NATURAL VIBRATIONS OF BEAM-LIKE TRUSSES

by

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ABSTRACT

The dynamics of the Structural Assembly Demonstration Experiment (SADE) truss, a beam-like space truss to be used in a proposed Space Shuttle flight experiment, are analyzed with finite element and continuum models. A two-dimensional model of the SADE truss and a more general two-dimensional truss are also analyzed. The results show that continuum models can be effective in estimating the modal frequencies obtained from the conventional finite element model with pin-jointed bar elements, even in higher modes. However, the continuum models and conventional pin-jointed finite element model do not account for the influence of individual bar vibrations on the global modes of a lattice structure, while the conventional finite element model with rigid-jointed bar elements and nodes only at bar endpoints does not accurately predict the closely spaced modes which are characterized by such bar vibrations. The results show that a refined rigid-jointed model with additional nodes at all bar midpoints is the simplest finite element model which accurately determines the dynamic modes of the lattice structures considered.

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NOMENCLATURE

An underbar denotes a matrix or vector.

A	Cross-sectional area
B	Y dimension of a truss bay
$\underline{C}$	Equivalent stiffness matrix
E	Young's Modulus
$\underline{f}$	Applied force vector
G	Shear modulus
H	Z dimension of a truss bay
$\underline{i}$	Unit vector
i_p	Polar mass moment of inertia per unit length
i_r	Rotary mass moment of inertia per unit length
I	Cross-sectional area moment of inertia
J	Cross-sectional polar area moment of inertia
$\underline{K}$	Finite element stiffness matrix
ℓ	Direction cosine
L	Length of a bar element or length of one bay
m	Mass per unit length
M	Applied moment
$\underline{M}$	Finite element mass matrix
N	Node number
P	Applied axial force
S	Applied shear force
t	Time
T	Applied torque
$\underline{T}$	Transformation matrix
u	Displacement in X direction
$\underline{u}$	Displacement vector
U	Strain energy
v	Displacement in Y direction
w	Displacement in Z direction
W	Work
xyz	Local Cartesian coordinates
XYZ	Global Cartesian coordinates

NOMENCLATURE (cont.)

δ	Denotes first variation
ε	Extensional or shearing strain
ε	Strain vector
θ	Rotation about X
κ	Curvature
Π	Total energy
ϕ	Rotation about Y
ψ	Rotation about Z
ω	Natural Frequency

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1. INTRODUCTION

Large space structures are becoming increasingly important to the exploration and development of space. Many of these structures, such as the proposed NASA Space Station, are lattice structures, and will be extremely large and flexible. Others, such as large antennae or the Space Telescope, require a high degree of pointing accuracy in addition to being very flexible, which may necessitate active control to suppress vibrations (reference 1).

Finding the dynamic characteristics of such structures is a challenging problem. Conventional finite element models are extremely large and therefore expensive to implement, which makes them undesirable or impractical, especially in an advanced design stage. Testing large space structures to find their dynamic characteristics is impractical for several reasons. Many will be larger than any existing testing facility, and most will not be able to support their own weight on Earth. Structures such as the Space Station will be built by various contractors which also makes testing impractical (reference 2).

Controlling large space structures is also a very difficult task. The problem is compounded by the occurrence of closely spaced dynamic modes which can cause instabilities if not considered in controlling the structure. In addition, the dynamic model chosen for the structure has a profound effect on the design and performance of the controller (reference 3).

This thesis is a study of the dynamics of the Structural Assembly Demonstration Experiment (SADE) truss. SADE is a proposed Space Shuttle experiment designed to test the assembly and deployment of structures in space. The SADE truss consists of seven cubical bays arranged linearly to form a beam-like structure extending from the Shuttle bay, with a tip mass at the free end. The truss resembles a shorter version of the long central mast in the current proposed Space Station design.

The purpose of this thesis is to evaluate several methods for finding the dynamic characteristics of the SADE truss and to determine the effects of individual bar vibrations on the dynamic modes of a beam-like truss. Several finite element models are set up first, to find the modes of the SADE truss. These results are considered to be a reliable basis for comparison with subsequent results. Next, several methods are employed to determine the stiffness properties of the truss. These stiffness properties are then used in continuum models, yielding bending, torsional, and axial frequencies for comparison with finite element results. The stiffness properties are also used to set up a stiffness matrix for one bay of the truss. This super-finite element is employed in constructing a much smaller finite element model of the SADE truss from which the dynamic modes of the truss are found again. To study the effects of bar vibrations on the modes of a beam-like truss, and to estimate the global modes of the SADE truss, a two-dimensional version of the SADE truss is considered. This planar truss is analyzed with two standard finite element models and with a refined finite element model with additional nodes at bar midpoints. Finally, a two-dimensional truss with no lumped masses is considered to more generally assess the effects of individual bar vibrations on the global modes of a truss. This truss is analyzed with two standard finite element models, a refined finite element model with additional nodes at bar midpoints, and a finite element model with exact dynamic stiffness coefficients.

2. FINITE ELEMENT METHODS

2.1 DESCRIPTION OF SADE TRUSS

The SADE truss consists of seven cubical bays arranged in a row to form a beam-like structure three-hundred and eighty-five inches long (see figure 1). The four nodes at the restrained end of the truss are pinned, so the structure resembles a cantilevered beam, although rotational degrees of freedom are allowed at these four restrained nodes. The arrangement of the bar elements can be seen in Figure 1 and is specified in Table 1.

The bar elements are made from 6061 aluminum. Each bar has a circular cross section with an outer diameter of 2.0 inches and a thickness of 0.072 inches. The length of the shorter bars is 55.0 inches (the length of one bay) while the length of the longer diagonal members is $55.0\sqrt{2}$ inches.

The mass of the truss is 442.00 lb_m. The mass of the bars is 259.65 lb_m. The shorter bars have a mass of 2.3411 lb_m while the mass of the longer bars is 3.3109 lb_m. The remaining 182.35 lb_m is allocated to the joints. The amount of joint mass placed at a specific node is proportional to the number of bar elements which are joined at that node (see Table 2). Thus, node 10 has twice as much joint mass as node 2 since eight bars are joined at node 10 as opposed to four bars at node 2. In addition, there is a tip mass of 220.46 lb_m at the free end of the truss. This tip mass is equally divided between the four nodes at the free end (nodes 29, 30, 31, and 32).

2.2 PIN-JOINTED MODEL

The first model of the SADE truss considered was a pin-jointed finite element model with a consistent mass matrix. This model allows three degrees of freedom at each node. The three degrees of freedom are the translational

FIGURE 1

The SADE Truss

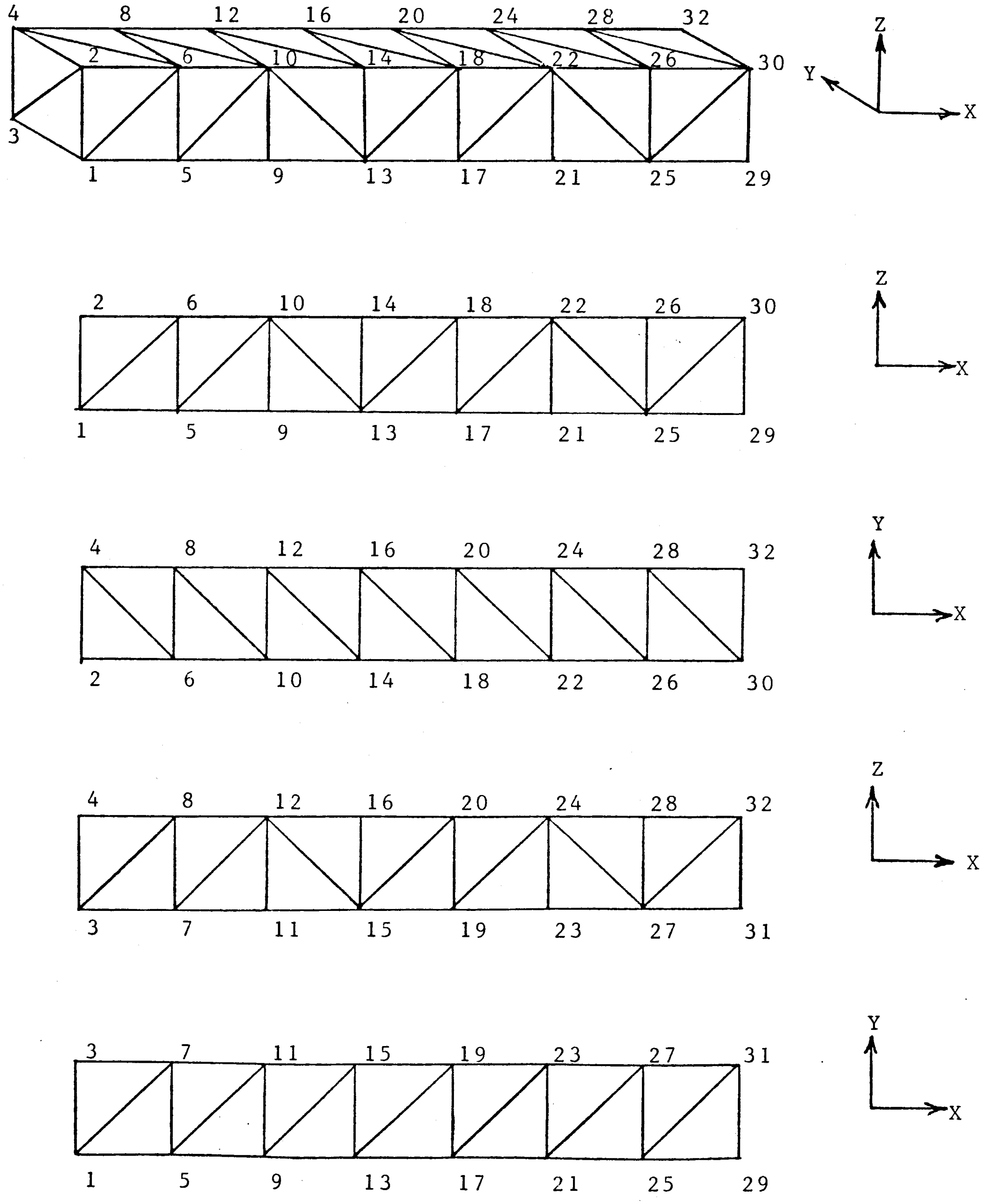

TABLE 1

SADE Bar Connectivities (From node i to node j)

1 - 2
1 - 3
1 - 5
1 - 6
1 - 7
2 - 3
2 - 4
2 - 6
3 - 4
3 - 7
3 - 8
4 - 6
4 - 8
5 - 6
5 - 7
5 - 9
5 - 10
5 - 11
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29 - 31
30 - 31
30 - 32
31 - 32

TABLE 2

SADE Joint Masses

Node	Joint Mass (lb-sec^2/in)	Node	Joint Mass (lb-sec^2/in)
1	1.2290x10^{-2}	17	1.4747x10^{-2}
2	9.832 x10^{-3}	18	1.7205x10^{-2}
3	1.2290x10^{-2}	19	1.7205x10^{-2}
4	9.832 x10^{-3}	20	1.4747x10^{-2}
5	1.4747x10^{-2}	21	1.2290x10^{-2}
6	1.7205x10^{-2}	22	1.9663x10^{-2}
7	1.7205x10^{-2}	23	1.4747x10^{-2}
8	1.4747x10^{-2}	24	1.7205x10^{-2}
9	1.2290x10^{-2}	25	1.7205x10^{-2}
10	1.9663x10^{-2}	26	1.4747x10^{-2}
11	1.4747x10^{-2}	27	1.9663x10^{-2}
12	1.7205x10^{-2}	28	1.2290x10^{-2}
13	1.7205x10^{-2}	29	7.374 x10^{-3}
14	1.4747x10^{-2}	30	1.4747x10^{-2}
15	1.9663x10^{-2}	31	1.2290x10^{-2}
16	1.2290x10^{-2}	32	9.832 x10^{-3}

TABLE 3

SADE Bar Properties

EI (bending stiffness) = 2.0263×10^{6} lb-in^{2}
GJ (torsional stiffness) = 1.5236×10^{6} lb-in^{2}
EA (axial stiffness) = 4.3610×10^{6} lb
m (mass per unit length) = 1.1016×10^{-4} lb-sec^{2}/in
I_p/A (polar area moment of inertia divided by cross sectional area) = 9.2930×10^{-1} in^{2}

Diagonal bar length is $55\sqrt{2}$ in. Shorter bar length is 55 in.

displacements in the X, Y, and Z coordinate directions. Therefore, this model does not take into account the bending or torsional stiffness of the bar elements.

Since the truss contains thirty-two nodes, there are ninety-six unrestrained degrees of freedom in the model. Thus, the unrestrained displacement vector $\underline{u}$ is 96x1 while the unrestrained stiffness and mass matrices are each 96x96. The X displacement at node N is degree of freedom 3N-2, while the Y displacement is degree of freedom 3N-1 and the Z displacement is degree of freedom 3N.

A typical bar element is shown in Figure 2. The local coordinates are lowercase and the global coordinates are uppercase. The degrees of freedom in local coordinates of the bar are numbered. The element stiffness matrix for a bar in local coordinates is

$$\underline{K}_{xyz} = EA/L \begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (2.1)$$

where E is Young's modulus, A is the cross sectional area of the bar, and L is the length of the bar.

The element stiffness matrix must be transferred to global coordinates. Let $\underline{i}_x$ be a unit vector along the bar from bar node 1 to bar node 2. $\underline{i}_y$ is a unit vector perpendicular to $\underline{i}_x$ and the unit vector $\underline{i}_z$ completes the right handed system for the local bar axes. Now define a 3x3 matrix $\underline{t}$:

$$\underline{t} = \begin{bmatrix} \underline{i}_x \cdot \underline{i}_X & \underline{i}_x \cdot \underline{i}_Y & \underline{i}_x \cdot \underline{i}_Z \\ \underline{i}_y \cdot \underline{i}_X & \underline{i}_y \cdot \underline{i}_Y & \underline{i}_y \cdot \underline{i}_Z \\ \underline{i}_z \cdot \underline{i}_X & \underline{i}_z \cdot \underline{i}_Y & \underline{i}_z \cdot \underline{i}_Z \end{bmatrix} \qquad (2.2)$$

where $\underline{i}_X$, $\underline{i}_Y$, and $\underline{i}_Z$ are unit vectors in the global coordinate system along the coordinate axes. To find the elements of the direction cosine matrix $\underline{t}$ above, let

$$\underline{i}_x = \alpha \underline{i}_X + \beta \underline{i}_Y + \gamma \underline{i}_Z \qquad (2.3)$$

where

$$\alpha = (X_2 - X_1)/L \qquad \beta = (Y_2 - Y_1)/L \qquad \gamma = (Z_2 - Z_1)/L$$

$$L = \quad (X_2 - X_1)^2 + (Y_2 - Y_1)^2 + (Z_2 - Z_1)^2$$

(X_2, Y_2, Z_2) is the global coordinate of bar node 2 while (X_1, Y_1, Z_1) is the global coordinate of bar node 1.
Also, let

$$\underline{i}_y = a\underline{i}_X + b\underline{i}_Y \qquad (2.4)$$

Here, $\underline{i}_y$ needs no Z component since it is perpendicular to $\underline{i}_x$, and is assumed to lie in the X-Y plane. Since $\underline{i}_y$ is a unit vector,

$$\sqrt{a^2 + b^2} = 1 \qquad (2.5)$$

or

$$b^2 = 1 - a^2 \qquad (2.6)$$

Since $\underline{i}_y$ is perpendicular to $\underline{i}_x$,

$$\underline{i}_y \cdot \underline{i}_x = 0 \qquad (2.7)$$

Therefore,

$$a\alpha + b\beta = 0 \tag{2.8}$$

which gives

$$a^2 = \frac{\beta^2}{\alpha^2 + \beta^2} \tag{2.9}$$

or

$$a = -\sqrt{\frac{(Y_2-Y_1)^2}{(X_2-X_1)^2+(Y_2-Y_1)^2}} \tag{2.10}$$

The negative sign is chosen as the convention here. From (2.8) we have

(2.11)

or

$$b = \frac{X_2-X_1}{Y_2-Y_1}\sqrt{\frac{(X_2-X_1)^2}{(Y_2-Y_1)^2+(X_2-X_1)^2}} \tag{2.12}$$

However, if $\beta=0$ then $a=0$ and we set $b=1$, which satisfies both the condition that $\underline{i}_y$ has unit magnitude and that $\underline{i}_y$ is perpendicular to $\underline{i}_x$. We have now defined $\underline{i}_x$ and $\underline{i}_y$ in terms of the global coordinates. For $\underline{i}_z$ we take the vector cross product of $\underline{i}_x$ and $\underline{i}_y$:

$$\begin{aligned} \underline{i}_z &= \underline{i}_x \times \underline{i}_y \qquad (2.13) \\ &= -b\gamma\underline{i}_X + a\gamma\underline{i}_Y + (b\alpha-a\beta)\underline{i}_Z \end{aligned}$$

Since we have defined $\underline{i}_x$, $\underline{i}_y$, and $\underline{i}_z$ in terms of the global coordinates we can rewrite (2.2) as

$$\underline{t} = \begin{bmatrix} \alpha & \beta & \gamma \\ a & b & 0 \\ -b\gamma & a\gamma & b\alpha-a\beta \end{bmatrix} \qquad (2.14)$$

where α, β, and γ are defined in (2.3), a is given by (2.10), and b is given by (2.12).

The element stiffness matrix in global coordinates $\underline{K}_{XYZ}$ can now be found from the element stiffness matrix in local coordinates. Let

$$\underset{6x6}{\underline{T}} = \left[\begin{array}{c|c} \underset{3x3}{\underline{t}} & \\ \hline & \underset{3x3}{\underline{t}} \end{array}\right] \qquad (2.15)$$

where the terms in the off-diagonal blocks are all zero (reference 4). Then

$$\underline{K}_{XYZ} = \underline{T}^T \, \underline{K}_{xyz} \, \underline{T} \qquad (2.16)$$

The element stiffness matrices in global coordinates are assembled to form the unrestrained structure stiffness matrix $\underline{K}$ by adding their stiffnesses to the proper elements of $\underline{K}$. This is done by considering the appropriate degrees of freedom.

The unrestrained structure consistent mass matrix $\underline{M}_C$ is formed in a similar manner. The element mass matrix in local coordinates is

$$\underline{M}_{xyz} = mL \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & 1/3 & 0 & 0 & 1/6 & 0 \\ 0 & 0 & 1/3 & 0 & 0 & 1/6 \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & 1/6 & 0 & 0 & 1/3 & 0 \\ 0 & 0 & 1/6 & 0 & 0 & 1/3 \end{bmatrix} \qquad (2.17)$$

where m is the mass per unit length of the bar, and allowance is made for bar rotations as well as bar stretching. This element mass matrix is converted to global coordinates with the same transformation used for the element stiffness matrix:

$$\underline{M}_{XYZ} = \underline{T}^T \underline{M}_{xyz} \underline{T} \qquad (2.18)$$

The element mass matrices in global coordinates are then assembled to form the unrestrained structure consistent mass matrix $\underline{M}_C$.

To get the unrestrained structure mass matrix $\underline{M}$, the joint masses (and tip masses) at the nodes must be added to $\underline{M}_C$. The joint masses are modeled as point masses. Therefore, the point mass at node N is added to elements (3N-2,3N-2), (3N-1,3N-1), and (3N,3N) of $\underline{M}_C$, corresponding to the three translational degrees of freedom at that node.

The structure is restrained in degrees of freedom 1 through 12. Therefore, the rows and columns of $\underline{K}$ and $\underline{M}$ corresponding to these degrees of freedom are removed to form the restrained structure stiffness matrix $\underline{K}$ and the restrained structure mass matrix $\underline{M}$. Also, the first twelve entries of the unrestrained displacement vector $\underline{u}$ are removed (and set equal to zero) to form the restrained displacement vector $\underline{u}$.

The equation of motion of the truss is

$$\underline{M\ddot{u}} + \underline{Ku} = \underline{0} \qquad (2.19)$$

Assuming $\underline{u}=\underline{U}\cos(\omega t+\phi)$, where ϕ is a phase angle, we obtain

$$(\underline{K}-\omega^2\underline{M})\ \underline{U} = \underline{0} \qquad (2.20)$$

The characteristic equation

$$\det\ (\underline{K}-\omega^2\underline{M}) = 0 \qquad (2.21)$$

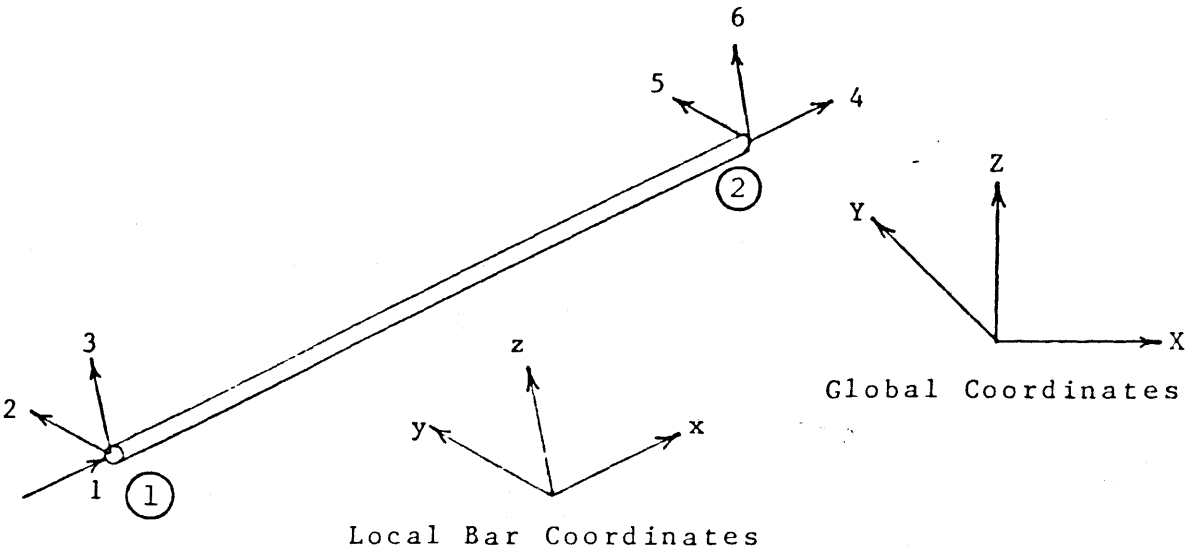
is solved using the method of subspace iteration to obtain the desired number of lowest eigenvalues (squared natural frequencies) ω_r^2. These are used in (2.20) to find the associated eigenvectors $\underline{U}$, which specify the mode shapes.

The computer program that assembles the restrained structure stiffness and mass matrices (which is designed to work for a general space truss) is presented in Appendix D. The natural frequencies and mode shapes were found by using the output from this program in the Finite Element Analysis Basic Library of the Aeroelastic and Structures Research Laboratory at MIT. The eigenvectors for the first twelve modes are presented in Appendix C. The natural frequencies for the first sixteen modes and mode shape descriptions for the first twelve modes are presented in Table 4. The modes are well spaced except that the bending modes occur in closely spaced pairs. Also, if the truss is considered as a beam, the neutral axis in bending passes through diagonally opposite nodes. Another way of stating this is that the direction of bending motion is at a forty-five degree angle to the X and Y axes (see Figure 3).

This analysis was also performed using a lumped mass matrix formed by concentrating one-half of the mass of each bar element at each of the nodes at its ends. This method yielded results which were very close to those obtained with the consistent mass matrix (with concentrated joint masses), especially in the lower modes.

FIGURE 2

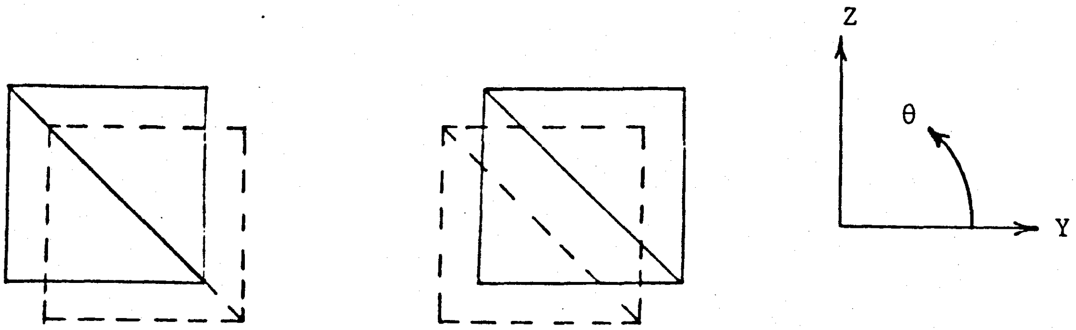
Pin-Jointed Bar Element



y axis is chosen to lie in a plane parallel to X-Y plane

FIGURE 3

First Bending Mode Pair of SADE Truss



First Mode of First Bending Mode Pair

Second Mode of First Bending Mode Pair

2.3 RIGID-JOINTED MODEL

The rigid-jointed model of the SADE truss allows six degrees of freedom at each node. Three of these are the translations in the coordinate directions. The other three degrees of freedom are the rotations about axes in the coordinate directions. Therefore, this model takes into account the bending and torsional stiffness of the bar elements.

There are now one-hundred and ninety-two degrees of freedom in the model since the truss contains thirty-two nodes. The X, Y, and Z displacements at node N are degrees of freedom 6N-5, 6N-4, and 6N-3, respectively. The rotations about axes in the X, Y, and Z directions are denoted by θ, ϕ, and ψ, and the degrees of freedom associated with these rotations at node N are 6N-2, 6N-1, and 6N, respectively.

A typical element is shown in Figure 4. The 12x12 element stiffness and consistent mass matrices (in local coordinates) are given by Craig (reference 5, pp. 391,392):

$$\underline{K}_{xyz} = \left[\begin{array}{c:c} \underline{k}_a & \underline{k}_b \\ \hdashline \underline{k}_c & \underline{k}_d \end{array}\right] \qquad (2.22)$$

where

$$\underline{k}_a = \begin{bmatrix} EA/L & & & & & \\ & 12EI_z/L^3 & & & & 6EI_z/L^2 \\ & & 12EI_y/L^3 & & -6EI_y/L^2 & \\ & & & GJ/L & & \\ & & -6EI_y/L^2 & & 4EI_y/L & \\ & 6EI_z/L^2 & & & & 4EI_z/L \end{bmatrix}$$

$$\underline{k}_b = \begin{bmatrix} -EA/L & & & & & \\ & -12EI_z/L^3 & & & & 6EI_z/L^2 \\ & & -12EI_y/L^3 & & -6EI_y/L^2 & \\ & & & -GJ/L & & \\ & & 6EI_y/L^2 & & 2EI_y/L & \\ & -6EI_z/L^2 & & & & 2EI_z/L \end{bmatrix}$$

$$\underline{k}_d = \begin{bmatrix} EA/L & & & & & \\ & 12EI_z/L^3 & & & & -6EI_z/L^2 \\ & & 12EI_y/L^3 & & 6EI_y/L^2 & \\ & & & GJ/L & & \\ & & 6EI_y/L^2 & & 4EI_y/L & \\ & -6EI_z/L^2 & & & & 4EI_z/L \end{bmatrix}$$

$$\underline{k}_c = \underline{k}_b^T$$

and I_y and I_z are the area moments of inertia of the bar cross section about the y and z axes, G is the shear stiffness, and J is the polar area moment of inertia of the bar cross section; also,

$$\underline{M}_{xyz} = mL/420 \left[\begin{array}{c|c} \underline{m}_a & \underline{m}_b \\ \hline \underline{m}_c & \underline{m}_d \end{array}\right] \qquad (2.23)$$

where

$$\underline{m}_a = \begin{bmatrix} 140 & & & & & \\ & 156 & & & & 22L \\ & & 156 & & -22L & \\ & & & 140I_p/A & & \\ & & -22L & & 4L^2 & \\ & 22L & & & & 4L^2 \end{bmatrix}$$

$$\underline{m}_b = \begin{bmatrix} 70 & & & & & \\ & 54 & & & & -13L \\ & & 54 & & 13L & \\ & & & 70I_p/A & & \\ & & -13L & & -3L^2 & \\ & 13L & & & & -3L^2 \end{bmatrix}$$

$$\underline{m}_d = \begin{bmatrix} 140 & & & & & \\ & 156 & & & & -22L \\ & & 156 & & 22L & \\ & & & 140I_p/A & & \\ & & 22L & & 4L^2 & \\ & -22L & & & & 4L^2 \end{bmatrix}$$

$$\underline{m}_c = \underline{m}_b^T$$

where I_p is the polar mass moment of inertia of the bar cross section. The element matrices are again transformed according to (2.16) and (2.18), except now

$$\underset{12\text{x}12}{\underline{T}} = \begin{bmatrix} \underset{3\text{x}3}{\underline{t}} & & & \\ & \underset{3\text{x}3}{\underline{t}} & & \\ & & \underset{3\text{x}3}{\underline{t}} & \\ & & & \underset{3\text{x}3}{\underline{t}} \end{bmatrix} \qquad (2.24)$$

The joint mass at a node is added to the diagonal elements of the unrestrained structure consistent mass matrix which correspond to the translational degrees of freedom at that node. Thus, the joint mass at node N is added to elements (6N-5,6N-5), (6N-4,6N-4), and (6N-3,6N-3) of $\underline{M}_C$. Since the joint masses are assumed to be point masses they have zero moment of inertia, and consequently no lumped quantities are added to the diagonal elements of $\underline{M}_C$ which correspond to rotational degrees of freedom.

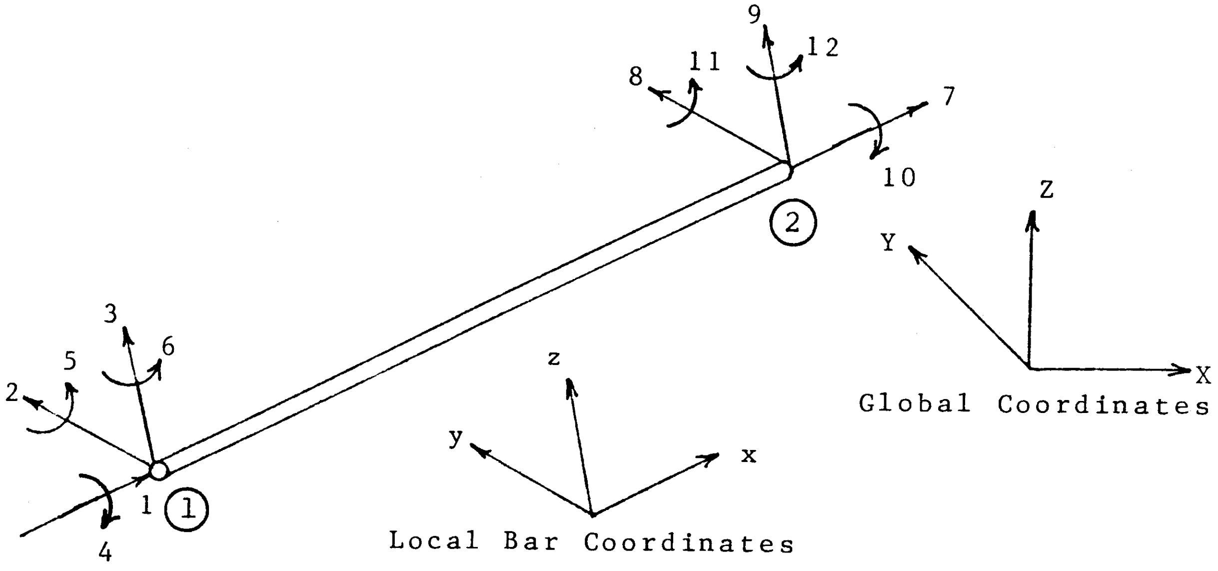
It is assumed that the four nodes at the base of the truss are pinned and not clamped. Therefore, degrees of freedom 1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, and 21 are now restrained. The rows and columns of the unrestrained structure stiffness and mass matrices and the rows of the unrestrained displacement vector corresponding to these degrees of freedom are removed to form the restrained system. The resulting equation of motion is then solved using the same techniques as in the pin-jointed analysis to obtain natural frequencies and mode shapes.

The natural frequencies for the lowest sixteen modes again are presented in Table 4 and the eigenvectors are given in Appendix C. The frequency results correlate very well with those from the pin-jointed analysis in the lower modes.

As the mode number increases, the frequencies obtained using the rigid-jointed model become somewhat lower than those from the pin-jointed model. This is probably due to the influence of local bar element natural frequencies on the global structural modes. Note, however, that the last several modes of the rigid-jointed model are now very closely spaced. This phenomenon is associated with the vibrations relative to the joints of the individual bar elements and will be addressed in Chapters 6 and 7.

FIGURE 4

Rigid-Jointed Bar Element



y axis is chosen to lie in a plane parallel to X-Y plane

3. CONTINUUM METHODS

3.1 GENERATION OF STIFFNESS PROPERTIES

If a beam-like truss is considered to be a beam in bending then a continuum equation of motion from beam theory can be used to find the natural frequencies for the bending modes of the truss. Similarly, continuum models for a torsional rod and an axial bar can be used to find the natural frequencies of the truss in its torsional and axial modes. However, to use these continuum models we must first define continuum stiffness and inertia properties for the truss. For bending, we need to define the bending stiffness EI and the shear stiffness GA if Timoshenko beam theory is used. The inertia properties needed are an equivalent mass per unit length m and an eqivalent rotary mass moment of inertia per unit length i_r. For torsion, a torsional stiffness GJ must be defined as well as a mass per unit length and a polar mass moment of inertia per unit length i_p. For axial vibrations, we need an axial stiffness EA as well as m.

Defining the equivalent inertia properties of the SADE truss is relatively straightforward. The analyses used to define m, i_r, and i_p are presented in sections 3.2 through 3.5 as these quantities are needed. However, methods for finding the continuum stifness properties are not as obvious. Two methods are presented in this section. The first finds the stiffness properties by performing a static pin-jointed finite element analysis of one bay of the truss. The second considers the energy of the bar elements in one bay and is based on the work of Noor, Anderson, and Greene (reference 6).

A. Static Pin-Jointed Truss Analysis for One Bay

Consider the bay at the base of the SADE truss. This bay extends from X=0 inches to X=55 inches (see Figure 1). It is bounded by nodes 1, 2, 3, and 4 at the fixed (X=0) end and by nodes 5, 6, 7, and 8 at the free (X=55 inches) end. To analyze the stiffness properties of the bay we restrain it at the fixed end by pinning nodes 1 through 4.

The restrained stiffness matrix $\underline{K}$ for the bay is found using the pin-jointed finite element procedure described in section 2.2. This matrix is 12x12 since there are four free nodes with three translational degrees of freedom at each node. Thus, degrees of freedom 1, 4, 7, and 10 are the X displacements at nodes 5, 6, 7, and 8, respectively. Degrees of freedom 2, 5, 8, and 11 are the Y displacements and degrees of freedom 3, 6, 9, and 12 are the Z displacements at these nodes.

Appropriate forces can be applied at the free end of the bay by way of a 12x1 force vector $\underline{f}$. We can find the corresponding 12x1 displacement vector $\underline{u}$ according to

$$\underline{Ku} = \underline{f} \quad \text{or} \quad \underline{u} = \underline{K}^{-1}\underline{f} \qquad (3.1)$$

Once $\underline{u}$ is determined, beam theory equations can be used to find the continuum stiffness properties of the bay.

For example, consider placing a pure moment M_Y (in the Y direction using the right hand rule) on the free end of the bay. This can be accomplished by letting

$$\underline{f}^T = [-100\ 0\ 0\ 100\ 0\ 0\ -100\ 0\ 0\ 100\ 0\ 0]\ \text{lb} \qquad (3.2)$$

The magnitude of the moment is chosen with regard to purely numerical considerations; any value would do in theory. The above value of $\underline{f}$ gives M_Y=11000 in-lb. We can find the corresponding displacement vector from (3.1):

$$\underline{u} = \begin{bmatrix} -1.26117E-03 \\ 1.26117E-03 \\ -1.26119E-03 \\ 1.26118E-03 \\ 1.26119E-03 \\ -1.26119E-03 \\ -1.26117E-03 \\ 1.26117E-03 \\ -1.26121E-03 \\ 1.26118E-03 \\ 1.26119E-03 \\ -1.26121E-03 \end{bmatrix} \text{ in} \qquad (3.3)$$

From beam theory we have

$$M_Y = EI_Y\kappa_Y - EI_{YZ}\kappa_Z \qquad (3.4)$$

$$M_Z = -EI_{YZ}\kappa_Y - EI_Z\kappa_Z \qquad (3.5)$$

where M_Z (which is zero for now) is the moment on the free end of the bay in the Z direction, EI_Y (EI_Z) is the bending stiffness about the Y (Z) axis, EI_{YZ} is a coupling term, and κ_Y (κ_Z) is the curvature about the Y (Z) axis. κ_Y and κ_Z are approximated by

$$\kappa_Y = \frac{u_{X_6} + u_{X_8}}{2HL} - \frac{u_{X_5} + u_{X_7}}{2HL} \qquad (3.6)$$

$$\kappa_Z = \frac{u_{X_6} + u_{X_5}}{2BL} - \frac{u_{X_8} + u_{X_7}}{2BL} \qquad (3.7)$$

where u_{XN} is the X displacement at node N, L is the X dimension of the bay, H is the Z dimension of the bay, and B is the Y dimension of the bay (L=H=B in this case). We now have κ_Y, κ_Z, and M_Y with $M_Z=0$. Similarly, we can determine κ_Y and κ_Z with $M_Z=11000$ in-lb and $M_Y=0$ by letting

$$\underline{f}^T = [100\ 0\ 0\ 100\ 0\ 0\ -100\ 0\ 0\ -100\ 0\ 0] \text{ lb} \qquad (3.8)$$

which yields

$$\underline{u} = \begin{bmatrix} 1.26117E-03 \\ 1.71793E-03 \\ -6.85117E-04 \\ 1.02256E-03 \\ 6.85117E-04 \\ -6.85117E-04 \\ -1.38048E-03 \\ 1.71793E-03 \\ 6.85145E-04 \\ -1.14187E-03 \\ 6.85117E-04 \\ 8.04447E-04 \end{bmatrix} \text{in} \qquad (3.9)$$

The system defined by (3.4) and (3.5) can be solved two separate times, the first time with M_Y=11000 in-lb and M_Z=0, using the corresponding values of κ_Y and κ_Z, and the second time with M_Z=11000 in-lb and M_Y=0 using the new values of κ_Y and κ_Z. Doing this yields $EI_Y=1.3192\text{x}10^{10}$ lb-in^2, $EI_Z=1.3847\text{x}10^{10}$ lb-in^2, and $EI_{YZ}=0$.

For the shear stiffness the equations to be solved are

$$-\frac{\psi}{2} + \frac{dv}{dX} = \frac{1}{GA_Y} S_Y + \frac{1}{GA_{YZ}} S_Z \qquad (3.10)$$

$$\frac{\phi}{2} + \frac{dw}{dX} = \frac{1}{GA_{YZ}} S_Y + \frac{1}{GA_Z} S_Z \qquad (3.11)$$

where ψ and ϕ are rotations about Z and Y, respectively, at X=L, and the terms on the left side of the equations are shear strains. Also, GA_Y (GA_Z) is the shear stiffness in the Y (Z) direction, $1/GA_{YZ}$ is the coupling term, S_Y (S_Z) is the shear force in the Y (Z) direction and v (w) is the deflection along the length of the bay in the Y (Z) direction. dv/dX, dw/dX, ϕ, and ψ are approximated by

$$\frac{dv}{dX} = \frac{u_{Y_5} + u_{Y_6} + u_{Y_7} + u_{Y_8}}{4L} \qquad (3.12)$$

$$\frac{dw}{dX} = \frac{u_{Z_5} + u_{Z_6} + u_{Z_7} + u_{Z_8}}{4L} \tag{3.13}$$

$$\Phi = \frac{u_{X_6} + u_{X_8}}{2L} - \frac{u_{X_5} + u_{X_7}}{2L} \tag{3.14}$$

$$\psi = \frac{u_{X_6} + u_{X_5}}{2L} - \frac{u_{X_8} + u_{X_7}}{2L} \tag{3.15}$$

The analysis procedure is the same as for the bending stiffness. We solve the system defined by (3.10) and (3.11) twice, once with S_Y=200 lb and S_Z=0, and next with S_Z=200 lb and S_Y=0. For S_Y=200 lb we set

$$\underline{f}^T = [0\ 50\ 0\ 0\ 50\ 0\ 0\ 50\ 0\ 0\ 50\ 0]\ \text{lb} \tag{3.16}$$

with which we can use (3.1) to find

$$\underline{u} = \begin{bmatrix} 0.00000E+00 \\ 5.68727E-03 \\ -9.73155E-04 \\ 1.14187E-03 \\ 4.54028E-03 \\ -9.73155E-04 \\ -1.32083E-03 \\ 5.05668E-03 \\ -2.88033E-04 \\ 5.96540E-05 \\ 5.17087E-03 \\ -2.28380E-04 \end{bmatrix} \text{in} \tag{3.17}$$

For S_Z=200 lb,

$$\underline{f}^T = [0\ 0\ 50\ 0\ 0\ 50\ 0\ 0\ 50\ 0\ 0\ 50]\ \text{lb} \tag{3.18}$$

$$\underline{u} = \begin{bmatrix} 0.00000E+00 \\ -1.14155E-04 \\ 5.31492E-03 \\ -1.20154E-03 \\ -1.11721E-03 \\ 4.68434E-03 \\ 2.98183E-05 \\ -1.14155E-04 \\ 5.60306E-03 \\ -1.29104E-03 \\ -1.11721E-03 \\ 4.94263E-03 \end{bmatrix} \text{ in} \qquad (3.19)$$

Using these results for $\underline{u}$ in (3.10) and (3.11) we find that $GA_Y=2.1511\text{x}10^6$ lb, $GA_Z=2.1417\text{x}10^6$ lb, and $GA_{YZ}=-1.7866\text{x}10^7$ lb.

We have now determined the stiffnesses needed for a continuum dynamic model of the beam-like truss in bending. However, such a model requires that the neutral axis for bending is defined. The finite element results of Chapter 2 showed that the neutral axes for the bending mode pairs are at a forty-five degree angle to the coordinate Y and Z directions. Bending about these neutral axes is assumed in the continuum model. A Mohr's circle transformation on the shear stiffnesses shows that the principal axes for shear are also at forty-five degree angles to the Y and Z axes, which is where the coupling term $1/GA_{YZ}$ equals zero. The first principal axis is at $\theta=135^o$, where θ is the angle about the X axis as measured from the Y axis (see Figure 3). The shear stiffness associated with this direction is $GA_1=1.9161\text{x}10^6$ lb. The other principal axis for shear is at $\theta=45^o$ and the shear stiffness associated with this direction is $GA_2=2.4394\text{x}10^6$ lb. The principal axes for the bending stiffnesses are the Y and Z axes since $EI_{YZ}=0$. If we denote EI_1 as the bending stiffness at $\theta=135^o$ and EI_2 as the bending stiffness at $\theta=45^o$, we find that $EI_1=EI_2=EI=1.3520\text{x}10^{10}$ lb-in^2. Thus, the change in the bending stiffness is less than three percent as the axes are rotated. This is due to the fact that most of the bending stiffness is provided by the longitudinal (X-X) bars. If we take a cross section of the truss and calculate the bending stiffness considering only the longitudinal bars, we find that $EI_Y = EI_Z = EI_1 = EI_2 = 1.3192\text{x}10^{10}$ lb-in^2.

The torsional stiffness GJ is found by using the equation

$$\theta = \frac{TL}{GJ} \qquad (3.20)$$

where θ is the angle of twist of the bay about the X axis. A torque T of $-1.1(10^6)$ in-lb produces an angle which is numerically reasonable for computational considerations. This torque can be produced by setting

$$\underline{f}^T = [0\ -5\ 5\ 0\ 5\ 5\ 0\ -5\ -5\ 0\ 5\ -5]\ \text{x}10^3\ \text{lb} \qquad (3.21)$$

which yields

$$\underline{u} = \begin{bmatrix} 0.00000\text{E}+00 \\ -5.57312\text{E}-01 \\ 4.05366\text{E}-01 \\ 5.96678\text{E}-03 \\ 3.71118\text{E}-01 \\ 3.42308\text{E}-01 \\ 1.29101\text{E}-01 \\ -4.94253\text{E}-01 \\ -5.31502\text{E}-01 \\ 1.23138\text{E}-01 \\ 4.34177\text{E}-01 \\ -4.71425\text{E}-01 \end{bmatrix} \text{in} \qquad (3.22)$$

θ can be found by considering the Y and Z components of $\underline{u}$ and ignoring the X components. We can then define θ to be the average of the angles of rotation in the Y-Z plane of each of the four nodes at the free end of the bay. The angle of rotation of a node can be determined with vector analysis. We know the components of the vector which extends from the center of the bay cross section (in the Y-Z plane at X=L) to the undisplaced node. Since the Y and Z components of $\underline{u}$ have been found, we also know the components of the vector which extends from the bay center to the displaced node. The angle between these two vectors is the angle of rotation of the node and can be found by taking the dot product of the vectors. Once this is done for all four nodes, θ is determined and from (3.20) we find $GJ=3.6923 \times 10^9$ lb-in^2.

For EA we use the equation

$$u = \frac{PL}{EA} \tag{3.23}$$

where P=400 lb is a force in the X direction. P is applied to the bay by setting

$$\underline{f}^T = [100\ 0\ 0\ 100\ 0\ 0\ 100\ 0\ 0\ 100\ 0\ 0]\ \text{lb} \tag{3.24}$$

which yields

$$\underline{u} = \begin{bmatrix} 1.26117E{-}03 \\ -8.04441E{-}04 \\ -6.85137E{-}04 \\ 1.02257E{-}03 \\ 6.85136E{-}04 \\ -6.85137E{-}04 \\ 1.14187E{-}03 \\ -8.04441E{-}04 \\ -1.83728E{-}03 \\ 1.38049E{-}03 \\ 6.85137E{-}04 \\ -1.71797E{-}03 \end{bmatrix} \text{in} \tag{3.25}$$

u is simply the average of the four X displacements in $\underline{u}$, so from (3.23) we find $EA=1.831 \times 10^7$ lb.

B. Energy of Bars Method for One Bay

The method for deriving the continuum stiffness properties of the SADE truss outlined below is based on the work of Noor, Anderson, and Greene (reference 6). First, we must place a new set of coordinate axes (x,y,z) on the bay considered in the previous article. These axes are placed at the stiffness center of the bay at the cross section at X=27.5 inches (the center of the bay). If we let a subscript 0 denote the location of the new coordinate system then X_0=27.5 inches and Y_0 and Z_0 will be determined later.

For now, the bay is assumed to be made up of a continuous medium. Let u, v, and w denote the translational displacements of a point on the cross section (in the xyz system) and θ, ϕ, and ψ denote the corresponding rotations about axes in the x, y, and z coordinate directions. Also, let $\varepsilon_2{}^0$ and $\varepsilon_3{}^0$ be the extensional strains in the y and z directions at y=z=0 and $2\varepsilon_{23}{}^0$ be the shearing strain in the y-z plane at y=z=0. For any point on the cross section we assume

$$\begin{aligned} u &= u^0 - y\psi + z\phi \\ v &= v^0 + y\varepsilon_2{}^0 + z(-\vartheta+\tfrac{1}{2}2\varepsilon_{23}{}^0) \\ w &= w^0 + y(\theta+\tfrac{1}{2}2\varepsilon_{23}{}^0) + z\varepsilon_3{}^0 \end{aligned} \qquad (3.26)$$

where a superscript 0 denotes y=z=0. It is also assumed that u^0, v^0, w^0, θ, ϕ, ψ, $\varepsilon_2{}^0$, $\varepsilon_3{}^0$, and $2\varepsilon_{23}{}^0$ are functions of x only and therefore constant in the cross section.

