Multi-Objective Optimization of Arch Bridges

By

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Abstract

Trussed arch bridges are commonly used to attain big spans. They are efficient structures that offer a wide range of geometries, materials, and topologies. This thesis studies the influence of the geometry and topology of arch bridges on both their structural performance relayed by the maximum deflection and their structural weight. Various materials are also considered to calculate the embodied carbon emission and investigate the environmental impact of arch bridges. Gustave Eiffel's Garabit Viaduct is used as a design precedent for this study. 2-D and 3-D parametric models of the arch bridge are realized using Grasshopper [8]. Changing the geometric parameters in addition to the topology enables the investigation of the bridge's performance. The cross sections are automatically optimized in each case. Furthermore, a multi-objective optimization process was run on the bridge to examine the tradeoffs between the deflection and the self-weight. The weight-oriented optimization allows saving more than 60% of the weight compared to the original structure. Analyzing the different resulting designs proves that increasing the depth at the arch's crown and the depth at the base of the arch leads to better deflection results. It also demonstrates that using a denser truss structure leads to a lighter structure.

Keywords: trussed arch bridges, geometry, topology, multi-objective optimization, deflection, self-weight, embodied carbon emission.

Thesis Supervisor: Caitlin T. Mueller

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Moreover, I would like to thank all the family and friends who trusted and encouraged me to be where I am today. Lastly, my special thanks go to my sisters, my brother in law, and most importantly my parents. Coming to MIT would have stayed a dream without their faith in me. They bore sending me away to a different continent when I was only 17 and a different and farther one this year because they believe that realizing my dream is the most important part of my life. During this whole time, they have been supporting me in every possible way. By being here, I hope that I finally realized my biggest dream by making them proud of me.

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Chapter 1: Introduction

1.1 Arch bridges

Trussed arch bridges are structurally efficient and aesthetically appealing structures. Designers usually choose them to span rivers and valleys that surpass 500m. Additionally, arch bridges are attractive architectural structures that have become historical landmarks over the years. Arch bridges have a rich history since they been used by many famous engineers such as Gustave Eiffel and Thomas Telford (Figure 1). This legacy consists of many bridges that were considered cutting-edge at the time because of their innovation, efficiency, and aesthetics. Those bridges still look as impressive today especially since those engineers could attain such complete structures without the new structural computational tools available now

Figure 1 Craigellachie Bridge, United Kingdom - Tomas Telford (1814)

These bridges are exciting and interesting to study because they can have different geometries and topologies and can be made from a wide range of materials such as wrought iron, steel, concrete, etc. However, most of these famous historical structures have not been studied using the new structural computational tools. For this reason, this thesis uses these tools to study and understand those bridges to learn from the past experience and attempt to contribute to future trussed arch bridges.

1.2 Bridges of Gustave Eiffel

Gustave Eiffel (1832-1923) was a famous French engineer mostly known for his design of the Eiffel Tower in Paris (1889) and the internal truss used as a frame for the Statue of Liberty in New York City (1886). Gustave Eiffel was also involved in designing bridges and creating new technologies surrounding them. His contributions include the creation of portable bridges that can be easily assembled and disassembled when needed. Gustave Eiffel designed many bridges some of which are famous while others are less known. Two of Eiffel's famous bridges are Maria Pia Bridge (Figure 2) in Portugal also known as the Douro Bridge (1877) and the Garabit Viaduct in France (1884). [1]

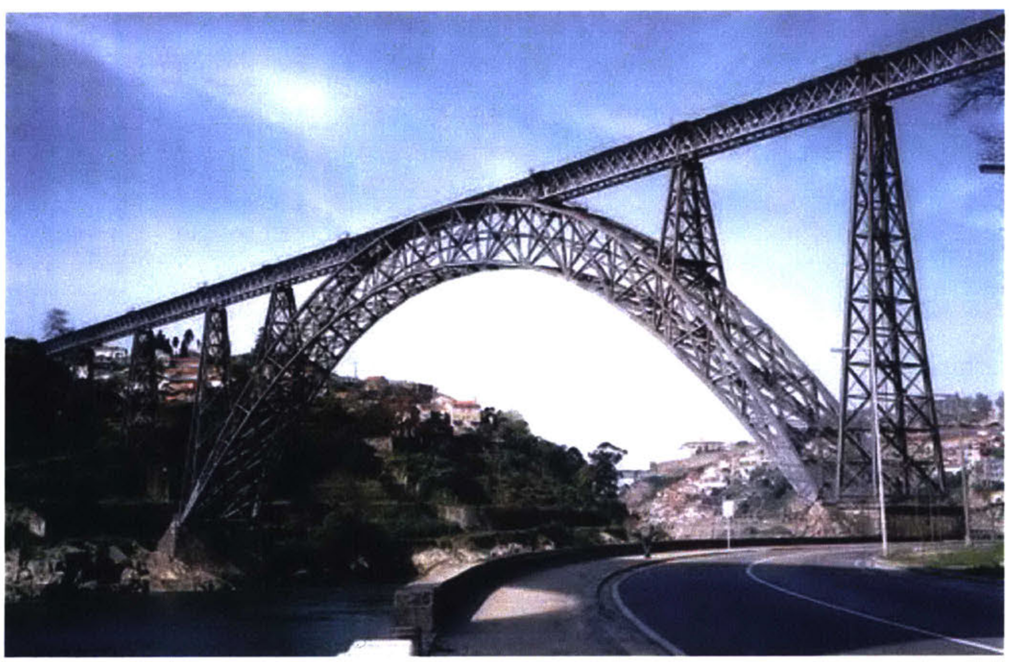


Figure 2 Maria Pia Bridge, Portugal – Gustave Eiffel (1877)

These two famous structures are both arch bridges. This solution was used in both situations to span a deep valley. While the Maria Pia bridge was designed by Eiffel and his partner at the time, Theophile Seyrig, the Garabit Viaduct was the result of the collaboration between Gustave Eiffel and Maurice Koechlin [1,2,5]. These two bridges that look strikingly alike are actually different. The Garabit Viaduct can be considered as an optimized version of the Maria Pia Bridge. The Garabit uses the same concept of the arch bridge but changed some key elements to make the structure more efficient. However, both bridges share one very important detail. Graphic statics was used to find the form of the arch. The forces were thus used to find the form. This leads us to conclude that geometry is a very important part when designing any structure. This thesis investigates the influence of the geometry and topology on the performance and efficiency of arch bridges.

1.3 Thesis scope and focus

This thesis analyzes the behavior of arch bridges if the geometry, topology, or materials were to be changed. The work is divided into two different parts: a 2-D study of a trussed arch and a 3-D study of an arch bridge. A parametric model is realized using Grasshopper [8] enabling the change of geometry and topology in each case to observe the structural performance and self-weight evolution under different values assigned to these parameters. A multi-objective optimization is also run on these models to create a design that satisfies the different objectives: maximum deflection, structural weight, and embodied carbon. However, the one and only optimum solution does not exist as every designer can assign a weight to each of the objectives which will have considerable effect on the resulting design.

Chapter 2: Literature Review

2.1 Historical context: Eiffel's drawings and calculations

In 1875, the Royal Portuguese Railway Company decided to realize a railway between Lisbon and Porto. However, this railroad had to cross over the Douro river which presented many challenges. The soil condition of the Douro river made it difficult and very expensive to resort to building piers inside the water as it represented a very thick layer of sand. So the bridge had to face a 160m span and a valley which is 61m deep while avoiding to use any piers. [2]

G. Eiffel and Cie was up against three other competitors who presented 5 different designs in total but still lost in front of Eiffel and Seyrig's design. The design consists of a two-hinge arch bridge made of wrought iron. Graphic static was used to decide the shape of the arch. The originality, efficiency and cost of this bridge made it an iconic and famous bridge that was considered as a revolution at the time. Gustave Eiffel got assigned the design and construction of the bridge through a very singular bid as he was directly sought by the Léon Boyer, the state engineer in charge of the railway between Marjevols and Neussargues, following the success of the Maria Pia bridge. [2]

The Garabit Viaduct is a one-way railway bridge. It is constructed as the continuity of the railway between Marjevols and Neussargues (figure 3). The bridge is 564.65m long. It is mostly made of wrought iron. However, around 116m is made out of masonry and it consists of both ends of the bridge. [5,6]

Figure 3 Geographical position of the Garabit viaduct

The main challenge that faced the designer was the crossing over the Truyère river. The bridge had to span for 165m over a 122.20m deep valley. This was challenging because of the soil conditions but also since piles in France at the time were never higher than 80m. Indeed, the higher the piles the more the influence of the wind load and it could have made the stability of the bridge in jeopardy. [5,6]

The deck is designed to support one railway only. It uses a very common truss system with an upper and lower horizontal chord linked by vertical beams making various adjacent boxes. In order to add stiffness to the truss, X-shaped beams are added in the middle of each box. However, the deck isn't as ordinary as it seems. Instead of being placed at the top of the deck, the railway was placed 1.66m under the top chord (figure 4). This fairly simple change made the train ride mode secure. On one hand, the surface of the train which is in direct contact with the wind got minimized so the wind load will have less influence. Besides, in case the train deviates from its trajectory it will be supported by the deck and is less likely to fall into the river which would end up being a very tragic accident especially that the Garabit Viaduct was the highest bridge at the time. The deck is supported by 5 piers on the shore. In order to cross the Truyère river, the deck beard on an arch at four different points. [5,6]

Figure 4 The railway positioned far from the top of the deck

The arch (figure 5) is a two-hinge arch that rests on two masonry blocks by a pin connection at the base. It is a trussed arch using the box system and the X-shaped beams to link the corners of each box. The arch is meant to be acting in compression under its self-weight. For this reason, graphic statics was used to make sure that the neutral chord of the arch was very close to the truss line as the latter is supposed to stay inside the arch. As a result, the chords have a parabolic shape with a 165m chord. The intrados has a rise equal to 51.858m. [5,6]

Looking at the bridge from the side, he depth of the arch gets bigger the closer we get to the crown we find the biggest depth of 10m. That part is there mostly to resist the vertical loads such as the self-weight of the bridge or the live load caused by the trains. And the moment resulting from these forces is higher is higher at the crown. Looking at the bridge from the bottom we realize that the depth at the crown is only equal to 6.28m while it gets bigger at the base of the arch to attend 20m. This shape also reflects the shape of the moment diagram. Indeed, the depth of the bridge is mainly resisting the wind load, the moment caused by the wind load is minimal at the crown but maximal at the base of the arch which is why a bigger depth is needed there. [5,6]

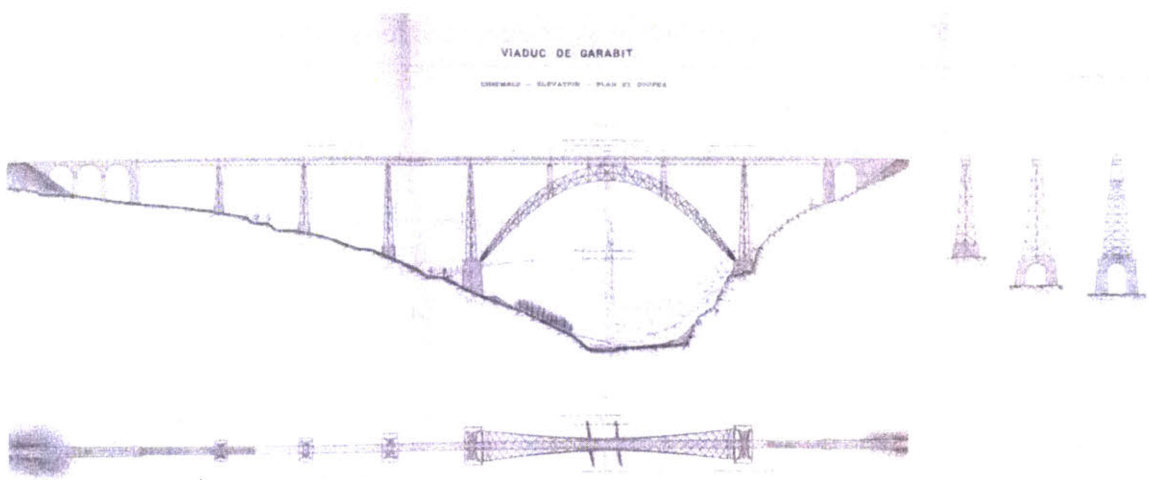


Figure 5 Original drawings of the bridge with dimensions [5]

Aside from having a very sturdy and stable structure, a bridge needs to be maintained in order to keep performing well. The design of the Garabit Viaduct included various access routes for maintenance. At the bottom of the deck, a track was installed to enable the movement of a small wagon that would transport maintenance workers along the bridge. Besides, every pile included an helicoidal ladder for potential visits. [5]

In 1965, the Garabit Viaduct was categorized as a French historical monument. The viaduct is now one of the main touristic attraction in the region of Auvergne-Rhône-Alpes. On January 24th, 2012, a joint nominating process was started in order to include the Garabit and Millau Viaducts in the list of the World Heritage Sites by UNESCO. [7]

The Maria Pia bridge was the reason this viaduct was assigned to the Eiffel company as Boyer contacted Eiffel after hearing about the Maria Pia bridge. If one were to consider the overall aesthetic aspect, the bridges are almost similar with only the position of the deck as difference. However, so many differences exist between the bridges such as the position of the railway, the trussed pier and box beam system, the position of the piers on the arch, etc. This all leads to one conclusion. The Garabit viaduct has a better design than the Maria Pia bridge and it owes nothing but its general concept to it. So there was an optimization process that led to the design of the Garabit Viaduct while taking the Maria Pia bridge as an inspiration. For this reason, new structural design tools should be used more in order to study old bridges and understand the optimization process like the Gaudi bridges for instance.

2.2 Structural optimization for arch bridges

Structural optimization is an approach used broadly in structural engineering research to explore minimal weight or maximal stiffness solutions for structural design. Many papers and journals were published about optimization and the different algorithms used for it. However, only a few documents tackled its application to study arch bridges.

2.2.1 Size optimization (cross sections)

Most papers focusing on the application of optimization on arch bridges consider cross sections are the only design variable. These papers aim at minimizing the structural weight in respect to two constraints. The internal stress in the elements (strength) and the serviceability (deflection) using different algorithms. [12,13]

2.2.1.1 Teaching-learning-base optimization algorithm

This algorithm is used to solve the problem mentioned above. The geometry is fixed and used as an input for the algorithm in addition to a catalogue of cross sections. The algorithm tries to minimize the weight defined by the following function:

$$W(A) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i$$

While W is the structural weight (objective function), A_k is the cross section assigned to each element, ρ_i is the density of the material and L_i is the length of the different elements. [12]

Each iteration of the optimization process has 2 defined phases. The teacher phase where the best design is considered as a reference and then all the other designs are changed to get closer to it using the following formula:

$$X_{new} = X_i + r.(X_{teacher} - T_F . X_{mean})$$

While r and T_F are random parameters equal to 0 or 1. The second phase is called the learner phase. In this phase all the different designs are compared to each other using the following formulas depending on their performance related to the objective function f. Every element X_i is compared to the elements X_j for which $i \neq j$. X_i is moved closer to X_j if $f(X_i) > f(X_j)$ and it is moved farther from it otherwise. [12]

$$X_{new} = X_i + r.(X_j - X_i) \quad if \quad f(X_i) > f(X_j)$$

$$X_{new} = X_i + r.(X_i - X_j) \quad if \quad f(X_i) < f(X_j)$$

This algorithm was applied to two different bridges. On bridge that was considered is Burro Creek Bridge. The bridge was modelled as a 2D trussed arch and the load applied is considered 5713.5 lb/ft for dead and live load to simplify the problem.

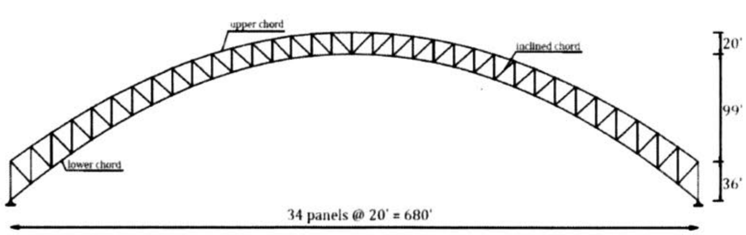


Figure 6 Elevation view of the Burro Creek Bridge [12]

As the bridge is symmetrical, only half of it was considered for the problem (Figure 7). The boundary conditions and loads considered for the Finite Element analysis are represented in the picture below.

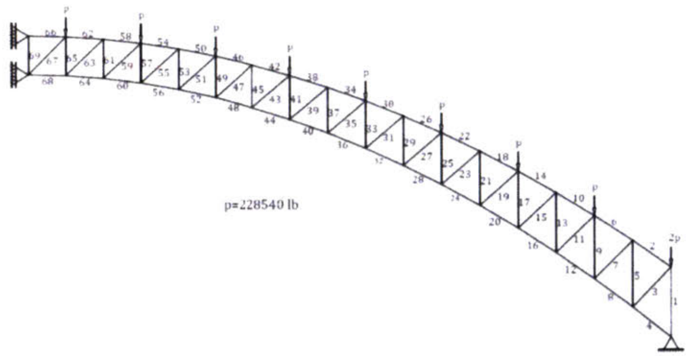


Figure 7 2D finite element model and element numbering [12]

This model is studied for 3 different cases. Each case has a different number of design variables. 4, 8, and 12 variables are considered for cases I, II and III respectively. The results giver by each case are then compared to each other (Figure 8). [12]

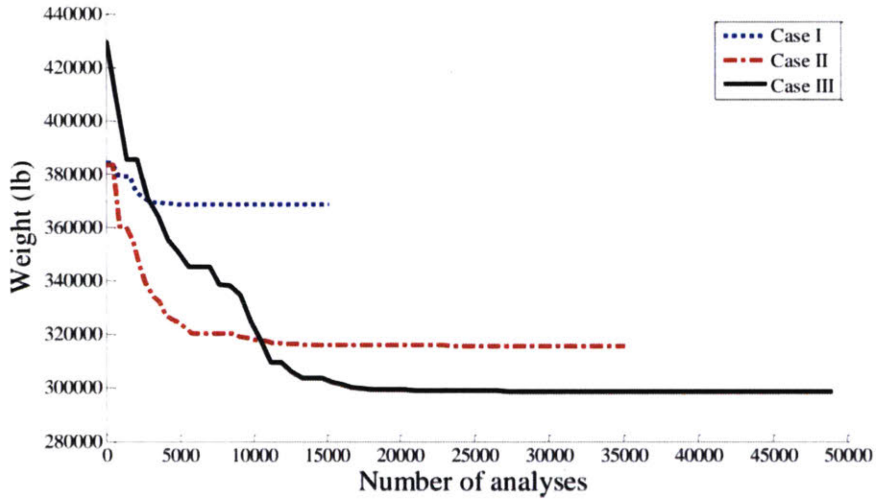


Figure 8 Comparison of the convergence rates for Burro Creek Bridge for three different cases

Even though increasing the number of variables makes the number of analysis needed in order to converge considerably bigger but it generates significant structural weight savings. [12]

2.2.1.2 Hybrid genetic algorithm

Jin Cheng [13] uses a different algorithm to solve the same optimization problem defined above. This optimization method functions according to the next flow chart (Figure 9).

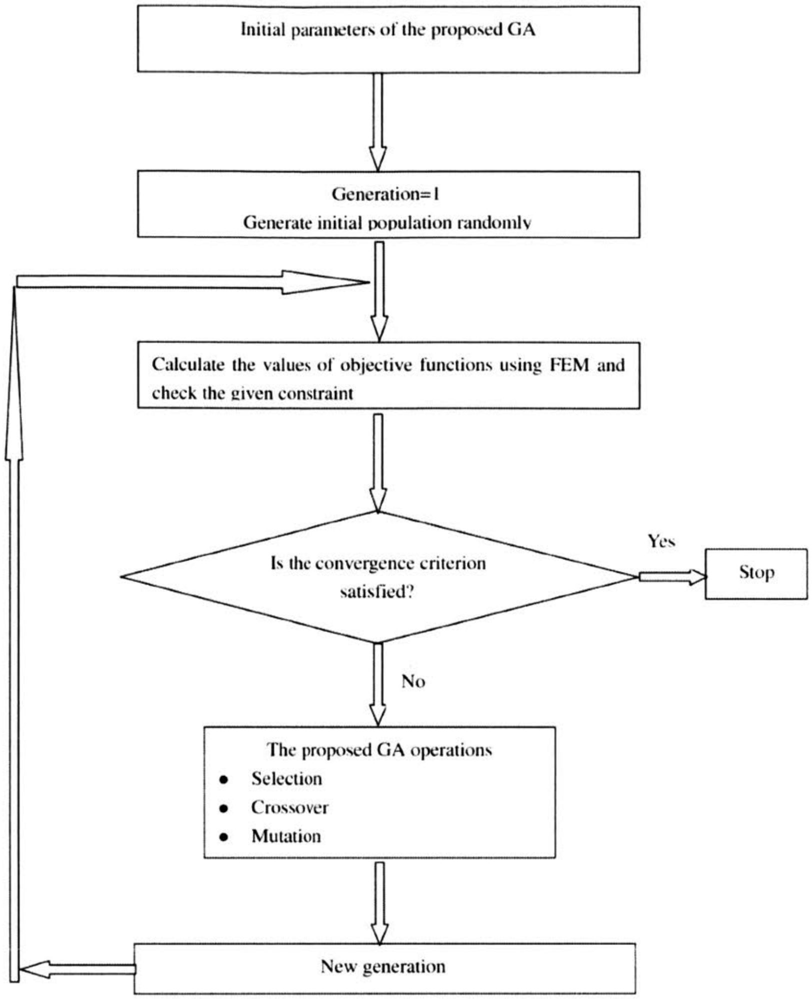


Figure 9 Flow chart of the hybred genetic algorithm based optimization [13]

The convergence criterion used in this study is defined as the maximum number of generations. So at the beginning of the process, the number of population, the penalty parameter, and the maximum number of generations are fixe. For the selection process, two random designs are chosen from the population and compared. If they both satisfy the constraints then the design with the least weight is chosen. If only one design satisfies the constraints then it is automatically chosen. However, if none of the chosen design satisfies the constraints then the closest one to satisfying them is chosen. [13]

An arithmetic crossover is used in this paper. Two parent c_1 and c_2 designs are chosen and generate two offsprings c_1' and c_2' using the following linear combination while w is a randomly-generated number in the interval [0,1]:

$$c_1' = \omega \cdot c_1 + (1 - \omega) \cdot c_2$$
$$c_2' = (1 - \omega) \cdot c_1 + \omega \cdot c_2$$

The paper uses the Chaotianmen Bridge as a study case. The bridge was once again modeled by a 2D truss. [13]

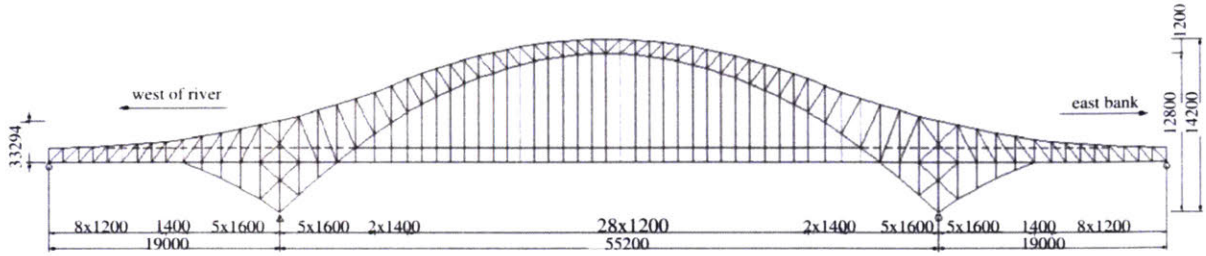


Figure 10 Elevation view of Chaotianmen Bridge (mm) [13]

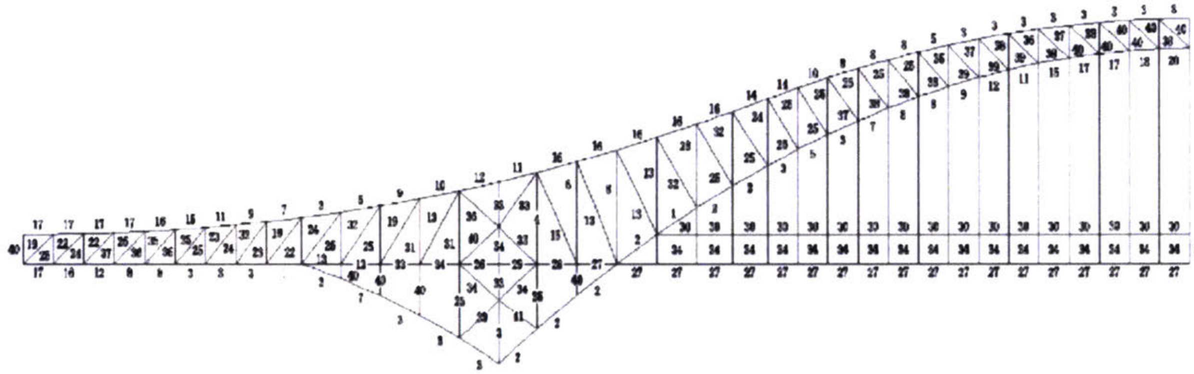


Figure 11 Element numbering (only half the bridge because of symmetry) [13]

The same analysis as earlier is run by using different number of variables

Table 1. Weight of optimal design by number of design variables

	Case I	Case II	Case III
Number of design variables	41	10	4
Total weight (kg)	5215.445	7959.982	9514.766

Once again, having more variables lead to a better result. So in order to get a better result it is recommendable to consider a larger choice of cross sections.

The work that has been done in the field of the optimization of arch bridges focuses the on cross sections only. This thesis is thus trying to explore other characteristics of the bridge such as the geometry and topology as they have the most impact. [13]

2.2.2 Shape/geometry optimization

There is very little information about the geometry optimization of arch bridges. A paper published in 2015 dwells on the optimization of open spandrel arch bridges [14]. This optimization process uses a gradient-based algorithm and is defined in MATLAB.

