Effect of Gas Path Heat Transfer on Turbine Loss

by

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Submitted to the Department of Aeronautics and Astronautics
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Abstract

This thesis presents an assessment of the impact of gas path, i.e., streamtube-to-streamtube, heat transfer on aero engine turbine loss and efficiency. The assessment, based on the concept of mechanical work potential [19], was carried out for two model problems to introduce the ideas. Three-dimensional RANS calculations were also conducted to show the application to realistic configurations. The first model problem, a constant area mixing duct, demonstrates the importance of selecting a fluid component loss metric appropriate to the purpose of the overall system in which the component resides. The phenomenon of thrust increase due to mixing is analyzed to show that system performance can increase even though there is a loss of thermodynamic availability. Gas path heat transfer affects mechanical work potential, and thus turbine loss, through a mechanism called thermal creation [19]. The second model problem, an inviscid heat exchanger, illustrates how thermal creation is due to enthalpy redistribution between flow regions with different local Brayton efficiency. Heat transfer across a static pressure difference, or between gases with different specific heat ratios, can cause turbine efficiency to increase or decrease depending on the direction of the heat flow. Three-dimensional RANS calculations have also been interrogated to define and determine the thermal creation, and thus the losses, in a modern two-stage cooled high pressure turbine. At representative engine operating conditions the effect of thermal creation was a 0.1% decrease in efficiency, with the thermal creation accounting for 1% of the overall lost work. Introducing coolant flow into the main gas path increased the loss from thermal creation in the first stage by 84% and decreased the loss from thermal creation in the second stage by 8%.

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Nomenclature

Acronyms

CFD  computational fluid dynamics
OTDF  overall temperature distribution function
PS  pressure side
RANS  Reynolds-Averaged Navier-Stokes
RTDF  radial temperature distribution function
SA  Spalart-Allmaras
SS  suction side
SST  shear stress transport
TSFC  thrust specific fuel consumption
1D  one-dimensional

Roman Symbols

\( A \)  area
\( b \)  availability
\( b \)  span
\( c_p \)  specific heat at constant pressure
\( c_x \)  axial chord
\( D \)  corrected flow function
\( DP \)  non-dimensionalized stagnation pressure difference
\( DT \)  non-dimensionalized stagnation temperature difference
\( F \)  gross thrust
\begin{align*}
F_{\text{net}} & \quad \text{net thrust} \\
h & \quad \text{enthalpy} \\
k & \quad \text{turbulent kinetic energy} \\
M & \quad \text{Mach number} \\
\dot{m} & \quad \text{mass flow rate} \\
m_f & \quad \text{mechanical work potential} \\
(m_f)_{\text{loss}} & \quad \text{mechanical work potential loss metric} \\
\bar{M}_i & \quad \text{representative inlet Mach number} \\
\text{nr} & \quad \text{number of rotor passages} \\
p & \quad \text{pressure} \\
PR & \quad \text{stagnation-to-static pressure ratio} \\
\bar{p}_t & \quad \text{arithmetic average stagnation pressure} \\
(p_t)_{\text{loss}} & \quad \text{stagnation pressure loss metric} \\
\bar{q} & \quad \text{heat flux vector} \\
R & \quad \text{specific gas constant} \\
r & \quad \text{radial coordinate} \\
s & \quad \text{entropy} \\
T & \quad \text{temperature} \\
TR & \quad \text{stagnation temperature ratio} \\
\bar{T}_t & \quad \text{arithmetic average stagnation temperature} \\
u & \quad \text{velocity magnitude} \\
u_c^* & \quad \text{ideal specific gross thrust} \\
(u_c^*)_{\text{loss}} & \quad \text{ideal specific gross thrust loss metric} \\
\bar{u}_i & \quad \text{representative inlet velocity} \\
u, v, w & \quad \text{x,y,z components of velocity} \\
V & \quad \text{volume} \\
w & \quad \text{work} \\
\dot{W} & \quad \text{turbine power} \\
w_{\text{lost}} & \quad \text{lost work} \\
w_{\text{shaft}} & \quad \text{shaft work}
\end{align*}
\[ [w_{\text{shaft}}]_{\text{max}} \] maximum shaft work

\( x, y, z \) cartesian coordinates

\( y^+ \) non-dimensional wall cell distance

**Greek Symbols**

\( \alpha \) projection angle

\( \beta \) proportionality constant

\( \gamma \) specific heat ratio

\( \bar{\gamma} \) arithmetic average specific heat ratio

\( \Delta \) change in a quantity

\( \Delta_{\text{therm}} \) thermal creation

\( \Delta h_{\text{fuel}} \) heat of combustion of fuel

\( \Delta s \) non-dimensional entropy rise

\( \Delta s_{\text{irreversible}} \) irreversible entropy change

\( \Delta \eta_{\text{tc}} \) change in turbine efficiency from thermal creation

\( \epsilon \) turbulent dissipation

\( \eta \) efficiency

\( \eta_{\text{flux}} \) turbine efficiency based on mechanical work potential fluxes

\( \eta_0 \) overall efficiency

\( \eta_p \) propulsive efficiency

\( \eta_{\text{source}} \) turbine efficiency based on mechanical work potential sources

\( \eta_{\text{th}} \) thermal efficiency

\( \eta_{\text{wa}} \) turbine efficiency based on work-averaged stagnation pressure

\( \eta_0 \) turbine efficiency with OTDF = 0

\( \theta \) circumferential coordinate

\( \mu \) dynamic viscosity

\( \nu_l \) laminar viscosity

\( \nu_t \) turbulent viscosity

\( \rho \) density
\( \sigma \)  bypass ratio
\( \Phi_{visc} \)  viscous dissipation
\( \overline{\Phi}_{visc} \)  volumetric average viscous dissipation
\( \omega \)  turbulence dissipation rate

**Subscripts**

- \( c \)  core
- \( e \)  exit station
- \( f \)  fan
- \( i \)  inlet station
- \( ref \)  reference state
- \( se \)  isentropic turbine exit state
- \( t \)  stagnation quantity
- \( 0 \)  surrounding state
- \( 1,2,ect. \)  stream number
- \( 1,2,ect. \)  station number

**Superscripts**

- \( a \)  averaged quantity
- \( ba \)  availability-averaged
- \( m \)  mass-averaged
- \( ta \)  thrust-averaged
- \( wa \)  work-averaged
Chapter 1

Introduction and Background

Large thermal gradients exist within the gas path of an aero engine high pressure turbine, due to hot streaks and coolant flows. These temperature non-uniformities create the potential for substantial heat transfer in the gas path\(^1\). At the turbine inlet, hot streaks from the combustor result variations of stagnation temperature by \(\pm 20\%\) of the mass-averaged value \([15]\). The coolant flows, which are used to keep the vane and rotor metal temperatures enough below melting so the material properties have acceptable values, are injected into the main gas path and exchange heat with the hot flow exiting the combustor. In this thesis we examine the turbine efficiency changes associated with gas path heat transfer and address the connection between entropy generation and turbine efficiency.

In Denton’s classic paper on loss mechanisms \([5]\), he makes the connection between losses in turbomachinery and increases in entropy; an increase in entropy from heat transfer between fluid streams with a finite temperature difference results in lost work and an efficiency decrease. In the same paper, however, Denton includes an model of a cooled turbine where the entropy generated from heat transfer between coolant streams and the main gas path flow has no impact on turbine efficiency. It is this issue on which we focus.

In a high pressure turbine, the entropy generated from gas path heat transfer

\(^1\)In this thesis, gas path heat transfer refers to the heat transferred streamtube-to-streamtube in the main gas path of a turbomachine.
can be of the same order as that generated from viscous dissipation. Jedamski [13] assessed the change in efficiency of a single stage turbine with increasing hot streak strength. He found that the efficiency change between a turbine with uniform inlet stagnation temperature and a turbine with a non-uniform inlet stagnation temperature was sensitive to the choice of efficiency definition. If a definition based on the availability averaged stagnation pressure was used, in which all increases in entropy contribute to loss, a turbine with an OTDF\(^2 = 0.6\) hot streak has 6% lower efficiency than one with uniform inlet. Jedamski argues that efficiency should instead be based on a mixed-work definition, using a work-averaged inlet stagnation pressure and a mixed-out average exit stagnation pressure for which the efficiency decrease due to the hot streak is less than 1%.

Lim et al. [17] assessed turbine loss associated with injecting film cooling air into the main gas path and found that turbine efficiency decreases by 8.0% if all entropy generation is considered a loss. They advocate adopting a “pragmatic approach” where irreversibility associated with heat transfer is neglected, however, and the decrease in turbine efficiency is 0.7%.

Young and Horlock [23] described several definitions for cooled turbine efficiency. They argue that the fully reversible mixed efficiency should be used, in which the actual turbine power is compared to the maximum power that could possibly be extracted from the flow. In such an ideal process, Carnot cycles are used to reversibly bring all inlet streams to a common stagnation pressure and stagnation temperature, after which the streams are mixed and expanded through an isentropic turbine. This definition is similar to the “rational efficiency” based on availability, and it also accounts for the increase in work possible when gases with different specific heat ratio are mixed. From this perspective, all irreversibility results in a loss of turbine efficiency [23].

Another ideal process was also proposed by Young and Horlock [23] to determine a weighted-pressure mixed efficiency. That efficiency considers thermal mixing of inlet streams to be inevitable, and recognizes that in aero engine turbines work cannot be

\[ \text{OTDF} = \frac{\max(T_{t,4}(r, \theta)) - T_{t,1}}{T_{t,1} - T_{t,3}} \]

2OTDF is a measure of temperature distortion and is defined as \(\frac{\max(T_{t,4}(r, \theta)) - T_{t,1}}{T_{t,1} - T_{t,3}}\).
extracted from the temperature difference between streams. The ideal process used is thus irreversible and includes the "minimum practical thermal entropy production." Only if the change in entropy is greater than the minimum practical thermal entropy production does lost work result.

To analyze cooled turbines, Hartsel [10] used an efficiency where the actual turbine work is compared to an ideal process where each inlet stream undergoes separate isentropic expansion to the exhaust static pressure. The ideal turbine work is given by the sum of the kinetic energies of each stream following expansion to the exhaust pressure. No explicit link between gas path heat transfer and changes in turbine efficiency was given.

Miller [19] has provided a rigorous definition for turbine efficiency based on the concept of mechanical work potential, defined as the turbine work possible when expanding a fluid isentropically to a dead state pressure with zero velocity. In this framework the actual turbine work is compared to the sum of the changes in mechanical work potential of each stream, which is equivalent to the turbine work obtained by expanding isentropically from the inlet state to the exit state. This efficiency definition is equal to the one proposed by Hartsel [10], and if the dead state pressure is taken equal to the turbine exit static pressure, the inlet mechanical work potential is equal to Hartsel's ideal work.

Miller [19] also showed that changes in mechanical work potential are possible by exchanging heat or work with the surroundings, and by two internal mechanisms: viscous dissipation and thermal creation. Only entropy generated from viscous dissipation contributes to lost turbine work. The effect of streamtube-to-streamtube heat transfer on turbine efficiency is captured by thermal creation (investigated in detail in Chapter 3), which can be positive or negative depending on the angle between the local static pressure gradient and the heat flux vector. Thermal creation can result in a mechanical work potential efficiency, or equivalently, Hartsel efficiency, that exceeds unity.
1.1 Research Questions

The goal of this work is to quantify the changes in turbine efficiency from gas path heat transfer. The selection of an appropriate metric to characterize turbine loss and the application of mechanical work potential to adiabatic high pressure turbine flows are addressed. The specific research questions are:

- What is the correct loss metric for aero engine high pressure turbines and what are the implications of the choice?
- Under what conditions is thermal creation significant compared to turbine work?
- How large is thermal creation in the main gas path of a high pressure turbine?
- What is the effect of inlet hot streaks and coolant flows on thermal creation in a high pressure turbine?

1.2 Methodology

To address the research questions, a combination of model problems and three-dimensional RANS CFD has been carried out. Analysis of a constant area mixing duct with three different downstream processes is used to highlight the importance of selecting the appropriate loss metric. Discussion of ideal processes relevant for aero engine turbines aids in this selection. Analysis of the inviscid heat exchanger proposed by Miller [19] isolates the effect of heat transfer on turbine work, and serves to help explain the thermal creation mechanism.

Thermal creation is evaluated in a high pressure turbine stage and compared to viscous dissipation and turbine work. The effect of inlet hot streaks on thermal creation is defined for an idealized hot streak with OTDF up to 0.6. The effect of a more representative turbine inlet profile, obtained from combustor CFD, on thermal creation is then examined for a two-stage high pressure turbine. The thermal creation due to the addition of cooling flows to the main gas path is also evaluated and compared with the other mechanisms for turbine loss.
1.3 Contributions

The main contributions of this work are:

- Miller's argument [19] that mechanical work potential should be used when analyzing aero engine turbines instead of approaches based on entropy, i.e. availability, is supported by several different model problems including gas path heat transfer. An example of thrust increase from mixing upstream of a nozzle is used to demonstrate the importance of using different loss metrics for fluid components depending on the purpose of the overall system.

- Changes in turbine efficiency due to thermal creation of ±10% are found for an inviscid heat exchanger with static pressure differences or specific heat ratio differences. The efficiency change is caused by a redistribution of the enthalpy of the flow between regions of differing Brayton efficiency [20] based on static pressure.

- The impact of thermal creation on turbine efficiency for a representative uncooled high pressure turbine stage is determined to be less than 0.01%, regardless of inlet stagnation temperature non-uniformity. With or without hot streaks, the highest local thermal creation is due to the rotor tip leakage flow.

- The impact of thermal creation on efficiency for a representative two-stage, cooled high pressure turbine is determined to be -0.1%. Adding coolant flow to the main gas path approximately doubles the change in efficiency from thermal creation in the first stage. Even with coolant flow, thermal creation comprises only 1% of the overall lost work in the high pressure turbine.

- The sensitivity of thermal creation to inlet turbulence has been shown. Using a turbine inlet profile taken from combustor CFD at engine operating conditions, with inlet turbulent to laminar viscosity ratio of 1,000, the thermal creation is approximately 10 times higher than when an inlet boundary condition with turbulent viscosity ratio of 100 is used.
1.4 Organization of Thesis

The first part of this thesis supports the use of mechanical work potential and thermal creation [19] to quantify the effect of gas path heat transfer on turbine loss. In Chapter 2, an analysis of mixing with three different processes occurring downstream is used to demonstrate the importance of selecting the correct loss metric. One process, thrust augmentation from mixing, shows a situation in which, even though there can be an entropy increase, the mixing can lead to enhanced performance (thrust). In Chapter 3, the mechanism of thermal creation, is investigated for an inviscid heat exchanger to show how heat transfer between streams with differing static pressure or specific heat ratio leads to changes in turbine work.

The second part of the thesis quantifies changes in turbine efficiency due to gas path heat transfer. In Chapters 4 and 5, mechanical work potential analysis is applied to three-dimensional turbine flows. The impact of thermal creation on efficiency is found for an uncooled high pressure turbine stage and for a cooled two-stage high pressure turbine. The impact of inlet turbulence and of coolant flows on thermal creation is assessed. Mechanisms for thermal creation in three-dimensional flows are discussed, and regions of the flows with highest thermal creation are identified.

Chapter 6 provides a summary of the key results from Chapters 2-5 and provides recommendations for future work on turbine efficiency changes due to heat transfer.
Chapter 2

Loss Assessment Methodologies

2.1 Introduction

A rigorous loss quantification is necessary to analyze and mitigate mechanisms that reduce system performance. As described by Cumpsty and Horlock, there are a number of choices for loss metrics in internal flow [4] that arise from comparing the real process to one that is idealized in some way. One choice is to consider a reversible process as the ideal, but other choices can be made in which irreversibility is included [23]. It is well known that an upper bound on loss exists, given by availability in the sense of the maximum amount of lost work for a given passage from one state to another [12] [14]. However, such a metric is not useful in many situations, including aero engine high pressure turbine flows [19], in which there is no opportunity for reversible heat exchange downstream of the process of interest. In what follows we demonstrate this by evaluating the mixing loss for two streams in a constant area duct with different downstream processes.

