Exploration of Configurations of Wave Energy Converters to Mechanically Drive a Seawater Uranium Harvester

by

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Abstract

Nuclear power accounts for about 20% of the electricity generated in the United States today [29], but conventional reserves of terrestrial uranium are estimated to be depleted within the century [19]. Fortunately, an estimated 4.5 billion tonnes of uranium exists as ions in the ocean [28], and a system of adsorbent polymers has been designed to extract the uranium. A proposed machine to harvest seawater uranium, the Symbiotic Machine for Ocean Uranium Extraction (SMORE), is fixed to a floating wind turbine and requires 550 kW to power four nets of shell enclosures containing the adsorbent and the chemical processing required to remove the uranium and reuse the polymer [16]. Given the high energy density of ocean waves, this thesis explores the potential of wave energy converters to provide the power requirements of SMORE. Each net requires 92 kW of power, or about 1100 kNm to drive continuous movement at 0.087 rad/s. This thesis found that harnessing that amount of power would require a heaving buoy of 11.5 m diameter and 1 m height, though the large geometry and range of motion caused structural concerns. In contrast, a pitching buoy of 4.7 m diameter and 2 m height could provide the same amount of power, and the structure could be more easily reinforced with only one moving body. Various configurations of pitching buoys are discussed as well. While this thesis defined a first order approximation of a future system, the modeling of realistic sea states and several mechanical optimizations need to be explored further. The integration of some electronics to power the chemical processing tanks and optimize the response control of the buoy may also provide benefit at a small increase in cost. Using a wave energy converter reduces not only the power load on the turbine, but also may decrease the incident wave loads and stabilization requirements of the turbine [13]. Further cost analysis is required, but a future implementation of this wave energy converter could add great value to both the uranium harvesting system and floating wind turbine.

Thesis Supervisor: Alexander Slocum
Title: Professor of Mechanical Engineering
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Chapter 1

Introduction

1.1 Motivations for Seawater Uranium Harvesting

The fission of one gram of uranium-235 can theoretically produce as much energy as burning 1.5 million grams of coal [3], which speaks to the potential that nuclear energy has to significantly reduce global carbon emissions. Currently, nuclear power accounts for about 20% of the 4.08 trillion kilowatthours (kWh) of electricity generated annually in the United States [29]. However, the 7.6 tonnes of conventional terrestrial reserves of uranium are estimated to be depleted within a century [19], after which uranium would have to be sourced from lower quality sites that require higher extraction costs and have a greater negative impact on the environment. Additionally, land resources of uranium are not well distributed, resulting in price uncertainty, while the reprocessing of spent fuel for use in breeder reactors has nuclear proliferation issues.

Fortunately, uranium is present at a concentration of 3 ppb in the ocean [21], which amounts to 4.5 billion tonnes of uranium over the total volume of the world’s oceans, or over 500 times the size of conventional terrestrial reserves [28]. While the extraction of uranium from the ocean has been researched for decades, a 2013 report by Kim et al. identified the most promising technology to date to be adsorption by chelating polymers [15]. When cycled into the ocean, the adsorbent polymers bind to the uranyl ions present in the water. After being submerged for a predetermined time, the adsorbents are removed and placed into chemical processing tanks where they are...
eluted in a strong acid bath or a bicarbonate bath to strip away the bound elements. Elution frees the ligand sites of the polymer, then an alkali wash regenerates the functional groups for reuse so that the polymer can be resubmerged to adsorb more uranyl. This project is part of a nationwide consortium of universities and national labs and is supported by the US Department of Energy’s Office of Nuclear Energy.

1.2 Current Uranium Harvester System

Developing the mechanical component of this multi-institution system is PhD candidate Maha Haji of the Precision Engineering Research Group at MIT under Professor Alexander Slocum. The functional requirements of her system are as follows [16].

1. Utilize the amidoxime polyethylene braid adsorbent developed by Oak Ridge National Laboratory known as A18 for the harvest of uranium.

2. Recover 1.2 tonnes of uranium from seawater annually, enough for a 5 MW nuclear power plant.

3. Bring the cost of uranium obtained via seawater extraction as close to that of terrestrial mining as possible (currently approximately $80/kg-U).

The current concept for the Symbiotic Machine for Ocean uRanium Extraction (SMORE) is shown below in Figure 1-1, and consists of ball-chains of adsorbent driven by rollers above the water, cycled 120 m down into the water and back up into a chemical elution tank located at the top platform. There are four belts equally spaced around the turbine. Because the chemical adsorbent is mechanically fragile, it is stored in small chunks within the shell enclosures shown in Figure 1-1a. The adsorbent is still exposed to the seawater, but is mechanically protected by the spherical enclosures. Thus the mechanical and chemical properties of the adsorbent and the ball-chain can be decoupled [11].

The first two functional requirements are achieved by optimizing the chemical adsorption process. According to Saito et al. (2014), the interaction between the uranyl and the adsorbent is characterized by a one-site ligand saturation model [24].
Figure 1-1: CAD of the Symbiotic Machine for Ocean Uranium Extraction (SMORE) [16]. (a) shows the shell enclosures (white) that protect the chemical adsorbent (purple) [11]. (b) shows the rollers attached to the floating wind turbine, with the rollers in closer view on the right. The spherical enclosures form a ball-chain belt that engages with the holes in the roller.
Uranium uptake $C_0$ after an exposure time $t$ days is given by

$$C_0 = \frac{\beta_{\text{max}} t}{K_D + t}$$  \hspace{1cm} (1.1)

where $\beta_{\text{max}}$ is the saturation capacity in grams of uranium per kilogram of adsorbent, and $K_D$ is the half-saturation time in days. Per Gill et al. in 2016, for adsorbent tested in $20^\circ\text{C}$ water, $\beta_{\text{max}} = 6.86$ g-U/kg-ads and $K_D = 37$ days [8]. Unfortunately, after pre-deployment conditioning of the adsorbent, the actual adsorbent capacity is then reduced to 90% of the theoretical capacity. Further, after each elution cycle, the adsorbent loses an additional $d\%$ of its capacity due to damage from the chemical bath, resulting in a capacity of

$$C_n = 0.9C_0(1 - d)^n$$ \hspace{1cm} (1.2)

after the $n^{\text{th}}$ elution cycle. For initial design calculations, $d$ was estimated to be about 5% per cycle.

The output of the adsorbent model can be seen in Figure 1-2. In the green model, the uranium was harvested after submersion periods of 30 days, whereas in the blue model, the uranium was harvested every 60 days. After 120 days, a 30 day cycle would adsorb 10.25 g-U/kg-ads while a 60 day cycle would adsorb 7.45 g-U/kg-ads. The adsorption kinetics are fastest when the adsorbent’s ligands are all free and initially submerged. The decrease in the average daily recovery rate is shown in Figure 1-2b. As a result, the 30 day submersion cycle extracts more uranium over time than the 60 day cycle. However, due to practical reasons and degradation it is not possible nor optimal to harvest the uranium every day. In particular, elution also requires time on the order of hours and causes degradation in the polymer. Assuming adsorbent degradation of 5% per elution cycle and an ocean temperature of $20^\circ\text{C}$, an optimal cycle to harvest 1.2 tonnes of uranium annually would last 23 days, occur about 15 times over the course of a year, and require approximately 45 tonnes of adsorbent [10].

The last functional requirement is achieved by reducing the capital and operations cost of the mechanical system. In order to reduce mooring cost, which was a large
Figure 1-2: Uranium recovery of the adsorbent A18 over time. In (a), the green lines represent a harvest cycle of 30 days and the blue lines represent a cycle of 60 days. The dashed lines show the kinetics of the adsorption reaction, whereas the solid lines show the total uranium extracted from the water. Discounting adsorbent preparation and degradation, after each 30 day cycle, 2.76 g-U/kg-ads are obtained, whereas after each 60 day cycle, 3.82 g-U/kg-ads are obtained. After 120 days, a 30 day cycle would adsorb 10.25 g-U/kg-ads while a 60 day cycle would adsorb 7.45 g-U/kg-ads. Reaction kinetics are fastest when all the ligands of the polymer are free at the beginning of the cycle. Thus the uranium adsorbed over time is greater for the 30 day cycle. Figure (b) also shows this by plotting the averaged daily recovery rate which decreases as cycle time increases.
proportion of the cost in the past, the system is attached to a deep-water wind turbine. Furthermore, the system autonomously cycles the adsorbent through the ocean and the elution tank continuously to reduce the need for frequent trips to retrieve and elute the adsorbent, hence reducing labor and fuel costs.

The current 1/10 physical scale prototype is powered by a battery, and the full design requires 550 kW for all mechanical and chemical systems. A single ball-chain belt requires approximately 92 kW to cycle continuously, so a total of 368 kW is required for each system just to drive the belts. Since the system is situated in open waters, a tangible method of reducing the electrical power requirement is to mechanically drive the belt via oscillating wave energy converters (WECs). Even though the buoy would have a sinusoidal velocity profile, the period of a wave is on the order of 10 sec, whereas the harvest time is 23 days and the chemical processing time is 27 hours. Therefore variations in submersion times would be on the order of seconds and be hardly noticeable. In the example velocity profile shown in Figure 1-3, the time-averaged velocity comes within 1% of the desired output velocity after about 50 seconds, which is much less than either the elution or the submersion times.

![Figure 1-3: Belt output velocity averaging over time. In this example of rectified sinusoidal output motion with a period of 10 seconds, the time-averaged velocity comes within 1% of the setpoint velocity after about 50 seconds.](image)

Furthermore, due to the high density of water relative to other mediums of renewable energy transfer, such as the wind, power is very concentrated in the ocean,
and a wave energy device would not have to be very large in order to achieve the drive power requirement. The uranium harvester is currently designed to attach to deep water spar buoy floating wind turbines, and the most viable location in the United States for such a wind farm is off the coast of northern California. In the region of consideration, using data obtained from National Data Buoy Center Station 46012, the power per unit width of the wave front is approximately 20 kW/m, and this number grows further from shore. Another 600 km away from shore the power per unit width of wave front approaches 25-30 kW/m [20]. Wave energy converters are generally capable of capturing the energy of a wave many times their own characteristic size if designed to interact with the wave optimally. Thus, it is valuable to explore whether or not an oscillating wave energy converter mechanically coupled to the ball-chain belt drive can feasibly and effectively offset power requirements.

1.3 Wave Energy Resource

Waves are formed by the transfer of energy from the wind blowing over the water surface. When waves are fully developed in deep waters, these surface motions can be described by circular patterns, referred to as orbital motion (illustrated in Figure 1-4). Also annotated on Figure 1-4 is the wavelength $\lambda$ and the height $H$ of the wave. Other important values with respect to identifying waves are the wave period $T$, wave number $k = 2\pi/\lambda$, and wave frequency $\omega = 2\pi/T$. Power is concentrated near the surface and is characterized by an exponentially decreasing function

$$I(z) = I_0 e^{2kz} = I_0 e^{4\pi z/\lambda},$$

(1.3)

where $I$ is the wave power flow intensity [7]. As a result, the size of the orbital motions decreases with increasing depth. The power in the top $z_0$ meters of the water can be found via integration of $I$ from 0 to $z_0$ so the power also scales with $e^{4\pi z/\lambda}$. Thus 50% of the wave’s power is concentrated in the top 0.055$\lambda$ of the wave. Meanwhile, above 0.25$\lambda$, over 95% of the wave’s power is accessible. Thus, a wave energy converter with a limited submersion depth can access most of a wave’s power.
Figure 1-4: Illustration of orbital motion in fully developed deep water waves. Wave intensity decreases exponentially with depth (green line) and as a result, very little motion occurs past about a third of a wavelength below the still water line.

Another phenomenon which contributes to the high potential power absorption density of a wave energy device is called the capture width. This will be discussed more in Section 2.5.3, but the notable result is that an oscillating device can actually absorb the power of a wave front width many times greater than its own width if it is well designed to destructively interfere with the incoming waves. Specifically in the seas of interest off the coast of California, the capture width of an optimal design can be up to 25 times the device size.

Though it is convenient to think of waves as perfect sinusoidal motions, waves are actually the sum of many sinusoids of varying heights, periods, phases, and directions superimposed upon each other. The combination of sinusoids at any point in time is known as the sea state, and due to its random nature, many of the important parameters involved in wave analysis are actually statistically calculated. An example of a 2D sea state of superimposed sinusoids is shown in Figure 1-5. The most notable parameters of a sea state are the significant wave height $H_s$, zero up-crossing period $T_z$, and dominant wave period. The significant wave height is defined as four times the standard deviation of the wave displacement from the still water level, or four
times the root mean square (rms) of the water surface. More practically, this is approximately the average height of the top third of waves. Thus, most waves are actually shorter than the significant wave heights, but the larger peaks are close to $H_s$ and tend not to significantly surpass $H_s$. The zero up-crossing period $T_z$, often called the average wave period, is the average time between successive upward crossings of the mean water level. Meanwhile, the dominant wave period is the period of the wave component with the most energy. The dominant wave period is often 25-50% longer than the average wave period. This data, including the distribution of significant wave heights with respect to wave periods, is accessible from the National Data Buoy Center, run by the National Oceanic and Atmospheric Administration [20].

### 1.4 Design Requirements

Based on the likely deployment location of this uranium harvesting system and from the system specifications of the most current prototype, the following design requirements have been specified for this project.

