

AUTOMATIC CONTROL BY ARITHMETICAL OPERATIONS

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INTRODUCTION

During the last five years Bush, Caldwell, Overbeck, and others in unpublished memoranda at the Massachusetts Institute of Technology have described methods of performing arithmetical operations with electronic devices. While no papers on this subject have appeared in the literature, workers elsewhere recently have made a number of contributions. The value of electronic calculating systems for accounting operations and for solving problems in engineering and science has been recognized from the beginning. Recently it has been proposed that electronic calculating systems can perform a valuable function in fire-control operations. It is the purpose of this thesis to describe the elements and operation of a calculating system for performing one of the operations in the control of anti-aircraft gunfire, which is, namely, the prediction of the future position of the target.

It is to be emphasized at the outset that little progress has been made toward the construction of automatic electronic calculating systems for any purpose. Effort to the present time has been confined to the development or even invention of fundamental elements of electronic calculating systems. The system described in this thesis must therefore be regarded as tentative, and, in a large measure, arbitrary because it is not based on experimental evidence. It can be proposed only that this thesis shows a possible approach to the design of a number of calculating system-elements and to the structure of an arithmetical predictor.

Because no published material on electronic calculating systems is available, it is not possible to embark directly on a description and discussion of an arithmetical predictor. In this introduction, equipment for performing the operations occurring in automatic calculating is described. This

equipment includes electronic switching elements, devices for multiplying two numbers, finding a function of a variable, recording numbers, translating mechanical displacements into numerical data, and for translating numerical data into mechanical displacements. A brief description of binary numeration and operations with binary numbers serves to recall the properties of the binary number system.

Binary Numeration

There are two digits in the binary number system: zero (0) and one (1). Any number written in the decimal system can be rewritten in the binary system by writing the number as a sum of powers of two and adding the binary representations of the powers. For the first few powers these representations are

<u>Decimal</u>	<u>Binary</u>
$1 = 2^0 = 1,$	
$2 = 2^1 = 10,$	
$4 = 2^2 = 100,$	
$8 = 2^3 = 1000,$	
etc.	

Thus,

$$13 = 8 + 4 + 1 = 1000 + 100 + 1 = 1101.$$

Table I shows the decimal powers of two and their binary representations over the range of practical interest. By referring to this table, the binary representation of any decimal number can be written. For example,

$$\begin{aligned} 3481 &= 2048 + 1024 + 256 + 128 + 16 + 8 + 1 \\ &= 2^{11} + 2^{10} + 2^8 + 2^7 + 2^4 + 2^3 + 2^0 \\ &= 110110011001, \end{aligned}$$

2^{-10}	0 0 0 9 7 6 5 6 2 5
2^{-9}	0 0 1 9 5 3 1 2 5
2^{-8}	0 0 3 9 0 6 2 5
2^{-7}	0 0 7 8 1 2 5
2^{-6}	0 1 5 6 2 5
2^{-5}	0 3 1 2 5
2^{-4}	0 6 2 5
2^{-3}	1 2 5
2^{-2}	2 5
2^{-1}	5
2^0	1
2^1	2
2^2	4
2^3	8
2^4	1 6
2^5	3 2
2^6	6 4
2^7	1 2 8
2^8	2 5 6
2^9	5 1 2
2^{10}	1 0 2 4
2^{11}	2 0 4 8
2^{12}	4 0 9 6
2^{13}	8 1 9 2
2^{14}	1 6 3 8 4

Decimal Representation

10^{-1010}	0 0 0 0 0 0 0 0 0 0 1
10^{-1001}	0 0 0 0 0 0 0 0 0 1
10^{-1000}	0 0 0 0 0 0 0 0 1
10^{-111}	0 0 0 0 0 0 1
10^{-110}	0 0 0 0 0 1
10^{-101}	0 0 0 0 1
10^{-100}	0 0 0 1
10^{-11}	0 0 1
10^{-10}	0 1
10^{-1}	1
10^0	1
10^1	1 0
10^{10}	1 0 0
10^{11}	1 0 0 0
10^{100}	1 0 0 0 0
10^{101}	1 0 0 0 0 0
10^{110}	1 0 0 0 0 0 0
10^{111}	1 0 0 0 0 0 0 0
10^{1000}	1 0 0 0 0 0 0 0 0
10^{1001}	1 0 0 0 0 0 0 0 0 0
10^{1010}	1 0 0 0 0 0 0 0 0 0 0
10^{1011}	1 0 0 0 0 0 0 0 0 0 0 0
10^{1100}	1 0 0 0 0 0 0 0 0 0 0 0 0
10^{1101}	1 0 0 0 0 0 0 0 0 0 0 0 0 0
10^{1110}	1 0 0 0 0 0 0 0 0 0 0 0 0 0 0

Binary Representation

Table I

Binary Representations of the Powers of Two

and

$$\begin{aligned}
 237.34 &= 128 + 64 + 32 + 8 + 4 + 1 + 0.25 + 0.0625 + 0.015625 + \\
 &\quad 0.0078125 + 0.00390625 + \dots \\
 &= 2^7 + 2^6 + 2^5 + 2^3 + 2^2 + 2^0 + 2^{-2} + 2^{-4} + 2^{-6} + 2^{-7} + 2^{-8} + \dots \\
 &= 11101101.01010111 \dots
 \end{aligned}$$

Two binary numbers are added in the same way that decimal numbers are added. For example,

	1 11	Carryover digits
53	= 110101	First term
<u>23</u>	= <u>10111</u>	Second term
76	= 1001100	Sum

A column of numbers are added similarly. The sum of each column of digits is formed, and the carryover digits are either distributed among the appropriate digit positions or are added to the sum of digits in the adjacent column. Two binary numbers are subtracted in the same way that decimal numbers are subtracted. If 1 is subtracted from 0, the difference is 1 and a 1 is "borrowed" from the adjacent digit position. If no ones are available, the difference is a complement. For example,

110101	Minuend
<u>10111</u>	Subtrahend
11110.	Difference

If the subtrahend is larger than the minuend,

10111	Minuend
<u>110101</u>	Subtrahend
...111100010.	Difference

The difference in the second example appears as a complement. A comparison of the differences in the two examples shows that the complement of a binary number is formed simply by replacing ones with zeros and zeros with ones,

except for the one in the smallest digit position occupied by a one which is left unchanged.

For lack of more appropriate names, the several digit positions of a binary number are referred to as the units, tens, hundreds, etc., positions, or as the tenths, hundredths, thousandths, etc., positions depending on the location of the digit position with respect to the decimal point.

Two binary numbers can be multiplied by repeated addition of the multiplicand. For example,

110101	Multiplicand
<u>10111</u>	Multiplier
110101	
110101	
110101	
<u>110101</u>	
10011000011.	Product

The multiplicand is written down once for each digit in the multiplier, and for a particular digit is written in a series of digit positions that correspond to multiplication by the power of ten associated with the position of the digit. The resulting columns of digits are added to give the product. The process of moving the multiplicand from one series of digit positions to another is referred to as "shifting" the multiplicand. If the multiplicand is a complement, the product can be obtained in the above manner. If the multiplier is a complement, it must be translated into the equivalent negative number.

Division of binary numbers can be performed in the same way that division of decimal numbers is performed. The process is difficult from the standpoint of switching circuits, however, and in an automatic calculating system, division is performed by multiplication by the reciprocal. The reciprocal is obtained from a table of functions.

The number of binary digits required to represent a decimal number

with a particular number of figures is shown by Table I. For most fire-control operations, twelve or thirteen digits are adequate to represent the values of a variable. A larger number of digit positions must be used in some parts of the system, however, because numbers ranging over two orders of magnitude (decimal) are used.

Electronic Switching Devices and Circuits

The fundamental electronic switching element is a vacuum triode. A single triode coupled to appropriate external resistance networks can operate like a set of switches connected in series, like a set of switches connected in parallel, or like a set of switches connected in a symmetric series-parallel circuit. By a symmetric circuit is meant, for example, a circuit that closes if any two switches in a set of four switches close.

If several voltage-sources are connected through high-value resistors to the grid of a triode, the voltage of the grid is the mean of the voltages of the sources. Let it be assumed that four voltage-sources are connected to a grid and that the voltage of each source shifts between 0 and 40 volts. Then if the voltage of one source is 40 volts and the voltage of the other sources is zero, the voltage of the grid is 10 volts. If the voltage of two sources is 40 volts, the voltage of the grid is 20 volts. If the voltage of three of the sources is 40 volts, the voltage of the grid is 30 volts, and if the voltage of all four sources is 40 volts, the voltage of the grid is 40 volts. Assume that the plate current cut-off voltage of the triode is -10 volts and that full current flows and the triode is "operated" if the voltage of the grid is raised to the voltage of the cathode which is zero voltage.

If the bias voltage of the grid is -40 volts, the voltages of all four of the voltage sources must be 40 volts to operate the triode, and the triode acts like a set of four switches connected in series.

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If the bias voltage of the grid is -10 volts, the triode operates if the voltage of any one of the sources is 40 volts, and the triode acts like a set of four switches connected in parallel.

If the bias voltage of the grid is -20 volts, the triode operates if the voltage of any two of the sources is 40 volts, and the triode acts like six parallel switch-paths, each path having two switches in series.

If the bias voltage of the triode is -30 volts, the triode operates if the voltage of any three of the sources is 40 volts, and the triode acts like four parallel switch-paths, each path having three switches in series.

If the voltage-shift of the sources is increased sufficiently, an indefinitely large number of sources can be connected to the grid of a triode and contribute a switching signal.

If the voltage shifts available for performing a switching operation are negative, the triode can be operated at positive bias voltage, and the sources can decrease the voltage of the grid to cut-off voltage. Since the series-resistance of the grid circuit is large, the triode saturates for positive grid voltages and a significant change of plate current results only when the voltage of the grid is shifted from cathode voltage to cut-off voltage.

When two voltage sources are connected to the grid of a triode, or, hereafter, switch, it is often convenient to speak of the voltage shift of one source priming the switch and the voltage shift of the other source operating the switch.

Trigger Circuits

A circuit with two or more conditions of stability is called a trigger circuit. Figure 1 shows a double triode trigger circuit that has two conditions of stability, corresponding to a maximum current flowing to one anode or the other. When the current flowing to one anode is a maximum, the current

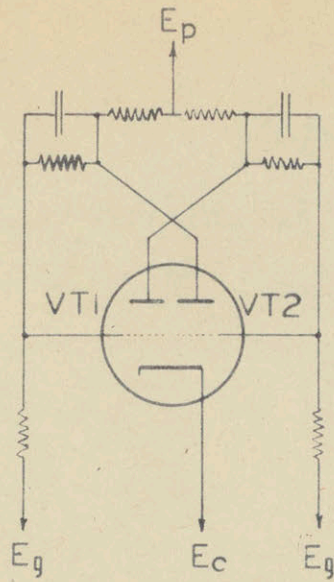


FIGURE 1

DOUBLE TRIODE TRIGGER CIRCUIT

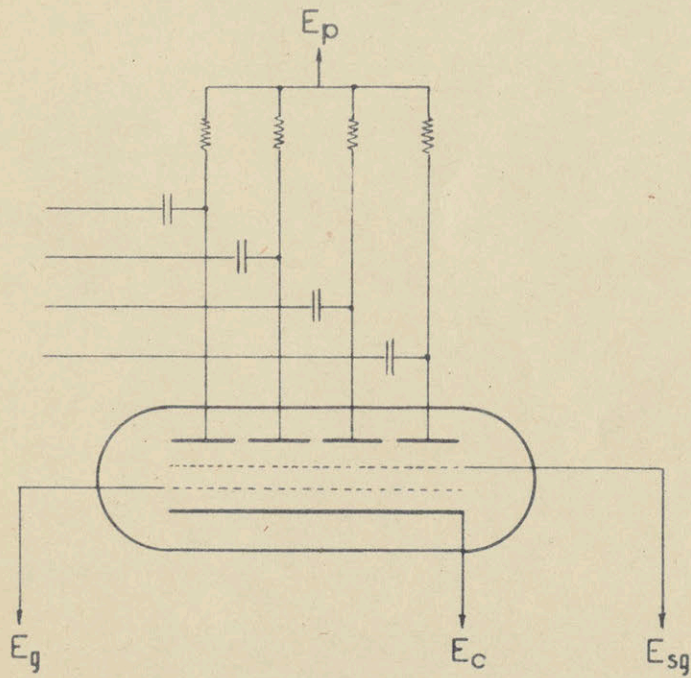


FIGURE 2

FOUR ELEMENT HOLDING SET

flowing to the other anode can be made zero.

To show that the circuit has two conditions of stability, let it be assumed that equal currents flow to the two anodes. If the voltage of one grid, say the grid of VT1 shifts positively a small amount for any reason, the current through VT1 increases and the anode voltage of VT1 decreases. The grid voltage of VT2 is decreased by the anode of VT1 and the current through VT2 decreases. The anode voltage of VT2 and therefore the grid voltage of VT1 increase. The reaction of the circuit thus is to propagate the original small shift of grid voltage. The circuit reaches equilibrium when a maximum current flows in VT1 and zero current flows in VT2.

If the original voltage shift of the grid of VT1 is negative, the circuit reaches the second stable condition, which is the condition where a maximum current flows to the anode of VT2 and zero current flows to the anode of VT1. If the grid voltage of VT2 shifts when equal currents are flowing to the two anodes, the reaction of the circuit is similar to the reaction described above except for a voltage shift in a given direction, the opposite stable condition is reached.

A trigger circuit of the kind shown in Figure 1 can be transferred from one stable state to the other by positive or negative voltage pulses applied to both grids simultaneously, or by voltage pulses of proper polarity applied to either grid individually. It is useful to distinguish the two stable states of a trigger circuit by the names active and passive. By convention, the active state of the circuit will correspond to the current flowing to the anode of VT2, and the passive state, to the current flowing to the anode of VT1. A positive voltage pulse applied to the grid of VT2 or a negative voltage pulse applied to the grid of VT1 transfers the trigger circuit from the passive state to the active state, and it is convenient to call the

grid terminal of VT2 the positive-active input terminal of the trigger circuit, and the grid terminal of VT1, the negative-active input terminal of the trigger circuit. When the trigger circuit is active, the anode of VT2 is at supply voltage, and the anode of VT1 is at some voltage less than supply voltage. It is convenient to refer to the anode terminal of VT2 as the positive-active output terminal of the circuit and to the anode terminal of VT1 as the negative-active output terminal of the circuit.

It is often convenient to distinguish between a trigger circuit with a pulsing connection common to the two grids and a circuit with separate pulsing connections to the two grids. A circuit with separate connections will be called a holding circuit, while the name trigger circuit will be used for the circuit with a common pulsing connection. A holding circuit is closely analogous to an electro-mechanical relay with a "holding" or "lock-in" circuit. It is useful to speak of turning a holding circuit on or of operating a holding circuit instead of transferring it from the passive state to the active state, and of turning it off or resetting it instead of transferring it from the active state to the passive state.

The trigger circuit shown in Figure 1 is only one of many kinds of trigger circuits that have been described and built. Another trigger circuit of particular interest in the design of automatic calculating systems is a circuit endowed with two conditions of stability by secondary emission from the anode. A circuit employing the old type 24 tetrode is an example of such a trigger circuit. Recently, Overbeck has built tubes with negative resistance produced by secondary emission that are particularly suited to some of the requirements of an automatic calculating system. To distinguish between a trigger circuit of the kind shown in Figure 1 and a secondary emission trigger circuit, the former will be referred to as a trigger-pair and the

latter, as a trigger-element where confusion might otherwise result.

Holding Set

A holding set is an array of trigger-elements for recording the digits of a binary number. When several trigger-elements are used as a group, the cathode and grids of the elements can be connected to common circuits, or, what is more practicable, the cathode and grids can themselves be common to a group of anodes. In a form of holding set that has actually been realized, ten anodes are disposed radially about a cathode and a pair of cylindrical grids. Figure 2 is a representation of a holding set with four anodes. Each anode is capable of assuming one of two voltages independently of the rest of the anodes and can be transferred from one voltage to the other by voltage pulses applied to it through a condenser. One stable voltage of an anode is approximately supply voltage while the other stable voltage is a few volts above cathode voltage. Because an anode can be shifted from its lower to its upper stable voltage more rapidly than it can be shifted in the opposite direction, and because it is convenient to apply voltage pulses in the positions of the ones of a binary number that is to be recorded in the holding set, the lower stable voltage is associated with the binary digit zero and the upper stable voltage with the binary number one. When an anode voltage is the lower stable value, the trigger-element that is the anode will be said to be passive, and when an anode voltage is the upper stable value, the trigger-element will be said to be active.

Overbeck has not yet had an opportunity to describe the most effective ways to use holding sets. For the purposes of this thesis, it will be enough to assume that two terminals in addition to the anode terminals are associated with a holding set. If the voltage of one terminal is momentarily shifted an appropriate amount, the holding set is cleared, and the number in

the holding set is, in effect, transferred from the holding set. The voltage of the anodes of the active trigger-elements drops to the voltage of the anodes of the passive elements, and the voltage of the anodes of the passive trigger-elements does not change. The voltage shifts momentarily produced in leads connected to the anodes through condensers indicate the values of the digits in the holding set.

If the voltage of the second terminal is shifted and maintained at an appropriate value, the holding set can receive a number transmitted to it in the form of voltage shifts of leads connected to the anodes through condensers.

The former terminal will be called the clearing terminal of the holding set, and the latter terminal, the priming terminal. The leads connected to the anodes will be called transfer leads.

