COUETTE FLOW MEASUREMENT OF EQUILIBRIUM AND ENERGIZATION CHARGING IN TRANSFORMER INSULATION

by

David John Lyon

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Submitted to the Department of Electrical Engineering and Computer Science on July 16, 1987 in partial fulfillment of the requirements for the degree of Master of Science.

Abstract

A number of failures of large oil cooled transformers has been attributed to flow-induced electrification currents. These currents are comprised of convected charges that are separated at liquid/solid interfaces by physical, chemical, and electrical means. A Couette "mixer" is developed which provides measurement of properties needed to predict charge generated by a transformer oil/cellulosic interface under both equilibrium (i.e. no current flows across the interface) and non-equilibrium conditions.

The measured equilibrium charge is used to predict the equilibrium wall-charge using an adaptation of the fully developed equilibrium duct-flow theory of Abedian and Sonin. The dependence of this charge on Reynold's number $R$ is that predicted over the measured range $280 < R < 5040$. The application of an external field simulates the effects of transformer energization and produces non-equilibrium currents, which give rise to charges that are compared to those predicted by a model developed in this work. This migration-based model predicts the effect of external field strength and frequency, as well as flow conditions, on the injection of charge from the boundary into the turbulent flow core. Experimental data displays energization currents which have dependences on frequency, Reynold's number and voltage sufficiently like those predicted to encourage refinement of the injection model to include the effects of what turbulent diffusion there is inside the "laminar sub-layer".
I would like to thank Professor James Melcher and Professor Markus Zahn for all the time and understanding they gave me during my work on this thesis. Most students would consider themselves fortunate to have a supervisor who is concerned about their needs and is both easy and inspiring to work with; I am at least doubly blessed, not only in having attention from two supervisors, but in having two of the absolute best. My rewards from working with these two men go much deeper and will last much longer than is indicated by anything between these two covers.

I am also extremely grateful for the pleasant and supportive work environment I have enjoyed for the past few years. In particular Paul Warren has been, as he is for all students in the lab, an invaluable resource, and has saved my experiment from disaster on more than one occasion. I am also indebted to my office-mates, both past and present, for their continuous support and friendly dispositions; of special note are Stephen Bart, Rayomond Kotwal, and Diane Gaylor. People like these make it all worthwhile, or at least bearable. MIT High Voltage Laboratory machinist Rocky Albano is especially acknowledged for his cheerful expertise in all construction aspects of the Couette Electrification Apparatus.

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Chapter 1
Introduction

1.1 Transformer Electrification

The flow of semi-insulating liquid coolant through power distribution equipment gives rise to electric fields through the convection of charges that have been separated at liquid/solid interfaces by chemical means. These fields, either alone or in consort with the fields imposed by energization of the equipment, can exceed those anticipated in design and contribute to insulation failure. Examples include freon cooled static power conditioning equipment\textsuperscript{1,2}, where the flow-induced fields can puncture the walls of insulating tubes, and oil cooled transformers\textsuperscript{3-16}, in which the field build-up leads to catastrophic discharges. Both to ensure reliable operation of existing equipment and to guide in the development of the next generation of power apparatus, it is desirable that flow electrification be reduced to an engineering science.

Flow electrification has been attributed to the failure of a number of large power transformers in the U.S. Electrification has been implicated in these failures for two primary reasons: first, the failures occurred only when the coolant had been continuously circulating through the unit for a length of time; and second, "autopsies" of the failed equipment revealed tracking on the solid insulation bearing evidence of partial electrical discharges. That the greatest proportion of these tracks were found at the bottom of the windings, where the oil first enters the ducts from the external cooling system, points both to the oil pumps and to the flow conditions peculiar to the inlet region as potential suspects for extreme charge generation.

The first reports associating electrification with problems in large forced-oil-cooled transformers came from the Japanese in 1979\textsuperscript{5,8,10,11}. They described the large-scale transformer models that were used in their tests, and gave evidence of electrification phenomena, including photographs of electrical discharges. In December, 1983 the first U.S. field transformer failure attributed to flow electrification occurred, and has been followed by about eleven more\textsuperscript{17,18}. Typically, these failures involved flashovers from high voltage windings to low-voltage leads across considerable oil gaps, and upon tear-down of the unit, significant tracking and other evidence of electrification could be found on the insulation. Although it is common in the transformer electrification literature to associate electrification problems with a particular transformer design (usually shell-form), a single manufacturer and a single oil refiner, the situation is not that straightforward: the twelve known failures have occurred in both shell-form and core-form transformers, and have involved at least six manufacturers and five oil refiners.\textsuperscript{18}
1.2 Identification of Practical Mechanisms

1.2.1 Four Stages of Electrification

The roles of electrical and mechanical forces in any system experiencing flow electrification can be analyzed in terms of four basic processes: charge generation, transport and field generation, accumulation, and leakage. These processes are perhaps most easily visualized in the Van de Graaff generator, where each is clearly separated from the others. The charge in Figure 1-1a is generated at the bottom, where it is sprayed onto a moving belt which transports the net charge to a metal dome. Charge accumulates on the metal surface and is allowed to leak off through an external resistor.

Figure 1-1: The basic processes of flow electrification in a) Van de Graaff generator, where each is spatially distinct, and b) power transformer, where processes overlap.

Figure 1-1b identifies the same processes in a power transformer, where they are clearly not spatially distinct. Charge generation occurs in both the pumps and in the ducts between coil windings, a region where all four processes can overlap. Here charge can accumulate on the surface of the solid insulation, and can also leak away through this insulation material. Finally, the plenum above the windings allows accumulation of volume charge, as long as the relaxation (leakage) currents in that space are exceeded by convection currents entering from below.
1.2.2 List of Influences and Effects

There are a number of attributes of an operating transformer that have been linked to the degree of electrification activity the unit will suffer through operational and service experience. These attributes are presented below in a summary based on a list detailed by D.W. Crofts 18:

1. **Charging Tendency** - The "charging tendency" of a particular oil is determined by a "standard" measurement technique, and indicates the average charge density created by forcing the oil past a paper filter in a proscribed fashion 14. While this technique does not use the solid insulation typically found in transformers, it is widely used to explore the effects of various parameters on electrification. The Couette charger developed here provides a more rigorous approach, which takes into account the role played by the "paper" as well.

2. **Flow Turbulence** - is generally considered to significantly augment electrification activity.

3. **Flow Rate** - in smooth ducts turbulence generally sets in for Reynold's numbers (proportional to the flow rate) above 2000-3000.

4. **Surface Characteristics of Solid Insulation** - Roughness of the solid surface enhances charge generation 8.

5. **Energization** - Transformer energization has been shown to increase electrification currents 10, 19.

6. **Dielectric Strength of Oil** - The dielectric strength of oil is obviously important in determining the harmful effects of electrification activity, but is also one mode of charge leakage.

7. **Conductivity of Solid Insulation** - This parameter is important as it controls the amount of leakage available. It is affected by the migration of moisture between the solid and the oil, which occurs as the relative moisture contents regain equilibrium after temperature or pressure changes; and by surface active agents. In addition to influencing the charging tendency of the oil, surfactants can contribute to a layer of surface conductivity on the solid insulation. This layer provides another path for leakage.

8. **External Charging** - Streaming currents emanating from a transformer duct are increased by currents entering the duct's inlet from upstream charge generators, namely the pumps. The mixing action of the flow through the pumps has been found to generate currents in the same manner as the oil ducts, increasing suspicion that the pumps may play a significant role in transformer electrification.

9. **Contaminants, Surface Active Agents and Particulate Matter** - Trace quantities of surfactants are known to significantly alter the charging tendency of an oil.

10. **Oil Temperature** - The temperature of the oil influences electrification activity through its effect on the oil viscosity and conductivity. Changes in the
temperature inside a transformer are also known to change the moisture equilibrium between the paper and oil, thus affecting the conductivity of the paper.

11. **Moisture Content of Oil** - High moisture levels in the oil tend to decrease the charging tendency; levels below 15 ppm produce the highest charging effects.

12. **Configuration** - The physical configuration of the transformer has a great impact on the degree of electrification to be expected. Location of the pumps, sizes of the plenums above and below the coil windings, the materials comprising the paper/oil interface, and the geometries of the oil ducts all play crucial roles in the electrification process.

### 1.2.3 Matrix of Stages and Effects

The properties listed above influence different aspects of the electrification process. Table 1-I indicates which of the four basic processes are affected by each property.

### 1.3 Isolation of Charge Generation Stage

Since the charge generation process provides the fuel, as it were, to the other basic processes, a thorough understanding of it is fundamental to an appreciation of the overall electrification problem. In addition, as indicated in Table 1-I, the generation process is the one affected by the most properties of the transformer system, and thereby reveals most clearly the effects of these properties. It is therefore natural to isolate this process and study it in a regime where the other electrification processes do not come into play.

As will be seen in the following chapters, one result of this isolation is the ability to add to the above list of influences and effects. The two most salient contributions are the effects of

1. **presence of obstacles** - There is a flow regime peculiar to the Couette configuration in which the even, laminar flow breaks up into cells, as described in Section 3.1.2. This flow is very similar to that encountered around obstacles to the flow, and thus provides an opportunity to investigate the effect of the complex flow around the many spacer blocks in the oil ducts. In any case, protuberances in ducts and secondary convection in transition regions result in charge entrainment similar to that caused by turbulence, but occurring at a much smaller Reynold’s number.

2. **charge injection** - The model proposed in Chapter 4 for studying the effects of AC energization assumes the existence of a charge injection process at the boundary. The verification of this model is of extreme importance to the study of electrification enhancement due to energization, and is a primary objective in this work.
### Table 1-I: The Stage-Effects Matrix, indicating which processes are affected by each property

<table>
<thead>
<tr>
<th>The Stage-Effects Matrix</th>
<th>generation</th>
<th>transport</th>
<th>accumulation</th>
<th>leakage and insulation failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;Charging Tendency&quot;</td>
<td>ECT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turbulence</td>
<td>Laminar sub-layer</td>
<td>Wall shear stress</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flow Rate</td>
<td>Streaming current</td>
<td>Reynold’s number</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Surface Characteristics</td>
<td>Turbulence, roughness</td>
<td>drag on wall</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Energization (AC field)</td>
<td>Charge injection</td>
<td></td>
<td></td>
<td>puncture and creep failure</td>
</tr>
<tr>
<td>Dielectric Strength</td>
<td></td>
<td></td>
<td></td>
<td>Wall current</td>
</tr>
<tr>
<td>Conductivity of Solid</td>
<td></td>
<td></td>
<td></td>
<td>Enhanced conduction</td>
</tr>
<tr>
<td>External Charging</td>
<td>Inlet Current</td>
<td></td>
<td></td>
<td>Conductivity, paper/moisture equilibrium</td>
</tr>
<tr>
<td>Contaminants, Surface Active Agents and Particulate Matter</td>
<td>Zeta Potential</td>
<td>Enhanced conduction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>Debye Length, Laminar sub-layer, Arrhenius</td>
<td>Viscosity</td>
<td>Conductivity, paper/moisture equilibrium</td>
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</tr>
<tr>
<td>Moisture</td>
<td></td>
<td></td>
<td></td>
<td>Paper</td>
</tr>
<tr>
<td>Configuration</td>
<td>Pumps, paper/oil interface</td>
<td>Pipes</td>
<td>Upper plenum, paper/oil interface</td>
<td>Paper/oil interface, paper bulk, oil bulk</td>
</tr>
</tbody>
</table>
1.3.1 Equilibrium and Non-equilibrium Charge Generation

While the transport process inherently overlaps all other processes due to the dependence of electrification on the existence of a flow, charge generation can be separated from accumulation and leakage if it has reached *equilibrium*. This electrical equilibrium exists if the generated streaming current has reached a steady-state value, referred to as $I_\infty$, and it implies that no current flows to the wall. When these conditions are met the charge density at the wall is the equilibrium value $\rho_w$, it has no gradient in the direction of the flow, and charge accumulation has therefore stopped.

Systems that are not in equilibrium, that have non-zero currents at the boundaries, can experience the accumulation and leakage phenomena. Charge can accumulate in these regimes by self-precipitating to the walls or by selective adsorption at the interface, leading to field generation not found in equilibrium charging regions. Leakage through the insulation is inherent to the presence of wall currents, but can be enhanced by accumulated charge, and can even find a new path along the surface due to a surface conductivity developed by this charge. Clearly, systems experiencing non-equilibrium charging find the four basic processes overlapping and prevent isolation of the charge generation phenomenon.

1.3.2 Charge Generated by Energization

In order to obtain a complete understanding of charge generation, effects of energization must be taken into account. Studies have indicated increased electrification activity due to transformer energization$^{20,19}$, and the present research demonstrates that the AC fields applied during energization augment the flow-induced streaming currents through the charge injection process. In the process hypothesized here, charge is injected into the flow from the boundary by the presence of the applied field; these charges are then acted upon by a combination of fluid and electric forces that can yield a net increase in axial current.

Since the charge injected by this means comes from an external source, it must be transmitted by a wall current, and equilibrium must therefore be destroyed. Indeed, many of the ideas germane to the analysis of flow-induced currents must now be abandoned, because the applied fields often overwhelm those intrinsic to the equilibrium double layer, disrupting the very source of the original generation process.

