STUDY OF THE TURBULENT BOUNDARY LAYER.

By

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S.B., Chiao-tung University, China
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Submitted in Partial Fulfillment of the
Requirement for the Degree of
Master of Science
From the
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1936

Signature Redacted
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Aero
Thesis
1936
September 1, 1936.

Professor G. W. Swett  
Secretary of the Faculty  
Massachusetts Institute of Technology  
Cambridge, Massachusetts

Dear Sir:

In accordance with requirements for the degree of Master of Science, I hereby submit a thesis entitled "A Study of the Turbulent Boundary Layer"

Respectfully submitted,
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This work was done in collaboration with Mr. W. H. Peters, a graduate student in the department of mechanical engineering.
These tests were made in order to compare results from the boundary layer research tunnel at the Massachusetts Institute of Technology with existing information available. The design of the tunnel was modified slightly and other modifications to improve the performance of the tunnel are recommended.

Tests were run with five different pressure gradients and were analyzed according to Gruschwitz' method.

The results obtained compare favorably with the work of others in this field. However, due to the physical limitations of the equipment in its present form the experiment is definitely limited in scope.
II. INTRODUCTION

While the effect of the viscosity of air is limited only to the thin boundary layer, many of the critical characteristics of air flow are determined by this thin layer of retarded air. This is the source of skin resistance of flow. Also, due to a pressure gradient, this layer gives the form resistance, and even may separate from the surface of the body and change the whole flow picture as in the case of the stalling of an airfoil.

To solve such important problems as mentioned a full understanding of the behavior of the boundary layer is necessary. The character of the laminar boundary layer was studied theoretically first by Pohlhausen and more recently by von Karman. It has been studied experimentally by Blaisdell. (Ref. 1). Von Karman's theory seems to check with experiment very closely.

However, the character of the turbulent boundary layer is not thoroughly understood. This is perhaps due to the fact that equations of motion of turbulent flow involve empirical factors which can be obtained only from experimental data while the equations of motion of laminar flow have been known since Navier
and Stokes. The first work on the turbulent boundary layer on a flat plate with a pressure gradient is that of Gruschwitz. (Ref. 2). His method will give one point in the velocity profile and the equivalent boundary layer thickness by solving two simultaneous linear differential equations. This method was checked with boundary layer measurements in free flight by Stüper. (Ref. 3). Gruschwitz' experiment was done on a rather short plate, 1 meter long and his empirical expression does not hold when separation occurs. The check of Stuper also did not include separation.

The Gruschwitz method was applied to a wing section tested at high Reynolds Number by H. Peters. (Ref. 4). The results show the approximate nature of this method and the difficulty of predicting separation.

An extension of the Gruschwitz method to curved two dimensionable flow was made by Schmidbauer. (Ref. 5). This experiment was also at rather small scale.

Another important problem of the boundary layer flow is the transition from a laminar to a turbulent flow. While there are theoretical studies by Tollmien and others with various simplifying assumptions, this problem in a flow with a pressure gradient is too complicated to be solved by theoretical calculations. The different experimental results on transition are scattered and no definite conclusion can be drawn.
Under such circumstances the Aeronautical Engineering Department of the Massachusetts Institute of Technology built a tunnel especially designed to carry out boundary layer research. It is the purpose of the present authors to try out the apparatus and obtain some preliminary results to be compared with previous works.
III. THEORY

\[ x = \text{distance from the stagnation point measured along the plate.} \]

\[ y = \text{distance normal to the surface of the plate.} \]

\[ v = \text{velocity at any point within the boundary layer.} \]

\[ V = \text{velocity at the outer edge of the boundary layer.} \]

\[ \delta = \text{thickness of the boundary layer, defined as } y = \delta \text{ when } \frac{V}{V} \to 1 \]

\[ p = \text{static pressure along the surface, which varies only with } x \text{ and is independent of } y. \]

\[ \rho = \text{mass density.} \]

\[ \tau_0 = \text{shearing stress intensity at the surface of the plate.} \]

\[ q_\delta = \frac{\rho V^2}{2} \]

\[ \Delta = \frac{1}{V^2} \int_0^\delta (V^2 - v^2) \, dy. \]

\[ \delta^* = \frac{1}{V} \int_0^\delta (V - v) \, dy. \]

\[ \theta = \Delta - \delta^* \]

\[ H = \frac{\delta^*}{\theta} \]

\[ \gamma = 1 - (\frac{V}{V}^2) \text{ at } y = \theta \]

\[ g_1 = \text{total pressure at } y = \theta \]

\[ R_\theta = \frac{V_\theta}{\rho} = \text{Reynolds Number based upon } \theta. \]

\[ r = \text{radius of curvature of surface.} \]
Von Karman applied the law of momentum to the boundary layer and derived the following equation:

\[
\frac{1}{V} \frac{d}{dx} \int_0^\delta y dy - \frac{1}{V^2} \frac{d}{dx} \int_0^\delta u^2 dy = \frac{1}{V^2} \delta \frac{dp}{dx} + \frac{T^o}{p V^2}
\]  

