting zone share of employment in non-tradable services as our dependent variable.[20] We present the results from this exercise in columns (3) and (4) of Table 2.6. In column (3) we report our estimates without controlling for the upstream exposure of non-tradable industries, and in column

[19]Ganong and Noel (2016) find that spending on non-durable goods and services drops by 6% at the onset of unemployment and continues to fall during the unemployment period. When unemployment insurance is exhausted, spending falls by an additional 11%.

[20]Non-tradable services include retail and other services, but exclude professional services, as in Autor and Dorn (2013).

Figure 2-4: Employment and its counterfactual behavior if no U.S. region produced durables, there were no upstream propagation to industries in affected regions, or there were no upstream propagation to industries in other regions. All series are expressed in percent deviations from their 2007 level.



Notes: This figure shows the observed U.S. non-farm private employment (in black circles), the counterfactual employment absent industries producing consumer durables (in yellow triangles), the counterfactual employment absent industries producing consumer durables and their upstream suppliers in the same commuting zones (in blue hollow diamonds), and the counterfactual employment absent industries producing consumer durables and all their upstream suppliers (in red squares) between 1988 and 2014. More details of the calculations of the counterfactuals in sections 2.4 and 2.5.1.

(4) we include the upstream exposure of retail and service industries to durables. We find that employment in non-tradable services is highly cyclical in commuting zones that host durable industries. In the absence of demand spillovers, and because non-tradables do not include consumer durable goods, we would expect employment in these industries to expand during recessions in commuting zones that host durable industries relative to other regions, as displaced workers laid off from durable industries reallocate to the non-tradable sector. That is, absent

demand spillovers, employment in non-tradables should be comparatively less cyclical in commuting zones with larger durable industries. We thus attribute our opposite results to local demand spillovers.[21] We find a similar effect when we control for the upstream linkages of non-tradables to durables, and find no strong upstream linkages between these two types of industries.

In both columns we find that, when slack in the U.S. labor market rises by one percentage point, non-tradable employment in the average commuting zone declines by 0.234% more than if it did not host any durables. Quantitatively, the negative spillover on non-tradable industries explains about a fifth of the decline in overall local employment associated with durable goods.[22]

Because of our empirical strategy, this spillover is a differential effect, measured by comparing different commuting zones with different shares of durable industries. Just as the high cyclicality of durable employment at the industry level found in section 2.3 could be mitigated through reallocation, spillovers due to local aggregate demand externalities need not be present at the national level. Whether they are still present depends in part on the monetary and fiscal policy adjustments used to stimulate aggregate demand. However, if fiscal stimuli are in part geographically directed, they are likely to target differentially those commuting zones particularly affected by a recession.[23] That is, we find evidence of more procyclical demand externalities in areas that are likely to already be benefiting more from counter-cyclical fiscal transfers.

On the other hand, a loosening of monetary policy as a result of worsening economic conditions may neutralize our negative spillover results in the aggregate. However, if nominal interest rates are already close to zero, as in the Great Recession, monetary policy may not have room to adjust. Moreover, if the demand channel is driven by complementarities between durable goods and non-tradable consumption, monetary policy will be ineffective against the increased cyclicality of non-tradable consumption. Furthermore, even if the demand spillovers are neutralized in aggregate, our evidence still suggests large distributional impacts of

[21] The fall in demand for non-tradables driving these results can be due to local aggregate demand externalities. However, we cannot rule out the possibility that it is driven by income effects due to strong complementarities between durable good consumption and consumption of non-tradables or non-homothetic preferences.

[22] The estimates need to be scaled down by the share of non-tradable employment at the commuting zone to obtain effects on total employment.

[23] Automatic fiscal stabilizers in the form of unemployment insurance, for example, are likely to flow differentially more to areas with larger drops in employment.

durable consumer goods across commuting zones. Summarizing, durable industries amplify the cyclicality of U.S. employment due to TFP or demand shocks by between 32% and 40%, depending on whether demand spillovers are present in the aggregate.

2.5.3 Lack of reallocation

Abstracting from the contribution of upstream propagation and demand externalities, we find that for every additional year of durability in the consumer goods produced in a commuting zone, its employment declines by 0.32% more when the slack in the economy rises by one percentage point. Starting from an estimate for β^C of -0.649, we have that one third of the effect is explained by the co-location of upstream suppliers in the same commuting zones that host durables and one fifth is explained by demand externalities. Our residual estimate with these adjustments is now below the comparable industry-level β^I, but still close.

These computations suggest that workers laid-off from durable industries are not reallocating to other tradable industries that are less cyclical and that are not affected by the upstream propagation. To test this idea we estimate equation (2.5) but use the log of employment in non-durable manufacturing industries as our dependent variable. We present the results from this exercise in columns (5) and (6) of Table 2.6. In column (5) we report our estimates without controlling for the upstream exposure of these nondurable industries, and in column (6) we include the upstream exposure of non-durable industries to durables at the commuting zone-level. In both columns, we estimate a positive impact of durable cyclicality on non-durable employment, but the estimated effect is economically small and not statistically significant. Notice that in order to compare the coefficient with the impact on aggregate employment, we need to scale it down by the average employment share of non-durable manufacturing, about 8.5% in 1988. In line with our previous findings, we estimate that one of the factors that keep these industries from expanding when durable industries shrink is their input-output linkages to durables and the upstream propagation these generate. However, even when we control for these linkages, we still find that even non-durable industries that are not affected by demand externalities nor input-output linkages fail to expand significantly when employment in durables, suppliers to durables, and non-tradable services shrinks. Our point estimate in column (6) suggests that unaffected non-durable industries only expand by about a tenth of the overall decline in employment, and this effect is not statistically significant. The lack of reallocation explains why the decline in the demand for durables has a negative effect on overall local employment comparable to our industry estimates, even after we control for other sources of propagation.

These findings raise the question of why workers are not fully reallocating from highly cyclical industries to less cyclical ones. One possibility is that, anticipating that the shocks to durables and their suppliers are only temporary, workers do not reallocate but remain "rest unemployed" until conditions improve. This is a hypothesis that we are currently investigating using other sources of data.

2.6 Quantitative implications and remarks

Consumer demand for durable goods is highly pro-cyclical. We find that this cyclicality has large implications for the volatility of U.S. aggregate employment. Consumer durables, and the propagation mechanisms highlighted above, explain between 32% and 40% of the business cycle volatility of aggregate employment. This effect can be decomposed into: a direct increase in volatility due to the cyclicality of durable industries of 10%, a subsequent effect through input-output linkages on suppliers to durable industries of 22%, and a spillover effect through aggregate demand externalities of 8% that may or may not be present nationally depending on the room for adjustment in monetary policy.

Much works remains to be done. We are currently analyzing the effect of consumer durables on measures of payroll and establishment counts, to decompose the intensive versus extensive margin of adjustment by firms. Moreover, the lack of reallocation of workers to less cyclical tradable sectors is surprising. We find that reallocation forces only mitigate up to 10% of the cyclicality of consumer durable employment, and plan to investigate this further.

Chapter 3

Fiscal Fragility (joint with Dejanir Silva)

3.1 Introduction

The importance of public debt in the economy is widely recognized in macroeconomics.[1] There are several strands of literature that study the effect of government debt in an economy, from its role as a tool to smooth the government's fiscal needs (Barro (1979)), to generating a burden (D'Erasmo et al. (2016)) and triggering recessions or slowing growth (Reinhart et al. (2012)). In recent years, there has been a renewed interest in this topic, in the light of the recent crisis that hit the world economy and had its epicenter in the developed countries. In response to the Great Recession and the Euro Crisis many economists called for government intervention of magnitudes not seen in the post-war era. This proposals coexist with many of the involved countries facing historically high levels of debt. However, we lack theoretical models that allow us to think about the interaction between stabilization policies and the level of government debt. Are stabilization policies effective in economies with a high level of government debt? Are negative shocks exacerbated in the presence of high levels of debt? Does the composition of the debt (short- or long- term, indexed) matter? These are the questions we address in this paper.

We study the role of public debt in a standard New Keynesian model in continuous time. This setting proves convenient since it allows us to obtain closed-

[1]See Elmendorf and Mankiw (1999) and the references therein.

form solutions for the equilibrium objects, which facilitates the interpretation of our results. However, our approach differs from the existing literature in two dimensions. First, we study a setting with non-Ricardian fiscal policy. In terms of Leeper (1991) terminology, this corresponds to the "active fiscal/passive monetary policy regime". Second, we introduce a distortionary sales-tax. These assumptions guarantee the existence of a unique equilibrium in this economy and generates comparative statics results that differ from the standard Fiscal Theory of the Price Level.

In this setting, the household's budget constraint becomes a relevant equilibrium condition and government debt affects the real economy through wealth effects that are not canceled out by offsetting tax policy, unlike in the standard New Keynesian model. The channel introduced by the wealth effects is different than the ones commonly analyzed in the literature. To fix ideas, consider the effect of an increase in government spending. Assume that government debt is short-term. The standard result implies that the fiscal multiplier is greater than one when the nominal interest rate is fixed, as in a liquidity trap scenario. The channel at work is mediated through the higher inflation that an increase in spending generates. The increase in government spending generates inflation, which reduces the real interest rate, and boosts current consumption. But now, we have two new effects: higher government spending increases agents' income and the increase in inflation reduces the real return on nominal assets. As in the "old-Keynesian" logic, an increase in government spending has a multiplier effect, as the increase in government spending raises income, increasing consumption, further increasing income and consumption. Since this effect is not mediated by inflation, we find that the fiscal multiplier is positive even when prices are fully rigid. When inflation responds to the increase in government spending, the multiplier effect is mitigated by a different wealth effect, as an increase in inflation will reduce the real return on government bonds. The strength of this second effect will depend on the amount of outstanding government debt.

This logic leads to the main result of the paper: the size and composition of government debt is important to understand the dynamic response of an economy to shocks. In particular, we show that fiscal and monetary policy are less effective in economies with a higher level of public debt, meaning that both the fiscal multiplier and the response to changes in the monetary policy are attenuated.

The maturity structure and asset composition also matters. Long-term bonds and indexed debt improve the efficacy of government policy. Moreover, the level of debt also has implications for how the economy responds to shocks. In response to a preference shock that pushes the economy into a liquidity trap, high-debt economies experience larger and more prolonged recessions. Therefore, more indebted economies and economies that rely less on long- term or indexed debt are, in this sense, more fragile.

We start the analysis by presenting an *irrelevance-of-debt* result in the steady state of the economy: the size and composition of debt does not affect the equilibrium allocation as long as it is compensated with lump-sum transfers. This result is a consequence of Ricardian equivalence[2], and it provides an important benchmark for our exercises. It shows that if we consider a cross-section of economies with similar characteristics and that only differ in the size of their public debt (compensated by lump-sum transfers), their steady state allocation coincides.

We then show that the impulse response to fiscal, monetary and real shocks depends on the outstanding stock of public debt. The intuition is closely related to the wealth effects these shocks have. Public debt is part of the household's wealth, and changes in the real return on agent's assets have an impact on their consumption decisions. When all government debt is short-term, increases in the inflation rate, for a given nominal interest rate, or reductions in the nominal rate in the presence of nominal rigidities, reduces the real return on public debt. This negative wealth effect is a force that reduces agent's consumption, reducing the fiscal multiplier. Moreover, the wealth effect is proportional to the stock the agent holds: the larger the stock, the larger the wealth effect.

Consider again the effect of government spending. Since an increase in government spending while keeping the nominal interest rate fixed generates an increase in inflation, the real return on government bonds decreases, and the wealth effect described before *reduces* the fiscal multiplier. Moreover, this reduction is larger when government debt is higher. Importantly, the effect of debt on the fiscal multiplier works exclusively through inflation. In particular, if prices are fully rigid the fiscal multiplier is independent of the level of government debt (though still positive).

The effect of monetary policy is even more direct. A reduction in the nominal

[2]See Barro (1974).

interest rate increases current consumption in the standard model with nominal rigidities. But now, it also reduces the real return on government bonds. Hence this negative wealth effect partially offsets the standard intertemporal substitution effect, and an expansive monetary policy has a smaller effect.

Next, we allow the government to issue long-term debt and indexed debt. Consider first the role of long-term bonds. Keeping fixed the level of public debt, the effect of a monetary shock becomes more effective when the share and the duration of long-term debt is higher. However, long-term debt has no impact on the fiscal multiplier. The intuition for this result is as follows. Long-term bonds can affect the household's wealth through two channels. First, through changes in the real interest rate. A higher interest rate increases the return of bonds, which translates into a positive wealth effect. This is the same channel as the one when debt is short-term. Second, long-term bonds also have a price effect on the outstanding stock. An increase in the nominal rate reduces the price of outstanding bonds, which generates a negative wealth effect to the households. The longer the duration of the bond, the more prevalent is the price effect. However, the price effect is only nominal; it only reacts to changes in the nominal interest rate. Therefore, long-term bonds improve the effectiveness of monetary policy but have no effect on the fiscal multiplier.

On the other hand, indexed bonds completely insulate their real return from changes in the inflation rate. This means that their return *and* price react one-to-one to changes in the real rate. As a result, more indexed debt makes government spending more powerful. Government spending generates inflation. With short-term debt only and fixed nominal rate, the real return on bonds decreases, attenuating the consumption response of the agents. With indexed bonds, increases in the inflation rate increase the nominal price of indexed bonds, *reinforcing* the positive wealth effect. Therefore, indexed bonds increase the effectiveness of both fiscal and monetary policy.

Finally, we show that the level of government debt has implications for how the economy behaves in a liquidity trap. We study a shock that temporarily increases the household's discount rate and makes the natural rate of interest to be negative for the duration of the shock. A recession generated by this preference shock is exacerbated by the level of debt. The reason is that the preference shock generates inflation in this model. Keeping the nominal interest rate fixed, this reduces the real rate of return, which has a negative wealth effect, contributing to the reduction of consumption.