1.3.3 The Duct Flow Approach

Many studies of transformer electrification perform experiments on pipe flow models that are designed to simulate the flow between coil windings in a transformer. Both the pipe flow and the flow in the actual oil ducts have development lengths that are even longer than the system, and therefore in both the development process must be taken into account. The wall currents that take part in developing the axial current density are typically detected in pipe flow experiments by measuring image currents either to segmented electrodes that are wrapped around the outer insulating cylinder or to a single electrode covering the entire pipe.

This approach enables researchers to examine streaming currents as they develop along the
length of the pipe. It also allows for the generation of substantial electric fields which can become great enough to create discharges both inside the tube and at its edge\textsuperscript{20,7,11,8}. However, it is difficult in these models to rigorously correlate the observed effects with parameters of the system, since the parameter(s) that determine the magnitude of the generated currents may not be the same as those affecting, say, the build up of surface charge or leakage paths. This obfuscation of pertinent parameters is inherent in systems where the basic electrification processes can overlap.

1.4 The Couette Flow Approach

The present research studies a model in which there is a development time, but no development length. All surfaces under examination are in essentially the same state. The spatial development length in pipe flow is here translated into a temporal transient, after which steady conditions are achieved. Although the temporal transient can be used to glean additional information from the experiment, emphasis here is on two types of steady-state measurements, as explained below.

1.4.1 Description of Configuration

The model used to achieve this steady state is depicted in Figure 1-2. Couette flow is produced by rotation of the inner cylinder, and the generated currents are sampled by a small flow into and out of the Couette apparatus. Variations in the Reynold's number are easily achieved by simply varying the rotational speed of the inner cylinder; the sampling flow rate can remain constant since it does not establish the flow conditions inside the device. Application of an external field is also facilitated, for use in non-equilibrium charge injection experiments.

![Figure 1-2: The Couette flow apparatus.](image-url)
1.4.2 Equilibrium Charge Density

As long as the sampling flow rate is small the system, after the start-up transient, is indeed in electrical and mechanical equilibrium: the symmetry in the \( \theta \)-direction allows for no spatial development of current, and therefore no net wall current is required, as long as the currents exiting by the sampling process are negligibly small. Thus, the currents measured are indicators of a purely equilibrium charging process, where no charge accumulation or leakage occurs.

Since it is not appropriate in the present model to think of developing currents convecting charge downstream, the Couette system is best understood in terms of an equilibrium double layer with a much larger Debye length, \( \lambda_T \), created by turbulent diffusion. Ignoring the small currents required to account for the measurements, no radial currents exist and the action of the turbulent flow is merely to bring charge from the diffuse double layer into the core of the flow. Since the turbulent core, which extends across almost the entire gap between the cylinders, is a mixed region, the charge density there is assumed to be a constant, \( \rho_o \); it is therefore this charge density that is sampled, for the measured current is approximately \( \rho_o Q_v \), with \( Q_v \) the sampling flow rate.

Mathematically, this idea of an enlarged charge double layer is achieved by adding to the molecular diffusion coefficient, \( D_m \), a turbulent diffusion coefficient \( D_T \), which represents the mixing action of the turbulent flow. \( D_T \) is small in the laminar sub-layer, but increases from there to a value \( D_{TC} \) that is uniform across the flow core and can be orders of magnitude larger than the molecular diffusion coefficient. As long as the laminar sub-layer is smaller than or comparable to the molecular Debye length, turbulent diffusion can act upon the double layer charges and spread them over a much larger distance, given by

\[
\lambda_T = \sqrt{\frac{\varepsilon D_T}{\sigma}}
\]

where the turbulent diffusion coefficient has replaced \( D_m \) in the familiar expression for the Debye length. If \( D_T \) is sufficiently large, the charge double layer can extend across the entire oil gap.

The ratio of the measured charge density \( \rho_o \) to \( \rho_w \), the density at the solid boundary, can be computed from the Reynold’s number of the flow, the liquid conductivity, and the thickness of a molecular diffusion-based laminar sub-layer, using the results of work done by A.A. Sonin and B. Abedian\(^{21}\). The present work investigates in part the validity of using \( \rho_w \) as a parameter indicative of the chemistry at the liquid/solid interface, which requires \( \rho_w \) to be independent of flow conditions and be determined only by measured properties of the oil and solid material. If its usefulness can be proven, \( \rho_w \) can be tabulated for all materials considered for use in power transformers and used as an indicator of their relative charging tendencies. However, \( \rho_w \) as a parameter can only be associated with equilibrium charging conditions, for the wall charge varies in developing situations. Thus, to unambiguously test the invariability of the wall charge requires an experiment that can truly measure equilibrium currents.
1.4.3 Non-equilibrium Charging in the Couette Mixer

There are, however, two methods for driving this system out of equilibrium. As will be further discussed in Section 2.1.1, significant wall currents can be required to supply the sampled current if the sampling flow rate is increased. Wall currents are also generated when an applied field is used to augment the flow-induced currents. Both these scenarios destroy the equilibrium that is attained by the unenergized Couette mixer when the sampling flow rate is small, but the symmetry of the Couette device allows these non-equilibrium charging conditions to be investigated in a controlled fashion. Although charge accumulation and leakage may now proceed, no fields tangential to the boundaries can still develop since the charge density varies only in the radial direction. In addition, the degree to which the system is taken out of equilibrium can be precisely controlled, either by the amount of increase in the flow rate through the device or by the electric field strength and frequency.

A major part of the present work is the study of the non-equilibrium charging effects induced by AC energization. Chapter 4 develops a theoretical model to describe the current enhancement that is based on an analysis of the trajectories of injected charges. A rectification process, where charge of only one polarity is injected, is assumed in order to allow a net charge to develop. Turbulence is also assumed to be a major contributor: charges whose trajectories reach into the turbulent core become "trapped" by the turbulence and are mixed across the entire gap. This is a significant effect both because charge over a much larger area is exposed to fluid convection, but also because the radial force exerted by turbulence draws more current from the wall.

A parameter analogous to $\rho_w$ is introduced in this analysis. The injected charge density at the wall is assumed to be a constant, $\rho_{\text{inj}}$, and can similarly be used as an indication of the degree of energization activity a particular material interface will contribute.
Chapter 2
Couette Electrification Apparatus

The Couette flow model used to study electrification activity is redrawn in Figure 2-1. Here, the role of the oil ducts between transformer windings is played by the annulus of a compact Couette device consisting of coaxial cylinders, the inner rotating. The choice of this geometry is motivated by a desire for versatility as well as a need for the ability to control and isolate the basic processes at work. The range of speeds at which the inner cylinder can rotate (up to 2000 rpm) provides the ability to study flows varying from the laminar to fully turbulent regimes, and (in the limit where the axial velocity due to the flow through the apparatus is negligible compared to that in the azimuthal direction) the symmetry of the experiment enforces uniformity in the circumferential direction, creating a flow that is fully developed in both mechanical and electrical senses.

In principle, the total volume of oil required for the experiment is little more than that required to fill the annulus. However, in the work described here, the charge is sampled with a continuous flow system which itself requires a much greater volume of oil.

![Figure 2-1: The Couette Flow Apparatus](image-url)
2.1 The Experimental System

The entire flow system is illustrated by Figure 2-2. Tanks pressurized with dry nitrogen provide the flow, a motor drive system rotates the inner cylinder, and the relaxation chamber on the inlet side traps incoming charge. The outlet of the system is electrically isolated from the fluid reservoir by a short piece of insulating tubing, and the output convection current is then measured by summing the currents from the ensuing metal pipe and the fluid reservoir itself. Through a brush mounted on the rotor, an ac voltage can be applied across the annulus so that effects of energization on the charging process can be studied.

![Figure 2-2: Couette electrification system showing dry nitrogen pressure drive, fluid reservoir and receiver, charge trap, Couette mixer where inner cylinder rotates, and measurement of current exiting receiver.](image)

2.1.1 The Sampling Approach

There are two flows associated with the system of Figure 2-2: the Couette flow in the central mixer, which establishes the flow conditions for electrification; and the sampling flow entering and exiting the mixer, that allows the charge generated in the device to be measured. As discussed in Section 1.4, these two flows combine to yield a system capable of measuring equilibrium charging currents. The Couette approach differs from pipe flow experiments in that it emphasizes the average charge density across a flow cross-section, rather than the convection current. The rotational flow in the Couette mixer establishes a uniform charge density in the core consistent with the flow conditions and the boundary condition on the charge density at the wall, \( \rho_w \). The flow through the mixer then creates a sampling current proportional to the product of the charge density and the sampling volume flow rate, as long as the current carried by the charge in the thin laminar sub-layers is much smaller than that carried by the core. This withdrawn current is truly a sample of the electrification inside the annulus if it is small compared to the current circulating the mixer.
In order to allow all currents to be accounted for through conservation of charge, the sampling flow is an open loop. Pressure in the fluid reservoir forces the oil through the mixer and into the receiving tank, where it accumulates until the reservoir is emptied. The pressure is then released, and the receiving tank is pressurized to return the oil to the reservoir. In this way, data taken is free from the ambiguities of possible reentrant streaming currents that would be generated in the return path if the loop were closed.

2.1.2 Charge Settling Chamber and Trap

In order for the measured current to accurately reflect charge generated solely in the Couette mixer, the current entering the device must be made negligibly small. This is accomplished in the present system by a relaxation chamber, or charge trap, placed immediately upstream of the mixer, which provides a fluid residence time large enough that any incoming charge can relax or self-precipitate to the grounded wall, greatly reducing the inlet charge density $\rho_{\text{in}}$. Previous analysis\(^1\) has included molecular diffusion and ohmic conduction to analyze the effectiveness of the charge trap. In the present analysis, diffusion is ignored but the effects of ohmic conduction and charge self-precipitation are combined to calculate $\rho_{\text{in}}$ as a function of the parameters of the system. The thrust of the analysis is a comparison of the fluid residence time in the trap, $\tau_{\text{rest}}$, to the dielectric relaxation time $\tau_e$ and the self-precipitation time $\tau_m$. If either $\tau_e$ or $\tau_m$ is small compared to $\tau_{\text{rest}}$, any charge entering the trap has time to leak away before exiting the trap and entering the Couette mixer.

This analysis is carried out in a previous paper\(^2\), and yields an expression for the ratio of the charge density in the fluid leaving the trap to the charge density entering. As the charge leaving the trap is the inlet charge to the Couette mixer, it is denoted $\rho_{\text{in}}$, while the charge entering the trap is $\rho_i$; the ratio $\rho_{\text{in}}/\rho_i$ is then

$$\lim_{\tau_m \gg \tau_{\text{rest}}/\tau_e} \frac{\rho_{\text{in}}}{\rho_i} = \frac{1}{1 + \tau_{\text{rest}}/\tau_e}$$

Here the migration time $\tau_m = \varepsilon/\rho\beta$ is assumed to be very large, since the charge densities encountered in the trap in our system are small. For values of $\tau_{\text{rest}}$ and $\tau_e$ typical of our system (computed in Section 2.1.4), the charge density exiting the trap is about 6% of what enters.

Section 2.2.1 describes how the trap efficiency can be experimentally verified. Although this has not been done, a ceiling can be placed on the current entering the mixer from the current measured with the flow on but with no rotation of the inner cylinder. Since this value is 20-100 times less than the currents measured with rotation, we are assured that rotation and energization effects are dominant. This check, made possible by the independence of the effective Reynold’s number of the flow used to sample the charge, is another advantage of the Couette approach. In a duct flow, the degree to which charge enters the duct is likely to depend on the same flow rate that determines the pipe Reynold’s number.
2.1.3 Mounting of Paper

An important attribute of the Couette system is the relative ease with which it allows for a meaningful examination of different materials: the small inventory of oil can easily be replaced or altered, and the small surface area exposed in the mixer means that different solid materials can be quickly wrapped on the cylinders. The solid insulation used in transformers consists primarily of sheets of pressboard and layers of kraft paper; in our experiments the kraft paper was chosen for its greater ease of application, though thin pressboards may be used in the future. The actual materials used were chosen to match those used in Westinghouse transformers: the paper is Manning Paper Company's grade 220 insulating kraft, and the glue is Imperial Vetak, supplied by EHV-Weidmann.

The two methods for mounting the paper on the inner and outer surfaces are drawn schematically in Figure 2-3. The inner cylinder has a groove cut down its length to allow one edge of the paper to be firmly wedged into place by a close-fitting block. The paper is then wrapped once around the cylinder and a strip of glue attaches it back onto itself. The more difficult placement of paper on the outer cylinder is achieved by first preparing a cylinder of paper around an extra, appropriately-sized cylinder and gluing it to itself. This hollow cylinder is cut into quarters and mounted on end caps during the wrapping process, as shown in Figure 2-3. Once the 8-9 layer thick paper roll is dried, the end caps are removed and the cylinder is collapsed, allowing the roll to be easily removed. The paper can then slide snugly into the outer cylinder and is held in place by two steel bands and two lipped sleeves in the inlet and outlet holes. The steel bands expand to press the paper to the metal wall, and the sleeves screw into the external fittings from the inside.

The paper thus installed in the Couette mixer can be oil-impregnated in a vacuum oven to remove the moisture it has naturally absorbed. This impregnation procedure, detailed in Appendix A, is a two-step process: the paper-lined mixer is placed in a vacuum-tight, heated environment for a prolonged period, and is then moved without exposure to air into a glovebox with a dry nitrogen atmosphere to be prepared for installation in the flow system. The vacuum-heating stage drives moisture from the paper and oil and provides for the moisture to be externally collected. Once treated, the glovebox enables the mixer to be capped without exposing the contained oil and paper to humid air.