1. Design for system deployment off of the coast of California in waters about 200m in depth.

2. Provide 1100 kNm of torque to drive the output shaft at 0.087 rad/s (0.83 rpm).
1.5 Thesis Overview

This chapter has outlined the motivation and function of the Symbiotic Machine for Ocean uRanium Extraction (SMORE). It has also highlighted the potential of wave energy conversion to satisfy the power requirements of SMORE. In Chapter 2, the basic principles of designing a wave energy converter will be explained using a prototypical cylindrical buoy that heaves, pitches, and/or surges in incident waves. Then in Chapter 3, three design concepts are compared using first order wave response and structural analysis, two of which are heaving buoy systems and one of which is a pitching buoy system. In Chapter 4, the pitching buoy system is chosen for configuration analysis with the SMORE, while further modeling and design optimizations are discussed. Finally, Chapter 5 explores how a design can be extended to power the processing of adsorbent in addition to the driving of the ball-chain belt. It also explains the potential benefit of a WEC to both the uranium harvester and the wind turbine.
Chapter 2

Wave Energy Conversion Design Theory

As explained intuitively by Johannes Falnes, a good wave absorber must also be a good wave maker [6]. In order to absorb energy, the waves generated by the motion of the oscillating device must destructively interfere with the incident wave such that the energy of the wave is reduced and transferred to the device.

Waves are defined by a coordinate system in which the direction of wave propagation is considered the $x$ direction, and is referred to as the surge direction. The $y$ direction is referred to as sway, and the $z$ direction is referred to as heave. Because a

Figure 2-1: Standard coordinate system of wave dynamics. Surge is defined in the direction of the incident wave.
real sea state consists of many wave directions, sway and surge are often defined at the discretion of the designer, and the directions of the waves are defined by an angle $\theta$ deviation from the defined $x$ axis.

For any single mode of oscillation, the maximum theoretical absorption of a symmetric body is 50% [6]. Though a fraction of the wave is absorbed, the rest is radiated outwards from the body as a result of its oscillation (consider the body as a wave generator now). This is shown in Figure 2-2, where (a) shows an undisturbed incident wave, (b) shows the optimal heave response, and (c) shows the optimal pitch response. The response in (d) requires both a heaving and pitching motion. Stephen Salter rose to great significance in the 1970’s because his nodding duck design (Figure 2-3), which had a sufficiently asymmetric pitching body that produced a coupled heave and pitch motion, came close to 90% efficiency using this principle.

Figure 2-2: Optimal heave and pitch response for an arbitrary body [6] (a) An undisturbed incident wave. (b) Wave response to motion of an axisymmetric body in heave, causing symmetric radiation. (c) Wave response to motion of an axisymmetric body in pitch, causing antisymmetric radiation. (d) Body response to an axisymmetric body heaving and pitching optimally to absorb 100% of the incident wave energy. Note that the wave response in (d) is the sum of the input wave (a) and both response waves (b) and (c).

However, the reactive control scheme necessary to reach these theoretical optimal efficiency levels would require full knowledge of the past, present, and future shape of the incident waves, which is difficult with the many environmental conditions that
Figure 2-3: Illustration of operation of Salter's Duck made by Green the Future [9]. With a sufficiently asymmetric pitching body, Salter was able to achieve nearly 90% efficiency with an incoming sinusoidal wave.

influence wave development. Additionally, though the theoretical efficiency of Salter's duck is impressive, the shape of the body must be optimized based on the shape of the incoming wave and may not work well in all sea states present.

Another way to maximize output is to design the body’s resonant frequency. A small device operating at resonance is capable of creating waves larger than the incident wave amplitude and can thus generate and absorb as much power as a larger device operating far from resonance. Large wave energy converters have broader bandwidths than smaller ones. Unfortunately, creating a system at a larger scale often requires greater manufacturing costs. As a result, several control schemes are also being developed to compensate by shifting the body’s natural frequency and widening its resonant range. Latching control, for example, is briefly discussed later in Section 4.4.3.
2.1 Wave Power Availability

In order to gauge how much power can be generated by a WEC of a specific characteristic dimension, it is valuable to understand how much energy is in the ocean. The energy per unit area of sea surface is described by

\[ E = \rho g \int_0^\infty S(f)df = \frac{\rho g H_s^2}{16}, \quad (2.1) \]

where the density of seawater \( \rho \) is approximately 1029 kg/m\(^3\) and \( S(f) \) is the wave spectrum in m\(^2\)/Hz. The spectrum describes the contribution of each wave frequency to the wave energy. There are several kinds of spectrum fits that can be used to describe a sea state, such as the Pierson-Moskowitz spectrum and the JONSWAP spectrum, but for the simplicity of the first order approximations to be calculated here, sea states are assumed to be 2D and the result of a single sinusoidal exciting wave.

The amount of power per unit width is then \( P = c_g E \) where \( c_g \) is the group velocity, or the velocity of energy transport. In deep waters, \( c_g = g/2\omega = gT/4\pi \), which means

\[ P = c_g E = \frac{\rho g^2 H_s^2 T_e}{64\pi}, \quad (2.2) \]

where \( T_e \) is the energy period of the spectrum. In reality, the energy period is slightly greater than the zero up-crossing period \([1]\), but the latter is more accessible, so \( T \) will be used instead with the understanding that \( T \) will slightly underestimate power.

Recall from Section 1.3 that wave intensity decreases exponentially with submersion depth, and that power above a depth \( z_0 \) is characterized by the integral of \( I(z) \),

\[ P(z_0) = \int_{-z_0}^0 I_0 e^{2kz}dz = \frac{I_0}{2k}(1 - e^{2kz_0}) = \frac{I_0}{4\pi} \lambda(1 - e^{4\pi \frac{z_0}{\lambda}}). \quad (2.3) \]

This relationship shows that a significant amount of the power is accessible by a small oscillating body. Wavelengths in this region tend to be on the order of 50-100 m, so a body of draft 5 m experiences between 50-70% of the power going into the wave, albeit without accounting for the actual dynamics of the system which is discussed...
later in Section 2.5.

![Figure 2-4: Power available above a non-dimensionalized depth, or $z_0/\lambda$](image)

2.2 National Data Buoy Center Station Data

In order to optimize a WEC for a specific operating location, it is important to understand the likely wave conditions in the area. As mentioned previously, the National Data Buoy Center has decades of data available from buoy stations all over the world [20].

In this project, the uranium harvester is planned to be deployed on a deep-sea wind farm off the coast of northern California. Deep sea wind farms are typically located in waters greater than 200 meters in depth. Thus, as a representative location, the design of this WEC will be specialized for NDBC Station 46012 at Half Moon Bay. The buoy is located at 37.363N 122.881W, or about 45 km SSW of San Francisco, in a water depth of 208.8 m. The annual significant wave height and average wave period are shown in Figure 2-5. Note the variation in wave height and period between the summer and winter.

Another valuable data visualization technique is to plot the frequency distribution of simultaneous significant wave heights and average wave periods. This is how a sea
Figure 2-5: NDBC monthly (a) Significant Wave Height and (b) Average Wave Period data for Station 46012. The edges of the boxes show the 25th, 50th, and 75th percentiles, while the black lines show the overall maximum and minimum values.

state is often represented, with power contours annotated over the plot. In Figure 2-6a the frequency concentration is shown in blue while the power contours are overlaid in green. Figure 2-6b shows a heat map of the power of each sea state multiplied by its probability of occurring. By adding up all of the power * probability terms, it is possible to determine the annual expected power output, which for this location is approximately 20 kW/m.
Figure 2-6: NDBC annual (a) sea state and (b) power distributions for Station 46012. In (a) the frequency of each sea state is labelled and contoured in blue, while the power of that sea state is contoured in green. In (b), the probability and power are multiplied to form a heat map of expected power. The darkest green regions show the intersection of the most frequent and most powerful sea states. The sum of values in the heat map produces the expected annual power, which is approximately 20 kW/m in this region.

2.3 Overview of Wave Energy Converter Types

There are multiple ways to categorize WECs, but the most basic is by size and orientation. Consider $L_x$ to be the characteristic length of the WEC in the direction of the incident wave and $L_y$ to be the characteristic length of the WEC perpendicular to the incident wave direction. Point absorbers often have $L_x, L_y << \lambda$ Otherwise, the WEC
Figure 2-7: Types of wave energy converters. Shown in yellow is a point absorber, in red is an attenuator, and in orange is a terminator.

can be considered a line absorber. Attenuators are characterized by $L_x \approx \lambda, L_y << \lambda$, and terminators by $L_y \approx \lambda, L_x << \lambda$. An attenuator reduces the height variations of the waves underneath it by damping changes in height. Meanwhile, terminators and point absorbers rotate and bob such that the wave exiting the terminator is much smaller than the incident wave (see curve (d) of Figure 2-2). Another key difference between the three WECs is that a point absorber is not directional, whereas attenuators and terminators have an optimal operating direction. However, the oscillation mode of attenuators and terminators has a greater wave capture potential, as discussed in Section 2.5.3. The most notable attenuator is probably the Pelamis (Figure 2-8a). Salter’s Duck is considered a terminator (Figure 2-8b). Finally, there are many implementations of point absorbers, but a typical concept is similar to Ocean Power Technology’s PowerBuoy (Figure 2-8c).

Another way to categorize WECs is by how their energy is transferred. Most WECs harvest energy from mechanical oscillation. Line absorbers oscillate in pitch and roll, while point absorbers most often oscillate in heave. The other two notable energy harvest types are overtoppers and oscillating water columns. Overtoppers

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<tr>
<th>WEC Type</th>
<th>$L_x$</th>
<th>$L_y$</th>
<th>Directional?</th>
<th>Oscillation Mode</th>
<th>Notable Example</th>
</tr>
</thead>
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<td>Point absorber</td>
<td>$\ll \lambda$</td>
<td>$\ll \lambda$</td>
<td>No</td>
<td>Heave</td>
<td>OPT PowerBuoy</td>
</tr>
<tr>
<td>Attenuator</td>
<td>$\approx \lambda$</td>
<td>$\ll \lambda$</td>
<td>Yes</td>
<td>Pitch &amp; Roll</td>
<td>Pelamis</td>
</tr>
<tr>
<td>Terminator</td>
<td>$\ll \lambda$</td>
<td>$\approx \lambda$</td>
<td>Yes</td>
<td>Pitch &amp; Roll</td>
<td>Salter’s Duck</td>
</tr>
</tbody>
</table>
Figure 2-8: Real examples of wave energy converters.
have a large basin into which wave crests can fall into, and a turbine turns when the water returns to the ocean. Oscillating water columns are vertical tubes exposed to the water at the bottom end and open to the air at the top with a Well’s turbine in between that rotates unidirectionally when the heave of the waves pushes the air in and out of the tube.

In the design presented in this thesis, it is important to keep the system small to keep costs low. Additionally, it is convenient to keep the transmission system small, which is possible with simple oscillating bodies which would move at a rate similar to the desired output speed. As a result, the WECs to be explored further will be point absorbers and limited-width pitching terminators.

2.4 Assumptions and Simplifications

In the initial design process, several assumptions and simplifications about the input waves will be made.

First is the deep water assumption, which is generally estimated as depth $> \frac{1}{3} \lambda$. Recall Figure 1-4, in which the intensity of the wave decreases exponentially with distance below the water line. By a third of a wavelength below the surface, only about 1.5% of the orbital motion continues to exist, so the interaction between the wave and the bottom of the ocean is negligible. Second, the long wavelength approximation, in which the body’s characteristic length $L \ll \lambda$, allows the scattering effects of waves to be neglected.

Next, the sea state is assumed to contain only linear waves, with no higher order terms. This is valid for sufficiently small amplitude waves. Waves generally start to break when the height to wavelength ratio exceeds $1/7$ [4]. In the location of study, the significant wave height is about 2 m while the wavelength is about 80 m, so the ratio is about $1/40$.

Finally, the major simplification in this design study is the assumption that all incident waves are sinusoidal. Recall the root mean square definition of $H_s$ where $H_s = 4 \times rms(\text{wave surface})$. With the assumption of a sinusoidal input, the root
mean square is equivalent to \( A/\sqrt{2} \) where \( A \) is the wave amplitude, so \( 2A = H = H_s/\sqrt{2} \).

### 2.5 Device Excitation Response

A structure placed in a sea of waves faces three types of load: drag, inertia, and diffraction - drag due to separation of wave flow, inertia due to pressure gradients that arise from fluid acceleration, and diffraction due to the scattering of incident waves when they interact with the body [4]. The equation of motion of a body of mass and moment of inertia matrix \( M \) in the water can be described by the sum of forces

\[
M\ddot{x} = F_{exc} + F_{rad} + F_{PTO} + F_b + F_m + F_{HO}
\]  

(2.4)

where \( \dot{x} \) is the displacement vector of individual components \((x, y, z)\), \( F_{exc} \) is the excitation force matrix, \( F_{rad} \) is the radiation damping force matrix, \( F_{PTO} \) is the power takeoff force matrix, \( F_b \) is the buoyancy restoring force matrix, \( F_m \) is the mooring force matrix, and \( F_{HO} \) are higher order forces required to fully characterize the response [17].

The excitation force is the integral of the fluid pressure over the wetted surface of the body

\[
F_{exc} = \iint_{SB} p\hat{n}ds,
\]

(2.5)

where \( p \) is the fluid pressure at a point, \( \hat{n} \) is the normal vector of the body surface at that point, and \( ds \) is an infinitesimal point on \( SB \), the body's surface boundary. Pressure can be found via \( p = -\rho \frac{\partial \phi}{\partial t} \) where \( \phi \) is the wave potential. For the assumption of a sinusoidal wave,

\[
\phi = \Re \left\{ \frac{igA}{\omega} e^{ikx + i\omega t} \right\}.
\]

(2.6)

Radiation damping resists body motion and is of the form

\[
F_{rad} = -A(\omega)\ddot{x} - B(\omega)\dot{x},
\]

(2.7)
where $A$ is the added mass matrix and $B$ is the wave damping radiation matrix. In a realistic randomized sea state, the radiation force would have to be calculated via convolution in order to account for the fluid memory effect on the body.

