MODELS TO PREDICT DYNAMIC RESPONSE OF MOTORCYCLES WITH A OUTRIGGER & TRAILER

by

Jimmie D. Harris, V

Submitted to the
Department of Mechanical Engineering
in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science in Mechanical Engineering

at the

Massachusetts Institute of Technology

June 2017

Signature redacted

Signature of Author: ______________________
Department of Mechanical Engineering
May 19, 2017

Signature redacted

Certified by: ______________________
Amos G. Winter, V
Assistant Professor of Mechanical Engineering
Thesis Supervisor

Signature redacted

Accepted by: ______________________
Rohit Karnik
Professor of Mechanical Engineering
Undergraduate Officer

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ABSTRACT

It is of interest, especially in the developing world, to explore the feasibility of using motorcycles in applications beyond personal transport. In particular, adding an outrigger wheel to a motorcycle may increase its capabilities for heavier duty operations like road haulage and agricultural mechanization. This thesis examines the feasibility of using motorcycles for low-speed high-weight towing and outrigger like attachments.

Two different configurations will be evaluated. The first looks at how the addition of a large trailer affects the turning ability, stability, and power delivery of a motorcycle. The trailer is modeled as a single body system consisting of a single axle and large load and to attached by a definable hitch mounted near the rear wheel of the motorcycle.

The second case examines the attachment of an outrigger like structure. This is of interest to farm like vehicles that wish to simply support a tool over a set of wheels using a motorcycle Here the motorcycle and third wheel are modeled as a single body system with a long simply supported beam that has a load applied from the the forces due to the side car.

A MATLAB model detailing the ability, stability, and power delivery of a motorcycle was created to determine these performances.

Thesis Supervisor: Amos G. Winter, V
Tile: Assistant Professor of Mechanical Engineering

Acknowledgements

I would like to thank every that has been a part of my life here at MIT. Your support and love has been incredibly important in my time here.

This thesis (and many of the other opportunities I have had at MIT) would not have been possible without guidance of Professor Amos Winter, so thank you for making MIT much more exciting, relevant, and fun.

Finally, I would like to thank Guillermo Fabian Diaz Lankenau for making this thesis happen (and thereby allowing me to graduate). Thank you so much for encouraging me and helping me!

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1. Introduction

This thesis examines the dynamic performance of a motorcycle with the addition of a trailer and an outrigger attachment. MATLAB scripts were created to characterize these performances.

1.1 Alternative Applications of Motorcycles

In many countries motorcycle ownership far outnumbers car ownership. In India, for example, there are six times as many households that own motorcycles over cars [1]. In countries where motorcycle usage is very common, motorcycles are often used for a variety of tasks beyond personal transport. Of particular interest is the application of motorcycles in small acreage farming. Small acreage farms are farms that are less than one hectare and make up 72 % of all farms worldwide [2]. These farms often employ animal labor (bullocks, horses, mules, camels, etc.) to transport their produce. However, bullocks are expensive to maintain and have a limited amount of work output per day. Motorcycles on the other hand, are relatively cheap to maintain, can work for the entire day, and are prevalent in developing market contexts. It is of interest then to consider how motorcycles could be implanted to improve the profit, efficiency, and decrease maintenance cost of traditional labor methods for small acreage farms.

Figure 1: Example of motorcycle with trailer attachment

1.2 Modifications of Interest

Two modifications of interest are examined within this framework. The first is simply a trailer attachment that is fixed to the rear of the motorcycle with a single 3 degree of freedom hitch (ball joint) that has a limited maximum angle of rotation. Trailers are being considered because often agricultural goods need to be transported from these small acreage farms to local markets. Currently, people use improvised tractors and carts to bring their grown goods to the market. A trailer could potentially carry much more weight for a small additional cost, allowing people to transport more goods and leading to a higher potential profit. The trailer is modeled as a single body that affixes at a single point and has a fixed mass. This model allows for a variety of mounting locations to be examined and different loads to be modeled.

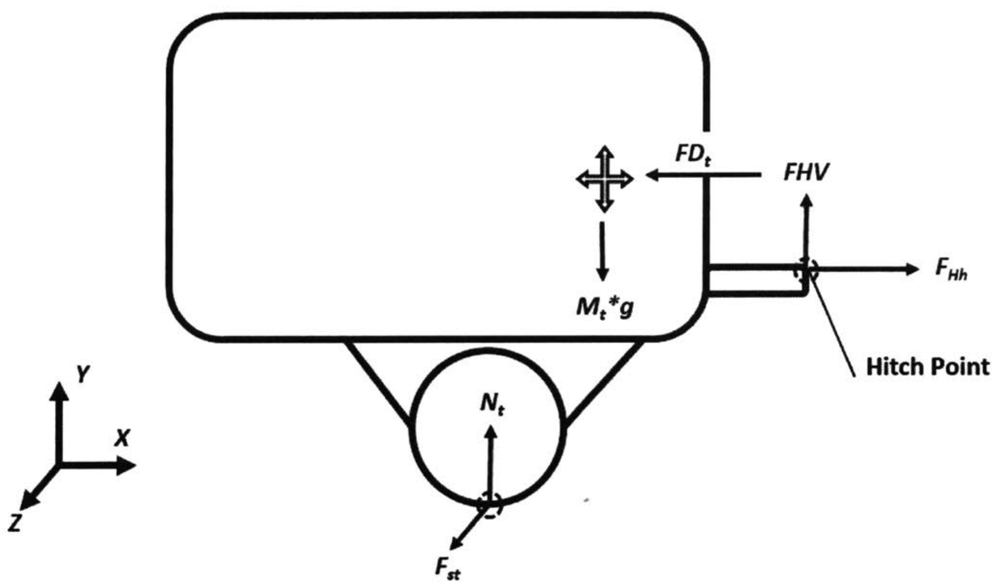


Figure 2: Trailer depiction used for the following models. Note that the center of mass, hitch point, and wheel contact point are all independent.

The second modification of interest is the addition of an outrigger wheel. Here, an idle third wheel extends laterally from the motorcycle to provide support. This wheel has its own set of forces acting upon it that must be accounted for.

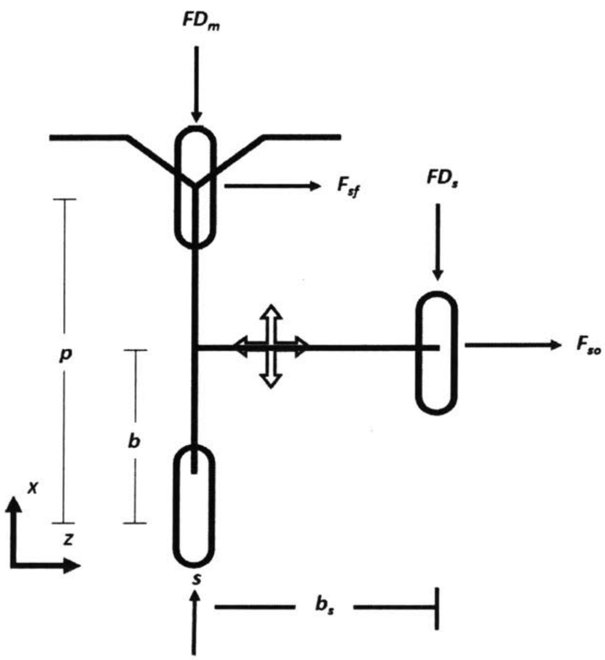


Figure 3: Outrigger depiction used for the following models. Note that the center of mass, attachment point, and outrigger wheel contact point are all independent.