Using the equations of elasticity, the strain in the cross section can be written as

$$\begin{aligned} \varepsilon_{11} &= \frac{du^0}{dx} - y\frac{d\psi}{dx} + z\frac{d\phi}{dx} \\ &= \varepsilon_1{}^0 - y\kappa_2{}^0 + z\kappa_3{}^0 \end{aligned} \qquad (3.27)$$

$$\varepsilon_{22} = \varepsilon_2{}^0 \qquad (3.28)$$

$$\varepsilon_{33} = \varepsilon_3{}^0 \qquad (3.29)$$

$$2\varepsilon_{12} = -\psi + \frac{dv^0}{dx} + y\frac{d\varepsilon_2^{\,0}}{dx} + z\left(-\frac{d\theta}{dx} + \tfrac{1}{2}\frac{d(2\varepsilon_{23}^{\,0})}{dx}\right)$$

$$= 2\varepsilon_{12}^{\,0} + y\frac{d\varepsilon_2^{\,0}}{dx} + z\left(-\kappa_1^{\,0} + \tfrac{1}{2}\frac{d(\varepsilon_{23}^{\,0})}{dx}\right) \qquad (3.30)$$

$$2\varepsilon_{13} = \phi + \frac{dw^0}{dx} + y\left(\frac{d\theta}{dx} + \tfrac{1}{2}\frac{d(2\varepsilon_{23}^{\,0})}{dx}\right) + z\frac{d\varepsilon_3^{\,0}}{dx} \qquad (3.31)$$

$$= 2\varepsilon_{13}^{\,0} + y\left(\kappa_1^{\,0} + \tfrac{1}{2}\frac{d(2\varepsilon_{23}^{\,0})}{dx}\right) + z\frac{d\varepsilon_3^{\,0}}{dx}$$

$$2\varepsilon_{23} = 2\varepsilon_{23}^{\,0} \qquad (3.32)$$

where the subscripts 1, 2, and 3 refer to the x, y, and z directions, respectively. If the distortion of the cross section is ignored, (3.30) and (3.31) can be rewritten as

$$2\varepsilon_{12} = 2\varepsilon_{12}^{\,0} - z\kappa_1^{\,0} \qquad (3.33)$$

$$2\varepsilon_{13} = 2\varepsilon_{13}^{\,0} + y\kappa_1^{\,0} \qquad (3.34)$$

The extensional strain ε in an arbitrary direction can be written as

$$\varepsilon = \sum_{i=1}^{3} \sum_{j=1}^{3} \varepsilon_{ij}\,\ell_i\,\ell_j \qquad (3.35)$$

where ℓ_i is the direction cosine from the arbitrary direction to the i direction and ε_{ij} are the strain components given in (3.27) to (3.34). Now if we assume the cross section does not change shape, we can set $\varepsilon_{22}=0$, $\varepsilon_{33} = 0$, and $2\,\varepsilon_{23}=0$, and we can write the extensional strain ε in any arbitrarily directed bar as

$$\varepsilon = \underline{a}^T \underline{\varepsilon} \qquad (3.36)$$

where

$$\underline{a}^T = \begin{bmatrix} \ell_1^{\,2} & \ell_1\ell_2 & \ell_1\ell_3 \end{bmatrix} \qquad (3.37)$$

$$\underline{\varepsilon}^T = \begin{bmatrix} \varepsilon_{11} & 2\varepsilon_{12} & 2\varepsilon_{13} \end{bmatrix} \tag{3.38}$$

Considering (3.27), (3.33), and (3.34), we have

$$\begin{aligned} \underline{\varepsilon} &= \begin{bmatrix} 1 & -y & z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -z \\ 0 & 0 & 0 & 0 & 1 & y \end{bmatrix} \begin{bmatrix} \varepsilon_1^0 \\ \kappa_2^0 \\ \kappa_3^0 \\ 2\varepsilon_{12}^0 \\ 2\varepsilon_{13}^0 \\ \kappa_1^0 \end{bmatrix} \\ &= \underline{E}_A \underline{\varepsilon}_A \end{aligned} \tag{3.39}$$

The potential energy U of the bay can be expressed by summing the energy of all the included bars as

$$\begin{aligned} U &= \tfrac{1}{2} \sum^{k \text{ bars}} E_k A_k L_k \varepsilon_k^2 \\ &= \tfrac{1}{2} \sum^{k} E_k A_k L_k \underline{\varepsilon}_k^T \underline{a}_k \underline{a}_k^T \underline{\varepsilon}_k \\ &= \tfrac{1}{2} \sum^{k} E_k A_k L_k \underline{\varepsilon}_A^T \underline{E}_{A_k}^T \underline{a}_k \underline{a}_k^T \underline{E}_{A_k} \underline{\varepsilon}_A \end{aligned} \tag{3.40}$$

where the quantities included in the summation are for the kth bar. U can also be expressed as

$$U = \tfrac{1}{2} L \underline{\varepsilon}_A^T \underline{C} \underline{\varepsilon}_A \tag{3.41}$$

where an L outside of the summation is the length of the bay and

$$\underline{C} = \begin{bmatrix} EA & & & & & \\ C_{21} & EI_Z & & \text{SYMMETRIC} & & \\ C_{31} & C_{32} & EI_Y & & & \\ C_{41} & C_{42} & C_{43} & GA_Y & & \\ C_{51} & C_{52} & C_{53} & C_{54} & GA_Z & \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & GJ \end{bmatrix} \tag{3.42}$$

where the off-diagonal elements are coupling terms. Comparing (3.40) and (3.41) yields

$$\underline{C} = \frac{1}{L} \sum^{k} E_k A_k L_k \underline{E}_{A_K}{}^{T} \underline{a}_k \underline{a}_k{}^{T} \underline{E}_{A_k} \tag{3.43}$$

To find Z_0, the Z location of the bay coordinate axes, we choose

$$C_{31} = \sum^{k} E_k A_k L_k z_k \ell_{1_k}{}^{4} = 0 \tag{3.44}$$

where $z_k = Z_k - Z_0$ and the reference point for every bar is its midpoint. This results in

$$Z_0 = \frac{\sum^{k} E_k A_k L_k Z_k \ell_{1_k}^{4}}{\sum^{k} E_k A_k L_k \ell_{1_k}^{4}} \qquad (3.45)$$

Similarly, for Y_0 we choose $C_{21}=0$, which yields

$$Y_0 = \frac{\sum^{k} E_k A_k L_k Y_k \ell_{1_k}^{4}}{\sum^{k} E_k A_k L_k \ell_{1_k}^{4}} \qquad (3.46)$$

For the bay of the SADE truss being considered,

$$\underline{C} = \begin{bmatrix} 2.36116E+07 & 6.93581E-08 & -6.93581E-08 & 0.00000E+00 & 3.08372E+06 & 8.48022E+07 \\ 6.93581E-08 & 1.55242E+10 & 0.00000E+00 & 0.00000E+00 & 1.08372E-08 & -2.33206E+09 \\ -6.93581E-08 & 0.00000E+00 & 1.55242E+10 & -8.48022E+07 & -1.09556E-08 & -7.62939E-07 \\ 0.00000E+00 & 0.00000E+00 & -8.48022E+07 & 3.08372E+06 & 0.00000E+00 & 1.73395E-08 \\ 3.08372E+06 & 1.08372E-08 & -1.09556E-08 & 0.00000E+00 & 3.08372E+06 & -1.73395E-08 \\ 8.48022E+07 & -2.33206E+09 & -7.62939E-07 & 1.73395E-08 & -1.73395E-08 & 4.66412E+09 \end{bmatrix}$$

(3.47)

where the units are in pounds, inches, and radians. The stiffness terms on the diagonal of the $\underline{C}$ matrix compare favorably with those obtained from the finite element analysis of one bay.

C. Comparison of the Two Methods for Stiffness Property Determination

A direct comparison between the two methods can be made by transforming the results obtained with the finite element anaysis to the form of the $\underline{C}$ matrix. We have already found the 12x12 restrained stiffness matrix for the bay using the methods of Chapter 2. A different restrained stiffness matrix can be derived for the bay if we consider (3.1) where now

$$\underline{f}^T = \begin{bmatrix} P & M_Z & M_Y & S_Y & S_Z & T \end{bmatrix} \qquad (3.48)$$

$$\underline{u}^T = \begin{bmatrix} u & \psi & \phi & v & w & \theta \end{bmatrix} \qquad (3.49)$$

Therefore, $\underline{K}$ is now 6x6. The elements of $\underline{K}^{-1}$ can be found by considering the forces in $\underline{f}$ to be applied one at a time, and determining each of the resulting components of $\underline{u}$. For example, P has been applied to the bay through (3.24). Equation (3.25) gives the corresponding deflections. These can be used to find the elements of the new 6x1 displacement vector $\underline{u}$. For u, v, or w, we simply average the X, Y, or Z deflections at the four free nodes. ϕ and ψ are given by (3.14) and (3.15). θ is found by considering the rotations of the four free nodes as is described in the derivation of GJ in article A.

We have now determined the first column of $\underline{K}^{-1}$:

$$(\underline{K}^{-1})_{i1} = u_i/P \qquad i=1,6 \qquad (3.50)$$

By applying the other five forces one at a time the other five columns of $\underline{K}^{-1}$ can be found:

$$\underline{K}^{-1} = \begin{bmatrix} 3.00381E-06 & -5.42318E-09 & 4.54546E-13 & -1.49130E-07 & -3.07845E-06 & -5.86831E-08 \\ -5.42318E-09 & 3.97197E-09 & 0.00000E+00 & 1.09229E-07 & 2.71300E-09 & 2.03531E-09 \\ 4.54546E-13 & 0.00000E+00 & 4.16917E-09 & 1.14653E-07 & -1.14654E-07 & -3.12397E-14 \\ -1.49130E-07 & 1.09229E-07 & 1.14653E-07 & 2.55689E-05 & -3.07840E-06 & 5.59704E-08 \\ -3.07845E-06 & 2.71300E-09 & -1.14654E-07 & -3.07840E-06 & 2.56812E-05 & 5.80120E-08 \\ -4.03524E-08 & 3.16811E-09 & -3.08491E-10 & 6.38888E-08 & 4.79282E-08 & 1.48957E-08 \end{bmatrix}$$

(3.51)

where the units are in pounds, inches, and radians. The symmetry is very good except for the sixth row and column. We replace the sixth row with the sixth column and round off the other elements where needed to obtain symmetry. The elements of the sixth column are more reliable than those of the sixth row because the terms in the sixth row represent bay rotations about the X axis. The bay does not rotate uniformly, except when a torque is applied. For example, when S_Y is applied node 6 rotates positively but node 8 rotates negatively, and the definition of bay rotation does not make sense.

The symmetric $\underline{K}^{-1}$ can be inverted to obtain

$$\underline{K}=\begin{bmatrix} 4.15378E{+}05 & -2.16980E{+}05 & 1.45828E{+}06 & 6.41260E{-}02 & 5.30286E{+}04 & 1.45955E{+}06 \\ -2.16980E{+}05 & 3.09879E{+}08 & 3.99367E{+}07 & -1.41664E{+}06 & 3.56055E{+}04 & -3.80114E{+}07 \\ 1.45828E{+}06 & 3.99367E{+}07 & 3.17855E{+}08 & -1.41665E{+}06 & 1.41966E{+}06 & 8.29681E{+}04 \\ 6.04642E{-}02 & -1.41664E{+}06 & -1.41665E{+}06 & 5.15143E{+}04 & -4.89010E{-}03 & -1.37428E{+}00 \\ 5.30286E{+}04 & 3.56055E{+}04 & 1.41966E{+}06 & -2.46706E{-}03 & 5.16232E{+}04 & 3.00011E{+}03 \\ 1.45955E{+}06 & -3.80114E{+}07 & 8.29711E{+}04 & -1.25984E{+}00 & 3.00033E{+}03 & 7.80655E{+}07 \end{bmatrix} \tag{3.52}$$

where the units are in pounds, inches, and radians.

$\underline{K}$ can be transformed to $\underline{C}$ by relating the strains $\underline{\varepsilon}_A$ in the energy method as defined in (3.39) to the displacements $\underline{u}$ in the finite element method as defined in (3.49). In matrix form, the strains $\underline{\varepsilon}_A$ are approximated by

$$\begin{bmatrix} \varepsilon_1^0 \\ \kappa_2^0 \\ \kappa_3^0 \\ 2\varepsilon_{12}^0 \\ 2\varepsilon_{13}^0 \\ \kappa_1^0 \end{bmatrix} = \begin{bmatrix} 1/L & & & & & \\ & 1/L & & & & \\ & & 1/L & & & \\ & -1/2 & & 1/L & & \\ & & 1/2 & & 1/L & \\ & & & & & 1/L \end{bmatrix} \begin{bmatrix} u \\ \psi \\ \phi \\ v \\ w \\ \theta \end{bmatrix}$$

or

$$\underline{\varepsilon}_A = \underline{Ru} \tag{3.53}$$

We already have an expression for the potential energy of the bay in (3.41). The potential energy can also be expressed as

$$U = \tfrac{1}{2}\underline{u}^T\underline{K}\underline{u} = \tfrac{1}{2}\underline{\varepsilon}_A^T\underline{R}^{-T}\underline{KR}\underline{\varepsilon}_A \qquad (3.54)$$

Comparing (3.41) and (3.54) yields

$$\underline{C} = \frac{1}{L}(\underline{R}^{-T}\underline{KR}^{-1}) \qquad (3.55)$$

Substituting the values for L, $\underline{R}$, and $\underline{K}$, we obtain

$$\underline{C} = \begin{bmatrix} 2.28458E+07 & -1.19338E+07 & -4.46836E+02 & 3.52693E+00 & 2.91657E+06 & 8.02752E+07 \\ -1.19338E+07 & 1.49007E+10 & -1.77292E+04 & 1.28000E+02 & 1.95829E+06 & -2.09062E+09 \\ -5.28000E+02 & -1.78735E+04 & 1.53347E+10 & -7.79157E+07 & 1.13600E+03 & 2.55830E+04 \\ 3.32553E+00 & 1.39636E+02 & -7.79157E+07 & 2.83328E+06 & -2.68955E-01 & -7.55852E+01 \\ 2.91657E+06 & 1.95830E+06 & 1.15433E+03 & -1.35688E-01 & 2.83928E+06 & 1.65006E+05 \\ 8.02753E+07 & -2.09062E+09 & 2.54100E+04 & -6.92912E+01 & 1.65018E+05 & 4.29360E+09 \end{bmatrix}$$

(3.56)

where the units are in pounds, inches, and radians. The diagonal elements of (3.56) are in excellent agreement with those of (3.45). The axial and bending stiffnesses differ by less than five percent while the torsional and shear stiffnesses differ by less than ten percent. The off-diagonal coupling elements also agree if we consider the coupling ratios,

$$q_{ij} = \left| \frac{c_{ij}}{\sqrt{c_{ii}c_{jj}}} \right| \quad i \neq j \qquad (3.57)$$

If q_{ij} is less than about 0.1, coupling between degrees of freedom i and j is unimportant. In both cases, $q_{ij} << 0.1$

except for C_{43}, C_{51}, C_{61}, and C_{62}. These important coupling terms are in excellent agreement between the two methods. Thus, the energy of bars method for one bay and the finite element pin-jointed truss analysis for one bay each produce about the same stiffnesses when the results from the truss analysis are transformed into the form of those from the energy method. However, the original stiffnesses from the finite element truss analysis of one bay produce frequencies which better match the finite element results of Chapter 2 than do those from the energy of bars method.

3.2 BENDING FREQUENCIES FROM BERNOULLI-EULER BEAM THEORY

Consider the equation of motion of a Bernoulli-Euler beam in bending,

$$EI\frac{\partial^4 w}{\partial X^4} + m\frac{\partial^2 w}{\partial t^2} = 0 \qquad (3.58)$$

where w is the transverse deflection of the beam. If we assume harmonic motion with frequency , and if we define $\bar{w}$=w/L, where L is the length of the beam, and ξ=X/L, we can write

$$\bar{w}^{(iv)} - \omega^2\left(\frac{mL^4}{EI}\right)\bar{w} = 0 \qquad (3.59)$$

where the derivatives are now with respect to ξ. The solution to this equation is

$$\bar{w} = C_1\cos\lambda\xi + C_2\sin\lambda\xi + C_3\cosh\lambda\xi + C_4\sinh\lambda\xi \qquad (3.60)$$

$$\lambda^4 = \omega^2\left(\frac{mL^4}{EI}\right) \qquad (3.61)$$

The geometric boundary conditions at X=0 are

$$\begin{aligned} w &= 0 \\ w' &= 0 \end{aligned} \qquad (3.62)$$

With a concentrated mass M_c and a concentratd moment of inertia I_c at the tip of the beam, the natural boundary conditions at X=L are

$$\begin{aligned} \bar{w}''' + \beta\bar{w} &= 0 \\ \bar{w}'' - \alpha\bar{w}' &= 0 \end{aligned} \qquad (3.63)$$

where

$$\beta = \omega^2 \frac{M_c L^3}{EI} = \lambda^4 \frac{M_c}{mL}$$

$$\alpha = \omega^2 \frac{I_c L}{EI} = \lambda^4 \frac{I_c}{mL^3} \qquad (3.64)$$

If we place (3.60) in (3.62) and (3.63), we obtain the characteristic equation in λ:

$$\lambda^3\{\sin\lambda - \sinh\lambda + \gamma(\cos\lambda + \cosh\lambda)\}$$

$$+ \beta\{\cos\lambda - \cosh\lambda - \gamma(\sin\lambda - \sinh\lambda)\} = 0 \qquad (3.65)$$

$$\gamma = \frac{\lambda(\cos\lambda + \cosh\lambda) - \alpha(\sin\lambda + \sinh\lambda)}{\lambda(\sin\lambda + \sinh\lambda) + \alpha(\cos\lambda - \cosh\lambda)}$$

We can solve this equation for λ and then use (3.61) to find the natural bending frequencies of the beam, ω.

For the SADE truss, the concentrated mass M_c is equal to the tip mass plus the joint mass at nodes 29, 30, 31, and 32, which adds up to 0.61479 lb-sec^2/in. To obtain the concentrated moment of inertia I_C, we average the joint mass at the four nodes at the free end of the truss and calculate the moment of inertia of these point masses about the neutral axis for bending, which passes through either nodes 29 and 32 or nodes 30 and 31. This moment of inertia is equal to 464.94 lb-sec^2-in. Note that Bernoulli-Euler beam theory does not distinguish between bending about the two neutral axes, which correspond to the pairs of closely spaced bending modes in the finite element analysis of Chapter 2. This is a result of $EI_1 = EI_2 = 1.3520 \times 10^{10}$ lb-in^2 and the fact that the shear stiffness is not accounted for.

The only quantity left to be found is the mass per unit length m. To calculate m we first average the joint mass over all joints (excluding the joint mass at nodes 29, 30, 31, and 32 at the tip of the truss). We then find the mass of a typical segment of the truss, such as the segment from X=27.5 inches to X=82.5 inches. In this segment there are eight 55 inch bars, five diagonal bars, and four average joints. We sum the mass of the bars and the joints and divide by 55 in. to obtain $m=2.733 \times 10^{-3}$ lb-sec^2/in^2. Since the length L of the SADE truss is 385 in., we then have $M_C/(mL)=0.58429$ and $I_C/(mL^3)=2.9811 \times 10^{-3}$.

The frequencies for the first four modes of the Bernoulli-Euler model are presented in Table 4. These results correspond to the first four bending mode pairs of the SADE truss.

3.3 BENDING FREQUENCIES FROM TIMOSHENKO BEAM THEORY

The equations of motion for a Timoshenko beam with no applied forces are

$$GA\left(\frac{\partial^2 w}{\partial X^2} + \frac{\partial \psi}{\partial X}\right) = m\ddot{w}$$

$$EI\frac{\partial^2 \psi}{\partial X^2} - GA\left(\frac{\partial w}{\partial X} + \psi\right) = i_R\ddot{\psi} \qquad (3.66)$$

where i_r is the rotary mass moment of inertia per unit length, w is the deflection of the midline, and ψ is the angle of rotation of the cross section (see Figure 5). If we assume harmonic motion with frequency ω and nondimensionalize by setting $\bar{w}=w/L$ and $\xi=X/L$, we can rewrite (3.66) as

$$\bar{w}'' + b^2 S\bar{w} + \psi' = 0$$

$$-\bar{w}' + S\psi'' - (1-b^2SR)\psi = 0 \qquad (3.67)$$

where a prime indicates differentiation with respect to ξ, and

$$b^2 = \omega^2\left(\frac{mL^4}{EI}\right)$$

$$S = \frac{EI}{GAL^2} \qquad (3.68)$$

$$R = \frac{i_R}{mL^2}$$

If $1-b^2SR > 0$ and we define

$$\lambda_{1,2} = \frac{b}{\sqrt{2}}\sqrt{\mp(S+R) + \sqrt{(S+R)^2 + 4\left(\frac{1}{b^2} - SR\right)}} \qquad (3.69)$$

we can rewrite the solution to (3.67) as

$$\bar{w} = C_1\cosh\lambda_1\xi + C_2\sinh\lambda_1\xi + C_3\cos\lambda_2\xi + C_4\sin\lambda_2\xi \tag{3.70}$$
$$\psi = C_1'\sinh\lambda_1\xi + C_2'\cosh\lambda_1\xi + C_3'\sin\lambda_2\xi + C_4'\cos\lambda_2\xi$$

However, from (3.67) we have

$$\psi' = -\bar{w}'' - b^2S\bar{w} \tag{3.71}$$

This equation can be used with (3.70) to give

$$\begin{aligned} C_1' &= -F_1C_1 \\ C_2' &= -F_1C_2 \\ C_3' &= F_2C_3 \\ C_4' &= -F_2C_4 \end{aligned} \tag{3.72}$$

$$F_1 = \frac{\lambda_1{}^2 + b^2S}{\lambda_1}$$

$$F_2 = \frac{\lambda_2{}^2 - b^2S}{\lambda_2}$$

There are four boundary conditions to consider. The two geometric boundary conditions at X=0 are

$$\begin{aligned} w &= 0 \\ \psi &= 0 \end{aligned} \tag{3.73}$$

The two natural boundary conditions at X=L are

$$-EI\frac{\partial\psi}{\partial X} = I_c\ddot{\psi}$$

$$-GA(\frac{\partial w}{\partial X} + \psi) = M_c\ddot{w} \qquad (3.74)$$

Using (3.68), we can rewrite the boundary conditions as

$$\bar{w} = 0$$

$$\psi = 0$$

$$\psi' - b^2\bar{I}_c\psi = 0 \qquad (3.75)$$

$$\bar{w}' - b^2 S\bar{M}_c\bar{w} + \psi = 0$$

$$\bar{I}_c = \frac{I_c}{mL^3}$$

$$\bar{M}_c = \frac{M_c}{mL}$$

If (3.70) are substituted into (3.75) the boundary conditions can be expressed in matrix form as

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \underline{0} \qquad (3.76)$$

where

$A_{11} = 1$ $\quad A_{12} = 0$ $\quad A_{13} = 1$ $\quad A_{14} = 0$

$A_{21} = 0$ $\quad A_{22} = -F_1$ $\quad A_{23} = 0$ $\quad A_{24} = -F_2$

$$A_{31} = F_1(b^2\bar{I}_c \sinh\lambda_1 - \lambda_1\cosh\lambda_1)$$

$$A_{32} = F_1(b^2\bar{I}_c \cosh\lambda_1 - \lambda_1\sinh\lambda_1)$$

$$A_{33} = F_2(\lambda_2\cos\lambda_2 - b^2\bar{I}_c \sin\lambda_2)$$

$$A_{34} = F_2(\lambda_2\sin\ {}_2 + b^2\bar{I}_c \cos\lambda_2)$$

$$A_{41} = (\lambda_1 - F_1)\sinh\lambda_1 - b^2 S\bar{M}_c \cosh\lambda_1$$

$$A_{42} = (\lambda_1 - F_1)\cosh\lambda_1 - b^2 S\bar{M}_c \sinh\lambda_1$$

$$A_{43} = (-\lambda_2 + F_2)\sin\lambda_2 - b^2 S\bar{M}_c \cos\lambda_2$$

$$A_{44} = (\lambda_2 - F_2)\cos\lambda_2 - b^2 S\bar{M}_c \sin\lambda_2$$

For the boundary conditions to be satisfied

$$\det(\underline{A}) = 0 \qquad (3.77)$$

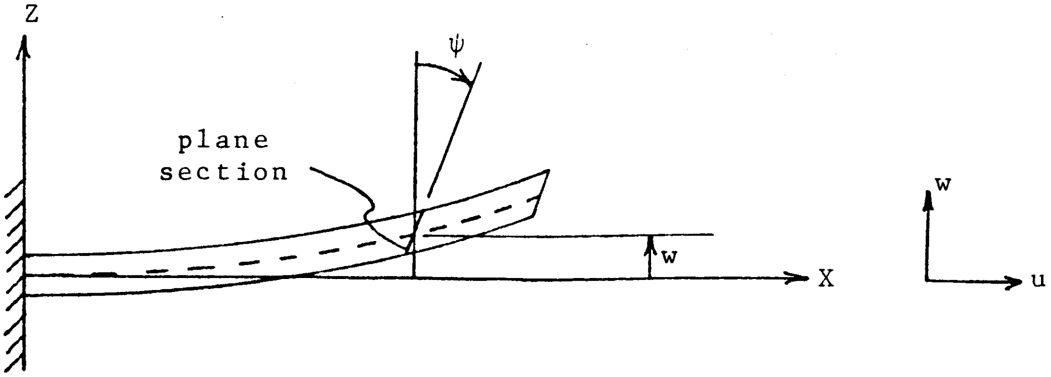
This is the characteristic equation, which must be solved for b^2. Once we have a value of b^2 we can use (3.68) to find the corresponding natural frequency, ω.

In section 3.1, article A, it was shown that the bending of the SADE truss occurs in two principal directions, with corresponding bending stiffnesses EI_1 and EI_2 and shear stiffnesses GA_1 and GA_2. In addition, I_c changes slightly depending on the bending direction. The joint masses of nodes 29, 30, 31, and 32 from Table 2 can be used with Figure 3 to calculate I_{c1}=472.37 lb-sec^2-in and I_{c2}=457.50 lb-sec^2-in. The length L of the truss is 385 inches. The only quantity left to be found is the rotary inertia per unit length i_r. For i_r, consider one segment of the truss, such as the segment from X=27.5 inches to X=82.5 inches. The planes about which bending occurs pass through diagonally opposite nodes, slicing the truss cross section into two triangles. The rotary inertia of the segment is the sum of the products of the infinitessimal masses in the segment and their perpendicular distances to the bending plane. The rotary inertias of the bars can be found using integration and inertia transfer theorems; calculating the rotary inertia of the joint masses is straightforward. Once the total inertia is determined it is divided by 55 inches, yielding $i_{r1}=1.7592$ lb-sec^2 and $i_{r2}=1.6807$ lb-sec^2.

The results are presented in Table 4. Bending in principal direction 1 corresponds to the lower of the two frequencies in each bending mode pair from the finite element analysis. Bending in principal direction 2 corresponds to the higher of the two frequencies in each bending pair.

FIGURE 5

Timoshenko Beam



3.4 TORSIONAL FREQUENCIES

The equation of motion for a rod in torsion is

$$GJ\frac{\partial^2\theta}{\partial X^2} - i_p\frac{\partial^2\theta}{\partial t^2} = 0 \qquad (3.78)$$

where θ is the angular deflection along the rod. If we assume harmonic motion with frequency ω, and if we define $\xi=X/L$, we can write

$$\theta'' + \omega^2\frac{i_pL^2}{GJ}\theta = 0 \qquad (3.79)$$

where the derivatives are now with respect to ξ. The solution to this equation is

$$\theta = C_1\cos\lambda\xi + C_2\sin\lambda\xi \qquad (3.80)$$

$$\lambda^2 = \omega^2\frac{i_pL^2}{GJ} \qquad (3.81)$$

The geometric boundary condition at X=0 is

$$\theta = 0 \qquad (3.82)$$

With a concentrated polar moment of inertia I_p at the tip, the natural boundary condition at X=L is

$$\theta' - \alpha\theta = 0$$

$$\alpha = \omega^2\frac{I_cL}{GJ} = \lambda^2\frac{I_c}{i_pL} \qquad (3.83)$$

Placing (3.80) in (3.82) and (3.83), we obtain the characteristic equation in λ :

$$\lambda\cos\lambda - \alpha\sin\lambda = 0 \tag{3.84}$$

We can solve this equation for λ and use (3.81) to find the torsional frequencies of the rod, ω.

For the SADE truss, the concentrated polar moment of inertia I_p is calculated about an axis in the X direction passing through the point Y=Z=27.5 inches using the tip mass and joint masses at nodes 29 through 32. It equals 929.87 lb-sec^2-in. For i_p consider a typical segment of the truss, such as the segment from X=27.5 inches to X=82.5 inches. We calculate the polar moment of inertia of the bars in this segment about an axis in the X direction passing through Y=Z=27.5 inches. We add to this the polar moment of inertia of four average joint masses (as derived in section 3.2) to obtain the total polar moment of inertia of this segment of the truss. Dividing this quantity by 55 inches yields the polar moment of inertia per unit length i_p=3.4399 lb-sec^2. GJ was found to be 3.6923x10^9 lb-in^2 in section 3.1, article A. With L=385 inches, $(I_pL)/(GJ)=0.70213$. We can now solve for the torsional frequencies of the SADE truss. The results are presented in Table 4.

3.5 AXIAL FREQUENCIES

The equation of motion of an axial bar is

$$EA\frac{\partial^2 u}{\partial X^2} - m\frac{\partial^2 u}{\partial t^2} = 0 \tag{3.85}$$

This equation is of the same form as (3.78) for a torsional rod. If we define $\bar{u}=u/L$, the analysis for the natural frequencies of the axial bar is exactly the same as for the torsional rod. We only need to substitute EA for GJ, m for i_p, $\bar{u}$ for θ, and M_c for I_p.

For the SADE truss, EA was found to be 1.8310×10^7 lb in section 3.1, article A. M_c and m were calculated in section 3.2, so we know the parameters needed to solve for the axial frequencies of the SADE truss.

The result is presented in Table 4. Only the first axial frequency is calculated since the second axial mode is the eighteenth overall mode of the SADE truss with a pin-jointed finite element analysis, for which no finite element results were obtained.

4. SUPER-FINITE ELEMENT METHOD

The displacement vector $\underline{u}$ in (3.49) can be used to define the degrees of freedom of a super-finite element for the SADE truss. This super beam element consists of one bay of the truss. There are twelve degrees of freedom for this element: three translations and three rotations at each end. Thus, (3.49) defines the degrees of freedom at one end of an element.

Since there are seven bays in the SADE truss, it takes seven super-finite elements to model the truss. Therefore, there are now eight nodes in the finite element model. Since there are six degrees of freedom at each node, the model of the unrestrained structure has forty-eight degrees of freedom. Thus, the super-finite element model is much smaller than the ninety-six degree of freedom pin-jointed and one-hundred-and-ninety-two degree of freedom rigid-jointed finite element models of Chapter 2.

The 12x12 element stiffness matrix $\underline{K}_{xyz}$ for one bay can be found by using some of the results from Chapter 3. From (3.55), we can solve for $\underline{K}$ in terms of the $\underline{C}$ matrix, which was obtained by considering the bar energy of one bay:

$$\underline{K} = L\, \underline{R}^T\, \underline{C}\, \underline{R} \qquad (4.1)$$

Let $\underline{K}_R$ be the matrix defined in (4.1) after the rows and columns of $\underline{K}$ are rearranged to correspond to the degrees of freedom in $\underline{u}$ (as given by (3.49)). $\underline{K}_R$ is the 6x6 stiffness matrix for one bay when that bay is restrained at X=0, as discussed in Chapter 3. This matrix is the same for all bays of the SADE truss since $\underline{C}$ is the same for all bays. Now let $\underline{u}_2$ be the 6x1 displacement vector at the free end (X=L) of the bay as defined in (3.49), with $\underline{f}_2$ the corresponding 6x1 force vector as defined in (3.48). Also, let $\underline{u}_1$ and $\underline{f}_1$ be the displacement vector and force vector at X=0. ($\underline{u}_1=\underline{0}$ for now, since we have restrained this end of the bay). Then

$$\begin{bmatrix} \underline{f}_1 \\ \\ \underline{f}_2 \end{bmatrix} = \left[\begin{array}{c|c} \underline{K}_R' & \underline{K}_S \\ \hline \underline{K}_S^T & \underline{K}_R \end{array}\right] \begin{bmatrix} \underline{u}_1 \\ \\ \underline{u}_2 \end{bmatrix}$$

$$= \underline{K}_{xyz} \begin{bmatrix} \underline{u}_1 \\ \\ \underline{u}_2 \end{bmatrix} \qquad (4.2)$$

where $\underline{K}_R'$ and $\underline{K}_S$ are yet to be determined.

To determine $\underline{K}_S$ set $\underline{u}_1=\underline{0}$. Also, let all elements of $\underline{u}_2$ be zero except the ith element, which is set equal to one. We can find the force vector $\underline{f}_2$ which corresponds to this unit displacement state from $\underline{f}_2=\underline{K}_R\underline{u}_2$ (since $\underline{u}_1=\underline{0}$). The corresponding $\underline{f}_1$ can then be found by imposing the six static equilibrium conditions on the bay. This $\underline{f}_1$ is then the ith column of $\underline{K}_S$. $\underline{K}_R'$ is found in the same way, that is, $\underline{u}_2$ is set equal to $\underline{0}$ and we find $\underline{f}_2$ from $\underline{f}_2=\underline{K}_S^T\underline{u}_1$ with only the ith column of $\underline{u}_1$ nonzero and equal to one. Then $\underline{f}_1$ is found from the conditions of static equilibrium to give the ith column of $\underline{K}_R'$.

Performing the above analysis yields

$$\underline{K}_R = \begin{bmatrix} 4.29302E+05 & 0.00000E+00 & 5.60676E+04 & 1.54186E+06 & 1.54186E+06 & 0.00000E+00 \\ 0.00000E+00 & 5.60676E+04 & 0.00000E+00 & 0.00000E+00 & -1.54186E+06 & -1.54186E+06 \\ 5.60676E+04 & 0.00000E+00 & 5.60676E+04 & 0.00000E+00 & 1.54186E+06 & 0.00000E+00 \\ 1.54186E+06 & 0.00000E+00 & 0.00000E+00 & 8.48021E+07 & 0.00000E+00 & -4.24011E+07 \\ 1.54186E+06 & -1.54186E+06 & 1.54186E+06 & 0.00000E+00 & 3.24659E+08 & 4.24011E+07 \\ 0.00000E+00 & -1.54186E+06 & 0.00000E+00 & -4.24011E+07 & 4.24011E+07 & 3.24659E+08 \end{bmatrix} \qquad (4.3)$$

$$\underline{K}_R' = \begin{bmatrix} 4.29302E+05 & 0.00000E+00 & 5.60676E+04 & 1.54186E+06 & -1.54186E+06 & 0.00000E+00 \\ 0.00000E+00 & 5.60676E+04 & 0.00000E+00 & 0.00000E+00 & -1.54186E+06 & 1.54186E+06 \\ 5.60676E+04 & 0.00000E+00 & 5.60676E+04 & 0.00000E+00 & -1.54186E+06 & 0.00000E+00 \\ 1.54186E+06 & 0.00000E+00 & 0.00000E+00 & 8.48021E+07 & 0.00000E+00 & -4.24011E+07 \\ -1.54186E+06 & -1.54186E+06 & -1.54186E+06 & 0.00000E+00 & 3.24659E+08 & -4.24011E+07 \\ 0.00000E+00 & 1.54186E+06 & 0.00000E+00 & -4.24011E+07 & -4.24011E+07 & 3.24659E+08 \end{bmatrix} \qquad (4.4)$$

$$\underline{K}_S = \begin{bmatrix} -4.29302E+05 & 0.00000E+00 & -5.60676E+04 & -1.54186E+06 & -1.54186E+06 & 0.00000E+00 \\ 0.00000E+00 & -5.60676E+04 & 0.00000E+00 & 0.00000E+00 & 1.54186E+06 & 1.54186E+06 \\ -5.60676E+04 & 0.00000E+00 & -5.60676E+04 & 0.00000E+00 & -1.54186E+06 & 0.00000E+00 \\ -1.54186E+06 & 0.00000E+00 & 0.00000E+00 & -8.48021E+07 & 0.00000E+00 & 4.24011E+07 \\ 1.54186E+06 & 1.54186E+06 & 1.54186E+06 & 0.00000E+00 & -2.39857E+08 & -4.24011E+07 \\ 0.00000E+00 & -1.54186E+06 & 0.00000E+00 & 4.24011E+07 & 4.24011E+07 & -2.39857E+08 \end{bmatrix} \tag{4.5}$$

where the units are in pounds, inches, and radians. Therefore, we now have the element stiffness matrix in local coordinates $\underline{K}_{xyz}$. In this case, the local coordinates are in the same directions as the global coordinates, so we can assemble seven element stiffness matrices $\underline{K}_{xyz}$ to determine the 48x48 unrestrained structure stiffness matrix.

To assemble a structure mass matrix we lump the mass and inertia of the bays at their endpoints. The bay endpoints are now the nodes of the finite element model. Thus, node 1 is at X=0, node 2 is at X=55 inches, etc., and node 8 is at X=385 inches (see Figure 1). All eight nodes are at Y=Z=27.5 inches. The first three diagonal elements of the 48x48 unrestrained structure mass matrix are the lumped mass at node 1. The next diagonal element (element (4,4)) is the lumped polar inertia at node 1, about an axis in the X direction through the nodes. Element (5,5) is the lumped rotary inertia at node 1, about a plane at Z=27.5 inches parallel to the X-Y plane (for bending about Y), and element (6,6) is the lumped rotary inertia at node 1, about a plane at Y=27.5 inches parallel to the X-Z plane (for bending about Z). This sequence of six lumped parameters is repeated for the remaining seven nodes to make up the forty-eight diagonal elements of the lumped unrestrained structure mass matrix.

The lumped mass at a node is made up of the four joint masses at the X location of that node as well as the bar masses on each side of it. Let X_N be the X coordinate of node N. Then we lump the bar mass from $X=X_N-L/2$ to $X=X_N+L/2$ at node N, where L is the length of the bay. For node 1 this quantity equals 0.10630 lb-sec^2/in. For nodes

2 through 7 it is 0.15522 lb-sec^2/in and for node 8 it is 0.67686 lb-sec^2/in (including the tip mass). The lumped polar inertia at node N is the sum of the polar moments of inertia of all bar and joint masses from X_N-L/2 to X_N+L/2. At node 1 this quantity equals 131.28 lb-sec^2-in, at nodes 2 through 7 it is 196.63 lb-sec^2-in, and at node 8 it is 994.23 lb-sec^2-in. The lumped rotary inertia, which is the same for bending about Y or Z, is the sum of the rotary moments of inertia of all bar and joint masses from X_N-L/2 to X_N+L/2. For node 1 it equals 65.642 lb-sec^2-in, for nodes 2 through 7 it is 98.314 lb-sec^2-in, and for node 8 it is 497.12 lb-sec^2-in.

The structure must now be totally restrained at its base (X=0). Therefore, rows and columns 1 through 6 are removed from the unrestrained structure stiffness and mass matrices to form the restrained structure stiffness and mass matrices, and rows 1 through 6 are removed from the 48x1 unrestrained displacement vector. The restrained system (2.19) has now been defined. The natural frequencies and mode shapes are solved for using the methods of Chapter 2. The results are presented in Table 4.

5. COMPARISON OF METHODS FOR FREQUENCY DETERMINATION

The results from chapters 2, 3, and 4 are presented in Table 4. The pin-jointed and rigid-jointed finite element results are in very good agreement through the third bending mode pair (modes 8 and 9). After mode 9, the rigid-jointed model encounters closely spaced modes caused by the vibrations of individual bar members of the truss (this phenomenon will be addressed in the two subsequent chapters). Therefore, up to mode 9, both standard finite element models can be used as a basis to judge the continuum and super-finite element results. Past mode 9, the continuum and super-finite element results should be judged against the pin-jointed finite element frequencies only, since the pin-jointed, continuum, and super-finite element models do not account for the closely spaced bar modes, while the rigid-jointed model does account for the closely spaced bar modes.

The first frequency from the Bernoulli-Euler continuum model provides a good estimate of the average of the finite element frequencies for the first bending mode pair. However, the subsequent frequency values diverge rapidly from the finite element results.

In contrast, the Timoshenko continuum model does an excellent job of finding the bending frequencies of the SADE truss. As discussed in section 3.3, the Timoshenko model distinguishes between the two modes in each bending mode pair. In the first two bending mode pairs, which correspond to overall structural modes 1, 2, 4, and 5, the Timoshenko continuum frequencies differ from both the pin-jointed and rigid-jointed finite element results by less than four percent. In modes 8 and 9, the Timoshenko continuum results differ by four percent from the pin-jointed frequencies and by less than nine percent from the rigid-jointed frequencies. In the modes 11 and 12, the difference from the pin-jointed frequencies is still less than seven percent.

The torsional continuum model provides good agreement with the finite element results in the first two torsional modes. The first torsional mode of the SADE truss is the third overall structural mode. In this mode the torsional continuum result differs from the finite element results by less than five percent. In the second torsional mode, which is the seventh overall mode, the torsional continuum frequency differs by less than seven percent from the finite element results. The results start to diverge in the third torsional mode, which is the tenth overall mode. Here the torsional continuum frequency differs from the pin-jointed result by fifteen percent.

The frequency obtained from the axial continuum model for the first axial mode of the SADE truss is in excellent agreement with the finite element results, differing by only three percent. Only the first axial frequency is calculated since the second axial mode is the eighteenth overall mode of the SADE truss with a pin-jointed finite element analysis, for which no finite element results were obtained.

The super-finite element model has two inherent advantages over the continuum models: it yields frequencies for all three mode types (bending, torsional, and axial, including all couplings) at once, and it provides eigenvectors to specify the mode shapes. In bending, the super-finite element model does as well as the Timoshenko continuum model in the lower modes, although it does slightly worse in higher modes, where the seven element discretization comes into play. For the first bending mode pair the super-finite element results differ from the finite element frequencies by less than two percent. In the second pair the difference is about five percent or less. For the third bending mode pair the frequencies from the super-finite element model differ from the pin-jointed results by less than seven percent and from the rigid-jointed results by less than thirteen percent. In the fourth bending mode pair, corresponding to the eleventh and twelfth overall modes of the SADE truss, the difference is less than thirteen percent from the pin-jointed results. The super-finite element model is not quite as good at matching the torsional finite element frequencies as is the torsional continuum model. It differs from the finite element results by twelve percent in the first torsional mode, by over eight percent in the second torsional mode, and by twelve percent from the pin-jointed frequency in the third torsional mode. The super-finite element result for the first axial mode is not in as good agreement as the axial continuum result, although it only differs from the finite element results by about six percent.

TABLE 4

SADE Natural Frequencies
(All frequencies in Hz)

Finite Element Methods		Continuum Methods				Super-Finite Element Method	Mode Number	Mode Shape*
Pinned Joints	Rigid Joints	Bernoulli -Euler	Timoshenko	Torsional	Axial			
4.05	4.07	4.55	4.21			4.12	1	1 Bending
4.50	4.51		4.28			4.57	2	"
12.6	12.7			13.2		14.1	3	1 Torsional
25.8	25.6	37.3	25.3			26.5	4	2 Bending
27.0	26.7		26.9			28.2	5	"
35.9	35.6				34.8	37.9	6	1 Axial
44.9	44.8			47.8		48.6	7	2 Torsional
59.5	55.8	100.	57.4			61.0	8	3 Bending
60.5	57.2		61.9			64.3	9	"
76.5	68.9			88.0		85.4	10	3 Torsional
80.2	69.8	182.	85.2			90.2	11	4 Bending
89.5	71.0		92.7			98.9	12	"
95.0	71.2					106.		
102.	72.8					116.		
110.	72.9					124.		
112.	73.5					132.		
118.	73.8					134.		

*Mode shapes apply to all methods except the rigid-jointed finite element model past the third bending mode pair, since the subsequent closely spaced modes obtained with this model are chacterized by local bar vibrations. No eigenvectors were obtained for modes past mode 12.

6. VIBRATIONS OF A TWO-DIMENSIONAL MODEL OF THE SADE TRUSS

6.1 STANDARD FINITE ELEMENT METHODS

To determine the effects of individual bar natural frequencies on the global modes of a space truss, a simpler, two-dimensional version of the SADE truss is considered. This truss has the same configuration as the general two-dimensional truss of Figure 6. The bar elements have the same dimensions and properties as the bars of the three-dimensional truss of Chapter 2. The joint masses of the SADE truss from Table 2 are averaged, and this average joint mass is placed at each node of the two-dimensional truss. Also, one-half of the SADE tip mass is divided equally between nodes 15 and 16. In addition, the mass of the bars in the three-dimensional truss which would enter the nodes of the two-dimensional truss from the Z dimension is accounted for. This extra mass is from those half bars which do not lie in the X-Y plane which remain after the three-dimensional truss is sliced down the middle at Z=27.5 inches. The mass of each half bar is lumped at that node of the two-dimensional model where the bar would enter from the Z dimension.

The two-dimensional truss is analyzed with three types of finite element models. First, a standard pin-jointed finite element model is employed (see section 2.2), using a consistent mass matrix to model the mass of the bars in the truss. This model has two translational degrees of freedom at each node. Nodes 1 and 2 are pinned to restrain the structure, so there are 28 degrees of freedom in the restrained model. Next, a standard rigid-jointed finite element model is used (see section 2.3). There are now three degrees of freedom, two translations and one rotation, per unrestrained node, plus the rotations at nodes 1 and 2, yielding 44 degrees of freedom in the restrained model. The results for these two standard finite element models are presented in Table 5 and discussed in the section below.

6.2 REFINED FINITE ELEMENT MODEL WITH ADDITIONAL NODES AT BAR MIDPOINTS

The truss is analyzed next with a refined rigid-jointed finite element model, with extra nodes at all bar midpoints. There are now forty-five nodes with three degrees of freedom per node. The two translational degrees of freedom at nodes 1 and 2 of Figure 6 are restrained, so the restrained model now contains 131 degrees of freedom.

The frequency results for the standard finite element models and the refined finite element model with additional nodes at bar midpoints are presented in Table 5, and the eigenvectors are in Appendix C. The standard pin-jointed frequencies provide good estimates of the pin-jointed bending and axial frequencies of the three dimensional SADE truss (compare Tables 4 and 5). The frequencies from the standard rigid-jointed model of the two-dimensional truss match the pin-jointed frequencies for the two-dimensional truss through the fourth mode. However, the fifth through eleventh modes of the rigid-jointed model of the two-dimensional truss are closely spaced. These closely spaced modes are characterized by the domination of vibrations, relative to the joints, of the seven individual diagonal bars in the truss. This phenomenon is explained more fully in Chapter 7.

The eigenvectors of the refined rigid-jointed model with additional nodes at bar midpoints validate the fact that the closely spaced modes are a result of individual bar vibrations. The frequencies obtained with this model match the results from the two standard finite element models up to the fourth mode. However, the refined rigid-jointed model more accurately locates the closely spaced modes than does the standard rigid-jointed model, placing them in a lower frequency band. Now modes 5 through 11 are characterized by vibrations of the seven diagonal bars. The eigenvectors for these modes show that the largest deflections occur at the midpoints of these bars. The diagonal bars also vibrate relative to the joints in modes 3 and 4, although not as severely as in the band of closely spaced modes from mode 5 to mode 11. In mode 12, the displacements of the shorter bar midpoints are about two orders of magnitude greater than the displacements of the diagonal bar midpoints. The displacements of the shorter bar midpoints are also larger than the joint displacements in mode 12, so it appears that a band of modes which are characterized by vibrations of the shorter bars begins with mode 12.

