Two-stage reverse osmosis: optimal element configuration and flux distribution, energy savings

by

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Abstract

In a two-stage reverse osmosis (RO) system of finite size, there are two degrees of
freedom not present in a single-stage RO system: distribution of RO elements between
the two stages (system design), and feed pressures (system operation). In this study,
we investigate the optimal system design and operation of a two-stage RO system with
a mass-balance model and establish a lower bound for the energy savings achieved
by the optimized two-stage system compared to a single-stage system. A two-stage
RO system may consume more or less energy than a single-stage RO system of the
same size and freshwater productivity, depending on the first-stage feed pressure and
second-stage feed pressure. To minimize energy consumption, feed pressures should be
chosen to minimize spatial variance in flux. The optimal element configuration places
at least half the elements in the first stage; the exact configuration depends on feed
salinity, recovery ratio, and membrane permeability. The greatest energy savings are
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least work of separation is larger. For a given feed salinity, energy savings from adding
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to high recovery ratios to achieve substantial energy savings; comparable savings can
be achieved at lower recovery ratios for higher salinity feeds. We find that significant
energy can be saved with the simplest two-stage RO design, at a system flux similar
to today’s RO plants and accounting for the effects of concentration polarization.

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Chapter 1

Introduction

This thesis is based off a paper by Wei et al. [1].

Although freshwater is a plentiful and renewable resource, many regions face freshwater scarcity due to uneven distribution and overconsumption. More than two billion people gained improved access to clean drinking water between 1990 and 2015, but about one in eleven people (660 million) still lack access to clean drinking water [2]. Demand for freshwater is only projected to increase as the world’s population grows larger and wealthier. At the same time, shifts in rainfall patterns and temperatures will make the freshwater supply less dependable. Freshwater scarcity will not affect all regions equally - the Middle East, North Africa, Central Asia, and parts of South Asia are projected to be hit the hardest. However, the potential fallout from lack of freshwater availability in these regions - stalled economic growth, conflict, and migration - is likely to spread and impact regions that have plenty of water [3].

There are a number of ways to mitigate the impending water crisis. Increased water efficiency in the agricultural and industrial sectors would help by decreasing the total demand for freshwater. Public recognition of water as a scarce and valuable resource will aid water conservation efforts. Many water resources cross political boundaries; successful cooperation between states will be critical to the sustainable management of such resources. Another strategy is to increase the amount of freshwater supply through wastewater recycling or desalination [3].

Coastal regions facing freshwater scarcity have direct access to seawater, a more
abundant resource than freshwater. As technology has improved, coastal populations have been able to turn to seawater desalination as a means to expand their freshwater supply. Desalination of brackish groundwater can serve as additional source of freshwater, and is not limited to coastal regions. Total worldwide desalination capacity more than doubled from 2005 to 2015. While thermal methods dominated the desalination industry in the 1980s, reverse osmosis (RO) plants currently account for the majority (65%) of total worldwide desalination capacity [4].

In reverse osmosis desalination, a salty feed stream is forced through a semipermeable membrane that allows the passage of water but rejects salt passage. This results in two output streams: a freshwater permeate stream and a concentrated brine stream. The RO process is energy intensive because the feed stream must be brought to a hydraulic pressure high enough to overcome the difference in osmotic pressures between the feed and permeate streams. For desalination of seawater, this osmotic pressure difference is approximately 28 bar.

Reverse osmosis desalination energy consumption has steadily improved since the 1970s due to the development of high permeability membranes and improved energy recovery devices [5, 6, 7]. However, RO water is still costly and sometimes not competitive with alternatives such as wastewater reuse or water importation [6]. A vast majority of the cost of water from RO is the result of capital costs and energy consumption [6]. Since many RO plants are powered by fossil fuels, this energy consumption has an environmental cost as well. Thus, both cost and environmental concerns motivate efforts to decrease the energy consumption of RO systems.

Large scale RO plants today have an energy consumption of roughly 3.5-4.2 kWh/m³, of which approximately 2.9-3.5 kWh/m³ is directly due to the RO system, with the balance being consumed by intake, pretreatment, and other auxiliary systems [6, 8, 9]. The RO energy may potentially be further reduced, but energy consumption can never fall below the thermodynamic least work of separation. The least work of separation for 50% recovery of a seawater feed (\(w_{s,f} = 35 \text{ g/kg}\)) is \(\approx 1\) kWh/m³, but with associated pumping and other factors, the practical limit will be higher than the thermodynamic limit [6, 10, 11, 12]. Increased membrane permeabil-
ity beyond today's typical values will not drastically reduce the energy consumption of RO - there is more potential to reduce RO energy consumption through innovative system design [6, 13].

1.1 Two-stage RO plant description

Energy inefficiency in RO systems may be understood in terms of entropy generation. The majority of entropy generation in an RO system can be attributed to permeate flow through the membrane [10]. One simple way to reduce this portion of the entropy generation, and thus energy consumption, is by adding a second stage to a standard single-pass RO process [14, 15, 16]. This is accomplished by inserting an intermediate pump between RO stages, as shown in Fig. 1-1. To avoid pressurizing the entire feed to a very high pressure, the feed is initially brought to the first-stage feed pressure. Some permeate is produced in the first stage, and then the brine is brought up to the second-stage feed pressure. The remainder of the permeate is produced in the second stage. Energy is recovered from the pressurized brine stream as it passes through the pressure exchangers. It is necessary to split up the feed streams and use booster pumps because pressure exchangers require that the low and high pressure flows have equal flow rates. Staging has already been implemented in many working brackish reverse osmosis plants [4]. Previous work has demonstrated similar benefits for staging of other desalination processes, such as humidification-dehumidification desalination systems [17].

1.2 Previous work

Previous studies have shown that staged RO has potential for significant energy savings [18, 19, 20, 11, 21]. These studies demonstrate the energetic benefits of staged RO for a range of feed salinities and recovery ratios. For a low salinity (brackish water) feed, very high recovery ratios are needed for staged RO to be cost beneficial [18, 19, 21]. In general, energy savings increase with recovery ratio. Most of these studies assume that each stage is allowed to reach a state of "thermodynamic restriction," i.e.,
Figure 1-1: System diagram of a two-stage reverse osmosis system with pressure exchangers in each stage. The initial feed stream is brought to the first-stage feed pressure, and some permeate is produced in the first stage. The brine from the first stage becomes the feed for the second stage and is brought to the second-stage feed pressure. The rest of the permeate is produced in the second stage. Pressure exchangers are used in each stage to extract energy from the pressurized brine stream.

that the brine osmotic pressure reaches the feed hydraulic pressure. No constraint is imposed on the membrane area in each stage, and the energetic benefits of staging are confounded with the benefits of added membrane area. In these studies and one which considers the kinetics (flux) of two-stage RO [22], the effects of concentration polarization\(^1\) are neglected since permeate flux vanishes at the “thermodynamic restriction.” In a real RO plant with limited membrane area, the effects of concentration polarization may be significant. However, these studies do establish a reasonable upper bound for the energy savings achievable with a two-stage RO system.