Cetina River bridge is used as a precedent for this study. The only objective function of this optimization process is the volume of the arch but as it is directly proportional to the structural weight then the objective function is no different than the previous cases. For this paper, there are three different constraints implemented in the MATLAB code. The geometry is added as a constraint in addition to the stress and deflection. Indeed, the arch geometry is being optimized, but the span and rise of the bridge are fixed following the Cetina River bridge dimensions. The arch crown height is thus not allowed to exceed a certain value. [14]

The deck and piers are modelled as fixed line elements in MATLAB while the arch geometry is defined parametrically following the next graphic.

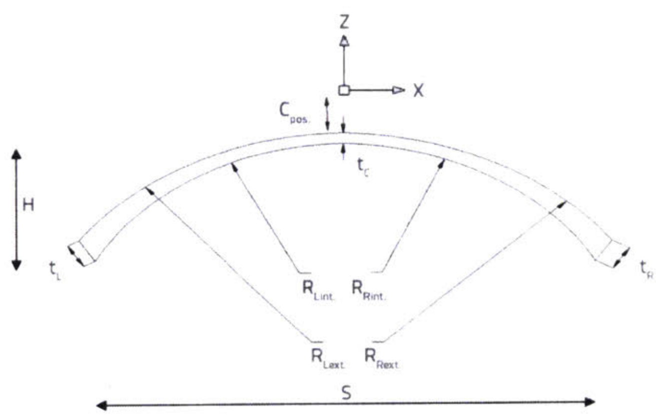


Figure 12 Arch geometry configuration, and definition of shape and design variables [14]

The web thickness is an important factor but was not considered as a design variable for the MATLAB model. The thickness is defined before the algorithm is run. This was done for different thicknesses in order to find the optimal solution.

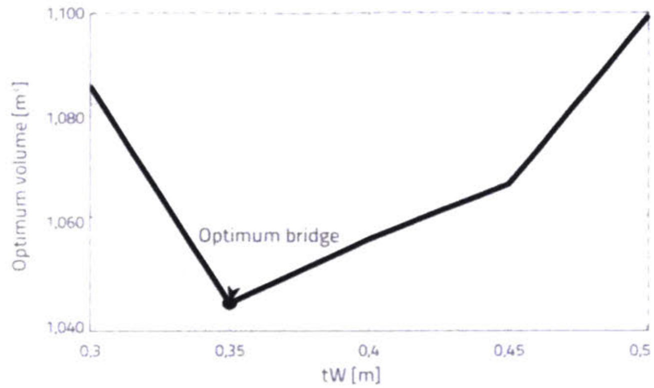


Figure 13 Optimum volume for different web thicknesses [14]

The optimal solution is obtained for a 0.35m thick web. The volume of the arch is 1054 m^3 after the optimization process compared to 1504 m^3 for the original bridge. [14]

This is a simple application of optimization of arch bridges. This thesis will further the analysis by considering more variables such as the deck height or the number of piers in addition to their spacing.

2.3.3 Topology optimization

The optimization of truss topology is a problem that has been addressed by many papers for the past few years. All these papers aim at optimizing the layout of the truss elements to link a certain number of nodes. Although the objective is the same, the papers use different algorithms while attempting to find a solution. The genetic algorithm is the most commonly used one in these papers [15,17,18]

In the Michell beam example [16]. A ground structure (Figure 14) is considered as a starting point and a global optimization process is run to minimize the relaxation (Figure 15) and to solve the discrete minimum compliance problem (Figure 16).

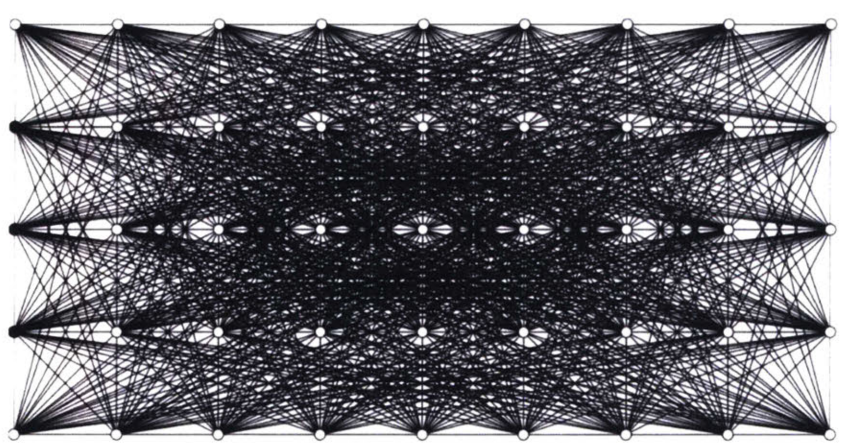


Figure 14 The ground structure (n=632) [16]

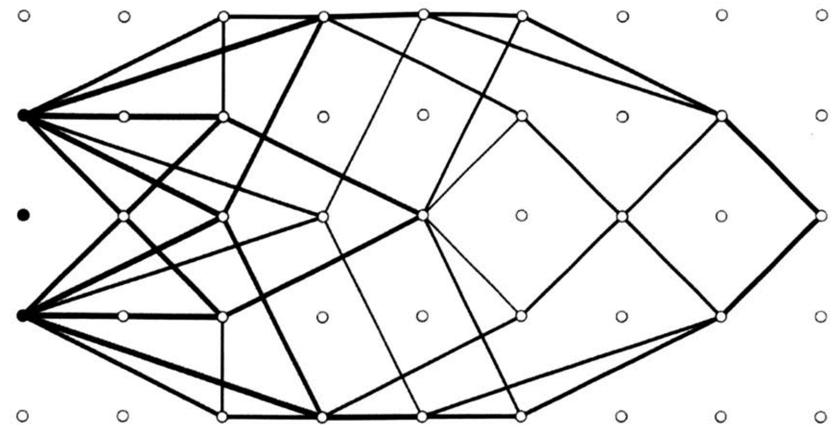


Figure 15 Minimum relaxation solution [16]

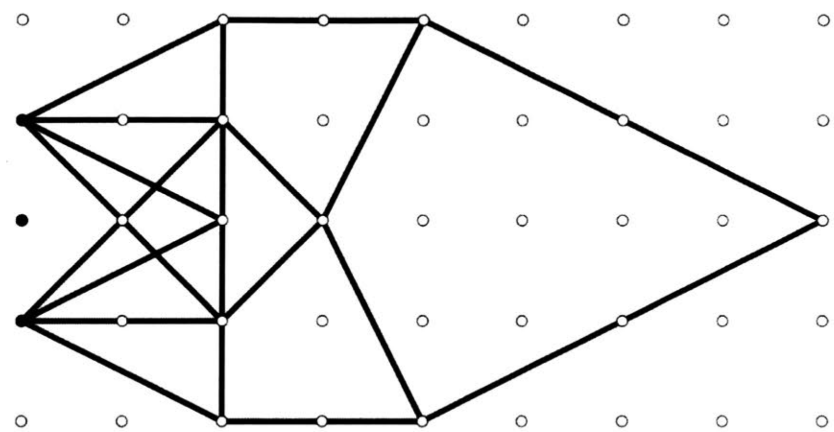


Figure 16 Solution for the discrete minimum compliance problem [16]

The results of all these different papers are interesting but these truss structures are not applicable to arch bridges. For this thesis, 2 different topologies are to be considered: Saint-Andre bracing and a single diagonal bracing.

2.3 Research question

Optimization of arch bridges is a very interesting, broad, and rich research topic. However, there haven't been a lot of work done in this field. Most existing papers focus on the structural weight optimization by varying the different cross sections without studying the impact of geometry on the structural and weight performance. In response, this thesis asks the following questions:

- What kind of structural insights can we gain about arch bridges like the Garabit Viaduct using new computational design tools?
- How to create an optimal structure that meets structural design goals including weight, structural stiffness, and embodied carbon?

Chapter 3: Methodology

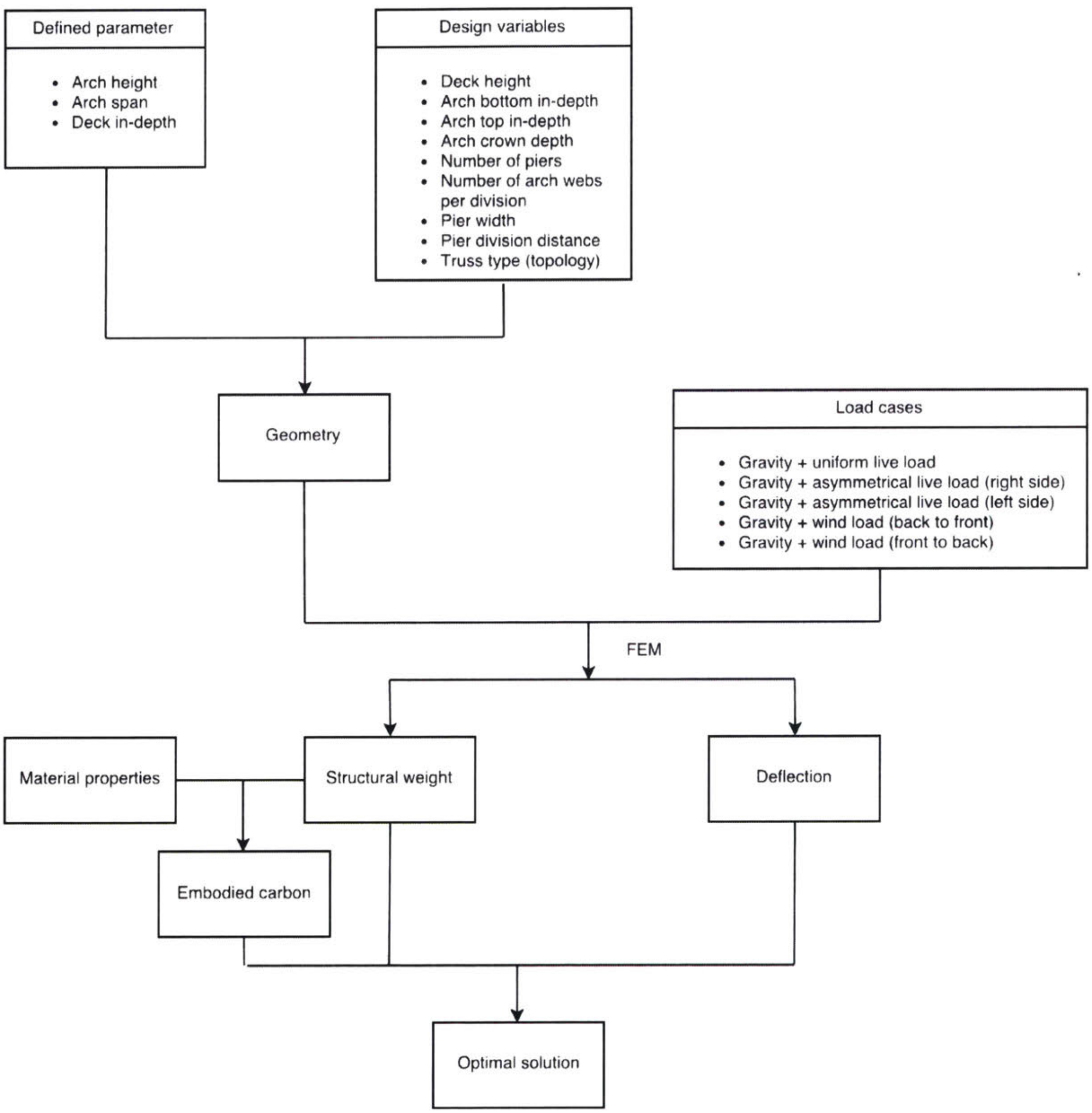
The use of new computational design tools generates a very accurate analysis of the structural behavior and the performance of arch bridges. This thesis analyzes different geometries and topologies of arch bridges while seeking optimal solutions and discusses their behavior. This chapter explains the methodology used to achieve the result.

3.1 Conceptual overview

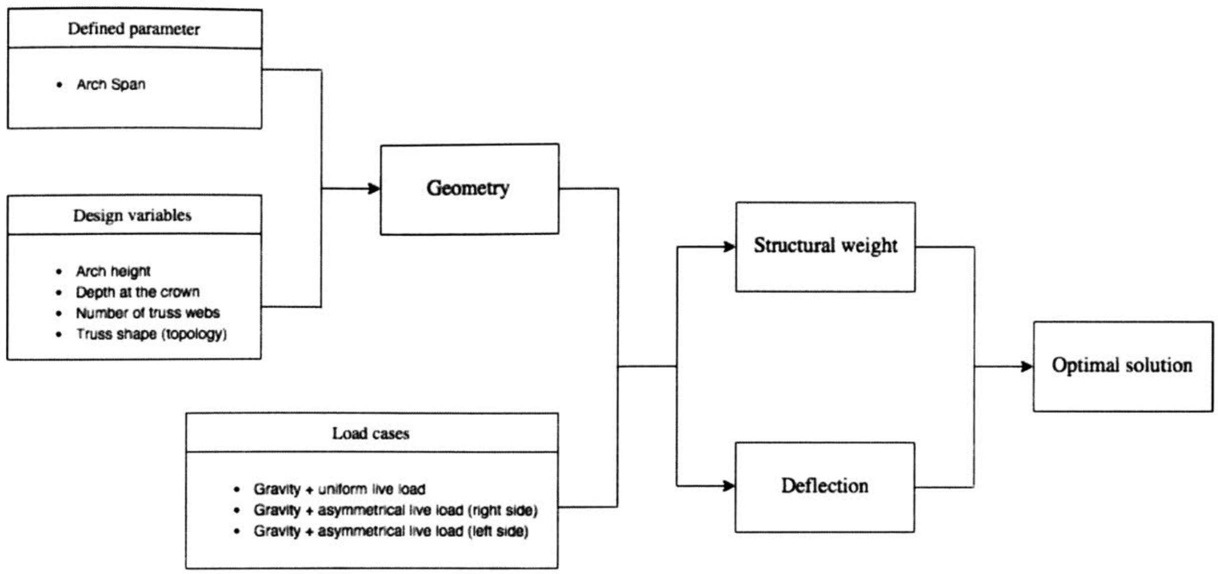
This thesis focuses on two different cases. A 2-D case focusing on a trussed arch and a 3-D case of a complete arch bridge. The same approach is used for both cases. The parametrized geometry is created in Grasshopper and is controlled by fixed parameters and design variables. 5 different load cases including different combinations of gravity, live load, and wind load are considered for the analysis of the 3-D bridge. However, 3 load cases are applied for the 2-D as the wind load is not considered. Karamba then runs a Finite Element Analysis for each of the different load cases and gives the different weight and deflection results. Moreover, this analysis is used as a result to obtain the optimal result using the Optimize Cross Section component in Karamba and different optimization algorithms such as MOO and Goat.


Karamba
parametric engineering
MOO

3.1.1 3-D case


Defined parameter
Arch height
Arch span
Deck in-depth
Design variables
Deck height
Arch bottom in-depth
Arch top in-depth
Arch crown depth
Number of piers
Number of arch webs per division
Pier width
Pier division distance
Truss type (topology)
Geometry
Load cases
Gravity + uniform live load
Gravity + asymmetrical live load (right side)
Gravity + asymmetrical live load (left side)
Gravity + wind load (back to front)
Gravity + wind load (front to back)
FEM
Material properties
Structural weight
Deflection
Embodied carbon
Optimal solution

3.1.2 2-D case



3.2 Project parameters

3.2.1 Loading

A total of 3 types of load are considered for this study but the combinations and representations in Grasshopper depend on the case-study.

- Gravity: it is the self-weight and is automatically implemented by Karamba after assigning the different cross sections to the elements and then multiplying by the density of the material used for the analysis.
- Live load: it is uniform load indicating the influence of a train on the bridge and is equal to 9.34 kN/m. For simplification, the point loads from the wheels are not considered.
- Wind load: it is the only lateral load considered for this study and is equal to 1.44 kN/m^2 ≈ 30psf, which is a reasonable value to use for wind loads simulations.

3.2.1.1 2-D case

In the case of the 2-D arch, out-of-plane wind load isn't considered as it only makes sense in a 3-D context. Thus, there is a total of 3 load combinations engaging gravity and live load. The gravity is calculated by Karamba as explained earlier. The live load is on the other hand modelled as point loads pointing downwards and applied on the upper chord nodes. The value of the load was assumed to be the same at every node and equal to $\frac{z*x}{y}$:

- x: arch span

- y: number of truss webs
- z: 9.34 kN/m it is the value of the uniform load applied by the passage of one train according to the European code.

In the load case 0 (Figure 17) we consider the self-weight and a uniform live load applied along the whole structure. As for load cases 1 & 2 (Figures 18 & 19), instead of applying the live load to all the structure we only apply it to the left side for load case 1 and to the right side for load case 2.

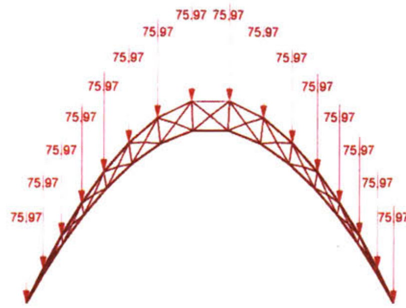


Figure 17 Load case 0

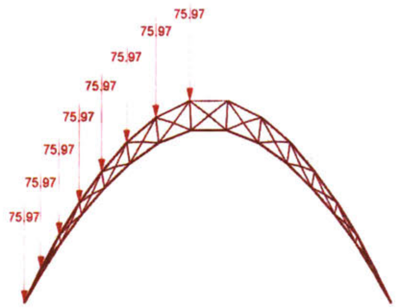


Figure 18 Load case 1

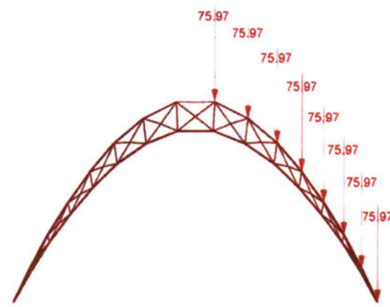


Figure 19 Load case 2

3.2.1.2 3-D case

A total of 5 load cases is considered for this case using all 3 different load types. Like the prior case, gravity is calculated by Karamba. However, the live load is directly modelled as a uniform load applied to the bottom chords of the deck. Besides, the wind load is modelled as a uniform load applied horizontally to each single element.

In the load case 0 (Figure 20) we consider the self-weight and a uniform live load applied along the whole structure. For load cases 1 & 2, instead of applying the live load to all the structure we only apply it to the left side for load case 1 and to the right side for load case 2. As for load cases 3 & 4 (figures 21 & 22), wind load is added to the gravity instead of the live load and it is applied towards the back for load case 3 and towards the front for load case 4.

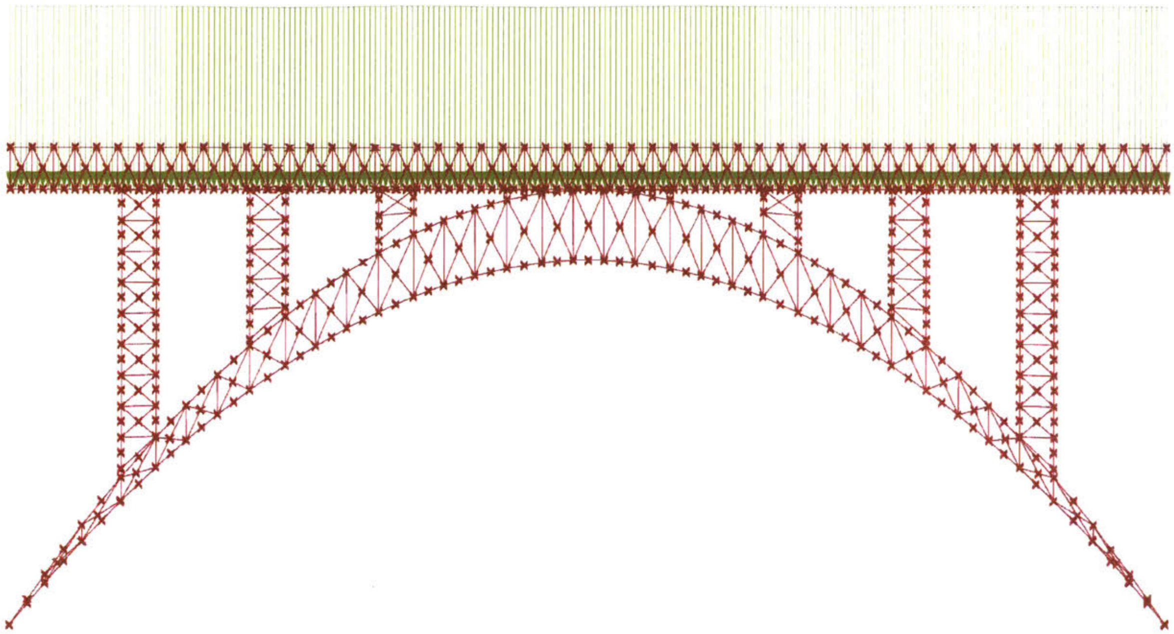


Figure 20 Load case 0

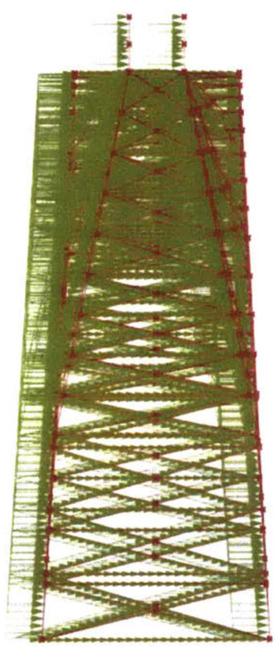


Figure 21 Load case 3

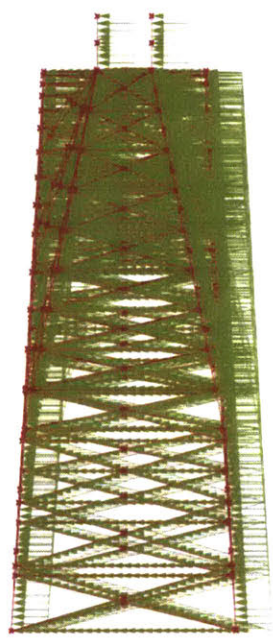


Figure 22 Load case 4

3.2.2 Materials

3.2.2.1 2-D case

The only material considered here is steel. It has the following properties:

Table 2. Material properties used for the 2D case

Material	Steel
E: Young modulus [kN/cm^2]	20,000
G: Shear modulus [kN/cm^2]	8,600
Gamma: Density [kN/cm^3]	78.45
Fy: Yield strength [kN/cm^2]	21.5

3.2.2.2 3-D case

Different materials were considered for this case to push the study even further since the embodied carbon emission is one of the objective functions.

Table 3. Material properties used for the 3D case

Material	Steel	Aluminum	Carbon Fiber
E: Young modulus [kN/cm^2]	20,000	6,890	18,100
G: Shear modulus [kN/cm^2]	8,600	2,600	6,205
Gamma: Density [kN/cm^3]	78.45	26.48	15.7
Fy: Yield strength [kN/cm^2]	21.5	27.6	13.79
ECC [kg CO_2 per kg material]	1.2	8.2	15

These materials are assigned to every element of the bridge except for the connections linking the deck to the piers which are modeled as an infinitely stiff material in Grasshopper. Figure 23 shows the definition of the different materials in Grasshopper.

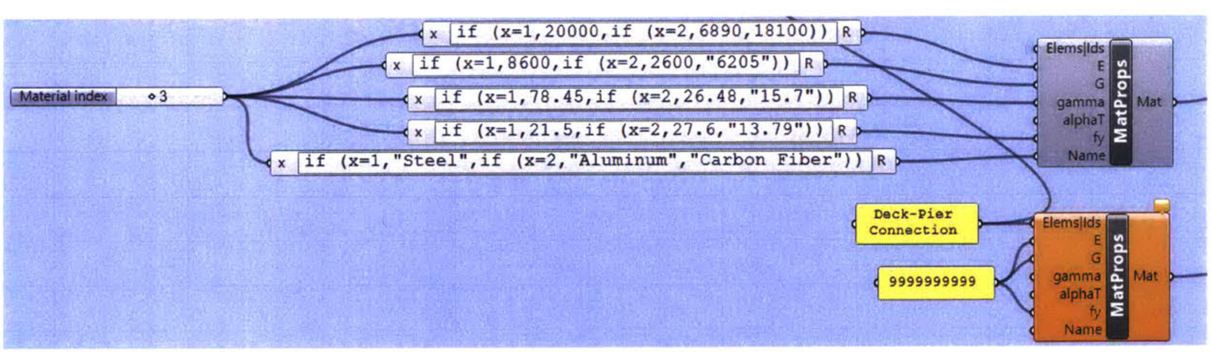


Figure 23 Materials definition in Grasshopper

3.2.3 Boundary conditions

For the 2-D case, the arch has a total of 2 supports located at each base and are pin connections.

For the 3-D case, the deck is initially supported by the piers which either link it to the arch or to the masonry base. Since this thesis is only interested in the part surrounding the arch. Only the piers connecting the deck to the arch are modeled. The deck spanning outside of the arch span is not included in the analysis and a fixed connection is placed at each end of the end of the two lower chords of the deck to replace the piers. On the base of the arch, 4 pin connections are used to fix it to the base.

3.2.4 Geometry and topology

The bridges considered here are all composed of a deck supported by various piers which link it to the arch. The geometry is controlled by two fixed parameters which are the bridge span fixed at 165m and the bridge height equal to 68m (Figure 24). These two values represent the dimensions of Gustave Eiffel's Garabit Viaduct. The geometry is parametric and depends on different design variables such as the number of piers, the depths at the crown and the base, the number of truss webs, and the height of the deck. These design variables will be discussed in detail later in this paper.