2.1.4 Critical Times and Lengths

Certain time constants associated with electrification phenomena help explain the transients evident in measurements. These are listed below:

- **Migration time, \( \tau_m \)** - The self-precipitation time of charge with density \( \rho \) and mobility \( b \) is

\[
\tau_m = \frac{\varepsilon}{\rho b}
\]

where the mobility can be found from an empirical rule for positive ions in insulating dielectrics with absolute viscosity \( \eta \):
Figure 2-3: Method of wrapping paper onto a) the inner cylinder and b) the outer cylinder. The quartered wrapping cylinder c) is also illustrated.
For our transformer oil at 70°F, \( \eta = 16 \text{ cp} = 0.016 \text{ N-s/m}^2 \) so that \( b = 10^{-9} \text{ m}^2/(\text{V-s}) \). Typical measured charge densities in transformers vary over the wide range of 1-1000 \( \mu \text{C/m}^3 \) so that the migration time varies from 19,500 seconds down to 19.5 seconds; however, the measured densities in the Couette apparatus are more on the order of 3-150 \( \mu \text{C/m}^3 \), indicating large migration times of order 130-6500 seconds.

* Dielectric relaxation time, \( \tau_e \) - The dielectric relaxation time \( \tau_e \), given by

\[
\tau_e = \frac{\varepsilon}{\sigma},
\]

(2.3)

can vary by orders of magnitude because of the large variation in ohmic conductivity of insulating oils as a function of parameters not easily controlled. Pure Gulf Transcrest H oil, as rated by the manufacturer, has a permittivity \( \varepsilon = 2.2\varepsilon_0 \) and an ohmic conductivity \( \sigma = 10^{-13} \text{ mhos/m at 25^0 C} \) yielding a relaxation time of 195 seconds. While our measurements of the oil in our system give the same value of the permittivity, measurements of the conductivity indicate a value \( \sigma = 1.2 \times 10^{-12} \text{ mhos/m} \). Thus the relaxation time of the oil in our experiments is comparatively short, \( \tau_e = 16 \) seconds.

* Residence time, \( \tau_{res} \) - The residence time in a chamber is the quotient of the volume of that chamber and the flow rate through it:

\[
\tau_{res} = \frac{V_{chamber}}{Q_v}
\]

In the Couette flow system, there are two residence times of interest: the residence time of the charge trap, \( \tau_{resT} \), and that of the Couette mixer itself, \( \tau_{resC} \).

**Trap** The volume of the charge trap is \( 1.85 \times 10^{-3} \text{ m}^3 \), yielding \( \tau_{resT} = 265 \) seconds for a volume rate of flow \( Q_v = 7 \times 10^{-6} \text{ m}^3/\text{sec} \)

**Mixer** The mixer's volume is \( 5.8 \times 10^{-4} \text{ m}^3 \), and has residence time \( \tau_{resC} = 83 \) seconds.

* Run time, \( \tau_{run} \) - The duration of a single run of the experiment is simply

\[
\tau_{run} = \frac{V_{liquid}}{Q_v}
\]

and, since the experiment contains approximately 6 liters of oil, lasts about 15 minutes for a \( Q_v = 7 \times 10^{-6} \text{ m}^3/\text{sec} \).
Turbulent development time, $\tau_{\text{dev}}$ - As a first-order approximation to the time required for the turbulent flow in the mixer to develop, we calculate a diffusion time using the coefficient of turbulent diffusion:

$$\tau_{\text{dev}} = \frac{d^2}{D_T}$$  \hspace{1cm} (2.4)

where $d=.5''$ is the spacing between inner and outer cylinders. We use an expression for the turbulent diffusion coefficient in the core from Abedian and Sonin's work\textsuperscript{21}:

$$D_{TC} = .05 v_* d$$

where $v_*$ is the friction velocity, given by

$$v_* = \sqrt{\frac{\tau_w}{\rho_m}}$$  \hspace{1cm} (2.5)

with $\tau_w$ the wall stress and $\rho_m$ the fluid mass density. The friction velocity can be computed with empirical correlations for particular flow geometries. G.I. Taylor has correlated the normalized wall stress for Couette flow\textsuperscript{24}:

$$\frac{\tau_w}{\rho_m (\Omega R_2)^2} = .1 R^{-1/2} \hspace{1cm} 300 \leq R \leq 10,000 \hspace{1cm} (R=\Omega R_2 d/v)$$  \hspace{1cm} (2.6)

in terms of the angular velocity $\Omega$ (rad/s) of the inner cylinder, the outer cylinder radius $R_2$, and the fluid Reynold's number $R$. The Reynold's number for Couette flow,

$$R = \frac{\Omega R_2 d}{v}$$

is 2.8 times the rotational speed in RPM for our system. We can now write

$$v_* = .32 \frac{v}{d} R^{3/4}$$

and determine the core turbulent diffusion coefficient:

$$D_{TC} = 1.6 \times 10^{-2} v R^{3/4}$$

Thus for our viscosity of $v=1.8 \times 10^{-5}$ m$^2$/s ($\rho_m=900$ kg/m$^3$), $D_{TC}=1.2 \times 10^{-4}$ m$^2$/s and $\tau_{\text{dev}}=1.4$ seconds at a Reynold's number of 3000.

There are also several important lengths:
**Debye length, \( \lambda \) -** The stagnant debye length results from a balance of the ion diffusion time, \( \lambda^2/D_m \), with the relaxation time, \( \epsilon/\sigma \):

\[
\lambda = \sqrt{\frac{\epsilon D_m}{\sigma}} = \sqrt{D_m \tau_c}
\]

The molecular diffusion coefficient \( D_m \) is found from the Einstein relation:

\[
\frac{D_m}{b} = \frac{kT}{q} = 25 \text{ mV}
\]

where our mobility yields \( D_m = 2.5 \times 10^{-11} \text{ m}^2/\text{s} \). For our conductivity \( \sigma = 1.2 \times 10^{-12} \text{ mhos/m} \), the Debye length is approximately 20 \( \mu \text{m} \).

**Turbulent Debye length, \( \lambda_T \) -** Modeling the effects of turbulence with a turbulent diffusion coefficient \( D_T \) gives rise to a new length, the turbulent debye length \( \lambda_T \):

\[
\lambda_T = \sqrt{\frac{\epsilon D_T}{\sigma}}
\]

Using the core turbulent diffusion coefficient found above and \( \sigma = 1.2 \times 10^{-12} \text{ mhos/m} \), we obtain \( \lambda_T = 4 \text{ mm} \) for a Reynold’s number of 3000.

**Diffusion-based laminar sub-layer thickness, \( \delta_m \) -** This distance defines a region near the wall inside which the flow can be approximated as laminar for the purposes of mass transport. It is thus dependent on the molecular diffusion coefficient \( D_m \), through the Schmidt number \( S^{21} \):

\[
\delta_m = \frac{11.7 v}{S^{1/3} \sqrt{\tau_w/\rho_m}}; \quad S = \frac{v}{D_m}
\]

Using Taylor’s wall stress correlation, we relate \( \delta_m \) to the Reynold’s number as

\[
\delta_m = \frac{36.6 d}{S^{1/3} R^{-3/4}}
\]

which is 12.8 \( \mu \text{m} \) at \( R = 3000 \).

**Viscous sub-layer thickness, \( \delta_v \) -** The viscous sub-layer is analogous to the laminar sub-layer, and is a zone near the wall where turbulence is negligible in determining momentum transport. It is, therefore, the sub-layer used in determining the velocity profile. Schlichting gives an empirical expression for this parameter based again on the friction velocity \( \nu^* \):

\[
\frac{\delta_v}{\nu^*} = 5
\]

which becomes
These times and lengths are summarized in Table 2-I, which includes typical values of each parameter in our system.

Other constants of the oil and the geometry are listed in Table 2-II for reference.

2.2 Absolute Measurement of Charge

As discussed in Section 2.1.1, the current measured from the Couette mixer samples the uniform charge density created by the turbulent flow inside. The rotation of the inner cylinder establishes the flow conditions, which if highly turbulent enough produce a uniform charge density \( \rho_0 \) across the gap, and the small sampling flow \( Q_v \) through the device yields the measured current, \( \rho_0 Q_v \). The open-loop nature of the overall flow system excludes any ambiguities in the measurements of this current, as can be seen by conservation of charge.

Figure 2-4 reillustrates the Couette electrification system and indicates a surface \( S \) with which conservation of charge can be used to find the current measured through the electrometer. The charged oil exiting the Couette mixer passes first through a short section of insulating tubing and then into a length of steel tubing that carries it into the fluid receiver. The steel tubing is insulated from the outside of the receiver, and is connected to an inner bucket that is electrically isolated from the outer container; thus, the only conductor that the surface \( S \) cuts is the wire to the electrometer. In addition, the mixer, steel tubing and fluid receiver are shielded in a Faraday cage, and thus the only currents passing through \( S \) are the current convected through the piece of insulating tubing, \( \rho_0 Q_v \), and the measured current through the electrometer. Hence, the measured current is truly the current exiting the Couette device, independent of currents generated by electrification in the metal tubing.

In addition, the frequency response is limited only by an RC time constant comprised of the resistance of the electrometer and the capacitance between the inner and outer cans. Since both these values are very low, the frequency response is quite good.

2.2.1 Other Currents in the Electrification System

The open-loop nature of the flow system also allows other pertinent currents to be measured in manners analogous to that used above to find the current exiting the mixer. The current entering the mixer can be found from the surface \( S_i \); the three currents exiting that surface must add to zero, and thus the inlet current \( I_i = \rho_i Q_v \) is the negative of the sum of the ground currents from the charge trap and the inner container in the fluid reservoir. The efficiency of the trap can thus be determined by measurements of the two ground currents.

Similarly, the difference between the currents exiting and entering the Couette mixer can be found from the surface \( S_d \). This difference, \( I_{\text{diff}} = Q_v (\rho_o - \rho_i) \), must be supplied by currents
Table 2-I: Various time constants in the Couette system.

<table>
<thead>
<tr>
<th>Time</th>
<th>Formula</th>
<th>Typical Values in Couette System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Migration Time</td>
<td>$\tau_m = \frac{\varepsilon}{\rho b}$</td>
<td>130 - 6500 sec for $\rho=3-150 \mu C/m^3$</td>
</tr>
<tr>
<td>Relaxation Time</td>
<td>$\tau_e = \frac{\varepsilon}{\sigma}$</td>
<td>16 seconds with $\sigma=1.2\times10^{-12}$ mhos/m</td>
</tr>
<tr>
<td>Residence Time</td>
<td>$\tau_{res} = \frac{V}{Q_v}$</td>
<td>265 sec in trap 83 sec in mixer when $Q_v=7$ ml/sec</td>
</tr>
<tr>
<td>Turbulent Development Time</td>
<td>$\tau_{dev} = \frac{d^2}{D_T}$</td>
<td>1.4 sec at R=3000</td>
</tr>
<tr>
<td>Run Time</td>
<td>$\tau_{run} = \frac{V_{liquid}}{Q_v}$</td>
<td>15 minutes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length</th>
<th>Formula</th>
<th>Typical Values in Couette System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Debye Length</td>
<td>$\lambda = \sqrt{\frac{\varepsilon D_m}{\sigma}}$</td>
<td>20 $\mu m$ for $\sigma=1.2\times10^{-12}$ mhos/m, $D_m=2.5\times10^{-11}$ m$^2$/s</td>
</tr>
<tr>
<td>Turbulent Debye Length</td>
<td>$\lambda_T = \sqrt{\frac{\varepsilon D_T}{\sigma}}$</td>
<td>4 mm for $D_T=1.2\times10^{-4}$ m$^2$/s $\sigma=1.2\times10^{-12}$ mhos/m</td>
</tr>
<tr>
<td>Laminar Sub-Layer Thickness</td>
<td>$\delta_m = \frac{11.7 V}{S^{1/3} \sqrt{\tau_s / \rho_m}}$</td>
<td>8 - 23 $\mu m$ for 500-1800 RPM</td>
</tr>
<tr>
<td>Viscous Sub-Layer Thickness</td>
<td>$\delta_v = 15.6 d R^{-3/4}$</td>
<td>330 - 866 $\mu m$ for 500-1800 RPM</td>
</tr>
</tbody>
</table>
Table 2-II: Constants of the oil and the geometry of the Couette system at 20°C.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil permittivity</td>
<td>ε</td>
<td>$2.19 \varepsilon_0$</td>
</tr>
<tr>
<td>Oil conductivity</td>
<td>σ</td>
<td>$1.2 \times 10^{-12}$ mhos/m</td>
</tr>
<tr>
<td>Ion mobility</td>
<td>b</td>
<td>$10^{-9}$ m²/(V-s)</td>
</tr>
<tr>
<td>Kinematic viscosity</td>
<td>ν</td>
<td>$1.8 \times 10^{-5}$ m²/s</td>
</tr>
<tr>
<td>Mass density</td>
<td>$\rho_m$</td>
<td>900 kg/m³</td>
</tr>
<tr>
<td>Molecular diffusion coefficient</td>
<td>$D_m$</td>
<td>$2.5 \times 10^{-11}$ m²/s</td>
</tr>
<tr>
<td>Inner cylinder radius</td>
<td>$R_1$</td>
<td>$2.54 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Outer cylinder radius</td>
<td>$R_2$</td>
<td>$3.81 \times 10^{-2}$ m</td>
</tr>
<tr>
<td>Height of Couette device</td>
<td>$l$</td>
<td>$2.29 \times 10^{-1}$ m</td>
</tr>
</tbody>
</table>

Figure 2-4: The Couette electrification system, showing surfaces for performing conservation of charge.

from ground to the inner and outer cylinders, and these currents can be measured as a check
on previous results. These wall currents indicate the degree to which the Couette mixer falls short of attaining true equilibrium conditions, since equilibrium demands zero current at the liquid/solid interface. However, as long as these wall currents, which are necessary to provide a measurement, are small compared to the convection current circulating the mixer, equilibrium is very closely approximated.
Chapter 3
Equilibrium Charging Theory and Experiments

This chapter summarizes the theoretical basis for the experimental results of the Couette electrification apparatus, and presents the equilibrium charging data taken. The experimental procedure is detailed, and the system parameters that are documented in the experiments are indicated. The goal of this section is to provide and examine a framework for understanding the data that relates the results to a boundary condition determined by the documented parameters.

