Force control in flapping foils using in-line motion and passive pitch

by

Fiona Grant

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Abstract
Flapping foils are the standard engineering approximation for many biological actuators in air and water, but these devices typically provide propulsion at the cost of parasitic oscillatory forces. The addition of in-line motion to a flapping foil trajectory can improve the control of the fluid force. Previous work has shown that by actuating the heave, surge, and pitch motions of a foil and iteratively optimizing the results, the lift and thrust forces on the foil can be precisely and independently controlled. In this thesis, the same experiment is modified to include solely passive pitch to determine if similar force control performance can be achieved without either a fully actuated pitch motion or an optimization process. In the new experiment, the fluid forces naturally drive the pitch motion for most of the flapping cycle, until the foil reaches a maximum pitch angle, which is set with a mechanical stopper. The hydrodynamic forces are recorded for a range of trajectories that include forwards in-line, backwards in-line, and no in-line motion. Lift force control improves over that of the fully actuated system, but thrust force control is not achieved to the same level of performance. Further work can be done to determine whether simple pitch angle control can be implemented to improve thrust force control without the addition of the optimization process.

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1. Introduction

Flapping foils are mechanical devices modeled on the swimming mechanism for cetaceans and many fish (Fig. 1-1). These devices have been widely researched, and it has been shown through experiments and simulation that flapping foils can be used to produce lift and thrust for the propulsion of a body [1], [2]. Further research has investigated the parameters that influence these propulsive forces and possibilities for controlling them [3], [4]. Work in [5] has shown that it is possible to independently control the lift and thrust forces on a foil. This is done by controlling the heave, surge, and pitch motion of the foil as it moves through the water. The force control is further improved by iterating the results through software and selecting the parameters that give the best control of the lift or thrust force for a given trajectory.

![Flapping Foil Inspiration](image)

Figure 1-1: Flapping Foil Inspiration – Natural flapping foils include fish, dolphin, and whale tails. These animal propulsors inspired the design and development of mechanical flapping foils. Images from [6], [7], [8], [9].

While the force control performance after the optimization process is compelling, this process suggests a complicated control scheme would have to be put in place to implement this force control in, for example, autonomous underwater vehicles (AUVs). The goal of this thesis is to determine if the actuated pitch motion and optimization process implemented in [5] can be replaced with passive pitch, while still maintaining similar force control performance. If so, this
would eliminate the need to optimize the foil trajectory over multiple trials and could suggest a less complex system for various propulsion applications.

2. Background

Traditional flapping foil devices have been found to be reasonably efficient in producing thrust for symmetric flapping trajectories that prescribe a sinusoidal angle of attack profile [3]. However, the symmetric trajectory affords little control over the forces on the foil. Improved force control can be achieved by adding in-line motion to the foil trajectory [5]. In-line motion describes the movement of the foil upstream or downstream during its motion cycle to generate a highly asymmetric trajectory (Fig. 2-1). This trajectory involves a force-generating downstroke, a power stroke, followed by a feathering upstroke of little force, a recovery stroke. If the in-line motion occurs during the downstroke and moves backwards relative to the direction of the trajectory, it is called backwards in-line motion; if the in-line motion occurs during the downstroke and moves forwards relative to the direction of the trajectory, it is called forwards in-line motion.

The addition of in-line motion to improve force control is biologically inspired. Birds use forwards in-line motion to support their weight while flying at low speeds [5]. The downstroke of the wing generates a large lift force, while the upstroke generates little to no force. Turtles use backwards in-line motion to generate thrust forces underwater. The flipper is rotated and moves backwards during the downstroke, producing thrust, and then feathers during the upstroke (Fig. 2-1). These instances of isolated, hydrodynamic force control on biological foils suggest that force control can be mechanized and implemented for flapping foil devices.
(a) Turtle-like motion: backwards in-line motion

(b) Bird-like motion: forwards in-line motion

Figure 2-1: Depiction of In-Line Motion in Bird and Turtle Locomotion — A comparison of (a) turtle-like and (b) bird-like motion. Turtles employ backwards in-line motion to generate primarily thrust force with the downstroke of their flippers. Birds employ forwards in-line motion to generate large lift forces with the downstroke of their wings. These isolated forces are indicated in the inertial reference frame plots with red arrows. Animal figures reproduced with permission from Izraelevitz, [5]. Animal art modified from public domain content at openclipart.org.

3. Methods

A series of foil trajectories are tested to determine the effectiveness of passive pitch motion coupled with in-line motion to control the forces on the foil.