(1)

As the flow is potential at the outer edge of the boundary layer, Bernoulli's law holds and therefore

\[- \frac{dp}{dx} = + \frac{p}{2} \frac{dV}{dx} = + \frac{dq}{d\delta}
\]

Let \[\frac{1}{V} \int_0^\delta (V - u^2) dy = \Delta\]

\[\frac{1}{V} \int_0^\delta (V - V) dy = \delta^*
\]

both have the dimension of length, then (1) reduces to

\[\frac{1}{V^2} \frac{d}{dx} (\Delta V^2) - \frac{1}{V} \frac{d}{dx} (\delta^* V) = \frac{T^o}{p V^2}
\]

By letting

\[\theta = \Delta - \delta^* = \int_0^\delta \left(1 - \frac{u}{V}\right) dy
\]

the following is true

\[\frac{d\theta}{dx} + \frac{2 + H}{\delta^*} \frac{\theta}{\frac{d\delta}{dx}} = \frac{T^o}{p V^2}
\]

(2)

where \[H = \frac{\delta^*}{\theta}
\]

Since \[\theta \] has a dimension of length it may be taken as a measure of the thickness of the boundary layer. Since \(V - u\) is the deficiency of velocity and
is the boundary velocity, \( \int_0^\delta (1 - \frac{v}{V}) \frac{V}{V} \, dy \)

is a measure of the deficiency of momentum.

Equation 2 is a linear differential equation in \( \theta \). It can be solved if \( \frac{1}{\delta} \frac{d\eta}{dx} \) is known, and if values of \( H \) and \( \frac{T_0}{C V^2} \) are assumed.

To find the relation between the velocity profile and \( x \) the energy deficiency, \( \{\eta\} \) at \( y = 0 \) is used.

Now \( H + 1 = (\frac{\theta}{\delta} + 1) = \frac{\int_0^\delta (1 - \frac{v}{V})(1 + \frac{V}{V}) \, dy}{\int_0^\delta (1 - \frac{v}{V}) \, dy} \)

\[ = 2 \frac{\int_0^\delta (1 - \frac{v}{V}) \left[ 1 - \frac{1}{2} \left( 1 - \frac{v}{V} \right) \right] \, dy}{\int_0^\delta (1 - \frac{v}{V}) \, dy} \]

Therefore \( H + 1 \) is twice the distance from the \( y \) axis to the center of gravity of the \( 1 - \frac{V}{V} \) area, and should be a good indication of the geometrical form of the velocity. Hence the definite relation between \( \eta \) and \( H \) (Fig. 18), confirms the proper choice of this parameter.

Consideration of the conditions at \( y = 0 \) indicates that the factors which should affect the change of energy at this point are \( v, v, \theta, \rho, \) and kinematic viscosity \( \rho \). That is

\[ \frac{d}{dx} \left( \frac{\rho V^2}{2} + \rho \right) = \frac{d\eta}{dx} = f(v, V, \theta, \rho, \rho) \]
Combining these into non-dimensional form gives:
\[
\frac{\frac{\theta}{\frac{\rho}{2} V^2}}{\frac{d}{dx}} = \frac{\theta}{q_{s}} \frac{d q_{s}}{dx} = f(\eta, \nu \frac{\rho}{\rho V^2})
\]

The experimental work of Gruschwitz showed negligible effect of Reynolds Number. The preceding then simplifies to
\[
\frac{\theta}{q_{s}} \frac{d q_{s}}{dx} = f(\eta)
\]

From experimental results Gruschitz found that
\[
\frac{\theta}{q_{s}} \frac{d q_{s}}{dx} = -\theta \left( \frac{d \eta}{dx} + \frac{\eta}{\frac{\rho}{2}} \frac{d \eta}{dx} \right) = 0.00894 \eta - 0.00461 \quad (3)
\]

This is a linear differential equation in \( \eta \) and is combined with equation 2 to give one set of simultaneous differential equations. To solve them constant values of \( \frac{\tau_{b}}{\rho V^2} \) and \( H \) are first assumed. Equation 2 is then solved for \( \theta \). \( \eta \) can then be found from equation 3. From the \( H - \eta \) curve the variation of \( H \) along \( x \) can be found. Similarly values for \( \frac{\tau_{b}}{\rho V^2} \) can be taken from the \( \frac{\tau_{b}}{\rho V^2} \) vs. \( R_{\theta} \) curve for a flat plate. It is then possible to obtain a second and closer approximation of \( \theta \) and \( \eta \). It is shown by Gruschwitz, H. Peters, and Schmidbauer that two approximations are enough.