Counter

A possible form of counter is an array of trigger-pairs so coupled that when a series of pulses is applied to the grids of the pair at one end of the array, the active pairs in the array are in the positions of the ones of the binary number that is the number of applied pulses. Figure 3 shows two counter stages and the coupling between the stages. Assume that both trigger-pairs are passive initially. The first voltage pulse applied to the units trigger-pair transfers the trigger-pair to the active state. The coupling tube is normally biased to cut-off, and the negative shift of the negative-active output terminal of the first trigger-pair produces no change in the anode voltage of the coupling tube. The second voltage pulse applied to the units trigger-pair transfers the trigger-pair from the active to the passive state. When the negative-active output terminal of the trigger-pair shifts

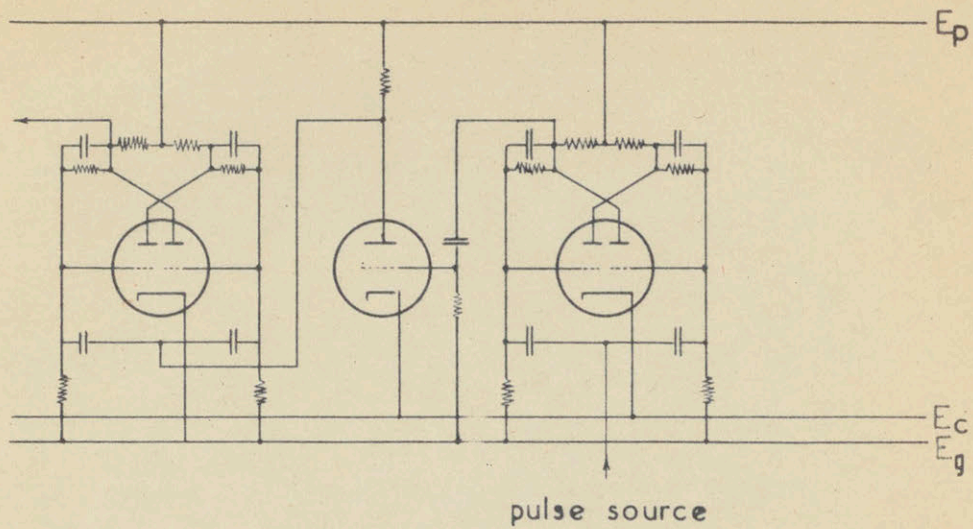


FIGURE 3
BINARY COUNTER STAGES

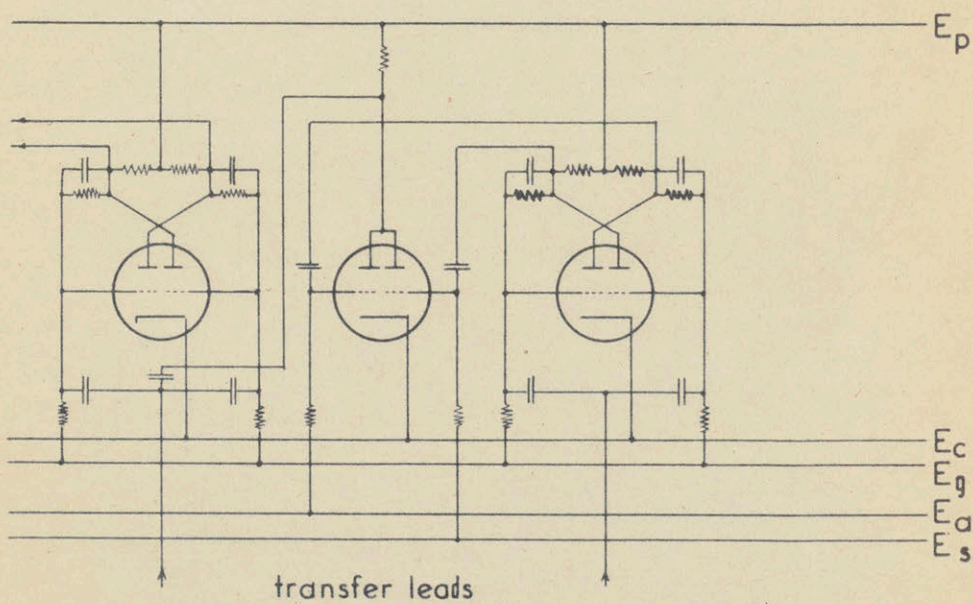


FIGURE 4
BINARY TOTALIZER STAGES

positively, the grid voltage of the coupling tube is raised to some value above cut-off voltage, and the anode voltage of the coupling tube drops. The anode of the coupling tube applies a negative pulse to the grids of the tens trigger-pair, and the tens trigger-pair is transferred from the passive to the active state. The units trigger-pair is transferred from the passive to the active state by the third pulse applied to its grids, but the tens trigger-pair is not affected. The units trigger-pair is transferred from the active state to the passive state by the fourth pulse, and the tens trigger-pair is likewise transferred from the active to the passive state by a pulse from the anode of the coupling tube. When the tens trigger-pair transfers from the active to the passive state, it causes a pulse to be applied to the hundreds trigger-pair, and the hundreds trigger-pair is transferred from the passive to the active state. At this point, four pulses have been applied to the array of trigger-pairs, and the hundreds trigger-pair is active and the rest of the trigger-pairs are passive. The array of trigger-pairs contains the binary number four (100), and the array thus performs in the manner proposed. A counter for practical applications may contain from ten to sixteen trigger-pairs. The performance of the trigger-pairs beyond the third pair is similar to the performance described above.

In some applications of counters, it is desirable to count backwards or negatively. In a counter that counts only backwards, the coupling tubes are connected to positive-active output terminals instead of to the negative-active output terminals. If a counter counts both forward and backwards, two coupling tubes are used, and switching leads to the coupling tube grids permit one tube or the other to transmit carryover signals. In some applications of counters that count backwards, numbers are placed in the counter before counting begins, and digit transfer leads are therefore connected to the input

terminal of each trigger-pair. A counter may be made to add pulses of one polarity and subtract pulses of the other polarity, but such counters are not required in the apparatus to be described.

Totalizer

The name totalizer is applied to a special form of counter wherein pulses to be added or subtracted may be applied to any trigger-pair. Figure 4 shows two trigger-pairs and a coupling tube of a totalizer. A transfer lead is connected to the input terminals of each trigger-pair. Pulses applied to the tens trigger-pair add tens to the totalizer, pulses applied to the hundreds trigger-pair add hundreds to the totalizer, and so on. The bias voltages of the double triode coupling tube are supplied by a pair of leads whose voltages shift oppositely and assume two values. One value is the cut-off voltage of the double triode, and the other value is approximately cathode voltage. When pulses are added to the totalizer, the coupling tube grid labelled 1 in Figure 4 is held at cathode voltage, and the grid labelled 2 is held at cut-off voltage. When the trigger-pair on the right in the figure transfers from the active state to the passive state, a negative pulse from the positive-active terminal reduces the voltage of grid 1 of the coupling tube to cut-off voltage. Since grid 2 is already cut-off, the anode voltage of the coupling tube rises to supply voltage and a pulse is applied to the input terminal of the trigger-pair on the left. When pulses are subtracted from the totalizer, the bias voltages of the coupling tube grids are reversed, and pulses are applied to the trigger-pair on the left when the trigger-pair on the right transfers from the passive state to the active state.

Stepping Switches

A stepping switch is an array of trigger-pairs so coupled that only

one of the pairs at a time is in the active state, and one after another of the pairs is transferred to the active state as voltage pulses are applied simultaneously to all of the pairs in the array. Figure 5 shows two sections of a stepping switch. Assume that the pair on the right is active and the pair on the left is passive. The stepping switch is stepped by a negative pulse applied to the trigger-pairs by the stepping lead. When the negative pulse is applied, the trigger-pair on the right is transferred from the active to the passive state, and the trigger-pair on the left is unaffected. As the current transfers in the right trigger-pair, the voltages of the positive-active output terminal rises, and a positive pulse is applied to the positive-active input terminal of the trigger-pair on the left. The left trigger-pair is thus transferred from the passive state to the active state, and the switch has stepped one position. Subsequent pulses cause successive trigger-pairs to become active. In most applications of stepping switches, the last trigger-pair on the left is coupled to the first trigger-pair on the right, and the switch is "rotated" continuously by stepping pulses.

Overbeck has recently devised a novel stepping switch where the elements of the switch are transferred from the passive to the active state in sequence by a series of stepping pulses, but where the elements are not returned to the passive state until all of the elements are active. This stepping switch is characterized by remarkable simplicity. Following Overbeck, the name "Steichotron" from the Greek work for step will be applied to this stepping switch. A Steichotron can be realized as a single multi-anode tube with a simple external resistance network.

Selector Switches

By selector switch may be understood any combination of trigger-pairs, switches, and resistance networks that shift the voltage of one terminal

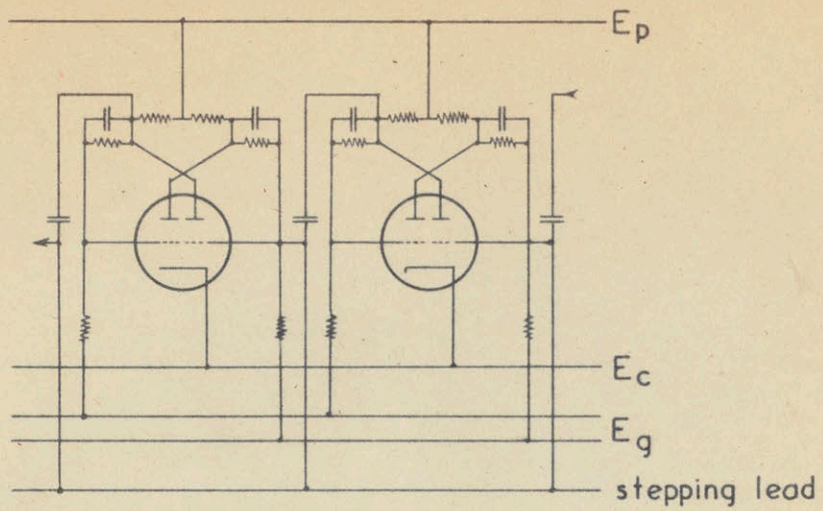


FIGURE 5

STEPPING SWITCH ELEMENTS

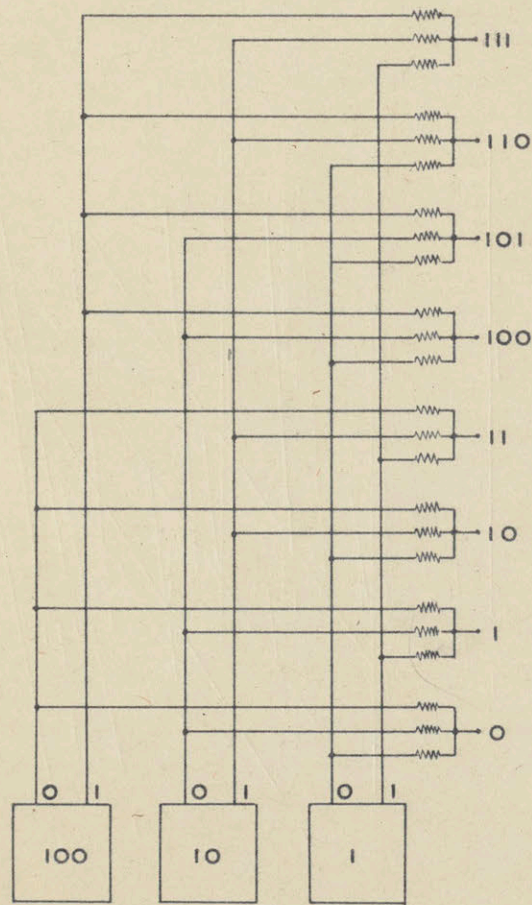


FIGURE 6

8-TERMINAL SELECTOR SWITCH

in a set of terminals more than it shifts the voltage of any other terminal. In practical applications of selector switches, the difference between the voltage of the selected terminal and any other terminal must exceed some definite value, say three or five volts. The stepping switch described in the previous section is one type of selector switch. The terminal-set whose members are selected is the set of output terminals of the stepping switch trigger-pairs. When a stepping switch selects the members of a terminal set, only the voltage of the selected terminal is shifted, once the switch is in position. The voltage of the selected terminal might differ from the voltage of the corresponding terminal of the other trigger-pairs by 150 volts or more.

A second type of selector switch is shown in Figure 6. A set of eight terminals are associated with the binary numbers from 0 to 111, and that when a binary number in this range is transferred to a set of three trigger-pairs, the terminal associated with the number is selected. The selector-switch consists of the three trigger-pairs and a resistance network that connects terminals in the terminal-set to one output terminal of each trigger-pair. Since there are three trigger-pairs, each with two terminals, there are $2^3 = 8$ possible sets of connections, one for each of the eight terminals. Assume that the voltage difference between the output terminals of a trigger-pair is 100 volts, and that if a trigger-pair contains a one, the voltage of the output lead labelled 1 is 100 volts and the voltage of the output lead labelled 0 is 0. If the trigger-pair contains 0, the voltages of the output leads is reversed. Assume that the number in the trigger-pairs is 000. Then the voltage of the terminal labelled 0 is 100 volts since the terminal is connected to the three output terminals whose voltage is 100 volts. The voltage of the output terminal labelled 1 is $66 \frac{2}{3}$ volts since it is connected to two output terminals whose voltage is 100 volts and to one

terminal whose voltage is 0 volts. It is clear that if the number that labels each terminal contains a single one, the terminal voltage is $66 \frac{2}{3}$ volts, if it contains two ones, the terminal voltage is $33 \frac{1}{3}$ volts, and if it contains three ones, it is 0 volts. For any other number in the three trigger-pairs, the performance of the network is similar, but the terminal voltages are different for each number. The terminal whose label is the number in the trigger-pairs always shifts 100 volts. Three other terminals shift $66 \frac{2}{3}$ volts, three more terminals shift $33 \frac{1}{3}$ volts, and the voltage of one terminal does not change. If the circuits to which the terminals in the terminal set are connected do not operate unless the terminal voltage changes an amount greater than $66 \frac{2}{3}$ volts, or in other words, if the threshold of the circuits is greater than $66 \frac{2}{3}$ volts, then for each number in the trigger-pairs the circuits connected to only one of the terminals operate, and the selective switching action desired is obtained.

If no limit is placed on the values of the resistances that connect the trigger-pair output terminals to the members of the terminal-set, there is no limit to the number of terminals that can be connected to the output terminals of an array of trigger-pairs. The distribution of voltages of the terminals in the terminal-set is given by the coefficients in the binomial expansion $(1 + 1)^n$ where n is the number of trigger-pairs. If $n = 2$, there are four terminals in the terminal set, and if the voltage difference between the output terminals of a trigger-pair is 100 volts, the voltage of one of the terminals is 100 volts, the voltage of two of the terminals is 50 volts, and the voltage of one of the terminals is 0 volts. The case $n = 3$ was described above. If $n = 4$, the voltage of the selected terminal is 100 volts, the voltage of four terminals is 75 volts, the voltage of six terminals is 50 volts, the voltage of four terminals is 25 volts, and the voltage of one

terminal is 0 volts. The distribution of voltages for other values of n follows directly.

In most applications of selector switches, a single circuit is operated by a number of terminals in a terminal-set. Let it be assumed that a particular circuit is operated if the grid of a switch is shifted a definite amount. There exists a definite limit to the number of terminals that may be connected to the grid of this switch for a given set of voltage differentials between the voltage of the selected terminal and the voltages of the rest of the terminals. There exist, therefore, definite limitations on the performance of a resistance network type of selector switch, and to overcome these limitations, intermediate switches in the selector network are necessary. Thus, the terminals of the resistance network in Figure 6 can be connected to the grids of a set of eight switches. If the bias voltage of the grids of the switches is less than $-66 \frac{2}{3}$ volts, only the switch connected to the selected terminal operates. If a particular circuit is connected to four terminals in the terminal-set, and if the voltage shift of the anode of the operated switch is 100 volts, a shift of 25 volts is available to operate the circuit. If the intermediate switches were not present, the voltage shift available for operating the circuit would depend on which other terminals in addition to the selected terminal the circuit is connected to. In extreme cases, where the circuit to be operated is connected to several terminals in the terminal-set, it may be impossible to produce a voltage differential that operates only the desired circuit.

If intermediate switches are connected to each of the terminals associated with n trigger-pairs, 2^n switches are required. A smaller number of switches are required if the trigger-pairs are divided in two or more groups, switches are associated with the individual groups, and the anode

circuits of the switches are coupled by a resistance network. Figure 7 shows a selector switch where four trigger-pairs are divided into two groups. Each group of trigger-pairs operates a group of four switches. The anode of each switch in one group is connected through high resistance to the anode of each switch in the other group. When a four digit binary number is transferred to the four trigger-pairs, one switch in each group operates. Assume that the voltage difference between the anode of an operated switch and an unoperated switch is 100 volts. Then the voltage of the midpoint of the resistor that joins the anodes of the two operated switches shifts 100 volts, the midpoints of the resistors joining the anodes of the operated switches to the anodes of unoperated switches shift 50 volts, and the voltage of the rest of the resistor midpoints does not change. Thus, in the set of sixteen terminals, when a number is transferred to the set of trigger-pairs, one terminal changes 100 volts, six terminals change 50 volts, and nine terminals do not change. The voltage change available to operate some circuit connected to several of the terminals in the set depends on to which and to how many of the terminals the circuit is connected. The change is smaller than if the circuit were connected to the anodes of a set of sixteen switches operated by the trigger-pairs, and greater than if the sixteen terminals were directly connected through a resistance network to the output terminals of the trigger-pairs.