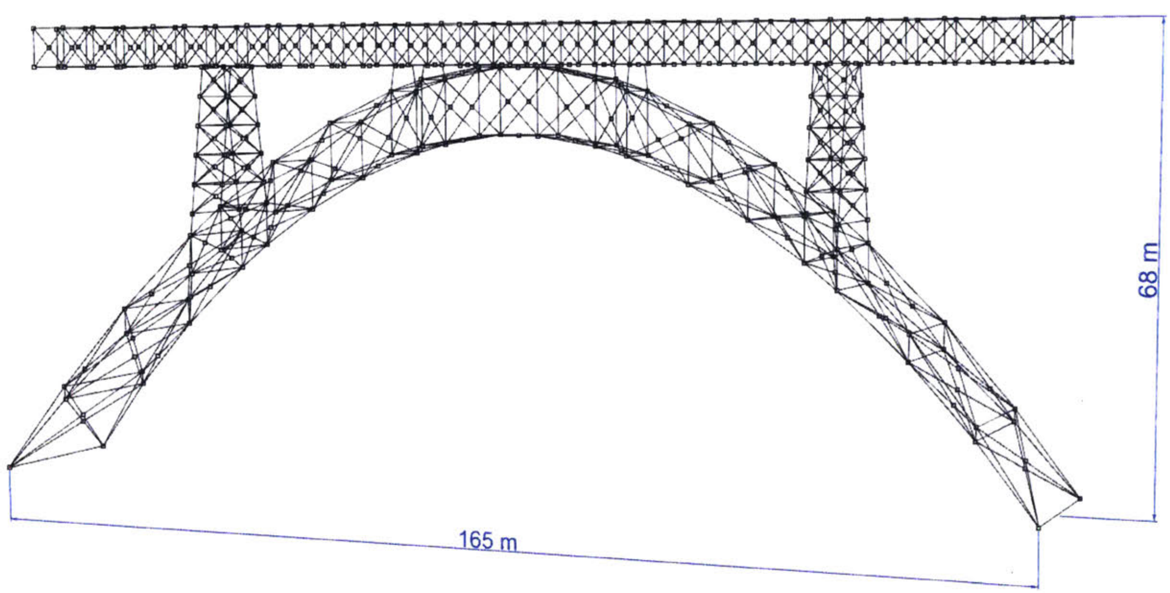


Figure 24 General geometry

In addition to the geometry, two different topologies are considered for the arch in this study. Truss shape 0 (Figure 25) is saint-andre crosses inside the truss webs. This type is the one used for the deck and the piers. The truss type 1 (Figure 26) considered for the arch is a one diagonal in every truss web.

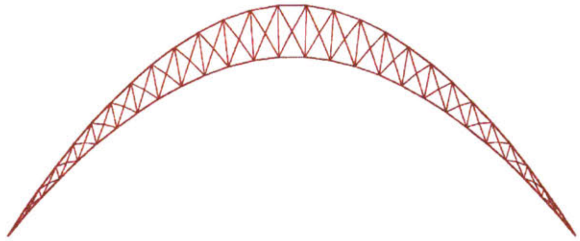


Figure 25 Arch with truss shape 0

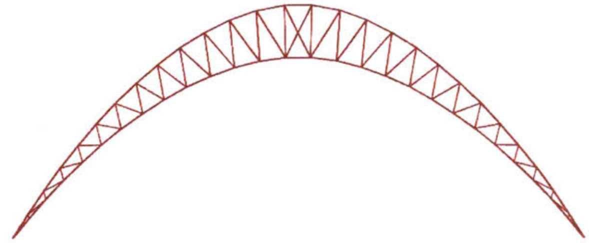


Figure 26 Arch with truss shape 1

are defined, a finite element analysis is run on the model using Karamba. Karamba is a plug-in of Rhinoceros that runs linear elastic analysis on the models based on the Eurocode. This tool generates different outputs such as the maximum deflection, weight of the structure, and the elastic energy as a result. Different algorithms can be used to run the FE analysis on Karamba. The algorithm used for this thesis is based on the first order theory.

The model is at initially assigned default cross sections. To obtain the lightest efficient structure, the Optimize Cross Section is usually in the Karamba script. This component assigns cross section to every element in the model by choosing it from a given list. This component uses an iterative procedure to make a choice. In each iteration, it calculates the section forces in the elements and assigns the lightest elements with a sufficient capacity then calculates the section forces again. The algorithm stops if the all the sections proves to be sufficient after the verification calculation or if the maximum number of iterations is used. The section forces are calculated according to the Eurocode 1993-1-1 for steel. This component different outputs such as the weight and the maximum deflection for each load case. It is possible to define a maximum deflection limit as an input in this component. In this case, the deflection was limited to L/360 while L is the span of the Arch.

In order to verify the results from Optimize Crosse Section, the utilization of the different sections can be observed on the Model View component. The lists of pre-defined sections in Karamba are usually used for buildings. Thus, those sections are too small for a bridge. For this thesis, a list of 40 different circular hollow cross sections (Appendix A) was created and implemented in Optimize Cross Section instead of using one of the existing list.

3.4 Optimization

3.4.1 Design variables (2D vs 3D cases)

3.4.1.1 2-D case

Before starting the analysis and optimization process, the arch span is fixed as an input characterizing the arch (Figure 27). The arch span is defined as the distance between the two supports of the bridge. There is a total of 4 design variables. 3 of these variables change the arch geometry:

1. The depth at the crown: the distance between the upper and lower chords vertexes.
2. The arch height: the height of the vertex of the upper cord of the arch.
3. The number of truss webs: the number of divisions in the arch.
4. The truss shape: it changes the topology of the arch by changing the type of the truss

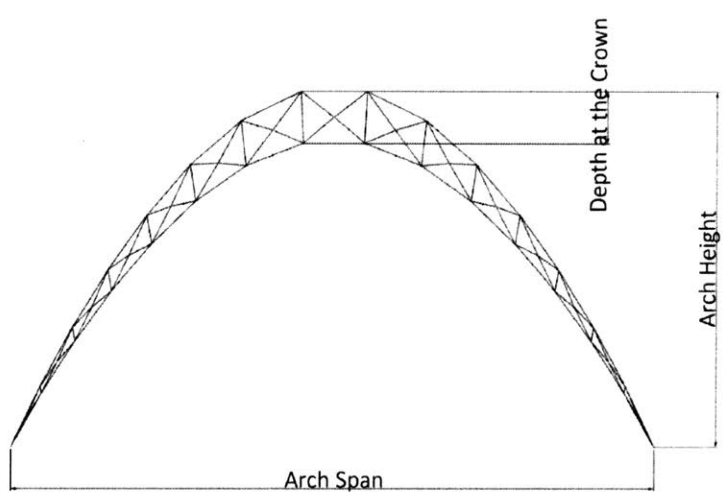


Figure 27 Graphical representation of the variables and inputs

3.4.1.2 3-D case

A total of 12 parameters control the geometry in addition to the changing topology of the arch. In addition to the bridge span defined earlier, the bridge height and the deck width are also fixed as an input at the beginning of the analysis. The bridge height represents the vertical distance between the supports at the base of the arch and the upper chord of the deck. The other 9 parameters are used in addition to the varying topology as design variables.

1. Deck height: the distance between the upper and lower chord of the deck
2. Arch bottom width: the transversal depth at the base of the arch

3. Arch top width: the transversal depth at the crown of the arch
4. Arch crown depth: the longitudinal depth at the crown of the arch
5. Number of piers: the number of piers connecting the deck to the arch
6. Number of arch webs per division: the piers divide the arch into different spans called divisions. This variable describes the number of truss webs in each division. This number is the same everywhere.
7. Deck-pier connection height: the height of the infinitely stiff element connecting the deck to the piers
8. Pier width: the width of the piers and it is the same everywhere
9. Pier division distance: the height of the truss webs in the piers
10. Truss type: the topology of the arch truss

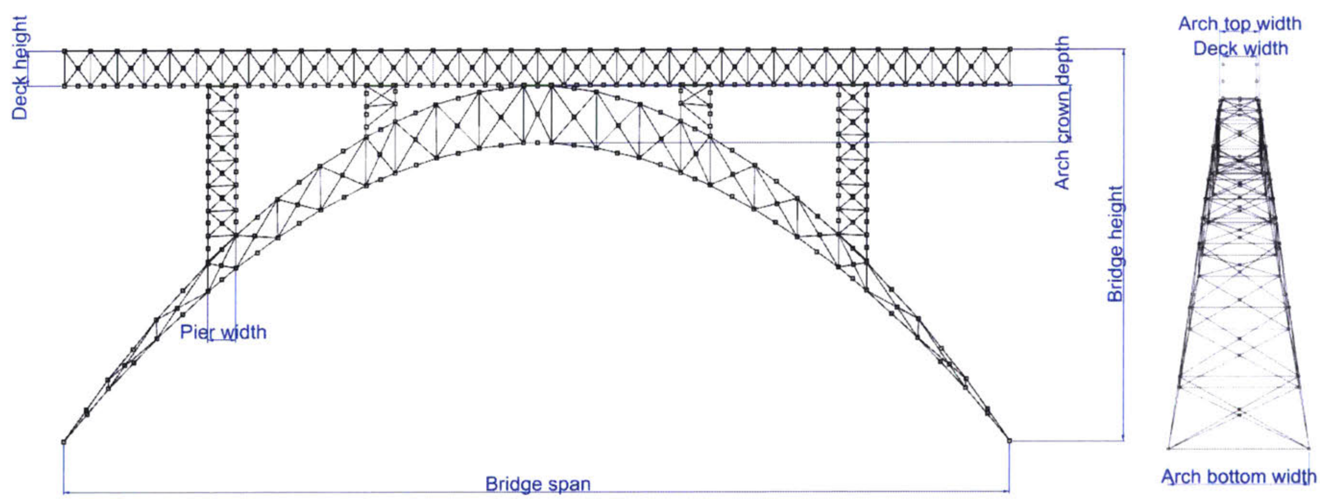


Figure 28 Graphical representation of the geometry design variables and fixed inputs

3.4.2 Objectives

3.4.2.1 2-D case

Two objectives are considered for this case in order to study their relationship and the presence of any trade-offs.

- Structural weight: the weight of the structure given by Karamba [9] after assigning cross sections to the different elements. The material used in this case is steel.
- Deflection: the maximum deflection obtained at any point of the structure.

3.4.2.2 3-D case

In addition to the 2 objectives mentioned earlier, 2 new objectives are considered.

- Structural weight: the weight of the structure given by Karamba [9] after assigning cross sections and materials to the different elements.
- Deflection: the maximum deflection obtained at any point of the structure.

3.4.3 Algorithms

Two different approaches are used for this optimization process. On one hand, Goat is used to optimize the structure for one objective at a time. On the other hand, MOO is used for the multi-objective optimization.

Goat [10] focuses on one objective only. It uses a mathematical approach to get closer to the optimal solution. Goat offers different algorithms when running. A global algorithm that uses bigger gradients to explore the design space randomly and covers an important part of the design space. The local algorithm on the other hand uses smaller gradients and explore a specific area of the design space while staying closer to the starting point of the analysis. For this analysis, a global algorithm is used to run Goat starting from a random point while giving it enough time to explore the design space. The local algorithm is used afterwards taking the result from the global algorithm as a starting point in order to get the best result possible.

MOO [11] is a tool for multi-objective optimization. It uses the NSGA-II algorithm which acts in a revolutionary way and is able to study many samples of the population in an attempt to find the Pareto front of the different objectives. In addition to the objective functions and the design variables. The size of the population and the number of generations must be fixed before running the tool. A design is chosen from the Pareto front for the comparison with the other types of optimization and is represented on the graphics showing the Pareto front.

Chapter 4: Results

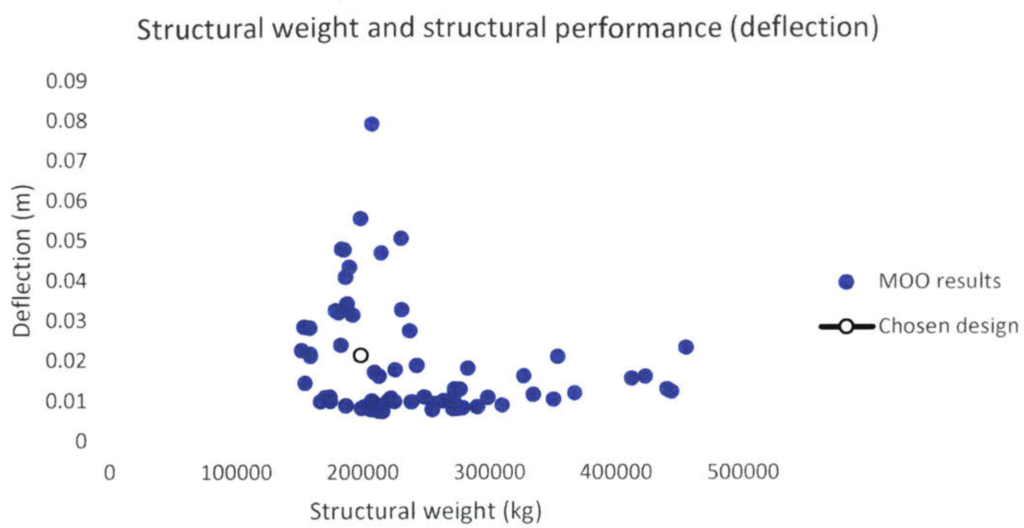
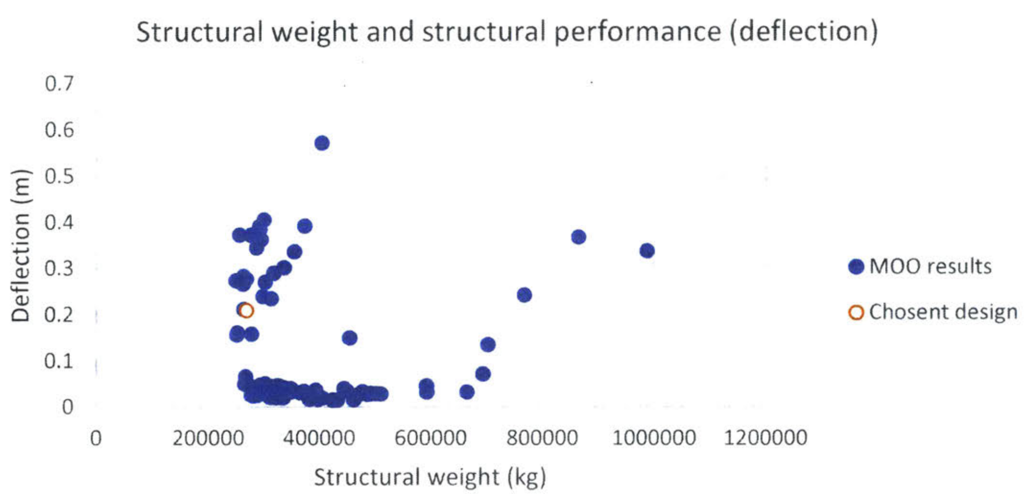
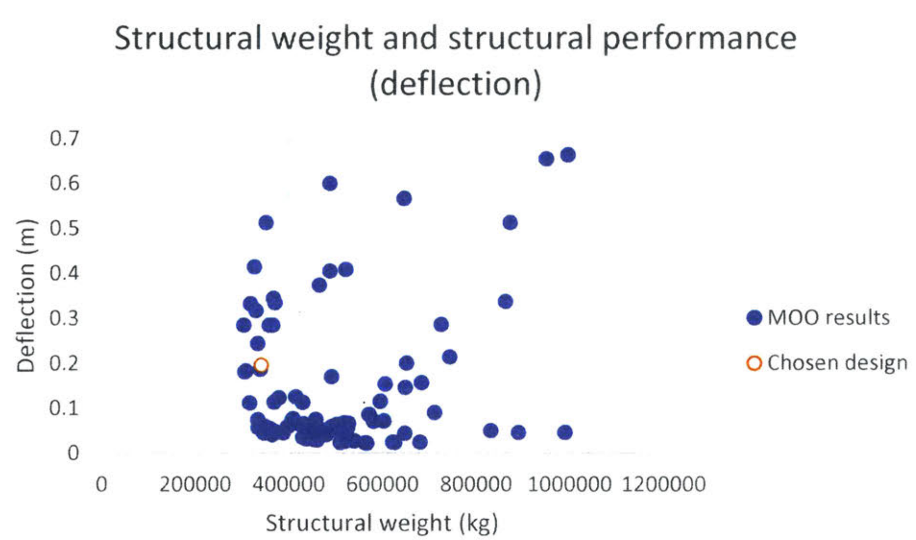
This chapter explores the results given at the end of the optimization processes. For each case, the Pareto front is first investigated while analyzing the logic behind. Furthermore, using different types of optimization on different design problems by changing the span of the bridge allows to compare the general geometry pattern and predict the overall geometry necessary for each case.

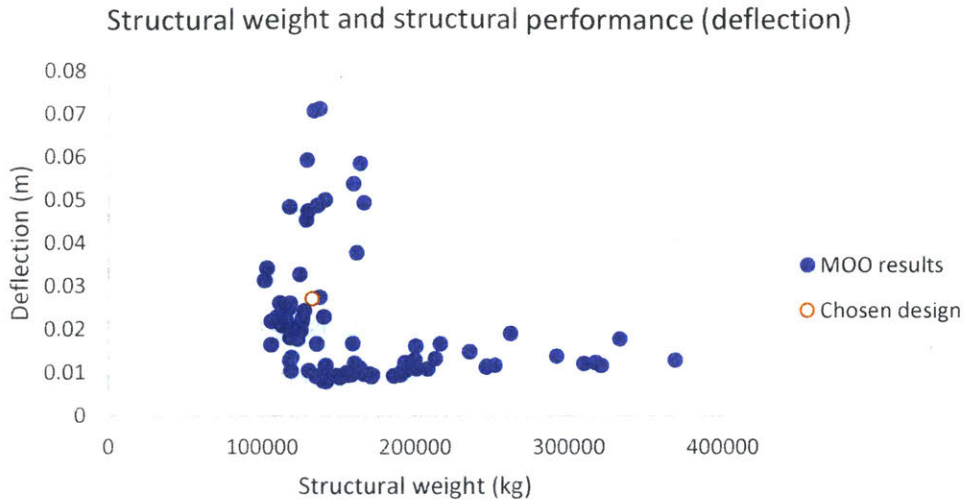
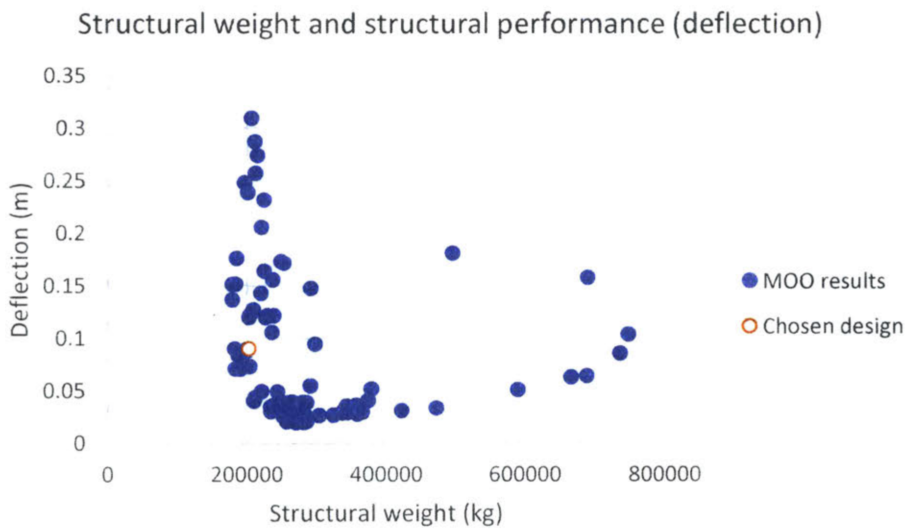
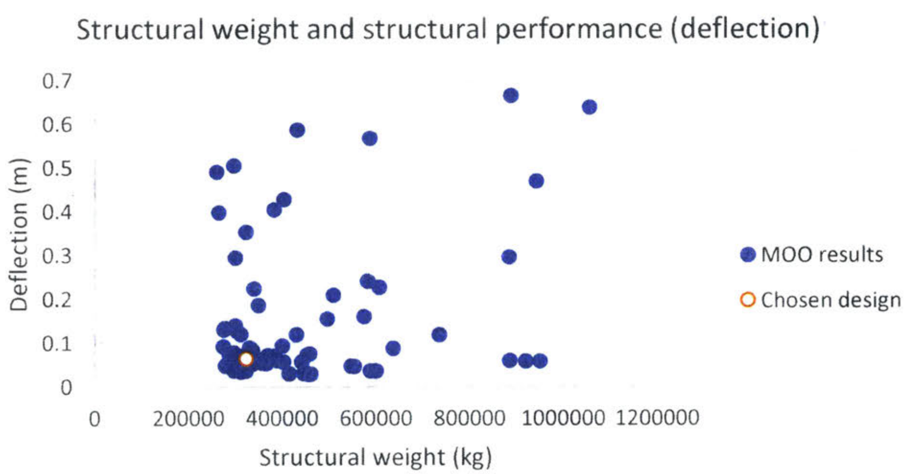
4.1 2D case

4.1.1 Data analysis

This section explores the results given by the multi-objective optimization done via MOO and it investigates the existence of the Pareto front and the implications of it.

Table 4. Evolution of the deflection and the structural weight

<table>
<tr><th>Arch Span</th><th>Truss type 0</th></tr>
<tr><td>100m</td><td>
</td></tr>
<tr><td>165m</td><td>
</td></tr>
<tr><td>200m</td><td>
</td></tr>
</table>

Arch Span	Truss type 1
100m	
165m	
200m	

All these plots share the same pattern. A trade-off exists between the structural weight and the deflection. Besides, each of the different cases has some Pareto optimum solutions representing the best designs possible considering both objectives for each case. These solutions are essentially located on the bottom left corner of the plots.

4.1.2 Optimization

This section investigates the different geometries given by MOO and Goat in order to study the patterns and compares the maximum deflection and self-weight in each case. Goat is used in two phases: a first run starting from a random point in the design space and using a global algorithm and a second run starting from the best design chosen by the global algorithm and exploring the small area of the design space surrounding that point using a local algorithm.