2.2 Mixing Duct Model

The model problem to be described is constant area mixing of two co-flowing streams as in Figure 2-1. This allows us to show how the interpretation of loss for the same physical process depends on the overall purpose of the system. The flow has two
streams of an ideal gas with stagnation pressures $p_{t,1}$ and $p_{t,2}$, stagnation temperatures $T_{t,1}$ and $T_{t,2}$, and areas $A_1$ and $A_2$ that enter an adiabatic, constant area duct. To focus on the mixing process we neglect wall shear and assume that streamline curvature induced by mixing is negligible so the pressure across the mixing duct is uniform. The flow mixes between station $i$ (inlet) and station $e$ (exit) with a uniform state at $e$ that has stagnation pressure $p_{t,e}$, stagnation temperature $T_{t,e}$, and Mach number $M_e$.

![Figure 2-1: Constant area mixing of two streams to a uniform exit state [5].](image)

### 2.2.1 The Mixed-Out State

Conditions at the mixed-out state can be found using a control volume from mixing duct inlet to mixing duct exit and applying the one-dimensional (1D) conservation equations of mass, momentum, and energy for a steady flow in an adiabatic constant area duct with wall shear neglected [5]. The mixed-out mass flow is the sum of inlet mass flows. Conservation of energy means that the mass flux of stagnation enthalpy at the inlet and exit are the same, so, the mixed out stagnation temperature is determined. The mixed-out Mach number is found from the 1D momentum equation. The mixed-out stagnation pressure and entropy are then given from the mass flow rate, stagnation temperature, area, and Mach number.

### 2.2.2 Boundary Conditions

We follow the procedure of Denton [5] to set the non-dimensional boundary conditions. In the examples to be worked out, we take the areas of the two streams to be equal. The inlet stagnation temperature boundary condition is set by choosing $DT$, the
 stagnation temperature difference between the two inlet streams non-dimensionalized by $\overline{T}_t$, the arithmetic average of the stagnation temperatures (Equations 2.1 and 2.2),

$$DT = \frac{T_{t,1} - T_{t,2}}{\overline{T}_t}, \quad (2.1)$$

$$\overline{T}_t = \frac{1}{2}(T_{t,1} + T_{t,2}). \quad (2.2)$$

The inlet stagnation pressure boundary condition is given by $DP$, the inlet stagnation pressure difference normalized by the difference between the arithmetic average inlet stagnation pressure $\overline{p}_i$ and the mixing duct inlet static pressure (Equations 2.3 and 2.4),

$$DP = \frac{p_{t,1} - p_{t,2}}{\overline{p}_i - p_i}, \quad (2.3)$$

$$\overline{p}_i = \frac{1}{2}(p_{t,1} + p_{t,2}). \quad (2.4)$$

The duct inlet static pressure and hence the Mach number of each stream is set by specifying a representative inlet Mach number, $\overline{M}_i$, based on the average stagnation pressure as in Equation 2.5,

$$\overline{p}_i = p_i \left(1 + \frac{\gamma - 1}{2} \overline{M}_i^2 \right)^{\frac{\gamma}{\gamma - 1}}. \quad (2.5)$$

Using Equations 2.3, 2.4, 2.5, and the total to static pressure ratio of each stream, the representative inlet Mach number is given as a function of the stream 1 and stream 2 Mach numbers in Equation 2.6,

$$\overline{M}_i = \sqrt{\frac{2}{\gamma - 1} \left[ \left( \frac{1}{2} \left( 1 + \frac{\gamma - 1}{2} M_1^2 \right)^{\frac{\gamma}{\gamma - 1}} + \frac{1}{2} \left( 1 + \frac{\gamma - 1}{2} M_2^2 \right)^{\frac{\gamma}{\gamma - 1}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]} \quad (2.6)$$

Given $\overline{M}_i$, stream inlet Mach numbers $M_1$ and $M_2$ can be found as a function of $DP$. With uniform duct inlet stagnation pressure ($DP = 0$) the Mach number in both streams is $\overline{M}_i$. Contours of $\overline{M}_i$ are provided in Figure 2-2 using $\gamma = 1.4$ and varying the individual stream Mach numbers from 0 to 0.75. The mixed-out state is
determined by specifying $DT$, $DP$, $\bar{M}_i$, and $\gamma$.

![Figure 2-2: The representative inlet Mach number, $\bar{M}_i$, as a function of stream 1 and stream 2 inlet Mach numbers, $M_1$ and $M_2$, for $\gamma = 1.4$.](image)

### 2.3 Mixing Loss Metrics

The mixing process can have different impacts on the overall system performance depending on the use of the flow downstream of the mixing region. The mixing duct will be considered here as a part of three different systems, and an appropriate loss metric will be defined for each system. The first class of problems considered is work extraction where both pressure and temperature differences between the fluid and the surroundings can be used to obtain work. An example is a co-generation power plant [12], where work is obtained from (i) a turbine expanding flow to the surrounding pressure, and (ii) downstream heat engines in which the flow can exchange heat with the surroundings. The second situation is one in which work can be obtained only
through a pressure difference between the fluid and surroundings. An example is an aircraft engine turbine, which generates work by expanding the flow to a lower pressure, but which does not provide an opportunity for generation of additional work through heat exchange with the surroundings. The third example is one in which the flow is used to produce thrust as in the flows immediately upstream of and internal to a propelling nozzle.

2.3.1 Availability

When the *maximum work that is possible* to extract from a flow for a given state change (or the minimum amount of work required) is of interest, the change in availability is the appropriate loss indicator [12]. Availability measures the maximum useful work that can be extracted from a flow undergoing a state change in a reversible process in which there is heat exchange with one or more heat reservoirs.

The availability for a steady flow can be defined from application of the first and second laws of thermodynamics to a control volume with heat addition and shaft work. Equation 2.7 states the first law for a control volume, where the shaft work output is equal to the heat input plus the decrease in stagnation enthalpy from state 1 to state 2,

\[ w_{\text{shaft}} = q + (h_{t1} - h_{t2}). \]  

(2.7)

For a fluid stream exchanging heat only with an environment at temperature, \( T_0 \), the change in entropy from state 1 to state 2 is given by the difference between (i) irreversible entropy generated inside the volume and (ii) heat rejected to the surroundings divided by the temperature of the surroundings\(^1\),

\[ (s_2 - s_1) - \frac{q}{T_0} = \Delta s_{\text{irreversible}} \geq 0. \]  

(2.8)

Combining Equations 2.7 and 2.8 yields an expression for shaft work as a function of the change in the quantity \( h_t - T_0 s \), from state 1 to state 2, and the irreversible

\(^1\)The boundary of the system at which the heat is exchanged is also at \( T_0 \).
entropy generated inside of the control volume multiplied by the temperature of the surroundings (Equation 2.9),

\[ w_{\text{shaft}} = (h_t - T_0 s)_1 - (h_t - T_0 s)_2 - T_0 \Delta s_{\text{irreversible}}. \]  

The quantity \( h_t - T_0 s \), denoted by the symbol \( b \), is defined as the flow availability [12],

\[ b = h_t - T_0 s. \]  

The irreversible entropy generation must be greater than or equal to zero, and Equation 2.9 thus means that maximum shaft work occurs when \( \Delta s_{\text{irreversible}} = 0 \), corresponding to a reversible process from states 1 to 2 and reversible heat transfer. The decrease in availability is equal to the shaft work extracted in this ideal process.

The difference between the shaft work extracted in the ideal process and the actual shaft work extracted for the change between state 1 and state 2 is the work lost due to irreversibility. Equation 2.11 expresses the actual shaft work in terms of the change in availability and the lost work, \( T_0 \Delta s_{\text{irreversible}} \),

\[ w_{\text{shaft}} = -\Delta b - T_0 \Delta s_{\text{irreversible}} = [w_{\text{shaft}}]_{\text{max}} - w_{\text{lost}}. \]  

To extract work reversibly between two states we can expand the flow from the initial pressure to the final pressure using an isentropic turbine, and then bring the flow to the final temperature using a Carnot cycle. For flow with uniform stagnation pressure and temperature this process is shown in an h-s diagram in Figure 2-3. The flow is first expanded from the inlet state, \( i \), to the isentropic turbine exit state, \( se \). It is then brought to the exit state, \( 0 \), using a series of Carnot cycles to exchange heat with the surroundings.

In the mixing duct there is no shaft work so, from Equation 2.11, the lost work is equal to the change in availability from inlet to exit. Further, because the mixing duct is adiabatic, entropy changes from inlet to exit must be due to irreversible entropy generation, and the entropy rise is proportional to the lost work. The entropy rise
Figure 2-3: An ideal process to obtain the maximum work (equal to the change in availability from state i to e) on an h-s diagram.

Coefficient in Equation 2.12 is a non-dimensional metric for entropy generation in the adiabatic mixing duct. It is defined by multiplying the change in entropy by the arithmetic average stagnation temperature \( T_{\text{av}} \) and dividing by a representative inlet specific kinetic energy \( \bar{u}_i^2 \) (Equation 2.13) based on a flow velocity defined by the arithmetic average stagnation temperature \( T_{\text{av}} \) and the reference inlet Mach number \( \bar{M}_i \).

\[
\Delta \bar{s} = \frac{T_{\text{av}}(s_e - s_i)}{\bar{u}_i^2}, \quad (2.12)
\]

\[
\bar{u}_i = \frac{\bar{M}_i \sqrt{\gamma RT_{\text{av}}}}{\sqrt{1 + \frac{\gamma - 1}{2} \bar{M}_i^2}}, \quad (2.13)
\]

Figure 2-4 shows the entropy rise coefficient for \( \bar{M}_i = 0.5 \) and \( \gamma = 1.4 \) as a function of inlet stagnation temperature difference and inlet stagnation pressure difference. Note that \( DT = 1 \) is equivalent to a stagnation temperature ratio \( T_{t,1}/T_{t,2} = 3 \).

For uniform inflow, there is no entropy change. As the stagnation pressure and stagnation temperature non-uniformities increase, the entropy generated in mixing tends to increase. With inlet stagnation temperature difference \( |DT| > 0.5 \), increas-
ing the inlet stagnation pressure difference can lead to lower entropy generation. This is due to the competition of the two processes for entropy generation: viscous dissipation, driven by differences in velocity, and internal heat transfer, driven by differences in static temperature. Increasing the stagnation pressure difference for positive \( DT \) decreases the static temperature difference and increases the velocity difference. For the higher values of \( DT \) shown, this trade-off leads to a decrease in mixed-out entropy.

![Figure 2-4: Entropy rise coefficient \( \Delta s = T_t(s_e - s_i)/\bar{u}_t^2 \) for two stream constant area mixing, with \( \bar{M}_t = 0.5 \) and \( \gamma = 1.4 \), as a function of stream inlet stagnation pressure and temperature differences.](image)

When there is a possibility for the system to take advantage of heat transfer with the surroundings to generate work, the change in availability is an appropriate loss benchmark and the lost work, proportional to irreversible entropy generation, is the appropriate loss metric.

### 2.3.2 Mechanical Work Potential

For situations in which no heat can be exchanged with the surroundings, it is not appropriate to use an ideal process which includes extracting work from the temperature
difference between system and surroundings for analysis of the maximum work that can be obtained. Put another way, the reversible processes used to define availability are not a useful benchmark for these situations. Following Miller [19], the ideal process for such adiabatic systems is taken to be work extraction through isentropic expansion to the pressure of the surroundings and zero velocity. The flow mechanical work potential per unit mass, $m_f$ [19], is defined in Equation 2.14. The mechanical work potential is a function of enthalpy $h$, pressure $p$, velocity $u$, and a chosen dead state pressure $p_0$, equal to the pressure of the surroundings which the fluid reaches at the end of the isentropic expansion,

$$m_f = h \left(1 - \left(\frac{p_0}{p}\right)^{\frac{k-1}{k}}\right) + \frac{u^2}{2}$$  \hspace{1cm} (2.14)

![Diagram showing the ideal process when using mechanical work potential analysis on an h-s diagram.](image)

Figure 2-5: The ideal process when using mechanical work potential analysis shown on an h-s diagram.

We can write Equation 2.14 for a perfect gas with constant specific heats in terms of stagnation quantities to give the flow mechanical work potential as the change in

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stagnation enthalpy from expansion to the isentropic exhaust state,

\[ m_f = c_p T_t \left( 1 - \left( \frac{P_0}{P_t} \right)^{\frac{\gamma-1}{\gamma}} \right) = h_t - h_{se}. \]  

(2.15)

This state change is indicated in an h-s diagram in Figure 2-5 by considering a fluid with initial state \( i \). A decrease in flow mechanical work potential represents a decrease in the isentropic work that can be obtained.

Figure 2-6 presents a second representation of the reversible ideal processes used to define mechanical work potential by modifying a commonly used representation of the availability ideal process. The work obtainable using a reversible Carnot cycle is not relevant to mechanical work potential; the ideal work only consists of work obtained by reversible processes in the control volume (i.e., with an isentropic turbine).

![Diagram](image)

Figure 2-6: The ideal process used to define availability [12], modified to show the mechanical work potential ideal process.

We can analyze the mixing duct problem using the decrease in mechanical work potential as the metric. A work potential loss metric, \( m_f \text{loss} \), is defined in Equation
2.16 as the change in mechanical work potential flux from mixing duct inlet to mixing duct exit normalized by the inlet mechanical work potential flux,

\[
(m_f)_{\text{loss}} = \frac{\int (m_f)_{\text{in}} \, dm - \int (m_f)_{\text{in}} \, dm}{\int (m_f)_{\text{in}} \, dm}.
\]  (2.16)

If we specify an ambient pressure \( p_0 \), and ratio of specific heats \( \gamma \), we can find the change in mechanical work potential for a control volume. For specified \( M_i \), \( \gamma \), and \( p_0 \), the loss in mechanical work potential is a function of the inlet stagnation temperature and stagnation pressure non-uniformities. The mechanical work potential loss, expressed as a percentage, is given in Figure 2-7 for \( M_i = 0.5 \), \( \gamma = 1.4 \), and \( p_0 = \frac{3}{4} p_i \).

![Figure 2-7: Loss of mechanical work potential [%] due to mixing for \( M_i = 0.5 \), \( \gamma = 1.4 \), and \( p_0 = \frac{3}{4} p_i \) as a function of inlet stagnation temperature difference and inlet stagnation pressure difference.](image)

Unlike loss based on availability, mechanical work potential loss does not always increase with increasing inlet stagnation temperature non-uniformity. Combinations of non-uniform stagnation temperatures and pressures (for example, points A and B in Figure 2-7) exist such that there is no mixing loss from the mechanical work potential metric, even though entropy is generated. For these conditions, the amount
of work that can be extracted from the flow is unchanged by the mixing process.

Miller [19] details the physical mechanisms that change the mechanical work potential. One mechanism is entropy generation from viscous dissipation, i.e., conversion of mechanical energy to internal energy. A second mechanism, termed thermal creation, arises when the heat flux vector and the static pressure gradient are non-orthogonal. For the mixing duct problem, the static pressure at the inlet face is assumed uniform and the static pressure gradient along the duct is not taken into account, so the change in mechanical work potential is due to viscous entropy generation only.

To estimate the viscous mixing losses, we examine the velocity difference between the streams at the inlet to the mixing duct. Figure 2-8 shows the velocity difference between the two streams at mixing duct inlet, normalized by the arithmetic average inlet velocity, as a function of inlet stagnation pressure non-uniformity and inlet stagnation temperature non-uniformity. As previously, $\overline{M_i} = 0.5$ and $\gamma = 1.4$. Points A and B are marked in the same locations as in Figure 2-7, and correspond to zero inlet velocity difference.

![Figure 2-8: Normalized inlet velocity difference $(u_1 - u_2)/\frac{1}{2}(u_1 + u_2)$ as a function of stagnation temperature and pressure difference for $\overline{M_i} = 0.5$ and $\gamma = 1.4.$](image-url)
The mechanical work potential loss observed in Figure 2-7 increases as the magnitude of the inlet velocity difference increases, suggesting that mechanical work potential losses are driven by inlet velocity differences. For matched inlet velocities \((u_1 = u_2)\), the mechanical work potential loss is exactly zero, even though the inlet stagnation pressure and stagnation temperature are non-uniform. In these cases, entropy is generated from streamtube-to-streamtube heat transfer between streams of differing static temperature, but there is no entropy generation from velocity mixing. There would be a loss calculated using an availability analysis, but there is no loss in terms of mechanical work potential.