The intermediate switches in a selector switch can be arranged in more than two groups. Thus, 64 terminals can be selected by three arrays of four switches each. The anodes of the switches in each array are connected through high resistance to the anodes of the switches in each of the other arrays. The resulting resistance network is a three-dimensional matrix of connections. When a switch in each array is operated, and if the voltage

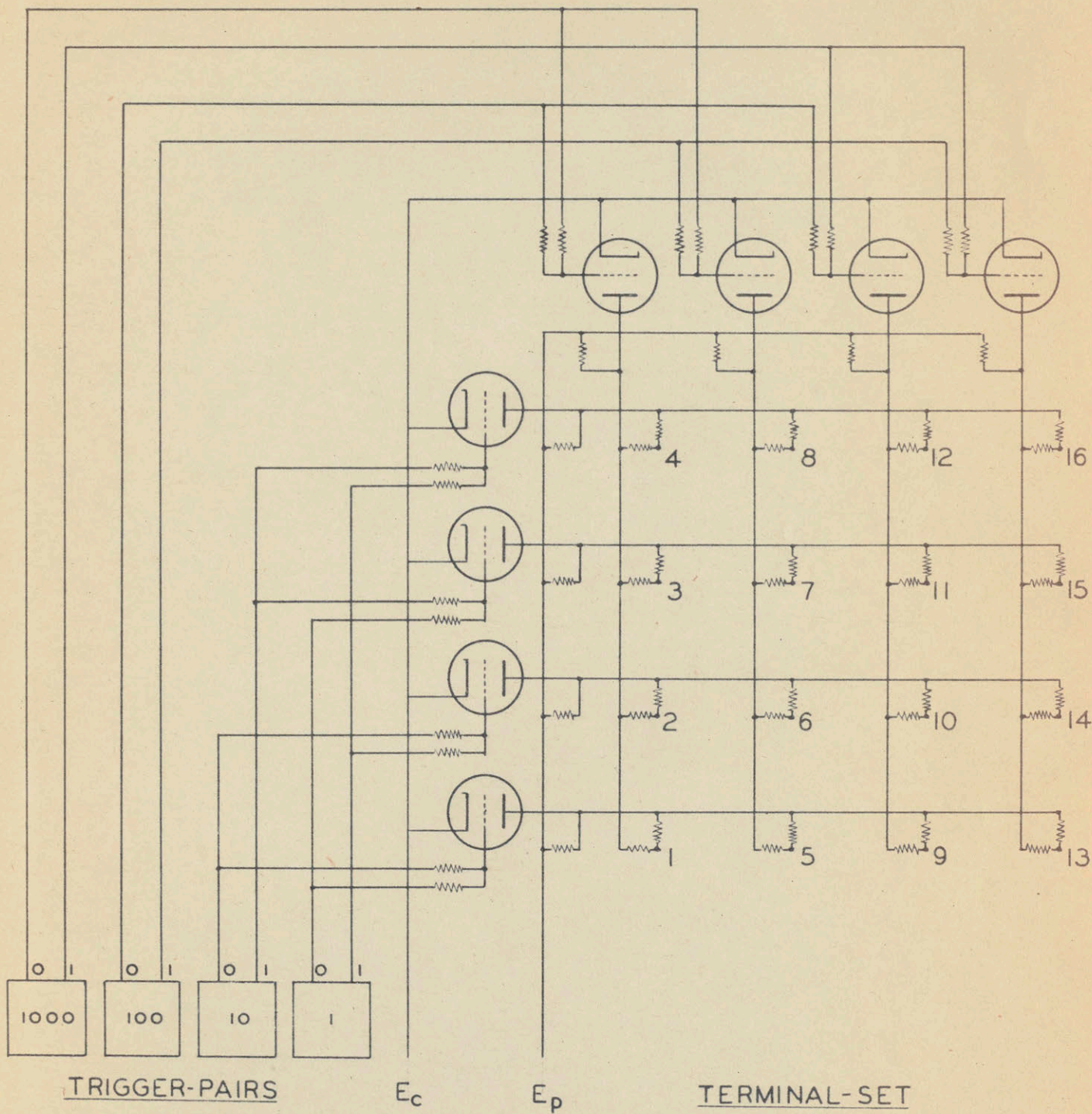


FIGURE 7

16-TERMINAL SELECTOR SWITCH

difference between the anode of an operated switch and the anode of an unoperated switch is 100 volts, the voltage of the selected terminal changes 100 volts, the voltages of nine terminals change $66 \frac{2}{3}$ volts, the voltages of 27 terminals change $33 \frac{1}{3}$ volts, and the voltages of the rest of the terminals do not change. It is possible to arrange the trigger-pairs that select the terminals in four groups or five groups and connect the anodes of the switches in one group to the anodes of the switches in each other group with four- or five-dimensional matrices of connections. When there is only one trigger-pair in a group, no intermediate switches are associated with the pair, and connections from the output terminals of the pair proceed directly to the resistance matrix. In the limiting case of dividing the six trigger-pairs in six groups, a six-dimensional resistance matrix without intermediate switches selects the terminals. This matrix is equivalent to the type of resistance network shown in Figure 6.

Multiplication of Binary Numbers

Several methods of multiplying binary numbers have been described. The present discussion is confined to a multiplier, or product network as it will be called, that appears to be simple and sufficiently fast for practical purposes, and which uses the secondary emission trigger-elements that have been developed by Overbeck. The network operates on a principle similar to the principle of multiplying two numbers on paper by the ordinary "long-hand" process. The details of the network are not given. The present section presents enough information to permit a product network to be used in a calculating system.

Figure 8 shows the structure of the product network. At the beginning of the interval during which multiplication takes place, or product interval as it is convenient to call it, the multiplier is transferred to the multiplier holding set and the multiplicand is transferred to the multiplicand holding set. A holding set is the array of trigger-elements for recording the digits of a binary number illustrated in Figure 2. During the product interval the digits of the multiplier and multiplicand are transferred repeatedly to the coincidence circuit by the multiplier and multiplicand stepping switches. These stepping switches can be the Steichotrons mentioned in an earlier section, or they can be arrays of trigger-pairs. Let it be assumed that they are Steichotrons. The transfer of the multiplicand and multiplier digits to the coincidence circuit is accomplished by changing momentarily the voltages of the anodes of the trigger-elements in the holding set. The shift produced in the voltage of the screen grid of the holding set depends on whether the trigger-elements whose anode voltage is changed is at its lower or upper voltage, or, in other words, whether the element contains a

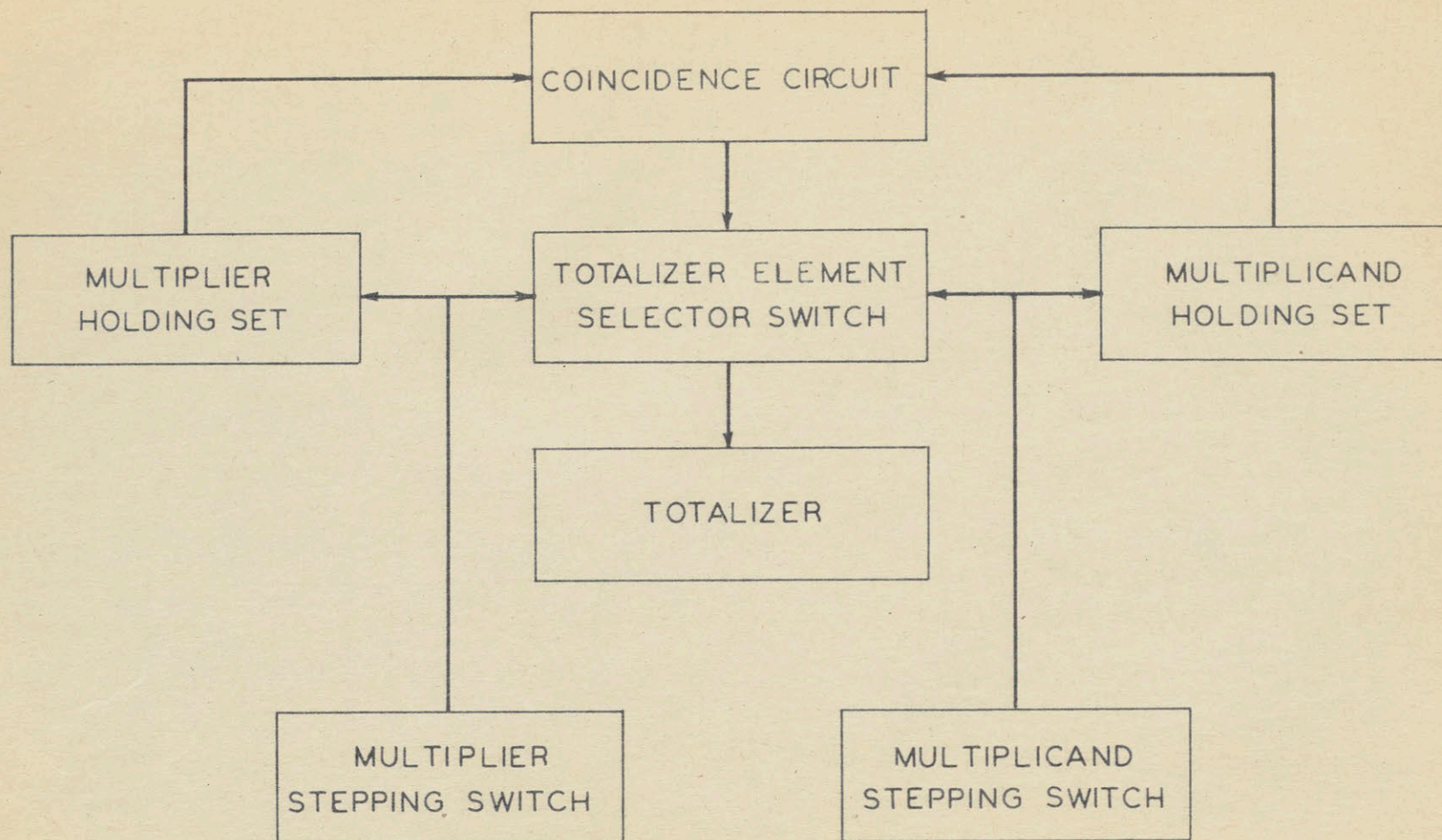


FIGURE 8

COMPONENTS OF PRODUCT NETWORK

zero or a one. The anodes of the two holding sets are pulsed in sequence, and the digits of the multiplier and multiplicand are transferred in sequence to the coincidence circuit.

A product interval is divided into a number of partial product intervals, and during each partial interval, a group of multiplier and multiplicand digits are transferred to the coincidence circuit. When a one in the multiplier and a one in the multiplicand are transferred to the coincidence circuit simultaneously, a one is transferred to an element of the totalizer. Each one transferred to the totalizer is added automatically to the number already in the totalizer unless the subtraction circuit of the totalizer is operated. If the subtraction circuit is operated, each one is automatically subtracted from the number in the totalizer. All the "coincident ones" occurring in a single partial interval are transferred to the same element of the totalizer. Between each partial interval, a stepping switch operates and connects the next totalizer element to the coincidence circuit. The order in which the multiplier and multiplicand digits and the totalizer elements are selected will be shown by an example.

Consider the multiplication of 1.0101 by 0.1011. Assume that the multiplicand and multiplier holding sets have six trigger-elements each, and let the elements be numbered from one to six. Assume that the totalizer has seven elements, and let the elements be numbered from zero to six. Assume that the decimal point lies between the fourth and fifth elements of the holding sets and the totalizer. Then, the multiplication can be represented as follows:

Multiplicand Trigger-element:	6	5	4	3	2	1	Multiplier Trigger-Element	Multiplier Digit
			0	1	0	1	0	1
			0	1	0	1	0	1
			0	1	0	1	0	1
Multiplicand Digits:	0	1	0	1	0	1	4	1
	0	1	0	1	0	1	5	0
	0	1	0	1	0	1	6	0
Product Digits:	0	0	0	1	1	1	0	0
Totalizer Element:	6	5	4	3	2	1	0	

During the first partial product interval, digits are transferred to the 0th totalizer element. The first step in the interval transfers the multiplicand digit in the 4th trigger-element and the multiplier digit in the 1st trigger-element to the coincidence circuit. One of the digits is zero, and there is no transfer to the totalizer. The second step transfers the multiplicand digit in the 3rd trigger-element, and the multiplier digit in the 2nd trigger-element to the coincidence circuit. Both digits are one and a one is transferred to the totalizer. The third step transfers the multiplicand digit in the 2nd trigger-element, and the multiplier digit in the 3rd trigger-element to the coincidence circuit. Both digits are zero and there is no transfer to the totalizer. Both digits transferred to the coincidence circuit in the fourth step are one, and a one is therefore transferred to the totalizer. At the end of the fourth step, all multiplicand digits have been transferred to the coincidence circuit, and the second partial product interval is ready to begin. Between the first and second partial product intervals, the stepping switch that connects the coincidence circuit to the totalizer elements operates and connects the circuit to the 1st totalizer element.

The first step in the second partial product interval transfers the multiplicand digit in the 5th trigger-element and the multiplier digit in the 1st trigger-element to the coincidence circuit. The remaining steps of the second partial product interval transfers the digits of the multiplicand to the coincidence circuit in decreasing order and the digits of the multiplier in increasing order. Table II shows the order of the transfers in the second partial product interval and in the rest of the partial intervals of the product interval. Table II shows also the totalizer element to which the coincident ones are transferred in each partial interval.

Table II

<u>Partial Interval</u>	<u>Totalizer Element</u>	<u>Holding Set</u>	<u>Trigger Elements</u>
1	0	Multiplicand Multiplier	5 4 3 2 1 1 2 3 4
2	1	Multiplicand Multiplier	5 4 3 2 1 1 2 3 4 5
3	2	Multiplicand Multiplier	5 4 3 2 1 2 3 4 5
4	3	Multiplicand Multiplier	5 4 3 1 2 3 4 5
5	4	Multiplicand Multiplier	5 4 1 2 3 4 5
6	5	Multiplicand Multiplier	5 1 2 3 4 5

It was assumed that the transfer of the multiplier and multiplicand digits is accomplished with Steichotrons. Let it be recalled that the trigger-elements of the Steichotron become and remain active as pulses are applied to the stepping lead of the Steichotron, and that all elements are transferred simultaneously to their passive states after all of them are active.

The operation of returning all of the trigger elements to their passive states will be called a resetting operation.

The easiest way to shift the transfer of multiplicand digits with respect to the transfer of multiplier digits from one partial product interval to another is to give one of the Steichotrons one more element than the other. The Steichotron with the extra element drops behind one step between each partial product interval, and the desired shift is obtained. However, this method of shifting requires that one of the Steichotrons be reset while the other Steichotron is still stepping. It takes longer to reset a Steichotron than to step it from one element to another, and the stepping of the Steichotrons is irregular. It is necessary, therefore, to employ a method of shifting that permits both Steichotrons to be reset simultaneously between partial product intervals. A method of shifting that meets with this requirement is to step the multiplier Steichotron uniformly and completely in each partial product interval except the first, and to delay the stepping of the multiplicand Steichotron until the multiplier Steichotron has reached the trigger-element appropriate to the interval as indicated in Table II.

In the first partial product interval, the multiplicand Steichotron starts first and steps from the 5th to the 4th element before the multiplier Steichotron begins to step. When the multiplicand Steichotron steps from the 4th to the 3rd element, the multiplier Steichotron steps from the 1st to the 2nd element, and for the rest of the interval, both steichotrons step together. After the 1st element of the multiplicand Steichotron becomes active, both Steichotrons are reset. Both Steichotrons step together throughout the second partial product interval. In the third partial product interval, the multiplier Steichotron steps from the 1st to the 2nd element before the multiplier Steichotron begins to step, in the fourth partial product interval, from

the 1st to the 3rd element, and so on. The delays required are shown in Table II. Each time the Steichotrons are reset the totalizer stepping switch is stepped once.

The delay in the starting of one Steichotron is accomplished by the co-operation of a set of switches connected to certain Steichotron elements and to the stepping switch that selects the totalizer elements. For example, consider the switching operations that delay the multiplicand Steichotron in the fourth partial product interval. The positive-active terminal of the 3rd element of the totalizer stepping switch and the 2nd element of the multiplier Steichotron are connected to the grid of a switch which connects the stepping pulse source to the multiplicand steichotron when it operates. The switch operates when the two trigger elements are both active. The next stepping pulse steps the multiplicand stepping switch, and for the rest of the interval, the two steichotrons step together. There is a switch for producing the delay required at the beginning of each partial product interval.

If the multiplicand is negative, it is in complementary form, and the 6th element of the multiplicand holding set contains a one. The presence of this one operates a circuit that causes ones to be transferred to the coincidence circuit during those parts of each partial product interval that the multiplicand digits are not transferred to the coincidence circuit. Reference to the example on page 22 shows that transferring ones to the coincidence circuit at these times fills the triangular region at the left of the staggered column of multiplicands with ones. If the multiplicand is a complement, this region is filled with ones which must be transferred to the coincidence circuit with the rest of the multiplicand digits. If the multiplier is negative, it is also in complementary form, but it is necessary to reverse the sense of each digit of the multiplier as it is transferred to the coinci-

dence circuit except for the smallest one that occurs in the multiplier. The smallest one is treated as a one, but the rest of the ones are treated as zeros by the coincidence circuit, and the zeros, as ones. It is to facilitate reversing the sense of the multiplier digits that the multiplier digits are transferred to the coincidence circuit in increasing order. The first one that reaches the coincidence circuit in each partial product interval is treated as a one, but if the multiplier is a complement, the sense of the rest of the digits is reversed. That the multiplier is a complement is indicated as before by the presense of a one in the 6th element of the multiplier holding set.

If the multiplier is positive, coincident ones are added to the totalizer regardless of the sign of the multiplicand. If the multiplier is negative, coincident ones are subtracted from the totalizer regardless of the sign of the multiplicand. If the product is negative as it is if either but not both multiplicand and multiplier are negative, the product appears in the totalizer as a complement.