3.1 Experimental set-up

The experiments are conducted on a foil in a small glass tank, 2.4 m by 0.75 m by 0.75 m, in the MIT Towing Tank Lab. The foil is a lightweight NACA0013 carbon fiber blade with a 55 mm chord length and an aspect ratio of 6.5. The foil hangs vertically in the tank and is drawn along the length of the tank (Fig. 3-1). A false bottom is added to the tank, approximately 0.25 m above the tank bottom, to eliminate three-dimensional flow effects around the foil.
Figure 3-1: Full Experimental Set-up – The foil is attached to the shaft of a disabled pitch motor, which sits in the pitch motor housing. The mechanical stopper, located between the pitch motor and the top of the foil, is used to set the maximum pitch angle for each trajectory. The force meter is secured between the housing and a beam that connects the entire foil jig to the linear actuators that move the foil in $x$ and $y$. The entire set-up is attached to the carriage, which tows the foil down the tank at a constant speed, $U$.

The $x$ and $y$ motions of the foil are controlled by two linear motors, as shown in Figure 3-1. The $x$ and $y$ motions are controlled through a Delta Tau PMAC2A-PC motion controller and amplified by two Copley Controls XENUS Digital Drives. The foil is attached to the pitch motor shaft, as shown in Figure 3-2, such that it pitches about its leading edge. An adjustable stopper allows for control of the maximum pitch angle of the foil for a given trajectory. The stopper consists of a set screw shaft collar, a $\frac{3}{4}$" screw, and a stationary steel post that screws into the base of the pitch motor housing (Fig. 3-2). The pitch motor housing is connected to an ATI Gamma force transducer which measures all forces and torques on the foil and housing. This entire set-up is mounted on a carriage that tows it down the tank at a set constant speed, $U$. 
The mechanical stopper consists of a shaft clamp secured with a screw that restricts the pitch motion when it comes in contact with the stationary post. The stopper is adjusted by loosening the screw, rotating the shaft clamp to the new angular position, and retightening the screw.

The angular position of the foil, $\theta$, is measured by the pitch motor encoder. In order to set the maximum pitch angle, the encoder first locates the angular position and the pitch motor turns the foil to this angle. The screw is then screwed into the shaft collar such that the angular motion of the foil is restricted at this maximum pitch angle (Fig. 3-2). The pitch motor is subsequently disabled to allow the foil to pitch passively during each trajectory. All data is logged in LabVIEW, and all data processing is done in MATLAB. The data is filtered with a fifth-order, low-pass Butterworth filter at 10 Hz to remove high frequency noise due to electrical components and mechanical vibrations.

3.2 Foil motion definitions

The foil is towed down the tank at a constant speed $U$ and has three degrees of freedom in surge, heave, and pitch, which are parameterized as $x(t)$, $y(t)$, and $\theta(t)$, respectively. The axes of these three motions are shown in Figure 3-3. The surge and heave motions are sinusoids with the same oscillation frequency and zero relative phase lag. The towing speed represents the mean speed of the animal body, and the motion in $x$ and $y$ represents the relative motion of the foil to the body. This is a simplified approximation for the animal motion [5].
Figure 3-3: Degrees of Freedom for Foil Motion – The foil is actuated in heave and surge motion by linear actuators. The $y$ motion is equivalent to the foil heave and the $x$ motion is equivalent to the foil surge. The foil pitch motion, $\theta$, is not actuated by the pitch motor, but a maximum pitch angle is set for each trajectory. The three axes are oriented as shown.

A fourth motion parameter, the stroke angle $\beta$, is used to define the foil trajectory. When the foil moves in $x$, $y$, and $\theta$, the foil travels along a straight line in the inertial reference frame, which is at an angle $\beta$ with respect to the horizontal (Fig. 3-4 a). Changing this angle changes the profile of the foil trajectory as shown in Fig. 3-4 b and c. Thus, the surge and heave motions are:

$$x(t) = \frac{h}{\tan(\beta)} \cos(2\pi ft)$$

$$y(t) = h \cos(2\pi ft)$$

where $h$ is the heave amplitude, $\beta$ is the stroke angle, and $f$ is the flapping frequency of the foil in Hz. The corresponding velocities are given by:

$$\dot{x}(t) = \frac{-2\pi fh}{\tan(\beta)} \sin(2\pi ft)$$

$$\dot{y}(t) = -2\pi fh \sin(2\pi ft)$$
Inertial reference frame: backwards in-line motion

(a) Carriage reference frame

(b) Inertial reference frame: backwards in-line motion

(c) Inertial reference frame: forwards in-line motion

Figure 3-4: Foil Motion Variables – The foil motion in heave, surge, and pitch is parameterized by $x(t)$, $y(t)$, and $\theta(t)$. A fourth motion parameter, the stroke angle $\beta$, defines the trajectory. The type of in-line motion depends on the value of the stroke angle. These parameters are shown in (a) the carriage reference frame and the inertial reference frame for (b) backwards in-line motion and (c) forwards in-line motion. The approximate angle of attack is also shown.

