Redesign of FlexLab Cantilever Beam for Reduced Resonance Frequencies and Increased Damping

by

Christopher Harmon

Submitted to the Department of Mechanical Engineering in Partial Fulfillment of the Requirements for the Degree of Bachelor of Science in Mechanical Engineering at the Massachusetts Institute of Technology

June 2017

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ABSTRACT

Thesis Supervisor: David Trumper
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Hands-on learning remains a key aspect of the educational path through MIT. It provides the practical experience and real-world tie-in that theoretical study and analysis alone could not uphold. Thus, it is necessary to ensure that a lack of lab resources, space, and time do not present barriers to prospective students. As such, the portable FlexLab/LevLab module serves to bring laboratory teaching beyond the lab. This paper presents the results of efforts to redesign the FlexLab portion’s cantilever beam to meet two design goals. First of all, that the frequency of the second natural mode of the beam fall below 100 Hz. Second, that the beam’s damping is increased such that the first peak gain is within an order of magnitude of the surrounding gain. After testing, a new beam geometry and damping mechanism that satisfied both goals is proposed.
Acknowledgments

To Professor Trumper:
Thank you for continuing to believe in me, despite the circumstances surrounding my completion, and despite your own challenges this year. I truly wish you and your wife the best of health during the road ahead.

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1. Introduction

*Mens et Manus.* The Massachusetts Institute of Technology motto, which translates from Latin to mean “Mind and Hand”, embodies a critical feature of the institute’s upheld educational values. Learning must not only engage the mind through theoretical revision, but must also incorporate practical, hands-on applications of subject material. These exercises, often taking place in laboratory/workshop settings, present an opportunity for students to supplement their theoretical understanding, test their comprehension, and gain valuable experience and confidence in their fields of study. However, laboratory exercises are often constrained by a number of limited resources, including (but not limited to): available workstations, open time available in the labs, number of lab and teaching assistants, among others. Especially with respect to the design and analysis of control feedback systems, students would benefit from increased logistical flexibility and nearly unfettered access to lab materials so that they may complete more in-depth assignments with a greater degree of understanding. Further, as more educational resources turn to take advantage of digital and web-based media, there exists a need now more than ever to partially decouple laboratory learning from the physical laboratory space.

To that end, the portable FlexLab/LevLab module offers a means to help move the lab experience into the hands of students, wherever they go. This module represents the work of another, larger research project, of which this constitutes only a portion. The research presented here is only concerned with the improvement of the FlexLab part of the system, specifically with respect to the natural performance characteristics of the cantilever beam. As a consequence, driving towards two primary design goals will remain the focus of my work. First of all, it is desired that the first and second natural modes of the redesigned beam occur at frequencies below 100 Hz. Above this threshold, as can be seen from measurements taken later, the data becomes somewhat noisy and potentially more susceptible to extraneous vibration modes from components external to the beam. The second goal concerns the large resonance peak at the first mode. The system is grossly underdamped, and this peak makes measurement more difficult as system response gains rise over two orders of magnitude. Thus, a reduction of these peaks to within an order of magnitude of the surrounding values is desired.

2. Background and Theory

There exists a number of concepts that are relevant to the understanding and improvement of the FlexLab module. General information on these concepts are outlined below.

2.1 Euler-Bernoulli Beam Theory

Also referred to as Classical Beam Theory, the Euler-Bernoulli Beam Theory is a subsection of the linear elastic theories of solid mechanics, specifically concerning a simplified model for analyzing beams under load. This theory provides methods for understanding how external forces and moments, internal forces and moments, stresses, strains, and deflections (displacements) are related to each other in beam-like geometry. These methods rely on a number of assumptions about the mechanics of a beam, including that “(i) the length of the member is significantly greater than the greatest dimension in the cross section; (ii) we are away from the regions of stress...
concentration; (iii) the variation of external loads or changes in the cross-sectional areas are gradual except in regions of stress concentration” [1].