These integrations start at the transition point. The constants of integration, \( \theta \) and \( \eta \) at the transition point can be found by either Pohlhausen's or von Karman's method. Gruschwitz suggested a value of \( \eta = 0.9 \) at this point.
Gruschwitz claims that when $\eta = 0.8$ separation is impending. However, the tests of H. Peters showed that Gruschwitz' empirical rule of $\frac{b}{q_s} \frac{d\theta}{dx} = f(\eta)$ does not apply when separation occurs and $f(\eta)$ is not linear. However some of these discrepancies must be due to the curvature of the wing surface. This can be seen from Schmidbauer's empirical equation

$$\frac{b}{q_s} \frac{d\theta}{dx} = (\eta + 0.1)(0.00894 - 0.315 \frac{\theta}{r}) - 0.0055 \quad (4)$$

where $r$ is the radius of curvature of the surface.
IV. DESCRIPTION OF THE APPARATUS

The wind tunnel used in this test was designed and built especially for the investigation of boundary layer characteristics. It is located in the large wind tunnel room in the Guggenheim Aeronautical Building at the Massachusetts Institute of Technology.

The tunnel consists, essentially, of four major parts: entrance section, test section, diverging section enclosing the fan, and an auxiliary chamber and blower for control of the boundary layer. The fan is at the downstream end of the test section so that the tunnel operates below atmospheric pressure.

The tunnel was made to operate at pressures below atmospheric in order to maintain a more exact control of the boundary layer at all parts of the test section than is possible with a high pressure tunnel.

A disadvantage of this type of tunnel is that separation occurring in the test section causes irregular flow on the fan blades. This, in turn, causes quite severe pressure fluctuations in the test section.

The entrance section, in the final form, is 54 inches high, 33 inches wide, and 8 feet long. The mouth of this section has a metal fairing and is covered with a single thickness of cheesecloth to reduce the
disturbance at this point. A honeycomb, made up of sheet metal tubes 3 inches in diameter and 2 feet long is located 2 inches in back of the point where the fairing becomes tangent to the sides of the section. The first of a series of 5 screens, spaced at 6 inch intervals, is located 6 inches in back of the honeycomb. These screens are of ordinary door screening. The last screen is 40 inches from the convergence into the test section.

The test section is 3 feet high and 11 feet long. The entrance of the test section is 10 inches wide. One side of the test section is a steel plate along which measurements of static and dynamic pressures are made. The other side is a false back which can be moved in order to obtain different pressure gradients along the flat plate.

The steel plate is 1/4 inch thick and stiffened on the outside with 1 1/2 x 1 1/2 inch angles. The inside of the plate is nickel-plated and polished. A detail of the leading edge of the plate is shown in Fig. 1. Holes, 1/2 inch in diameter, are located in the plate as shown on the elevation in Fig. 1.

Each of these holes is filled with a brass plug (Fig. 2). A 3/16 inch hole is drilled in the plug to within about 1/16 inch of the inside surface. A No. 76 (.020 inch) drill is used to drill through to the inside,
SCHEMATIC DIAGRAM OF BOUNDARY LAYER RESEARCH TUNNEL

FIGURE 1
forming the static pressure orifice. A piece of 3/16 inch brass tubing is soldered into the larger hole. The inner surface of the plug is flush with the surface of the plate. Each plug was finished in place with a No. 0 file.

This construction makes possible the accurate measurement of the static pressure at the surface of the plate. Each plug may be removed to allow the insertion of a pitot tube for measuring the dynamic pressure.

The pitot tube is inserted through a plug of the same outside dimensions as the static plug. Into this plug an insulating bushing has pressed (Fig. 3).

The false back of the test section is of 3/8 inch plywood, stiffened at top and bottom by 1 x 1 3/8 inch stringers. The false back is held in any desired position by fastening the stiffeners to the top and bottom of the tunnel with wood screws.

The convergence from the entrance section to the test section is about 5 to 1. The top, bottom, and back of the convergent section are made of 1/16 inch plywood. The side ahead of the steel plate is a brass plate, the downstream end of which laps under the leading edge of the test plate.

This lap is opened to form a slot, so that the boundary layer from entrance section and convergence can be removed. This insures a stagnation point at the leading edge of the plate. The top, bottom, and back of the test
DETAIL OF STATIC PLUG

FIGURE 2

DETAIL OF INSULATED PLUG

FIGURE 3

DETAIL OF CONTROL SLOT

FIGURE 4
section each has two slots to control the thickness of the boundary layers on these three sides so that they will not interfere with the layer on the test plate. Since this tunnel operates below atmospheric pressure an auxiliary blower and system of ducts are used to remove these boundary layers. The location and construction of the slots and ducts are shown in Fig. 1 and Fig. 4.

The space between the false back of the test section and the removable back panels of the tunnel acts as the trunk of the boundary layer removal system. Branches extend from this space to the control slot at the leading edge of the test plate and to the four control slots on the top and bottom of the tunnel. The two control slots in the false back open directly into the trunk of the system.