Assuming the width of the foil wake is equal to the total heave displacement, the Strouhal number can be defined as:

$$St = \frac{2fh}{U}$$

(3.5)

The Strouhal number is a dimensionless parameter that characterizes the flapping frequency of the foil with respect to the size of its wake. For trajectories with no in-line motion ($\beta = 90^\circ$), high-efficiency thrust occurs for $0.2 < St < 0.4$ [5]. The angle of foil motion is defined as:

$$\theta_m(t) = \arctan\left(\frac{\dot{y}(t)}{\dot{x}(t)+U}\right)$$

(3.6)
The angle of flow is the angle of the on-coming fluid flow with respect to horizontal in the reference frame of the foil. As suggested in [5], the angle of flow can be approximated with the angle of foil motion, and therefore, the approximate angle of attack can be calculated as:

\[
\alpha(t) = \theta(t) - \theta_m(t) = \theta(t) - \arctan\left(\frac{\dot{y}(t)}{x(t)+U}\right) \quad (3.7)
\]

The pitch angle is limited by the maximum pitch angle, \(\theta_{\text{max}}\), which is calculated as a function of the motion parameters:

\[
\theta_{\text{max}} = \alpha(t_{\text{max}}) + \arctan\left(\frac{\dot{y}(t_{\text{max}})}{x(t_{\text{max}})+U}\right) \quad (3.8)
\]

where \(t_{\text{max}}\) is the time at which \(\alpha(t)\) is at its maximum value for the desired trajectory. This is calculated using the equations for \(\alpha\) corresponding to symmetric and asymmetric flapping profiles described in [5]. The maximum angle of attack in all experiments is set to 25°, which gives \(t_{\text{max}}\) equal to the quarter period, \(T/4\). Since \(x(t)\) is a function of the stroke angle, \(\beta\), the maximum pitch angle is unique to each trajectory.

A foil trajectory is defined by first setting the maximum angle of attack and the stroke angle, calculating the surge and heave trajectories, and finally calculating the maximum pitch angle. For all trajectories, the foil is restricted in pitch during the downstroke of the flap and allowed to feather during the upstroke (Fig. 3-5).
Backwards in-line motion trajectory

Forwards in-line motion trajectory

Figure 3-5: Restriction of Pitch Angle for Backwards (a) and Forwards (b) In-Line Motion – The forwards and backwards in-line motion trajectories are shown in the inertial reference frame. The location of the stopper is shown for the part of the trajectory where the foil reaches its maximum pitch angle. The stopper is indicated by the red octagon.

The parameter space for the experiment is as follows:

\[
\begin{align*}
    0.3 &< St < 0.5 \\
    45^\circ &< \beta < 135^\circ \\
    h/c &= 1 \\
    Re &= 11000
\end{align*}
\] (3.9)

This Strouhal range is within the range for high-efficiency, high-thrust foil motions. The chosen Reynolds number matches the regime for animal propulsion and fits the capabilities of the tank equipment [5].

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3.3 Propulsive Performance

The propulsive performance of the foil trajectory is determined by two metrics: the propulsive efficiency and a non-dimensional parasitic force ratio, which measures the ratio of the magnitude of parasitic oscillatory forces to the magnitude of the mean propulsive force. The definition of efficiency and the parasitic force ratio proposed in [5] are used so the performance of this passive system may be properly compared to the performance of the pitch-actuated system. In each trial, the forces and moments on the foil are recorded:

\[ \mathbf{F}(t) = \begin{bmatrix} F_x(t) \\ F_y(t) \\ M_\theta(t) \end{bmatrix} \]  \hspace{1cm} (3.10)

where \( F_x \) is the thrust force on the foil, \( F_y \) is the lift force on the foil, and \( M_\theta \) is the torque on the foil due to the fluid forces about the axis of rotation. These forces are non-dimensionalized with the dynamic pressure and the projected area of the foil (the foil span times its chord, \( c \)), and the moment is non-dimensionalized with the dynamic pressure and the projected area times the foil chord. The following are the thrust, lift, and moment coefficients:

\[ C_x(t) = \frac{F_x(t)}{0.5\rho U^2 S}, \quad C_y(t) = \frac{F_y(t)}{0.5\rho U^2 S}, \quad C_m(t) = \frac{M_\theta(t)}{0.5\rho U^2 Sc} \]  \hspace{1cm} (3.11)