The models presented in this paper are built using the models developed and discussed in *Motorcycle Dynamics* [3]

2. Modeling of the System

This section will give an overview of the way the motorcycle and attachments were modeled in order to analyze as well as some of the criteria of interest. The model allows the user to input various motorcycle dimensions as well as forces.

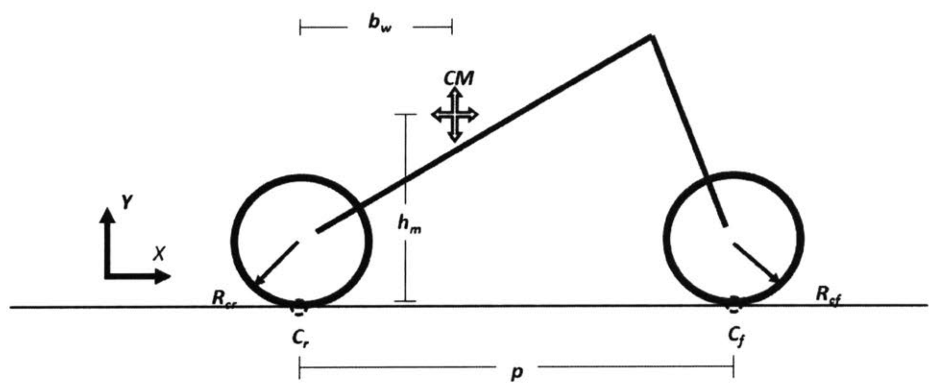
In order to model the system, it was decided that a multi-body model would be used to analyze the different performance characteristics (load transfer, braking behavior, acceleration behavior, and turning ability) of interest. This model could then be evaluated using equilibrium conditions during acceleration, deceleration, steady driving, and steady turning events.

2.1 The Motorcycle Model

2.1.1 Geometric Model

For the analysis, the motorcycle was modeled as a single body with contact points at the front and rear wheels, C_f and C_r, center of mass *CM,* and wheelbase *p*. This simplification to a single body system was made in order in to allow for easy estimation and modeling of changes in the full system. In this model, the vehicles and riders mass has been accounted for in the CM position. The wheel sizes and moments are given by R_{cr}, R_{cf}, and I_{mwr} and I_{mwf}. The lean angle of the rider which measures the amount the rider and motorcycle have tipped against the ground coordinate system is given by φ. The kinematic steering angle which is the angle between the ground coordinate system and the direction that the motorcycle is heading is Δ. The height from the ground plane to the motorcycle CM is h_m.

$$C_f = Front\ Wheel\ Contact\ Point$$
$$C_r = Rear\ Wheel\ Contact\ Point$$
$$CM = Center\ of\ Mass$$
$$p = Wheelbase$$
$$R_r = Rear\ Wheel\ Radius$$
$$R_r = Rear\ Wheel\ Radius$$
$$I_{m\omega r} = Moment\ of\ inertia\ for\ the\ front\ wheel$$
$$I_{m\omega f} = Moment\ of\ inertia\ for\ the\ rear\ wheel$$
$$\varphi = Rider\ lean\ angle$$
$$\Delta = Kinematic\ Steering\ Angle$$
$$h_m = CM\ height\ from\ ground$$



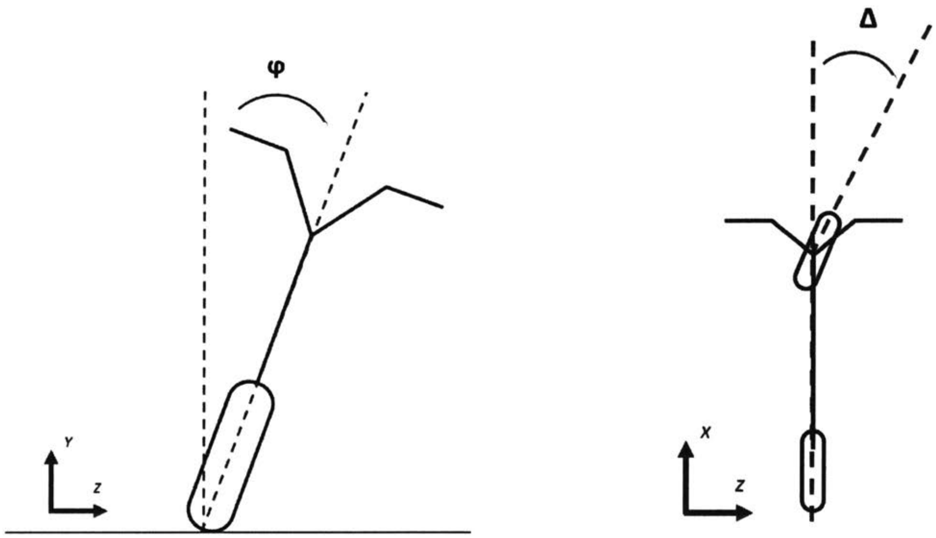


Figure 4: Figures depicting various motorcycle geometric parameters. Here the *CM* depicts the center of mass, *p* is the wheel base, h_m is the height of the *CM*, R_{cr} and R_{cf} are the rear and front wheel, φ is the lean angle, and Δ is the kinematic steering angle.

2.1.2 Force Model

In addition to the geometry of the motorcycle model, the forces on the motorcycle also need to be modeled. The thrust force, produced on the motorcycle and located at the rear contact patch, is given by *S*. It is important to note that parasitic losses within the motorcycle were not modeled, instead *S* captures the amount of force delivered to the ground and should be adjusted to reflect so. The lateral forces experienced by the motorcycle on each tire patch are given by Fsf and Fsr for the front and rear tires respectively. The drag force FDm is modeled using a simple script that uses the basic drag equation - $F_D = \frac{1}{2} C_d \rho v^2 A$ -to predict various drag forces under different driving conditions and acts on the center of mass of the motorcycle. There are also normal reaction forces occurring on both tires of the motorcycle Nf and Nr. The mass of the motorcycle is given by Mm. Finally, the forces due to attaching either a trailer or side-car will be discussed in the following models.

$$S = Thrust\ Force$$
$$F_{sf} = Front\ Tire\ Lateral\ Force$$
$$F_{sr} = Rear\ Tire\ Lateral\ Force$$
$$FD_m = Motorcycle\ Drag\ Force$$
$$N_f = Normal\ Force\ on\ front\ tire$$
$$N_r = Normal\ Force\ on\ rear\ tire$$
$$M_m = Mass\ of\ the\ motorcycle$$