### 2.3.3 Ideal Specific Gross Thrust

The third process is related to thrust generation in propulsion systems. For a flow that generates thrust by expanding through a propelling nozzle, the *ideal specific gross thrust* furnishes a useful loss metric. As in mechanical work potential analysis, the ideal process is isentropic, but no shaft work is extracted by the nozzle. The ideal gross thrust per unit mass flow, \(u_e^*\), is equal to the exit velocity attained from isentropic nozzle expansion to ambient pressure, \(p_0\), as in Equation 2.17,

\[
u_e^* = \sqrt{2c_pT_e \left( 1 - \left( \frac{p_0}{p_t} \right)^{\frac{\gamma - 1}{\gamma}} \right)}.
\]  

(2.17)

A loss metric can be defined as in Equation 2.18 as the change in the flux of ideal specific gross thrust from mixing duct inlet to mixing duct exit normalized by the flux of inlet ideal specific gross thrust,

\[
(u_e^*)_\text{loss} = -\frac{\int (u_e^*)_1 d\ln - \int (u_e^*)_e d\ln}{\int (u_e^*)_i d\ln}.
\]

(2.18)

For the same conditions used previously (\(\bar{M_i} = 0.5, \gamma = 1.4\), and \(p_0 = \frac{3}{4}p_t\)), the ideal specific gross thrust change, expressed as a percentage, is given for a range of inlet stagnation temperature and stagnation pressure non-uniformities in Figure 2-9.

Combining Equations 2.17 and Equation 2.15, we can write the ideal specific
Figure 2-9: Loss of ideal specific gross thrust [%] due to mixing for $M_i = 0.5$, $\gamma = 1.4$, and $p_0 = \frac{3}{4}p_t$. Negative loss indicates an increase in ideal specific gross thrust due to mixing.

gross thrust in terms of the mechanical work potential. This is done in Equation 2.19, which states that the exit kinetic energy per unit mass flow at the nozzle exit following isentropic expansion is equal to the work that can be extracted with an isentropic turbine,

$$u_e^* = \sqrt{2m_f}. \quad (2.19)$$

The ideal specific gross thrust is proportional to the square root of the mechanical work potential, but processes that lead to changes in ideal specific gross thrust do not necessarily have the same effect on changes in mechanical work potential. To maximize thrust, the exit momentum flux should be maximized, not the exit kinetic energy flux. In the mixing duct, work and heat exchange between two streams with different inlet mechanical work potential increases the mechanical work potential of one stream at the expense of the other. From Figure 2-7, this results in either no change or a net decrease in mechanical work potential. Work and heat exchange between two streams with different ideal specific gross thrust increases the ideal specific
gross thrust of one stream at the expense of the other, but the net effect can be to increase or to decrease the specific gross thrust.

An important result is that the contours of ideal specific gross thrust loss are qualitatively different from the contours of mechanical work potential loss. Most notably, the ideal specific gross thrust is increased due to mixing with large normalized stagnation temperature non-uniformity and small stagnation pressure non-uniformity. The thrust increases because, for given heat exchange, the exit velocity of the cold stream (receiving heat) increases more than the exit velocity of the hot stream (losing heat) decreases. This mechanism is responsible for the thrust increase provided by gas turbine engine mixer nozzles that mix cold fan flow and hot core flow [7] [15] [3].

Changes in ideal specific gross thrust are also caused by stagnation pressure losses, from both viscous dissipation and heat transfer. The contours of zero ideal specific gross thrust loss represent the loci of conditions at which the stagnation pressure losses balance the thrust increase from heat transfer.

2.4 Relation of Loss Metrics to Averaged Stagnation Pressures

To describe the loss for the three types of downstream processes, three different metrics were required. We can also illustrate the loss in terms of stagnation pressure loss due to mixing in the different situations. Cumpsty and Horlock [4] describe the selection of appropriate stagnation pressure averaging schemes and suggest scenarios in which each scheme is useful. They define four averaging methods which we use to compute stagnation pressure loss from mixing duct inlet to mixing duct exit. When the amount of work extracted with a non-adiabatic flow is of interest, they state that the availability-averaged stagnation pressure, defined in Equation 2.20, is the most appropriate choice,

\[
ln(p_{in}^a) = \frac{1}{m} \int ln(p_i) dbi - \frac{\gamma}{(\gamma - 1)m} \int ln(T_i/T_{in}) dbi.
\] (2.20)
The mass-averaged stagnation temperature in Equation 2.20, $T_t^{m}$, is defined as,

$$T_t^{m} = \frac{1}{m} \int T_t \, dm.$$  \hfill (2.21)

For work extraction in adiabatic flows, a work-averaged stagnation pressure, as defined in Equation 2.22, provides a meaningful indicator of loss,

$$p_{t,w}^{wa} = \left[ \frac{\int T_t \, dm}{\int T_t / \rho_t^{\gamma} \, dm} \right]^{\gamma-1}. \hfill (2.22)$$

A thrust-averaged stagnation pressure, as in Equation 2.23, can be used to represent a flow expanded through a propelling nozzle to atmospheric pressure $p_0$,

$$p_{t,i}^{ta} = p_0 \left[ 1 - \left( \frac{1}{m} \int \sqrt{\frac{T_t}{T_t^{m}} \left( 1 - \left( \frac{p_0}{\rho_t} \right)^{\frac{\gamma - 1}{\gamma}} \right)^2} \, dm \right)^{\frac{\gamma - 1}{\gamma}} \right]. \hfill (2.23)$$

Finally a mass-averaged stagnation pressure "is without rational basis" [4] but is commonly used to represent an entropy change in an adiabatic flow,

$$p_{t,i}^{m} = \frac{1}{m} \int p_t \, dm.$$  \hfill (2.24)

These four stagnation pressure averaging techniques have been applied to the mixing duct solution with $M_i = 0.5$, $\gamma = 1.4$, and $p_0 = \frac{3}{4} p_t$, to determine the stagnation pressure loss. The exit stagnation pressure is uniform, but the average stagnation pressure of the non-uniform duct inlet depends on the averaging technique applied. The change in stagnation pressure from $i$ to $e$ is normalized by the difference between the inlet stagnation and static pressures.

The stagnation pressure loss from mixing, using $p_{t,i}^{wa}$ to denote the inlet averaged stagnation pressure, is,

$$\left( p_t \right)_{loss} = \frac{p_{t,i}^{wa} - p_{t,e}}{p_{t,i}^{wa} - p_t}. \hfill (2.25)$$

The results are given in figures 2-10 to 2-13, which present contours of stagnation pressure loss, expressed as a percentage, for availability-averaging, work-averaging,
thrust-averaging, and mass-averaging:

- The *availability-averaged stagnation pressure loss* in Figure 2-10 and the entropy rise plotted in Figure 2-4 have qualitatively the same contour shapes.
- The *work-averaged stagnation pressure loss* in Figure 2-11 and the mechanical work potential loss in Figure 2-7 have qualitatively the same contour shapes. The magnitude of mechanical work potential loss is dependent on the specified downstream static pressure $p_0$, but the work-averaged stagnation pressure loss metric has no such dependency.
- The *thrust-averaged stagnation pressure loss* in Figure 2-12 and the ideal specific gross thrust loss in Figure 2-9 have qualitatively the same contour shapes. The magnitude of both ideal specific gross thrust loss and thrust-averaged stagnation pressure loss are dependent on $p_0$.
- The *mass-averaged stagnation pressure loss* plotted in Figure 2-13 is qualitatively different from the result of any of the three preceding situations. Mass-averaged stagnation pressure should not be applied as a metric for flows in any of the three categories described in the previous section.

Figures 2-10, 2-11, and 2-12 emphasize that the qualitative relationship between mixing loss and inlet non-uniformity is dependent on the downstream use of the flow. Selection of the appropriate analysis framework, or equivalently, the stagnation pressure averaging scheme, is critical to ensure that component performance is characterized in a way appropriate for the system it is a part of.

### 2.5 Gross Thrust Changes from Mixing

As demonstrated in the previous section, the ideal specific gross thrust can increase through mixing. We will use the example of thrust augmentation to demonstrate how the appropriate loss methodology can be utilized to inform design changes that increase system performance. To increase thrust from mixing, the nozzle exit area must be set to accommodate the mixed flow. If the nozzle exit area is unchanged
Figure 2-10: Availability-averaged stagnation pressure loss [%] due to mixing for $\bar{M}_i = 0.5$, and $\gamma = 1.4$.

Figure 2-11: Work-averaged stagnation pressure loss [%] due to mixing for $\bar{M}_i = 0.5$, and $\gamma = 1.4$. 
Figure 2-12: Thrust-averaged stagnation pressure loss [%] due to mixing for $\overline{M}_i = 0.5$, $\gamma = 1.4$, and $p_0 = \frac{3}{4}p_i$.

Figure 2-13: Mass-averaged stagnation pressure loss [%] due to mixing for $\overline{M}_i = 0.5$, and $\gamma = 1.4$. 
from that needed to pass an unmixed flow, there will be no increase in gross thrust; mixing out the non-uniform flow will increase the exit velocity, but the mass flow rate will decrease and there will be no increase in gross thrust. We show this in section 2.5.1 and 2.5.2.

2.5.1 Thrust Balance for Constant Area Nozzles

We consider a subsonic isentropic nozzle that produces thrust by expanding unmixed flow at station \( i \) to atmospheric pressure \( p_0 \). The stagnation pressures of stream 1 and 2 are equal \( (DP = 0) \), but the stagnation temperatures of the streams can differ. For the unmixed flow, the gross thrust is given by Equation 2.26,

\[
F = \dot{m}_1 u_{e,1} + \dot{m}_2 u_{e,2}.
\]  

(2.26)

The exit velocity for isentropic flow as a function of exit Mach number and stagnation temperature is,

\[
u_e = M_e \sqrt{\frac{\gamma RT_i}{1 + \frac{\gamma - 1}{2} M_e^2}}. 
\]  

(2.27)

The mass flow can be expressed in terms of area, Mach number, stagnation pressure, stagnation temperature, and gas properties,

\[
\dot{m}_e = \frac{D(M, \gamma) A_e p_t \sqrt{\gamma}}{\sqrt{RT_i}}.
\]  

(2.28)

The corrected flow function \( D(M, \gamma)^2 \) [8] is a function of Mach number \( M \) and ratio of specific heats \( \gamma \) and increases monotonically for subsonic flow \( (M < 1) \). The exit Mach number that matches pressure \( p_0 \) after expanding through an isentropic nozzle is given by Equation 2.29,

\[
M_e = \left[ \frac{2}{\gamma - 1} \left( \frac{p_t}{p_0} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right]^{1/2}.
\]  

(2.29)

\( ^2 D(M, \gamma) = M/(1 + \frac{\gamma - 1}{2} M^2)^{\frac{\gamma + 1}{2(\gamma - 1)}} \)
The exit Mach number is independent of stream stagnation temperature and is uniform for a flow with uniform inlet stagnation pressure. For a given stagnation pressure, the stream exit velocity is proportional to the square root of the stream stagnation temperature,

\[ u_c \propto \sqrt{T_i}. \quad (2.30) \]

The Munk and Prim Substitution Principle states that for a given upstream stagnation pressure distribution in an inviscid, non-conducting fluid, the streamlines, and therefore the ratio of stream exit areas, will be independent of stagnation temperature [21]. For given nozzle exit area and stagnation pressure, the stream mass flow rate is thus inversely proportional to the stream stagnation temperature,

\[ \dot{m}_c \propto \frac{1}{\sqrt{T_i}}. \quad (2.31) \]

The gross thrust from each stream is equal to the exit velocity multiplied by mass flow and, is thus independent of stream stagnation temperature. The unmixed gross thrust is set by specifying inlet stagnation pressure distribution, atmospheric pressure, and nozzle contraction ratio.

Now suppose we have the same nozzle downstream of station e in Figure 2-1 that isentropically expands the mixed-out flow to atmospheric pressure \( p_0 \) with the same exit area. The stagnation pressure loss in the mixing duct scales approximately with inlet Mach number squared \( ((p_{t,i} - p_{t,e})/p_{t,i} \sim M_i^2) \). For small inlet Mach number such that \( M_i^2 << 1 \), the mixed-out stagnation pressure is approximately the same as the duct inlet stagnation pressure with Equations 2.30 and 2.31 applying to the overall flows. The gross thrust depends on only the stagnation pressure and area, which are unchanged.

When the stagnation pressure loss from mixing is non-negligible, the stagnation pressure of the flow entering the isentropic nozzle decreases, and from Equation 2.29, the exit Mach number also decreases. From Equations 2.27 and 2.28, the nozzle exit velocity and mass flow decrease, leading to a decrease in gross thrust.

Figure 2-14 shows the gross thrust from nozzle expansion after mixing, divided by
the gross thrust from unmixed expansion, as a function of inlet stagnation temperature difference and inlet Mach number, with nozzle expansion ratio $p_0/p_{t,i} = 3/4$.

![Figure 2-14: Gross thrust with mixing, normalized by unmixed gross thrust, for a fixed area subsonic nozzle with uniform inlet stagnation pressure, $p_0 = \frac{3}{4}p_{t,i}$, and $\gamma = 1.4$.](image)

The results shown in Figure 2-14 contrast with the results for ideal specific gross thrust in Figure 2-9 because although the specific gross thrust (exit velocity) increases, the mass flow rate has decreased. To realize the benefits of mixing, the nozzle area needs to be modified.

### 2.5.2 Thrust Increase for Constant Mass Flow Nozzles

We now allow the exit nozzle area to vary so the mass flow in the mixed-out case is equal to that in the unmixed case. For constant mass flow, changes in exit velocity are proportional to changes in gross thrust. For $M_i^2 << 1$, where we can neglect stagnation pressure changes from mixing, the exit velocity is proportional to the square root of stagnation temperature. When the two fluid streams reach the mixed-out stagnation temperature, the velocity increase from raising the stagnation
temperature of the cold stream is larger than the velocity decrease from lowering the stagnation temperature of the hot stream, resulting in an increase in ideal specific gross thrust. An analytic expression for the change in gross thrust due mixing two low Mach number streams with constant mass flow is given in [9], see Appendix A for derivation. The analytic, low Mach number result is compared with calculations for inlet Mach numbers up to 0.5 in Figure 2-15 for equal inlet areas, uniform inlet stagnation pressure, nozzle expansion ratio $p_0/p_{t,i} = 3/4$, and $\gamma = 1.4$.

![Figure 2-15: Gross thrust with mixing, normalized by unmixed gross thrust, for a fixed mass flow, variable area subsonic nozzle with uniform inlet stagnation pressure, $p_0 = \frac{3}{4}p_{t,i}$, and $\gamma = 1.4$. Comparison of low Mach number analysis and numerical evaluation for inlet Mach numbers up to 0.5.](image)

There is a competition between thrust augmentation from mixing and mixing loss. At low inlet Mach number the increase in thrust from mixing dominates the effect of stagnation pressure loss, with a nearly 4% increase in thrust for $DT = 1$. As the inlet Mach number is increased, viscous losses become significant and the thrust increase at $DT = 1$ is reduced to below 1.5%. 

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2.6 Summary and Conclusions

There are different loss metrics for fluid components depending on the downstream use of the flow. For a component that is part of a system that can use heat transfer from the exhaust stream to create work, an availability analysis is appropriate; in terms of availability, irreversible entropy generation from internal heat transfer and viscous dissipation both lead to loss. Entropy generated from internal heat transfer lowers the work that can be extracted using a downstream heat engine [19] [12], and altering the component design to reduce the source of entropy generation increases system performance.

For a component that is part of a system in which heat transfer with the surroundings cannot result in work, mechanical work potential is the appropriate metric. This includes components in aero engines, where there are “no downstream Carnot cycles” [6] and, the entropy generated due to internal heat transfer is irrelevant to loss. The path to reduce loss is (i) minimizing viscous dissipation and (ii) utilizing thermal creation by transferring heat from low static pressure regions to high static pressure regions.