The primary physical system under consideration in the FlexLab consists of a cantilever beam under oscillating actuation at the unsupported end and gravity. To ultimately understand how a driven oscillatory force on a beam affects its deflection characteristics, one must first understand the dependence of deflection on the input force. For the static case, it can be shown that [2]:

\[ EI \frac{\partial^4 w}{\partial x^4} = q(x) \]  

where \( q \) is a distributed load that is a function of \( x \), the product \( EI \) is the flexural rigidity (product of Elastic Modulus and area moment of inertia of the cross section) of the beam and is a measure of bending stiffness, and \( w \) is the deflection as a function of \( x \). This can be solved for various boundary conditions. However, for the dynamic case analysis becomes more complicated. It can be shown that the Euler-Bernoulli equation for the dynamic bending of slender, isotropic, homogeneous beams of constant cross-section under an applied load is [2]:

\[ EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = q(x, t) \]  

where \( m \) is mass per unit length of the beam. If a damping force is present along the beam (such as from some sort of viscous fluid), then the addition of a damping term that is dependent upon the rate of change of deflection further complicates the solving process:

\[ EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} + b \frac{\partial w}{\partial t} = q(x, t) \]  

Whitney [4] proposes separation of variables to analytically and separately solve for solutions to such cases, however non trivial geometry and forcing conditions quickly complicate these efforts. Numerical solvers, such as SolidWorks’ frequency finite element analysis tool, may then be used to converge upon solutions to these complicated situations.

On another note, when a beam is made up of multiple materials in a layered fashion, it is considered a composite. For each section having Elastic Modulus \( E \) and area moment of inertia \( I \), the total flexural rigidity is just the sum of that of each section [3]:

\[ EI_{eff} = E_1 I_1 + E_2 I_2 + \cdots + E_n I_n \]  

For a beam with a constant rectangular cross-section of width \( b \) and height/thickness \( h \), one can use the following to determine \( I \) [1]:

\[ I = \int \int y^2 dA = \frac{bh^3}{12} \]  

In this manner, adding layers of material on top of a beam will increase the bending stiffness of the beam. This added stiffness is, of course, relative to the original beam stiffness.
2.2 Magnetic Force

On the FlexLab module, the actuation of the end of the beam is provided through the use of magnetic forces. Within the circuit, an alternating current is sent through the actuator coil beneath the end of the beam, embedded within the PCB. This creates an oscillating magnetic force which acts on the neodymium magnets fixed to the end of the beam, causing the beam and magnets to vibrate in an approximately vertical motion, assuming small angle displacements. A relationship between the force from the coil acting on the magnet and distance between these two components can be modeled as [6]:

\[ f = C \frac{i}{x^2} \]  

Here, \( C \) is "a constant that is determined by the strength of magnet, the number-of-turns of the coil and the geometric parameters", and \( i \) is the current through the coil.

2.3 Damping Ratio

In second order systems, the magnitude due to a second order, underdamped pair of poles [5] is:

\[ |H(j\omega)| = \frac{1}{\sqrt{(1 - \frac{\omega}{\omega_0})^2 + (2\zeta \frac{\omega}{\omega_0})^2}} \]  

Through the use of differentiation, it can be shown that the location of the peak gain associated with this pair of poles is:

\[ \omega_r = \omega_0\sqrt{1 - 2\zeta^2} \]  

Substituting (7) into (6), one obtains the following relationship between the magnitude at the peak and damping ratio, \( \zeta \) [5]:

\[ |H(j\omega)| = \frac{1}{2\zeta \sqrt{1 - \zeta^2}} \]  

For damping ratios that are very small, i.e. \( \zeta << 1 \), the portion of the term beneath the radical approaches unity. Thus, in these situations, (8) may be approximated by:

\[ |H(j\omega)| = \frac{1}{2\zeta} \]  

For very lightly damped systems, (8) and (9) yield very similar results, with minimal error arising from the simpler approximation [5].

3 Beam Geometry Selection

As my initial goal concerning the first and second natural modes of the cantilever beam is entirely affected by beam dimensions and material properties, which are continuous properties, there likely exists an infinite number of physical solutions. Therefore, in restricting the scope to
allow only minimal modifications to the FlexLab module, as well as maintaining some level of ease for sourcing materials, I was able to narrow the set of solutions to a discrete set. The beam geometry remained simple and rectangular in cross-section, and the beam width was already controlled by the fixed diameter of the cylindrical magnets and of the actuator coils already in use. Thus, the remaining degrees of freedom lay with the beam material, length, and thickness. Next, of the five mounting holes already in the PCB, two are necessary for a sturdy rigid mount. This leaves just four possible beam lengths, which are shown below.