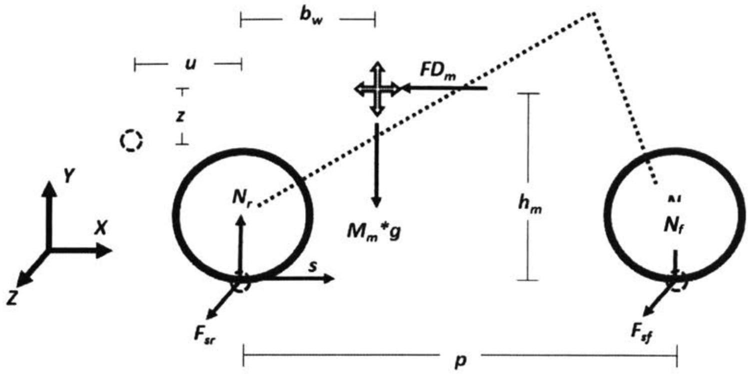


Figure 5: Figure depicting various motorcycle forces. Here *Mm* is the mass of the motorcycle, FD_m is the drag force, F_{sr} and F_{sf} are the rear and front lateral forces, N_r and N_f, are the rear and front normal forces, and *s* is the thrust force.

2.3 Trailer Model

2.3.1 Geometric Model

The trailer is modeled as a single body system that attaches at a hitch point to the motorcycle and has a single axle. The length of the trailer is given by p_t. The center of mass of the trailer is CM_t.The distance between the front hitch and the center of mass of the motorcycle is given by b_t. The trailers wheels have a radius r_t and are located a distance w_t away from the hitch. It was assumed that the trailers lean angle would be negligible due to having two laterally opposed wheels. The height of the trailer from the ground is ht.

$$p_t = length\ of\ the\ trailer$$
$$CM_t = Centor\ of\ Mass\ of\ the\ trailer$$
$$b_t = Distance\ from\ front\ hitch\ to\ CMt$$
$$R_t = Radius\ of\ trailer\ wheels$$
$$h_t = Vertical\ distance\ from\ ground\ plane\ to\ CM$$

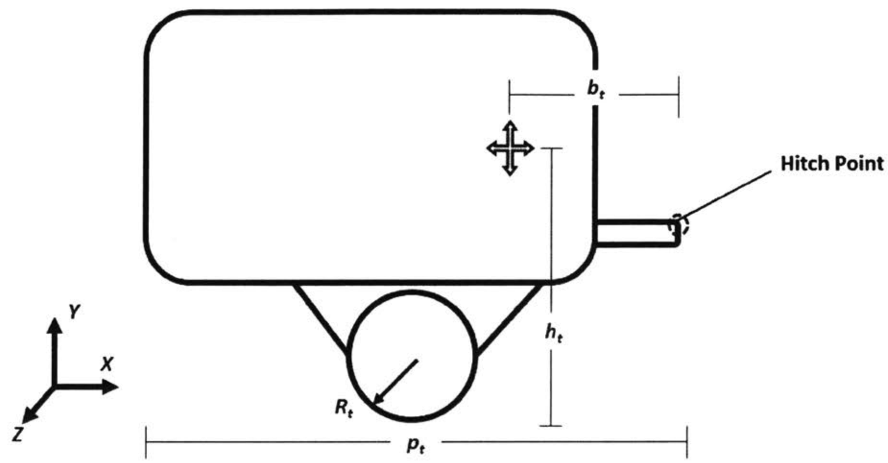


Figure 6: Figure depicting various trailer geometric parameters. Here h_t is the height of the *CM*, R_t is the trailers wheel radius, and p_t is the trailer wheelbase.

2.3.2 Force Modeling

The trailer in the system also experiences forces that change the way it behaves relative to the motorcycle. The drag force on the trailer was calculated using a simple drag equation script and is given by FDt. This drag force is modeled as acting on the center of mass of the motorcycle and varies based on speed. The lateral force experienced by the trailers tires is Fst. For this model, the trailer was simplified to have a single tire directly below CM_t *and a roll angle of zero*, while in reality they would most likely have two laterally opposed tires. The normal force felt by the trailer tire is given by Nt. Finally, the forces due to the connection with the motorcycle are treated as point forces acting about the hitch location. FHh is the horizontal component of this force and FHv is the vertical component. The mass of the trailer is Mt.

$$\begin{aligned}
&FD_t = Trailer\ drag\ force\\
&F_{st} = Trailer\ Lateral\ Force\\
&N_t = Trailer\ Normal\ Force\\
&F_{Hh} = Horizontal\ force\ from\ hitch\\
&F_{Hv} = Veritcal\ force\ from\ hitch\\
&M_t = Mass\ of\ the\ trailer
\end{aligned}$$

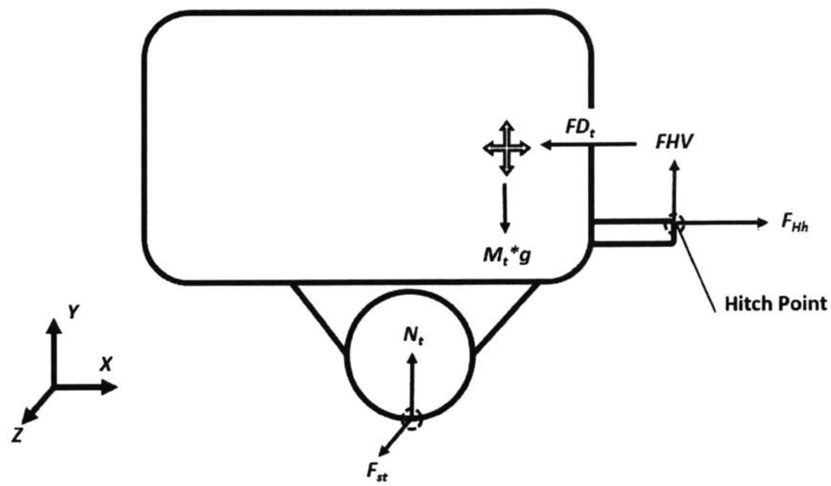


Figure 7: Figure depicting various trailer forces. Here M_t is the mass of the motorcycle, FD_t is the drag force, F_{st} is the lateral force, N_t is the tire normal force, F_{Hh} is the horizontal hitch force, and F_{Hv} is the vertical hitch force.