For mixing upstream of thrust-producing nozzles, an ideal specific gross thrust metric should be used. In this framework, heat transfer between parts of the flow with different stagnation temperatures can either increase or decrease thrust, depending on viscous losses.
Chapter 3

Inviscid Model for Turbine Work
Changes from Heat Transfer

3.1 Introduction

As described in Chapter 2, in analysis of aero engine turbines the mechanical work potential framework should be used. Through the ‘lens’ of mechanical work potential it is only irreversible increases in entropy due to viscous dissipation that contribute to loss. Entropy generated from gas path heat transfer does not contribute to loss, although it can change mechanical work potential through the effect of thermal creation as described in this chapter.

Viscous dissipation is well understood [5], but the role of thermal creation is a more recent finding [19]. The large thermal gradients present in the gas turbine, due to coolant flows and hot streaks, create a potential for substantial gas path heat transfer. Depending on the heat transfer processes, thermal creation from gas path heat transfer can cause turbine work output to increase, decrease, or remain unchanged. In this chapter we demonstrate that the ability of heat transfer to change turbine work is due to the redistribution of enthalpy between regions of the flow that have differing isentropic Brayton thermodynamic efficiency. Heat transfer accompanied by a difference in static pressure or a difference in specific heat ratio changes the work that can be extracted with a turbine. We present both numerical examples and con-
ceptual guides to illustrate the magnitude of changes in turbine work from thermal creation.

### 3.2 Low Mach Number Heat Transfer

To quantify the changes in turbine work due to internal heat transfer, we present results from analysis of a model problem. As first proposed by Miller [19], a frictionless, co-flow heat exchanger can be used to isolate the effect of heat transfer on turbine work. The physical situation considered is presented in Figure 3-1. Two streams of perfect gasses enter the heat exchanger with given stagnation pressure, stagnation temperature, Mach number, and specific heat ratio. The gas constant, $R$, for the two streams is the same and the area ratio of the two streams is set so the mass flows are equal. In this section (3.2), the stream areas are constant over the length of the heat exchanger. The heat exchanger is inviscid, insulated from the surroundings, and long enough that the streams reach an equilibrium static temperature at the exit.

In addition, for this section the inlet Mach numbers are taken as $M^2 << 1$, so the heat transfer can be considered to occur at constant pressure. Downstream of the heat exchanger, each stream is expanded through an isentropic turbine to the same specified exhaust static pressure, $p_0$. The kinetic energy of the streams at the turbine exit are taken as negligible. The two streams are not mixed, so the effects of mixing streams with different specific heat ratio, identified by Young and Horlock [23], are not considered. The conditions at the heat exchanger exit state are found by applying the one-dimensional continuity, momentum, and energy equations.

The isentropic expansion of each stream to a specified static pressure with zero velocity matches the ideal process in the mechanical work potential. In each stream, the turbine power per unit mass flow extracted downstream of the heat exchanger is equal to the mechanical work potential at the heat exchanger exit. The inlet flow mechanical work potential is equal to the turbine power per unit mass flow when the heat exchanger is bypassed and the flow is directly expanded from the inlet state to the exhaust state. The change in turbine work due to the heat exchanger is equivalent
Figure 3-1: Inviscid heat exchanger upstream of isentropic turbines exhausting to the same static pressure, $p_0$.

The isentropic turbine power per unit mass flow $w$, or equivalently the mechanical work potential, can be expressed as a function of the static pressure $p$, static enthalpy $h$, velocity $u$, specific heat ratio $\gamma$, and exhaust pressure $p_0$, as in Equation 3.1,

$$w = h \left( 1 - \left( \frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} \right) + \frac{u^2}{2}. \tag{3.1}$$

An increment of enthalpy added to the flow upstream of the turbine expansion increases the work that can be extracted. The additional work, normalized by the change in enthalpy of the flow, is equal to the Brayton efficiency based on static pressure,

$$\frac{\Delta w}{\Delta h} = 1 - \left( \frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}}. \tag{3.2}$$

Equation 3.2 is the Brayton thermodynamic efficiency between local static pressure $p$ and exhaust pressure $p_0$. If heat is transferred between streams with different local Brayton efficiency, changes in net turbine work are possible. Miller [20] was the first to recognize the link between changes in work and stream Brayton efficiencies based
on static pressure.

3.2.1 Entropy Generation with No Change in Turbine Work

As in the mixing duct example of Chapter 2, there are conditions for the heat exchanger where entropy is generated but mechanical work potential, and thus net turbine work, remains unchanged. If so, the entropy generated from heat transfer is not useful as a loss metric. From a mechanical work potential framework, the only role of internal heat transfer is to redistribute the enthalpy of the flow. Heat transfer that occurs between two streams of equal static pressure and specific heat ratio has no effect on the turbine work that can be extracted.

An h-s diagram for heat transfer between streams of equal mass flow, static pressure, and specific heat ratio is presented in Figure 3-2. The isentropic turbine work, neglecting kinetic energy at the inlet and exit, is given by the change in enthalpy from isentropic expansion to the exhaust pressure $p_0$. The isentropic turbine work that could be realized without heat transfer is represented by the dashed blue lines at the inlet temperatures $T_1$ and $T_2$. Suppose that heat transfer occurs from stream 1 to stream 2 until both streams reach a common temperature $T_e$, and the streams are then expanded isentropically to the exhaust pressure (dashed red line at $T_e$). The work obtained from expansion of stream 1 decreases, but the decrease in work is exactly matched by the increase in work obtained from isentropic expansion of stream 2. Entropy is generated but turbine work is unchanged [19].

3.2.2 Heat Transfer with Static Pressure Difference

When the two heat exchanger streams have different static pressures, net changes in turbine work are possible. Figure 3-3 presents contours of change in isentropic turbine work due to heat transfer, normalized by the isentropic work without heat transfer, as a function of stream inlet stagnation temperature difference and inlet stagnation pressure difference. The inlet stagnation temperature is normalized by the arithmetic average stagnation temperature as in the previous chapter. The stagnation pressure
difference is normalized by the arithmetic average stagnation pressure. The stream inlet Mach numbers, mass flow rates, ratio of specific heats, and specific gas constants are equal and the exhaust pressure is set to 1/4 of the stream 1 inlet stagnation pressure.

There is no change in work due to heat transfer at $p_{t,2i} = p_{t,1i}$, where stream stagnation pressures, Mach numbers, and static pressures are equal throughout the heat exchange. With high amounts of heat transfer and large differences in pressure, changes in work on the order of 1%-10% are possible for the range of parameters shown. Defining the nature and magnitude of these effects in practical situations is important, and will be the focus of Chapter 4.

Figure 3-4 shows an h-s diagram of heat transfer between two streams with a static pressure difference and equal specific heat ratio. As in Figure 3-2, isentropic work before heat transfer is the sum of the change in enthalpy for isentropic expansions (dashed blue lines at $T_{1i}$ and $T_{2i}$). Heat transfer occurs at constant pressure between the streams until the streams reach the same temperature. The ideal work at the exit of the heat exchanger is again the sum of the change in enthalpy for isentropic
expansions, starting from $T_e$ (shown by the dashed red lines). The net work after heat transfer is larger than the net work before heat transfer occurs. The work obtained from stream 1, which gives up heat to stream 2, decreases, but the work obtained from stream 2, which receives heat and is higher pressure, increases more than the work from stream 1 decreases. Stream 2, at higher pressure than stream 1, can convert heat into work more efficiently.

By transferring heat from a stream at lower pressure to a stream at higher pressure, and then expanding the streams, more turbine work is extracted from the flow than if no heat transfer occurs. If the heat transfer went the other way, from a high pressure to a lower pressure stream, the heat transfer would decrease the turbine work. Whether heat transfer increases or decreases work, heat transfer with a finite temperature difference increases the entropy of the system and decreases availability. However, the entropy generated by this internal heat transfer does not affect the ideal turbine work.
3.2.3 Heat Transfer with Specific Heat Ratio Difference

Heat transfer without a static pressure difference can also lead to changes in turbine work if there is a difference in the ratio of specific heats, $\gamma$. For an isentropic expansion across a given pressure ratio, the specific heats ratio sets the inlet to exit temperature ratio. Expanding a gas with higher $\gamma$ thus increases the isentropic turbine work that can be extracted for the same pressure ratio.

Figure 3-5 presents changes in work due to heat transfer, normalized by the work with no heat transfer, as a function of normalized specific heat ratio difference and inlet stagnation temperature difference for equal pressure streams. The average specific heat ratio is constant at $\bar{\gamma} = \frac{1}{2}(\gamma_1 + \gamma_2) = 1.3$. As in the previous situation of heat transfer with a pressure difference, the exhaust pressure is $1/4$ of the stream 1 inlet stagnation pressure. With equal specific heat ratio and equal pressure, there is no change in work due to heat transfer, but there can be either a positive or negative change in work from heat transfer due to the difference in specific heat ratio.

Figure 3-6 shows an h-s diagram for heat transfer between two streams at equal
static pressure but different specific heat ratio. To more easily illustrate the changes in work possible from heat transfer between streams with different specific heat ratio, consider streams with different gas constant $R$ but equal specific heat $c_p$. At the end of heat exchange, when the streams have the same temperature, both streams will have the same enthalpy. The two gases are assumed to have the same reference entropy, $s_{\text{ref}}$, at reference temperature, $T_{\text{ref}}$, and reference pressure, $p_{\text{ref}} = p_0$. The isobars on this h-s diagram are dependent on static pressure and specific heat ratio, and because of the choice of $p_{\text{ref}} = p_0$, there are three isobars corresponding to the two pressures.

As in previous figures, the red dashed lines show the isentropic turbine work available before heat transfer, and the blue dashed lines represent the isentropic turbine work after heat transfer. Heat transfer from lower $\gamma$ to higher $\gamma$ increases the turbine work; the increase in work obtained from stream 1, with higher $\gamma$, is greater than the decrease in work obtained from stream 2, with lower $\gamma$. When heat transfers to regions of higher specific heat ratio the transferred enthalpy can be more efficiently converted to turbine work.

Figure 3-5: Change in work due to heat transfer with specific heat ratio difference. $\frac{p_{1i}}{p_0} = 4$, $p_1 = p_2$, $\bar{\gamma} = 1.3$, $M_{1i} = M_{2i} << 1$, $\dot{m}_1 = \dot{m}_2$.
Figure 3-6: h-s diagram for heat transfer at constant static pressure between streams with a difference in specific heat ratio and equal static pressure.

### 3.3 Constant Pressure Heat Exchanger

For Mach numbers that are not negligible compared to unity we need to account for the changes in static pressure of each stream due to heat transfer. For heat transfer to a stream, the static pressure varies with area change and stagnation temperature as in Equation 3.3 [8],

\[
\frac{dp}{p} = \frac{\gamma M^2}{1 - M^2} \frac{dA}{A} - \frac{\gamma M^2 (1 + \frac{\gamma - 1}{2} M^2)}{1 - M^2} \frac{dT_t}{T_t} \tag{3.3}
\]

For \( M^2 << 1 \) and constant area, as in previous calculations, there is negligible change in pressure with heat transfer, but as the inlet Mach number increases, heat transfer to a subsonic stream decreases the static pressure. To examine the effect of these static pressure differences we consider a situation where static pressure is held constant by varying stream area to maintain the inlet static pressures. Flow quantities along the heat exchanger can be found from integrating the influence coefficients for one dimensional compressible flow [8].
If the changes in isentropic turbine work are normalized by the heat transferred instead of by the work with no heat transfer, the contour plot in Figure 3-3 becomes a function of static pressure difference only [20] and the contour plot in Figure 3-5 becomes a function of specific heat ratio difference only. Following Equation 3.2, the net change in work normalized by the heat transferred is equal to the difference in Brayton efficiency between the two streams.

The change in work due to heat transfer normalized by the heat transferred is shown as a function of inlet static pressure ratio for several inlet Mach numbers in Figure 3-7. For inlet Mach numbers ranging from 0.01 to 0.9, there is a small sensitivity of changes in work to inlet Mach number. As inlet Mach number increases, larger changes in work are achieved for the same amount of heat transfer at the same stream pressure difference.

![Figure 3-7: Change in turbine work normalized by heat transferred for a constant pressure heat exchanger with inlet pressure difference, $\frac{p_{i1} - p_{i2}}{\frac{1}{2}(p_{i1} + p_{i2})}$](image)

For constant inlet stagnation pressure and exit static pressure, increasing the inlet Mach number decreases the inlet static pressure and the inlet static pressure to turbine exhaust static pressure ratio. As inlet Mach number increases, the Brayton efficiency of streams 1 and 2 decrease with decreasing $p/p_0$. However, the low efficiency stream
decreases in efficiency more than the high efficiency stream, so the difference between the two efficiencies increases and larger changes in work result.

The decrease of \( \frac{p}{P_0} \) with increasing inlet Mach number is also responsible for the Mach number sensitivity in the case of heat transfer between two streams with a specific heat ratio difference. As Mach number increases, the changes in work due to heat transfer with a difference in specific heat ratio become smaller, as seen in Figure 3-8. The high efficiency stream decreases in efficiency more than the low efficiency stream, and the difference in efficiency between the two streams diminishes.

![Figure 3-8](image)

**Figure 3-8:** Change in turbine work normalized by heat transferred for a constant pressure heat exchanger with specific heat ratio difference, \( \frac{p_{1i}}{p_0} = 4, p_1 = p_2, \gamma = 1.3, M_{1i} = M_{2i}, \dot{m}_1 = \dot{m}_2. \)

### 3.4 Constant Area Heat Exchanger

When terms \( O(M^2) \) are not negligible, the static pressure will vary along a constant area duct with heat transfer, and normalizing the change in isentropic work due to heat transfer by the heat transferred does not remove the dependency on inlet stagnation temperature difference. Figure 3-9 presents the changes in isentropic turbine work from heat transfer, normalized by work with no heat transfer, as a function of
inlet stagnation pressure difference and inlet stagnation temperature difference for an inlet Mach number of 0.3. Each stream has equal mass flow and ratio of specific heats of 1.3.

Comparing Figure 3-9 to Figure 3-3, for which \( M^2 << 1 \), we see that increasing inlet Mach number lowers the turbine work extracted downstream of the heat exchanger. The effect of lowering \( p/p_0 \) with increasing inlet Mach number observed in the constant pressure heat exchanger case is still present, but a second effect, from the change in static pressure with heat transfer, also exists. As noted by Miller [19], flow mechanical work potential is decreased due to the reduction of static pressure as heat is added.

![Figure 3-9](image)

**Figure 3-9:** Change in work from heat transfer with pressure difference, inlet Mach number = 0.3 in constant area heat exchanger. \( p_1/p_0 = 4 \), \( \gamma_1 = \gamma_2 = 1.3 \), \( M_{1i} = M_{2i} = 0.3 \), \( \dot{m}_1 = \dot{m}_2 \).

We also observe a decrease in turbine work when inlet Mach number is increased for a constant area heat exchanger with equal inlet static pressure streams but different specific heat ratios. Figure 3-10 shows the change in work due to heat transfer, normalized by the work obtained with no heat transfer, for streams of equal inlet static pressure as a function of inlet stagnation temperature difference and specific heat ratio difference. The conditions are the same as in Figure 3-5, except that the
inlet Mach number is 0.3.

![Graph showing the change in work from heat transfer with specific heat ratio difference, inlet Mach number = 0.3 in constant area heat exchanger.](image)

Figure 3-10: Change in work from heat transfer with specific heat ratio difference, inlet Mach number = 0.3 in constant area heat exchanger. $\frac{\dot{m}_{1i}}{\rho_0} = 4$, $p_{t,1i} = p_{t,2i}$, $\gamma = 1.3$, $M_{1i} = M_{2i} = 0.3$, $\dot{m}_1 = \dot{m}_2$.