\(^1\)Concentration polarization is an effect that reduces the permeate flux due to increased salt concentrations at the membrane surface relative to the bulk feed.
1.3 Present work

A two-stage RO system has two degrees of freedom that are not present in a single-stage system. Firstly, membrane area can be distributed between the two stages in any proportion. Previous studies do not address this aspect of staged RO system design. In an actual RO plant, membrane area varies in discrete amounts, via spiral wound RO membrane elements. For a fixed-size two-stage RO system with eight membrane elements, the elements can be distributed between the two stages in seven distinct element configurations. In this work, we seek to identify the optimal element configuration for a two-stage RO system.

The second degree of freedom relates to the operating state of each stage. System operation can be viewed in terms of: first-stage and second-stage recovery ratios, brine concentrations, or feed pressures. Zhu et al. found that the maximum energy savings are obtained when the first-stage and second-stage recovery ratios are equal to each other, in the case when both stages are able to reach the state of "thermodynamic restriction" [18]. Thiel et al. derived an expression for the optimal intermediate brine concentration, accounting for density variations and with no constraint on each stage’s membrane area [11]. Optimal operation of a staged RO system will minimize entropy generation in the system, and lead to lower energy consumptions. Thermodynamic equipartition indicates that irreversible losses in a system can be attributed to two distinct factors: finite driving force and spatial and/or temporal variance in driving force [16]. Therefore, we expect that the energy consumption in a given two-stage RO system can be minimized by optimizing feed pressures such that spatial variance in driving force is minimized.

We have chosen to present the net energy savings of two-stage RO compared to a corresponding single-stage RO system, rather than the percentage savings. The capital costs of adding a second stage to an RO system are relatively fixed: the costs of an additional pressure vessel, high pressure pump, pressure exchanger, and booster pump. These additional costs must be compared to the energetic benefits and associated cost savings of adding a second stage to an RO system. Consequently,
the percentage savings are not relevant when making this decision. We note that the second stage pumps require a special seal due to the high pressure feed stream; they will likely be more expensive than the first stage pumps.

In this work, we investigate the optimal element configuration and feed pressures of a fixed-size two-stage RO system. We separate the energetic benefits of staging from the energetic benefits of additional membrane area by considering the energy savings of the optimized two-stage RO system relative to a single-stage RO system with the same size and same freshwater production. Both RO systems have an average system flux and membrane permeability comparable to today's typical values. Our spatially-discretized model of a spiral wound RO membrane also accounts for the effects of concentration polarization. Finally, we investigate the energy savings achieved with an optimized two-stage RO system for various feed salinities, membrane permeabilities, and recovery ratios.
Chapter 2

Methods

We implemented a mass-balance model of a spiral wound reverse osmosis element in MATLAB. Water flux is predicted with the solution-diffusion model and concentration polarization with the film theory model [6, 23]. This model allows us to evaluate the energetic performance of various single-stage and two-stage RO systems. For a RO system with given feed flow rate, desired recovery ratio (i.e. permeate flow rate), and membrane area (number of elements), the model calculates the required feed pressure. Once the required feed pressure is known, the specific energy consumption is found by a simple calculation. We validated our model by comparison to the Q+ Projection Software by NanoH2O (recently acquired by LG). Model validation data can be found in Appendix C.

Our model assumes that salt is perfectly rejected by the membranes, a simplifying assumption for today’s most permeable commercial membranes, which have a high salt rejection of > 99%[24]. In reality, the permeate salinity will be greater than zero as limited amounts of salt pass through the membrane. Salt passage will slightly decrease the overall energy consumption of the RO system due to the increased osmotic pressure on the permeate side.

Pump and pressure exchanger efficiencies are assumed to be 100% in our model. Therefore, the results presented here serve as a lower bound to the energy savings that can be achieved by moving to an equal-sized two-stage RO system, as shown in Section 3.3.
2.1 Model

Each membrane element is discretized spatially into up to 32 cells. A mass balance is performed between the feed channel portion in each pair of adjacent cells. Feed (NaCl and water) flows into each cell from the previous cell. Permeate (pure water) passes through the membrane into the permeate channel. The concentrate (NaCl and remaining water) flows into the feed channel of the next cell. The assumption of perfect salt rejection allows us to express the brine salinity, $w_{s,b}$, as a function of the instantaneous recovery ratio, $RR$, as shown by Eq. 2.1:

$$w_{s,b}(RR) = \frac{w_{s,f}}{1 - RR}$$  (2.1)

where $w_{s,f}$ is the salinity of the incoming feed stream.

The rejection of salt leads to an increased salt concentration on the membrane surface relative to the bulk feed. This effect, concentration polarization, reduces permeate flux. The permeate flux and concentration polarization factor (CPF) in each cell are governed by Eqs. 2.2 – 2.4:

$$J_v = A(P - \pi_m)$$  (2.2)

$$w_{s,m} = CPFw_{s,f}$$  (2.3)

$$CPF = e^{\frac{J_v}{k}}$$  (2.4)

where $J_v$ is the permeate flux, $A$ is the membrane permeability, $P$ is the feed pressure, $\pi_m$ is the osmotic pressure at the membrane surface, $w_{s,m}$ is the salt concentration at the membrane surface, CPF is the concentration polarization factor, $w_{s,f}$ is the salt concentration in the bulk feed stream, and $k$ is the mass transfer coefficient. The permeate flux is defined implicitly in terms of CPF, and vice versa. This prevents a straightforward calculation of the permeate flux. However, the maximum possible value of permeate flux occurs when $CPF = 1$. This bounds the range of possible
permeate flux values, so we can use a simple brute-force approach to evaluate the permeate flux and CPF. We create a vector spanning the range of possible permeate flux values, and use it to calculate a vector of possible CPFs with Eq. 2.4. Next we use the vector of possible CPFs to calculate a vector of corresponding permeate flux values with Eq. 2.2. Finally, we can compare both permeate flux vectors to determine the matching values of permeate flux and CPF. A more traditional approach, such as Newton’s method, is difficult to implement in this case because an expression for the necessary derivative is not readily available.

With a finite membrane area, an iterative solution is needed to calculate the feed pressure needed to achieve a certain recovery ratio. The model makes an initial guess at the necessary feed pressure based on the desired brine osmotic pressure. Permeate flux is calculated in every single cell, and summed up. The obtained recovery ratio is compared to the desired recovery ratio. The feed pressure is then adjusted according to the error in recovery ratio, and this process is repeated until the desired recovery ratio is achieved.

The chosen number of cells can have an effect on the calculated feed pressure. Permeate flux decreases monotonically along the length of the element. With a finite number of cells, we overestimate the amount of permeate that would be obtained at a given feed pressure. In order to achieve grid independence the model iterates over the entire process, doubling the number of cells in each simulation until the feed pressure varies by less than 1%.