The power takeoff force refers to the amount of force that the power output system exerts on the body. This requires characterization of the power takeoff, but then the generalized power absorbed from the wave is

$$P_{PTO} = -F_{PTO} \ddot{x}. \quad (2.8)$$

The buoyancy force is the hydrostatic restoring force, which can be modelled as a spring

$$F_b = -C \ddot{x}, \quad (2.9)$$

where $C$ is the hydrostatic restoring coefficient matrix.

Mooring forces are necessary because WECs need to be connected to a reference against which the body moves, which is often the seabed. However, a large rigid structure is susceptible to extremely destructive forces in storms, which is why cables with carefully designed amounts of slack are used instead. They can be modelled as

$$F_m = -K_m \ddot{x} - C_m \ddot{x}. \quad (2.10)$$

In this case, the WEC will be attached to a wind turbine and the structure needs to be rigid in order to avoid being thrown into the turbine. The attachment method would theoretically not limit the body in the oscillating mode direction assuming the waves are small enough not to hit any hard limits of motion, so this term will be ignored for now. The turbine has its own mooring forces as well, which should also be considered in future models in addition to forces on the structure between the turbine and the WEC, but again will be neglected for now.

The higher order terms $F_{HO}$ are further contributions that are often necessary to correlate analytical models to experimental results, but they include quadratic damping forces that complicate the system. In particular, the fluid viscosity adds drag to the system and will cause some overestimation of energy absorption, and the
Morison Effect says that fluid forces on a slender body also include pressure effects from potential flow. These forces will not be discussed further at this stage, though they should not be ignored later in the design process.

By rearranging the non-neglected forces above, the linearized equation of motion is taken to be

\[(M + A)\ddot{x} + B\dot{x} + Cx = F_{exc} + F_{PTO}.\] (2.11)

Unfortunately, the most reliable way to obtain the hydrodynamic coefficients \(A\), \(B\), and \(C\) of an arbitrary shape is by experimentation or by numerical analysis via panel methods with a software such as WAMIT. A cylinder is one of the only shapes with mostly analytically determinable hydrodynamic properties. Additionally, cylinders and other rounded shapes have significantly less damping than those with corners at which wave-body interaction discontinuities appear, which allows them to have the greatest response amplitude. Thus, the rest of the calculations below will assume a cylindrical buoy is the oscillator in this WEC.

Figure 2-9: Parameters of cylindrical buoy under consideration, where \(D\) is the buoy diameter, \(H\) is the buoy height, and \(T\) is the buoy draft or submersion depth.

### 2.5.1 Oscillation in Heave

To begin, consider a floating buoy with no power takeoff system attached. For an axisymmetric floating buoy, heave motion \(\xi_3\) is uncoupled from all other oscillation
modes, so the equation of motion simplifies to

\[(M + A_{33})\ddot{\xi}_3 + B_{33}\dot{\xi}_3 + C_{33}\xi_3 = X_3\]  

(2.12)

where \(M\) is the body mass, \(A_{33}\) is the 3D heave added mass, \(B_{33}\) is the damping coefficient, \(C_{33}\) is the hydrostatic restoring coefficient, and \(X_3 = F_z\) the heave excitation force.

First, the heave exciting force can be found by the definition of exciting force as written in Equation 2.5, which expands in the \(z\) direction to

\[F_z = \Re \left\{ -i\omega \rho \int_{SB} (\phi_I e^{i\omega t} + \phi_D e^{i\omega t}) n_3 ds \right\},\]  

(2.13)

where \(SB\) is the wetted surface of the body, \(\phi_I\) is the incident wave potential and \(\phi_D\) is the diffraction wave potential, both with the time dependent terms already pulled out.

From our existing definition of the incident wave potential in Equation 2.6, we can write

\[\phi_I = \frac{igA}{\omega} e^{kz-ikx}\]  

(2.14)

\[-i\omega \phi_I = -i\omega \frac{igA}{\omega} e^{kz-ikx} = gA e^{kz-ikx}.\]  

(2.15)

but since the body is in deep water with long waves, \(k \to 0\), Equation 2.14 can be approximated such that \(-i\omega \phi_I \approx gA\). And because \(\int_{SB} \phi_I n_3 ds = A_w = \frac{\pi D^2}{4}\), the first term of \(F_z\), known as the Froude-Krylov force, is

\[F_z^{FK} = \Re \left\{ \rho g A e^{i\omega t} \frac{D^2}{4} \right\}.\]  

(2.16)

To find the diffraction component, or latter portion, of the force, the body boundary condition on the buoy surface must be satisfied, where

\[\nabla (\phi_D + \phi_I) \cdot \vec{n} = 0.\]  

(2.17)
Thus,

\[
\frac{\partial \phi_D}{\partial n} = -\frac{\partial}{\partial n} (\phi_I) = -(\vec{n} \cdot \vec{\nabla}) \left( \frac{igA}{\omega} e^{kz-ikx} \right),
\]

(2.18)

and if \( e^{kz-ikx} \) is approximated by the Taylor series to only keep terms with low orders of \( k \),

\[
e^{kz-ikx} \approx 1 + (kz - ikx),
\]

(2.20)

which gives the result

\[
\frac{\partial \phi_D}{\partial n} = -\frac{igA}{\omega} \left( n_1 \frac{\partial}{\partial x} (1 + (kz - ikx)) + n_3 \frac{\partial}{\partial z} (1 + (kz - ikx)) \right)
\]

(2.21)

\[
= -\frac{igA}{\omega} (-ikn_1 + kn_3)
\]

(2.22)

\[
\rightarrow \frac{\partial \phi_D}{\partial n} \propto k.
\]

(2.23)

This leads to the result that \( \phi_D \) will also scale with \( k \) so by the long-wavelength approximation of \( k \to 0 \), then \( \phi_D \to 0 \) as well.

Thus, the total heave excitation force is

\[
F_z = \Re \left\{ \rho g A e^{i\omega t} \pi \frac{D^2}{4} \right\}.
\]

(2.24)

Next, the remaining coefficients in the equation of motion (Equation 2.12) can be calculated for the buoy based on its dimensions.

The added mass of a floating buoy in long waves \( A_{33} \) can be approximated as half of the surge added mass of a 2D disk of the same diameter.

\[
A_{33} = 0.0525 \pi \rho D^3.
\]

(2.25)

The hydrostatic restoring coefficient is

\[
C_{33} = \rho g A_2 = \rho g \pi \frac{D^2}{4},
\]

(2.26)
and the damping coefficient can be approximated by the Haskind relation

\[
B_{33} = \frac{k}{4\rho g c_g} \left| \frac{X_3}{A} \right|^2, \tag{2.27}
\]

where \(c_g = g/2\omega\), \(A\) is the wave amplitude, and

\[
X_3 = F_z = \Re \{X_3 e^{i\omega t}\} \rightarrow X_3 = \rho g A \pi \frac{D^2}{4} \tag{2.28}
\]

Since the forcing is assumed to be sinusoidal, the body’s response in heave is of the form

\[
\begin{align*}
\xi_3 &= \Re \{\Xi_3 e^{i\omega t}\} \tag{2.29} \\
\dot{\xi}_3 &= \Re \{i\omega \Xi_3 e^{i\omega t}\} \tag{2.30} \\
\ddot{\xi}_3 &= \Re \{-\omega^2 \Xi_3 e^{i\omega t}\} \tag{2.31}
\end{align*}
\]

which allows the equation of motion (Equation 2.12) to be rewritten as

\[
(-\omega^2 (M + A_{33}) + i\omega B_{33} + C_{33}) \Xi_3 = X_3, \tag{2.32}
\]

where \(\Xi_3\) is the response amplitude operator, and can be isolated to predict the response of the system in the frequency domain.

\[
\Xi_3 = \frac{X_3}{-\omega^2 (M + A_{33}) + i\omega B_{33} + C_{33}}. \tag{2.33}
\]

The undamped, freely vibrating system \((B_{33} = 0)\) has a maximum response when the remainder of the response denominator \(-\omega^2 (M + A_{33}) + C_{33}\) reaches zero. Thus the system’s resonant frequency is then

\[
\omega_n = \left( \frac{C_{33}}{M + A_{33}} \right)^{1/2}, \tag{2.34}
\]

and this will be the frequency at which the system has the greatest response. The normalized response amplitude at this frequency is

\[
\frac{\Xi_3}{A} = \frac{|X_3|}{\omega \frac{k}{4\rho g c_g} \left| \frac{X_3}{A} \right|^2} = \frac{2\rho g^3 A}{\omega^4 |X_3|}. \tag{2.35}
\]

38
It is interesting to note that the modulus of the heave response is inversely proportional to the modulus of the heave exciting force because damping increases quadratically with excitation force.

### 2.5.2 Oscillation in Surge and Pitch

For an axisymmetric floating buoy, surge and pitch are coupled to each other due to hydrostatic and inertia effects, such that

\[
(M + A_{11})\ddot{\xi}_1 + (A_{15} + I_{15})\ddot{\xi}_5 + B_{11}\dot{\xi}_1 + B_{15}\dot{\xi}_5 = X_1 \tag{2.37}
\]

\[
(I_{55} + A_{55})\ddot{\xi}_5 + (A_{51} + I_{51})\ddot{\xi}_1 + B_{55}\dot{\xi}_5 + B_{51}\dot{\xi}_1 + C_{55}\xi_5 = X_5 \tag{2.38}
\]

where \( X_1 = F_x \) is the surge exciting force and \( X_5 = M_5 \) is the pitch exciting moment.

First, the surge exciting force can be found via G.I. Taylor’s formula using strip theory, because the body is partially submerged. This amounts to

\[
dF_x = \left( \rho V + a_{11} \right) \frac{\partial u}{\partial t} \bigg|_{x=0} \tag{2.39}
\]

where \( a_{11} \) is the 2D surge added mass of the body and \( u \) is the velocity of the wave and can be found from the velocity potential described in Equation 2.6 by the relationship

\[
u = \frac{\partial \phi}{\partial x}, \tag{2.40}
\]

Because the buoy is operating in deep water, the dispersion relation is given by \( \omega^2 = kg \) such that

\[
u = \Re \left\{ \frac{igA}{\omega} e^{ikz-ikx+i\omega t} \right\}. \tag{2.41}
\]

The 2D surge added mass for a cylindrical buoy is given by \( a_{11} = \pi \rho \left( \frac{D}{2} \right)^2 \) and the 2D volume of the cylinder is given by \( \forall = \pi \left( \frac{D}{2} \right)^2 \). Combining these into Equation 39.
2.39 yields
\[ dF_x = \frac{\rho \pi D^2}{2} \Re \{ i \omega^2 A e^{i\omega t} e^{kz} \}, \]  
(2.43)
and the full surge force can be found by integration along the draft of the buoy
\[ F_x = \int_{-T}^{0} \frac{\rho \pi D^2}{2} \Re \{ i \omega^2 A e^{i\omega t} e^{kz} \} dz \]  
(2.44)
\[ = \frac{\rho \pi D^2}{2} \Re \{ i \omega^2 A e^{i\omega t} \left( \frac{1 - e^{-kT}}{k} \right) \}. \]  
(2.45)

Meanwhile, the pitch exciting moment is given by
\[ M_5 = \int dF_x z dz \]  
(2.46)
\[ = \int_{-T}^{0} \frac{\rho \pi D^2}{2} \Re \{ i \omega^2 A e^{i\omega t} e^{kz} \} z dz \]  
(2.47)
\[ = \frac{\rho \pi D^2}{2} \Re \{ i \omega^2 A e^{i\omega t} \left( \frac{e^{-kT}}{k^2} \right) \left( 1 - kT - e^{kT} \right) \}. \]  
(2.48)

Next, the coefficients of the equation of motion (Equation 2.38), can be found. The 3D surge added mass can be approximated by vertical strip theory, such that
\[ A_{11} \approx \int_{-T}^{0} a_{11} dz = \pi \rho \left( \frac{D}{2} \right)^2 T, \]  
(2.49)
and the 3D pitch added mass can similarly be found, but with an additional \( z^2 \) factor to account for the moment about the origin
\[ A_{55} \approx \int_{-T}^{0} a_{11} z^2 dz = \pi \rho \left( \frac{D}{2} \right)^2 T^3. \]  
(2.50)
Finally, the cross coupled added mass can be found by
\[ A_{15} = A_{51} = -\int_{-T}^{0} a_{11} z dz = \pi \rho \left( \frac{D}{2} \right)^2 T^2. \]  
(2.51)

The mass moment of inertia of a cylinder pitching about its diameter (say the \( y \)-axis), is given by
\[ I_{55} = \frac{1}{4} M \left( \frac{D}{2} \right)^2 + \frac{1}{12} M H^2, \]  
(2.52)
assuming the mass is evenly distributed throughout the buoy. Meanwhile, the cross-
coupled moment of inertia is given by

\[ I_{15} = I_{51} = mz_G, \]  

(2.53)

where \( z_G \) is the distance from the center of mass to the still water line.

