The Mechanics of Tractor Performance and Their Impact on Historical and Future Device Designs

by

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Abstract

This thesis utilizes a terramechanics-based farm tractor model to predict machine performance. This model is used to reflect on tractor evolution throughout the last century and the physics-based principles that govern tractor performance. Insights from this model and reflection can help designers create new farm tractor embodiments, especially for markets where farming practices and industrial context differ significantly from those that shaped the conventional tractor's major evolutionary steps.

It is shown how the small tractor evolved to its conventional modern form in in the early 1900s in USA pushed not only by suitability to domestic agriculture at the time but also efficiency in contemporary mass manufacturing and symbiosis with the burgeoning automotive industry. The farm tractor model as suggested in this thesis is proven to be in good agreement with published experimental data and historical standarized tractor testing. Inline drive wheels and mounting soil working implements between front and rear axles are identified as high potential design options for adapting the small tractor to modern emerging markets where draft animals are the dominant source of draft power.

Thesis Supervisor: Amos G. Winter, V
Title: Assistant Professor of Mechanical Engineering

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Chapter 1

Introduction

1.1 Contributions

This thesis provides tools and background to facilitate the creation of a novel farm tractor specialized to small farms (<2Ha), which represent 72% of the world's farms (Table 1.1). Contributions include:

- Description of evolution process to arrive at current conventional tractor (Chapter 1).
- Tractor mathematical model for exploring large design space with minimal experiments (Chapter 2 and Chapter 3).
- Physics-based observations on past tractor designs (Chapter 4).
- Design suggestions for creation of tractors well suited to small farmers in developing countries (Chapter 4).

1.2 Tractors are ubiquitous in high productivity farming

Tractors are an icon of industrialized, modern farming and their presence has been noted as a differentiator between farming in developed vs. developing countries [1]

[2]. In 1950, the USA Census Bureau stated the benefits of mechanizing American agriculture [3]:

" The increased use of mechanical power on farms has influenced agriculture more than any other factor during the present century. The changes from horses and mules to tractors for farm work has made available an acreage of cropland greater than the total increase in cropland during the half century for the production, directly or indirectly, of meat, milk, eggs, and other food. The use of the tractor and related equipment for farm work, and the use of farm trucks for hauling and automobiles for traveling have increased the rate at which farm work is done and has increased the capacity of agricultural workers, enabling considerable numbers of farm workers to leave the farms or to engage in non-farm work, notwithstanding considerable increases in total farm production. Tractors and power-operated equipment have made an increase in the size of farms possible. The substitution of tractors for animal power has also made available additional power for farm use. With a tractor, the farmer of 1950 probably turned out twice the amount of farm products for market as his father did with a team of horses 50 years earlier. Moreover, less of the farmer's time was required to care for the tractor than to raise feed for and to produce and care for horses that were replaced by the tractor."

There is high correlation worldwide between farm productivity and available tractor power [4] [5] [6] [1].

1.3 Brief History of the Farm Tractor

The conventional small tractor produced today found its form mostly in the USA between 1910 and 1940. This "conventional tractor" or "dominant tractor design" is a four wheeled tractor, with front steering, rear wheel drive, and a trailing implement behind the rear axle. The design has changed little in the following 80 years [7] [8] [8] [9] [10].

In 1903 the term "tractor" was first coined by the Hart Parr Gasoline Engine company of Charles City, Iowa. In the USA, the Homestead Act of 1862 was still ongoing with minor revisions and motivated farmers to extend westward from the northeastern cities, rapidly expanding the amount of available arable land [7]. At the time, horses and mules were the primary source of draft power in the burgeoning American farming industry. Tractors were more capable at opening new land for farming but also unwieldy and expensive.

During the late 1910's the agricultural industry in the USA became highly profitable as food exports increased dramatically to feed Europe and Russia during and after WWI. Farming land prices were on a sharp rise as farmers had surplus money and looked to increase production by expanding their properties. Farms grew in number and size yet farm labor was more scarce as the rural youth went to fight in WWI and later returned preferring an urban lifestyle [citation needed]. Farm tractors became an attractive way to multiply the capacity of each laborer [3].

In the year 1920, 166 companies in the USA manufactured farm tractors and had a combined year production of 203,207 tractors. These were dramatic increases from 1910, when only 15 farm tractor companies were in business and had a combined production of 4,000 tractors [11]. These 166 companies were fighting to define the shape of the "farm tractor" and to distinguish themselves through innovative designs [12] [13]. A sample of 24 production tractor layout designs from the 1910 to 1920 period with their respective manufacturers can be found in Figure 1-1. Layouts varied widely in traction gear (tracks, wheels, or drums mostly), number of axles, driver position, tool position, and overall dimensions.

In the 1910's the rapidly expanding tractor industry was learning from the more readily availabe feedback from farmers and taking some engineering lessons from its younger but more refined cousin, the automobile [12]. Important obstacles to more tractor adoption were initial cost and need for more versatility in usage. The demand for a less expensive, smaller, lighter tractor became apparent during this decade [14] [15]. It was often the case that the farmer who owned a tractor still had to own horses, which were more manageable and smaller, for cultivation operations [13][12]. Very large tractors that had been used to open large fields in the expanding West would lay rusting with little or no use after that initial heavy ploughing operation

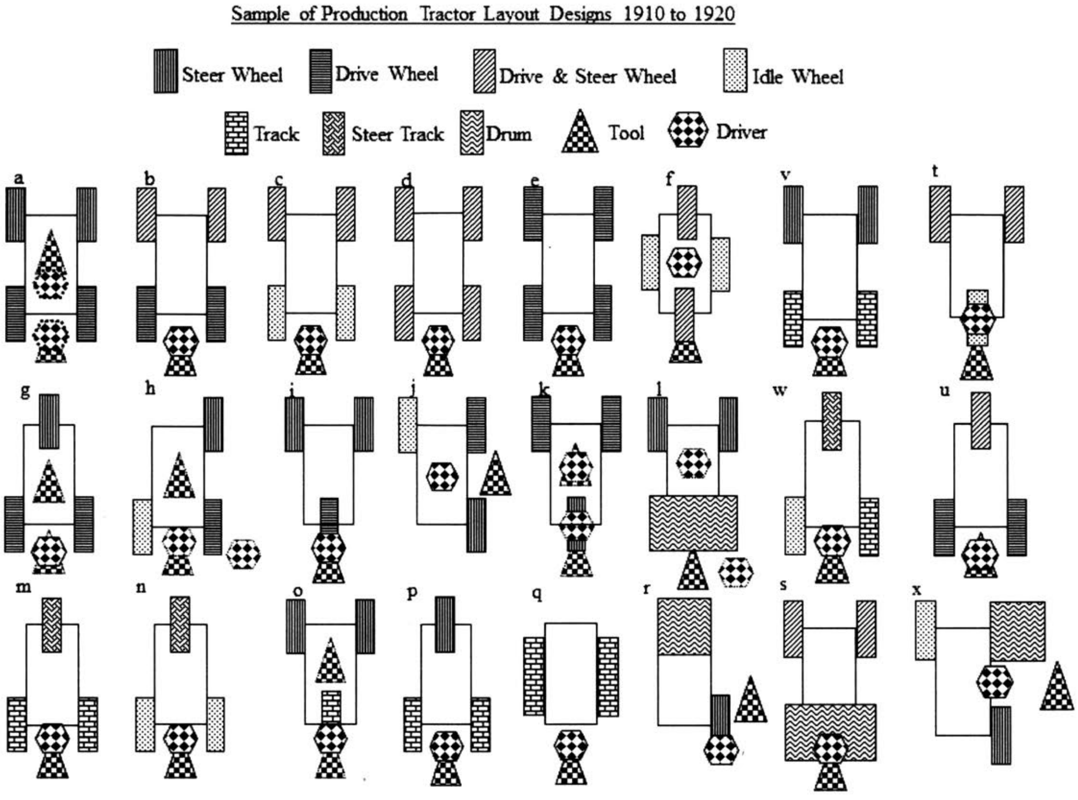


Figure 1-1: **Sample of tractor design layouts from 1910 to 1920. A list of some production tractors using each layout can be found in Appendix A.**

[7] [16]. The first tractor to meet the demands of the common farmer in size and price was offered by the "Bull Tractor Company" in 1913. This lightweight tractor had three wheels with a single drive-wheel and had an initial price comparable to a team of horses. By 1914 it was the best-selling tractor in the country [7]. The tractor industry still had reliability issues and production volume challenges that it would learn to solve partially from automobile experts becoming involved. In 1917 Henry and Edsel Ford introduced Fordson tractors, and immediately started producing at a volume and price that left them with little competition [7].

By 1921, a dramatic shift had occurred. European and Russian agriculture had recovered enough to make them largely independent of imported of food. Farmers in the USA had misjudged demand and severe food overproduction caused the prices of agricultural produce to plummet. Farmers suddenly found themselves unprofitable and with outstanding bank loans used to purchase farmland that had since collapsed in value. Farm tractor production plunged from 203,277 in 1920 to 68,029 in 1921 [11].

The great depression and stock market crash would keep American farmers in a difficult position through the 1920's and forced tractor manufacturers to adapt to a low cash flow style of farming. In February 1922, the "Tractor Price Wars" started when Ford (known then as Fordson Tractors) slashed the price of its Model F from $625 to $395. Over the next 20 years a fiercely price-competitive tractor market would see manufacturers converge on similar designs [7]. Many manufacturers would disappear in this "war", from 166 manufacturers in 1920 to only 38 in 1930. However, combined production had rebounded to 196,297 in 1930, very similar to the level of 1920[11]. Yearly total production of American tractors would keep rising until reaching a peak in 1951, when 564,000 tractors were manufactured. By 1950 there were over 3.6 million tractors operating in American farms (about 1 tractor for every 6 people living on a farm) and the internal combustion engine had become the primary source of draft power for farmers [3].

Some major innovations between 1920 and 1940 that shaped the modern small tractor are [10]:

1921 - First Nebraska Tractor Test is performed. These tests would go on to become the national, and later international, standard for tractor testing. The test's prominence would make it a major quantifiable target for tractor manufacturers.

1922 - International Harvester introduces the Power Take Off (PTO), allowing the tractor's engine to power farming implements through a rigid shaft instead of using a belt. Implement manufacturers rush to take advantage of this innovation.

1925 - International Harvester introduces its Farmall "General Purpose" (GP) tractor. The Farmall series would become the best-selling tractor series ever in the USA. Compared to most other tractors on the market it:

- Was Lighter
- Had higher ground clearance

- Utilized smaller front wheels (enabling tighter turns)
- Had adjustable track width
- Was advertised for cultivating, plowing, and cutting.

1927 - John Deere introduces "Power Lift", allowing the farmer to use the engine's power to raise and lower farming implements. This reduced the drudgery of tractor usage and increased tractors' field capacity.

1932 - Firestone and Allis Chalmers introduce the pneumatic rubber tire. This allowed tractors on the growing network of paved roads (where steel, lugged wheel were not permitted) and enabled farmers to operate at higher speeds more comfortably.

Circa 1935 - Diesel engines are advanced enough to become standard in farm tractors, lowering fuel costs.

1937 - Henry Ford licenses the now standard "three point hitch" design from Harry Ferguson. The "N series" tractors created after this agreement would culminate in the 8N, which is the best-selling single tractor model ever in the USA.

After the 1940s, a small tractor design now common to major brands would rapidly overtake the market and finish replacing animal power in farms. By 1945, more work was being performed by tractors on farms than horses and mules combined. By 1955 the total number of tractors on farms was greater than the total number of horses and mules combined. This common design was as mentioned at the beginning of Section 1.3: four wheeled, with front steering, rear wheel drive, and a trailing implement behind the rear axle. This tractor design from the 1940's still remains as the standard layout for modern small tractors [7] [8] [8] [9] [10]

1.4 Evolution of small tractor to consolidated design

The modern, conventional small tractor is four wheeled, with front steering, rear wheel drive, and a trailing implement but its predecessors came in larger variety of configurations. A sample of these configurations is shown in Figure 1-1. This section exemplifies some important milestones and configurations in the evolution towards the modern small tractor. Figure 1-2 illustrates tractors related by engineering desing from 1902 to 1939. A written description of these relationships is provided as well.