![Diagram of possible beam sizes constrained to PCB holes.](image)

**Figure 1:** The 4 possible beam sizes are constrained to the 5 holes on the PCB. The independent variable for each beam (3 of the 4 shown here) is dimensioned above.

Finally, as for the beam thickness, I simply chose the eight thinnest easily obtained options for bronze sheets from McMaster-Carr to be my starting point for further study. With four beam lengths and eight thicknesses, I modeled the bronze cantilever beam and neodymium magnets in SolidWorks and prepared a design study with the Frequency Finite Element Analysis tool to numerically solve for the first four natural modes of each beam configuration. See the figures below for the SolidWorks model used and for the results of the simulation.
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Table 1: Results of SolidWorks frequency analysis design study, sorted by Frequency 2; selected beam geometries are highlighted in blue.
As shown in the table, only four of the potential geometric configurations seemed to be reasonable candidates for further study. Hereafter, the four beams will be referred to as Beam 1, 2, 3, and 4. Beam 1 will represent the 0.010” thick beam, whereas Beam 2, Beam 3, and Beam 4 will represent the 0.008” thick beams of variable length at 1.196”, 0.906”, and 0.615”, respectively.

4. Experimental Setup

To conduct my experiments and analysis, I relied on LabView, on National Instruments’ myRio embedded hardware device, on fabricated beam test samples, and on hardware designed for the main research associated with the educational FlexLab/LevLab module. This module consists of a single printed PCB, to which a single bronze cantilever beam is fixed and suspended over a set of two magnetic actuator coils. Pairs of magnets sit on the beam just above both actuator coil locations, offering the ability to model both a 2nd order and 4th order system. Pairs of hall effect sensors capture displacement information from each of the magnet pairs. See the figure below for more details.
For more detailed information about the hardware configuration of the FlexLab, including PCB design considerations, please see *FlexLab and LevLab: A Portable Lab for Dynamics and Control Teaching* [5]. For a full circuit schematic, please see Appendix A [5].

### 4.1 Fabrication

In order to test the suggestions informed from the mode analysis, I first needed to fabricate the potentially successful beam geometry candidates. The stock material used consisted of 12” by 12” sheets of very high strength Bronze 510 Alloy at two thicknesses: 0.008” and 0.010”. I used a waterjet to cut the contours of the beam designs out from the stock, making multiple copies at once in case of defects or future misplacement of parts. From the 0.010” thick sheet, I cut the sizes of Beam 1 and Beam 2 (the latter was purely cut in the event I might need them later; they were, however, never used for experimentation), and from the 0.008” thick sheet, I cut the sizes of Beam 1, Beam 2, and Beam 3. The resulting bronze beams are shown in the figure below.
Figure 4: Beams arranged on table after water jetting process

The water jetting process left raised edges on the underside of the beams, however. Quick post-processing with the use of a buffing wheel cleaned up these edges well without removing too much material too quickly. Before and after images are shown below.

Figure 5: On the left, beam was left with rough and jagged raised edges from the waterjet; On the right, beam after mild buffing to remove these deformities

In addition to fabrication of the bronze beams, I also needed to prepare a means to test damping through eddy current dissipation. For this, I prepared small, thin cards of folded aluminum foil, held together by adhesives and pressure. Each card could then be fixed in close
proximity to and directly above the end magnets for maximum effect. The cards contained varying numbers of folds of aluminum foil, including 1, 7, 15, 30, and 60.

Figure 6: Examples of three of the prepared foil cards to facilitate eddy current damping of the system; the numbers below each card reflect the number of folds of aluminum present on the card; each card has been flattened under high pressure.

Thin string was taped to the back to facilitate the attachment of each card to a small plastic 3D-printed fixture that suspends the foil in the appropriate position.

Figure 7: Plastic fixture (shown on the right) suspends aluminum card in proper position during testing.

4.2 Data Acquisition

LabView, in conjunction with the myRio module, was utilized to both control inputs to the FlexLab system and streamline the process of taking measurements of the system response. For system identification and analysis, this response data was then compared to the input data. The LabView virtual instrument (hereafter VI) used to collect data is primarily based upon Tyler
Hamer’s Dynamic Signal Analyzer (hereafter DSA) VI, which excites a system at a series of frequencies, then compares the response and the input to plot system gain and phase lead/lag at said various frequencies. On its own, however, this DSA was insufficient to make the necessary measurements across all frequencies of interest, which include system gains spanning over 5 orders of magnitude. Thus, I developed a modified VI around the DSA (hereafter DSA2) to add functionality to offer me more finely tuned control over the system identification process. The LabView front panel for this is shown below.