Table 5. Optimized designs for both structural weight and deflection (MOO)

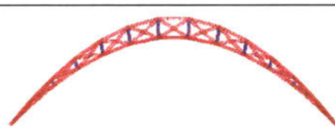
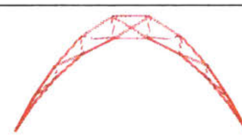
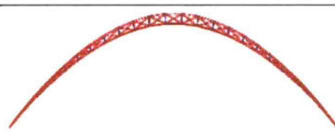
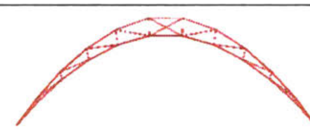
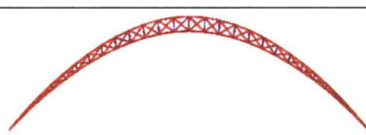
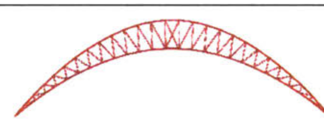
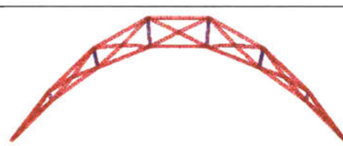
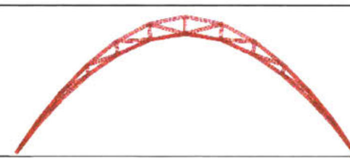
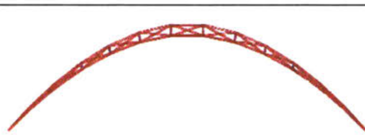
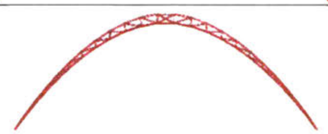
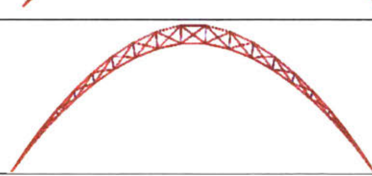
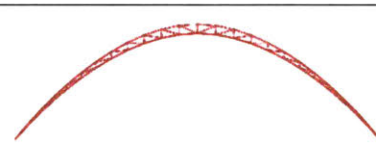
Arch Span	Truss type 0		Truss type 1	
100m	H=100m D=33m N=6 W=158266 kg CO_2=197832kg Def=0.02 m		H=51m D=10m N=9 W=132709 kg CO_2=165886kg Def=0.03m	
165m	H=57m D=5m N=25 W=268459 kg CO_2=335574 kg Def=0.21m		H=6m D=10m N=11 W=202856 kg CO_2=253570kg Def=0.09m	
200m	H=63m D=8m N=28 W=340895 kg CO_2=426119 kg Def=0.20m		H=61m D=18m N=27 W=323223 kg CO_2=404028 kg Def=0.06m	

Table 6. Optimized designs for structural weight only (Goat)

	Truss type 0		Truss type 1	
Arch Span	Global optimization method			
100 m	H=38m D=9m N=7 W=155371 kg CO_2=194214 kg Def=0.016m		H=40m D=5m N=12 W=116613 kg CO_2=145767 kg Def=0.046m	
165 m	H=49m D=5m N=13 W=237451 kg CO_2=296814 kg Def=0.16m		H=63m D=5m N=23 W=202090kg CO_2=252612 kg Def=0.30m	
200 m	H=81m D=10m N=19 W=349241 kg CO_2=436552kg Def=0.22m		H=63m D=5m N=24 W=226814 kg CO_2=283517kg Def=0.47m	

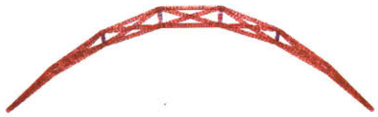
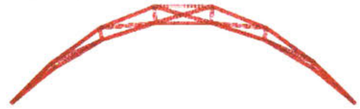
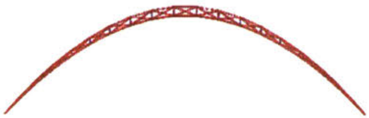
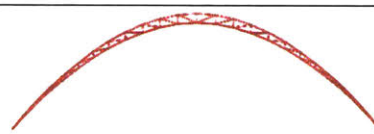
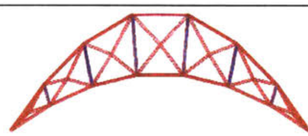
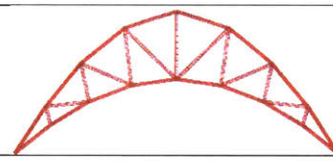
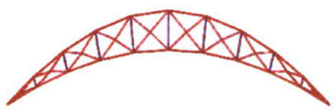
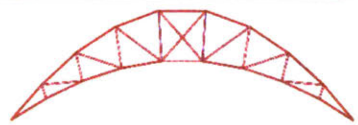
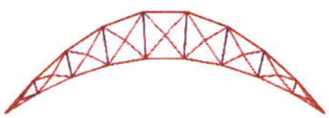
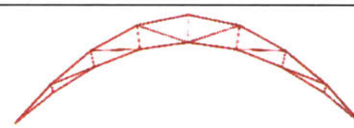
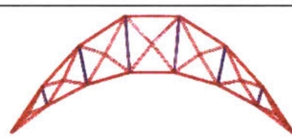
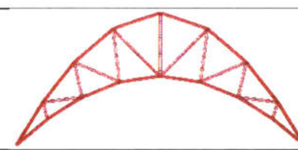
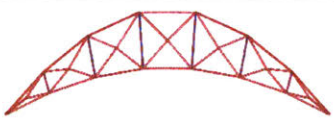
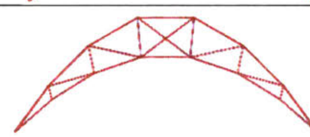
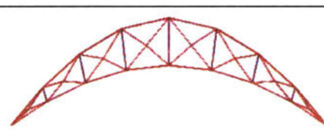
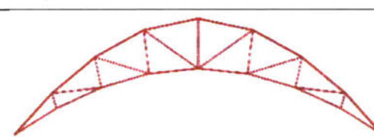
Arch Span	Local optimization method	
100 m	H=29m D=5m N=7 W=139709 kg CO_2=174637 kg Def=0.025m 	H=29m D=5m N=7 W=103842 kg CO_2=129802 kg Def=0.03m 
165 m	H=47m D=5m N=13 W=239866 kg CO_2=299832 kg Def=0.17m 	H=45m D=5m N=16 W=175308 kg CO_2=219135 kg Def=0.18m 
200 m	H=60m D=5m N=19 W=299623 kg CO_2=374528kg Def=0.36m 	H=63m D=5m N=24 W=226814 kg CO_2=283517kg Def=0.47m 

Table 7. Optimized designs for deflection only (Goat)

	Truss type 0	Truss type 1
Arch Span	Global optimization method	
100 m	H=40m D=21m N=7 W=179401 kg CO_2=224251 kg Def=0.007m 	H=44m D=21m N=8 W=133387 kg CO_2=166734 kg Def=0.008m 
165 m	H=48m D=20m N=10 W=274822 kg CO_2=343528 kg Def=0.026 m 	H=54m D=25m N=9 W=232693 kg CO_2=290866 kg Def=0.025 m 
200 m	H=65m D=30m N=9 W=502588 kg CO_2=628235 kg Def=0.024 m 	H=63m D=16m N=8 W=446584 kg CO_2=558230 kg Def=0.04m 

Arch Span	Local optimization method	
100 m	H=43m D=21m N=7 W=181886 kg CO_2=227358 kg Def=0.007m 	H=46m D=22m N=8 W=136037 kg CO_2=170047 kg Def=0.008m 
165 m	H=54m D=30m N=7 W=395187 kg CO_2=493984 kg Def=0.017 m 	H=65m D=23m N=7 W=304342 kg CO_2=380427 kg Def=0.022 m 
200 m	H=68m D=30m N=8 W=566008 kg CO_2=707510 kg Def=0.023 m 	H=63m D=27m N=8 W=456364 kg CO_2=570455 kg Def=0.026 m 

From an aesthetic perspective, a pattern characterizing the different types of optimization can be noticed. The structures optimized for the structural weight and embodied carbon tend to be slender as they have more truss webs and a narrower crown. On the other hand, the designs optimized for defection only tend to have fewer truss webs but the depth at the crown is bigger. Finally, the designs given by MOO which are optimized in consideration for both the structural weight and the maximum deflection have a medium number of truss webs and depth at the crown. The conclusion can thus be made that the more the deflection weighs for the optimization, the broader the depth at the crown and the more truss webs there are, while the opposite would be the case if the optimization is leaning more towards the structural weight. In addition, for the structural weight optimization, the truss type 1 tends to have more truss webs than the truss type 0.

The global and local algorithms generate very similar arches from an aesthetic perspective. This is probably due to the narrowness of the design space used by Goat as a gradient-based optimization tool. Especially for local algorithms. It is thus only natural to get similar designs since the resulting design of the global algorithm is used as a starting point for the local one. Besides, all the offspring created by the local algorithm has a better performance than the parent design. Mixing these two algorithms is thus the best way to use Goat for optimization.

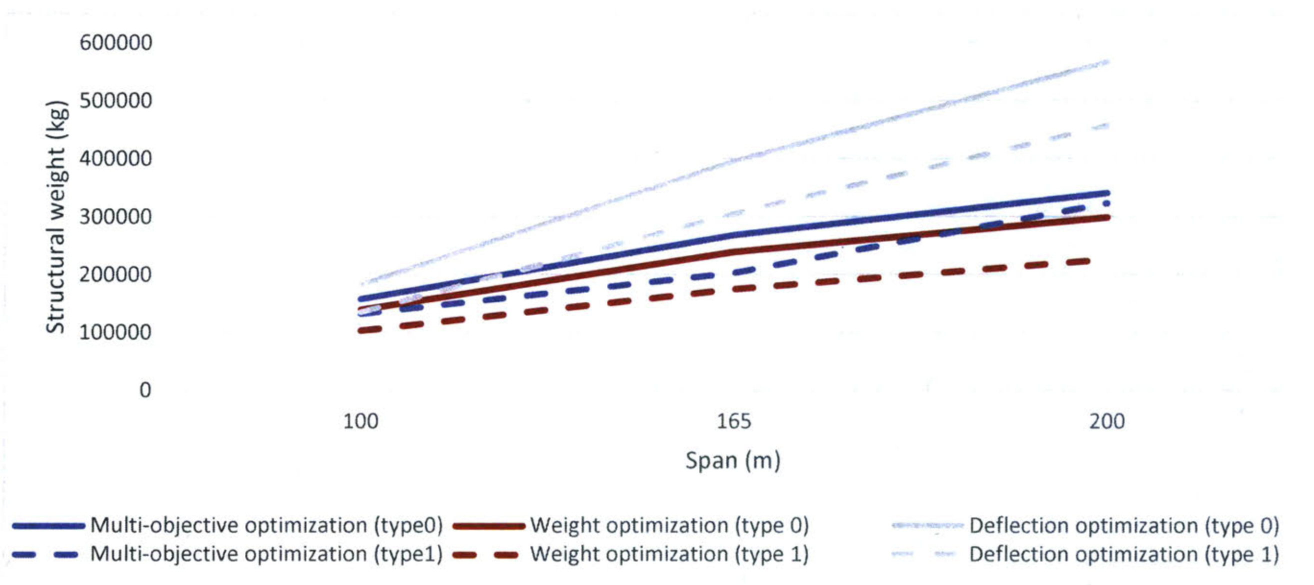


Figure 29 Weight of Optimal Designs by Span

In general, the weight increases when the span is increased. However, except for the multi-objective optimization of a type 1 truss, all the other processes show a decrease in the slope for the second half of the plot, which means that the weight increased at a lower speed when the span is increased.

This plot proves once again that the weight of the objectives does matter in the optimization process. If only one type of trusses is considered (solid lines for type 0 and dash lines for type 1), it can be seen that the weight optimization has the least weight which is expected as this process only aims at reducing the weight without any regard to the deflection or any other available objective. The deflection oriented optimization on the other hand has the highest weight which gets considerably higher when the span is increased because this kind of optimization does not take weight into consideration and seeks reducing the deflection only. Finally, the multi-optimization process scores the middle weight as it represents a trade-off between the structural weight and the deflection while looking for the optimum design.

For every single optimization process, the structural weight of the arches with a type 0 truss is higher than for the arches with a type 1 truss. This is probably the result of the type 1 truss using fewer elements than type 0 trusses. However, the result might be the exact opposite for the deflection.

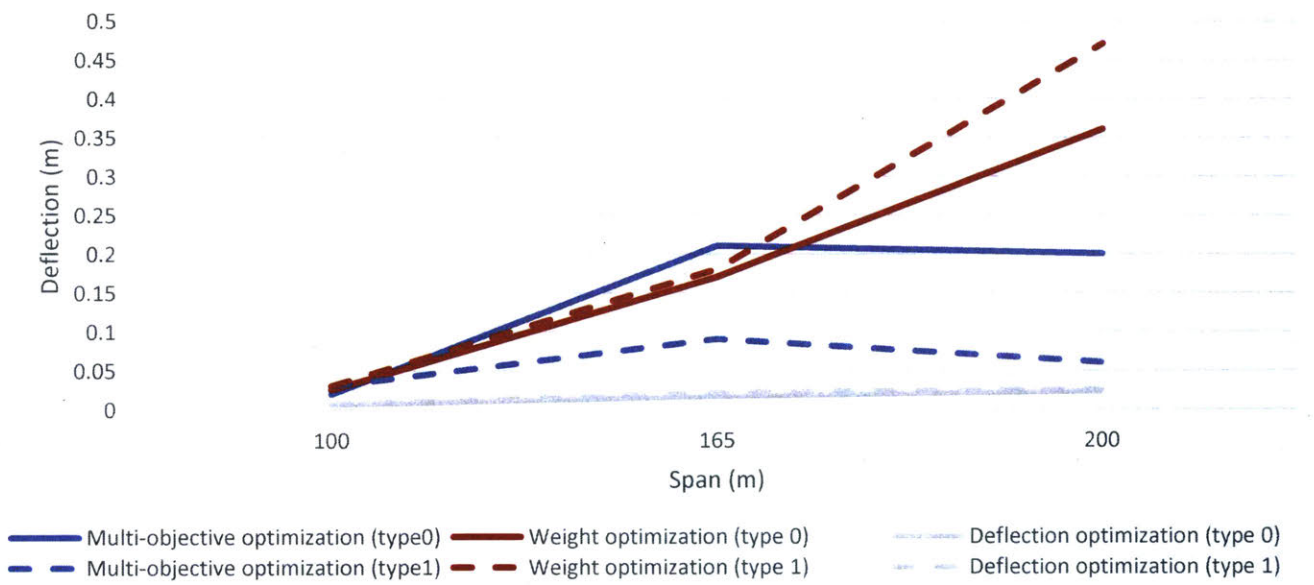


Figure 30 Deflection of Optimal Design by Span

The general placement of the different plots follows the expected pattern by having the deflection oriented optimization as the lowest while the structural weight oriented optimization has the highest deflection and the multi-optimization in the middle. However, the multi-optimization of a type 0 truss generated the highest deflection for a 165-m span. This result is different from the expectation, but since there is only 4cm which represents around 8.9% of the 45.8cm allowable deflection this situation can be considered exceptional but also without impact on the rest of the results.

The most interesting part of this plot is the evolution pattern that changes according to the optimization process. For the deflection orientated optimization, the deflection stays very low and increases at a very slow rate so it almost seems constant. While it increases then decreases for the multi-objective optimization. Finally, the deflection increases at an increasing rate when the span is bigger for the structural weight orientated optimization.

There is almost no difference between both truss types for the deflection orientated optimization. However, the deflection is higher for truss type 1 resulting from the structural weight oriented optimization. Getting a lower weight was possible for this truss type because it has fewer elements, but fewer elements also means less bracing so the deflection can be higher. On the other hand, a different pattern is shown by the multi-objective optimization, as the deflection is considerably lower for a truss type 1. This means that the truss type 1 has a very good potential but it's more apparent when there is a trade-off between both objectives for the optimization process.

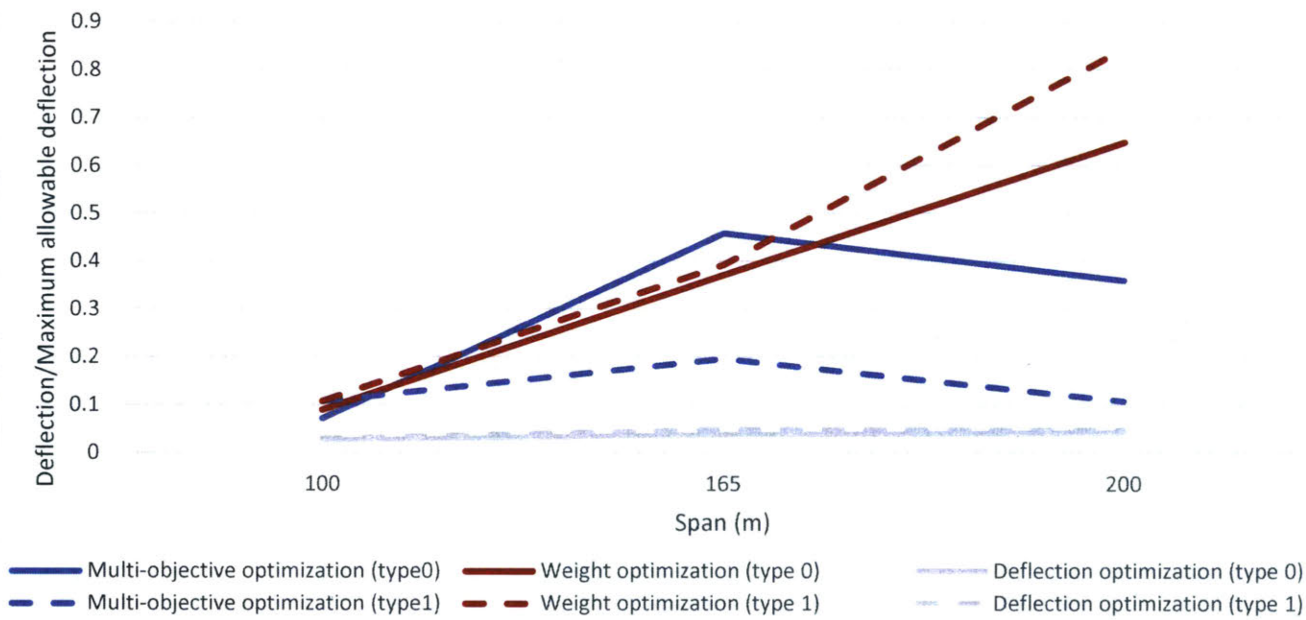


Figure 31 Deflection/Maximum allowable deflection ratio by span

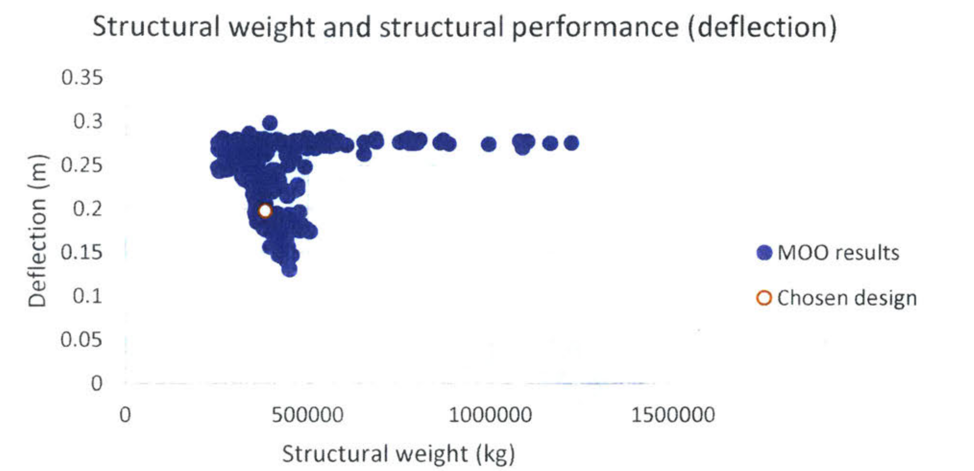
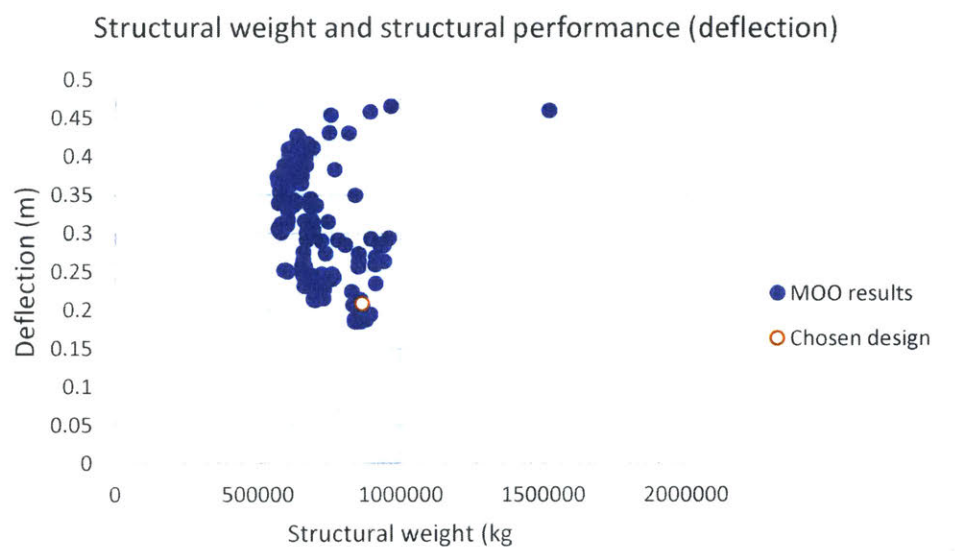
The maximum allowable deflection depends on the defined span L of the bridge and is equal to L/360. The graphic above (Figure 31) has the same shape as the Graphic of the optimal deflection by span. It is noticeable that the deflection/maximum allowable deflection ratio increases at a faster rate for the weight oriented optimization and gets closer to 1. Focusing on the weight only leads to getting closer to the maximum allowable deflection as the span increases. However, as expected for the deflection oriented optimization, the ratio is low and almost constant and has an average of 3.8%.

4.2 3D case study #1 – steel trussed arch bridge

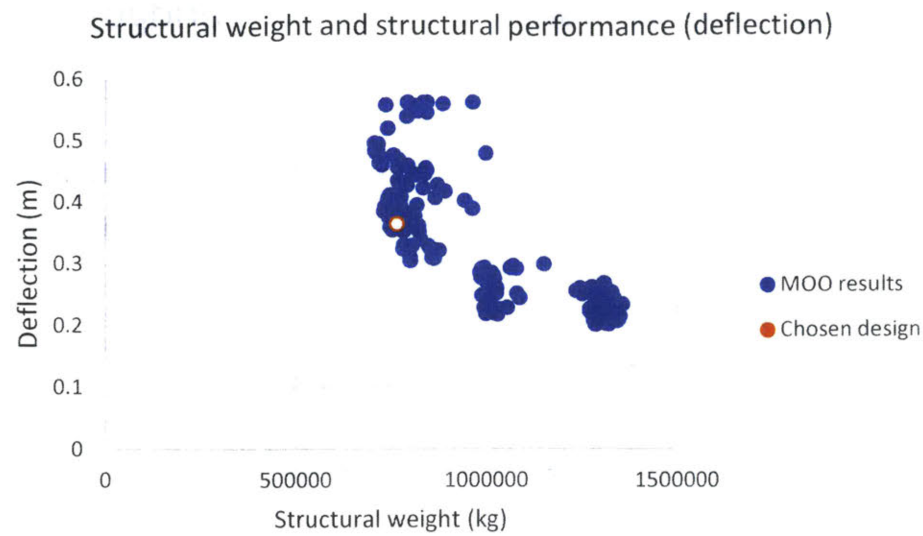
4.2.1 Data analysis

This section investigates the result of the multi-objective optimization run on the 3D model using MOO. It investigates the possible existence of a trade-off between the structural weight and the deflection.

Table 8. Evolution of the deflection and the structural weight

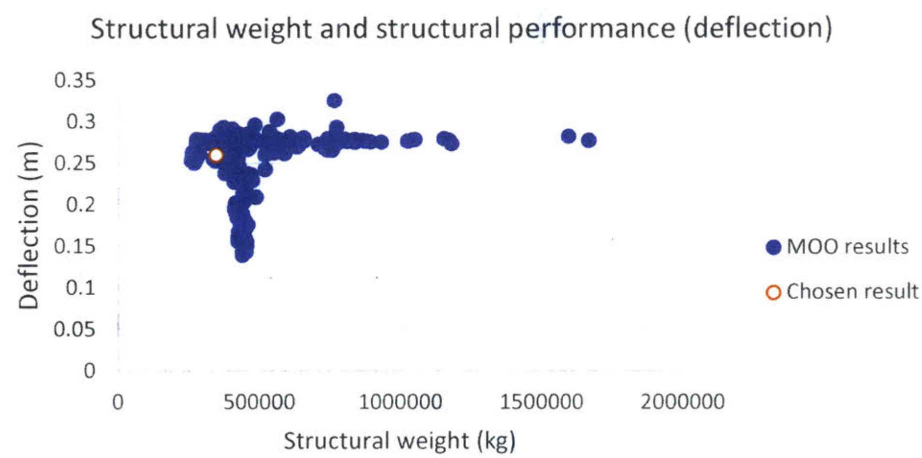
<table>
<tr><th>Arch Span</th><th>Truss type 0</th></tr>
<tr><td>100m</td><td>
</td></tr>
<tr><td>165m</td><td>
</td></tr>
</table>

200m

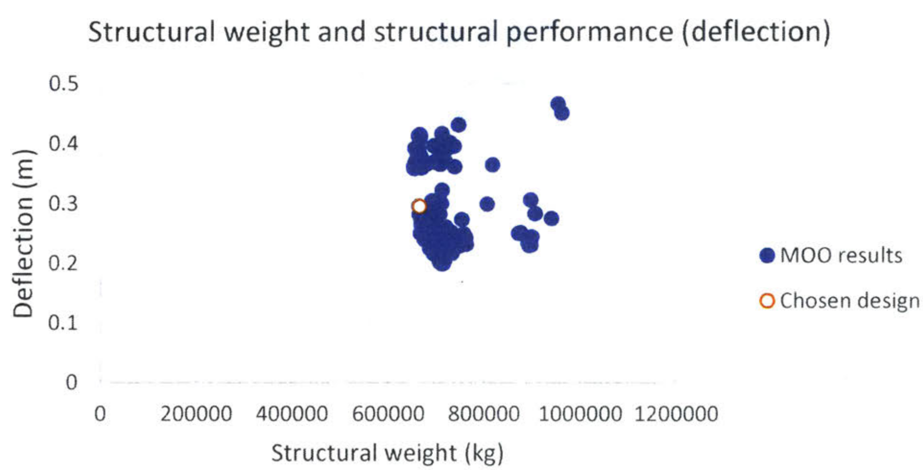


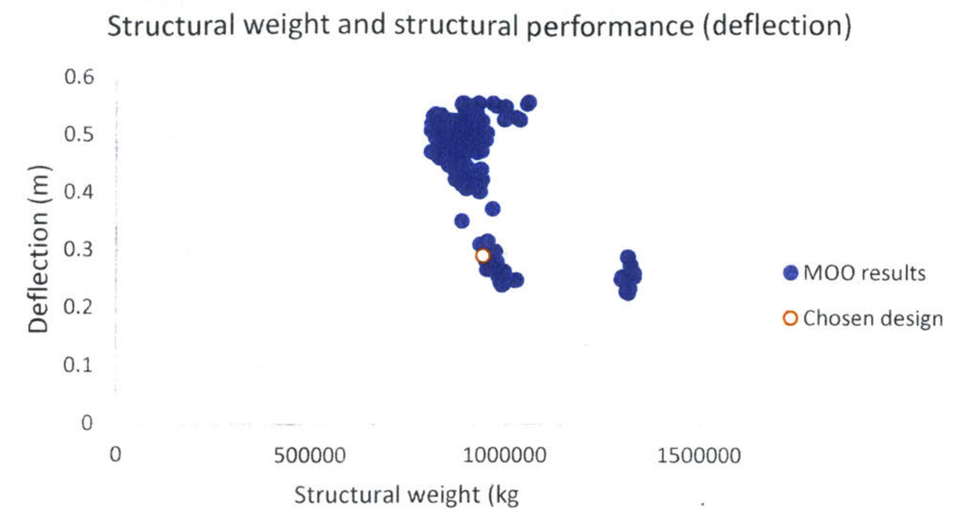
Truss type 1

100m



165m



200m	

Similarly to the 2D case, MOO is run on the 3D model of the arch bridge while using the maximum deflection and the structural weight as objectives. In all cases, the graphics show a Pareto front. Thus, there is a trade-off between the weight and deflection. Trying to optimize one of the two will generally lead to worse results in the other. The bottom left corner of each graphic consists of the optimum solutions that satisfy both objectives at the same time.

4.2.2 Optimization

This section investigates the differences between the results of each of the different types of optimization mentioned earlier: a multi-objective optimization between weight and deflection, a weight orientated optimization and a deflection orientated optimization. It analyzes the different geometries to find characteristic properties for the optimization processes. It also studies the weight, deflection, and embodied carbon for different study cases by changing the spans, truss types or used materials.