Literature Review This paper is related to several strands of literature. First, the paper is connected to the literature that studies the real effects of government debt. Elmendorf and Mankiw (1999) provides a great review of many important topics. For example, Ball and Mankiw (1995) study the crowding-out effect of government debt. Reinhart et al. (2012) argue that high levels of debt are associated with lower long-run growth. Our paper identifies the relationship between public debt and macroeconomic stabilization policies as a new channel through which government debt can have a real effect in the economy.

Second, our paper is related to the literature that studies fiscal multipliers. There is a vast theoretical and empirical literature that explores the determinants and size of fiscal multipliers. See Farhi and Werning (2016) and the references therein. However, there is a little work on the role of government debt. One exception is Ilzetzki et al. (2013), who empirically identify that the fiscal multiplier is smaller in economies with higher public debt. Our paper contributes to this literature by providing a theoretical exploration of a channel that can explain these results.

Finally, our paper contributes to the theoretical literature on New Keynesian models and the Fiscal Theory of the Price Level (FTPL). Leeper (1991), Sims (1994) and Woodford (2001) are early developments of the FTPL. Kim (2003) provides an analysis combining studying the effects of the FTPL in a New Keynesian model. We extend his analysis and focus on the role of government debt in shaping the effectiveness of macroeconomic stabilization policies. Moreover, Cochrane (2001) identifies the importance of the maturity in determining the path of inflation under the FTPL. Our analysis differs from his in that we study the interaction between the composition of debt and macroeconomic stabilization policies.

Outline The rest of the paper is organized as follows. In section 2, we present the basic model with short-term debt only and present the *irrelevance-of-debt* result. Section 3 studies the equilibrium dynamics and shows how the economy reacts to fiscal and monetary shocks. Section 4 presents the main results of the paper: the relationship between the level of debt and macroeconomic stabilization policies. In section 5, we extend the basic model to include long-term and indexed debt.

Section 6 studies the interaction bewteen government debt and the severity of a liquidity trap scenario. Section 7 concludes.

3.2 The Basic Model

Time is continuous and denoted by $t \in \mathbb{R}_{+}$. There are two types of agents in the economy: a large number of identical, infinitely-lived households, and a infinitely-lived government. Moreover, there is a continuum of mass one of firms. We follow the literature and consider the Dixit-Stiglitz model of monopolistic competition. Each firm is the single monopoly producer of a differentiated good, and households' preferences are such that final consumption is a CES aggregator of the purchases of each of the differentiated goods.

As is standard in the literature, we will later log-linearize the model around it's steady state equilibrium in order to study a first-order approximation of the equilibrium response of the economy to exogenous shocks. In this context, the presence of risk is irrelevant since the log-linearized version satisfies a certainty equivalence property. Therefore, without loss of generality, we consider a model of perfect foresight that suffers a one-time unexpected shock to current and future paths of the exogenous variables that is realized at the beginning of period 0.

3.2.1 Households

The representative household has preferences given by

$$\int_0^{\infty} e^{-\rho t}\left[\frac{C_t^{1-\sigma}}{1-\sigma}-\frac{N_t^{1+\phi}}{1+\phi}\right] dt,$$

where C_t is a Dixit-Stiglitz aggregator of a continuum of differentiated goods

$$C_t=\left(\int_0^1 C_t(j)^{\frac{\epsilon}{\epsilon-1}} dj\right)^{\frac{\epsilon-1}{\epsilon}},$$

where $C_t(j)$ is the amount of variety j consumed in period t, N_t is hours of labor supplied in period t, and $\rho > 0$ is the instantaneous discount factor. Households derive utility from the aggregate consumption good, C_t, while they get disutility from work, N_t.

Households face a per-period budget constraint given by

$$\dot{B}_t = i_t B_t + W_t N_t + \Pi_t + \hat{T}_t - \int_0^1 P_t(j) C_t(j) dj,$$

where i_t represents the nominal interest rate, B_t short-term (instantaneous) nominal assets, Π_t aggregate nominal profits, and $\hat{T}_t$ government lump-sum transfers. Moreover, they are subject to the usual No-Ponzi condition

$$\lim_{t\to\infty} e^{-\int_0^t i_s ds} B_t \geq 0.$$

In the optimum, the household's demand for each variety is given by

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t,$$

where

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

is the ideal price index. Therefore, the household's budget constraint can be written as

$$\dot{B}_t = i_t B_t + W_t N_t + \Pi_t + \hat{T}_t - P_t C_t.$$

3.2.2 Firms

Each differentiated good is produced using labor as the only input

$$Y_t(j) = A N_t(j)^{\frac{1}{\varphi}},$$

where $\varphi \geq 1$.

We assume Calvo pricing, where firms are allowed to reset their prices with Poison intensity ρ_δ. Moreover, we assume that the government chooses a sales tax τ_t, and a payroll tax τ_t^W. Therefore, firms optimally choose the price of their product by solving the following problem

$$\max_{P_t^*(j)} \int_0^\infty e^{-\int_t^s (i_{t+z}+\rho_\delta)dz} \left[(1-\tau_{t+s}) P_t^*(j) Y_{t+s|t} - (1+\tau_t^W) W_{t+s} \left(\frac{Y_{t+s|t}}{A_{t+s}}\right)^{\varphi}\right] ds,$$

where $Y_{t+s|t}$ represents the demand function the producer faces at period $t+s$

$$Y_{t+s|t} = \left(\frac{P_t^*(j)}{P_{t+s}} \right)^{-\epsilon} Y_{t+s},$$

and Y_t denotes aggregate demand at period t. Firms take the sequences for W_t, Y_t, P_t and fiscal policy as given when solving their problem.

3.2.3 Government

Government consumption is given by the same aggregate of individual goods as the household's,

$$G_t = \left(\int_0^1 G_t(j)^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}.$$

Given an aggregate amount G_t, government purchases of each individual good is made in order to minimize the total costs.

Combining the household's intertemporal budget constraint and the market clearing condition $C_t + G_t = Y_t$, we obtain

$$D_t^g = \int_t^\infty e^{-\int_t^s i_z dz} P_s \left[\tau_s Y_s + \tau_s^W W_s N_s - G_s - T_s \right] ds,$$

where $T_t \equiv \hat{T}_t / P_t$ denotes transfers in real terms, and $D_t^g = B_t$ is the market clearing condition in the bonds market. Using the agent's optimality condition to substitute for the nominal interest rate, the price level, and the nominal wage, we get

$$\frac{D_0^g}{P_0} = \int_0^\infty e^{-\rho t} \left(\frac{C_t}{C_0} \right)^{-\sigma} \left[\tau_t Y_t + \tau_t^W C_t^\sigma N_t^{1+\phi} - G_t - T_t \right] dt.$$

Since P_0 is predetermined in this continuous time setting, we normalize it to one, i.e., $P_0 = 1$.

An important feature for the determination of equilibrium is that the fiscal policy is described by a non-Ricardian rule, in the sense that primary surplus does not automatically adjust in order to satisfy the budget constraint *for any* sequence of inflation rates. Moreover, we assume that monetary policy is characterized by a given path for the interest rate $\{i_t\}$. Typically, the choice of an exogenous process for the nominal interest rate is not enough to determine equilibrium uniquely. However, assuming a non-Ricardian fiscal policy gives us determinacy in this model.

3.2.4 Equilibrium Conditions

Given a rule for $\{G_t, \tau_t, \tau_t^W, T_t, i_t\}$, an equilibrium for this economy can be characterized by the following system of equations

$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}(i_t - \pi_t - \rho),$$

$$N_t = \left(\frac{Y_t}{A_t}\right)^{\varphi} \Delta_t,$$

$$\pi_t = \frac{\rho_\delta}{\epsilon - 1}\left[1 - \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\epsilon(\varphi-1)}}\right],$$

$$Y_t = C_t + G_t,$$

$$D_0^g = \int_0^\infty e^{-\rho t}\left(\frac{C_t}{C_0}\right)^{-\sigma}\left[\tau_t Y_t + \tau_t^W C_t^\sigma N_t^{1+\phi} - G_t - T_t\right] dt,$$

where Δ_t, F_t and K_t are defined in the appendix.

3.2.5 Irrelevance of Debt in Steady State

Let's study first the properties of the economy in a steady state with zero inflation. In this equilibrium, policy is such that: i) the fiscal variables are constant, i.e., $G_t = \overline{G}$, $\tau_t = \overline{\tau}$, $\tau_t^W = \overline{\tau}^W$ and $T_t = \overline{T}$ for all t; ii) the nominal interest rate is $i_t = \rho$ for all t. The steady state allocation satisfies

$$\overline{Y} = \left[\frac{(1-\overline{\tau})(\epsilon - 1)}{\varphi\epsilon(1+\overline{\tau}^W)}\overline{C}^{-\sigma}\right]^{\frac{1}{\varphi-1+\varphi\phi}} \tag{3.1}$$

$$\overline{C} = \overline{Y} - \overline{G} \tag{3.2}$$

$$\overline{N} = \overline{Y}^{\varphi} \tag{3.3}$$

$$\overline{D^g} = \frac{\overline{\tau}\overline{Y} + \overline{\tau}^W \overline{C}^\sigma \overline{N}^{1+\phi} - \overline{G} - \overline{T}}{\rho}. \tag{3.4}$$

These equations lead to the following result.

Proposition 16. *Given $\overline{G}$, $\overline{\tau}$ and $\overline{\tau}^W$, the steady state level of output, consumption and labor are independent of the size of debt.*

The steady state level of output, consumption and labor of the economy are determined by equations (3.1)-(3.3), which are independent of the level of debt. Equation (3.4) determines the combination of lump-sum transfers and debt levels consistent with the government's budget constraint. For example, a higher level of steady state debt is associated with a higher level of lump-sum transfers, which are used to pay the interest on debt. However, a Ricardian Equivalence result holds, in the sense that the timing of the lump-sum transfers does not matter for the equilibrium allocation. The following corollary provides a benchmark for our analysis below.

Corollary 16.1. *Consider two economies like the one described here, with the same preferences, technologies and price-setting frictions. If the steady state level of government spending and distortionary taxes coincide, then their steady state level of output, consumption and labor also coincide.*

This result provides an important benchmark for the exercises we perform in the next sections. It says that two economies that differ only in their steady state level of debt feature the same steady state allocation. However, we will show that, despite of this, their behavior after fiscal and monetary shocks, as well as in a liquidity trap scenario, is different.

In order to simplify exposition, we make the following assumptions.

Assumption 5. *The payroll tax is fixed and given by*

$$\tau_t^W = \overline{\tau}^W = (1-\overline{\tau})\frac{\epsilon-1}{\epsilon} - 1 < 0.$$

Moreover, the lump-sum transfer, T_t, has two components: $T_t = \overline{T} + T_t^W$, where

$$T_t^W = \overline{\tau}^W C_t^\sigma N_t^{1-\phi}.$$

Assumption 5 implies that the payroll tax is chosen in order to eliminate the monopoly and tax distortions in the steady state, while the lump-sum tax has a component that automatically adjusts to fund this subsidy in all periods of time. Under this assumption, the budget constraint of the government can be rewritten as

$$D_0^g = \int_0^\infty e^{-\rho t}\left(\frac{C_t}{C_0}\right)^{-\sigma}\left[\tau_t Y_t - G_t - \overline{T}\right]dt.$$

3.3 Equilibrium Dynamics

To study the dynamics of the economy, we log-linearize the equilibrium conditions around a steady state that features a constant path for the policy variables and zero inflation. Define

$$c_t = \log(C_t) - \log(\overline{C}), \quad y_t = \log(Y_t) - \log(\overline{Y}), \quad g_t = \log(G_t) - \log(\overline{G}),$$

so that up to first order

$$y_t = \varsigma_c c_t + \varsigma_g g_t,$$

where $\varsigma_c \equiv \overline{C}/\overline{Y}$ is the level of consumption-over-GDP in steady-state, and $\varsigma_g \equiv \overline{G}/\overline{Y}$ is the level of government spending-over-GDP in steady-state. Moreover, define

$$\hat{\tau}_t \equiv -\log\left(\frac{1-\tau_t}{1-\overline{\tau}}\right).$$

Given the path of interest rates, $\{i_t\}$, government spending $\{g_t\}$, and taxes, $\{\hat{\tau}_t\}$, the equilibrium is characterized by

$$\dot{c}_t = \sigma^{-1}(i_t - \pi_t - \rho), \tag{3.5}$$

$$\dot{\pi}_t = \rho\pi_t - \kappa(\omega_c c_t + \omega_g g_t + \hat{\tau}_t), \tag{3.6}$$

where κ, ω_c and ω_g are positive constants defined in the appendix, and the intertemporal budget constraint

$$\int_0^\infty e^{-\rho t}\varsigma_c c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})(y_t - \hat{\tau}_t) + \sigma\rho\varsigma_d(c_t - c_0)\right]dt + \varsigma_d d_0^g, \tag{3.7}$$

where ς_d is the debt-to-gdp ratio in steady state. For simplicity, we assume that the deviation of the initial level of debt with respect to its steady state value is zero, so that $d_0^g = 0$.

It is useful to define the following two numbers (which are the eigenvalues of the system given by (3.5) and (3.6))

$$\overline{\omega} = \frac{\rho + \sqrt{\rho^2 + 4\kappa\omega_c\sigma^{-1}}}{2}, \quad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\kappa\omega_c\sigma^{-1}}}{2}.$$

Given the linearity of the system given by (3.5) and (3.6), we can decompose the values of consumption and inflation at any point t as a term that responds to fiscal shocks, a term that responds to monetary shocks, and a term with the initial condition, respectively:

$$c_t = c_t^f + c_t^r + e^{\underline{\omega} t} c_0,$$
$$\pi_t = \pi_t^f + \pi_t^r + e^{\underline{\omega} t} \pi_0.$$

Let's focus on consumption. It has been common in the literature to assume that $c_t = 0$ for all $t \geq T$, for some T (possibly large), and use that as initial condition to solve the system above.[3] We follow a different path and use the budget constraint (3.7) to pin down the initial condition. The next proposition characterizes the solution in closed-form.