![Figure 8: Front Panel for my DSA2 VI; Controls C0 – C8 are breakpoints on the frequency of excitation, and I can control the excitation amplitude between these points by modifying G0 – G9; sample rate is 1 kHz](image)

There are two main additions to the original DSA. First, a feedback loop from the system response to the excitation input signal was added for tighter control over the oscillation of the beam. The original DSA only supports open loop control. Second, nested case structures in the DSA2 offer optional functionality to apply different input amplitudes over specific ranges of the excitation frequencies. Thus, in high gain regions I can assign lower driving amplitudes to avoid mechanical saturation of the system response (i.e. the magnets slamming into the PCB), and in low gain regions I can assign higher driving amplitudes to avoid un-measureable responses.

5. Results and Discussion

In this section I discuss the results of my experiments, and make informed analyses as they pertain to my two overarching goals, outlined previously.
5.1 Beam Selection

Informed by system modeling and finite element analysis (discussed above), I employed the DSA2 to obtain Bode plots of the selected FlexLab beam configurations. Especially at the lower frequencies, the gain and phase data came out rather clean; underdamped resonance peaks in the gain plots were all very well defined, as well as the gain asymptotes, and steps in the phase plots. The four beams were tested in the following order: Beam 1 \((t = 0.010\text{"}, l = 1.196\text{"})\), Beam 2 \((t = 0.008\text{"}, l = 1.196\text{"})\), Beam 3 \((t = 0.008\text{"}, l = 0.906\text{"})\), and Beam 4 \((t = 0.008\text{"}, l = 0.615\text{"})\). Data was gathered on the tip deflection for both the second order case (only one magnet set at the free end of the beam) and the fourth order case (two sets of magnets – at the free end and partway down the beam). The figure below shows these four beams.

![Image of four beams]

**Figure 9:** The four beams used for testing and mode analyses; U.S. quarter for reference

To begin with, as a second order system, Beam 1 had a peak gain of about 27.225 at 11.358 Hz. Using (10), I estimate the damping ratio at this peak to be about 0.019. As a fourth order system, the first peak gain was about 35.064 at approximately 13.657 Hz and with a damping ratio of about 0.014, whereas that of the second peak was about 0.199 at 83.438 Hz. Above 100 Hz, the data appears to become a little noisy. This is possibly the result of higher frequency vibrations elsewhere in the PCB.

As for Beam 2, the second order peak gain occurred with a value of 8.398 at 12.987 Hz, resulting in a damping ratio of 0.060. With both sets of magnets, the first peak gain was recorded to be 8.247 at 12.771 Hz. The damping ratio for this was 0.061. For the second peak, the gain was 0.129 at 65.989 Hz. Again, uncontrolled vibrations elsewhere may contribute to noise above 100 Hz.
Testing on Beam 3, the second order peak in the gain data was 3.864, at about 14.604 Hz, and the fourth order first and second peaks were 3.334 at 14.604 Hz and 0.047 at 87.740 Hz, respectively. The estimated damping ratio for the second orders system was thus 0.132, and for the fourth order system’s first peak was 0.153. The small irregularities in the gain plot between 20 Hz and 50 Hz are simply symptoms of large step changes in the gain used to drive the system and maintain coherent results as the gain drastically changes throughout the experiment.

Finally, the second order peak gain value was 4.949 at 15.101 Hz, resulting in a damping ratio of 0.103. For the fourth order system, the first and second peak gain values were 4.656 at 14.603 Hz and 0.051 at 86.282 Hz, respectively. The damping ratio for the first peak was 0.109. However, while the fourth order gain data above 100 Hz was relatively clean (compared to that of some of the other data), it also revealed an unexpected, slightly smaller third peak of about 0.037 at about 131.180 Hz. Repeated trials of data acquisition over this produced similar results. Given information about the other two peaks, and from the previous mode analysis, this peak occurs at a frequency far too low to be the third mode of this cantilever beam. Likely, this third resonance is a symptom of vibrational characteristics of the PCB, PCB to myRio interface, or other that clamping one end of the PCB could not attenuate.