The air is removed from the space behind the false back by a centrifugal blower which is driven through a V-belt by a 2 horsepower, D.C. motor.

Since the pressure difference against which this blower has to work is greater than originally estimated, the blower is enclosed within a large box which opens into the diverging section of the tunnel on the upstream of the main fan. This materially reduces the load on the blower.

The divergent section is directly attached to the test section 16 inches downstream from the end of the
test plate. It changes the cross-section of the tunnel from a 22 x 36 inch rectangle to a 46 inch diameter circle in 47 inches. Beyond this is a cylindrical section, 5 feet long, in which the axial fan is located. From this section the air is discharged into the room.

The fan has eight adjustable pitch blades, a hub diameter of 26 inches and overall diameter of 45-3/4 inches. The fan is mounted on a 1-15/16 inch diameter shaft which extends beyond the end of the tunnel, to be supported in an outboard bearing. Between the end of the tunnel and the outboard bearing is a 4-groove V-belt pulley driven through V-belts by a 20 horsepower, D.C., shunt motor located at the side of the tunnel. This motor has a separately excited field. The speed is regulated by variation of the armature voltage. A variable resistance in the field circuit gives fine speed adjustment.

For reference, the dynamic pressure of the main air stream at the leading edge of the test plate is obtained with a Prandtl type of pitot-static tube connected to an N.A.C.A. type of micromanometer which has graduations of 0.001 inch.

The static pressures at the plate and the dynamic pressures at each station are read on a Prandtl manometer. This manometer has graduations of 0.05 millimeter.
Two pitot tubes are used in this investigation. They are shown in Figures 5 and 6. The shorter tube is for obtaining the regular velocity profiles. The longer tube, to be used in station 1, extends to the leading edge of the test plate and is to be used in measuring the velocity distribution at that point only. Both tubes are of brass. The ends have been annealed, flattened into a rectangular cross-section, and filed so that the center line of the opening is a minimum distance from the surface of the plate.

The distance of the pitot tube from the surface of the plate is adjusted by a micrometer screw shown in Plate 2. The precision of this adjustment is 0.02 millimeter.
V. DEVELOPMENT OF THE APPARATUS.

The preceding description is of the apparatus in its final form. After the construction of the tunnel had been completed and several tests had been run, certain changes were made. These greatly improved the operation of the tunnel.

Originally the entrance section was but two feet long and had neither honeycomb nor screens. A single layer of cheesecloth was stretched across the mouth of the entrance section. Since the avoidance of turbulence due to a honeycomb is desirable for the study of transition, it had been decided to try the tunnel without the honeycomb. However, data taken under these conditions showed large, long period fluctuations of the dynamic pressure at any setting of the pitot tube. A yaw meter and hot-wire anemometer indicated that rotation existed and that the strength of the rotation varied with time. Installation of the honeycomb and screens eliminated this rotation and as a result the fluctuation in dynamic pressure was less.

It was noticed that the drop in static pressure from the test section to the boundary layer control duct at the leading edge fluctuated considerably. Allowing more clearance between the edge of the flap and the wall of the duct, and inserting a fairing at this point reduced
the variation in static pressure drop. It also reduced the fluctuation in dynamic pressure within the test section.

These two modifications reduced the worst fluctuation in the boundary layer velocities from about 20% to less than 1%. 
VI. TEST PROCEDURE.

The following method was used in obtaining each velocity profile.

The pitot tube, supported by the insulate plug, was placed in the tunnel and the micrometer screw was adjusted until the edge of the pitot tube just touched the test plate. An electrical circuit, comprised of lamp, battery, pitot tube, and test plate, was closed when the tube touched the plate.

The main stream velocity at the leading edge of the plate was maintained at a constant value indicated on the N.A.C.A. manometer connected to the pitot-static tube. The fine adjustment of motor speed to keep this velocity constant was obtained with a variable resistance in the field circuit.

The pitot tube was moved across the tunnel to the edge of the boundary layer. At regular intervals of travel the dynamic pressure was read on the Prandtl manometer one side of which was connected to the pitot tube. The other side of the manometer was connected to the static plug in the station immediately upstream from that in which the pitot tube was located. When the pitot tube was at station 1, the static pressure was taken at station 26.
MOUTH OF TUBE
SCALE-12MM=1MM.

PITOT TUBE NO. 1
FIGURE 5

SCALE-FULL SIZE

$\frac{3}{4}$ DIAM.


PITOT TUBE NO. 2
FIGURE 6

SCALE-FULL SIZE

$\frac{3}{8}$ DIAM.
Corrections were made for the distance between the end of the pitot tube and the static plug by reading the difference between the static pressures at these two points from a faired curve of static pressure plotted against the distance from the leading edge.

The static pressure distribution along the plate was measured by referring all stations to station 1 which was arbitrarily made zero. The measurements were made on the Prandtl manometer.