Proposition 17. *The equilibrium path for consumption is given by*

$$c_t = c_t^f + c_t^r + e^{\underline{\omega} t} c_0,$$

where

$$c_t^f = -\frac{e^{\underline{\omega} t}}{\sigma(\overline{\omega} - \underline{\omega})} \left[\int_0^t \left(e^{-\underline{\omega} s} - e^{-\overline{\omega} s}\right) \kappa(\omega_g g_s + \hat{\tau}_s) ds + \left(e^{(\overline{\omega} - \underline{\omega})t} - 1\right) \int_t^\infty e^{-\overline{\omega} s} \kappa(\omega_g g_s + \hat{\tau}_s) ds \right],$$
$$c_t^r = \frac{e^{\underline{\omega} t}}{\sigma(\overline{\omega} - \underline{\omega})} \left[\int_0^t \left(\overline{\omega} e^{-\underline{\omega} s} - \underline{\omega} e^{-\overline{\omega} s}\right) (i_s - \rho) ds + \underline{\omega} \left(e^{(\overline{\omega} - \underline{\omega})t} - 1\right) \int_t^\infty e^{-\overline{\omega} s} (i_s - \rho) ds \right],$$

and the initial value of c_0 is given by

$$c_0 = \int_0^\infty e^{-\rho t} \left(\chi_{g,t}^c g_t + \chi_{\tau,t}^c \hat{\tau}_t + \chi_{r,t}^c (i_t - \rho) \right) dt,$$

where

$$\chi_{g,t}^c = \overline{\omega} \frac{\overline{\tau} \varsigma_c - \rho \sigma \varsigma_d}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} \frac{\omega_g}{\omega_c} (1 - e^{\underline{\omega} t}) + \overline{\omega} \frac{1 - \overline{\tau}}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} \varsigma_g,$$
$$\chi_{\tau,t}^c = \overline{\omega} \frac{\overline{\tau} \varsigma_c - \rho \sigma \varsigma_d}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} \frac{1}{\omega_c} (1 - e^{\underline{\omega} t}) - \overline{\omega} \frac{1 - \overline{\tau}}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d},$$
$$\chi_{r,t}^c = -\sigma^{-1} \frac{\overline{\tau} \varsigma_c - \rho \sigma \varsigma_d}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} e^{\underline{\omega} t}.$$

Note that given $\{g_t, \hat{\tau}_t, i_t\}$, initial consumption, c_0, is uniquely determined.

[3] See, for example, Werning (2011), Farhi and Werning (2016), McKay et al. (2016).

In what follows, we make the following assumption.

Assumption 6. $\overline{\tau}\varsigma_c > \rho\sigma\varsigma_d$.

The left-hand side captures the first-order effect of an increase in future consumption on tax revenues. The right-hand side captures the first order effect of an increase in consumption on the interest payments on the debt. An increase in future consumption pushes interest rates up by σ, while the interest payments on debt in steady state is given by $\rho\varsigma_d$. Hence, Assumption 6 implies that a boom in consumption increases government revenues by more than it increases financing costs, so that it improves government finances. A sufficient condition is that $\sigma \leq 1$. Under this condition we get the following proposition.

Proposition 18. *Suppose Assumption 6 holds. Then,*

$$\frac{\partial c_0}{\partial g_t} > 0, \quad \frac{\partial c_0}{\partial i_t} < 0.$$

Moreover, if we further impose $\sigma \leq 1$, then

$$\frac{\partial c_0}{\partial \hat{\tau}_t} < 0.$$

These results work through two different channels. The first channel is the standard substitution effect through changes in the real interest rate. This effect is immediate when the central bank increases the nominal interest rate and there are nominal rigidities. On the other hand, an increase in government spending increases initial inflation, which reduces the real rate for a fixed nominal rate, thus increasing initial consumption. The second effect is an income and wealth effect coming from the non-Ricardian fiscal policy. Consider an increase in government spending. This increases the output in the period in which the increase in government spending takes place, so that the household's *permanent income* increases. This naturally leads to an increase in initial consumption. Note that in the background we need that the government's budget constraint is satisfied. Assumption 6 guarantees this: a boom in consumption increases tax revenues by more than it increases real rates, so that a boom large enough will produce the revenues necessary to satisfy the government's budget.

It is interesting to note that the implied fiscal multipliers are different than the ones found with the standard equilibrium selection.[4] First, while the literature has found completely forward looking fiscal multipliers, in this economy past government spending has an impact on current consumption. Second, the multiplier is not always increasing in the time in which the spending actually happens. In particular, we show that the fiscal multiplier is decreasing in time when government spending happens in the distant future. Third, the effect of government spending on consumption is not exclusively mediated through the effect of government spending on inflation. In particular, the fiscal multipliers are different than zero even if prices are rigid, i.e., $\kappa = 0$. And, relatedly, fourth, the contemporaneous fiscal multiplier is different than zero. The next proposition summarizes these results.

Proposition 19. *The effect on consumption in period 0 of an increase in government spending in period t, $\frac{\partial c_0}{\partial g_t}$, has the following properties:*

1. ***Backward looking:*** $\frac{\partial c_t}{\partial g_s} \neq 0$ *for* $s < t$;
2. ***No back-loading:*** $\frac{\partial c_0}{\partial g_t}$ *is increasing in t at* $t = 0$, *but decreasing in t as* $t \to \infty$;
3. ***Wealth-effect:*** *the effect is not (only) mediated through inflation,* $\frac{\partial c_0}{\partial g_t} > 0$ *even if* $\kappa = 0$;
4. ***Contemporaneous effect:*** $\frac{\partial c_0}{\partial g_0} > 0$.

Here we mostly focused on the effects of policy changes in period-0 consumption. Recall that

$$c_t = c_t^f + c_t^r + e^{\underline{\omega} t} c_0 \tag{3.8}$$

From Proposition 17 we know that c_t^f and c_t^r are independent of ς_d. Since the objective of this paper to study how the response of the economy to shocks depends on the level of debt, it is sufficient to study the response of c_0 to these shocks and then use (3.8) to determine the whole path for consumption. Therefore, in what follows, we will focus on the analysis of c_0.

3.3.1 Rigid prices

It is useful to study the rigid prices case in order to grasp a better understanding on the mechanics of the model. To do this, we set $\kappa = 0$ in the system of equations

[4] See Farhi and Werning (2016).

above. In this case, $\underline{\omega} = 0, \overline{\omega} = \rho$, and hence $c_t^f = 0$ for all t, while $c_t^r = \sigma^{-1} \int_0^t (i_s - \rho)ds$, so that

$$c_t = c_0 + \sigma^{-1} \int_0^t (i_s - \rho)ds$$

and

$$c_0 = \int_0^\infty e^{-\rho t} \left(\chi_{g,t} g_t + \chi_{\tau,t} \hat{\tau}_t + \chi_{i,t}(i_t - \rho) \right) dt,$$

where

$$\chi_{g,t} \equiv \rho \frac{1-\overline{\tau}}{\overline{\tau}\varsigma_c} \varsigma_g,$$
$$\chi_{\tau,t} \equiv -\rho \frac{1-\overline{\tau}}{\varsigma_c},$$
$$\chi_{i,t} \equiv -\sigma^{-1} \frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c}.$$

Unlike the result with the standard equilibrium selection in the New Keynesian model (see Farhi and Werning (2016)), the fiscal multiplier is positive even when inflation is zero. The reason for this result is that, with non-Ricardian fiscal policy and distortionary taxes, an increase in government spending produces an income effect reminiscent to the "old Keynesian" multiplier.

To see this, suppose that there are no monetary shocks (i.e., $i_t = \rho$ for all t), so the budget constraint can be written as

$$\int_0^\infty e^{-\rho t} \varsigma_c c_t dt = (1-\overline{\tau}) \int_0^\infty e^{-\rho t} y_t dt - (1-\overline{\tau}) \int_0^\infty e^{-\rho t} \hat{\tau}_t dt. \tag{3.9}$$

Define

$$C \equiv \int_0^\infty e^{-\rho t} \varsigma_c c_t dt,$$
$$G \equiv \int_0^\infty e^{-\rho t} \varsigma_g g_t dt,$$
$$Y \equiv \int_0^\infty e^{-\rho t} y_t dt,$$
$$T \equiv \int_0^\infty e^{-\rho t} \hat{\tau}_t dt.$$

Then, we can rewrite (3.9) as

$$C = (1-\overline{\tau})(Y - T),$$

Using the resource constraint $Y = C + G$, we get

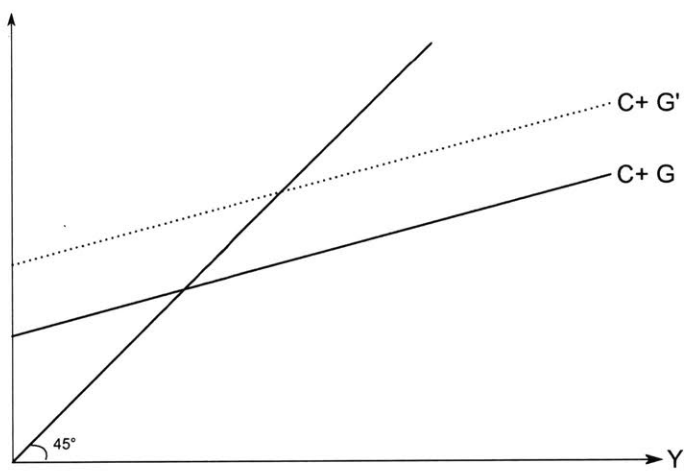
$$Y = (1 - \overline{\tau})(Y - T) + G,$$

and solving for Y

$$Y = \frac{1}{\overline{\tau}}(G - T),$$

which implies a fiscal multiplier of $1/\overline{\tau} > 1$. This system of equations can be represented by a (present value) Keynesian Cross, as in Figure 1. An increase in

Figure 3-1: (Present Value) Keynesian Cross



government spending, G, increases disposable income, $(1 - \overline{\tau})Y$, which increases consumption, C, with a multiplier of $1/\overline{\tau} > 1$. The increase in government spending is paid for by an increase in revenues from the consumption boom that the policy generates.

3.3.2 No distortionary taxes

Let's study what would happen in this economy if there were only lump-sum transfers, that is, if we set $\overline{\tau} = \hat{\tau}_t = 0$ for all t. Given the path of interest rates i_t and government spending g_t, the equilibrium dynamics is determined by

$$\dot{c}_t = \sigma^{-1}\left(i_t - \pi_t - \rho\right),$$
$$\dot{\pi}_t = \rho\pi_t - \kappa\left(\omega_c c_t + \omega_g g_t\right),$$

and the intertemporal budget constraint

$$\int_0^\infty e^{-\rho t}\varsigma_c c_t dt = \int_0^\infty e^{-\rho t}\left[y_t + \sigma\rho\varsigma_d(c_t - c_0)\right] dt.$$

Solving this system as before, we get the following expression for initial consumption

$$c_0 = \int_0^\infty e^{-\rho t}\left[\left(\rho\frac{\overline{\omega}}{\underline{\omega}}\frac{\omega_g}{\omega_c}(1 - e^{\underline{\omega}t}) + \frac{\overline{\omega}}{\underline{\omega}}\frac{\varsigma_g}{\sigma\varsigma_d}\right)g_t - \frac{\rho}{\sigma\underline{\omega}}e^{\underline{\omega}t}i_t\right] dt.$$

We obtain the following comparative statics.

Proposition 20. *The effect of government policy on consumption is summarized by the following:*

1. *There exists $\bar{t} > 0$ such that*

$$\frac{\partial c_0}{\partial g_t} < 0, \quad \forall t < \bar{t};$$

2. *For all $t \geq 0$*

$$\frac{\partial c_0}{\partial i_t} > 0.$$

If there are no distortionary taxes, Assumption 6 is not satisfied (since $\overline{\tau} = 0$). This means that changes in the consumption affect government expenditures through the effect on interest payments but there is no effect on revenues.[5] Therefore, all the results on the effect of government policy are reversed.

3.4 Debt Level and Stabilization Policies

Now, we want to understand how the level of debt affects the effectiveness of policy intervention, that is, the effect of the size of debt on the fiscal multiplier and on the reaction of the economy to monetary shocks. Up to a first-order approximation, one could think of this exercise as the reaction of the government to some exogenous shock, trying to stabilize the economy (for example, close the output gap or lower inflationary pressures) through fiscal and monetary policy. Since up to first-order the responses are linear in the shocks and the change in policy, we

[5] In the presence of sales taxes, revenues change when output changes because of the change of the tax base.

can just focus on the effect of the latter.[6]

Consider two economies that have the same technology, preferences, government spending and distortionary taxes, but they differ in the steady state level of debt. As we showed in Proposition 16, both economies have the same equilibrium allocation in steady state. However, in this section we show that the size of the intervention to achieve a predetermined objective depends on the size of the debt.

Consider first how the fiscal multiplier changes with the level of debt. In the previous analysis we showed that an increase in government spending, g_t, increases initial consumption, c_0, if Assumption 6 is satisfied. This result relies on two reinforcing mechanisms. First, an increase in government spending increases production, which increases the household's income. We illustrated this force showing that this effect is reminiscent to an "old Keynesian" multiplier logic. Second, an increase in government spending increases initial inflation, which reduces the real rate of the economy (for a fixed path of nominal interest rates), introducing a force to front-load consumption.