2.4 Outrigger Modeling

2.4.1 Geometric Modeling

The outrigger of the motorcycle is modeled as a single body that is connected by a pinned joint some location along the centerline of the motorcycle. Here b_s refers to the length from the motorcycle to the outrigger. The height of the outrigger CM is given by h_s.

$$b_s = Distance\ from\ motorcycle\ to\ outrigger$$
$$h_s = height\ of\ the\ outrigger$$

2.4.2 Force Modeling

In addition to the normal motorcycle forces, the outrigger experiences a drag force FD_s which acts at the CM of the system. The mass of the outrigger is M_s and acts at the CM of the system as well. Finally, there is a lateral force on the outrigger tire, F_{so}.

$$FD_s = Outrigger\ drag\ force$$
$$M_s = Mass\ of\ the\ outrigger$$
$$F_{so} = Lateral\ tire\ force\ on\ outrigger\ wheel$$

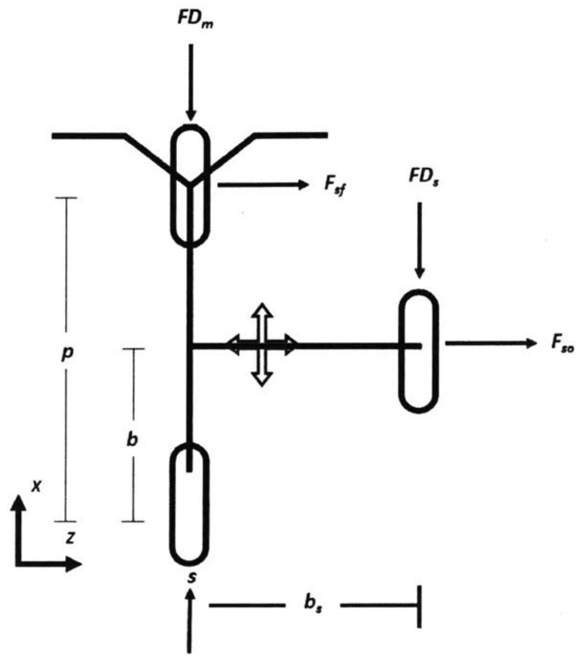


Figure 8: Figure depicting the outrigger setup. Here b_S is the horizontal distance from the *CM* to the outrigger wheel. FD_t is the drag force and F_{so} is the lateral outrigger wheel force.

3. Motion Studies

The following will explore how to use the previously devised model inputs to determine information regarding the acceleration, deceleration, steady driving, and steady turning states of a motorcycle and attachment.

3.1 Constant Velocity

Using the models presented, it can be determined what the steady state load transfer is within the motorcycle system for steady driving (no acceleration events). This is useful to determine the maximum speed, design expected loading cases, and the optimal geometries for motorcycle systems.

3.1.1 Constant Velocity Motorcycle with Trailer

In the case of a constant velocity motorcycle with trailer we are able to write equations of motions for the system that have an equal number of unknowns and equations. The following are our inputs and outputs to the system.

$\underline{Inputs}$

$FD_t, M_t, g, b_w, b_t, q, FD_m, M_m, z, u, h_m, p$

$\underline{Unknowns}$

$F_{Hh}, F_{Hv}, N_f, N_r, N_t, s$

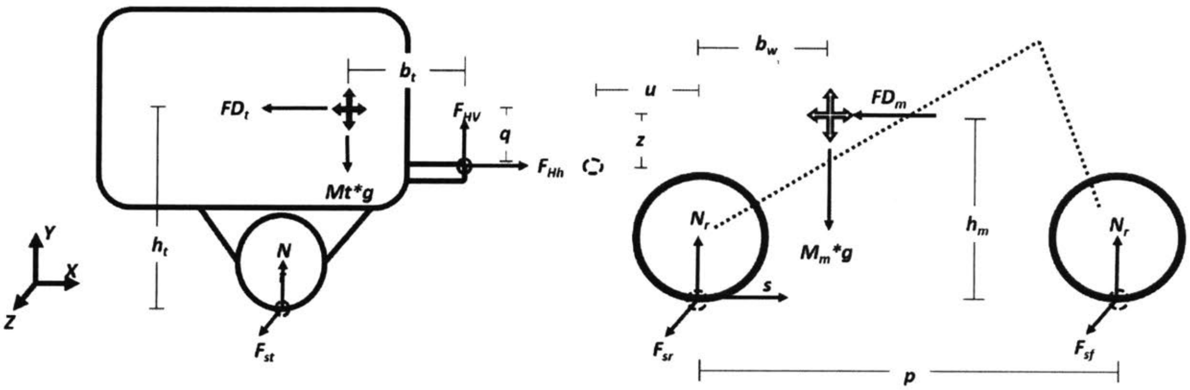


Figure 9: Figure depicting the motorcycle and trailer setup under steady driving conditions.

Writing out the equations of motion yields in the XY plane (as pictured) for both the motorcycle and the trailer yields:

$$\sum Fx_m = -FD_m + F_{Hh} + s = 0$$
$$\sum Fy_m = N_r + N_f - M_m * g - F_{Hv} = 0$$
$$\sum M_m = -z * F_{Hh} + F_{Hv} * (u + b) + s * h_m - (N_r * b) + N_f * (p - b) = 0$$
$$\sum Fx_t = -FD_t + F_{Hh} = 0$$
$$\sum Fy_t = N_t + F_{Hv} - M_t * g = 0$$
$$\sum M_t = -b_w * N_t + b_t * F_{Hv} + q * F_{Hh} = 0$$

Equation 1

A linear equation solver script was implemented in MATLAB to solve these equations and the inputs can be varied to reflect changes in geometry and forces. It may also be of interest to substitute s as a known parameter and thereby change the drag forces to unknowns.

3.1.2 Constant Velocity Motorcycle with Outrigger

The motorcycle outrigger wheel was modeled as a single body to reflect the rigid connection between the two entities. The unknowns and equations are therefore as follows:

$$\underline{Inputs}$$
$$FD_m, FD_s, p, b, b_t$$

$$\underline{Unknowns}$$
$$s, FZ_s, F_{sf}$$

$$\sum Fx = s - FD_m - FD_s = 0$$
$$\sum Fy = F_{sf} + FZ_s = 0 \qquad \text{Equation 2}$$
$$\sum M = F_{sf} * (p - b) - FD_s * b_t = 0$$

These equations govern the characteristics of sidecar performance.

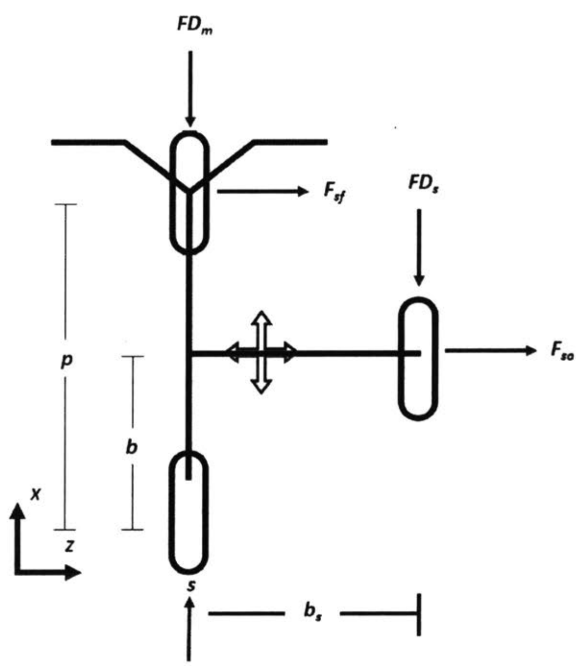


Figure 10: Figure depicting the motorcycle and outrigger setup.