Feed streams are approximated by aqueous NaCl solutions. The osmotic pressure and density of aqueous NaCl is calculated based on equations developed by Pitzer et al. [25]. The MATLAB implementation of those equations was developed by Thiel et al. [11]. The pressure dependence of density and osmotic pressure of aqueous NaCl solutions is neglected. Mass transfer coefficients and pressure losses are determined using correlations developed by Schock and Koutsou for flow in spiral wound membranes [26, 27].
Chapter 3

Results

The addition of a second stage to a reverse osmosis system does not guarantee energy savings. A two-stage RO system may consume more energy than the corresponding single-stage system if not properly designed or operated. Therefore, we must carefully optimize our two-stage RO system in order to achieve the desired energy savings.

A two-stage RO system has two degrees of freedom that are not present in a single-stage system. For a fixed size system with eight membrane elements, the elements can be shared between the two stages in seven distinct element configurations. Next, one must decide how much of the system's permeate is produced in the first stage, with the rest produced in the second stage. This permeate flux distribution is determined by the first-stage and second-stage feed pressures. Element configuration and flux distribution together have a significant effect on the final energy consumption, so an optimized system will feature the right combination of both factors.

Flux distribution informs system operation: to achieve the lowest energy consumption with an existing two-stage RO system, feed pressures should be chosen such that spatial variance of flux throughout the system is minimized. Element configuration informs system design: we find that the optimal design for a two-stage RO system places at least half of the elements in the first stage.

In the following sections, we optimize the design and operation of a two-stage seawater \( (w_{\text{seawater}} = 35 \text{ g/kg}) \) RO system. Our system reflects the size (8 membrane elements), membrane permeability \( (A = 1 \text{ L/m}^2\text{-h-bar}) \), and system flux \( (J_{\text{sys}} = 15 \)
L/m²-h) typical of today’s RO systems. Our system operates at a recovery ratio \((RR = 0.7)\) higher than today’s typical systems. We compare the energy consumption of this two-stage RO system to that of a single-stage seawater RO system with the same size, membrane permeability, system flux, and recovery ratio. Note that \(J_{sys}\) is defined as the total permeate volumetric flow rate divided by the total system size, although in some cases a portion of the system area may produce very little permeate.

### 3.1 Optimal flux distribution

A poorly operated two-stage RO system can consume more energy than a single-stage system. When the first-stage and second-stage feed pressures are properly chosen, significant energy can be saved. Here, we observe the energy consumption of a two-stage system versus flux distribution. We measure flux distribution in terms of \(J_1^*\), the permeate flux in the first stage normalized by the system flux:

\[
J_1^* = \frac{J_1}{J_{sys}}
\]

When \(J_1^* = 0\), no permeate is produced in the first stage of the system. This occurs when the first-stage feed pressure is at its lowest value \(\pi_f\) and the second-stage feed pressure is at its highest value. When \(J_1^* = 1\), permeate flux is perfectly balanced between the first and second stages. When \(J_1^*\) is at its maximum value, all permeate is produced in the first stage. This occurs when the first-stage feed pressure is at its highest value, and the second-stage feed pressure is at its lowest value \(\pi_b\).

We calculate the spatial variance of permeate flux using the flux values obtained in every cell along the length of the RO system:

\[
V = \frac{1}{N - 1} \sum_{i=1}^{N} |J_{v,i} - J_{sys}|^2
\]

where \(V\) is the spatial variance in flux, \(N\) is the total number of cells in the RO system, and \(J_{v,i}\) is the permeate flux in the \(i\)th cell.

In Fig. 3-1, we show the energy consumptions of a two-stage RO system as the
Figure 3-1: Effect of flux distribution on specific energy consumption. First stage flux is normalized by the overall system flux. Flux is balanced between the first and second stages when the normalized first stage flux is unity. Depending on the flux distribution, a two-stage RO system can consume more or less energy than the corresponding single-stage system. The lowest energy consumption occurs nearly where the variance in flux is minimized.
flux distribution varies. The dotted horizontal line shows the energy consumption of the corresponding single-stage system for comparison. Here, six elements are in the first stage and two elements are in the second stage. This particular choice of element configuration is arbitrary; an analysis of the optimal element configuration follows in Section 3.1. As flux distribution varies, the two-stage system can consume more or less energy than the corresponding single-stage system.

When $\bar{J}_1^* = 0$, all permeate flux occurs in the two elements of the second stage. The six elements in the first stage do not produce any permeate. This requires a relatively high second-stage feed pressure, and therefore results in a greater energy consumption than the single-stage RO system.

When $\bar{J}_1^* = 1$, permeate flux is balanced between the first and second stages. An ideal RO system would be able to constantly change feed pressure to follow the osmotic pressure curve and keep the permeate flux constant. In this ideal situation, equivalent to having infinite stages, flux is perfectly balanced throughout the system - the variance of flux would be zero [21]. In our two-stage system, we cannot achieve zero variance of flux. However, the minimum energy consumption occurs nearly when variance of flux throughout the system is minimized. We would expect the minimum energy consumption to coincide perfectly with minimum flux variance if the relationship between flux and driving force were linear; this is not the case here due to concentration polarization. However, values for the concentration polarization factor (CPF) are close to unity for today's typical membrane permeabilities, so the flux-force relationship is nearly linear in practice.

When $\bar{J}_1^* = 1.33$, all permeate flux occurs in the six elements of the first stage and the energy consumption is only slightly greater than the single-stage system. In this situation, one may consider switching to a single-stage system with just six elements. The energy consumption would be nearly the same as the single-stage system with eight elements, but with a lower capital cost and system footprint. However, the optimized two-stage system will have much lower maximum fluxes than either single-stage system. The benefits of a two-stage system are two-fold: decreased energy consumption and decreased maximum fluxes. Both factors should be considered when
3.2 Optimal element configuration

Any given element configuration has a wide range of possible energy consumptions. This is due to the flexibility in flux distribution, as described in Section 3.1. We repeated the flux distribution analysis for every single element configuration in order to find the optimal two-stage system. For today’s typical membrane permeabilities, the optimal element configuration places slightly more elements in the first stage than the second stage.

Figure 3-2 shows all the possible energy consumptions of a two-stage seawater \( w_{s,f} = 35 \text{ g/kg} \) RO system with 8 total elements, at \( J_{\text{sys}} = 15 \text{ L/m}^2\text{-h} \), \( A = 1 \text{ L/m}^2\text{-h-bar} \), \( w_{s,f} = 35 \text{ g/kg} \), \( RR = 0.7 \). For each element configuration \( J_1^* \) is varied from 0 to its maximum value to obtain the range of specific work shown. The optimal element distribution front-loads the first stage (five elements in the first stage - 2.20 kWh/m\(^3\)), although similar energy consumption may be achieved with six or four elements in the first stage (2.21 and 2.26 kWh/m\(^3\)).

designing an RO system.
Table 3.1: Minimum specific energy consumption (SEC) of different element configurations of a two-stage seawater RO system, compared to the single-stage energy consumption. Both systems have the same number of membrane elements, system flux, membrane permeability, feed salinity, and recovery ratio (8 total elements, $\dot{J}_{sys} = 15$ L/m$^2$-h, $A = 1$ L/m$^2$-h-bar, $w_{s,f} = 35$ g/kg, RR = 0.7)

<table>
<thead>
<tr>
<th># of elements in 1st stage</th>
<th>Two-stage</th>
<th>Single-stage</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Minimum SEC</td>
<td>2.78</td>
<td>2.53</td>
</tr>
</tbody>
</table>

maximum value to obtain the range of specific work shown. The black bars indicate energy consumption greater than the corresponding single-stage system, and the gray bars indicate energy consumption lower than the single-stage system.