Now, compare the response of two economies with different levels of steady state debt and the same increase in government spending. It turns out that if $\kappa > 0$, then the fiscal multiplier is lower in the economy with higher debt, that is,

$$\frac{\partial^2 c_0}{\partial g_t \partial \varsigma_d} < 0.$$

In order to understand this result, recall that the household's budget constraint can be written as

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t}(1-\overline{\tau}) y_t dt + \int_0^\infty e^{-\rho t} \sigma \rho \varsigma_d (c_t - c_0) dt.$$

The direct effect of an increase in government spending on household's income is the same in both economies, since this only depends on $\overline{\tau}$ and ς_g. However, the effect of inflation differs in the two economies. As inflation increases, the real rate decreases, so the return on government bonds decreases.[7] This has a negative wealth effect for the household, and this effect is larger the more debt they are holding. Therefore, the increase in consumption in period 0 after an increases in

[6] It is important to note that we are not considering *policy rules* that react to exogenous shocks, but surprise changes in policy that can occur together with other shocks.

[7] Recall that government bonds are short-term. We study how this conclusion changes with the inclusion of long-term bonds in the next section.

government spending is smaller the larger the debt-to-GDP ratio of the economy. Note that this effect works exclusively through inflation. A different way to see this is that if prices were rigid (i.e., $\kappa = 0$), then the fiscal multiplier would not be affected by the size of the debt.

Consider now the effect of monetary policy. In the previous section, we showed that if Assumption 6 is satisfied, an increase in the nominal interest rate reduces consumption in period 0. This effect works exclusively through its effect on the real rate. When the nominal interest rate increases, the real rate increases, generating a substitution of present consumption towards future consumption. Moreover, there is a wealth effect that goes in the opposite direction: as the interest rate increases, the return from government bonds increases.[8] Following this logic, it is immediate to see that if the stock of debt is larger, the wealth effect is stronger, so the offsetting force of the wealth effect is larger. As a result, an increase in the nominal interest rate has a smaller impact in economies with higher debt.

The next proposition summarizes these results.

Proposition 21. *Suppose Assumption 6 holds. Then*

1. *If $\kappa > 0$, the fiscal multiplier is decreasing in the level of debt,*

$$\frac{\partial^2 c_0}{\partial g_t \partial \varsigma_d} < 0.$$

If prices are fully rigid (i.e., $\kappa = 0$), then the fiscal multiplier is independent of the level of debt,

$$\frac{\partial^2 c_t}{\partial g_t \partial \varsigma_d}|_{\kappa=0} = 0.$$

2. *The effect of monetary policy is attenuated with more debt,*

$$\frac{\partial^2 c_0}{\partial i_t \partial \varsigma_d} > 0.$$

3.5 Long Term Bonds and Indexed Debt

So far we have considered an economy in which the only asset the government issues is a short-term (instantaneous) nominal bond. Let's suppose now that the government has the possibility of issuing long-term nominal debt as well.

[8]Once again, note that this effect relies on the fact that government debt is short-term.

The long-term bond is a perpetuity with exponentially decaying coupons. Formally, one unit of the bond at date t corresponds to a promise to pay $e^{-\rho_L(s-t)}$ in nominal terms at every date $s \geq t$. Note that the decaying coupon acts as if the bond "depreciates" at rate ρ_L every period.

The price of the bond is given by

$$q_{L,t} = \int_t^{\infty} e^{-\int_t^s i_z dz} e^{-\rho_L(s-t)} ds = \int_t^{\infty} e^{-\int_t^s (i_z+\rho_L)dz} ds.$$

In differential form, we have

$$\dot{q}_{L,t} = -1 + (i_t + \rho_L) q_{L,t}, \tag{3.10}$$

and rearranging

$$\frac{1}{q_{L,t}} + \frac{\dot{q}_{L,t}}{q_{L,t}} - \rho_L = i_t$$

This equation says that the short term interest rate must equal the return on the long-term bond, which is given by the dividend yield, $1/q_{L,t}$, plus the capital gain, $\frac{\dot{q}_{L,t}}{q_{L,t}} - \rho_L$.

For future reference, note that the duration of the bond is given by

$$D_t = \int_t^{\infty} (s-t) \frac{e^{-\rho_L(s-t)} e^{-\int_t^s i_z dz}}{q_{L,t}} ds = \frac{1}{q_{L,t}} \int_t^{\infty} \frac{e^{-\int_t^s (i_z+\rho_L)dz}}{i_t + \rho_L} ds = \frac{1}{i_t + \rho_L}.$$

Hence, for a given short-term interest i_t, by varying ρ_L from zero to infinity, the duration varies from $1/\rho$ to zero. Therefore, by changing ρ_L we can study how the results change with the "average maturity" of the bond.

If we denote by l_t the issuance of new bond at period t, the evolution of the stock of debt can be written as

$$\dot{B}_{L,t} = -\rho_L B_{L,t} + l_t \tag{3.11}$$

The household's per-period budget constraint can then be written as

$$\dot{B}_{S,t} + q_{L,t} l_t = i_t B_{S,t} + B_{L,t} + W_t N_t + \Pi_t + T_t - P_t C_t.$$

Using the law of motion for the stock of long term debt (3.11) and the law of motion for $q_{L,t}$ (3.10), we get

$$\dot{A}_t = i_t A_t + W_t N_t + \Pi_t + T_t - P_t C_t$$

where

$$A_t \equiv B_{S,t} + q_{L,t} B_{L,t}$$

The term A_t represents the agent's total financial wealth, comprising both the amount invested in the short- and long-term bonds. Since the model is non-stochastic, the return on wealth is simply i_t.

Following an analogous derivation as in the previous sections, we obtain the intertemporal budget constraint

$$D_0^g = \int_0^\infty e^{-\rho t} \left(\frac{C_t}{C_0}\right)^{-\sigma} \left[\tau_t Y_t - G_t - \overline{T}_t\right] dt \tag{3.12}$$

where $D_0^g = A_0 = B_{S,0} + q_{L,0} B_{L,0}$. Importantly, the initial debt now depends on the price of the long-term bond. This is the only difference with the previous model.

In steady state, the debt-to-GDP ratio is now given by

$$\varsigma_d = \varsigma_S + \frac{\varsigma_L}{\rho + \rho_L},$$

where ς_S is the fraction of debt that is short-term, and ς_L is the *face value* of the long-term debt, divided by GDP. In order to get the *fraction* of debt that is long-term, we need to multiply ς_L by the steady state price of long-term debt, which is given by $\frac{1}{\rho+\rho_L}$. This difference is going to be important below, when we analyze the role of long-term debt in this economy. The following result provides the benchmark for this economy with long-term bonds.

Proposition 22. *The composition of debt is irrelevant for the steady state allocation.*

This result is saying two things. First, as before, the steady state *level* of debt, ς_d, is irrelevant for the steady state allocation. Second, the maturity of debt, that is, the fraction of debt that is short-term, ς_S, and the fraction of long-term debt, $\frac{\varsigma_L}{\rho+\rho_L}$, does not affect the steady state allocation.

The log-linearized budget constraint can be written as

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t} \left[(1-\overline{\tau})(y_t - \hat{\tau}_t) + \sigma \rho \varsigma_d (c_t - c_0)\right] dt + d_0^g \varsigma_d.$$

Let's log-linearize the expression for the initial government liabilities:

$$d_0^g = \log\left[\frac{B_{S,0} + q_{L,0}B_{L,0}}{\overline{D}^g}\right] = \log\left[\frac{\overline{B}_S}{\overline{D}^g}e^{b_{S,0}} + \frac{\overline{q}_L\overline{B}_L}{\overline{D}^g}e^{\hat{q}_{L,0}+b_{L,0}}\right]$$
$$\approx \varsigma_S b_{S,0} + \varsigma_L\left[\hat{q}_{L,0} + b_{L,0}\right]$$

And log-linearizing the price of the long term bond:

$$\hat{q}_{L,t} = \log\left[q_{L,t}/\overline{q}_L\right] = \log\left[(\rho+\rho_L)\int_t^\infty e^{-\int_t^s (i_z+\rho_L)dz}ds\right]$$
$$= -\int_t^\infty e^{-(\rho+\rho_L)s}(i_s-\rho)ds$$

Plugging these expressions into the budget constraint, we get

$$\int_0^\infty e^{-\rho t}c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})(y_t-\hat{\tau}_t) + \sigma(c_t-c_0)\rho\varsigma_d\right]dt +$$
$$+\left[\varsigma_S b_{S,0} + \varsigma_L\left(b_{L,0} - \int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt\right)\right]\varsigma_d.$$

Assuming, as before, that the deviation of the initial value of debt, both short- and long-term, is zero, we get

$$\int_0^\infty e^{-\rho t}c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})(y_t-\hat{\tau}_t) + \sigma\rho\varsigma_d(c_t-c_0)\right]dt +$$
$$-\varsigma_L\int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt\varsigma_d.$$

Hence, the budget constraint has an extra term that depends on the nominal interest rate, i_t, the fraction of long-term bonds, ς_L, and a measure of the bonds' duration, ρ_L.

Solving the new system of equations we get

$$c_0 = \int_0^\infty e^{-\rho t}\left[\chi_{g,t}g_t + \chi_{\tau,t}\hat{\tau}_t + \chi_{i,t}(i_t-\rho)\right]dt - \frac{\overline{\omega}\varsigma_L\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d}\int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt,$$

where $\chi_{g,t}$, $\chi_{\tau,t}$ and $\chi_{i,t}$ are defined as in Proposition 17. Hence, the presence of long-term bonds only affect the effect of monetary shocks. The next proposition states the result of this section.

Proposition 23. *Suppose Assumption 2 holds. Then*

1. *The fiscal multiplier is independent of the fraction of long-term debt in the economy, that is,*

$$\frac{\partial^2 c_0}{\partial g_t \partial \varsigma_L} = 0.$$

2. *The effect of monetary policy increases with the fraction of long-term debt, that is,*

$$\frac{\partial^2 c_0}{\partial i_t \partial \varsigma_L} < 0.$$

Moreover, the effect of monetary policy increases with the bond duration

$$\frac{\partial^2 c_t}{\partial i_t \partial \rho_L} > 0.$$

3.5.1 Indexed Bonds

Suppose the government is allowed to issue *indexed* debt. In particular, suppose the government has also the chance of issuing long-term indexed bonds. The price of the bond is given by

$$q_{I,t} = \int_t^\infty e^{-\int_t^s (i_z - \pi_z)dz} e^{-\rho_L(s-t)} ds = \int_t^\infty e^{-\int_t^s (i_z - \pi_z + \rho_L)dz} ds$$

A similar derivation as before shows that the initial value of government liabilities can be written as

$$d_0^g = B_{S,0} + q_{I,0} B_{I,0}.$$

In log-linear terms, we have

$$d_0^g \approx \varsigma_S b_{S,0} + \varsigma_I \left[\hat{q}_{I,0} + b_{I,0}\right]$$

The log-linear expression for the indexed bond is

$$\begin{aligned}\hat{q}_{I,t} &= \log\left[q_{I,t}/\bar{q}_L\right] = \log\left[(\rho + \rho_L)\int_t^\infty e^{-\int_t^s (i_z - \pi_z + \rho_L)dz} ds\right] \\ &\approx -\int_t^\infty e^{-(\rho+\rho_L)s}(i_s - \pi_z - \rho)ds\end{aligned}$$

The log-linear budget constraint can then be written as

$$c_0 = \int_0^\infty e^{-\rho t} \left[\chi_{g,t} g_t + \chi_{\tau,t} \hat{\tau}_t + \chi_{i,t}(i_t - \rho)\right] dt - \frac{\overline{\omega}\varsigma_I \varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d} \int_0^\infty e^{-(\rho+\rho_I)t}(i_t - \pi_t - \rho)dt,$$

where we set $b_{S,0} = b_{I,0} = 0$.

Using the Euler equation to replace $\sigma\dot{c}_t = i_t - \pi_t - \rho$, we can write the term involving the real interest rate as

$$\begin{aligned}\int_0^\infty e^{-(\rho+\rho_I)t}\sigma\dot{c}_t dt &= -\sigma c_0 + \sigma(\rho+\rho_I)\int_0^\infty e^{-(\rho+\rho_I)t} c_t dt \\ &= \frac{\sigma\underline{\omega}_e}{\overline{\omega}_e + \rho_I} c_0 + \sigma(\rho+\rho_I)\int_0^\infty e^{-(\rho+\rho_I)t}\left(c_t^r + c_t^f\right) dt,\end{aligned}$$

were c_t^f and c_t^r are defined as in Proposition 17. We can express initial consumption as

$$c_0 = \int_0^\infty e^{-\rho t} \left[\tilde{\chi}_{g,t} g_t + \tilde{\chi}_{\tau,t} \hat{\tau}_t + \tilde{\chi}_{i,t}(i_t - \rho)\right] dt,$$

where the coefficients are given by

$$\begin{aligned}\tilde{\chi}_{g,t} &= \left[(\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d)\frac{\omega_g}{\omega_c}(1 - e^{\underline{\omega}t}) - \kappa\omega_g\varsigma_I\varsigma_d(\rho+\rho_I)\frac{e^{-\rho_I t} - e^{\underline{\omega}t}}{(\overline{\omega}+\rho_I)(\underline{\omega}+\rho_I)} + \varsigma_g(1-\overline{\tau})\right]\Omega, \\ \tilde{\chi}_{\tau,t} &= \left[(\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d)\frac{1}{\omega_c}(1 - e^{\underline{\omega}t}) - \kappa\varsigma_I\varsigma_d(\rho+\rho_I)\frac{e^{-\rho_I t} - e^{\underline{\omega}t}}{(\overline{\omega}+\rho_I)(\underline{\omega}+\rho_I)} - \varsigma_g(1-\overline{\tau})\right]\Omega, \\ \tilde{\chi}_{i,t} &= -\left[\frac{\overline{\tau}\varsigma_c - \sigma\rho\varsigma_d}{\sigma\overline{\omega}} e^{\underline{\omega}t} + (\rho+\rho_I)\varsigma_I\varsigma_d\frac{\rho_I e^{-\rho_I t} + \underline{\omega}e^{\underline{\omega}t}}{(\overline{\omega}+\rho_I)(\underline{\omega}+\rho_I)}\right]\Omega\end{aligned}$$

where

$$\Omega \equiv \frac{\overline{\omega}(\overline{\omega}+\rho_I)}{(\overline{\omega}+\rho_I)(\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d) + \varsigma_I\varsigma_d\sigma\underline{\omega}\overline{\omega}}.$$

Indexed bonds adjust their price with the inflation rate. Therefore, an increase in government spending now has an impact on prices, which occurs through its effect on the *real* rate. In particular, this channel increases the fiscal multiplier. The increase in inflation associated to government spending *increases* the nominal price of indexed bonds. Therefore, consumption increases by more than if bonds were nominal or short-term. Interestingly, in the particular case in which the indexed bonds are consols, a higher fraction of indexed bonds is mathematically equivalent to a reduction in the debt-to-gdp ratio when debt is short-term, with respect to its impact on the effectiveness of government policy. The next proposition summarizes these results.