In all, all four beams exhibited first and second modes below the desired threshold of 100 Hz, and thus meet the primary design goal of the experiment. Moreover, as expected, the frequencies of the first and second vibration modes correlated positively with beam thickness, and negatively with beam. For convenience, see the table below for a summary of this data for the first peak of all four beams, as well as additional relevant information. Figures containing commented Bode plots of the raw data are presented below the table.

<table>
<thead>
<tr>
<th>Beam</th>
<th>$G_1$</th>
<th>$\omega_1$ (Hz)</th>
<th>peak height</th>
<th>$\zeta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam 1</td>
<td>27.22495</td>
<td>11.357823</td>
<td>26.6624</td>
<td>0.018753</td>
</tr>
<tr>
<td>Beam 2</td>
<td>8.397689</td>
<td>12.987282</td>
<td>8.295297</td>
<td>0.060271</td>
</tr>
<tr>
<td>Beam 3</td>
<td>3.86409</td>
<td>14.603722</td>
<td>3.798823</td>
<td>0.13162</td>
</tr>
<tr>
<td>Beam 4</td>
<td>4.948784</td>
<td>15.101474</td>
<td>4.866493</td>
<td>0.102743</td>
</tr>
</tbody>
</table>

**Table 2:** Key data on underdamped peak, second order system
Table 3: Key data on first and second underdamped peaks, fourth order system

<table>
<thead>
<tr>
<th>Beam</th>
<th>G₁</th>
<th>ω₁ (Hz)</th>
<th>peak height</th>
<th>ζ₁</th>
<th>G₂</th>
<th>ω₂ (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35.064139</td>
<td>13.656895</td>
<td>34.73405</td>
<td>0.014395</td>
<td>0.199365</td>
<td>83.438187</td>
</tr>
<tr>
<td>2</td>
<td>8.247364</td>
<td>12.771455</td>
<td>8.146685</td>
<td>0.061383</td>
<td>0.129203</td>
<td>65.989418</td>
</tr>
<tr>
<td>3</td>
<td>3.333623</td>
<td>14.603722</td>
<td>3.27066</td>
<td>0.152874</td>
<td>0.046507</td>
<td>87.740186</td>
</tr>
<tr>
<td>4</td>
<td>4.655701</td>
<td>14.603722</td>
<td>4.578006</td>
<td>0.109218</td>
<td>0.050648</td>
<td>86.282089</td>
</tr>
</tbody>
</table>

Figure 10: 2nd (left) and 4th (right) order Bode plots for Beam 1
Figure 11: 2\textsuperscript{nd} (left) and 4\textsuperscript{th} (right) order Bode plots for Beam 2

Figure 12: 2\textsuperscript{nd} (left) and 4\textsuperscript{th} (right) order Bode plots for Beam 3
As all four configurations yielded favorable results below my 100 Hz threshold, I decided to select the thickest beam (0.010") to proceed with for further analysis. This was done because the added thickness would significantly increase the stiffness of the beam, and thus better protect it from plastic deformation due to future wear and tear. From (5) in the section on the Euler-Bernoulli Beam Theory, the flexural rigidity for a uniform rectangular cross section beam scales with the third power of thickness. Therefore, a 25% increase (125% change) in beam thickness (from 0.008" to 0.010") nearly doubles the overall stiffness of the beam ($1.25^3 = 1.953125 = 195.3125\%$ change). As shown in the images below, I already began to notice this stiffness disparity among the test samples.

**Figure 13:** 2\textsuperscript{nd} (left) and 4\textsuperscript{th} (right) order Bode plots for Beam 4

![Bode plots for Beam 4](image)

**Figure 14:** Comparison of two cantilever beams; the thinner beam (on bottom) has sustained greater deformation under similar loadings

![Comparison of beams](image)
5.2 Damping Considerations/Methods

With a beam geometry finally selected, I proceeded to test the damping methods considered during system modeling. Concerned primarily with the first peak, and lacking a specific target for damping, I attempt to bring this peak below 10 and closer to the DC gain of the system. Unmodified, the beam exhibited a peak gain of about 145.6068. At low frequencies (measured at 1 Hz), the system exhibited a gain of 0.5625. Taking this to be approximately equal to the DC gain, the resulting resonance peak height was about 145.1. Using (10) again, I estimated that the damping ratio was equivalent to about 0.00345. The full bode plot for this beam is already presented, but a close up around the first peak (frequency range of 10 – 14 Hz) is presented below.