The finite element models of the two-dimensional truss do not accurately predict the closely spaced bar modes in the three-dimensional truss. For the three-dimensional SADE truss, the bar modes begin with mode 10 at 68.9 Hz, according to the standard rigid-jointed model (see Table 4). In contrast, the standard rigid-jointed model for the two-dimensional truss shows the bar modes beginning at 82.0 Hz. (The correct value for the two-dimensional truss, as given by the refined rigid-jointed finite element model with nodes at bar midpoints, is actually 59.5 Hz.) The difference between the three-dimensional and two-dimensional cases appears to be due to the presence of many more diagonal bars in the three-dimensional truss than in the two-dimensional truss. This would result in a greater number of closely spaced modes characterized by vibrations of the diagonal bars for the three-dimensional truss. Since there are more modes, it makes sense that the frequency band containing these modes is larger than the two-dimensional band, and thus, the band for the three-dimensional truss begins at a lower frequency than does the band for the two-dimensional truss.

TABLE 5

Natural Frequencies of a Two-Dimensional Model of SADE Truss
(All frequencies in Hz)

Finite Element Model			Mode No.	Mode Shape for Pin-Jointed Model
Pin-Jointed	Rigid-Jointed	Rigid-Jointed with Additional Nodes at Bar Midpoints		
4.17	4.18	4.19	1	1 Bending
25.7	25.7	25.6	2	2 Bending
34.8	34.7	34.6	3	1 Axial
58.9	57.9	55.5	4	3 Bending
87.8	81.9	61.4	5	4 Bending
103.	82.0	61.5	6	5 Bending
125.	83.8	62.4	7	2 Axial
137.	84.9	63.5	8	6 Bending
155.	88.6	64.2	9	7 Bending
173.	93.5	68.6	10	8 Bending
190.	94.7	70.7	11	3 Axial
196.	99.0	82.8	12	9 Bending
224.	101.	84.5		
257.	105.	88.8		
318.	111.	91.5		
340.	113.	100.		
352.	130.	103.		

VIBRATIONS OF A GENERAL TWO-DIMENSIONAL TRUSS

7.1 STANDARD FINITE ELEMENT METHODS

A simple two-dimensional truss with no lumped masses is considered to illustrate the effects of individual bar natural frequencies on the global modes of the truss. The truss is shown in Figure 6. The only mass in the truss is from the bars, which have the same dimensions and properties as the bars of the three dimensional truss of Chapter 2 (see Table 3).

The truss is first analyzed with a standard pin-jointed finite element model (see section 2.2), using a consistent mass matrix to model the bar mass. There are two translational degrees of freedom at each node, which yields 28 restrained degrees of freedom after nodes 1 and 2 are pinned. The results are presented in Table 6 and Appendix C. The frequencies are substantially higher than those of the two-dimensional SADE model of Chapter 6, due to the absence of the joint masses and tip mass.

The truss is also analyzed using the continuum methods of Chapter 3. Using the methods of section 3.1, part A, we find that the equivalent stiffness properties of the truss are $EI=6.5960\times10^{9}$ lb-in^{2}, $GA=1.0693\times10^{-1}$ lb, and $EA=8.7220\times10^{6}$ lb. The equivalent mass properties are $m=4.8627\times10^{-4}$ lb-sec^{2}/in^{2} and $i_r = 1.9438\times10^{-1}$ lb-sec^{2}. The resulting continuum frequencies are also shown in Table 6. The Bernoulli-Euler continuum bending frequencies are not reliable, except in the first mode. However, the axial continuum frequencies and Timoshenko continuum bending frequencies compare very well with the pin-jointed finite element results, even in the higher modes.

The truss is next analyzed with a standard rigid-jointed finite element model (see section 2.3), with nodes only at bar endpoints. There are now three degrees of freedom, two translations and one rotation, at each node. The translational degrees of freedom at nodes 1 and 2 are restrained to yield 44 degrees of freedom in the restrained model. The results are presented in Table 6 and Appendix C.

The results obtained with the standard rigid-jointed model are in good agreement with the pin-jointed results in the first two modes. However, the results start to diverge in the third mode. The third natural frequency is near the first natural bending frequency of the longer, diagonal bars in the truss. The first natural bending frequency of the diagonal bars is 79.8 Hz with clamped endpoints and 35.2 Hz with pinned endpoints (using standard Bernoulli-Euler beam theory). The first natural bending frequency of the shorter, 55 inch bars is 160. Hz with clamped endpoints and 70.4 Hz with pinned endpoints. The seven closely spaced modes from 82.0 Hz to 99.2 Hz obtained with the standard rigid-jointed model appear to be characterized by vibrations, relative to the joints, of the diagonal bars.

The pin-jointed finite element model completely missed this phenomenon. After mode 2, the results obtained with this model (or the continuum models) are completely unreliable. The element stiffness matrices in the pin-jointed model contain only extensional stiffnesses. Therefore, the pin-jointed model cannot account for bending vibrations relative to the joints of the bar elements of the truss. The continuum models are inherently unable to account for the bending vibrations of individual elements of the truss. However, the rigid-jointed element stiffness matrices do account for the bending stiffnesses of the individual bars in the truss. The rigid-jointed model therefore is influenced by first bending mode vibrations of the diagonal bars, near the first clamped-clamped natural frequency of these bars (79.8 Hz). The standard rigid-jointed model of the three-dimensional SADE truss also found closely spaced modes from 68.9 Hz characterized by vibrations of the diagonal bars (see Table 4). However, the results in the next two sections show that although the standard rigid-jointed model finds the closely spaced bar modes, it does not place these modes in the correct frequency band.

7.2 REFINED FINITE ELEMENT METHOD WITH ADDITIONAL NODES AT BAR MIDPOINTS

The general two dimensional truss of Figure 6 is now analyzed with a refined rigid-jointed finite element model, with an extra node at all bar midpoints. Therefore, the refined model contains forty-five nodes. Since there are three degrees of freedom per node (two translations and one rotation), the restrained model contains 131 degrees of freedom, after nodes 1 and 2 in Figure 6 are pinned. The first bending modes of the individual bars can now be modeled more accurately since the refined model with nodes at bar midpoints takes the deflection at the bar midpoints into account. This model can also accurately account for the second bending mode of the bars, since it also considers the rotations at the bar midpoints.

The results from the refined rigid-jointed model are presented in Table 6 and Appendix C. The frequency for the first mode at 12.3 Hz matches those from the pin-jointed and standard rigid-jointed finite element analyses. This mode is the usual first bending mode of the truss. Table 7 shows that the maximum deflection at a diagonal bar midpoint, the maximum deflection at a joint, and the maximum deflection at a shorter, 55 inch bar midpoint, are about the same for the first mode. The maximum deflections occur at the free end of the truss, as would be expected in the first bending mode. The first mode is plotted in Figure 6, using the eigenvector from the refined rigid-jointed finite element model with additional nodes at bar midpoints. The actual physical truss is 385 inches long, and the eigenvectors are normalized to a maximum deflection of one inch in a coordinate direction. In the figure, the truss is 4.2 inches long and the maximum deflection of any node in a coordinate direction is shown as 0.2 inches. The eigenvectors were scaled accordingly to make the plots.

The frequency for the second mode from the refined rigid-jointed model, at 49.8 Hz, is slightly lower than the corresponding second mode frequencies from the pin-jointed and standard rigid-jointed models, but this mode is still the second bending mode, as the eigenvector plot of Figure 6 shows. However, notice that the diagonal bars have started to vibrate relative to the joints and that the maximum deflections occur at the diagonal bar midpoints. Table 7 shows that the maximum of the diagonal bar midpoint deflections is about twice as great as the maximum of the joint deflections or the maximum of the shorter bar midpoint deflections.

The third natural frequency from the refined rigid-jointed model, at 61.3 Hz, is not at all close to the corresponding frequencies from the pin-jointed and standard rigid-jointed models. With the pin-jointed and standard rigid-jointed models, the third mode is an axial mode, but the refined rigid-jointed model shows that the third mode of the structure is not an axial mode. The plot of the third mode (at 61.3 Hz) in Figure 6 reveals that this mode is characterized by severe vibrations of the diagonal bars, while the deflections of the joints is imperceptible. While some of the shorter bar midpoints show a deflection, these deflections only occur in those bays of the truss where the diagonal bars are most excited. Table 7 shows that the maximum deflection of a diagonal bar midpoint is two orders of magnitude greater than the maximum deflection at a joint and significantly greater than the maximum deflection at a shorter bar midpoint. The deflection of the shorter bar midpoints appears to be a secondary phenomenon caused by the need for the boundary conditions to be satisfied at the rigid joints. While a joint is free to rotate, the relative angles at a joint between the bars which are connected at that joint must remain constant.

The next six modes from the refined rigid-jointed model are similar to the third mode, at 61.3 Hz. Table 7 and the eigenvectors in Appendix C reveal that all seven of the modes in the closely spaced band from 61.3 Hz to 74.2 Hz are characterized by vibrations of the diagonal bars, although the joint deflections become more significant as frequency is increased. As discussed above, the pin-jointed finite element model and the continuum models completely miss this band, while the standard rigid-jointed finite element model found the band, but placed it in the wrong location (see Table 6). The analysis of the truss with a finite element model employing exact dynamic stiffness coefficients presented in the next section validates the results of the refined rigid-jointed finite element model. The first natural bending frequency of the diagonal bars is 35.2 Hz with pinned endpoints and 79.8 Hz with clamped endpoints, so the band of diagonal bar modes lies somewhere between these two values, but closer to that obtained with the clamped (rigid) endpoints.

The next three modes, modes 10, 11, and 12, from the refined rigid-jointed model are very similar to each other. They are characterized by severe vibrations relative to the joints of the shorter, 55 inch bars. Mode 10, at 82.7 Hz, is plotted in Figure 6. Note the symmetry and the second bending mode shape of the diagonal bars, which deflect slightly to satisfy the boundary conditions at the joints. However, Table 7 shows that the deflections of the shorter bar midpoints are two orders of magnitude greater than the deflections of the joints and one order of magnitude greater than the deflections of the diagonal bar midpoints. Thus, it appears that a band of modes characterized by vibrations of the shorter bars begins at 82.7 Hz. The first natural bending frequency of the shorter bars is 70.4 Hz with pinned endpoints and 160. Hz with clamped endpoints. Therefore, the band of shorter bar modes begins after the first natural pinned bending frequency of the shorter bars. Since there are twenty-two shorter bars in the truss, there are most likely twenty-two modes in the band of shorter bar modes, so this band may extend to near the first natural clamped bending frequency of the shorter bars.

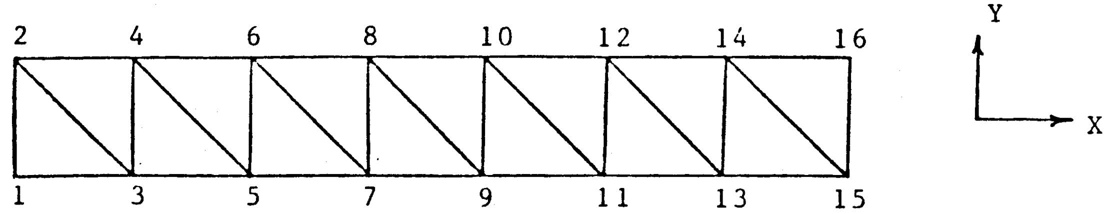
7.3 FINITE ELEMENT MODEL WITH EXACT DYNAMIC STIFFNESS COEFFICIENTS

To validate the natural frequencies obtained for the general two-dimensional truss with the refined rigid-jointed finite element model with additional nodes at bar midpoints, a finite element analysis with exact dynamic stiffness coefficients is performed. The dynamic stiffness coefficients modify the terms in the rigid-jointed bar element stiffness matrix of section 2.3 (see (2.22)), making these terms frequency dependent. Note that for the general two-dimensional truss of Figure 6 in the X-Y plane, the rows and columns of the element stiffness matrix (2.22) corresponding to the translational degree of freedom in Z and the rotational degrees of freedom about X and Y are not needed, but the more general case is presented here.

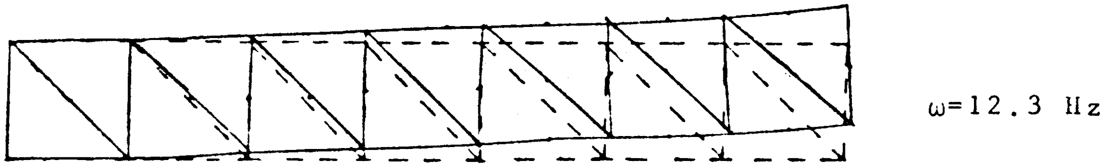
The dynamic stiffness coefficients used, which neglect transverse shear and rotary inertia, are those of Anderson (reference 7), as obtained from Howson (reference 8). For nonzero terms $K_{i,j}$ of the element stiffness matrix (2.22), a dynamic stiffness coefficient $F_{i,j}$ is multiplied by $K_{i,j}$, and the resulting frequency dependent product replaces $K_{i,j}$ in the element stiffness matrix. We assume that there is no prestressing and use the notation of Chapter 2, where L is the length of the bar element, and define

FIGURE 6

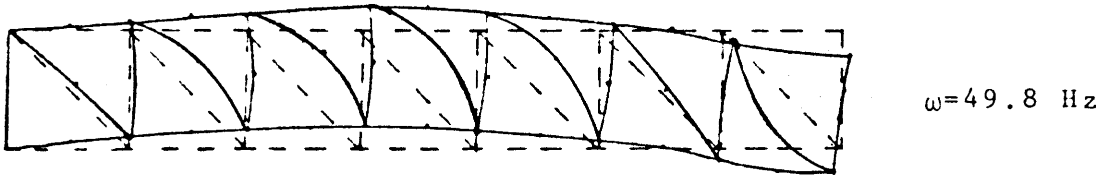
General Two-Dimensional Truss and Mode Shapes with Additional Nodes at Bar Midpoints



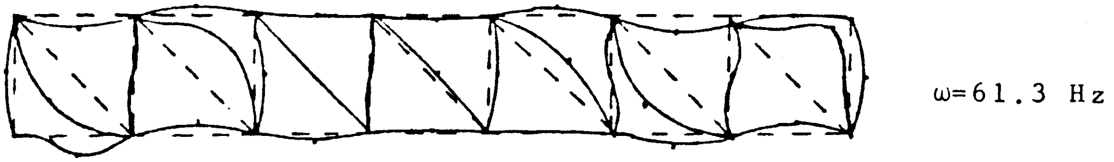
General Truss
(Nodes at bar midpoints are not labelled)



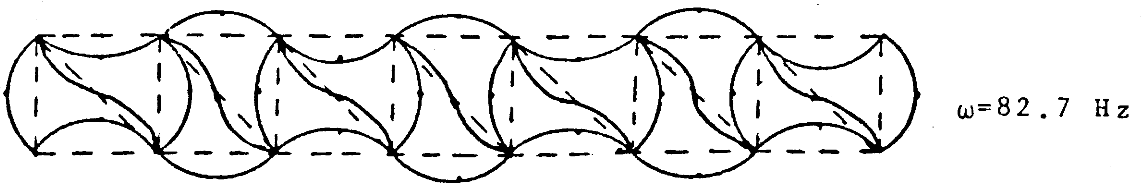
First Bending



Second Bending



First of Seven Closely Spaced Modes Characterized by Diagonal Bar Vibrations



First of the Closely Spaced Modes Characterized by Shorter 55 inch Bar Vibrations

$$g^2 = \omega^2 \frac{mL^2}{EA} \qquad h^2 = \omega^2 \frac{mL^2}{GJ} \tag{7.1}$$

$$d_k{}^4 = \omega^2 \frac{mL^4}{EI_k} \tag{7.2}$$

where k can be 2 or 3, with k=2 corresponding to EI_Y and k=3 corresponding to EI_Z. Also let

$$\Delta_k = 1-\cosh d_k \cos d_k \tag{7.3}$$

Then for the nonzero elements in the upper triangle (j>i) of (2.22),

$$F_{1,1} = g\cot g$$

$$F_{1,7} = g\csc g$$

$$F_{2,2} = \frac{d_3{}^2(\cosh d_3 \sin d_3 + \sinh d_3 \cos d_3)}{12\Delta_3}$$

$$F_{2,6} = \frac{d_3{}^3 \sinh d_3 \sin d_3}{6\Delta_3}$$

$$F_{2,8} = \frac{d_3{}^3(\sin d_3 + \sinh d_3)}{12\Delta_3}$$

$$F_{2,12} = \frac{d_3{}^2(\cosh d_3 - \cos d_3)}{6\Delta_3}$$

$$F_{3,3} = \frac{d_2{}^3(\cosh d_2 \sin d_2 + \sinh d_2 \cos d_2)}{12\Delta_2}$$

$$F_{3,5} = \frac{d_2{}^2 \sinh d_2 \sin d_2}{6\Delta_2}$$

$$F_{3,9} = \frac{d_2^{\,3}(\sin d_2 + \sinh d_2)}{12\Delta_2}$$

$$F_{3,11} = \frac{d_2^{\,2}(\cosh d_2 - \cos d_2)}{6\Delta_2}$$

$$F_{4,4} = h\coth \qquad (7.4)$$

$$F_{4,10} = h\operatorname{csch}$$

$$F_{5,5} = \frac{d_2(\cosh d_2 \sin d_2 - \sinh d_2 \cos d_2)}{4\Delta_2}$$

$$F_{5,9} = F_{3,11}$$

$$F_{5,11} = \frac{d_2(\sinh d_2 - \sin d_2)}{2\Delta_2}$$

$$F_{6,6} = \frac{d_3(\cosh d_3 \sin d_3 - \sinh d_3 \cos d_3)}{4\Delta_3}$$

$$F_{6,8} = F_{2,12}$$

$$F_{6,12} = \frac{d_3(\sinh d_3 - \sin d_3)}{2\Delta_3}$$

and

$$F_{i+6,j+6} = F_{i,j}$$

The frequency dependent restrained structure stiffness matrix for the general two-dimensional truss of sections 7.1 and 7.2 is assembled in the same manner as is the standard rigid-jointed restrained stiffness matrix of section 7.1. Therefore, the node numbering scheme is that of Figure 6, and the frequency dependent restrained structure stiffness matrix $\underline{K}(\omega)$ is of order 44. No mass matrix is needed, since the mass of the bars is accounted for in the frequency dependent dynamic stiffness coefficients. We need only to solve

$$\det (\underline{K}(\omega)) = 0 \tag{7.5}$$

to obtain the natural frequencies of the truss. If lumped inertias were present at the nodes, they would be placed in a diagonal mass matrix $\underline{J}$ and the equation to be solved for the natural frequencies would be

$$\det (\underline{K}(\omega) - \omega^2 \underline{J}) = 0 \tag{7.6}$$

For the general two dimensional truss, which contains no lumped inertias, the results are presented in Table 6. The maximum difference in the first twelve modes between frequencies from the finite element model with exact dynamic stiffness coefficients and the refined rigid-jointed finite element model with additional nodes at bar midpoints is 1.1 percent in mode 6. The maximum difference in the first eighteen modes is 2.5 percent in mode 17. Therefore, the results from the finite element model with exact dynamic stiffness coefficients validate the frequencies obtained with the refined rigid-jointed model.

TABLE 6

Natural Frequencies of General Two-Dimensional Truss
(All frequencies in Hz)

Continuum Methods			Finite Element Methods				Mode No.	Mode Shape for Continuum and Pin-Jointed Finite Element Models
Bernoulli -Euler	Timoshenko	Axial	Pinned Joints	Rigid Joints	Rigid Joints with Additional Nodes at Bar Midpoints	Rigid Joints with Exact Dynamic Stiffness Coefficients		
13.9	12.6		12.2	12.2	12.3	12.2	1	1 Bending
87.1	55.8		55.3	53.2	49.8	49.5	2	2 Bending
		87.0	88.1	76.6	61.3	60.7	3	1 Axial
244.	120.		123.	82.0	61.4	60.8	4	3 Bending
478.	184.		195.	82.8	62.4	61.8	5	4 Bending
790.	248.		254.	85.1	63.3	62.6	6	5 Bending
		261.	276.	87.9	64.1	63.5	7	2 Axial
1180.	308.		336.	92.1	64.6	64.0	8	6 Bending
1650.	356.		379.	94.7	74.7	74.1	9	7 Bending
2200.	388.		404.	99.2	82.7	82.1	10	8 Bending
		435.	421.	105.	84.4	83.8	11	3 Axial
2820.	442.		498.	111.	91.2	90.4	12	9 Bending
			572.	128.	99.6	98.8		
			656.	133.	102.	101.		
			718.	149.	108.	107.		
			751.	157.	116.	114.		
			757.	188.	123.	120.		

TABLE 7

Node Deflections of General Two-Dimensional Truss

Mode No.	Modal Frequency (Hz)	Magnitude of Node Deflection		
		Largest Deflection at a Joint	Largest Deflection at a Diagonal Bar Midpoint	Largest Deflection at a 55 inch Bar Midpoint
1	12.3	1.005	0.938	1.000
2	49.8	0.657	1.148	0.657
3	61.3	0.092	1.330	0.432
4	61.4	0.062	1.412	0.466
5	62.4	0.053	1.396	0.412
6	63.3	0.072	1.406	0.354
7	64.1	0.127	1.313	0.394
8	64.6	0.227	1.297	0.496
9	74.7	0.422	1.170	0.537
10	82.7	0.062	0.127	1.001
11	84.4	0.059	0.236	1.000
12	91.2	0.099	0.362	1.002

8. CONCLUSIONS

The dynamic modes of the SADE truss have been obtained using finite element and continuum models, and the results are presented in Table 4 and Appendix C. The dynamics of a two-dimensional model of the SADE truss and a general two-dimensional truss have also been analyzed, and these results are presented in Tables 5 and 6 and Appendix C.

The results of the truss analyses accomplished in this study yield two major conclusions. Firstly, continuum models of beam-like trusses can produce global natural frequencies which are very close to standard pin-jointed finite element results. However, the results from refined finite element models reveal that the modes obtained with standard finite element and continuum models are inaccurate except in the lowest modes, since the standard finite element and continuum models do not adequately account for the effects of individual bar vibrations on the global modes of the truss.

The Timoshenko continuum model, the torsional continuum model, and the axial continuum model yield results which are close to the bending, torsional, and axial frequencies, respectively, obtained with a standard finite element model with pin-jointed bar elements (three degrees of freedom per node). The Bernoulli-Euler continuum model produces an accurate bending frequency only for the first bending mode.

The standard finite element model with rigid-jointed bar elements (six degrees of freedom per node) with nodes only at bar endpoints yields results which are very close to the pin-jointed finite element results in the lower modes. However, above a frequency corresponding to the lowest natural bending frequency of the individual bars in the truss, assuming pinned endpoints, the standard rigid-jointed finite element model yields a group of closely spaced bar modes characterized by vibrations, relative to the joints, of the individual bars in the truss. These bar modes occur between the first natural bending frequency of a bar with pinned endpoints and the first natural bending frequency of a bar with clamped endpoints. For the trusses considered in this study, there are two types of bars in the truss (with the same cross-sectional properties but different lengths), so the standard rigid-jointed model finds two groups of closely spaced bar modes characterized by first bending mode vibrations of the individual bars.

The standard pin-jointed finite element and continuum models completely miss these groups of bar modes. Therefore, it appears that continuum models such as those of Noor, Anderson, and Greene (reference 6) and Berry, Yang, and Skelton (reference 9) may not be directly applicable to determining the modes of lattice structures, except in the lowest modes and perhaps those modes with frequencies which are relatively far from any bar natural frequencies. These continuum methods do not adequately model the individual bars of the truss.

While a standard rigid-jointed finite element model with nodes only at bar endpoints finds the closely spaced bar modes, it does not place these modes at the right frequencies. Therefore, analyses of lattice structures which take advantage of the periodicity of such structures and which employ standard rigid-jointed bar elements with nodes only at bar endpoints, such as that of Leung (reference 10), may yield inaccurate results. A refined rigid-jointed finite element model with additional nodes at all bar midpoints places the closely spaced bar modes in a significantly lower frequency band than does the standard rigid-jointed model, although the band still lies between the first pinned natural bending frequency and the first clamped natural bending frequency of that type of bar. The frequencies from the refined rigid-jointed model are validated by a finite element analysis employing exact dynamic stiffness coefficients. Related analyses of these types of trusses with individual bar vibration effects have been done by Schroeder (reference 11). However, even the refined rigid-jointed finite element model may yield inaccurate results if the slenderness ratios of the bars are high enough, due to buckling and eccentricity effects, unless the stiffness of the bars is modified as is shown by Regelbrugge and Park (reference 12). But in general, it appears that a refined rigid-jointed model with additional nodes at all bar midpoints is the simplest finite element model which accurately determines the modes of a lattice structure.

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APPENDIX A

SADE Sensor Placement

This appendix outlines a scheme for instrumenting the Structural Assembly Demonstration Experiment (SADE) truss to obtain structural dynamics data on a Space Shuttle flight. The type, number, and placement of sensors is considered. Since the truss would extend from the Shuttle bay, impulse force inputs could be applied to the base of the truss by the Shuttle attitude control system for lower mode excitation. Actuators could be used on the truss to excite higher modes.

To obtain the global modes of the truss, sensors should be placed only at the joints of the truss. Placing sensors on the bars would not give results which would be useful for obtaining global modes except for the lowest few modes, since in many modes the bars vibrate relative to the joints, as explained in Chapters 6 and 7. Therefore, sensors for obtaining global modes would be placed only at cross-sectional stations along the truss (along the X dimension in Figure 1) where joints occur. The first bending mode deflections of the bars could be obtained by placing two accelerometers at the center of each bar, normal to the bar and to each other. However, the bar deflections could probably be measured adequately and less expensively by placing strain guages at the centers of the bars. To find the bar vibration modes, sensors would probably be needed at only a few of the longer diagonal bars and a few of the shorter, 55 inch bars (see Figure 1). However, these modes might be impossible to separate since the rigid-jointed finite element models indicate that they are closely spaced.

Six accelerometers would be needed at a given cross-sectional station to determine the global mode characteristics at that station. Three mutually orthogonal accelerometers would be placed at each of two diagonally opposite joints at the station. For example, for the station at the free end of the truss, at X=385, inches (see Figure 1), three accelerometers would be placed at each of nodes 29 and 32 or nodes 30 and 31. The accelerometers at a given joint would be aligned with the X, Y, and Z directions. The two X accelerometers at diagonally opposite joints of a station would sense axial motion and be able to distinguish an axial mode from a bending mode. For an axial mode, the readings from the two X accelerometers would be in phase but for a bending mode they would be out of phase, except when the neutral axis for bending passed through the two joints with the accelerometers. To accomodate this case, accelerometers at successive stations could be placed at joints which define orthogonal axes. For example, if the accelerometers at X=330 inches were placed at diagonally opposite nodes 26 and 26, the accelerometers at X=385 inches would be placed at diagonally opposite nodes 29 and 32. The Y and Z accelerometers at a station would yield the magnitude and direction of bending at that station and an average torsional deflection for that cross-section which would be used to obtain the torsional modes. However, more accelerometers would be needed if the state of distortion of the cross section were desired.

The number of stations to be instrumented would depend on cost versus accuracy and the number of modes desired. There are seven stations where accelerometers could be placed (discounting the station at the base of the truss, at X=0), so the maximum number of accelerometers which could be used for obtaining the global modes would be forty-two. Vander Velde and Carignan (reference 13) and Juang and Rodriguez (reference 14) present methods for obtainig optimum sensor locations for a given number of sensors, based on the minimization of the state estimation error. These methods could be used to determine the best X stations for accelerometer placement if less than seven stations were instrumented. However, for this relatively short truss, it might be more practical to place the sensors at stations equal distances apart, as was done for the Astromast (reference 15). In any case, it would certainly be desirable to place sensors at the station at the free end of the truss because large deflections could be expected there in the lower modes, especially since the truss supports a relatively large tip mass.

APPENDIX B

Coupled Timoshenko Equations for a Cantilevered Beam-Like Structure

The strain energy U for a beam of length L can be expressed as

$$U = \tfrac{1}{2}\int_0^L \underline{\varepsilon}^T \underline{C}\underline{\varepsilon}\, dX \tag{B.1}$$

This equation is the continuum analog of (3.39), which is applicable to a discretized structure. $\underline{C}$ is given by (3.40), and $\underline{\varepsilon}$ is defined by

$$\underline{\varepsilon}^T = \begin{bmatrix} \frac{du}{dX} & \frac{d\psi}{dX} & \frac{d\phi}{dX} & (\frac{dv}{dX} - \psi) & (\frac{dw}{dX} + \phi) & \frac{d\theta}{dX} \end{bmatrix} \tag{B.2}$$

where u, v, and w are the translations in the X, Y, and Z directions, respectively, and θ, ϕ, and ψ are the rotations about X, Y, and Z, respectively. Also let p_X, p_Y, and p_Z be forces per unit length and m_X, m_Y, and m_Z be moments per unit length with the obvious senses. Let P_X, P_Y, and P_Z be concentrated forces at X=L, and let M_X, M_Y, and M_Z be concentrated moments at X=L.

The first variation of the total energy of the system must equal zero:

$$\delta\Pi = \delta U - \delta W = 0 \tag{B.3}$$

where Π is the total energy. δU, the first variation of the strain energy, can be found by expanding (B.1), taking the first variation of the resulting scalar integral expression, and then integrating by parts. The resulting expression contains an integral portion and boundary terms. The first variation of the work W is

$$\delta W = \int_0^L (p_X \delta u + p_Y \delta v + p_Z \delta w + m_X \delta\theta + m_Y \delta\phi + m_Z \delta\psi)\, dX$$
$$+ P_X \delta u(L) + P_Y \delta v(L) + P_Z \delta w(L) + M_X \delta\theta(L) + M_Y \delta\phi(L) + M_Z \delta\psi(L) \qquad \textbf{(B.4)}$$

Substituting for δU and δW in (B.3) yields the six coupled Timoshenko equations for the cantilevered beam and six natural boundary conditions at X=L:

$$C_{11}\frac{d^2u}{dX^2}+C_{12}\frac{d^2\psi}{dX^2}+C_{13}\frac{d^2\phi}{dX^2}+C_{14}\frac{d}{dX}(\frac{dv}{dX}-\psi)+C_{15}\frac{d}{dX}(\frac{dw}{dX}+\phi)+C_{16}\frac{d^2\theta}{dX^2} = -p_X$$

$$C_{14}\frac{d^2u}{dX^2}+C_{24}\frac{d^2\psi}{dX^2}+C_{34}\frac{d^2\phi}{dX^2}+C_{44}\frac{d}{dX}(\frac{dv}{dX}-\psi)+C_{45}\frac{d}{dX}(\frac{dw}{dX}+\phi)+C_{46}\frac{d^2\theta}{dX^2} = -p_Y$$

$$C_{15}\frac{d^2u}{dX^2}+C_{25}\frac{d^2\psi}{dX^2}+C_{35}\frac{d^2\phi}{dX^2}+C_{45}\frac{d}{dX}(\frac{dv}{dX}-\psi)+C_{55}\frac{d}{dX}(\frac{dw}{dX}+\phi)+C_{56}\frac{d^2\theta}{dX^2} = -p_Z$$

$$C_{16}\frac{d^2u}{dX^2}+C_{26}\frac{d^2\psi}{dX^2}+C_{36}\frac{d^2\phi}{dX^2}+C_{46}\frac{d}{dX}(\frac{dv}{dX}-\psi)+C_{56}\frac{d}{dX}(\frac{dw}{dX}+\phi)+C_{66}\frac{d^2\theta}{dX^2} = -m_X$$

$$C_{13}\frac{d^2u}{dX^2}-C_{15}\frac{du}{dX}+C_{23}\frac{d^2\psi}{dX^2}-C_{25}\frac{d\psi}{dX}+C_{33}\frac{d^2\phi}{dX^2}+C_{34}\frac{d}{dX}(\frac{dv}{dX}-\psi)-C_{35}\frac{d\phi}{dX}$$
$$+C_{35}\frac{d}{dX}(\frac{dw}{dX}+\phi)+C_{36}\frac{d^2\theta}{dX^2}-C_{45}(\frac{dv}{dX}-\psi)-C_{55}(\frac{dw}{dX}+\phi)-C_{56}\frac{d\theta}{dX} = -m_Y$$

$$C_{12}\frac{d^2u}{dX^2}+C_{14}\frac{du}{dX}+C_{22}\frac{d^2\psi}{dX^2}+C_{23}\frac{d^2\phi}{dX^2}+C_{24}\frac{d\psi}{dX}+C_{24}\frac{d}{dX}(\frac{dv}{dX}-\psi)+C_{25}\frac{d}{dX}(\frac{dw}{dX}+\phi)$$
$$+C_{26}\frac{d^2\theta}{dX^2}+C_{34}\frac{d\phi}{dX}+C_{44}(\frac{dv}{dX}-\psi)+C_{45}(\frac{dw}{dX}+\phi)+C_{46}\frac{d\theta}{dX} = -m_Z$$

At X=L,

$$C_{11}\frac{du}{dX}+C_{12}\frac{d\psi}{dX}+C_{13}\frac{d\phi}{dX}+C_{14}\left(\frac{dv}{dX}-\psi\right)+C_{15}\left(\frac{dw}{dX}+\phi\right)+C_{16}\frac{d\theta}{dX} = P_X$$

$$C_{14}\frac{du}{dX}+C_{24}\frac{d\psi}{dX}+C_{34}\frac{d\phi}{dX}+C_{44}\left(\frac{dv}{dX}-\psi\right)+C_{45}\left(\frac{dw}{dX}+\phi\right)+C_{46}\frac{d\theta}{dX} = P_Y$$

$$C_{15}\frac{du}{dX}+C_{25}\frac{d\psi}{dX}+C_{35}\frac{d\phi}{dX}+C_{45}\left(\frac{dv}{dX}-\psi\right)+C_{55}\left(\frac{dw}{dX}+\phi\right)+C_{56}\frac{d\theta}{dX} = P_Z$$

$$C_{16}\frac{du}{dX}+C_{26}\frac{d\psi}{dX}+C_{36}\frac{d\phi}{dX}+C_{46}\left(\frac{dv}{dX}-\psi\right)+C_{56}\left(\frac{dw}{dX}+\phi\right)+C_{66}\frac{d\theta}{dX} = M_X$$

$$C_{13}\frac{du}{dX}+C_{23}\frac{d\psi}{dX}+C_{33}\frac{d\phi}{dX}+C_{34}\left(\frac{dv}{dX}-\psi\right)+C_{35}\left(\frac{dw}{dX}+\phi\right)+C_{36}\frac{d\theta}{dX} = M_Y$$

$$C_{12}\frac{du}{dX}+C_{22}\frac{d\psi}{dX}+C_{23}\frac{d\phi}{dX}+C_{24}\left(\frac{dv}{dX}-\psi\right)+C_{25}\left(\frac{dw}{dX}+\phi\right)+C_{26}\frac{d\theta}{dX} = M_Z$$

Appendix C

Eigenvectors for Finite Element Models

The following pages contain the eigenvectors for the finite element models discussed in Chapters 2, 4, 6, and 7. All eigenvectors are normalized to one. The structural modes are numbered in reverse order in the subspace iteration program output. Thus, the first mode appearing in a listing is the first structural mode, even though it is not numbered as such. The eigenvectors for the three-dimensional models contain zero entries for the restrained degrees of freedom. The eigenvectors for the two-dimensional models do not include the restrained degrees of freedom.

Pin-Jointed Model of SADE (Section 2.2)

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*******************************
EIGENVECTORS INCLUDING RESTRAINED DOF'S:


OMODE NO.  24     :  LAMBDA= 0.64832D+03          FREQ (RAD/SEC)= 0.25462D+02          FREQ (HERTZ)= 0.40524D+01
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.000D+00   0.000D+00   0.148D-02  -0.706D-01   0.536D-01  -0.450D-01  -0.494D-01   0.507D-01   0.480D-01  -0.654D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.321D-01  -0.825D-02  -0.519D-01   0.258D-01   0.519D-02  -0.171D+00   0.150D+00  -0.863D-01  -0.137D+00   0.150D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.847D-01  -0.168D+00   0.111D+00  -0.128D-01  -0.142D+00   0.108D+00   0.112D-01  -0.292D+00   0.263D+00  -0.117D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE    -0.261D+00   0.265D+00   0.116D+00  -0.288D+00   0.243D+00  -0.148D-01  -0.264D+00   0.243D+00   0.120D-01  -0.435D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.429D+00  -0.142D+00  -0.400D+00   0.426D+00   0.139D+00  -0.431D+00   0.395D+00  -0.199D-01  -0.402D+00   0.391D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.136D-01  -0.590D+00   0.610D+00  -0.160D+00  -0.550D+00   0.610D+00   0.153D+00  -0.587D+00   0.570D+00  -0.236D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.554D+00   0.569D+00   0.177D-01  -0.749D+00   0.799D+00  -0.169D+00  -0.709D+00   0.800D+00   0.164D+00  -0.747D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.763D+00  -0.239D-01  -0.711D+00   0.764D+00   0.178D-01  -0.906D+00   0.100D+01  -0.174D+00  -0.869D+00   0.999D+00
O  DOF            91          92          93          94          95          96
 VALUE     0.166D+00  -0.905D+00   0.963D+00  -0.266D-01  -0.870D+00   0.961D+00
OMODE NO.  23     :  LAMBDA= 0.79801D+03          FREQ (RAD/SEC)= 0.28249D+02          FREQ (HERTZ)= 0.44960D+01
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.000D+00   0.000D+00   0.497D-01   0.385D-01   0.291D-01  -0.319D-02   0.191D-02   0.237D-01  -0.989D-02   0.324D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.594D-01  -0.521D-01   0.556D-02   0.575D-01   0.880D-01   0.124D+00   0.950D-01  -0.924D-03   0.671D-01   0.949D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.155D-01   0.121D+00   0.156D+00  -0.986D-01   0.733D-01   0.159D+00   0.123D+00   0.243D+00   0.226D+00   0.575D-02
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.195D+00   0.224D+00  -0.151D-01   0.237D+00   0.260D+00  -0.129D+00   0.199D+00   0.260D+00   0.145D+00   0.407D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.361D+00   0.624D-02   0.345D+00   0.356D+00  -0.212D-01   0.402D+00   0.415D+00  -0.156D+00   0.349D+00   0.411D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.158D+00   0.594D+00   0.509D+00   0.766D-02   0.524D+00   0.509D+00  -0.258D-01   0.591D+00   0.578D+00  -0.174D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE     0.528D+00   0.579D+00   0.168D+00   0.792D+00   0.677D+00   0.121D-01   0.726D+00   0.677D+00  -0.257D-01   0.789D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.739D+00  -0.181D+00   0.729D+00   0.739D+00   0.168D+00   0.100D+01   0.842D+00   0.125D-01   0.932D+00   0.841D+00
O  DOF            91          92          93          94          95          96
 VALUE    -0.291D-01   0.998D+00   0.907D+00  -0.184D+00   0.933D+00   0.906D+00
OMODE NO.  22     :  LAMBDA= 0.62896D+04          FREQ (RAD/SEC)= 0.79307D+02          FREQ (HERTZ)= 0.12622D+02
OEIGENVECTOR:
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O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE   0.000D+00   0.000D+00   0.237D-01  -0.172D+00   0.139D+00   0.128D-01   0.135D+00   0.983D-01   0.275D-01  -0.136D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE  -0.179D+00   0.301D-01   0.176D+00  -0.140D+00   0.420D-01  -0.341D+00   0.257D+00   0.217D-01   0.281D+00   0.257D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE   0.568D-01  -0.306D+00  -0.328D+00   0.614D-01   0.321D+00  -0.327D+00   0.949D-01  -0.481D+00   0.440D+00   0.698D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE   0.440D+00   0.442D+00   0.503D-01  -0.447D+00  -0.445D+00   0.535D-01   0.479D+00  -0.446D+00   0.105D+00  -0.592D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE   0.593D+00   0.787D-01   0.610D+00   0.559D+00   0.820D-01  -0.560D+00  -0.610D+00   0.830D-01   0.647D+00  -0.577D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE   0.113D+00  -0.713D+00   0.705D+00   0.858D-01   0.748D+00   0.704D+00   0.112D+00  -0.685D+00  -0.731D+00   0.112D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE   0.781D+00  -0.729D+00   0.146D+00  -0.812D+00   0.848D+00   0.120D+00   0.873D+00   0.850D+00   0.112D+00  -0.787D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE  -0.809D+00   0.111D+00   0.901D+00  -0.811D+00   0.148D+00  -0.876D+00   0.953D+00   0.123D+00   0.987D+00   0.941D+00
O  DOF            91          92          93          94          95          96
 VALUE   0.136D+00  -0.865D+00  -0.912D+00   0.137D+00   0.100D+01  -0.900D+00
OMODE NO.  21        :  LAMBDA= 0.26226D+05           FREQ (RAD/SEC)= 0.16194D+03          FREQ (HERTZ)= 0.25774D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE   0.000D+00   0.000D+00  -0.360D-01  -0.384D+00   0.389D+00  -0.164D+00  -0.311D+00   0.331D+00   0.112D+00  -0.319D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE   0.312D+00  -0.550D-01  -0.363D+00   0.247D+00  -0.648D-01  -0.732D+00   0.727D+00  -0.215D+00  -0.657D+00   0.720D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE   0.921D-01  -0.683D+00   0.684D+00  -0.889D-01  -0.697D+00   0.673D+00  -0.509D-01  -0.950D+00   0.994D+00  -0.123D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE  -0.818D+00   0.100D+01   0.114D-01  -0.928D+00   0.876D+00  -0.596D-01  -0.844D+00   0.884D+00  -0.841D-01  -0.870D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE   0.100D+01   0.134D-02  -0.834D+00   0.995D+00  -0.156D+00  -0.873D+00   0.957D+00  -0.479D-01  -0.841D+00   0.959D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE  -0.122D+00  -0.595D+00   0.742D+00   0.163D+00  -0.621D+00   0.736D+00  -0.316D+00  -0.604D+00   0.780D+00  -0.241D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE  -0.599D+00   0.780D+00  -0.174D+00  -0.188D+00   0.301D+00   0.278D+00  -0.209D+00   0.297D+00  -0.460D+00  -0.214D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE   0.272D+00  -0.104D-01  -0.183D+00   0.274D+00  -0.184D+00   0.248D+00  -0.349D+00   0.360D+00   0.265D+00  -0.331D+00
O  DOF            91          92          93          94          95          96
 VALUE  -0.517D+00   0.235D+00  -0.354D+00   0.308D-01   0.280D+00  -0.331D+00
OMODE NO.  20        :  LAMBDA= 0.28787D+05           FREQ (RAD/SEC)= 0.16967D+03          FREQ (HERTZ)= 0.27003D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE   0.000D+00   0.000D+00   0.608D-01   0.324D+00   0.300D+00  -0.387D-01   0.135D+00   0.241D+00  -0.604D-01   0.265D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE   0.396D+00  -0.164D+00   0.203D+00   0.338D+00   0.112D-01   0.704D+00   0.589D+00  -0.665D-01   0.486D+00   0.583D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE  -0.108D+00   0.648D+00   0.722D+00  -0.200D+00   0.534D+00   0.705D+00  -0.863D-01   0.972D+00   0.695D+00  -0.569D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE   0.790D+00   0.713D+00  -0.993D-01   0.949D+00   0.898D+00  -0.997D-01   0.828D+00   0.907D+00  -0.256D+00   0.994D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
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 VALUE      0.753D+00  -0.864D-01   0.926D+00   0.747D+00  -0.901D-01   0.100D+01   0.820D+00   0.310D-01   0.941D+00   0.814D+00
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE     -0.400D+00   0.730D+00   0.622D+00  -0.126D+00   0.794D+00   0.616D+00  -0.652D-01   0.750D+00   0.547D+00   0.182D+00
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE      0.775D+00   0.531D+00  -0.514D+00   0.272D+00   0.197D+00  -0.181D+00   0.326D+00   0.206D+00  -0.580D-01   0.311D+00
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE      0.207D+00   0.264D+00   0.297D+00   0.208D+00  -0.545D+00  -0.365D+00  -0.188D+00  -0.203D+00  -0.260D+00  -0.177D+00
O  DOF             91          92          93          94          95          96
 VALUE     -0.172D-01  -0.344D+00  -0.263D+00   0.312D+00  -0.276D+00  -0.248D+00
OMODE NO.  19        :  LAMBDA= 0.50757D+05           FREQ (RAD/SEC)= 0.22529D+03           FREQ (HERTZ)= 0.35857D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.000D+00   0.000D+00   0.209D+00   0.153D-01  -0.578D-01   0.856D-01   0.202D-01  -0.568D-01   0.145D+00  -0.177D-01
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.190D-01   0.128D+00  -0.446D-01  -0.481D-01   0.380D+00   0.589D-01  -0.651D-01   0.216D+00  -0.224D-01  -0.644D-01
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.289D+00   0.275D-01  -0.126D-01   0.257D+00  -0.447D-01  -0.116D-01   0.514D+00   0.924D-01   0.238D+00   0.365D+00
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.355D-02   0.245D+00   0.460D+00   0.656D-01   0.312D+00   0.442D+00  -0.162D-01   0.318D+00   0.620D+00   0.125D+00
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE      0.176D+00   0.527D+00   0.138D-01   0.193D+00   0.574D+00   0.103D+00   0.282D+00   0.600D+00  -0.619D-02   0.257D+00
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.713D+00   0.137D+00   0.549D-01   0.683D+00   0.202D-01   0.536D-01   0.679D+00   0.120D+00   0.153D+00   0.755D+00
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE     -0.894D-03   0.150D+00   0.778D+00   0.115D+00   0.799D-01   0.793D+00  -0.233D-01   0.823D-01   0.788D+00   0.103D+00
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE      0.210D+00   0.895D+00  -0.438D-01   0.214D+00   0.866D+00   0.461D-01  -0.142D+00   0.905D+00  -0.896D-01  -0.128D+00
O  DOF             91          92          93          94          95          96
 VALUE      0.876D+00   0.414D-01   0.208D-02   0.100D+01  -0.998D-01   0.140D-02
OMODE NO.  18        :  LAMBDA= 0.79518D+05           FREQ (RAD/SEC)= 0.28199D+03           FREQ (HERTZ)= 0.44880D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.000D+00   0.000D+00   0.619D-01  -0.433D+00   0.413D+00   0.382D-01   0.449D+00   0.300D+00   0.852D-01  -0.352D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.510D+00   0.844D-01   0.575D+00  -0.408D+00   0.887D-01  -0.769D+00   0.727D+00   0.492D-01   0.845D+00   0.711D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.159D+00  -0.722D+00  -0.855D+00   0.159D+00   0.920D+00  -0.830D+00   0.139D+00  -0.909D+00   0.954D+00   0.104D+00
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.987D+00   0.982D+00   0.161D+00  -0.917D+00  -0.919D+00   0.152D+00   0.100D+01  -0.941D+00   0.101D+00  -0.731D+00
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE      0.906D+00   0.113D+00   0.923D+00   0.953D+00   0.131D+00  -0.792D+00  -0.750D+00   0.127D+00   0.882D+00  -0.804D+00
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.521D-01  -0.349D+00   0.692D+00   0.120D+00   0.683D+00   0.675D+00   0.508D-01  -0.446D+00  -0.456D+00   0.594D-01
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE      0.587D+00  -0.446D+00  -0.104D+00   0.616D-01   0.114D+00  -0.481D-02   0.193D+00   0.113D+00   0.521D-01  -0.499D-01
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     -0.141D+00   0.651D-01   0.665D-01  -0.145D+00  -0.124D+00   0.392D+00  -0.383D+00  -0.131D-01  -0.374D+00  -0.323D+00
O  DOF             91          92          93          94          95          96
 VALUE     -0.678D-01   0.331D+00   0.361D+00  -0.590D-01  -0.446D+00   0.294D+00
OMODE NO.  17        :  LAMBDA= 0.13963D+06           FREQ (RAD/SEC)= 0.37367D+03           FREQ (HERTZ)= 0.59471D+02
OEIGENVECTOR:
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O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.000D+00   0.000D+00  -0.352D-01  -0.357D+00   0.633D+00  -0.145D+00  -0.416D+00   0.528D+00   0.202D-01  -0.290D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.638D+00  -0.107D+00  -0.490D+00   0.523D+00  -0.877D-01  -0.426D+00   0.873D+00  -0.486D-01  -0.560D+00   0.830D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.118D+00  -0.402D+00   0.100D+01  -0.107D+00  -0.556D+00   0.945D+00  -0.132D+00  -0.203D+00   0.601D+00   0.111D+00
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE   -0.220D+00   0.630D+00  -0.295D+00  -0.229D+00   0.592D+00  -0.518D-01  -0.174D+00   0.621D+00  -0.351D-01   0.630D-01
O  DOF           51          52          53          54          55          56          57          58          59          60
 VALUE   -0.216D+00   0.224D+00   0.253D+00  -0.972D-01  -0.249D+00   0.286D-01  -0.314D+00   0.586D-01   0.293D+00  -0.177D+00
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE    0.136D+00   0.162D+00  -0.728D+00   0.149D+00   0.453D+00  -0.692D+00  -0.354D-01   0.147D+00  -0.985D+00   0.438D-01
O  DOF           71          72          73          74          75          76          77          78          79          80
 VALUE    0.447D+00  -0.924D+00   0.294D+00   0.979D-01  -0.588D+00  -0.781D-01   0.227D+00  -0.621D+00   0.209D+00   0.108D+00
O  DOF           81          82          83          84          85          86          87          88          89          90
 VALUE   -0.723D+00  -0.108D+00   0.184D+00  -0.757D+00   0.407D+00   0.274D-01   0.186D+00  -0.267D+00  -0.119D+00   0.136D+00
O  DOF           91          92          93          94          95          96
 VALUE    0.322D+00   0.204D-01   0.190D+00  -0.305D+00  -0.165D+00   0.106D+00
OMODE NO.   16        :  LAMBDA= 0.14440D+06         FREQ (RAD/SEC)= 0.38000D+03         FREQ (HERTZ)= 0.60478D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.000D+00   0.000D+00  -0.271D-01   0.626D+00   0.395D+00   0.233D-01   0.495D+00   0.312D+00  -0.994D-01   0.528D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.379D+00  -0.582D-01   0.638D+00   0.310D+00  -0.169D+00   0.870D+00   0.564D+00  -0.275D-02   0.987D+00   0.537D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.957D-01   0.859D+00   0.388D+00   0.793D-01   0.100D+01   0.355D+00  -0.270D+00   0.546D+00   0.252D+00  -0.789D-01
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE    0.675D+00   0.270D+00  -0.285D-01   0.635D+00   0.233D+00   0.203D+00   0.574D+00   0.244D+00  -0.176D+00  -0.245D+00
O  DOF           51          52          53          54          55          56          57          58          59          60
 VALUE   -0.791D-01  -0.920D-01  -0.873D-01  -0.474D-01   0.501D-01  -0.137D+00  -0.122D+00   0.189D+00  -0.221D+00  -0.899D-01
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE    0.443D-01  -0.775D+00  -0.330D+00  -0.127D-01  -0.767D+00  -0.315D+00   0.307D-01  -0.709D+00  -0.203D+00   0.727D-02
O  DOF           71          72          73          74          75          76          77          78          79          80
 VALUE   -0.805D+00  -0.171D+00   0.289D+00  -0.676D+00  -0.265D-01   0.157D+00  -0.599D+00  -0.478D-01  -0.809D-01  -0.726D+00
O  DOF           81          82          83          84          85          86          87          88          89          90
 VALUE   -0.223D+00  -0.175D+00  -0.557D+00  -0.233D+00   0.404D+00   0.186D+00   0.290D-01   0.299D+00   0.111D+00   0.203D-01
O  DOF           91          92          93          94          95          96
 VALUE   -0.250D+00   0.134D+00   0.581D-01  -0.282D+00   0.153D+00   0.466D-01
OMODE NO.   15        :  LAMBDA= 0.23108D+06         FREQ (RAD/SFC)= 0.48070D+03         FREQ (HERTZ)= 0.76506D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.000D+00   0.000D+00  -0.256D-01   0.250D+00  -0.339D+00   0.320D-03  -0.181D+00  -0.270D+00  -0.537D-01   0.215D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.131D+00  -0.301D-01  -0.225D+00   0.110D+00  -0.241D-01   0.303D+00  -0.475D+00  -0.302D-01  -0.294D+00  -0.441D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.546D-01   0.317D+00   0.170D+00  -0.532D-01  -0.286D+00   0.159D+00   0.161D-01   0.138D+00  -0.252D+00  -0.199D-01
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE   -0.150D+00  -0.263D+00  -0.366D-01   0.211D+00   0.130D+00  -0.400D-01  -0.891D-01   0.140D+00   0.127D-02  -0.187D+00
O  DOF           51          52          53          54          55          56          57          58          59          60
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 VALUE      0.122D+00  -0.435D-01   0.969D-01   0.617D-01  -0.733D-02  -0.837D-01  -0.526D-01   0.434D-01   0.177D+00  -0.264D-01
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.242D-01  -0.445D+00   0.324D+00  -0.400D-01   0.272D+00   0.302D+00  -0.686D-01  -0.362D+00  -0.254D+00   0.159D+00
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE      0.333D+00  -0.206D+00   0.958D-01  -0.305D+00   0.331D+00  -0.838D-01   0.173D+00   0.304D+00  -0.210D+00  -0.350D+00
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     -0.365D+00   0.368D+00   0.138D+00  -0.393D+00   0.173D+00   0.100D+01   0.817D+00  -0.151D+00  -0.492D+00   0.444D+00
O  DOF             91          92          93          94          95          96
 VALUE     -0.550D+00   0.547D+00  -0.334D+00   0.512D+00  -0.929D+00  -0.816D+00
OMODE NO.  14         :  LAMBDA= 0.25405D+06          FREQ (RAD/SEC)= 0.50403D+03          FREQ (HERTZ)= 0.80219D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.000D+00   0.000D+00  -0.278D-02  -0.405D+00   0.736D+00   0.864D-02   0.607D+00   0.591D+00   0.955D-01  -0.348D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.365D+00   0.863D-01   0.751D+00  -0.311D+00  -0.578D-01  -0.543D+00   0.100D+01   0.159D-01   0.870D+00   0.922D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.105D+00  -0.571D+00  -0.508D+00   0.152D+00   0.831D+00  -0.465D+00  -0.180D+00  -0.345D+00   0.355D+00  -0.105D+00
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.207D+00   0.387D+00   0.836D-01  -0.482D+00  -0.307D+00   0.133D+00   0.547D-01  -0.334D+00  -0.117D+00   0.241D+00
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE     -0.502D+00  -0.866D-01  -0.599D+00  -0.328D+00  -0.961D-01   0.669D-01   0.344D+00  -0.488D-01  -0.680D+00   0.220D+00
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE     -0.670D-01   0.810D+00  -0.932D+00  -0.112D+00  -0.771D+00  -0.861D+00  -0.173D+00   0.726D+00   0.638D+00  -0.961D-01
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE     -0.686D+00   0.585D+00  -0.262D-01   0.776D+00  -0.608D+00  -0.175D+00  -0.314D+00  -0.669D+00  -0.798D-01   0.869D+00
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE      0.504D+00   0.798D-01  -0.129D+00   0.546D+00  -0.535D-01   0.824D-01   0.573D+00  -0.380D+00  -0.388D-01   0.289D+00
O  DOF             91          92          93          94          95          96
 VALUE      0.209D+00   0.383D-01  -0.336D+00   0.446D+00  -0.733D-01  -0.406D+00
OMODE NO.  13         :  LAMBDA= 0.31608D+06          FREQ (RAD/SEC)= 0.56221D+03          FREQ (HERTZ)= 0.89479D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.000D+00   0.000D+00   0.139D+00  -0.428D+00  -0.345D+00   0.572D-01   0.509D-01  -0.302D+00   0.112D+00  -0.381D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.718D+00   0.510D-01   0.511D-01  -0.599D+00   0.247D+00  -0.336D+00  -0.235D+00   0.206D-01   0.853D-02  -0.210D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.118D+00  -0.389D+00  -0.598D+00  -0.102D+00   0.103D-01  -0.518D+00   0.244D+00   0.114D+00   0.214D+00   0.576D-01
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.176D+00   0.248D+00   0.114D+00   0.423D-01   0.129D+00  -0.930D-01   0.161D+00   0.148D+00   0.824D-01   0.560D+00
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE      0.321D+00   0.845D-01   0.245D+00   0.377D+00  -0.917D-01   0.592D+00   0.711D+00  -0.109D+00   0.165D+00   0.680D+00
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.114D+00   0.287D+00  -0.107D-01   0.133D+00   0.113D+00  -0.109D-01  -0.153D+00   0.501D+00   0.504D+00  -0.101D+00
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE      0.286D-01   0.486D+00   0.378D+00  -0.205D+00  -0.197D+00   0.121D-01  -0.121D+00  -0.254D+00  -0.301D+00   0.132D-01
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     -0.199D+00  -0.421D+00  -0.228D+00  -0.219D+00   0.100D+01  -0.129D+00   0.161D-01  -0.158D-01  -0.596D-01   0.765D-02
O  DOF             91          92          93          94          95          96
 VALUE     -0.318D+00  -0.476D-01  -0.195D+00  -0.782D+00  -0.174D+00  -0.138D+00
OMODE NO.  12         :  LAMBDA= 0.35609D+06          FREQ (RAD/SEC)= 0.59673D+03          FREQ (HERTZ)= 0.94972D+02
OMODE NO.  11         :  LAMBDA= 0.40944D+06          FREQ (RAD/SFC)= 0.63987D+03          FREQ (HERTZ)= 0.10184D+03
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OMODE NO.  10     :  LAMBDA= 0.48149D+06        FREQ (RAD/SEC)= 0.69390D+03        FREQ (HERTZ)= 0.11044D+03
OMODE NO.   9     :  LAMBDA= 0.49088D+06        FREQ (RAD/SEC)= 0.70063D+03        FREQ (HERTZ)= 0.11151D+03
OMODE NO.   8     :  LAMBDA= 0.55126D+06        FREQ (RAD/SEC)= 0.74247D+03        FREQ (HERTZ)= 0.11817D+03
OMODE NO.   7     :  LAMBDA= 0.61112D+06        FREQ (RAD/SEC)= 0.78174D+03        FREQ (HERTZ)= 0.12442D+03
OMODE NO.   6     :  LAMBDA= 0.73176D+06        FREQ (RAD/SEC)= 0.85543D+03        FREQ (HERTZ)= 0.13615D+03
OMODE NO.   5     :  LAMBDA= 0.81437D+06        FREQ (RAD/SEC)= 0.90242D+03        FREQ (HERTZ)= 0.14363D+03
OMODE NO.   4     :  LAMBDA= 0.10010D+07        FREQ (RAD/SEC)= 0.10005D+04        FREQ (HERTZ)= 0.15923D+03
OMODE NO.   3     :  LAMBDA= 0.11390D+07        FREQ (RAD/SEC)= 0.10672D+04        FREQ (HERTZ)= 0.16985D+03
OMODE NO.   2     :  LAMBDA= 0.12711D+07        FREQ (RAD/SEC)= 0.11274D+04        FREQ (HERTZ)= 0.17944D+03
OMODE NO.   1     :  LAMBDA= 0.17488D+07        FREQ (RAD/SEC)= 0.13224D+04        FREQ (HERTZ)= 0.21047D+03
O MESSAGE SUMMARY: MESSAGE NUMBER - COUNT
O                             208       43
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Rigid-Jointed Model of SADE (Section 2.3)

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EIGENVECTORS INCLUDING RESTRAINED DOF'S:


OMODE NO.  24      :  LAMBDA= 0.65270D+03         FREQ (RAD/SEC)= 0.25548D+02         FREQ (HERTZ)= 0.40661D+01
OEIGENVECTOR:
O   DOF            1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00  -0.112D-04  -0.465D-03  -0.643D-03   0.000D+00   0.000D+00   0.000D+00  -0.110D-04
O   DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    -0.178D-03  -0.233D-03   0.000D+00   0.000D+00   0.000D+00   0.458D-04  -0.586D-04  -0.417D-03   0.000D+00   0.000D+00
O   DOF           21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00  -0.851D-04  -0.190D-03  -0.465D-03   0.168D-02  -0.699D-01   0.534D-01  -0.381D-03  -0.130D-02  -0.141D-02
O   DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    -0.449D-01  -0.492D-01   0.505D-01  -0.339D-03  -0.109D-02  -0.103D-02   0.478D-01  -0.649D-01   0.323D-01  -0.237D-03
O   DOF           41          42          43          44          45          46          47          48          49          50
 VALUE    -0.950D-03  -0.129D-02  -0.845D-02  -0.516D-01   0.261D-01  -0.383D-03  -0.966D-03  -0.115D-02   0.555D-02  -0.169D+00
O   DOF           51          52          53          54          55          56          57          58          59          60
 VALUE     0.149D+00  -0.566D-03  -0.185D-02  -0.185D-02  -0.861D-01  -0.136D+00   0.149D+00  -0.703D-03  -0.180D-02  -0.173D-02
O   DOF           61          62          63          64          65          66          67          68          69          70
 VALUE     0.843D-01  -0.167D+00   0.111D+00  -0.555D-03  -0.180D-02  -0.175D-02  -0.131D-01  -0.141D+00   0.108D+00  -0.591D-03
O   DOF           71          72          73          74          75          76          77          78          79          80
 VALUE    -0.198D-02  -0.183D-02   0.117D-01  -0.290D+00   0.263D+00  -0.409D-03  -0.250D-02  -0.229D-02  -0.116D+00  -0.259D+00
O   DOF           81          82          83          84          85          86          87          88          89          90
 VALUE     0.265D+00  -0.514D-03  -0.240D-02  -0.214D-02   0.116D+00  -0.286D+00   0.244D+00  -0.382D-03  -0.247D-02  -0.219D-02
O   DOF           91          92          93          94          95          96          97          98          99         100
 VALUE    -0.153D-01  -0.262D+00   0.244D+00  -0.492D-03  -0.257D-02  -0.223D-02   0.126D-01  -0.431D+00   0.429D+00  -0.530D-03
O   DOF          101         102         103         104         105         106         107         108         109         110
 VALUE    -0.315D-02  -0.262D-02  -0.141D+00  -0.396D+00   0.426D+00  -0.597D-03  -0.296D-02  -0.241D-02   0.138D+00  -0.427D+00
O   DOF          111         112         113         114         115         116         117         118         119         120
 VALUE     0.396D+00  -0.481D-03  -0.293D-02  -0.254D-02  -0.205D-01  -0.399D+00   0.392D+00  -0.670D-03  -0.292D-02  -0.253D-02
O   DOF          121         122         123         124         125         126         127         128         129         130
 VALUE     0.143D-01  -0.584D+00   0.610D+00  -0.574D-03  -0.332D-02  -0.277D-02  -0.160D+00  -0.546D+00   0.610D+00  -0.764D-03
O   DOF          131         132         133         134         135         136         137         138         139         140
 VALUE    -0.327D-02  -0.268D-02   0.153D+00  -0.582D+00   0.571D+00  -0.661D-03  -0.325D-02  -0.267D-02  -0.242D-01  -0.549D+00
O   DOF          141         142         143         144         145         146         147         148         149         150
 VALUE     0.570D+00  -0.779D-03  -0.339D-02  -0.275D-02   0.184D-01  -0.743D+00   0.799D+00  -0.504D-03  -0.356D-02  -0.285D-02
O   DOF          151         152         153         154         155         156         157         158         159         160
 VALUE    -0.168D+00  -0.703D+00   0.800D+00  -0.666D-03  -0.346D-02  -0.273D-02   0.163D+00  -0.740D+00   0.764D+00  -0.595D-03
O   DOF          161         162         163         164         165         166         167         168         169         170
 VALUE    -0.349D-02  -0.277D-02  -0.245D-01  -0.705D+00   0.764D+00  -0.796D-03  -0.359D-02  -0.284D-02   0.184D-01  -0.898D+00
O   DOF          171         172         173         174         175         176         177         178         179         180
 VALUE     0.100D+01  -0.548D-03  -0.356D-02  -0.270D-02  -0.174D+00  -0.861D+00   0.999D+00  -0.652D-03  -0.343D-02  -0.265D-02
O   DOF          181         182         183         184         185         186         187         188         189         190
 VALUE     0.166D+00  -0.897D+00   0.963D+00  -0.683D-03  -0.350D-02  -0.266D-02  -0.272D-01  -0.863D+00   0.962D+00  -0.765D-03
O   DOF          191         192
 VALUE    -0.346D-02  -0.270D-02
OMODE NO.  23      :  LAMBDA= 0.80197D+03         FREQ (RAD/SEC)= 0.28319D+02         FREQ (HERTZ)= 0.45071D+01
OEIGENVECTOR:
O   DOF            1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.131D-03  -0.191D-03   0.170D-03   0.000D+00   0.000D+00   0.000D+00   0.407D-05
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O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.856D-04  -0.112D-04   0.000D+00   0.000D+00   0.000D+00   0.330D-04  -0.451D-03   0.176D-03   0.000D+00   0.000D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.000D+00   0.353D-04  -0.389D-03  -0.129D-03   0.496D-01   0.386D-01   0.288D-01   0.720D-03  -0.921D-03   0.115D-02
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.295D-02   0.235D-02   0.234D-01   0.568D-03  -0.767D-03   0.610D-03  -0.100D-01   0.325D-01   0.588D-01   0.480D-03
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE   -0.121D-02   0.104D-02  -0.519D-01   0.604D-02   0.570D-01   0.547D-03  -0.125D-02   0.709D-03   0.877D-01   0.125D+00
O  DOF           51          52          53          54          55          56          57          58          59          60
 VALUE    0.942D-01   0.106D-02  -0.179D-02   0.184D-02  -0.508D-03   0.678D-01   0.941D-01   0.109D-02  -0.174D-02   0.176D-02
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE   -0.158D-01   0.121D+00   0.155D+00   0.912D-03  -0.167D-02   0.184D-02  -0.982D-01   0.740D-01   0.157D+00   0.894D-03
O  DOF           71          72          73          74          75          76          77          78          79          80
 VALUE   -0.173D-02   0.184D-02   0.122D+00   0.243D+00   0.224D+00   0.803D-03  -0.236D-02   0.258D-02   0.623D-02   0.196D+00
O  DOF           81          82          83          84          85          86          87          88          89          90
 VALUE    0.222D+00   0.868D-03  -0.227D-02   0.250D-02  -0.155D-01   0.237D+00   0.258D+00   0.606D-03  -0.220D-02   0.248D-02
O  DOF           91          92          93          94          95          96          97          98          99         100
 VALUE   -0.128D+00   0.200D+00   0.258D+00   0.621D-03  -0.226D-02   0.250D-02   0.144D+00   0.407D+00   0.357D+00   0.111D-02
O  DOF          101         102         103         104         105         106         107         108         109         110
 VALUE   -0.259D-02   0.322D-02   0.683D-02   0.346D+00   0.353D+00   0.113D-02  -0.248D-02   0.292D-02  -0.217D-01   0.402D+00
O  DOF          111         112         113         114         115         116         117         118         119         120
 VALUE    0.411D+00   0.807D-03  -0.277D-02   0.315D-02  -0.155D+00   0.350D+00   0.407D+00   0.912D-03  -0.278D-02   0.297D-02
O  DOF          121         122         123         124         125         126         127         128         129         130
 VALUE    0.158D+00   0.595D+00   0.505D+00   0.131D-02  -0.288D-02   0.346D-02   0.833D-02   0.524D+00   0.505D+00   0.144D-02
O  DOF          131         132         133         134         135         136         137         138         139         140
 VALUE   -0.284D-02   0.341D-02  -0.263D-01   0.591D+00   0.573D+00   0.973D-03  -0.282D-02   0.342D-02  -0.174D+00   0.529D+00
O  DOF          141         142         143         144         145         146         147         148         149         150
 VALUE    0.574D+00   0.117D-02  -0.289D-02   0.348D-02   0.167D+00   0.792D+00   0.672D+00   0.113D-02  -0.301D-02   0.373D-02
O  DOF          151         152         153         154         155         156         157         158         159         160
 VALUE    0.128D-01   0.727D+00   0.672D+00   0.142D-02  -0.295D-02   0.364D-02  -0.262D-01   0.789D+00   0.732D+00   0.821D-03
O  DOF          161         162         163         164         165         166         167         168         169         170
 VALUE   -0.292D-02   0.363D-02  -0.180D+00   0.730D+00   0.733D+00   0.101D-02  -0.299D-02   0.370D-02   0.167D+00   0.100D+01
O  DOF          171         172         173         174         175         176         177         178         179         180
 VALUE    0.835D+00   0.120D-02  -0.282D-02   0.361D-02   0.132D-01   0.932D+00   0.834D+00   0.148D-02  -0.275D-02   0.355D-02
O  DOF          181         182         183         184         185         186         187         188         189         190
 VALUE   -0.296D-01   0.998D+00   0.900D+00   0.871D-03  -0.286D-02   0.359D-02  -0.183D+00   0.934D+00   0.898D+00   0.120D-02
O  DOF          191         192
 VALUE   -0.277D-02   0.352D-02
OMODE NO.   22        :  LAMBDA= 0.63537D+04          FREQ (RAD/SEC)= 0.79710D+02          FREQ (HERTZ)= 0.12686D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00  -0.103D-02  -0.935D-03  -0.136D-02   0.000D+00   0.000D+00   0.000D+00   0.498D-04
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.653D-03   0.763D-03   0.000D+00   0.000D+00   0.000D+00  -0.381D-03   0.139D-02  -0.694D-03   0.000D+00   0.000D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.000D+00  -0.495D-03   0.931D-03   0.159D-02   0.237D-01  -0.172D+00   0.140D+00  -0.672D-02  -0.138D-02  -0.197D-02
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.129D-01   0.135D+00   0.985D-01  -0.408D-02  -0.118D-02   0.123D-02   0.276D-01  -0.136D+00  -0.179D+00  -0.548D-02
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE    0.179D-02  -0.160D-02   0.302D-01   0.176D+00  -0.140D+00  -0.508D-02   0.189D-02   0.170D-02   0.420D-01  -0.341D+00
O  DOF           51          52          53          54          55          56          57          58          59          60
 VALUE    0.258D+00  -0.115D-01  -0.181D-02  -0.213D-02   0.218D-01   0.281D+00   0.257D+00  -0.103D-01  -0.191D-02   0.874D-03
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE    0.569D-01  -0.306D+00  -0.328D+00  -0.104D-01   0.124D-02  -0.170D-02   0.615D-01   0.321D+00  -0.327D+00  -0.115D-01
O  DOF           71          72          73          74          75          76          77          78          79          80
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  VALUE    0.156D-02   0.119D-02   0.949D-01  -0.482D+00   0.440D+00  -0.169D-01  -0.198D-02  -0.911D-03   0.700D-01   0.440D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
  VALUE    0.442D+00  -0.157D-01  -0.172D-02   0.173D-02   0.504D-01  -0.448D+00  -0.445D+00  -0.159D-01   0.133D-02  -0.492D-03
O  DOF            91          92          93          94          95          96          97          98          99         100
  VALUE    0.536D-01   0.479D+00  -0.446D+00  -0.168D-01   0.135D-02   0.236D-02   0.105D+00  -0.593D+00   0.593D+00  -0.227D-01
O  DOF           101         102         103         104         105         106         107         108         109         110
  VALUE   -0.165D-02  -0.137D-02   0.789D-01   0.610D+00   0.559D+00  -0.203D-01  -0.114D-02   0.122D-02   0.821D-01  -0.561D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
  VALUE   -0.611D+00  -0.215D-01   0.176D-02  -0.752D-03   0.831D-01   0.647D+00  -0.577D+00  -0.212D-01   0.137D-02   0.192D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
  VALUE    0.113D+00  -0.713D+00   0.706D+00  -0.266D-01  -0.157D-02  -0.172D-02   0.860D-01   0.748D+00   0.704D+00  -0.255D-01
O  DOF           131         132         133         134         135         136         137         138         139         140
  VALUE   -0.174D-02   0.646D-03   0.112D+00  -0.685D+00  -0.731D+00  -0.258D-01   0.838D-03  -0.868D-03   0.112D+00   0.781D+00
O  DOF           141         142         143         144         145         146         147         148         149         150
  VALUE   -0.730D+00  -0.265D-01   0.108D-02   0.128D-02   0.146D+00  -0.813D+00   0.848D+00  -0.307D-01  -0.156D-02  -0.896D-03
O  DOF           151         152         153         154         155         156         157         158         159         160
  VALUE    0.120D+00   0.873D+00   0.850D+00  -0.296D-01  -0.152D-02   0.997D-03   0.112D+00  -0.788D+00  -0.810D+00  -0.298D-01
O  DOF           161         162         163         164         165         166         167         168         169         170
  VALUE    0.102D-02  -0.154D-03   0.111D+00   0.901D+00  -0.812D+00  -0.307D-01   0.969D-03   0.208D-02   0.148D+00  -0.877D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
  VALUE    0.952D+00  -0.338D-01  -0.616D-03  -0.168D-03   0.123D+00   0.987D+00   0.941D+00  -0.329D-01  -0.140D-03  -0.313D-03
O  DOF           181         182         183         184         185         186         187         188         189         190
  VALUE    0.136D+00  -0.866D+00  -0.913D+00  -0.334D-01   0.417D-03   0.372D-03   0.137D+00   0.100D+01  -0.901D+00  -0.332D-01
O  DOF           191         192
  VALUE   -0.149D-03   0.198D-03
OMODE NO.  21        :  LAMBDA= 0.25899D+05            FREQ (RAD/SEC)= 0.16093D+03            FREQ (HERTZ)= 0.25613D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
  VALUE    0.000D+00   0.000D+00   0.000D+00   0.109D-02  -0.432D-02  -0.392D-02   0.000D+00   0.000D+00   0.000D+00  -0.213D-03
O  DOF            11          12          13          14          15          16          17          18          19          20
  VALUE   -0.170D-02  -0.170D-02   0.000D+00   0.000D+00   0.000D+00   0.858D-03  -0.178D-02  -0.271D-02   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
  VALUE    0.000D+00  -0.120D-02  -0.255D-02  -0.390D-02  -0.345D-01  -0.375D+00   0.388D+00   0.263D-02  -0.771D-02  -0.709D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
  VALUE   -0.163D+00  -0.305D+00   0.330D+00  -0.144D-02  -0.411D-02  -0.385D-02   0.110D+00  -0.312D+00   0.314D+00   0.939D-03
O  DOF            41          42          43          44          45          46          47          48          49          50
  VALUE   -0.573D-02  -0.560D-02  -0.567D-01  -0.355D+00   0.249D+00  -0.340D-02  -0.574D-02  -0.642D-02  -0.636D-01  -0.713D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
  VALUE    0.727D+00   0.470D-02  -0.588D-02  -0.585D-02  -0.214D+00  -0.642D+00   0.720D+00  -0.275D-02  -0.400D-02  -0.334D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
  VALUE    0.902D-01  -0.666D+00   0.687D+00  -0.110D-02  -0.354D-02  -0.329D-02  -0.908D-01  -0.681D+00   0.676D+00  -0.689D-02
O  DOF            71          72          73          74          75          76          77          78          79          80
  VALUE   -0.649D-02  -0.555D-02  -0.515D-01  -0.924D+00   0.992D+00   0.614D-02  -0.386D-02  -0.199D-02  -0.122D+00  -0.798D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
  VALUE    0.999D+00  -0.142D-04  -0.989D-03   0.304D-03   0.102D-01  -0.902D+00   0.880D+00  -0.238D-03  -0.107D-02  -0.574D-03
O  DOF            91          92          93          94          95          96          97          98          99         100
  VALUE   -0.603D-01  -0.822D+00   0.889D+00  -0.918D-02  -0.387D-02  -0.304D-02  -0.870D-01  -0.844D+00   0.100D+01   0.696D-02
O  DOF           101         102         103         104         105         106         107         108         109         110
  VALUE   -0.801D-03   0.176D-02   0.698D-03  -0.810D+00   0.995D+00   0.742D-04   0.575D-02   0.449D-02  -0.156D+00  -0.847D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
  VALUE    0.959D+00  -0.214D-02   0.309D-03   0.188D-02  -0.464D-01  -0.816D+00   0.961D+00  -0.821D-02   0.215D-02   0.156D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
  VALUE   -0.127D+00  -0.575D+00   0.743D+00   0.538D-02   0.597D-02   0.556D-02   0.160D+00  -0.600D+00   0.736D+00   0.763D-03
O  DOF           131         132         133         134         135         136         137         138         139         140
  VALUE    0.731D-02   0.625D-02  -0.314D+00  -0.585D+00   0.777D+00  -0.281D-02   0.737D-02   0.533D-02  -0.199D-01  -0.577D+00
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O  DOF           141         142         143         144         145         146         147         148         149         150
 VALUE     0.777D+00  -0.531D-02   0.650D-02   0.535D-02  -0.181D+00  -0.179D+00   0.297D+00   0.119D-02   0.107D-01   0.874D-02
O  DOF           151         152         153         154         155         156         157         158         159         160
 VALUE     0.274D+00  -0.200D+00   0.293D+00   0.178D-02   0.105D-01   0.706D-02  -0.456D+00  -0.205D+00   0.268D+00  -0.136D-02
O  DOF           161         162         163         164         165         166         167         168         169         170
 VALUE     0.100D-01   0.734D-02  -0.486D-02  -0.175D+00   0.271D+00  -0.172D-02   0.109D-01   0.809D-02  -0.191D+00   0.241D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
 VALUE    -0.353D+00  -0.145D-02   0.109D-01   0.735D-02   0.354D+00   0.259D+00  -0.335D+00  -0.363D-03   0.994D-02   0.591D-02
O  DOF           181         182         183         184         185         186         187         188         189         190
 VALUE    -0.512D+00   0.229D+00  -0.359D+00  -0.190D-03   0.985D-02   0.424D-02   0.366D-01   0.274D+00  -0.337D+00   0.618D-03
O  DOF           191         192
 VALUE     0.107D-01   0.690D-02
OMODE NO.   20      :  LAMBDA= 0.28103D+05         FREQ (RAD/SEC)= 0.16764D+03          FREQ (HERTZ)= 0.26681D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.894D-03  -0.347D-02   0.300D-02   0.000D+00   0.000D+00   0.000D+00   0.429D-03
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.161D-02   0.114D-02   0.000D+00   0.000D+00   0.000D+00  -0.482D-03  -0.352D-02   0.231D-02   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00  -0.759D-03  -0.348D-02   0.123D-02   0.617D-01   0.328D+00   0.290D+00   0.348D-02  -0.667D-02   0.760D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.349D-01   0.141D+00   0.232D+00   0.537D-02  -0.380D-02   0.417D-02  -0.612D-01   0.267D+00   0.384D+00  -0.248D-02
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE    -0.640D-02   0.520D-02  -0.161D+00   0.209D+00   0.329D+00  -0.393D-03  -0.580D-02   0.408D-02   0.131D-01   0.709D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.570D+00   0.596D-02  -0.482D-02   0.665D-02  -0.612D-01   0.495D+00   0.564D+00   0.110D-01  -0.279D-02   0.557D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE    -0.107D+00   0.653D+00   0.700D+00  -0.668D-02  -0.358D-02   0.380D-02  -0.194D+00   0.544D+00   0.683D+00   0.309D-02
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.532D-02   0.692D-02  -0.836D-01   0.977D+00   0.671D+00   0.409D-03  -0.280D-02   0.452D-02  -0.531D-01   0.797D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.688D+00   0.149D-01  -0.420D-03   0.203D-02  -0.963D-01   0.954D+00   0.869D+00  -0.923D-02  -0.104D-03   0.105D-02
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.946D-01   0.836D+00   0.879D+00  -0.157D-03  -0.305D-02   0.453D-02  -0.250D+00   0.994D+00   0.725D+00  -0.435D-03
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.269D-02   0.152D-02  -0.841D-01   0.928D+00   0.719D+00   0.137D-01   0.446D-02  -0.263D-02  -0.836D-01   0.100D+01
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.790D+00  -0.108D-01   0.199D-02  -0.371D-02   0.353D-01   0.943D+00   0.784D+00   0.139D-02   0.357D-02  -0.172D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.392D+00   0.726D+00   0.597D+00  -0.337D-03   0.442D-02  -0.460D-02  -0.126D+00   0.789D+00   0.591D+00   0.825D-02
O  DOF           131         132         133         134         135         136         137         138         139         140
 VALUE     0.718D-02  -0.715D-02  -0.557D-01   0.746D+00   0.522D+00  -0.101D-01   0.613D-02  -0.903D-02   0.184D+00   0.770D+00
O  DOF           141         142         143         144         145         146         147         148         149         150
 VALUE     0.507D+00   0.254D-02   0.423D-02  -0.591D-02  -0.502D+00   0.264D+00   0.186D+00  -0.227D-02   0.617D-02  -0.109D-01
O  DOF           151         152         153         154         155         156         157         158         159         160
 VALUE    -0.181D+00   0.319D+00   0.194D+00   0.300D-02   0.794D-02  -0.105D-01  -0.457D-01   0.302D+00   0.194D+00  -0.376D-02
O  DOF           161         162         163         164         165         166         167         168         169         170
 VALUE     0.818D-02  -0.119D-01   0.264D+00   0.290D+00   0.196D+00   0.479D-03   0.705D-02  -0.953D-02  -0.531D+00  -0.371D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
 VALUE    -0.182D+00  -0.163D-02   0.495D-02  -0.913D-02  -0.204D+00  -0.267D+00  -0.172D+00  -0.353D-02   0.774D-02  -0.954D-02
O  DOF           181         182         183         184         185         186         187         188         189         190
 VALUE    -0.478D-02  -0.351D+00  -0.257D+00   0.122D-02   0.652D-02  -0.113D-01   0.310D+00  -0.284D+00  -0.243D+00  -0.122D-02
O  DOF           191         192
 VALUE     0.574D-02  -0.102D-01
OMODE NO.   19      :  LAMBDA= 0.49913D+05         FREQ (RAD/SEC)= 0.22341D+03          FREQ (HERTZ)= 0.35557D+02
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OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.929D-04   0.959D-03  -0.717D-03   0.000D+00   0.000D+00   0.000D+00  -0.244D-03
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.821D-03  -0.518D-04   0.000D+00   0.000D+00   0.000D+00   0.130D-03   0.474D-03  -0.448D-03   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00   0.607D-03   0.956D-03  -0.329D-04   0.210D+00   0.101D-01  -0.601D-01   0.178D-03   0.283D-02  -0.792D-03
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.867D-01  -0.209D-01  -0.591D-01  -0.571D-03  -0.820D-03  -0.135D-02   0.147D+00  -0.223D-01  -0.250D-01   0.159D-02
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.268D-02   0.679D-03   0.130D+00  -0.460D-01  -0.535D-01   0.113D-02   0.575D-03   0.380D-03   0.383D+00   0.500D-01
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    -0.662D-01   0.252D-02  -0.397D-02  -0.201D-02   0.218D+00  -0.256D-01  -0.661D-01   0.911D-03  -0.716D-02  -0.108D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.293D+00   0.188D-01  -0.191D-01   0.149D-02  -0.144D-02   0.399D-02   0.260D+00  -0.482D-01  -0.184D-01  -0.129D-04
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.739D-02   0.219D-02   0.518D+00   0.837D-01   0.237D+00   0.295D-02   0.148D-03  -0.165D-02   0.369D+00   0.186D-03
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.243D+00   0.483D-02  -0.530D-02  -0.355D-02   0.463D+00   0.561D-01   0.304D+00  -0.106D-02   0.228D-02   0.386D-02
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.445D+00  -0.195D-01   0.309D+00  -0.930D-03  -0.434D-02   0.345D-02   0.625D+00   0.119D+00   0.173D+00   0.271D-02
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE     0.529D-02  -0.266D-02   0.532D+00   0.114D-01   0.190D+00   0.507D-02  -0.158D-02  -0.500D-02   0.578D+00   0.958D-01
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.275D+00  -0.150D-02   0.768D-02   0.432D-02   0.603D+00  -0.842D-02   0.250D+00   0.175D-02  -0.340D-03   0.368D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.718D+00   0.136D+00   0.544D-01   0.434D-02   0.530D-03  -0.474D-02   0.688D+00   0.184D-01   0.522D-01   0.476D-02
O  DOF           131         132         133         134         135         136         137         138         139         140
 VALUE    -0.786D-02  -0.514D-02   0.683D+00   0.118D+00   0.152D+00  -0.924D-03   0.480D-02   0.672D-02   0.757D+00  -0.281D-02
O  DOF           141         142         143         144         145         146         147         148         149         150
 VALUE     0.148D+00   0.250D-02  -0.838D-02   0.367D-02   0.782D+00   0.117D+00   0.755D-01   0.294D-02   0.101D-01  -0.560D-02
O  DOF           151         152         153         154         155         156         157         158         159         160
 VALUE     0.796D+00  -0.259D-01   0.774D-01   0.489D-02  -0.186D-02  -0.897D-02   0.792D+00   0.105D+00   0.210D+00   0.640D-03
O  DOF           161         162         163         164         165         166         167         168         169         170
 VALUE     0.116D-01   0.364D-02   0.898D+00  -0.470D-01   0.213D+00   0.366D-02   0.128D-02   0.564D-02   0.870D+00   0.508D-01
O  DOF           171         172         173         174         175         176         177         178         179         180
 VALUE    -0.146D+00   0.237D-02   0.836D-02  -0.809D-02   0.907D+00  -0.960D-01  -0.131D+00   0.146D-02  -0.105D-01  -0.124D-01
O  DOF           181         182         183         184         185         186         187         188         189         190
 VALUE     0.879D+00   0.465D-01   0.891D-02   0.425D-02   0.125D-01   0.119D-01   0.100D+01  -0.106D+00   0.816D-02   0.376D-02
O  DOF           191         192
 VALUE    -0.825D-02   0.428D-02
OMODE NO.  18         :  LAMBDA= 0.79397D+05          FREQ (RAD/SEC)= 0.28177D+03          FREQ (HERTZ)= 0.44846D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00  -0.551D-02  -0.296D-02  -0.405D-02   0.000D+00   0.000D+00   0.000D+00   0.139D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.303D-02   0.362D-02   0.000D+00   0.000D+00   0.000D+00  -0.124D-02   0.612D-02  -0.137D-02   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00  -0.221D-02   0.281D-02   0.764D-02   0.609D-01  -0.443D+00   0.411D+00  -0.226D-01  -0.636D-02  -0.933D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.391D-01   0.453D+00   0.298D+00  -0.914D-02  -0.315D-02   0.950D-03   0.866D-01  -0.361D+00  -0.525D+00  -0.129D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.962D-02  -0.257D-02   0.866D-01   0.579D+00  -0.420D+00  -0.119D-01   0.342D-02   0.101D-01   0.875D-01  -0.786D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.725D+00  -0.307D-01  -0.326D-02  -0.115D-01   0.491D-01   0.847D+00   0.708D+00  -0.219D-01  -0.683D-02  -0.338D-02
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O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE    0.161D+00  -0.738D+00  -0.876D+00  -0.233D-01   0.151D-02   0.253D-02   0.160D+00   0.922D+00  -0.850D+00  -0.262D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    0.199D-02   0.691D-02   0.137D+00  -0.926D+00   0.950D+00  -0.313D-01  -0.703D-03  -0.537D-02   0.102D+00   0.986D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE    0.978D+00  -0.287D-01  -0.350D-02  -0.823D-02   0.160D+00  -0.934D+00  -0.940D+00  -0.285D-01   0.139D-02   0.583D-02
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    0.149D+00   0.100D+01  -0.962D+00  -0.347D-01  -0.187D-03   0.824D-02   0.968D-01  -0.742D+00   0.906D+00  -0.269D-01
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE   -0.530D-02  -0.113D-02   0.108D+00   0.922D+00   0.953D+00  -0.257D-01   0.617D-02  -0.103D-01   0.127D+00  -0.804D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE   -0.760D+00  -0.276D-01   0.220D-02   0.873D-02   0.120D+00   0.882D+00  -0.814D+00  -0.284D-01  -0.112D-01   0.271D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    0.439D-01  -0.352D+00   0.694D+00  -0.162D-01   0.602D-02   0.404D-02   0.112D+00   0.682D+00   0.677D+00  -0.121D-01
O  DOF           131         132         133         134         135         136         137         138         139         140
 VALUE    0.610D-02  -0.747D-02   0.422D-01  -0.450D+00  -0.456D+00  -0.246D-01  -0.276D-02   0.988D-02   0.481D-01   0.586D+00
O  DOF           141         142         143         144         145         146         147         148         149         150
 VALUE   -0.445D+00  -0.139D-01  -0.532D-02   0.285D-03  -0.117D+00   0.631D-01   0.109D+00  -0.966D-03   0.934D-02   0.331D-02
O  DOF           151         152         153         154         155         156         157         158         159         160
 VALUE   -0.167D-01   0.190D+00   0.108D+00  -0.390D-02   0.770D-02  -0.106D-01   0.400D-01  -0.479D-01  -0.140D+00  -0.635D-02
O  DOF           161         162         163         164         165         166         167         168         169         170
 VALUE   -0.532D-02   0.204D-02   0.510D-01   0.625D-01  -0.143D+00   0.499D-03  -0.413D-02  -0.107D-01  -0.138D+00   0.396D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
 VALUE   -0.387D+00   0.138D-01   0.286D-02   0.242D-02  -0.277D-01  -0.379D+00  -0.327D+00   0.773D-02   0.104D-02   0.159D-02
O  DOF           181         182         183         184         185         186         187         188         189         190
 VALUE   -0.823D-01   0.335D+00   0.366D+00   0.111D-01  -0.243D-02  -0.215D-02  -0.759D-01   0.451D+00   0.299D+00   0.104D-01
O  DOF           191         192
 VALUE   -0.145D-02  -0.146D-02
OMODE NO.   17        :  LAMBDA= 0.12286D+06           FREQ (RAD/SEC)= 0.35051D+03            FREQ (HERTZ)= 0.55785D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.263D-02  -0.171D-01   0.107D-01   0.000D+00   0.000D+00   0.000D+00   0.675D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE   -0.811D-02   0.196D-02   0.000D+00   0.000D+00   0.000D+00  -0.137D-02  -0.544D-02   0.131D-02   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE    0.000D+00  -0.146D-01  -0.187D-01   0.265D-02  -0.456D-01   0.314D+00   0.720D+00   0.144D-01  -0.404D-01   0.245D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE   -0.647D-01   0.189D+00   0.585D+00   0.439D-01   0.529D-02   0.757D-02  -0.670D-01   0.268D+00   0.679D+00  -0.430D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE   -0.152D-01  -0.431D-04  -0.104D+00   0.268D+00   0.556D+00  -0.225D-01  -0.133D-01   0.429D-02  -0.194D+00   0.460D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    0.100D+01   0.275D-01  -0.100D-01   0.803D-02  -0.276D-01   0.520D+00   0.947D+00   0.651D-01   0.180D-01  -0.106D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE   -0.138D+00   0.465D+00   0.881D+00  -0.814D-01   0.124D-01  -0.124D-01   0.214D-01   0.535D+00   0.820D+00   0.267D-02
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE   -0.111D-01   0.932D-02  -0.299D+00   0.296D+00   0.567D+00  -0.863D-02   0.647D-02  -0.987D-02   0.916D-03   0.436D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE    0.596D+00   0.479D-01   0.157D-01  -0.133D-01  -0.180D+00   0.355D+00   0.499D+00  -0.550D-01   0.234D-01  -0.114D-01
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    0.155D+00   0.376D+00   0.525D+00  -0.195D-02   0.948D-02   0.662D-03  -0.156D+00  -0.222D+00  -0.190D+00  -0.944D-02
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    0.288D-01  -0.187D-01   0.565D-01   0.499D-01  -0.946D-01  -0.115D-01   0.170D-01  -0.122D-01  -0.864D-01  -0.153D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE   -0.311D+00   0.596D-02   0.137D-01  -0.134D-01   0.201D+00  -0.377D-01  -0.210D+00   0.194D-01   0.189D-01  -0.146D-01
O  DOF           121         122         123         124         125         126         127         128         129         130
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 VALUE      0.128D+00  -0.580D+00  -0.686D+00  -0.227D-01   0.135D-01  -0.130D-01   0.810D-01  -0.405D+00  -0.649D+00  -0.538D-01
O  DOF            131         132         133         134         135         136         137         138         139         140
 VALUE     -0.120D-01  -0.362D-02   0.159D-01  -0.542D+00  -0.737D+00   0.573D-01  -0.107D-02   0.696D-02   0.316D-01  -0.440D+00
O  DOF            141         142         143         144         145         146         147         148         149         150
 VALUE     -0.674D+00  -0.533D-02   0.151D-01  -0.111D-01   0.412D+00  -0.492D+00  -0.340D+00   0.949D-02  -0.410D-02   0.105D-01
O  DOF            151         152         153         154         155         156         157         158         159         160
 VALUE      0.990D-01  -0.370D+00  -0.375D+00  -0.500D-01  -0.121D-01   0.132D-01   0.633D-01  -0.536D+00  -0.567D+00   0.452D-01
O  DOF            161         162         163         164         165         166         167         168         169         170
 VALUE     -0.241D-01   0.165D-01  -0.198D+00  -0.355D+00  -0.594D+00   0.274D-02  -0.521D-02   0.145D-02   0.545D+00   0.206D+00
O  DOF            171         172         173         174         175         176         177         178         179         180
 VALUE      0.145D+00   0.464D-02  -0.110D-02   0.547D-02   0.112D+00   0.527D-01   0.110D+00  -0.207D-02  -0.226D-01   0.107D-01
O  DOF            181         182         183         184         185         186         187         188         189         190
 VALUE     -0.160D-01   0.158D+00   0.190D+00  -0.101D-03  -0.666D-02   0.213D-01  -0.383D+00   0.662D-01   0.132D+00  -0.457D-02
O  DOF            191         192
 VALUE     -0.778D-02   0.202D-01
OMODE NO.  16        :  LAMBDA= 0.12932D+06          FREQ (RAD/SEC)= 0.35961D+03          FREQ (HERTZ)= 0.57234D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00  -0.931D-02   0.302D-02   0.128D-01   0.000D+00   0.000D+00   0.000D+00   0.526D-02
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.213D-02   0.765D-02   0.000D+00   0.000D+00   0.000D+00  -0.791D-02   0.153D-02   0.102D-01   0.000D+00   0.000D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE      0.000D+00   0.344D-02   0.206D-02   0.144D-01   0.904D-02   0.662D+00  -0.291D+00  -0.358D-01   0.138D-01   0.291D-01
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.131D+00   0.618D+00  -0.253D+00   0.242D-01  -0.433D-02  -0.272D-02  -0.756D-01   0.550D+00  -0.289D+00  -0.242D-01
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.109D-01   0.143D-01   0.547D-01   0.755D+00  -0.236D+00   0.197D-01   0.129D-02   0.188D-01  -0.289D-01   0.840D+00
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE     -0.398D+00  -0.334D-01   0.172D-02   0.713D-02   0.364D-01   0.100D+01  -0.380D+00   0.317D-01  -0.147D-02  -0.134D-01
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.420D-01   0.820D+00  -0.582D+00  -0.166D-01  -0.838D-02  -0.127D-01   0.134D+00   0.100D+01  -0.556D+00   0.440D-01
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE      0.697D-02   0.659D-02  -0.420D-01   0.445D+00  -0.348D+00  -0.195D-01  -0.292D-02  -0.198D-01  -0.138D+00   0.529D+00
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     -0.363D+00  -0.642D-02  -0.125D-01  -0.202D-01   0.227D+00   0.520D+00  -0.347D+00  -0.991D-02  -0.695D-02  -0.217D-01
O  DOF             91          92          93          94          95          96          97          98          99         100
 VALUE      0.155D+00   0.435D+00  -0.365D+00   0.214D-01  -0.618D-02  -0.133D-01  -0.619D-01  -0.216D+00   0.142D+00   0.234D-01
O  DOF            101         102         103         104         105         106         107         108         109         110
 VALUE     -0.190D-01  -0.196D-01  -0.235D+00  -0.278D+00   0.618D-01  -0.130D-01  -0,541D-02  -0.921D-02   0.234D+00  -0.131D+00
O  DOF            111         112         113         114         115         116         117         118         119         120
 VALUE      0.191D+00   0.147D-01  -0.142D-01  -0.117D-01   0.549D-01  -0.379D+00   0.986D-01  -0.487D-02  -0.117D-01  -0.216D-01
O  DOF            121         122         123         124         125         126         127         128         129         130
 VALUE     -0.810D-01  -0.557D+00   0.429D+00   0.342D-01  -0.772D-02  -0.988D-02   0.126D+00  -0.787D+00   0.406D+00  -0.138D-01
O  DOF            131         132         133         134         135         136         137         138         139         140
 VALUE      0.980D-03   0.566D-02   0.461D-01  -0.517D+00   0.696D+00   0.260D-02   0.546D-02   0.157D-02  -0.299D-01  -0.796D+00
O  DOF            141         142         143         144         145         146         147         148         149         150
 VALUE      0.662D+00  -0.325D-01  -0.134D-01  -0.990D-02  -0.853D-01  -0.419D+00   0.461D+00   0.210D-01   0.541D-02   0.163D-01
O  DOF            151         152         153         154         155         156         157         158         159         160
 VALUE      0.150D+00  -0.490D+00   0.477D+00   0.130D-01   0.129D-01   0.142D-01  -0.213D+00  -0.459D+00   0.469D+00   0.122D-02
O  DOF            161         162         163         164         165         166         167         168         169         170
 VALUE      0.145D-01   0.166D-01   0.136D-02  -0.429D+00   0.492D+00  -0.208D-01   0.749D-02   0.132D-01  -0.115D+00   0.920D-01
O  DOF            171         172         173         174         175         176         177         178         179         180
 VALUE     -0.147D+00  -0.151D-02   0.129D-01   0.117D-01   0.373D+00   0.166D+00  -0.111D+00   0.429D-02   0.132D-01   0.497D-02
O  DOF            181         182         183         184         185         186         187         188         189         190
 VALUE     -0.384D+00   0.678D-01  -0.144D+00  -0.685D-02   0.115D-01  -0.400D-02   0.111D+00   0.223D+00  -0.833D-01  -0.237D-04
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O  DOF           191         192
 VALUE     0.130D-01   0.106D-01
OMODE NO.  15      :  LAMBDA= 0.18745D+06         FREQ (RAD/SEC)= 0.43296D+03         FREQ (HERTZ)= 0.68908D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.816D+00  -0.231D+00  -0.499D+00   0.000D+00   0.000D+00   0.000D+00  -0.316D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.107D+00  -0.199D+00   0.000D+00   0.000D+00   0.000D+00  -0.714D+00  -0.224D+00   0.426D+00   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00   0.694D+00   0.319D+00   0.130D+00   0.171D+00  -0.343D+00  -0.317D+00   0.828D+00   0.150D+00  -0.122D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.135D-01  -0.323D+00  -0.381D+00  -0.898D+00  -0.154D+00   0.207D+00   0.775D-01  -0.194D+00  -0.182D+00  -0.558D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.220D+00   0.187D+00   0.138D-01  -0.190D+00  -0.286D+00   0.100D+01   0.151D+00  -0.315D+00   0.191D+00   0.425D-01
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    -0.820D-02   0.736D+00  -0.192D+00   0.422D-01   0.157D-01  -0.134D+00  -0.115D+00  -0.875D+00  -0.212D+00   0.728D-01
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.202D-01   0.164D-01   0.584D-01  -0.368D+00   0.515D-01   0.551D-01  -0.230D-01   0.123D+00  -0.758D-02   0.832D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.150D+00   0.144D+00   0.913D-01   0.310D+00   0.207D+00   0.647D+00   0.104D+00   0.979D-01   0.807D-01   0.463D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.182D+00  -0.260D+00  -0.990D-01  -0.116D+00   0.807D-02   0.486D+00   0.233D+00  -0.765D+00   0.211D+00  -0.768D-01
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.833D-01   0.530D+00   0.194D+00   0.533D+00   0.564D-01   0.136D-02  -0.188D-01  -0.354D-01   0.257D+00   0.351D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.211D-01  -0.978D-01   0.227D-01   0.366D+00   0.220D+00  -0.323D+00  -0.848D-01   0.133D+00   0.417D-01   0.116D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.229D-01  -0.332D+00   0.391D-01   0.942D-02   0.131D+00   0.381D+00  -0.405D-01   0.444D+00  -0.281D-01  -0.181D+00
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.282D-01  -0.430D+00   0.226D+00   0.164D+00   0.202D-01   0.409D-02  -0.853D-02  -0.933D-02   0.198D+00  -0.275D+00
O  DOF           131         132         133         134         135         136         137         138         139         140
 VALUE    -0.717D-01   0.310D-01   0.129D-01  -0.342D+00  -0.963D-01  -0.111D+00   0.704D-01   0.423D-01   0.302D-01  -0.136D-01
O  DOF           141         142         143         144         145         146         147         148         149         150
 VALUE    -0.101D+00   0.220D+00  -0.403D-02  -0.217D-01  -0.627D-02  -0.430D+00   0.104D+00   0.120D+00   0.259D-01   0.488D-01
O  DOF           151         152         153         154         155         156         157         158         159         160
 VALUE    -0.512D-02  -0.134D+00   0.774D-01  -0.129D+00   0.325D-02   0.141D-01  -0.127D+00  -0.432D+00  -0.250D+00  -0.441D-01
O  DOF           161         162         163         164         165         166         167         168         169         170
 VALUE    -0.117D-01  -0.488D-01  -0.135D+00  -0.150D+00  -0.282D+00   0.854D-01   0.244D-01  -0.135D-01   0.127D-02   0.127D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
 VALUE    -0.849D-02   0.376D-01   0.134D-01  -0.535D-02   0.344D-01   0.773D-02  -0.252D-02  -0.511D-01  -0.464D-01   0.247D-01
O  DOF           181         182         183         184         185         186         187         188         189         190
 VALUE    -0.321D+00   0.820D-01   0.666D-01  -0.152D-01   0.230D-01   0.177D-01  -0.293D+00   0.852D-02   0.328D-01   0.177D-01
O  DOF           191         192
 VALUE    -0.135D-01   0.111D-01
OMODE NO.  14      :  LAMBDA= 0.19210D+06         FREQ (RAD/SEC)= 0.43829D+03         FREQ (HERTZ)= 0.69756D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00  -0.582D+00   0.201D+00   0.336D+00   0.000D+00   0.000D+00   0.000D+00   0.267D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.166D+00   0.856D-01   0.000D+00   0.000D+00   0.000D+00   0.482D+00   0.270D+00  -0.269D+00   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.000D+00  -0.534D+00  -0.319D+00  -0.102D+00  -0.219D+00   0.224D+00   0.678D+00  -0.464D+00  -0.246D+00   0.782D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.742D-01   0.287D+00   0.656D+00   0.666D+00   0.183D+00  -0.771D-01  -0.127D+00   0.147D+00   0.411D+00   0.283D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
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  VALUE   -0.297D+00  -0.108D+00  -0.370D-01   0.215D+00   0.440D+00  -0.664D+00  -0.996D-01   0.222D+00  -0.277D+00  -0.136D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
  VALUE    0.444D+00  -0.237D+00   0.104D+00  -0.557D-02  -0.369D-01   0.203D+00   0.439D+00   0.442D+00   0.223D+00  -0.179D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
  VALUE   -0.129D+00  -0.781D-01   0.138D+00  -0.105D-01   0.692D-01  -0.675D-01   0.504D-01   0.669D-01   0.115D+00  -0.255D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
  VALUE    0.232D+00   0.719D-02  -0.204D+00  -0.552D+00  -0.192D+00   0.254D+00  -0.163D+00  -0.245D+00  -0.160D+00  -0.447D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
  VALUE   -0.289D+00  -0.205D+00   0.604D-01   0.499D-01  -0.167D+00  -0.527D+00  -0.438D+00  -0.412D-01  -0.287D+00   0 134D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
  VALUE   -0.980D-01  -0.497D+00  -0.534D+00   0.238D+00   0.105D+00  -0.391D-01   0.222D-01  -0.302D+00  -0.391D+00   0.537D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
  VALUE    0.691D-01  -0.122D+00  -0.133D+00  -0.582D+00  -0.454D+00  -0.536D+00  -0.705D-01   0.802D-01  -0.136D+00  -0.361D+00
O  DOF           111         112         113         114         115         116         117         118         119         120
  VALUE   -0.191D+00  -0.272D+00   0.111D+00   0.191D+00  -0.226D+00  -0.430D+00  -0.341D+00   0.525D+00   0.214D+00  -0.556D-01
O  DOF           121         122         123         124         125         126         127         128         129         130
  VALUE   -0.414D-01   0.627D+00   0.853D-02   0.649D+00  -0.186D+00   0.886D-01  -0.151D+00   0.690D-01  -0.103D+00  -0.646D+00
O  DOF           131         132         133         134         135         136         137         138         139         140
  VALUE   -0.139D+00  -0.353D-02  -0.155D+00   0.478D+00   0.259D+00  -0.365D+00   0.601D-01   0.521D-01  -0.150D+00   0.354D+00
O  DOF           141         142         143         144         145         146         147         148         149         150
  VALUE    0.150D+00   0.698D+00  -0.186D+00   0.170D+00  -0.262D+00   0.807D+00  -0.192D-02   0.677D+00   0.555D-01   0.117D-01
O  DOF           151         152         153         154         155         156         157         158         159         160
  VALUE   -0.211D+00   0.527D+00  -0.305D-01  -0.205D+00   0.102D-01  -0.112D+00   0.213D-01   0.100D+01   0.511D+00  -0.814D+00
O  DOF           161         162         163         164         165         166         167         168         169         170
  VALUE    0.182D+00  -0.109D+00   0.158D+00   0.604D+00   0.449D+00   0.604D+00   0.193D+00  -0.138D-01  -0.390D+00  -0.297D+00
O  DOF           171         172         173         174         175         176         177         178         179         180
  VALUE    0.726D-01   0.344D+00   0.115D+00  -0.264D-01  -0.360D+00  -0.327D-01   0.296D-01  -0.416D+00  -0.302D+00   0.182D+00
O  DOF           181         182         183         184         185         186         187         188         189         190
  VALUE    0.321D+00  -0.170D+00  -0.113D-01  -0.292D+00   0.237D+00   0.255D-01   0.403D+00   0.671D-01  -0.268D-01   0.451D+00
O  DOF           191         192
  VALUE   -0.268D+00  -0.285D+00
OMODE NO.   13       :  LAMBDA= 0.19918D+06          FREQ (RAD/SEC)= 0.44630D+03          FREQ (HERTZ)= 0.71031D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
  VALUE    0.000D+00   0.000D+00   0.000D+00   0.130D+00   0.463D+00  -0.532D+00   0.000D+00   0.000D+00   0.000D+00   0.231D-01
O  DOF            11          12          13          14          15          16          17          18          19          20
  VALUE   -0.813D+00  -0.844D+00   0.000D+00   0.000D+00   0.000D+00  -0.302D+00   0.800D+00   0.100D+01   0.000D+00   0.000D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
  VALUE    0.000D+00   0.899D-01  -0.387D+00   0.339D+00  -0.513D-01   0.163D+00  -0.143D+00   0.444D-01  -0.287D+00   0.338D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
  VALUE   -0.657D-01  -0.550D-01  -0.124D+00  -0.262D+00   0.389D+00   0.579D+00  -0.108D+00   0.140D+00  -0.591D-01   0.133D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
  VALUE   -0.567D+00  -0.477D+00  -0.581D-01   0.527D-01  -0.674D-01   0.396D+00   0.133D-01  -0.476D+00  -0.422D-01   0.267D+00
O  DOF            51          52          53          54          55          56          57          58          59          60
  VALUE   -0.220D+00  -0.150D+00   0.255D+00  -0.140D+00  -0.468D-01   0.829D-01  -0.179D+00   0.602D-01  -0.228D+00  -0.205D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
  VALUE   -0.846D-01   0.262D+00   0.579D-01   0.295D-01   0.311D+00   0.182D+00  -0.966D-01   0.465D-01   0.314D-01  -0.165D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
  VALUE    0.142D+00   0.249D+00  -0.236D-01   0.191D+00  -0.108D-01  -0.135D+00  -0.667D-01   0.164D+00  -0.534D-01   0.175D+00
O  DOF            81          82          83          84          85          86          87          88          89          90
  VALUE    0.116D-01   0.494D-01   0.149D+00   0.114D+00  -0.631D-01   0.196D+00   0.155D-02   0.205D+00  -0.265D+00  -0.160D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
  VALUE   -0.676D-01   0.180D+00   0.206D-01  -0.169D+00  -0.183D-01  -0.108D+00  -0.354D-01   0.977D-02   0.159D+00  -0.173D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
  VALUE    0.306D-01  -0.275D-01  -0.477D-01   0.193D+00   0.161D+00   0.221D+00   0.476D-02  -0.949D-01  -0.137D-02   0.390D-01
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O  DOF          111          112          113          114          115          116          117          118          119          120
 VALUE   0.970D-02    0.781D-01    0.749D-01   -0.114D-01   -0.908D-02    0.118D+00    0.311D-01   -0.232D+00    0.480D-01    0.130D+00
O  DOF          121          122          123          124          125          126          127          128          129          130
 VALUE   0.238D-01   -0.300D+00    0.114D+00   -0.110D+00    0.720D-03   -0.133D-01    0.402D-02    0.545D-02    0.126D+00    0.108D+00
O  DOF          131          132          133          134          135          136          137          138          139          140
 VALUE   0.349D-01   -0.217D-01    0.401D-01   -0.262D+00   -0.136D+00    0.927D-01   -0.492D-01   -0.710D-02    0.324D-01   -0.626D-01
O  DOF          141          142          143          144          145          146          147          148          149          150
 VALUE  -0.967D-01   -0.144D+00   -0.204D-01   -0.471D-01    0.105D+00   -0.299D+00    0.157D+00   -0.625D-01   -0.703D-02   -0.181D-01
O  DOF          151          152          153          154          155          156          157          158          159          160
 VALUE   0.781D-01   -0.124D+00    0.168D+00    0.268D-01    0.764D-02    0.117D-01    0.870D-02   -0.350D+00   -0.780D-01    0.103D+00
O  DOF          161          162          163          164          165          166          167          168          169          170
 VALUE  -0.394D-02    0.428D-01    0.214D-01   -0.155D+00   -0.748D-01   -0.728D-01   -0.246D-02    0.110D-01    0.170D+00    0.137D+00
O  DOF          171          172          173          174          175          176          177          178          179          180
 VALUE  -0.431D-01   -0.223D-01    0.347D-02    0.134D-01    0.184D+00   -0.187D-01   -0.279D-01    0.241D-01    0.153D-01   -0.159D-01
O  DOF          181          182          183          184          185          186          187          188          189          190
 VALUE  -0.910D-01    0.853D-01    0.223D-01    0.242D-01   -0.103D-01    0.577D-02   -0.143D-02   -0.428D-01   -0.158D-02   -0.327D-01
O  DOF          191          192
 VALUE   0.652D-02    0.313D-01
OMODE NO.  12     :  LAMBDA= 0.20026D+06          FREQ (RAD/SEC)= 0.44751D+03          FREQ (HERTZ)= 0.71223D+02
OMODE NO.  11     :  LAMBDA= 0.20917D+06          FREQ (RAD/SEC)= 0.45736D+03          FREQ (HERTZ)= 0.72790D+02
OMODE NO.  10     :  LAMBDA= 0.21000D+06          FREQ (RAD/SEC)= 0.45825D+03          FREQ (HERTZ)= 0.72933D+02
OMODE NO.   9     :  LAMBDA= 0.21332D+06          FREQ (RAD/SEC)= 0.46187D+03          FREQ (HERTZ)= 0.73509D+02
OMODE NO.   8     :  LAMBDA= 0.21518D+06          FREQ (RAD/SEC)= 0.46387D+03          FREQ (HERTZ)= 0.73828D+02
OMODE NO.   7     :  LAMBDA= 0.22405D+06          FREQ (RAD/SEC)= 0.47334D+03          FREQ (HERTZ)= 0.75334D+02
OMODE NO.   6     :  LAMBDA= 0.22700D+06          FREQ (RAD/SEC)= 0.47645D+03          FREQ (HERTZ)= 0.75829D+02
OMODE NO.   5     :  LAMBDA= 0.23679D+06          FREQ (RAD/SEC)= 0.48661D+03          FREQ (HERTZ)= 0.77446D+02
OMODE NO.   4     :  LAMBDA= 0.24200D+06          FREQ (RAD/SEC)= 0.49193D+03          FREQ (HERTZ)= 0.78293D+02
OMODE NO.   3     :  LAMBDA= 0.25495D+06          FREQ (RAD/SEC)= 0.50493D+03          FREQ (HERTZ)= 0.80361D+02
OMODE NO.   2     :  LAMBDA= 0.26536D+06          FREQ (RAD/SEC)= 0.51513D+03          FREQ (HERTZ)= 0.81985D+02
OMODE NO.   1     :  LAMBDA= 0.27973D+06          FREQ (RAD/SEC)= 0.52889D+03          FREQ (HERTZ)= 0.84176D+02
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Super-Finite Element Model of SADE (Chapter 4)