3.1 Couette Fluid Mechanics

The first step towards an appreciation of the results that will be presented in Section 3.4 is to gain an understanding of the fluid mechanics in the Couette mixer. The flow between concentric rotating cylinders has been well-studied\textsuperscript{24,26,27,28} and has taken the name of the first person to detail the mathematics, Couette. The flow conditions experienced when the inner cylinder rotates and the outer one is fixed pass through three regimes, including a region of cellular convection between the typical laminar and turbulent flows that is peculiar to Couette flow. These are listed below and illustrated in Figure 3-1:

1. \textbf{laminar flow} - the flow pictured in Figure 3-1a applies to low rotational speeds and is perfectly smooth, with no radial velocity component.

2. \textbf{cellular convection} - as the speed passes through a (relatively low) critical value, the flow experiences a type of instability not found in pipe flows: the smooth laminar flow breaks up into three-dimensional cells. These are depicted in Figure 3-1b.

3. \textbf{full turbulence} - as the rotational speed increases past the critical value, the cells begin to break up and eventually give way to fully turbulent flow.

3.1.1 Laminar Flow

The smooth flow characteristic of low rotational speeds has zero radial component, and is given by

$$V_\theta(r) = \frac{R_1 R_2}{R_2^2 - R_1^2} V_1 \left( \frac{r}{R_2} - \frac{r}{R_1} \right)$$

(3.1)

where \(R_1\) and \(R_2\) are the radii of the inner and outer cylinders, respectively, and \(V_1\) is the velocity at the surface of the inner cylinder.
3.1.2 Cellular Convection

At surprisingly low speeds, the laminar flow of Section 3.1.1 breaks up into cells. A condition for the incipience of cellular convection with the outer cylinder of radius $R_2$ stationary and the inner cylinder of radius $R_1$ rotating at angular speed $\Omega$ is given in terms of a parameter $P$, which is related to the Taylor number $T$:

$$P_c = 0.0571 \left( 1 - 0.652 \frac{d}{R_1} \right) + 0.00056 \left( 1 - 0.652 \frac{d}{R_1} \right)^{-1}$$  \hspace{1cm} (3.2)

where $d$ is the oil gap spacing and $T$ is the Taylor number. Thus, the critical rotational speed $\Omega_c$ is

$$\Omega_c = \sqrt{\frac{\pi^4 \nu^2 (R_1 + R_2)}{2P_c d^3 R_1^2}}$$  \hspace{1cm} (3.3)

For the values in our system, this speed is
\[ \Omega_c = 4.39 \text{ rad/sec} = 42 \text{ RPM} \]

and the critical value of \( P \) is \( P_c = 0.039 \). Thus, at very low rotational speeds, the flow in the Couette mixer goes unstable and forms Taylor cells.

S. Chandrasekhar has computed the Taylor number \( T \) in terms of a parameter \( a = kd \), where \( k \) is the wavenumber associated with the spatial periodicity of the cells. This expression for \( T \) is then minimized to yield a value \( a_{\text{min}} \) that gives the wavenumber at incipience of the instability (i.e. when \( T = T_c \)):

\[ a_{\text{min}} = k \cdot d = 3.12 \]

This wavenumber can be used to predict the frequency of any oscillations in the streaming current data that would reflect the existence of Taylor cells. This frequency should be the product of the critical wavenumber and the axial flow velocity through the mixer, \( U_{\text{axial}} \):

\[ \omega_{\text{osc}} = k_c U_{\text{axial}} = \frac{3.12 Q_v}{d \pi (R_2^2 - R_1^2)} \]

where \( Q_v \) is the sampling flow rate and the axial velocity is \( U_{\text{axial}} = Q_v / \pi (R_2^2 - R_1^2) \). Using the parameters of our system, we obtain \( \omega_{\text{osc}} = 0.64 \text{ rad/sec} \) at \( Q_v = 7 \times 10^{-6} \text{ m}^3/\text{s} \), corresponding to a period of

\[ t_{\text{osc}} = \frac{1}{f} = 10 \text{ sec} \]

### 3.1.3 Turbulence

When the flow has reached full turbulence, Taylor measures the velocity distribution as shown in Figure 3-1c. To a very high accuracy, the product of the average velocity \( U \) and the radius \( r \) is constant throughout the core of the flow. However, the velocity departs from this behavior in the viscous sub-layers near the walls: near the inner cylinder the flow increases to the value \( \Omega R_1 \) with slope \( \tau_w / \eta \) (wall stress divided by the absolute viscosity), while it tapers to zero with the same slope as it approaches the outer, stationary cylinder. The thickness of these sub-layers we recall from Equation 2.7:

\[ \delta_v = 15.6 d R^{-3/4} \]

We approximate the turbulent velocity profile by piecing together three linear regions: the two sub-layers have slopes \( \tau_w / \eta \) and thicknesses \( \delta_v \), and the region in-between simply connects the two. Thus, the velocity profile is given by
where the slope of the velocity in the core is

\[ \frac{2\delta_y \tau_w}{\eta - \Omega R_1} \]

3.1.4 Sampling Model

Figure 3-2 illustrates the fluid mechanics model used to calculate the sampled current. Since the outlet of the Couette mixer is very near the top plate of the device, the fluid surrounding the exit hole is assumed to be fully mixed due to the secondary convection created by the zero-velocity boundary condition at the top plate; the fluid there is thus assumed to have an average charge density \( \rho_0 \). The current exiting the Couette mixer, \( \rho_0 Q \), must equal the current flowing up into the mixed region, and that current is found by integrating the product of the charge density and the velocity below the mixed region over the cross-section:

\[ I = \int_{R_1}^{R_2} \rho(r) v_z(r) 2\pi r dr \]  

Here \( \rho(r) \) is the charge density and \( v_z(r) \) is the axial flow velocity profile beneath the mixed sampling region.

The shape of the assumed axial velocity profile is determined by the rotation of the inner cylinder: turbulence is presumed to dominate the central part, as it does in the circumferential velocity profile, and two boundary layers near the walls are assumed to have linearly decreasing profiles. The thickness of these two linear regions is determined by the viscous sub-layer thickness \( \delta_v \) associated with the rotational Reynold’s number, as determined in Equation 2.7:

\[ \delta_v = 15.6 d R^{-3/4} \]

The mean velocity \( V \) in the core, which is uniform with radius since neither wall has a vertical velocity component, is then determined by integrating the velocity profile over the cross-section and equating to the volume rate of flow through the mixer:

\[ Q_v = \int_{R_1}^{R_2} v_z(r) 2\pi r dr \]

Note that we assume turbulence throughout the entire operating range (280 < \( R < 5040 \)). The region between Reynold’s number 110 and around 2800, however, is ambiguous, since the
cellular convection that arises there can be thought of as both laminar and turbulent: the velocity profile is well-defined, but the cellular action enhances mass transport in a way similar to turbulence. Taylor helps decide the debate, since his turbulent wall stress correlation is valid throughout the range $R=300$ to $R=10,000$.  

We now write a summary of the axial velocity profile:

$$v_z = \begin{cases} 
\frac{V}{\delta_y(r-R_1)} & R_1 < r < R_1 + \delta_y \\
V & R_1 + \delta_y < r < R_2 - \delta_y \\
\frac{V}{\delta_y(R_2-r)} & R_2 - \delta_y < r < R_2 
\end{cases}$$  \hspace{1cm} (3.7)$$

which yields an expression for $V$ in terms of the volume rate of flow:

$$V = \frac{Q_\nu}{\pi(R_2-R_1-\delta_y)(R_1+R_2)}$$

The next section discusses an adaptation of a model which allows calculation of a uniform flow-induced charge density in the core. Once this is found, it can be plugged into Equation 3.6 to predict the measured current.
3.2 Adaptation of Theory of Abedian and Sonin

As described above, the expression we would like to evaluate to calculate the sampled current is Equation 3.6; unfortunately, we do not have an expression for the charge density as a function of radial position across the gap. The approximation we will use assumes that the current carried by the laminar sub-layers is negligibly small compared to that in the core. This assumption is supported by the facts that 1) the sub-layers are much thinner than the core, typically by a factor of $10^3$; and 2) the average velocity is much smaller than the mean velocity in the core. If the sub-layer currents can be ignored, then the entire current is attributed to that carried by the mixed turbulent core carrying uniform charge density $\rho_0$; the current is then $\rho_0 Q_\nu$.

For an expression for this average charge density $\rho_0$, we turn to a model which calculates the steady-state streaming current in a pipe, $I_\infty$, normalized to the product of $\rho_w$ and the volume rate of flow through the pipe. This is, then, an average over the cross-section of the charge density weighted by the velocity profile, and is normalized to $\rho_w$. The weighting factor causes the charge in high velocity regions to make a much greater contribution to the total current than the charge near the wall.

It is in the limit where duct flows are fully developed in both a fluid mechanical and in an electrical sense that electrokinetic charging has been given a rigorous treatment. With semi-insulating walls, this is difficult to achieve in duct flows of reasonable length, but can be attained by the symmetry of the Couette configuration. In principle, the radial equilibrium between molecular diffusion, turbulent diffusion and migration is only disturbed by the rate of convection through the apparatus and that can be made as small as the instrumentation allows.

The most general streaming current analysis valid for laminar and turbulent flows in a circular pipe of radius $a$, including the effects of molecular and turbulent charge diffusion added to ohmic and convection currents, is the work of B. Abedian and A. Sonin, which considers a laminar sub-layer whose thickness $\delta_m$ may be larger or smaller than the Debye length $\lambda=\sqrt{e D_m/\sigma}$. Because the inner core region of the flow is turbulent, any charge in the Debye layer extending beyond the laminar sub-layer will become mixed into the core and be swept away, providing the bulk of the charge sampled at the outlet. Their results for fully developed flow with mean velocity $V_f$ for the streaming current $i_c(z)$ as a function of position $z$ down the pipe were:

$$i_c(z) = I_\infty (1 - e^{-zL}) + i_c(z=0)e^{-zL}, \quad L = \frac{eV_f/\sigma}{1 + 2\lambda^2/\delta_m^2}$$

where $I_\infty$ is the resulting streaming current for a very long pipe with $z \gg L$:

$$\frac{I_\infty}{\rho_w \pi a^2 V_f} = \frac{R \tau_w \lambda^2}{\rho_m V_f^2 a^2} \left[ 1 - \frac{\delta_m/\lambda}{\sinh \delta_m/\lambda} \right] + \frac{\delta_m/\lambda}{\sinh \delta_m/\lambda} \frac{1}{1 + a \delta_m/2\lambda^2}$$

(3.8)

with $\rho_w$ the charge density at the wall, $\rho_m$ the fluid mass density, $R=V_f 2a/v$ the fluid
Reynold's number and $\tau_w$ the wall shear stress. For turbulent pipe flow, the Blasius correlation for the wall stress is used, both in the expression for $I_\infty$ and in the formula for $\delta_m$:21,29

$$\frac{\tau_w}{\rho V_f^2} = 0.0396R^{-1/4} \quad R = V_f 2a/v < 10^5$$

This analysis was used to successfully correlate a wide range of measurements of hydrocarbon liquids in conducting pipes, picking values of the wall charge density $\rho_w$ that gave a best fit to the measurements. This analysis in part was also used for electrification phenomena of liquid Freon flowing through insulating pipes in HVDC valves1,2.

Since $I_\infty$ is the equilibrium charging current, we can adapt Equation 3.8 for use as a first approximation to currents generated in the Couette device. This equation can be thought of as giving the velocity-averaged charge density across the gap, $\rho_0$, normalized to $\rho_w$. Since, as discussed in Section 2.1.1, the Couette mixer provides measurements of the product of this average charge density and the sampling flow rate $Q_v$, we can rewrite Equation 3.8 in terms of parameters in our system:

$$\frac{I}{\rho_w Q_v} = 0.4 R^{1/2} \frac{\lambda^2}{d^2} \left[ 1 - \frac{\delta_m \lambda}{\sinh \delta_m \lambda} \right] + \frac{\delta_m \lambda}{\sinh \delta_m \lambda} \frac{1}{1 + \delta_m^4 \lambda^2}$$

where $I$ is our sampled current, and we have substituted $d/2$ for $a$. We have also used Taylor's wall stress correlation from Equation 2.6.

### 3.3 Experimental Technique

The Couette electrification system is redrawn here in Figure 3-3. This section will list the parameters of the system we are able to record and the experimental procedure that is followed when taking data.