![Bode plot for the underdamped Beam 1 alone, zoomed in around the first peak; ω_{min} = 10 Hz, ω_{max} = 14 Hz](image)

In the following subsections, I analyze the results of experimenting with various damping methods and make comparisons to this original underdamped case.

5.2.1 Eddy Current Damping

As I increased the number of folds of aluminum foil suspended above the magnets at the end of the beam, I noticed significant decreases in peak gains around the resonant frequency of the first natural mode. Over fold counts as low as two and up to sixty, the peak resonant gains ultimately decreased from 145.61 in the original case to just 2.23, reflecting a total 98.47% decrease in peak height from 5.48 to 1.67. The following table encapsulates all of this information for each successive aluminum sheet fold count. Note that all percent changes are referenced to the original case of no eddy current damping.
Table 4: Key response gain data at various levels of eddy current damping; eddy current damping scaled with thickness of the aluminum; percent changes are compared to original

<table>
<thead>
<tr>
<th>Fold Count</th>
<th>$G_1$</th>
<th>$\omega_1$ [Hz]</th>
<th>% Change</th>
<th>$G_{DC}$</th>
<th>peak height</th>
<th>% Change</th>
<th>$\zeta$</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>145.6068</td>
<td>11.93312</td>
<td>n/a</td>
<td>0.5625</td>
<td>145.0443</td>
<td>n/a</td>
<td>0.003447</td>
<td>n/a</td>
</tr>
<tr>
<td>2</td>
<td>42.75614</td>
<td>12.05542</td>
<td>-70.6359</td>
<td>0.5625</td>
<td>42.19364</td>
<td>-70.9098</td>
<td>0.01185</td>
<td>243.7586</td>
</tr>
<tr>
<td>7</td>
<td>11.22993</td>
<td>11.93312</td>
<td>-92.2875</td>
<td>0.5625</td>
<td>10.66743</td>
<td>-92.6454</td>
<td>0.046872</td>
<td>1259.693</td>
</tr>
<tr>
<td>15</td>
<td>17.77802</td>
<td>12.01451</td>
<td>-87.7904</td>
<td>0.5625</td>
<td>17.21552</td>
<td>-88.1309</td>
<td>0.029044</td>
<td>742.5204</td>
</tr>
<tr>
<td>30</td>
<td>6.04525</td>
<td>11.89264</td>
<td>-95.8482</td>
<td>0.5625</td>
<td>5.48275</td>
<td>-96.2199</td>
<td>0.091195</td>
<td>2545.466</td>
</tr>
<tr>
<td>60</td>
<td>2.234638</td>
<td>11.77199</td>
<td>-98.4653</td>
<td>0.5625</td>
<td>1.672138</td>
<td>-98.8472</td>
<td>0.299018</td>
<td>8574.182</td>
</tr>
</tbody>
</table>

In all, I was able to increase the damping ratio by about a factor of about 100, bringing the underdamped peak at the first natural mode down into the same order of magnitude as the system response gains at neighboring frequencies. The figures below present the gain plot data for each of these cases, zoomed in around the peak, as well as the full range gain plot for the case where the fold count was sixty. Notice in the plots how the diminishing and gradual smoothing of the first peak occurs as fold count (and thus thickness) of the aluminum increases.

Figure 16: Bode Gain plot for Beam 1 with 2 folds of aluminum, zoomed in around the first peak; $\omega_{\text{min}} = 10$ Hz, $\omega_{\text{max}} = 14$ Hz
Figure 17: Bode Gain plot for Beam 1 with 7 folds of aluminum, zoomed in around the first peak; \( \omega_{\text{min}} = 10 \text{ Hz}, \omega_{\text{max}} = 14 \text{ Hz} \)

Figure 18: Bode Gain plot for Beam 1 with 15 folds of aluminum, zoomed in around the first peak; \( \omega_{\text{min}} = 10 \text{ Hz}, \omega_{\text{max}} = 14 \text{ Hz} \)
**Figure 19:** Bode Gain plot for Beam 1 with 30 folds of aluminum, zoomed in around the first peak; $\omega_{\text{min}} = 10 \text{ Hz}$, $\omega_{\text{max}} = 14 \text{ Hz}$