Between twelve and fourteen binary digits are required to represent the variables occurring in fire control. The holding sets and the totalizer therefore have at least this number of elements. The multiplier holding set must have several additional elements because it is desirable to be able to multiply by reciprocals. The variables whose reciprocals are required vary over a ten-fold range. The multiplier holding set must therefore have three or four additional elements, which makes a total of sixteen or seventeen elements. The variables that occur in the totalizer do not vary over such a wide range, but it is necessary to place one or two extra elements in the totalizer to permit the product to be rounded-off. It is to be observed that the totalizer receives only the significant figures of the

product plus the rounding-off figures.

The speed of operation of the product network is of the order of the quotient of the stepping signal frequency and the square of the number of elements in the totalizer. The minimum stepping signal frequency that it is reasonable to expect is of the order of 100 Kc, and the maximum, 500 Kc. The square of the number of totalizer elements is in the neighborhood of 300 or 400. The operating speed of the network is therefore of the order of 1000 products per second. It is convenient to assume that the operating speed of the network is 1024 products per second.

Recording of Coefficients and Functions

Constants that appear as coefficients in the computations of control operations are recorded in permanently wired resistance networks that couple the members of a terminal-set to the multiplier holding set of the product network. Functions of one or two variables can be recorded in the same way, but are recorded more satisfactorily on photographic film that is scanned by electron multipliers. A resistance network for recording coefficients will be called a coefficient matrix, and an instrument for recording functions, a function unit.

Coefficient Matrix

Each recorded coefficient is associated with a member of a terminal set. The terminals are connected through high resistances to the grids of a set of switches whose anodes are connected to the trigger-elements of the multiplier holding set of the product network. Connections are made from each terminal to switches of trigger-elements that occupy the same positions with respect to the decimal point as do the ones in the coefficient associated with the terminal. When the voltage of the terminal is shifted by a selector switch, the switches connected to the terminal operate and transfer the ones of the coefficient to the multiplier holding set. The switches that operate the trigger-elements of the holding set will be called output switches.

Figure 9 represents a few of the connections in a coefficient matrix. In a typical calculating system for control operations, the matrix may operate sixteen or seventeen output switches and may be operated by fifty to two hundred terminals. The probability that the digit of a coefficient in each decimal position is a one is $1/2$. If the number of coefficients is reasonably large, the grid of each output switch is connected to one half of the terminals.

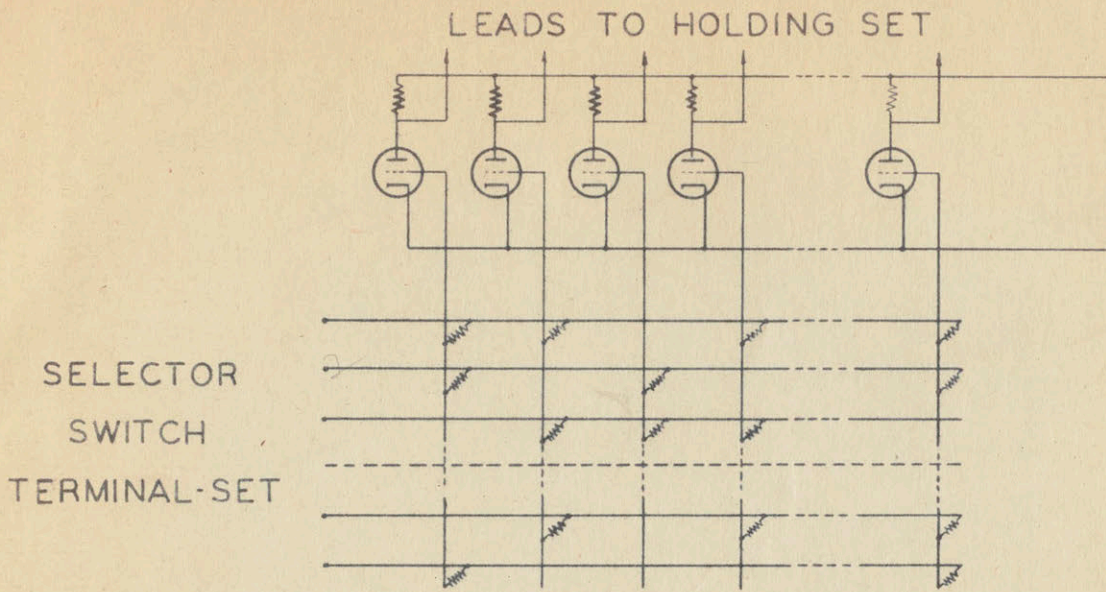


FIGURE 9

COEFFICIENT MATRIX

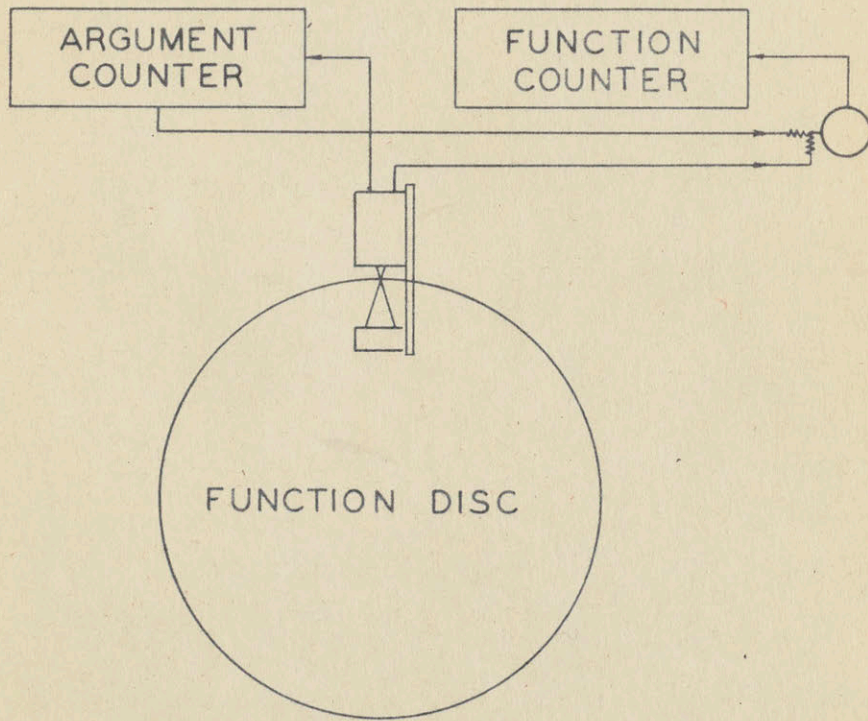


FIGURE 10

COMPONENTS OF FUNCTION UNIT

To obtain a first approximation of the voltage shift of an amplifier grid when a terminal is selected, it is assumed that if the voltages of groups of terminals shift various amounts when a particular terminal is selected, the grid of an amplifier is connected to one half of the terminals in each group.

As an example of a coefficient matrix consider a matrix coupling 64 terminals to a set of output switches. Let it be assumed that a five volt shift of an output switch grid is required to operate the switch and transfer a digit to the holding set. Consider first a selector switch with 64 triode switches in a single array which are operated by six trigger-pairs. When a six digit number is transferred to the trigger-pairs, one of the switches in the array operates, and the anode voltage of the switch shifts. The anode voltages of the rest of the switches do not change. On the average, an amplifier grid connected to the selected terminal is connected to 31 other terminals. To a first approximation, the voltage of the grid is the mean of the voltages of the terminals to which it is connected. If the voltage of the selected terminal changes by unity, the voltage of the amplifier grid changes $1/32$. If this change must correspond to a change of five volts, the voltage change of the anode of the operated switch must be $5 \times 32 = 160$ volts. To a first approximation, the voltages of amplifier grids not connected to the selected terminal do not change.

Consider next a selector switch with sixteen switches in a double array that are operated by six trigger-pairs divided in two groups of three trigger-pairs each. The anode of each switch in one array is connected to the anode of each switch in the other array in the manner of the 16-terminal selector switch shown in Figure 7. The midpoints of the 64 resistors that connect the anodes are the 64 terminals of the selector switch. When a six digit number is transferred to the trigger-pairs, a switch in each array

operates. If the voltage shift of the midpoint of the resistor joining the anodes of the operated switches is unity, the voltage shift of 14 other resistor-midpoints is $1/2$, and the voltage shift of the rest of the resistor-midpoints is zero. The voltage shift of an output-switch grid connected to the selected terminal is $(1+1/2 \times 1/2 \times 14)/32 = 9/64$, and the voltage shift of an output-switch grid that is not connected to the selected terminal is $(1/2 \times 1/2 \times 14)/32 = 7/64$. The voltage shift of the switches is 160 volts, as before. The amplifiers normally have a positive grid bias, and the five volt shift required to operate the trigger-elements might be a shift from zero bias to a bias of -5 volts. Then for the selector switch with sixteen switches in a double array, the bias is 17.5 volts. For the selector switch with switches in a single array, the bias is zero.

It might be possible to operate a 64-coefficient matrix with a selector switch with twelve switches in a triple array. However, the saving in switches is small, and for such a small number of terminals, the assumption that the grids of the amplifiers are connected to half of the terminals in each voltage group would break down. Positive operation could not be secured. A square array of switches is probably the most efficient form of selector switch for a coefficient matrix.

In most applications of coefficient matrices, the trigger-pairs that operate the selector switch form a binary counter, and the coefficients are selected in sequence as the binary counter is stepped over its range. In other applications where the number of terminals is small, the terminals are selected by switches in the control network of the calculating system.

Function Unit

Functions of a single variable are recorded on photographic film that is scanned by electron multipliers. Two series of apertures are scanned

simultaneously by a pair of multipliers, and the apertures scanned in each series, or track, are counted. The spacing of the apertures in one track is related to the change of value of the function and the spacing of the apertures in the other track is related to the change of value of the argument. When a function of an argument is to be obtained, the value of the argument is transferred to one counter, the argument counter, and the initial value of the function is transferred to a second counter, the function counter. Let it be assumed that the function increases in the range of interest. Pulses from the argument electron multiplier, or argument scanning pole, as it will be called, are subtracted from the argument counter and reduce the reading of the counter toward zero. Pulses from the function scanning pole are added to the function counter. The spacing of the apertures in the two tracks is such that when the reading of the argument counter is zero, the reading of the function counter is the value of the function of the argument.

To gain an idea of how the apertures in the function and argument tracks are spaced, consider a record of the sine of an angle between 0 and $\pi/2$. For small values of the angle, the sine is equal to the angle, and near the origin, the apertures in both tracks are equally spaced. The spacing of the apertures is the least that can be resolved by the optical system and output circuits. As the angle increases, the sine increases more slowly, and the spacing between the apertures in the function track increases. The apertures in the argument track are uniformly spaced at the minimum possible spacing over the whole length of the track.

If the angle whose sine is to be found is normalized so that it varies from 0 to 1 as the sine varies from 0 to 1, the sine at first changes more rapidly than the angle and the spacing of the apertures in the function

track is therefore smaller than the spacing of the apertures in the argument track. In this part of the table, the spacing of the apertures in the function track is uniform, and is the minimum possible spacing, while the spacing of the apertures in the argument track varies. When the change of the sine becomes the same as the change of the function, the apertures in both tracks are uniformly spaced at the minimum spacing. In the rest of the table, the spacing of the argument apertures is uniform, and the spacing of the function apertures increases.

The initial value of a function is transferred to the function counter by a resistance network of the type described in the previous section. The terminals of the network are operated by switches in the control network of the calculating system. If a function decreases throughout its range, the subtraction circuit of the function counter is operated by a switch in the control network, and pulses from the function scanning pole are subtracted. If a function both increases and decreases, the function counter must be built to add pulses of one polarity and subtract pulses of the other polarity. By making the apertures in the function track variable width or variable density, the scanning light beam can be chopped abruptly by one side of the aperture and relatively gradually by the other side. Then the polarity of the pulse transmitted to the counter through a condenser depends on which side of the aperture first transmits the beam. The aperture-number¹ of variable width or area apertures is smaller than the aperture-number of uniform apertures, and the scanning light beam must be narrower to reduce aperture distortion and insure discrimination between the two edges of the aperture. Tables of functions that increase and decrease are therefore longer than tables of

¹ Number of apertures per inch.

functions that only increase or only decrease.

Functions like the sine and cosine require special switching operations to provide correct signs or quadrant indications if a complete table is not to be recorded. A complete table of the sine would be four times longer than would be necessary if special circuits were used to revise the function produced by a table of the sine for a single quadrant. If the values of an angle are to be correct to 11 binary digits in each quadrant, 13 digits are required to specify the angle completely. A switching circuit would examine the two largest digits of an argument to find the quadrant of the angle. If the argument were in the first quadrant, the performance of the system would be as described. If the argument were in the second quadrant, the subtraction circuit of the argument counter would not be operated and the counter would count positively. The operation of the function counter would be stopped when the argument counter reading reached 4096. If the angle were in the third quadrant, the argument counter would count backwards until it reached 4096. If the argument were in the fourth quadrant, the argument counter would positively count to 8192. In the second and third quadrants, the subtraction circuit of the function counter would be operated, and the function would be obtained as a complement.

It is reasonable to expect that the function and argument counters can count at the rate of 500,000 pulses per second, and that uniform apertures may be spaced 250 to the inch. If a table has 2048 apertures, it can be recorded in approximately 8 inches of film. The table is scanned in 4 milliseconds or product intervals. The argument track and function track should be recorded side-by-side on the same piece of film and scanned with a single light beam which is directed to separate electron multipliers by a prism. In most applications of function units, the film is mounted around

the periphery of a disc. The scanning speed is 2048 inches per second, which is approximately the peripheral speed of a disc two feet in diameter rotated 30 revolutions per second.

Two means are available of reducing the time required to obtain a function. The table can be divided in two halves which are scanned simultaneously by two scanning poles, or the interval of the table can be tens or hundreds instead of units, and a separate table can contain apertures so spaced that the number of apertures in an interval is the contribution to the units position of the function. When the hundreds of the argument have been reduced to zero, and the hundreds of the function added, signals from a second argument scanning pole can reduce the units and tens of the argument to zero, while units are added the function counter. In this thesis it is assumed that the entire function is recorded in a single table.

Recording of Intermediate Results

Intermediate results that are recorded for a few product intervals are recorded in an array of holding sets. Intermediate results that are recorded for longer periods are recorded on magnetic tapes. The array of holding sets is called a holding network, and a magnetic tape with its associated recording and reproducing equipment is called a recording unit. Both the holding network and the recording unit record an intermediate result some definite number of product intervals, and automatically produce the intermediate result at the proper time.

Holding Network

Figure 11 shows the connections of a holding network with four holding sets. The holding sets in the holding network are cleared in sequence by a stepping switch. One holding set is cleared between each product interval. The holding sets are primed by the co-operation of the stepping switch and a delay selector switch. The holding set that will delay a number for a given number of product intervals is selected when the proper delay selector switch is operated, regardless of the position of the stepping switch. To secure this performance, an output terminal of each stepping switch element is connected through high resistance to the anode of each delay selector switch. The midpoints of resistors lying along diagonals are connected to common leads to the priming terminals of the holding sets. It is assumed that the voltage shift of the lead connected to the midpoint of the resistor joining the output terminal of the active stepping switch element to the anode of the operated delay selector switch is sufficient to prime the holding set to whose priming terminal it is connected.

That the action of the network is the action desired is shown by an

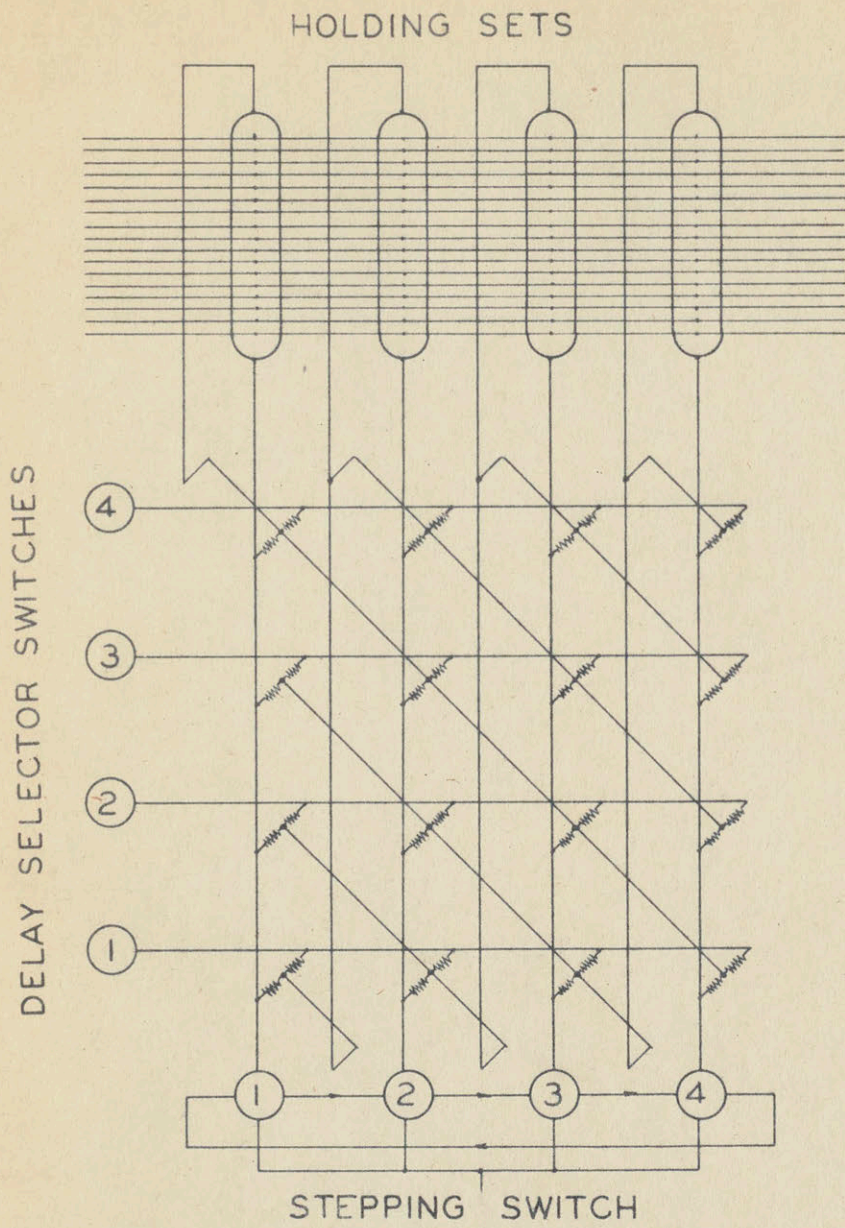


FIGURE II
HOLDING NETWORK

example. Assume that the 2nd stepping switch element is active and that a transferred number is to be delayed three product intervals. Then the control network operates the 3rd delay selector switch. The midpoint of the resistor joining the 2nd stepping switch element to the 3rd delay selector switch is connected to the lead that proceeds along the diagonal to the upper left corner of the network and thence to the priming terminal of the first holding set. The voltage shift of the midpoint of the resistor is sufficient to prime the first holding set and cause it to receive the transferred number. The desired performance is thus secured. The performance of the rest of the stepping switch elements and delay selector switches is similar.