Proposition 24. *Suppose Assumption 2 holds. Then*

1. *Fiscal and monetary policy are more effective in economies with more indexed debt,*

$$\frac{\partial c_0^2}{\partial g_t \partial \varsigma_I} > 0, \quad \frac{\partial^2 c_0}{\partial i_t \partial \varsigma_I} < 0;$$

2. *If $\rho_L = 0$, then*

$$\frac{\partial c_0}{\partial g_t}|_{\varsigma_I = \overline{\varsigma}_I, \varsigma_d = \overline{\varsigma}_d} = \frac{\partial c_0}{\partial g_t}|_{\varsigma_I = 0, \varsigma_d = (1-\overline{\varsigma}_I)\overline{\varsigma}_d}.$$

$$\frac{\partial c_0}{\partial i_t}|_{\varsigma_I = \overline{\varsigma}_I, \varsigma_d = \overline{\varsigma}_d} = \frac{\partial c_0}{\partial i_t}|_{\varsigma_I = 0, \varsigma_d = (1-\overline{\varsigma}_I)\overline{\varsigma}_d}.$$

3.6 A Liquidity Trap

In this section we study how the level of debt shapes the dynamics of an economy in a liquidity trap. We do this in stages. We first study the response of an economy to a preference shock. Then, we consider a preference shock large enough to make the natural rate to be negative for some time.

3.6.1 Preference Shock

Suppose the household is subject to a preference shock, where the discount rate between dates t and s is now given by $e^{-\rho(t-s)+(\xi_t - \xi_s)}$. The equilibrium dynamics can now be described by the following system:

$$\dot{c}_t = \sigma^{-1}\left(i_t - \pi_t - \rho + \dot{\xi}_t\right)$$
$$\dot{\pi}_t = \rho\pi_t - \kappa\left(\omega_c c_t + \omega_g g_t + \hat{\tau}_t\right)$$
$$\int_0^\infty e^{-\rho t}\varsigma_c c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})(y_t - \hat{\tau}_t) + \sigma\rho\varsigma_d(c_t - c_0) + \dot{\xi}_t\right] dt$$

By defining $\tilde{i}_t \equiv i_t + \dot{\xi}_t$ the system involving inflation and consumption is analogous to the one solved above, so the solution is analogous to the one in the previous sections. We can write initial consumption as the following function of disturbances:

$$c_0 = \int_0^\infty e^{-\rho t} \left(\chi_{g,t} g_t + \chi_{\tau,t} \hat{\tau}_t + \chi_{i,t} (i_t - \rho) + \chi_{\xi,t} \dot{\xi}_t \right) dt,$$

where $\chi_{g,t}$, $\chi_{\tau,t}$ and $\chi_{i,t}$ are defined as in Proposition 17, and

$$\chi_{\xi,t} \equiv \chi_{i,t} - \frac{\overline{\omega}\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} = -\sigma^{-1} \frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d} e^{\underline{\omega}t} - \frac{\overline{\omega}\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} < 0.$$

The following proposition characterizes the reaction of the economy to a preference shock.

Proposition 25. *Suppose Assumption 6 holds. Then*

1. *The preference shock is contractionary,*

$$\frac{\partial c_0}{\partial \dot{\xi}_t} < 0;$$

2. *The effect of the preference shock is exacerbated with more debt,*

$$\frac{\partial^2 c_0}{\partial \dot{\xi}_t \partial \varsigma_d} |_{\kappa > 0} < 0, \quad \frac{\partial^2 c_0}{\partial \dot{\xi}_t \partial \varsigma_d} |_{\kappa = 0} = 0.$$

In order to understand this result, let's start by studying the case in which prices are fully rigid. In this case, the real rate of the economy is determined by the nominal rate. Hence, as long as the central bank keeps the nominal interest rate fixed, the fiscal cost of debt does not change after the shock, so the household's wealth does not change. Therefore, a reduction in the discount factor has the only effect of increasing the slope of the consumption path, while keeping the present value of the consumption path fixed. This can only happen if initial consumption, c_0, decreases. And since there is no wealth effect, the reaction of the economy to the preference shock is independent of the level of debt.

When prices are not fully rigid, a preference shock generates inflation in this model. As a result, the real rate of return decreases, generating a negative wealth effect. And since this negative effect is increasing in the level of government debt, more indebted economy will suffer a deeper drop in initial consumption and a larger recession.

Note that the preference shock enters the household's Euler equation in a analogous way as the nominal interest; however, they have different effects. Both, preference and monetary shocks, have similar impacts on the timing of the path of consumption. However, while monetary shocks are attenuated with higher debt, preference shocks are exacerbated in high debt economies. The reason is that preference shocks do not have the wealth effect that comes from higher rate of return on debt.

3.6.2 A Liquidity Trap Exercise

Consider a shock that pushes the economy into a liquidity trap. Suppose that $\dot{\xi}_t = \Delta > \rho$ for $t \in [0, T]$ and $\dot{\xi}_t = 0$ for $t > T$. Suppose the central bank sets the nominal interest rate to zero for the duration of the trap. After that, it sets the interest rate to be equal to the discount rate. Specifically, assume that the nominal interest is $i_t = 0$ for $t \in [0, T]$ and $i_t = \rho$ for $t > T$.

Consumption at period 0 is then given by

$$c_0 = \int_0^T e^{-\rho t} \left(-\chi_{i,t}\rho + \chi_{\xi,t}\Delta \right) dt.$$

Solving the integral, we get

$$c_0 = -\sigma^{-1} \frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d} \frac{1 - e^{-\overline{\omega}t}}{\overline{\omega}} (\Delta - \rho) - \frac{\overline{\omega}\varsigma_d}{\overline{\tau}\varsigma_c - \sigma\underline{\omega}\varsigma_d} \frac{1 - e^{-\rho t}}{\rho} \Delta < 0.$$

The next proposition establishes that the recession caused by a liquidity trap is more severe in economies with higher debt.

Proposition 26. *Suppose Assumption 6 holds. Suppose there is a preference shock $\Delta > \rho$ from $t = 0$ to $t = T$ and zero afterwards, and the central bank sets the interest rate to zero until T and ρ afterwards. Then, initial consumption decreases. Moreover, consumption drops by more in an economy with higher debt.*

3.7 Conclusion

This paper presented a model in which the level, maturity structure and characteristics of government debt affects the severity of crises and the effectiveness of stabilization policies. We show that fiscal and monetary policy are less powerful in high debt economies; monetary policy is more effective in economies with a larger share of long-term debt; stabilization policies are more powerful in economies with a larger share of indexed bonds; and a liquidity trap generated by a preference shock is deeper and more prolonged in high-debt economies.

Our analysis opens the room for a deeper analysis of the role of debt in shaping monetary and fiscal policy. Some questions for future work are: should monetary policy be implemented differently depending on the debt level of a country? should fiscal policy have a precautionary motive, where fiscal policy respond less strongly in order to avoid future debt increases? how should debt management policy be designed depending on the different shocks the economy is subject to? These questions will require an approach that takes into account the non-linearities of the model.

3.8 Appendix

3.8.1 Model's Equilibrium Conditions

Demand Block

Household optimization

Demand for each variety is given by

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} C_t$$

where

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$$

The consumer's budget constraint can now be rewritten as

$$\dot{B}_t = i_t B_t + W_t N_t + \Pi_t + \hat{T}_t - P_t C_t \tag{3.13}$$

We can now obtain an expression for aggregate demand. The total demand for good j can be written as

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} (C_t + G_t)$$

Define $Y_t \equiv \left[\int_0^1 Y_t(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$, then aggregate demand is given by

$$Y_t = C_t + G_t$$

The household's problem can then be written as

$$\max_{[C_t, N_t, B_t]_{t=0}^{\infty}} \left[\int_0^{\infty} e^{-\rho t} \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}\right] dt\right]$$

subject to (3.13) and the No-Ponzi condition.

The (current-value) Hamiltonian is given by

$$\mathcal{H}_t = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} + \mu_t \left[i_t B_t + W_t N_t + \Pi_t + \hat{T}_t - P_t C_t \right]$$

The first-order conditions are

$$\begin{aligned}
\mathcal{H}_{C,t} &= 0 \Rightarrow C_t^{-\sigma} = \mu_t P_t \\
\mathcal{H}_{N,t} &= 0 \Rightarrow N_t^{\phi} = \mu_t W_t \\
\mathcal{H}_{B,t} &= \rho\mu_t - \dot{\mu}_t \Rightarrow i_t - \rho = -\frac{\dot{\mu}_t}{\mu_t}
\end{aligned}$$

and the transversality condition:

$$\lim_{t\to\infty} e^{-\rho t} \mu_t B_t = 0$$

Combining the conditions above, we get

$$\begin{aligned}
C_t^{\sigma} N_t^{\phi} &= \frac{W_t}{P_t} \\
\frac{\dot{C}_t}{C_t} &= \sigma^{-1} \left(i_t - \pi_t - \rho \right) \\
\lim_{t\to\infty} e^{-\int_0^t i_s ds} B_t &= 0
\end{aligned}$$

Note that aggregate nominal profits are given by

$$\begin{aligned}
\Pi_t = \int_0^1 \Pi_t(j) dj = &\int_0^1 \left[(1-\tau_t) P_t(j) Y_t(j) - (1+\tau_t^W) W_t N_t(j) \right] dj \\
=&(1-\tau_t) P_t Y_t - (1+\tau_t^W) W_t N_t
\end{aligned}$$

Hence, the dynamic budget constraint and the transversality condition combined are equivalent to the intertemporal budget constraint for the household:

$$B_t = \int_t^{\infty} e^{-\int_t^s i_z dz} \left[P_s C_s - (1-\tau_s) P_s Y_s + \tau_t^W W_t N_t - \hat{T}_s \right] ds$$

Government Solvency

Combining the intertemporal budget constraint for the household and market clearing condition, we obtain:

$$D_t^g = \int_t^\infty e^{-\int_t^s i_z dz} P_s \left[\tau_s Y_s + \tau_s^W C_s^\sigma N_s^{1+\phi} - G_s - T_s\right] ds$$

where $T_t \equiv \hat{T}_t / P_t$ denote transfers in real terms.

Using the Euler equation to eliminate the interest rate, we obtain

$$\frac{C_t^{-\sigma} D_t^g}{P_t} = \int_t^\infty e^{-\rho(s-t)} C_s^{-\sigma} \left[\tau_s Y_s + \tau_s^W C_s^\sigma N_s^{1+\phi} - G_s - T_s\right] ds$$

Define real government debt (in utility terms) as $d_t^g \equiv C_t^{-\sigma} D_t^g / P_t$. We can then write the constraint as:

$$\dot{d}_t^g = \rho d_t^g + C_t^{-\sigma}\left[G_t + T_t - \tau_s^W C_s^\sigma N_s^{1+\phi} - \tau_t Y_t\right]$$
$$d_0^g = C_0^{-\sigma} D_0^g / P_0$$

Supply Block

Let's derive now the optimal price choice for a domestic producer. The first order condition is given by

$$\mathbb{E}_t \left[\int_0^\infty e^{-(\rho+\rho_\delta)s} C_{t+s}^{-\sigma} P_{t+s}^{-1} \left[(1-\tau_{t+s}) \frac{P_t^*(j)^{-\epsilon}}{P_{t+s}^{-\epsilon}} Y_{t+s} - \frac{\varphi\epsilon}{\epsilon-1}(1+\tau_{t+s}^W) W_{t+s} \frac{P_t^*(j)^{-\varphi\epsilon-1}}{P_{t+s}^{-\varphi\epsilon}} \left(\frac{Y_{t+s}}{A_{t+s}}\right)^\varphi\right] ds\right] = 0$$

where we used the Euler equation to eliminate the nominal interest rate.