The damping coefficients can again be found by the Haskind relations:

\[ B_{11} = \frac{k}{8\rho gc_g} \left| \frac{X_1}{A} \right|^2, \]  

(2.54)

\[ B_{55} = \frac{k}{8\rho gc_g} \left| \frac{X_5}{A} \right|^2, \]  

(2.55)

where

\[ X_1 = \Re \{ X_1 e^{i\omega t} \} \rightarrow X_1 = \rho A \pi i \omega^2 \left( \frac{1 - e^{-kT}}{k} \right) \frac{D^2}{2} \]  

(2.56)

\[ X_5 = \Re \{ X_5 e^{i\omega t} \} \rightarrow X_5 = \rho A i \omega D^2 \left( \frac{e^{-kT}}{k^2} \right) \left( 1 - kT - e^{kT} \right). \]  

(2.57)

The cross coupled damping term must be calculated by strip theory

\[ B_{15} = B_{51} = - \int_{-T}^{0} b_{11} z dz, \]  

(2.58)

where

\[ b_{11} = \frac{|X_1|}{2\rho gc_g} \]  

(2.59)

\[ \rightarrow B_{15} = B_{51} = \frac{|X_1|}{2\rho gc_g} \frac{T^2}{2}. \]  

(2.60)

Finally, the restoring coefficient \( C_{55} \) is made up of two components due to the body's waterplane area and the body's inertia. The inertia component arises because the rotation of the buoy causes the buoyancy and gravity forces to go out of line, creating a moment on the buoy. If \( z_B \) is the distance from the center of buoyancy to the free surface and \( z_G \) is the distance from the center of gravity to the free surface, then the moment arm from the buoyancy force is \( z_B \sin \xi_5 \) and from gravity is \( z_G \sin \xi_5 \), though for small angles \( \sin(\xi_5) \approx \xi_5 \). Thus the sum of moments acting on the body is
Figure 2-10: Sketch depicting how the inertial hydrostatic pitch restoring moment $C_{55}^I$ is derived. The blue dot is the center of gravity, while the green dot is the center of buoyancy. The pair of forces create a moment on the buoy whenever the buoy is tilted by an angle $\theta$.

\[
M_0 = F_{B}r_{B} - F_{G}r_{G} = \rho g \delta (z_B - z_G) \xi_5
\]
\[
= -C_{55}^{H} \xi_5
\]
\[
\rightarrow C_{55}^{I} = \rho g \delta (z_B - z_G) \rightarrow C_{55}^{I} = \rho g \pi \frac{D^2}{4} T (z_B - z_G).
\]

The restoring coefficient due to the waterplane area is determined by

\[
C_{55}^{H} = \rho g \int_A \int_{A_{wp}} r^2 ds
\]
\[
= \rho g \int_{-\pi}^{\pi} \int_{0}^{D/2} (r \cos(\theta))^2 r dr d\theta
\]
\[
= \rho g \int_{-\pi}^{\pi} \cos^2 \theta d\theta \int_{0}^{D/2} r^3 dr
\]
\[
\rightarrow C_{55}^{H} = \frac{\rho g \pi}{4} \left( \frac{D}{2} \right)^4.
\]

Thus the total restoring coefficient in pitch is

\[
C_{55} = C_{55}^{H} + C_{55}^{I}
\]
\[
= \frac{\rho g \pi D^2}{4} \left( \left( \frac{D}{4} \right)^2 + T (z_B - z_G) \right)
\]

The resonant frequencies of the surge and pitch oscillations can again be found
by analyzing the case of undamped, free vibrations

$$
\begin{pmatrix}
-\omega^2 & \begin{bmatrix} M + A_{11} & I_{15} + A_{15} \\ I_{51} + A_{51} & I_{55} + A_{55} \end{bmatrix} + \begin{bmatrix} C_{11} & C_{15} \\ C_{51} & C_{55} \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Xi_1 \\ \Xi_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
\end{pmatrix}
$$

Noting that $C_{11}, C_{51}, C_{51} = 0$ and solving for non-trivial solutions,

$$
det \begin{bmatrix}
-\omega_n^2(M + A_{11}) & -\omega_n^2(I_{15} + A_{15}) \\
-\omega_n^2(I_{51} + A_{51}) & -\omega_n^2(I_{55} + A_{55}) + C_{55}
\end{bmatrix} = 0
$$

$$
\rightarrow \omega_n^4(M + A_{11})(I_{55} + A_{55}) - \omega_n^2(M + A_{11})C_{55} - \omega_n^4(I_{51} + A_{51})^2 = 0
$$

$$
\rightarrow \omega_n^2 \left\{ \omega_n^2 \left[ (M + A_{11})(I_{55} + A_{55}) - (I_{51} + A_{51})^2 \right] - (M + A_{11})C_{55} \right\} = 0
$$

which yields the solutions

$$
\omega_n = 0, \sqrt{\frac{(M + A_{11})C_{55}}{(M + A_{11})(I_{55} + A_{55}) - (I_{51} + A_{51})^2}}.
$$

### 2.5.3 Power Capture Width

Even though the power per unit width of wave front can be estimated from wave parameters, the value does not fully represent how much energy can be absorbed by an oscillating body of that unit width. A better measure is the capture width, which is the width of the incoming wavefront that has the same power as is absorbed by the device. The capture width is often larger than the physical dimensions of the WEC because an optimized WEC oscillates such that it produces destructive interference with the full surrounding waters, and not just within the width of the device.

The theoretical maximum energy flux that can be absorbed by a wave energy device is

$$
P_{\text{max},i} = \frac{1}{8} \frac{|X_i|^2}{B_{ii}},
$$

where $X_i$ is the modulus of the wave excitation force and $B_{ii}$ is the damping coefficient.
Assuming that viscous forces are negligible, the Haskind relations can be used to represent $B_{ij}$. Thus, the maximum energy flux in heave, surge, and pitch, respectively, are given by

$$P_{\text{max},3} = \frac{1}{8} \frac{|X_3|^2}{k} \frac{1}{4 \rho g c_g} = \frac{1}{k} \left( \frac{1}{2 \rho g A^2 c_g} \right) \rightarrow P_{\text{max},3} = \frac{\lambda}{2 \pi} P_{\text{in}}, \quad (2.76)$$

$$P_{\text{max},1} = \frac{1}{8} \frac{|X_1|^2}{k} \frac{1}{\delta \rho g c_g} \frac{1}{X_1 A^2} = \frac{2}{k} \left( \frac{1}{2 \rho g A^2 c_g} \right) \rightarrow P_{\text{max},1} = \frac{\lambda}{\pi} P_{\text{in}}, \quad (2.77)$$

$$P_{\text{max},5} = \frac{1}{8} \frac{|X_5|^2}{k} \frac{1}{\delta \rho g c_g} \frac{1}{X_5 A^2} = \frac{2}{k} \left( \frac{1}{2 \rho g A^2 c_g} \right) \rightarrow P_{\text{max},5} = \frac{\lambda}{\pi} P_{\text{in}}. \quad (2.78)$$

Since the wavelengths in the region of interest are on average 80m long, in heave the maximum possible energy absorption of the WEC is about 13 times the power in the portion of the wave front approaching the WEC directly. Or in other words, the device can harvest the power of a wave with width 13 times its own characteristic dimension. Meanwhile, in surge and pitch, the maximum possible energy absorption is over 25 times the power of the directly incoming wave, i.e. the device can harvest the power of a wave of width 25 times its own size.

This increased capture width makes devices that oscillate in surge and pitch extremely attractive, except that systems in surge and pitch often have increased stability challenges, in which the device can capsize in storms if a failsafe is not designed in. However, if the system is designed robustly, these types of WECs have a great potential to harvest large magnitudes of power in a small package.

### 2.6 Power Output Model

Most WECs output their energy to an electric generator with controllable current draw characteristics that determine the power output of the system and the force
exerted on the buoy. These electrical systems often have a force profile of

\[ F_{PTO} = -K_{PTO}x - C_{PTO}\dot{x} \]  

(2.79)
similar to a linear spring damper system, though some may also impose an inertia term depending on the control strategy. Using this force profile in Equation 2.8 and noting that \( x \) is perfectly out of phase with \( \dot{x} \),

\[ P_{PTO} = K_{PTO}x\dot{x} + C_{PTO}\dot{x}^2 = C_{PTO}\dot{x}^2. \]  

(2.80)

However, in the designs presented in this thesis, the power takeoff force is physically coupled to the mechanical system being driven, and should instead be modelled as the combination of forces required to drive the ball-chain belt.

\[ \tau_{drive} = I\dot{\omega} + \frac{1}{2}C_D\rho\omega^2 + \tau_{friction}, \]  

(2.81)

where \( \tau_{drive} \) is related to \( F_{PTO} \) via the mechanical transmission system. In the real system, the torque required to move is dominated by friction, which allows for a convenient simplification of the system to \( \tau_{drive} \approx \tau_{friction} \).

Recall the given design requirement \( \tau_{req} = 1100 \text{ kNm} \), which was actually the value of \( \tau_{friction} \). In the simplest typical case, if the velocity profile looks like a rectified sinusoid with no velocity smoothing, similar to that in Figure 1-3, then the peak angular velocity required to reach an rms velocity of 0.087 rad/s is 0.123 rad/s, and the maximum angular acceleration assuming a motion period of \( T_z = 7.2 \text{ s} \) is 0.107 rad/s². The ball-chain rollers are made of 316 Stainless Steels tubes of outer diameter 2.3 m, inner diameter 2.1 m, and length 6.6 m [16], leading to a total mass moment of inertia on the order of 90,000 kg m². This means that at maximum, the inertial term would contribute about 9.5 kNm to the total torque requirement, or less than 1% of the friction term. Similarly, \( c_D \approx 1 \) and \( A \approx 2 \text{ m}^2 \) for the ball-chain belt, so the drag term is only on the order of 15 Nm, or much less than 0.01% of the friction term.

Thus, in the equation of motion, the ball-chain belt drive will be modelled as a constant \( F_{PTO} \) opposing the direction of motion. All WEC designs in consideration
are fully rectified at the output, meaning that both the positive direction motion and
the negative direction motion cause forward driving output. This means that $F_{PTO}$
will always be opposite the sign of $\xi_t$, and if the sum of the exciting force and existing
system forces is not greater than the required driving force to move in that direction,
the system will not move.

Unfortunately, this model means the system response cannot be analytically de-
termined. Instead an iterative function will update the values of $\xi_t, \dot{\xi}_t, \ddot{\xi}_t$. An example
of a response profile is shown in Figure 2-11. The vertical jumps in the resultant force
are of size $F_{PTO}$.

![Figure 2-11: Example device response to system PTO, where the exciting force is
the sinusoid caused by the incident wave, and the resultant force is the force that is
applied to the buoy after the ball-chain belt force requirements are met.]

For first order approximations, a mechanical transmission efficiency of 90% would
be reasonable between the buoy and the drive output. This is typical for various belt
and chain drive systems, and would bring the torque required to drive the belt up
from 1100 kNm to 1222 kNm. The power output can still be calculated from Equation
2.8 using the torque or force requirement and the body's average positive velocity,
since all motion will be rectified when transmitted from the buoy to the output drive
 shaft. The relationship between $\tau_{\text{drive}}$ and $F_{\text{PTO}}$ will be discussed in more detail in the next chapter when specific mechanical implementations are discussed.

2.7 Key Implementation Challenges of Wave Energy Converters

Even though the high power density of ocean waves makes it an attractive energy resource, there are a few large challenges that have kept wave energy from being widely commercialized.

The first major design limitation is that while the average power level of a location may be known, the nominal power capacity of the WEC must be much higher to account for the large variation in the power at any point in time. Increased power capacity comes at a high cost because the components must be better rated, e.g. an electric generator would need higher voltage and current ratings, yet the increased capacity will be unused for most of the time. Falters argues that it is more economical to keep the nominal operating point near the average, even if it results in more downtime where the buoy is unable to oscillate fully [5]. However, this also means the WEC structure must be able to take greater storm forces when the oscillating body has reached its physical limits.

This variation comes from both a difference in waves across seasons and high power peaks in storms. In Section 2.2, annual expected power output was calculated, but if the same calculation is done with only the summer months and only the winter months, the power output can range from approximately 10 kW per unit width of wave front in the summer to 31 kW per unit width in the winter. Furthermore, if the 50th percentile $H_s$ and $T_z$ conditions from Figure 2-5 are compared to 75th percentile conditions, which would likely be the result of strong winds or small storms, the power more than doubles. Unfortunately, a significant amount of expected wave power exists in these storm waves because of the high power they contain, even if they occur at low frequencies. For example, if only waves below the 75th percentile in height
Figure 2-12: Challenges in WEC design: the power available at any time in the seas varies significantly more than traditional thermal and wind energy plants [23].

are considered for energy conversion, only about 70% of the expected power output remains. However, even if the higher power operating conditions are inaccessible to the WEC, the system must still be robust enough to withstand the forces of its strongest waves.