Table 9. Optimized designs for both structural weight and deflection (MOO)

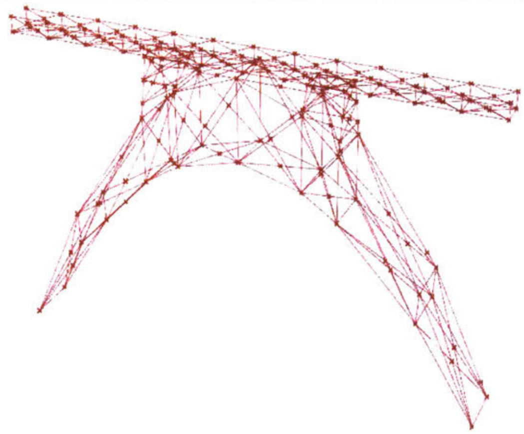
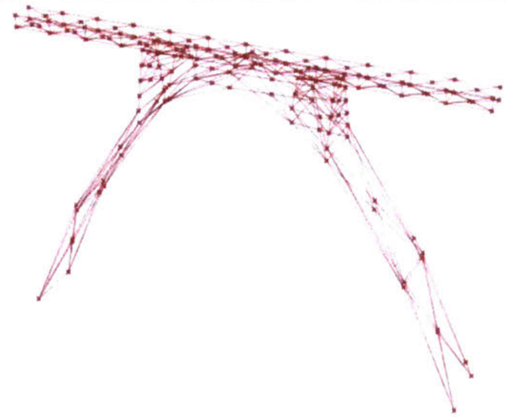
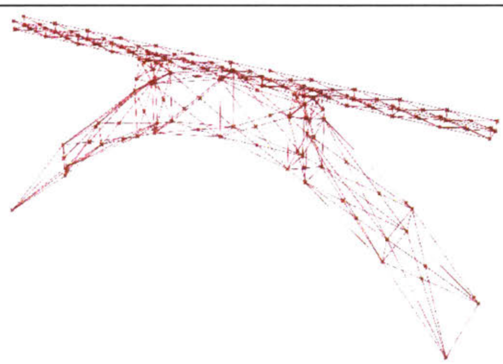
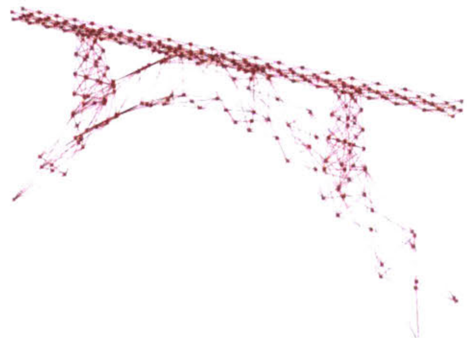
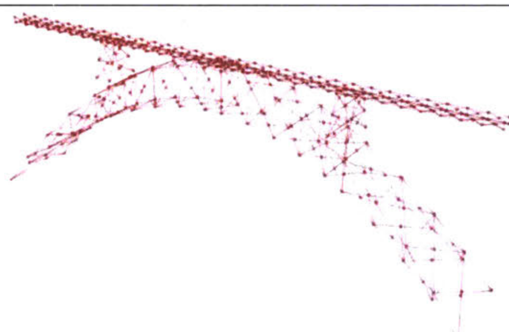
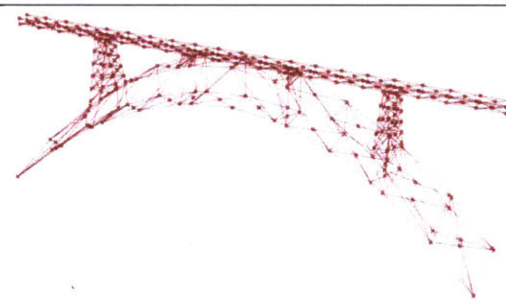
Arch Span	Truss type 0	Truss type 1
100m		
165m		
200m		

Table 10. Optimized designs for structural weight only (Goat)

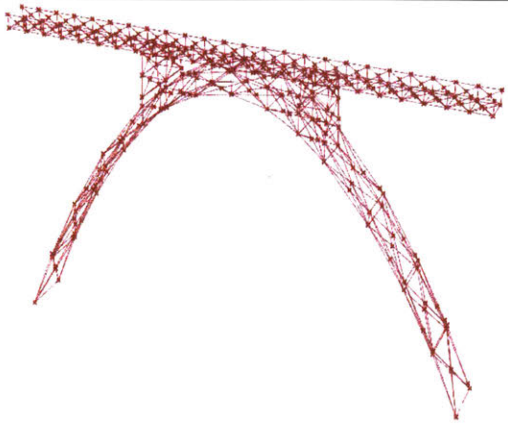
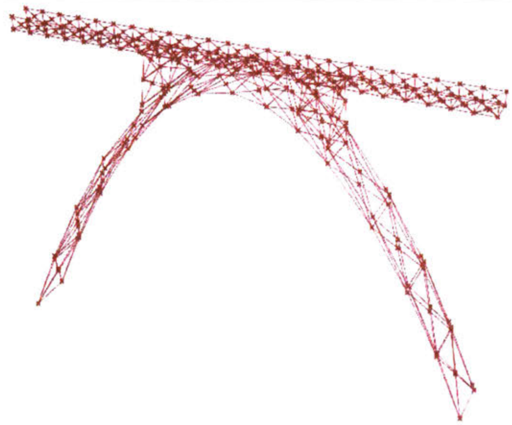
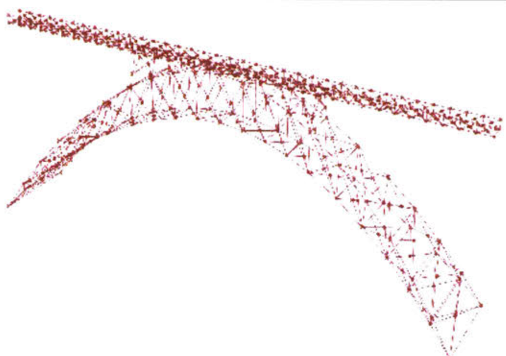
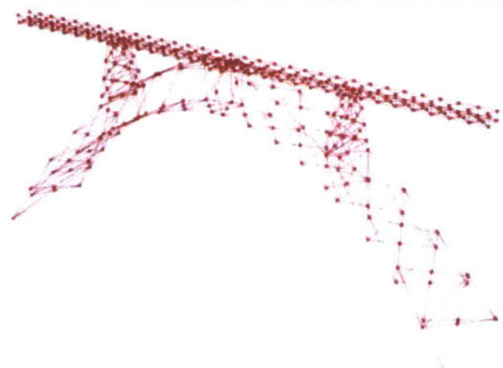
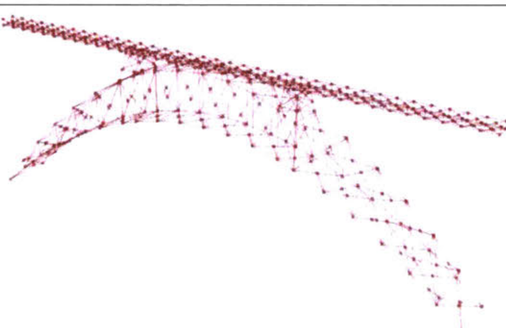
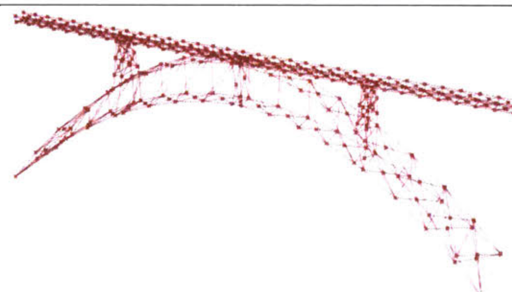
Arch Span	Truss type 0	Truss type 1
100m		
165m		
200m		

Table 11. Optimized designs for Deflection only only (Goat)

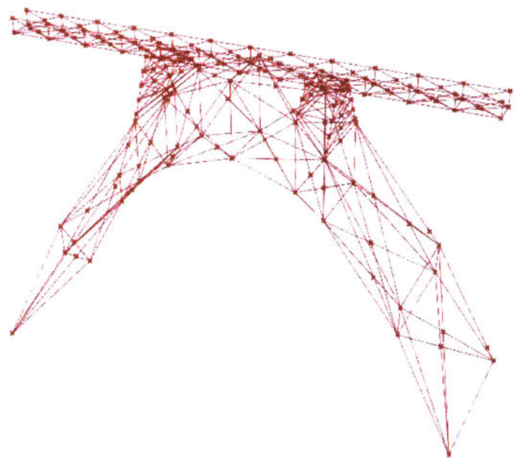
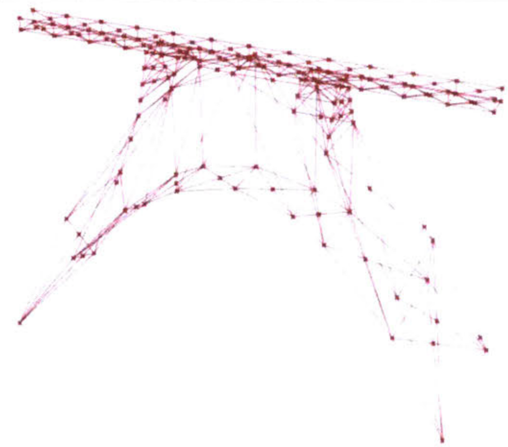
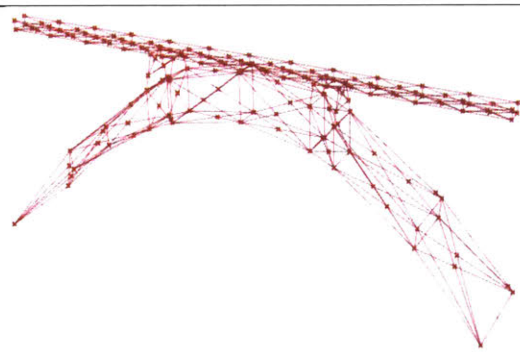
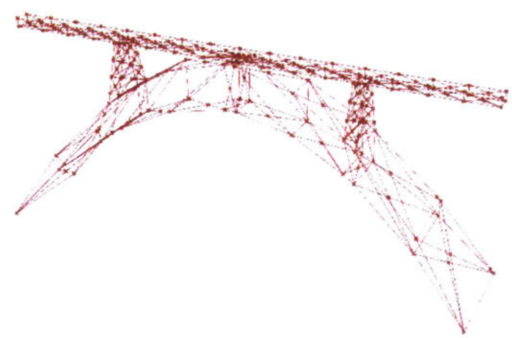
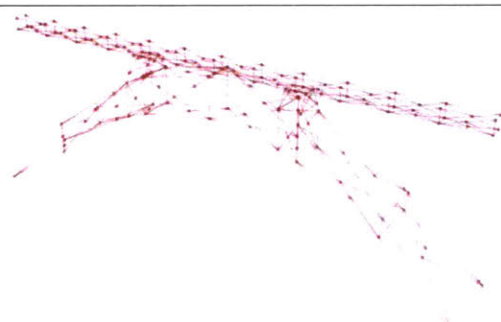
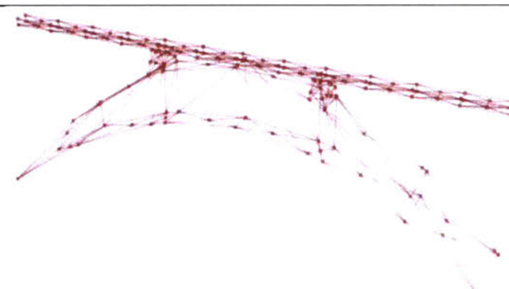
Arch Span	Truss type 0	Truss type 1
100m		
165m		
200m		

Table 12. Numerical results of the different types of optimization

Span [m]	Truss type	Deck height [m]	Arch bottom width [m]	Arch top width [m]	Arch crown depth [m]	Number of piers	Number of truss webs per division	Pier width [m]	Pier division distance [m]	Weight [kg]	Deflection [m]
Multi-objective optimization											
100	Truss type 0	3.218	10.298	10.081	17.159	2	2	7.350	4.583	382155.801	0.198
165	Truss type 0	3.107	29.972	8.037	19.199	2	2	6.422	6.472	863108.771	0.208
200	Truss type 0	2.150	32.096	10.394	15.693	3	4	6.927	6.870	765868.525	0.363
100	Truss type 1	2.592	12.456	10.659	3.191	2	2	7.401	3.121	345435.213	0.260
165	Truss type 1	2.689	25.833	9.361	14.255	4	2	5.254	6.726	663991.233	0.295
200	Truss type 1	2.601	32.011	11.824	16.350	4	2	5.010	5.054	941465.750	0.291
Structural weight only											
100	Truss type 0	2.873	10.085	6.710	4.690	2	4	5.391	4.493	251907.740	0.247
165	Truss type 0	2.776	28.616	7.585	15.833	2	6	7.619	6.685	566575.506	0.372
200	Truss type 0	2.493	19.277	11.771	19.587	2	6	6.445	6.210	707010.095	0.494
100	Truss type 1	2.739	10.029	9.166	3.722	2	4	6.268	5.116	256692.381	0.254
165	Truss type 1	2.541	31.908	9.622	13.066	3	4	3.940	5.494	651511.967	0.362
200	Truss type 1	2.595	29.614	7.236	16.255	3	4	3.574	6.658	807268.035	0.506
Deflection only											
100	Truss type 0	3.132	32.569	12.817	15.957	2	2	6.205	2.763	449133.830	0.130
165	Truss type 0	3.119	30.655	8.063	15.807	2	2	8.095	6.545	841868.402	0.184
200	Truss type 0	4.762	30.553	11.711	16.248	2	2	7.966	6.779	1287862.692	0.202
100	Truss type 1	2.593	32.423	10.223	24.997	2	2	7.291	3.173	439121.587	0.140
165	Truss type 1	2.862	34.183	9.849	15.018	3	2	3.777	5.513	711391.904	0.199
200	Truss type 1	2.671	30.007	8.971	24.322	2	2	6.219	4.684	1315041.576	0.227

Just like the 2D case, the arch crown depth is the largest for the deflection oriented optimization as the crown tries to contain the moment from the vertical forces. Furthermore, the truss tends to be denser for the weight oriented optimization. Thus, having more elements sometimes means a smaller weight, since each element needs to have less capacity and is consequently lighter. Overall, the 2D and 3D results are compatible.

In most cases, the arch bottom width is significantly higher than the arch top depth. This shape is expected as the depth of the arch maintains the stability under the different load cases and especially the wind loads. Since wind loads induce a maximum moment at the base of the arch, the depth at the base should be larger compared to the depth of the arch which is the case here. It is noticeable that in average, the top depth is 67.6% smaller than the bottom depth for the deflection-oriented optimization. For the multi-objective optimization and the weight-oriented optimization, the ratio is around 65% for the 165m and the 200m spans. However, the ratio average is decreased to 50% if the 100m span is considered. The total load applied by the wind in this case is smaller since the area to which the load is applied is smaller. If reducing the deflection is not the priority, it is then possible to reduce the difference between the top and bottom arch depth if the span gets considerably small.

In general, only two piers are used for truss type 0 and it is the minimum number of piers allowable by the design space. On the other hand, truss type 1 uses three piers in average. This is due to the density of the truss. Since truss type 0 is denser as it has two diagonal elements, it needs fewer points to transfer the load from the deck to the arch while truss type 1 needs more points to transfer the load as there is only one diagonal in each truss web. Moreover, the piers are wider and have a denser truss when the objective is to reduce the deflection while they are mode slender and have less dense trusses when reducing the self-weight is considered as the only objective.

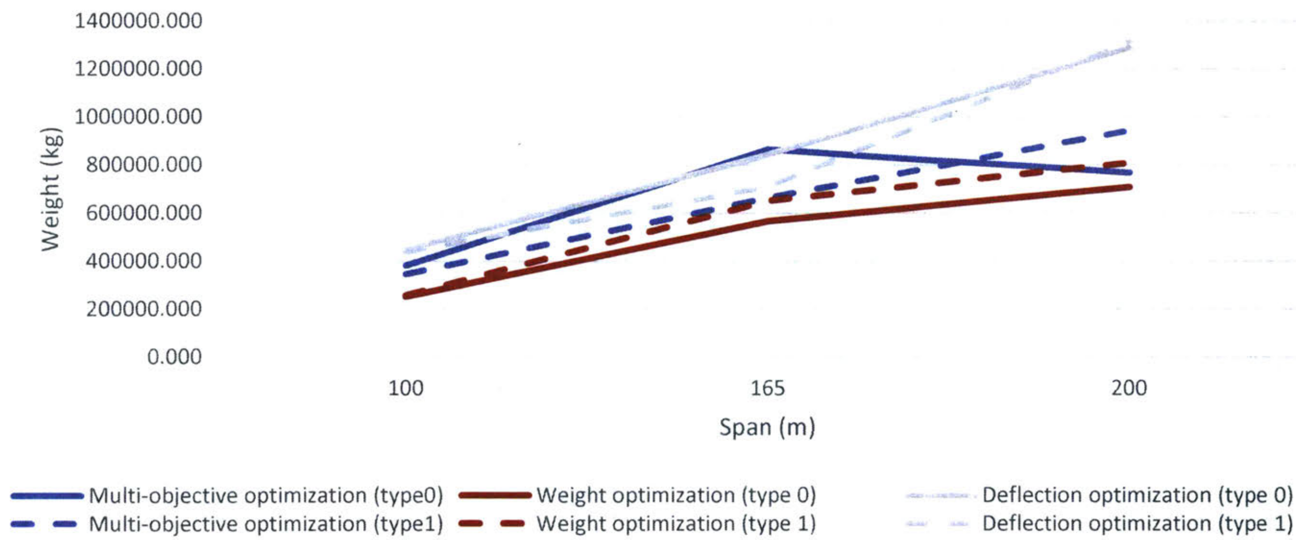


Figure 33 Weight of Optimal Design by Span

The deflection is overall higher for the deflection-oriented optimization and gets lower for the multi-objective optimization. The weight oriented optimization has the lowest weight as expected. For the weight oriented optimization, the truss type 0 gives lighter results than truss type 1 result while it is the opposite for the other two types of optimization. For the 200m span of the weight optimization, truss type 1 has a result 14% heavier than truss type 0 (Figure 32) but the deflection is almost the same (Figure 33). For this specific case, truss type 0 is the best choice. However, truss type 1 is a better choice because designers don't aim at reducing the weight only in reality.

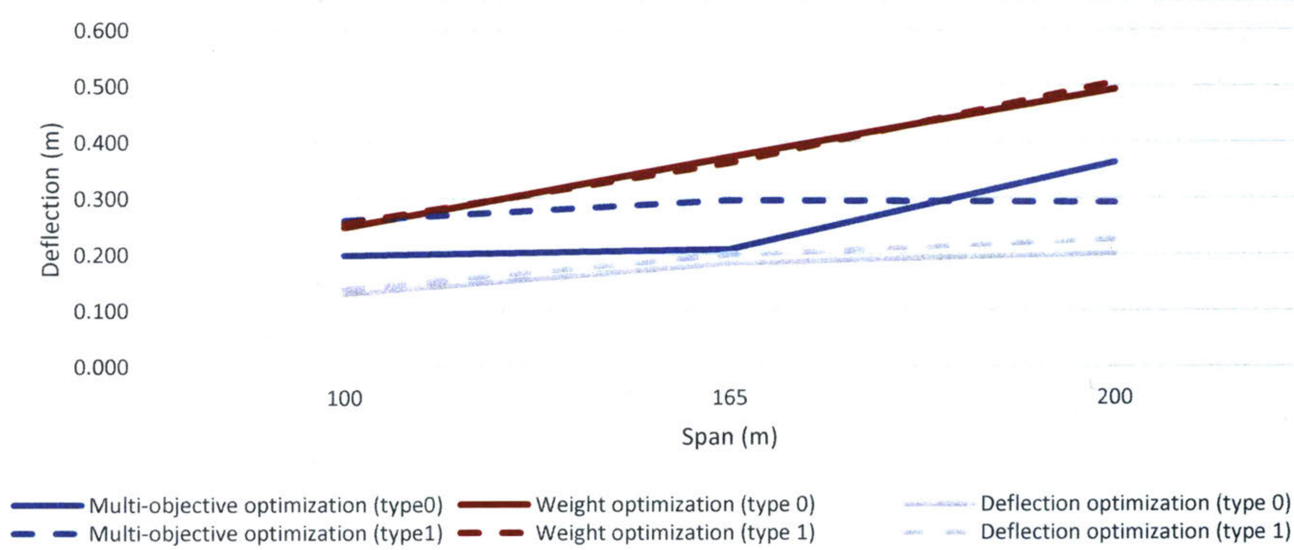


Figure 32 Deflection of Optimal Designs by Span

The shape of this graphic (Figure 33) is interesting. The order of the curves follows the expected pattern by the deflection oriented optimization giving the best results and the weight-oriented optimization giving the worse results while not exceeding the maximum allowable deflection. However, is these two cases, there is almost no difference in the results generated by the different truss types. The multi-objective optimization results don't follow a specific pattern following the truss type.

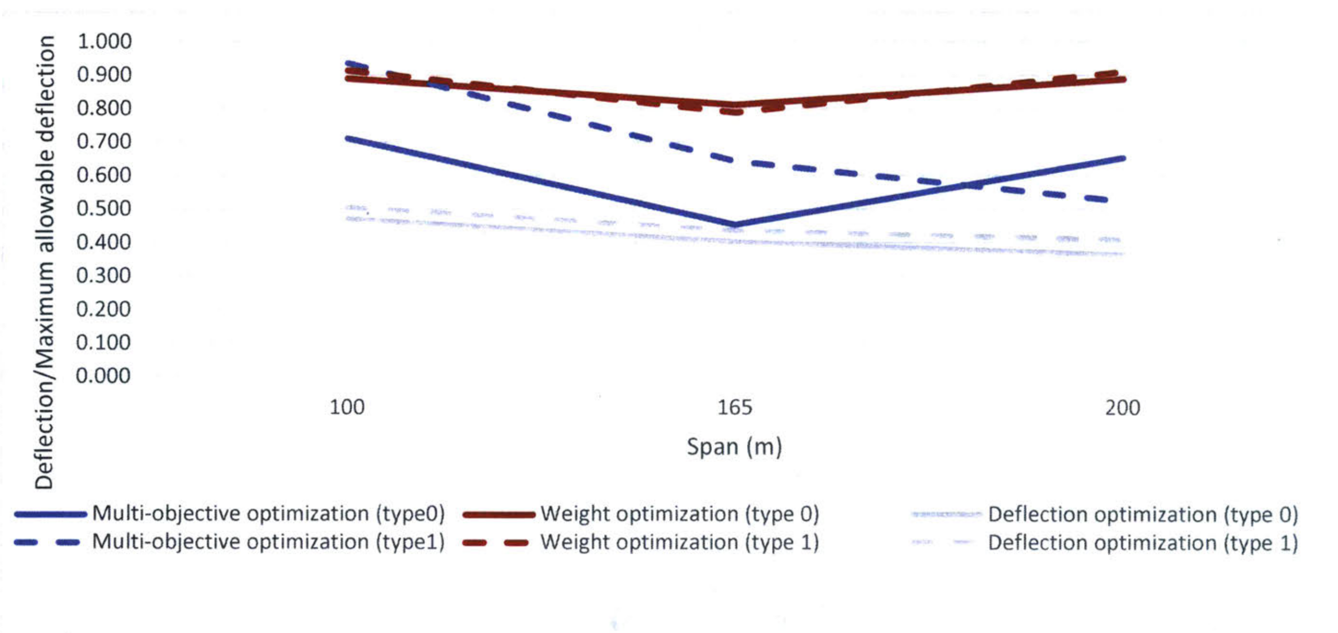


Figure 34 Deflection/Maximum allowable deflection ratio by span

None of the results given by the different optimization processes exceeds the maximum allowable deflection. The order of the results by optimization type follows the same order as the deflection with the deflection-oriented optimization giving the best result. However, the average ratio for this type of optimization is 40% while it is 3.8% for the 2D case (Figure 31). The controlling load case causes this difference. For the 2D case, the live load generates the maximum deflection in the structure while the wind load generates it for the 3D case.

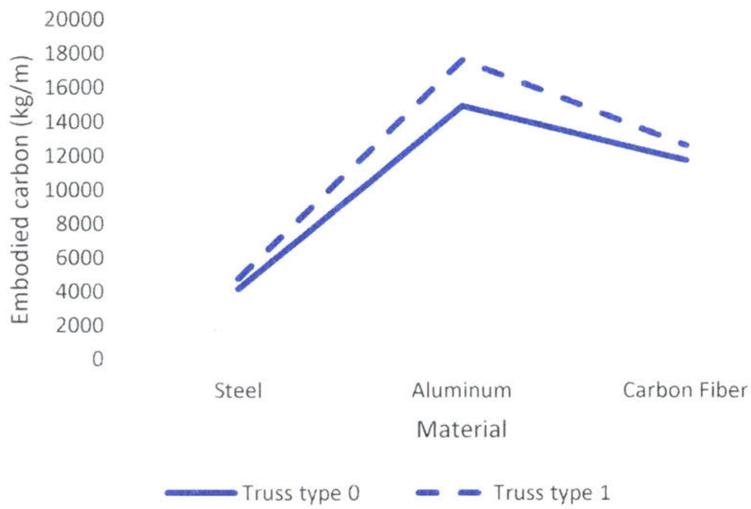


Figure 35 Embodied carbon (kg/m) by material for a span = 165m

The results of the graphic above (Figure 35) are the results of the analysis of the structure for two additional materials: aluminum and carbon fiber. The results of the study are available in Appendix C and Appendix D. The steel option generates a high amount of carbon emissions 4120.6kg/m for truss type 0 and 4738.3kg/m for truss type 1. However, it generates 63% less carbon in average compared to carbon fiber and 72.6% compared to aluminum. Choosing steel is thus more considerate towards the environment as it emits less carbon.