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EIGENVECTORS INCLUDING RESTRAINED DOF'S:


OMODE NO.   24        :  LAMBDA= 0.66899D+03          FREQ (RAD/SEC)= 0.25865D+02          FREQ (HERTZ)= 0.41165D+01
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.563D-03  -0.526D-01   0.394D-01  -0.311D-03
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.967D-03  -0.613D-03  -0.136D-02  -0.132D+00   0.128D+00  -0.564D-03  -0.178D-02  -0.112D-02  -0.236D-02  -0.231D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.256D+00  -0.758D-03  -0.243D-02  -0.151D-02  -0.351D-02  -0.344D+00   0.416D+00  -0.897D-03  -0.293D-02  -0.181D-02
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.477D-02  -0.467D+00   0.599D+00  -0.984D-03  -0.328D-02  -0.201D-02  -0.604D-02  -0.593D+00   0.796D+00  -0.103D-02
O  DOF           41          42          43          44          45          46          47          48
 VALUE   -0.350D-02  -0.212D-02  -0.725D-02  -0.719D+00   0.100D+01  -0.103D-02  -0.359D-02  -0.215D-02
OMODE NO.   23        :  LAMBDA= 0.82475D+03          FREQ (RAD/SEC)= 0.28718D+02          FREQ (HERTZ)= 0.45707D+01
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.438D-02   0.235D-01   0.354D-01   0.643D-03
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.723D-03   0.104D-02  -0.835D-02   0.103D+00   0.106D+00   0.119D-02  -0.130D-02   0.190D-02  -0.119D-01   0.229D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.204D+00   0.164D-02  -0.175D-02   0.259D-02  -0.149D-01   0.391D+00   0.322D+00   0.199D-02  -0.205D-02   0.311D-02
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.175D-01   0.580D+00   0.452D+00   0.225D-02  -0.224D-02   0.346D-02  -0.195D-01   0.786D+00   0.588D+00   0.241D-02
O  DOF           41          42          43          44          45          46          47          48
 VALUE   -0.232D-02   0.367D-02  -0.210D-01   0.100D+01   0.724D+00   0.250D-02  -0.230D-02   0.374D-02
OMODE NO.   22        :  LAMBDA= 0.78359D+04          FREQ (RAD/SEC)= 0.88520D+02          FREQ (HERTZ)= 0.14088D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.162D+00   0.181D+00  -0.111D+00  -0.338D-01
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.123D-02   0.246D-02   0.323D+00   0.456D+00  -0.157D+00  -0.678D-01  -0.224D-02   0.329D-02   0.481D+00   0.732D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE   -0.147D+00  -0.101D+00  -0.303D-02   0.269D-02   0.631D+00   0.933D+00  -0.885D-01  -0.133D+00  -0.359D-02   0.911D-03
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.770D+00   0.100D+01   0.780D-02  -0.163D+00  -0.392D-02  -0.171D-02   0.893D+00   0.893D+00   0.131D+00  -0.190D+00
O  DOF           41          42          43          44          45          46          47          48
 VALUE   -0.402D-02  -0.483D-02   0.999D+00   0.598D+00   0.267D+00  -0.213D+00  -0.389D-02  -0.810D-02
OMODE NO.   21        :  LAMBDA= 0.27703D+05          FREQ (RAD/SEC)= 0.16644D+03          FREQ (HERTZ)= 0.26490D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.405D-01  -0.190D+00   0.317D+00  -0.511D-03
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.278D-02  -0.110D-02  -0.786D-01  -0.331D+00   0.691D+00  -0.578D-03  -0.302D-02  -0.129D-02  -0.109D+00  -0.380D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.958D+00  -0.423D-03  -0.113D-02  -0.824D-03  -0.127D+00  -0.331D+00   0.100D+01  -0.310D-03   0.206D-02  -0.427D-04
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.130D+00  -0.206D+00   0.767D+00  -0.439D-03   0.552D-02   0.718D-03  -0.122D+00  -0.493D-01   0.293D+00  -0.872D-03
O  DOF           41          42          43          44          45          46          47          48
 VALUE    0.829D-02   0.122D-02  -0.109D+00   0.922D-01  -0.325D+00  -0.153D-02   0.974D-02   0.136D-02
OMODE NO.   20        :  LAMBDA= 0.31435D+05          FREQ (RAD/SEC)= 0.17730D+03          FREQ (HERTZ)= 0.28218D+02
OEIGENVECTOR:
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O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.396D-01   0.286D+00   0.151D+00   0.332D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.589D-03   0.283D-02  -0.696D-01   0.657D+00   0.284D+00   0.492D-02  -0.190D-03   0.305D-02  -0.882D-01   0.940D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.345D+00   0.488D-02   0.740D-03   0.114D-02  -0.961D-01   0.100D+01   0.314D+00   0.365D-02   0.168D-02  -0.208D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.966D-01   0.778D+00   0.207D+00   0.190D-02   0.220D-02  -0.552D-02  -0.938D-01   0.304D+00   0.667D-01   0.339D-03
O  DOF            41          42          43          44          45          46          47          48
 VALUE     0.215D-02  -0.821D-02  -0.918D-01  -0.319D+00  -0.631D-01  -0.546D-03   0.162D-02  -0.952D-02
OMODE NO.   19        :  LAMBDA= 0.56849D+05            FREQ (RAD/SEC)= 0.23843D+03            FREQ (HERTZ)= 0.37947D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.156D+00  -0.439D-01   0.176D-01   0.165D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.249D-02   0.418D-03   0.310D+00  -0.248D-01   0.132D+00   0.301D-02  -0.354D-02   0.604D-03   0.461D+00   0.349D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.250D+00   0.384D-02  -0.330D-02   0.461D-03   0.611D+00   0.949D-01   0.295D+00   0.404D-02  -0.223D-02  -0.166D-04
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.755D+00   0.114D+00   0.233D+00   0.369D-02  -0.954D-03  -0.694D-03   0.887D+00   0.669D-01   0.886D-01   0.310D-02
O  DOF            41          42          43          44          45          46          47          48
 VALUE    -0.711D-04  -0.133D-02   0.100D+01  -0.365D-01  -0.800D-01   0.259D-02   0.138D-03  -0.171D-02
OMODE NO.   18        :  LAMBDA= 0.93314D+05            FREQ (RAD/SEC)= 0.30547D+03            FREQ (HERTZ)= 0.48618D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.213D+00   0.597D+00   0.374D-01  -0.427D-01
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.128D-02   0.760D-03   0.389D+00   0.100D+01   0.125D+00  -0.779D-01  -0.796D-03  -0.320D-02   0.480D+00   0.928D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.216D+00  -0.951D-01   0.498D-03  -0.793D-02   0.461D+00   0.448D+00   0.281D+00  -0.881D-01   0.211D-02  -0.965D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.338D+00  -0.102D+00   0.283D+00  -0.578D-01   0.394D-02  -0.646D-02   0.148D+00  -0.332D+00   0.174D+00  -0.116D-01
O  DOF            41          42          43          44          45          46          47          48
 VALUE     0.590D-02   0.896D-03  -0.555D-01  -0.399D-01  -0.647D-01   0.380D-01   0.754D-02   0.970D-02
OMODE NO.   17        :  LAMBDA= 0.14696D+06            FREQ (RAD/SEC)= 0.38335D+03            FREQ (HERTZ)= 0.61012D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.915D-01  -0.194D+00   0.681D+00   0.148D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.222D-02  -0.182D-03  -0.131D+00  -0.141D+00   0.100D+01   0.194D-02   0.123D-02   0.463D-03  -0.935D-01   0.265D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.620D+00   0.255D-03   0.558D-02   0.989D-03  -0.723D-02   0.952D-01  -0.191D+00  -0.252D-02   0.592D-02   0.127D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.619D-01   0.388D-02  -0.777D+00  -0.392D-02   0.901D-03   0.193D-02   0.678D-01  -0.929D-01  -0.646D+00  -0.244D-02
O  DOF            41          42          43          44          45          46          47          48
 VALUE    -0.638D-02   0.339D-02   0.223D-01  -0.290D-02   0.140D+00   0.978D-03  -0.115D-01   0.522D-02
OMODE NO.   16        :  LAMBDA= 0.16318D+06            FREQ (RAD/SEC)= 0.40395D+03            FREQ (HERTZ)= 0.64291D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.494D-01   0.674D+00   0.170D+00   0.731D-02
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.172D-02   0.222D-02  -0.579D-01   0.100D+01   0.139D+00   0.831D-02   0.304D-02  -0.115D-02  -0.359D-01   0.600D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.628D-02   0.516D-02   0.202D-02  -0.547D-02  -0.105D-01  -0.247D+00  -0.346D-01   0.189D-02  -0.534D-03  -0.591D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
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 VALUE      0.502D-02  -0.832D+00   0.581D-01   0.631D-03  -0.190D-02  -0.129D-02   0.210D-01  -0.653D+00   0.119D+00   0.111D-03
O  DOF             41          42          43          44          45          46          47          48
 VALUE     -0.352D-03   0.529D-02   0.496D-01   0.163D+00  -0.196D-01  -0.186D-02   0.294D-02   0.945D-02
OMODE NO.   15         :  LAMBDA= 0.28816D+06              FREQ (RAD/SEC)= 0.53681D+03              FREQ (HERTZ)= 0.85436D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.146D+00   0.100D+01   0.187D+00  -0.509D-01
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.268D-02  -0.521D-02   0.148D+00   0.842D+00   0.254D+00  -0.659D-01   0.416D-02  -0.118D-01  -0.549D-01  -0.150D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE      0.318D+00  -0.281D-01   0.290D-02  -0.867D-02  -0.312D+00  -0.659D+00   0.268D+00   0.336D-01   0.996D-03   0.272D-02
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.383D+00  -0.132D+00  -0.609D-01   0.670D-01  -0.146D-02   0.983D-02  -0.183D+00   0.429D+00  -0.364D+00   0.444D-01
O  DOF             41          42          43          44          45          46          47          48
 VALUE     -0.804D-02   0.422D-02   0.140D+00   0.200D-02  -0.211D-01  -0.135D-01  -0.188D-01  -0.906D-02
OMODE NO.   14         :  LAMBDA= 0.32120D+06              FREQ (RAD/SEC)= 0.56675D+03              FREQ (HERTZ)= 0.90201D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.688D-01   0.106D+00  -0.965D+00  -0.103D-02
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.467D-03   0.704D-03  -0.154D-01  -0.175D-01  -0.765D+00   0.158D-02  -0.557D-02   0.158D-02  -0.203D+00   0.591D-01
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE      0.417D+00   0.495D-02  -0.672D-02   0.254D-02  -0.282D+00   0.294D+00   0.100D+01   0.217D-02  -0.127D-02   0.263D-02
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.171D+00   0.203D+00   0.304D+00  -0.507D-02   0.604D-03   0.381D-02  -0.207D-01  -0.163D+00  -0.510D+00  -0.681D-02
O  DOF             41          42          43          44          45          46          47          48
 VALUE     -0.825D-02   0.985D-02   0.263D-01  -0.913D-01  -0.109D+00   0.799D-03  -0.234D-01   0.208D-01
OMODE NO.   13         :  LAMBDA= 0.38633D+06              FREQ (RAD/SEC)= 0.62155D+03              FREQ (HERTZ)= 0.98923D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00   0.000D+00  -0.146D+00   0.100D+01   0.177D+00   0.146D-01
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.460D-02   0.631D-03  -0.164D+00   0.606D+00  -0.294D-01   0.107D-01   0.242D-02  -0.512D-02  -0.127D+00  -0.751D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.859D-02  -0.133D-02  -0.462D-02  -0.550D-02  -0.899D-01  -0.960D+00   0.244D+00  -0.883D-02  -0.556D-02   0.562D-03
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.206D-01   0.221D+00   0.914D-01  -0.110D-01  -0.682D-03   0.230D-02   0.431D-01   0.867D+00  -0.271D+00  -0.759D-02
O  DOF             41          42          43          44          45          46          47          48
 VALUE     -0.116D-02  -0.467D-02   0.265D-01  -0.418D-01  -0.129D-01   0.326D-02  -0.994D-02  -0.128D-01
OMODE NO.   12         :  LAMBDA= 0.44257D+06              FREQ (RAD/SEC)= 0.66526D+03              FREQ (HERTZ)= 0.10588D+03
OMODE NO.   11         :  LAMBDA= 0.53366D+06              FREQ (RAD/SEC)= 0.73052D+03              FREQ (HERTZ)= 0.11627D+03
OMODE NO.   10         :  LAMBDA= 0.60298D+06              FREQ (RAD/SEC)= 0.77652D+03              FREQ (HERTZ)= 0.12359D+03
OMODE NO.    9         :  LAMBDA= 0.68696D+06              FREQ (RAD/SEC)= 0.82883D+03              FREQ (HERTZ)= 0.13191D+03
OMODE NO.    8         :  LAMBDA= 0.71081D+06              FREQ (RAD/SEC)= 0.84309D+03              FREQ (HERTZ)= 0.13418D+03
OMODE NO.    7         :  LAMBDA= 0.79439D+06              FREQ (RAD/SEC)= 0.89129D+03              FREQ (HERTZ)= 0.14185D+03
OMODE NO.    6         :  LAMBDA= 0.94921D+06              FREQ (RAD/SEC)= 0.97427D+03              FREQ (HERTZ)= 0.15506D+03
OMODE NO.    5         :  LAMBDA= 0.98793D+06              FREQ (RAD/SEC)= 0.99394D+03              FREQ (HERTZ)= 0.15819D+03
OMODE NO.    4         :  LAMBDA= 0.13228D+07              FREQ (RAD/SEC)= 0.11501D+04              FREQ (HERTZ)= 0.18305D+03
OMODE NO.    3         :  LAMBDA= 0.19187D+07              FREQ (RAD/SEC)= 0.13852D+04              FREQ (HERTZ)= 0.22046D+03
OMODE NO.    2         :  LAMBDA= 0.23145D+07              FREQ (RAD/SEC)= 0.15213D+04              FREQ (HERTZ)= 0.24213D+03
OMODE NO.    1         :  LAMBDA= 0.27549D+07              FREQ (RAD/SEC)= 0.16598D+04              FREQ (HERTZ)= 0.26416D+03
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Two-Dimensional Model of SADE with Pinned Joints (Section 6.1)
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OMODE NO.  24        :  LAMBDA= 0.68738D+03          FREQ (RAD/SEC)= 0.26218D+02          FREQ (HERTZ)= 0.41727D+01
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.284D-01    0.412D-01   -0.238D-01    0.457D-01    0.523D-01    0.134D+00   -0.431D-01    0.139D+00    0.716D-01    0.266D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.580D-01    0.270D+00    0.866D-01    0.427D+00   -0.686D-01    0.431D+00    0.973D-01    0.608D+00   -0.753D-01    0.612D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.104D+00    0.801D+00   -0.785D-01    0.804D+00    0.107D+00    0.999D+00   -0.786D-01    0.100D+01
OMODE NO.  23        :  LAMBDA= 0.26156D+05          FREQ (RAD/SEC)= 0.16173D+03          FREQ (HERTZ)= 0.25740D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.112D+00    0.299D+00   -0.113D-01    0.361D+00    0.156D+00    0.693D+00    0.354D-01    0.741D+00    0.140D+00    0.954D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.120D+00    0.982D+00    0.844D-01    0.996D+00    0.220D+00    0.100D+01    0.101D-01    0.770D+00    0.312D+00    0.751D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE   -0.575D-01    0.304D+00    0.371D+00    0.272D+00   -0.969D-01   -0.292D+00    0.393D+00   -0.309D+00
OMODE NO.  22        :  LAMBDA= 0.47744D+05          FREQ (RAD/SEC)= 0.21850D+03          FREQ (HERTZ)= 0.34776D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.936D-01   -0.453D-02    0.218D+00   -0.390D-01    0.220D+00   -0.132D+00    0.398D+00   -0.162D+00    0.373D+00   -0.261D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.542D+00   -0.282D+00    0.545D+00   -0.322D+00    0.661D+00   -0.330D+00    0.719D+00   -0.274D+00    0.765D+00   -0.268D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.875D+00   -0.122D+00    0.863D+00   -0.107D+00    0.100D+01    0.787D-01    0.962D+00    0.876D-01
OMODE NO.  21        :  LAMBDA= 0.13676D+06          FREQ (RAD/SEC)= 0.36981D+03          FREQ (HERTZ)= 0.58857D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.129D+00    0.593D+00    0.426D-01    0.718D+00    0.853D-01    0.992D+00    0.164D+00    0.100D+01   -0.460D-01    0.626D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.216D+00    0.530D+00   -0.118D+00   -0.151D+00    0.136D+00   -0.266D+00   -0.600D-01   -0.737D+00   -0.486D-01   -0.781D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.990D-01   -0.611D+00   -0.223D+00   -0.566D+00    0.245D+00    0.107D+00   -0.314D+00    0.149D+00
OMODE NO.  20        :  LAMBDA= 0.30414D+06          FREQ (RAD/SEC)= 0.55149D+03          FREQ (HERTZ)= 0.87772D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE   -0.149D-01    0.613D+00   -0.746D-02    0.746D+00   -0.249D+00    0.610D+00    0.245D-01    0.548D+00   -0.480D+00   -0.348D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.812D-01   -0.445D+00   -0.513D+00   -0.998D+00   -0.216D+00   -0.897D+00   -0.429D+00   -0.496D+00   -0.104D+00   -0.196D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE   -0.491D+00    0.344D+00    0.355D+00    0.632D+00   -0.818D+00    0.824D-01    0.100D+01    0.242D+00
OMODE NO.  19        :  LAMBDA= 0.42087D+06          FREQ (RAD/SEC)= 0.64874D+03          FREQ (HERTZ)= 0.10325D+03
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.229D+00    0.904D+00    0.325D+00    0.966D+00    0.173D+00    0.510D+00    0.485D+00    0.286D+00    0.182D+00   -0.793D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.273D+00   -0.933D+00    0.426D+00   -0.768D+00    0.203D-01   -0.609D+00    0.581D+00    0.683D+00    0.145D-01    0.797D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.350D+00    0.100D+01   -0.466D-01    0.775D+00    0.105D+00   -0.413D-01   -0.444D+00   -0.354D+00
OMODE NO.  18        :  LAMBDA= 0.62103D+06          FREQ (RAD/SEC)= 0.78805D+03          FREQ (HERTZ)= 0.12542D+03
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OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.356D+00 -0.139D+00  0.516D+00 -0.272D+00  0.779D+00 -0.186D+00  0.827D+00 -0.186D+00  0.100D+01  0.134D+00
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.933D+00  0.156D+00  0.896D+00  0.843D-01  0.800D+00  0.484D-01  0.528D+00 -0.347D+00  0.406D+00 -0.280D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE    0.194D-01 -0.243D+00  0.384D-01 -0.290D-01 -0.646D+00 -0.758D-01 -0.107D+00  0.250D+00
OMODE NO.  17       :  LAMBDA= 0.73668D+06          FREQ (RAD/SEC)= 0.85830D+03          FREQ (HERTZ)= 0.13660D+03
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.667D-01  0.100D+01  0.212D+00  0.995D+00 -0.218D+00 -0.262D+00  0.721D-01 -0.469D+00 -0.134D+00 -0.799D+00
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE   -0.132D+00 -0.603D+00  0.354D-01  0.535D+00  0.394D-01  0.708D+00 -0.146D+00  0.415D+00  0.163D+00  0.217D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE   -0.237D+00 -0.916D+00 -0.217D-01 -0.908D+00 -0.285D-03 -0.210D+00  0.385D-01  0.389D+00
OMODE NO.  16       :  LAMBDA= 0.95034D+06          FREQ (RAD/SEC)= 0.97485D+03          FREQ (HERTZ)= 0.15515D+03
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.143D+00  0.100D+01  0.295D+00  0.878D+00 -0.794D-01 -0.733D+00  0.419D-01 -0.839D+00  0.163D+00 -0.268D-01
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.101D-01  0.238D+00  0.100D+00  0.828D+00  0.146D+00  0.653D+00 -0.184D+00 -0.825D+00 -0.175D+00 -0.854D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE    0.230D-01  0.258D+00 -0.954D-01  0.501D+00 -0.941D-01  0.727D+00  0.962D-01 -0.727D+00
OMODE NO.  15       :  LAMBDA= 0.11838D+07          FREQ (RAD/SEC)= 0.10880D+04          FREQ (HERTZ)= 0.17316D+03
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE   -0.233D-01  0.566D+00  0.374D-01  0.478D+00 -0.240D+00 -0.633D+00 -0.228D+00 -0.573D+00 -0.163D-01  0.602D+00
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE   -0.296D-01  0.675D+00 -0.115D+00  0.393D-01 -0.730D-02 -0.189D+00  0.412D-01 -0.522D+00 -0.113D+00 -0.355D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE    0.308D+00  0.100D+01  0.160D+00  0.876D+00  0.992D-01 -0.528D+00 -0.108D+00  0.344D+00
OMODE NO.  14       :  LAMBDA= 0.14180D+07          FREQ (RAD/SEC)= 0.11908D+04          FREQ (HERTZ)= 0.18952D+03
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.325D-01  0.408D+00  0.166D+00  0.291D+00 -0.896D-01 -0.699D+00 -0.167D-01 -0.488D+00  0.104D+00  0.100D+01
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.285D+00  0.953D+00 -0.241D+00 -0.910D+00 -0.177D-01 -0.847D+00  0.287D-01  0.621D+00  0.187D+00  0.720D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE   -0.219D+00 -0.605D+00 -0.429D-01 -0.610D+00 -0.244D-01  0.183D+00  0.223D-01 -0.879D-01
OMODE NO.  13       :  LAMBDA= 0.15152D+07          FREQ (RAD/SEC)= 0.12309D+04          FREQ (HERTZ)= 0.19591D+03
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE   -0.632D+00 -0.245D+00  0.297D-01  0.592D-01 -0.959D+00 -0.156D+00  0.347D+00  0.239D+00 -0.813D+00 -0.244D+00
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.696D+00  0.136D+00 -0.421D+00 -0.110D+00  0.100D+01  0.272D+00 -0.755D-01 -0.177D+00  0.912D+00  0.170D+00
O  DOF           21         22         23         24         25         26         27         28
 VALUE    0.685D-01 -0.305D-01  0.408D+00  0.211D+00 -0.865D-01  0.698D-01 -0.187D+00 -0.362D-01
OMODE NO.  12       :  LAMBDA= 0.19834D+07          FREQ (RAD/SEC)= 0.14083D+04          FREQ (HERTZ)= 0.22414D+03
OMODE NO.  11       :  LAMBDA= 0.26135D+07          FREQ (RAD/SEC)= 0.16166D+04          FREQ (HERTZ)= 0.25729D+03
OMODE NO.  10       :  LAMBDA= 0.39892D+07          FREQ (RAD/SEC)= 0.19973D+04          FREQ (HERTZ)= 0.31788D+03
OMODE NO.   9       :  LAMBDA= 0.45663D+07          FREQ (RAD/SEC)= 0.21369D+04          FREQ (HERTZ)= 0.34010D+03
OMODE NO.   8       :  LAMBDA= 0.49021D+07          FREQ (RAD/SEC)= 0.22141D+04          FREQ (HERTZ)= 0.35238D+03
OMODE NO.   7       :  LAMBDA= 0.51108D+07          FREQ (RAD/SEC)= 0.22607D+04          FREQ (HERTZ)= 0.35980D+03
OMODE NO.   6       :  LAMBDA= 0.58022D+07          FREQ (RAD/SEC)= 0.24088D+04          FREQ (HERTZ)= 0.38337D+03
OMODE NO.   5       :  LAMBDA= 0.64986D+07          FREQ (RAD/SEC)= 0.25492D+04          FREQ (HERTZ)= 0.40572D+03
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Two-Dimensional Model of SADE with Rigid Joints (Section 6.1)
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OMODE NO.  24      :  LAMBDA= 0.68813D+03          FREQ (RAD/SEC)= 0.26232D+02          FREQ (HERTZ)= 0.41750D+01
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.217D-03  0.307D-03  0.284D-01  0.412D-01  0.108D-02 -0.238D-01  0.457D-01  0.120D-02  0.523D-01  0.134D+00
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE   0.191D-02 -0.431D-01  0.139D+00  0.198D-02  0.717D-01  0.266D+00  0.253D-02 -0.580D-01  0.270D+00  0.260D-02
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE   0.866D-01  0.427D+00  0.298D-02 -0.687D-01  0.431D+00  0.306D-02  0.973D-01  0.608D+00  0.328D-02 -0.754D-01
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE   0.612D+00  0.335D-02  0.104D+00  0.801D+00  0.345D-02 -0.785D-01  0.804D+00  0.354D-02  0.107D+00  0.999D+00
O  DOF          41         42         43         44
 VALUE   0.338D-02 -0.787D-01  0.100D+01  0.342D-02
OMODE NO.  23      :  LAMBDA= 0.25996D+05          FREQ (RAD/SEC)= 0.16123D+03          FREQ (HERTZ)= 0.25661D+02
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.225D-02  0.340D-02  0.112D+00  0.299D+00  0.488D-02 -0.114D-01  0.361D+00  0.688D-02  0.157D+00  0.693D+00
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE   0.372D-02  0.355D-01  0.741D+00  0.661D-02  0.141D+00  0.955D+00  0.110D-03  0.120D+00  0.982D+00  0.409D-02
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE   0.850D-01  0.996D+00 -0.423D-02  0.221D+00  0.100D+01 -0.921D-04  0.105D-01  0.769D+00 -0.834D-02  0.313D+00
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE   0.750D+00 -0.496D-02 -0.571D-01  0.301D+00 -0.108D-01  0.372D+00  0.270D+00 -0.922D-02 -0.962D-01 -0.294D+00
O  DOF          41         42         43         44
 VALUE  -0.987D-02  0.394D+00 -0.311D+00 -0.914D-02
OMODE NO.  22      :  LAMBDA= 0.47581D+05          FREQ (RAD/SEC)= 0.21813D+03          FREQ (HERTZ)= 0.34716D+02
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.307D-03  0.475D-03  0.930D-01 -0.672D-02 -0.192D-02  0.218D+00 -0.417D-01 -0.837D-03  0.219D+00 -0.137D+00
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE  -0.405D-02  0.398D+00 -0.168D+00 -0.115D-02  0.373D+00 -0.268D+00 -0.389D-02  0.542D+00 -0.289D+00 -0.754D-05
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE   0.545D+00 -0.330D+00 -0.299D-02  0.660D+00 -0.338D+00  0.248D-02  0.720D+00 -0.279D+00 -0.212D-02  0.763D+00
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE  -0.273D+00  0.549D-02  0.876D+00 -0.123D+00 -0.142D-02  0.861D+00 -0.108D+00  0.900D-02  0.100D+01  0.777D-01
O  DOF          41         42         43         44
 VALUE  -0.754D-02  0.960D+00  0.869D-01  0.748D-02
OMODE NO.  21      :  LAMBDA= 0.13214D+06          FREQ (RAD/SEC)= 0.36351D+03          FREQ (HERTZ)= 0.57855D+02
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.773D-02  0.103D-01  0.131D+00  0.593D+00  0.369D-02  0.449D-01  0.717D+00  0.248D-01  0.885D-01  0.992D+00
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE  -0.168D-01  0.168D+00  0.100D+01  0.138D-01 -0.439D-01  0.625D+00 -0.251D-01  0.220D+00  0.529D+00 -0.649D-02
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE  -0.117D+00 -0.156D+00 -0.149D-01  0.138D+00 -0.269D+00 -0.195D-01 -0.606D-01 -0.741D+00  0.591D-02 -0.486D-01
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE  -0.783D+00 -0.160D-01  0.986D-01 -0.608D+00  0.218D-01 -0.221D+00 -0.562D+00  0.664D-02  0.244D+00  0.113D+00
O  DOF          41         42         43         44
 VALUE   0.126D-01 -0.308D+00  0.156D+00  0.135D-01
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OMODE NO.  20        :  LAMBDA= 0.26508D+06         FREQ (RAD/SEC)= 0.51486D+03         FREQ (HERTZ)= 0.81942D+02
OEIGENVECTOR:
O DOF             1          2          3          4          5          6          7          8          9         10
 VALUE   -0.363D+00  0.100D+01 -0.227D-01 -0.358D+00 -0.345D+00 -0.950D-01 -0.432D+00 -0.819D+00  0.540D-01 -0.216D+00
O DOF            11         12         13         14         15         16         17         18         19         20
 VALUE    0.686D+00 -0.806D-01 -0.864D-01  0.441D+00  0.167D+00  0.539D+00 -0.613D+00  0.624D-01  0.514D+00 -0.525D-01
O DOF            21         22         23         24         25         26         27         28         29         30
 VALUE    0.606D-01  0.247D+00  0.317D+00  0.772D-01  0.188D+00 -0.671D-01  0.713D-01 -0.247D-01 -0.132D+00  0.329D-01
O DOF            31         32         33         34         35         36         37         38         39         40
 VALUE   -0.124D+00  0.568D-01  0.134D+00 -0.258D+00  0.288D-01 -0.112D+00 -0.307D+00 -0.329D-01  0.263D+00  0.102D-02
O DOF            41         42         43         44
 VALUE    0.122D-01 -0.269D+00 -0.195D-01  0.247D-01
OMODE NO.  19        :  LAMBDA= 0.26571D+06         FREQ (RAD/SEC)= 0.51547D+03         FREQ (HERTZ)= 0.82039D+02
OEIGENVECTOR:
O DOF             1          2          3          4          5          6          7          8          9         10
 VALUE   -0.126D+00  0.495D+00  0.815D-01  0.157D+00 -0.250D+00  0.796D-01  0.156D+00 -0.303D+00  0.101D+00  0.146D+00
O DOF            11         12         13         14         15         16         17         18         19         20
 VALUE    0.372D+00  0.165D+00  0.148D+00  0.495D-01  0.160D+00  0.171D+00 -0.358D+00  0.245D+00  0.109D+00  0.337D+00
O DOF            21         22         23         24         25         26         27         28         29         30
 VALUE    0.132D+00 -0.416D+00  0.503D-01  0.103D+00 -0.468D+00 -0.645D+00  0.267D+00 -0.329D-01  0.382D+00  0.120D+00
O DOF            31         32         33         34         35         36         37         38         39         40
 VALUE    0.648D-01  0.723D+00  0.277D+00  0.261D+00 -0.809D+00  0.572D-01  0.199D+00 -0.356D+00  0.166D+00  0.724D-01
O DOF            41         42         43         44
 VALUE    0.100D+01 -0.728D-01  0.113D+00 -0.349D+00
OMODE NO.  18        :  LAMBDA= 0.27727D+06         FREQ (RAD/SEC)= 0.52657D+03         FREQ (HERTZ)= 0.83806D+02
OEIGENVECTOR:
O DOF             1          2          3          4          5          6          7          8          9         10
 VALUE    0.162D-02  0.192D-01 -0.238D-01 -0.600D+00 -0.597D-01 -0.343D-01 -0.719D+00 -0.226D-01  0.155D+00 -0.651D+00
O DOF            11         12         13         14         15         16         17         18         19         20
 VALUE    0.130D+00 -0.102D+00 -0.574D+00 -0.187D+00  0.382D+00  0.462D+00  0.101D+00  0.674D-01  0.587D+00  0.300D+00
O DOF            21         22         23         24         25         26         27         28         29         30
 VALUE    0.365D+00  0.100D+01 -0.320D+00  0.231D+00  0.866D+00  0.310D-01  0.212D+00  0.960D-01  0.757D-01  0.756D-01
O DOF            31         32         33         34         35         36         37         38         39         40
 VALUE   -0.154D+00 -0.113D+00  0.305D+00 -0.550D+00  0.343D-01 -0.294D+00 -0.705D+00  0.145D-01  0.593D+00 -0.322D-01
O DOF            41         42         43         44
 VALUE    0.888D-02 -0.710D+00 -0.859D-01  0.296D-01
OMODE NO.  17        :  LAMBDA= 0.28425D+06         FREQ (RAD/SEC)= 0.53315D+03         FREQ (HERTZ)= 0.84854D+02
OEIGENVECTOR:
O DOF             1          2          3          4          5          6          7          8          9         10
 VALUE   -0.161D-01  0.303D+00 -0.388D-01  0.520D+00 -0.242D+00 -0.688D-01  0.627D+00 -0.126D-01 -0.309D+00  0.385D+00
O DOF            11         12         13         14         15         16         17         18         19         20
 VALUE    0.164D+00 -0.138D+00  0.361D+00 -0.213D+00 -0.500D+00 -0.101D+00  0.335D-01 -0.187D+00 -0.138D+00  0.209D+00
O DOF            21         22         23         24         25         26         27         28         29         30
 VALUE   -0.596D+00 -0.682D+00 -0.178D+00 -0.286D+00 -0.635D+00 -0.225D+00 -0.572D+00 -0.552D+00  0.386D+00 -0.186D+00
O DOF            31         32         33         34         35         36         37         38         39         40
 VALUE   -0.281D+00 -0.207D+00 -0.526D+00  0.424D+00 -0.190D-02  0.364D+00  0.696D+00  0.336D+00 -0.802D+00  0.139D-01
O DOF            41         42         43         44
 VALUE   -0.381D+00  0.100D+01  0.114D+00 -0.225D-01
OMODE NO.  16        :  LAMBDA= 0.30967D+06         FREQ (RAD/SEC)= 0.55648D+03         FREQ (HERTZ)= 0.88566D+02
OEIGENVECTOR:
O DOF             1          2          3          4          5          6          7          8          9         10
 VALUE    0.735D-01  0.463D+00  0.190D+00  0.950D+00 -0.524D+00  0.242D+00  0.100D+01  0.254D+00 -0.246D-01  0.258D+00
O DOF            11         12         13         14         15         16         17         18         19         20
 VALUE   -0.725D-02  0.213D+00  0.101D+00 -0.342D+00 -0.581D-01 -0.612D+00  0.320D+00  0.546D-01 -0.643D+00 -0.143D+00
O DOF            21         22         23         24         25         26         27         28         29         30
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 VALUE    0.140D+00  -0.260D+00  -0.151D-01   0.365D-01  -0.157D+00   0.274D+00   0.196D+00   0.288D+00  -0.266D+00   0.648D-01
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.312D+00   0.788D-01   0.657D-01   0.129D+00   0.732D-01  -0.932D-02   0.850D-01  -0.214D+00   0.119D-01   0.183D-01
O  DOF           41          42          43          44
 VALUE    0.171D+00  -0.742D-01   0.740D-03   0.217D-01
OMODE NO.  15        :  LAMBDA= 0.34513D+06          FREQ (RAD/SEC)= 0.58747D+03          FREQ (HERTZ)= 0.93499D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE   -0.206D-02  -0.981D-01  -0.157D+00  -0.239D+00   0.855D-01  -0.199D+00  -0.205D+00  -0.732D-01  -0.233D+00  -0.128D-01
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.121D+00  -0.259D+00   0.797D-01  -0.966D-01  -0.315D+00   0.316D+00   0.147D-01  -0.146D+00   0.366D+00   0.935D-01
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE   -0.451D+00   0.656D-01  -0.656D-01  -0.238D-01   0.328D-01  -0.838D-02  -0.529D+00  -0.621D+00  -0.817D-01   0.413D-01
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.534D+00  -0.396D-01  -0.466D+00  -0.577D+00   0.288D+00   0.287D+00  -0.271D+00  -0.414D+00  -0.427D+00   0.170D+00
O  DOF           41          42          43          44
 VALUE    0.218D+00   0.100D+01   0.485D+00   0.130D+00
OMODE NO.  14        :  LAMBDA= 0.35381D+06          FREQ (RAD/SEC)= 0.59482D+03          FREQ (HERTZ)= 0.94669D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.100D+01  -0.758D+00   0.799D-01  -0.441D-01  -0.671D+00   0.921D-01  -0.124D+00   0.655D+00   0.110D+00  -0.227D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.521D+00   0.262D-01  -0.240D+00  -0.399D+00   0.206D+00  -0.747D-01  -0.734D+00  -0.166D-01  -0.857D-01   0.567D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.168D+00   0.161D-01   0.677D+00  -0.171D+00   0.296D-01  -0.746D+00   0.239D+00   0.559D+00  -0.393D+00  -0.139D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.504D+00   0.596D+00   0.184D+00   0.476D+00   0.468D+00  -0.277D+00   0.223D+00  -0.419D+00   0.276D+00  -0.316D-01
O  DOF           41          42          43          44
 VALUE   -0.911D+00  -0.645D+00  -0.422D+00   0.932D+00
OMODE NO.  13        :  LAMBDA= 0.38716D+06          FREQ (RAD/SEC)= 0.62222D+03          FREQ (HERTZ)= 0.99029D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.100D+01  -0.665D+00   0.589D-01  -0.353D+00  -0.640D+00   0.134D+00  -0.494D+00   0.347D+00   0.188D+00  -0.377D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.469D+00   0.197D+00  -0.247D+00   0.101D+00   0.278D+00   0.571D-01  -0.537D+00   0.289D+00   0.226D-01  -0.286D-01
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.163D+00  -0.784D-01   0.352D-01   0.299D+00   0.486D-02   0.482D+00  -0.342D-01  -0.381D+00  -0.178D-01   0.179D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.392D+00  -0.584D+00  -0.198D+00  -0.469D+00  -0.214D+00   0.191D+00  -0.179D+00   0.524D+00  -0.468D+00  -0.296D-01
O  DOF           41          42          43          44
 VALUE    0.744D+00   0.437D+00   0.370D+00  -0.998D+00
OMODE NO.  12        :  LAMBDA= 0.40363D+06          FREQ (RAD/SEC)= 0.63532D+03          FREQ (HERTZ)= 0.10111D+03
OMODE NO.  11        :  LAMBDA= 0.43592D+06          FREQ (RAD/SEC)= 0.66024D+03          FREQ (HERTZ)= 0.10508D+03
OMODE NO.  10        :  LAMBDA= 0.48970D+06          FREQ (RAD/SEC)= 0.69979D+03          FREQ (HERTZ)= 0.11137D+03
OMODE NO.   9        :  LAMBDA= 0.50101D+06          FREQ (RAD/SEC)= 0.70782D+03          FREQ (HERTZ)= 0.11265D+03
OMODE NO.   8        :  LAMBDA= 0.66673D+06          FREQ (RAD/SEC)= 0.81654D+03          FREQ (HERTZ)= 0.12996D+03
OMODE NO.   7        :  LAMBDA= 0.73128D+06          FREQ (RAD/SEC)= 0.85515D+03          FREQ (HERTZ)= 0.13610D+03
OMODE NO.   6        :  LAMBDA= 0.75529D+06          FREQ (RAD/SEC)= 0.86908D+03          FREQ (HERTZ)= 0.13832D+03
OMODE NO.   5        :  LAMBDA= 0.10388D+07          FREQ (RAD/SEC)= 0.10192D+04          FREQ (HERTZ)= 0.16221D+03
OMODE NO.   4        :  LAMBDA= 0.10955D+07          FREQ (RAD/SEC)= 0.10467D+04          FREQ (HERTZ)= 0.16658D+03
OMODE NO.   3        :  LAMBDA= 0.14973D+07          FREQ (RAD/SEC)= 0.12237D+04          FREQ (HERTZ)= 0.19475D+03
OMODE NO.   2        :  LAMBDA= 0.16826D+07          FREQ (RAD/SEC)= 0.12971D+04          FREQ (HERTZ)= 0.20645D+03
OMODE NO.   1        :  LAMBDA= 0.18455D+07          FREQ (RAD/SEC)= 0.13585D+04          FREQ (HERTZ)= 0.21621D+03
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; Two-Dimensional Model of SADE with Nodes at Bar Midpoints (Section 6.2)