3.2 Acceleration

When driving it is important to be able to accelerate up to speed in a somewhat timely manner so as to not cause a traffic impediment or be able to do simple maneuvers like change lanes. In a motorcycle, all power is normally delivered through the rear wheel so it is important to look at how the load changes in the motorcycle during an acceleration event to deliver the right amount of power to that wheel. As a guideline for this study we are taking an average semi-truck acceleration to be the minimum acceptable acceleration which is approximately $.36\,\frac{m}{s^2}$ or a 23 second 0 to 30 $\frac{km}{h}$ time [4].

3.2.1 Acceleration Motorcycle with Trailer

Using the motorcycle and trailer model a system of equations was created to determine the acceleration characteristics of a motorcycle with trailer system. A thrust force can then be input into the model and the resulting forces at each point in time can be found. In this case, a predetermined thrust is inputted and the resulting acceleration times are recorded. For a constant thrust case, it can be seen that as you increase in velocity drag forces will increase and change the weight distribution of the motorcycle system. This could also be used to find the maximum amount of thrust that can be applied at any given state, maximizing performance.

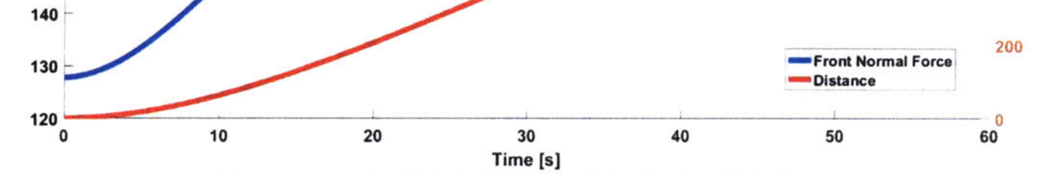


Figure 11: Graph of Motorcycle and Trailer Acceleration. A 1000N thrust is applied to a typical motorcycle and trailer system.

3.2.2 Acceleration Motorcycle with Outrigger

The same model can be applied to the motorcycle with outrigger system. The model shows the transient weight distribution response within the outrigger system changes.

3.3 Deceleration

Braking is perhaps the most important performance characteristic to analyze in an automobile system [5]. To determine braking characteristics two different methods were analyzed, the maximum braking performance under a locked brake condition and the maximum braking performance under a tire slip condition. It is also of interest to consider the "jackknifing" case for a motorcycle and trailer system.

3.3.1 Deceleration Motorcycle with Trailer

Braking in a motorcycle and trailer system can be accomplished by using either only brakes in the motorcycle or in both the motorcycle and trailer. Both methods will be examined here.

Using the motorcycle trailer system model, we have previously discussed (Figure 9), we can determine the weight transfer found within the system for a given acceleration. Given an allotted amount of braking force for each wheel in the motorcycle trailer system an integration method can be applied to the equations of motion that will determine the stopping distance and time of a motorcycle trailer.

$$\text{Velocity} = \iint_0^t Acceleration \qquad \text{Equation 3}$$

In this case, the equations of motion for the previous motorcycle trailer system [Equation1] are still true, however, instead of a positive thrust force a negative thrust force would be implemented at each wheel. In this case, these thrust forces would be known – they are equal to either the maximum braking force or the maximum shear force the motorcycles tires are able to apply to the road. These thrust forces are represented with s_f, s_r, s_t signifying the front, rear, and trailer tires.

$$\begin{aligned}
&\sum Fx_m = -FD_m + F_{Hh} + s_f + s_r = 0\\
&\sum Fy_m = N_r + N_f - M_m * g - F_{Hv} = 0\\
&\sum M_m = -z * F_{Hh} + F_{Hv} * (u + b) + (s_r + s_f) * h_m - (N_r * b) + N_f\\
&\qquad\qquad * (p - b) = 0\\
&\sum Fx_t = -FD_t + F_{Hh} + s_t = 0 \qquad \text{Equation 4}\\
&\sum Fyt = N_t + F_{Hv} - M_t * g = 0\\
&\sum Mt = -b_w * N_t + b_t * F_{Hv} + q * F_{Hh} + s_t * h_t = 0
\end{aligned}$$

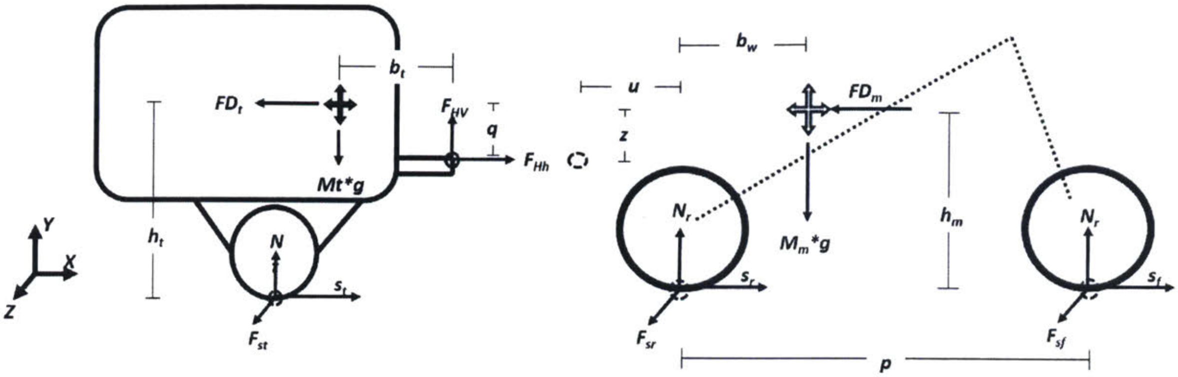


Figure 12: Figure depicting the motorcycle and trailer setup undergo braking.

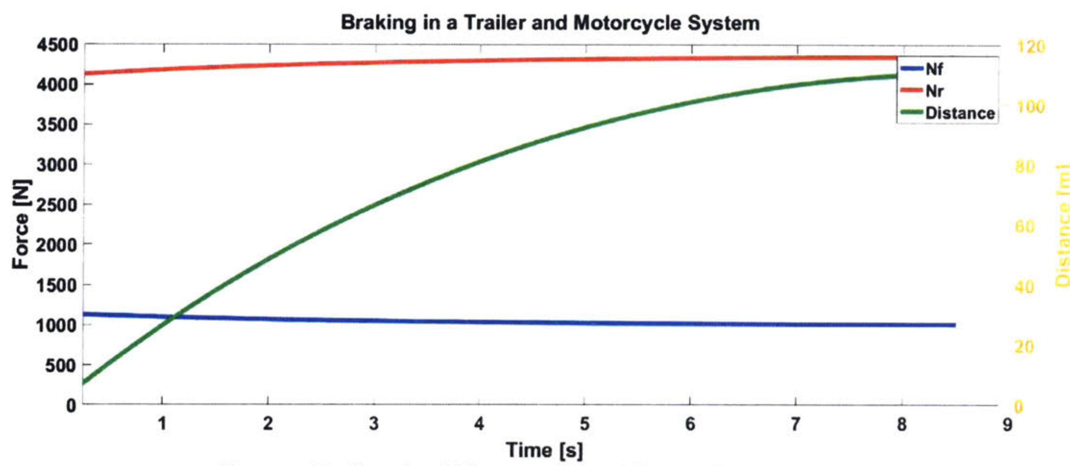


Figure 13: Graph of Motorcycle and Trailer Braking. A MATLAB script was written to determine the braking time, distance, and relation between braking and load transfer in a motorcycle trailer system.