4.3 Discussion and recommendations

The arch of the Garabit Viaduct weighs 1,185,701 kg and the deck weighs 2,140,713 kg. However, the deck is 564.65m long in total [16,17]. The deck above the arch is 165m long so it weighs 625,551.5 kg. The part considered of the bridge weighs 1,811,252.5 kg originally. The weight-oriented optimization gives a structure that weighs 566,575.5 kg. This represents almost 69% of savings. The big gap in the weight is the result of the optimization process and the use of steel which is stronger than wrought iron used in the original bridge.

Both the 2D and 3D case showed that a bigger depth at the crown and more truss webs should be used for better results for the deflection-oriented optimization. Besides, the piers should be wider and have a denser truss for this type of optimization. Doing the opposite of these recommendations will lead to increasing the deflection but the bridge will get lighter.

To resist the wind loads, the bottom arch width should be around 65% larger than the top arch depth for all different cases if the span is higher than 100m. Finally, the truss type 1 is usually a better choice as it generates a lighter structure in most cases while respecting the maximum allowable deflection. To make the use of this type more efficient, more piers should be used to transfer the load from the deck to the arch.

Chapter 5: Conclusions

5.1 Summary of contributions

This thesis tackles a new way of optimizing trussed arch bridges in comparison with the existing literature by increasing the design space and the design variables. It analyzes the influence of changing the geometry and topology on the structural performance and self-weight of arch bridges. The two parametric models constructed using Grasshopper enabled their finite element analysis and optimization using Goat and MOO.

Comparing the different resulting designs from an aesthetic perspective showed a clear pattern characterizing each type of optimization. When the optimization leans more towards decreasing the deflection, the design has larger depth at the crown and fewer truss webs. However, the more weight is important in the optimization process, the narrower the crown is and the more truss webs there are.

This thesis proves once again that there is a trade-off relationship between structural weight and deflection as we can not improve one without deteriorating the performance of the other. This effect is shown by the plot positions for the different objectives.

It is easier to get a lighter design by using the type 1 truss because it uses fewer elements. However, more care should be put into using this type of truss as the deflection can easily get higher if neglected while choosing the design. This process usually leads to better results as it was demonstrated by the multi-objective optimization since the type 1 had better results for both weight and deflection using this optimization process for the design.

The optimization enables to save up to 69% of weight compared to the original material. Moreover, steel is the best material for trussed arch bridges. Steel is structurally efficient but it also helps to save 63% of carbon emissions in average compared to carbon fiber and 72.6% compared to aluminum.

5.2 Potential impact

This thesis demonstrates that geometry has a major influence on the performance of arch bridges. The results and conclusions made at the end of the analysis show the different geometrical patterns according to single-objective or multi-objective optimization. These results can be used as a starting point to design trussed arch bridges in the future.

5.3 Future work

The work realized in this thesis could be deepened by studying the impact of every single parameter on the maximum deflection and the structural weight of the arch bridge. It is also possible to replicate an exact 3D model of the Garabit Viaduct and analyze it using the same tools to compare the results. In addition, considering wrought iron and replicating the loads used for the analysis of the Garabit Viaduct will give new interesting results as it is a more direct comparison.

5.4 Concluding remarks

This thesis explores the use of new computational tools to study trussed arch bridges such as Grasshopper, Karamba, MOO, and Goat. These tools enable the optimization of the trussed arch bridges generating considerable amounts of weight and embodied carbon savings. These computational tools should be explored more to study more structures to visualize their behavior and understand how the difference parameters influence the structures' weight and structural stiffness.

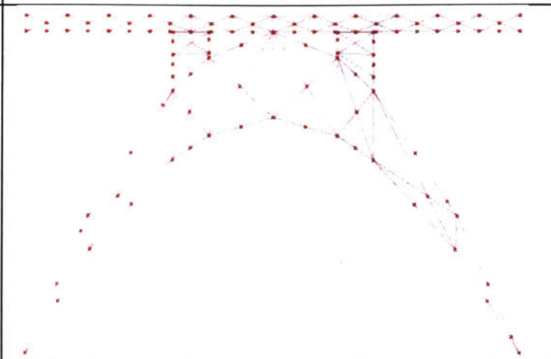
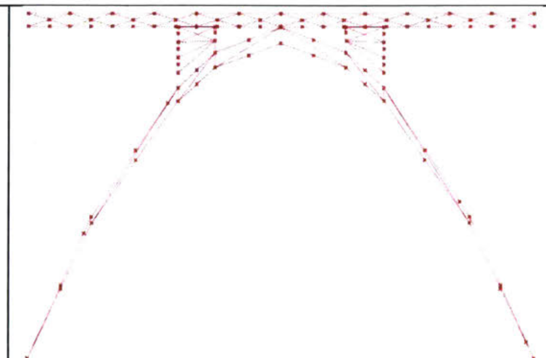
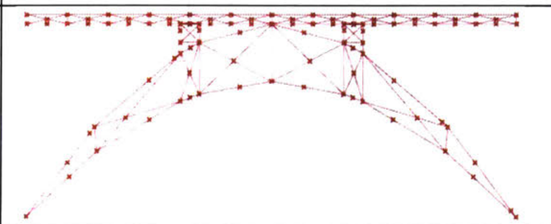
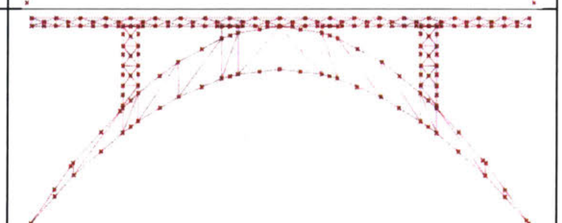
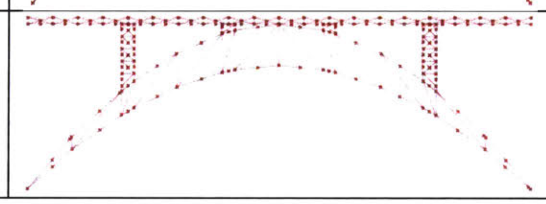
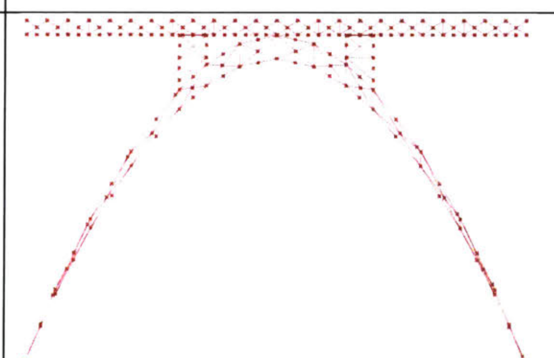
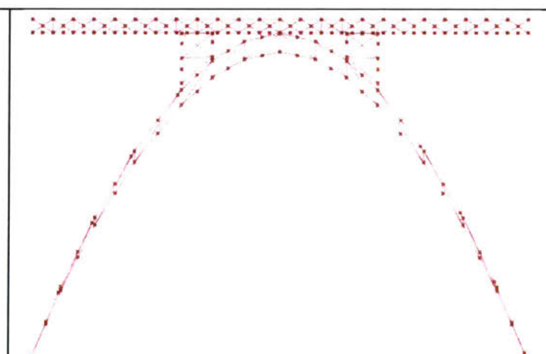
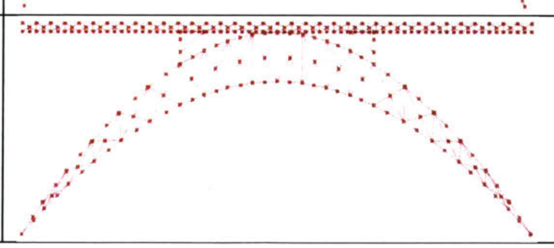
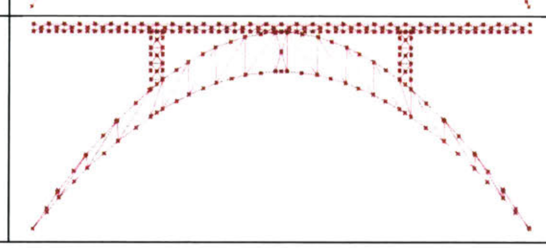
Appendix A: Cross sections catalog

Table 13. List of used cross sections

Profile	Diameter [cm]	Thickness [cm]	Diameter [cm]	Thickness [cm]
Hollow circle	5	0.25	105	5.25
	10	0.5	110	5.5
	15	0.75	115	5.75
	20	1	120	6
	25	1.25	125	6.25
	30	1.5	130	6.5
	35	1.75	135	6.75
	40	2	140	7
	45	2.25	145	7.25
	50	2.5	150	7.5
	55	2.75	155	7.75
	60	3	160	8
	65	3.25	165	8.25
	70	3.5	170	8.5
	75	3.75	175	8.75
	80	4	180	9
	85	4.25	185	9.25
	90	4.5	190	9.5
	95	4.75	195	9.75
	100	5	200	10

Appendix B: Case study #1 – steel trussed arch bridge (extra views)

Table 14. Front view

	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		
200 m		
Arch Span	Weight only	
100 m		
165 m		

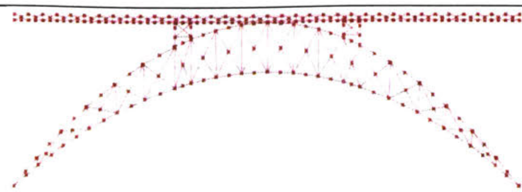
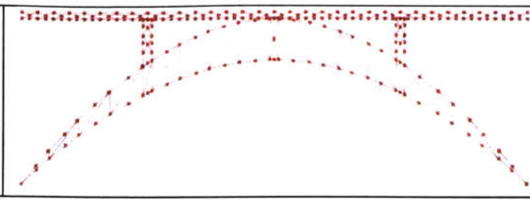
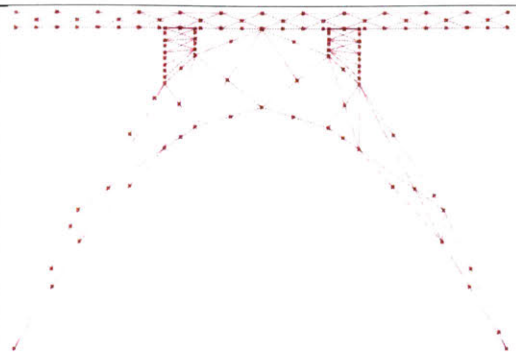
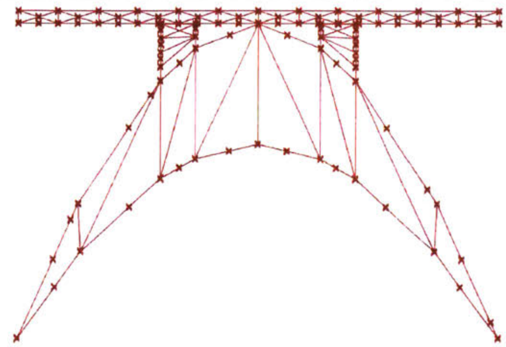
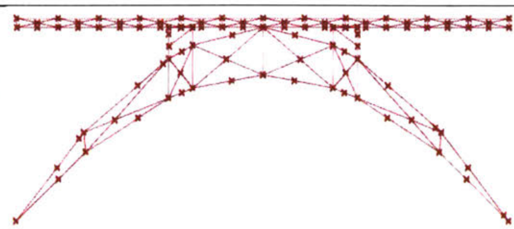
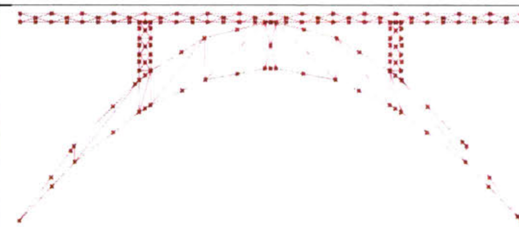
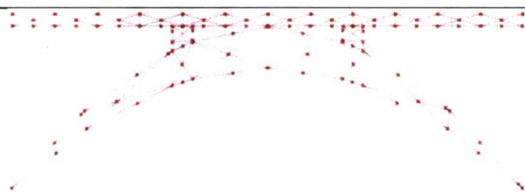
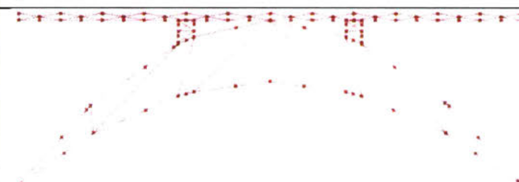
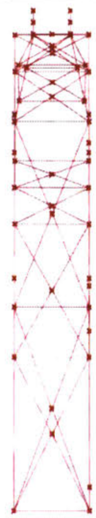
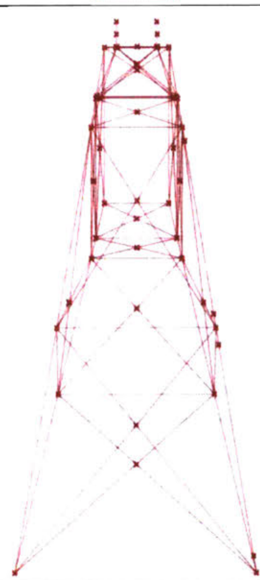
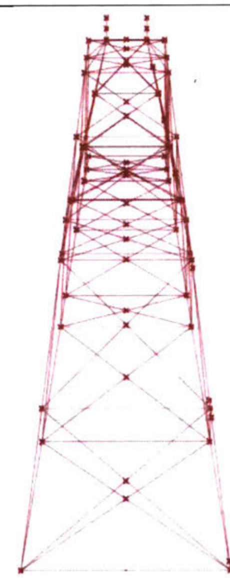
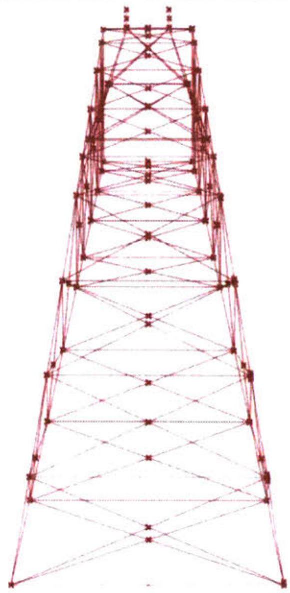
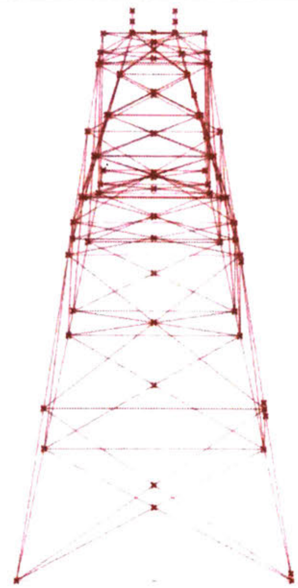
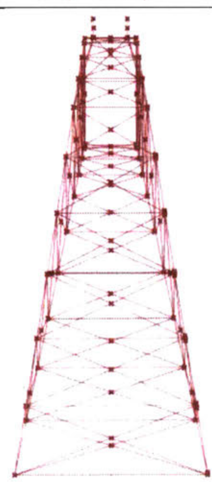
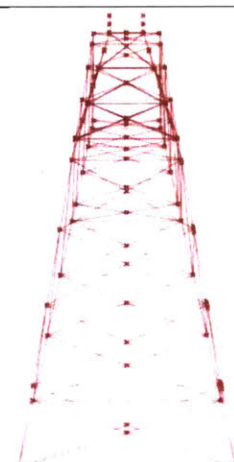
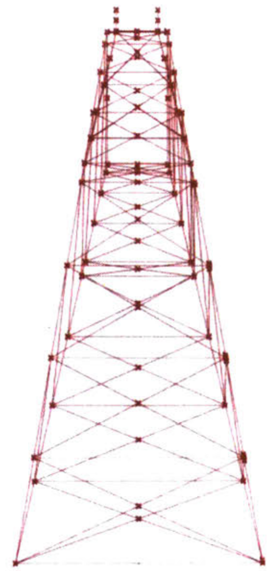
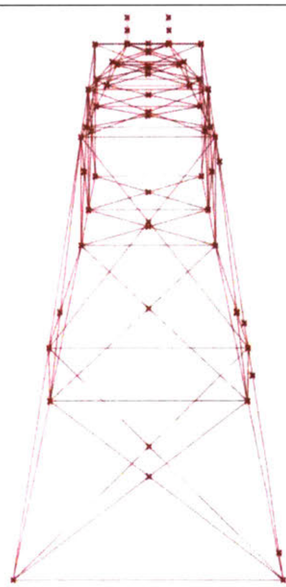
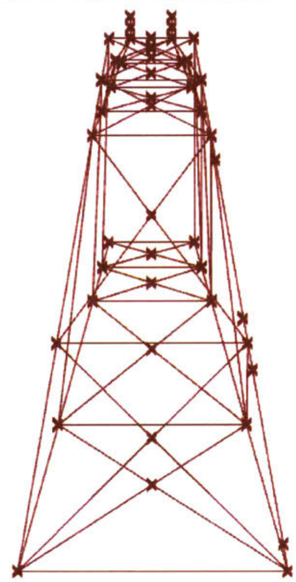
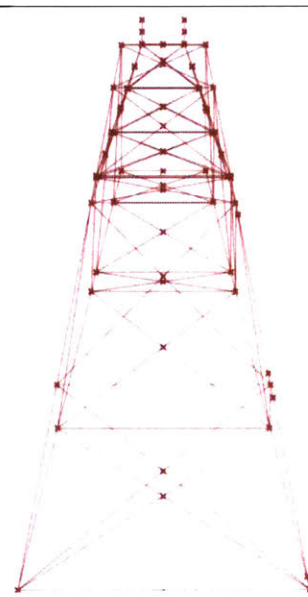
200 m		
Arch Span	Deflection only	
100 m		
165 m		
200 m		

Table 15. Side view

	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		

200 m		
Arch Span	Weight only	
100 m		
165 m		

200 m		
Arch Span	Deflection only	
100 m		
165 m		

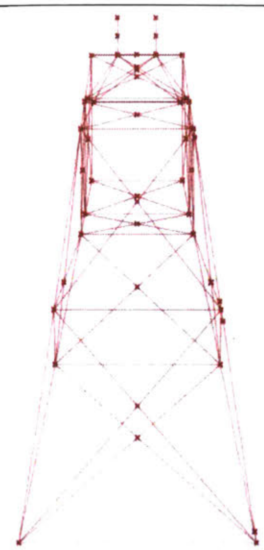
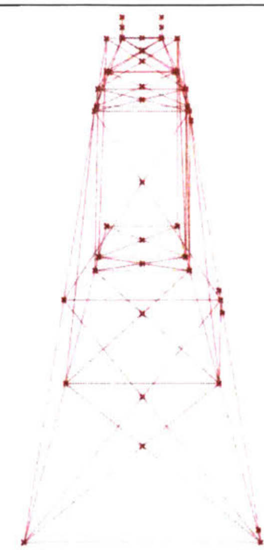
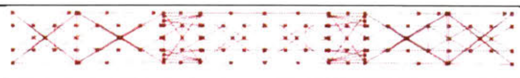
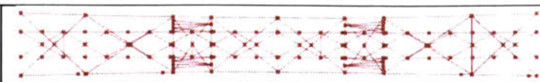
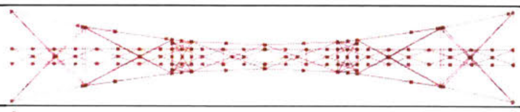
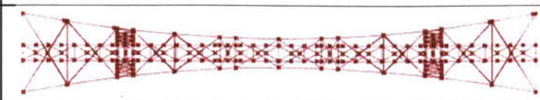
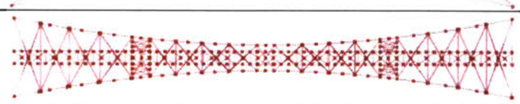
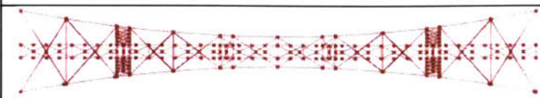
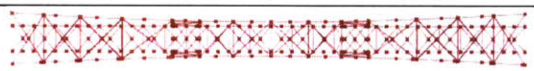
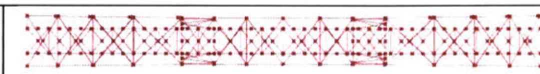
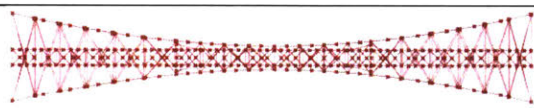
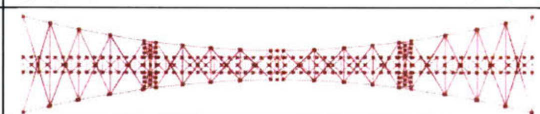
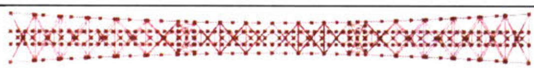
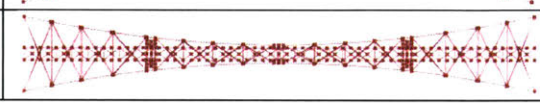
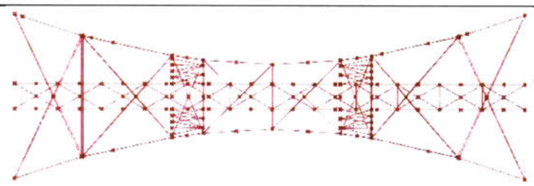
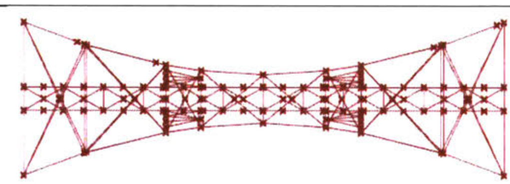
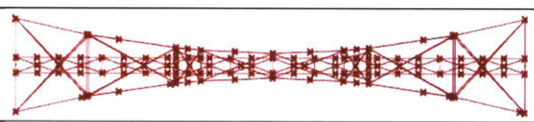
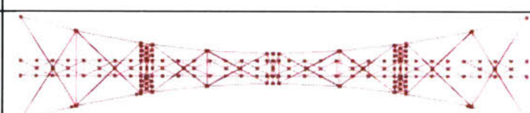
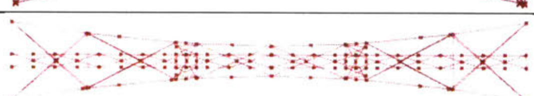
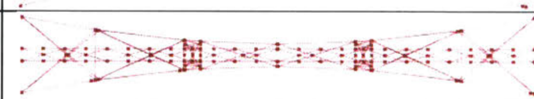
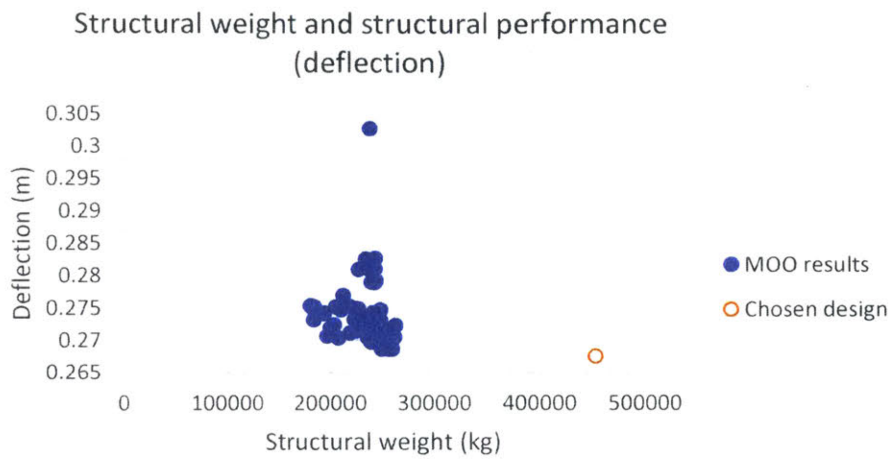
200 m		

Table 16. Top view

	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		
200 m		
Arch Span	Weight only	
100 m		
165 m		
200 m		
Arch Span	Deflection only	
100 m		
165 m		
200 m		