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OMODE NO.  24       :  LAMBDA= 0.69454D+03          FREQ (RAD/SEC)= 0.26354D+02          FREQ (HERTZ)= 0.41944D+01
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.216D-03  0.621D-03  0.438D-21 -0.131D-03  0.306D-03  0.142D-01  0.147D-01  0.799D-03  0.896D-02  0.153D-01
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE   0.603D-03 -0.119D-01  0.166D-01  0.866D-03  0.284D-01  0.411D-01  0.107D-02  0.318D-02  0.433D-01  0.855D-03
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE  -0.238D-01  0.456D-01  0.120D-02  0.403D-01  0.820D-01  0.179D-02  0.951D-02  0.852D-01  0.147D-02 -0.335D-01
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE   0.868D-01  0.174D-02  0.523D-01  0.134D+00  0.191D-02  0.506D-02  0.136D+00  0.163D-02 -0.431D-01  0.139D+00
O  DOF          41         42         43         44         45         46         47         48         49         50
 VALUE   0.198D-02  0.620D-01  0.196D+00  0.248D-02  0.109D-01  0.199D+00  0.217D-02 -0.506D-01  0.200D+00  0.244D-02
O  DOF          51         52         53         54         55         56         57         58         59         60
 VALUE   0.716D-01  0.266D+00  0.253D-02  0.732D-02  0.268D+00  0.226D-02 -0.580D-01  0.270D+00  0.260D-02  0.791D-01
O  DOF          61         62         63         64         65         66         67         68         69         70
 VALUE   0.343D+00  0.301D-02  0.122D-01  0.346D+00  0.271D-02 -0.634D-01  0.347D+00  0.297D-02  0.866D-01  0.427D+00
O  DOF          71         72         73         74         75         76         77         78         79         80
 VALUE   0.298D-02  0.946D-02  0.429D+00  0.273D-02 -0.687D-01  0.431D+00  0.306D-02  0.920D-01  0.516D+00  0.338D-02
O  DOF          81         82         83         84         85         86         87         88         89         90
 VALUE   0.136D-01  0.519D+00  0.310D-02 -0.721D-01  0.519D+00  0.333D-02  0.973D-01  0.608D+00  0.328D-02  0.115D-01
O  DOF          91         92         93         94         95         96         97         98         99        100
 VALUE   0.610D+00  0.305D-02 -0.754D-01  0.611D+00  0.335D-02  0.101D+00  0.704D+00  0.358D-02  0.149D-01  0.707D+00
O  DOF         101        102        103        104        105        106        107        108        109        110
 VALUE   0.333D-02 -0.770D-01  0.707D+00  0.353D-02  0.104D+00  0.801D+00  0.345D-02  0.133D-01  0.802D+00  0.324D-02
O  DOF         111        112        113        114        115        116        117        118        119        120
 VALUE  -0.786D-01  0.804D+00  0.354D-02  0.106D+00  0.901D+00  0.368D-02  0.169D-01  0.904D+00  0.346D-02 -0.787D-01
O  DOF         121        122        123        124        125        126        127        128        129        130
 VALUE   0.904D+00  0.361D-02  0.107D+00  0.999D+00  0.338D-02  0.145D-01  0.999D+00  0.337D-02 -0.787D-01  0.100D+01
O  DOF         131
 VALUE   0.342D-02
OMODE NO.  23       :  LAMBDA= 0.25836D+05          FREQ (RAD/SEC)= 0.16074D+03          FREQ (HERTZ)= 0.25582D+02
OEIGENVECTOR:
O  DOF           1          2          3          4          5          6          7          8          9         10
 VALUE   0.198D-02  0.733D-02 -0.514D-19 -0.126D-02  0.302D-02  0.494D-01  0.120D+00  0.564D-02  0.531D-01  0.135D+00
O  DOF          11         12         13         14         15         16         17         18         19         20
 VALUE   0.319D-02 -0.484D-02  0.142D+00  0.640D-02  0.987D-01  0.263D+00  0.426D-02  0.593D-01  0.290D+00  0.362D-03
O  DOF          21         22         23         24         25         26         27         28         29         30
 VALUE  -0.967D-02  0.317D+00  0.614D-02  0.118D+00  0.458D+00  0.763D-02  0.126D+00  0.525D+00  0.376D-02  0.110D-01
O  DOF          31         32         33         34         35         36         37         38         39         40
 VALUE   0.502D+00  0.611D-02  0.138D+00  0.608D+00  0.316D-02  0.108D+00  0.630D+00  0.621D-03  0.318D-01  0.651D+00
O  DOF          41         42         43         44         45         46         47         48         49         50
 VALUE   0.597D-02  0.131D+00  0.771D+00  0.554D-02  0.186D+00  0.853D+00  0.242D-02  0.692D-01  0.798D+00  0.335D-02
O  DOF          51         52         53         54         55         56         57         58         59         60
 VALUE   0.124D+00  0.838D+00 -0.925D-04  0.147D+00  0.851D+00 -0.444D-03  0.107D+00  0.862D+00  0.380D-02  0.998D-01
O  DOF          61         62         63         64         65         66         67         68         69         70
 VALUE   0.913D+00  0.198D-02  0.222D+00  0.100D+01 -0.245D-03  0.151D+00  0.927D+00 -0.543D-03  0.753D-01  0.874D+00
O  DOF          71         72         73         74         75         76         77         78         79         80
 VALUE  -0.392D-02  0.168D+00  0.876D+00 -0.232D-02  0.195D+00  0.878D+00  0.394D-04  0.426D-01  0.826D+00 -0.262D-02
O  DOF          81         82         83         84         85         86         87         88         89         90
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 VALUE     0.225D+00   0.899D+00  -0.354D-02   0.236D+00   0.825D+00  -0.497D-02   0.989D-02   0.674D+00  -0.746D-02   0.171D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.666D+00  -0.436D-02   0.276D+00   0.658D+00  -0.422D-02  -0.198D-01   0.501D+00  -0.696D-02   0.197D+00   0.545D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.651D-02   0.302D+00   0.490D+00  -0.848D-02  -0.495D-01   0.264D+00  -0.968D-02   0.155D+00   0.250D+00  -0.589D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.328D+00   0.236D+00  -0.811D-02  -0.667D-01  -0.508D-02  -0.976D-02   0.134D+00   0.451D-03  -0.841D-02   0.338D+00
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.206D-01  -0.997D-02  -0.839D-01  -0.260D+00  -0.868D-02   0.141D+00  -0.268D+00  -0.763D-02   0.347D+00  -0.275D+00
O  DOF           131
 VALUE    -0.804D-02
OMODE NO.  22        :  LAMBDA= 0.47388D+05           FREQ (RAD/SEC)= 0.21769D+03           FREQ (HERTZ)= 0.34646D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.241D-03   0.122D-02   0.211D-18  -0.164D-03   0.408D-03   0.375D-01   0.987D-02   0.156D-03   0.605D-01   0.193D-01
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.129D-02   0.890D-01  -0.117D-01  -0.941D-03   0.750D-01  -0.725D-02  -0.165D-02   0.142D+00  -0.217D-01  -0.226D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.178D+00  -0.361D-01  -0.628D-03   0.126D+00  -0.526D-01  -0.172D-02   0.218D+00  -0.364D-01  -0.919D-04   0.251D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.930D-01  -0.253D-02   0.177D+00  -0.116D+00  -0.345D-02   0.286D+00  -0.129D+00  -0.296D-02   0.324D+00  -0.141D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE    -0.836D-03   0.240D+00  -0.182D+00  -0.126D-02   0.356D+00  -0.140D+00  -0.392D-03   0.382D+00  -0.211D+00  -0.260D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.303D+00  -0.224D+00  -0.336D-02   0.422D+00  -0.233D+00  -0.300D-02   0.440D+00  -0.241D+00   0.200D-03   0.373D+00
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE    -0.271D+00   0.134D-03   0.497D+00  -0.204D+00   0.210D-03   0.488D+00  -0.294D+00  -0.174D-02   0.443D+00  -0.274D+00
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.266D-02   0.558D+00  -0.278D+00  -0.248D-02   0.536D+00  -0.281D+00   0.236D-02   0.515D+00  -0.274D+00   0.234D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.647D+00  -0.171D+00   0.131D-02   0.578D+00  -0.288D+00  -0.329D-03   0.586D+00  -0.233D+00  -0.203D-02   0.691D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.230D+00  -0.163D-02   0.620D+00  -0.227D+00   0.480D-02   0.650D+00  -0.183D+00   0.444D-02   0.797D+00  -0.355D-01
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE     0.223D-02   0.660D+00  -0.194D+00   0.466D-03   0.713D+00  -0.104D+00  -0.145D-02   0.822D+00  -0.975D-01  -0.131D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.699D+00  -0.913D-01   0.821D-02   0.764D+00   0.202D-01   0.678D-02   0.100D+01   0.228D+00   0.359D-02   0.739D+00
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.141D-02   0.749D-03   0.814D+00   0.624D-01  -0.729D-02   0.947D+00   0.662D-01   0.120D-02   0.779D+00   0.699D-01
O  DOF           131
 VALUE     0.644D-02
OMODE NO.  21        :  LAMBDA= 0.12172D+06           FREQ (RAD/SEC)= 0.34888D+03           FREQ (HERTZ)= 0.55527D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.354D-02   0.122D-01  -0.434D-12  -0.220D-02   0.508D-02   0.274D-01   0.172D+00   0.593D-02   0.190D+00   0.283D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.321D-02   0.110D-01   0.793D-01   0.259D-02   0.548D-01   0.241D+00  -0.309D-03   0.180D+00   0.266D+00  -0.326D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.220D-01   0.290D+00   0.167D-01   0.470D-01   0.489D+00   0.819D-02   0.685D+00   0.100D+01   0.129D-02   0.474D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.443D+00  -0.429D-02   0.391D-01   0.399D+00  -0.144D-01   0.278D+00   0.401D+00  -0.410D-03   0.727D-01   0.401D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.123D-01   0.123D-01   0.394D+00   0.352D-02   0.635D+00   0.930D+00  -0.265D-02   0.826D-01   0.463D+00  -0.840D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    -0.144D-01   0.246D+00  -0.162D-01   0.168D+00   0.227D+00   0.134D-02   0.923D-01   0.207D+00  -0.803D-03  -0.299D-01
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O  DOF          61           62           63           64           65           66           67           68           69           70
 VALUE   0.268D-01   -0.311D-02    0.159D+00    0.202D+00   -0.432D-02    0.742D-01    0.137D+00   -0.601D-02   -0.454D-01   -0.733D-01
O  DOF          71           72           73           74           75           76           77           78           79           80
 VALUE  -0.671D-02   -0.313D-01   -0.962D-01    0.179D-02    0.560D-01   -0.119D+00   -0.114D-01   -0.344D-01   -0.333D+00   -0.665D-02
O  DOF          81           82           83           84           85           86           87           88           89           90
 VALUE  -0.358D+00   -0.587D+00   -0.289D-02    0.176D-01   -0.249D+00    0.467D-03   -0.233D-01   -0.307D+00    0.706D-02   -0.184D+00
O  DOF          91           92           93           94           95           96           97           98           99          100
 VALUE  -0.316D+00    0.140D-02   -0.208D-01   -0.323D+00   -0.128D-01    0.876D-02   -0.387D+00   -0.384D-02   -0.535D+00   -0.830D+00
O  DOF         101          102          103          104          105          106          107          108          109          110
 VALUE   0.160D-02   -0.547D-01   -0.444D+00    0.543D-02    0.408D-01   -0.245D+00    0.147D-01   -0.126D+00   -0.235D+00   -0.754D-03
O  DOF         111          112          113          114          115          116          117          118          119          120
 VALUE  -0.885D-01   -0.225D+00    0.240D-02    0.700D-01   -0.313D-01    0.348D-02   -0.697D-01   -0.161D+00    0.524D-02   -0.104D+00
O  DOF         121          122          123          124          125          126          127          128          129          130
 VALUE  -0.118D+00    0.617D-02    0.990D-01    0.528D-01    0.436D-02   -0.146D-03    0.619D-01    0.351D-02   -0.120D+00    0.708D-01
O  DOF         131
 VALUE   0.590D-02
OMODE NO.  20       :  LAMBDA= 0.14877D+06          FREQ (RAD/SEC)= 0.38571D+03          FREQ (HERTZ)= 0.61387D+02
OEIGENVECTOR:
O  DOF           1            2            3            4            5            6            7            8            9           10
 VALUE  -0.615D-02    0.345D+00   -0.662D-12   -0.763D-02    0.358D-01    0.116D-02    0.122D+00    0.701D-02    0.998D+00    0.100D+01
O  DOF          11           12           13           14           15           16           17           18           19           20
 VALUE  -0.415D-02   -0.117D-03    0.471D+00   -0.365D-02    0.232D-02    0.560D-02   -0.205D-01   -0.453D-02    0.471D-02    0.108D-01
O  DOF          21           22           23           24           25           26           27           28           29           30
 VALUE  -0.234D-03    0.379D-02   -0.212D-01    0.143D-02   -0.339D+00   -0.274D-03   -0.750D+00   -0.746D+00   -0.312D-04    0.170D-03
O  DOF          31           32           33           34           35           36           37           38           39           40
 VALUE  -0.249D+00    0.291D-02    0.533D-03    0.293D-02    0.213D-01   -0.909D-01    0.444D-02   -0.808D-02    0.572D-03    0.593D-02
O  DOF          41           42           43           44           45           46           47           48           49           50
 VALUE   0.101D-01    0.108D-02    0.302D+00   -0.151D-02    0.442D+00    0.450D+00    0.130D-02    0.170D-02    0.112D+00   -0.192D-02
O  DOF          51           52           53           54           55           56           57           58           59           60
 VALUE   0.162D-02    0.122D-01   -0.144D-01    0.102D+00    0.112D-01    0.423D-02    0.282D-02    0.101D-01   -0.217D-02    0.414D-03
O  DOF          61           62           63           64           65           66           67           68           69           70
 VALUE  -0.155D+00    0.191D-02   -0.128D+00   -0.124D+00   -0.114D-02    0.232D-02   -0.127D-01    0.268D-03   -0.795D-03   -0.101D-02
O  DOF          71           72           73           74           75           76           77           78           79           80
 VALUE   0.553D-02   -0.451D-01   -0.159D-02   -0.144D-02    0.181D-02   -0.215D-02   -0.221D-03   -0.444D-03    0.535D-01   -0.114D-02
O  DOF          81           82           83           84           85           86           87           88           89           90
 VALUE   0.247D-01    0.194D-01    0.424D-03    0.896D-03   -0.669D-02   -0.838D-04   -0.922D-04   -0.674D-02   -0.172D-02    0.132D-01
O  DOF          91           92           93           94           95           96           97           98           99          100
 VALUE  -0.717D-02    0.453D-03   -0.248D-04   -0.758D-02   -0.552D-04    0.691D-03   -0.282D-01    0.237D-03   -0.163D-01   -0.242D-01
O  DOF         101          102          103          104          105          106          107          108          109          110
 VALUE  -0.163D-03   -0.109D-02   -0.710D-02    0.112D-03    0.147D-02   -0.677D-02    0.792D-03   -0.863D-02   -0.659D-02   -0.361D-04
O  DOF         111          112          113          114          115          116          117          118          119          120
 VALUE  -0.216D-02   -0.640D-02   -0.249D-03    0.236D-02    0.386D-03   -0.554D-04   -0.103D-01   -0.135D-01    0.185D-03   -0.276D-02
O  DOF         121          122          123          124          125          126          127          128          129          130
 VALUE  -0.634D-02    0.239D-03    0.323D-02    0.114D-02    0.294D-03   -0.118D-02    0.136D-02    0.603D-04   -0.334D-02    0.157D-02
O  DOF         131
 VALUE   0.196D-03
OMODE NO.  19       :  LAMBDA= 0.14933D+06          FREQ (RAD/SEC)= 0.38643D+03          FREQ (HERTZ)= 0.61503D+02
OEIGENVECTOR:
O  DOF           1            2            3            4            5            6            7            8            9           10
 VALUE   0.612D-03   -0.620D-01   -0.147D-12    0.163D-02   -0.694D-02   -0.216D-02   -0.409D-01   -0.169D-02   -0.217D+00   -0.220D+00
O  DOF          11           12           13           14           15           16           17           18           19           20
 VALUE   0.376D-03   -0.245D-02   -0.746D-01    0.113D-02   -0.431D-02   -0.105D-01    0.481D-02   -0.344D-01   -0.108D-01   -0.156D-02
O  DOF          21           22           23           24           25           26           27           28           29           30
 VALUE  -0.489D-02   -0.112D-01    0.131D-02   -0.480D-02    0.499D-01   -0.521D-03    0.534D-01    0.476D-01    0.415D-03   -0.700D-02
O  DOF          31           32           33           34           35           36           37           38           39           40
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 VALUE    -0.756D-02  -0.469D-03  -0.529D-02  -0.107D-01  -0.281D-02   0.195D-01  -0.103D-01   0.659D-03  -0.908D-02  -0.988D-02
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.656D-03  -0.582D-02  -0.623D-01  -0.351D-04  -0.750D-01  -0.743D-01  -0.108D-02  -0.103D-01   0.747D-01   0.243D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    -0.634D-02  -0.410D-02   0.367D-02  -0.117D+00  -0.306D-02   0.158D-02  -0.114D-01  -0.201D-02  -0.926D-02  -0.672D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.719D-02  -0.131D-02  -0.238D+00  -0.222D+00   0.192D-02  -0.951D-02  -0.220D+00  -0.187D-02  -0.709D-02   0.149D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE     0.354D-02   0.116D+00   0.166D-01  -0.572D-02  -0.756D-02   0.182D-01   0.188D-01  -0.959D-02   0.155D+00   0.239D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.564D+00   0.588D+00  -0.169D-02  -0.722D-02   0.379D+00   0.137D-02  -0.121D-01   0.915D-02  -0.135D-01  -0.111D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.746D-02   0.986D-02  -0.686D-02   0.575D-02  -0.255D-01  -0.136D-01  -0.318D+00  -0.348D-02  -0.927D+00  -0.914D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.337D-03  -0.466D-02  -0.370D+00   0.127D-02  -0.150D-01  -0.132D-02   0.258D-01  -0.580D-01   0.119D-03  -0.122D-01
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.243D-02   0.156D-02   0.201D-01  -0.151D-01   0.512D+00   0.271D-02   0.995D+00   0.100D+01   0.443D-02  -0.111D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.104D+00  -0.723D-02  -0.151D-01  -0.960D-02  -0.373D-01   0.352D+00  -0.107D-01   0.744D-02   0.210D-03  -0.117D-01
O  DOF           131
 VALUE     0.663D-02
OMODE NO.   18        :  LAMBDA= 0.15381D+06           FREQ (RAD/SEC)= 0.39218D+03           FREQ (HERTZ)= 0.62417D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.548D-03   0.116D+00  -0.270D-12  -0.332D-02   0.135D-01  -0.214D-03   0.961D-01   0.328D-02   0.474D+00   0.474D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.382D-03  -0.772D-03   0.951D-01  -0.398D-02  -0.427D-03   0.800D-03  -0.121D-01   0.115D+00   0.129D-03   0.265D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE    -0.154D-02  -0.543D-03   0.192D-02  -0.194D-02  -0.157D+00   0.122D-02  -0.860D-01  -0.898D-01  -0.242D-02  -0.420D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.184D+00   0.464D-02  -0.345D-02  -0.121D-01   0.596D-02  -0.231D+00  -0.122D-01   0.403D-02  -0.684D-02  -0.123D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE    -0.213D-01  -0.219D-02  -0.691D-01  -0.455D-02  -0.662D+00  -0.660D+00   0.245D-02  -0.496D-02  -0.389D+00  -0.451D-03
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE    -0.926D-03   0.889D-02   0.140D-01   0.961D-01   0.117D-01  -0.102D-01  -0.308D-02   0.145D-01   0.260D-01  -0.182D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.352D+00   0.346D-02   0.981D+00   0.100D+01   0.220D-03  -0.224D-02   0.325D+00  -0.383D-02  -0.272D-02   0.175D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.265D-01   0.124D+00   0.153D-01   0.968D-02  -0.141D-02   0.130D-01  -0.112D-01  -0.569D-02  -0.372D+00   0.117D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE    -0.570D+00  -0.563D+00  -0.265D-02  -0.282D-02  -0.258D-01   0.429D-02  -0.866D-02  -0.890D-02   0.191D-01  -0.230D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.890D-02  -0.302D-02  -0.423D-02  -0.887D-02  -0.784D-02  -0.725D-02   0.147D+00  -0.482D-02  -0.168D+00  -0.168D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE     0.216D-02  -0.296D-02  -0.168D+00  -0.772D-03  -0.583D-02  -0.189D-02   0.415D-03   0.892D-01  -0.110D-02  -0.326D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.168D-02  -0.302D-03   0.118D-01  -0.578D-02   0.116D+00   0.346D-02   0.470D+00   0.471D+00   0.523D-03  -0.131D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.879D-01  -0.336D-02  -0.571D-02  -0.405D-02  -0.141D-01   0.119D+00  -0.433D-02   0.328D-02  -0.944D-03  -0.459D-02
O  DOF           131
 VALUE     0.796D-03
OMODE NO.   17        :  LAMBDA= 0.15937D+06           FREQ (RAD/SEC)= 0.39921D+03           FREQ (HERTZ)= 0.63536D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.763D-03   0.184D+00   0.320D-12  -0.607D-02   0.228D-01   0.488D-02   0.215D+00   0.658D-02   0.915D+00   0.928D+00
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O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.634D-03   0.588D-02   0.579D-01  -0.962D-02   0.974D-02   0.357D-01  -0.224D-01   0.356D+00   0.365D-01   0.917D-03
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.117D-01   0.371D-01   0.186D-01   0.689D-02  -0.633D-01   0.796D-02   0.630D+00   0.646D+00  -0.259D-02   0.115D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.355D+00  -0.364D-03   0.403D-02   0.915D-02  -0.114D-01  -0.716D-01   0.539D-02   0.817D-02   0.112D-01   0.160D-02
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE    -0.211D-01   0.391D-02  -0.310D+00  -0.434D-02  -0.899D+00  -0.917D+00  -0.155D-02   0.942D-02  -0.120D+00   0.688D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.378D-02  -0.210D-01   0.250D-01  -0.267D+00  -0.206D-01  -0.447D-02   0.766D-02  -0.201D-01  -0.801D-02   0.854D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.222D+00  -0.494D-02  -0.798D-01  -0.994D-01   0.359D-02   0.966D-02  -0.291D+00  -0.387D-02   0.133D-01   0.194D-02
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.325D-02   0.257D+00   0.330D-02  -0.575D-02   0.116D-01   0.466D-02   0.258D-01   0.140D-01   0.173D+00   0.694D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.100D+01   0.990D+00  -0.586D-03   0.999D-02   0.240D+00  -0.614D-02   0.146D-01   0.255D-02  -0.237D-01   0.187D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.297D-03   0.699D-02   0.832D-02  -0.315D-02  -0.278D-02   0.134D-01  -0.294D+00   0.277D-02  -0.249D+00  -0.272D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.233D-02   0.234D-02   0.116D+00   0.498D-02   0.122D-01  -0.204D-01   0.104D-01  -0.236D+00  -0.213D-01   0.258D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.365D-02  -0.222D-01  -0.187D-01   0.156D-01  -0.940D-01  -0.717D-02  -0.765D+00  -0.779D+00   0.249D-03  -0.579D-02
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.163D+00   0.573D-02   0.189D-01   0.985D-02   0.208D-01  -0.165D+00   0.112D-01  -0.465D-02  -0.790D-02   0.125D-01
O  DOF           131
 VALUE     0.201D-03
OMODE NO.  16       :  LAMBDA= 0.16283D+06          FREQ (RAD/SEC)= 0.40352D+03          FREQ (HERTZ)= 0.64223D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.130D-02  -0.117D+00   0.259D-13   0.428D-02  -0.153D-01  -0.255D-02  -0.169D+00  -0.496D-02  -0.658D+00  -0.679D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.836D-03  -0.166D-02   0.216D-02   0.748D-02  -0.510D-02  -0.461D-01   0.155D-01  -0.298D+00  -0.493D-01   0.977D-03
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE    -0.331D-02  -0.524D-01  -0.195D-01   0.205D-02  -0.650D-01  -0.810D-02  -0.791D+00  -0.837D+00   0.110D-02  -0.122D-02
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.235D+00   0.507D-02   0.919D-02  -0.330D-01   0.174D-01  -0.116D+00  -0.294D-01  -0.495D-02   0.863D-03  -0.256D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.276D-02   0.165D-01   0.209D+00  -0.682D-03   0.244D+00   0.234D+00   0.267D-02   0.854D-02  -0.150D+00  -0.453D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.238D-01   0.289D-01  -0.792D-02   0.270D+00   0.316D-01  -0.326D-02   0.162D-01   0.341D-01   0.214D-01   0.235D-01
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.166D+00   0.821D-02   0.983D+00   0.100D+01   0.572D-03   0.206D-01   0.905D-01  -0.958D-02   0.231D-01   0.396D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.228D-01   0.350D+00   0.355D-01   0.182D-02   0.249D-01   0.313D-01   0.155D-01   0.193D-01  -0.912D-01   0.682D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.577D+00   0.562D+00  -0.240D-02   0.184D-01   0.246D+00  -0.224D-02   0.154D-01  -0.224D-01  -0.105D-01  -0.139D-01
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.292D-01   0.647D-02   0.119D-01  -0.359D-01  -0.142D-01   0.179D-01  -0.291D+00  -0.299D-02  -0.682D+00  -0.745D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.138D-02   0.112D-02  -0.104D-01   0.820D-02   0.204D-01  -0.555D-01   0.185D-01  -0.313D+00  -0.558D-01   0.120D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.962D-02  -0.559D-01  -0.198D-01   0.296D-01  -0.572D-01  -0.849D-02  -0.847D+00  -0.882D+00   0.120D-02  -0.134D-01
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.199D+00   0.691D-02   0.386D-01   0.153D-01   0.222D-01  -0.161D+00   0.181D-01  -0.453D-02  -0.172D-01   0.209D-01
O  DOF           131
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 VALUE    0.146D-02
OMODE NO.  15      :  LAMBDA= 0.18589D+06           FREQ (RAD/SEC)= 0.43115D+03           FREQ (HERTZ)= 0.68620D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.342D-03   0.123D-01   0.468D-12  -0.545D-03   0.176D-02   0.498D-04   0.229D-01   0.395D-03   0.125D+00   0.119D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.239D-03   0.459D-02  -0.497D-01  -0.256D-02   0.994D-04  -0.125D-01  -0.325D-02   0.881D-01  -0.149D-01  -0.103D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.917D-02  -0.171D-01   0.624D-02   0.176D-02   0.165D-01   0.245D-02   0.377D+00   0.345D+00   0.329D-03   0.102D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.837D-01  -0.526D-02   0.342D-02  -0.350D-01  -0.869D-02   0.186D+00  -0.369D-01  -0.101D-02   0.111D-01  -0.386D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.117D-01   0.554D-02  -0.119D-01   0.535D-02   0.633D+00   0.580D+00   0.298D-03   0.829D-02  -0.997D-01  -0.769D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.765D-02  -0.494D-01  -0.136D-01   0.268D+00  -0.508D-01  -0.698D-03   0.543D-02  -0.520D-01   0.165D-01   0.841D-02
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE    -0.328D-01   0.791D-02   0.855D+00   0.794D+00   0.342D-03  -0.179D-02  -0.105D+00  -0.970D-02   0.914D-02  -0.559D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.177D-01   0.328D+00  -0.573D-01  -0.151D-03  -0.900D-02  -0.584D-01   0.203D-01   0.775D-02  -0.476D-01   0.996D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.100D+01   0.947D+00   0.353D-03  -0.207D-01  -0.806D-01  -0.106D-01   0.634D-02  -0.507D-01  -0.202D-01   0.340D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.523D-01   0.840D-03  -0.323D-01  -0.537D-01   0.212D-01   0.251D-02  -0.613D-01   0.107D-01   0.977D+00   0.950D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE     0.296D-03  -0.480D-01  -0.140D-01  -0.920D-02  -0.134D-02  -0.337D-01  -0.194D-01   0.262D+00  -0.366D-01   0.267D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE    -0.635D-01  -0.395D-01   0.159D-01  -0.638D-02  -0.956D-01   0.866D-02   0.599D+00   0.611D+00   0.716D-04  -0.838D-01
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.111D+00  -0.316D-02  -0.114D-01  -0.108D-01  -0.116D-01   0.158D-01  -0.136D-01   0.592D-02  -0.104D+00  -0.163D-01
O  DOF           131
 VALUE    -0.113D-02
OMODE NO.  14      :  LAMBDA= 0.19710D+06           FREQ (RAD/SEC)= 0.44396D+03           FREQ (HERTZ)= 0.70659D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.144D-03   0.401D-01  -0.861D-12  -0.126D-02   0.474D-02  -0.731D-02   0.336D-01  -0.117D-04   0.388D+00   0.344D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.541D-05   0.426D-02  -0.198D+00  -0.950D-02  -0.146D-01  -0.103D+00  -0.115D-01   0.251D+00  -0.116D+00  -0.228D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.850D-02  -0.129D+00   0.177D-01  -0.743D-02  -0.877D-01   0.662D-02   0.100D+01   0.836D+00   0.544D-03  -0.207D-03
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.259D+00  -0.125D-01  -0.238D-03  -0.190D+00  -0.236D-01   0.403D+00  -0.193D+00   0.364D-03  -0.891D-02  -0.194D+00
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.232D-01   0.124D-01  -0.265D+00   0.123D-01   0.999D+00   0.825D+00  -0.616D-05  -0.169D-01  -0.115D+00  -0.716D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.250D-01  -0.136D+00  -0.180D-01   0.268D+00  -0.128D+00   0.280D-02  -0.249D-01  -0.120D+00   0.127D-01   0.291D-01
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE    -0.224D+00   0.921D-02   0.377D+00   0.311D+00  -0.156D-04  -0.234D-01   0.817D-01   0.173D-02   0.330D-01  -0.192D-02
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.272D-02  -0.355D-02   0.907D-02   0.330D-02  -0.219D-01   0.200D-01  -0.395D-02   0.242D-01  -0.544D-01   0.884D-03
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE    -0.382D+00  -0.312D+00  -0.188D-03  -0.873D-02   0.195D+00   0.801D-02   0.152D-01   0.113D+00   0.120D-01  -0.226D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.119D+00   0.120D-02   0.444D-02   0.125D+00  -0.154D-01  -0.262D-02   0.132D+00  -0.670D-02  -0.701D+00  -0.576D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.930D-04   0.215D-01   0.973D-01   0.586D-02  -0.204D-01   0.109D+00   0.134D-01  -0.186D+00   0.108D+00  -0.275D-02
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O  DOF          111          112          113          114          115          116          117          118          119          120
 VALUE   0.384D-01    0.106D+00   -0.927D-02   -0.368D-01    0.153D+00   -0.766D-02   -0.324D+00   -0.267D+00   -0.167D-02    0.520D-01
O  DOF          121          122          123          124          125          126          127          128          129          130
 VALUE   0.222D-02   -0.340D-03   -0.529D-01   -0.634D-02    0.349D-02   -0.438D-01   -0.867D-02   -0.369D-02    0.654D-01   -0.110D-01
O  DOF          131
 VALUE  -0.228D-02
OMODE NO.  13        :  LAMBDA= 0.27035D+06           FREQ (RAD/SEC)= 0.51995D+03           FREQ (HERTZ)= 0.82753D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE  -0.605D-01    0.100D+01   -0.320D-12    0.472D-02    0.425D-01   -0.411D-03   -0.998D+00    0.469D-02    0.310D-01    0.315D-01
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE  -0.259D-01   -0.122D-03    0.775D+00   -0.128D-02   -0.820D-03    0.138D-03    0.426D-01   -0.775D+00    0.129D-02   -0.141D-02
O  DOF           21           22           23           24           25           26           27           28           29           30
 VALUE  -0.244D-03    0.244D-02   -0.374D-01    0.902D-03    0.780D+00   -0.131D-02    0.234D-01    0.276D-01    0.227D-01    0.266D-02
O  DOF           31           32           33           34           35           36           37           38           39           40
 VALUE  -0.684D+00    0.109D-02    0.262D-02    0.828D-02   -0.367D-01    0.691D+00    0.768D-02    0.719D-03    0.554D-02    0.705D-02
O  DOF           41           42           43           44           45           46           47           48           49           50
 VALUE   0.337D-01    0.138D-02   -0.674D+00    0.543D-03   -0.303D-01   -0.306D-01   -0.207D-01    0.532D-02    0.652D+00   -0.432D-03
O  DOF           51           52           53           54           55           56           57           58           59           60
 VALUE   0.126D-03   -0.201D-02    0.336D-01   -0.639D+00   -0.228D-02   -0.262D-03    0.506D-02   -0.255D-02   -0.331D-01    0.160D-02
O  DOF           61           62           63           64           65           66           67           68           69           70
 VALUE   0.641D+00   -0.157D-03    0.579D-03   -0.734D-02    0.201D-01    0.653D-02   -0.660D+00   -0.384D-03    0.307D-02   -0.509D-02
O  DOF           71           72           73           74           75           76           77           78           79           80
 VALUE  -0.333D-01    0.667D+00   -0.483D-02   -0.419D-03    0.797D-02   -0.455D-02    0.343D-01    0.226D-02   -0.665D+00   -0.330D-03
O  DOF           81           82           83           84           85           86           87           88           89           90
 VALUE   0.227D-01    0.109D-01   -0.210D-01    0.704D-02    0.683D+00    0.552D-03    0.144D-02   -0.962D-02    0.340D-01   -0.683D+00
O  DOF           91           92           93           94           95           96           97           98           99          100
 VALUE  -0.887D-02    0.603D-03    0.607D-02   -0.808D-02   -0.368D-01    0.245D-02    0.678D+00    0.966D-03   -0.333D-01   -0.424D-01
O  DOF          101          102          103          104          105          106          107          108          109          110
 VALUE   0.225D-01    0.881D-02   -0.765D+00   -0.924D-03    0.345D-02   -0.505D-03   -0.367D-01    0.771D+00    0.193D-02   -0.155D-02
O  DOF          111          112          113          114          115          116          117          118          119          120
 VALUE   0.115D-01    0.436D-02    0.417D-01    0.567D-04   -0.764D+00   -0.143D-02   -0.431D-01   -0.465D-01   -0.258D-01    0.129D-01
O  DOF          121          122          123          124          125          126          127          128          129          130
 VALUE   0.988D+00    0.472D-02   -0.334D-02   -0.304D-02    0.419D-01   -0.974D+00    0.169D-02    0.411D-02    0.141D-01    0.642D-02
O  DOF          131
 VALUE  -0.595D-01
OMODE NO.  12        :  LAMBDA= 0.28212D+06           FREQ (RAD/SEC)= 0.53115D+03           FREQ (HERTZ)= 0.84535D+02
OMODE NO.  11        :  LAMBDA= 0.31138D+06           FREQ (RAD/SEC)= 0.55801D+03           FREQ (HERTZ)= 0.88810D+02
OMODE NO.  10        :  LAMBDA= 0.33053D+06           FREQ (RAD/SEC)= 0.57492D+03           FREQ (HERTZ)= 0.91501D+02
OMODE NO.   9        :  LAMBDA= 0.39518D+06           FREQ (RAD/SEC)= 0.62864D+03           FREQ (HERTZ)= 0.10005D+03
OMODE NO.   8        :  LAMBDA= 0.42134D+06           FREQ (RAD/SEC)= 0.64910D+03           FREQ (HERTZ)= 0.10331D+03
OMODE NO.   7        :  LAMBDA= 0.48992D+06           FREQ (RAD/SEC)= 0.69994D+03           FREQ (HERTZ)= 0.11140D+03
OMODE NO.   6        :  LAMBDA= 0.58703D+06           FREQ (RAD/SEC)= 0.76618D+03           FREQ (HERTZ)= 0.12194D+03
OMODE NO.   5        :  LAMBDA= 0.60606D+06           FREQ (RAD/SEC)= 0.77850D+03           FREQ (HERTZ)= 0.12390D+03
OMODE NO.   4        :  LAMBDA= 0.64173D+06           FREQ (RAD/SEC)= 0.80108D+03           FREQ (HERTZ)= 0.12750D+03
OMODE NO.   3        :  LAMBDA= 0.66070D+06           FREQ (RAD/SEC)= 0.81284D+03           FREQ (HERTZ)= 0.12937D+03
OMODE NO.   2        :  LAMBDA= 0.96436D+06           FREQ (RAD/SEC)= 0.98202D+03           FREQ (HERTZ)= 0.15629D+03
OMODE NO.   1        :  LAMBDA= 0.15704D+09           FREQ (RAD/SEC)= 0.12532D+05           FREQ (HERTZ)= 0.19945D+04


 EIGENVECTORS INCLUDING RESTRAINED DOF'S:
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 General Two-Dimensional Truss with Pinned Joints (Section 7.1)