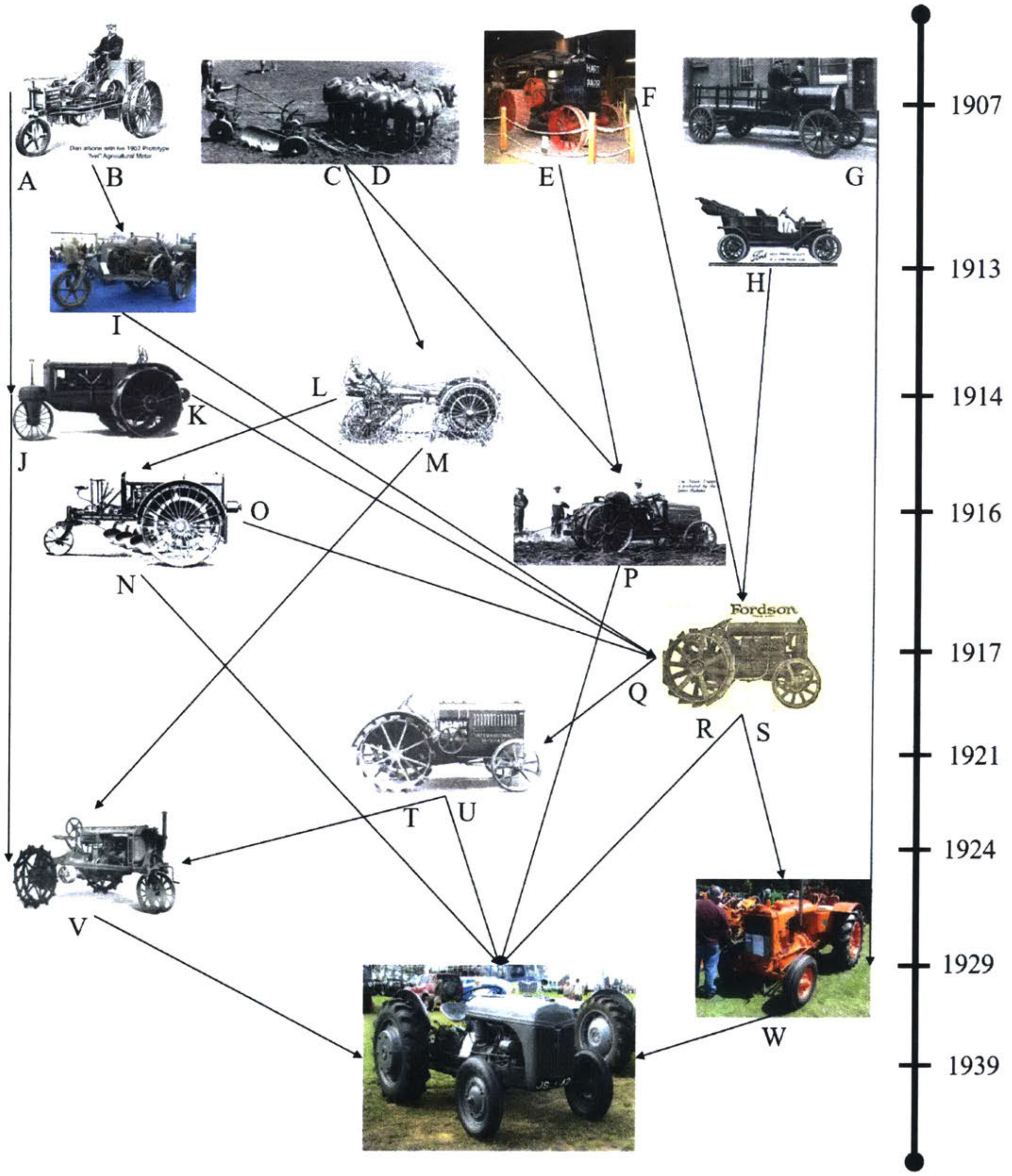


Figure 1-2: **Graphic chronology of tractor evolution into conventional small tractor design. Connections (marked by letters) are described in section 1.4. More data on vehicles may be found in Appendix B. The photo credits are referenced from the origin of each connection: A [17], C [18], E [19], G [20], H [21], I [22], J [23], L [24], N [25], P [26], Q [27], Q [28], V [29], W [30], Bottom [31]**

A) 1902 Ivel Tractor to 1914 Wallis Cub: Lightweight three-wheeled vehicles.

B) 1902 Ivel Tractor to 1913 Bull: Lightweight, mechanically simple, three-wheeled vehicles. Exposed "I-beam" frame.

C) Horse Plow to 1914 Moline Universal: Driver is behind or above implements which facilitates supervising field operation. Advertising boasted of user-friendly layout being similar to horses.

D) Horse Plow to 1916 Nilson: "Fulcrum and lever" attachment system increases downward force on rear wheels as draft force increases. Compared to other tractors, tool is attached higher above the rear axle but pulled same distance behind tractor. This increases the downward vertical force generated by pulling the implement, it also increases the lever-arm that the backward forces have to rotate the vehicle around the rear wheels ground contact point. Nilson advertises that tool pulling angle is similar to that of horses pulling a plow. More details on this design can be found in Chapter 4.

E) 1907 Hart-Parr 30-60 to Nilson 1916: Similar layout with two front steering wheels. Nilson attempts to achieve high pulling force of heavier vehicles (like the Hart-Parr) in lightweight vehicle by taking advantage of draft force and increasing wheel rear area (the Nilson has three rear wheels, the central one being a drum). F) 1907 Hart-Parr 30-60 to 1917 Fordson: The Fordson miniaturized the "Praire Style" four wheel tractors like the Hart-Parr. This smaller tractor was more versatile and less expensive.

G) 1909 Avery Farm & City to 1929 Allis-Chalmers U: When the Avery was made, the farm "tractor" still did not have a definite shape or use on the farm. The Farm & City placed high importance on road haulage as well as plowing and made a compromise between both. The tractor was supplied with wooden "plugs" that could be placed on steel wheels to prevent them from damaging paved roads (which made the wheels road-legal and more comfortable).

H) 1908 Ford Model T to 1917 Fordson: Ford's automobile engineering and assembly line manufacturing expertise (along with the the brand's fame) made it a near-immediate leader in the tractor industry.

I) 1913 Bull Tractor to 1917 Fordson: The Bull tractor was novel in its low cost and light weight. It became the selling tractor in the country within a year of launch. Four years later, the Fordson will repeat the same feat due to the similar characteristics but in a more reliable package.

J) 1914 Wallis Cub to 1924 International Harvester (IHC) Farmall: Both vehicles shared a tricycle layout. The Farmall utilized the U-frame design that had been pioneered by Wallis. This design utilized the transmission and engine castings as structural components, which yielded a lighter and less expensive tractor.

K) 1914 Wallis Cub to 1917 Fordson: The Fordson utilized the U-frame design that had been pioneered by Wallis. This design utilized the transmission and engine castings as structural components, which yielded a lighter and less expensive tractor.

L) 1914 Moline Universal to 1916 Square Turn: Like the Moline, the Square-Turn offers a similar driver experience to horses. The Square-Turn takes the experience further by allowing each drive wheel to be controlled independently, this meant the wheels could be driven in opposite directions for in-place turning. This tight turning was important in the smaller 80-100 acre farms that were common in Nebraska, where the Square-Turn was designed and made.

M) 1914 Moline Universal to 1924 International Harvester (IHC) Farmall: The Moline was the first popular tractor that attempted to be useful for all farm operations. This was a tractor that could not only do plowing but also cultivating once crops were growing. The Farmall would later become the best-selling tractor series ever in the USA due in good part to its "General Purpose" design. The Moline was also a thoroughly modern tractor for its time, using unusually advanced electric features. The Farmall would later also be a very modern tractor for its time.

N) 1916 Square Turn to 1939 Ford 9N: The Square Turn provided powered control of the tillage tool's vertical position (mounted under the frame between both axles). The use could control the tool even when the tractor was static, which was unusual at the time. The Fordson was the first popular tractor with hydraulics, which allowed the user a high level of control over the tillage tool's position.

O) 1916 Square Turn to 1917 Fordson: Both of these tractors were intended to be useful to the small farmer as well as the large farmer. The Square Turn achieved this through tight turning, the Fordson did it through a compact overall size, a large dealer network, and a low selling price.

P) 1916 Nilson to 1939 Ford 9N: The Nilson's "Fulcrum and Lever" rear tool attachment system utilized tillage draft forces to increase downward load on the tires and therefore traction. The "Three-Point Hitch" in the Fordson would utilize the same principle but include powered-tool control. More details on this evolution can be found in Chapter 4.

Q) 1917 Fordson to 1921 International Harvester (IHC) 15-30: The success of the Fordson tractor pushed other manufacturers to make smaller, less expensive, four-wheel tractors. The 15-30 was IHC's first significant response to the Fordson.

R) 1917 Fordson to 1939 Ford 9N: Ford would stop American production of the Fordson in 1928 but would come back to take advantage of the lessons learned then and novel technologies in 1939 with the Ford 9N.

S) 1917 Fordson to 1929 Allis-Chalmers U: When Ford stopped making the Fordson in 1928, suddenly hundreds of dealers were left without a product to sell. These dealers formed the "United Tractor and Equipment Corporation" and contracted Allis-Chalmers to make a small, modern tractor similar to the Fordson they could sell (the model name "U" stands for "United").

T) 1921 International Harvester (IHC) 15-30 to 1924 Farmall: The 15-30 was IHC's quick response to the Fordson's market success but eventually they released the highly novel Farmall which would go on to replace the Fordson as the best-selling tractor. The Farmall included a Power Take Off (PTO) shaft, which had first been introduced to the market by the 15-30.

U) 1921 International Harvester (IHC) 15-30 to 1939 Ford 9N: The IHC 15-30 was the first tractor with a Power Take OFF (PTO) shaft. This was a very popular feature that would become standard in the industry and was included in the Ford 9N.

V) 1924 International Harvester (IHC) Farmall to 1939 Ford 9N: The Ford 9N was a "General Purpose" tractor, a category that the Farmall had made the biggest one in the American tractor market.

W) 1929 Allis-Chalmers U to 1939 Ford 9N: The Allis-Chalmers U was the first farm tractor with pneumatic rubber tires. A feature that would become a market standard and that was present in the Ford 9N.

1.5 There is an untapped market for which tractors could be designed

In developing countries, more acres of farmland are tended to with animals than farm tractors. Adoption of conventional small tractors has been delayed by an initial cost higher than animals but also by a functionality that differs starkly from that of animals and that is not as well suited to remote, small farms [32]. This may not come as a surprise when one considers the small tractor has remained largely unchanged worldwide since its evolution mainly in the USA during the first half of the 20th century (as described in Section 1.3). Farms in early 20th century USA were smaller than those in the country today but were nonetheless significantly larger than those found in most developing countries (Table 1.1 and Table 1.2). In addition to this farm size discrepancy, driving the conventional tractor's form was not only farming operations but also suitability in a different era for manufacturing and distribution at low cost and high volume, success at standardized testing, and use on paved roads. The conventional tractor shape is not necessarily well suited to the style and scale of farming currently practiced in developing countries with farm animals.

Farm Size	Percentage of World Farms
<1Ha	72%
1-2Ha	12%
2-5Ha	10%
5-10Ha	3%
10-20Ha	1%
>20Ha	2%

Table 1.1: Data from FAO for farm sizes worldwide [citation needed]. The sample is from 460million farms in 111 countries.

Year	Farms <1.2 Ha	Farms >1.2, <3.6 Ha	Avg. Farm (Ha)	Total Farms
1900	0.73%	3.95%	59.48	5,739,000
1910	0.28%	5.25	55.85	6,366,000
1920	0.32%	4.50%	59.89	6,454,000
1930	0.70%	5.75%	63.54	6,295,000
1940	0.61%	8.35%	70.82	6,102,000
1950	1.45%	7.62%	87.41	5,388,437
1960	2.13%	4.45%	122.62	3,710,503
2012	11% (aprox. 0% of farmland)		175.6	2,109,303

Table 1.2: Data from USDA agricultural census [citation needed][citation needed].

1.6 We want to know how tractors work parametrically, reflecting on the strengths and drawbacks of previous designs

A new tractor layout, developed from the ground up for small farmers (<5acres) currently using animals, may help accelerate developing world tractor adoption. To significantly lower the purchase cost of the conventional tractor configuration, it is necessary to reduce the tractor's weight and power but that renders a conventional tractor incapable of generating sufficient drawbar pull [33]. Empirical models are common in tractor design and based on hundreds of experimental datapoints from production tractors [34][35]. However, empirical models are most applicable to designs of similar dimensions to those tested since they are based on experiments and not the fundamental physics of the tire-soil interaction. Chapter 2 will describe a terramechanics-based farm tractor model to parametrically study the performance of the conventional tractor and to enable the identification of novel tractor configurations in a large design space not constrained by similarity to existing tractors.

Chapter 2

Tractor Theory

Important performance improvements can be attained in off-road vehicles by predicting soil-tire interactions. In the case of farm tractors this usually means minimizing power losses and damage to soil while maximizing tractive force. The modelling of a tractor on soil can be separated into two related parts: calculating the distribution of forces at all tires (which hold the tractor afloat and propel it forward) and calculating the tire deformation, tire sinkage, and tire slippage at each individual tire. For force distribution: This thesis contributes a strategy for distributing the forces among tires without requiring iterative calculations in parallel with solving for individual tire-soil performance. Reducing the number of calculations per iteration facilitates design exploration by allowing more configurations to be tested in the same amount of time. For individual tire-soil interactions: This thesis uses a semi-empirical model proposed by J.Y. Wong [36].

2.1 Qualitative description of importance of soil-tire interaction in tractor design

Converting engine power to drawbar power (i.e. pulling force times forward speed) results from converting engine power into traction force at the wheel-soil interface and overcoming all internal mechanical losses plus motion resistance at the tire-soil interface. While drivetrain mechanical losses in a small tractor can be under 5% , power conversion at the tire-soil interface usually involves losses of 30 to 60% [37]. A refined terramechanic design can reduce the power lost at the soil-tire interfaces, something especially critical for tractors that may initially appear to be underpowered. The two major causes of power loss are soil deformation and slippage at the tire-soil interface [36]. The effects of soil deformation from wheeled vehicles are observed in the ruts they leave behind. As the wheel rolls forward it deforms soil ahead of it (known as "bulldozing"), this deformation requires energy but achieves no useful work. Slippage occurs when the tangential speed of the tire contact points is faster than the forward speed of the vehicle. Presence of at least minimal slippage is unavoidable because for a thrust force to occur the tire must exert a shear force on the soil (therefore causing soil deformation). When the shear strength of the soil is low relative to the traction being generated, the shear stress may result in large shear deformation and thus higher slippage (which is a source of energy loss). (Figure 2-1) synthetizes published experimental studies on the flow of granular, frictional soil under a driven, rigid wheel [36] [38] [39]. In it, the flow of soil downward (sinkage), forward (bulldozing), and backward (slippage) can be observed.