Appendix C: Case study #2 – aluminum trussed arch bridge

Table 17. Evolution of the deflection and the structural weight

<table>
<tr><th>Arch Span</th><th>Truss type 0</th></tr>
<tr><td>100m</td><td>
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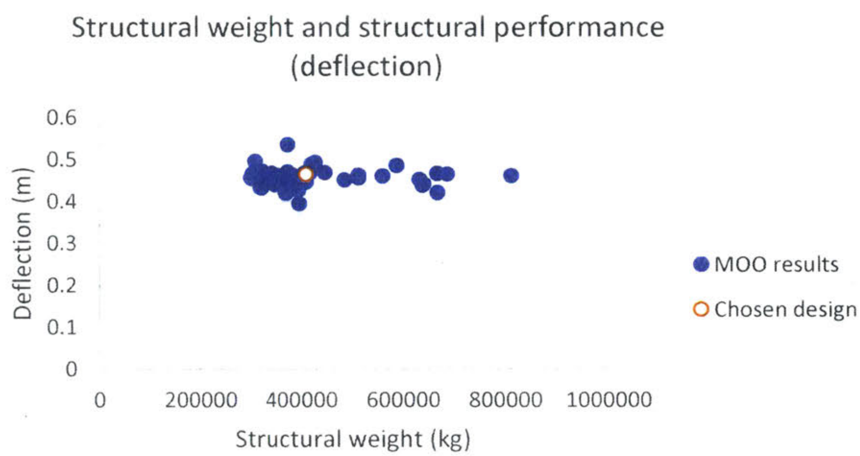
<tr><td>165m</td><td>
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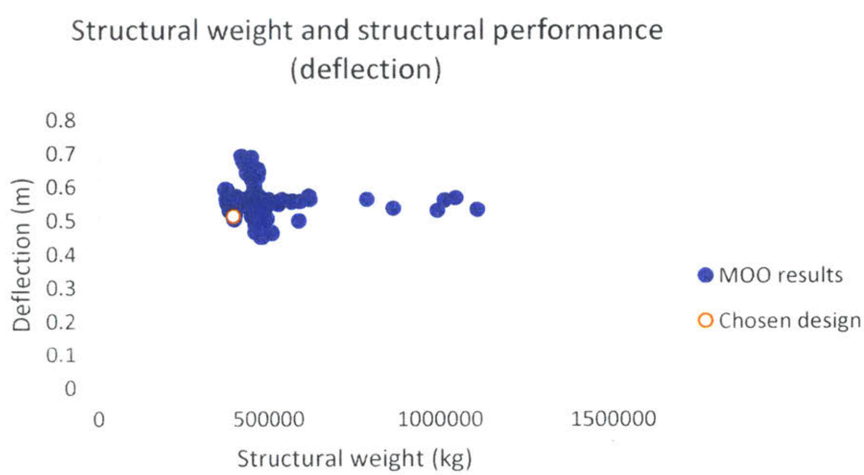
<tr><td>200m</td><td>
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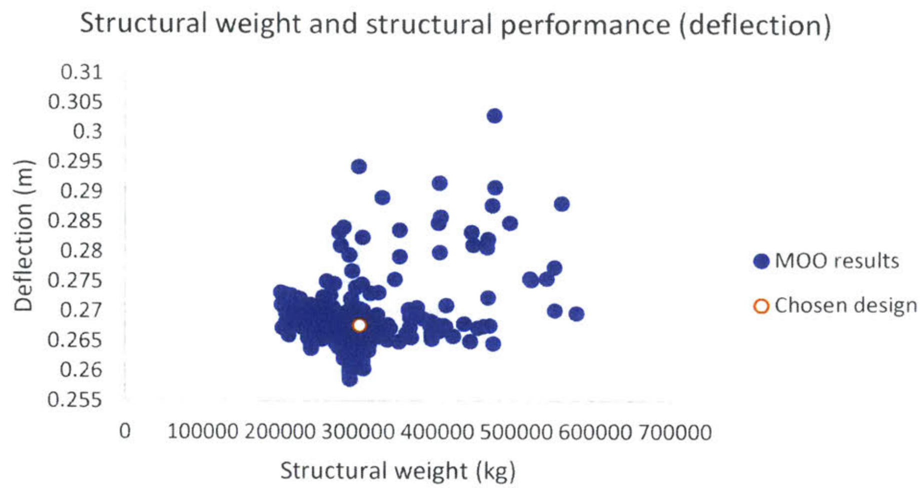
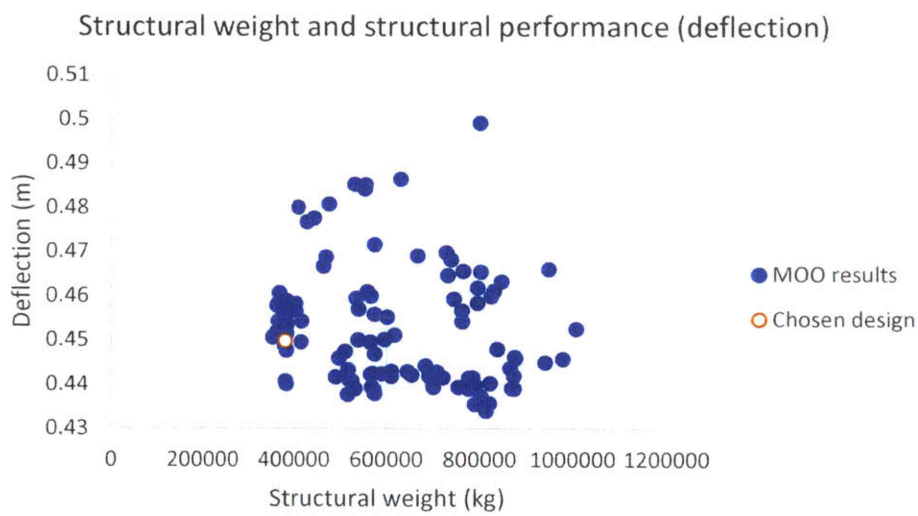
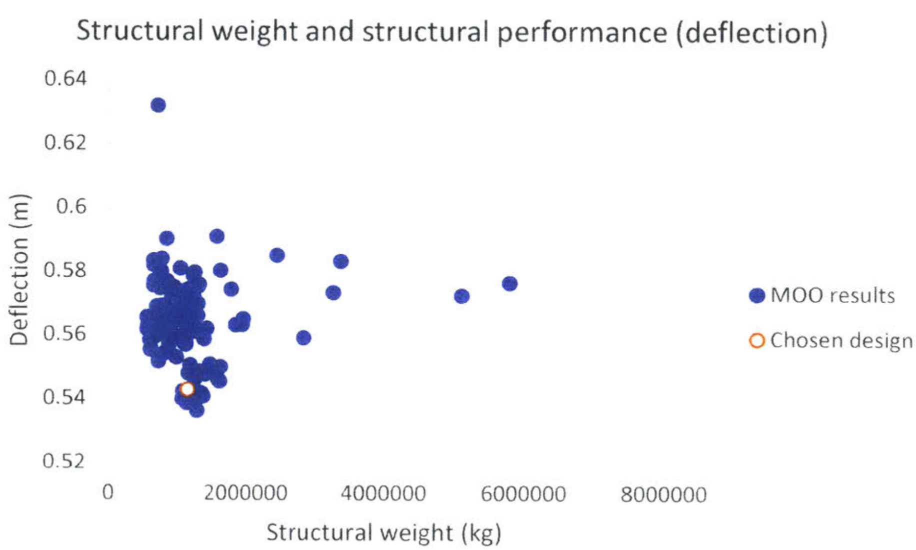
	Truss type 1
100m	
165m	
200m	

Table 18. Optimized designs for both structural weight and deflection (MOO)

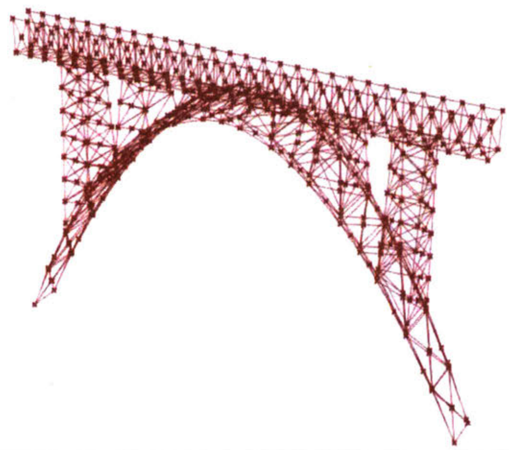
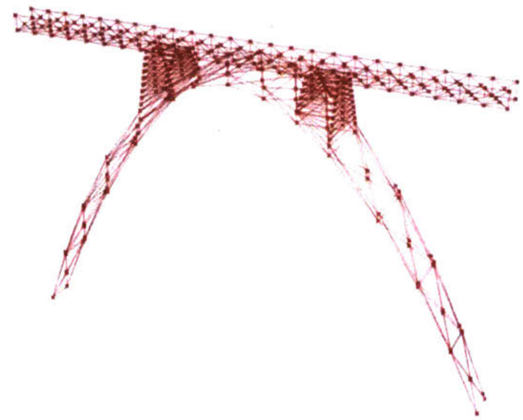
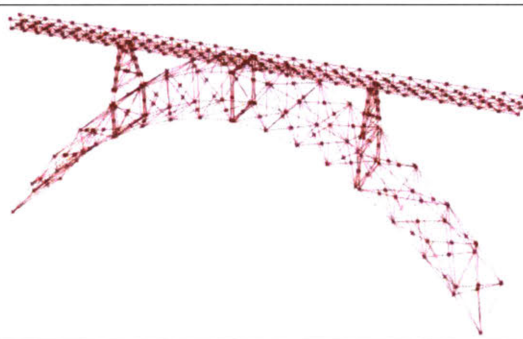
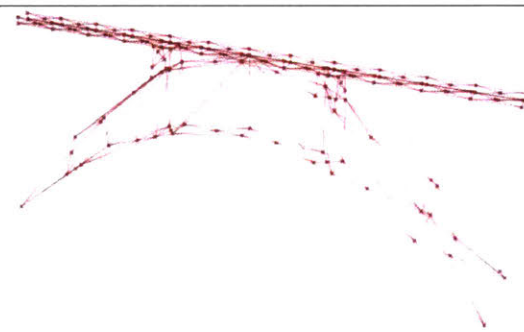
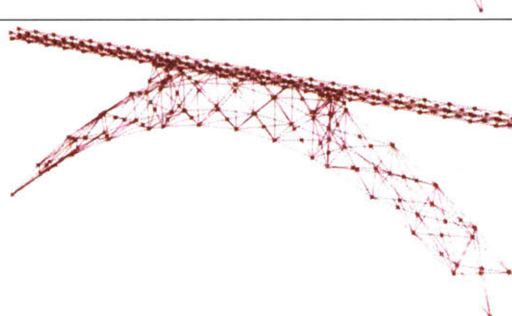
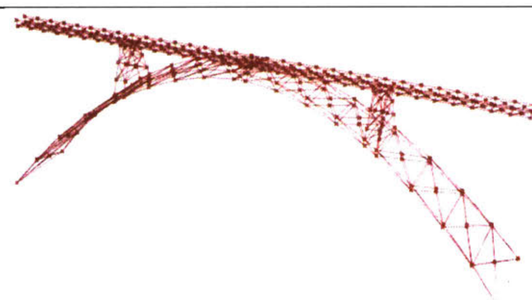
Arch Span	Truss type 0	Truss type 1
100 m		
165 m		
200 m		

Table 19. Optimized designs for structural weight only (Goat)

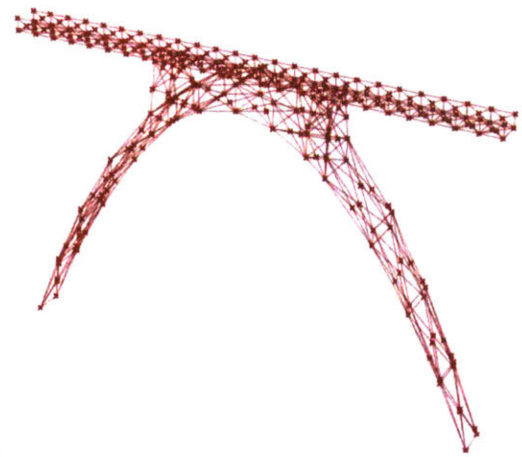
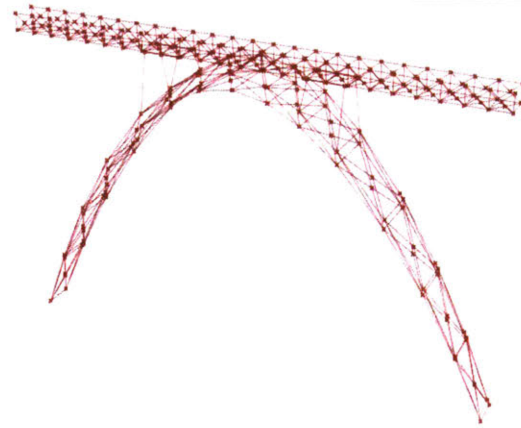
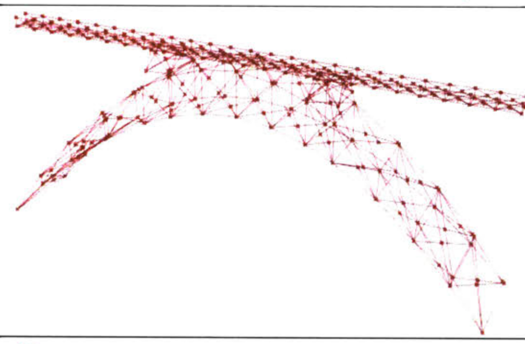
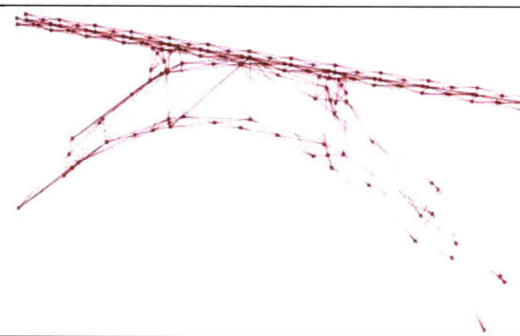
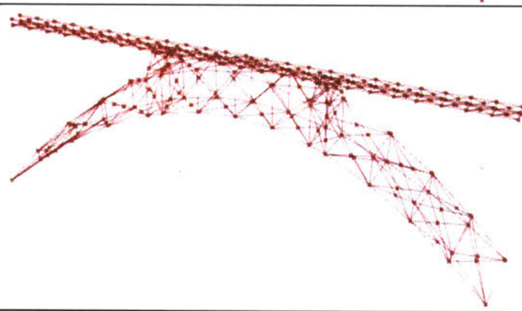
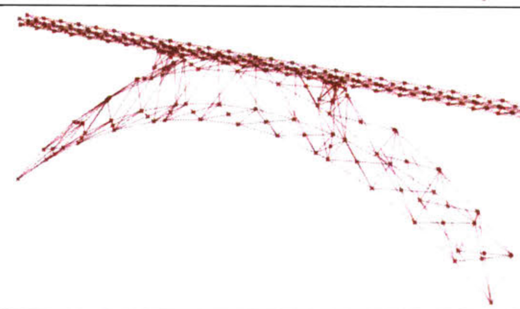
Arch Span	Truss type 0	Truss type 1
100m		
165m		
200m		

Table 20. Optimized designs for Deflection only only (Goat)

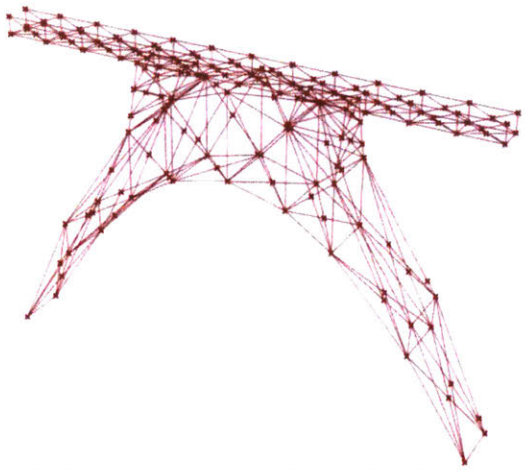
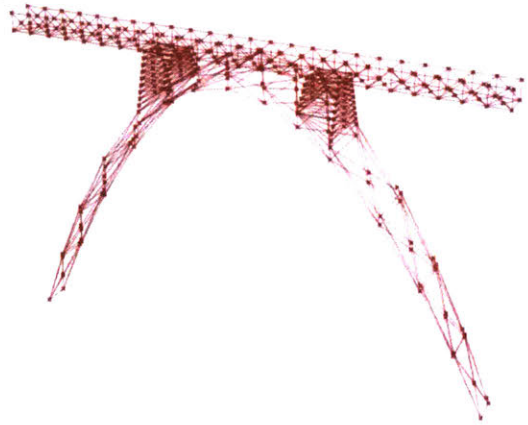
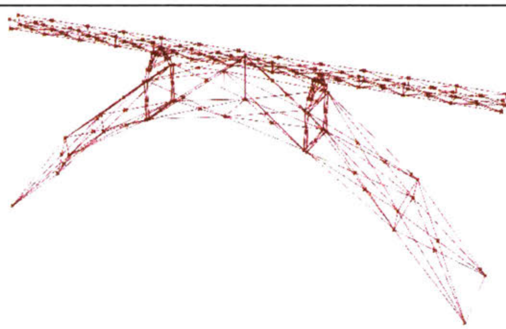
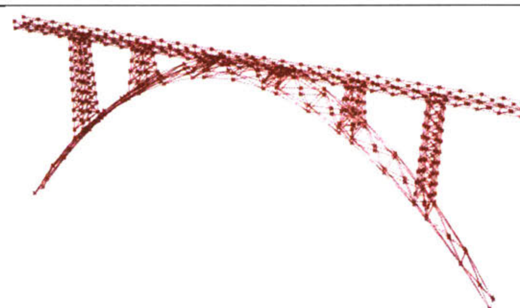
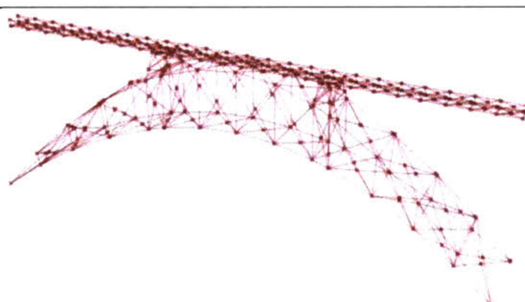
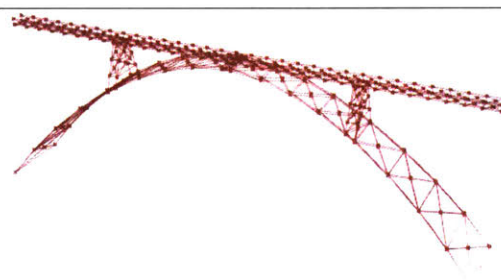
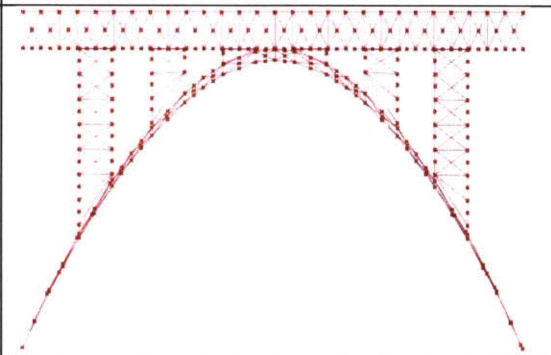
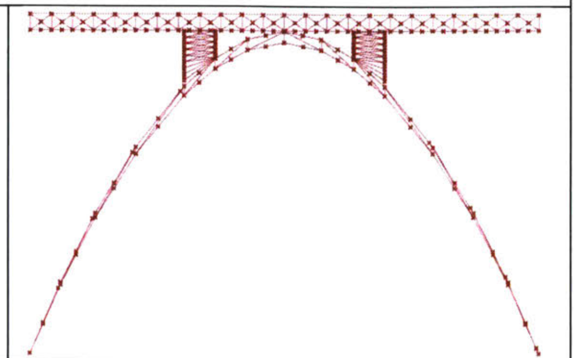
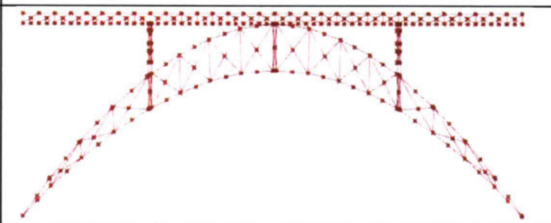
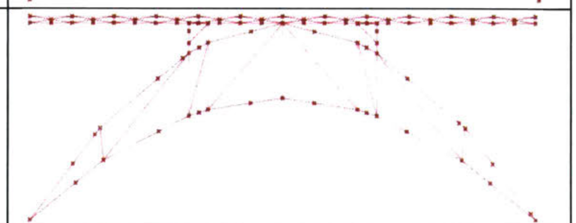
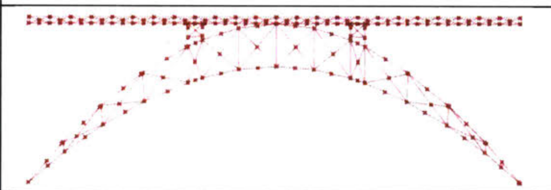
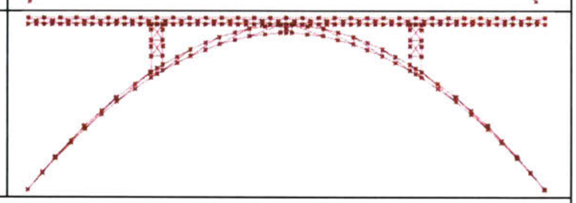
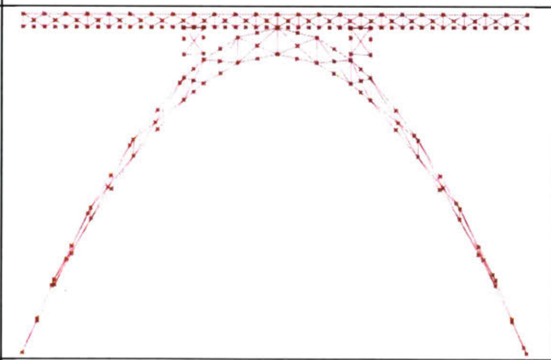
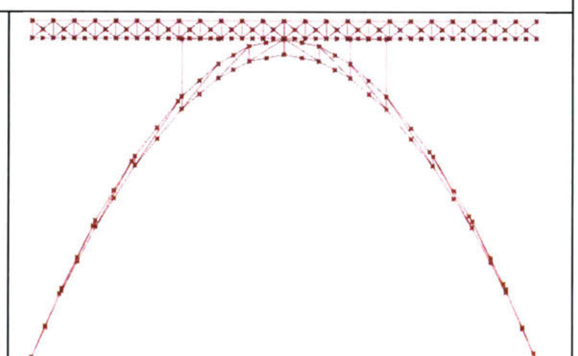
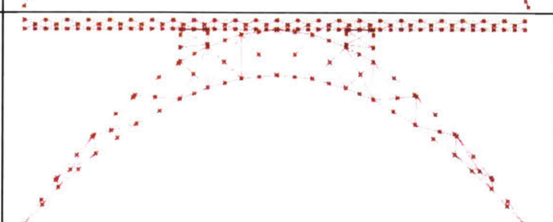
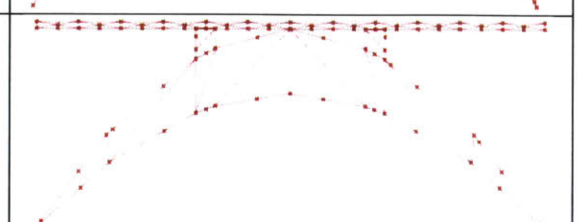
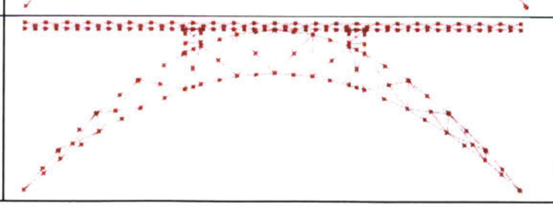
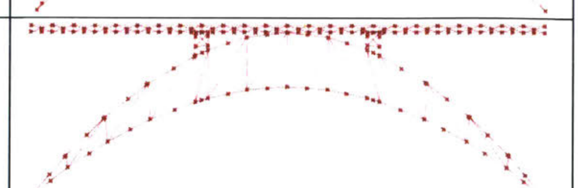
Arch Span	Truss type 0	Truss type 1
100 m		
165 m		
200 m		

Table 21. Front view

	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		
200 m		
Arch Span	Weight only	
100 m		
165 m		
200 m		