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OMODE NO.  24      :  LAMBDA= 0.59197D+04        FREQ (RAD/SEC)= 0.76939D+02        FREQ (HERTZ)= 0.12245D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.312D-01    0.484D-01   -0.250D-01    0.544D-01    0.563D-01    0.153D+00   -0.440D-01    0.158D+00    0.754D-01    0.294D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.573D-01    0.299D+00    0.888D-01    0.459D+00   -0.655D-01    0.464D+00    0.971D-01    0.638D+00   -0.697D-01    0.642D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.101D+00    0.821D+00   -0.709D-01    0.823D+00    0.102D+00    0.100D+01   -0.709D-01    0.100D+01
OMODE NO.  23      :  LAMBDA= 0.12065D+06        FREQ (RAD/SEC)= 0.34735D+03        FREQ (HERTZ)= 0.55283D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE   -0.106D+00   -0.335D+00   -0.335D-02   -0.410D+00   -0.129D+00   -0.722D+00   -0.722D-01   -0.775D+00   -0.831D-01   -0.890D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.176D+00   -0.909D+00    0.798D-03   -0.737D+00   -0.279D+00   -0.724D+00    0.872D-01   -0.283D+00   -0.351D+00   -0.250D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.147D+00    0.351D+00   -0.380D+00    0.381D+00    0.169D+00    0.992D+00   -0.383D+00    0.100D+01
OMODE NO.  22      :  LAMBDA= 0.30613D+06        FREQ (RAD/SEC)= 0.55329D+03        FREQ (HERTZ)= 0.88059D+02
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.138D+00    0.391D-01    0.265D+00    0.315D-02    0.303D+00   -0.594D-01    0.482D+00   -0.921D-01    0.489D+00   -0.164D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.647D+00   -0.184D+00    0.675D+00   -0.186D+00    0.768D+00   -0.187D+00    0.838D+00   -0.970D-01    0.857D+00   -0.847D-01
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.951D+00    0.541D-01    0.917D+00    0.679D-01    0.100D+01    0.179D+00    0.939D+00    0.183D+00
OMODE NO.  21      :  LAMBDA= 0.59806D+06        FREQ (RAD/SEC)= 0.77334D+03        FREQ (HERTZ)= 0.12308D+03
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.127D+00    0.702D+00    0.603D-01    0.845D+00    0.401D-01    0.100D+01    0.176D+00    0.970D+00   -0.113D+00    0.357D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE    0.164D+00    0.204D+00   -0.146D+00   -0.577D+00   -0.230D-01   -0.697D+00   -0.471D-02   -0.836D+00   -0.262D+00   -0.816D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE    0.195D+00   -0.112D+00   -0.396D+00   -0.867D-02    0.297D+00    0.947D+00   -0.409D+00    0.984D+00
OMODE NO.  20      :  LAMBDA= 0.14999D+07        FREQ (RAD/SEC)= 0.12247D+04        FREQ (HERTZ)= 0.19492D+03
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE    0.819D-01    0.892D+00    0.152D+00    0.100D+01   -0.136D+00    0.445D+00    0.183D+00    0.243D+00   -0.242D+00   -0.835D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.400D-01   -0.910D+00   -0.768D-01   -0.627D+00   -0.138D+00   -0.373D+00   -0.325D-02    0.647D+00    0.101D+00    0.845D+00
O  DOF           21           22           23           24           25           26           27           28
 VALUE   -0.234D+00    0.583D+00    0.351D+00    0.481D+00   -0.452D+00   -0.806D+00    0.378D+00   -0.880D+00
OMODE NO.  19      :  LAMBDA= 0.25554D+07        FREQ (RAD/SEC)= 0.15986D+04        FREQ (HERTZ)= 0.25442D+03
OEIGENVECTOR:
O  DOF            1            2            3            4            5            6            7            8            9           10
 VALUE   -0.365D+00   -0.465D+00   -0.488D+00   -0.360D+00   -0.556D+00    0.154D+00   -0.573D+00    0.325D+00   -0.717D+00    0.349D+00
O  DOF           11           12           13           14           15           16           17           18           19           20
 VALUE   -0.356D+00    0.291D+00   -0.654D+00   -0.382D+00   -0.944D-01   -0.409D+00   -0.220D+00   -0.235D+00    0.332D+00   -0.406D-01
O  DOF           21           22           23           24           25           26           27           28
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 VALUE     0.180D+00   0.360D+00   0.834D+00   0.453D+00   0.288D+00  -0.185D+00   0.100D+01  -0.202D+00
OMODE NO.  18       :  LAMBDA= 0.29979D+07          FREQ (RAD/SEC)= 0.17315D+04          FREQ (HERTZ)= 0.27557D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.165D+00   0.952D+00  -0.133D+00   0.100D+01  -0.642D+00  -0.321D+00  -0.450D+00  -0.543D+00  -0.562D+00  -0.687D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.545D+00  -0.443D+00  -0.205D+00   0.903D+00  -0.502D-01   0.962D+00  -0.775D-01   0.139D+00   0.205D+00  -0.235D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE     0.347D+00  -0.832D+00   0.113D+00  -0.802D+00   0.868D+00   0.718D+00   0.178D+00   0.844D+00
OMODE NO.  17       :  LAMBDA= 0.44708D+07          FREQ (RAD/SEC)= 0.21144D+04          FREQ (HERTZ)= 0.33652D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.923D-01  -0.756D+00  -0.375D+00  -0.625D+00   0.110D+00   0.895D+00  -0.188D+00   0.867D+00  -0.594D-01  -0.214D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.298D+00  -0.603D+00   0.241D+00  -0.307D+00  -0.330D+00  -0.222D+00   0.523D+00   0.100D+01  -0.991D-01   0.658D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE     0.407D+00  -0.576D+00  -0.452D+00  -0.972D+00   0.665D+00   0.280D+00  -0.566D+00   0.312D+00
OMODE NO.  16       :  LAMBDA= 0.56705D+07          FREQ (RAD/SEC)= 0.23813D+04          FREQ (HERTZ)= 0.37899D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.276D+00   0.772D+00  -0.281D+00   0.463D+00   0.207D+00  -0.574D+00  -0.899D+00  -0.859D+00   0.576D+00   0.981D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.809D+00   0.578D+00   0.546D+00  -0.814D-01  -0.884D+00  -0.828D+00   0.917D+00   0.220D+00  -0.688D+00  -0.138D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE     0.100D+01   0.707D+00  -0.424D+00  -0.772D-04   0.724D+00  -0.605D+00  -0.566D+00  -0.911D+00
OMODE NO.  15       :  LAMBDA= 0.64585D+07          FREQ (RAD/SEC)= 0.25414D+04          FREQ (HERTZ)= 0.40447D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.345D+00  -0.344D+00  -0.206D+00  -0.379D+00   0.660D+00   0.858D+00  -0.218D+00   0.459D+00   0.343D+00  -0.711D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.703D+00  -0.936D+00   0.386D+00   0.100D+01  -0.400D+00   0.739D+00  -0.586D-01  -0.726D+00  -0.381D+00  -0.973D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE     0.207D+00   0.575D+00   0.248D+00   0.647D+00   0.811D-02  -0.374D+00   0.393D+00  -0.520D+00
OMODE NO.  14       :  LAMBDA= 0.69843D+07          FREQ (RAD/SEC)= 0.26428D+04          FREQ (HERTZ)= 0.42061D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.678D+00   0.196D+00   0.306D+00  -0.110D+00   0.890D+00  -0.102D+00  -0.410D-02  -0.235D+00   0.465D+00   0.294D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.418D+00   0.141D+00  -0.355D+00  -0.351D+00  -0.639D+00  -0.320D+00  -0.581D+00   0.365D+00   0.425D-02   0.386D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE    -0.423D+00  -0.555D+00   0.574D+00  -0.382D+00   0.229D+00   0.376D+00   0.100D+01   0.582D+00
OMODE NO.  13       :  LAMBDA= 0.97732D+07          FREQ (RAD/SEC)= 0.31262D+04          FREQ (HERTZ)= 0.49755D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.130D+00  -0.223D+00   0.844D+00   0.199D-01  -0.168D+00  -0.266D+00   0.945D+00   0.307D+00  -0.470D+00  -0.299D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.260D+00   0.258D+00  -0.640D+00   0.405D-01  -0.370D+00   0.196D+00  -0.257D+00   0.655D-01  -0.523D+00  -0.177D+00
O  DOF            21          22          23          24          25          26          27          28
 VALUE     0.631D+00   0.531D+00  -0.256D+00  -0.145D+00   0.100D+01  -0.199D+00  -0.322D+00  -0.473D+00
OMODE NO.  12       :  LAMBDA= 0.12919D+08          FREQ (RAD/SEC)= 0.35944D+04          FREQ (HERTZ)= 0.57206D+03
OMODE NO.  11       :  LAMBDA= 0.16992D+08          FREQ (RAD/SEC)= 0.41221D+04          FREQ (HERTZ)= 0.65606D+03
OMODE NO.  10       :  LAMBDA= 0.20370D+08          FREQ (RAD/SEC)= 0.45133D+04          FREQ (HERTZ)= 0.71831D+03
OMODE NO.   9       :  LAMBDA= 0.22286D+08          FREQ (RAD/SEC)= 0.47208D+04          FREQ (HERTZ)= 0.75133D+03
OMODE NO.   8       :  LAMBDA= 0.22620D+08          FREQ (RAD/SEC)= 0.47560D+04          FREQ (HERTZ)= 0.75695D+03
OMODE NO.   7       :  LAMBDA= 0.24148D+08          FREQ (RAD/SEC)= 0.49141D+04          FREQ (HERTZ)= 0.78210D+03
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   General Two-Dimensional Truss with Rigid Joints (Section 7.1)

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OMODE NO.  24       :  LAMBDA= 0.59211D+04           FREQ (RAD/SEC)= 0.76948D+02          FREQ (HERTZ)= 0.12247D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.269D-03   0.387D-03   0.313D-01   0.484D-01   0.121D-02  -0.250D-01   0.544D-01   0.140D-02   0.564D-01   0.153D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.204D-02  -0.440D-01   0.158D+00   0.223D-02   0.755D-01   0.294D+00   0.256D-02  -0.574D-01   0.299D+00   0.286D-02
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.889D-01   0.460D+00   0.286D-02  -0.656D-01   0.464D+00   0.328D-02   0.972D-01   0.639D+00   0.297D-02  -0.697D-01
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.642D+00   0.349D-02   0.101D+00   0.821D+00   0.298D-02  -0.708D-01   0.823D+00   0.378D-02   0.102D+00   0.100D+01
O  DOF           41          42          43          44
 VALUE    0.226D-02  -0.708D-01   0.100D+01   0.274D-02
OMODE NO.  23       :  LAMBDA= 0.11163D+06           FREQ (RAD/SEC)= 0.33411D+03          FREQ (HERTZ)= 0.53175D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE   -0.370D-02  -0.518D-02  -0.109D+00  -0.336D+00  -0.381D-02  -0.534D-02  -0.410D+00  -0.138D-01  -0.134D+00  -0.726D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.357D-02  -0.763D-01  -0.778D+00  -0.151D-01  -0.900D-01  -0.897D+00   0.118D-01  -0.182D+00  -0.917D+00  -0.111D-01
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE   -0.584D-02  -0.746D+00   0.174D-01  -0.286D+00  -0.734D+00  -0.189D-02   0.821D-01  -0.291D+00   0.181D-01  -0.357D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.258D+00   0.614D-02   0.144D+00   0.350D+00   0.153D-01  -0.385D+00   0.381D+00   0.238D-01   0.166D+00   0.992D+00
O  DOF           41          42          43          44
 VALUE   -0.877D-02  -0.386D+00   0.100D+01   0.171D-02
OMODE NO.  22       :  LAMBDA= 0.23180D+06           FREQ (RAD/SEC)= 0.48145D+03          FREQ (HERTZ)= 0.76625D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.658D-02  -0.279D-02   0.135D+00   0.644D-01  -0.677D-02   0.271D+00   0.349D-01   0.192D-01   0.288D+00  -0.368D-01
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.291D-01   0.498D+00  -0.705D-01   0.192D-01   0.455D+00  -0.207D+00  -0.229D-01   0.668D+00  -0.235D+00   0.134D-01
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.633D+00  -0.325D+00  -0.305D-01   0.790D+00  -0.336D+00   0.481D-01   0.799D+00  -0.341D+00  -0.419D-01   0.870D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE   -0.335D+00   0.203D-02   0.932D+00  -0.212D+00   0.302D-02   0.934D+00  -0.191D+00   0.975D-01   0.100D+01  -0.745D-01
O  DOF           41          42          43          44
 VALUE   -0.163D+00   0.964D+00  -0.683D-01   0.677D-01
OMODE NO.  21       :  LAMBDA= 0.26561D+06           FREQ (RAD/SEC)= 0.51538D+03          FREQ (HERTZ)= 0.82025D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE   -0.359D+00   0.100D+01  -0.365D-02  -0.236D+00  -0.345D+00  -0.466D-03  -0.312D+00  -0.807D+00   0.519D-01  -0.303D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.650D+00   0.560D-01  -0.233D+00   0.455D+00   0.183D+00   0.815D-01  -0.602D+00   0.158D+00   0.561D-01  -0.134D+00
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.177D+00  -0.106D-01   0.389D+00   0.206D+00   0.240D-01  -0.295D-02   0.231D+00   0.157D+00  -0.225D+00   0.303D+00
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.144D+00   0.111D+00   0.210D+00  -0.237D-01   0.225D-01   0.335D+00  -0.376D-01  -0.529D-01   0.204D+00  -0.211D+00
O  DOF           41          42          43          44
 VALUE   -0.134D-01   0.342D+00  -0.219D+00   0.420D-01
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OMODE NO.  20      :  LAMBDA= 0.27035D+06        FREQ (RAD/SEC)= 0.51995D+03        FREQ (HERTZ)= 0.82753D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.108D-01   0.836D-01   0.723D-01  -0.237D+00  -0.118D+00   0.265D+00  -0.340D+00  -0.629D-02   0.236D+00  -0.652D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.123D+00   0.419D+00  -0.708D+00  -0.183D+00   0.513D+00  -0.573D+00  -0.204D-01   0.590D+00  -0.548D+00   0.356D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE      0.733D+00  -0.406D+00  -0.179D+00   0.694D+00  -0.397D+00  -0.357D+00   0.914D+00   0.359D-01   0.299D+00   0.871D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE      0.121D+00   0.446D+00   0.989D+00   0.352D+00  -0.587D+00   0.100D+01   0.316D+00  -0.136D-01   0.906D+00   0.158D-01
O  DOF             41          42          43          44
 VALUE      0.366D+00   0.987D+00   0.179D-01  -0.168D+00
OMODE NO.  19      :  LAMBDA= 0.28603D+06        FREQ (RAD/SEC)= 0.53481D+03        FREQ (HERTZ)= 0.85118D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE     -0.265D-01   0.462D+00   0.244D-02   0.412D+00  -0.389D+00  -0.196D+00   0.510D+00  -0.229D-01  -0.202D+00   0.570D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.315D+00  -0.370D+00   0.634D+00  -0.394D+00  -0.356D+00   0.968D+00   0.119D+00  -0.346D+00   0.100D+01   0.504D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.613D+00   0.566D+00  -0.551D+00  -0.434D+00   0.435D+00  -0.106D+00  -0.856D+00  -0.553D+00   0.444D+00  -0.704D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.622D+00  -0.554D+00  -0.767D+00  -0.551D+00   0.291D+00  -0.788D+00  -0.491D+00   0.271D+00  -0.650D+00  -0.105D+00
O  DOF             41          42          43          44
 VALUE     -0.377D+00  -0.771D+00  -0.852D-01   0.510D-01
OMODE NO.  18      :  LAMBDA= 0.30527D+06        FREQ (RAD/SEC)= 0.55251D+03        FREQ (HERTZ)= 0.87935D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE     -0.532D-02  -0.148D+00   0.615D-01   0.417D+00   0.183D+00  -0.484D-01   0.538D+00  -0.175D-01   0.248D-01   0.968D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE     -0.104D+00   0.408D-01   0.100D+01   0.297D+00  -0.174D+00   0.632D+00  -0.306D+00   0.154D-01   0.521D+00   0.320D-01
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.364D+00  -0.374D+00   0.114D+00  -0.212D+00  -0.482D+00  -0.336D+00  -0.319D+00  -0.728D+00   0.303D+00  -0.412D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.711D+00  -0.836D-01  -0.147D+00  -0.169D+00  -0.354D-01  -0.463D+00  -0.937D-01   0.181D+00  -0.758D-01   0.460D+00
O  DOF             41          42          43          44
 VALUE     -0.109D+00  -0.463D+00   0.486D+00  -0.494D-01
OMODE NO.  17      :  LAMBDA= 0.33490D+06        FREQ (RAD/SEC)= 0.57870D+03        FREQ (HERTZ)= 0.92104D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.169D+00   0.255D+00   0.200D+00   0.888D+00  -0.454D+00   0.147D+00   0.100D+01   0.381D+00   0.430D-01   0.864D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE     -0.160D+00   0.178D+00   0.809D+00  -0.448D-01  -0.150D+00   0.794D-01   0.296D-01   0.515D-01  -0.279D-01  -0.193D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     -0.172D+00  -0.470D+00   0.234D+00  -0.111D+00  -0.496D+00  -0.122D+00  -0.555D-01  -0.343D+00  -0.797D-02  -0.197D+00
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.306D+00   0.949D-01   0.302D-01   0.690D-01  -0.341D-01  -0.235D+00   0.111D+00   0.717D-02   0.527D-01   0.468D+00
O  DOF             41          42          43          44
 VALUE      0.998D-02  -0.241D+00   0.479D+00  -0.168D-01
OMODE NO.  16      :  LAMBDA= 0.35404D+06        FREQ (RAD/SEC)= 0.59501D+03        FREQ (HERTZ)= 0.94700D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.872D+00  -0.878D+00  -0.925D-01  -0.399D+00  -0.390D+00  -0.995D-01  -0.469D+00   0.402D+00  -0.843D-01  -0.421D+00
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.682D+00  -0.211D+00  -0.386D+00  -0.492D+00   0.136D-01   0.439D-01  -0.685D+00  -0.172D+00   0.771D-01   0.716D+00
O  DOF             21          22          23          24          25          26          27          28          29          30
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 VALUE    -0.694D-01   0.150D+00   0.491D+00  -0.245D+00   0.136D+00  -0.653D+00  -0.743D-01   0.255D+00  -0.486D+00  -0.239D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.254D+00   0.552D+00  -0.112D+00   0.320D+00   0.728D+00  -0.293D+00   0.285D+00  -0.816D+00   0.251D-03   0.607D+00
O  DOF            41          42          43          44
 VALUE    -0.640D+00  -0.209D+00   0.514D+00   0.100D+01
OMODE NO.  15       :  LAMBDA= 0.38833D+06            FREQ (RAD/SEC)= 0.62316D+03          FREQ (HERTZ)= 0.99179D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.100D+01  -0.776D+00  -0.469D-01  -0.384D+00  -0.527D+00  -0.284D-02  -0.503D+00   0.219D+00   0.201D-02  -0.450D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.643D+00  -0.141D-01  -0.352D+00  -0.578D-01   0.108D+00  -0.755D-01  -0.460D+00   0.609D-01  -0.108D+00  -0.631D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.102D+00   0.547D-01   0.116D+00   0.147D+00   0.154D+00   0.460D+00   0.225D-01   0.225D-01  -0.531D-01   0.165D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.727D-01  -0.492D+00  -0.562D-01  -0.423D+00  -0.344D+00   0.161D+00  -0.377D+00   0.682D+00  -0.160D+00  -0.679D+00
O  DOF            41          42          43          44
 VALUE     0.534D+00   0.762D-01  -0.594D+00  -0.930D+00
OMODE NO.  14       :  LAMBDA= 0.43828D+06            FREQ (RAD/SEC)= 0.66203D+03          FREQ (HERTZ)= 0.10537D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.305D-02   0.299D-01   0.499D-01  -0.511D-01  -0.365D-01   0.139D+00  -0.999D-01   0.433D-01   0.118D+00  -0.250D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.940D-01   0.202D+00  -0.302D+00   0.109D+00   0.191D+00  -0.439D+00  -0.107D+00   0.168D+00  -0.480D+00   0.108D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.255D+00  -0.488D+00  -0.119D+00   0.463D-01  -0.503D+00   0.771D-01   0.298D+00  -0.265D+00  -0.193D-01  -0.114D+00
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.238D+00  -0.139D-01   0.318D+00   0.276D+00   0.945D-01  -0.232D+00   0.319D+00  -0.122D+00   0.322D+00   0.981D+00
O  DOF            41          42          43          44
 VALUE     0.158D+00  -0.257D+00   0.100D+01  -0.652D-01
OMODE NO.  13       :  LAMBDA= 0.49028D+06            FREQ (RAD/SEC)= 0.70020D+03          FREQ (HERTZ)= 0.11144D+03
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.100D+01  -0.409D+00  -0.471D-01  -0.469D+00  -0.519D+00  -0.460D-01  -0.710D+00  -0.242D+00   0.307D-01  -0.693D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.112D+00  -0.111D+00  -0.524D+00   0.682D+00   0.985D-01  -0.251D-01   0.342D+00  -0.147D+00  -0.237D-01  -0.686D+00
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.130D+00   0.212D+00  -0.600D+00  -0.852D-01   0.222D+00   0.287D+00   0.944D-01   0.715D+00   0.773D+00   0.515D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.891D+00   0.225D-02   0.590D-01   0.873D+00  -0.190D+00   0.242D+00   0.647D+00  -0.572D+00   0.553D-01   0.115D+00
O  DOF            41          42          43          44
 VALUE    -0.327D+00   0.346D+00  -0.205D-02   0.974D+00
OMODE NO.  12       :  LAMBDA= 0.64992D+06            FREQ (RAD/SEC)= 0.80618D+03          FREQ (HERTZ)= 0.12831D+03
OMODE NO.  11       :  LAMBDA= 0.69413D+06            FREQ (RAD/SEC)= 0.83315D+03          FREQ (HERTZ)= 0.13260D+03
OMODE NO.  10       :  LAMBDA= 0.87607D+06            FREQ (RAD/SEC)= 0.93599D+03          FREQ (HERTZ)= 0.14897D+03
OMODE NO.   9       :  LAMBDA= 0.97553D+06            FREQ (RAD/SEC)= 0.98769D+03          FREQ (HERTZ)= 0.15720D+03
OMODE NO.   8       :  LAMBDA= 0.13927D+07            FREQ (RAD/SEC)= 0.11801D+04          FREQ (HERTZ)= 0.18782D+03
OMODE NO.   7       :  LAMBDA= 0.19454D+07            FREQ (RAD/SEC)= 0.13948D+04          FREQ (HERTZ)= 0.22199D+03
OMODE NO.   6       :  LAMBDA= 0.22169D+07            FREQ (RAD/SEC)= 0.14889D+04          FREQ (HERTZ)= 0.23697D+03
OMODE NO.   5       :  LAMBDA= 0.29001D+07            FREQ (RAD/SEC)= 0.17030D+04          FREQ (HERTZ)= 0.27104D+03
OMODE NO.   4       :  LAMBDA= 0.30087D+07            FREQ (RAD/SEC)= 0.17346D+04          FREQ (HERTZ)= 0.27606D+03
OMODE NO.   3       :  LAMBDA= 0.39715D+07            FREQ (RAD/SEC)= 0.19929D+04          FREQ (HERTZ)= 0.31718D+03
OMODE NO.   2       :  LAMBDA= 0.45129D+07            FREQ (RAD/SEC)= 0.21244D+04          FREQ (HERTZ)= 0.33810D+03
OMODE NO.   1       :  LAMBDA= 0.69016D+07            FREQ (RAD/SEC)= 0.26271D+04          FREQ (HERTZ)= 0.41811D+03
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General Two-Dimensional Truss with Nodes at Bar Midpoints (Section 7.2)

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OMODE NO.  24        :  LAMBDA= 0.59536D+04          FREQ (RAD/SEC)= 0.77160D+02           FREQ (HERTZ)= 0.12280D+02
OEIGENVECTOR:
O  DOF             1            2           3           4           5           6           7           8           9          10
 VALUE    0.269D-03    0.818D-03  -0.723D-18  -0.164D-03   0.388D-03   0.156D-01   0.179D-01   0.953D-03   0.105D-01   0.190D-01
O  DOF            11           12          13          14          15          16          17          18          19          20
 VALUE    0.692D-03   -0.125D-01   0.204D-01   0.104D-02   0.313D-01   0.484D-01   0.121D-02   0.447D-02   0.514D-01   0.885D-03
O  DOF            21           22          23          24          25          26          27          28          29          30
 VALUE   -0.250D-01    0.544D-01   0.140D-02   0.438D-01   0.955D-01   0.203D-02   0.130D-01   0.101D+00   0.160D-02  -0.345D-01
O  DOF            31           32          33          34          35          36          37          38          39          40
 VALUE    0.101D+00    0.193D-02   0.564D-01   0.153D+00   0.204D-02   0.758D-02   0.156D+00   0.167D-02  -0.440D-01   0.158D+00
O  DOF            41           42          43          44          45          46          47          48          49          50
 VALUE    0.223D-02    0.660D-01   0.221D+00   0.271D-02   0.172D-01   0.228D+00   0.229D-02  -0.507D-01   0.226D+00   0.257D-02
O  DOF            51           52          53          54          55          56          57          58          59          60
 VALUE    0.755D-01    0.294D+00   0.256D-02   0.113D-01   0.297D+00   0.227D-02  -0.573D-01   0.299D+00   0.286D-02   0.822D-01
O  DOF            61           62          63          64          65          66          67          68          69          70
 VALUE    0.378D+00    0.317D-02   0.220D-01   0.386D+00   0.277D-02  -0.615D-01   0.382D+00   0.297D-02   0.889D-01   0.460D+00
O  DOF            71           72          73          74          75          76          77          78          79          80
 VALUE    0.286D-02    0.147D-01   0.462D+00   0.268D-02  -0.656D-01   0.464D+00   0.329D-02   0.931D-01   0.553D+00   0.343D-02
O  DOF            81           82          83          84          85          86          87          88          89          90
 VALUE    0.270D-01    0.563D+00   0.305D-02  -0.676D-01   0.556D+00   0.316D-02   0.972D-01   0.639D+00   0.296D-02   0.176D-01
O  DOF            91           92          93          94          95          96          97          98          99         100
 VALUE    0.641D+00    0.294D-02  -0.696D-01   0.642D+00   0.350D-02   0.993D-01   0.736D+00   0.350D-02   0.312D-01   0.747D+00
O  DOF           101          102         103         104         105         106         107         108         109         110
 VALUE    0.317D-02   -0.702D-01   0.736D+00   0.312D-02   0.101D+00   0.821D+00   0.297D-02   0.210D-01   0.822D+00   0.301D-02
O  DOF           111          112         113         114         115         116         117         118         119         120
 VALUE   -0.707D-01    0.823D+00   0.379D-02   0.102D+00   0.922D+00   0.356D-02   0.412D-01   0.937D+00   0.327D-02  -0.707D-01
O  DOF           121          122         123         124         125         126         127         128         129         130
 VALUE    0.926D+00    0.319D-02   0.102D+00   0.100D+01   0.225D-02   0.193D-01   0.100D+01   0.348D-02  -0.707D-01   0.100D+01
O  DOF           131
 VALUE    0.273D-02
OMODE NO.  23        :  LAMBDA= 0.97779D+05          FREQ (RAD/SEC)= 0.31270D+03           FREQ (HERTZ)= 0.49767D+02
OEIGENVECTOR:
O  DOF             1            2           3           4           5           6           7           8           9          10
 VALUE    0.227D-02    0.102D-01  -0.849D-14  -0.149D-02   0.359D-02   0.336D-01   0.121D+00   0.469D-02   0.111D+00   0.179D+00
O  DOF            11           12          13          14          15          16          17          18          19          20
 VALUE    0.262D-02    0.315D-02   0.911D-01   0.344D-02   0.671D-01   0.204D+00   0.172D-02   0.107D+00   0.227D+00  -0.133D-02
O  DOF            21           22          23          24          25          26          27          28          29          30
 VALUE    0.630D-02    0.249D+00   0.102D-01   0.757D-01   0.414D+00   0.723D-02   0.373D+00   0.673D+00   0.244D-02   0.288D-01
O  DOF            31           32          33          34          35          36          37          38          39          40
 VALUE    0.396D+00    0.531D-03   0.841D-01   0.439D+00  -0.439D-02   0.203D+00   0.455D+00  -0.103D-02   0.513D-01   0.470D+00
O  DOF            41           42          43          44          45          46          47          48          49          50
 VALUE    0.120D-01    0.714D-01   0.603D+00   0.653D-02   0.542D+00   0.993D+00   0.662D-03   0.838D-01   0.600D+00  -0.327D-02
O  DOF            51           52          53          54          55          56          57          58          59          60
 VALUE    0.586D-01    0.540D+00  -0.102D-01   0.253D+00   0.546D+00  -0.148D-02   0.116D+00   0.551D+00   0.973D-02   0.333D-01
O  DOF            61           62          63          64          65          66          67          68          69          70
 VALUE    0.587D+00    0.342D-02   0.563D+00   0.100D+01  -0.216D-02   0.147D+00   0.615D+00  -0.644D-02   0.807D-02   0.446D+00
O  DOF            71           72          73          74          75          76          77          78          79          80
 VALUE   -0.135D-01    0.235D+00   0.442D+00  -0.210D-02   0.178D+00   0.438D+00   0.311D-02  -0.187D-01   0.344D+00  -0.105D-02
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O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.401D+00   0.639D+00  -0.477D-02   0.199D+00   0.380D+00  -0.817D-02  -0.454D-01   0.168D+00  -0.128D-01   0.177D+00
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE     0.158D+00  -0.346D-02   0.220D+00   0.148D+00  -0.277D-02  -0.645D-01  -0.542D-01  -0.516D-02   0.160D+00   0.553D-01
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE    -0.671D-02   0.228D+00   0.639D-01  -0.559D-02  -0.834D-01  -0.221D+00  -0.963D-02   0.236D-01  -0.231D+00  -0.191D-02
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.235D+00  -0.240D+00  -0.178D-01  -0.903D-01  -0.621D+00  -0.108D-01  -0.396D+00  -0.890D+00  -0.826D-02   0.236D+00
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE    -0.613D+00  -0.550D-02  -0.971D-01  -0.609D+00   0.962D-02  -0.604D-02  -0.612D+00  -0.114D-01   0.236D+00  -0.613D+00
O  DOF           131
 VALUE    -0.149D-02
OMODE NO.   22        :  LAMBDA= 0.14838D+06          FREQ (RAD/SEC)= 0.38520D+03          FREQ (HERTZ)= 0.61306D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE     0.481D-02  -0.234D+00  -0.579D-14   0.484D-02  -0.237D-01   0.579D-02  -0.560D-01  -0.402D-02  -0.628D+00  -0.623D+00
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE     0.342D-02   0.944D-02  -0.333D+00   0.201D-02   0.116D-01   0.231D-01   0.133D-01   0.642D-01   0.255D-01  -0.850D-02
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.189D-01   0.277D-01   0.189D-01   0.159D-01   0.314D+00   0.220D-02   0.758D+00   0.770D+00   0.591D-03   0.283D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE     0.192D+00  -0.472D-02   0.201D-01   0.358D-01  -0.205D-01   0.205D+00   0.347D-01   0.476D-02   0.376D-01   0.335D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.950D-04   0.215D-01  -0.181D+00   0.324D-02  -0.621D-02  -0.165D-01  -0.199D-02   0.438D-01   0.409D-01  -0.322D-03
O  DOF            51          52          53          54          55          56          57          58          59          60
 VALUE     0.227D-01   0.638D-02   0.468D-02  -0.116D-01   0.473D-02  -0.141D-02   0.498D-01   0.308D-02  -0.219D-02   0.260D-01
O  DOF            61          62          63          64          65          66          67          68          69          70
 VALUE     0.718D-01  -0.692D-03   0.122D+00   0.714D-01   0.137D-02   0.540D-01  -0.142D+00  -0.372D-02   0.292D-01  -0.254D-01
O  DOF            71          72          73          74          75          76          77          78          79          80
 VALUE    -0.548D-02   0.207D+00  -0.271D-01  -0.279D-02   0.580D-01  -0.287D-01   0.132D-01   0.328D-01  -0.358D-01   0.230D-02
O  DOF            81          82          83          84          85          86          87          88          89          90
 VALUE     0.418D+00   0.323D+00  -0.238D-02   0.579D-01   0.227D+00   0.105D-02   0.363D-01  -0.661D-01  -0.797D-02  -0.569D-01
O  DOF            91          92          93          94          95          96          97          98          99         100
 VALUE    -0.681D-01   0.705D-02   0.575D-01  -0.700D-01  -0.218D-01   0.421D-01  -0.320D+00  -0.349D-02  -0.708D+00  -0.833D+00
O  DOF           101         102         103         104         105         106         107         108         109         110
 VALUE     0.110D-03   0.588D-01  -0.424D+00   0.606D-03   0.478D-01  -0.746D-01   0.206D-01   0.561D-01  -0.722D-01  -0.106D-01
O  DOF           111         112         113         114         115         116         117         118         119         120
 VALUE     0.600D-01  -0.695D-01   0.195D-01   0.530D-01   0.370D+00   0.399D-02   0.100D+01   0.877D+00   0.429D-02   0.616D-01
O  DOF           121         122         123         124         125         126         127         128         129         130
 VALUE     0.981D-02  -0.690D-02   0.581D-01  -0.580D-01  -0.343D-01   0.428D+00  -0.577D-01   0.643D-02   0.631D-01  -0.572D-01
O  DOF           131
 VALUE     0.874D-02
OMODE NO.   21        :  LAMBDA= 0.14890D+06          FREQ (RAD/SEC)= 0.38587D+03          FREQ (HERTZ)= 0.61413D+02
OEIGENVECTOR:
O  DOF             1           2           3           4           5           6           7           8           9          10
 VALUE    -0.601D-02   0.342D+00  -0.626D-14  -0.761D-02   0.356D-01   0.426D-02   0.124D+00   0.711D-02   0.997D+00   0.100D+01
O  DOF            11          12          13          14          15          16          17          18          19          20
 VALUE    -0.397D-02   0.519D-02   0.466D+00  -0.362D-02   0.850D-02   0.140D-01  -0.201D-01   0.118D-01   0.135D-01   0.103D-01
O  DOF            21          22          23          24          25          26          27          28          29          30
 VALUE     0.104D-01   0.129D-01  -0.201D-01   0.103D-01  -0.300D+00   0.434D-03  -0.647D+00  -0.644D+00   0.493D-03   0.161D-01
O  DOF            31          32          33          34          35          36          37          38          39          40
 VALUE    -0.277D+00   0.121D-02   0.120D-01   0.147D-01   0.185D-01  -0.411D-03   0.161D-01  -0.912D-02   0.217D-01   0.174D-01
O  DOF            41          42          43          44          45          46          47          48          49          50
 VALUE     0.159D-01   0.143D-01   0.324D+00   0.334D-04   0.661D+00   0.657D+00   0.652D-03   0.268D-01   0.202D+00  -0.259D-02
O  DOF            51          52          53          54          55          56          57          58          59          60
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 VALUE    0.166D-01   0.136D-01  -0.187D-01   0.130D+00   0.119D-01   0.609D-02   0.317D-01   0.101D-01  -0.661D-02   0.175D-01
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE   -0.216D+00   0.214D-02  -0.205D+00  -0.231D+00  -0.835D-03   0.347D-01  -0.142D+00  -0.163D-02   0.182D-01  -0.119D-01
O  DOF           71          72          73          74          75          76          77          78          79          80
 VALUE    0.761D-02   0.573D-01  -0.124D-01  -0.518D-02   0.376D-01  -0.130D-01   0.104D-01   0.203D-01   0.123D+00   0.171D-03
O  DOF           81          82          83          84          85          86          87          88          89          90
 VALUE    0.412D+00   0.357D+00  -0.448D-03   0.381D-01   0.167D+00   0.115D-03   0.224D-01  -0.365D-01  -0.110D-01   0.136D-01
O  DOF           91          92          93          94          95          96          97          98          99         100
 VALUE   -0.382D-01   0.591D-02   0.385D-01  -0.398D-01  -0.138D-01   0.254D-01  -0.267D+00  -0.144D-02  -0.486D+00  -0.564D+00
O  DOF          101         102         103         104         105         106         107         108         109         110
 VALUE   -0.682D-03   0.394D-01  -0.259D+00   0.451D-03   0.283D-01  -0.496D-01   0.151D-01   0.975D-02  -0.482D-01  -0.713D-02
O  DOF          111         112         113         114         115         116         117         118         119         120
 VALUE    0.401D-01  -0.467D-01   0.113D-01   0.315D-01   0.248D+00   0.192D-02   0.618D+00   0.535D+00   0.295D-02   0.412D-01
O  DOF          121         122         123         124         125         126         127         128         129         130
 VALUE   -0.136D-01  -0.439D-02   0.345D-01  -0.443D-01  -0.220D-01   0.278D+00  -0.442D-01   0.388D-02   0.422D-01  -0.439D-01
O  DOF          131
 VALUE    0.608D-02
OMODE NO.  20       :  LAMBDA= 0.15374D+06          FREQ (RAD/SEC)= 0.39210D+03          FREQ (HERTZ)= 0.62405D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE   -0.354D-03   0.125D+00   0.854D-14  -0.370D-02   0.148D-01   0.368D-02   0.114D+00   0.385D-02   0.528D+00   0.532D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE   -0.137D-03   0.558D-02   0.988D-01  -0.439D-02   0.735D-02   0.144D-01  -0.130D-01   0.151D+00   0.144D-01   0.225D-02
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.111D-01   0.143D-01   0.389D-02   0.843D-02  -0.120D+00   0.235D-02   0.484D-01   0.478D-01  -0.207D-02   0.147D-01
O  DOF           31          32          33          34          35          36          37          38          39          40
 VALUE    0.173D+00   0.274D-02   0.947D-02   0.513D-02   0.288D-02  -0.137D+00   0.438D-02   0.305D-02   0.183D-01   0.362D-02
O  DOF           41          42          43          44          45          46          47          48          49          50
 VALUE   -0.157D-01   0.122D-01  -0.354D-01  -0.277D-02  -0.399D+00  -0.412D+00   0.215D-02   0.236D-01  -0.339D+00  -0.257D-02
O  DOF           51          52          53          54          55          56          57          58          59          60
 VALUE    0.149D-01   0.551D-02   0.792D-02   0.177D+00   0.637D-02  -0.914D-02   0.288D-01   0.720D-02   0.261D-01   0.165D-01
O  DOF           61          62          63          64          65          66          67          68          69          70
 VALUE    0.273D+00   0.405D-02   0.100D+01   0.974D+00  -0.614D-03   0.306D-01   0.328D+00  -0.372D-02   0.180D-01  -0.116D-01
O  DOF           71          72          73          74          75          76          77          78          79          80
 VALUE   -0.256D-01   0.123D+00  -0.146D-01   0.986D-02   0.322D-01  -0.177D-01  -0.143D-01   0.191D-01  -0.412D+00   0.481D-03
O  DOF           81          82          83          84          85          86          87          88          89          90
 VALUE   -0.617D+00  -0.674D+00  -0.222D-02   0.318D-01  -0.131D+00   0.378D-02   0.201D-01  -0.431D-01   0.202D-01  -0.160D+00
O  DOF           91          92          93          94          95          96          97          98          99         100
 VALUE   -0.428D-01  -0.473D-02   0.312D-01  -0.423D-01  -0.305D-02   0.248D-01   0.157D+00  -0.378D-02   0.476D-01  -0.211D-01
O  DOF          101         102         103         104         105         106         107         108         109         110
 VALUE    0.221D-02   0.327D-01  -0.164D+00  -0.182D-02   0.293D-01  -0.341D-01  -0.451D-02   0.171D+00  -0.331D-01  -0.187D-02
O  DOF          111         112         113         114         115         116         117         118         119         120
 VALUE    0.342D-01  -0.320D-01   0.112D-01   0.313D-01   0.199D-01   0.430D-02   0.459D+00   0.395D+00   0.140D-03   0.349D-01
O  DOF          121         122         123         124         125         126         127         128         129         130
 VALUE    0.440D-01  -0.318D-02   0.333D-01  -0.291D-01  -0.117D-01   0.151D+00  -0.288D-01   0.255D-02   0.355D-01  -0.285D-01
O  DOF          131
 VALUE    0.150D-02
OMODE NO.  19       :  LAMBDA= 0.15803D+06          FREQ (RAD/SEC)= 0.39753D+03          FREQ (HERTZ)= 0.63269D+02
OEIGENVECTOR:
O  DOF            1           2           3           4           5           6           7           8           9          10
 VALUE    0.393D-03  -0.887D-01   0.878D-14   0.255D-02  -0.103D-01   0.396D-02  -0.732D-01  -0.238D-02  -0.378D+00  -0.377D+00
O  DOF           11          12          13          14          15          16          17          18          19          20
 VALUE    0.374D-03   0.714D-02  -0.514D-01   0.381D-02   0.791D-02   0.102D-01   0.997D-02  -0.954D-01   0.113D-01  -0.193D-02
O  DOF           21          22          23          24          25          26          27          28          29          30
 VALUE    0.143D-01   0.123D-01  -0.317D-02   0.123D-01   0.133D+00  -0.127D-02   0.537D-01   0.574D-01   0.207D-02   0.235D-01
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O DOF             31          32          33          34          35          36          37          38          39          40
 VALUE    -0.195D+00  -0.476D-02   0.168D-01   0.262D-01  -0.327D-02   0.251D+00   0.266D-01  -0.560D-02   0.327D-01   0.269D-01
O DOF             41          42          43          44          45          46          47          48          49          50
 VALUE     0.233D-01   0.180D-01   0.196D+00   0.655D-02   0.100D+01   0.988D+00  -0.161D-03   0.372D-01   0.202D+00  -0.708D-02
O DOF             51          52          53          54          55          56          57          58          59          60
 VALUE     0.192D-01   0.185D-02  -0.249D-01   0.249D+00  -0.208D-02   0.560D-02   0.416D-01  -0.599D-02   0.630D-03   0.201D-01
O DOF             61          62          63          64          65          66          67          68          69          70
 VALUE    -0.308D+00   0.268D-02  -0.113D+00  -0.175D+00  -0.357D-02   0.400D-01   0.152D+00   0.409D-02   0.208D-01  -0.556D-01
O DOF             71          72          73          74          75          76          77          78          79          80
 VALUE     0.816D-02  -0.219D+00  -0.583D-01   0.320D-02   0.383D-01  -0.608D-01  -0.225D-01   0.275D-01  -0.146D+00  -0.666D-02
O DOF             81          82          83          84          85          86          87          88          89          90
 VALUE    -0.752D+00  -0.851D+00   0.149D-02   0.387D-01  -0.352D+00   0.307D-02   0.341D-01  -0.627D-01   0.169D-01  -0.352D-02
O DOF             91          92          93          94          95          96          97          98          99         100
 VALUE    -0.606D-01  -0.729D-02   0.389D-01  -0.583D-01   0.109D-01   0.414D-01   0.207D+00   0.407D-03   0.541D+00   0.453D+00
O DOF            101         102         103         104         105         106         107         108         109         110
 VALUE     0.174D-02   0.412D-01  -0.593D-01  -0.493D-02   0.485D-01  -0.315D-01  -0.151D-01   0.279D+00  -0.302D-01   0.106D-02
O DOF            111         112         113         114         115         116         117         118         119         120
 VALUE     0.434D-01  -0.287D-01   0.115D-01   0.505D-01  -0.825D-01   0.655D-02   0.464D+00   0.393D+00  -0.455D-03   0.441D-01
O DOF            121         122         123         124         125         126         127         128         129         130
 VALUE     0.643D-01  -0.276D-02   0.524D-01  -0.171D-01  -0.877D-02   0.138D+00  -0.169D-01   0.233D-02   0.447D-01  -0.165D-01
O DOF            131
 VALUE     0.580D-03
OMODE NO.  18        :  LAMBDA= 0.16234D+06         FREQ (RAD/SEC)= 0.40292D+03         FREQ (HERTZ)= 0.64126D+02
OEIGENVECTOR:
O DOF              1           2           3           4           5           6           7           8           9          10
 VALUE     0.130D-02   0.133D+00   0.242D-14  -0.478D-02   0.172D-01   0.876D-02   0.184D+00   0.525D-02   0.758D+00   0.763D+00
O DOF             11          12          13          14          15          16          17          18          19          20
 VALUE     0.109D-02   0.183D-01  -0.144D-01  -0.908D-02   0.175D-01   0.287D-01  -0.185D-01   0.364D+00   0.279D-01  -0.118D-02
O DOF             21          22          23          24          25          26          27          28          29          30
 VALUE     0.366D-01   0.270D-01   0.210D-01   0.215D-01   0.170D-01   0.863D-02   0.883D+00   0.863D+00  -0.129D-02   0.479D-01
O DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     0.222D+00  -0.540D-02   0.255D-01  -0.650D-02  -0.189D-01   0.170D+00  -0.114D-01   0.514D-02   0.590D-01  -0.163D-01
O DOF             41          42          43          44          45          46          47          48          49          50
 VALUE    -0.478D-02   0.312D-01  -0.280D+00   0.790D-03  -0.225D+00  -0.315D+00  -0.236D-02   0.644D-01   0.265D-01   0.339D-02
O DOF             51          52          53          54          55          56          57          58          59          60
 VALUE     0.369D-01  -0.680D-01   0.905D-02  -0.132D+00  -0.707D-01   0.511D-03   0.695D-01  -0.732D-01  -0.146D-01   0.484D-01
O DOF             61          62          63          64          65          66          67          68          69          70
 VALUE    -0.108D+00  -0.553D-02  -0.464D+00  -0.607D+00   0.786D-03   0.755D-01  -0.259D+00   0.230D-02   0.598D-01  -0.829D-01
O DOF             71          72          73          74          75          76          77          78          79          80
 VALUE     0.108D-01   0.398D-01  -0.814D-01  -0.466D-02   0.812D-01  -0.797D-01   0.497D-02   0.715D-01   0.940D-01   0.763D-03
O DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     0.422D+00   0.270D+00   0.208D-02   0.889D-01  -0.227D+00  -0.628D-02   0.830D-01  -0.594D-01  -0.111D-01   0.390D+00
O DOF             91          92          93          94          95          96          97          98          99         100
 VALUE    -0.573D-01  -0.321D-02   0.964D-01  -0.551D-01   0.221D-01   0.887D-01   0.633D-02   0.830D-02   0.100D+01   0.851D+00
O DOF            101         102         103         104         105         106         107         108         109         110
 VALUE    -0.514D-03   0.101D+00,  0.593D-01  -0.743D-02   0.942D-01  -0.521D-01  -0.203D-01   0.352D+00  -0.525D-01   0.309D-02
O DOF            111         112         113         114         115         116         117         118         119         120
 VALUE     0.106D+00  -0.527D-01   0.699D-02   0.953D-01  -0.198D+00   0.619D-02   0.399D+00   0.237D+00  -0.815D-03   0.107D+00
O DOF            121         122         123         124         125         126         127         128         129         130
 VALUE    -0.386D-01  -0.290D-02   0.961D-01  -0.674D-01  -0.545D-02   0.195D+00  -0.677D-01   0.408D-03   0.108D+00  -0.677D-01
O DOF            131
 VALUE     0.262D-02
OMODE NO.  17        :  LAMBDA= 0.16496D+06         FREQ (RAD/SEC)= 0.40615D+03         FREQ (HERTZ)= 0.64641D+02
OEIGENVECTOR:
O DOF              1           2           3           4           5           6           7           8           9          10
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 VALUE  -0.124D-02  -0.106D+00   0.142D-12   0.390D-02  -0.139D-01   0.101D-03  -0.158D+00  -0.472D-02  -0.598D+00  -0.627D+00
O DOF           11          12          13          14          15          16          17          18          19          20
 VALUE  -0.708D-03   0.154D-01   0.685D-02   0.671D-02   0.201D-03  -0.587D-01   0.131D-01  -0.261D+00  -0.663D-01   0.964D-03
O DOF           21          22          23          24          25          26          27          28          29          30
 VALUE   0.307D-01  -0.737D-01  -0.202D-01   0.128D-01  -0.137D+00  -0.911D-02  -0.825D+00  -0.942D+00   0.239D-03   0.441D-01
O DOF           31          32          33          34          35          36          37          38          39          40
 VALUE  -0.232D+00   0.587D-02   0.254D-01  -0.102D+00   0.175D-01  -0.145D+00  -0.103D+00  -0.389D-02   0.575D-01  -0.104D+00
O DOF           41          42          43          44          45          46          47          48          49          50
 VALUE  -0.590D-02   0.440D-01   0.211D-01  -0.433D-02  -0.119D+00  -0.276D+00   0.176D-02   0.732D-01  -0.255D+00  -0.615D-03
O DOF           51          52          53          54          55          56          57          58          59          60
 VALUE   0.624D-01  -0.883D-01   0.824D-03   0.176D+00  -0.859D-01  -0.368D-02   0.887D-01  -0.833D-01   0.106D-01   0.782D-01
O DOF           61          62          63          64          65          66          67          68          69          70
 VALUE   0.244D-01   0.399D-02   0.613D+00   0.450D+00   0.119D-02   0.105D+00  -0.157D+00  -0.677D-02   0.938D-01  -0.593D-01
O DOF           71          72          73          74          75          76          77          78          79          80
 VALUE  -0.131D-01   0.403D+00  -0.572D-01  -0.226D-02   0.122D+00  -0.550D-01   0.188D-01   0.103D+00  -0.120D-01   0.872D-02
O DOF           81          82          83          84          85          86          87          88          89          90
 VALUE   0.100D+01   0.826D+00   0.282D-03   0.135D+00  -0.709D-01  -0.979D-02   0.112D+00  -0.577D-01  -0.206D-01   0.493D+00
O DOF           91          92          93          94          95          96          97          98          99         100
 VALUE  -0.580D-01  -0.600D-03   0.149D+00  -0.580D-01   0.188D-01   0.115D+00  -0.124D+00   0.863D-02   0.957D+00   0.745D+00
O DOF          101         102         103         104         105         106         107         108         109         110
 VALUE  -0.154D-02   0.156D+00  -0.390D-02  -0.803D-02   0.119D+00  -0.972D-01  -0.173D-01   0.383D+00  -0.993D-01   0.124D-02
O DOF          111         112         113         114         115         116         117         118         119         120
 VALUE   0.163D+00  -0.101D+00   0.761D-02   0.120D+00  -0.200D+00   0.595D-02   0.567D+00   0.297D+00  -0.223D-03   0.165D+00
O DOF          121         122         123         124         125         126         127         128         129         130
 VALUE  -0.163D+00  -0.540D-02   0.120D+00  -0.154D+00  -0.120D-01   0.346D+00  -0.155D+00  -0.969D-04   0.166D+00  -0.155D+00
O DOF          131
 VALUE   0.739D-02
OMODE NO.  16       :  LAMBDA= 0.22024D+06          FREQ (RAD/SEC)= 0.46929D+03          FREQ (HERTZ)= 0.74690D+02
OEIGENVECTOR:
O DOF            1           2           3           4           5           6           7           8           9          10
 VALUE  -0.926D-04   0.982D-02  -0.361D-13  -0.240D-03   0.991D-03  -0.938D-02  -0.618D-02  -0.115D-02   0.202D+00   0.155D+00
O DOF           11          12          13          14          15          16          17          18          19          20
 VALUE  -0.158D-03   0.165D-01  -0.166D+00  -0.661D-02  -0.187D-01  -0.112D+00  -0.787D-02   0.158D+00  -0.128D+00  -0.167D-02
O DOF           21          22          23          24          25          26          27          28          29          30
 VALUE   0.330D-01  -0.143D+00   0.857D-02  -0.125D-01  -0.167D+00   0.233D-02   0.654D+00   0.437D+00   0.271D-04   0.326D-01
O DOF           31          32          33          34          35          36          37          38          39          40
 VALUE  -0.374D+00  -0.109D-01  -0.630D-02  -0.262D+00  -0.177D-01   0.337D+00  -0.275D+00  -0.109D-02   0.320D-01  -0.286D+00
O DOF           41          42          43          44          45          46          47          48          49          50
 VALUE   0.176D-01   0.110D-01  -0.376D+00   0.797D-02   0.981D+00   0.637D+00   0.152D-03   0.184D-01  -0.470D+00  -0.121D-01
O DOF           51          52          53          54          55          56          57          58          59          60
 VALUE   0.283D-01  -0.339D+00  -0.214D-01   0.412D+00  -0.345D+00   0.616D-03   0.474D-02  -0.350D+00   0.217D-01   0.499D-01
O DOF           61          62          63          64          65          66          67          68          69          70
 VALUE  -0.466D+00   0.115D-01   0.100D+01   0.638D+00   0.838D-03  -0.152D-01  -0.395D+00  -0.841D-02   0.712D-01  -0.294D+00
O DOF           71          72          73          74          75          76          77          78          79          80
 VALUE  -0.177D-01   0.338D+00  -0.293D+00   0.320D-02  -0.352D-01  -0.291D+00   0.171D-01   0.898D-01  -0.387D+00   0.111D-01
O DOF           81          82          83          84          85          86          87          88          89          90
 VALUE   0.609D+00   0.363D+00   0.202D-02  -0.536D-01  -0.179D+00  -0.100D-02   0.108D+00  -0.125D+00  -0.641D-02   0.140D+00
O DOF           91          92          93          94          95          96          97          98          99         100
 VALUE  -0.119D+00   0.514D-02  -0.718D-01  -0.113D+00   0.649D-02   0.119D+00  -0.156D+00   0.599D-02  -0.834D-01  -0.105D+00
O DOF          101         102         103         104         105         106         107         108         109         110
 VALUE   0.216D-02  -0.828D-01   0.189D+00   0.879D-02   0.129D+00   0.127D+00   0.113D-01  -0.188D+00   0.134D+00   0.668D-02
O DOF          111         112         113         114         115         116         117         118         119         120
 VALUE  -0.933D-01   0.140D+00  -0.124D-01   0.133D+00   0.235D+00  -0.160D-02  -0.807D+00  -0.557D+00   0.446D-02  -0.950D-01
O DOF          121         122         123         124         125         126         127         128         129         130
 VALUE   0.333D+00   0.129D-01   0.136D+00   0.399D+00   0.248D-01  -0.272D+00   0.402D+00   0.236D-02  -0.962D-01   0.404D+00
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O  DOF          131
 VALUE   -0.831D-02
OMODE NO.   15        :  LAMBDA= 0.26976D+06          FREQ (RAD/SEC)= 0.51938D+03          FREQ (HERTZ)= 0.82662D+02
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.542D-01 -0.895D+00 -0.143D-12 -0.422D-02 -0.382D-01  0.311D-02  0.895D+00 -0.417D-02 -0.370D-01 -0.382D-01
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.233D-01  0.449D-02 -0.690D+00  0.136D-02  0.621D-02  0.382D-02 -0.379D-01  0.700D+00  0.246D-02  0.119D-02
O  DOF           21         22         23         24         25         26         27         28         29         30
 VALUE    0.896D-02  0.110D-02  0.331D-01  0.767D-02 -0.693D+00  0.941D-03 -0.424D-01 -0.527D-01 -0.204D-01  0.103D-01
O  DOF           31         32         33         34         35         36         37         38         39         40
 VALUE    0.622D+00 -0.585D-03  0.909D-02 -0.356D-02  0.335D-01 -0.617D+00 -0.374D-02 -0.629D-03  0.116D-01 -0.391D-02
O  DOF           41         42         43         44         45         46         47         48         49         50
 VALUE   -0.314D-01  0.134D-01  0.610D+00 -0.806D-03 -0.317D-02 -0.194D-01  0.188D-01  0.146D-01 -0.598D+00  0.480D-03
O  DOF           51         52         53         54         55         56         57         58         59         60
 VALUE    0.176D-01  0.877D-03 -0.300D-01  0.604D+00  0.627D-03 -0.231D-04  0.176D-01  0.371D-03  0.300D-01  0.201D-01
O  DOF           61         62         63         64         65         66         67         68         69         70
 VALUE   -0.591D+00 -0.356D-03 -0.304D-01 -0.485D-01 -0.187D-01  0.183D-01  0.626D+00  0.998D-03  0.224D-01  0.368D-02
O  DOF           71         72         73         74         75         76         77         78         79         80
 VALUE    0.316D-01 -0.606D+00  0.353D-02  0.603D-03  0.189D-01  0.337D-02 -0.335D-01  0.269D-01  0.610D+00 -0.401D-04
O  DOF           81         82         83         84         85         86         87         88         89         90
 VALUE   -0.732D-01 -0.882D-01  0.198D-01  0.218D-01 -0.636D+00  0.467D-03  0.313D-01  0.171D-01 -0.300D-01  0.654D+00
O  DOF           91         92         93         94         95         96         97         98         99        100
 VALUE    0.176D-01 -0.694D-03  0.246D-01  0.181D-01  0.334D-01  0.333D-01 -0.615D+00 -0.156D-02 -0.519D-01 -0.584D-01
O  DOF          101        102        103        104        105        106        107        108        109        110
 VALUE   -0.211D-01  0.254D-01  0.769D+00  0.277D-02  0.351D-01  0.286D-01  0.372D-01 -0.723D+00  0.279D-01  0.177D-02
O  DOF          111        112        113        114        115        116        117        118        119        120
 VALUE    0.260D-01  0.270D-01 -0.430D-01  0.391D-01  0.787D+00  0.765D-03 -0.905D-01 -0.888D-01  0.253D-01  0.291D-01
O  DOF          121        122        123        124        125        126        127        128        129        130
 VALUE   -0.930D+00 -0.372D-02  0.429D-01  0.453D-01 -0.382D-01  0.100D+01  0.431D-01 -0.502D-02  0.321D-01  0.406D-01
O  DOF          131
 VALUE    0.586D-01
OMODE NO.   14        :  LAMBDA= 0.28141D+06          FREQ (RAD/SEC)= 0.53048D+03          FREQ (HERTZ)= 0.84428D+02
OEIGENVECTOR:
O  DOF            1          2          3          4          5          6          7          8          9         10
 VALUE    0.610D-01 -0.997D+00 -0.131D-13 -0.548D-02 -0.402D-01 -0.642D-03  0.100D+01 -0.534D-02 -0.984D-02 -0.642D-02
O  DOF           11         12         13         14         15         16         17         18         19         20
 VALUE    0.247D-01 -0.212D-02 -0.684D+00  0.292D-02 -0.128D-02  0.552D-02 -0.401D-01  0.682D+00  0.493D-02  0.285D-02
O  DOF           21         22         23         24         25         26         27         28         29         30
 VALUE   -0.423D-02  0.431D-02  0.296D-01 -0.518D-02 -0.667D+00  0.304D-02 -0.995D-01 -0.886D-01 -0.179D-01 -0.758D-02
O  DOF           31         32         33         34         35         36         37         38         39         40
 VALUE    0.462D+00 -0.335D-02 -0.904D-02  0.416D-02  0.284D-01 -0.454D+00  0.650D-02 -0.311D-02 -0.109D-01  0.880D-02
O  DOF           41         42         43         44         45         46         47         48         49         50
 VALUE   -0.164D-01 -0.120D-01  0.444D+00 -0.326D-02  0.115D+00  0.136D+00  0.975D-02 -0.125D-01 -0.183D+00  0.334D-02
O  DOF           51         52         53         54         55         56         57         58         59         60
 VALUE   -0.149D-01  0.661D-02 -0.157D-01  0.166D+00  0.487D-02  0.331D-02 -0.140D-01  0.311D-02  0.307D-02 -0.187D-01
O  DOF           61         62         63         64         65         66         67         68         69         70
 VALUE   -0.159D+00  0.367D-02 -0.915D-01 -0.744D-01 -0.137D-02 -0.159D-01 -0.926D-01 -0.420D-02 -0.224D-01 -0.563D-02
O  DOF           71         72         73         74         75         76         77         78         79         80
 VALUE    0.449D-03  0.899D-01 -0.395D-02 -0.350D-02 -0.177D-01 -0.224D-02  0.123D-01 -0.261D-01 -0.118D+00 -0.326D-02
O  DOF           81         82         83         84         85         86         87         88         89         90
 VALUE    0.159D+00  0.174D+00 -0.757D-02 -0.203D-01  0.334D+00  0.230D-02 -0.297D-01 -0.156D-01  0.108D-01 -0.373D+00
O  DOF           91         92         93         94         95         96         97         98         99        100
 VALUE   -0.176D-01  0.299D-02 -0.228D-01 -0.195D-01 -0.230D-01 -0.318D-01  0.341D+00  0.391D-02 -0.197D-01 -0.169D-01
O  DOF          101        102        103        104        105        106        107        108        109        110
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 VALUE      0.150D-01  -0.242D-01  -0.640D+00  -0.437D-02  -0.337D-01  -0.316D-01  -0.275D-01   0.602D+00  -0.299D-01  -0.317D-02
O  DOF            111         112         113         114         115         116         117         118         119         120
 VALUE     -0.255D-01  -0.282D-01   0.386D-01  -0.372D-01  -0.664D+00  -0.212D-02   0.104D+00   0.101D+00  -0.229D-01  -0.285D-01
O  DOF            121         122         123         124         125         126         127         128         129         130
 VALUE      0.882D+00   0.432D-02  -0.406D-01  -0.423D-01   0.343D-01  -0.949D+00  -0.401D-01   0.546D-02  -0.314D-01  -0.377D-01
O  DOF            131
 VALUE     -0.560D-01
OMODE NO.   13       :  LAMBDA= 0.32844D+06          FREQ (RAD/SEC)= 0.57310D+03          FREQ (HERTZ)= 0.91212D+02
OEIGENVECTOR:
O  DOF              1           2           3           4           5           6           7           8           9          10
 VALUE      0.600D-01  -0.962D+00   0.316D-12  -0.776D-02  -0.308D-01   0.512D-02   0.931D+00  -0.857D-02   0.728D-01   0.643D-01
O  DOF             11          12          13          14          15          16          17          18          19          20
 VALUE      0.189D-01   0.859D-02  -0.353D+00   0.729D-02   0.102D-01  -0.663D-02  -0.285D-01   0.335D+00  -0.137D-01   0.708D-02
O  DOF             21          22          23          24          25          26          27          28          29          30
 VALUE      0.171D-01  -0.207D-01   0.106D-02   0.169D-01  -0.256D+00   0.883D-02  -0.220D+00  -0.263D+00   0.187D-02   0.227D-01
O  DOF             31          32          33          34          35          36          37          38          39          40
 VALUE     -0.344D+00  -0.837D-02   0.235D-01  -0.251D-01  -0.684D-02   0.438D+00  -0.217D-01  -0.648D-02   0.281D-01  -0.181D-01
O  DOF             41          42          43          44          45          46          47          48          49          50
 VALUE      0.308D-01   0.306D-01  -0.493D+00  -0.748D-02   0.156D+00   0.111D+00  -0.212D-01   0.317D-01   0.804D+00   0.464D-02
O  DOF             51          52          53          54          55          56          57          58          59          60
 VALUE      0.375D-01  -0.562D-02   0.371D-01  -0.841D+00  -0.739D-02   0.275D-02   0.352D-01  -0.910D-02  -0.472D-01   0.464D-01
O  DOF             61          62          63          64          65          66          67          68          69          70
 VALUE      0.869D+00   0.241D-02  -0.417D-01  -0.879D-01   0.299D-01   0.411D-01  -0.890D+00   0.332D-02   0.550D-01   0.740D-02
O  DOF             71          72          73          74          75          76          77          78          79          80
 VALUE     -0.448D-01   0.938D+00   0.660D-02   0.248D-02   0.468D-01   0.576D-02   0.364D-01   0.601D-01  -0.804D+00   0.346D-02
O  DOF             81          82          83          84          85          86          87          88          89          90
 VALUE     -0.238D+00  -0.273D+00  -0.217D-01   0.512D-01   0.543D+00  -0.543D-02   0.648D-01   0.359D-01   0.350D-01  -0.387D+00
O  DOF             91          92          93          94          95          96          97          98          99         100
 VALUE      0.411D-01  -0.596D-02   0.554D-01   0.460D-01  -0.115D-01   0.689D-01   0.446D+00  -0.860D-02   0.842D-01   0.754D-01
O  DOF            101         102         103         104         105         106         107         108         109         110
 VALUE      0.414D-02   0.607D-01   0.244D+00   0.105D-01   0.725D-01   0.642D-01   0.656D-03  -0.170D+00   0.592D-01   0.731D-02
O  DOF            111         112         113         114         115         116         117         118         119         120
 VALUE      0.656D-01   0.539D-01  -0.274D-01   0.758D-01   0.367D+00   0.631D-02  -0.235D+00  -0.251D+00   0.169D-01   0.697D-01
O  DOF            121         122         123         124         125         126         127         128         129         130
 VALUE     -0.802D+00  -0.782D-02   0.787D-01   0.594D-01  -0.249D-01   0.100D+01   0.575D-01  -0.836D-02   0.734D-01   0.553D-01
O  DOF            131
 VALUE      0.569D-01
OMODE NO.   12       :  LAMBDA= 0.39158D+06          FREQ (RAD/SEC)= 0.62576D+03          FREQ (HERTZ)= 0.99593D+02
OMODE NO.   11       :  LAMBDA= 0.41308D+06          FREQ (RAD/SEC)= 0.64271D+03          FREQ (HERTZ)= 0.10229D+03
OMODE NO.   10       :  LAMBDA= 0.46158D+06          FREQ (RAD/SEC)= 0.67940D+03          FREQ (HERTZ)= 0.10813D+03
OMODE NO.    9       :  LAMBDA= 0.52781D+06          FREQ (RAD/SEC)= 0.72651D+03          FREQ (HERTZ)= 0.11563D+03
OMODE NO.    8       :  LAMBDA= 0.59290D+06          FREQ (RAD/SEC)= 0.77000D+03          FREQ (HERTZ)= 0.12255D+03
OMODE NO.    7       :  LAMBDA= 0.63175D+06          FREQ (RAD/SEC)= 0.79483D+03          FREQ (HERTZ)= 0.12650D+03
OMODE NO.    6       :  LAMBDA= 0.68254D+06          FREQ (RAD/SEC)= 0.82616D+03          FREQ (HERTZ)= 0.13149D+03
OMODE NO.    5       :  LAMBDA= 0.69070D+06          FREQ (RAD/SEC)= 0.83109D+03          FREQ (HERTZ)= 0.13227D+03
OMODE NO.    4       :  LAMBDA= 0.98838D+06          FREQ (RAD/SEC)= 0.99417D+03          FREQ (HERTZ)= 0.15823D+03
OMODE NO.    3       :  LAMBDA= 0.10268D+07          FREQ (RAD/SEC)= 0.10133D+04          FREQ (HERTZ)= 0.16128D+03
OMODE NO.    2       :  LAMBDA= 0.12545D+07          FREQ (RAD/SEC)= 0.11200D+04          FREQ (HERTZ)= 0.17826D+03
OMODE NO.    1       :  LAMBDA= 0.15704D+09          FREQ (RAD/SEC)= 0.12532D+05          FREQ (HERTZ)= 0.19945D+04