#### 3.3.1 Documentation of Parameters

It is postulated that the extrapolated wall charge $\rho_w$ is a function only of mechanical, chemical and electrical properties of the system, notably the paper and the oil, and is independent of the flow conditions established. If so, this will allow $\rho_w$ to be considered itself a parameter of the oil/paper interface, indicative of the degree of charging to be expected from that interface under documented conditions. In order for this hypothesis to be examined, the measurable parameters of the system that are considered primary influences on the wall charge must be documented and correlated to the calculated values of $\rho_w$; we regularly record three parameters associated with the oil:

- **viscosity** - The kinematic viscosity $\nu$ is measured in the lab and has the nominal value $1.8 \times 10^{-5}$ m$^2$/s. This parameter is measured only infrequently, as it is not believed to vary significantly from run to run.
Figure 3-3: The Couette electrification system.

- **moisture content** - The moisture content is measured by a Mitsubishi moisture meter injected with oil samples taken with a hypodermic needle from a sampling section on the flow path out of the fluid reservoir. This measurement can be taken every run, and spans the range 5-50 ppm.

- **conductivity** - A capacitance cell is used in conjunction with a capacitance bridge to measure the conductivity of the oil in the fluid reservoir on a daily basis. As a check on this measurement, samples were periodically taken for independent readings by other methods. Unfortunately, this check implied that measurements taken with the particular cell-bridge combination in the experiment are unreliable, since both independent checks yielded nearly identical values that differed from our readings by as much as a factor of eight. In addition, the values returned by the cell-bridge apparatus declined significantly over the course of a week from about $9 \times 10^{-12}$ mhos/m to around $3 \times 10^{-12}$ mhos/m while the checks yielded measurements that did not vary with time. Thus, the measurements taken in the reservoir are ignored and the accepted value is that from the independent readings: $1.2 \times 10^{-12}$ mhos/m.

### 3.3.2 Procedure

This section outlines the experimental procedure followed in a given run of the system. This procedure has several steps:

1. **document parameters** - Before a run begins, the conductivity of the oil in the fluid reservoir is measured with the capacitance bridge discussed in Section 3.3.1. Then the moisture content of the oil is measured by taking a sample from a port on the outlet of the reservoir, and analyzing it in a Mitsubishi moisture meter.
2. initialize recording equipment - The electrometer and chart recorder used to measure and record the streaming currents exiting the Couette mixer are turned on and adjusted before the system flow begins.

3. start system flow - Once the recording equipment is ready the system flow is begun by pressurizing the fluid reservoir; the fluid receiver (the right tank) is kept open to the atmosphere through a one-way check valve. For most experiments, the pressure is brought to 5 psi, yielding a flow rate of 7 ml/sec, though different pressures can be used to test linearity with flow rate. The flow created by a 5 psi difference is large enough to produce easily measurable currents, but small enough to be a negligible disturbance in the Couette mixer.

At this point in the run, the system flow is passing through the Couette mixer, although there is no rotation of the inner cylinder. The currents generated by this flow through the connecting tubing and the mixer is small compared to the currents measured when the inner cylinder rotates (typically 5-10 pA compared to 200-500 pA), and so is treated as a background level providing a reference. The current measured in this state undergoes a transient, and the rotational flow in the Couette mixer is not started until this transient has leveled off; the steady-state value of the current measured with only system flow is subtracted from measurements taken with the inner cylinder rotating.

4. establish flow conditions in Couette mixer - Once the transient due to system flow has steadied out, the primary flow conditions for electrification in the mixer are initialized by starting the rotation of the inner cylinder. The inner cylinder is quickly brought up to the first desired speed, and the transient (approximately 90 seconds) in the resulting measured current is observed. When this transient has vanished, the rotational speed can be changed to a new value, and this pattern repeated until the fluid in the reservoir has been depleted, usually 12-15 minutes.

5. turn-off - A run is ended by turning off the applied field (if any), the rotation of the inner cylinder, and the system flow.

6. refill fluid reservoir - To prepare for the next run, fluid is pumped from the fluid receiver to the reservoir by pressurizing the receiver and opening the reservoir to the atmosphere through a check valve.

3.4 Experimental Observations

This section will first describe the dependence of the measured current on the rotational speed of the inner cylinder, and compare its dependence to predictions based on Equation 3.9. Based on the theory, \( \rho_w \) will be deduced from the data. Then, the effect on \( \rho_w \) of increasing the moisture content of the paper and oil is deduced.
3.4.1 Streaming Current Dependence on Rotational Speed

The data reported in this section comes under two categories: data taken with dry, oil-impregnated paper, and data taken after the paper had been exposed to humid air (roughly 60% relative humidity) for 36 hours. Each run of the experiment could produce only about five data points due to the relatively short run time of the system, and thus two runs are required to piece together an entire span of the rotation range 100-1800 rpm. Once the data was taken, a computer program provided an appropriate value of the wall charge density $\rho_w$ to best fit the expression in Equation 3.9 to a particular data curve.

Figure 3-4 shows both the data from runs taken with the dry paper, and a plot of Equation 3.9 matched to one of the data sets with a value $\rho_w=2.3\times10^{-3}$ C/m$^3$. The legend indicates for each set what day and which run number is represented. The theoretical curve fits the data quite well, with the exception of the first run on June 5 (6/5, #1). The lower values on this curve can best be understood by the realization that this run was the very first taken with the newly treated paper, and thus was likely to be suffering a transient.

3.4.2 Effect of Moisture

After complete sets of data (including the AC energization experiments of the next chapter) were taken with the dry paper, the Couette mixer was disassembled, emptied of oil, and exposed to humid air for 36 hours. The motivation for “wetting” the paper came from previous experience with untreated paper which, upon installation into the Couette device, first produced large negative currents, that gradually increased over the course of a week until they leveled off at positive values close to those measured with dry paper. One hypothesis was that the large moisture content of the untreated paper was gradually being expelled into the oil until an equilibrium between the two could be reached. During this period, the electrochemistry of the interface could be in a state of flux.

The ability to dry the paper in a vacuum oven gave an opportunity to test our hypothesis for the transient: after experiments had been performed on the dry paper, the same paper could be moistened and then reinstalled and checked for a transient of the nature we had previously experienced. The moisture content of the paper was inferred from measurements of the moisture content of the oil, since the two numbers are related when the oil and paper have reached equilibrium.

To a certain degree, this experiment was a success. After the dry paper was installed in the system, the measured current negative achieved a negative value, and with the exception of the very first run, produced very consistent values. Runs taken after the paper had been exposed, however, produced currents with negative values that, over the course of the run, eventually changed sign and came to relatively low positive values. This transient was much shorter than that experienced before, though, and was for the most part completed within a single day, at the end of which the currents were comparable to those measured with dry paper.

As might be expected from the significantly shorter transient suffered, the moisture content of the oil did not increase dramatically, though it did go up measurably and reliably.
Moisture content readings of the oil when the treated paper was in the system yielded values consistently between 6.0-7.0 ppm, whereas after the paper was exposed, the moisture gradually increased over the course of two days from 6 ppm to steady values of between 11 and 12 ppm. It is felt that both the relatively small increase in oil moisture content and the short current transient are indications that the paper was unable to absorb much moisture from the air due to the oil film covering it. Dry paper (i.e. with no oil at all) absorbs very large amounts of water compared to the amount the oil is able to hold-- at room temperature paper can hold up to about 17% of its mass in water, whereas oil saturates at about 50 ppm. This fact was indicated in the transients suffered earlier when untreated paper was first used in the system: within five runs the moisture content of the oil had gone from an initial value of 15 ppm, to a peak of 50 ppm, and settled to a value around 35 ppm.

Since the change in equilibrium moisture content was so small, the measured currents did not change significantly after exposure. Figure 3-5 shows the data taken with wetted paper as well as a theoretical curve matched with a $\rho_w=2.2\times10^{-3}$. The curves are perceptibly lower.
than those in Figure 3-4 and a couple of points fall below zero, but the difference is not large enough to make a strong correlation between moisture content and the magnitudes of the generated currents. Further comment on these results is reserved for Chapter 5.

Figure 3-5: Data and theoretical curve for streaming current dependence on rotation rate; for wetted paper. Theoretical curve was matched to run #8 (7/11) with a value $\rho_w=2.2\times10^{-3}$ C/m$^3$. 
Chapter 4
Injection Charging Due to AC Energization

This chapter presents data and a theoretical model for the enhancement of convection current due to the AC energization of a transformer. A previous model, which links the enhancement to modulation of the double-layer charge, is described and critiqued, setting the stage for the need for the charge injection model proposed in this work.

4.1 Previously Proposed Model

A model for the effect of transformer energization on electrification phenomena has been proposed by Tanaka et al. Their experiments measured leakage currents along the wall of a pressboard tube, and added energization effects by applying a voltage across electrodes surrounding the tube. When energized, they measured currents to the ground electrodes after filtering out the AC displacement current component. Their model associates the observed enhancement with a modulation of the charge distribution near the wall in response to an AC applied field: as the double-layer charges are moved away from the wall, they experience a higher velocity and thus generate more streaming current.

The charge distribution near the wall under either stagnant or laminar flow conditions, approximated as an exponential

$$\rho(z,x) = \frac{q(z)}{\lambda} e^{-\frac{x}{\lambda}}$$

is assumed to respond to an external field by merely suffering a displacement \(x_d\):

$$\rho(z,x) = \frac{q(z)}{\lambda} e^{-(x-x_d)/\lambda}$$

where \(x_d\) is the integral of the migration velocity \(bE_0\sin(2\pi ft)\):

$$x_d = \frac{bE_0}{2\pi f} [1 - \cos 2\pi ft]$$

The motion of double layer charges is assumed to be dominated by the external field (i.e. ignoring self-field and diffusion effects), yet their relative distribution is presumed unchanged, though that distribution was determined by a balance between the same diffusive and self-field forces that are now being ignored. The field due to space charge in the double layer can be approximated by the quotient of the zeta potential and the Debye length. Since the zeta potential is typically no greater than 100 mV, double layer fields are on the order of \(10^4\) V/m for a 10\(\mu\)m Debye length-- easily an order of magnitude less than the applied fields, which are commonly between \(10^5 - 10^6\) V/m. Diffusion and self-field migration are not able
to maintain the Debye distribution themselves in the face of such high external fields, and it is unreasonable to assume that small disturbances and unrecognized side effects (such as external field gradients) will not quickly destroy the exponential distribution.

Thus we turn to a migration-dominated model in which charges are supplied by the wall due to the presence of the applied field.

4.2 Turbulent Convection Model

The model used in the present work to understand energization convection currents presumes a charge injection process at the boundary and relies on turbulence for a significant effect. This migration model analyzes the motion of an injected charge under the influence of an externally applied field, ignoring effects of diffusion and self-field migration, which are negligible under the relatively high field strengths encountered.

4.2.1 Steady State Charge Density Across the Gap

Our model for the injected charge distribution begins with a picture of the simplest case, laminar flow, in Figure 4-1. Here, ion trajectories perpendicular to the wall (x-direction) are determined solely by the electric field, as the flow is taken to be parallel to the wall. A charge injected into the oil from the wall by the presence of the electric field moves away from the wall as long as the sinusoidally-varying field is positive (assuming positive charge injection), and then reverses direction and returns to the wall during the next part of the AC cycle. Charges injected at different times enter at different parts of the cycle and thus penetrate different distances into the flow. The position of a charge as a function of time and of the time $t_0$ it was injected is found by integrating the velocity $v=bE$:

$$x = \int (bE_0 \cos \omega t) \, dt = \frac{bE_0}{\omega} [\sin \omega t - \sin \omega t_0]$$

Figure 4-1: Injected charge trajectories in laminar flow.

Our model postulates a simple charge injection boundary condition which assumes that the
charge density at x=0 (the wall) is $\rho_{\text{inj}}$ (a property of the interface) as long as the field is positive. If charge relaxation effects are insignificant the charge density along a trajectory is constant, and thus any point in the x-t plane that intersects a trajectory that emanates from the wall (i.e. a trajectory that intersects the t-axis) has charge density $\rho_{\text{inj}}$ (for more on this "method of characteristics", see 31.)

The trajectory that carries a charge the farthest from the wall originates at $t_0 = -\pi/2\omega$, and that charge obtains a maximum distance $x_i$:

$$x_i = \frac{2bE_0}{\omega}$$

Thus, the region under this curve contains $\rho = \rho_{\text{inj}}$, while elsewhere the fluid is uncharged.

### 4.2.1.1 Effect of Turbulence

Charge trajectories that enter the core of a turbulent flow will be altered by the flow, as the turbulence has a force component in the x-direction. The two lengths of interest are thus $x_i$, the maximum injection distance, and $\delta_b$, the thickness of the migration-based laminar sub-layer in which x-directed turbulent forces are negligible compared to coulombic forces exerted by the external field. If $\delta_b$ is larger than $x_i$ the results are those computed above, since no trajectories enter the turbulent core. If, however, $\delta_b$ is less than $x_i$ (as shown in Figure 4-2), charge is injected into the turbulent core of the flow where it is immediately mixed across the thickness of the core. This mixing action reduces the charge density in the core to a value $\rho_c$, which we now proceed to calculate.