The propulsive efficiency is defined as the ratio of power output by the foil flap to input power due to the fluid forces on the foil. The expressions for output and input power and propulsive efficiency are given by:

\[ P_{out}(t) = \frac{1}{T} \int_0^T F_x(t) U dt \]  \hspace{1cm} (3.13)

\[ P_{in}(t) = \frac{1}{T} \int_0^T [F_x(t)\dot{x}(t) + F_y(t)\dot{y}(t) + M_\theta(t)\dot{\theta}(t)] dt \]  \hspace{1cm} (3.14)

\[ \eta = \frac{P_{out}}{P_{in}} \]  \hspace{1cm} (3.15)

The non-dimensional parasitic force ratio, the second metric used to measure propulsive performance, is defined as the root-mean-square of the undesired oscillatory forces over the magnitude of the mean propulsive force, whose components are in \( \dot{x} \) and \( \dot{y} \). The main objective
in altering the foil trajectory to include in-line motion is to gain greater control over the lift and thrust forces on the foil. Thus, any component of the force on the foil that is perpendicular to the desired force is a parasitic force. In a trajectory designed to produce lift, the parasitic force is in the \( \hat{x} \) direction, and in a trajectory designed to produce thrust, the parasitic force is in the \( \hat{y} \) direction. For any given trajectory, \( F_\perp(t) \) is the instantaneous parasitic force and \( F_\parallel(t) \) is the instantaneous desired force [5]. The magnitude of the mean force is denoted by \( \| F_m \| \). The non-dimensional parasitic force ratio, \( \sigma^* \), is therefore defined as:

\[
RMS(F_\perp) = \sqrt{\frac{1}{T} \int_0^T F_\perp(t)^2 \, dt} \\
\sigma^* = \frac{RMS(F_\perp)}{\| F_m \|}
\]

(3.3)

(3.17)

4. Experimental Results

Three types of trajectories are tested: forwards in-line (bird-like) motion, no in-line motion (\( \beta = 90^\circ \)), and backwards in-line (turtle-like) motion. The trajectories with in-line motion are tested over a range of stroke angles in increments of 15°, from \( 45^\circ \leq \beta < 90^\circ \) and from \( 90^\circ < \beta \leq 135^\circ \), respectively. The results for each trajectory are averaged over the total number of flapping cycles for 3 trials. The total number of cycles per trial is 4 at \( St = 0.3 \), 6 at \( St = 0.4 \), and 7 at \( St = 0.5 \). Table 1 summarizes the trajectories selected for comparison and analysis. The performance of these trajectories is presented below.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>Stroke angle ( \beta )</th>
<th>Strouhal number ( St )</th>
<th>Max AοA</th>
<th>Max pitch angle ( \theta_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trajectory I</td>
<td>90°</td>
<td>0.3</td>
<td>25°</td>
<td>18.3°</td>
</tr>
<tr>
<td>Trajectory IIa</td>
<td>45°</td>
<td>0.3</td>
<td>25°</td>
<td>0.903°</td>
</tr>
<tr>
<td>Trajectory IIb</td>
<td>60°</td>
<td>0.3</td>
<td>25°</td>
<td>6.43°</td>
</tr>
<tr>
<td>Trajectory IIc</td>
<td>60°</td>
<td>0.5</td>
<td>25°</td>
<td>14.5°</td>
</tr>
<tr>
<td>Trajectory IIId</td>
<td>45°</td>
<td>0.5</td>
<td>25°</td>
<td>6.43°</td>
</tr>
<tr>
<td>Trajectory IIIa</td>
<td>135°</td>
<td>0.3</td>
<td>25°</td>
<td>61.6°</td>
</tr>
<tr>
<td>Trajectory IIIb</td>
<td>135°</td>
<td>0.4</td>
<td>25°</td>
<td>76.7°</td>
</tr>
<tr>
<td>Trajectory IIIc</td>
<td>135°</td>
<td>0.5</td>
<td>25°</td>
<td>85.0°</td>
</tr>
</tbody>
</table>
As the pitch is not actuated, it is expected that the pitch angle will be equal to the angle of flow throughout the trajectory, except when it is limited by $\theta_{\text{max}}$. Using (3.7) and (3.8), it can be shown that the desired angle of attack increases to $\alpha_{\text{max}}$ at the quarter period and is zero for the remainder of the trajectory.

4.1 Trajectory I – no in-line motion

Trajectory I, which is defined by $\beta = 90^\circ$, is used mainly as a control since it lacks in-line motion. The results for this trajectory are averaged over 3 trials, each with 3 cycles, and are shown in Figure 4-1. The thrust efficiency, $\eta = 18.2\%$, is significantly lower than typical results for a symmetrically flapping foil with this angle of attack and Strouhal number, but this result is to be expected as the foil is free to feather within a pitch angle range less than $\theta_{\text{max}}$.