Recording Unit

The magnetic tapes of the recording unit are mounted on the outer edges of discs. Recording, reproducing, and erasing poles mounted on arms ride directly on the tape. A digit is recorded on the tape in the form of a short train of sinusoidal variations in the intensity of magnetization. Available data on magnetic recording suggests that sixteen digit binary numbers can be recorded in approximately one inch of tape when the tape is scanned at such a velocity that a complete number is recorded or reproduced in one product interval. Since a product interval is $1/1024$ seconds, the tape velocity is 1024 inches per second. This velocity is much greater than has ever been used in magnetic recording systems. If it should prove to be unattainable, two or more tapes operating in parallel at slower speeds can supply numbers at the rate required by the product network.

There are two methods of using a magnetic recording unit in a calculating system. In one method the numbers on the tape are continuously reproduced, and the tape is continuously erased. Numbers are recorded on the tape

by a group of recording poles disposed around the disc between the erasing pole and the reproducing pole. When a number is recorded for a particular number of product intervals, say eight product intervals, the number is recorded during the first product interval and reproduced during the seventh product interval. The number is then available for use in the eighth product interval. It was assumed that one number is recorded in an inch of tape. The separation of the recording pole and the erasing pole in inches along the tape is the number of product intervals in the delay less one. The delay that can be obtained with this method of operation is limited by the permissible circumference of the disc. For fire-control equipment, it is probable that the maximum circumference of the disc is in the neighborhood of 60 inches. Since the recording pole for the longest delay must be separated from the reproducing pole by at least the interval occupied by the erasing pole, the possible delay is no greater than 64 product intervals and is probably 48 intervals.

The second method of operating the recording unit is used when numbers must be delayed for periods longer than are possible with a single disc that is continuously erased. In this method, the numbers are recorded on the tape by a single recording pole, and the disc rotates several times before the numbers are reproduced. The disc does not rotate an integral number of times in the delay period since the positions of the recording and reproducing poles cannot coincide. When the required capacity of the disc and the delay are known, it is necessary to find the speed of rotation of the disc, the circumference of the disc, and the separation of the recording and reproducing poles that will give the desired capacity and delay. The product of the angular velocity and the circumference of the disc is constant since the peripheral speed of the disc is fixed. In most applications of this

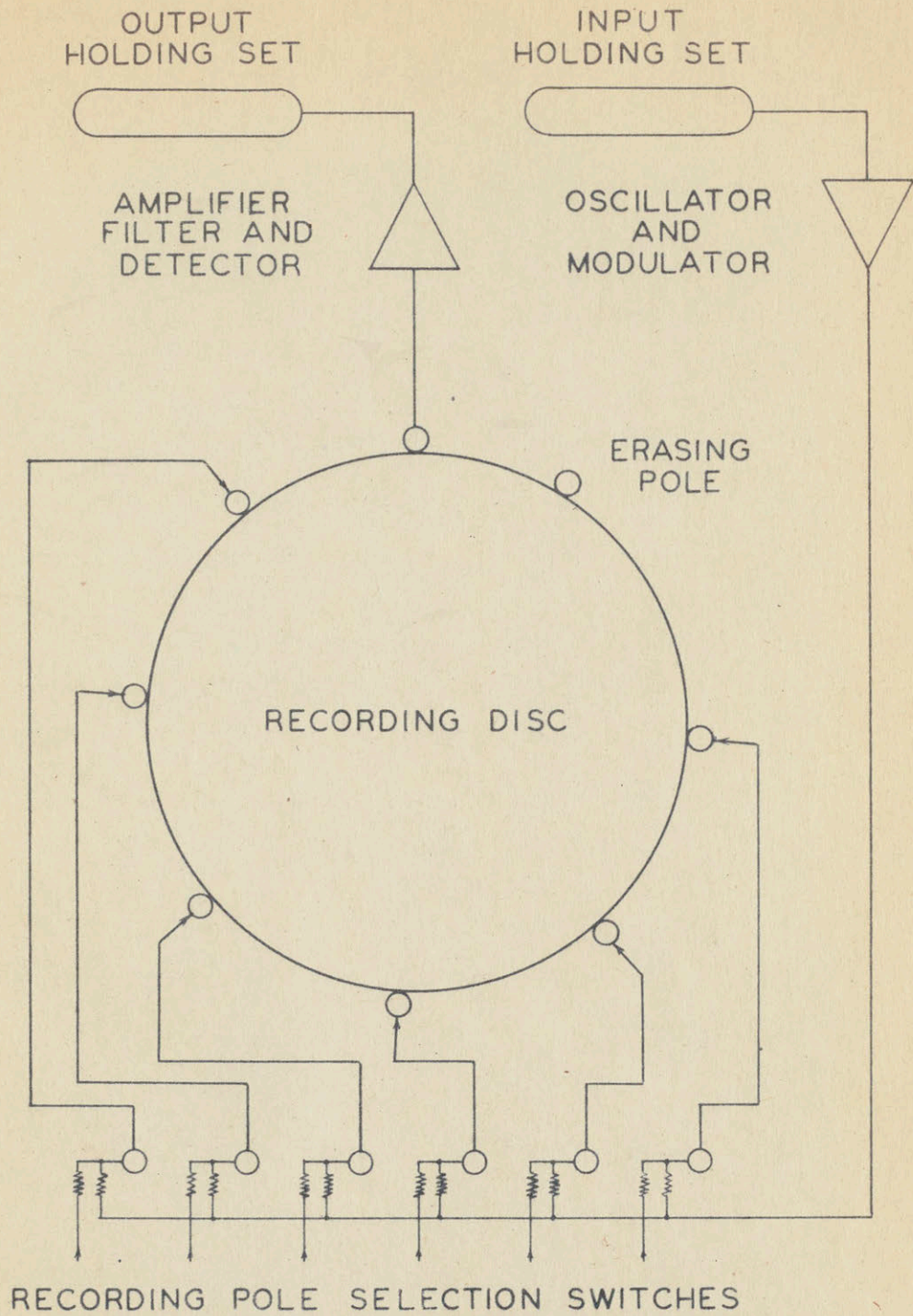


FIGURE 12

COMPONENTS OF RECORDING UNIT

method of recording, numbers reproduced by the reproducing pole are immediately re-recorded by the recording pole. The tape is erased between the time it leaves the reproducing pole and the time it reaches the recording pole.

The method of calculating the diameter and angular velocity of the disc and the separation of the recording and reproducing pole that gives a desired capacity and delay will be shown by an example. The example is a recording disc used in the predictor described later. It is desired to record twenty-four numbers for 128 product intervals. Since one number is recorded per inch, the separation of the recording and reproducing pole must be at least twenty-four inches. The delay of 128 product intervals is obtained when a number recorded in a particular product interval, say the 1st product interval, is reproduced in the 127th product interval. The tape speed is 1024 inches per second and a product interval is $1/1024$ th seconds.

If the reproducing pole and recording pole must be eight inches away from each other to limit direct magnetic coupling and leave room for the erasing pole, the circumference of the disc is approximately 32 inches. Since a point on the surface of the disc travels 127 inches in the delay period, the disc rotates approximately four times in the delay period. Since the angular separation of the recording and reproducing pole is approximately 90° , let the approximate rotation of a recorded signal in the delay period be 3.75 turns. Since the delay period of 128 product intervals is $1/8$ th second, the angular velocity of the disc is 30 revolutions per second. The correct rotation of a recorded signal is $(127/1024)30 = 3.7207$ turns = 3 turns + 259.453° . The separation of the recording and the reproducing pole is therefore 259.453 degrees. At 30 revolutions per second, the desired peripheral speed of 1024 inches per second is obtained if the circumference of the disc is 34 inches. The lineal separation of the recording and repro-

ducing pole is 24.504 inches, which exceeds the minimum separation by an adequate margin.

Figure 12 shows the circuits associated with a continuously-erased recording unit. A number to be recorded is transferred to the input holding set. The digits of the number are transferred in sequence from the holding set over a lead connected to the screen grid of the holding set when a stepping switch whose elements are connected to the anodes of the holding set operates. A one transferred from the holding set closes a switch in a connection between an oscillator and the recording poles. The switch in the connection between the oscillator switch and the recording pole that produces the desired delay is closed by the control network of the system. As the stepping switch steps, the digits of the number in the holding set are recorded on the tape in sequence.

When the recorded number reaches the reproducing pole, its digits are reproduced in sequence. The reproduced signals are amplified, filtered, and detected. The detector output is applied to the priming terminal of the output holding set, while the voltages of the anodes are shifted in sequence by a stepping switch. The stepping switch steps in synchronism with the rotation of the recording disc, and the reproduced digits are transferred in sequence to the elements of the holding set.

The recording poles of the recording unit can be spaced to give any desired delay within the capacity of the unit. For most applications, it is convenient to space the poles to give delays of 8, 16, 24, 32, etc., product intervals. By recording a number in the holding network one or more times and then recording it in the recording unit, it is possible to delay the number any number of product intervals.

Measurement of Displacements

In applications of arithmetical equipment to fire control, it is necessary to translate angular or linear displacements into numerical data. Many instruments for making this translation have been proposed, and it is difficult to predict which will prove to be effective. The instrument described below in effect remeasures completely a displacement when called upon to produce a measure of the displacement. The measurement is performed by counting the number of apertures in an array of apertures that is exposed by a shutter displaced by the quantity to be measured. This instrument is relatively simple, or at least employs a small number of components, and possesses the advantage that distortion in the transmission of the data can produce inaccurate results but not wholly incorrect results.

Figure 13 shows the essential components of the position indicator. The shaft whose displacement is to be measured rotates the shutter disc SD. An edge of an opaque tape is mounted on the rim of the shutter disc, and the tape extends half way around the disc. The tape extends from the plane of the shutter disc to cover the apertures in the aperture track AT which is mounted around the rim of the fixed aperture disc, AD. The aperture track completely encircles the aperture disc, but the train of apertures extends only half way around the disc. As the angular displacement of the shutter disc changes in accordance with the measured quantity, a varying number of apertures is exposed by the shutter to a light beam that scans the aperture track at high speed. An image of the aperture track is directed on a fixed photocell by a right angle prism rotating with the light beam and mounted on the axis of rotation. The voltage pulses produced by the photocell are counted by a binary counter at the calculating system.

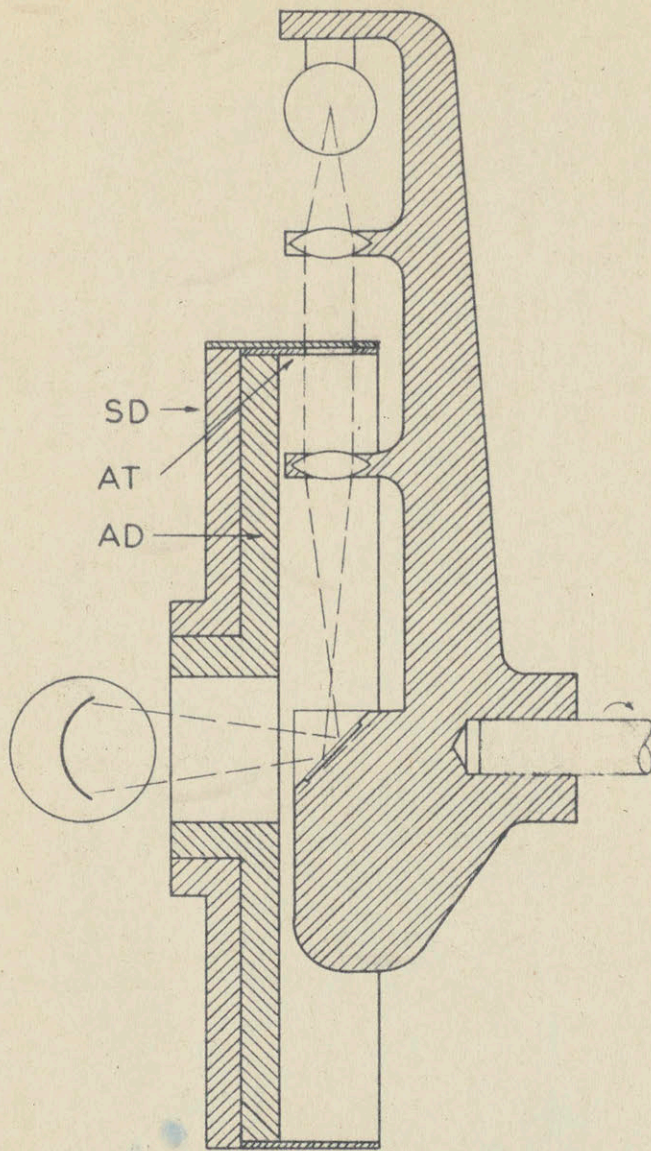


FIGURE 13

POSITION INDICATOR SECTION

If possible, the rotation of the scanning lead beam is synchronized with the rotation of the function discs and recording discs at the calculating system and therefore with the control signals of the calculating system. If the rotation of the light beam is not synchronized with the calculating system, it takes longer to obtain readings since counting cannot begin until the light beam reaches the beginning of the aperture track.

In a practical form of position indicator, a single light beam scans and measures a number of displacements. A quantity to be measured displaces a tape which may or may not be a closed loop depending on whether or not the displacing quantity can move continuously in one direction. The tape has a slit along its center which exposes the apertures in the aperture track. Each of several tapes exposes a variable interval of a sector of the aperture track, and as the scanning light beam rotates, it scans in sequence the parts of the sectors exposed by the several tapes.

Control of Displacements

The shafts of a ballistic computer must be caused to rotate in accordance with the data produced by an arithmetical predictor. The problem of controlling the displacements of these shafts is essentially a servo-mechanism problem, and it is not feasible to undertake an investigation of the problem for this thesis. It is proposed only that position indicators be operated by the shafts whose displacements are controlled and that a series of observed shaft positions be compared with a series of calculated shaft positions and that a numerical error signal be produced by arithmetical operations. When transferred to a holding set whose trigger-elements are appropriately coupled to the grid of a vacuum tube, the numerical control signal can produce a flow of current in the plate circuit of the tube and operate a motor connected to the shaft whose displacement is controlled. The frequency at which control signals must be obtained to produce adequate following, as well as the optimum number of calculated and observed positions from which the control signal is derived will be assumed.

A holding set for translating a number into a grid voltage will be called an interpolating holding set. It may be assumed that the contribution of a particular trigger-element of the holding set to the grid voltage is proportional to the power of two associated with the position of the trigger-element. Thus, if the trigger-element in the lowest digit position contributes unit voltage, the second trigger-element contributes two units, the third trigger-element four units, and so on. The contributions of the trigger-elements may vary in any other manner desired.

BASIS OF PREDICTION

The design of an arithmetical calculating system for predicting the future position of an airplane will be described. The calculating system employs electronic circuits and devices and performs ordinary arithmetical operations at high speeds. The operation of the system is automatic. It is intended that the arithmetical predictor co-operate with a mechanical ballistic computer of conventional design to produce the data required to aim and fire an anti-aircraft battery.

The organization of a fire-control installation employing an arithmetical predictor of the type contemplated is shown in Figure 14. It may be assumed that optical tracking instruments supply to the predictor the present range, azimuth, and elevation of the target. The ballistic computer supplies to the predictor the time of flight of the projectile. The predictor supplies to the ballistic computer the future range, azimuth, and elevation of the target. The ballistic computer supplies to the gun the corrected azimuth and elevation of the gun and the fuse setting of the projectile.