![Figure 4-2: Charge trajectories in turbulent flow.](image)

The laminar sub-layer thickness $\delta_b$ is the distance at which the average x-directed turbulent velocity equals the migration velocity $bE$:

$$bE = v_{x(rms)}$$

where $v_{x(rms)}$ has been empirically determined.
\[ v_{x(m)} = A v_*(y^*)^2 \]
\[ A = 0.008 \]
\[ y^* = \frac{y v_*}{v} \]

where \( v_* \) is given by Equation 2.5. Thus we require

\[ bE = v_* A \left( \frac{\delta_b v_*}{v} \right)^2 \]

obtaining

\[ \delta_b = 63dR^{-9/8} \sqrt{\frac{bEd}{v}} \]

Here \( E \) is the peak field strength applied. Note that this laminar sub-layer thickness differs from (is smaller than) the viscous sub-layer thickness \( \delta_v \). This is because \( \delta_b \) represents the distance at which radial turbulence forces can compete with the coulombic force exerted by the external field, while \( \delta_v \) is the distance at which turbulence is important to momentum transport. Thus, \( \delta_b \) is used when discussing where charge enters the turbulent core, while \( \delta_v \) is used to determine the overall velocity profile.

There are several assumptions necessary for the present model to achieve a net current enhancement due to AC energization:

1. There must be a charge injection process at the wall. In this model the boundary condition is that \( \rho(x=0) = \rho_{inj} \) while \( E > 0 \).

2. A rectification of the injected current must take place to keep the injection processes from each wall from cancelling each other. This rectification can be achieved by different mobilities for the positive and negative ions (making \( J = \rho_{inj} bE \) different for the two species) or by simply assuming only one sign of charge is injected; the present work assumes the latter.

3. For the enhancement to be significant, charge trajectories must penetrate the turbulent core, thus charging the entire gap. The thickness of the laminar sub-layers is so small that the current carried by the charge in them alone is small compared to that carried by the entire charged gap.

4. Our model assumes migration dominated currents, where the effect of the applied field is much larger than that of the self-field or that due to molecular diffusion. Though this condition is not necessary, it greatly simplifies the mathematics and is an accurate reflection of the physics at work.

5. The turbulent core is assumed to be fully mixed. This again is a simplifying assumption, which allows the charge density in the core to be approximated as uniform.
If the laminar sub-layer thickness $\delta_b$ is less than the injection length $x_i$, some charge trajectories penetrate into the turbulent core. As shown in Figure 4-2, those trajectories that enter the core emanate from the wall between the times $t=-\pi/2\omega$ and $t=t_1$,

$$t_1 = \frac{1}{\omega} \sin^{-1} \left( \frac{\delta_b \omega}{bE_o} \right)$$

where the $t_1$ trajectory carries a charge out a distance $\delta_b$ before reversing direction. Then, the time-average current injected across the laminar sublayer and into the turbulent core is

$$J_{in} = \frac{1}{T} \int_{-\pi/2\omega}^{t_1} \rho_{inj} bE_o \cos \omega \tau \, d\tau$$

with solution

$$J_{in} = \begin{cases} \frac{\rho_{inj} bE_o}{2\pi} (2 - \frac{\delta_b \omega}{bE_o}) ; & \frac{\delta_b \omega}{bE_o} < 2 \\ 0 \end{cases}$$

There is no injected current if the frequency is so high that the electric field reverses before any charge can reach beyond the laminar sub-layer.

Ions are removed from the turbulent core and impacted on the boundary in the same fashion. However, the appropriate charge density is that in the turbulent core rather than that injected from the wall. The expression for this precipitated current density is Equation 4.1 with $\rho_{inj}$ replaced by the core charge density, $\rho_c$. Thus, with $J_{inj}$ defined as the net injected current density,

$$J_{inj} = \begin{cases} \frac{bE_o}{2\pi} (2 - \frac{\delta_b \omega}{bE_o}) (\rho_{inj} - \rho_c) ; & \frac{\delta_b \omega}{bE_o} > 2 \\ 0 \end{cases}$$

To solve for $\rho_c$ we must perform conservation of charge on the surface $S$ shown in Figure 4-3. This surface is drawn just inside the laminar sub-layer so that it encloses the entire region that has charge density $\rho_c$. Any increase of $\rho_c$ with time must be supplied by the difference of currents into and out of the enclosed volume; currents in consist of the injected currents calculated above and any convection current entering the mixer, while currents out of the volume are comprised by the sampled current and leakage currents driven by the self-field of the core charge. This is expressed by the following rate equation:

$$V_c \frac{d\rho_c}{dt} = A_2 \left| J_2 \right| + A_1 \left| J_1 \right| + Q_v \rho_{inj} - Q_c \rho_c - \oint_S (\rho_c b + \sigma) \mathbf{E} \cdot \mathbf{n} \, da$$

The undefined terms are specified below:

$V_c$ - the volume enclosed by the surface $S$.

$A_1$ and $A_2$ - the surface areas of the inner and outer cylinders, respectively.
Figure 4-3: Conservation of charge in the Couette mixer.

$\mathbf{J}_1$ and $\mathbf{J}_2$ - the injected current densities $\mathbf{J}_{\text{inj}}$ evaluated at the inner and outer cylinders, respectively. Rewriting Equation 4.2 somewhat we have

$$
\left| \mathbf{J}_2 \right| = \frac{(\rho_{\text{inj}} - \rho_c)}{\pi} f \left[ \frac{bV_o}{R_2 \ln(R_2/R_1)} - \frac{\delta_{b2} \omega}{2} \right]
$$

$$
\left| \mathbf{J}_1 \right| = \frac{(\rho_{\text{inj}} - \rho_c)}{\pi} f \left[ \frac{bV_o}{R_1 \ln(R_2/R_1)} - \frac{\delta_{b1} \omega}{2} \right]
$$

$$
f(u) \equiv \begin{cases} 
  u & u>0 \\
  0 & u<0 
\end{cases}
$$

Here, $\delta_{b1}$ and $\delta_{b2}$ are slightly different laminar sub-layer thicknesses, taking into account the difference in field strength at the two cylinders.

$\rho_{\text{in}}$ - is the charge density entering the Couette mixer. If the charge trap is functioning efficiently, $\rho_{\text{in}}$ should be zero.

$Q_c$ - the fraction of the flow rate $Q_v$ coming from the turbulent core, i.e.

$$
Q_c \equiv \int_{R_1+\delta_{b1}}^{R_2-\delta_{b2}+\delta_{b1}} v_2(r) \frac{2 \pi r \, dr}{3 \delta_v}
$$

$$
= \frac{\pi V}{3 \delta_v} \left[ (R_1+\delta_{b1})^2(R_1-2\delta_{b1}) - (R_1+\delta_v)^3 + (R_2-\delta_{b2})^2(R_2+2\delta_{b2}) - (R_2-\delta_v)^3 \right]
$$

Since the electric field in the last term of Equation 4.3 is the self-field created by $\rho_c$, that term can be simplified with Gauss' Law:
We normalize the charge density to $\rho_{\text{inj}}$,

$$\rho \equiv \frac{\rho}{\rho_{\text{inj}}}$$

and rewrite Equation 4.3 in terms of time constants:

$$\frac{d\rho_c}{dt} = -\frac{\rho_c^2}{\tau_s} - \rho_c \left( \frac{1}{\tau_c} + \frac{1}{\tau_2} + \frac{1}{\tau_1} + \frac{1}{\tau_{cr}} \right) + \left[ \frac{1}{\tau_2} + \frac{1}{\tau_1} \right]$$

with the time constants defined as follows:

$$\tau_s \equiv \frac{\varepsilon}{\rho_{\text{inj}} b}$$

$$\tau_c \equiv \frac{\varepsilon}{\sigma}$$

$$\tau_2 \equiv \frac{2R_2 l}{V_c} \left[ \frac{bV_o}{R_2 \ln(R_2/R_1)} - \frac{\delta_{b2} \omega}{2} \right]$$

$$\tau_1 \equiv \frac{2R_1 l}{V_c} \left[ \frac{bV_o}{R_1 \ln(R_2/R_1)} - \frac{\delta_{b1} \omega}{2} \right]$$

$$\tau_{cr} \equiv \frac{Q_c}{V_c}$$

where $l$ is the height of the Couette mixer.

In the steady state, where $d/dt=0$, the normalized core charge density is in quadratic form, with solution

$$\rho_{c0} = -\frac{\tau_s}{2} \left( \frac{1}{\tau_c} + \frac{1}{\tau_2} + \frac{1}{\tau_1} + \frac{1}{\tau_{cr}} \right) + \sqrt{\frac{\tau_s^2}{4} \left( \frac{1}{\tau_c} + \frac{1}{\tau_2} + \frac{1}{\tau_1} + \frac{1}{\tau_{cr}} \right)^2 + \tau_s \left( \frac{1}{\tau_2} + \frac{1}{\tau_1} \right) (4.4)}$$

4.2.1.2 Charge Density in the Laminar Sub-layers

The charge density in the laminar sub-layers is found from the method of characteristics described earlier. Since this value depends on the relative magnitudes of the laminar sub-layer thickness $\delta_b$ and the injection length $x_i$, we divide the problem into two cases.

For $\delta_b > 2bE_o/\omega$:

Here the charge in the sub-layer is unaffected by the turbulent core, and we use Figure 4-4 as a guide in calculating the time averaged charge density as a function of position $x$. The shaded region is bounded by the curve
Figure 4-4: Charge density in the laminar sub-layer.

\[ x = \frac{bE_0}{\omega} (\sin \omega t + 1) \]

and contains charge density \( \rho_{\text{inj}} \); the rest of the sub-layer is uncharged. Thus, the time-averaged charge density is given by

\[ <\rho(x)> = \rho_{\text{inj}} \left( \frac{\omega}{2\pi} \right) (t_b - t_a) \]

where \( t_b \) and \( t_a \) are the times at which the bounding trajectory has position \( x \) out from the wall. Taking advantage of these times’ symmetry about \( t = \pi/2\omega \), we can relate them as follows:

\[ \omega t_a = \pi - \omega t_b \]

Noting also that

\[ \cos (\omega t_b - \frac{\pi}{2}) = \sin \omega t_b \]

and that

\[ \sin \omega t_b = x \left( \frac{\omega}{bE_o} \right) - 1 \]

we write

\[ \frac{<\rho>}{\rho_{\text{inj}}} = \frac{1}{\pi} \cos^{-1} (2x - 1) \]

(4.5)

where \( x \) has been normalized to the injection length, \( x_i \):

\[ x \equiv \frac{x}{x_i} = \frac{x}{(2bE_0/\omega)} \]
For $\delta_b < 2bE_u/\omega$:

Figure 4-5: Charge density in the laminar sub-layer when it is thinner than the injection length.

Figure 4-5 depicts the laminar sub-layer when it is shorter than the injection length $x_i$. The sub-layer is now filled with charge, the two shaded areas distinguishing between regions filled with $\rho_{\text{inj}}$ and with $\rho_c$. The second region is filled with $\rho_c$ because the trajectories there originate at the boundary between the laminar sub-layer and the turbulent core, and thereby carry the core charge $\rho_c$ with them.

The normalized time-averaged charge density as a function of $x$ is now given by

$$\langle \varphi \rangle = \frac{\omega_t - \omega_{t_a}}{2\pi} + \rho_c \left[ 1 - \frac{\omega_t - \omega_{t_a}}{2\pi} \right]$$

Recognizing that

$$\sin \omega_{t_a} = 2x - 1$$
$$\sin \omega_{t_c} = 2x - (2\delta - 1)$$

we can write

$$\langle \varphi \rangle = \left[ \frac{1}{2} - \frac{1}{2\pi} \sin^{-1} [2x - (2\delta - 1)] - \frac{1}{2\pi} \sin^{-1} (2x - 1) \right] (1 - \rho_c) + \rho_c$$

(4.6)

4.2.2 Current Entering Fully Mixed Sampling Region

With the velocity profile defined in Section 3.1.4 and the charge densities from Section 4.2.1, we can now perform the integration of Equation 3.6. Since the boundaries of the laminar sub-layers do not coincide with the viscous sub-layers, this integration must be performed in five regions:
The terms on the right represent the currents from, respectively, the laminar sub-layer near
the inner cylinder; the region between the inner laminar layer and the outer edge of the
viscous sub-layer, which contains charge density \( \rho_{ci} \); the region of uniform velocity, also of
charge density \( \rho_{ci} \); the space between the inner edge of the outer viscous sub-layer and the
outer laminar sub-layer, again carrying \( \rho_{ci} \); and the outer laminar sub-layer. If the laminar
sub-layer thickness is greater than the injection length \( x_i \), only the first and last terms are
non-zero, as the core is uncharged. However, if the injection zone is penetrated by the
turbulent core then all terms must be kept. The problem is thus again divided into two:

For \( \delta_b > 2bE_\omega/\omega \):

If the laminar sub-layer thickness is larger than the injection length there are only two
integrations to be performed, each extending the injection length out from their respective
walls. Since \( x_i << d \), we approximate the two regions as planar, with \( x \) the distance from the
wall. The velocities in these regions are now the same:

\[
v_z = \frac{V}{\delta_v} x = \frac{2bE_1 V}{\omega \delta_v} x
\]

The distance \( x \) is now normalized to \( 2bE_\omega/\omega \) to avoid confusion between the field at the inner
wall (subscript 1) and that at the outer wall (subscript 2). This leaves two injection lengths,
\( x_{i1} \) and \( x_{i2} \), one for each wall:

\[
x_{i1} = \frac{2bE_1}{\omega} ; \quad x_{i1} = 1
\]

\[
x_{i2} = \frac{2bE_2}{\omega} ; \quad x_{i2} = \frac{E_2}{E_1} \frac{R_1}{R_2} \equiv \frac{1}{\gamma}
\]

The current can now be expressed as

\[
I = \frac{V}{\delta_v} \left[ 2\pi R_1 \int_0^{x_{i1}} <p>x dx + 2\pi R_2 \int_0^{x_{i2}} <p>x dx \right]
\]
or in normalized terms,

\[
I = \frac{8\pi \rho_{i1} V}{\delta_v} \left( \frac{bE_1}{\omega} \right)^2 \left[ R_1 \int_0^1 <p>x dx + R_2 \int_0^{1/\gamma} <p>x dx \right]
\]

Since \( \delta_b > 2bE_\omega/\omega \), we use the charge density from Equation 4.5:
where the term $2\gamma x$ in the second integral represents $x/x_i$, since the injection length near the outer wall is less than that at the inner one. From integral tables we note that

$$\int x \cos^{-1}(ax+b) \, dx = \frac{1}{4a^2} \left[ 2(a^2x^2-b^2-\frac{1}{2}) \cos^{-1}(ax+b)-(ax-3b)\sqrt{1-(ax+b)^2} \right]$$

and after some simplification obtain

$$I = \frac{3\pi R_1 V \rho_{inj}}{2\delta_y} \left( \frac{bE_1}{\omega} \right)^2 \left( 1 + \frac{1}{\gamma} \right)$$

For $\delta_b < 2bE_0/\omega$:

Our current now has five terms, since the core is charged, and the charge densities in the laminar sub-layers are now given by Equation 4.6. The integrals across the laminar sub-layers are still taken in rectangular coordinates, but the other three terms use the cylindrical system as they span much greater distances.