The two-stage RO system may consume either more or less energy than the single stage system in all element configurations, but the lowest energy consumption occurs when there are five elements in the first stage (2.20 kWh/m$^3$, see Table 3.1). Similar energy consumptions may be achieved with six or four elements in the first stage (2.21 and 2.26 kWh/m$^3$). When an extreme configuration (one or seven elements in the first stage) is operated poorly, the two-stage system can consume more than double the energy of a single-stage system.

### 3.3 Optimized two-stage seawater RO system

In the previous section, we found that the optimal two-stage seawater RO system puts five elements in the first stage. Significant savings are achievable with our optimized two-stage RO system. Figure 3-3 shows the energy consumption of a single-stage RO system and our optimized two-stage RO system for different pump and pressure exchanger efficiencies, compared to the thermodynamic least work of separation at RR = 0.7. The single-stage and two-stage systems both have 8 elements total, $\dot{J}_{sys} = 15$ L/m$^2$-h, $A = 1$ L/m$^2$-h-bar, $w_{s,f} = 35$ g/kg, and RR = 0.7.

In the realistic case ($\eta_P = 0.85$, $\eta_{PX} = 0.92$), moving from a single-stage RO system to a two-stage RO system cuts energy consumption down by 1.04 kWh/m$^3$, 

30
a 26.3% savings. In the ideal case switching to our optimized two-stage system cuts energy consumption down by 0.89 kWh/m³, a 28.6% savings. This is 53.3% of the potential savings, the difference between the single-stage energy consumption and the least work of separation. Further energy savings could be achieved by adding more stages, but the energy saved would decrease with each successive stage.

Figure 3-4 shows that the energy savings associated with moving from a single-stage RO system to a two-stage RO system are actually larger in the realistic case than in the ideal case. If all pumps have the same efficiency, then the rate of work saved by moving from a single-stage RO system to a two-stage RO system is given by Eq. 3.3:

$$\Delta W_{1S \rightarrow 2S} = \frac{PQ_f\left|_{1S} - P_2Q_f\left|_{2S} + (P_2 - P_1)Q_{p1}\left|_{2S} \right.}{\eta_P}$$

(3.3)

where $Q_f$ is the volumetric flow rate of the entire feed stream and $Q_{p1}$ is the volumetric flow rate of the permeate produced in the first stage of the two-stage RO system. If the single-stage and two-stage RO systems have equal membrane area, average system flux, and recovery ratio then $Q_{f,1S} = Q_{f,2S}$. In an optimized two-stage RO system, $P_{2,2S} \approx P_{1S}$. In this case, the rate of work saved is approximately:

$$\Delta W_{1S \rightarrow 2S} \approx \frac{(P_2 - P_1)Q_{p1}\left|_{2S} \right.}{\eta_P}$$

(3.4)

Energy is saved in a two-stage RO system because the permeate produced in the first stage is not raised to the second-stage feed pressure. These savings are increased when you account for realistic pump efficiencies; an actual single-stage RO system must provide even more work to pressurize the entire feed stream.
Figure 3-3: Energy consumptions of a single-stage RO system and our optimized two-stage RO system for different pump and pressure exchanger efficiencies, compared to the thermodynamic least work of separation at $RR = 0.7$. Energy consumptions are larger in the realistic case ($\eta_P = 0.85, \eta_{PX} = 0.92$) than in the ideal case ($\eta_P = \eta_{PX} = 1$). The single-stage and two-stage systems both have 8 elements total, $\dot{J}_{sys} = 15$ L/m$^2$-h, $A = 1$ L/m$^2$-h-bar, $w_{s,f} = 35$ g/kg, $RR = 0.7$.

Figure 3-4: Energy savings achieved by moving from a single-stage RO system to a two-stage RO system for different pump and pressure exchanger efficiencies. The energy savings associated with moving from a single-stage RO system to a two-stage RO system are actually larger in the realistic case ($\eta_P = 0.85, \eta_{PX} = 0.92$) than in the ideal case ($\eta_P = \eta_{PX} = 1$).
Chapter 4

Factors affecting energetics of staged reverse osmosis

When should we add a second stage to a single-stage RO system? Here we look at the energy savings achieved for different recovery ratios, feed salinities, and membrane permeabilities. At a certain point, the energy savings will outweigh the cost of a second stage. This threshold will depend on a number of plant-specific factors.

Energy savings from adding a second stage grow as recovery ratio increases. However, increased energy savings does not indicate a lower specific energy consumption. The least work of separation of an RO process also rises with recovery ratio. Energy savings are increased at higher recovery ratios or feed salinities in part because more energy is available to be saved. In general, more energy can be saved by adding a stage when the least work of separation is large.

This can be seen if we observe the performance of a brackish RO system \((w_{s,f} = 3 \text{ g/kg})\) in Table 4.1. A two-stage RO system operating at a relatively high recovery ratio \((RR = 0.98)\) can consume more energy than a single-stage RO system operating at a lower recovery ratio \((RR = 0.9)\). However, any two-stage RO system will consume less energy than the corresponding single-stage RO system operating at the same recovery ratio. We also see that the energy savings achieved by adding a second stage are much greater at \(RR = 0.98\) than at \(RR = 0.94\) or 0.9.

High specific energy consumption does not automatically lead to high overall costs.
Table 4.1: Energy consumptions of both single-stage and two-stage RO systems increase with recovery ratio, as does the thermodynamic least work of separation. The energy savings associated with adding a second stage also increase with recovery ratio. Lower salinity feeds must be taken to very high recovery ratios before significant energy savings are realized.

<table>
<thead>
<tr>
<th>Salinity</th>
<th>Brackish water 3</th>
<th>Seawater 35</th>
<th>Produced water 95</th>
<th>[g/kg]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Recovery ratio</td>
<td>0.9 0.94 0.98</td>
<td>0.4 0.6 0.8</td>
<td>0.2 0.4 0.6</td>
<td>[-]</td>
</tr>
<tr>
<td>Least work</td>
<td>0.17 0.20 0.26</td>
<td>1.02 1.24 1.71</td>
<td>2.74 3.27 4.21</td>
<td>[kWh/m³]</td>
</tr>
<tr>
<td>Single-stage SEC</td>
<td>0.71 1.153 4.33</td>
<td>1.58 2.20 5.41</td>
<td>3.435 4.69 8.85</td>
<td>[kWh/m³]</td>
</tr>
<tr>
<td>Two-stage SEC</td>
<td>0.63 0.688 1.01</td>
<td>1.52 1.85 3.00</td>
<td>3.314 4.08 6.08</td>
<td>[kWh/m³]</td>
</tr>
<tr>
<td>Energy savings</td>
<td>0.08 0.465 3.32</td>
<td>0.06 0.35 2.41</td>
<td>0.121 0.61 2.77</td>
<td>[kWh/m³]</td>
</tr>
</tbody>
</table>

Many factors contribute to the final cost of water. Although the RO process requires more energy at high recovery ratios, pretreatment and brine disposal costs should decrease. Pressure exchangers may not be installed at high recovery ratios, since there will be less energy to extract as the brine flow shrinks. A full economic analysis of the final cost of water is out of the scope of this paper. Here we simply present the energy that would be saved by moving to a two-stage system at specific operating points.