The prediction at time t_0 of the position of the airplane at some future time, $t_0 + t_f$, is based on the assumption that the most probable behavior of the airplane is to continue the course and maintain the speed observed at time t_0 . The components of the position of the plane in the polar coordinates of the tracking instruments are transformed into components in a rectangular coordinate system. The velocity of the plane along each rectangular axis and the smoothed present position on each axis are obtained. The position of the plane on each axis at the end of an interval equal to the time of flight of the projectile is obtained by adding to the present position on the axis the product of the time of flight and the velocity along the axis. The components of the future position of the plane in rectangular

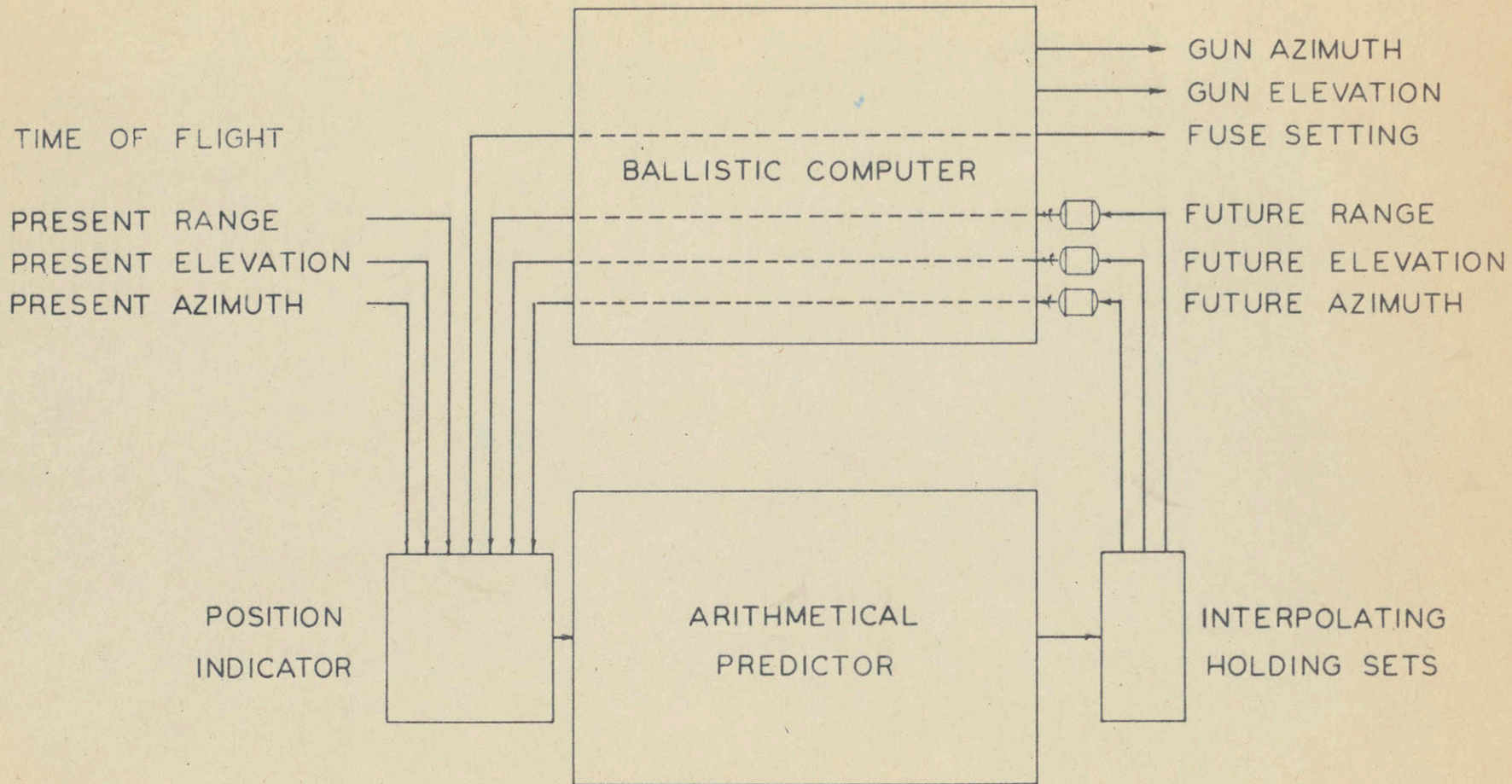


FIGURE 14

coordinates are transformed into components in polar coordinates. The future range, azimuth, and elevation shafts of the ballistic computer are rotated in accordance with the calculated position of the plane.

Arithmetical prediction is particularly well suited to the prediction of the future position of fast-moving aircraft flying at extreme altitudes. The arithmetical procedure outlined below is based on the requirements of a fire-control system for this class of targets. Observations of the position of the plane are made at intervals of one-half second, and the future position of the plane is predicted after each set of observations. The velocities of the plane are calculated from the set of new observations and the last fifteen sets of observations. The smoothed present positions are obtained from the new set of observations and the last seven sets of observations. The positions of the future range, azimuth, and elevation shafts of the ballistic computer are measured every one-eighth second, and error signals are calculated and applied to the shaft motors after each set of observations. The error signals are calculated from four sets of observed shaft positions and four sets of calculated shaft positions. Figure 15 shows the schedule of operations of the calculating system during a prediction cycle. The prediction operations require only the first one quarter of the interval between observations.

The first step in the prediction cycle is the transformation of the newest observed position of the target into the position of the target in rectangular coordinates. Figure 16 shows the two coordinate systems. When $\sin \phi$, $\cos \phi$, $\sin \theta$, and $\cos \theta$ are obtained from a function unit, the operations

$$R_s \cos \theta$$

$$R_s \cos \theta \cos \phi = x_0,$$

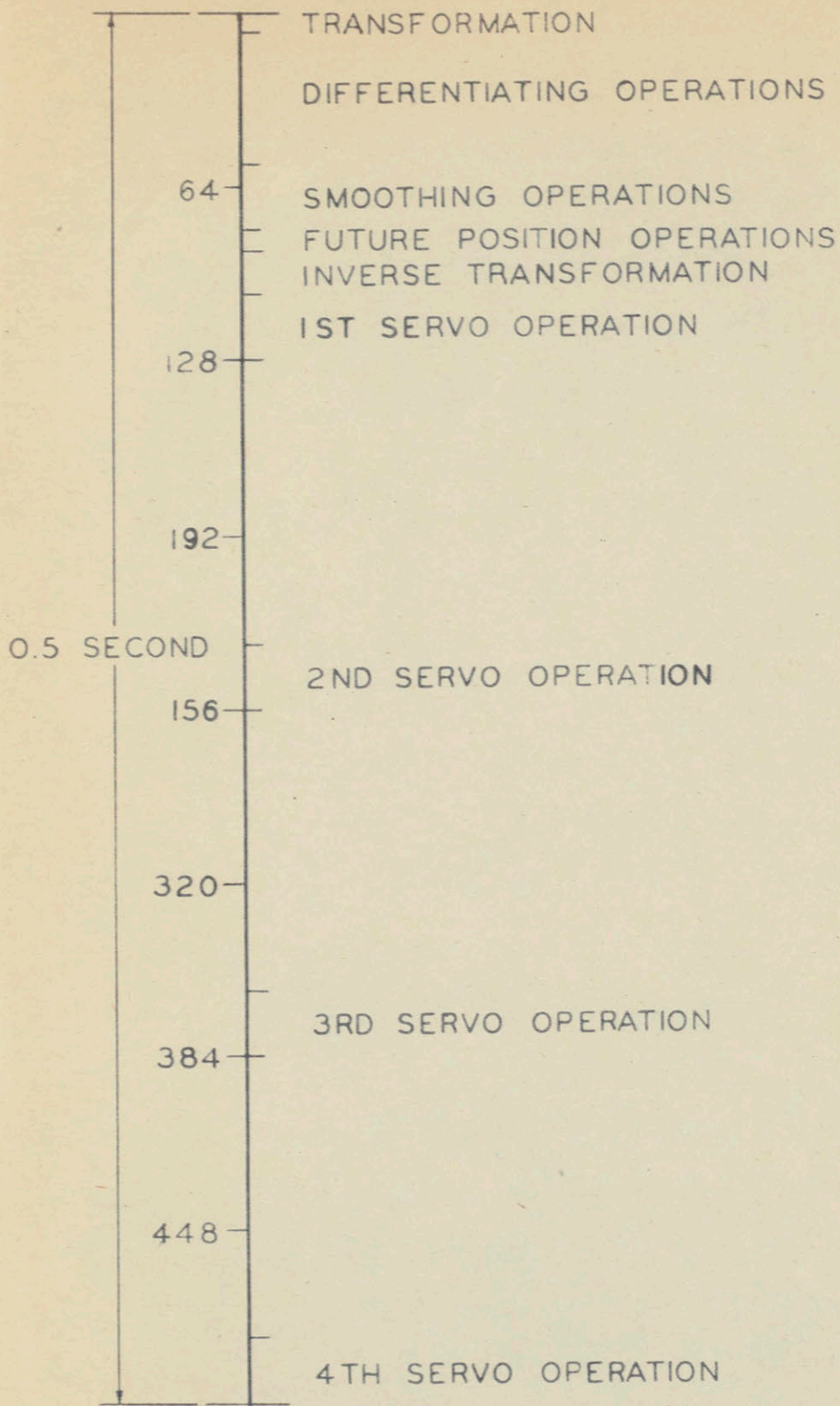


FIGURE 15

SCHEDULE OF OPERATIONS

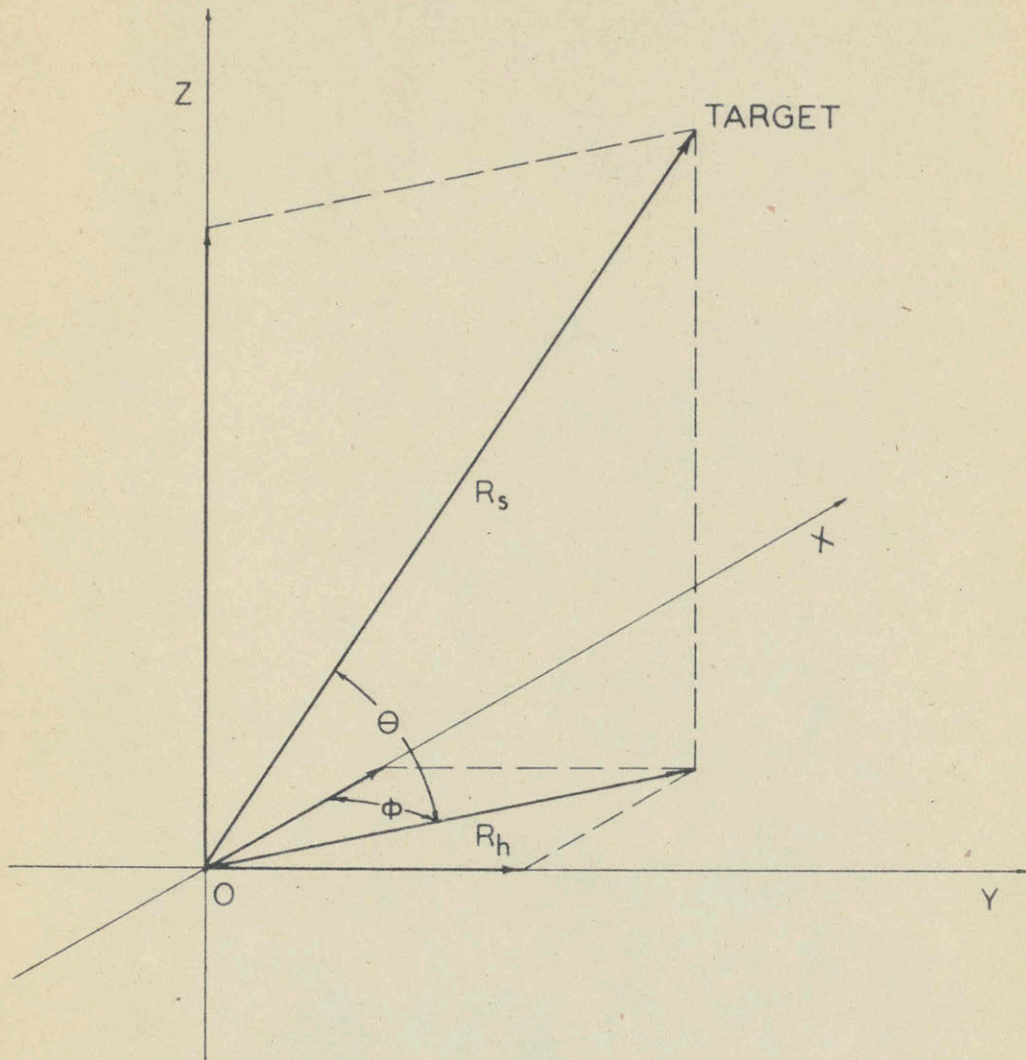


FIGURE 16

COORDINATE SYSTEMS

$$R_s \cos \theta \sin \phi = y_0, \text{ and}$$

$$R_s \sin \theta = z_0$$

are performed. The velocities of the plane at time $t = t_0$ are obtained by the operations

$$\sum_{i=-15}^0 D_i x_i = \dot{x},$$

$$\sum_{i=-15}^0 D_i y_i = \dot{y}, \text{ and}$$

$$\sum_{i=-15}^0 D_i z_i = \dot{z},$$

where the D_i are coefficients that give for the velocities the slopes at time $t = t_0$ of second degree algebraic polynomials fitted to the observed positions by least squares. The observed positions may be weighted in some manner. The smoothed values of the components of the position of the plane at time $t = t_0$ are obtained by the operations

$$\sum_{i=-7}^0 S_i x_i = \bar{x}_0,$$

$$\sum_{i=-7}^0 S_i y_i = \bar{y}_0, \text{ and}$$

$$\sum_{i=-7}^0 S_i z_i = \bar{z}_0,$$

where the bars denote smoothed values and the S_i are coefficients that give for the present position the value at time $t = t_0$ of the second degree algebraic polynomial fitted to the observed positions by least squares. The future position of the plane is obtained by the operations

$$\bar{x}_0 + \dot{x}t_f = x_f,$$

$$\bar{y}_0 + \dot{y}t_f = y_f, \text{ and}$$

$$\bar{z}_0 + \dot{z}t_f = z_f,$$

where x_f , y_f , and z_f are the components of the future position of the plane, and t_f is the time of flight of the projectile. The reading of the time of

flight shaft of the ballistic computer is used directly as the value of the time of flight. The future position of the plane in polar coordinates is obtained from the future position in rectangular coordinates by the operations

$$x_f^2 + y_f^2 = R_h^2$$

$$\sin^{-1} y_f/R_h = \phi$$

$$R_h^2 + z_f^2 = R_s^2$$

$$\sin^{-1} z_f/R_s = \theta$$

It is not desirable to provide an automatic calculating system with equipment for dividing two numbers. The values of $1/R_h$ and $1/R_s$ are obtained by entering a table of the reciprocal of the square root with the arguments R_h^2 and R_s^2 . The value of R_s is obtained by multiplying R_s^2 by $1/R_s$.

The calculation of the error signals to be applied to the motors of the future range, azimuth, and elevation shafts of the ballistic computer is divided into two parts. In the first part, a set of four coefficients multiply a set of four calculated shaft positions. Since error signals are calculated four times with only one set of calculated shaft positions, four sets of coefficients are required, one for each set of error signals. The second part of the calculation of the error signals is the multiplication of a set of four coefficients by a set of four observed shaft positions. Since the shaft positions are observed before each calculation of the error signal, the same set of four coefficients multiply each set of observed shaft positions. It is assumed that the mechanical properties of each of the three shafts are the same and that the same coefficients can be used to obtain the error signal for each motor. The error signals are produced by the operations

$$\sum_{i=-3}^0 C_{ij} R_{si} + \sum_{i=-3}^0 \overset{\curvearrowright}{C}_i \overset{\curvearrowright}{R}_{si} = \epsilon_{Rj},$$

$$\sum_{i=-3}^0 C_{ij} \theta_i + \sum_{i=-3}^0 \overset{\curvearrowright}{C}_i \overset{\curvearrowright}{\theta}_i = \epsilon_{\theta j}, \text{ and}$$

$$\sum_{i=-3}^0 C_{ij} \phi_i + \sum_{i=-3}^0 \overset{\curvearrowright}{C}_i \overset{\curvearrowright}{\phi}_i = \epsilon_{\phi j},$$

where the arrows indicate the values of shaft positions or coefficients of shaft positions and where the subscript $j = 1, 2, 3, 4$ indicates the number in the prediction cycle of the servo operation.

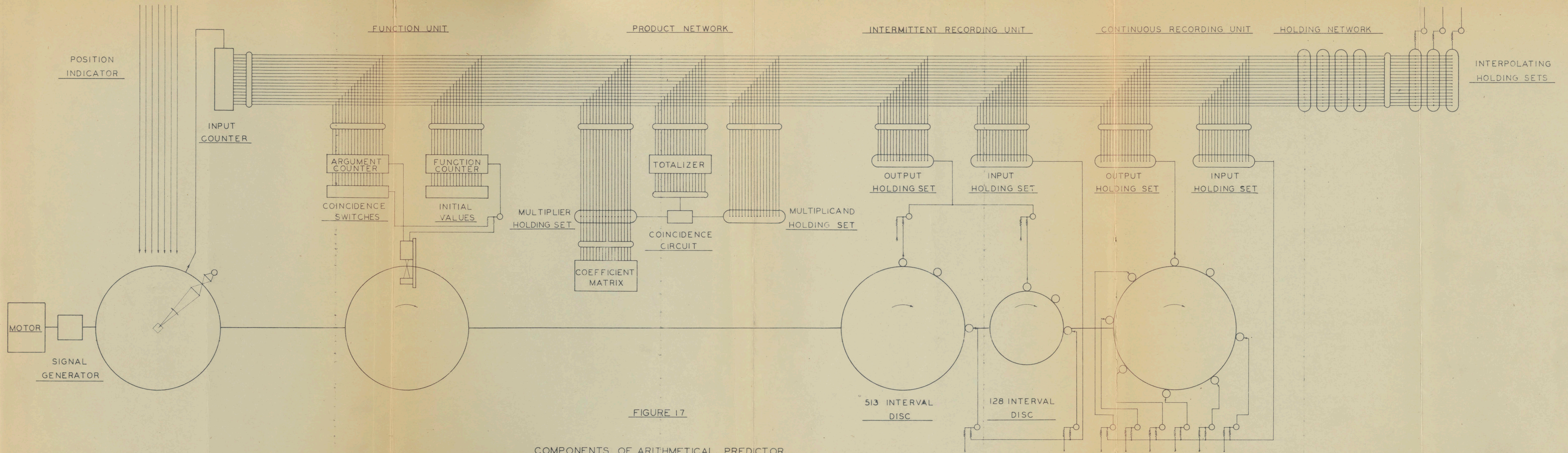


FIGURE 17

COMPONENTS OF ARITHMETICAL PREDICTOR

COMPONENTS OF PREDICTOR

Figure 17 shows schematically the components of an arithmetical predictor. The general design and operation of each of the components illustrated was described in the introduction. The product network, holding network, and interpolating holding sets require no special comment. The position indicator, function unit, coefficient matrix, and recording units are especially designed to perform the operations described in the previous section and are described below. It is assumed that the speed of operation of the product network is 1024 products per second. Stepping switches for selecting the trigger-elements of the holding sets in the product network and recording units, and the elements of the totalizer in the product network are not shown in the figure. The control wiring is likewise not shown.