For a motorcycle trailer system that does not have trailer brakes, this term is dropped from the equations.

3.3.2 Motorcycle Trailer "Fishtailing"

Large segmented vehicles are prone to the phenomena known as fishtailing and jackknifing. In these situations, the vehicle loses the ability to steer due to the large momentum force of the trailer body [6]. This can be corrected through proper steering techniques but is quite dangerous. It is important to consider how this may be avoided in a motorcycle trailer system.

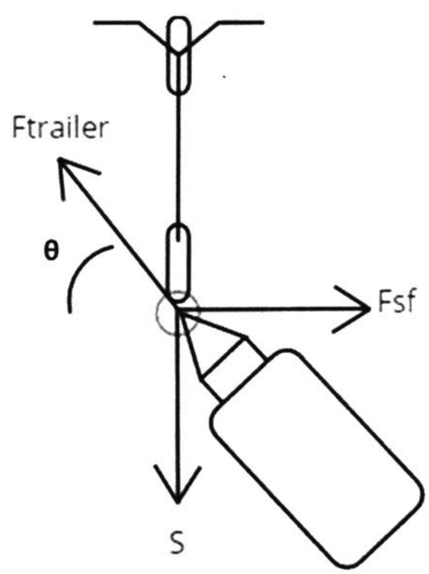
In these phenomena, the loss of control is caused by the heavy body applying an inertial force in opposition to the braking force applied by the front member. An easy remedy to this problem is to install brakes on both (or just the rear trailer) systems. However, it is not always practical to install brakes on the rear trailer of a vehicle especially in resource constrained environments. Another solution is to limit the amount of angle allowed by the hitch mechanism. This would prevent the rear trailers inertial force from dominating the braking and turning forces supplied by the cab. This angle can be estimated using the following equation

$$\sum Fx = F_{sf} - \frac{s}{\tan(\theta)} = 0 \qquad \text{Equation 5}$$

Where Fsf can be calculated from the equilibrium equations and the hitch angle can be sized appropriately.

$$F_{sf} \leq N_r * \mu \qquad \text{Equation 6}$$

Figure 14: Figure of motorcycle trailer fishtailing.



3.3.3 Deceleration Motorcycle with Outrigger

Braking in a motorcycle outrigger system can be difficult and non-intuitive for a motorcycle driver. If the outrigger system does not have brakes built in, the motorcycle will tend to turn away from the outrigger as you brake due to the inertial forces from the outrigger as it continues forward. In this analysis, it will be assumed that the driver is able to counter the turning imposed by the outrigger system. The systems inputs, unknowns, and equations of motion are therefore:

$$\underline{Inputs}$$
$$FD_m, FD_s, p, b, b_t$$

$$\underline{Unknowns}$$
$$s, FZ_s, F_{sf}$$

$$\begin{aligned} \sum Fx &= s_f + s_r + s_o - FD_m - FD_s = 0 \\ \sum Fy &= F_{sf} + FZ_s = 0 \\ \sum M &= F_{sf} * (p - b) - FD_s * b_t = 0 \end{aligned} \quad \text{Equation 7}$$

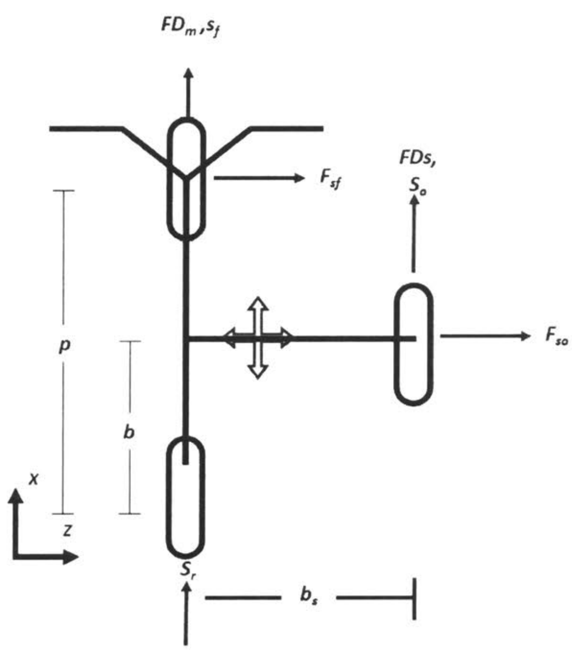


Figure 15: Figure of motorcycle with outrigger undergoing braking.

A negative braking force, s, can then be applied and the results integrated in MATLAB to determine the ideal amount of braking force, weight distribution, and minimum stopping distance for this system.

3.4 Steady State Turning

One of the most important parameters to consider is the minimum turn radius expected for different motorcycle systems. In order for motorcycle attachments to be able to be used in agriculture and in transport settings, the motorcycle must be able to turn through a predefined radius. The following will examine a method devised to predict the turning behavior of a motorcycle system.

3.4.1 Steady Turning Motorcycle Trailer

Turning in a motorcycle trailer system is governed by many different variables, in this method, the following simplifications are made in order to predict an estimated performance.

- *It is assumed that the roll of the trailer in the system is negligible.*
- *The motorcycle is able to go through a turn only if all normal forces are positive.*

We then have the following inputs, unknowns, and equations

$$\underline{Inputs: Geometric}$$

$$\Delta = Kinematic\ steering\ anlge$$
$$\varphi = Rider\ lean\ angle$$
$$i = Max\ Hitch\ angle$$
$$\lambda = Sideslip\ angle$$

$$bm = Horizontal\ distance\ from\ rear\ wheel\ to\ motorcycle\ CM$$
$$bw = Horizontal\ distance\ from\ trailer\ CM\ to\ wheel$$
$$ht = Height\ of\ the\ trailer\ CM\ from\ ground\ plane$$

$$\underline{Inputs: Moments}$$
$$I_{mxg} = Mass\ moment\ of\ the\ motorcycle\ in\ the\ x\ direction$$
$$I_{myg} = Mass\ moment\ of\ the\ motorcycle\ in\ the\ y\ direction$$
$$I_{mzg} = Mass\ moment\ of\ the\ motorcycle\ in\ the\ z\ direction$$

$$I_{txg} = Mass\ moment\ of\ the\ trailer\ in\ the\ x\ direction$$
$$I_{tyg} = Mass\ moment\ of\ the\ trailer\ in\ the\ y\ direction$$
$$I_{tzg} = Mass\ moment\ of\ the\ trailer\ in\ the\ z\ direction$$

$$I_{wf} = Spin\ moment\ of\ inertia\ of\ the\ front\ wheel$$
$$I_{wr} = Spin\ moment\ of\ inertia\ of\ the\ rear\ wheel$$
$$I_{wt} = Spin\ moment\ of\ inertia\ of\ the\ trailer\ wheel$$