As shown in Fig. 4-1, the foil does not have enough time to fully recover at the end of the downstroke, which results in an angle of attack around $40^\circ$ just after the halfway point of the stroke cycle. This then results in imperfect feathering during the subsequent downstroke, which is why the desired angle of attack profile is not achieved. It is also worth noting that while the lift force coefficient, $C_y$, does follow a sinusoidal shape, it does not integrate to zero over a cycle. Similarly, the thrust force coefficient, $C_x$, exhibits two peaks that indicate thrust production on the downstroke and the upstroke, but the magnitude of these peaks is smaller than expected. Therefore, the overall result of not actuating the pitch motion for this stroke cycle is a lower thrust efficiency, due to the reduced thrust production and non-negligible lift force, and a larger than expected maximum angle of attack. This control trajectory is particularly useful because it illuminates the trends that appear in the remaining two trajectories that detract from and contribute to the force control on the foil.
Figure 4-1: Trajectory I – This trajectory is defined by $\beta = 90^\circ$ with $St = 0.3$. As it lacks in-line motion, this trajectory acts as a control case. The thrust efficiency, $\eta = 18.2\%$, is low compared to the efficiency of the typical symmetric trajectory, which may be due to the insufficient recovery time at the bottom of the downstroke. For this and following plots, the top right shows the measured and desired AoA profiles, the top left shows the averaged force and moment coefficients (where the grey is the standard deviation between the trials), and the bottom shows the measured force on the foil as it moves down the tank.

4.2 Trajectory II – forwards in-line motion

The following trajectories are compared to both their pitch-actuated and optimized counterparts in [5]. The optimized trajectories are also pitch-actuated, but have been iterated through software to select the parameters that produce the best performance. These comparisons are used to determine if optimization is indeed necessary to gain good control over the force on the foil.

Trajectory IIa-d (bird-like trajectories), which are defined by $45^\circ \leq \beta < 90^\circ$, exhibit forwards in-line motion. This produces large lift during the downstroke and ideally feathers on
the upstroke. As such, all of these trajectories have a low thrust efficiency by design. Trajectory Ila, at $St = 0.3$ and a stroke angle $\beta = 45^\circ$, produces a mean lift force coefficient $\bar{C_y} = 2.72$ and a mean thrust force coefficient $\bar{C_x} = 0.153$, with a parasitic force ratio $\sigma^* = 0.127$. This is a slightly better performance than the corresponding pitch-actuated trajectory, which produces $\bar{C_y} = 2.31$, $\bar{C_x} = 0.224$, and $\sigma^* = 0.282$ [5]. As in Trajectory I, the foil does not have enough time to recover completely before the upstroke, which may contribute to its improved performance over the pitch-actuated case. In this case, negative lift and a small amount of thrust are generated during most of the upstroke (Fig. 4-2). However, in the pitch-actuated case, when the foil is forced to recover, more thrust is generated during the upstroke. There is less unwanted thrust produced by the passive system, which may account for its lower parasitic force ratio.
its pitch-actuated counterpart because more lift and less thrust is produced when the foil does not fully recover at the bottom of the downstroke.

Trajectory IIb has $St = 0.3$ and $\beta = 60^\circ$, and produces $C_y = 1.54$, $C_x = 0.286$, and $\sigma^* = 0.211$. This is comparable to the corresponding optimized trajectory at $St = 0.3$ and $\beta = 57^\circ$, which produces $C_y = 2.1$, $C_x = 0.014$, and $\sigma^* = 0.0641$ [5]. The passive case nearly reaches a mean lift coefficient of 2, but it does so with a parasitic force ratio that is an order of magnitude larger than that of the optimized case. Here, the inability of the foil to recover after the downstroke is detrimental to force control on the foil, because a significant amount of thrust is produced during the upstroke, which contributes to a larger $\sigma^*$. Thrust is also produced during the downstroke because the foil pitch angle remains at $\theta_{max}$, which is not perfectly parallel to the oncoming flow, $U$ (Fig. 4-3).