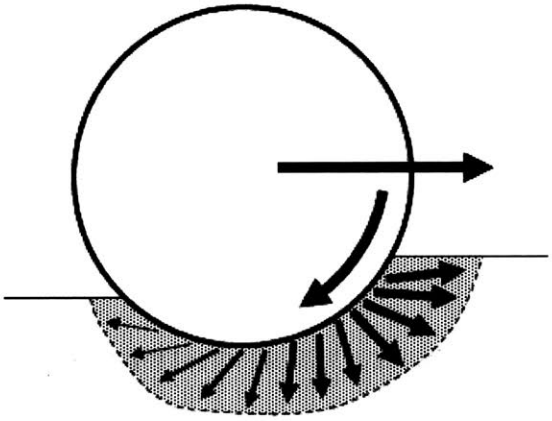


Figure 2-1: **Granular material flow under driven rigid tire. Arrows under soil represent flow speed and direction. The dotted line represents shear interface.**

An efficient terramechanic design must strike a balance between sinkage and slippage The amount of power lost to slippage and lost to bulldozing are both correlated to ground pressure but with opposite effects [40][36]. As ground pressure increases the shear strength of soils with a frictional component (most natural soils) increases and thus less shear deformation is provoked by a given shear stress. This reduces energy lost to slippage. The counterpoint is that as ground pressure increases so usually does the sinkage of the tire into the soil. This increases the energy lost to bulldozing. Ground pressure is affected by tire vertical load but also by contact patch dimensions. A larger contact patch will reduce contact pressure. This larger area can be obtained by increasing tire diameter, and thus contact length for a given sinkage, increasing tire width, and/or increasing tire compliance in the case of deformable tires. Increasing tire width will increase the frontal area of the tire sinkage pattern, this will be observed as wider tire rut which is a sign of more tire bulldozing occurring for a given rut depth. Increasing tire diameter yields similar smaller contact patch benefits than increasing width [36] but does not increase rut width, on the other hand, larger diameter tires can come with packaging and inertial challenges. Increasing tire compliance can increase both contact patch length and width for a given tire load but usually comes at the cost of more mechanical losses within the tire and higher slippage at the tire-soil interface due to tire deformation. A final note on contact patch shape: if two tire-soil ruts have the same cross-sectional areas, the deeper rut will experience higher bulldozing resistance (despite being less wide) all else being equal because soil strength increases with depth. The ideal tire properties (vertical load, width, diameter, compliance) minimize the sum of power consumed by slippage, bulldozing and tire internal losses, while of course remaining mechanically and economically practical with the tractor system.

2.2 Analytical model for interaction of single drive tire with soil

The tire-soil model suggested here is as described by J.Y. Wong [36]. This model is commonly accepted in terramechanics and has been improved for accuracy by several groups, but often at the expense of requiring more experimental data [41].

In Figure 2-2, a stress distribution at the soil-tire interface is shown for a rigid smooth wheel in a homogeneous soil. It is helpful to separate this stress into normal stress (normal to the wheel perimeter) and shear stress (tangent to wheel perimeter). Note that the soil is deformed plastically, as the wheel moves through it and thus the soil surface is lower behind the wheel than ahead of it. All weight-bearing wheels generate a normal stress on the soil that supports the vehicle vertically. Only braked or powered wheels generate significant shear stress on the soil. Calculating the net vertical force (for flotation) and the net horizontal force (for thrust if driven or resistance if idle) for a specific tire configuration (including torque and air pressure) will be the goal of this section.

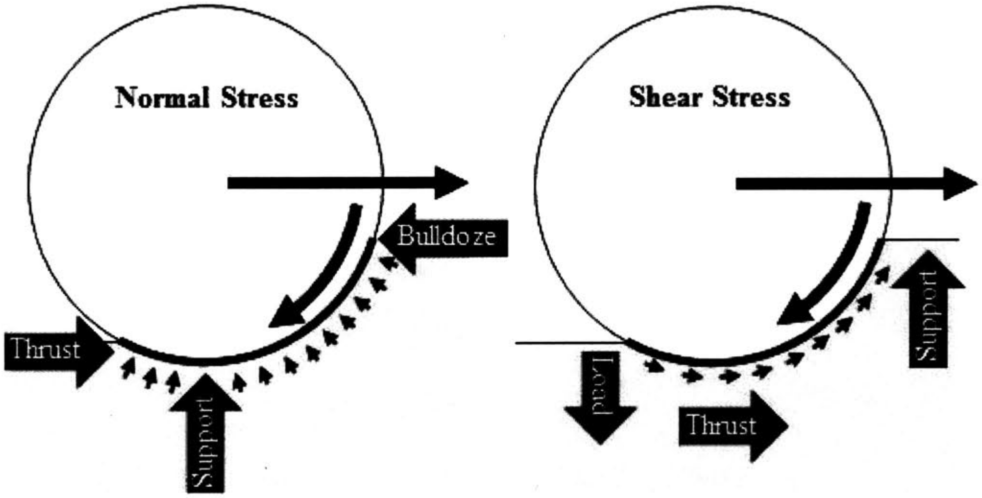


Figure 2-2: **Stress under rigid tire.**

In agricultural soils, the pressure required to penetrate into the ground increases with depth. Soil pressure as a function of depth is commonly expressed in terramechanics using Bekker's or Reece's equation [42][43]. Reece's equation is

$$p = (ck'_c + w\gamma_s k'_\phi)(z/w)^n, \tag{2.1}$$

where,
p =soil pressure
c =soil cohesion
k'_c =cohesion constant
w =tire width
γ_s =soil bulk density
k'_ϕ =friction constant
z =depth below soil surface
w =width
n =depth exponent

The shear strength of frictional soil increases with pressure. This means that a wheel operating on soil may be interacting with soil of different strengths along its perimeter that is sunk into soil. Mohr-Coulomb shear strength equation as a function of normal pressure is

$$s = (c + ptan(\phi))(1 - e^{(} - j(i)/k)), \tag{2.2}$$

where,
p =soil pressure
c =soil cohesion
ϕ =soil friction angle
k =shear modulus
j =shear deformation
i =slip at interface

To calculate the total reaction forces experienced by the tire when contacting soil, the shear and normal stresses must be integrated along the tire's casing. If the tire is compliant and assumed to take the idealized form shown in when deformed, it can be separated into three sections: front circular arc of tire, flat horizontal section at bottom of tire (the depth at which the tire total pressure matches the soil pressure), and rear circular arc of tire.

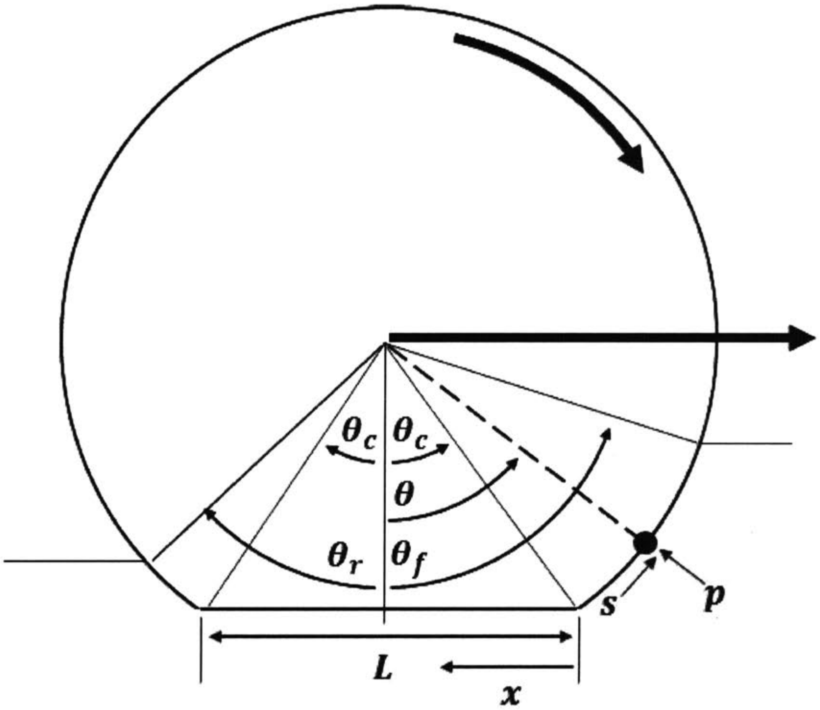


Figure 2-3: **Parameters of tire perimeter for calculation of forces at interface.**

The net vertical force may then be mathematically expressed as

$$\begin{aligned} V =& wR\int_{\theta_c}^{\theta_f}[p(\theta)cos(\theta) + s(theta, i)sin(theta)]d\theta + w2RP_t sin(\theta) \\ &+ wR\int_{\theta_c}^{\theta_r}[p(\theta)cos(\theta) - s(\theta, i)sin(\theta)]d\theta] \end{aligned} \tag{2.3}$$

The horizontal force will be expressed as

$$
\begin{aligned}
H =& wR\int_{\theta_c}^{\theta_f}[-p(\theta)sin(\theta)+s(theta,i)cos(theta)]d\theta \\
&+w\int_{0}^{L}s(\theta)dx \\
&+wR\int_{\theta_c}^{\theta_r}[p(\theta)sin(\theta)+s(\theta,i)cos(\theta)]d\theta]
\end{aligned}
\tag{2.4}
$$

For both equations variables are define as

H =drawbar pull

V =vertical ground reaction

w =tire width

R =tire radius

P_t =tire pressure

2.3 Multi-pass effect for tires

The pressure exerted on agricultural soil by tires affects the soil's mechanical properties. In the farming context, this is often discussed as undesirable soil compaction that hinders crop growth, but it can also be leveraged to improve tire traction. Each tire pass strengthens (compacts) the patch of soil it runs on, making it a better surface for trailing tires [44] [45][36]. If all tire passes are identical, compaction will be highest after the first pass and become asymptotically less after subsequent passes. It is less detrimental to crop yields to drive over the same patch of soil in the field multiple times than it is to drive over more areas of soil on the field only once; leveraging this is called "Controlled Traffic" and has been proven in farm fields across the world [46] [47] [48] [49] [50] [51]. Figure 2-4 is an idealized diagram showing the interactions of inline drive tires on soil during loading, unloading and reloading.

Soil response to reloading can be elucidated by the simple experiment of a rigid plate applying pressure on natural terrain (Figure 2-5).

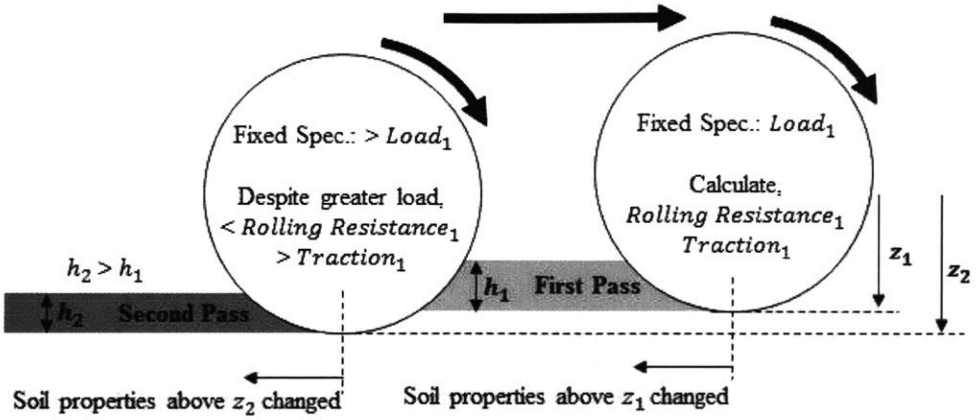


Figure 2-4: **Illustration of tire-soil interaction and multi-pass effect used in analysis**

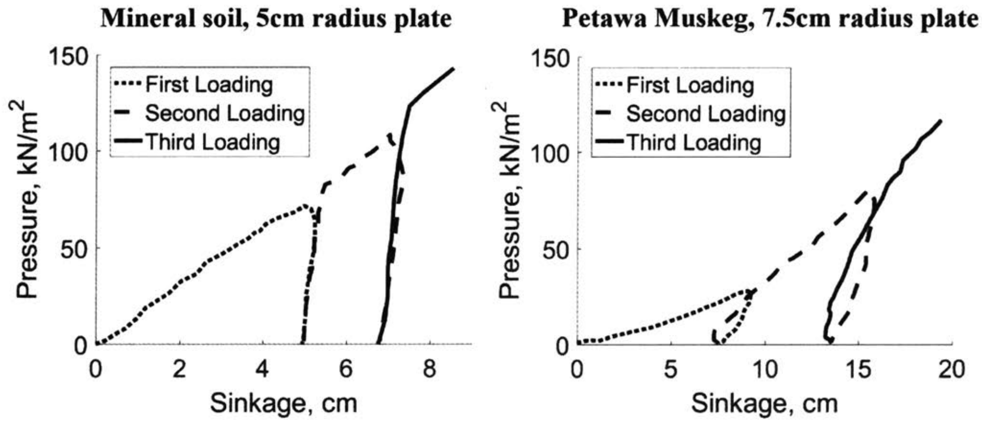


Figure 2-5: **Effect of repetitive loading on natural soil deformation and stiffness. Data from [36]**

2.4 Conventional tractor dimensions and relevant forces

As discussed in sections Section 2.1, 2.2 and 2.3, the response of the soil to the tractor on it depends on the forces the tractor is applying to the ground, tire/wheel dimensions, and soil history. Calculation of the net forces to solve for at the tire-soil interface requires modelling of the full tractor system. Under the assumption of the tractor being a rigid body and its wheels will rest on the same plane with zero roll angle (roll being the the tractor's longitudinal axis), the tractor free-body diagram can be simplified to include only:

- overall center of mass location and magnitude
- draft tool force direction, magnitude, and origin (center of pressure)
- location of ground contact points
- tractor orientation with respect to gravity (ground slope)

An example of this dimensions for a conventional tractor is shown in Figure 2-6.