$\underline{Inputs: Stiffnesses}$

$K_{\lambda r} = Cornering\ stiffness\ coefficient$

$K_{\varphi f} = Cambering\ stiffness\ coefficient$

$\underline{Inputs: Forces}$

$FDm = Motorcycle\ Drag\ force$

$FDt = Trailer\ drag\ force$

$\underline{Inputs: Other}$

$v = Vehicle\ velocity$

$\Omega_f = Front\ wheel\ angular\ velocity$

$\Omega_r = Rear\ wheel\ angular\ velocity$

$\Omega_t = Trailer\ wheel\ angular\ velocity$

$g = Gravity$

$Mm = Motorcycle\ Mass$

$Mt = Trailer\ mass$

$\underline{Unknowns}$

$s = Thrust\ force$

$Fsf = Front\ tire\ lateral\ force$

$Fsr = Rear\ tire\ lateral\ force$

$Fst = Trailer\ tire\ lateral\ force$

$Nf = Front\ tire\ normal\ force$

$Nr = Rear\ tire\ normal\ force$

$Nt = Trailer\ tire\ normal\ force$

$Rcr = Radius\ of\ curvature$

$\underline{Coordinate\ System\ Transformations}$

$Xmg, Ymg, Imyz, Ymr, Ytr, Ytg, Imxz, Xmr, Zmg, Itxz, Xtr, Xtg, Ztg,$

$For\ a\ description\ please\ see\ Appendix\ A$

$\underline{Equations}$

$$\sum Fx = s - Fsf * \sin(\Delta) + Mm * Xmg * \Omega^2 - FDt = 0$$

$$\sum Fy = Fsf * \cos(\Delta) + Fsr + Mm * Ymg * \Omega^2 = 0$$

$$\sum Fz = -Nf - Nr + Mm * g - Nt + Mt * g = 0$$

$$\sum Mx = -Imyz * \Omega^2 - (Nf + Nr) * Ymr + Mm * g * Ymg + (I\omega f * \omega f + I\omega r) * \Omega * \cos(\varphi) - Ityz * \Omega^2 - Nt * Ytr + Mt * g * Ytg + I\omega t * \omega t * \Omega * \cos(\varphi) = 0$$

$$\sum My = -Imxz * \Omega^2 - Nf * (p + Xmr) + Mm * g * Xmg + FDm * Zmg - Nr * Xmr + Itxz * \Omega^2 - Nt * Xtr + Mt * g * Xtg + FDt * Ztg = 0$$

$$\sum Mz = (p + Xmr) * Fsf * \cos(\Delta) + Ymr * Fsf * \sin(\Delta) + Fsr * Xmr - s * Ymr + FDm * Ymg + Xtr * Fst * \cos(i) + Ytr * Fst * \sin(i) + Fst * Xtr + FDt * Ytg = 0$$

Equation 8

It can be noted that the equations of motion are not linear, in addition there are more unknown than known variables so this is a non-linear problem and a non-linear MATLAB script was created to evaluate the unknowns.

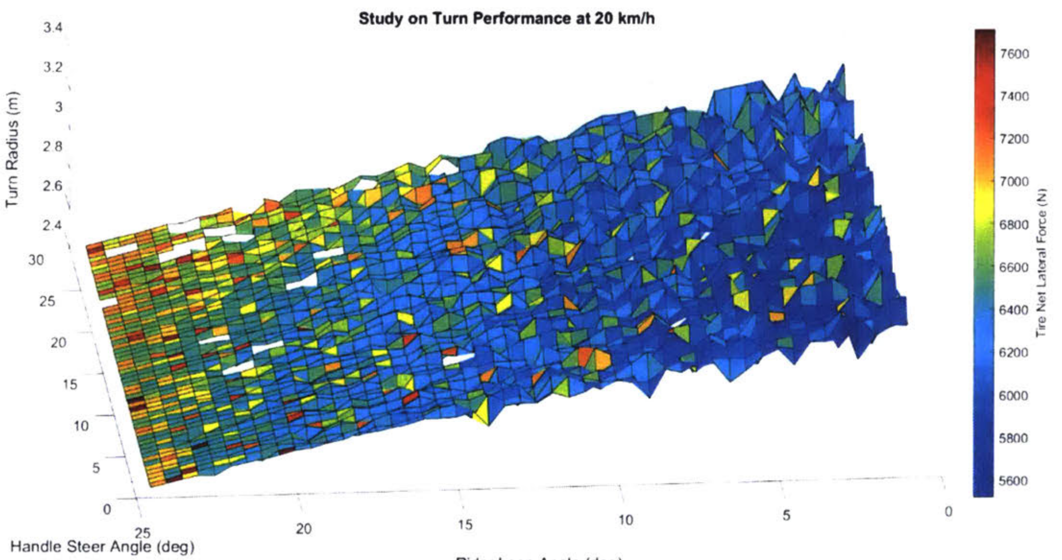


Figure 16: Graph of a motorcycle trailer in turn. A test system was implemented and a model was created in MATLAB to solve the equations of motion. The roughness of the graph is due to the non-linearity of the system and the inability of the solver to find the global minima.

3.4.2 Steady turning in a motorcycle outrigger system

This same methodology in section 3.4.1 can be applied to the motorcycle outrigger system and the unknowns solved for. The equations of motion will become slightly more complicated as many dynamic phenomena occur.

4. Summary and Conclusions

4.1 Constant Velocity Driving

In the case of constant velocity driving for a motorcycle trailer system it can be seen that the placement of the hitch and the CM of the trailer are very important to the weight transfer of the system.