Additionally, the sea state does not occur in clean sinusoids as simple models assume. The main dynamic challenge of optimizing wave energy converter response is the randomized sea state. When the sea is a single frequency harmonic, device response can be tuned to this frequency so that energy extraction is straightforward. However, chaotic seas reduce the ability to exploit resonance for maximum power absorption. Instead, other control algorithms have been developed, such as latching control (discussed in Section 4.4.3), to maximize device response to randomized seas.
Figure 2-13: NDBC summer months (a) sea state frequency and (b) power distributions for Station 46012. The expected average power output in the summer is about 10 kW/m.
Figure 2-14: NDBC winter months (a) sea state frequency and (b) power distributions for Station 46012. The expected average power output in the summer is about 31 kW/m.
Finally, another major challenge is the harsh operating environment of the ocean. Between corrosion due to the water's salinity and biofouling on open surfaces, it can be hard to design a system with a long lifetime that is still cost effective. Flat, relatively stationary surfaces will undergo biofouling, whereby various plants, algae, or organisms will attach themselves to open surfaces and grow on them. The ocean is also relatively conductive due to its salinity, so over time materials with dissimilar electrode potentials will undergo galvanic corrosion. This can be limiting for material choices. Corrosion also occurs between the metals and the ions in the water, slowly degrading the structural integrity of the metal. Material treatments to prevent biofouling and corrosion exist to varying effectiveness, but can significantly increase material costs.

2.8 Distinctions Between This Thesis and Other WEC Designs

While many of the challenges mentioned above are still relevant to this design project, there are some key differences that may change the direction of design optimization. The biggest difference is that this buoy will not be operating for full power absorption. Instead, the design will be for the smallest device that can achieve the power requirement. The priority is to have the simplest design that can survive the harsh conditions in order to keep costs and risks low. For this reason, a shape like Salter’s Duck would not be an optimal design because of the costs associated with the complicated shape, and because the only way to change the power absorption is by changing device width. In other words, a duck designed for power requirements this low would still need a large profile, but be very thin in width, which makes it susceptible to sway forces.

Furthermore, while most WECs include some degree of control over the current draw rate of the electric generator to maximize overall power absorption, the output of this system is purely mechanical, which changes its design optimization method.
Additionally, the design requirements are written as a torque and angular velocity, but since the input and output are coupled, the requirement can also be read as a requirement for the buoy to traverse or rotate a certain total distance over the time of each submersion cycle.
Chapter 3

Initial Design Concepts

To start the exploration of potential designs, three types of mechanisms were considered. The oscillating body in each mechanism is a cylindrical buoy to keep calculations analytically possible and comparable across designs.

3.1 Refined Design Requirements

In order to overcome the challenges mentioned in Section 2.7, two additional design constraints are imposed on the system.

1. Avoid the debilitating effects of biofouling by constraining all motion to be via rotary joints.

2. Survive storm waves up to the 99th percentile in power.

The first design requirement limits the use of linear joints, such as slides that require a smooth surface to translate. With the messy environment of the ocean, it is easy for objects to get caught and jam the slide. Furthermore, biofouling slowly accumulates over free surfaces. For example, if a linear slide were designed for a point absorber, biofouling would start at the upper and lower limits of the slide which are not traversed as frequently but then grow into the operating zone and shrink the range of motion. The need to avoid this problem also extends to rotational surfaces,
such that any mechanical transmission that requires a precise alignment or fit should not be used if it is exposed to the water, such as roller chain for example.

As a result of this requirement, three general types of mechanisms will be explored in this section: a buoy operating in heave attached to cables that drive pulleys that interface to the output, a buoy operating mostly in heave attached to the turbine by a four bar linkage, and a purely pitching buoy.

The second design requirement exists because the biggest wave in an area can carry 10-15 times more energy than an average wave. However, there is also a notable difference between a 99th percentile wave and the biggest wave ever seen. For example, consider the cumulative significant wave height and dominant wave period charts for Station 46012 (Figure 3-1). There are a few waves that make it up to 7-10 m in significant height, but they are few and far between, while 99% of waves are below 4.5-5 m significant height. Since power scales with $H_s^2$ this means power in a 100th and a 99th percentile wave varies by a factor of 4. Since these major storms would likely cause other problems with the uranium harvester, designing for the extremely infrequent maximum wave condition would be inconvenient and excessive.

Another technical detail to note is that the dominant wave period will be used for storm condition calculations instead of the average wave period. The dominant wave period is defined as the period of the wave component with the most energy. Since the dominant wave period is often around 50% greater than the average wave period, using the dominant wave period and the significant wave height serves as a useful upper bound on power conditions. Thus, the extreme conditions that the device will be expected to survive are $H_s$ up to 5 m and $T_s$ up to 17 s.

Furthermore, in order to achieve any motion at all, the buoy must be excited by a force greater than the force from the ball-chain belt that resists the motion. The following force and moment contours show the relationship between buoy dimensions and the incident wave's exciting force (Figure 3-2) and moment (Figure 3-3). Any set of parameters that would not lead to a force high enough to move the buoy is not shown, so the ragged edge along the bottom right of each contour shows the absolute design limit. In the case of the heave force, the “Gear Ratio” $N$ is the relation
Figure 3-1: NDBC cumulative (a) Significant Wave Height, (b) Average Wave Period, and (c) Dominant Wave Period data for Station 46012.
$F_{PTO} = N \tau_{drive}$ and $\omega_{drive} = N \omega_{buoy}$. This will be discussed in further detail later with respect to individual mechanisms. Note that the exciting force scales quadratically with only the buoy diameter, while the force required to move scales with $N$, which is what causes the design limit to change with respect to $N$, while the response amplitude will later scale with buoy diameter.

![Contour of heave force required to achieve motion with respect to buoy diameter and gear ratio.](image)

Figure 3-2: Contour of heave force required to achieve motion with respect to buoy diameter and gear ratio.

The pitching device design threshold also changes with respect to $N$, but the pitch exciting moment depends on the wave frequency and the buoy draft in addition to buoy diameter and gear ratio. To simplify the visual, the moment contour is shown for the average wave frequency $T_z = 7.2$ s or $\omega = 0.87$ rad/s and the design-optimized gear ratio $N = 0.235$ (see Section 3.4) where $\tau_{PTO} = N \tau_{drive}$ and $\omega_{drive} = N \omega_{buoy}$. In this contour, the ragged edge along the bottom is strictly due to buoy dimensions that are not great enough to achieve motion with the given wave frequency and buoy to belt gear ratio.
The primary metric to gauge each system is the size of the buoy required to meet the power requirements. A bigger buoy requires more material and more space, which may be difficult to get near the wind turbine because of the uranium harvesting belts. The optimization for buoy size occurred by minimizing size, while keeping the power output between the middle 50% of average wave periods close to the power requirement. Additionally, to minimize risk in storm conditions, the response must drop off in the 15-25 second period range. Using the power distribution across frequencies and the minimized volume as priorities, the following first order models were made.

3.2 Cable Driving Buoy

The first design concept is a buoy that oscillates in heave. The buoy is attached to cables, which are tensioned such that they force the pulleys to rotate. The pulleys feed motion into a gearbox and full wave rectifier (a double ratchet and pawl system)
which connect directly to the output shaft of the ball-chain belt drive.

Figure 3-4: Sketch of the cable-driving buoy. (a) Side view. (b) Top view. The light blue cylinder is the heaving buoy, while the dark blue and green cylinders are the driven pulleys. The dark red arrows show the resultant motion of the system.

3.2.1 Buoy Model Results

After optimizing the buoy size by 1) placing resonance at the average \( T_z \), 2) producing an average velocity output of 0.087 rad/s with the average \( H_s \), and 3) minimizing volume, it was determined that a buoy of diameter 11.5 m was necessary. However, recalling Equation 2.24, it is important to note that due to the long wavelengths and the small size of the buoy relative to the wavelength, the draft is not a factor in the exciting force or any coefficient in the equation of motion. Instead, the draft must be chosen for practical purposes. A draft of 1 m was chosen so that the proportions of the buoy would not cause significant bending of the buoy itself. Recall that in beam bending situations with distributed loads, deflection scales quartically with beam length, while the area moment of inertia scales cubically with beam height. In this system, the cables attach to the buoy near its circumference, but if the wavelength
was short and the force was concentrated at the center of the buoy, the buoy would distort.

The design flexibility of the gear ratio between the pulleys and the output shaft is important to the ability to optimize the buoy size. Suppose the transmission system was designed such that \( \omega_{\text{drive}} = Nv_{\text{in}} \) where \( v_{\text{in}} \) is the buoy velocity. Note that there is not only a gearbox, but also a transition from linear to rotary motion, and \( N \) includes both. Hence \( N = \frac{GR}{\tau_{\text{pulley}}} \). Accordingly, the force required to move the pulley would be \( F_{\text{in}} = N\tau_{\text{drive}} \), where \( F_{\text{in}} = F_{\text{PTO}} \) as discussed in Section 2.6. A convenient result of this is that even though the torque required to turn the ball-chain belt is high, \( N \) can be designed to be less than 1 so that the force resisting the buoy, \( F_{\text{PTO}} \) is reduced and the response is increased. With a buoy of diameter 11.5 m and draft 1 m, \( N \) was optimized at 0.284. The ratio of the gearbox itself would be \( N\tau_{\text{pulley}} \).

Using these results, Figure 3-5 was the response to an input exciting wave of height \( H_{s,\text{avg}}/\sqrt{2} \) and period of \( T_{z,\text{avg}} \).

Figure 3-6 shows the frequency domain response, and it is plotted against wave period on the \( x \)-axis rather than wave frequency because most of the reported wave data is with respect to period. Note that the response to 25th, 50th, and 75th percentile wave heights (1.4 m, 1.9 m, and 2.6 m) are plotted, and that the 25th, 50th, and 75th percentile periods are 6.1 s, 7.2 s, and 8.3 s respectively. The actual ball-chain belt will be driven at \( Nv_{\text{response}} \), where \( N = 0.39 \), and is shown on the right \( y \)-axis.

A convenient result of this frequency response is that high power storm waves with long periods, say the top 10\% of average wave periods from 10 s up to 17 s, respond almost half as much as the 50th percentile average wave period. However, storm waves also tend to have higher wave amplitudes. An example of a storm wave with \( H_s = 5 \) m and \( T_z = 10 \) s is shown in Figure 3-7.

To begin discussion of storm wave response, recall the heave oscillation equation of motion (Equation 2.12). It is recreated below with all of the hydrodynamic coefficients substituted in, and ignoring \( F_{\text{PTO}} \) because it is negligibly small compared to the
Figure 3-5: Time response of the 11.5 m diameter, 1 m draft cable driving buoy to an exciting wave of average $H_s$ and $T_s$.

exciting forces of storms. The resulting motion is also essentially sinusoidal.

\[
\left(0.0525\pi \rho D^3 + \pi \rho \frac{D^2}{4} T\right) \ddot{x} + \frac{\omega^3}{2\rho g^3} \left(\rho g \pi \frac{D^2}{4}\right)^2 \dot{x} + \left(\rho g \pi \frac{D^2}{4}\right) x = \rho g A \pi \frac{D^2}{4} \tag{3.1}
\]

\[
\rightarrow \left(0.0525D + \frac{T}{4}\right) \ddot{x} + \frac{\omega^3 \pi D^2}{32g} \dot{x} + \frac{g}{4} x = \frac{gA}{4}. \tag{3.2}
\]

The first important note is that the damping term scales cubically with wave frequency. Storm waves occur at wave frequencies about 1/2 of typical wave frequencies, which means that the damping of the buoy in storms would be about 1/8 of typical damping. This means the response amplitude will increase dramatically. This is not obvious from Figure 3-6 because the response amplitude is divided by the longer response period. However, there is a greater chance of hitting the physical limits of the cable system, and the force will be greater if the unhindered response amplitude is
Figure 3-6: Wave frequency response of the cable driving buoy, but plotted against the wave period for consistency with reported data. The 25th, 50th, and 75th percentile $H_s$ responses are shown, and the 25th, 50th, and 75th percentile $T$ are 6.1 s, 7.2 s, and 8.3 s respectively.

much greater than the physical height of the system.

The second point to note is the resonant frequency of the buoy. Written with respect to the geometric parameters,

$$\omega_n = \sqrt{\frac{g}{4}} \frac{1}{\sqrt{0.0525D + 0.25T}},$$

which indicates that the resonant frequency is more than twice as sensitive to the buoy draft than the diameter. Changing $T$ can have a great effect on the resonant frequency of the buoy without significantly changing the exciting forces and overall power, which may be useful for shifting the resonance away from storm frequencies if they are a big problem.
3.2.2 Structural Considerations

A system with large range of motion would reduce the storm wave forces on the structure in the heave direction and increase the response capability of the buoy. Unfortunately, the taller the system, the more cable tension is required to keep the buoy in place. Instead, the range will be made to accommodate for 90% of the occurring significant wave heights, or about 3 m. Since the significant wave height is defined as approximately the average of the top third of waves, that means about 5/6 of 90%, or about 75%, of waves will be under 3 m. Statistically, there will be some waves with $H_s < 3$ m with individual wave crests above 3 m, but there will also be waves with $H_s > 3$ m and individual crests below 3 m. Taking the expected value of the probability vs. power distribution with only waves of $H_s$ less than or equal to 3 gives about 14.5 kW/m, or about 73% of the original prediction.

For this 3 m range system, if three cables are used like in the sketch in Figure 3-4, based on a force balance in the case of storm surge and sway forces which can
reach 120 kN, each cable would have to withstand about 125 kN of tension in order to limit drifting of the buoy to less than 0.25 m. If the range of the system were increased or the allowable parasitic motion decreased, the cable tension would also increase approximately proportionally. If the required supporting forces can no longer be satisfied purely by the cable’s stretching, the rest must be compensated by cable pretension.

An example system implementation may use 1.5" Novablue marine grade rope, which has an average tensile strength of 78,000 pounds, which is equivalent to 347 kN. Based on the material properties of the rope, to have a resisting force of 125 kN the rope would have to be stretched 7.5%. The elongation of the rope if the buoy drifts 0.25 m is only about 1.4%, so the rest must be accounted for by the preload. The cable would need about 61 kN of preload.