Some preliminary tests had to be made to calibrate and adjust some of the equipment. The first of these was the calibration of the pitot tube. The center of the pitot tube is 0.18 millimeter from the surface of the plate when the electrical circuit is closed. At small distances from the plate the disturbance between the pitot tube and the plate causes an appreciable error in the measured dynamic pressure. To correct for this a velocity profile at station 1, on a test in which data from the first three stations gave laminar profiles, was compared with a true velocity profile for laminar flow. This comparison gave correction factors which were assumed to be functions of the distance from the surface of the plate only. These factors can be applied to this pitot tube only. The method of arriving at the correction factor curve, Fig. 7, is shown in detail in Appendix I.
To be certain that the flow along the test plate was two-dimensional, velocity profiles were taken at three stations arranged vertically such as numbers 3, 21, and 26. Plots of these, Fig. 8, at two different values of $x$ indicate how closely the flow approaches the two-dimensional. At the downstream stations, Fig. 8b, the flow is not two-dimensional. This is due to the very low ratio of height of test section to width. This condition was not present in the other series.

To make sure that the velocity distribution was steady and that the boundary layer thickness was a minimum at the leading edge of the test plate a traverse was made with pitot tube number 2, Fig. 6, located at station 1. The optimum opening of the slot at the leading edge was found to be about $\frac{3}{8}$ inch. A pressure drop across the slot of 0.60 inch of alcohol was found to be the most satisfactory when the velocity at the leading was about 60 miles per hour, $q_o = 2.20$ inches of alcohol. This combination of slot opening and pressure drop reduced the thickness of the boundary layer at the leading edge to about .60 millimeter.

The pressure drop was read on an ordinary U-tube manometer connected between the test section and the boundary layer control duct. This differential pressure remained quite constant throughout each test.
Five series of tests were run after the tunnel had been modified to give better data. Each series was run with a different pressure gradient. All were run with a reference dynamic pressure of 2.20 inches of alcohol at the leading edge. The variation of pressure gradient was obtained by changing the angle between the test plate and the movable false back. In all series the width of the channel at the convergence was 10 inches. The differential pressure at the leading edge control slot was kept at 0.60 inch of alcohol for all series.

Series 5, Fig. 9a

The maximum angle of divergence possible with an initial channel width of 10 inches and the use of the full length of the test plate was used in this series.

Series 6,

This series was not completed. The entire length of the false back was parallel to the test plate. The separation caused by the very rapid divergence at the end of the test section produced a very unsteady flow in the test section.

To overcome this condition only one half of the length of the test plate was used for the rest of the series. This modification allowed a smaller angle of divergence beyond the active test section. The boundary
FIGURE 9  SCHEMATIC DIAGRAM SHOWING CHANNEL FORMS

(a) SERIES 5  (b) SERIES 7  (c) SERIES 8  (d) SERIES 9  (e) SERIES 10
layer control slot in this diverging part was opened wider than previously to aid in the prevention of separation.

Series 7, Fig. 9b
This series was run on a convergent channel but with only the upstream half of the plate used.

Series 8, Fig. 9c
A parallel channel down to and including station 12 was used for this series.

Series 9, Fig. 9d
A slightly greater angle of convergence than in series 7 was used.

Series 10, Fig. 9e
This series was run with the maximum angle of divergence possible with an initial channel width of 10 inches and the use of only half the plate. No separation was obtained with this channel.
VII. DISCUSSION OF RESULTS

The numerical results obtained in this investigation are shown in curves, Fig. 10 to 20 inclusive, and in Table I. The velocity profiles, Fig. 10 to 12, are self-explanatory. The experimental points fall quite closely to a faired curve. The worst scattering was obtained at all stations when the ratio of the dynamic pressure in the stream was between 0.6 to 0.8. In this region the fluctuation of dynamic pressure, previously referred to, was the most troublesome. The cause of this fluctuation is problematical. The magnitude of this fluctuation was reduced from about 20% to less than 2% by the addition of the honeycomb and screen, and the improvement of the leading edge slot.

Possibly the remaining fluctuation is due to wandering of the transition point. This would cause a varying boundary layer thickness. However, it is questionable that this effect would continue throughout the length of the tunnel. The fact that there is still a fluctuation in static pressure drop through the leading edge slot suggests that the leading edge of the test plate may be a source of the disturbance. This fluctuation of pressure drop was materially reduced by adding a fillet in the boundary layer removal duct.

, obtained from graphical integration above the \((v/V)\) and \((v/V)^2\) curves, plotted against \(x\), the distance
from the leading edge, gives a comparatively fair curve (Fig. 13 to 17 inclusive) beyond the third station. Between the leading edge and station No. 1 there is a radical change of slope of the $\theta$ curve. As no data can be taken in this region with existing equipment, it is difficult to fair these points into the curve. Since the pressure gradients in these tests are rather small, it is expected that there will be little change in the shape of the laminar profiles from leading edge to station No. 1. Therefore the values of $\theta$ will be proportional to $\delta$ which in turn increases as the square root of $x$. The curve is actually faired in this region with this point in mind. It is, however, this uncertainty that makes the points in the $\frac{\tau_x}{\rho V}$ vs $R \theta$ curve scatter in this region.