Interestingly, Trajectory IIa with $\beta = 45^\circ$ produces a greater mean lift force coefficient ($C_y = 2.72$) than this optimized trajectory and outperforms the passive trajectory at $\beta = 60^\circ$ in terms of the parasitic force ratio ($\sigma^* = 0.127$). This is likely due to the fact that $\theta_{max}$ is smaller for Trajectory IIa and brings the foil essentially parallel to $U$ during the downstroke, producing less thrust than Trajectory IIb (Fig. 4-2). However, a significant amount of thrust is produced during the upstroke in Trajectory IIa, so the optimized trajectory still has a much lower cost due to the parasitic force ratio.
Figure 4-3: Trajectory IIb – This trajectory, with $\beta = 60^\circ$ and $St = 0.3$, produces $\bar{C}_y = 1.54$, $\bar{C}_x = 0.286$, and $\sigma^* = 0.211$. This produces comparable lift to the optimized case, where $\bar{C}_y = 2.1$, but with a parasitic force ratio that is nearly an order of magnitude greater. The insufficient recovery time after the downstroke leads to unwanted thrust during the upstroke, increasing the magnitude of the parasitic force.

The second optimized trajectory in [5] has $St = 0.5$ and $\beta = 59^\circ$, and produces $\bar{C}_y = 4.05$, $\bar{C}_x = 0.989$, and $\sigma^* = 0.135$. This is compared to Trajectory IIc, which has $St = 0.5$ and $\beta = 60^\circ$, and produces $\bar{C}_y = 2.97$, $\bar{C}_x = 0.954$, and $\sigma^* = 0.155$. Although this trajectory does not reach a mean lift force coefficient of 4, it does produce a comparably low thrust force coefficient and parasitic force ratio to the optimized case. As shown in Fig. 4-4, the foil still does not have sufficient time to recover, which may contribute to the slightly larger parasitic force ratio in this trajectory compared to the optimized case.
Trajectory IIc with stroke angle $\beta = 60^\circ$ at $St = 0.5$ reaches $\overline{C_y} = 2.97$ with $\overline{C_x} = 0.954$, and $\sigma^* = 0.155$. Although the mean lift coefficient does not reach 4, the parasitic force is similarly low to the corresponding optimized trajectory. Both $\overline{C_x}$ and $\sigma^*$ are slightly higher than the optimized case because the foil cannot recover fully at the bottom of the downstroke.

Trajectory IIId with stroke angle $\beta = 45^\circ$ at $St = 0.5$ reaches $\overline{C_y} = 4.83$ with $\overline{C_x} = 0.476$, and $\sigma^* = 0.218$. This trajectory reaches a higher mean lift force coefficient for a lower mean thrust force coefficient than the optimized case presented in [5]. However, the parasitic force ratio, $\sigma^* = 0.218$, is about 1.5 times higher than both the optimized case and the passive case at $\beta = 60^\circ$, which suggests that the magnitude of the overall mean force is smaller for Trajectory IIId (Fig. 4-5).
4.3 Trajectory III – backwards in-line motion

Trajectory IIIa-c (turtle-like trajectories), which are defined by $90^\circ < \beta \leq 135^\circ$, exhibit backwards in-line motion and are designed to generate primarily thrust force during the downstroke and little to no lift force during the upstroke. These results were more difficult to achieve with the passive system than the results for Trajectory II. The reasons for this will be further explored in this section.

Figure 4-5: Trajectory IIId – The fourth bird-like trajectory, with $\beta = 45^\circ$ at $St = 0.5$, produces $\bar{C_y} = 4.83$, $\bar{C_x} = 0.476$, and $\sigma^* = 0.218$. This is a higher mean lift coefficient for a lower mean thrust coefficient than the optimized case, which produces $\bar{C_y} = 4.05$, $\bar{C_x} = 0.989$, and $\sigma^* = 0.135$. However, the parasitic force ratio of Trajectory IIId is larger than that of the optimized case, which suggests this trajectory produces a smaller mean force.
Trajectory IIIa has $St = 0.3$ and $\beta = 135^\circ$, and produces $\bar{C}_x = 0.0686$, $\bar{C}_y = -0.410$, and $\sigma^* = 0.977$. The thrust efficiency is also low ($\eta = 11\%$), but not much lower than that of Trajectory I ($\eta = 18\%$). In both cases, the foil does not have enough time to fully recover after the downstroke, which leads to a large lift force relative to the thrust force. For comparison, the corresponding pitch-actuated case presented in [5] produces $\bar{C}_x = 0.116$, $\bar{C}_y = 0.416$, and $\sigma^* = 1.54$, with a thrust efficiency $\eta = 27\%$. Although the passive case has a better parasitic force ratio, the thrust efficiency is half that of the pitch-actuated case. The mean lift force coefficient has approximately the same magnitude, but opposite sign, as that of the pitch-actuated case. The lift on the passive foil is negative because the lag in foil recovery leads to imperfect feathering during the upstroke, which in turn produces a negative lift force on the foil (Fig. 4-6).
Figure 4-6: Trajectory IIIa – The first turtle-like trajectory, with $\beta = 135^\circ$ and $St = 0.3$, produces $\bar{C}_x = 0.0686$, $\bar{C}_y = -0.410$, and $\sigma^* = 0.977$. Although the pitch actuated case has a higher parasitic force ratio ($\sigma^* = 1.54$), it is twice as efficient as the passive case.