Due to the nature of cable drives, there would also be an amount of hysteresis associated with the difference in tension above and below the buoy associated with the buoy changing directions. While hysteresis would decrease for systems with greater pre-tension, the power dissipated due to friction in the pulleys would increase. However, the most important design factor remains keeping the cables on the pulley.

The other important structural consideration is the ability to survive storm waves when the buoy is forced in heave to the top and bottom limits. A 99th percentile storm wave as defined in the refined design requirements are $H_s = 5$ m and $T_s = 17$ s, and without any motion limitations would have a response amplitude of approximately 3.7 m. Fortunately, this is not significantly more than the allowable range, which means the force on the structure will only be a fraction of the magnitude of the excitation force, which is nearly 1850 kN.

To estimate the amount of structure required to survive this force, consider the free body diagram in Figure 3-8. The area of concern for yield failure is the rigid attachment to the turbine, where the stress due to bending moment is highest. Approximate the system as having the cross section shown in Figure 3-8b at the turbine interface. A rectangular cross section was chosen so that the structure can also act as an enclosure for the power transmission system including the pulleys and the belt.
or chain that connects to the uranium ball-chain belt drive output.

The horizontal neutral axis of the cross section with the dimensions shown is the vertical centroid of the profile:

\[
\begin{align*}
z_{NA} &= \frac{z_{NA1}A_1 + z_{NA2}A_2}{A_1 + A_2} \\
&= \frac{(H/2 + D)(2(b + h)t - 2t^2) + H/2(2(B + H)T - 2T^2)}{(2(b + h)t - 2t^2) + (2(B + H)T - 2T^2)},
\end{align*}
\]

where \(z_{NA1}\) is the position of each beam’s neutral axis (thin dotted line) and \(A_i\) is each beam’s area, while \(z_{NA}\) is the position of the overall neutral axis denoted by the thick dashed line.

Accordingly, the area moment of inertia that resists bending across the horizontal
neutral axis is:

\[ I_{xx} = \int \int (z - z_{NA})^2 dA \]  
\[ = 2 \int_y^{y+h} t z^2 dz + \int_y^{y+t} (b - 2t)z^2 dz + \int_{y+h-t}^{y+h} (b - 2t)z^2 dz \]  
\[ + 2 \int_Y^{Y+H} T z^2 dz + \int_Y^{Y+T} (B - 2T)z^2 dz + \int_{Y+H-T}^{Y+H} (B - 2T)z^2 dz \]  
\[ = \frac{2}{3} t((h + y)^3 - y^3) + \frac{2}{3} T((H + Y)^3 - Y^3) \]  
\[ + \frac{1}{3} (b - 2t)((y + t)^3 - y^3 + (y + h)^3 - (y + h - t)^3) \]  
\[ + \frac{1}{3} (B - 2T)((Y + T)^3 - Y^3 + (Y + H)^3 - (Y + H - T)^3) \]  

where \( y = z_{NA1} - z_{NA} - h/2 \) and \( Y = z_{NA} - H \). Since this design would likely have a truss on both sides of the buoy, the \( I_{xx} \) will be double that of Equation 3.8. A similar calculation can be done for \( I_{yy} \) which resists sway forces if the two trusses are 2W distance apart:

\[ I_{yy} = \int \int (x - x_{NA})^2 dA \]  
\[ = 2 \frac{3}{3} (2t((W + b)^3 - W^3 + 2T((W + B)^3 - W^3)) \]  
\[ + \frac{2}{3} (h - 2t)((W + t)^3 - W^3 + (W + b)^3 - (W + b - t)^3) \]  
\[ + \frac{2}{3} (H - 2T)((W + T)^3 - W^3 + (W + B)^3 - (W + B - T)^3) \]  

Treating the truss as a cantilevered member with the area moment of inertia approximated as that of the cross section shown, the stress can be found via

\[ \sigma = \frac{M_{c,max}}{I} \]  

where \( c \) is the distance from the neutral axis. The material used will likely be 316 Stainless Steel or a similar alloy, because this stainless steel is typical in marine environments which require corrosion resistance. The tensile yield stress of 316 SS is about 290 MPa. Constraining the maximum stress to less than a third of the yield stress requires dimensions on the order of \( \theta = 40^\circ \), \( B = b = 0.15 \text{ m (6 in)} \), \( H = h = 0.30 \text{ m (12 in)} \), and \( T = t = 0.013 \text{ m (0.5 in)} \). The storm heave force on these beams...
would produce a deflection of about 1.3 mm (0.05 in). In reality, the diagonal member can support greater loads because of the compressive component of the force, so its dimensions would be greater in a fully designed system. With all four triangular struss segments, this comes out to around 11,000 kg of steel.

3.3 Four Bar Linkage

The next design concept is also an oscillator in heave which is constrained by a four bar linkage. Note that the buoy is attached to the linkages such that it is the heave forces that move the linkages rather than the pitching moment acting on the linkages themselves.

Figure 3-9: Side view of a buoy contrained by a four bar linkage. The buoy (light blue cylinder) is shown in two potential location along its arc. The incident wave shown puts the buoy at the upper location, while the upcoming trough would bring the buoy to the lower location.
3.3.1 Buoy Model Results

Unsurprisingly, the model optimization resulted in the same size buoy as in the previous design. However, there is a small change in how $\tau_{\text{drive}}$ relates to $F_{PTO}$.

$$\omega_{\text{drive}} = GR\omega_{\text{buoy}} = GRv_{\text{buoy}}/L,$$

where $GR$ is the gear ratio. Thus, conversely

$$F_{PTO} = \tau_{\text{buoy}}/L = GR\tau_{\text{drive}}/L.$$

Again, a 1 m buoy draft was chosen for practical reasons, so the optimized buoy is again 11.5 m in diameter with $N = GR/L = 0.284$. Figure 3-10 shows the response to an input exciting wave of height $H_{s,\text{avg}}/\sqrt{2}$ and period of $T_{z,\text{avg}}$.

![Time response of the 11.5 m diameter, 1 m draft buoy on the four bar linkage to an exciting wave of average $H_s$ and $T_z$.](image)

Figure 3-10: Time response of the 11.5 m diameter, 1 m draft buoy on the four bar linkage to an exciting wave of average $H_s$ and $T_z$.

Figure 3-11 shows the frequency domain response, again plotted against wave period on the x-axis rather than frequency. The actual ball-chain belt would be driven at $Nv_{\text{response}}$, where $N = 0.39$, and is shown on the right y-axis.
Figure 3-11: Wave frequency response of the buoy on the four bar linkage, but plotted against the wave period for consistency with reported data. The 25th, 50th, and 75th percentile $H_s$ responses are shown, and the 25th, 50th, and 75th percentile $T_z$ are 6.1 s, 7.2 s, and 8.3 s respectively.

### 3.3.2 Structural Considerations

Along a similar vein as discussed with the Cable Driving Buoy (Section 3.2), the range of motion is set at 3 m. The main challenge associated with this design is the matrix of tradeoffs required to optimize a design. For example, the buoy cannot get closer than about 4.8 m from the outside of the turbine to avoid hitting the uranium adsorbent belts. To limit the motion of the buoy, it would be ideal to have hardstops that meet the buoy (see the black hardstops in Figure 3-9) to create a closed structural loop, rather than somewhere along the linkage, which would cause high cantilevering forces. The hardstops thus would need to be at least 4.8 m long. In order to minimize the effect of buoy pitching on the linkages, the buoy should be attached to the linkages at its center. However, if the buoy ever needs to reach the lower position depicted in Figure 3-9, the linkages would have to be at least a little longer than the buoy radius.
The proportions would look more like Figure 3-12.

Figure 3-12: Figure 3-9 updated to have more realistic proportions. The dashed blue line represents the still water level. The large buoy size requires that many of the structural components are similarly large.

Figure 3-13: Variation on the four bar linkage system that is limited in range to the top right quadrant. The linkages (green) can get shorter while the static supporting member (grey) gets longer.

Another option is to change the range of motion of the buoy to only one quadrant, as in Figure 3-13, which would reduce the length of the linkages while increasing the length of the static supporting member. However, the angular range is limited by
the collision of the non-zero thickness linkages, in which case the linkages would need to be longer to decrease the angular range or the linkages would need to be thinner. Both options decrease the system’s stiffness. Since the incident waves are currently approximated in 2D, consider the storm sway forces to be equivalent to the storm surge forces but rotated by 90°. The system needs to withstand up to 1850 kN of heave forces and 120 kN of sway force.

The optimal linkage configuration maximizes linkage height while minimizing linkage length. The range of motion $2\theta$ needs to sweep a vertical distance of 3 m, so $\frac{3}{L} = \sin 2\theta$. Meanwhile, since the buoy has a draft of 1 m and the total height would be slightly greater, the vertical distance between the linkage hinges must be less than 1 m. This limits $h$ to $\frac{1}{\cos \theta}$ in the worst case situation, where the linkages are horizontal at the lower hardstop. At the lower limit, illustrated in Figure 3-14b, the maximum linkage thickness is such that the linkages collide, so the constraint at the lower limit $t < h \cos \theta$ and the upper limit $t < h \sin \theta$ must be satisfied.

![Figure 3-14: Sketches of linkage geometry. (a) shows the linkages centered in its range of motion, with $\theta$ degrees of freedom in either direction. (b) shows the physical constraint of the linkages when they collide. The thickness of each linkage must satisfy $t < h \cos \theta$.](image)

Using these constraints, a possible four bar linkage could sweep 70° and have length 3.2 m (125.7 in) and up to height ($T$) 0.63 m (24.8 in). In order to survive the sway forces with a yield safety factor of 2, the beam would have to have width 0.225 m (9 in) and tube thickness 0.025 m (1 in). Under storm sway forces, the linkages
would deflect about 50 mm (2 in). These linkages would require about 1,200 kg of steel to construct.

In this linkage configuration, the upper and lower boundaries face different forces. When the buoy is at its lower limit, the triangular truss must support the weight of the buoy, which is about 106,000 kg or over 1,000 kN when calculated using the geometry’s displaced water mass. However, it must also support the impact force of the buoy. Even in storm forces, the buoy’s response amplitude does not significantly surpass 3 m. Thus, assume the impact force is about half of the storm heave exciting force, for a total force of 1,900 kN. The location of the hinges is approximately 10.75 m away from the turbine, due to the 4.8 m length hardstop and 5.95 m radius buoy. Using the same truss profile defined in Figure 3-8b but with only one triangular truss located below the buoy, in order to keep the stress a factor of 3 below the yield stress, the profile would need to be on the order of $\theta = 30^\circ$, $B = b = 0.2$ m (8 in), $H = h = 0.5$ m (20 in), and $T = t = 0.025$ m (1 in). With this storm heave force, the truss would deflect about 19 mm (0.74 in). In comparison to the 11,000 kg of steel required in the previous design, this would only require about 6,200 kg from the bottom truss and 1,200 kg from the linkages for a total of 8,400 kg. Since the height of the rectangular tube is the dominant dimension of the structure, it may be worth using I beams instead, at least on the diagonal profile which would not be used to protect any power transmission belts.

3.4 Pinwheel Buoy

The final design under consideration is a pitching device that simply rotates about an axle. The challenge with pitching devices, however, is the likelihood of capsizing, especially for a small device acting in waves with heights almost as large or larger than it.

It would have been possible to update the previous four bar linkage design such that the main surface being excited by the waves was one of the linkages that changes angle with time, and the hardstops to prevent capsizing were the linkages hitting each
other. However, this still creates large forces on all of the moving elements. This was the key reason for the choice to move the pitching axle to the middle of the buoy and making the buoy symmetric over the water line. A huge force can overturn the buoy, but the buoy would still be able to operate as normal.

### 3.4.1 Buoy Model Results

Because there is no surge motion with the pitching axle fixed at the middle of the buoy, the equation of motion in pitch is no longer cross coupled and simplifies to

\[
(I_{55} + A_{55})\ddot{\xi}_5 + B_{55}\dot{\xi}_5 + C_{55}\xi_5 = X_5.
\]  

where the buoy height in $I_{55}$ is $H = 2T$ due to the vertical symmetry of the buoy. Note that in pitch oscillation, the buoy draft and height more actively contribute to the response compared to the buoy oscillating in heave.

Furthermore, the "$F_{PTO}$" that opposes the motion of the buoy is actually a torque in this situation, since $X_5$ is also a moment. The power output equation changes form to $P_{out} = \tau_{PTO}\omega_{buoy}$ where $\tau_{PTO} = N\tau_{drive}$, and $N$ is simply the gear ratio.

Based on the optimization of the velocity output of the buoy at the wave frequencies of interest, the optimal buoy diameter would be approximately 4.7 m with
a draft of 1 m and a total height of 2 m, while $N$ is about 0.235. Figure 3-16 was the response to an input exciting wave of height $H_{s\text{,avg}}/\sqrt{2}$ and period of $T_{z\text{,avg}}$. Figure

![Figure 3-16](image)

**Figure 3-16:** Time response of the 4.7 m diameter, 1 m draft pinwheel buoy to an exciting wave of average $H_s$ and $T_z$.

3-17 shows the frequency domain response, again plotted against wave period on the $x$-axis rather than frequency. The actual ball-chain belt would be driven at $N\omega_{buoy}$, where $N = 0.235$, and is shown on the right $y$-axis.