Since $\eta$ is read from the $(v/V)^2$ curve at $y = \theta$, and since the slope of this curve is quite steep at this point, a scattering of points on the $\eta$ vs $x$ curve would be expected. (Fig. 13 to Fig. 17).

The points plotted on the $H-\eta$ coordinates (Fig. 18) indicates only a satisfactory grouping about part of Gruschwitz' curve. An insufficient range of pressure gradients were used to check all parts of his curve. The range of pressure gradients is limited by the existing equipment. A longer active part of the test plate should be made available without having trouble from separation beyond the test section.
\[ \frac{t_o}{pV^2} = \frac{0.208}{R_g} \]

\[ R_g = \frac{Vg}{\nu} \]

Fig. 20
The plot of \( \frac{\Delta \theta}{q_0 \frac{dx}{x}} \) vs \( \gamma \) (Fig. 19) shows that a curve through the experimental points is practically a straight line. This agrees with the linear approximation of Gruschwitz in the limited range of \( \gamma \) obtained.

Comparison with Gruschwitz' curve shows that the value of \( \gamma \) for \( \frac{\Delta \theta}{q_0 \frac{dx}{x}} = 0 \) is the same as that obtained by him. However, the slope of the curve is considerably greater. It would appear that the form of the profile is less sensitive to pressure gradient than Gruschwitz' results would indicate.

The points on the \( \frac{\tau_0}{\rho V^2} \) vs \( R_e \) curve are badly scattered. This scattering is due to the difficulty in finding the correct value of \( \frac{\Delta \theta}{dx} \), especially at low Reynolds' number, corresponding to short distances from the leading edge, where the actual shape of the \( \theta \) vs \( x \) curve is questionable.

H. Peters, in his work with a wing section, found that a positive pressure gradient was accompanied by lower values of \( \frac{\tau_0}{\rho V^2} \) than those for a flat plate with no pressure gradient. Results of the present tests do not verify this.

Table I shows quite satisfactorily that the critical Reynolds' Number, based upon \( x \), for transition from laminar boundary layer to turbulent boundary layer, increases as the pressure gradient decreases from positive to negative. Comparison of the critical Reynolds' Number
for the parallel section with that from tests on a similar section before the screens and honeycomb were added shows that the flow in the present tunnel is more turbulent.

Table I

Critical Reynolds' Numbers and Maximum Reynolds' Number in the Tests

<table>
<thead>
<tr>
<th>( \frac{1}{q_c} \frac{dp}{dx} )</th>
<th>( V, \text{mph.} )</th>
<th>( (R_x)_{\text{cri.}} )</th>
<th>( (R_\infty)_{\text{cri.}} )</th>
<th>( (R_\infty)_{\text{max.}} )</th>
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<td>+0.0275</td>
<td>54.5</td>
<td>246,000</td>
<td>485</td>
<td>7660</td>
<td>10</td>
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<tr>
<td>+0.0108</td>
<td>56.0</td>
<td>253,000</td>
<td>415</td>
<td>12300</td>
<td>5</td>
</tr>
<tr>
<td>-0.0014</td>
<td>58.0</td>
<td>284,000</td>
<td>516</td>
<td>4390</td>
<td>8</td>
</tr>
<tr>
<td>-0.0071</td>
<td>60.6</td>
<td>320,000</td>
<td>514</td>
<td>3820</td>
<td>9</td>
</tr>
<tr>
<td>-0.0075</td>
<td>59.1</td>
<td>298,000</td>
<td>473</td>
<td>4650</td>
<td>7</td>
</tr>
</tbody>
</table>
VIII. CONCLUSIONS

The results obtained from the existing tunnel are definitely limited by the present equipment. They cover only a small range of pressure gradients and do not include any separation profiles. Within this limited range the results from this tunnel compare favorably with the work of other investigators.

The correlation of experimental results with results obtained by Gruschwitz substantiates his method of attacking the problem. However, no definite conclusions can be based upon results from this tunnel until the recommended changes have been made.
IX. RECOMMENDATIONS

These preliminary tests on this tunnel have made apparent the need for certain additional modifications that should be made in order to obtain more satisfactory operation.

In order to use the entire length of the test plate when the channel is parallel or convergent, a divergent section, not less than four feet long, should be installed between the present diverging section and the test section. The rear panel of this section should be movable and equipped with slots for boundary layer control. The construction of this section should be identical with that of the present test section and low pressure space except for the substitution of plywood for the steel plate.