In an attempt to obtain better results for this trajectory, the Strouhal number was increased to 0.4 for the same stroke angle and carriage speed (Trajectory IIIb). Increasing the Strouhal number increases the flapping frequency, which increases the foil velocity. If the foil is moving more quickly, the fluid forces on the foil should increase, producing a large thrust force during the downstroke of the trajectory. If the thrust force increases more than the lift force, the thrust efficiency of the trajectory should improve. This is indeed the case, as shown in Fig. 4-7. At $St = 0.4$, the mean thrust force coefficient increases to 0.331, about 4 times the value of $\bar{C}_x$ for $St = 0.3$, and the mean lift force coefficient remains about the same ($\bar{C}_y = -0.419$). This results in an efficiency $\eta = 27\%$ that is comparable to the pitch-actuated case at $St = 0.3$. However, the pitch-actuated case in [5] has a lower parasitic force ratio ($\sigma^* = 1.54$) than the passive case at $St = 0.4$ ($\sigma^* = 2.19$). Nevertheless, this comparison leads to the important insight that increasing the flapping frequency for trajectories with backwards in-line motion, can substantially increase thrust production and improve thrust efficiency.
AOA Profile

$\beta = 135^\circ$
$St = 0.4$
$h/c = 1$
$\theta_{max} = 77^\circ$

Mean $C_x = 0.331$
Mean $C_y = -0.419$

$\sigma^* = 2.19$
Thrust $\eta = 0.271$

**Figure 4-7:** Trajectory IIIb – Increasing the Strouhal number to 0.4 for $\beta = 135^\circ$ increases the thrust production to $C_x = 0.331$ and the thrust efficiency to $\eta = 27\%$. The parasitic force ratio is higher than that of the pitch-activated case, but increasing the flapping frequency increases thrust production and efficiency in the passive case.

The third turtle-like trajectory (Trajectory IIIc) has $St = 0.5$ and $\beta = 135^*$, and produces $C_x = 0.654$, $C_y = -0.601$, and $\sigma^* = 1.88$, at a thrust efficiency $\eta = 30\%$. This is the largest mean thrust force coefficient produced for any of the turtle-like trajectories, and one of the highest efficiencies. While increasing the flapping frequency improves the thrust production, the cost due to the parasitic forces is still high ($\sigma^* = 1.88$). Although the mean thrust force clearly increases compared to the previous two trajectories, the unwanted lift force also increases (Fig. 4-8).
The corresponding optimized case, with \( St = 0.5 \) and \( \beta = 135^\circ \), produces \( \bar{C}_x = 1.02 \), \( \bar{C}_y = -0.0589 \), and \( \sigma^* = 0.829 \) with a thrust efficiency \( \eta = 47\% \) [5]. For Trajectory IIIc, the optimized system clearly outperforms the passive system, with half the parasitic force ratio and an efficiency that is nearly 20 percentage points greater. The magnitude of the mean lift force for the optimized system is also an order of magnitude smaller than that of the passive system.

It seems the performance tradeoff for the passive system on a backwards in-line motion trajectory is between thrust production and the parasitic force ratio. It is possible to increase the thrust production to values that are comparable to the optimized system, but this corresponds to a detrimental increase in the parasitic force ratio. As with all the other passive trajectories, the foil fails to recover in time for the upstroke. This leads to large lift forces on the foil during the majority of the upstroke that only diminish as the foil realigns with the trajectory at the end of the upstroke (Fig. 4-8).
Figure 4-8: Trajectory IIIc – The final turtle-like trajectory at $St = 0.5$ and $\beta = 135^\circ$ produces the largest mean lift force ($C_L = 0.654$) at a high thrust efficiency ($\eta = 30\%$), but with much greater parasitic lift force ($C_Y = -0.601$). Therefore, the optimized case still far outperforms the passive case.

IV. Summary of Results

The results of Trajectory I, with no in-line motion, illustrate the challenges to gaining force control over a flapping foil with passive pitch motion. In general, the insufficient recovery time at the bottom of the downstroke leads to an increase in unwanted forces during the upstroke and low thrust efficiencies for backwards in-line motion trajectories.