4.1 Effect of feed salinity

Staged reverse osmosis is not limited to a seawater feed. Brackish water reverse osmosis plants have already implemented staging in their designs. A hypothetical 2-stage RO system could also perform well at high salinities [16].

We chose three different NaCl concentrations to represent a brackish feed (3 g/kg), a seawater feed (35 g/kg), and a representative produced water feed (95 g/kg).

In Fig. 4-1, we consider the maximum energy savings achievable with a two-stage RO system for various feeds, when the brine stream is completely saturated with NaCl. The highest energy savings are achievable when a brackish water feed is taken all the way to saturation; in this case the variation in osmotic pressure is greater than if a seawater or produced water feed were taken all the way to saturation. It
Brackish water, $w_{s,f} = 3 \text{ g/kg}$

Seawater, $w_{s,f} = 35 \text{ g/kg}$

Produced water, $w_{s,f} = 95 \text{ g/kg}$

Energy Savings [kWh/m$^3$]

Figure 4-1: Maximum energy savings achievable with a two-stage RO system for various feeds, when the brine stream reaches NaCl saturation. The potential to save energy with staging grows with the variation in osmotic pressure.

is very energy inefficient to take a brackish feed all the way to NaCl saturation with a single-stage: the entire feed stream must be pressurized to the osmotic pressure at saturation. The energetic benefits of adding a second stage are the greatest in this extreme case.

Very little energy is saved when a brackish water feed is taken to high recoveries ($0.08 \text{ kWh/m}^3 \text{ at } RR = 0.9$), as seen in Table 4.1. On the other hand, significant energy savings are achievable at lower recovery ratios when starting with a produced water feed ($2.77 \text{ kWh/m}^3 \text{ at } RR = 0.6$). Full parametric plots of the energy saved at selected feed salinities for varying recovery ratios or brine salinities can be found in Appendix A.

4.2 Effect of membrane permeability

Here we consider the effect of membrane permeability on energy savings associated with adding a second stage.

In Table 4.2, we compare the energy savings achieved with a two-stage brackish RO system at various recovery ratios with high permeability membranes ($A = 3$ and 10 L/m$^2$-h-bar) relative to a system with permeability typical of today's membranes ($A = 1$ L/m$^2$-h-bar). At low recoveries, the energy savings achieved at all permeabilities are quite small ($0.02 \text{ kWh/m}^3 \text{ at } RR = 0.60, A = 10 \text{ L/m}^2\text{-h-bar}$). At very high recoveries close to saturation ($RR \geq 0.98$), energy savings are nearly the same. Increasing
membrane permeability decreases the energy needed for over-pressurization, and this energy is small compared to the least work of separation at high recoveries.

The additional energy savings gained due to increased membrane permeability peaks at $RR = 0.91$. We consider the energetic performance of a two-stage brackish RO system with various membrane permeabilities at this recovery ratio in Fig. 4-2. Increasing membrane permeability from 3 to 10 L/m$^2$-h-bar does not significantly improve energetic performance; at very high membrane permeabilities, concentration polarization grows and becomes the dominant resistance to mass transfer. Therefore, a modest increase in membrane permeability (up to $A = 3$ L/m$^2$-h-bar) will slightly extend the viable range of staged RO.

The energetic benefits of increased membrane permeability in two-stage RO diminish as feed salinity increases; when a higher salinity feed is taken to the high recovery ratios where staging saves more energy, the least work of separation becomes much larger than the over-pressurization energy. Full parametric plots of the energy saved at selected membrane permeabilities for varying recovery ratios can be found in Appendix A.

Increased permeability will not drastically decrease the energy consumption of RO or the viability of staged RO [13, 28]. There is a greater potential to save cost by decreasing plant footprint size with high permeability membranes [13]. With high permeability membranes, system fluxes can theoretically be increased to nearly four times today’s typical values without increasing energy consumption [28]. In Fig. 4-3, we compare a typical seawater RO system ($A = 1$ L/m$^2$-h-bar, $J_{sys} = 15$ L/m$^2$-h, $w_{s,f} = 35$ g/kg, $RR = 0.7$) to a hypothetical RO system with very high permeability membranes ($A = 10$ L/m$^2$-h-bar, $J_{sys} = 30$ L/m$^2$-h, $w_{s,f} = 35$ g/kg, $RR = 0.7$). The RO plant with very high permeability membranes can produce the same amount of freshwater with half the membrane area (4 vs. 8 membrane elements) and less energy (2.12 vs. 2.20 kWh/m$^3$).
A = 1 L/m²-h-bar

A = 3 L/m²-h-bar

A = 10 L/m²-h-bar

Least work

Specific work [kWh/m³]

Figure 4-2: Comparison of the specific energy consumption of a two-stage brackish RO system (8 total elements, $w_{s,f} = 3$ g/kg, $J_{sys} = 15$ L/m²-h, $RR = 0.91$) at various membrane permeabilities to the least work of separation at $RR = 0.91$.

Figure 4-3: Evolution of osmotic pressure in optimized two-stage reverse osmosis systems at different membrane permeabilities. A hypothetical RO plant with very high permeability membranes ($A = 10$ L/m²-h-bar) can desalinate the same volume of water as today’s typical plants ($A = 1$ L/m²-h-bar) with half the membrane area (4 vs. 8 membrane elements) and less energy (2.12 vs. 2.20 kWh/m³).
Table 4.2: Energy savings achieved by adding a second stage to a brackish feed RO system (8 total elements, \( w_{s,f} = 3 \text{ g/kg} \), \( \bar{J}_{sys} = 15 \text{ L/m}^2\text{-h} \)) at various recovery ratios and membrane permeabilities. Energy savings are small at low recoveries since energy consumption is low to start. At high recoveries, energy savings do not change much since the energy needed for over-pressurization is small compared to the least work of separation. The arrival of membranes with higher permeabilities will slightly increase the viability of staged RO at moderate recovery ratios.