Thermal creation is due to heat transfer between regions of a flow with differing local Brayton efficiency, and we can examine the Brayton efficiency to illustrate the effect of Mach number on turbine work. Figure 3-11 and Figure 3-12 show the contours of Brayton efficiency based on static pressure (Equation 3.2) plotted as a function of specific heat ratio $\gamma$ and local static-to-exhaust pressure ratio $p/p_0$. Figure 3-11 presents the case of heat transfer from stream 1 with 20% Brayton efficiency to stream 2 with 30% Brayton efficiency. Heat transfer between the two streams increases the turbine work that can be extracted from the flow. However, as heat is transferred at non-negligible, subsonic Mach number, the static pressure of the stream gaining heat drops and the static pressure of the stream losing heat rises. Stream 1 increases in efficiency, and stream 2 decreases in efficiency with conditions moving from 1 to 1' and 2 to 2'. The benefit of increased turbine work is therefore reduced compared to the heat transfer with low Mach number, i.e., the gain in turbine work from heat transfer is reduced as inlet Mach number increases in a constant area heat exchanger.

Figure 3-12 shows the case of heat transfer from stream 1 with 30% Brayton
Figure 3-11: Contours of Brayton efficiency as a function of pressure ratio and $\gamma$. Changes in Brayton efficiency of streams 1 and 2 due to heat transfer from low efficiency to high efficiency in a constant area heat exchanger with non-negligible inlet Mach number are tracked.
efficiency to stream 2 with 20% Brayton efficiency. The efficiency of stream 1, which loses heat, increases (moving from 1 to 1’) while the efficiency of stream 2, which gains heat, decreases (moving from 2 to 2’). As a result of the changes in static pressure, the efficiency difference between the streams is larger and the heat transfer leads to a larger decrease in work. In this model problem, the increasing inlet Mach number effects the changes in turbine work from heat transfer by changing the static pressure difference at which the heat transfer occurs.

![Figure 3-12: Contours of Brayton efficiency as a function of pressure ratio and γ. Changes in Brayton efficiency of streams 1 and 2 due to heat transfer from high efficiency to low efficiency in a constant area heat exchanger are tracked.](image)

### 3.5 Thermal Creation with Uniform Inlet Flow

A non-uniform inlet temperature is not required for thermal creation to occur as illustrated by the following example of thermal creation in an inviscid flow with a uniform inlet state. Streams expanded isentropically to different static temperatures can exchange heat even if the inlet state is uniform. A sketch of the physical situation considered is provided in Figure 3-13.

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Two streams enter a duct with equal stagnation pressure, stagnation temperature, Mach number, specific heat ratio, and area. Between stations A and B, stream 1 area increases from 1/2 to 3/4 of the total duct area, and stream 2 area decreases from 1/2 to 1/4 of the total duct area. The static pressure and static temperature vary isentropically from station A to B, resulting in a difference in static temperature and static pressure between the two streams. From station B to C, irreversible heat transfer occurs between the streams at constant area until the static temperatures equilibrate. The heat transfer occurs from low static pressure to high static pressure, so there is negative thermal creation. Between stations C and D, the stream areas return to their initial values, with static pressure and temperature varying isentropically. At station D, stream 2 (which received heat from stream 1) is at a higher static temperature than stream 1. From D to E, heat transfers irreversibly from stream 2 back to stream 1.

The changes in mechanical work potential for the situation described, with inlet Mach number $M_i = 0.3$ and specific heat ratio $\gamma = 1.3$, are given in Table 3.1. The reference static pressure used to define mechanical work potential was 1/2 of the inlet stagnation pressure. The isentropic processes occurring A-B and C-D do not change mechanical work potential. For heat transfer between B and C, thermal creation is negative and mechanical work potential decreases by 1.71% of the inlet work potential.
For heat transfer between D and E, thermal creation is positive and the mechanical work potential increases by 0.08% of the inlet mechanical work potential, recovering 4.7% of the mechanical work potential lost from B-C.

For a uniform inlet with temperature differences caused by differential isentropic expansion, thermal creation will always be negative because changes in static pressure follow those in static temperature. Heat transfer thus occurs from high pressure regions of the flow to low pressure regions.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.75</td>
<td>0.5</td>
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</tr>
<tr>
<td>$A_2/(A_1 + A_2)$</td>
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<td>0.25</td>
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</tr>
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<td>$M_1$</td>
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<td>0.1941</td>
<td>0.1889</td>
<td>0.2914</td>
<td>0.3001</td>
</tr>
<tr>
<td>$M_2$</td>
<td>0.3000</td>
<td>0.8332</td>
<td>0.8709</td>
<td>0.3037</td>
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</tr>
<tr>
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<td>1.0000</td>
<td>0.9829</td>
<td>0.9829</td>
<td>0.9837</td>
</tr>
</tbody>
</table>

Table 3.1: Streamtube area, Mach number, and mass average mechanical work potential at each station for $\gamma = 1.3$, $M_i = 0.3$, $p_0 = \frac{1}{2}p_{i,i}$.

3.6 Summary and Conclusions

Internal heat transfer in turbines redistributes the enthalpy of the flow. Thermal creation, which can increase or decrease the amount of work that can be extracted by a turbine, occurs when heat is transferred between regions of flow that have different local Brayton efficiency. Changes in work can be due to heat transfer with static pressure differences or differences in specific heat ratio, both of which are present in real turbine flows. Numerical examples have been presented for a constant area heat exchanger, constant pressure heat exchanger, and a heat exchanger operating from differential isentropic expansion. These show that thermal creation can result in work increases or decreases of up to 10% for certain conditions, although the changes in turbine work from gas path heat transfer for realistic turbine flows have been found to be two orders of magnitude smaller. In Chapters 3 and 4 we will examine the quantitative values of thermal creation to be expected in the gas path of uncooled
and cooled turbines.
Chapter 4

Quantifying Thermal Creation in an Uncooled Turbine

4.1 Introduction

The significance of thermal creation in high pressure turbines has not been defined in depth for actual geometries and operating conditions. Miller [19] analyzed a high pressure turbine vane with inlet hot streaks [22] defined by combustor test rig measurements, and found the impact of thermal creation on high pressure turbine efficiency to be -0.006%. The geometry included two vanes and one burner, with the hot streak clocked to the leading edge of one of the vanes. Regions of positive thermal creation were observed in the vane passage clocked with the hot streak, and negative thermal creation was observed in the vane passage without the hot streak. The cancellation of positive and negative thermal creation led to small efficiency changes.

In this chapter, mechanical work potential analysis is used to assess the importance of thermal creation in a single stage high pressure turbine with 2% rotor tip gap and inlet hot streaks. A cooled two-stage high pressure turbine with representative inlet boundary conditions is then examined in the next chapter.

The models analyzed here, which idealize hot streaks using a radial temperature distribution function (RTDF), were created by Jedamski [13], who assessed turbine efficiency changes from inlet hot streaks. Jedamski’s definition of turbine efficiency
neglects thermal creation and only accounts for lost work from viscous dissipation. The analysis in this chapter justifies his assumption that thermal creation is small in the uncooled high pressure turbine flow.

4.2 Computational Approach

The proprietary Rolls-Royce CFD code HYDRA was used to obtain a steady RANS solution in high pressure turbine flow with a mixing plane. HYDRA is an unstructured, node-based RANS solver. In this steady analysis a mixing plane was used so the rotor inlet flow is a circumferential average of the vane exit flow. Mass flow and mass flux of stagnation enthalpy are conserved over the mixing plane but mass flux of mechanical work potential is not. The one equation Spalart-Allmaras (SA) turbulence model was used.

4.2.1 Geometry and Mesh

The single stage turbine geometry is representative of a high pressure turbine for an aero engine application. The vane domain includes two airfoils to accommodate the vane to burner ratio of 2:1. No cooling holes or slots are included, the annulus inner and outer radius is smooth, and the rotor has a 2% tip gap. The structured mesh consists of 8.8 million nodes. To accurately resolve the boundary layers with the SA turbulence model, the average wall $y^+$ was 1 \cite{13}. The mesh resolution in the rotor domain is shown in Figure 4-1.

4.2.2 Boundary Conditions

All surfaces are modeled as adiabatic. Inlet turbulent viscosity was initialized to set a uniform turbulent to laminar viscosity ratio ($\nu_t/\nu_l$) of 100, where laminar viscosity is computed as a function of static temperature based on the Sutherland model. At the actual combustor-turbine interface, the viscosity ratio is non-uniform with experimental measurements of peak viscosity ratio as high as 8,000 \cite{1}. A uniform viscosity
ratio boundary condition was chosen for simplicity, because the results of Huang [11] indicate that tip leakage losses are insensitive to inlet viscosity ratio. Inlet stagnation pressure is uniform. The stagnation temperature profile was set using a radial temperature distribution function (RTDF) to generate idealized inlet hot streaks. The intensity of the hot streak is represented by the overall temperature distribution function (OTDF), defined in Equation 4.1, where $T_i^m$ denotes mass averaging, station 3 is the combustor inlet station, and station 4 is the turbine inlet station,

$$OTDF = \frac{\max(T_{i,4}(r, \theta)) - T_i^m}{T_{i,4}^m - T_{i,3}^m}. \quad (4.1)$$

Computations were carried out for cases of OTDF = 0 (uniform stagnation temperature), OTDF = 0.2, 0.4, and 0.6. Figure 4-2 shows the hot streak at inlet in terms of contours of stagnation temperature normalized by mass-averaged inlet stagnation temperature for OTDF = 0.6.
4.3 Methods to Compute Mechanical Work Potential Efficiency

For an adiabatic turbine, we can measure the efficiency by comparing the actual turbine work to the change in mechanical work potential. The change in mechanical work potential can be estimated in two ways: (i) by integrating the mass flux of mechanical work potential over the boundaries, and (ii) by integrating source terms inside the control volume. In the flux-based calculation, flow mechanical work potential $m_f$ is computed directly from primitive variables, then integrated over the inlet and outlet. The source-based method involves first differentiating the primitive variables everywhere in the flow, then integrating over the entire volume. The source-based method is more susceptible to numerical errors than the flux-based method, however, the source-based method allows the turbine losses to be linked to local mechanisms.

4.3.1 Integrating Fluxes

For an adiabatic turbine, the turbine power is computed from the change in stagnation enthalpy flux from inlet to outlet,

$$\dot{W} = \int \int_{\text{inlet}} h_i d\dot{m} - \int \int_{\text{outlet}} h_i d\dot{m}. \tag{4.2}$$
Comparing the actual turbine work to the change in mechanical work potential flux provides the turbine efficiency based on mechanical work potential,

$$
\eta_{\text{flux}} = \frac{W}{\int_{\text{inlet}} mfdm - \int_{\text{outlet}} mfdm}.
$$

(4.3)

The mechanical work potential efficiency is dependent on the choice of \( p_0 \), the dead state static pressure from the definition of mechanical work potential efficiency, which is at the discretion of the analyst. One choice is to set \( p_0 \) equal to the atmospheric pressure, but this does not seem appropriate when analyzing a high pressure turbine which does not exhaust directly to the atmosphere. In the present analysis, \( p_0 \) was set equal to the area-averaged exit static pressure.

### 4.3.2 Evaluation of Volumetric Source Terms

Miller [19] derived a mechanical work potential balance equation to identify the mechanisms that lead to changes in the mechanical work potential. For an adiabatic turbine, the changes in mechanical work potential can be split into the actual turbine work and lost turbine work. The lost work is comprised of viscous dissipation, \( \Phi_{\text{visc}} \), and thermal creation, \( \Delta_{\text{therm}} \). The turbine efficiency, based on mechanical work potential source terms, is written in Equation 4.4,

$$
\eta_{\text{source}} = \frac{W}{\int_{\text{source}} \left( \frac{\Phi_{\text{visc}} - \Delta_{\text{therm}}}{p} \right) dV + \int_{\text{volume}} \left( \frac{\Phi_{\text{visc}} - \Delta_{\text{therm}}}{p} \right) dV}.
$$

(4.4)

The viscous dissipation can be computed from viscosity and velocity gradients [16], as in Equation 4.5,

$$
\Phi_{\text{visc}} = \mu \left[ 2 \left( \frac{\partial u}{\partial x} \right)^2 + 2 \left( \frac{\partial v}{\partial y} \right)^2 + 2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 - 2 \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right)^2 \right].
$$

(4.5)

In evaluating the terms in Equation 4.5, velocity gradients at each node are computed using a first order differencing scheme. The viscosity is equal to the sum of the laminar
viscosity, computed as a function of static temperature according to Sutherland's law, and the turbulent viscosity obtained from the SA turbulence model.

Miller has also provided an expression for local thermal creation in a flow with constant specific heat ratio, $\gamma$ [19]. Thermal creation, $\Delta_{\text{therm}}$, with units of power per unit volume, is defined in Equation 4.6,

$$\Delta_{\text{therm}} = \left( \frac{\gamma - 1}{\gamma} \right) \frac{\nabla p}{p} \cdot \vec{q}. \quad (4.6)$$

Thermal creation is proportional to the dot product of the static pressure gradient and the heat flux vector. For heat transfer from low to high pressure, i.e. along the pressure gradient, thermal creation is positive. For heat transfer from high to low pressure, opposite the pressure gradient, thermal creation is negative. If the pressure gradient and heat flux are orthogonal, there is no thermal creation.

The heat flux is computed from thermal conductivity and static temperature gradient, assuming a constant turbulent Prandtl number. Static temperature gradient is calculated at each node using a first order differencing scheme. The change in mechanical work potential flux over the mixing plane is included as lost work when analyzing the turbine stage.

4.3.3 Comparison to Work-Averaged Stagnation Pressure Efficiency

A third approach to compute turbine efficiency while accounting for the effect of thermal creation is to use the work-averaged stagnation pressure. The denominator for turbine efficiency is equal to the isentropic turbine work obtained expanding from the work-averaged inlet stagnation pressure to the work-averaged outlet stagnation pressure. Using this approach the turbine efficiency is given in Equation 4.7 as,

$$\eta_{wa} = \frac{\dot{W}}{\dot{m} c_p T_{i,\text{in}} \left( 1 - \frac{T_{o,\text{out}}}{T_{o,\text{in}}} \right) \left( \frac{p_{o,\text{out}}}{p_{o,\text{in}}} \right)^{\frac{\gamma - 1}{\gamma}}}. \quad (4.7)$$
Unlike the efficiencies based on mechanical work potential, the efficiency based on work-averaged stagnation pressure does not depend on the choice of dead state pressure $p_0$. To assess the effect of dead state pressure on the results, the turbine efficiency was calculated with the three methods defined in Equations 4.3, 4.4, and 4.7 for the OTDF = 0.6 case. Figure 4-3 presents the difference between mechanical work potential efficiency and work-averaged stagnation pressure efficiency as a function of dead state static pressure normalized by exit work-averaged stagnation pressure. The area-average exit static pressure used for the dead state pressure in this analysis is marked with a dashed line.

The range of dead state pressures included show the consequences of choosing dead state pressure equal to the exit stagnation pressure. When dead state pressure is chosen to be exactly equal to the work-averaged exit stagnation pressure, the flux-based mechanical work potential efficiency is exactly equal to the efficiency based on work-averaged stagnation pressure. The offset of 0.8% efficiency between flux-based and source-based calculations is due to numerical errors accumulated when differentiating source terms.

Jedamski [13] found that as inlet hot streak intensity increased (higher OTDF), turbine efficiency decreased. Figure 4-4 compares the three previously discussed efficiency definitions plus the efficiency definition used by Jedamski, where thermal creation is neglected and lost work is computed based only on viscous dissipation. To compare changes in efficiency due to increasing inlet non-uniformity across efficiency definitions, we have plotted the quantity $\eta - \eta_0$, where $\eta_0$ is the efficiency with OTDF = 0, as a function of OTDF. The four efficiency definitions are tightly clustered, showing that the differences are small compared to the change in efficiency from increasing inlet OTDF. The implication is that all the methods for assessing efficiency capture the behavior in terms of loss increase, and that the source-based method can be applied to track efficiency changes.
Figure 4-3: Difference in turbine efficiency [%] between mechanical work potential efficiency and work-averaged stagnation pressure efficiency as a function of dead state pressure chosen to define mechanical work potential normalized by work-averaged exit stagnation pressure.