A blower of larger capacity should take the place of the one now used for boundary layer removal. As previously stated, the present blower will not remove sufficient air from the tunnel and discharge it into the room, and must discharge back into the diverging section just ahead of the fan. This arrangement is not conducive to smooth flow on to the fan. Unsteady flow on to the fan causes unsteady flow in the test section. Furthermore, the blower now in use has a high-pitched whine which, according to Hoerner (Ref.6) increases the turbulence in the boundary layer.
The flow in the present tunnel is too turbulent to allow a satisfactory investigation of the effect of pressure gradient upon the transition region. According to Dryden (Ref. 7) the turbulence decreases with increasing distance between the honeycomb and the test section and with increasing area reduction at the convergence into the test section. The present location of the tunnel makes the first of these methods impossible. Since the ratio of the tunnel height to width should not be smaller, the second method would involve considerable construction work. The substitution of a quieter blower also would probably reduce the turbulence. It would be well to measure the turbulence of the stream with a small sphere, about one inch in diameter, whose critical Reynolds' number has been calibrated by tests in free air.

The static pressure distribution along the test plate should be improved by installing more boundary layer control slots on top, bottom, and back. These would then be opened a smaller amount. This feature is needed especially when a diverging test section is used and more boundary layer must be removed than for the other shapes.

In the present tests it was found that opening a slot in the diverging section about 18 inches directly downstream from the end of the test plate produced a steadier flow in the test section. This probably
eliminated or reduced any separation that occurred in that region and allowed a smoother flow on to the fan. This would indicate the desirability of installing some type of automatic regulating valve at this point.
X. REFERENCES

(1) A.H. Blaisedell: Boundary Layer Flow over Flat and Concave Surfaces. Trans. A.S.M.E. Vol.58, No. 5, July, 1936

(2) E. Gruschwitz: Die turbulent Reibungsschicht in ebener Stromung bei Druckabfall und Druckanstieg. Ingenieur-Archivev 1931


(3) J. Stüper: Untersuchung von Reibungsschichten am fliegenden Flugzeug. Luftfahrtforschung, Band 11, Nr.1, May 1934


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APPENDIX I

Computations for Calibration of Pitot Tube

The velocity profile Fig. 21 from test No. 71 \((V_0 = 45 \text{ mph}, x = 4 \text{ inches}, \text{parallel section})\) was used for comparison with the velocity profile for a true laminar flow. The true boundary layer data was taken from Blasius' solution given in Table 1, page 88, "Aerodynamic Theory," Vol. III. In the experimental profile the boundary layer extends to \(y = 1.7 \text{ mm}\). In the true profile the boundary layer extends to \(\eta = 6.5\). Here \(\eta = y/\sqrt{\frac{\nu x}{u_0}}\). The other values of \(\eta\) were multiplied by the ratio of scales. The ordinate was plotted directly. The square of the true laminar profile was then plotted on the same sheet with the square of the experimental profile. The ratio of the true to the experimental velocity square curves at the same \(y\) is the correction factor. These factors are plotted against \(y\), the distance from the plate, (Fig. 7).
APPENDIX II

Sample Computations Using Series No.5, Station 8 (Test No.151)

(1) Determination of \((v/V)^2\)

Use of Static Pressure Distribution - At \(y = 0.50\) mm the dynamic pressure read from the manometer is 1.18 cm. alcohol. To correct this, using the static pressure at the end of the Pitot tube instead of a Station No. 7, reference is made to the static pressure distribution, Fig.13. The difference between the static pressure at Station No.7 and 2 inches, the length of the Pitot tube (Fig.5), upstream from Station No.8 is + 0.08 cm. alcohol. This value is then subtracted from the dynamic pressure reading.

Application of Pitot Tube Correction Factor - From Fig.7, the correction factor at \(y = 0.50\) mm is found to be 0.810. The corrected dynamic pressure is then

\[
0.810 \times (1.18 - 0.08) = 0.891 \text{ cm. alcohol.}
\]

The ratio of \((v/V)^2\), in which \(v = \) local velocity and \(V = \) stream velocity is then obtained from the ratio of dynamic pressures, for \(y = 0.50\) mm, \((v/V)^2 = 0.891 \div 3.88 = 0.230\).

These values are then plotted as the lower curve in Fig.22. The \((v/V)\) curve is plotted directly from the square root of the faired \((v/V)^2\) curve.
SERIES 5
Divergent Section
\( x = 30^\circ \)

Fig. 22
(2) Determination of $\theta$, $\eta$ and $H$

Planimetering the areas above the $(v/V)^2$ and $(v/V)$ curves and applying the scale correction gives $\Delta$ and $\delta^*$ respectively

$$\Delta = 7.10 \text{ (sq.in.)} \times 0.80 \text{ (mm per sq.in.)} = 5.680 \text{ mm}$$
$$\delta^* = 4.16 \text{ (sq.in.)} \times 0.80 \text{ (mm per sq.in.)} = 3.328 \text{ mm}$$

Subtracting $\Delta - \delta^* = \theta = 2.352 \text{ mm}$.