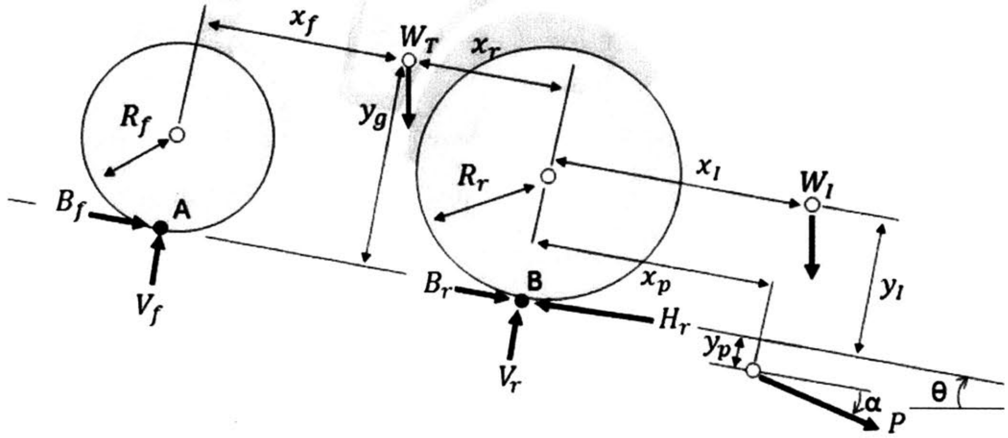


Figure 2-6: **Full body diagram for farm tractor in 2D**

The torque around the center of rotation at point B is then:

$$T = P(y_p cos(\alpha) - x_p sin(\alpha)). \tag{2.5}$$

Overall, the net vertical force on both front wheels is:

$$\begin{aligned} V_f = \frac{1}{x_f + x_r}(W_T(x_r cos(\theta) - y_g sin(\theta)) \\ + P(y_p + cos(\alpha) - x_p sin(\alpha)) \\ + W_I(-x_I cos(\theta) - y_I sin(\theta))), \end{aligned} \tag{2.6}$$

and the net vertical force on both rear wheels is:

$$\begin{aligned} V_r = \frac{1}{x_f + x_r}(W_T(x_r cos(\theta) + y_g sin(\theta)) \\ + P(-y_p + cos(\alpha) + x_p sin(\alpha)) \\ + W_I((x_I + x_r + x_f)cos(\theta) + y_I sin(\theta))). \end{aligned} \tag{2.7}$$

It is assumed in the conventional tractor configuration that only the rear wheels are driven. To move the tractor forward at a constant speed, the rear tires must provide the net horizontal force:

$$H = B_f + B_r + Pcos(\alpha) + (W_T + W_I)sin(\theta). \tag{2.8}$$

The calculation of the actual wheel torque necessary to achieve H_r and the calculation of resistance forces B_f and B_r, requires an analysis as described in section 3.1, 3.2 and 3.3.

2.5 Proposed generalized vertical load per tire calculation

The calculation of tire vertical reaction forces shown in section 2.6 can be generalized for any statically stable configuration. A stable configuration in 3D space has three or more wheels (support points in this study) on the ground with the vehicle's effective center of gravity located inside the convex hull of all contact points. For three support points, the problem is statically determinate and can be solved as a simple rigid body problem where the net forces and moments over the vehicles are zero. With four or more contact points (a fairly common occurrence) the problem becomes statically indeterminate. An attempt to solve this as a rigid body will yield four or more unknowns but only three equations. The conventional way of solving this is to calculate the stiffness of the structure and contact points to find the configuration of reaction forces that yields the least work (i.e. the least energy spent deforming tractor and soil). In the case of a tractor on deformable soil (potentially with pneumatic tires), a useful simplification for computer calculations is to assume deformation will only occur at the wheels and soil, and that all support points have equal stiffness properties. The problem is then simplified to finding all solutions that generate zero net force and zero net moment around vehicle, and then identifying the solution with the least energy spent deforming soil and tires. To efficiently estimate the energy spent deforming tires and soil at each contact point, it is recommended to select an exponent C. The energy spent deforming tires and soil is then:

$$E = \sum_{i=1}^{n} V_i^C, \tag{2.9}$$

where,

E =Energy spent deforming support points

V_n =vertical ground reaction force at tire "n"

C =exponent to estimate energy deformation. Use 2 if idealizing support points as springs.

If multiple solutions exist for the reaction forces at the tires (which will likely be the case for four or more tires) then the solution recommended is the one that minimizes the energy. The preferred solution V comes from Equation 2.10.

$$\min_{V} E \tag{2.10}$$

where,

V =vector with vertical ground reaction forces for all tires

We can calculate the set of solutions for reaction forces at each tire for any vehicle with three or more tires. This can be stated as a linear system:

$$pb + R = 0 \tag{2.11}$$

where,

p =particular solution to problem. nx1 vector.

b =Forces and torques not caused by support points. 3x1 vector

R =Forces and torques caused by support points. 3x1 vector

p will be solved for from R and b. The elements in R and b respectively are:

$$b_1 = \text{nx1 vector of ones} \tag{2.12}$$

$$b_2 = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \tag{2.13}$$

$$b_3 = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \tag{2.14}$$

$$b = (-1) \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}, \tag{2.15}$$

and

$$F = g(m_I + m_T + m_d) + P_z, \tag{2.16}$$

$$M_y = x_I g m_I + x_P P_z + z_P P_x, \tag{2.17}$$

$$M_x = x_I g m_I + x_P P_z + z_P P_y, \tag{2.18}$$

$$R = \begin{bmatrix} F \\ M_y \\ M_x \end{bmatrix}, \tag{2.19}$$

where,

n =number of tires (support points)

g =gravity acceleration

m_I =mass of implement

m_T =mass of tractor

m_d =mass of driver

M_x =moment around vehicle CG about x axis

M_y =moment around vehicle CG about y axis

P_x =longitudinal draft force from tillage tool

P_y =lateral draft force from tillage tool

P_z =vertical draft force from tillage tool

x_P =longitudinal distance from vehicle CG to tillage tool COP

z_P =vertical distance from vehicle CG to tillage tool COP

x_I =longitudinal distance from vehicle CG to tillage tool CG

x_i =longitudinal distance from vehicle CG to tire "i" CG

y_i =lateral distance from vehicle CG to tire "i" CG

Since the equation system may be indeterminate, the null space is then calculated. This can be achieved through singular value decomposition or use of the "null()" function in MATLAB,

$$Z = null(R). \tag{2.20}$$

All mathematically valid solutions for the vertical reaction forces V at each tire can then be obtained by inputting values of q in

$$V = p + Zq. \tag{2.21}$$

The preferred solution V is that which minimizes deformation energy (Equation 2.10) as defined in Equation 2.9.

2.6 Proposed generalized horizontal load per tire calculation

For soils with a frictional component (most natural soils), there is an approximately linear relationship between a large range of values for normal pressure and for maximum traction generated (Figure 2-7). This can be leveraged to accelerate calculations of tractor slip, sinkage, and power by distributing the drawbar pull amongst all tires proportionally to the vertical reaction force on each tire.

The drawbar pull that each drive tire will be "assigned" to pull can then be calculated as:

$$H_n = P_x \frac{D_n}{\sum_{i=1}^{k} D_i} \tag{2.22}$$

where,

H_n = Drawbar pull tire "n" must generate.

D_n = Vertical reaction force at tire "n"

k =number of drive tires in vehicle.

This idealizes the power distribution among the tires to get an estimate of the maximum drawbar pull a tractor configuration could achieve. For example, if a tractor has a simple differential between two drive tires, the drawbar pull distribution strategy suggested here would not consider that both tires must receive the same torque.

2.7 Variable sensitivity of tractor-soil model as implemented

The model presented in this section is highly non-linear and accepts many tractor parameters as inputs. To give the reader a better feel for the influence of key design parameters on a small tractor's performance, sensitivity studies for drawbar pull and tractive efficiency are shown in Figure 2-7 and 2-8. Soil data used for this sensitivity analysis was obtained from [52].

Tractive efficiency is the efficiency in converting power at the wheel axle into useful work. It is defined as:

$$\eta_{tract.} = \frac{F_pulling * V_forward}{P_wheels}, \tag{2.23}$$

where,

$eta_{tract.}$ =tractive efficiency

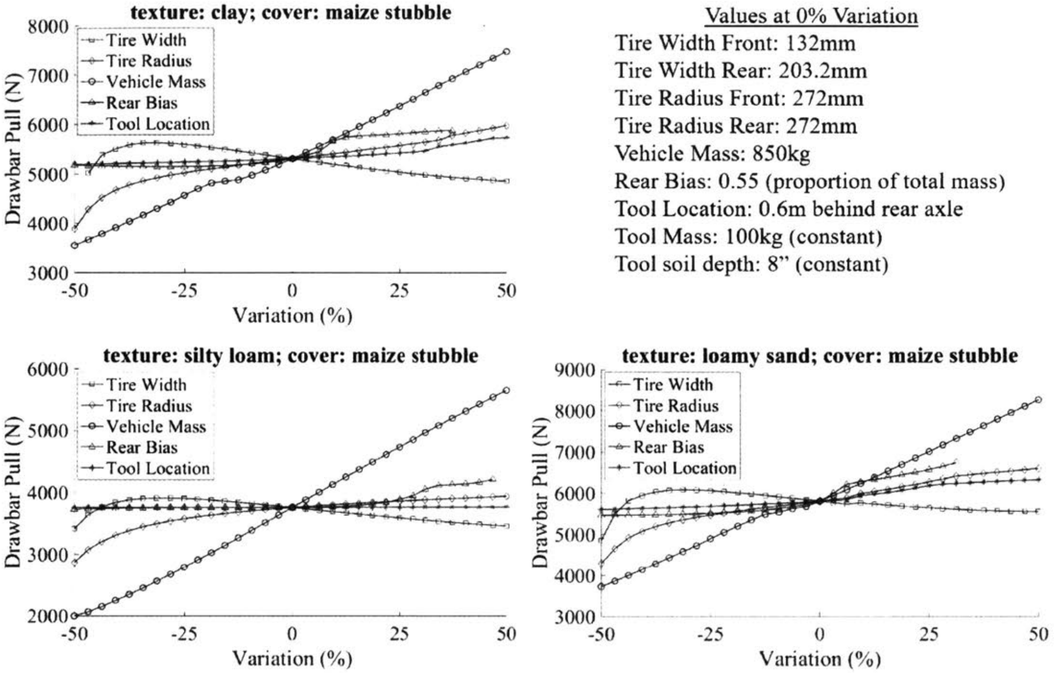
$F_pulling$ =pulling force generated by tire

$V_forward$ =actual forward speed of vehicle

P_wheels =power delivered to the wheel

Some behaviors that are worth highlighting:

- Drawbar pull increases approximately linearly with vehicle mass. This behavior becomes non-linear with mass is increased or decreased too much.
- Tractive efficiency is highest when mass is optimized to a happy medium.
- For a vehicle operating below its maximum drawbar pull, tractive efficiency may be improved by increasing tire width or radius.
- Drawbar pull increases with tire radius.
- Drawbar pull is maximized by an optimal tire width (not too much or too little).



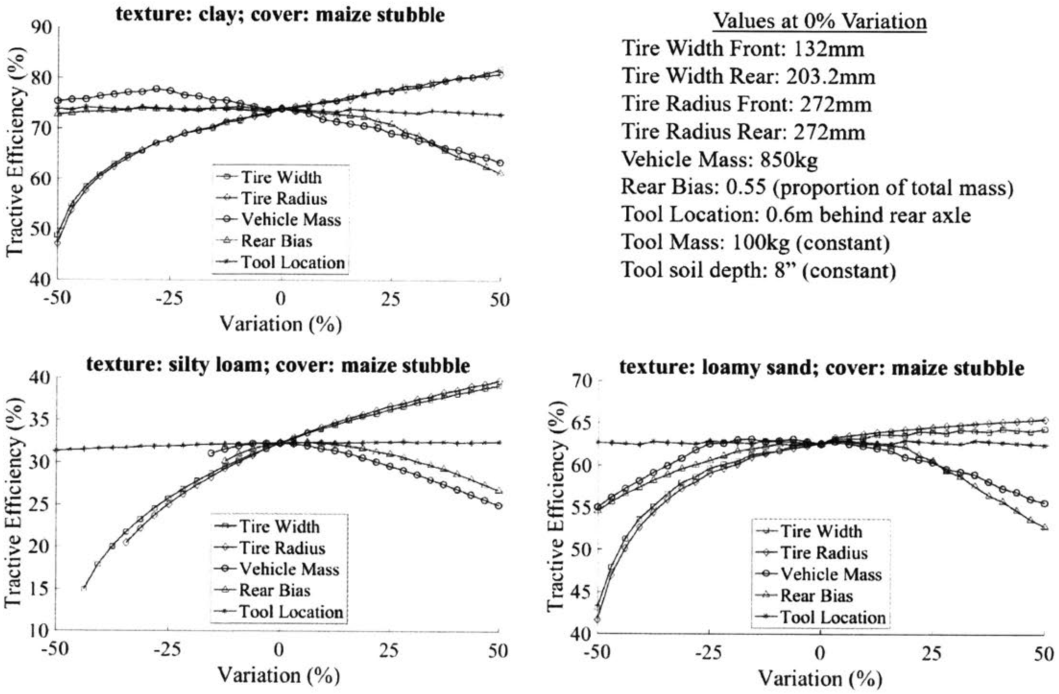
Values at 0% Variation

Tire Width Front: 132mm
Tire Width Rear: 203.2mm
Tire Radius Front: 272mm
Tire Radius Rear: 272mm
Vehicle Mass: 850kg
Rear Bias: 0.55 (proportion of total mass)
Tool Location: 0.6m behind rear axle
Tool Mass: 100kg (constant)
Tool soil depth: 8" (constant)

Figure 2-7: **Sensitivity analysis of drawbar pull at 15% slip for a conventional small tractor [32]. Data generated using model created for this thesis. Note that drawbar pull (net horizontal force) is approximately linearly related to weight for a large range of values and different soils.**

2.8 Flowchart of tractor design exploration model

The algorithm implemented to combine the tractor force distribution and tire-soil solver is shown in Figure 2-9. Note that the vehicle's weight and tool draft are distributed among all tires in the initial steps and then held constant throughout the analysis.