 EIGENVECTORS INCLUDING RESTRAINED DOF'S:
```

APPENDIX D

A Generalized Finite Element Truss Analysis Program

The Fortran program described in this appendix assembles the restrained stiffness and mass matrices for a finite element model of a space truss. The model can be either pin-jointed (three translational degrees of freedom per node) or rigid-jointed (three translational and three rotational degrees of freedom per node). The bar elements of the truss are modeled using consistent mass matrices, although this feature can be overridden by specifying the inertia properties of the bars to be zero and lumping their masses and moments of inertia at the nodes. The program has the capability of including concentrated masses, concentrated moments of inertia, and stiffnesses to ground at the nodes. The output appears as follows:

```
NRDOF          (number of restrained degrees of freedom)
IR(1)          (first restrained degree of freedom)
IR(2)          (second restrained degree of freedom)
 .
 .
 .
IR(NRDOF)      (NRDOFth restrained degree of freedom)

RESTRAINED STIFFNESS MATRIX IN LOWER TRIANGULAR FORM
RESTRAINED MASS MATRIX IN LOWER TRIANGULAR FORM
```

The input to the program is described on the next page through the example of the SADE truss of section 2.1. The program listing follows. The notation in the input description is that of Chapter 2.

```
32
 1   0.0000000E+00  0.0000000E+00  0.0000000E+00
 2   0.0000000E+00  0.0000000E+00  0.5500000E+02
 3   0.0000000E+00  0.5500000E+02  0.0000000E+00
 4   0.0000000E+00  0.5500000E+02  0.5500000E+02
 5   0.5500000E+02  0.0000000E+00  0.0000000E+00
 6   0.5500000E+02  0.0000000E+00  0.5500000E+02
 7   0.5500000E+02  0.5500000E+02  0.0000000E+00
 8   0.5500000E+02  0.5500000E+02  0.5500000E+02
 9   0.1100000E+03  0.0000000E+00  0.0000000E+00
10   0.1100000E+03  0.0000000E+00  0.5500000E+02
11   0.1100000E+03  0.5500000E+02  0.0000000E+00
12   0.1100000E+03  0.5500000E+02  0.5500000E+02
13   0.1650000E+03  0.0000000E+00  0.0000000E+00
14   0.1650000E+03  0.0000000E+00  0.5500000E+02
15   0.1650000E+03  0.5500000E+02  0.0000000E+00
16   0.1650000E+03  0.5500000E+02  0.5500000E+02
17   0.2200000E+03  0.0000000E+00  0.0000000E+00
18   0.2200000E+03  0.0000000E+00  0.5500000E+02
19   0.2200000E+03  0.5500000E+02  0.0000000E+00
20   0.2200000E+03  0.5500000E+02  0.5500000E+02
21   0.2750000E+03  0.0000000E+00  0.0000000E+00
22   0.2750000E+03  0.0000000E+00  0.5500000E+02
23   0.2750000E+03  0.5500000E+02  0.0000000E+00
24   0.2750000E+03  0.5500000E+02  0.5500000E+02
25   0.3300000E+03  0.0000000E+00  0.0000000E+00
26   0.3300000E+03  0.0000000E+00  0.5500000E+02
27   0.3300000E+03  0.5500000E+02  0.0000000E+00
28   0.3300000E+03  0.5500000E+02  0.5500000E+02
29   0.3850000E+03  0.0000000E+00  0.0000000E+00
30   0.3850000E+03  0.0000000E+00  0.5500000E+02
31   0.3850000E+03  0.5500000E+02  0.0000000E+00
32   0.3850000E+03  0.5500000E+02  0.5500000E+02
96,1,1
1,2,4.361033259E6,1.101597E-4,1.523568E6,2.026346E6,2.026346E6,.929296054
1,3
1,5
1,6
1,7
2,3
2,4
2,6
3,4
3,7
3,8
4,6
4,8
5,6
5,7
5,9
5,10
5,11
6,7
6,8
6,10
```

NNODE=Number of nodes in truss

NDUM, X, Y, Z

NDUM=Node number

X=X coordinate of node

Y=Y coordinate of node

Z=Zcoordinate of node

NEL,LL,LLT

NEL=Number of bars in truss

LL=0 if bars have different cross sectional properties

LL=1 if all bars have same cross sectional properties

LLT=0 if pinned joints

LLT=1 if rigid joints

N1, N2, EA, m, GJ, EI_Y, EI_Z, I_p/A

N1,N2 are the node numbers at the endpoints of a bar. If all bars have the same properties (LL=1), the properties are listed only for the first bar. If LL=0, the properties must be listed for every bar.

EA=Axial stiffness of bar

m=Mass per unit length of bar

GJ=Torsional stiffness of bar

$EI_{Y\ (Z)}$*=Bending stiffness about Y (Z)*

I_p/A*=Polar area moment of inertia divided by cross sectional area*

```
7,8
7,11
7,12
8,10
8,12
9,10
9,11
9,13
9,15
10,11
10,12
10,13
10,14
11,12
11,15
12,14
12,15
12,16
13,14
13,15
13,17
13,18
13,19
14,15
14,16
14,18
15,16
15,19
15,20
16,18
16,20
17,18
17,19
17,21
17,22
17,23
18,19
18,20
18,22
19,20
19,23
19,24
20,22
20,24
21,22
21,23
21,25
21,27
22,23
22,24
22,25
22,26
23,24
23,27
24,26
```

(More endpoints defining bar locations)

```
24,27
24,28
25,26
25,27
25,29
25,30
25,31
26,27
26,28
26,30
27,28
27,31
27,32
28,30
28,32
29,30
29,31
30,31
30,32
31,32
32
1,.01229,0.,0.,0.
2,.009832,0.,0.,0.
3,.01229,0.,0.,0.
4,.009832,0.,0.,0.
5,.014747,0.,0.,0.
6,.017205,0.,0.,0.
7,.017205,0.,0.,0.
8,.014747,0.,0.,0.
9,.01229,0.,0.,0.
10,.019663,0.,0.,0.
11,.014747,0.,0.,0.
12,.017205,0.,0.,0.
13,.017205,0.,0.,0.
14,.014747,0.,0.,0.
15,.019663,0.,0.,0.
16,.01229,0.,0.,0.
17,.014747,0.,0.,0.
18,.017205,0.,0.,0.
19,.017205,0.,0.,0.
20,.014747,0.,0.,0.
21,.01229,0.,0.,0.
22,.019663,0.,0.,0.
23,.014747,0.,0.,0.
24,.017205,0.,0.,0.
25,.017205,0.,0.,0.
26,.014747,0.,0.,0.
27,.019663,0.,0.,0.
28,.01229,0.,0.,0.
29,.150012,0.,0.,0.
30,.157385,0.,0.,0.
31,.154928,0.,0.,0.
32,.15247,0.,0.,0.
0
12
```

NNODCM= Number of nodes where there are concentrated masses or moments of inertia

NODECM, CM, CIX, CIY, CIZ

NODECM=Node number

CM=Concentrated mass at node NODECM

CIX=Concentrated moment of inertia about X at node NONECM

CIY=Concentrated moment of inertia about Y at node NODECM

CIZ=Concentrated moment of inertia about Z at node NODECM

NDOFCK=Number of degrees of freedom where there are stiffnesses to ground. If NDOFCK were not equal to zero in this example, the next lines would be of the form

NDFCK, CK

NDFCK=Degree of freedom number where stiffness to ground is located

CK=Stiffness to ground at degree of freedom NDFCK

NR OF=Number of degrees of freedom which are to be restrained

```
1
2
3
7
8
9
13
14
15
19
20
21
```

IR(I)=Restrained degrees of freedom. Degrees of freedom must be in rigid-jointed numbering system, even if pinned joints are specified (LLT=0). The X translation at node N is degree of freedom (dof) 6N-5. The Y translation at node N is dof 6N-4, and the Z translation at node N is dof 6N-3. The rotation about X, Y, or Z at node N is dof 6N-2, 6N-1, or 6N, respectively. Thus, in this example for the SADE truss, the translations at nodes 1, 2, 3, and 4 (see Figure 1) are restrained with a rigid-jointed model (LLT=1). If we wanted to restrain these translations with a pin-jointed model (LLT=0), the restrained degrees of freedom would be exactly as they are here. That is,we would restrain degrees of freedom 1, 2, 3, 7, 8, 9, 13, 14, 15, 19, 20, and 21 as if the model had six degrees of freedom per node instead of three.

```
C  THIS PROGRAM SETS UP STIFFNESS AND MASS MATRICES FOR SPACE TRUSSES
C  USING EITHER BAR ELEMENTS (PINNED JOINTS) OR FRAME ELEMENTS (RIGID
C  JOINTS).  IT ALSO ALLOWS FOR LUMPED MASSES AND STIFFNESSES TO
C  GROUND.  THE PROGRAM THEN TAKES OUT THE ROWS AND COLUMNS
C  CORRESPONDING TO THE RESTRAINED DEGREES OF FREEDOM OF THE STRUCTURE.
C
C  THE FOLLOWING VECTORS MIGHT NEED THEIR DIMENSION CHANGED DEPENDING
C  ON THE PROBLEM SIZE:
C  KV,MV,KRV,MRV,X,Y,Z, AND IR.
C
C  FOR FRAME ELEMENTS, THE NUMBER OF DEGREES OF FREEDOM (NDOF) EQUALS
C  6 TIMES THE NUMBER OF NODES IN THE TRUSS.  THE DIMENSION OF KV,MV,
C  KRV, AND MRV IS NDOF*(NDOF+1)/2.  THE DIMENSION OF X,Y, AND Z IS
C  EQUAL TO THE NUMBER OF NODES IN THE TRUSS.  THE DIMENSION OF IR IS
C  NDOF.
C
C  FOR BAR ELEMENTS, NDOF EQUALS 3 TIMES THE NUMBER OF NODES IN THE
C  TRUSS.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ND1(6),ND2(6),IR(192)
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT
      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV
C  READ IN NODE COORDINATES
      READ(5,*)NNODE
      DO 10 I=1,NNODE
   10 READ(5,*)NDUM,X(I),Y(I),Z(I)
      READ(5,*)NEL,LL,LLT
C  IF BAR ELEMENTS, GO TO 500
      IF(LLT.EQ.0)GO TO 500
      NDOF=6*NNODE
      NDOFV=NDOF*(NDOF+1)/2
      DO 40 I=1,NDOFV
      KV(I)=0.
   40 MV(I)=0.
      DO 45 I=1,12
      DO 45 J=1,12
   45 T(I,J)=0.
C  READ IN DATA FOR FIRST ELEMENT
      READ(5,*)N1,N2,EA,PA,GJ,EIY,EIZ,RIPA
C  THIS LOOP SETS UP THE ELEMENT STIFFNESS AND MASS MATRICES FOR EACH
C  ELEMENT, TRANSFORMS THEM TO THE GLOBAL COORDINATE SYSTEM, AND ADDS
C  THEM TO THE STRUCTURE SIZE MATRICES.
      DO 100 I=1,NEL
      IF(I.EQ.1)GO TO 47
      IF(LL.NE.0)READ(5,*)N1,N2
      IF(LL.EQ.0)READ(5,*)N1,N2,EA,PA,GJ,EIY,EIZ,RIPA
   47 DX=X(N2)-X(N1)
      DY=Y(N2)-Y(N1)
      DZ=Z(N2)-Z(N1)
      RL=DSQRT(DX*DX+DY*DY+DZ*DZ)
C  SET UP ELEMET MATRICES
      DO 105 II=2,12
```

```
      DO 105 J=1,12
      IF(J.GE.II)GO TO 105
      KE(II,J)=0.
      ME(II,J)=0.
  105 CONTINUE
      EAL=EA/RL
      GJL=GJ/RL
      EIY2L=2.*EIY/RL
      EIYL2=3.*EIY2L/RL
      EIYL3=2.*EIYL2/RL
      EIZ2L=2.*EIZ/RL
      EIZL2=3.*EIZ2L/RL
      EIZL3=2.*EIZL2/RL
      RIPA14=140.*RIPA
      RL22=22.*RL
      RL13=13.*RL
      RL24=4.*RL*RL
      RL23=-.75*RL24
      DO 110 J=1,7,6
      J1=J+1
      J2=J+2
      J3=J+3
      J4=J+4
      J5=J+5
      KE(J,J)=EAL
      KE(J1,J1)=EIZL3
      KE(J2,J2)=EIYL3
      KE(J3,J3)=GJL
      KE(J4,J4)=2.*EIY2L
      KE(J5,J5)=2.*EIZ2L
      ME(J,J)=140.
      ME(J1,J1)=156.
      ME(J2,J2)=156.
      ME(J3,J3)=RIPA14
      ME(J4,J4)=RL24
  110 ME(J5,J5)=RL24
      KE(5,3)=-EIYL2
      KE(6,2)=EIZL2
      KE(7,1)=-EAL
      KE(8,2)=-EIZL3
      KE(8,6)=-EIZL2
      KE(9,3)=-EIYL3
      KE(9,5)=EIYL2
      KE(10,4)=-GJL
      KE(11,3)=-EIYL2
      KE(11,5)=EIY2L
      KE(11,9)=EIYL2
      KE(12,2)=EIZL2
      KE(12,6)=EIZ2L
      KE(12,8)=-EIZL2
      ME(5,3)=-RL22
      ME(6,2)=RL22
      ME(7,1)=70.
      ME(8,2)=54.
      ME(8,6)=RL13
```

```
      ME(9,3)=54.
      ME(9,5)=-RL13
      ME(10,4)=.5*RIPA14
      ME(11,3)=RL13
      ME(11,5)=RL23
      ME(11,9)=RL22
      ME(12,2)=-RL13
      ME(12,6)=RL23
      ME(12,8)=-RL22
      PARL42=PA*RL/420.
      DO 120 II=1,12
      DO 120 J=1,12
      IF(J.GT.II) GO TO 120
      ME(II,J)=PARL42*ME(II,J)
  120 CONTINUE
      DO 130 II=1,12
      DO 130 J=2,12
      IF(J.LE.II) GO TO 130
      KE(II,J)=KE(J,II)
      ME(II,J)=ME(J,II)
  130 CONTINUE
C  PLACE   TRANSFORMS ELEMENT MATRICES TO GLOBAL COORDINATES AND ADDS
C  THEM TO THE STRUCTURE MATRICES
      CALL PLACE(DX,DY,DZ,RL,N1,N2,6)
  100 CONTINUE
C  ADDCM   ADDS LUMPED MASSES TO STRUCTURE MASS MATRIX
      CALL SCM(6)
C  ADDCK   ADDS STIFFNESSES TO GROUND TO STRUCTURE STIFFNESS MATRIX
      READ(5,*)NDOFCK
      IF(NDOFCK.EQ.0)GO TO 193
      DO 190 I=1,NDOFCK
      READ(5,*)NDFCK,CK
      CALL ADDCK(NDFCK,CK)
  190 CONTINUE
C  REST   TAKES OUT ROWS AND COLUMNS OF THE STRUCTURE MATRICES
C  CORRESPONDING TO RESTRAINED DEGREES OF FREEDOM
  193 CALL REST(NRDOF,6,NRDOFT,NDOF,NDOFV)
C  WRITE NUMBER OF RESTRAINED DEGREES OF FREEDOM, THE RESTRAINED
C  DEGREES OF FREEDOM, AND NUMBER OF DEGREES OF FREEDOM IN
C  RESTRAINED PROBLEM
      WRITE(6,1002)NRDOF
      IF(NRDOF.EQ.0)GO TO 300
      DO 235 I=1,NRDOF
  235 WRITE(6,1002)IR(I)
  290 NDOFR=NDOF-NRDOF
      WRITE(6,1002)NDOFR
 1002 FORMAT(1X,I8)
C  WRITE RESTRAINED STIFFNESS AND MASS MATRICES OF TRUSS
      I1=1
      DO 240 I=1,NDOFR
      I2=I*(I+1)/2
      WRITE(6,1000)(KRV(J),J=I1,I2)
 1000 FORMAT(1X,5E14.6)
  240 I1=I2+1
      I1=1
```

```
      DO 245 I=1,NDOFR                                                  FRA01660
      I2=I*(I+1)/2                                                      FRA01670
      WRITE(6,1000)(MRV(J),J=I1,I2)                                     FRA01680
  245 I1=I2+1                                                           FRA01690
      STOP                                                              FRA01700
  300 WRITE(6,1002)NDOF                                                 FRA01710
      I1=1                                                              FRA01720
      DO 250 I=1,NDOF                                                   FRA01730
      I2=I*(I+1)/2                                                      FRA01740
      WRITE(6,1000)(KV(J),J=I1,I2)                                      FRA01750
  250 I1=I2+1                                                           FRA01760
      I1=1                                                              FRA01770
      DO 255 I=1,NDOF                                                   FRA01780
      I2=I*(I+1)/2                                                      FRA01790
      WRITE(6,1000)(MV(J),J=I1,I2)                                      FRA01800
  255 I1=I2+1                                                           FRA01810
      STOP                                                              FRA01820
C  THIS SECTION OF THE PROGRAM SETS UP MATRICES USING BAR               FRA01830
C  ELEMENTS. IT HAS THE SAME FORM AS THE SECTION USING FRAME ELEMENTS.  FRA01840
  500 NDOF=3*NNODE                                                      FRA01850
      NDOFV=NDOF*(NDOF+1)/2                                             FRA01860
      DO 540 I=1,NDOFV                                                  FRA01870
      KV(I)=0.                                                          FRA01880
  540 MV(I)=0.                                                          FRA01890
      DO 545 I=1,6                                                      FRA01900
      DO 545 J=1,6                                                      FRA01910
  545 T(I,J)=0.                                                         FRA01920
      READ(5,*)N1,N2,EA,PA                                              FRA01930
      DO 600 I=1,NEL                                                    FRA01940
      IF(I.EQ.1)GO TO 547                                               FRA01950
      IF(LL.NE.0)READ(5,*)N1,N2                                         FRA01960
      IF(LL.EQ.0)READ(5,*)N1,N2,EA,PA                                   FRA01970
  547 DX=X(N2)-X(N1)                                                    FRA01980
      DY=Y(N2)-Y(N1)                                                    FRA01990
      DZ=Z(N2)-Z(N1)                                                    FRA02000
      RL=DSQRT(DX*DX+DY*DY+DZ*DZ)                                       FRA02010
      DO 605 II=1,6                                                     FRA02020
      DO 605 J=1,6                                                      FRA02030
      KE(II,J)=0.                                                       FRA02040
  605 ME(II,J)=0.                                                       FRA02050
      EAL=EA/RL                                                         FRA02060
      KE(1,1)=EAL                                                       FRA02070
      KE(4,4)=EAL                                                       FRA02080
      KE(4,1)=-EAL                                                      FRA02090
      KE(1,4)=-EAL                                                      FRA02100
      PAL=PA*RL/6.                                                      FRA02110
      PAL2=2.*PAL                                                       FRA02120
      DO 610 J=1,6                                                      FRA02130
      ME(J,J)=PAL2                                                      FRA02140
      IF(J.GT.3)GO TO 610                                               FRA02150
      ME(J,J+3)=PAL                                                     FRA02160
      ME(J+3,J)=PAL                                                     FRA02170
  610 CONTINUE                                                          FRA02180
      CALL PLACE(DX,DY,DZ,RL,N1,N2,3)                                   FRA02190
  600 CONTINUE                                                          FRA02200
```

```
      CALL SCM(3)
      READ(5,*)NDOFCK
      IF(NDOFCK.EQ.0)GO TO 693
      DO 690 I=1,NDOFCK
      READ(5,*)NDFCK,CK
      IF(MOD(NDFCK,6).EQ.0.OR.MOD(NDFCK+1,6).EQ.0.OR.MOD(NDFCK+2,6)
     1.EQ.0)GO TO 690
      NDFCKT=NDFCK-NDFCK/6*3
      CALL ADDCK(NDFCKT,CK)
  690 CONTINUE
  693 CALL REST(NRDOF,3,NRDOFT,NDOF,NDOFV)
      WRITE(6,1002)NRDOFT
      IF(NRDOF.EQ.0)GO TO 300
      DO 280 I=1,NRDOF
      IF(IR(I).EQ.0)GO TO 280
      WRITE(6,1002)IR(I)
  280 CONTINUE
      NRDOF=NRDOFT
      GO TO 290
      END
C  ADDCM   ADDS LUMPED MASSES TO STRUCTURE MASS MATRIX
      SUBROUTINE ADDCM(N,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ND1(6),ND2(6),IR(192)
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT
      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV
      I=N*(N+1)/2
      MV(I)=MV(I)+C
      RETURN
      END
C  ADDCK   ADDS STIFFNESSES TO GROUND TO STRUCTURE STIFFNESS MATRIX
      SUBROUTINE ADDCK(N,C)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ND1(6),ND2(6),IR(192)
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT
      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV
      I=N*(N+1)/2
      KV(I)=KV(I)+C
      RETURN
      END
C  PLACE   TRANSFORMS ELEMENT MATRICES TO GLOBAL COORDINATES AND ADDS
C  THEM TO STRUCTURE MATRICES
      SUBROUTINE PLACE(DX,DY,DZ,RL,N1,N2,N)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION ND1(6),ND2(6),IR(192)
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT
      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV
      ALFA=DX/RL
      BETA=DY/RL
```

```
      GAMA=DZ/RL
      IF(BETA.EQ.0.) THEN
      A=0.
      B=1.
      ELSE
      A=-DSQRT(BETA*BETA/(ALFA*ALFA+BETA*BETA))
      B=-A*ALFA/BETA
      END IF
      LAM(1,1)=ALFA
      LAM(1,2)=BETA
      LAM(1,3)=GAMA
      LAM(2,1)=A
      LAM(2,2)=B
      LAM(2,3)=0.
      LAM(3,1)=-B*GAMA
      LAM(3,2)=A*GAMA
      LAM(3,3)=B*ALFA-A*BETA
      DO 150 I=1,3
      I3=I+3
      I61=I+6
      I9=I+9
      DO 150 J=1,3
      RLAM=LAM(I,J)
      T(I,J)=RLAM
      T(I3,J+3)=RLAM
      IF(N.EQ.3)GO TO 150
      T(I61,J+6)=RLAM
      T(I9,J+9)=RLAM
  150 CONTINUE
      ND=2*N
      DO 151 I=1,ND
      DO 151 J=1,ND
      KET(I,J)=0.
  151 MET(I,J)=0.
      DO 152 I=1,ND
      DO 152 J=1,ND
      DO 152 K=1,ND
      KET(I,J)=KE(I,K)*T(K,J)+KET(I,J)
  152 MET(I,J)=ME(I,K)*T(K,J)+MET(I,J)
      DO 153 I=1,ND
      DO 153 J=1,ND
      IF(J.GT.I)GO TO 153
      KEG(I,J)=0.
      MEG(I,J)=0.
  153 CONTINUE
      DO 154 I=1,ND
      DO 154 J=1,ND
      IF(J.GT.I)GO TO 154
      DO 154 K=1,ND
      KEG(I,J)=T(K,I)*KET(K,J)+KEG(I,J)
      MEG(I,J)=T(K,I)*MET(K,J)+MEG(I,J)
  154 CONTINUE
      N16=N*N1-N
      N26=N*N2-N
      DO 160 I=1,N
```

```
      ND1(I)=N16+I                                                      FRA03310
  160 ND2(I)=N26+I                                                      FRA03320
      DO 170 I=1,N                                                      FRA03330
      I6=I+N                                                            FRA03340
      ND22=ND2(I)*(ND2(I)-1)/2                                          FRA03350
      DO 170 J=1,N                                                      FRA03360
      I12=ND22+ND1(J)                                                   FRA03370
      KV(I12)=KEG(I6,J)+KV(I12)                                         FRA03380
      MV(I12)=MEG(I6,J)+MV(I12)                                         FRA03390
      IF(J.GT.I) GO TO 170                                              FRA03400
      I1=ND1(I)*(ND1(I)-1)/2+ND1(J)                                     FRA03410
      I2=ND22+ND2(J)                                                    FRA03420
      KV(I1)=KEG(I,J)+KV(I1)                                            FRA03430
      MV(I1)=MEG(I,J)+MV(I1)                                            FRA03440
      KV(I2)=KEG(I6,J+N)+KV(I2)                                         FRA03450
      MV(I2)=MEG(I6,J+N)+MV(I2)                                         FRA03460
  170 CONTINUE                                                          FRA03470
      RETURN                                                            FRA03480
      END                                                               FRA03490
C  SCM   READS IN NODE NUMBERS WHERE LUMPED MASSES AND INERTIAS ARE     FRA03500
C  TO BE ADDED AND ADDS THEM TO STRUCTURE MASS MATRIX                   FRA03510
      SUBROUTINE SCM(N)                                                 FRA03520
      IMPLICIT REAL*8(A-H,O-Z)                                          FRA03530
      DIMENSION ND1(6),ND2(6),IR(192)                                   FRA03540
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1FRA03550
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),                       FRA03560
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT                     FRA03570
      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV       FRA03580
      READ(5,*)NNODCM                                                   FRA03590
      IF(NNODCM.EQ.0)RETURN                                             FRA03600
      DO 180 I=1,NNODCM                                                 FRA03610
      READ(5,*)NODECM,CM,CIX,CIY,CIZ                                    FRA03620
      NCM6=N*NODECM-N                                                   FRA03630
      DO 185 II=1,3                                                     FRA03640
      ND1(II)=NCM6+II                                                   FRA03650
      CALL ADDCM(ND1(II),CM)                                            FRA03660
  185 CONTINUE                                                          FRA03670
      IF(N.EQ.3)GO TO 180                                               FRA03680
      ND4=NCM6+4                                                        FRA03690
      CALL ADDCM(ND4,CIX)                                               FRA03700
      ND4=NCM6+5                                                        FRA03710
      CALL ADDCM(ND4,CIY)                                               FRA03720
      ND4=NCM6+6                                                        FRA03730
      CALL ADDCM(ND4,CIZ)                                               FRA03740
  180 CONTINUE                                                          FRA03750
      RETURN                                                            FRA03760
      END                                                               FRA03770
C  REST   TAKES OUT ROWS AND COLUMNS CORRESPONDING TO RESTRAINED        FRA03780
C  DEGREES OF FREEDOM                                                   FRA03790
      SUBROUTINE REST(NRDOF,N,NRDOFT,NDOF,NDOFV)                        FRA03800
      IMPLICIT REAL*8(A-H,O-Z)                                          FRA03810
      DIMENSION ND1(6),ND2(6),IR(192)                                   FRA03820
      REAL*8 KV(36585),MV(36585),KRV(36585),MRV(36585),KE(12,12),ME(12,1FRA03830
     12),LAM(3,3),T(12,12),KEG(12,12),MEG(12,12),                       FRA03840
     2X(45),Y(45),Z(45),KET(12,12),MET(12,12),DSQRT                     FRA03850
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      COMMON ND1,ND2,IR,LAM,T,KE,KET,KEG,KV,KRV,ME,MET,MEG,MV,MRV       FRA03860
      READ(5,*)NRDOF                                                    FRA03870
      NRDOFT=0                                                          FRA03880
      IF(NRDOF.EQ.0)RETURN                                              FRA03890
      DO 200 I=1,NRDOF                                                  FRA03900
      READ(5,*)IR(I)                                                    FRA03910
      IR1=IR(I)                                                         FRA03920
      IF(N.EQ.6)GO TO 205                                               FRA03930
      IF(MOD(IR1,6).EQ.0.OR.MOD(IR1+1,6).EQ.0.OR.MOD(IR1+2,6).EQ.0)     FRA03940
     1THEN                                                              FRA03950
      IR(I)=0                                                           FRA03960
      GO TO 200                                                         FRA03970
      END IF                                                            FRA03980
      NRDOFT=NRDOFT+1                                                   FRA03990
      IR1=IR1-IR1/6*3                                                   FRA04000
      IR(I)=IR1                                                         FRA04010
  205 IR11=IR1*(IR1-1)/2                                                FRA04020
      DO 210 J=1,NDOF                                                   FRA04030
      IF(J.GT.IR1)GO TO 210                                             FRA04040
      IRV=IR11+J                                                        FRA04050
      KV(IRV)=1.D71                                                     FRA04060
      MV(IRV)=1.D71                                                     FRA04070
  210 CONTINUE                                                          FRA04080
      DO 220 J=1,NDOF                                                   FRA04090
      IF(IR1.GT.J)GO TO 220                                             FRA04100
      JRV=J*(J-1)/2+IR1                                                 FRA04110
      KV(JRV)=1.D71                                                     FRA04120
      MV(JRV)=1.D71                                                     FRA04130
  220 CONTINUE                                                          FRA04140
  200 CONTINUE                                                          FRA04150
  225 II=0                                                              FRA04160
      DO 230 I=1,NDOFV                                                  FRA04170
      RK=KV(I)                                                          FRA04180
      IF(RK.GT.1.D70)GO TO 230                                          FRA04190
      II=II+1                                                           FRA04200
      KRV(II)=RK                                                        FRA04210
      MRV(II)=MV(I)                                                     FRA04220
  230 CONTINUE                                                          FRA04230
      RETURN                                                            FRA04240
      END                                                               FRA04250
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