**Figure 20:** Bode Gain plot for Beam 1 with 60 folds of aluminum, zoomed in around the first peak; $\omega_{\text{min}} = 10 \text{ Hz}$, $\omega_{\text{max}} = 14 \text{ Hz}$
**First Peak, All Fold Cases**

![Bode plot for Beam 1 and zoomed in around the first peak; all fold cases are compared simultaneously; \( \omega_{\text{min}} = 10 \text{ Hz}, \omega_{\text{max}} = 14 \text{ Hz} \)](image_url)

**Figure 21**: Bode plot for Beam 1 and zoomed in around the first peak; all fold cases are compared simultaneously; \( \omega_{\text{min}} = 10 \text{ Hz}, \omega_{\text{max}} = 14 \text{ Hz} \)
5.2.2 Viscous Adhesive Damping

I tested two different types of adhesive laminates in the pursuit of alternative damping methods that were not external to the beam itself (as was the case for eddy current damping): post-it notes and packaging tape. For each type, I also varied the number of layers covering the beam from 1 to 4. Different types of adhesives and/or viscous fluids, such as various oils, were also considered as a possible coating for the bronze beams. However, there is a decent level of difficulty associated with manually applying a thin yet even layer of such fluids reliably over a large number of thin beams. Further, without protection, these coatings may easily wipe or wash off over time through normal future use. Consequently, different types of tape present viscous damping advantages in a readily available and easily applicable manner. The figure below shows beams with the chosen methods applied.
Figure 23: Beams being tested for alternatives to eddy current damping; the beam on the far side has been layered with post-it notes, whereas the beam on the near side with packaging tape.

Both types of tape were successful in reducing the height of the first peak on the gain plot down below a magnitude of 10 when using a sufficient number of layers. For the beam with the additional Post-It note layer(s), the data came out very clean with clear definition of the resonant peak in each of the three cases. With a single layer of post-it note across the top of beam, the peak gain dropped to about 13.719 at a frequency of 12.511 Hz. I added another layer of post-it note to the underside of the beam for the double layer case. This time, the peak gain was about 4.670 at about 15.115 Hz. Lastly, in the case of the quadruple layer (two on top, two on the underside), the peak gain dropped to 1.896 at 16.671 Hz. Thus, the calculated damping ratios for the single layer, double layer, and quadruple layer beams were 0.038004, 0.12172, and 0.374814, respectively. For the beam covered with packaging tape instead, the peak gain for the single layer case was 4.187 at 16.671 Hz. The double and quadruple layer cases saw peak gains of 3.250 at 16.211 Hz and 2.466 at 17.025 Hz, respectively. Again, the calculated damping ratios for the single layer, double layer, and quadruple layer beams in this case were 0.137951, 0.186026, and 0.26264, respectively. For convenience, this information, as well as additional data, is presented in the table below.

<table>
<thead>
<tr>
<th>Layers</th>
<th>$G_1$ [Hz]</th>
<th>$\omega_1$ [Hz]</th>
<th>% Change</th>
<th>$G_{DC}$</th>
<th>peak height</th>
<th>% Change</th>
<th>$\zeta$</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>Post-it Note</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>13.71896</td>
<td>12.51131</td>
<td>-90.5781</td>
<td>0.5625</td>
<td>13.15646</td>
<td>-90.9293</td>
<td>0.038004</td>
<td>1002.457</td>
</tr>
<tr>
<td>2</td>
<td>4.670283</td>
<td>15.11478</td>
<td>-96.7925</td>
<td>0.5625</td>
<td>4.107783</td>
<td>-97.1679</td>
<td>0.12172</td>
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<tr>
<td>4</td>
<td>1.896496</td>
<td>16.67138</td>
<td>-98.6975</td>
<td>0.5625</td>
<td>1.333996</td>
<td>-99.0803</td>
<td>0.374814</td>
<td>10772.92</td>
</tr>
<tr>
<td>Packaging Tape</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.186987</td>
<td>16.67138</td>
<td>-97.1245</td>
<td>1.5625</td>
<td>2.624487</td>
<td>-98.1906</td>
<td>0.190513</td>
<td>5426.577</td>
</tr>
<tr>
<td>2</td>
<td>3.250291</td>
<td>16.21096</td>
<td>-97.7678</td>
<td>0.5625</td>
<td>2.687791</td>
<td>-98.1469</td>
<td>0.186026</td>
<td>5296.413</td>
</tr>
<tr>
<td>4</td>
<td>2.46625</td>
<td>17.02526</td>
<td>-98.3062</td>
<td>0.5625</td>
<td>1.90375</td>
<td>-98.6875</td>
<td>0.26264</td>
<td>7518.873</td>
</tr>
</tbody>
</table>