Values at 0% Variation
Tire Width Front: 132mm
Tire Width Rear: 203.2mm
Tire Radius Front: 272mm
Tire Radius Rear: 272mm
Vehicle Mass: 850kg
Rear Bias: 0.55 (proportion of total mass)
Tool Location: 0.6m behind rear axle
Tool Mass: 100kg (constant)
Tool soil depth: 8" (constant)

Figure 2-8: **Sensitivity analysis for tractive efficiency at a drawbar pull of 3000N. Data generated using model created for this thesis. Note that making a vehicle too heavy can be detrimental to efficiency.**

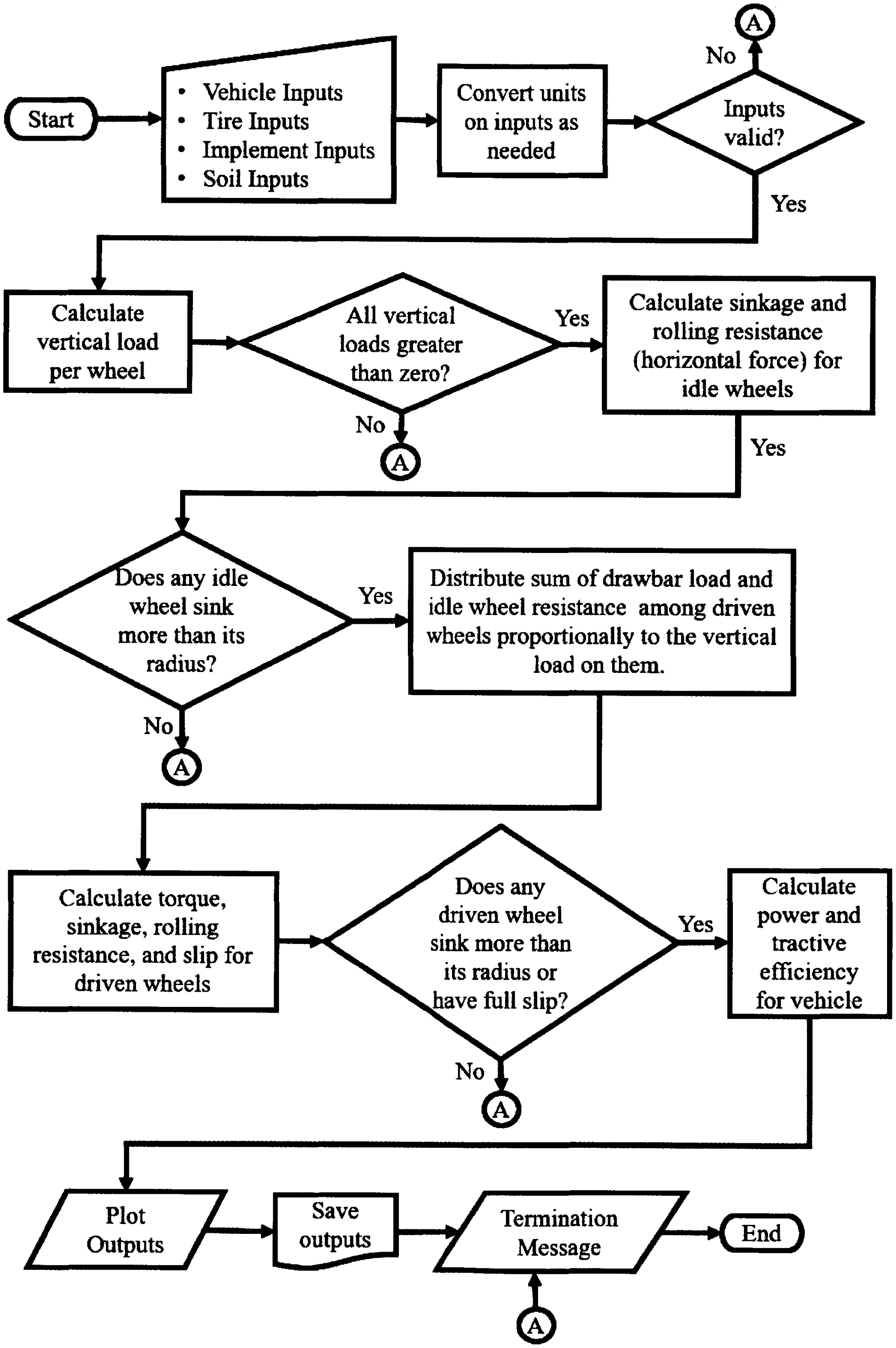


Figure 2-9: **Process flow in implemented MATLAB model to simulate a farm tractor on soil.**

Chapter 3

Validation of Tractor Model with Published Data

The model discussed in Chapter 2 may be used to evaluate and inform design of farm tractors. To verify the model's accuracy, in this chapter its outputs are compared to existing data on conventional tractors. By conventional we mean a four wheeled, rear wheel drive tractor with a trailing implement.

3.1 Comparison to specific tractor tests

The model presented in Chapter 2 has been evaluated against published experiments. The model magnitudes and trends show useful agreement to the experimental data available for specific tractor configurations (Figure 3-1). In particular, the model has its best accuracy between 5% and 20% slip, which is the range recommended for farm tractor operation [36][37][40][32].

Experiment data was obtained from [53][54], where four different-sized production tractors were tested in various soil conditions. To test a tractor's drawbar pull performance, it would tow a "braking" tractor behind via an instrumented cable. The braking tractor would be operated and adjusted to generate only the desired horizontal drawbar pull force on the tractor being evaluated.

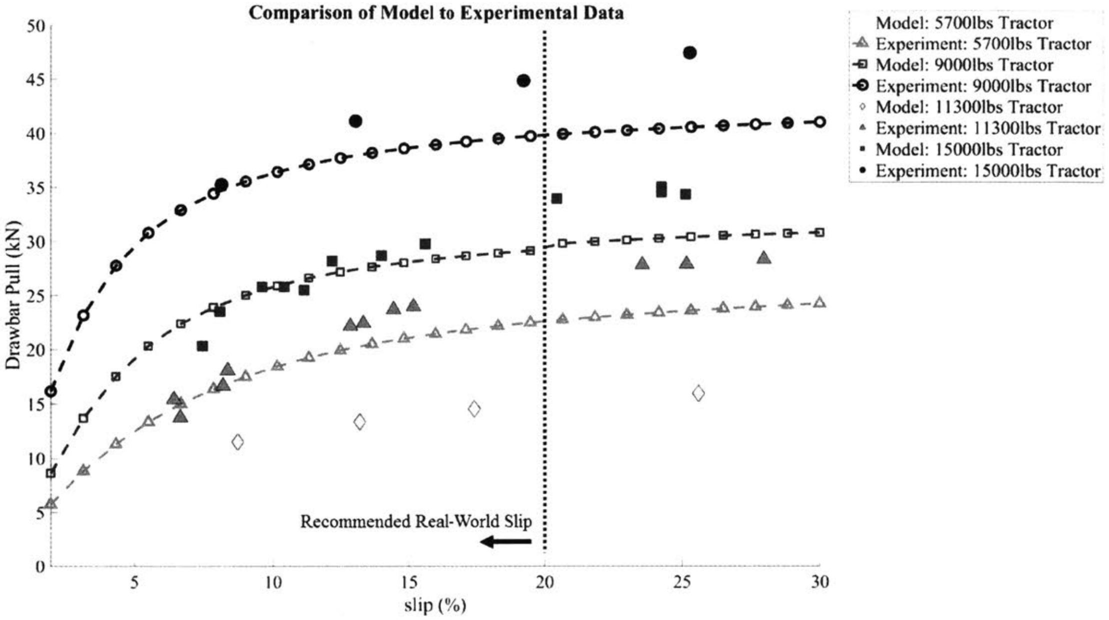


Figure 3-1: **Comparison of tractor model as described in Chapter 2 to published tractor experiments. Model has its best accuracy between 5% and 20% slip, which is the range recommended for farm tractor operation [36][37][40][32].**

3.2 Comparison to design trends

Through modeling and review of historical data, it is found that for a conventional small tractor (four wheels, rear wheel drive, trailing farm implement) the ideal weight distribution for efficient drawbar pull occurs at about 30% weight on the front wheels and 70% weight on the rear wheels when doing a towing operation.

Data Tractor Model. Figure 3-2 summarizes a study for the effect on tractive efficiency of weight, weight distribution, and draft magnitude. The weight distribution shown in Figure 3-2 does not account for weight transfer during operation (i.e. it is a statically measured weight distribution at zero drawbar pull). The effective weight distribution during operation is accounted for during simulation calculations, however. Note that when moving along the "Weight distribution on rear axle (%)" axis, power required to move (shading value) is reduced by shifting weight backwards until it asymptotes at around 70% of the tractor weight on back wheels. Placing more weight on the back wheels is not recommendable, as it does not improve efficiency but does increase the risk of flipping backwards if the tractor is of light weight for the amount of drawbar pull (draft pulling) being generated.

Data from Documented Tractor Testing. The Nebraska Tractor Tests are a standardized testing method to evaluate the performance of farm tractors. Before 1950 the tests were performed on soil instead of the concrete track now used. During that earlier period, it was also more common to test vehicles under 30hp. These two facts make the Nebraska prior to the 1950s the most informative ones for this work. Farm tractors below 25hp tested between 1941 and 1950 were selected for comparison to outputs from the Chapter 2 model. For tests, engineers employed by the manufacturer whose tractor was being tested were allowed to setup the vehicles as they preferred before testing began. Comparing Figure 3-2 and Figure 3-3, it can be observed that the engineers would generally setup their tractors to maximize drawbar performance by increasing vehicle mass and setting 70 to 80% of the tractor's weight on the rear wheels. This adjustments are in agreement with the outputs from the tractor model proposed in Chapter ??. The mass increase would improve drawbar pull as seen in the sensitivity analysis of Figure 2-7. The weight distribution is at the point where its benefits asymptote in Figure 3-2.

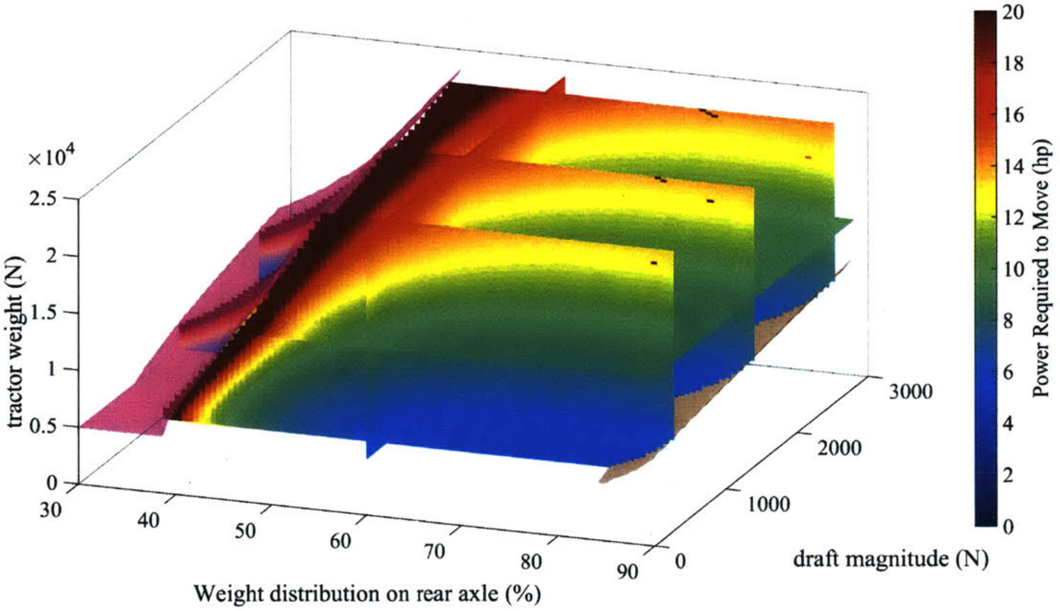


Figure 3-2: **Simulation data for one million tractor configurations. Demonstrates that optimal weight distribution for drawbar pull is about 70% on rear wheels. The semi-transparent purple frontier on the left on left represents where tractor wheels slip fully without generating progress or where any wheel sinks past its radius. The semi-transparent brown frontier on the right represents when the tractor flips backwards.**