Position Indicator

The position indicator co-operates with a counter to supply to the calculating system the setting of the range finder, the positions of the azimuth and elevation tracking telescopes, and the positions of the time of flight, future range, future azimuth, and future elevation shafts of the ballistic computer. The position indicator is located at the predictor, and the positions of the tracking instruments and ballistic computer shafts are transmitted to the position indicator by servo-mechanisms of conventional design. The scanning light beam of the position indicator rotates with the recording and function discs of the predictor, and its rotation is therefore synchronized with the operation of the predictor. The aperture track of the position indicator is divided into eight equal intervals, seven of which are used. The observed displacements are assigned to the intervals in the order $R_s, \theta, \phi, \bar{R}_s, \bar{\theta}, \bar{\phi}$, blank, and t_f . Neglecting for a moment the two

azimuth angles, the possible number of values of the variables does not exceed 2048. If the number of apertures per inch in the aperture disc is assumed to be in the neighborhood of 250, the length of the intervals of the aperture track can be 8 inches. The circumference of the aperture track is therefore 64 inches. The angular velocity of the scanning light beam is 16 revolutions per second. Each interval therefore is scanned in 8 milliseconds. The possible number of values of the azimuth is 8192. The quadrant of an azimuth angle is indicated to the counter by a pair of photocells that scan a circle graduated in quadrants by two broken concentric lines, and the value of the angle within a quadrant is measured by counting.

Function Unit

The function unit contains tables of the sine, cosine, reciprocal, and inverse sine. Each table is recorded in duplicate on a single disc. The order of the tables and the arguments with which the tables are entered are $\sin \phi$, $\cos \phi$, $\cos \theta$, $\sin \theta$, $(R_h)^{-1}$, $(R_s)^{-1}$, $\sin^{-1} y_f/R_h$, and $\sin^{-1} z_f/R_s$. By virtue of the duplication of the tables, all function values are obtained by a single scanning pole. The length of each table is assumed to be 8 inches. The tables follow one another without interruption, and the circumference of the disc is therefore 64 inches. The scanning speed is 2048 inches and the angular velocity of the disc is therefore 32 revolutions per second.

The initial values of the reciprocal, cosine and inverse sine are transferred to the function counter by a resistance network. The sine of the azimuth is obtained by the technique proposed in the introduction for finding the sine of an angle that varies through 2π radians, and the coincidence switches associated with the argument counter are assumed to include circuits that apprehend the quadrant of the angle of azimuth, operate the subtraction circuits of the function and argument counter if either counter is to count

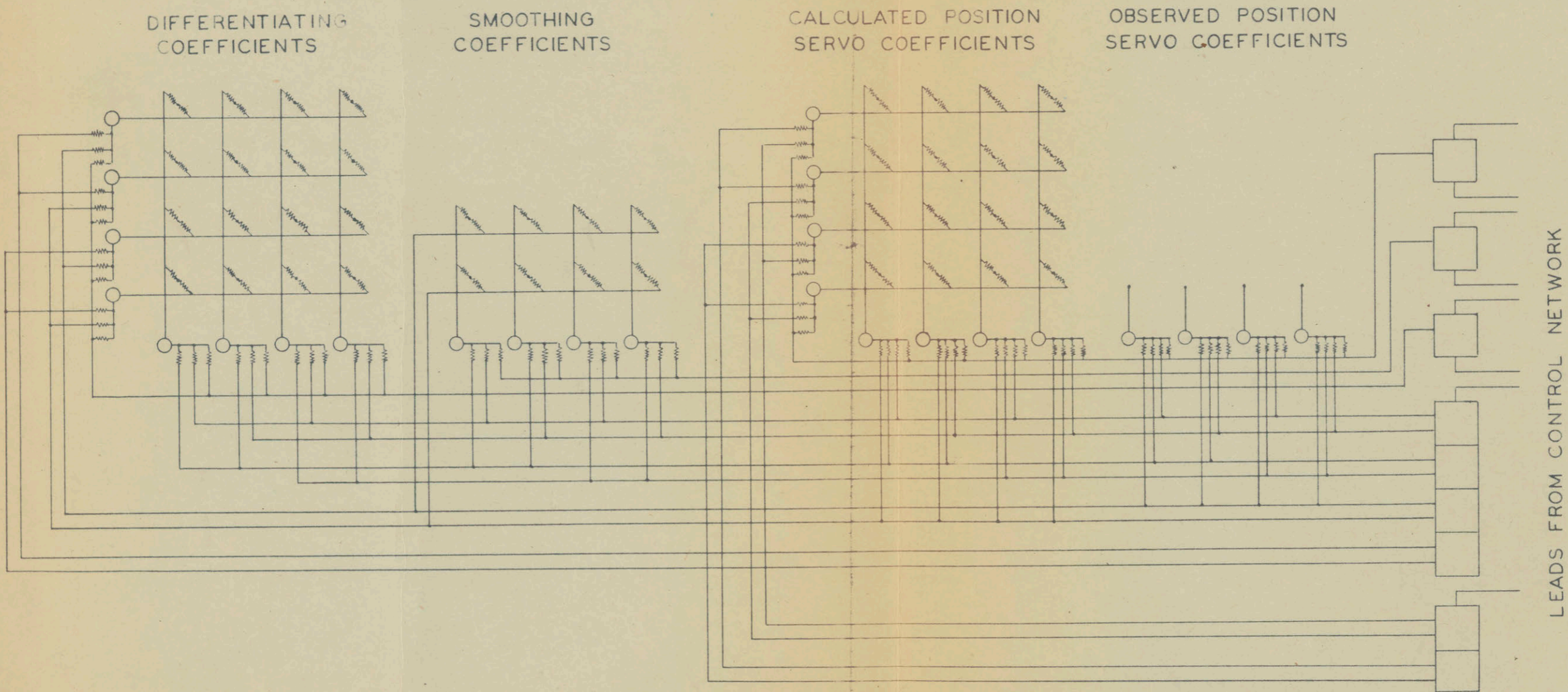
negatively, and select the coincidence switch whose operation indicates the end of the argument interval. When the angle of azimuth is found by entering a table of the inverse sine with y_f/R_h , the sign of y_f/R_h indicates whether the angle is in the first or second quadrants or in the third or fourth quadrants. Since the initial value of the angle of azimuth transferred to the function counter and the direction of the counters depends on the quadrant of the angle, it is necessary to transfer the sign of x_f to the function unit. It is assumed that this is done by a lead to the highest digit position of a holding set in the holding network in which x_f is recorded in the transfer period when y_f/R_h is transferred to the function unit.

Sets of switches are inserted in the connections between the elements of the function and argument counters and the transfer leads of the system in order to prevent the counters from being affected by the voltage shifts of the leads when they are in operation. These switches, together with the corresponding switches in the other components are called transfer switches.

Coefficient Matrix

The differentiating, smoothing, and error signal coefficients are recorded in a 44-terminal resistance network. Figure 18 shows the grouping and the manner of selecting the terminals. The members of each group of terminals are selected by trigger-pairs connected to form a binary counter. The counter selects a new terminal in each group at the beginning of each product interval. The members of each group of terminals are selected in sequence continually. When the coefficients recorded by a particular group of terminals are to be transferred to the multiplier holding set, the holding circuit associated with the group is operated and permits the switches of the group to be operated by the counter. The holding circuit remains operated until all

FIGURE 18
COEFFICIENT MATRIX



of the coefficients in the group have been transferred to the multiplier holding set the required number of times. A single holding circuit places the two groups of servo coefficients in action. The connections between the trigger-pairs and the switches in each group are so arranged that if all trigger-pairs are initially in their zero positions, four calculated position coefficients are first transferred to the multiplier holding set and then the four observed position coefficients are transferred. The grids of the horizontal column of switches in the calculated point coefficient group are connected to the negative-active terminal of the hundreds trigger-pair. When the hundreds trigger-pair is passive, or in other words, when it contains a zero as it does during the first four steps of the counter, the set of four calculated position coefficients is selected. The particular set of coefficients selected depends on the positions of the counter elements that select the rows of terminals. One after another of the horizontal rows is selected as successive sets of error signals are calculated from a single set of calculated shaft positions. When four calculated position coefficients have been selected, the hundreds trigger-pair becomes active, and the switches of the calculated point coefficient terminals are not operated for the next four product intervals. During these four intervals, the observed position coefficients are selected, since the switches of the observed position coefficient terminals are connected to the positive-active terminal of the hundreds trigger-pair. The three error signals are calculated one after another, and the servo coefficient terminals are selected in succession three times.

Intermittent Recording Unit

The intermittent recording unit has two recording discs. One disc records the observed positions of the target for 513 product intervals, and

the other records the calculated and observed positions of the future range, azimuth, and elevation shafts for 128 product intervals. Simultaneous transfers of numbers to or from the discs do not occur, and a single pair of holding sets suffice for both discs. The recording and reproducing poles of the two discs are connected to the output and input holding sets by two pairs of switches.

The capacity of the 513 interval recording disc is sufficient to record the observed positions of the target if the interval between the recording and reproducing poles measured along the periphery of the disc exceeds 47 inches. The required delay is obtained by a disc with a circumference in the proper range if the angular velocity of the disc is 15.5 revolutions per second and the angular displacement between the reproducing pole and the recording pole is 270° . A recorded number rotates through this angle plus seven complete rotations before it is reproduced. The desired peripheral speed of 1024 inches per second is obtained if the circumference of the disc is 66.0645 inches. The linear interval between the reproducing pole and the recording pole is 49.5484 inches. It is not essential that the peripheral speed of the disc be exactly 1024 inches per second. If the circumference of the disc is made 64 inches, which is the circumference of the function disc, the differences in peripheral speed and the wavenumber of the recorded signal are not significant.

The capacity of the 128 interval recording disc is sufficient to record the observed and calculated shaft positions if the separation along the tape of the recording and reproducing poles exceeds 23 inches. The required delay is obtained if the angular velocity of the disc is 30 revolutions per second and the angular separation of the recording and reproducing poles is 259.453° . A recorded signal rotates through this angle and makes three com-

plete revolutions in addition before it is reproduced. A peripheral speed of 1024 inches per second is obtained if the circumference of the disc is 34 inches. For a circumference of this value, the linear separation along the tape of the recording and the reproducing poles is 24.504 inches.

It was pointed out that both the 513 and the 128 interval discs rotate several times between the recording and reproduction of a number. During the period when no numbers are transferred to or from the discs, the switch in the leads from the two reproducing poles are open. Switches in the leads to the erasing poles are open also.

Continuous Recording Unit

The continuous recording unit records intermediate results for periods from 8 to 48 product intervals in steps of eight intervals. Only one number is recorded on the disc at a time, and a single pair of input and output holding sets is used. The digit transfer lead of the input holding set is connected to the grids of a set of switches in the input connections of the recording poles. Each switch is operated by a holding circuit which in turn is operated by the control network of the calculating system. The reproducing pole is directly connected to the output holding circuit. The reproducing pole reproduces continually the numbers recorded on the disc, and the tape is continuously erased after it leaves the recording pole. It can be assumed that the continuous recording disc is mounted on the same shaft with the 513 interval recording disc and has the same diameter. The six recording poles are disposed around the periphery of the disc at intervals of eight inches, beginning seven inches from the reproducing pole.

OPERATION OF PREDICTOR

A prediction cycle is composed of 512 product intervals and the same number of transfer periods. In each product interval, the product network multiplies two numbers. One product interval is required to record and to reproduce a number in a recording unit. The function unit requires four product intervals to produce the value of a function. The position indicator requires eight product intervals to measure a displacement.

The number-holding components of the system are divided into two groups depending on whether numbers are transmitted to or from the component. The components from which numbers are transferred are called transmitters, and the components to which numbers are transferred, receivers. The transmitters of the system are the position-indicator counter, the function counter, the totalizer, the coefficient matrix, the output holding sets of the two recording units, and the holding sets of the holding network. The receivers of the system are the argument counter, the multiplier and multiplicand holding sets of the product network, the input holding sets of the recording units, the holding sets of the holding network, and the interpolating holding sets.

In each transfer period, numbers are transmitted from each transmitter in sequence. The first transmitter in each transfer period is one of the holding sets in the holding network. The second transmitter is the output holding set of the continuous recording unit. The third transmitter is the output holding set of the intermittent recording unit. The fourth transmitter is the product network or the input counter. The fifth transmitter is the function unit or the input counter. In the predictor designed, numbers are never transmitted simultaneously from the product network, function unit, and

input counter, and it is therefore unnecessary to allot a transmission time to each transmitter in each transfer period.

The transmitters are holding sets, counters, or totalizers. A number is transmitted by applying a clearing signal to the transmitter. If a transmitter is isolated from the transfer leads by a set of transfer switches, the switches are closed before the transmitter is cleared.

If a receiver is isolated from the transfer leads by transfer switches the receiver receives a transmitted number when the transfer switches are operated. If the receiver is connected directly to transfer leads, the receiver receives a number when it is primed. Thus, the argument counter, the multiplier and multiplicand holding sets, and the input holding sets of the function unit receive transmitted numbers when their transfer switches are operated. An interpolating holding set receives numbers when the transfer switches of the group of holding sets is operated and when the individual holding set is primed. A holding set in the holding network receives a number when it is primed.

The receivers of a transferred number are selected by a selector switch with at least as many terminals as there are transfer periods in the prediction cycle in which numbers are transferred. It will be assumed that the selector switch, or interval switch as it will be called, is formed with a nine element binary counter and two arrays of switches. One array contains 32 switches that are operated by five of the counter elements, and the other array contains 16 switches that are operated by the other four counter elements. The interval switch has potentially 512 terminals, one for each transfer period in the prediction cycle. The interval switch is stepped one position before each transfer period.

The transmitters are cleared in sequence in each transfer period by

a five element stepping switch. This switch will be called the transmitter switch. When a receiver is to receive a number from a transmitter, a switch in the priming circuit of the holding set or in the operating circuit of the transfer switch of the receiver is operated by the interval switch and the transmitter switch. The switch is primed throughout the transfer period by the interval switch and is operated by the transmitter switch at the time the transmitter is cleared. Each receiver has one of these receiver switches for each transmitter from which it may receive a number. In a completely general calculating system, each receiver has a receiver switch for each transmitter, but in a calculating system designed to perform a single arithmetical process, a receiver has only as many receiver switches as there are transmitters from which it receives numbers.

Schedule of Operations

Table III is the schedule of operations of the calculating system during a prediction cycle. The general sequence of operations follows the abbreviated schedule presented earlier. In Table III each transmitter and receiver is associated with a vertical column. The transmitters are represented by vertical solid lines. Transferred numbers are represented by horizontal lines. A number in the holding network or in the continuous recording unit and the function unit in action are represented by sloping lines between the column of the receiver and the column of the transmitter. The transmitters participating in a transfer period are indicated by small circles, and the transmitted number is specified. The receivers of a transmitted number are indicated by dots, and the number of the transmitter is specified. The progress of any number through the system can be followed by moving from its transmitter to the receiver with the number of the transmitter.