Table 5: Key data concerning the first gain peak for both the post-it note and packaging tape methods.
Figure 24: Bode Gain plot for post-it note, 1 layer

First Peak, Single Post-It Note Layer

Figure 25: Bode Gain plot for post-it note, 2 layers

First Peak, Double Post-It Note Layer
Figure 26: Bode Gain plot for post-it note, 4 layers
First Peak, All Post-it Note Layer Cases

Figure 27: Bode Gain plot for post-it note, all layer cases
Figure 28: Bode Gain plot for packaging tape, 1 layer

Figure 29: Bode Gain plot for packaging tape, 2 layers
Figure 30: Bode Gain plot for packaging tape, 4 layers
Figure 31: Bode Gain plot for packaging tape, all layer cases

However, in all cases, the addition of viscous adhesive damping layers necessarily increased the thickness and the effective stiffness of the beam. Further, while neither of these types of tape have the characteristics to make significant alterations to the overall flexural rigidity of the beam, I have gone ahead and checked to be certain. Specifically, the second peak of the fourth order system is close enough to the desired threshold frequency warrant retesting. As such, I collected frequency response data over a range of 50 Hz to 150 Hz for all of these damping cases. In all six scenarios, the second gain peak remained below 100 Hz, albeit by small margins in the quadruple layer case for both tapes; these peaks occur less than 10 Hz from the threshold.
Figure 32: Verification of second peak for single, double, and quadruple layer post-it note case
Figure 33: Verification of second peak for single, double, and quadruple layer packaging tape case
6. Conclusion

For all four of the beams suggested from the results of the SolidWorks frequency analysis tool, the first and second modes were experimentally determined to lay below the design goal threshold of 100 Hz. In particular, Beam 1, 2, 3, and 4 had a secondary peak gains of 0.199365, 0.129203, 0.046507, and 0.050648, respectively. Further, these gains were located at frequencies of 83.438187, 65.989418, 87.740186, and 86.282089 Hz, again, all comfortably below the desired threshold. For the best protection against future wear and tear, the thickest beam ($t = 0.010''$, $l = 1.196''$) was selected for further study.

As for damping, the design goal involved reducing the first peak to values within an order of magnitude of low frequency response gains. Two main types of damping were considered: through use of eddy current losses (external to the beam) and through use of viscous losses in an adhesive beam layer (internal to the beam). The results for eddy current damping yielded a maximum gain reduction of over 98%, from 145.6068 (no added damping) down to 2.2346 (60 folds of aluminum). The corresponding damping ratio over the range rose by a factor of about 100, from 0.00345 to 0.2990. While both the packaging tape and post-it note layers also significantly increased the damping ratio, only the quadruple layer of post-it notes did so better than the 60-fold count aluminum card. However, I still recommend the eddy current method for a number of reasons. First of all, paper post-it notes on the vibrating beam may wear over time, especially during handling, which may negatively impact performance. Further, the eddy current damping can be further improved by increasing the number of folds in the aluminum cards. Even at 60 folds, the cards still compress relatively flat. If, conversely, keeping all of the beam dynamics internal to the beam geometry is more desirable than relying on external damping methods, then the use of post-it notes, for damping capabilities, may be better protected from damage if further wrapped in something similar to the packaging tape (or laminated by some other means). In this way, one may retain desired damping characteristics while ensuring a longer beam lifetime while in use.

This work on improving the dynamic characteristics of the FlexLab cantilever beam will help pave the way forward for the FlexLab/LevLab module to be incorporated in labs pertaining to the design and analysis of feedback control systems for coming semesters.
Appendix

Appendix A: FlexLab/LevLab Circuit Schematic

Figure 34: FlexLab/LevLab Circuit Schematic
References