<table>
<thead>
<tr>
<th>Recovery ratio</th>
<th>0.60</th>
<th>0.84</th>
<th>0.91</th>
<th>0.98</th>
<th>[-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Least work</td>
<td>0.10</td>
<td>0.14</td>
<td>0.17</td>
<td>0.25</td>
<td>[kWh/m(^3)]</td>
</tr>
<tr>
<td>Energy savings for ( A = 1 \text{ L/m}^2\text{-h-bar} )</td>
<td>0.01</td>
<td>0.02</td>
<td>0.12</td>
<td>3.32</td>
<td>[kWh/m(^3)]</td>
</tr>
<tr>
<td>Energy savings for ( A = 3 \text{ L/m}^2\text{-h-bar} )</td>
<td>0.01</td>
<td>0.10</td>
<td>0.36</td>
<td>3.40</td>
<td>[kWh/m(^3)]</td>
</tr>
<tr>
<td>Energy savings for ( A = 10 \text{ L/m}^2\text{-h-bar} )</td>
<td>0.02</td>
<td>0.17</td>
<td>0.40</td>
<td>3.40</td>
<td>[kWh/m(^3)]</td>
</tr>
</tbody>
</table>
Chapter 5

Implications

In this work, we have separated the energetic benefits of staging from the benefits of adding membrane area, accounting for the effects of concentration polarization. Throughout, we compare two-stage and single-stage RO systems with equal membrane area and average system flux. In an actual two-stage RO system, one must decide how to distribute membrane elements between the two stages and at which two feed pressures to operate. The optimal system will place more membrane elements in the first stage of the system and minimize variance in flux throughout the system, balancing the permeate flux between the first and second stages.

For membranes and system operation typical of seawater RO today ($A = 1 \text{ L/m}^2\text{-h-bar}, J_{\text{sys}} = 15 \text{ L/m}^2\text{-h-bar}, w_{s,f} = 35 \text{ g/kg, } n = 8 \text{ elements}$), an energy savings of $0.89 \text{ kWh/m}^3$ is possible at $RR = 0.7$, a recovery ratio much higher than today’s typical RO plants ($RR \approx 0.4$). At current recovery ratios, the potential for saving energy with two-stage seawater RO is much smaller ($0.06 \text{ kWh/m}^3$ at $RR = 0.4$). Further membrane innovations, particularly high-pressure and anti-scaling membranes, will be needed before significant energy savings can be achieved.

When a larger variation of osmotic pressure occurs through the RO system, more energy can be saved by adding a stage. Therefore, for any given feed, the potential to save energy with staged RO increases with recovery ratio as brine concentrations approach saturation. Brackish feeds have the greatest potential for energy savings through staging, although significant energy savings may also be achieved for high-
Modest increases in membrane permeability (up to $A \approx 3 \text{ L/m}^2\text{-h-bar}$) will slightly increase the energetic benefits of staged RO at moderate recovery ratios. At low recovery ratios, the energetic benefits of staged RO are not significant. The benefits of increased membrane permeability diminish at high recovery ratios and feed salinities, since the least work of separation is much larger than the over-pressurization energy. Large increases in membrane permeability ($A = 10 \text{ L/m}^2\text{-h-bar}$) do not yield significant energetic benefits as concentration polarization grows. Greater potential to save costs with very high permeability membranes arises by decreasing system size (membrane area), without sacrificing energetic performance.

Membrane manufacturers often denote a critical flux: it is important to keep permeate fluxes below this level to avoid excessive fouling. Maximum flux will occur at the beginning of a stage when the difference in feed and osmotic pressures is greatest. A properly optimized two-stage system will have lower permeate fluxes than the corresponding single-stage system. A two-stage RO system can help to avoid fouling in addition to saving energy.

Additional energy can be saved by moving to three stages or more, but the savings diminish with each additional stage [20, 21]. Since the capital costs of adding a stage remain relatively constant, significant energy must be saved by adding a second stage in order to justify adding a third stage, and so on. We recognize that in some cases it may be beneficial to move to three stages or beyond. However, in many cases it is not obvious that adding a second stage will decrease the overall costs of RO. Here, we focus on informing the latter decision by investigating the energetic benefits of moving from a single-stage system to a two-stage system of the same size and freshwater production.

There is room for more complex designs of staged RO. For example, two first-stages could be placed in parallel and fed into a single second-stage in order to increase the second-stage feed flow. This may yield additional benefits; however, we have established that significant energy savings are achievable with the simplest two-stage RO design.
Appendix A

Energy savings of a two-stage RO system

Here, we present full parametric plots of the energy savings achieved with optimized two-stage RO systems as recovery ratio, feed salinity, and membrane permeability vary. In order to compute the energy savings, we first calculated the SEC of a single-stage RO system with 8 membrane elements and $J_{sys} = 15$ L/m$^2$-h. The single-stage SEC was compared to the SEC of an optimized two-stage RO system with 8 total membrane elements and $J_{sys} = 15$ L/m$^2$-h. The optimal element configuration depends on recovery ratio, feed salinity, and membrane permeability. In the optimized two-stage systems presented here, between four and seven elements are in the first stage.

In Fig. A-1, we compare the energy savings versus recovery ratio for different feed salinities. A brackish feed stream must be taken to very high recovery ratios, close to saturation, before significant energy savings are achieved. For higher salinity feeds, large energy savings can be achieved at lower recovery ratios. However, when taken to saturation, lower salinity feeds have the greatest potential for saving energy. Curves of higher salinity feed terminate earlier as they reach NaCl saturation.

In Fig. A-2, we present energy savings versus brine salinity for different feed salinities. For a fixed brine salinity, the highest energy savings are achieved when starting at a low feed salinity. It is very inefficient to take a brackish feed all the way
to NaCl saturation with a single-stage: the entire feed stream must be pressurized to the osmotic pressure at saturation. The energetic benefits of adding a stage to an RO system are greatest in this extreme case, when the difference in feed and brine osmotic pressures is high.

In Fig. A-3, we compare the energy savings at various recovery ratios with a high permeability membrane ($A = 3 \text{ L/m}^2\text{-h-bar}$) versus today’s typical membranes ($A = 1 \text{ L/m}^2\text{-h-bar}$) for a brackish water feed. The energy savings achieved by adding a second stage to an RO system are nearly identical for different membrane permeabilities when the recovery ratio is relatively low ($RR < 0.5$) or high ($RR > 0.95$). At low recoveries energy consumption is small, so there is not much energy to save. At high recoveries the energy needed to drive a productive permeate flow becomes small compared to the least work of separation. Higher membrane permeabilities will slightly increase the energy savings associated with two-stage RO at moderate recovery ratios.
Figure A-1: Energy savings for a two-stage RO system compared to an equivalent single-stage RO system ($J_{\text{sys}} = 15 \text{ L/m}^2\text{-h}, A = 1 \text{ L/m}^2\text{-h-bar}$) at different water recovery rates and feed salinities. Staged RO becomes viable at relatively low recovery ratios for higher salinity feeds, but the greatest energy savings are achievable for lower salinity feeds.
Figure A-2: Energy savings for a two-stage RO system compared to an equivalent single-stage RO system ($J_{sys} = 15 \text{ L/m}^2\text{-h}, A = 1 \text{ L/m}^2\text{-h-bar}$) at different brine salinities and feed salinities. The greatest energy savings are available when the difference in brine salinity and feed salinity is high; the difference in feed and brine osmotic pressure is also high in these cases.
Figure A-3: Energy savings achieved by adding a second stage to an RO system at various recovery ratios and membrane permeabilities ($\bar{J}_{\text{sys}} = 15$ L/m$^2$-h, $w_{s,f} = 3$ g/kg). Energy savings are nearly identical for all membrane permeabilities when the recovery ratio is relatively low ($RR < 0.5$) or high ($RR > 0.95$). Increased membrane permeability will slightly increase energy savings at moderate recoveries.
Appendix B

Energy consumption in single-stage and two-stage RO

A straightforward view of the energy consumption of an RO system is obtained by observing the evolution of osmotic pressure versus recovery ratio. Recovery ratio is proportional to permeate flow, so the area under the feed pressure curve is proportional to the energy spent to pressurize the permeate flow.