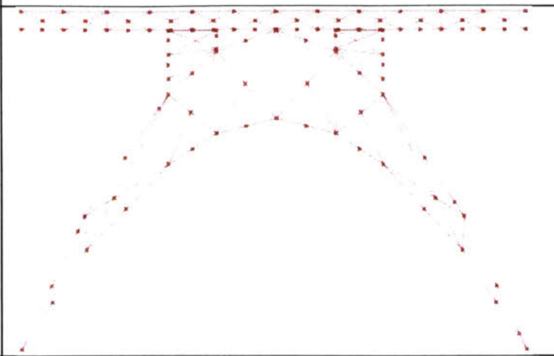
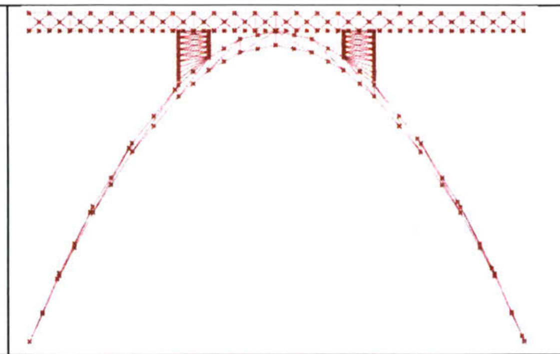
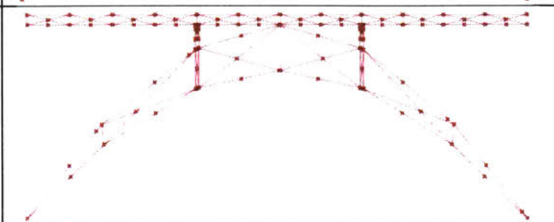
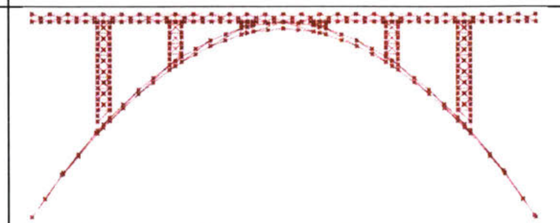
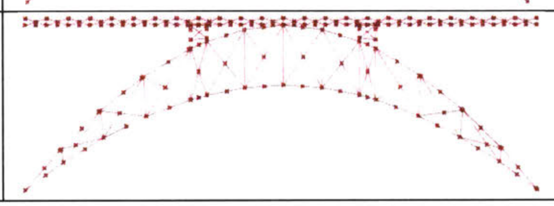
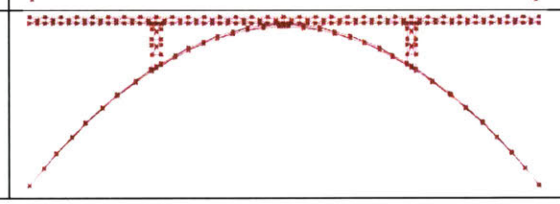
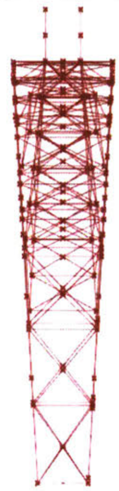
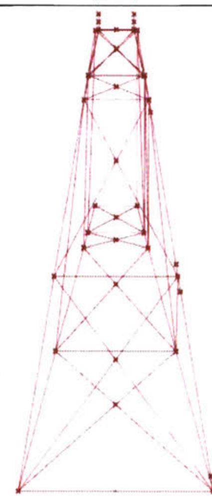
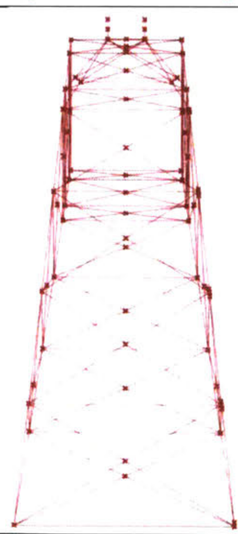
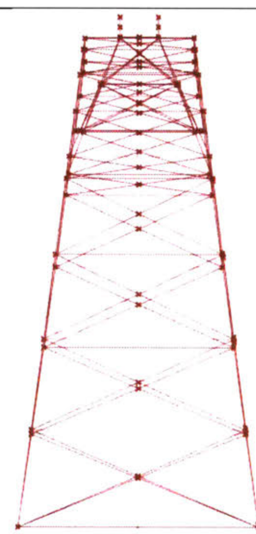
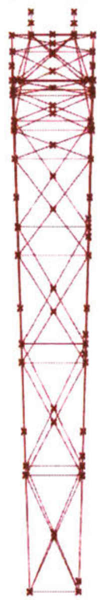
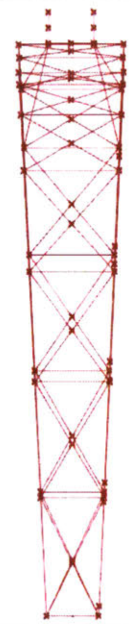
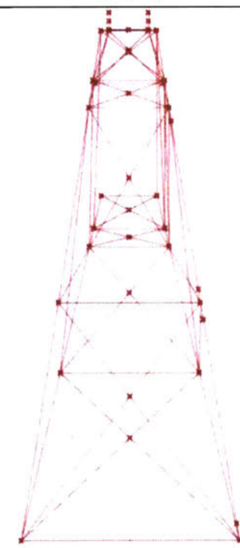
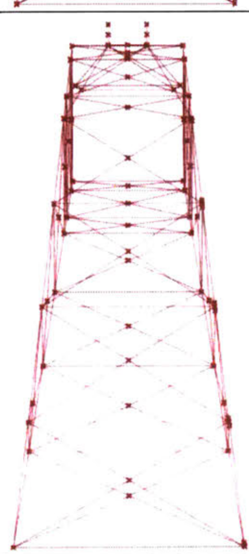
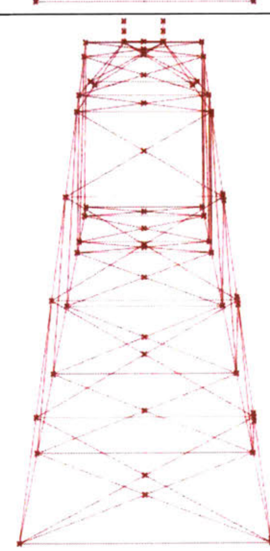
Arch Span	Deflection only	
100 m		
165 m		
200 m		

Table 22. Side view

	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		
200 m		

Arch Span	Weight only	
100 m		
165 m		
200 m		

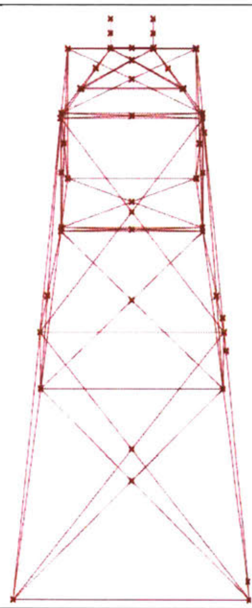
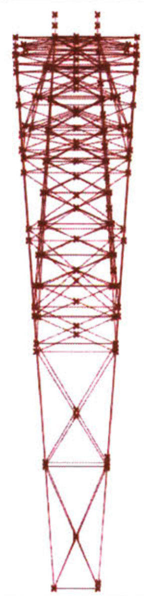
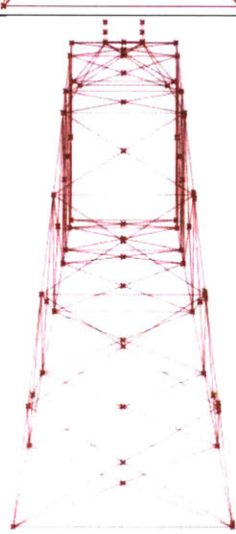
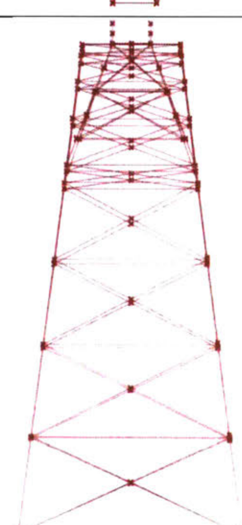
Arch Span	Deflection only	
100 m		
165 m		
200 m		

Table 23. Top view

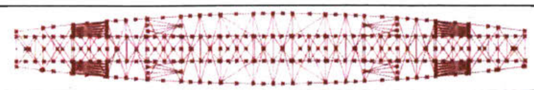
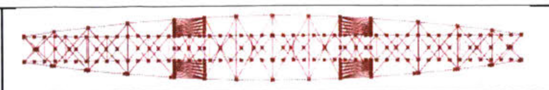
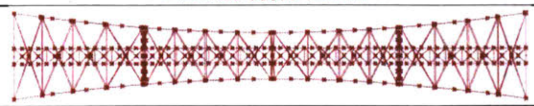
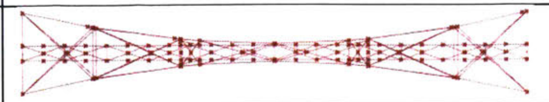
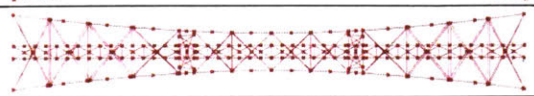
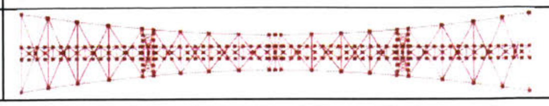
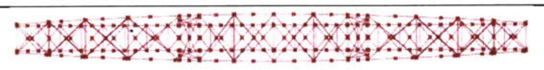
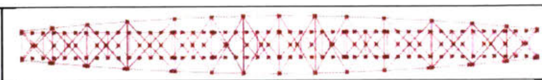
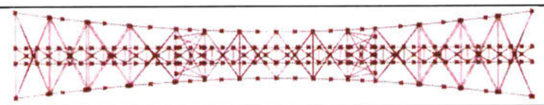
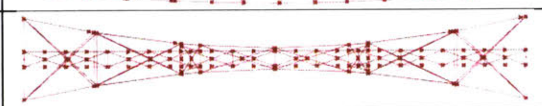
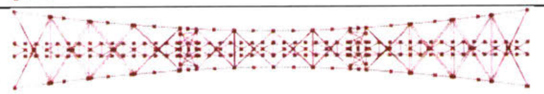
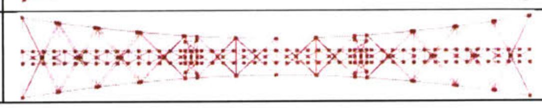
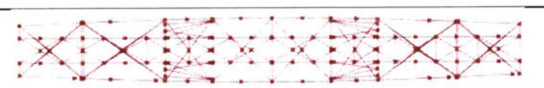
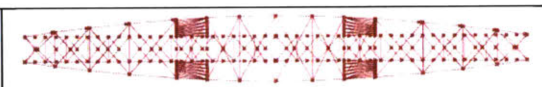
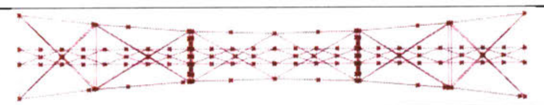
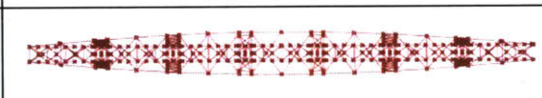
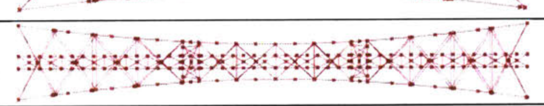
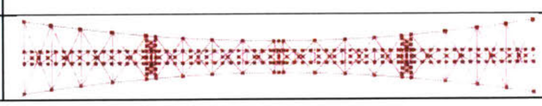
	Truss type 0	Truss type 1
Arch Span	Multiobjective optimization	
100 m		
165 m		
200 m		
Arch Span	Weight only	
100 m		
165 m		
200 m		
Arch Span	Deflection only	
100 m		
165 m		
200 m		

Table 24. Numerical results of the different types of optimization

Span [m]	Truss	Deck height [m]	Arch bottom width [m]	Arch top width [m]	Arch crown depth [m]	Number of piers	Number of truss webs per division	Pier width [m]	Pier division distance [m]	Weight [kg]	Deflection [m]
Multi-objective optimization											
100	Truss type 0	7.445	7.171	14.103	2.201	6	2	6.502	5.121	453344.502	0.267
165	Truss type 0	3.432	28.020	16.680	15.600	3	4	1.050	6.942	409724.453	0.463
200	Truss type 0	2.647	29.630	14.848	17.551	2	4	5.896	6.675	395248.115	0.513
100	Truss type 1	3.212	5.349	13.293	2.241	2	4	6.114	1.260	301813.283	0.268
165	Truss type 1	2.243	27.276	5.909	24.971	2	2	6.261	6.669	380468.481	0.450
200	Truss type 1	2.685	31.358	14.257	3.165	3	4	4.439	6.676	906585.923	0.524
Structural weight only											
100	Truss type 0	2.653	5.722	9.450	5.313	2	4	3.900	5.522	181545.803	0.275
165	Truss type 0	3.064	28.021	16.680	15.114	2	4	9.071	6.942	299143.327	0.455
200	Truss type 0	2.686	29.957	14.317	17.560	2	4	6.300	6.679	371252.201	0.590
100	Truss type 1	3.547	5.748	11.566	3.153	2	4	7.175	0.012	200385.308	0.273
165	Truss type 1	2.242	27.486	7.026	21.255	2	2	6.261	6.984	353810.166	0.451
200	Truss type 1	2.685	32.151	14.753	21.415	2	4	4.751	6.677	509286.496	0.573
Deflection only											
100	Truss type 0	3.690	10.128	12.780	17.752	2	2	9.471	5.119	262952.748	0.244
165	Truss type 0	3.420	27.573	15.425	15.210	2	2	1.086	5.872	396670.447	0.393
200	Truss type 0	2.685	29.590	14.107	23.860	2	4	6.344	6.686	479912.101	0.450
100	Truss type 1	3.703	5.220	13.293	2.899	2	4	6.114	1.260	289826.423	0.259
165	Truss type 1	2.590	5.179	14.190	2.188	6	2	3.962	3.695	814561.780	0.434
200	Truss type 1	2.654	29.223	12.852	1.145	3	4	3.622	6.731	1097309.891	0.486

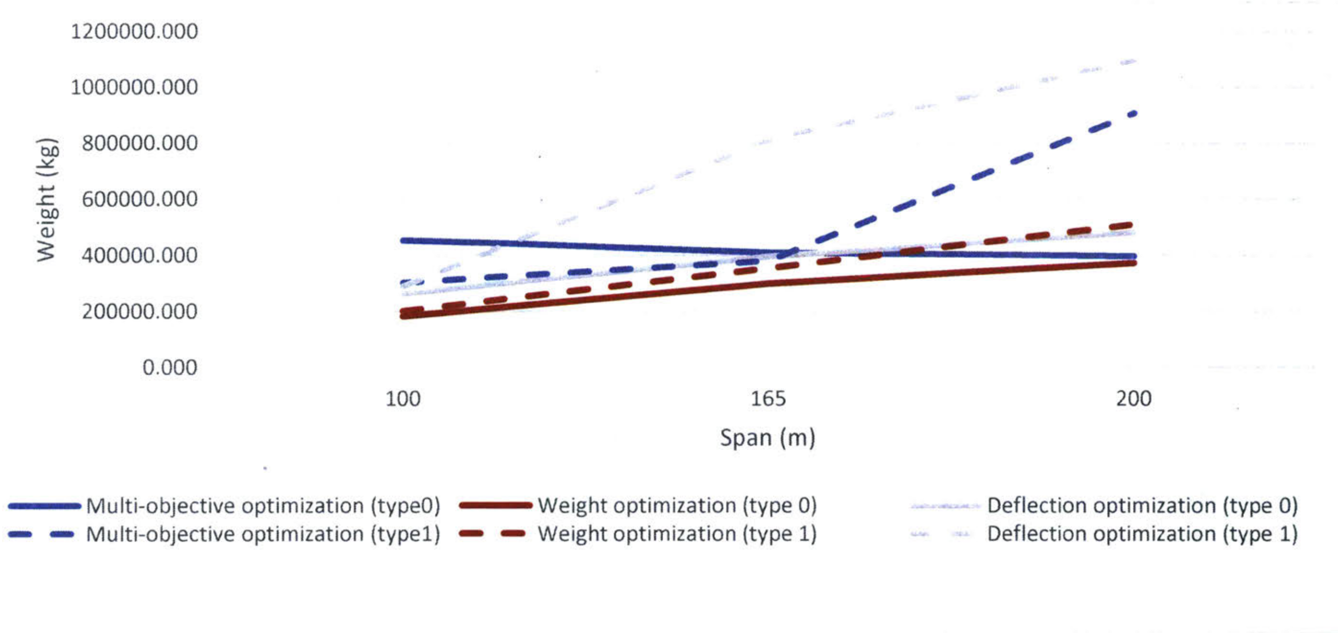


Figure 37 Weight of Optimal Design by Span

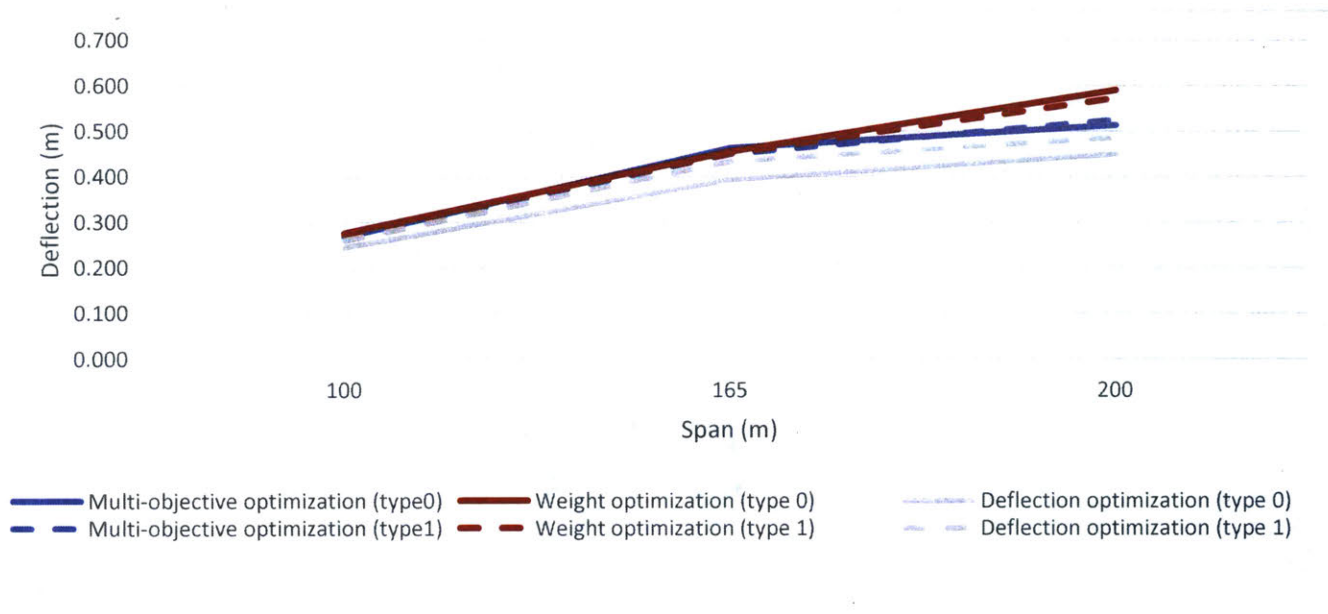


Figure 36 Deflection of Optimal Desgin by Span

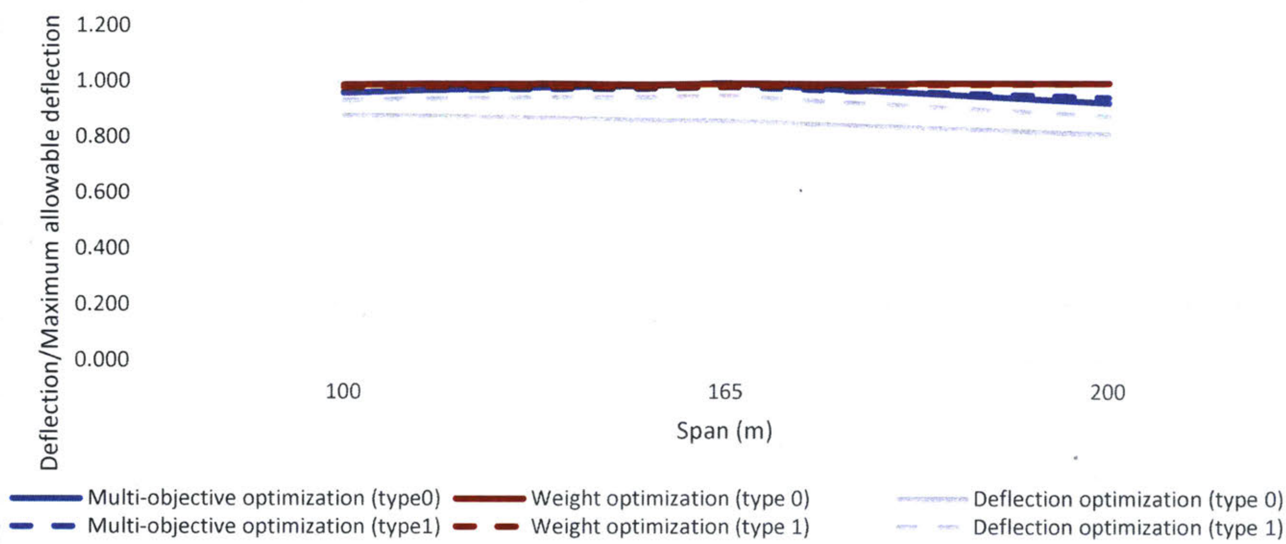


Figure 38 Deflection/Maximum allowable deflection ratio by span

Appendix D: Case study #3 – carbon fiber trussed arch bridge

Table 25. The results of the weight-oriented optimization for the span = 165m

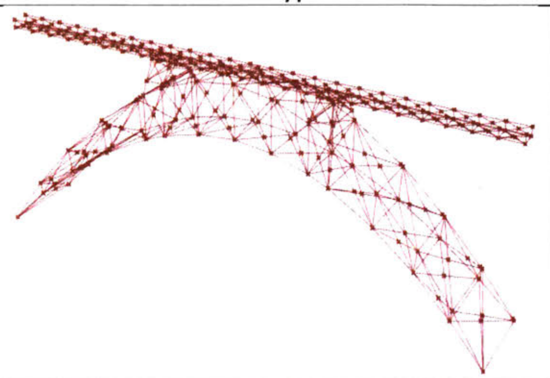
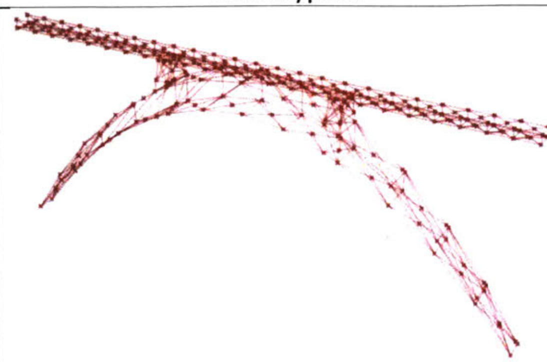
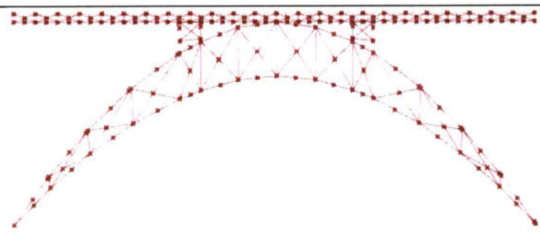
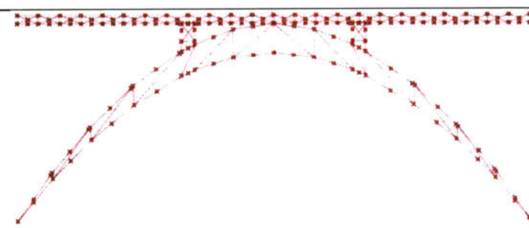
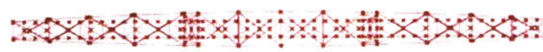
	Truss type 0	Truss type 1
Perspective		
Front view		
Side view		
Top view		

Table 26. Numerical results for weight-oriented optimization (span = 165m)

Span [m]	Truss	Deck height [m]	Arch bottom width [m]	Arch top width [m]	Arch crown depth [m]	Number of piers	Number of truss webs per division	Pier width [m]	Pier division distance [m]	Weight [kg]	Deflection [m]
Structural weight only											
165	type 0	2.732	26.666	13.914	17.530	2	4	6.646	6.867	128864.375	0.238
165	type 1	3.547	5.748	11.566	3.153	2	4	7.175	0.012	200385.308	0.273

References

1. Prévost, Jean. Eiffel. Paris: les Editions Rieder, 1929.
2. Thrall, A.p., D.p. Billington, and K.l. Bréa. "The Maria Pia Bridge: A Major Work of Structural Art." Engineering Structures 40 (2012): 479-86. doi:10.1016/j.engstruct.2012.02.032.
3. Kurrer, Karl-Eugen. "The History of the Theory of Structures: From Arch Analysis to Computational Mechanics." Berlin: Ernst & Sohn, 2008.
4. Giedion, Sigfried. "Space, Time and Architecture". 1941.
5. Eiffel, Gustave. "Mémoire Sur Le Viaduc De Garabit; Description, Calculs De Résistance, Montage, Epreuves, Et Renseignements Divers. "Paris: Baudry, 1889.
6. Eiffel, Gustave. "Notice Sur Le Viaduc Du Garabit: (près Saint-Flour) Ligne De Marvejols a Neussargues". Paris: Imprimerie Administrative & Des Chemins De Fer De Paul Dupont ..., 1888.
7. Donnaes, Philippe. "Les viaducs de Garabit et Millau frappent à la porte de l'Unesco." Le Moniteur. January 25, 2012. Accessed December 23, 2016. http://www.lemoniteur.fr/article/les-viaducs-de-garabit-et-millau-frappent-a-la-porte-de-l-unesco-16308391.
8. http://www.grasshopper3d.com/
9. Preisinger, C. (2013), Linking Structure and Parametric Geometry. Archit Design, 83: 110-113. doi: 10.1002/ad.1564.
10. http://www.rechenraum.com/en/goat/overview.html
11. http://digitalstructures.mit.edu/page/tools#moo
12. Makiabadi, M. H., et al. "Optimal design of truss bridges using teaching-learning-based optimization algorithm." Iran University of Science & Technology 3.3 (2013): 499-510.
13. Cheng, Jin. "Optimum design of steel truss arch bridges using a hybrid genetic algorithm." Journal of Constructional Steel Research 66.8 (2010): 1011-1017.
14. Pouraminian, Majid, and Mohsen Ghaemian. "Shape optimisation of concrete open spandrel arch bridges." GRADEVINAR 67.12 (2015): 1177-1185.
15. Ruiyi, Su, Gui Liangjin, and Fan Zijie. "Truss topology optimization using genetic algorithm with individual identification." Proceedings of the world congress on engineering. Vol. 2. 2009.
16. Achtziger, Wolfgang, and Mathias Stolpe. "Global optimization of truss topology with discrete bar areas—Part II: Implementation and numerical results." Computational Optimization and Applications 44.2 (2009): 315-341.
17. Šešok, Dmitrij, and Rimantas Belevičius. "Global optimization of trusses with a modified genetic algorithm." Journal of Civil Engineering and management 14.3 (2008): 147-154.
18. Šešok, D., and R. Belevičius. "Modified genetic algorithm for optimal design of truss structures." Mechanika,-Kaunas: Technologija 6 (2007): 68.
19. O'Brien, Eugene J., Damien L. Keogh, and Alan J. O'Connor. Bridge deck analysis. Boca Raton: CRC Press/Taylor & Francis Group, 2015. Print.
20. Guo, X., G. Cheng, and K. Yamazaki. "A new approach for the solution of singular optima in truss topology optimization with stress and local buckling constraints." Structural and Multidisciplinary Optimization 22.5 (2001): 364-373.

21. Stolpe, Mathias. "Truss topology optimization with discrete design variables by outer approximation." Journal of Global Optimization 61.1 (2015): 139-163.

22. Lute, Venkat, Akhil Upadhyay, and Krishna Kumar Singh. "Genetic algorithms-based optimization of cable stayed bridges." Journal of Software Engineering and Applications 4.10 (2011): 571.

23. Guan, Hong, et al. "Bridge topology optimisation with stress, displacement and frequency constraints." Computers & structures 81.3 (2003): 131-145.