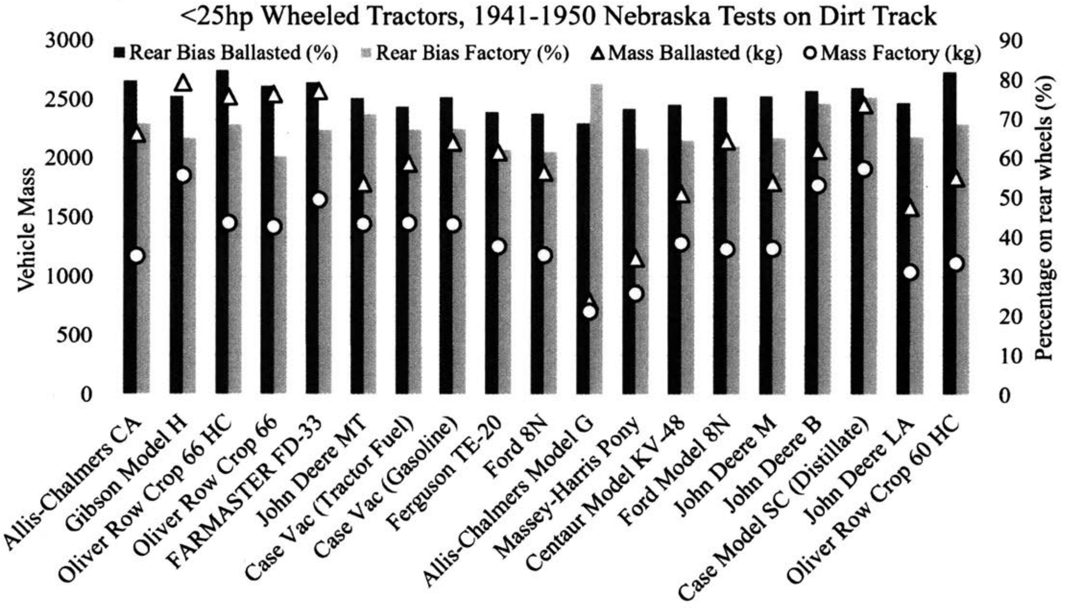


Figure 3-3: **Data compiled from Nebraska Tractor Test archives. Notice that, for testing, company engineers would ballast their tractors place about 70 to 80% of the total weight on the rear wheels.**

Chapter 4

Insights into small tractor design

4.1 Comments on past and current tractor designs

This section will discuss some layout innovations and challenges that guided tractor design between 1900 and 1940 to deliver the conventional small tractor we are familiar with today.

4.1.1 "Fulcrum and Lever" towing adjusts drive tire vertical loading in proportion to pulling force.

In 1916 Nilson Tractor Company introduced their "Fulcrum and Lever" hitch system. The goal was to use the draft force from the implement being towed to increase the downward force on the drive wheels. This would in turn increase the maximum pulling force the drive wheels could generate (the relationship between normal pressure on soil and shear strength at the tire-soil interface is better described in Chapter 2. Nilson's system attached implement towing bar or chain (which was under pure tension), much higher above the ground than other similar sized tractors had before. The effects of doing so on the vertical load at the rear wheel can be quantified from the free body diagram in Figure 4-1 (where variables are labeled) as

$$\sum F_y = -W + -Tsin(\phi) + V_F + V_R, \tag{4.1}$$

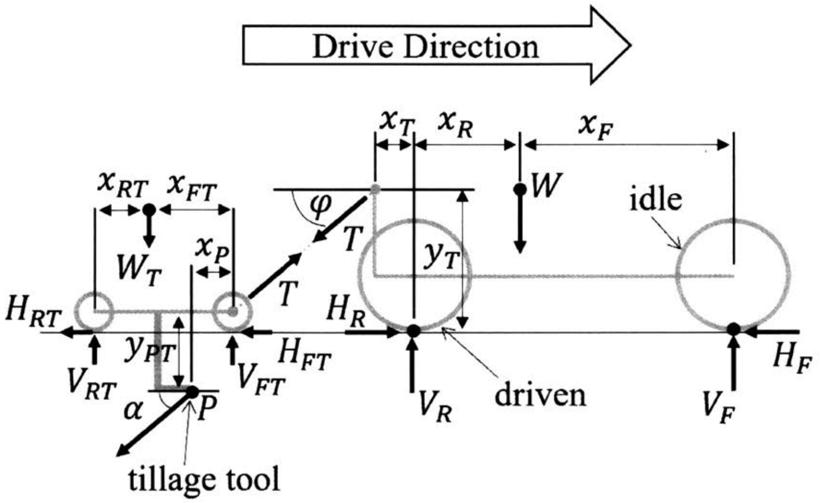


Figure 4-1: **When a tractor tows a high draft tool, the angle and position of the towing chain can have a significant effect on performance.**

$$\sum F_x = -Tcos(\phi) - H_F + H_R \tag{4.2}$$

$$\sum M_R = -Wx_R + V_F(x_R + x_F) + T(x_T cos(\phi) + y_T sin(\phi)) \tag{4.3}$$

From which it may be solved that,

$$V_F = \frac{Wx_R - T(x_T cos(\phi) + y_T sin(\phi))}{(x_R + x_F)} \tag{4.4}$$

$$V_R = W + Tsin(\phi) - \frac{Wx_R - T(x_T cos(\phi) + y_T sin(\phi))}{(x_R + x_F)} \tag{4.5}$$

$$H_R = Tcos(\phi) + H_F \tag{4.6}$$

Where,

$$T = \frac{Pcos(\alpha) + H_{FT} + H_{RT}}{cos(\phi)}. \tag{4.7}$$

Since T is a function of H_{FT} and H_{RT}, which in turn will be a affected by V_{FT}

and V_{RT}, respectively, it is useful to calculate these values as well. When focused on the towed cart:

$$\sum F_y = -Psin(\alpha) - W_T + V_{FT} + V_{RT} + Tsin(\phi), \tag{4.8}$$

$$\sum F_x = -Pcos(\phi) - H_{FT} - H_{RT} + Tcos(\phi), \tag{4.9}$$

$$\sum M_{FT} = Px_Psin(\phi) - Py_Psin(\phi) + Wx_{FT} - V_{RT}(x_{RT} + x_{FT}), \tag{4.10}$$

from which it may be solved that,

$$V_RT = \frac{-Px_Psin(\phi) + Py_Psin(\phi) - Wx_{FT}}{x_{RT} + x_{FT}} \tag{4.11}$$

$$V_FT = Psin(\alpha) + W_T - Tsin(\phi) - \frac{-Px_Psin(\phi) + Py_Psin(\phi) - Wx_{FT} + V_{RT}}{x_{RT} + x_{FT}} \tag{4.12}$$

The vertical reaction forces calculated at each tire, $V_{FT}/2$ and $V_{RT}/2$, are important because they will affect the motion resistance and thrust experienced by the vehicle. Since drawbar pull increases approximately linearly with vertical tire loading (as seen in Chapter ?? and Figure 2-7), it is proposed in this thesis that a first approach for drawbar pull capacity of a tractor distributes total required drawbar pull among all drive tires proportionally to their vertical vertical load

From the equations presented above, several important mechanical design observations may be made about raising the towing attachment point y_T on the tractor:

- **It will likely increase traction.** Raising the attachment point will increase the vertical reaction force on the rear tire V_R, which in turn increases the maximum shear strength of the soil and usually increases the generated drawbar pull (see Chapter 2). The 1916 Nilson tractor featured three rear wheels: a wide drum-wheel with a less wide wheel on each side. This wide contact patch helps reduce the risk of increasing the vertical load on the rear wheels to the point that it is detrimental to maximum drawbar pull (due to excessive wheel sinkage).

- **It will worsen safety of operation.** Raising the attachment point will decrease the vertical reaction force on the front tire V_F. This reduces the steering authority of the front wheels and increases the risk of the tractor flipping backwards during operation and crushing the driver.

- **It will reduce the draft of the trailing tool.** When a tillage tool is towed by a tractor, the tractor must overcome the horizontal tillage force but also the horizontal force generated by rolling the implement wheels in soil (see $\sum F_x$. Raising the tractor attachment point y_T decreases V_FT, which in turn reduces wheel sinkage and therefore H_FT (see Chapter 2).

It may be recalled from Chapter 2 that farm tractors usually have a tractive efficiency of about 0.3 to 0.7 on agricultural soils. Where tractive efficiency is defined as $eta_{tract.} = \frac{P_force*V_speed}{P_wheels}$ (Equation 2.23 in Chapter 2.

Rearranging the terms and using the pulling force (P_Force) generated by the tractor in Figure 4-1 this becomes:

$$P_wheels = \frac{Tcos(\phi)V_speed}{\eta_{tract.}} \tag{4.13}$$

It may be observed that rigidly mounting the tillage tool behind the tractor would eliminate the need for implement wheels and would place the implement's weight and the vertical component of P directly on the tractor's wheels. This has several important effects on the tractor's drawbar pull performance:

- Eliminating the implement wheels eliminates the terms H_{FT} and H_{RT} from the calculation of T, thus reducing the required pulling force which is proportional to the required pulling power.

- Placing the implement's weight W_T directly on the vehicle would increase the magnitude of W and reduce the distance x_R. This can then increase the vertical load on the rear drive wheels V_R (increasing it is usually beneficial) without increasing the vertical load front idle wheels V_F (increasing it is usually detrimental).

- Likewise, the downward component of draft force P can increase V_R without increasing V_F.

The case of the implement being rigidly attached to the tractor will be more carefully studied in the next subsection.

4.1.2 Central tool mounting increases safety but can be detrimental to traction in conventional tractors.

In 1939 Ford released the 9N tractor which featured the "Three-Point Hitch" trailing tool attachment system patented by Harry Ferguson. An updated version of this tractor with the same attachment system, the Ford 8N released in 1948, would go on to become the single best selling tractor model in the USA. The "three point hitch" is the standard implement attachment system for tractors. In 1948, a much lighter and differently designed tractor was also released: the Allis-Chalmers Model G. The Model G featured a tubular frame, an engine mounted behind the rear wheels, and a tool attachment point in front of the driver between the front and rear axles.

Table 4.1 shows a comparison on specifications for both these tractors.

—	**Ford 8N**	**Allis Chalmers G**
Mass	1,232kg	702kg
Engine Power	27hp	10hp
Weight Front/Rear %	35/65	18/82
Tool Control	Hydraulics	Manual Lever
Engine Location	On/Behind Front Axle	Behind Rear Axle
Tool Location	Behind Rear Axle	Behind Front Axle
Operator Location	Ahead of Rear Axle	Ahead of Rear Axle
Construction	Structural Drivetrain Castings	Welded Tubular Frame
Selling Price (2017)	$1,404 ($12,900)	$970 ($8,830)

Table 4.1: A comparison on some aspects of the 1948 to 1952 Ford 8N and the 1948 to 1955 Allis Chalmers G

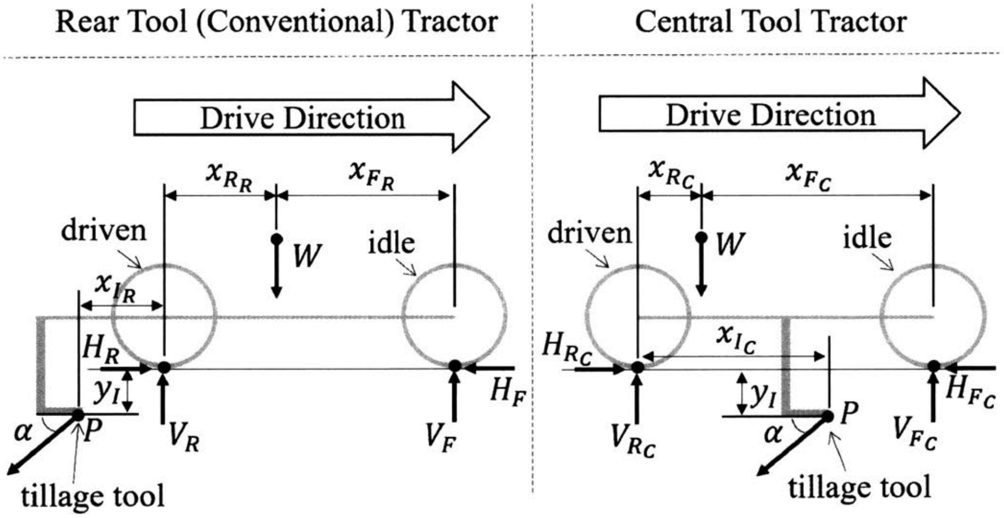


Figure 4-2: **Labeled standard conventional tractor like the Ford 8N (four wheels, rear wheel drive, and rigidly attached trailing tool) and conventional tractor with centrally mounted tool like the Allis Chalmers G.**

Free body diagrams are shown in Figure 4-2 to aid the comparison between rigidly attaching the tillage too behind or ahead the rear axle on a rear wheel drive tractor.