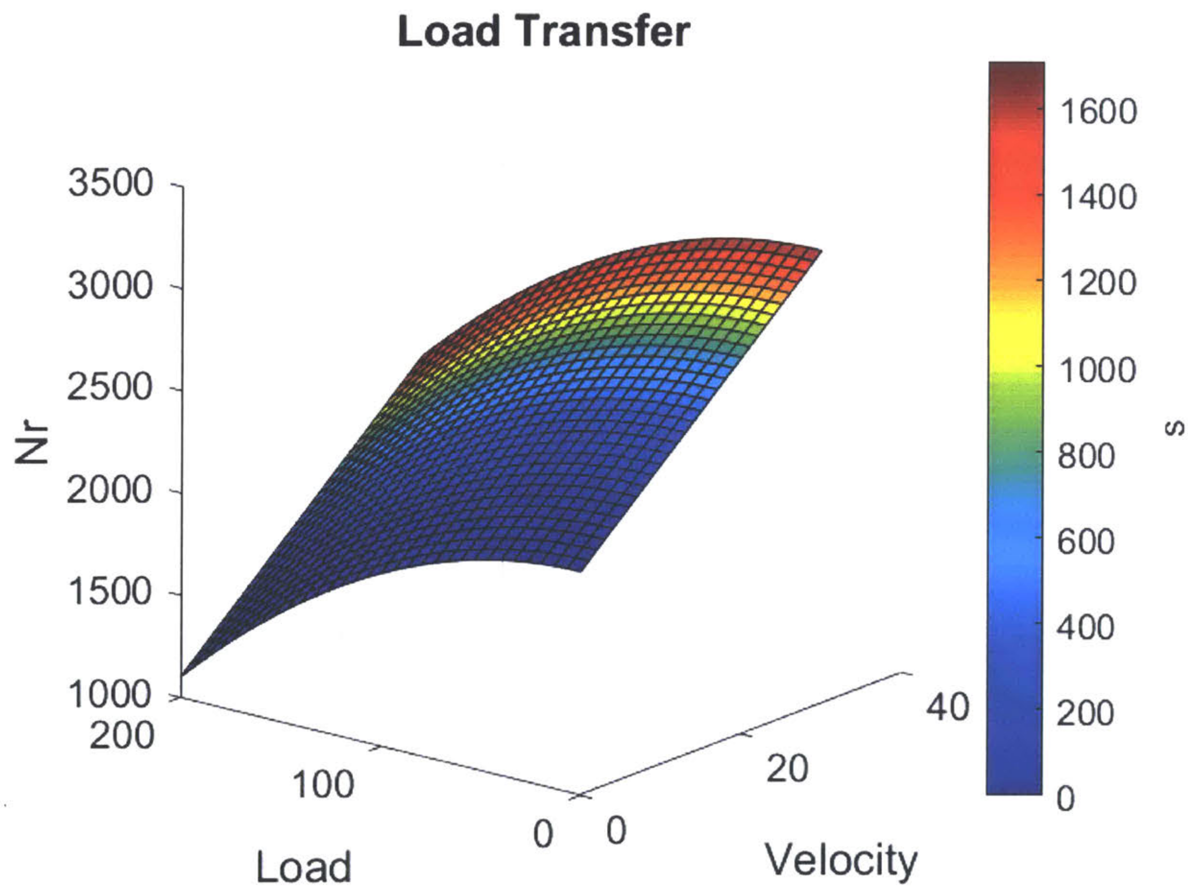


Figure 17: Graph of load transfer in motorcycle trailer system. Here the velocity and load are varied and the resulting normal force and thrust are plotted against each other. Analyses such as these can inform intelligent designs of motorcycle trailer systems and safe operating conditions.

The above figure shows how the thrust and normal force varies with load and velocity. In this example, it can be seen thrust and normal force both vary with respect to load and velocity. The MATLAB script used to create these figures can be used to determine optimal trailer motorcycle geometries by changing the input parameters to reflect what the geometries of interest are.

4.2 Acceleration

For accelerating in a motorcycle and attachment system it is useful to know where the weight is in the system. By analyzing this one can find the maximum amount of force that can be delivered to the ground. In addition, in order to know how quickly the system can accelerate and the maximum speed is important to determine how road worthy the vehicle is. The MATLAB code can be used to determine this information and optimize the design parameters. From the data, one can see that as the speed of the motorcycle increases (for a constant thrust in this configuration), more weight is shifted to the front wheel of the motorcycle.

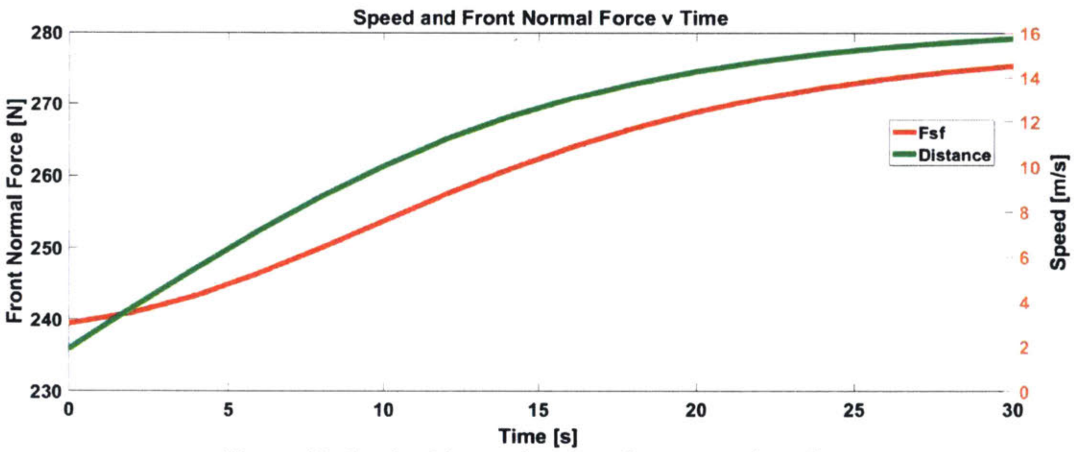


Figure 18: Graph of the acceleration of a motorcycle and trailer system. Here one can see how the front wheel normal force changes with speed.

4.3 Deceleration

Braking is perhaps the most important aspect of the motorcycle and attachment system. The MATLAB model developed can determine the weight transfer as the system decelerates. This would allow for one to predict the minimum stopping distance and maximum breaking force for the rear, front, and attachment brakes. In the future, more research should be done into how the different geometries affect the different braking parameters.

4.4 Turning

Using the system of equations for turning in a motorcycle and attachment system one can find trends that show how variables within the system affect one another. It is important to be able to turn around a relatively small radius to be able to drive and maneuver safely in the streets with a motorcycle and attachment vehicle. In order to attain accurate results for the system, the

MATLAB script should be tested and correlated. Also, the results from the MATLAB script could be improved by refining the non-linear system of equations solver used as well as tweaking the limits placed on the system. The graphs do show however, that increasing the rider lean angle strongly decreases the minimum turn radius for the system. This information can be used to appropriately design the joint and attachment mechanism between the trailer and the motorcycle.

5. Appendices

5.A Coordinate System Transformations

The equations found in the steady turning regime are made simple through the use of coordinate transformation to relocate different points in space. The transformations used here are modeled directly after the transformations found in *Motorcycle Dynamics.* [3]

$$Xmg = b - Rcr * \sin(\lambda_r)$$
$$Ymg = hm * \sin(\varphi) - Rcr * \cos(\lambda_r)$$
$$Zmg = -hm * \cos(\varphi)$$
$$Xtg = -Rcr * \sin(\lambda_r) - bt$$
$$Ytg = -Rcr * \cos(\lambda_r) + \sin(i) * bt$$
$$Ztg = -ht$$
$$Imxz = Mm * Xmg * Zmg + Imxzg * \cos(\varphi)$$
$$Imyz = Mm * (Ymg * Zmg) + (Imzg - Imyg) * \cos(\varphi) * \sin(\varphi)$$
$$Itxz = Mt * Xtg * Ztg + Itxzg * \cos(\varphi)$$
$$Xmr = -Rcr * \sin(\lambda_r)$$
$$Ymr = -Rcr * \cos(\lambda_r)$$
$$Xtr = Xmr - \cos(i) * bw$$
$$Ytr = Ymr - \sin(i) * bw$$

6. References

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