Rearranging expression above, we get

$$\left(\frac{P_t^*(j)}{P_t}\right)^{1-\epsilon} \left[\int_0^\infty e^{-\int_t^{t+s} r_{f,z} dz} (1-\tau_{t+s}) C_{t+s}^{-\sigma} Y_{t+s} ds\right] =$$
$$\left(\frac{P_t^*(j)}{P_t}\right)^{-\varphi\epsilon} \left[\int_0^\infty e^{-\int_t^{t+s} r_{k,z} dz} \frac{\varphi\epsilon}{\epsilon-1} C_{t+s}^{-\sigma} (1+\tau_{t+s}^W) \frac{W_{t+s}}{P_{t+s}} \left(\frac{Y_{t+s}}{A_{t+s}}\right)^\varphi ds\right]$$

where

$$r_{f,t} \equiv \rho + \rho_\delta - (\epsilon-1)\pi_t; \qquad r_{k,t} \equiv \rho + \rho_\delta - \varphi\epsilon\pi_t$$

We can isolate the term $P_t^*(j)/P_t$ and write it as the ratio of two integrals:

$$\frac{P_t^*(j)}{P_t} = \left(\frac{K_t}{F_t}\right)^{\frac{1}{1+\epsilon(\varphi-1)}}$$

where

$$K_t \equiv \int_t^{\infty} e^{-\int_t^s r_{k,z} dz} \frac{\varphi\epsilon}{\epsilon-1}(1+\tau_s^W) N_s^{\phi} \left(\frac{Y_s}{A_s}\right)^{\varphi} ds \tag{3.14}$$

$$F_t \equiv \int_t^{\infty} e^{-\int_t^s r_{f,z} dz}(1-\tau_s) C_s^{-\sigma} Y_s ds \tag{3.15}$$

Equivalently, we can write K_t and F_t in recursive form:[9]

$$\dot{K}_t = r_{k,t} K_t - \frac{\varphi\epsilon}{\epsilon-1}(1+\tau_s^W) N_t^{\phi} \left(\frac{Y_t}{A_t}\right)^{\varphi} \tag{3.16}$$

$$\dot{F}_t = r_{f,t} F_t - (1-\tau_t) C_t^{-\sigma} Y_t \tag{3.17}$$

This allow us to obtain the relative price for price-setters at period t. Note that all firms who can set their price at period t will choose the same price. We can then write $P_t^*(j) = P_t^*$.

Applying an appropriate law of large numbers, it is possible to show that the aggregate price level is an average of prices set in different periods:

$$P_t = \left(\int_0^{\infty} \rho_\delta e^{-\rho_\delta s} (P_{t-s}^*)^{1-\epsilon} ds\right)^{\frac{1}{1-\epsilon}} \iff P_t^{1-\epsilon} = \int_{-\infty}^{t} \rho_\delta e^{-\rho_\delta (t-s)} (P_s^*)^{1-\epsilon} ds$$

Differentiating the expression above, we get

$$(1-\epsilon) P_t^{1-\epsilon} \frac{\dot{P}_t}{P_t} = \rho_\delta (P_t^*)^{1-\epsilon} - \rho_\delta P_t^{1-\epsilon}$$

rearranging

$$\pi_t = \frac{\rho_\delta}{\epsilon-1}\left[1 - \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\epsilon(\varphi-1)}}\right]$$

Expression above is the non-linear version of the New Keynesian Phillips Curve.

[9] If K_t and F_t satisfy (3.14) and (3.15), respectively, then they also satisfy (3.16) and (3.17). Conversely, if K_t and F_t satisfy (3.16) and (3.17), respectively, and a boundary condition, then they also satisfy (3.14) and (3.15).

Equilibrium

The market clearing condition for good j imply

$$N_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon\varphi} \left(\frac{Y_t}{A_t}\right)^{\varphi}$$

Integrating over j, we obtain

$$N_t = \Delta_t \left(\frac{Y_t}{A_t}\right)^{\varphi}$$

where

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon\varphi} dj$$

Applying an exact law of large numbers, we can express the price dispersion Δ_t as

$$\Delta_t \equiv \frac{1}{P_t^{-\epsilon\varphi}} \int_{-\infty}^{t} \rho_\delta e^{-\rho_\delta(t-s)} (P_s^*)^{-\epsilon\varphi} ds$$

Differentiating with respect to time:

$$\dot{\Delta}_t = \epsilon\varphi\pi_t\Delta_t + \rho_\delta \left(\frac{P_t^*}{P_t}\right)^{-\epsilon\varphi} - \rho_\delta\Delta_t$$

rearranging

$$\dot{\Delta}_t = -(\rho_\delta - \epsilon\varphi\pi_t)\Delta_t + \rho_\delta \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon\varphi}{1-\epsilon(1-\varphi)}}$$

Equivalently, we can write the expression above in terms of the domestic inflation:

$$\dot{\Delta}_t = -(\rho_\delta - \epsilon\varphi\pi_t)\Delta_t + \rho_\delta \left[1 - \frac{\epsilon - 1}{\rho_\delta}\pi_t\right]^{\frac{\epsilon\varphi}{\epsilon-1}}$$

3.8.2 Non-linear problem

The equilibrium conditions can be grouped in three different blocks:

The aggregate supply block

$$\pi_t = \frac{\rho_\delta}{\epsilon - 1}\left[1 - \left(\frac{F_t}{K_t}\right)^{\frac{\epsilon-1}{1+\epsilon(\varphi-1)}}\right] \tag{3.18}$$

$$\dot{K}_t = (\rho + \rho_\delta - \varphi\epsilon\pi_t)\, K_t - \frac{\varphi\epsilon}{\epsilon - 1}(1 + \tau_t^W) N_t^\phi \left(\frac{Y_t}{A_t}\right)^\varphi \tag{3.19}$$

$$\dot{F}_t = (\rho + \rho_\delta - (\epsilon - 1)\pi_t)\, F_t - (1 - \tau_t) C_t^{-\sigma} Y_t \tag{3.20}$$

$$N_t = \left(\frac{Y_t}{A_t}\right)^\varphi \Delta_t \tag{3.21}$$

$$\dot{\Delta}_t = -(\rho_\delta - \epsilon\varphi\pi_t)\Delta_t + \rho_\delta\left[1 - \frac{\epsilon - 1}{\rho_\delta}\pi_t\right]^{\frac{\epsilon\varphi}{\epsilon-1}} \tag{3.22}$$

where Δ_0 is given and K_0, F_0 are given by (3.14) and (3.15), respectively, evaluated at $t = 0$.

The aggregate demand block

$$Y_t = C_t + G_t \tag{3.23}$$

$$\frac{\dot{C}_t}{C_t} = \sigma^{-1}(i_t - \pi_t - \rho) \tag{3.24}$$

$$i_t \geq 0 \tag{3.25}$$

where the last constraint is the zero lower bound.

Government solvency condition is given by

$$\dot{d}_t^g = \rho d_t^g + C_t^{-\sigma}\left[G_t + \overline{T}_t - \tau_t^W C_t^\sigma N_t^{1+\phi} - \tau_t Y_t\right] \tag{3.26}$$

$$d_0^g = \frac{C_0^{-\sigma} D_0^g}{P_0} \tag{3.27}$$

where D_0^g and P_0 are given.

In order to solve for an equilibrium, we need to determine 8 process given the 8 conditions above.[10]

[10]The computation of equilibrium takes as given the sequence of fiscal variables $[G_t, \tau_t]_{t=0}^\infty$ and the process for the exogenous variables.

Steady state solution

We will look for a stationary solution of the problem above, i.e., fiscal variables are constant: $G_t = \overline{G}$, $\tau_t = \overline{\tau}$, and $d_t^g = d^g$; constant inflation and nominal interest rate: $\pi_t = 0$, $i_t = \rho$, $\Delta_t = A_t = 1$, and constant consumption $C_t = \overline{C}$.

The equations in the aggregate supply block gives us

$$\begin{aligned}
\overline{K} &= \frac{\varphi\epsilon}{\epsilon-1}(1+\overline{\tau}^W)\frac{\overline{N}^{1+\phi}}{\rho+\rho_\delta} \\
\overline{F} &= \frac{1-\overline{\tau}}{\rho+\rho_\delta}\overline{C}^{-\sigma}\overline{Y} \\
\overline{F} &= \overline{K} \\
\overline{N} &= \overline{Y}^{\varphi}
\end{aligned}$$

combining the first three conditions, we get

$$\overline{Y} = \left[\frac{(1-\overline{\tau})(\epsilon-1)}{\varphi\epsilon(1+\overline{\tau}^W)}\overline{C}^{-\sigma}\right]^{\frac{1}{\varphi-1+\varphi\phi}} = \left[\frac{(1-\overline{\tau})(\epsilon-1)}{\varphi\epsilon(1+\overline{\tau}^W)}\varsigma_c^{-\sigma}\right]^{\frac{1}{\varphi(1+\phi)-1+\sigma}} \tag{3.28}$$

The demand block gives us

$$\overline{Y} = \overline{C} + \overline{G} \tag{3.29}$$

Combining (3.29) and (3.28), we obtain the steady state value of consumption (given $\overline{G}$ and $\overline{\tau}$).

Government debt is given by

$$\overline{d}^g = \frac{1}{\rho}\left[\overline{\tau}\overline{Y} + \overline{\tau}^W\overline{C}^{\sigma}\overline{N}^{1+\phi} - \overline{T} - \overline{G}\right]\overline{C}^{-\sigma}$$

We will focus below in the case of an efficient steady state, so $\overline{\tau}^W$ will be chosen to eliminate the monopoly distortion:

$$\overline{\tau}^W = (1-\overline{\tau})(1-\epsilon^{-1}) - 1 < 0$$

Part of the lump-sum tax will be used to finance this subsidy:

$$\overline{T}^W = \overline{\tau}^W \overline{C}^\sigma \overline{N}^{1+\phi}$$

Therefore, government debt is given by

$$\varsigma_d = \frac{1}{\rho} \left[\overline{\tau} - \varsigma_g - \overline{T} \right]$$

3.8.3 Approximate Solution

Let's consider a first-order approximation for the equilibrium conditions.

First-order approximation

Let's take a first order approximation of the aggregate supply block. Consider first the evolution of the price dispersion:

$$\dot{\hat{\Delta}}_t = -(\rho_\delta - \epsilon\varphi\pi_{H,t}) + \rho_\delta \left[1 - \frac{\epsilon - 1}{\rho_\delta} \pi_t \right]^{\frac{\epsilon\varphi}{\epsilon - 1}} e^{-\hat{\Delta}_t}$$

where $\hat{\Delta}_t \equiv \log \Delta_t$.

Taking a first-order approximation, we get

$$\dot{\hat{\Delta}}_t = -\rho_\delta \hat{\Delta}_t$$

Hence, if we start at $\hat{\Delta}_0 = 0$, then $\hat{\Delta}_t = 0$ up to first-order. This is the usual result that price dispersion have only second-order effects.

The employment condition can be written as

$$n_t = \varphi(y_t - a_t) + \hat{\Delta}_t \Rightarrow n_t = \varphi(y_t - a_t)$$

using the fact that $\hat{\Delta}_t$ is second-order and the definitions $n_t \equiv \log N_t/\overline{N}$, $y_t \equiv \log Y_t/\overline{Y}$, $a_t \equiv \log A_t$.

The evolution of the state variables K_t and F_t can be written:

$$\dot{k}_t = \rho + \rho_\delta - \varphi\epsilon\pi_t - (\rho+\rho_\delta)e^{\hat{\tau}_t^W + \phi n_t + \varphi(y_t - a_t) - k_t}$$
$$\dot{f}_t = \rho + \rho_\delta - (\epsilon - 1)\pi_t - (\rho+\rho_\delta)e^{-\hat{\tau}_t - \sigma c_t + y_t - f_t}$$

where $k_t \equiv \log K_t/\overline{K}$, $f_t \equiv \log F_t/\overline{F}$, $\hat{\tau}_t \equiv -\log(1-\tau_t)/(1-\overline{\tau})$, $\hat{\tau}_t^W = \log(1+\tau_t^W)/(1+\overline{\tau}^W)$ $c_t \equiv \log C_t/\overline{C}$.

The first-order approximation gives us

$$\dot{k}_t = (\rho+\rho_\delta)k_t - \varphi\epsilon\pi_t - (\rho+\rho_\delta)\left[\hat{\tau}_t^W + (\phi+1)\, n_t\right]$$
$$\dot{f}_t = (\rho+\rho_\delta)f_t - (\epsilon-1)\pi_t - (\rho+\rho_\delta)\left(-\hat{\tau}_t - \sigma c_t + y_t\right)$$

We can write expression (3.18) as

$$\pi_t = \frac{\rho_\delta}{\epsilon-1} - \frac{\rho_\delta}{\epsilon-1}\exp\left[\left(\frac{\epsilon-1}{1+\epsilon(\varphi-1)}\right)(f_t - k_t)\right]$$

Taking a first order approximation, we get

$$\pi_t = \frac{\rho_\delta}{1+\epsilon(\varphi-1)}(k_t - f_t)$$

Differentiating with respect to time, we obtain

$$\dot{\pi}_t = \frac{\rho_\delta}{1+\epsilon(\varphi-1)}(\dot{k}_t - \dot{f}_t)$$

The difference $\dot{k}_t - \dot{f}_t$ is given by

$$\begin{aligned}\dot{k}_t - \dot{f}_t &= (\rho+\rho_\delta)(k_t - f_t) - (1+\epsilon(\varphi-1))\pi_t - (\rho+\rho_\delta)\left[(\phi+1)\, n_t + \hat{\tau}_t + \hat{\tau}_t^W + \sigma c_t - y_t\right] \\ &= \frac{\rho}{\rho_\delta}(1+\epsilon(\varphi-1))\pi_t - (\rho+\rho_\delta)\left[(\varphi(\phi+1)-1)\, y_t - \varphi(\phi+1)a_t + \hat{\tau}_t + \hat{\tau}_t^W + \sigma c_t\right]\end{aligned}$$

The evolution of inflation is then given by

$$\dot{\pi}_t = \rho\pi_t - \frac{(\rho+\rho_\delta)\rho_\delta}{1+\epsilon(\varphi-1)}\left[(\varphi(\phi+1)-1)\, y_t - \varphi(\phi+1)a_t + \hat{\tau}_t + \hat{\tau}_t^W + \sigma c_t\right] \quad (3.30)$$

The Euler equation is given by

$$\dot{c}_t = \sigma^{-1} (i_t - \pi_t - \rho)$$

Consider now the aggregate demand equation:

$$e^{y_t} = \frac{\overline{C}}{\overline{Y}} e^{c_t} + \frac{\overline{G}}{\overline{Y}} e^{g_t}$$

Approximating this equation, we obtain

$$y_t = \varsigma_c c_t + \varsigma_g g_t$$

where $\varsigma_c \equiv \overline{C}/\overline{Y}$, $\varsigma_g \equiv \overline{G}/\overline{Y}$, $g_t \equiv \log G_t/\overline{G}$.