Figure 4-4: Computed changes in turbine efficiency [%] as a function of inlet stagnation temperature non-uniformity (OTDF) for four efficiency definitions.
4.4 Changes in Turbine Efficiency due to Thermal Creation

Using the source-based mechanical work potential efficiency, we now quantify the impact of thermal creation on efficiency. The change in efficiency from thermal creation is defined as the difference between the source-based mechanical work potential efficiency and source-based mechanical work potential efficiency with $\Delta_{\text{therm}} = 0$, Equation 4.8,

$$\Delta \eta_{tc} = \eta_{\text{source}} - \eta_{\text{source}}(\Delta_{\text{therm}} = 0).$$  \hspace{1cm} (4.8)

The effect of thermal creation in the vane and in the rotor on efficiency can also be assessed independently. To quantify the effect of thermal creation in the vane, the source-based efficiency is compared to the source-based efficiency with rotor thermal creation set to zero. To quantify the effect of thermal creation in the rotor, source-based efficiency is compared to source-based efficiency with vane thermal creation set to zero. Figure 4-5 presents the change in efficiency due to overall thermal creation, thermal creation in the vane, and thermal creation in the rotor, as a function of inlet hot streak OTDF.

In the vane, thermal creation is always negative and increases slightly with increasing OTDF. In the rotor, thermal creation is negative for uniform inlet stagnation temperature but becomes positive with inlet non-uniformity. The maximum change in efficiency from thermal creation is -0.006% for OTDF = 0. At OTDF = 0.4, thermal creation results in an increase of efficiency of +0.001%. The impact of thermal creation on turbine efficiency is thus negligible for the uncooled turbine with idealized hot streaks.

4.5 Identifying Thermal Creation Mechanisms

A metric for the importance of thermal creation is the ratio of lost work from thermal creation to the lost work from viscous dissipation. The local thermal creation, $\Delta_{\text{therm}}$,
Figure 4-5: Change in turbine efficiency [%] due to thermal creation in the vane and rotor as a function of inlet hot streak OTDF.

can be compared to a measure of the overall viscous loss, the volumetric averaged viscous dissipation, \( \Phi_{\text{visc}} \), defined in Equation 4.9,

\[
\Phi_{\text{visc}} = \frac{1}{V} \int \int \int \Phi_{\text{visc}} dV.
\] (4.9)

Figure 4-6 presents isosurfaces of local thermal creation normalized by volumetric average viscous dissipation for the OTDF = 0 (left) and OTDF = 0.6 (right) cases. The blue isosurfaces represent regions of negative thermal creation with magnitude of at least 10% \( \Phi_{\text{visc}} \). Red isosurfaces represent regions of positive thermal creation with magnitude of at least 10% \( \Phi_{\text{visc}} \).

With or without inlet hot streaks, the highest thermal creation is in the rotor tip gap flow. Both positive and negative structures are observed, leading to cancellation similar to that observed by Miller for thermal creation in vanes. Thermal creation in the tip gap flow is higher in the OTDF = 0.6 case, likely due to the increased tip mass flow identified by Jedamski [13] and associated with the changes in velocity triangles with OTDF. There is a small region of positive thermal creation just outside
Figure 4-6: Isosurfaces of thermal creation normalized by volumetric average viscous dissipation, $\Delta_{\text{therm}}/\bar{\Omega}_{\text{visc}}$, showing regions of largest positive and negative thermal creation for (i) OTDF = 0 and (ii) OTDF = 0.6.
the boundary layer near the suction side leading edge, where the hot, unexpanded boundary layer fluid transfers heat into the cooler expanded flow. The static pressure in the boundary layer follows the free stream static pressure. As the flow is expanded around the suction side of the airfoil, the pressure drops toward the surface. Heat is transferred away from the hotter fluid near the blade surface, from low pressure to high pressure, and thermal creation is positive.

To help identify regions of the flow that may lead to thermal creation, the thermal creation can be decomposed into a term dependent only on static pressure, a term only dependent on static temperature, and a term that describes the projection of pressure gradient onto heat flux. Changes to upstream stagnation temperature result in only small changes to the static pressure field \[7\], and so the magnitude of static pressure gradient is nearly independent of the temperature non-uniformity and thus the heat flux. The thermal creation can be found from the pressure gradient magnitude, heat flux magnitude, and the angle between the pressure gradient and heat flux vector \(\alpha\), as in Equation 4.10,

\[
\Delta_{\text{therm}} \propto \frac{||\nabla p||}{p} ||\vec{q}|| \cos \alpha.
\]  

(4.10)

To non-dimensionalize the pressure contribution to thermal creation, we use the pressure gradient magnitude divided by static pressure and multiplied by vane axial chord, \(c_x\). To non-dimensionalize the heat flux contribution to thermal creation, we use the heat flux magnitude divided by the turbine power per rotor passage planar area, \(\dot{W}/(n_r c_x b)\).

Figure 4-7 shows contours of thermal creation normalized by viscous dissipation (\(\Delta_{\text{therm}}/\Phi_{\text{visc}}\)), the non-dimensionalized pressure term (\(||\nabla p||/c_x\)), and the non-dimensionalized heat flux contribution (\(||\vec{q}||/\dot{W}/(n_r c_x b)\)) for a mid-span radial slice at OTDF = 0. The highest heat flux magnitudes occur in the van and rotor wakes, but the pressure gradient in these locations is small and so is the thermal creation. The small pressure gradient at the vane inlet indicates that even when there is freestream heat transfer ahead of the vane, thermal creation in this region is small. The highest pressure gradients are observed around the suction side leading edge, where a small
region of positive thermal creation is observed. Although immediately outside of the boundary layer thermal creation is positive, this is overpowered by the negative thermal creation in the expanded flow away from the boundary layer and thus the net thermal creation in the vane is negative.

Figure 4-7: Contours on mid-span radial slice with OTDF = 0: (i) thermal creation normalized by volumetric average viscous dissipation ($\Delta_{therm}/\bar{\Phi}_{visc}$), (ii) static pressure term ($\frac{|\nabla p|}{p}c_{u}$), (iii) heat flux term ($\frac{|\nabla T|}{W/(r_c h)}$).

The third component of thermal creation that must be considered is the projection of pressure gradient onto heat flux. The projection of pressure gradient onto heat flux on a mid-span radial slice for OTDF = 0 and OTDF = 0.6 is shown in Figure 4-8. For uniform inlet stagnation temperature, expansion of inviscid flow leads to negative thermal creation because in isentropic flow, static pressure gradients and static temperature gradients align. Some heat, however, is transferred from high pressure to low pressure as the flow expands, leading to negative thermal creation. Near the boundary layers and in the airfoil wakes, positive, negative, and zero thermal creation are possible. With a hot streak, positive thermal creation is observed in the vane on which the hot streak impinges. The vane without the hot streak has negative thermal creation, however, causing the cancellation effect previously observed.
Figure 4-8: Contours of the projection of pressure gradient on heat flux vector on a mid-span radial slice for OTDF = 0 (left) and OTDF = 0.6 (right).

4.6 Summary and Conclusions

In an uncooled turbine stage with idealized hot streaks, thermal creation was found to be negligible. An inlet temperature non-uniformity is not required for thermal creation to occur, and the magnitude of thermal creation with uniform inlet stagnation temperature was similar to that with high inlet stagnation temperature non-uniformity (OTDF = 0.6). The effect of cancellation between regions of flow with positive and negative thermal creation, observed by Miller [19], was seen in free stream heat transfer in the vane, as well as in the rotor tip leakage flow. The tip leakage flow had the highest local thermal creation, and changes to the tip leakage mass flow with OTDF led to changes in overall rotor thermal creation. The overall thermal creation, however, in this turbine is two orders of magnitude smaller than viscous dissipation.
Chapter 5

Quantifying Thermal Creation in a Cooled Turbine

5.1 Introduction

In the idealized high pressure turbine stage analyzed in Chapter 4, thermal creation was negligible. There are, however, differences between the idealized situation and the actual high pressure turbine environment that can change the thermal creation in the main gas path:

- Cold flow ejected into the hot main gas path through airfoil cooling holes or trailing edge slots increases gas path heat transfer and may increase or decrease thermal creation.

- Cold flow from the high pressure compressor that enters the main gas path through cavities in the case and hub increases gas path heat transfer and may increase or decrease thermal creation.

- Increased turbulence level, compared to the value used in the idealized stage computation, can increase thermal conductivity for constant turbulent Prandtl number. For the same temperature gradients, therefore, the gas path heat flux increases and thermal creation may increase or decrease.

- Non-uniform inlet stagnation temperature, swirl, and turbulence can affect pres-
sure gradients and heat flux in the endwall region resulting in an increase or decrease of thermal creation.

In line with the approximate substitution principle [7] the pressure gradient terms are unlikely to vary significantly between the situation in the ideal case and a more realistic configuration. Large changes to heat flux are possible, however, through the introduction of coolant flows and by increasing inlet turbulence. We thus evaluate the impact of thermal creation on high pressure turbine efficiency for a two-stage high pressure turbine with (i) representative cooling flows and (ii) representative combustor exit conditions. Previous studies have quantified the change in turbine efficiency from film cooling holes, but the effect of gas path heat transfer was neglected [10] [17]. As in Chapter 4, heat transfer between the main gas path and the internal air system, through the blade metal, is not considered.

5.2 Computational Approach

The Rolls-Royce CFD code HYDRA was used to obtain a steady RANS solution for the flow through the turbine, as in Chapter 4. Mixing planes were used to circumferentially average flow passing between rotating and non-rotating blade rows. Instead of the one equation SA turbulence model used in the calculation described in the previous chapter, Menter's two equation $k - \omega$ SST turbulence model was employed. Free stream turbulence should be better captured with this model, which uses the Wilcox $k - \omega$ model near the wall and the $k - \epsilon$ model in the outer wake and free stream [18]. Wall functions, based on the assumption of a universal logarithmic region of the turbulent boundary layer, are used to resolve the wall shear without fine boundary layer meshes.

5.2.1 Geometry and Mesh

A two-stage cooled turbine geometry representative of an industrial aero engine high pressure turbine was examined. The first stage rotor has tip clearance equal to 2%
rotor span. The second stage rotor is shrouded. Trailing edge slots were modeled in the first and second stage vanes. The film cooling holes in the first stage vane and rotor were not meshed, and a stripping model of the type described by Crawford, Kays, and Moffet [2] was used. Portions of the hub and case cavities were meshed so the calculation captured the effect of cavity inflows on the main flow through the turbine.

A meridional view of the two-stage turbine is shown in Figure 5-1. The dashed lines represent the locations of mixing planes between blade rows. The white cutouts in the vanes depict the trailing edge cooling slots.

![Image](image.png)

Figure 5-1: Meridional view of the two-stage high pressure turbine with cavities and slot cooling.

The turbine first stage vane to burner ratio is 9:4. Nine first stage vane passages (one quarter annulus) are thus modeled so the inlet boundary conditions from the combustor can be applied without the need for circumferential averaging. At the first mixing plane the flow was circumferentially averaged and only one passage is required for downstream blade rows. Table 5.1 summarizes the number of nodes in the structured mesh used to represent each blade row.

### 5.2.2 Boundary Conditions

All wall surfaces were modeled as adiabatic. The turbine inlet stagnation temperature, stagnation pressure, flow angles, turbulence kinetic energy $k$, and turbulence dissipation rate $\omega$ were taken from Rolls-Royce combustor CFD results for engine op-
Blade Row | Instances | Millions of Nodes
--- | --- | ---
Stage 1 Vane | 9 | 23.6
Stage 1 Rotor | 1 | 2.1
Stage 2 Vane | 1 | 1.5
Stage 2 Rotor | 1 | 2.1

Table 5.1: Nodes in the two-stage turbine mesh.

Operating conditions. Figure 5-2 shows the inlet stagnation temperature normalized by mass average stagnation temperature. Four hot spots are observed, corresponding to the burner locations, with OTDF ≈ 0.3. The circumferential and radial temperature non-uniformities are not as smooth as those modeled with RTDF profiles in Chapter 4.

![Inlet stagnation temperature normalized by mass-average inlet stagnation temperature, \( T_i/T_{in} \).](image)

Figure 5-2: Inlet stagnation temperature normalized by mass-average inlet stagnation temperature, \( T_i/T_{in} \).

Figure 5-3 shows the calculated turbulent to laminar viscosity ratio, \( \nu_t/\nu_l \), at the turbine inlet. The area-averaged inlet viscosity ratio is 1,040, and the maximum viscosity ratio is 1,600, closer to experimentally measured turbulence levels [1] than the value of 100 used by Jedamski [13].

Three types of coolant flow injection into the main gaspath were included in the two-stage model. The first is flow used for film cooling in the first stage vane and rotor, with mass flow equal to 9.7% of combustor exit mass flow, \( m_4 \). Film cooling air is taken from the high pressure compressor, used for internal blade cooling, then
ejected through small holes on the airfoil surface to create a film of cold air that protects the vane and rotor from the hot free stream. A film stripping model [2] with a continuous strip of flow is used in lieu of resolving discrete holes. At each node that intersects the strip, mass, momentum, and energy source terms were added to the RANS equations. The source terms require mass flow, stagnation temperature, and stagnation pressure, which are given from the engine operation point, and flow angles, which are set by the hole geometry.

Another type of coolant is flow ejected from the trailing edge of the first and second stage vanes, with mass flow equal to 6.3% of the combustor exit mass flow, \( \dot{m}_4 \). The flows taken from the high pressure compressor gain heat from cooling the vane before leaving at the trailing edge and mixing with the vane wake. In the trailing edge slot an inflow boundary condition with uniform stagnation pressure and stagnation temperature was applied based on the engine operating point.

The third type of cooling is flow that enters the main gas path through cavities in the turbine hub and case, comprising 4.9% of the combustor exit mass flow, \( \dot{m}_4 \). The total coolant flow is 20.9% of the mass flow entering the turbine from the combustor. Table 5.2 provides a list of the 8 sources of cooling flow included in the two-stage turbine calculation.
Table 5.2: Sources of coolant flow, as a percentage of combustor exit mass flow $\dot{m}_4$.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Blade Row</th>
<th>Cooling Mass Flow [% $\dot{m}_4$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upstream Hub Cavity</td>
<td>Stage 1 Vane</td>
<td>0.3</td>
</tr>
<tr>
<td>Upstream Case Cavity</td>
<td>Stage 1 Vane</td>
<td>0.2</td>
</tr>
<tr>
<td>Film Holes</td>
<td>Stage 1 Vane</td>
<td>4.8</td>
</tr>
<tr>
<td>Trailing Edge Slot</td>
<td>Stage 1 Vane</td>
<td>3.3</td>
</tr>
<tr>
<td>Upstream Hub Cavity</td>
<td>Stage 1 Rotor</td>
<td>2.4</td>
</tr>
<tr>
<td>Upstream Case Cavity</td>
<td>Stage 1 Rotor</td>
<td>2.0</td>
</tr>
<tr>
<td>Film Holes</td>
<td>Stage 1 Rotor</td>
<td>4.9</td>
</tr>
<tr>
<td>Trailing Edge Slot</td>
<td>Stage 2 Vane</td>
<td>3.0</td>
</tr>
<tr>
<td>Total</td>
<td>-</td>
<td>20.9%</td>
</tr>
</tbody>
</table>

5.3 Efficiency Changes from Thermal Creation

The changes in turbine efficiency from thermal creation have been found using the source-based method of computing mechanical work potential efficiency outlined in Section 4.3.2. A calculation with no coolant flows was used to establish a baseline with which to compare the cooled results. The effect of using the actual combustor exit conditions was also determined by comparing the uncooled case with the result from Chapter 4. The effect of thermal creation on the efficiency of each turbine stage was assessed in addition to the overall effect on the two-stage turbine.

The changes in turbine efficiency due to thermal creation, $\Delta \eta_{hc}$, in the first stage, second stage, and in the overall two-stage turbine are given in Table 5.3. The efficiency change is defined as in Chapter 4. For the "actual" two-stage cooled high pressure turbine the thermal creation is negative and the overall impact of thermal creation on turbine efficiency is -0.107%.

<table>
<thead>
<tr>
<th></th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uncooled</td>
<td>-0.077%</td>
<td>-0.086%</td>
<td>-0.080%</td>
</tr>
<tr>
<td>Cooled</td>
<td>-0.142%</td>
<td>-0.079%</td>
<td>-0.107%</td>
</tr>
</tbody>
</table>

Table 5.3: Change in efficiency from thermal creation, $\Delta \eta_{hc}$, in the two-stage turbine with and without cooling flows.