The five integrals will be performed separately, since they result in rather long expressions:

$$2\pi R_1 \int_{\delta_b}^{\delta'_b} \rho_x V \, dx = \frac{8\pi R_1 V \rho_{inj}}{\delta_y} \left( \frac{bE_1}{\omega} \right)^2 \int_{\delta_b}^{\delta'_b} \left[ \frac{(1-\rho_c)}{2} \left[ 1 - \frac{1}{\pi} \sin^{-1}[2x-(2\delta_b-1)] - \frac{1}{\pi} \sin^{-1}(2x-1) \right] + \rho_c \right] \, dx$$

$$= \frac{4\pi R_1 V \rho_{inj}}{\delta_y} \left( \frac{bE_1}{\omega} \right)^2 \left[ \frac{(1+\rho_c)}{2} \delta_b^2 - \frac{(1-\rho_c)}{8\pi} \left( 4\delta_b-3 \right) \right]$$

$$= \frac{\pi}{2} \sin^{-1}(1-2\delta_b) - (2\delta_b-3)\sqrt{4\delta_b(1-\delta_b)}$$

$$\int_{R_1+\delta_b}^{R_1+\delta'_b} \rho_c V \, 2\pi r \, dr = \frac{2\pi \rho_{inj} \rho_c V}{\delta_y} \int_{R_1+\delta_b}^{R_1+\delta'_b} r(r-R_1) \, dr$$

$$= \pi \rho_{inj} \rho_c V \left[ 3R_1(\delta_b^2-\delta_{b1}^2) + 2(\delta_b^3-\delta_{b1}^3) \right]$$

$$\int_{R_1+\delta_b}^{R_2+\delta_b} \rho_c V \, 2\pi r \, dr = \pi \rho_{inj} \rho_c V[R_2^2-R_1^2-2R_1R_2(\delta_b+\delta_{b1})]$$

$$\int_{R_1+\delta_b}^{R_2+\delta_b} \rho_c V \, 2\pi r \, dr = \pi \rho_{inj} \rho_c V[R_2^2-R_1^2-2R_1R_2(\delta_b+\delta_{b1})]$$
4.2.3 Dependence of Current on System Variables

As the dependence of the above expressions on system variables is not entirely transparent, they were entered into a computer program and plotted as functions of various parameters. This program computes the relation between \( \delta_b \) and \( x_i \) and then uses Equation 4.9 if \( \delta_b \) is larger, or Equations 4.10 - 4.14 if it is smaller than \( x_i \).

Figures 4-6 - 4-8 illustrate on arbitrary scales the general shape of the current dependencies on frequency, rotational speed and flow rate, respectively. Chapter 5 presents more detailed plots at the operating conditions of our experiment, for direct comparison with the actual experimental results shown in the next section.

Separate calculations of Equations 4.10 - 4.14 allow the contributions of the laminar sub-layers and the regions with charge density \( \rho_c \) to be compared. The combined currents carried by the two laminar sub-layers is at most about 1.2 pA, and decreases to .06 pA as frequency rises and the field strength goes down. Over this same range of field parameters the total current goes from almost 1200 pA under highly energized conditions to .06 pA when the sub-layers are thicker than the injection length, in which case all the current is, as expected, carried by the laminar sub-layers. Thus, when the core is charged, the sub-layers carry about .1% of the total current.
These calculations also explain the shape of the curve in Figure 4-6. As can be seen, the current decreases with increasing frequency and experiences two discontinuities in its slope at higher frequencies. These discontinuities mark the points at which the inner and outer laminar sub-layer thicknesses (which differ due to the difference in field strength at the two walls) become larger than their respective injection lengths. The first change in slope marks the fact that the outer sub-layer has extended beyond the outer injection length and is preventing the core from receiving any charging current from the outer wall. The second discontinuity occurs when both sub-layers exceed the injection lengths and thus the total is the very small current carried by the sub-layers.

The rotational speed at which the curve in Figure 4-7 begins to rise is similarly explained as that speed which yields a laminar sub-layer thickness that is less than \(2bE/\omega\). The curve starts to level off as the increasing charge in the core makes relaxation currents compete more and more strongly with injected currents.

Figure 4-8 shows a curve that is almost linear in flow rate, but starts to level off as \(Q_v\) rises. The linear region at low flows indicates that the sampling flow rate is not affecting the charge density \(\rho_c\) in the core of the rotational flow, and thus the current \(\rho_c \rho_{\text{inj}} Q_c\) is nearly linear in \(Q_v\). However, as \(Q_v\) increases, the sampled current begins to compete with the injected currents and thereby reduces the charge density in the core, causing the current to tend to saturate.

4.3 Experimental Observations

This section presents the AC energization data taken with the Couette electrification apparatus. The currents listed represent the difference between the total measured current at a given setting and the current generated by the same rotation of the device when unenergized. The data taken falls into four categories: data showing the dependence of the energized current on electric field strength, on frequency, on rotational speed of the inner cylinder, and on the sampling flow rate through the mixer. The graphs displayed here will be compared to the theoretical predictions of Section 4.2.2 in Chapter 5.

4.3.1 Experimental Technique

The procedure followed in the applied field measurements consist of establishing a steady-state current at the desired rotational speed and flow rate, and then applying the AC field at varying frequencies. The data point then taken at a certain frequency is the difference between the total current at that point and the unenergized current established before the field was applied. In somewhat more detail:

1. **document parameters** - Before a run begins, the conductivity and moisture content of the oil should be known. These are measured as described in Section 3.3.1.

2. **initialize recording equipment** - The electrometer and chart recorder are turned on and adjusted before the system flow begins.
Figure 4-6: Dependence of the AC field induced convection current on frequency of the applied voltage.

3. establish unenergized flow conditions - A run begins by starting the rotation of the inner cylinder and pressurizing the fluid reservoir, thereby establishing the sampling flow rate. The current thus measured is allowed to settle out before applying a field.

4. apply electric field - Once the current due to rotation and flow alone has leveled off, the external field is applied across the two cylinders, and the resulting current transient observed. Successive frequencies or voltages can be tested as soon as the transient decays.

The applied AC fields are generated by passing the output of a sine-wave generator through a high-voltage amplifier, and are typically in the ranges 2 kV (E=1.6x10^5) to 8 kV (E=6.3x10^5), and 0.5 Hz to 10 Hz.

To ensure that the applied fields are dominant, we must calculate the field due to the space charge resulting from the injected currents. The approximate charge density across the gap is simply the measured current divided by the flow rate; as will be shown in Chapter 4, the largest currents measured with
applied fields at $Q_v=7\times10^{-6}$ m$^3$/s are about 1300 pA, yielding a charge density $ho_0=2\times10^{-4}$ C/m$^3$. The electric field implied by this charge density is just under $6.5\times10^4$ V/m, which is almost an order of magnitude less than the fields typically applied.

5. **vary designated parameters** - After the current has again reached steady-state, a parameter may be altered to add another data point. Most often this is the frequency, but in some runs it is the flow rate that changes.

6. **turn-off** - More data points are taken until the experiment run time is up, at which point the flow must be shut off.

7. **refill fluid reservoir** - After the reservoir is depressurized, it can be refilled by pressurizing the fluid receiver and pumping the oil back through a return loop.
Figure 4-8: Dependence of the energization-induced current on the sampling flow rate through the Couette mixer.

4.3.2 Data Plots

The plots shown in this section are of four types: current versus frequency, at different field strengths; current versus frequency with the rotational speed as a parameter; current versus rotational speed with frequency as a parameter; and current vs flow rate, at different rotational speeds. The plots of current against rotational speed are necessarily replots of the same data displayed in the plots against frequency with rotational speed as a parameter; this is because the rotational speed cannot be changed during an individual run of the experiment, since doing so would alter the base current to be subtracted from the total.

This data falls into two general groups. The first sets were taken with dry, vacuum-impregnated paper, while the second sets were taken after the paper had been exposed to humid air for about 36 hours. As will be found in Chapter 5, this difference in paper moisture content will imply a difference in the value of $\rho_{\text{inj}}$ necessary to match theory to experiment.
Dry paper:

Figure 4-9 shows the frequency dependence of the energization current taken at six different field strengths. The applied voltage indicated is the peak value of the applied sinusoid. The lack of a 0.5 Hz data point on the 7.5 kV curve is due to the frequency response of the high voltage amplifier used, which prohibited that high a field strength at 0.5 Hz. In contrast to the curve from Figure 4-6, the data has an almost exponential dependence, and certainly does not share the slight negative curvature of the theoretical plot. The different curves do separate nicely from each other, though, and as will be seen in later comparisons, the theoretical curves have approximately the same frequency "breakpoint" (point at which the laminar sub-layers no longer penetrate the injection zone) when matched in magnitude.

![Figure 4-9: The frequency response of the experimental data for dry paper, taken at the peak voltages indicated. All the curves were taken at the same rotational speed, 1400 RPM.](image)

A similar plot is presented in Figure 4-10, but here the parameter distinguishing each curve is its rotational speed. Each curve was taken with the maximum applied voltage that can span the entire frequency range, 6 kV peak.
Figure 4-10: Frequency response of the experimental data for dry paper, taken at the indicated rotational speeds. All curves taken at 6 kV peak.

In order to clearly visualize the effect of the rotation, the data from Figure 4-10 was replotted versus rotational speed in Figure 4-11, where the parameter is now the frequency. The glitch in the .5 and 1 Hz curves around 500-800 RPM is felt to be experimental error caused by the fact that the points were not taken in chronological order, which allowed the experiment to drift.

If that glitch is ignored, however, the data is seen to have a fairly close resemblance with the predictions of Figure 4-7, except that the data slopes down toward the origin and not toward a point further out the rpm-axis. This problem becomes more severe for the higher frequency curves discussed in Chapter 5.

Wet paper:

After the above data was taken with the dry, oil-impregnated paper, the Couette mixer was opened, emptied of oil, and exposed to the humid room air (= 60% relative humidity) for 36 hours. Identical sets of data were then taken to see if a change in the value of $\rho_{\text{inj}}$ was
implied. As will be shown presently, the currents did indeed rise above the values taken with dry paper, implying a change in $p_{\text{inj}}$ that is calculated in Chapter 5.

The frequency dependence of the injected current with wet paper is shown at six field strengths in Figure 4-12. The data is entirely analogous to that with dry paper except for the increase in magnitude of about one-third; it is also similarly missing a point at .5 Hz on the $V_o=7.5$ kV curve.

The frequency dependence at different rotational speeds is displayed in Figure 4-13, and is replotted against the rotational speed in Figure 4-14. Again, we note that the currents have increased by almost one-third, but this time Figure 4-14 shows no "glitch": this is probably due to the fact that this time the runs were taken consecutively, each at a lower rotational speed, which prevented any experimental drift from producing anomalies in the data.

Figure 4-15 shows curves that were not taken with the dry paper: the current is measured as a function of the sampling flow rate through the system. The sampling approach at first leaves the impression that the measured current should be linear with the flow rate-- if $Q_v$ is...
doubled, then the sampled current $p_0Q_v$ should also double. However, as is indicated in Figure 4-8, the flow rate also affects the charge density, in that the sampled current drives the system slightly out of equilibrium by requiring steady-state currents through the walls. Indeed, if there were no sampled current, and effects of charge relaxation could be ignored, the turbulent core would eventually become charged to $p_c=p_{inj}$, as long as some charge trajectories penetrated into the turbulent core. This would make the current injected into the core from Equation 4.2 zero, yielding the true equilibrium where no current into the core is required since none is removed. The sampling current, however, is a departure from that equilibrium, and thus produces a non-linear current dependence.

**Figure 4-12:** Frequency dependence of the injected current with wet paper, taken at different field strengths. All curves taken at 1400 RPM.
Figure 4-13: Frequency dependence at different rotational speeds; wet paper. Applied voltage is $V_0 = 6$ kV.
Figure 4-14: Current against rotational speed, at different AC frequencies. Data in Figure 4-13, $V_0 = 6$ kV.
Figure 4-15: Dependence of the current on the sampling flow rate, \( Q_v \).
Each curve taken with \( V_0 = 4 \text{ kV} \) and \( f = 1 \text{ Hz} \).
Chapter 5
Data Correlations, Conclusions, and Recommendations

This chapter brings together the experimental results and the theoretical predictions in an examination of both the performance of the Couette electrification system and the validity of the analytic models presented. The contributions of the present approach are discussed in light of these results, and recommendations for future work are proposed.