However, this does not preclude the passive system from performing comparably or outperforming the pitch-actuated and optimized systems presented in [5]. The bird-like trajectories (Trajectory IIa-d), with forwards in-line motion, perform well in this regard. Trajectory IIa, with $\beta = 45^\circ$ and $St = 0.3$, outperforms the corresponding pitch-actuated case with a greater mean lift coefficient and a lower parasitic force ratio. Trajectory IIb, with $\beta = 60^\circ$ and $St = 0.3$, is comparable to the corresponding optimized case in lift force production, but at a higher parasitic force ratio. Trajectory IIc, with $\beta = 60^\circ$ and $St = 0.5$, reduces the mean thrust coefficient more effectively than the corresponding optimized case, but it does not produce the same lift and has a greater parasitic force ratio. Finally, Trajectory IId, with $\beta = 45^\circ$ and $St = 0.5$, produces lift and reduces thrust more effectively than the optimized case at $\beta = 59^\circ$ and $St = 0.5$, but does so at a higher parasitic force ratio. These results indicate that good lift force control can be achieved on the foil with passive pitch and no optimization.

It is much more difficult to obtain good performance with the turtle-like trajectories (Trajectory IIIa-c), with backwards in-line motion, given the passive system. Trajectory IIIa, with $\beta = 135^\circ$ and $St = 0.3$, and the corresponding pitch-actuated case both perform poorly, but the pitch-actuated system exhibits a much better thrust efficiency. Trajectory IIIb, with $\beta = 135^\circ$ and $St = 0.4$, produces a much larger mean thrust coefficient than Trajectory IIIa and the corresponding pitch-actuated case, at a comparable thrust efficiency to the pitch-actuated case, but at a higher parasitic force ratio. Trajectory IIIc, with $\beta = 135^\circ$ and $St = 0.5$, has the greatest thrust production and highest efficiency of the three trajectories, but it is far outperformed by the corresponding optimized case. This suggests that passive pitch control may not be sufficient to achieve control over the thrust force on the foil through a backwards in-line motion trajectory.
The following figures plot the parasitic force ratio and the thrust efficiency as functions of the stroke angle and the Strouhal number for every tested trajectory. These plots show the relationship between the performance metrics and the parameter space. For all Strouhal numbers, the parasitic force ratio remains below 1 until $\beta = 90^\circ$, reaches a maximum around $\beta = 120^\circ$, and then begins to decrease at $\beta = 135^\circ$ (Fig. 4-9). This supports the observation that, for the passive system, it is significantly easier to reproduce good force control for forwards in-line motion trajectories than for backwards in-line motion trajectories. The thrust efficiency appears to follow a distinct curve with respect to the stroke angle (Fig. 4-10). For $St = 0.3$, the efficiency peaks at $\beta = 75^\circ$ and $\beta = 105^\circ$. For both $St = 0.4$ and $St = 0.5$, the thrust efficiency appears to increase, plateau between $\beta = 75^\circ$ and $\beta = 120^\circ$, and then begin to decrease at $\beta = 135^\circ$. These efficiency curves suggest that there is a stroke angle or stroke angle range at each Strouhal number for which thrust efficiency can be maximized.

![Figure 4-9: Parasitic Force Ratio as a Function of Stroke Angle](image-url)

The parasitic force ratio is plotted for all tested trajectories. Three distinct scatter plots are shown for $St = 0.3$ (blue), $St = 0.4$ (green), and $St = 0.5$ (red). The increase in parasitic force ratio for the turtle-like trajectories underscores the general result that force control in a passive system is more difficult for backwards in-line motion trajectories.
The thrust efficiency is plotted for all tested trajectories. Three distinct scatter plots are shown for \( St = 0.3 \) (blue), \( St = 0.4 \) (green), and \( St = 0.5 \) (red). The plots tend to peak at \( \beta = 75^\circ \) and \( \beta = 120^\circ \), and decrease for smaller and larger stroke angles.

5. Conclusion and Future Work

Force control of a flapping foil is possible for a passive foil system, in which a restricted maximum pitch angle replaces fully actuated pitch motion control. However, this force control is limited to the range of stroke angles that correspond to forwards in-line motion trajectories, or lift force control. While the passive system can generate substantial thrust force for larger stroke angles, or backwards in-line motion trajectories, the performance of the force control is poor in this regime compared to that of the optimized, fully actuated system. Therefore, while optimization is wholly unnecessary for lift force control, it may be required for thrust force control. In general, poor performance of the passive system compared to the fully actuated system appears to correlate with the insufficient recovery time the passive foil experiences after the downstroke of each trajectory.
It may be possible to obtain better thrust force control by using a lighter foil or by implementing a restricted maximum pitch angle along with actuation that forces the foil to recover fully at the bottom of the downstroke. This could eliminate unwanted forces during the upstroke and greatly improve the force control performance for all trajectories. Future work could also include a further exploration of the parameter space and its relationship to the force control performance metrics.
References