From those diagrams it can be seen that:

$$\sum F_{y.} = -W + -Psin(\alpha) + V_{F.} + V_{R.} \tag{4.14}$$

$$\sum F_{x.} = -Pcos(\alpha) - H_{F.} + H_{R.} \tag{4.15}$$

$$\sum M_{R_R} = -Wx_{R_R} + V_{F_R}(x_{R_R} + x_{F_R}) + P(x_{I_R}cos(\alpha) - y_I sin(\alpha)) \tag{4.16}$$

$$\sum M_{R_C} = -Wx_{R_C} + V_{F_C}(x_{R_C} + x_{F_C}) + P(-x_{I_C}cos(\phi) - y_I sin(\phi)) \tag{4.17}$$

From which it may be solved that,

$$V_{F_R} = \frac{Wx_{R_R} - P(x_{I_R}cos(\alpha) + y_{I_R}sin(\alpha))}{(x_{R_R} + x_{F_R})} \tag{4.18}$$

$$V_{F_C} = \frac{Wx_{R_R} - P(-x_{I_R}cos(\alpha) + y_{I_R}sin(\alpha))}{(x_{R_R} + x_{F_R})} \tag{4.19}$$

$$V_{R_R} = W + Psin(\alpha) - \frac{Wx_{R_R} - P(x_{I_R}cos(\alpha) - y_I sin(\alpha))}{(x_{R_R} + x_{F_R})} \tag{4.20}$$

$$V_{R_C} = W + Psin(\phi) - \frac{Wx_{R_C} - P(-x_{I_C}cos(\alpha) - y_I sin(\phi))}{(x_{R_C} + x_{F_C})} \tag{4.21}$$

$$H_{R.} = Pcos(\alpha) + H_{F.} \tag{4.22}$$

It may be noted from Table ?? that as represented in Figure 4-2 for the case of these production tractors,

$$\frac{x_{R_C}}{x_{F_C} + x_{R_C}} < \frac{x_{R_R}}{x_{F_R} + x_{R_R}} \tag{4.23}$$

Some important observations may be made about the effects on tractor performance when mounting the tool ahead or behind of the rear axle. The relationship between vertical and horizontal forces at the wheel is described in detail in Chapter 2.

- Regardless of if the tool is mounted ahead or behind the rear axle, the horizontal component of draft force W increases the vertical reaction force at the front tires $V_{F_c dot}$. This has the benefits of increasing steering authority and reducing the risk of the tractor flipping backwards. It also has the important disadvantage of increasing sinkage at the front idle wheels and therefore the magnitude of the progress-opposing force $H_{F_c dot}$.

- When the tool is mounted behind the rear axle, the vertical component of the draft force P increases the rear vertical reaction force V_{R_R} and decreases the front vertical reaction force V_{F_R}. This has the benefit of decreasing the progress-opposing force H_{F_R} while also increasing the thrust force H_{R_R}, these two effects improve the tractor's drawbar pull. The disadvantage is that reducing V_{F_R} has a negative effect on steering authority and increases the risk of the tractor flipping backwards.

- When the tool is mounted ahead the rear axle but behind the front axle, the vertical component of the draft force P increases the vertical reaction force at both wheels V_{F_C} and V_{R_C}. If x_{I_C} is greater than x_{R_C}, then the effective weight distribution will be shifted forward and vice versa. To maximize drawbar pull it is preferable to minimize x_{I_C}. So long as draft force P acts in front of the rear axle, there will be little risk of the tractor flipping backward during normal operation on flat ground.

Some important observations can also be made about the effects on user experience when placing the tool behind the rear axle or between axles.

User experiences advantages of tool between axles

- Placing the implement (tillage tool) in between both axles, in front of the operator (especially in a tubular frame that allows good ground visibility) makes it easier for the operator to keep a close eye on the quality of the operation without needing to look over their shoulder. This visual advantage also facilitates the operator having manual control over the tool's position since they can more easily supervise it and make small adjustments.

- Placing the implement in between both axles, prevents the tractor from flipping backwards, which is a major source of injuries and fatalities in farming [55]. This is not only safer but also allows the operator to utilize the maximum drawbar pull from their vehicle without this becoming a potential hazard.

- Placing the implement between both axles can reduce the total length of the vehicle. This can be beneficial for operations in close quarters.

- Placing the implement between both axles enables mounting the engine behind the rear axle while still maintaing good weight distribution (see Figure 3-2 in Chapter 2). Putting the engine behind the operator (who usually sits ahead of the rear axle in modern tractors) allows better forward visibility and prevents engine heat, fumes, and noise from being blown into the operator's face while driving.

User experiences advantages of tool behind rear axles

- Placing the implement (tillage tool) behind the rear axle, especially in combination with hydraulics, makes it easier to pick up and drop off many farming implements. The driver need only reverse the implement-less tractor towards and implement, lock the implement attachment points, and use the hydraulics to lift the implement and drive away. When selling the 8N, Ford advertised an implement could be mounted in less than one minute.

- Placing the implement behind the rear axle does not constrain the length of the implement. This in an important advantage for more powerfull tractors that can pull several ground engaging "bottoms" at once.
- Placing a ground-engaging implement behind the rear axle (and therefore behind the driver) minimizes the amount of dirt that is kicked up into the driver and into the tractor.
- Placing the implement behind the driver places the driver further away from the moving parts of the implement. This can be especially important for implements that have moving parts powered by the engine.

Finally, some comments on manufacturing when the farming implement is placed behind the rear axle or between axles. For more information on preferred weight distributions see (see Figure 3-2 in Chapter 2).

- Placing the implement (tillage tool) behind the rear axle makes it beneficial for weight distribution to have the engine near the front axle. Since power is delivered to the ground at the rear wheels this means the engine and transmission castings together span the full length of the tractor. Using the engine and transmission casting as the structural "frame" of the tractor minimizes the amount of components, which facilitates fabrication and reduces mass, both of which help lower production costs and the latter can improve performance [7].
- Placing the implement (tillage tool) between axle makes it beneficial for weight distribution to have the engine near the rear axle. This can reduce the structural demands on the tractor since torque is being generated (at the engine) where it is needed (the rear axle). This then can lighten the tractor frame ahead of the rear axle.
- Placing the implement (tillage tool) behind the rear axle can facilitate packaging. The drive axle, hydraulics, and power take-off (PTO) shaft are all around the same location where engine power is being delivered. Additionally, the space

behind the rear axle can be fully dedicated to the implement, its attachment linkages, and its power sources (hydraulics and PTO). The space between both axles must be share with other objects including the operator, the operator controls (along with their transmission lines), and the tractor frame.

4.1.3 Inline and side-by-side drive wheels each have their advantages.

In this section the side-by-side drive wheels are qualitatively compared against inline drive wheels. To facilitate clear discussion, three default configurations have been selected:

- **Side-by-side drive wheels** are represented by the configuration of a conventional tractor: four wheels, rear wheels are driven, trailing implement.
- **Inline drive wheels (I)** is an all-wheel-drive, four wheeled tractor with front wheel steering and a trailing implement.
- **Inline drive wheels (II)** is a configuration akin to an all-wheel-drive, two-wheeled motorcycle with outrigger support wheels, and a trailing implement. This layout would be similar to "f" in Figure 4-3 but with only front wheel steering.

Advantages of side-by-side drive wheels

- **Tighter turning.** This happens in a conventional layout for two reasons: smaller front wheels (advantage vs. Inline A), and ability to steer with brake pedals (advantage vs. Inline B). For a tractor to take best advantage of inline drive wheels, all drive wheels must be of similar size. In a four wheeled tractor, the front wheels are usually a small diameter to prevent them from hitting the tractor frame during tight turning (during a steered turn, the rearmost part of the inside front tire swings toward the tractor). Making the front wheels larger to match the size of the rear wheels would increase the tractor's turning radius if nothing else was changed. Most tractors are equipped with a rear differential and the two rear drive wheels can be braked separately. Applying the brake on just one wheel causes the tractor to make a tight turn around that wheel.

- **Simpler construction.** Side-by-side drive wheels allow easy use of a standard differential and a drivetrain layout similar to automobiles. When front (steered) wheels are driven, additional linkages must be added to the powerline to enable steering. Since most implements are attached closely behind the rear axle, having all torque delivered to that axle shortens the load path between the implement and the traction force.

- **Well suited to weight transfer from rear mounted tool.** A tillage tool attached behind the tractor will cause the effective weight distribution of the tractor to shift rearward (as described in the previous subsection). This added vertical load on the rear wheels can increase their maximum tractive force. The opposite effect is occurring to the front wheels, which can reduce the benefits of inline drive wheels.

Advantages of inline drive wheels

- **Higher tractive efficiency.** The front drive wheel will compact the soil for the trailing drive wheel. This allows the trailing drive wheel to generate higher thrust while also sinking less into the soil [44]. This is described in more detail in Chapter 2.

- **Smaller track width.** A two-wheeled motorcycle-type layout (inline layout I) minimizes the vehicles width while still having two full size drive wheels.

- **Less damage to crops.** Inline drive wheels will usually cause less damage to crop soil than other comparable configurations. In the case of inline I: a lighter vehicle can achieve the same drawbar pull due to all wheels being driven [44]. In the case of inline I: the amount of drive wheel "traffic" lanes can be reduced since only one lane is created by each fresh tractor pass. Minimizing traffic lanes is the essence traffic control strategies [46] [47].

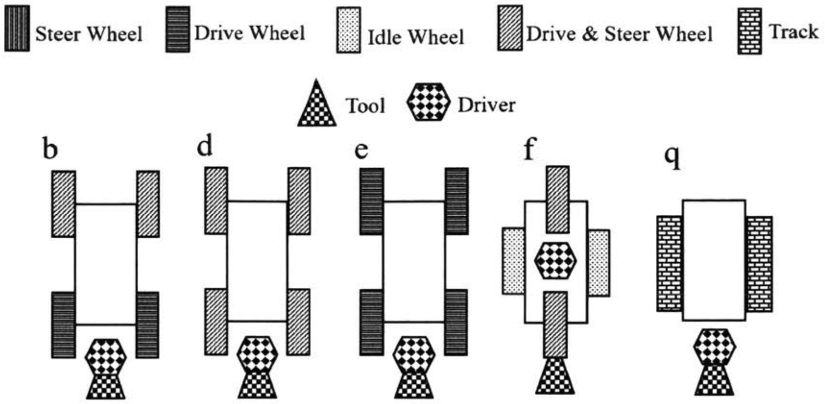


Figure 4-3: **Examples of tractor layouts with inline wheels. Layouts are selection from those of Figure 1-1.**

4.1.4 Hypothetical inline drive wheels with centrally mounted tool

This section presents an example of a quantitative comparison between side-by-side drive wheels and inline drive wheels. The comparison was made using the model from Chapter 2.

It has been selected to compare the configurations in a lightweight tractor version. Since one objective of this thesis is to identify design opportunities that may increase farm tractor adoption in developing countries. It is estimated that a tractor of cost comparable to draft animals would have to weigh about 500kg [33]. In a further comparison to draft animals, the pulling force of inline drive wheels and side-by-side wheels is compared to the pulling force of a team (pair) of bullocks (Figure 4-4).

In Figure 4-4, the two drive wheel layout configurations are compared the performance of a team (pair) of bullocks. It is important to note here that two levels of performance are used for the animals, a steady pull and a maximum pull. While the steady, sustained pull is about 15% of the animal's weight, the maximum pull can be about 50% of the animal's weight [56][57]. The maximum pull will keep the animals from becoming stuck when pulling a tillage tool through a harder patch of soil, a tractor that cannot perform that same maximum pull may become stuck, however. It can be seen in Figure 4-4 that the inline drive wheels can better match the performance of the bullocks in a lightweight package than the side-by-side wheels. Vehicle values for for side-by-side drive wheels and inline drive wheels were based on the Mahindra Yuvraj and the ROKON Scout, respectively (Table 4.2)

Further study is required but initial qualitative and quantitative observations suggest that inline drive wheels are a configuration that should be considered when designing a lightweight tractor.

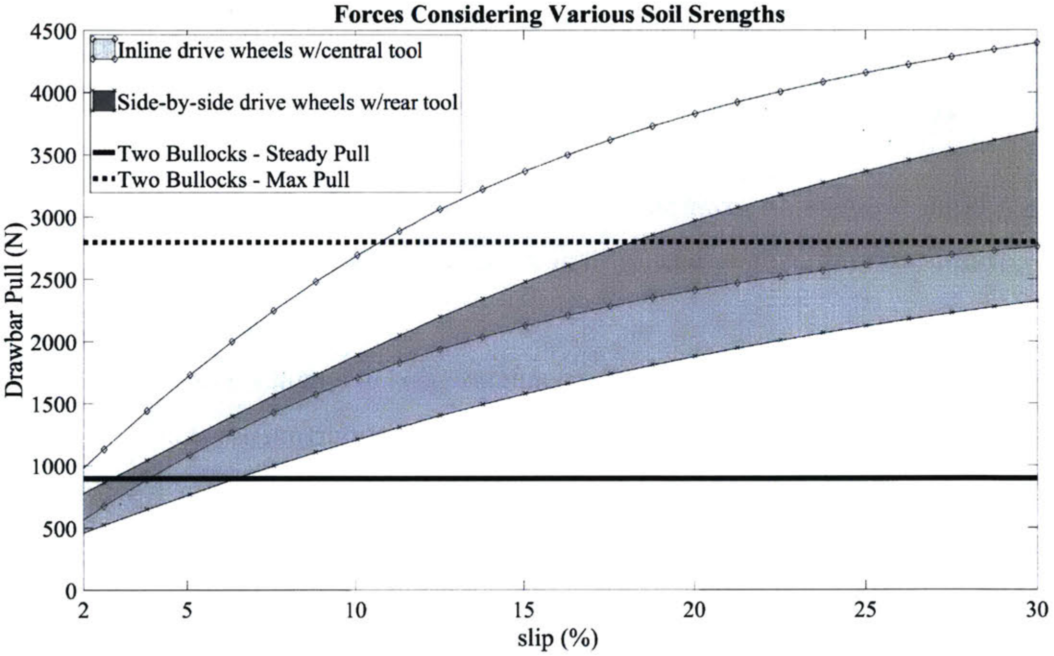


Figure 4-4: **A comparison of the drawbar pull of inline wheels vs. side-by-side wheels in a 500kg. In both cases, the horizontal drag from rolling idle wheels through the soil has been ignored. The model described in Chapter 2 was used. The drawbar pull of a bullock team (two animals) has been added for reference.**

—	**Side-by-Side Wheels**	**Inline Drive wheels**
Vehicle Mass (kg)	500	500
Rider Mass (kg)	60	60
Weight Front/Rear (%)	45/55	50/50
Wheelbase (m)	1.5	1.3
Tool Horz. from CG (m)	-1.5	0
Plow depth (m)	0.13	Behind Rear Axle
Tire Section (m)	0.085	0.165
Tire diameter (m)	0.64	0.64
Tire width (m)	0.2	0.2
Tire Pressure (psi)	8.7	8.7

Table 4.2: Values for side-by-side wheels and inline wheels used in Figure 4-4. Side-by-side wheels based on Mahindra Yuvraj and inline wheels based on ROKON Scout.