The new data employed in a given prediction cycle is collected at

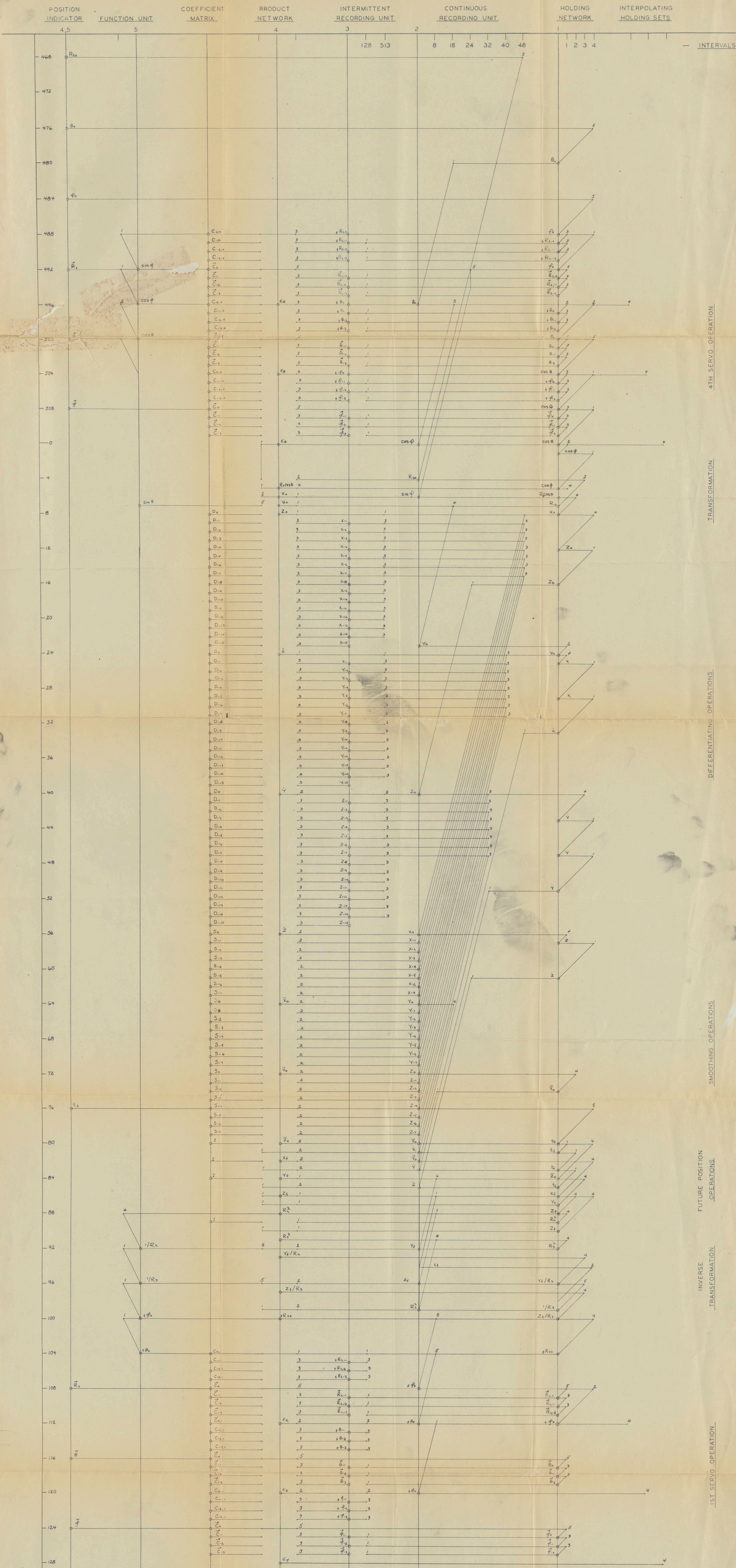
the end of the earlier prediction cycle. As the numbers are received, they are recorded in the holding network and continuous recording unit. The holding network is used to hold the numbers until they can be recorded in the continuous recording unit in such positions that they are produced at the desired time. Numbers may be held also in the holding network after they leave the recording unit, but during the 24 intervals preceding the origin of the prediction cycle, the holding network is engaged in holding observed and calculated shaft positions for one interval before they are recorded on the 128 interval disc, and the few transfer periods available are used to hold some of the trigonometric functions between the time they are produced and the time they are used by the product network.

The position indicator is so synchronized with the calculating system that the positions of the shafts of the ballistic computer are transferred directly from the input counter to the product network.

It is possible to produce the numbers participating in the transformation from polar to rectangular coordinates at the required time only by resorting to three irregular procedures. $\sin \phi$ is held in the input holding set of the continuous recording unit for two intervals before it is recorded. $\sin \theta$ is left in the function counter of the function unit from the 504th to the 7th product interval, and $\cos \theta$ is left in the multiplier holding set of the product network for four intervals before it is multiplied by R_s . These operations require special switching connections from the interval switch.

The operations of differentiating and smoothing require no special comment. The first eight values of x , y , and z used in the differentiating operations are recorded in the continuous recording unit to be used in smoothing. It is to be noticed that the first fourteen values of x , y , and z reproduced from the 513 interval disc are immediately re-recorded for use in the

TABLE III
SCHEDULE OF OPERATIONS



4TH SERVO OPERATION

TRANSFORMATION

DIFFERENTIATING OPERATIONS

SMOOTHING OPERATIONS

FUTURE POSITION OPERATIONS

INVERSE TRANSFORMATION

1ST SERVO OPERATION

next prediction cycle. The fifteenth values are not re-recorded.

The time of flight is available in the input counter at the 76th transfer period and is held in the holding network until the 81st transfer period when it multiplies \dot{x} . There is no provision for transferring numbers directly to the totalizer and \bar{x}_0 , \bar{y}_0 , and \bar{z}_0 are placed in the totalizer by multiplying them by one. The one is placed in the multiplier holding set by a lead from the interval switch. Also, there is no provision for transferring a number from the totalizer without clearing the totalizer. Therefore, when R_h^2 is obtained, it is transferred to the interval holding set in the 88th transfer period, to the multiplier holding set in the 89th transfer period, and is multiplied by one in the 89th product interval.

The sign of x_f must be available at the 96th transfer period when y_f/R_h is transferred to the argument counter. x_f is recorded in the continuous recording unit from the 86th to the 94th transfer period and is then transferred to the holding set that is the 4 interval holding set at the 94th transfer period. A lead from the 4 interval holding set to the switching circuits of the function unit indicates to the function unit the sign of x_f which, with the sign of y_f/R_h , determines the quadrant of ϕ .

The first servo operation of the prediction cycle begins with the 104th product interval and continues for 24 product intervals. In this operation, the calculated shaft positions are not delayed in the holding network before they are recorded in the 128 interval recording unit since the same calculated shaft positions are used in the succeeding three servo operations. Three observed positions of each shaft are delayed one interval in the holding network before they are recorded on the 128 interval disc. The fourth and oldest observed position of each shaft is not re-recorded.

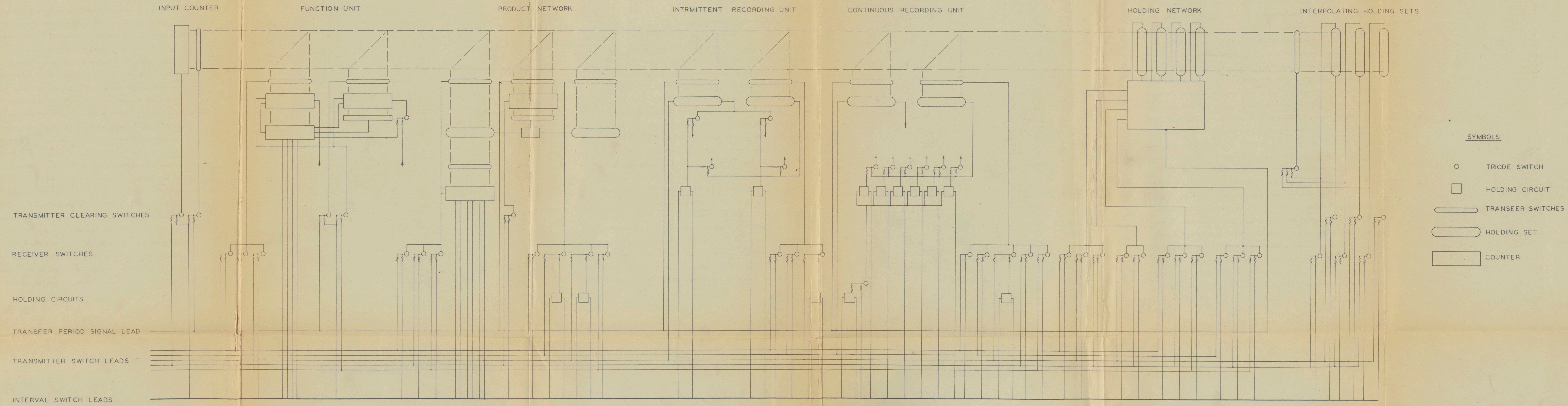
Control Wiring

Figure 19 shows schematically the control wiring of the calculating system. Individual connections between the grids of the receiver switches and the terminals of the interval switch are not shown. It is to be understood that, in general, a receiver switch grid is connected to as many interval switch terminals as there are transfers of numbers to the receiver operated by the receiver switch from the transmitter associated with the switch. This statement does not apply to a receiver that receives many numbers in succession from a single transmitter because a holding circuit operated by the interval switch can prime the grid of the receiver switch and thus cause the receiver to receive all numbers transmitted by the transmitter associated with the receiver switch. Examples of this type of operation include the reception by the multiplier holding set of coefficients transmitted from the coefficient matrix and the reception by the multiplicand holding set of numbers transmitted by the intermittent recording unit.

Not shown also in the figure are the stepping switches that select the elements of the holding sets in the recording unit and the product network and the elements of the totalizer. The special switching and control circuits mentioned in the previous section likewise are not shown.

At the beginning of each transfer period, the voltage of the transfer period signal lead shifts. The transfer switches of the totalizer and the output holding sets of the recording units are operated directly by the transfer period signal. The transfer switches of the function counter and the input counter are operated by the co-operation of the transfer period signal and the interval switch. The transfer switches are closed when numbers are transferred from the counters, but are left open when the counters are in operation. The transfer period signal steps the counter of the coefficient

CONTROL WIRING SCHEMATIC



SYMBOLS

- TRIODE SWITCH
- HOLDING CIRCUIT
- ▭ TRANSEER SWITCHES
- HOLDING SET
- ▭ COUNTER

matrix at the beginning of each transfer period. If the switches of a group of coefficient terminals are operated, a coefficient is transferred to the multiplier holding set when the counter operates. The transfer period signal resets the holding circuits that establish connections to the recording poles of the continuous recording unit unless the switch in the path of the signal is open, as it is when a series of numbers are recorded by a particular pole.

In each transfer period, the transmitter stepping switch steps through its five positions and clears in sequence a holding set in the holding network, the output holding set of the continuous recording unit, the output holding set of the intermittent recording unit, the totalizer or input counter, and the function counter or input counter. As mentioned above, the product network, function unit and input counter are cleared only if switches in the clearing leads are primed by the interval switch. At each position of the stepping switch, all receiver switches whose grids are connected to the lead of active element of the transmitter switch and which are primed by the interval switch operate and, in so doing, operate the transfer switches or prime the holding sets to which they are connected. For example, the transfer switches of the argument counter are operated by three receiver switches. The grid of one receiver switch is connected to the lead of the first stepping switch element, the grid of a second receiver switch is connected to the lead of the third stepping switch element, and the grid of the third receiver switch is connected to the lead of the fifth stepping switch element. The grids of the receiver switches are connected also to terminals of the interval switch that are active during the transfer periods in which the argument counter is the receiver of transmitted numbers. The receiver switches of the rest of the receivers in the system are operated similarly by the transmitter switch and the interval switch. The number of receiver switches required by

a receiver as well as the connections of the receiver switches are obtained from the schedule of operations.

The connections from the interval switch to the coincidence switch circuits associated with the argument counter indicate to the circuits the function to be obtained, and, when the function is the sine or inverse sine, the argument. It is to be recalled that special operations are required to obtain the sine of the angle of azimuth and to obtain the angle of azimuth from a table of the inverse sine. Connections from the coincidence switch circuits to the initial value resistance network of the function counter operate the terminals of the resistance matrix and transfer initial values of functions to the function counter.

Most of the control wiring of the product network is internal and is not indicated. The internal control wiring includes circuits to operate the subtraction circuit of the totalizer, to control the stepping of the selector switches, to invert the sense of the multiplier digits when the multiplier is a complement, to transfer ones to the coincidence switch when the multiplicand is a complement, and to clear the multiplicand and multiplier holding sets. It is frequently desirable to keep a number in either the multiplier or multiplicand holding set. A switch in the path of the clearing signal operated by the interval switch can accomplish this result. Long sequences of multiplicands are transmitted to the multiplicand holding set from the intermittent and continuous recording units. The receiver switches associated with the transmitters of these components are coupled to the interval switch through holding circuits.

The input and output holding sets of the intermittent recording unit are connected through switches to the poles of each disc. The switches are operated by a pair of holding circuits which are operated and reset by the

interval switch. The numbers reproduced from the 513 interval disc are transferred from the output holding set to the multiplicand holding set and to the input holding set. The receiver switch of the input holding set associated with the third transmitter switch element is therefore primed by a holding circuit which is operated by the interval switch.

The recording poles of the continuous recording unit are connected to the input holding set by switches that are operated by holding sets. The holding circuit of the pole that is to record a number is operated in the transfer period and remains operated during the product interval. At the end of the recording interval, the holding circuit is reset by the transfer period signal unless the holding circuit operated switch in the transfer period signal lead is open. The receiver switch associated with the intermittent recording unit transmitter is primed by a holding circuit.

The stepping switch of the holding network is stepped one position at the beginning of each transfer period by the first element of the transmitter stepping switch. One of the holding sets is cleared when the stepping switch steps. One or more of the holding sets are primed by the co-operation of sets of receiver switches and the set of delay selector switches.

When the error signal in an interpolating holding set is to be changed, the holding set is cleared by the third element of the transmitter switch. During the clearing operation, the transfer switches of the group of holding sets are open. The fourth transmitter switch element co-operates with the interval switch to close the transfer switches and to prime one of the interpolating holding sets. The new error signal is transmitted to the holding set from the totalizer.

DISCUSSION

Components and a basis of operation of an automatic calculating system have been described. An automatic calculating system has been designed to predict the future position of an airplane.

The novelty of arithmetical calculating equipment has not justified a detailed presentation of the specifications and operation of the equipment. The effort has been to present an over-all view of a calculating system that can provide a basis for the development of calculating systems for control operations.

The prediction process described is believed to be an effective one, but it may well be more extensive than what is demanded in present-day fire control. The process draws heavily upon a unique ability of arithmetical equipment, namely, the ability of record data for indefinite periods. The effectiveness of the process is enhanced by the ability of arithmetical equipment to operate on data without loss of precision.

The system described is incomplete in the sense that the prediction operations use only one quarter of its capacity. Three possible extensions of the system are described below. As described, the system employs in the neighborhood of 500 vacuum tubes and instruments for measuring displacements and recording functions, coefficients, and intermediate results. When the operations of the system are extended to use all of the available capacity, the number of vacuum tubes increases slightly, but the numbers of recorded functions, coefficients, and intermediate results increase greatly. At the end of this section possible forms of extended function units and recording units are described.

The first extension of the system and an extension that must be made to produce an effective fire-control instrument is the performance by

the system of ballistic computations. By ballistic computations are meant the calculation of the time of flight of the projectile and the azimuth and elevation of the gun corrected for wind, non-standard atmosphere, wear of the gun barrel, and other special conditions.

The principle changes in the calculating system brought about by an extension of the arithmetical process to include ballistic computations are in the function unit and the recording unit. The function unit must include a table of the time of flight of the projectile as a function of the future slant range and future elevation of the target. The recording unit must include a second continuous recording disc because the number of intermediate results recorded and brought to the product network together is larger in the ballistic computations than in prediction.

The position indicator, product network, holding network, and interpolating holding sets are changed little by the extension to ballistic computations. The position indicator measures six instead of seven displacements: the setting of the range finder, the positions of the azimuth and elevation tracking telescopes, the azimuth and elevation of the gun, and the position of the fuse-setter.

If the calculating system produces error signals used to displace the gun and fuse setter, modifications of the servo operations are necessary. So little is known about the problem of controlling displacements in accordance with numerical data produced at discrete intervals that it is not feasible to say anything about the modifications, but it is reasonable to assume that the error signals must be calculated from a larger number of calculated and observed positions.

The performance of ballistic computations has not been studied sufficiently carefully to permit an effective arithmetical process to be

described. Because the variation of the time of flight with the angle of elevation is small compared to the variation with slant range it may be possible to obtain the time of flight as a function of the slant range only and modify the time of flight so obtained with the sum of a quadratic form of the slant range and the elevation. In this way, a record of a function of two variables is avoided.

A second extension of the system is to a predictor whose operation is based on a more detailed formulation of the prediction problem. An example of a more detailed and effective formulation than that presented here is the statistical prediction proposed by Wiener.¹ The problem of carrying out statistical prediction with arithmetical equipment has not been studied carefully, but it appears possible to say that it can be carried out effectively. The intrinsic coordinate system proposed by Wiener can be employed by arithmetical equipment.

A third extension of the system is to permit a single installation of arithmetical calculating equipment to perform the fire-control operations of more than one battery. This extension lies far in the future and does not deserve detailed consideration at the present time.

It is worthwhile to describe alternative forms of the function unit and recording unit that will be useful in a calculating system that performs ballistic computations or employs a more extensive prediction process.

The physical dimensions of the function and recording discs are limited by available space. When more functions must be recorded than can be recorded on a single disc, or where delays longer than can be provided by a single disc are necessary, several discs must be employed, but, unless some means are provided of moving the scanning poles of the function discs or the

¹ N. Wiener, "The Extrapolation, Interpolation, and Smoothing of Time Series", National Defense Research Committee, 1942.

recording, reproducing, and erasing poles of the recording discs, each disc must be provided with poles. Since not all of the poles are in use simultaneously, equipment is wasted.

Consider first an alternative form of function unit that permits two poles to scan the equivalent of several discs. The photographic film on which the functions are recorded is wrapped in the form of a helix around a drum. The cross-section of the surface of the drum in a plane that includes the axis of the drum is a semi-circle. The two poles are mounted on the ends of an arm that rotates at the center of the semi-circular section of the drum. One pole at a time scans the function and argument tracks on the drum surface. Images of the tracks are projected to fixed electron multipliers. As one pole emerges from the track at one rim of the drum surface, the other pole enters the track at a point on the opposite rim. The poles thus scan in sequence and with uniform circular motion the equivalent of several function discs.

The alternative form of the recording unit is similar. The recording track describes a helix about the interior of a semi-circular drum, and the track is scanned by a pair of reproducing poles on a rotating arm. The track is continuously erased by a pair of erasing poles also rotating on an arm. Numbers are recorded by any desired number of recording pole-pairs mounted on rotating arms. The recording poles are spaced along the track to give the desired recording intervals. By use of a recording unit of this type, recording intervals up to 512 or more product intervals can be secured. The recording unit need occupy little more space than two or three discs of the type described in the thesis. Since all numbers are reproduced by a single pair of poles, the quantity of switching equipment is reduced.