Note that the optimized diameter of this buoy is less than half the size of the heaving buoys, as was predicted by the relationships in Section 2.5.3. As a result, the pitching buoy is nearly 2 times smaller in volume than the heaving buoys.
Figure 3-17: Wave frequency response of the pinwheel buoy, but plotted against the wave period for consistency with reported data. The 25th, 50th, and 75th percentile $H_s$ responses are shown, and the 25th, 50th, and 75th percentile $T_z$ are 6.1 s, 7.2 s, and 8.3 s respectively.

### 3.4.2 Structural Considerations

Despite the buoy being much smaller, the buoy must survive a huge heave force because it is stationary at the still water line. The maximum heave force with a 5 m incident storm wave would cause 310 kN of force on the buoy and its structure (updated free body diagram in Figure 3-18). However, since the buoy is the only body in motion, the structure can more easily be increased in size as necessary to support the loads.

The moment acting on the structure-turbine interface is less than 10% that of either heaving buoy. As a result, a potential profile would be $\theta = 30^\circ$, $B = b = 0.05$ m (2 in), $H = h = 0.13$ m (5 in), and $T = t = 0.006$ m (0.25 in). The heave storm force would cause a deflection of about 3.2 mm (0.13 in), while the sway storm force would cause a deflection of about 0.7 mm (0.026 in). These beam profiles would amount to
$L = 5.8 \text{ m}$

$310 \text{ kN}$

Figure 3-18: Free body diagram of a storm heave force on the pinwheel system structure.

450 kg of steel as opposed to 11,000 kg or 8,400 kg from the two heaving systems. The estimation method may be underestimating the interface stress because the profiles seem suspiciously small, especially for this pitching buoy design. However, all three systems have been estimated in the same manner, and the difference in material required between the pitching buoy structure and either heaving buoy structure is between a factor of 18 and 24.

3.5 Concept Comparison

Ultimately, the pinwheel buoy appears to be the most viable option in comparison to both heaving buoys. It has the smallest volume by a factor of three, and the smallest water plane footprint by a factor of six. It also has the fewest moving pieces with a structure that faces more than ten times lower bending moments, making it the path of least risk.

In particular, the major weaknesses of the cable driving buoy is the large structure and high cable pre-tension necessary, leading to either or both power losses in friction and hysteresis in the output. The structure occupies a huge footprint and would need to be well-reinforced due to the high cable tensions.

Meanwhile, the beams in the four bar linkage design would need to be at least 3
m in length, which puts them at risk to deformation under external forces, such as storm sway forces. Additionally, if the linkage dimensions are designed such that the linkages get close to contacting each other at the ends of their range, the system runs the risk of having its motion limited by biofouling growth on the linkage faces coming in contact. The linkages would need to be designed with special materials and chemicals that limit biofouling, or the linkages would have to be periodically cleaned.

Meanwhile, the pitching buoy is the only moving component of the "pinwheel" design. While the pitching buoy does run the risk of being directionally limited, if all of the output shafts for each of the four ball-chain belt systems and their corresponding buoys are coupled, the motion should smooth out. If the diameters of each buoy is increased slightly, the extra exciting moment should be able to compensate for any buoy that is not aligned with the wave direction. Many pitching devices are at the mercy of the seas if they capsize, but ideally, the symmetry in this case should prevent that from being a problem. If it is found that the buoy cannot fully return to its horizontal equilibrium position after a large wave, a declutching mechanism can be added such that the forces from the ball-chain belt no longer resist its motion until it has returned. The next chapter will address further design considerations with respect to this pinwheel design.

Figure 3-19: Comparison of buoy volume and structural mass for each design concept.
Chapter 4

Detailed Design Discussion

4.1 Overall Expected System Output

While the previous section assumed that all waves were of the average amplitude and period, the waves occupy a spectrum as shown in Figure 2-6a. However, Figure 4-1 below shows how the significant wave height also affects the output response. Usually response scales with incident wave amplitude so the response magnitude is normalized by wave amplitude when presented. However, due to the non-linearity of the power takeoff forcing function, there is a kink in the response curve which moves with the wave amplitude. Note how the 1 m height wave does not have enough power to evoke much buoy response, and then how the kink occurs at different wave frequencies for each wave amplitude.

Based on the expected velocity outputs from various amplitude and period waves and their expected occurrence frequencies, the annual expected velocity is approximately 0.0872 rad/s, only a few percent greater than the design requirement of 0.0870 rad/s. However, every month is characterized by a different spectrum of sea states. Figure 4-2 shows approximately how the average velocity changes each month, and assuming the adsorbent submersion time varies inversely with belt velocity, Figure 4-3 shows the approximate average recovery rate each month based on the calculations from Figure 1-2b.

The monthly average belt velocity deviates from the design requirement by an
average of 27%, but because the uranium adsorption kinetics have slowed down at the uranium harvester’s designed cycle lengths, the monthly average uranium adsorption rate only deviates from the design goal by an average of 11%. When averaged over the twelve months in the year, the annual average uranium adsorption rate reaches about 0.100 g/kg-ads/day while the design goal is to average about 0.103 g/kg-ads/day, which is a difference of only about 2.4%. 

Figure 4-1: Wave frequency response of the pinwheel buoy across various wave amplitudes.
4.2 Further Modeling Necessary

A higher fidelity measurement of the actual uranium adsorption rate requires a higher fidelity model of the system. Due to the complexity of modeling wave-body interactions, which often needs to be done numerically rather than analytically, and the randomness of sea states, it is fairly difficult and impractical to start with a complete model. However, two simple first steps can be made to improve the current first order model. The first is to tune the mass distribution of the pitching buoy. Note that the
natural frequency of the buoy is

\[ \omega_n = \left( \frac{C_{55}}{I_{55} + A_{55}} \right)^{1/2}, \]  

(4.1)

and that the moment of inertia of a body is defined as

\[ I = \int r^2 dm. \]  

(4.2)

Very often, wave energy converters must have a very large profile to have a resonant frequency near its input wave frequencies. A large profile also helps to expand the bandwidth of the body near resonance.

This may explain why the buoys in the initial concept designs are so large. The optimization process forced resonance to occur at the average incident \( T_z \) because of the mechanical coupling between the buoy and the output. It is a priority to have a controllable and relatively stable velocity output at the most frequently occurring sea state conditions. The uranium adsorbent is fragile, which is the primary reason it is enclosed in spheres, and ideally its speed has a definable maximum so that it is not damaged by fast motion in big waves.

A way to significantly change the natural frequency without drastically increasing the buoy's size is to redistribute its mass so that its \( I \) increases and \( \omega_n \) decreases. This is not such an accessible option for a buoy in heave, which is another reason a pitching buoy is attractive for future work.

The next most accessible model upgrade would be to add higher fidelity power output modeling. Even though the inertia term of the belt drive was ignored in the power takeoff characterization (Equation 2.81), it can play a large role if the moment of inertia is increased. While this would increase the magnitude of exciting moment necessary to first excite the buoy into motion, a large output inertia could smooth out the velocity response much like a flywheel in a car powertrain. In fact, it might be preferable to increase the inertia of the output system in order to reduce spikes in velocity and to maintain consistent buoy motion.

However, the most important upgrade to ensure model fidelity is to measure system response to randomized sea states. Wave energy converters react fairly differently
to variations in incident waves, and these random sea states and their accompanying storm waves are the biggest practical challenge for WEC designers.

Additionally, the model presented in this thesis utilizes several first order assumptions to expedite analysis. For example, the models assume the buoy draft is constant over time, which is not necessarily true, especially if the buoy is much smaller than the wavelength and amplitude. This is particularly notable for the pitching buoy, since at the crest of the wave, the buoy with its current dimensions would be nearly submerged if the axis of rotation is at the still water line. Higher order and nonlinear effects, such as viscosity and drag, that were mentioned in Section 2.5 but ignored here should eventually also be accounted for because they can shift the resonant frequency of the entire system.

Finally, a crucial question that remains is how the existence of the wind turbine and the uranium harvester’s ball-chain belts affect the waves that approach and are reflected in the vicinity of the turbine. It is ideal to keep the buoy as close as possible to turbine to reduce the amount of structure necessary to support it, but if the turbine and belt dampen the surrounding waves, the buoy would need to be supported further out and/or increased in size. However, it is also possible that the waves reflected off the turbine increase the wave excitation of the buoy, which may also require a design change. Conversely, it is important to understand how the existence of pitching buoys would affect both the wind turbine and the uranium adsorbing belts. It could cause high disturbance forces on the belt, or it could reduce the wave loads hitting the belt and the turbine. This analysis requires having an idea of the buoy configuration around the turbine and belts.

### 4.3 Buoy and Belt Configuration Discussion

Though calculations were made assuming one full size buoy per belt, this is not the only option, and there are multiple ways to arrange four buoys around four belts. The main risk associated with pitching buoys is if the incident waves do not align relatively perpendicular to the pitching axis, causing the exciting moment to be too
low to move the buoy. Coupling the belt shafts together provides a base level of risk management if each of the buoys is oriented in a different direction. A top view of several potential buoy configurations are shown in Figure 4-4. The inner circle represents the turbine diameter, while the larger circle would be the platform for the adsorbent processing tanks. The solid rectangles are the profiles of the rollers, while the dashed boundaries represent the space occupied by the ball-chain.

Figure 4-4: Potential one buoy per belt configurations. The inner circle represents the wind turbine, while the larger circle would be the platform for the adsorbent processing tanks. The solid rectangles are the profiles of the rollers, while the dashed boundaries represent the space occupied by the ball-chain.

Another important result to understand from the model is that buoy size does not scale linearly with power absorption. Even though it may seem attractive to split the load over more buoys to reduce buoy size, the opposite occurs. The buoy size optimized to share load over 4 buoys is a diameter of 4.7 m and height of 2 m. Sharing over 8 buoys is a diameter of 7.4 m and height of 1.5 m, and over 2 buoys is a diameter of 3.3 m and height of 10.8 m.

With two buoys, the configurations would change like in Figure 4-6.

The damping of a buoy scales quartically with its diameter, so decreasing the
Figure 4-5: Buoy size variation with respect to number of buoys per turbine. The changes arise to keep the body's resonant frequency at about the same as the incident wave frequency.

diameter causes excessive response which the optimization function limits for control and safety purposes. A greater portion of wave conditions will not excite the buoy at all, while the excited buoy experiences sharp velocity spikes, which results in great imbalance in the output response of the buoy. A buoy with a large torque requirement can take advantage of this response, though it would need greater draft as well. If the height of the buoy is greater than the diameter, the height of the buoy is the factor limiting how far away from the turbine the pitching axis must be mounted. Additional modeling may show that the buoy's horizontal position becomes its equilibrium position, which would change its operating characteristics. Unfortunately, reducing the number of buoys also limits the system to waves coming from fewer directions, though it would take a better understanding of the 3D sea state to determine the effect.

Based on these observations, it is best to remain at a one buoy per belt configuration, or four buoys total configuration. These four buoys require the smallest individual operating volumes and are capable of sharing the load over four different incident wave directions.
Figure 4-6: Potential one buoy per two belts configurations. The inner circle represents the wind turbine, while the larger circle would be the platform for the adsorbent processing tanks. The solid rectangles are the profiles of the rollers, while the dashed boundaries represent the space occupied by the ball-chain. The area occupied by the buoy when fully horizontal (light blue rectangular profile) is also shown for reference.
Even though placing the four buoys in the corners between the rollers rather than outside of them would minimize the system footprint and the distances that the parts would need to be supported, it may be difficult to actually fit structure in that limited space without interfering with the belt system. For example, a support on either side of the buoy in the corners would necessarily intersect with the belts if the structure were built straight back into the turbine (see Figure 4-7, which would represent the implementation of Figure 3-15). Note that because the rollers and the buoys are located at different heights, the space underneath the rollers is usable for building structures, though the belts would still have to be avoided.

![Diagram](image)

Figure 4-7: Potential difficulties in designing the buoy structure. The buoy rotates about the black dashed line and is supported on either side. If the structure were extended straight back to the turbine, it would necessarily intersect with the belts.

There are still ways to support the system (example in Figure 4-8), but to have a closed loop structure would require connecting all four buoys. Additionally, the rotation would have to be transferred between the buoy and the belt drive shaft at 45 degree angles. While this is entirely possible, there is a 3D component which is not captured by these configuration sketches, in which the top roller is actually about 10 m above the still water line, or higher than the $H_s$ of the region. This is not to say that putting the buoys on the sides is necessarily better, but that it is also an important option because the transmission would be more straightforward.
Another benefit of coupling the drive shafts and the presence of more buoys is that the velocity of each belt is smoothed, especially if the buoys are placed such that they are likely to move out of phase with each other. When the drive shafts are coupled, they will all move as fast as the fastest buoy. However, since this fastest buoy will now have the most load on it, it will be slowed down while the other buoys catch up in speed because they no longer have a resisting force. Ultimately, all of the buoys would share the load based on which buoy has the most incident power. In order to understand how this would change the load and response dynamics of the buoys, a new model would have to be used to distribute the torque requirement across multiple buoys rather than having a fixed torque requirement. It is possible to make a simple model of this, but without a model of the 3D sea state, it is difficult to determine what kinds of incident waves would hit each buoy.

4.4 Potential Methods for Optimization

While basic shapes and configurations were considered for a first order design analysis, there is a lot of design space for improving the WEC performance while maintaining the core benefits of the design.
4.4.1 Potential Shape Variations

The most notable optimization method would be to change the shape of the system such that the pitching is about an asymmetric axis. As mentioned in the introduction of Chapter 2, a symmetric pitching device is limited to only half of the power in a wave if designed optimally. However, by pitching about an asymmetric axis, the coupled heave and pitch motions can theoretically access all of the power in a wave.