4.2 Recommendations for future designs

Some insights into advantageous small tractor design have been identified:

Large diameter, skinny tires are preferable. A longer, less wide tire path can achieve equivalent flotation with less soil resistance. Additionally, the soil gets stronger as more of the wheel rolls over it, increasing the shear strength of the soil for the rear part of the wheel (this is particularly meaningful for deformable tires that have a flat contact patch at their lowest point). This is also beneficial to the farm field as it limits the amount of soil that is compacted.

Inline drive wheels are more efficient. This effect is similar to the preference for larger diameter, less wide tires. By rolling a trailing drive tire fully within the track width of a leading drive tire, the trailing tire can roll over stronger soil that has already been compacted by the leading tire. This allows the trailing tire to generate more thrust while having to overcome less resistance due to sinkage (since it sinks less).

A longitudinally central tool is safer, easier to manually adjust, and is efficient for inline drive wheels. In a conventional small tractor, a tillage tool is mounted behind the rear wheels and its vertical position is controlled via the tractor's hydraulic system. The downward reaction force felt by the tool underground at the soil working center-of-pressure and the tool's weight combine to increase the normal reaction force in the rear (drive) tires. This then usually increases the maximum drawbar pull of the tractor.

This configuration can pose some challenges for smaller tractors, especially those with no hydraulic system. In smaller tractors the front wheels may become so unweighted that they offer no significant steering authority. In the more critical cases, the tractor may even flip over backward around its rear wheels, crushing the driver. Additionally, in the absence of a hydraulic with "Automatic Draft Control" the operator must keep an eye on the tool and use manual power to lower or raise it as needed. This operation forces the driver to look exactly opposite of the direction they are driving in.

Mounting the tool longitudinally central (i.e. between the front and rear axles, ahead of and beneath the driver) means that the tool's weight and soil reaction forces increase the normal reaction force on both the front and rear tires. This can be disadvantageous for traction generation if only the rear wheels are driven, as is often the case, but when both front and rear wheels are driven the extra load on the front tires can be beneficial. In all drivetrain configurations, central tool mounting makes the tractor safer and easier to operate. The tool's location makes it easier to observe by the operator and basically eliminates the possibility of flipping the tractor backward during tillage operations.

Track width, turning radius, and off-roadability are primary concerns. The dimensions of a conventional small tractor and its offroad-capabilities do not allow it to operate in some spaces and/or terrains that are accessible to animals. An animal moves via intermittent steps that allow it to step over natural obstacles or farm field objects like furrow ridges, bunds, and vegetable crops. The tractor must drive over any object that is directly in its tire's path. The animal's width allows it to walk in between rows of growing crops to do intercultivating activities like mechanical deweeding. The animal's stepping motion, low width, and high maneuverability negate the need for the significant headland that tractors require to make 180 urns at the end of each row.

A small tractor that replaces animals must match their maximum pulling force, not only their average force. The steady state pulling force of a bullock working on a farm field is about 15% of its body weight. However, a bullock is capable of pulling about 50% of its body weight for a few moments when conditions require it [56][57]. To smoothly work the same fields as a bullock team, an engine-powered, wheeled off-road vehicle must match the team's maximum drawbar pull. Otherwise, the tractor may become stuck upon hitting a stronger patch of soil or other submerged obstacle with an underground tool in a situation where the animals would have applied a brief increase in force.

Initial cost is a major obstacle to tractor adoption. Small tractors have lower year round maintenance and operation costs than animals, and most farmers that we have spoken to in India are aware of this. However, tractors usually have an initial cost at least three times higher than animals [32]. Many farmers cannot afford to self-finance themselves for a few years until the total ownership of a tractor becomes less than that of animals bought during the same period [32]. To make the investment proposition riskier, some farmers will find that they must modify their farming practices (including row spacing and use of arable land for headland) when replacing their animals with a tractor.

Chapter 5

Conclusions

This thesis has described how the conventional small farm tractor evolved mostly in the USA between 1910 and 1940. Emerging markets in the contemporary world are not necessarily best served by a product configuration that evolved in different context 80 years ago. The tractor model proposed in this thesis and the observations made on the physics of farm tractors allow engineers to parametrically explore novel tractor designs that are created specifically to suit the needs of small farmers in developing countries.

Results shown in this thesis include:

- The conventional small tractor evolved in a different context than what occurs today in emerging markets. Farms were larger during the tractor's inception, this means the conventional small tractor may be too expensive and unwieldy for the majority of the world's farms (Chapter 1). Simply reducing the size of the conventional tractor may render it uncapable of matching the maximum drawbar pull performance of a draft animal team (Chapter 4).

- The mathematical tractor model proposed (Chapter 2) is shown to have good accuracy (Chapter 3). The model can be used to guide design decisions and create novel tractor designs. Design recommendations for a tractor better suited to small farmers have been drawn from model (Chapter 4).

- A high potential configuration is a narrow tractor with inline drive wheels and a tillage tool attached near the center of the vehicle (longitudinally between front and rear axles). This is shown in Chapter 4 based on the theory of Chapter 2.

- Quantifiable advantages and disadvantages of the modern conventional tractor layout are discussed in Chapter 4.

Appendix A

1910 to 1920 Production Vehicles Matched to Layouts in Ch. 1

<table>
<tr><th colspan="5">Layout A</th></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Twin City</td><td>12/20</td><td>1919 to 1926</td><td>27hp</td><td>2268kg</td></tr>
<tr><td>Russell</td><td>Model C 20/40</td><td>1919 to 1924</td><td>40hp</td><td>3450kg</td></tr>
<tr><td>Huber</td><td>30/60</td><td>1912 to 1916</td><td>60hp</td><td>5000kg</td></tr>
<tr><td colspan="5">Layout B</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Fitch Four Drive</td><td>20/30</td><td>1915 to 1918</td><td>30hp</td><td>1360kg</td></tr>
<tr><td colspan="5">Layout C</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>S.L. Allen</td><td>Planet Jr.</td><td>1920 to 1935</td><td>2.31hp</td><td>250kg</td></tr>
<tr><td>Moline</td><td>Universal</td><td>1914 to 1918</td><td>27hp</td><td>1630kg</td></tr>
<tr><td>Allis-Chalmers</td><td>6-12</td><td>1919 to 1926</td><td>12hp</td><td>1134kg</td></tr>
<tr><td colspan="5">Layout D</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Heer
Nelson</td><td>20-28
20-28</td><td>1912 to 1916
1917 to 1924</td><td>30hp
30hp</td><td>2̃000kg
2̃000kg</td></tr>
</table>

<table>
<tr><th colspan="5">Layout E</th></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Samson</td><td>Iron Horse D</td><td>1918 to 1923</td><td>26hp</td><td>850kg</td></tr>
<tr><td>Olmstead</td><td>Four Wheel Pull</td><td>1914 to 1920</td><td>50hp</td><td>3̃000kg</td></tr>
<tr><td colspan="5">Layout F</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Post</td><td>12-20</td><td>1918 to 1920</td><td>20hp</td><td>1500kg</td></tr>
<tr><td colspan="5">Layout G</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Hart Parr</td><td>20-40</td><td>1912 to 1914</td><td>40hp</td><td>6̃000kg</td></tr>
<tr><td>Samson</td><td>Sieve Grip</td><td>1914 to 1918</td><td>25hp</td><td>2630kg</td></tr>
<tr><td>Wallis</td><td>Cub</td><td>1913 to 1917</td><td>44hp</td><td>3855kg</td></tr>
<tr><td colspan="5">Layout H</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Bull</td><td>Little Bull</td><td>1913 to 1915</td><td>12hp</td><td>1800kg</td></tr>
<tr><td>Case</td><td>10/20</td><td>1914 to 1918</td><td>20hp</td><td>2304kg</td></tr>
<tr><td colspan="5">Layout I</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Hart-Parr</td><td>Little Devil</td><td>1914 to 1916</td><td>22hp</td><td>3015kg</td></tr>
<tr><td>Common Sense</td><td>15/25</td><td>1914 to 1918</td><td>25hp</td><td>2700kg</td></tr>
<tr><td>Emerson Brantingham</td><td>Model L</td><td>1916 to 1918</td><td>20hp</td><td>2500kg</td></tr>
<tr><td colspan="5">Layout J</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Rumely</td><td>Ideal Pull</td><td>1916 to 1917</td><td>16hp</td><td>1̃500kg</td></tr>
<tr><td colspan="5">Layout K</td></tr>
<tr><td>Manufacturer</td><td>Model</td><td>Years</td><td>Engine</td><td>Mass</td></tr>
<tr><td>Lawter</td><td>18/38</td><td>1914 to 1918</td><td>38hp</td><td>2950kg</td></tr>
</table>

Boring	12/25	1916 to 1922	25hp	2050kg
Hackney	Auto-Plow	1916 to 1922	36hp	3630kg
Layout L				
Manufacturer	Model	Years	Engine	Mass
Gray Tractor	Model B	1914 to 1918	25hp	2500kg
Layout M				
Manufacturer	Model	Years	Engine	Mass
Killen Strait	15-30	1913 to 1917	30hp	4300kg
Layout N				
Manufacturer	Model	Years	Engine	Mass
Bean	Track-Pull 6/10	1918 to 1920	10hp	1400kg
Layout O				
Manufacturer	Model	Years	Engine	Mass
Bean	Track-Pull 6/10	1918 to 1920	10hp	1400kg
Layout O				
Manufacturer	Model	Years	Engine	Mass
Beltrail	Model B 12-20	1917 to 1920	20hp	1̃500kg
Tom Thumb	12-20	1917 to 1920	20hp	1900kg
Layout P				
Manufacturer	Model	Years	Engine	Mass
Yuba	20-35	1911 to 1916	35hp	3̃500kg
Blewett	Webfoot 53	1920 to 1922	53hp	4500kg
Holt	75	1913 to 1924	75hp	10432kg
Layout Q				
Manufacturer	Model	Years	Engine	Mass
Bullock	Creeping Grip	1916 to 1919	20hp	3270kg
Layout R				
Manufacturer	Model	Years	Engine	Mass

Victor	Victor	1919	34hp	1950kg
Layout S				
Manufacturer	Model	Years	Engine	Mass
John Deere	Dain	1918-1919	24hp	2086kg
Layout V				
Manufacturer	Model	Years	Engine	Mass
Acme	12-24	1918-1919	24hp	1450kg
Buckeye	Junior	1912-1915	2̃5hp	2̃500kg
Layout X				
Manufacturer	Model	Years	Engine	Mass
Killen Strait	30hp	1917-1919	30hp	2600kg
Layout Y				
Manufacturer	Model	Years	Engine	Mass
Rumely	8-16	1917-1919	16hp	2600kg

Appendix B

Evolution Steps, matched to vehicles in Ch. 1

Manufacturer	Model	Years	Engine	Mass	Price (2017)	Units Made
Ivel	“tractor”	1902-1920	8hp	1814kg	£300 ($43,630)	500
Hart-Parr	30-60	1907-1918	60hp	9120kg	$2,600 ($64,030)	3,798
Ford	T	1908-1927	22hp	660kg	$360 ($5,067)	14,689,525
Avery	Farm & City	1909-1915	36hp	2100kg	$2,500 ($67,750)	N/A
Bull	Little Bull	1913-1915	12hp	1315kg	$335 ($8,450)	3,800
Wallis	Cub	1914-1918	44hp	3855kg	$2,480 ($55,495)	660
Moline	Universal	1915-1918	27hp	1630kg	$1,325 ($18,105)	20,000
Nilson	20-40	1916-1929	40hp	2380kg	$925 ($33,230)	N/A
Square Turn	18-35	1917-1925	35hp	3538kg	$1,875 ($22,900)	approx.700
Fordson (Ford)	F	1917-1928	20hp	1215kg	$395 ($5,760)	755,278
Int. Harvester	15-30	1921-1928	30hp	2653kg	$1250 ($17,905)	157,366
Int. Harvester	Farmall Reg.	1924-1932	20hp	1655kg	$925 ($13,530)	134,647
Allis-Chalmers	U	1929-1952	20hp	2086kg	$125 ($21,240)	19,009
Ford	9N	1939-1942	20hp	970kg	$585 ($10,291)	99,002

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