Eliminating output from (3.30), we get

$$\dot{\pi}_t = \rho\pi_t - \kappa \left(\omega_c c_t + \omega_g g_t + \hat{\tau}_t + \hat{\tau}_t^W + u_t \right)$$

where

$$\begin{aligned}
\omega_c &\equiv (\varphi (\phi + 1) - 1) \varsigma_c + \sigma \\
\omega_g &\equiv (\varphi (\phi + 1) - 1) \varsigma_g \\
u_t &\equiv -\varphi (\phi + 1) a_t
\end{aligned}$$

Let's consider now government solvency:

$$\dot{\hat{d}}_t^g = \rho - \frac{\rho}{\overline{\tau} - \varsigma_g} \exp \left(-\sigma c_t - \hat{d}_t^g + \log \left(e^{y_t} - (1 - \overline{\tau}) e^{y_t - \hat{\tau}_t} - \varsigma_g e^{g_t} + \varphi^{-1} ((1 + \overline{\tau}^W) e^{\hat{\tau}_t^W} - 1) e^{\sigma c_t + \varphi(1+\phi) y_t} - (\hat{T}_t + \varphi^{-1} \overline{\tau}_t^W) \right) \right)$$

where we used the definition $\hat{T}_t \equiv (T_t - \overline{T})/\overline{Y}$ and the fact that

$$\frac{\overline{YC}^{-\sigma}}{\overline{d}^g} = \frac{\rho}{\overline{\tau} - \varsigma_g}$$

The first order approximation is given by

$$\dot{\hat{d}}_t^g = \rho \hat{d}_t^g + \rho \left(\sigma c_t + \frac{\varsigma_g g_t + \hat{T}_t - \overline{\tau} y_t - (1 - \overline{\tau}) \hat{\tau}_t - \varphi^{-1} \overline{\tau}^W (\sigma c_t + \varphi(1 + \phi) y_t) - \varphi^{-1} (1 + \overline{\tau}^W) \hat{\tau}_t^W}{\overline{\tau} - \varsigma_g} \right)$$

If we want to ignore the effects of the taxes on labor, we can assume the following condition holds:

$$\hat{T}_t - \varphi^{-1}\overline{\tau}^W(\sigma c_t + \varphi(1+\phi)y_t) - \varphi^{-1}(1+\overline{\tau}^W)\hat{\tau}_t^W = 0$$

In this case, the evolution of government debt is

$$\dot{\hat{d}}_t^g = \rho \hat{d}_t^g + \rho\left(\sigma c_t + \frac{\varsigma_g g_t - \overline{\tau} y_t - (1-\overline{\tau})\hat{\tau}_t}{\overline{\tau} - \varsigma_g}\right)$$

The level of government debt is given by

$$\hat{d}_t^g = -\sigma c_t + \hat{D}_t^g - p_t$$

where we defined $\hat{D}_t^g \equiv \log D_t^g / \overline{D}^g$, $p_t \equiv \log P_t$.

Equilibrium Dynamics

Given the path of interest rates i_t, government spending g_t, and tax rates τ_t, equilibrium dynamics is determined by

$$\dot{c}_t = \sigma^{-1}\left(i_t - \pi_t - \rho\right)$$
$$\dot{\pi}_t = \rho\pi_t - \kappa\left(\omega_c c_t + \omega_g g_t + \tau_t + u_t\right)$$

and the intertemporal budget constraint:

$$\int_0^\infty e^{-\rho t}\varsigma_c c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})\left(y_t - \tau_t\right) + \sigma\left(c_t - c_0\right)\rho\varsigma_d\right] + \varsigma_d d_0^g. \tag{3.31}$$

The system above can be written in matrix form:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{\tilde{c}}_t \end{bmatrix} = \begin{bmatrix} \rho & -\hat{\kappa} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{c}_t \end{bmatrix} + \begin{bmatrix} f_t \\ r_t \end{bmatrix}$$

where

$$\hat{\kappa} \equiv \kappa\omega_c\sigma^{-1}$$
$$\tilde{c}_t \equiv \sigma c_t$$
$$f_t \equiv -\kappa\left(\omega_g g_t + \hat{\tau}_t + u_t\right)$$
$$r_t \equiv i_t - \rho$$

Let the eigenvalues of the coefficient matrix be denoted by

$$\overline{\omega} = \frac{\rho + \sqrt{\rho^2 + 4\hat{\kappa}}}{2}; \qquad \underline{\omega} = \frac{\rho - \sqrt{\rho^2 + 4\hat{\kappa}}}{2}.$$

The matrix of coefficients can be decomposed as

$$\begin{bmatrix} \rho & -\hat{\kappa} \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\overline{\omega}^{-1} & -\underline{\omega}^{-1} \end{bmatrix} \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -\overline{\omega}^{-1} & -\underline{\omega}^{-1} \end{bmatrix}^{-1}.$$

Note that $\overline{\omega} + \underline{\omega} = \rho$, $\overline{\omega}\underline{\omega} = -\hat{\kappa}$, $\overline{\omega} - \underline{\omega} = \sqrt{\rho^2 + 4\hat{\kappa}}$, and that if prices are rigid, i.e. $\kappa = 0$, then $\underline{\omega} = 0$.

Define the following transformation of our original variables

$$Z_t = \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} \equiv \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} \begin{bmatrix} -\underline{\omega}^{-1} & -1 \\ \overline{\omega}^{-1} & 1 \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{c}_t \end{bmatrix}$$

The system in the new coordinates can be written as

$$\begin{bmatrix} \dot{Z}_{1,t} \\ \dot{Z}_{2,t} \end{bmatrix} = \begin{bmatrix} \overline{\omega} & 0 \\ 0 & \underline{\omega} \end{bmatrix} \begin{bmatrix} Z_{1,t} \\ Z_{2,t} \end{bmatrix} + \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix}$$

where

$$\begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix} \equiv \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} \begin{bmatrix} -\underline{\omega}^{-1} & -1 \\ \overline{\omega}^{-1} & 1 \end{bmatrix} \begin{bmatrix} f_t \\ r_t \end{bmatrix}$$

Solving the decoupled system, we obtain

$$Z_{1,t} = Z_{1,T}e^{-\overline{\omega}(T-t)} - \int_t^T e^{-\overline{\omega}(s-t)}\eta_{1,s}ds,$$
$$Z_{2,t} = Z_{2,T}e^{-\underline{\omega}(T-t)} - \int_t^T e^{-\underline{\omega}(s-t)}\eta_{2,s}ds.$$

Since we are focusing on bounded solutions, we can solve the first equation forward and the second backward to get

$$Z_{1,t} = -\int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds,$$
$$Z_{2,t} = Z_{2,0}e^{\underline{\omega}t} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds.$$

In terms of the original variables, we have

$$\begin{bmatrix}\pi_t \\ \tilde{c}_t\end{bmatrix} = \begin{bmatrix}1 & 1 \\ -\overline{\omega}^{-1} & -\underline{\omega}^{-1}\end{bmatrix}\begin{bmatrix}-\int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds \\ Z_{2,0}e^{\underline{\omega}t} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds\end{bmatrix},$$

or

$$\pi_t = Z_{2,0}e^{\underline{\omega}t} + \int_0^t e^{\underline{\omega}(t-s)}\eta_{2,s}ds - \int_t^\infty e^{-\overline{\omega}(s-t)}\eta_{1,s}ds,$$
$$\tilde{c}_t = -\frac{Z_{2,0}e^{\underline{\omega}t}}{\underline{\omega}} - \int_0^t \frac{e^{\underline{\omega}(t-s)}\eta_{2,s}}{\underline{\omega}} + \int_t^\infty \frac{e^{-\overline{\omega}(s-t)}\eta_{1,s}}{\overline{\omega}}ds.$$

Evaluating in $t = 0$ we get

$$\pi_0 = Z_{2,0} - \int_0^\infty e^{-\overline{\omega}t}\eta_{1,t}dt,$$
$$\tilde{c}_0 = -\frac{Z_{2,0}}{\underline{\omega}} + \int_0^\infty \frac{e^{-\overline{\omega}t}\eta_{1,t}}{\overline{\omega}}dt,$$

and therefore, we can rewrite the system as

$$\pi_t = \pi_0 e^{\underline{\omega}t} + e^{\underline{\omega}t}\int_0^t \left(e^{-\underline{\omega}s}\eta_{2,s} + e^{-\overline{\omega}s}\eta_{1,s}\right)ds - \left(e^{\overline{\omega}t} - e^{\underline{\omega}t}\right)\int_t^\infty e^{-\overline{\omega}s}\eta_{1,s}ds,$$
$$\tilde{c}_t = \tilde{c}_0 e^{\underline{\omega}t} - e^{\underline{\omega}t}\int_0^t \left(\frac{e^{-\underline{\omega}s}\eta_{2,s}}{\underline{\omega}} + \frac{e^{-\overline{\omega}s}\eta_{1,s}}{\overline{\omega}}\right)ds + \frac{e^{\overline{\omega}t} - e^{\underline{\omega}t}}{\overline{\omega}}\int_t^\infty e^{-\overline{\omega}s}\eta_{1,s}ds.$$

Writing the system in terms of the original shocks, we obtain

$$c_t = e^{\underline{\omega}t}c_0 + c_t^r + c_t^f, \tag{3.32}$$
$$\pi_t = e^{\underline{\omega}t}\pi_0 + \pi_t^r + \pi_t^f, \tag{3.33}$$

where

$$c_t^r = \frac{\hat{\kappa}}{\sigma(\overline{\omega} - \underline{\omega})} \left[e^{\underline{\omega} t} \int_0^t \left(\frac{e^{-\overline{\omega} s}}{\overline{\omega}} - \frac{e^{-\underline{\omega} s}}{\underline{\omega}} \right) r_s ds - \frac{e^{\overline{\omega} t} - e^{\underline{\omega} t}}{\overline{\omega}} \int_t^\infty e^{-\overline{\omega} s} r_s ds \right],$$

$$c_t^f = \frac{1}{\sigma(\overline{\omega} - \underline{\omega})} \left[e^{\underline{\omega} t} \int_0^t \left(e^{-\underline{\omega} s} - e^{-\overline{\omega} s} \right) f_s ds + \left(e^{\overline{\omega} t} - e^{\underline{\omega} t} \right) \int_t^\infty e^{-\overline{\omega} s} f_s ds \right],$$

$$\pi_t^r = \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} \left[e^{\underline{\omega} t} \int_0^t \left(e^{-\underline{\omega} s} - e^{-\overline{\omega} s} \right) r_s ds + \left(e^{\overline{\omega} t} - e^{\underline{\omega} t} \right) \int_t^\infty e^{-\overline{\omega} s} r_s ds \right],$$

$$\pi_t^f = \frac{\hat{\kappa}}{\overline{\omega} - \underline{\omega}} \left[e^{\underline{\omega} t} \int_0^t \left(\frac{e^{-\underline{\omega} s}}{\overline{\omega}} - \frac{e^{-\overline{\omega} s}}{\underline{\omega}} \right) f_s ds + \left(e^{\overline{\omega} t} - e^{\underline{\omega} t} \right) \int_t^\infty \frac{e^{-\overline{\omega} s}}{\underline{\omega}} f_s ds \right].$$

It remains to determine c_0 and π_0. Plugging (3.32) in the budget constraint (3.31), we get

$$c_0 = -\frac{\overline{\omega} \varsigma_d}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} \left[\rho \int_0^\infty e^{-\rho t} \left[\left(\frac{\overline{\tau} \varsigma_c}{\overline{\tau} - \varsigma_g} - \sigma \right) \left(c_t^r + c_t^f \right) + \frac{1 - \overline{\tau}}{\overline{\tau} - \varsigma_g} (\tau_t - \varsigma_g g_t) \right] dt - d_0^g \right].$$

Let's compute the integral terms. First, the term involving the fiscal shocks is equal to

$$\int_0^\infty e^{-\rho t} c_t^f = \frac{1}{\sigma \hat{\kappa}} \int_0^\infty \left(e^{-\rho t} - e^{-\overline{\omega} t} \right) f_t dt.$$

Second, the term involving interest rate shocks is equal to

$$\int_0^\infty e^{-\rho t} c_t^r dt = \int_0^\infty \frac{e^{-\overline{\omega} t}}{\sigma \overline{\omega}} r_t dt.$$

The intertemporal budget constraint can then be written as

$$c_0 = -\frac{\overline{\omega} \varsigma_d}{\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d} \left[\rho \int_0^\infty e^{-\rho t} \left[\left(\frac{\overline{\tau} \varsigma_c}{\overline{\tau} - \varsigma_g} - \sigma \right) \left(\frac{e^{\underline{\omega} t}}{\sigma \overline{\omega}} r_t + \frac{1 - e^{\underline{\omega} t}}{\sigma \hat{\kappa}} f_t \right) + \frac{1 - \overline{\tau}}{\overline{\tau} - \varsigma_g} (\tau_t - \varsigma_g g_t) \right] dt - d_0^g \right],$$

or

$$c_0 = \int_0^\infty e^{-\rho t} \left(\chi_{g,t}^c g_t + \chi_{\tau,t}^c \tau_t + \chi_{u,t}^c u_t + \chi_{r,t}^c r_t \right) dt - \frac{\overline{\omega}(\overline{\tau} - \varsigma_g)}{\rho(\overline{\tau} \varsigma_c - \underline{\omega} \sigma \varsigma_d)} d_0,$$

where

$$\chi^c_{g,t} = \overline{\omega}\frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d}\frac{\omega_g}{\omega_c}(1 - e^{\underline{\omega}t}) + \overline{\omega}\frac{1-\overline{\tau}}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d}\varsigma_g,$$
$$\chi^c_{\tau,t} = \overline{\omega}\frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d}\frac{1}{\omega_c}(1 - e^{\underline{\omega}t}) - \overline{\omega}\frac{1-\overline{\tau}}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d},$$
$$\chi^c_{r,t} = -\sigma^{-1}\frac{\overline{\tau}\varsigma_c - \rho\sigma\varsigma_d}{\overline{\tau}\varsigma_c - \underline{\omega}\sigma\varsigma_d}e^{\underline{\omega}t}.$$

3.8.4 A Model with Long Term Debt

Following an analogous derivation to the one in the case with only one asset, we obtain the intertemporal budget constraint Using the Euler equation to eliminate the interest rate, we obtain

$$\frac{C_t^{-\sigma}D_t^g}{P_t} = \int_t^\infty e^{-\rho(s-t)}C_s^{-\sigma}\left[\tau_s Y_s + \tau_s^W C_s^\sigma N_s^{1+\phi} - G_s - \overline{T}_s\right] ds \tag{3.34}$$

where $D_t^g = A_t = B_{S,t} + q_{L,t}B_{L,t}$.