In Fig. B-1, we compare the energy consumption of a single-stage system to a two-stage system. In both systems, the blue area under the osmotic pressure curve represents the least work of separation. The orange area under the feed pressure is excess energy, some of which can be saved. In the two-stage system (b), the green area between the single-stage feed pressure and the first-stage feed pressure is approximately the energy saved when moving from the single-stage RO system to the two-stage RO system. The actual energy saved will be slightly less because the second-stage feed pressure is a bit higher than the single-stage feed pressure in this case. This small difference corresponds to the portion of the orange area between the single-stage feed pressure and the second-stage feed pressure. Note that this portion of the orange area is small compared to the green area. Figure B-1 neglects salt water’s variation in density due to increasing salinity, so the actual energy used to achieve the final feed pressure is slightly more than pictured. Still, these figures provide a sense of intuition for the energy consumption in a two-stage RO system.
Figure B-1: Energy consumption in a single-stage system (a) and a two-stage system (b). Energy consumption is illustrated when pressure is plotted against recovery ratio. Recovery ratio is proportional to permeate flow, so the area under the feed pressure curve is equal to the energy spent to pressurize the permeate flow.
Appendix C

Model validation

We validated our model by comparison to the Q+ Projection Software from NanoH2O (recently acquired by LG). For a single-stage RO system with eight membrane elements in series, we compared the required feed pressure needed to achieve certain recovery ratios. As seen in Fig. C-1, our model is in good agreement with the Q+ model. Errors in the required feed pressure are less than 7\%, and are much lower for typical RO operating conditions (< 3\% error at $RR = 0.4$). Furthermore, our model predicts feed pressure values for recovery ratios greater than 0.61; the Q+ software fails to calculate these values.
Figure C-1: Feed pressure required for a single-stage RO system (8 elements, $\bar{J}_{\text{sys}} = 15$ L/m$^2$-h, $A = 1.8$ L/m$^2$-h-bar, $w_{p,f} = 35$ g/kg) to achieve various recovery ratios according to Q+ Projection Software and our model. Our model agrees with the Q+ Projection Software, and predicts feed pressure values beyond the operating range of the Q+ software.
Appendix D

Spiral wound reverse osmosis element model for MATLAB

```matlab
function [pv]=crossflow(num_elements,rr_m,m_in,w_in,A,nx_fixed,nx_n)

%% Spiral-wound RO membrane pressure vessel model
% This script models one pressure vessel composed of spiral wound RO elements. For a given number of RO elements, desired recovery ratio, feed salinity, and feed flow rate, the function will return the required pressure and values along the length of the pressure vessel.

%% Inputs
% Number of elements in stage (-)
% Desired recovery ratio (m_permeate/m_feed)
% Feed flow rate (kg/h)
% Feed salinity - mass fraction (g/kg)
% Membrane permeability (L/m^2-h-bar)
% nx_fixed - Option to specify number of cells
% nx_n - Number of cells, if nx_fixed == 1.

%% Outputs
% feed pressure [bar]
% number of cells per element [-]
% feed flow along length [kg/hr]
% feed flow along length [L/hr]
% feed molality along length [mol/kg]
% feed osmotic pressure (bulk) along length [bar]
% feed osmotic pressure (membrane wall) along length [bar]
% permeate flux along length [L/m^2-h]
% concentration polarization factor along length [-]
```
% permeate produced along length [kg/hr]
% hydraulic pressure along length [bar]

%%% Assumptions
% The model is based on mass conservation between each differential
volume. We assume perfect salt rejection. Water temperature is
assumed constant.

%%% Set-up the model
load('NaCl_data.mat'); % density and osmotic pressure of aqueous NaCl
if ~exist('nx_fixed','var')
    nx_fixed=0;
end

% Membrane properties
area_metric=400*12^2+2.54^2/100^2; % element area (m^2)
l_metric=40*2.54/100; % element length (m)
t=0.7/1000; % spacer/channel thickness (m)
w=area_metric/(2*l_metric); % channel width (m)
area_c=w*t; % channel cross-sectional area (m^2)
d_h=2*area_c/(w+t); % hydraulic diameter (m)

% Water properties
t_w=25+273; % water temperature (K)

% Get feedwater properties
b_in=w2b(w_in); % feed molality (mol/kg)
rho_in=interpl(b_mat,rho_mat,b_in); % density (kg/m^3)
rho_p =interpl(b_mat,rho_mat,0); % density (kg/m^3)
m_h2o = m_in*(1-w_in/1000); % initial mass flow of water (kg/h)
mol_nacl = b_in*m_h2o; % initial molar flow of NaCl (mol/h)
q_in=m2q(m_in,b_in); % volumetric flow rate of feedwater (m^3/h)

% Parameters for concentration polarization
mu=1.01*10^-3; % dynamic viscosity (kg/m-s)
dif=1.99*10^-9; % mass diffusivity (m^2/s)

Mp_target=m_in*rr_m; % desired permeate flow rate (kg/s)
P_prev=0;
x=2;

%%% The first loop, to achieve grid independence
while 1
nx=nx*2; % Double the number of cells until we get grid independence
dx=l_metric/nx;

if nx_fixed==1 % Flag to fix number of cells in each element.
    nx=nx_n;
end

dA=area_metric/nx;
Nx=num_elements*nx;

if P_prev == 0; % Initial guess at required feed pressure
    w_brine=w_in/(1-rr_m);
    b_brine=w2b(w_brine);
    p_out=interp1(b_mat,pi_mat,b_brine);
    p=p_out;
else
    p=P_prev;
end

% Initialize matrices
M_h2o=[m_h2o zeros(1,Nx)]; % Mass of H2O in the feed [kg/hr]
B_f=[b_in zeros(1,Nx)]; % Molality in the feed [mol/kg]
Pi_f=[interp1(b_mat,pi_mat,b_in) zeros(1,Nx)]; % Feed osmotic pressure (bulk) [bar]
M_f=[m_in zeros(1,Nx)]; % feed flow [kg/hr]
Q_f=[q_in zeros(1,Nx)]; % feed flow [L/hr]
CP=[0 1 zeros(1,Nx-1)]; % Concentration polarization factor [-]
Pi_m=[0 interp1(b_mat,pi_mat,b_in) zeros(1,Nx-1)]; % Feed osmotic pressure (membrane) [bar]
J_v=[0 A*(p-Pi_m(2)) zeros(1,Nx-1)]; % Permeate flux [L/m^2-h]
M_p=[0 zeros(1,Nx)]; % permeate flow [kg/hr]