When cooling flows are included, the loss of efficiency attributed to thermal cre-
ation increases from 0.077% to 0.142% in the first stage and decreases from 0.086% to 0.079% in the second stage. Even for the uncooled turbine, the impact of thermal creation is much larger than computed for the turbine stage in Chapter 4, where efficiency changes from thermal creation were smaller than 0.01% regardless of inlet stagnation temperature profile. The change in thermal creation between the single stage uncooled turbine with idealized hot streaks and the two-stage uncooled turbine with representative combustor exit conditions is larger than the change in thermal creation from adding coolant flows in the two-stage turbine. Thermal creation in the first and second stage is the same order of magnitude.

The significance of thermal creation can also be assessed by comparing the lost work from thermal creation to the lost work from viscous dissipation and the decrease in mechanical work potential over mixing planes. Figure 5-4 shows breakdown of lost work in the cooled high pressure turbine first stage from (i) viscous dissipation, (ii) thermal creation, and (iii) loss of mechanical work potential in the mixing plane. Viscous dissipation dominates the lost work, comprising over 95% of the overall loss. Thermal creation is 1% of the lost work in the first stage. The contribution of thermal creation to lost work is even smaller in the uncooled case and in the second stage.

5.4 Sources of Highest Thermal Creation

To identify the regions of highest thermal creation in the cooled turbine, we examine axial slices through each blade row with contours of thermal creation on an $r - \theta$ plane. To compare local thermal creation in the cooled high pressure turbine to the overall viscous loss, we normalize thermal creation by the volumetric average viscous dissipation, $\Phi_{\text{visc}}$.

Axial slices of thermal creation, normalized by volumetric average viscous dissipation, in the cooled first stage vane are presented in Figure 5-5. Suction side (SS) and pressure side (PS) of the vane are labeled, and the vane leading edge and trailing edge positions are denoted using dashed lines.

One axial chord upstream of the leading edge, the heat flux is large but the pressure
Figure 5-4: Contributions to lost work in the mixing plane calculation of the cooled turbine first stage.

Figure 5-5: Contours of thermal creation normalized by volumetric average viscous dissipation, \( \Delta_{\text{therm}} / \bar{\Phi}_{\text{visc}} \), on four \( r-\theta \) planes in the cooled first stage vane. The axial locations are: (i) one axial chord upstream of vane leading edge, (ii) 1% axial chord upstream of vane leading edge, (iii) half axial chord, and (iv) 1% axial chord downstream of vane trailing edge. Leading edge and trailing edge positions are indicated with dashed lines, and the pressure side (PS) and suction side (SS) are identified.
gradients are small, so the thermal creation is small. One percent of axial chord upstream of the vane leading edge, pressure gradients from the upstream influence of the vane and heat flux from the film cooling air used to protect the vane leading edge from combustor hot spots coexist, resulting in high thermal creation. On the suction side thermal creation is positive and on the pressure side thermal creation is negative, resulting in cancellation of the effects on efficiency. At mid-chord, negative thermal creation from expanded flow near the suction side (as previously observed in the single stage turbine) is seen. At the vane trailing edge, coolant flow is injected into the airfoil wake, giving positive thermal creation as heat transfers from the hot, lower pressure flow on the suction side of the trailing edge to the cooler and higher pressure wake. There is a slight passage-to-passage variation of thermal creation in the vane endwall flow due to the non-uniform inlet boundary conditions. In some passages thermal creation in the endwall region is positive and in others thermal creation is negative, and the overall impact of inlet circumferential non-uniformity on thermal creation is reduced through cancellation.

Figure 5-6 presents contours of thermal creation, normalized by volumetric average viscous dissipation, on axial slices of the cooled first stage rotor. Circumferential non-uniformities are removed with the mixing plane ahead of the rotor, so passage-to-passage variations are not captured. The highest thermal creation, both positive and negative, at the rotor leading edge, half chord, and trailing edge is associated with streamwise vorticity in hub endwall and tip leakage flows. The highest local thermal creation is positive, but the regions of positive thermal creation are surrounded by regions of negative thermal creation, and the thermal creation integrated over the entire volume is negative.

Axial slices of thermal creation, normalized by volumetric average viscous dissipation, are presented for the cooled second stage vane in Figure 5-7. Positive thermal creation is seen from slot cooling at the vane trailing edge, as in the first vane. The free stream in the second stage of the turbine is cooler than in the first stage, however, so the temperature difference between the coolant flow and the free stream is lower and the positive thermal creation at the second stage vane trailing edge is lower
than in the first stage. Without film cooling or cooling flow from cavities, there are fewer regions of high local thermal creation, but the magnitude of the net thermal creation in the second stage vane is similar to that of the thermal creation in the first stage vane. This shows the strength of the cancellation encountered in the first stage vane, when flow features that promote local heat transfer generate higher positive and negative thermal creation but yield only small changes in net thermal creation.

Figure 5-7: Contours of thermal creation normalized by volumetric average viscous dissipation, $\Delta_{\text{therm}}/\overline{\Phi_{\text{visc}}}$, on three $r - \theta$ planes in the cooled second stage vane. The axial locations are: (i) 1% axial chord upstream of vane leading edge, (ii) half axial chord, and (iii) 1% axial chord downstream of vane trailing edge. Leading edge and trailing edge positions are indicated with dashed lines, and pressure side (PS) and suction side (SS) are identified.
Axial slices of thermal creation, normalized by volumetric average viscous dissipation, in the second stage rotor for the cooled turbine are presented in Figure 5-8. With no coolant flows entering the main gas path, heat fluxes and thermal creation are smaller than in the first stage rotor. Unlike the first stage rotor, the endwall flows do not generate significant amounts of thermal creation, and the regions of highest thermal creation are near the suction side blade surface leading edge, as observed in the uncooled turbine stage in Chapter 4. One chord downstream of the rotor, the pressure gradients are small and so is thermal creation. The impact of thermal creation on second stage efficiency is roughly half (56%) of the impact of thermal creation on first stage efficiency.

![Figure 5-8: Contours of thermal creation normalized by volumetric average viscous dissipation, Δ_therm/Δ_visc, on four r - θ planes in the cooled second stage rotor. The axial locations are: (i) 1% axial chord upstream of rotor leading edge, (ii) half axial chord, (iii) 1% downstream of rotor trailing edge, and (iv) one axial chord downstream of rotor trailing edge. Leading edge and trailing edge positions are indicated with dashed lines, and pressure side (PS) and suction side (SS) are identified.](image)

### 5.4.1 Impact of Cooling Flows on Thermal Creation

To assess the changes to thermal creation from adding coolant flow to the main gas path, thermal creation in the cooled turbine is compared to the baseline uncooled calculation. Contours of thermal creation on a mid-span radial slice through the turbine, and isosurfaces of thermal creation in the flow around the first stage rotor, allow us to identify the mechanisms for thermal creation caused by cooling flows.
Figure 5-9 shows contours of thermal creation, normalized by volumetric average viscous dissipation, on a mid-span slice through the two-stage turbine with no coolant flows. Figure 5-10 shows contours of thermal creation, normalized by volumetric average viscous dissipation, on a mid-span radial slice through the cooled two-stage turbine.

Figure 5-9: Contours of thermal creation normalized by volumetric average viscous dissipation, $\Delta_{\text{therm}}/\bar{\Psi}_{\text{visc}}$, on a mid-span radial slice through the uncooled two-stage turbine.

Figure 5-10: Contours of thermal creation normalized by volumetric average viscous dissipation, $\Delta_{\text{therm}}/\bar{\Psi}_{\text{visc}}$, on a mid-span radial slice through the cooled turbine.

To describe the three dimensional features, the first stage rotor will be examined in detail. In the first stage rotor, 44% of the overall coolant flow is added to the main gas path: 2.4% of $\dot{m}_4$ is injected in the upstream hub cavity, 2.0% of $\dot{m}_4$ is injected
in the upstream case cavity, and 4.9% of $\dot{m}_4$ is injected through film cooling holes, primarily on the suction side of the airfoil and around the leading edge.

Figure 5-11 shows isosurfaces of thermal creation equal to ±10% of $\overline{\Phi_{visc}}$, for the high pressure turbine first stage rotor without cooling flow (left) and with cooling flow (right). The isosurfaces are shaded between 10% and 20% to highlight the regions with largest thermal creation. Figure 5-12 presents the same isosurfaces with a view of the rotor pressure side.

![Figure 5-11: Isosurfaces of ± 0.1 thermal creation normalized by volumetric average viscous dissipation, $\Delta_{therm}/\overline{\Phi_{visc}}$, showing regions of highest positive and negative thermal creation around the first stage rotor suction side (i) without and (ii) with cooling flow.](image)

The first type of cooling flow considered is film cooling air, which is used in the first stage vane and rotor. Comparing Figure 5-9 with Figure 5-10, there is an increase in negative thermal creation near the vane and rotor leading edge suction side, corresponding to the region where the majority of the film cooling air is added to the main gas path. In Figure 5-11, a positive region of thermal creation near the suction side boundary layer is observed for the uncooled blade, associated with heat transfer between the hot boundary layer fluid and the cooler expanded fluid outside of the boundary layer. When cooling flows are added, the hot boundary
layer fluid is replaced with cold film cooling air, the heat flux reverses direction, and the thermal creation in this region becomes negative. The film cooling added on the pressure side of the vane and rotor can be observed in Figure 5-10, but the region of thermal creation is confined to the injection site. Figure 5-12 shows that the changes to thermal creation near the pressure side surface from cooling flows are minor compared to the changes near the suction side surface. The pressure gradients on the pressure side are less severe than near the suction side leading edge, so the increased heat flux due to film cooling air mixing with the main gas path results in small thermal creation. The impact of film cooling air is to increase the negative thermal creation, leading to higher loss.

The next type of coolant examined is trailing edge slot coolant. As described in the previous section, mixing coolant flow with the hot wake leads to positive thermal creation. This is seen in the first stage, when comparing Figure 5-9 to Figure 5-10, where positive thermal creation is largest close to the vane trailing edge and decreases as the pressure gradients and heat flux decrease downstream of the vane. For the second stage vane positive thermal creation is seen in Figure 5-7, but Figure 5-10 shows the region of positive thermal creation at the trailing edge is small. In the second stage, coolant flow and main gas path air are closer in temperature than
in the first stage, so heat flux in the wake is lower.

The last type of coolant considered is introduced into the main gas path by cavity flows. Figure 5-11 and Figure 5-12 show that the addition of cold cavity flows to the gas path upstream of the rotor causes increased thermal creation near the endwalls. Both positive and negative regions of thermal creation are produced in the hub endwall flow; the cancellation between them again reduces the impact of hub cavity flow on thermal creation. Casing cavity flow is pulled into the tip region and causes a reduction in the positive thermal creation observed in the uncooled tip leakage flow. Negative thermal creation near the blade tip region is greatly increased when the case cavity flow is added, resulting in a net decrease of efficiency. Adding coolant flow to the main gas path at mid span has a smaller impact on thermal creation than introducing it near the hub or tip, where higher pressure gradients are present. Overall, the impact of casing cavity flow is to increase negative thermal creation, contributing to loss, whereas the impact of hub cavity flow is to increase both positive and negative thermal creation, with the degree of cancellation making it difficult to make a direct statement about the contribution to loss.

5.4.2 Impact of Inlet Turbulence on Thermal Creation

The turbine stage in Chapter 4 and the high pressure turbine first stage considered in this chapter have similar pressure ratio, aspect ratio, and inlet stagnation temperature non-uniformity (OTDF \( \approx 0.3 \) at the turbine inlet). The thermal creation in the uncooled two-stage turbine, however, is an order of magnitude larger than thermal creation in the single stage turbine. This difference is due to the inlet turbulent viscosity boundary condition. The two-stage turbine inlet boundary conditions are taken from combustor CFD, where turbulent viscosity is non-uniform (Figure 5-3) and, on average 10, times larger than the boundary condition applied by Jedamski [13] for the single stage.

The contribution of pressure to thermal creation, measured by the pressure gradient magnitude divided by static pressure, is almost identical in the two cases. The contribution of heat flux to thermal creation, measured by the heat flux magnitude,
however, is 10 times larger in the uncooled two-stage turbine than in the single stage turbine. This increase in heat flux magnitude can be traced back to an increase in turbulent thermal conductivity, which is caused by higher turbulent viscosity in the free stream.

The increased free stream turbulence does not have a large effect on viscous dissipation, which is primarily generated in the boundary layers. Thermal creation in the free stream can dominate thermal creation in boundary layers, as seen in the uncooled vane in Chapter 4, where the small region of high, positive thermal creation immediately outside the boundary layer is dominated by a larger region of negative thermal creation away from the boundary layer. Thermal creation is more sensitive to free stream turbulence levels than viscous dissipation; increasing inlet turbulent viscosity increases the importance of thermal creation relative to viscous dissipation.

5.5 Summary and Conclusions

RANS computations show that the overall change in efficiency due to thermal creation in a cooled, two-stage high pressure turbine is -0.107%. Thermal creation is thus a small but nonzero portion of the overall loss. It should be taken into consideration when accounting changes to turbine efficiency but it may not be large enough to warrant consideration in the aerodynamic design of aero engine turbines. Even in the cooled first stage, the contribution of viscous dissipation to lost work is roughly 95 times that of thermal creation. It was found that the cooled turbine shows twice the impact of thermal creation on first stage efficiency compared to the uncooled turbine. Film cooling air, especially when injected into the main gas path near the suction side leading edge, increases negative thermal creation and thus loss. Coolant flow from the casing cavity upstream of the first stage rotor also increases negative thermal creation. A small amount of positive thermal creation is generated when trailing edge slot coolant mixes with the wake in the first stage vane. Changes in thermal creation due to coolant flows are small compared to the change in thermal creation from increasing inlet turbulent viscosity: increasing the magnitude of the
inlet turbulent viscosity by a factor of 10 resulted in a factor of 10 increase in thermal creation.
Chapter 6

Summary, Conclusions, and Recommendations for Future Work

6.1 Summary and Conclusions

An analysis of loss metrics for high pressure turbines has been carried out using both model problems and three dimensional RANS calculations to quantify the thermal creation property in these flows. The key results are:

- As described by Miller [19], for an aero engine turbine, in which the purpose is to extract work from the flow and in which no work can be obtained by heat transfer with the surroundings, the *mechanical work potential* is the correct loss metric. Entropy generated from gas path heat transfer is not linked to aero engine turbine loss, although heat transfer can increase or decrease turbine loss through thermal creation.

- A mixing duct model problem was used to demonstrate that the relevant loss metric for a fluid system component depends on the downstream use of the flow, i.e., the overall purpose of the system the component is a part of. This is illustrated by the example of thrust increase due to mixing; in the context of availability and mechanical work potential there is a loss but in the context of thrust there is a benefit.
• An analysis of an inviscid heat exchanger was used to demonstrate how changes in turbine work are possible from heat transfer between fluid streams with different static pressure or different specific heat ratio. The changes in work are due to the property denoted as thermal creation [19], which results when heat is transferred between streams with differing Brayton efficiencies based on static pressure.

• Numerical examples have been presented to show that increases or decreases in turbine efficiency are possible from heat transfer with a pressure difference or specific heat ratio difference; in high pressure turbine flows both are present.

• Thermal creation in an uncooled turbine was found to result in efficiency changes of less than 0.01% regardless of inlet hot streak intensity. The hot streak generates both positive and negative regions of thermal creation, resulting in a cancellation and thus small changes in efficiency.

• Adding cooling flows to the first stage of a high pressure turbine doubled thermal creation compared to the stage without cooling. Depending on the location where coolant is added, the local thermal creation can be positive or negative.

• In a two-stage cooled high pressure turbine at representative engine operating conditions with non-uniform inlet flow, thermal creation was found to result in a decrease in efficiency of 0.1%. In the RANS calculation of the first stage of the cooled turbine, thermal creation comprises 1% of the lost work.