For example, the design rationale behind Salter's Duck is to perfectly oppose the incident waves while creating no aftward waves. While the concept has allowed up to 90% wave power absorption in laboratory tests, its optimal operating range is limited to the wave it is designed for. Two of the figures from Salter's Patent (U.S. 4,134,023) are included in Figure 4-9 [25]. These figures were used to explain the geometric optimizations he performed for the duck shape. In particular, he used "FIG. 3" or Figure 4-9b to explain how the the duck's curve should be determined. Recall the wave orbital diagram from Figure 1-4. The radius of orbit at depth $z$ for a wave of amplitude $A$ is $Ae^{-2\pi z/\lambda}$. If $O$ is the nodding axis of the duck, then as the line moves through an angle $\theta$ it sweeps out an area $\theta RdR$. Based on the reaction wave created by this shape to perfectly destructively interfere with the incoming wave, Salter determined that the radius of the duck's lower profile at a depth $z$ should be

$$R = Ke^{-2\pi z/\lambda},$$  \hfill (4.3)

where $K$ is fixed for $z = 2R$ (after which the radius of curvature is constant so that no aftward waves are created) for some ratio $\lambda/R$. While this is an interesting result, the point that needs to be made is that this curvature is clearly optimized for a given wave amplitude and period without consideration of the rest of the sea state.

As a result, Salter's Duck has not taken root as the universal solution to wave energy conversion. Additionally, Salter has acknowledged that the duck's specific shape has high manufacturing costs [2], and the profile cannot be scaled down easily for lower power deployment prototypes because of its wave-specific design. However, in the same paper he has also shown that a submerged cylinder rotating on an axis which is not its axis of symmetry can provide efficiencies comparable to the duck.
While this is attractive, it is now important to note that one of the key benefits of a pinwheel buoy's symmetric design is that its capsize does not affect its functionality. One of the key challenges with pitching devices is avoiding capsize, especially if the device is designed to respond dramatically to a wave to absorb the most power. If another shape can be designed to be capsize-safe like a symmetric pitching oscillator, it could arguably be more practical than a very efficient but risky device. If physical limits are placed to prevent capsize, it would be important that these hardstops be very well reinforced to survive storm waves and the collision with the oscillator.

One potential shape would be a pendulum-type design, as shown in Figure 4-10. The pendulum always eventually settles so that its concentrated mass hangs below its pivot, so even after a storm wave, the system would return to a functional state. Though there may be a resisting force due to the coupling of the pendulum to the output shaft, a declutching mechanism can be incorporated such that if the pendulum is forced past a certain angle, the pendulum is no longer coupled to the belt drive and can return without resistance.
Figure 4-10: A potentially stable asymmetric pitching device resembling a pendulum, which would always return to its functional state due to gravity even after a large storm wave.

4.4.2 Reducing Mooring Stiffness

Another useful finding that Salter made was that compliance in the mooring of a pitching device can significantly increase its power absorption. For his specific test case, he found that for wavelengths 25 times the duck diameter, a carefully designed mooring stiffness could double the power output [26]. In fact, this is why his ducks were then mounted on a large flexing spine and then the Pelamis was created because the compliant spine was optimal for harvesting power at long wavelengths.

Figure 4-11: Structural compliance in heave can also increase power output.

Another benefit of allowing the buoy's vertical position to follow the profile of
the waves rather than being fixed is that the draft of the buoy can be optimized by changing the buoy's total mass. Instead of being limited to half of the total height of the buoy, the buoy can be submerged further and face greater exciting moments. Meanwhile, the heave forces on the structure would also decrease because the structure would no longer have to fully counteract vertical forces.

However, the difficulty in maintaining a large moving linkage would return. Furthermore, Salter mentioned that creating a compliant spine reduced the power that the ducks could absorb at long wavelengths because the spine would absorb that power instead. Salter was able to harvest the power from both the ducks and the spine, but if the uranium harvester powering system remains fully mechanical, it can be difficult to determine how the angular velocities of these components should be related and coupled together, or if the buoy should only passively follow the waves while only absorbing power from the pitching motion.

4.4.3 Mechanical Latching Control

Because WECs often need a large geometric footprint to have resonant frequencies in the range of wave frequencies, many WEC designers use control methods to force the system to resonate at the incident wave frequency. By exerting a little effort, a lot more power can be absorbed. One type of control that is practically achievable with significant results is latching control. The goal of latching control is to align the time of maximum force to the time of maximum velocity. This allows the buoy to mimic the response of a buoy with a natural frequency equal to the incident wave frequency, as shown below in Figure 4-12.

Latching control can result in a power absorption increase of about five times at peak frequencies, as well as increasing the operational bandwidth of a buoy [5, 7]. As can be visually interpreted from Figure 4-12, the optimal latching time is approximately $\frac{T_{\text{wave}} - T_{n, \text{buoy}}}{2}$. It turns out that higher order effects from the shape of the new buoy response curve (c) are negligible in tuning the latching time, and in fact underlatching (latching for less time than determined from the equation) may be slightly beneficial [27].
Figure 4-12: A graphical illustration of latching control, which was originally used to explain latching in the heave direction but can also be applied to a pitch moment [5]. Curve a shows the profile of the exciting moment, curve b (solid) shows the response of a buoy whose natural frequency is the same as the frequency of the exciting moment, and curve c (dashed) shows the response of a buoy whose natural frequency is higher, but is fixed at the top of its motion using latching control. The goal of latching control is to align the time of maximum force to the time of maximum velocity.

While latching control would be easiest to execute and most efficient with an electronic system which is fully controllable, it also would be fairly straightforward to create mechanically. The requirement is simply to prevent the buoy from going in the other direction until a certain timing requirement is met. The mechanism could be similar to that of a ratching wrench, where a switch determines the direction the wrench can move in. After a time determined by a mechanical clock (e.g. a pendulum), the switch can be flipped and the buoy can be allowed to rotate in the other direction.

There are multiple implementations possible with varying mechanical complexities. For example, if a basic pendulum is used, the simplest case would be to have a fixed latching time which can be chosen based on the buoy resonant frequency and the average incident wave frequency. A pendulum can also be created such that the resulting swing time depends on the input exciting moment, which scales quadratically with the incident wave frequency. Both mechanical and numerical models would be necessary to decide whether the added complexity is worth the extra power, as is a common theme in these discussion topics.
4.4.4 Disengage Input and Output

Finally, the design of an input and output decoupling system would also bring more control over the safety of the input and output systems with the additional complexity. As the most fundamental safety mechanism, the output can be disengaged if the velocity of the ball-chain belt would be damaging to the fragile adsorbent.

With respect to safety in storms, an input declutch system could allow the buoy to go into a least energy state, say the completely vertical position, such that the storm heave forces would decrease. Whether or not the buoy would actually respond in such a way would depend on modelling of the extreme sea state response. Furthermore, it may turn out that a declutch mechanism is necessary in order to allow the buoy to return to its functional position (horizontal for the current design) without the torque resistance of the belt drive. Even though symmetry allows both sides of the buoy to be fully functional, there is the possibility of the buoy getting stuck in an in-between position where it cannot get enough exciting force to return to horizontal.

This could be solved by a declutching system that is mostly simply defined by an angular threshold above which disengagement occurs. A declutch system could be implemented similarly to a car mirror's safety declutch, in which a sufficiently high force can compress the spring and unengage the teeth that hold the mirror in place.

This could also be a useful strategy to smooth out the average output velocity throughout the year. In the winter months when the output velocity tends to be higher, the input could disengage to limit the angular response per wave. If the average velocity of the output during the rest of the year can be increased while this limitation is placed at a certain response angle that frequently is reached in the winter but not in the summer, the extra output of the winter can be removed. Though there would be power that is necessarily given up, this could be the easiest way to achieve a more even uranium harvest output if it is decided to be necessary.
Chapter 5

Conclusion

The uranium harvesting system proposed by Haji (2017), known as the SMORE, requires nearly 400 kW of power to drive the nets of adsorbent shells that cycle the chemical adsorbent through the ocean. This power requirement can be achieved by mechanically transferring the power of the waves to the ball-chain belt due to the high power density of deep water ocean waves, as suggested in Chapter 1. Chapter 2 explained the dynamics of wave energy converters, but focused on the process for analyzing cylindrical heaving and pitching buoys. In Chapter 3, three mechanical designs were explored to estimate the requirements of such a power transfer system. To achieve the declared torque and velocity requirements of 1100 kNm and 0.087 rad/s with a cylindrical buoy oscillating in heave would need to be 11.5 m in diameter and 1 m in draft, while a cylindrical buoy oscillating in pitch would only need to be 4.7 m in diameter and 1 m in draft (2 m in height). This suggests that even though pitching devices are more directionally limited than heaving ones, the potential to fit a pitching device in a tighter footprint and with less structure makes it more attractive as a solution. Variations on the pitching buoy and how it could be further optimized and explored are discussed in Chapter 4. Ultimately there are more modeling and design iterations that must be considered before an accurate feasibility report can be made. There are also significant cost and time investments that must be factored into this decision.
5.1 Further Work

As discussed more at length in Section 4.2, further modeling is necessary before the system response can be well understood. Model upgrades include inputting randomized sea states, exploring buoy shape and mass distribution, and adding higher order wave-body interactions from the buoy as well as the turbine and the ball-chain belts. Steps to better understand the output system include making a higher fidelity model of the ball-chain belt drive output and analyzing the impact of coupling all of the drive shafts to each other.

Mechanical implementation also requires determining the best way to survive the harsh ocean environment, specifically corrosion and biofouling. Many solutions exist in the marine engineering community, including materials or coatings with varying costs and levels of corrosion and biofouling resistance. Furthermore, the commercialization of the Salter Duck was prevented in part by the high manufacturing cost of the complicated optimal shape, but Salter found that a simple cylinder could achieve similar results [2]. Manufacturing costs must be considered against design optimization in future work.

Mechanical methods of system optimization were also discussed in Section 4.4, including shape optimization, heave compliance, mechanical alternatives to latching control, and input and/or output disengagement for safety and control. One of the initial design goals implicit to the simplicity of the system was to have fully mechanical power transfer. This constrained the buoy designs to respond maximally to average incident wave frequencies, but as discussed previously, buoys with resonant frequencies in the incident wave range often have large footprints. This constraint was maintained in order to protect the system by preventing excessive response at other frequencies. Analysis of the damping terms of the equation of motions in heave and pitch showed that damping decreases significantly with smaller diameter buoys, and can thus produce greater responses. Thus the volume limiting parameter was not the power requirement but actually the resonant frequency range.

However, the last two mechanical optimization concepts in particular require ad-
ditional mechanical complexity to achieve, while a viable alternative is some level of integration of electronics and control to the system. Beyond the 386 kW necessary to power the ball-chain belt drives, the uranium harvester needs an additional 160 kW to chemically process the adsorbent. This part of the system already needs electronic control and energy storage, so adding some control and energy storage to the WEC output would not significantly increase system costs and risks. While traditional latching control would require the ability to stop and start the response of the buoy and may need to be implemented at the level of the buoy, the controls can be limited to tuning the inertia, damping, and stiffness characteristics of the power takeoff system, i.e. the belt drive and the chemical tanks. Salter noted that a negative inertia was helpful for short wave response, while a negative stiffness was helpful for long wave response, but both made his duck more susceptible to capsize [26].

An example flow chart of the power distribution system is shown in Figure 5-1, and the generator's current draw control to charge the battery can be utilized to tune the power takeoff characteristics with few geometric design changes.

![Figure 5-1: Potential flow chart of power distribution with chemical tanks included. Currently only the buoy to drive shaft to adsorbent belt has been designed for, but the addition of a generator and battery to power the chemical tanks would provide the opportunity to optimize the power-takeoff characteristics to obtain more power from the existing buoy design.](image)

The addition of a controller on the output drive shaft would be able to smooth out its velocity to an extent, as well as limit it to a safe maximum if necessary. Con-
trol with complete mechanical isolation between the input buoy and the output belt drive would have several additional benefits, notably that the buoy can be optimized independently of the output drive system. Meanwhile, implementing a control algorithm such as latching control could increase the power absorption by a factor of five and stop requiring the buoy's resonant frequency to coincide with the frequency of incident waves. However, the electronics system would need to be robust enough to handle at least 600 kW of power if this were to happen, whereas including the electronics for only the chemical tanks (< 200 kW) and supplementally for buoy control optimization would not be as demanding.

5.2 Why not power the harvester directly from the wind turbine?

A next generation deep sea wind turbine such as the OC3-Hywind system sponsored by the National Renewable Energy Laboratory is rated for 5 MW [14]. The full uranium harvesting system requires 0.55 MW of power, or 11% of the wind turbine output. This would require an extra turbine for every 9-10 uranium harvesters on the wind farm in order to maintain the same total output power. Wave energy may be an attractive alternative to reduce the extra materials and manufacturing required.

Wave energy is significantly more concentrated than wind energy because water is 1000 times denser than air. In theory, a well designed device would not need to be very large. For example, a potential configuration of this wave energy system would require four buoys of 4.7 m diameter and 2 m height, whereas each turbine has a base diameter of 6.5 m and rotor diameter of 126 m. Furthermore, attaching a wave energy device to a wind turbine can reduce the wave loads on the turbine, thereby possibly reducing the amount of structural and stabilization material required for the turbine [13]. The next step should be to determine how adding certain levels of electronics and control to the buoy would affect its dimensions and performance, and then consider how the costs of the system compare to the value added.
Bibliography


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