Referring to the $(v/V)^2$ curve in Fig. 22, $\eta = 1 - (v/V)^2 = 0.550$ when $y = \theta = 2.352 \text{ mm}$.

$H$ is defined as $\delta^*/\theta = 3.328/2.352 = 1.413$.

(3) Determination of $\frac{\theta}{q_\delta}\frac{d\delta}{dx}$

$\xi$, the total pressure at $y = \theta$, is given by

$$(1 - \eta)q_\delta + (p/q_c) q_c$$

in which $q_\delta$ = dynamic pressure of stream, cm. alcohol

$p/q_c$ = ratio of static pressure to stream dynamic pressure at Station No. 1 (Fig. 13)

$q_c$ = Stream dynamic pressure at Station No. 1, cm. alcohol.

$$\xi = (1 - 0.55) \times 3.93 + 0.235 \times 4.98 = 2.92 \text{ cm. alcohol.}$$

The slope of the $\xi$ vs. $x$ curve is measured at each station and multiplied by $\theta/q_\delta$. For Station No. 8

$$\frac{\theta}{q_\delta}\frac{d\delta}{dx} = (2.352/3.88) \times 0.000711 = 0.000430.$$ 

These values are plotted against $\eta$, Fig. 19.

(4) Determination of $\frac{C_d}{q} V^2$

d$\theta$/dx are read directly from the $\theta$ vs. $x$ curves.
\( \frac{dq}{dx} \) are taken as \( \frac{dp}{dx} \) and are found from the \( p/q_0 \) vs. \( x \) curves. These values are then substituted into the momentum equation,

\[
\frac{d\theta}{dx} + \frac{\theta}{q_0} \frac{2 + H}{2} \frac{dq}{dx} = \frac{V_0}{\varphi v^2}.
\]

In the units in which they were measured these values are

\( \frac{d\theta}{dx} = 0.085 \text{ mm per inch} \)

\( \theta = 2.352 \text{ mm} \)

\( q_0 = 3.88 \text{ cm alcohol} \)

\( \frac{dq}{dx} = -0.037 \text{ cm alcohol per inch} \).

To have \( \frac{V_0}{\varphi v^2} \) dimensionless both terms on the left side of the equation must be divided by 25.4.

Then

\[
\frac{V_0}{\varphi v^2} = \frac{0.085 + \frac{2.352}{3.88} \frac{2 + 1.413}{2} (-0.037)}{25.4}
\]

\( = 0.00184 \)

This value is plotted against Reynolds' Number Fig. 20.

(5) Determination of Reynolds' Number \( R_\theta \)

The Reynolds' Number at each station is based upon \( \theta \). To find \( V \), the stream velocity:

\( q_0 \), the dynamic pressure is changed from cm. of alcohol to pounds per square foot.

\( q_0 \), lbs./ft.\(^2\) = \( q_0 \) cm.alcohol

\[
x \frac{\text{Sp.Gr.} \times \text{Density of Water}}{12 \times 2.54} = q_0 \text{ cm.alcohol} \times \frac{0.807 \times 62.4}{12 \times 2.54}
\]
This is substituted into \( \frac{\rho v^2}{2} \), which can be solved for \( V \), where \( \rho \) = standard mass density = 0.00237 slugs per ft.\(^3\) for air at 59°F and 29.92 inches of Hg.

For these tests a chart of \( q \) vs. \( V \) was used to find \( V \). From the chart \( V = 49.8 \) mph.

\[
R_\theta = 29.65 \times V(\text{mph}) \times \theta \text{ (mm)}
\]

29.65 is a constant including the conversion factors for changing from feet per second to miles per hour, and feet to millimeters and the viscosity, assumed a constant value corresponding to an average temperature of 75°F during the tests.

\[
R_\theta = 29.56 \times 49.8 \times 2.352 = 3470.
\]

(6) Determination of \((v/V_c)\)

The value of \( v/V_c \) plotted in Fig. 10 to 12 is obtained by taking the square root of the ratio of \( q \), the dynamic pressure at any point, to \( q_c \), the dynamic pressure of the stream at Station No. 1.

For example, Series 5, Station No. 8 (Test 151), \( y = 0.50 \) mm from original corrected data,

\[
q = 0.892 \text{ cm. alcohol}
\]

\[
q_c = 4.98 \text{ cm. alcohol},
\]

\[
(v/V_c) = \sqrt{0.892/4.98} = 0.423.
\]
Plate I  Entrance Section and Convergence to Test Section
Plate II  Test Section showing Test Plate, Diverging Section and Driving Motor
Plate III  Boundary Layer Control Slot on False Back of Test Section
Plate IV  Pitot Tube Set Up and Static Hole on the Test Plate
Plate V  Leading Edge Slot and Pitot Tube