% Start solving model of pressure vessel.
% We need to guess the required pressure to achieve the desired recovery. We run the entire model and then check on permeate production. We adjust the pressure and repeat. We break out of the loop when the error drops below the target value.

dj=10000;
while 1
    p_h=[p zeros(1,Nx)];

    % Find the correct CPF in the first cell
\[ v = \frac{Q_f(1)}{area_c/1000/60/60}; \quad \% \text{Flow velocity [m/s]} \]
\[ re = \rho_{\text{in}} \cdot v \cdot d_h / \mu; \quad \% \text{Reynolds number [-]} \]
\[ sc = \mu / \text{dif} / \rho_{\text{in}}; \quad \% \text{Schmidt number [-]} \]
\[ sh = 0.2 \cdot re^{0.57} \cdot sc^{0.4}; \quad \% \text{Sherwood number [-]} \]
\[ f = 6.23 \cdot re^{-0.3}; \quad \% \text{Friction factor [-]} \]
\[ \delta = d_h / sh; \quad \% \text{Mass-transfer boundary layer thickness [m]} \]

\[ \delta_p = 0.5 \cdot f \cdot \rho_{\text{in}} \cdot v^2 \cdot dx / d_h; \quad \% \text{frictional pressure drop [Pa]} \]
\[ p_h(2) = p_h(1) - \delta_p \cdot e^{-5}; \]

\% Match permeate flux and concentration polarization predictions using a brute-force approach
\[
J_{\text{guess}_-\text{mat}} = \frac{J_v(2)}{dj:J_v(2)/dj:J_v(2)};
\]
\[ CP_{\text{guess}} = \exp\left(J_{\text{guess}_-\text{mat}/1000/60/60*\delta\_\text{dif}}\right);
\]
\[ J_{CP} = A \cdot (p_h(1) - \text{interpl}(b_{\text{mat}};\pi_{\text{mat}};CP_{\text{guess}} \cdot B_f(1))); \]
\[ \text{error}_{J_v} = \text{abs}\left(J_{CP} - J_{\text{guess}_-\text{mat}} / J_{\text{guess}_-\text{mat}}\right); \]
\[ [~,b] = \text{min}\left(\text{error}_{J_v}\right); \]
\[ CP(2) = CP_{\text{guess}}(b); \]
\[ \pi_{\text{m}}(2) = \text{interpl}(b_{\text{mat}};\pi_{\text{mat}};CP(2) \cdot B_f(1)); \]
\[ J_v(2) = A \cdot (p_h(1) - \pi_{\text{m}}(2)); \]
\[ M_p(2) = J_v(2) \cdot dA \cdot \rho_p / 1000; \]

\% Iterate through the rest of the cells in the pressure vessel using the same procedure.

\text{for } ii=2:Nx

\% Calculate values in the next cell
\[ M_{h2o}(ii) = M_{h2o}(ii-1) - M_p(ii); \]
\[ M_f(ii) = M_{h2o}(ii) + \text{mol}_{\text{nacl}} \cdot 58.44 / 1000; \]
\[ B_f(ii) = \text{mol}_{\text{nacl}} / M_{h2o}(ii); \]
\[ \pi_f(ii) = \text{interpl}(b_{\text{mat}};\pi_{\text{mat}};B_f(ii)); \]
\[ Q_f(ii) = m2q(M_f(ii);B_f(ii)); \]
\[ rho = \text{interpl}(b_{\text{mat}};\rho_{\text{mat}};B_f(ii)); \]
\[ v = Q_f(ii) / area_c / 1000 / 60 / 60; \]
\[ sc = \mu / \text{dif} / \rho; \]
\[ re = rho \cdot v \cdot d_h / \mu; \]
\[ sh = 0.2 \cdot re^{0.57} \cdot sc^{0.4}; \]
\[ \delta = d_h / sh; \]
f=6.23*re^(-0.3);

delta_p=0.5*f*rho_in*v^2*dx/d_h;
p_h(ii+1)=p_h(ii) - delta_p*1e-5;

% Brute force approach to calculating CPF
J_guess = A*(p_h(ii-1)-Pi_f(ii));
J_guess_mat = 0:J_guess/dj:J_guess;
CP_guess = exp(J_guess_mat/1000/60/60*delta/dif);
J_CP = A*(p_h(ii)-interp1(b_mat,pi_mat,CP_guess*B_f(ii)));
error_Jv = abs(J_CP-J_guess_mat)./J_guess_mat;
[-,b]=min(error_Jv);

CP(ii+1)=CP_guess(b);
Pi_m(ii+1)=interp1(b_mat,pi_mat,CP(ii+1)*B_f(ii));
J_v(ii+1)=J_guess_mat(b);

M_p(ii+1)=J_v(ii+1)*dA*rho_p/1000;

end

% Calculate values for the final cell
M_h2o(end)=M_h2o(end-1)-M_p(end);
M_f(end)=M_h2o(end)+mol_nacl*58.44/1000;
B_f(end)=mol_nacl/M_h2o(end);
Pi_f(end)=interp1(b_mat,pi_mat,B_f(end));
Q_f(end)=m2q(M_f(end),B_f(end));

% see whether desired recovery is achieved and calculate error
Mp_actual=sum(M_p);
error=(Mp_actual-Mp_target)/Mp_target;

% break out of the loop if error is small enough
if abs(error) < .001
    break
end

% update pressure using log mean average of osmotic pressure
Pi_ave=(Pi_f(end-1)-Pi_f(2))/(log(Pi_f(end-1))-log(Pi_f(2)));
p_old=p_h(1);
p=p_h(1)-error*(abs(p_h(1)-Pi_ave));

end
% Make sure things we achieve grid independence. The initial
value of P_prev is 0, so the initial error_P is 1. We must go
through at least two iterations.
error_P = (p-P_prev)/p;

% exit the loop immediately if the number of cells is specified
if nx_fixed==1
    error_P = 0;
end

% If error is small enough, exit function and pass outputs.
if error_P < 0.01
    pv.p=p; % feed pressure [bar]
    pv.nx=nx; % number of cells per element [-]
    pv.M_f=M_f; % feed flow along length [kg/hr]
    pv.Q_f=Q_f; % feed flow along length [L/hr]
    pv.B_f=B_f; % feed molality along length
    pv.Pi_f=Pi_f; % feed osmotic pressure (bulk) along length [bar]
    pv.Pi_m=Pi_m; % feed osmotic pressure (membrane wall) along
                  length [bar]
    pv.J_v=J_v; % permeate flux along length [L/m²*h]
    pv.CP=CP; % concentration polarization factor along length
    [ ]
    pv.M_p=M_p; % permeate produced along length [kg/hr]
    pv.p_h=p_h; % hydraulic pressure along length [bar]
    break
end
end

P_prev=p;
end
Bibliography