Defining total real government debt (in utility terms) as $d_t^g \equiv C_t^{-\sigma}D_t^g/P_t$, we can then write the constraint as:

$$\dot{d}_t^g = \rho d_t^g + C_t^{-\sigma}\left[G_t + \overline{T}_t - \tau_s^W C_s^\sigma N_s^{1+\phi} - \tau_t Y_t\right] \tag{3.35}$$

$$d_0^g = \frac{C_0^{-\sigma}}{P_0}\left[B_{S,0} + q_{L,0}B_{L,0}\right] \tag{3.36}$$

In steady state, the total value of government debt is given by

$$\frac{\overline{C}^\sigma}{\overline{Y}}\overline{d}^g = \frac{1}{\rho}\left[\overline{\tau} - \varsigma_g\right] = \overline{b}_S + \frac{1}{\rho+\rho_L}\overline{b}_L \tag{3.37}$$

where $\overline{b}_S \equiv \overline{B}_S/\overline{Y}$ is the ratio of short term debt to GDP in steady state and similarly $\overline{b}_L \equiv \overline{B}_L/\overline{Y}$ and the steady state price of the long term bond:

$$\overline{q}_L = \frac{1}{\rho+\rho_L} \tag{3.38}$$

The log-linearized budget constraint can be written as[11]

$$\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty e^{-\rho t}\left[(1-\overline{\tau})(y_t - \hat{\tau}_t) + \sigma(c_t - c_0)\rho\overline{D}\right] dt + d_0^g \overline{D}$$

Let's log-linearize the expression for the initial government liabilities:

$$\begin{aligned}\hat{D}_0^g &= \log\left[\frac{B_{S,0} + q_{L,0}B_{L,0}}{\overline{D}^g}\right] = \log\left[\frac{\overline{B}_S}{\overline{D}^g} e^{b_{S,0}} + \frac{\overline{q}_L \overline{B}_L}{\overline{D}^g} e^{\hat{q}_{L,0} + b_{L,0}}\right] \\ &\approx \varsigma_S b_{S,0} + \varsigma_L\left[\hat{q}_{L,0} + b_{L,0}\right] \end{aligned} \tag{3.39}$$

Log-linearizing the price of the long term bond:

$$\begin{aligned}\hat{q}_{L,t} &= \log\left[q_{L,t}/\overline{q}_L\right] = \log\left[(\rho + \rho_L)\int_t^\infty e^{-\int_t^s (i_z + \rho_L) dz} ds\right] \\ &\approx -(\rho + \rho_L)\int_t^\infty e^{-(\rho+\rho_L)s}\int_t^s (i_z - \rho) dz ds \\ &= -(\rho + \rho_L)\int_t^\infty \int_z^\infty e^{-(\rho+\rho_L)s}(i_z - \rho) ds dz \qquad (3.40) \\ &= -\int_t^\infty e^{-(\rho+\rho_L)s}(i_s - \rho) ds \qquad (3.41)\end{aligned}$$

Plugging these expressions into the budget constraint, we get

$$\begin{aligned}\int_0^\infty e^{-\rho t} c_t dt = \int_0^\infty & e^{-\rho t}\left[(1-\overline{\tau})(y_t - \hat{\tau}_t) + \sigma(c_t - c_0)\rho\overline{D}\right] dt + \\ & + \left[\varsigma_S b_{S,0} + \varsigma_L\left(b_{L,0} - \int_0^\infty e^{-(\rho+\rho_L)t}(i_t - \rho) dt\right)\right]\overline{D}\end{aligned}$$

We can rearrange this expression to obtain,

$$\begin{aligned}\frac{1}{\overline{\omega}_e}\left[\overline{\tau} - \underline{\omega}_e \sigma \overline{D}\right] c_0 = &-\frac{\overline{\tau} - \sigma\rho\overline{D}}{\sigma}\int_0^\infty e^{-\rho t}\left[\tilde{c}_t^r + \tilde{c}_t^f\right] dt + \int_0^\infty e^{-\rho t}(1-\overline{\tau})(g_t - \hat{\tau}_t) dt \\ &- \varsigma_L \overline{D}\int_0^\infty e^{-(\rho+\rho_L)t}(i_t - \rho) dt + \left[\varsigma_S b_{S,0} + \varsigma_L b_{L,0}\right]\overline{D} \end{aligned} \tag{3.42}$$

Using the fact that

$$\int_0^\infty e^{-\rho t}\tilde{c}_t^r dt = \int_0^\infty e^{-\rho t}\frac{e^{\underline{\omega}_e t}}{\overline{\omega}_e} r_t dt \tag{3.43}$$

[11] Note the slight change in notation compared to rest of these notes. It is consistent with the presentation notation.

we can compute the derivative

$$\frac{\partial c_0}{\partial i_t} = -e^{-\rho t} \frac{(\overline{\tau} - \sigma\rho\overline{D})e^{\underline{\omega}_e t} + \sigma\overline{\omega}_e \varsigma_L \overline{D} e^{-\rho_L t}}{\overline{\tau} - \underline{\omega}_e \sigma \overline{D}} \frac{1}{\sigma} < 0 \tag{3.44}$$

Note that the monetary policy becomes more powerful the higher the share of long term bonds or the larger the duration of those bonds:

$$\frac{\partial^2 c_t}{\partial \varsigma_L \partial i_t} < 0; \qquad \frac{\partial^2 c_t}{\partial \rho_L \partial i_t} > 0; \tag{3.45}$$

Indexed Bonds

Suppose now the government has also the chance of issuing long-term *indexed* bonds.

The price of the indexed bond is given by

$$q_{I,t} = \int_t^\infty e^{-\int_t^s (i_z - \pi_z) dz} e^{-\rho_L (s-t)} ds = \int_t^\infty e^{-\int_t^s (i_z - \pi_z + \rho_L) dz} ds \tag{3.46}$$

A similar derivation as above show the initial value of government liability can be written as

$$d_0^g = \frac{C_0^{-\sigma}}{P_0} \left[B_{S,0} + q_{L,0} B_{L,0} + q_{I,0} B_{I,0} \right] \tag{3.47}$$

In log-linear terms, we have

$$\hat{D}_0^g \approx \varsigma_S b_{S,0} + \varsigma_L \left[\hat{q}_{L,0} + b_{L,0} \right] + \varsigma_I \left[\hat{q}_{I,0} + b_{I,0} \right] \tag{3.48}$$

The log-linear expression for the indexed bond is

$$\begin{aligned} \hat{q}_{I,t} &= \log\left[q_{I,t} / \overline{q}_L \right] = \log\left[(\rho + \rho_L) \int_t^\infty e^{-\int_t^s (i_z - \pi_z + \rho_L) dz} ds \right] \\ &\approx - \int_t^\infty e^{-(\rho + \rho_L) s} (i_s - \pi_z - \rho) ds \end{aligned} \tag{3.49}$$

The log-linear budget constraint can then be written as

$$\frac{1}{\overline{\omega}_e}\left[\overline{\tau}-\underline{\omega}_e\sigma\overline{D}\right]c_0=-\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}\int_0^\infty e^{-\rho t}\left[\tilde{c}_t^r+\tilde{c}_t^f\right]dt+\int_0^\infty e^{-\rho t}(1-\overline{\tau})(g_t-\hat{\tau}_t)dt-$$
$$\varsigma_L\overline{D}\int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt-\varsigma_I\overline{D}\int_0^\infty e^{-(\rho+\rho_I)t}(i_t-\pi_t-\rho)dt+\left[\varsigma_S b_{S,0}+\varsigma_L b_{L,0}+\varsigma_I b_{I,0}\right]\overline{D}$$

Using the Euler equation, we can write the term involving the real interest rate as

$$\begin{aligned}\int_0^\infty e^{-(\rho+\rho_I)t}\sigma\dot{c}_t dt &= -\sigma c_0+\sigma(\rho+\rho_I)\int_0^\infty e^{-(\rho+\rho_I)t}c_t dt\\ &= -\sigma c_0+\sigma(\rho+\rho_I)\int_0^\infty e^{-(\rho+\rho_I)t}\left(\frac{\tilde{c}_t^r+\tilde{c}_t^f}{\sigma}+c_0 e^{\underline{\omega}_e t}\right)dt\\ &= \frac{\sigma\underline{\omega}_e}{\overline{\omega}_e+\rho_I}c_0+\sigma(\rho+\rho_I)\int_0^\infty e^{-(\rho+\rho_I)t}\left(\frac{\tilde{c}_t^r+\tilde{c}_t^f}{\sigma}\right)dt\end{aligned} \tag{3.50}$$

We can express the budget constraint as

$$\frac{1}{\overline{\omega}_e}\left[\overline{\tau}-\underline{\omega}_e\sigma\left(1-\frac{\overline{\omega}_e}{\overline{\omega}_e+\rho_L}\varsigma_I\right)\overline{D}\right]c_0=-\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}\int_0^\infty e^{-\rho t}\left[\tilde{c}_t^r+\tilde{c}_t^f\right]dt+\int_0^\infty e^{-\rho t}(1-\overline{\tau})(g_t-\hat{\tau}_t)dt$$
$$-\varsigma_L\overline{D}\int_0^\infty e^{-(\rho+\rho_L)t}(i_t-\rho)dt-\varsigma_I\overline{D}\int_0^\infty e^{-(\rho+\rho_I)t}(i_t-\pi_t-\rho)dt+\left[\varsigma_S b_{S,0}+\varsigma_L b_{L,0}+\varsigma_I b_{I,0}\right]\overline{D}$$

where

$$\chi_{g,t}=\omega_g\left[\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}+(\rho+\rho_I)\varsigma_I\overline{D}e^{-\rho_I t}\right](1-e^{\underline{\omega}_e t})+(1-\overline{\tau}) \tag{3.51}$$

$$\chi_{\tau,t}=\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}(1-e^{\underline{\omega}_e t})-(1-\overline{\tau}) \tag{3.52}$$

$$\chi_{u,t}=\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}(1-e^{\underline{\omega}_e t}) \tag{3.53}$$

$$\chi_{r,t}=-\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}\frac{e^{\underline{\omega}_e t}}{\overline{\omega}_e}-\varsigma_L e^{-\rho_L t}\overline{D} \tag{3.54}$$

(3.55)

Note that

$$\int_0^\infty e^{-(\rho+\rho_I)t}\tilde{c}_t^r dt = \int_0^\infty \frac{e^{-\rho s}}{\overline{\omega}_e}\left[\frac{\overline{\omega}_e}{\overline{\omega}_e+\rho_I}e^{-\rho_I s} + \kappa\frac{e^{-\rho_I s}-e^{\underline{\omega}_e s}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)}\right] r_s ds,$$

$$\int_0^\infty e^{-(\rho+\rho_I)t}\tilde{c}_t^f dt = -\int_0^\infty e^{-\rho s}\frac{e^{-\rho_I s}-e^{\underline{\omega}_e s}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)} f_s ds$$

The coefficients are now given by

$$\chi_{g,t} = \omega_g\left[\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}(1-e^{\underline{\omega}_e t}) - (\rho+\rho_I)\varsigma_I\overline{D}\frac{e^{-\rho_I t}-e^{\underline{\omega}_e t}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)}\right] + (1-\overline{\tau}) \tag{3.56}$$

$$\chi_{\tau,t} = \left[\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}(1-e^{\underline{\omega}_e t}) - (\rho+\rho_I)\varsigma_I\overline{D}\frac{e^{-\rho_I t}-e^{\underline{\omega}_e t}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)}\right] - (1-\overline{\tau}) \tag{3.57}$$

$$\chi_{u,t} = \left[\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}(1-e^{\underline{\omega}_e t}) - (\rho+\rho_I)\varsigma_I\overline{D}\frac{e^{-\rho_I t}-e^{\underline{\omega}_e t}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)}\right] \tag{3.58}$$

$$\chi_{r,t} = -\left[\frac{\overline{\tau}-\sigma\rho\overline{D}}{\sigma}\frac{e^{\underline{\omega}_e t}}{\overline{\omega}_e} + \frac{(\rho+\rho_I)\varsigma_I\overline{D}}{\overline{\omega}_e}\left[\frac{\overline{\omega}_e}{\overline{\omega}_e+\rho_I}e^{-\rho_I s} + \kappa\frac{e^{-\rho_I s}-e^{\underline{\omega}_e s}}{(\overline{\omega}_e+\rho_I)(\underline{\omega}_e+\rho_I)}\right]\right] - \varsigma_L e^{-\rho_L t}\overline{D} \tag{3.59}$$

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