• Increasing turbulence in the free stream results in larger free stream heat flux for the same temperature gradients, and larger overall thermal creation. In an uncooled turbine first stage with representative combustor turbulence and average inlet viscosity ratio of 1,000, the change in efficiency from thermal creation was 0.08% compared to 0.006% in an uncooled turbine with uniform inlet turbulent to laminar viscosity ratio of 100.
6.2 Recommendations for Future Work

The highest local thermal creation in both the cooled and uncooled turbine was observed in the endwall flows and tip leakage flows. Miller [19] showed that a vortex with uniform inlet stagnation temperature leads to negative thermal creation, however the RANS calculations included positive and negative regions of thermal creation. Analysis of a model problem of a vortex with varying inlet stagnation temperature profiles can quantify the degree of cancellation between positive and negative regions of thermal creation and help provide information on situations, if any, that lead to net positive thermal creation.

The importance of freestream turbulence to thermal creation was demonstrated and the sensitivity of thermal creation to physical turbulence levels and to turbulence models should be studied. For the latter an explicit free stream turbulence model [22] may need to be implemented to properly capture the heat flux in the free stream.

The mixing plane calculation of the cooled turbine that has been carried out should be supplemented by an unsteady calculation. Maintaining the circumferential stagnation temperature non-uniformities will increase heat flux in the first stage rotor and in the second stage, and it would be useful to see if this higher heat flux leads to increased thermal creation.

In the RANS calculations presented here, the specific heat ratio of the flow exiting the combustor and the coolant flow were equal. Changes in efficiency due to differing specific heat ratio are possible through mixing [23] and from gas path heat transfer. Future calculations could quantify the lost turbine work in a cooled high pressure turbine flow from 4 physical mechanisms: (i) viscous dissipation, (ii) thermal creation from heat flux with pressure gradient, (iii) thermal creation with heat flux from specific heat ratio difference, and (iv) effect from mixing gases with different specific heat ratio. Unlike thermal creation from static pressure difference, no mechanical work potential source term for thermal creation from specific heat ratio difference has been identified.

The calculations in this thesis assumed adiabatic walls. In the real situation, heat
transfers from the main gas path through the vane and rotor and into the internal cooling flow. The internal cooling air is at higher pressure than the flow through the main gas path, and the heat transfer results in positive thermal creation. Miller [19] identified a potential +0.5% increase in turbine efficiency from this process. Further, the specific heat ratio of coolant air is higher than the specific heat ratio of the hot combustion products in the main gas path. Thermal creation from heat transfer due to the specific heat ratio difference could result in an additional increase in turbine efficiency. It would be useful to assess this situation.
Appendix A

Thrust Increase from Mixing

Abstract

This note presents a description of the physical ideas that underpin the thrust benefits of mixing of two streams of different stagnation temperatures. The context is mixing of the core and fan streams in an aero engine and the commensurate thrust specific fuel consumption benefit, but the ideas apply to the effect of mixing on exit momentum and kinetic energy flux for general nozzle flows.

A.1 Introduction

In section 2.5, it was observed that mixing of two streams with different stagnation temperatures can provide a thrust benefit. In a turbofan, the core and fan streams can be mixed via lobed mixers, as sketched in Figure A-1, which imply a three-dimensional flow with streamwise vorticity as the cause of the rapid mixing. For the present purpose, however, the mixing process will not be described and we will consider the flow as two parallel streams, denoted by subscript $f$ and subscript $c$ (for fan and core) that mix between the entry and exit stations of a constant area duct, prior to passage through a nozzle.

The configuration to be analyzed is given in Figure A-2. The fan to core mass flow ratio, $\dot{m}_f/\dot{m}_c$, known as the bypass ratio, is denoted by $\sigma$. 

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Figure A-1: Mixing of fan and core streams in an aero engine propulsion system.

Figure A-2: Sketch of the mixing duct and nozzle, with nomenclature.
We present two different views of the mixing process. In the first we show the change in thrust for a small increment of mixing, in other words, a small change in the exit conditions from the unmixed situation when an increment of mixing is allowed. In the second we derive the ratio of thrust with complete mixing to thrust with no mixing, for the general situation. Finally, we show how changes in propulsive behavior due to mixing impact the overall efficiency and its component parts, the thermal and propulsive efficiencies.

A.2 Incremental Thrust Changes Due to Mixing

To present the process with minimum complexity, we assume the mixing takes place at Mach numbers small compared to unity, specifically \( M^2 << 1 \). The stagnation pressure changes associated with mixing scale as \( \Delta p_t < \rho u^2 = \gamma p M^2 \), where \( u \) is a representative velocity at the duct inlet. The fractional changes in stagnation pressure are thus \( \Delta p_t/p << 1 \) and the mixing duct exit stagnation pressure can be taken equal to the mixing duct inlet stagnation pressure; the consequence is that, for an ideally expanded nozzle, the exit Mach number is unchanged between the non-mixing and mixing situations. We also take the stagnation pressure of the two streams to be the same. This is not a necessity, but doing so shows the crux of the issues with fewer manipulations. The stagnation temperature ratio between the core and fan streams, \( T_{t,c}/T_{t,f} \), is denoted by \( TR \), and the working fluid is a perfect gas with constant specific heats.

As a metric for increase in thrust, we use the gross thrust, the sum of the momentum flux at nozzle exit. The thrust with no mixing, i.e., with separate nozzles, is denoted by \( F \),

\[
F = \dot{m}_c u_c + \dot{m}_f u_f, \tag{A.1}
\]

where \( u_c \) and \( u_f \) are the velocities of the core and fan streams at the nozzle exit station, \( e \).

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\(^1\)The stagnation pressure changes actually scale with the square of the difference in velocities rather than the absolute velocities so this limitation is conservative.
We now let a small amount of mixing occur, keeping the mass flows in the two streams constant\(^2\), and determine the incremental thrust change due to this mixing. The stagnation temperature of each stream at the exit of the mixing duct, station 2, also change. This results in a change in nozzle exit velocities. The gross thrust with a small amount of mixing is,

\[
F + dF = \dot{m}_c(u_c + du_c) + \dot{m}_f(u_f + du_f). \tag{A.2}
\]

The thrust change due to mixing is,

\[
dF = \dot{m}_c du_c + \dot{m}_f du_f. \tag{A.3}
\]

The exit velocity can be expressed in terms of the exit Mach number and the stagnation temperature of each stream. The stagnation-to-static pressure ratio at the exit is the same for both streams so the exit Mach number is also the same, \(M_e = M\). The velocity is related to the stagnation temperature as, for the core stream,

\[
u_c = \frac{M}{\sqrt{1 + \frac{\gamma - 1}{2} M^2}} \sqrt{\gamma R T_{t,c}}. \tag{A.4}
\]

A similar expression exists for the fan stream with the exit Mach number, \(M\), the same for both streams. Taking logarithmic derivatives of Equation A.4 leads to relations between the fractional changes in exit velocity and stagnation temperature, for core and fan stream, from the mixing,

\[
\frac{du_c}{u_c} = \frac{dT_{t,c}}{2T_{t,c}} \tag{A.5}
\]

\[
\frac{du_f}{u_f} = \frac{dT_{t,f}}{2T_{t,f}}. \tag{A.6}
\]

The changes in core and fan stream stagnation temperature are related by the steady

\(^2\)We need to specify how the comparison is made; there are different possibilities. For comparison at constant nozzle area, for example, the mass flow will change and there will not be a thrust benefit for the mixed nozzle (Section 2.5). For comparison at constant mass flow, as done here, the nozzle area at station e must change.
flow energy equation as
\[ \dot{m}_c T_{t,c} + \dot{m}_f T_{t,f} = \dot{m}_c (T_{t,c} + dT_{t,c}) + \dot{m}_f (T_{t,f} + T_{t,f}) , \] (A.7)

or, using the definition of bypass ratio,
\[ dT_{t,c} = -\sigma dT_{t,f} . \] (A.8)

We can rewrite the expression for the increment in thrust, \( dF \), as

\[ \frac{dF}{\dot{m}_f u_f} = \frac{1}{\sigma} \frac{du_c}{u_f} + \left( \frac{du_f}{u_f} \right) . \] (A.9)

From Equation A.4,

\[ \frac{u_f}{u_c} = \sqrt{\frac{T_{t,f}}{T_{t,c}}} = \sqrt{\frac{1}{TR}} . \] (A.10)

Using Equations A.5, A.6, A.8, and A.10 in Equation A.9 yields an expression for non-dimensional thrust,

\[ \frac{dF}{\dot{m}_f u_f} = \frac{dT_{t,f}}{2T_{t,f}} \left( 1 - \sqrt{\frac{1}{TR}} \right) . \] (A.11)

The change in fan stagnation temperature is positive, and \( TR \) is greater than one, so there is an increase in thrust due to the mixing.

The reason for the thrust increase can be seen by comparing the two ‘components’ of \( dF \), \( \dot{m}_c du_c \), and \( \dot{m}_f du_f \):

\[ \dot{m}_f du_f = \left( \frac{\dot{m}_f u_f}{2} \right) \left( \frac{dT_{t,f}}{T_{t,f}} \right) ; \] (A.12)

\[ \dot{m}_c du_c = -\left( \frac{\dot{m}_f u_f}{2} \right) \left( \sqrt{\frac{1}{TR}} \frac{dT_{t,f}}{T_{t,f}} \right) . \] (A.13)

The exit momentum flux of the cold fan flow increases more than the exit momentum.

\[^{3}\text{We use the fan stream quantities in the non-dimensionalization because most of the thrust is associated with fan stream momentum.}\]
flux of the hot core stream decreases.

### A.3 Thrust Benefit for Complete Mixing

Using the assumptions made in Section A.2, we can also determine the change in thrust for complete mixing of the two streams, which is the case of maximum thrust benefit for given stagnation temperature ratio, $TR$. To do this we carry out a control volume analysis for complete mixing from station 1 to station 2 (mix), in Figure A-2.

Conservation of mass is

$$m_{mix} = m_c + m_f.$$  \hspace{1cm} (A.14)

Conservation of energy is, for a perfect gas with constant specific heats,

$$m_{mix} T_{t, mix} = m_c T_{t, c} + m_f T_{t, f}.$$  \hspace{1cm} (A.15)

Combining Equations A.14 and A.15 yields

$$\frac{T_{t, mix}}{T_{t, f}} = \frac{TR + \sigma}{1 + \sigma}$$  \hspace{1cm} (A.16)

where $TR$ is the ratio of core stream stagnation temperature to fan stream stagnation temperature, $T_{t, c}/T_{t, f}$, and $\sigma$ is the bypass ratio.

The assumption of mixing at Mach numbers much less than unity means that the changes in both static and stagnation pressure are small compared to the absolute level of stagnation pressure and can be neglected in defining the exit Mach number. Conservation of momentum thus reduces to the statement that the stagnation pressure at station 2 can be taken as equal to the stagnation pressure at station 1.

The exit Mach number is a function of the ratio of exit stagnation pressure to exit static pressure, which is not changed by the mixing. Following from Equation A.5, the nozzle exit velocity is thus $u_c \propto \sqrt{T_{t, c}}$ and $u_f \propto \sqrt{T_{t, f}}$ for the unmixed core and
fan streams and $u_{\text{mix}} \propto \sqrt{T_{t,\text{mix}}}$ for the mixed nozzle. The mixed thrust is

$$F_{\text{mixed}} = m_{\text{mix}} u_{\text{mix}} \propto m_{\text{mix}} \sqrt{T_{t,\text{mix}}}.$$  (A.17)

The thrust in the unmixed configuration is

$$F_{\text{unmixed}} = m_c u_c + m_f u_f \propto m_c \sqrt{T_{t,c}} + m_f \sqrt{T_{t,f}}.$$  (A.18)

Substituting Equations A.14 and A.15 in Equation A.17 and A.18 gives the ratio of thrust for the mixed nozzle flow to thrust for the unmixed nozzle flow,

$$\frac{F_{\text{mixed}}}{F_{\text{unmixed}}} = \frac{\sqrt{(T R + \sigma)(1 + \sigma)}}{\sqrt{T R + \sigma}}.$$  (A.19)

Equation A.19 is the analytic, low Mach number expression used to plot thrust increase from mixing in Figure 2-15 with $\sigma$ set as a function of the stream stagnation temperature ratio. For a temperature ratio, $T R$, equal to unity, Equation A.19 shows there is no thrust increase. The ratio of mixed thrust to unmixed thrust is independent of exit Mach number, but it should be recalled that the assumption is that pressure changes due to mixing are very much less than the static pressure drop in the nozzle, so that the former can be neglected. The assumption is not appropriate if the pressure change in the nozzle is of the same order as the pressure change in the mixing duct.

### A.4 Effect of Mixing on Efficiency

#### A.4.1 Overall Efficiency

The overall efficiency of the engine, $\eta_O$, is characterized by the ratio of thrust power for vehicle propulsion, $u_0 F_{\text{net}}$, to the chemical energy input from burning of the fuel, $m_{\text{fuel}} \Delta h_{\text{fuel}}$, the product of fuel flow and the heat of combustion per unit mass,

$$\eta_O = \frac{u_0 F_{\text{net}}}{m_{\text{fuel}} \Delta h_{\text{fuel}}}.$$  (A.20)
The thrust used is the net thrust, i.e., the net force the engine exerts on the vehicle. From Equation A.20 we see that if the thrust increases and the fuel flow remains the same, the overall efficiency increases; mixing increases overall efficiency.\footnote{The overall efficiency is inversely proportional to the thrust specific fuel consumption (TSFC); higher overall efficiency means lower TSFC.}

The overall efficiency change can be separated into the two different processes of (i) conversion of chemical energy into mechanical energy and (ii) use of the mechanical energy to propel the vehicle [15] [3]. The first of these is referred to as the thermal efficiency, $\eta_{th}$, and the second as the propulsive efficiency, $\eta_p$.

$$\eta_o = \eta_{th} \eta_p = \left( \frac{\text{Net Mechanical Energy}}{\dot{m}_{fuel} \Delta h_{fuel}} \right) \left( \frac{u_0 F_{net}}{\text{Net Mechanical Energy}} \right). \quad (A.21)$$

The net mechanical energy produced is the difference between the incoming and the exit kinetic energy flow. This is, neglecting the small fuel mass flow, \( \dot{m}(u_e^2 - u_0^2)/2 \), where the velocities are those at exit and far upstream stations, respectively. It is useful to examine the changes in both $\eta_{th}$ and $\eta_p$ due to mixing.

### A.4.2 Thermal Efficiency

For constant mass flow through the engine the rate of fuel burn and the rate of incoming kinetic energy are fixed. Hence the only quantity that can change the thermal efficiency is a change in exit kinetic energy flow due to mixing. For each of the streams, and for the mixed stream, the kinetic energy per unit mass is related to the stagnation temperature by

$$u_e^2 = 2c_p(T_t - T_e) = T_t \left[ 2c_p \left( 1 - \frac{1}{PR \frac{\gamma}{\gamma - 1}} \right) \right]. \quad (A.22)$$

In Equation A.22 the quantity $PR$ is the stagnation to static pressure ratio at the exit, which is constant based on the assumptions we have made. The quantity in brackets is unchanged between unmixed and mixing situations, and we can thus write Equation A.22 as $u_e^2 = \beta T_t$, where $\beta$ represents the (constant) bracketed quantity.
From Equation A.15 and Equation A.22, the mass flow of kinetic energy is unaffected by the mixing process and the thermal efficiency is thus not changed due to mixing.

A.4.3 Propulsive Efficiency

From the previous section, the net mechanical energy, which is the denominator in the definition of propulsive efficiency, is unchanged by mixing. As a result, the propulsive efficiency varies directly as the net thrust. We have seen that mixing increases the gross thrust; the fractional net thrust increase will be larger than that for the gross thrust. For the situation examined, mixing increases the overall efficiency of the engine through an increase in propulsive efficiency, rather than any change in thermal efficiency, and the increase in thrust occurs at a constant rate of mechanical energy production. The behavior is thus different from the set of conditions in which mixing does not occur and the only way to increase thrust for a given mass flow is to increase the exit kinetic energy per unit mass. In that (much more familiar example) both the thrust and the mechanical energy increase, leading to a decrease in propulsive efficiency, according to the Froude form of the propulsive efficiency [3],

\[
\eta_{prop} = \frac{m u_0 (u_e - u_0)}{m (u_e^2 - u_0^2)/2} = \frac{2}{1 + (u_e/u_0)}.
\]  

(A.23)
Bibliography