5.1 Flow-induced Currents

As shown in Section 3.4, the measured currents due to the rotation of the inner cylinder are quite reproducible and align themselves fairly closely to the curves calculated in Section 3.2 (the plots are redrawn here in Figure 5-1). The analysis, however, still lacks a rigorous treatment accounting for the differences between the pipe flow studied by Abedian and Sonin and the flow conditions encountered in the Couette mixer. The most obvious of these is the geometry: a thorough analysis would repeat conservation of charge equations in cylindrical coordinates, again treating effects due to a turbulent diffusion profile and charge relaxation. An average charge density could then be found for use in calculating the sampled current.

The simple adaptation used here, though, seems to be adequate for the time being. That it predicts the relatively large currents encountered at rotational speeds below 1000 RPM, where there is a question of the degree of turbulence, suggests that the flow can be treated as essentially turbulent over the Reynold’s number range measured. The view that the cellular convection in this regime has a "turbulent" effect is reinforced by the degree to which Taylor’s expression for the turbulent wall stress correlates data to Reynold’s number of 300.

The equations work because the cellular convection in this regime, which is responsible for the current enhancement, is accounted for to a degree by Taylor’s turbulent wall stress correlation, which is valid down to Reynold’s numbers of 300.

Currents generated by the Couette system are large enough to be easily measured, and have been used to calculate values of the wall charge density $\rho_w$, a parameter that should prove to be an indication of the amount of electrification activity expected from a particular liquid/solid interface. Beyond the dependence on Reynold’s number the present work does not verify this parameterization, but by developing a working system capable of producing meaningful results, it sets the stage for this to be done in a device which can truly provide all the parameters necessary to characterize equilibrium and non-equilibrium charge generation.

The Couette electrification system developed in this work presents an extremely powerful approach to the study of electrification phenomenon. By use of a reentrant flow, it enforces a uniformity that eliminates any spatial development length; this prevents the existence of electric fields tangential to the interface, and significantly reduces the required size of the experiment. The sampling approach used also yields physical and practical advantages: the
charging conditions established inside the mixer can now be viewed in terms of an average charge density, which is sampled to produce the measured current; and the independence of the internal Reynold's number and the sampling flow allow the electrification conditions to be established solely by the rotation of the inner cylinder, without affecting the oil flow rate.

These advantages combine to produce the quintessential device for measuring charge generation due to electrification. It stands alone as a device capable of measuring truly equilibrium charging effects: in the absence of an applied field, only the sampling flow disturbs this equilibrium, and this can be made negligibly small. Indirect evidence of the significance of the sampling flow rate is obtained from the dependence on flow rate of the AC energization currents shown in Section 4.3. Though not strictly valid for unenergized currents, these curves indicate that the normal operating flow rate $Q_v=7 \times 10^{-6} \text{ m}^3/\text{s}$ is in a linear region where it has no effect on the sampled charge density.

This same device can also be used to measure charging under carefully controlled non-equilibrium conditions. By either taking the flow rate beyond the linear regime or by application of an external field, significant wall currents can be induced, thus destroying the equilibrium discussed above. The non-equilibrium conditions established here can be controlled and studied while avoiding complications inherent to pipe flow situations, in that spatial uniformity is still enforced, and in that the non-equilibrium is studied in the same device as the equilibrium effects. The Couette mixer is thus best suited to measuring the entire range of electrification charging effects.

Practical advantages of the present system also make it particularly appealing. The charging conditions discussed above make the Couette mixer a badly needed charge source for use in a variety of experimental situations-- a wide range of charge densities can be obtained by simply varying the rotational speed, the field strength or frequency. Its compact nature also makes for easy testing of different materials, and does not require a large reserve of oil. Since the amount of oil required for operation of the system is in principle little more than the volume inside the mixer, future work will perhaps lead to the development of a portable device.

**5.2 Currents Due to AC Energization**

We now turn to comparisons of theory and experiment for currents generated by the application of an external electric field. Since the present work goes into more detail in the analysis of these currents than those due to rotation alone, the data in this section will provide a more substantial test of our modeling ideas.

**5.2.1 Frequency Dependence**

Figures 5-2 and 5-3 compare the theoretical predictions and experimental results showing the frequency dependence of the measured current for dry and wet paper, respectively. Here, different curves represent different field strengths all taken at 1400 RPM; Figures 5-4 and 5-5 show analogous plots where the rotational speed is different between each curve, all of which are taken at 6 kV peak.
Figure 5-1: Theoretical predictions (solid lines) and measured currents due to rotation for a) dry paper and b) wet paper. Matched wall charge densities are $\rho_w = 2.3 \times 10^{-3}$ and $2.2 \times 10^{-3}$ C/m$^3$, respectively.
As expected, the currents decrease with increasing frequency, since the ratio of the injection lengths to the laminar sub-layer thicknesses is decreasing. Also as expected, the currents decrease with decreasing field strength and rotational speed: the injection length has a stronger field dependence than does the sub-layer thickness, and thus their ratio again goes down; as the rotational speed decreases, the sub-layer thickness enlarges, allowing fewer trajectories into the turbulent core.

There are, however, several marked discrepancies in the theoretical correlation. In addition to the fact that the calculated curves do not accurately predict the magnitude differences between different field strengths or rotational speeds, their general curvature and final values are notably different than those displayed by the data. Where the data shows an almost exponential dependence on frequency, the curvature of the plotted lines is actually negative throughout much of the frequency range. The curvature goes positive, and abruptly so, only at a frequency where one of the laminar sub-layers has stopped injecting charge into the core. At the higher frequency where the second sub-layer can no longer inject, the theoretical curve suffers another discontinuity in slope and plunges to values just above zero: the calculated current due solely to convection in the laminar sub-layers is small enough to be inconsequential. The data quite clearly, however, demonstrates easily measurable values throughout the entire frequency range.

The discrepancy in the high-frequency charge is the most easily understood, since the presumption of a purely laminar region followed by an immediately fully mixed turbulent core is rather crude. Effects of turbulence can be felt throughout the laminar sub-layer and are much more complicated than envisioned here. This "fuzziness" between the sub-layer and the turbulent core could probably account for the existence of currents even when the "laminar" sub-layer is larger than the injection length. Whether it can also account for the difference in curvature is not so clear, since the frequency dependence is deeply embedded in the equations.

5.2.2 Dependence on Rotational Speed

The theoretically predicted dependence of the current on rotational speed is presented for comparison with experimental results for dry and wet paper in Figures 5-6 and 5-7, respectively. This data is the same as that in Figures 5-4 and 5-5, just plotted against rotational speed instead of frequency.

Once again, the match is not too precise, especially in that the theory again predicts negligible currents where experiment shows that not to be the case. These breakpoints where the current starts to become significant indicate when the first laminar sub-layer shrinks beneath the thickness of the injection zone; at these points, injected charge is first able to penetrate into the core. The shapes of the lowest two frequency curves are fairly well predicted by the model, but the higher frequency results do not seem to match as well. One correlation worth noting, however, is that the increasing breakpoint at which the calculated curves leave the rpm-axis correlates roughly with the increasing frequencies at which the two higher frequency experimental curves increase their slopes. These two curves increase more or less linearly for low speeds, and then apparently plateau before assuming almost a constant upward slope at higher speeds.
Figure 5-2: Experimental results (a) and theoretical predictions (b) of the frequency dependence of the current at different field strengths for dry paper. Theoretical curves plotted with $\rho_{\text{inj}}=6.5 \times 10^{-4} \text{ C/m}^3$. All curves taken at 1400 RPM.
Figure 5-3: Experimental results (a) and theoretical predictions (b) of the frequency dependence of the current at different field strengths for wet paper. Theoretical curves plotted with $\rho_{inj}=8\times10^{-4}$ C/m$^3$. 1400 RPM.
Figure 5-4: Experimental results (a) and theoretical predictions (b) of the frequency dependence at different rotational speeds, for dry paper. Matched $\rho_{\text{inj}} = 6.5 \times 10^{-4}$ C/m$^3$, 6 kV peak.
Figure 5.5: Experimental results (a) and theoretical predictions (b) of the frequency dependence at different rotational speeds, for wet paper. Matched $\rho_{\text{inj}}=9 \times 10^{-4} \text{ C/m}^3$; 6 kV peak.
It is quite possible that much of the difficulty correlating the data could be accounted for by the presence of the cellular convection flow regime. More likely is the sensitivity of the response in this range to the details of the boundary layer structure alluded to above. The turbulence does not experience the sharp spatial cut-off we have assumed. Both of these hypotheses provide for charge to be brought into the turbulent core even when the "laminar" sub-layer is computed to extend beyond the injection zone, and would increase the currents in the regions where the calculated curves indicate almost zero values.

5.2.3 Dependence on Sampling Flow Rate

The predicted curves in Figure 5-8 match the shape of the experimental results very well, though the relative magnitudes between individual curves do not correlate too strongly. The linear region at low flow rates is a range in which the sampled current is too small to significantly affect the charging of the turbulent core; the current then is just $I_0 \rho_{inj} \Omega_v$. As the flow rate increases the sampled current starts to compete with the injection currents and the curves start to level off as indicated. Thus, the linear region on these curves indicates the flow rates at which the sampling process does not disturb the electrification processes at work inside the Couette mixer.

5.2.4 Discussion

The data correlation above provides a foundation for critiquing the charge injection model proposed in this work. This model rests on two basic tenets: the existence of a charge injection boundary condition at the wall, and the dominance of migration in the motion of charged particles.

One of the effects of a migration-based model is to oppose the view put forth in a previous model in which the Debye distribution of charge is supposed to suffer merely a displacement in the face of high external fields. A migration model would claim that the high field strengths applied make diffusion unimportant in the face of coulombic forces, and thus insists that the exponential charge distribution created by a balance between diffusion and self-field migration no longer exists. Unfortunately, it is not quite clear what effect the double-layer modulation model would have on the data that would allow it to be distinguished from migration-based theories. Though the previously-proposed model did not include effects of turbulence, it would be possible to construct a model along the lines of that in Section 3.2 in which the displacement of double-layer charges is accounted for in context with turbulent diffusion and charge relaxation. Such a model might then yield quite different results than that proposed here, which expects turbulence to account for almost the entirety of generated currents. However, such a comparison cannot be made at present.

There are other effects of the migration model which are tested by the available data, and they receive a mixture of support and criticism from the correlations performed. The ability to envision characteristic lines that describe charge motion is wholly dependent on the dominance of migration forces, for these lines assume that the motion of an individual particle can be predicted without regard for the particles around it. In turn these lines, which allow calculation of the charge density in the core, are the basis for the vast majority of the
Figure 5-6: Experimental results (a) and theoretical predictions (b) of the current dependence on rotational speed, at different frequencies, for dry paper. Matched $\rho_{\text{inj}}=6.5 \times 10^{-4}$ C/m$^3$; 6 kV peak.
Figure 5-7: Experimental results (a) and theoretical predictions (b) of the current dependence on rotational speed, at different frequencies, for wet paper. Matched $\rho_{\text{inj}}=9\times10^{-4}$ C/m$^3$; 6 kV peak.
Figure 5-8: Experimental results (a) and theoretical predictions (b) of the current dependence on sampling flow rate, at different rotational speeds, for wet paper. Matched $\rho_{\text{inj}}=4\times10^{-4}$ C/m$^3$. $4$ kV peak, $1$Hz.
computed current, since currents carried by the sub-layers are insignificant. Thus migration is central to the dependence on turbulence of the curves calculated by the present model.

As is evident from the theoretical curves above, the model of charge entering the core through a ratio of laminar sub-layer thicknesses and injection lengths predicts sharp changes in behavior when this ratio passes through unity. As is also shown above, the data does not possess this behavior, though many of the general shapes are apparent. The most likely explanation for this departure lies with the simplified model of turbulence adopted here: the region inside the sub-layer is assumed to be perfectly laminar, while just outside turbulent diffusion becomes infinite and totally mixes any charge escaping. Many models, including that of Abedian and Sonin, use a finite coefficient of turbulent diffusion which is not necessarily a step function; in their case, it increases linearly from the edge of a molecular-diffusion based laminar sub-layer to a point one quarter of the radius from the wall, and then becomes uniform. However, treating diffusion in conjunction with migration due to external fields in a rigorous fashion can be quite difficult mathematically, and has been avoided so far in the present work. What may prove even more difficult is to take into account the fact that effects of turbulence can be felt all the way up to the wall. There is no truly laminar region, just a zone in which forces due to migration or molecular diffusion dominate those from the turbulence. This fact ensures that some charge will always be carried into the core of the flow, even under conditions where the injection length is very small. However, it appears that to account for these effects, stochastic processes in which an individual particle has a certain probability of being transported into the turbulent core must be analyzed, another step which the present work has yet to take.

The second premise associated with the charge injection model pertains to a boundary condition in which currents generated by energization are supplied from the material comprising the solid insulation. The picture here views the interstitial regions inside the paper as filled with charge due to separation processes at the fibers. These mobile charges are then available to be injected by the presence of a strong external field. Previous work comprising a bachelor’s thesis examined this charge without applied fields: the experiments provided for the measurement of currents produced by forcing oil directly through a layer of transformer paper. These experiments need to be refined, but yielded a first estimate of this charge density on the order \( \rho = 2 \times 10^{-4} \) C/m\(^3\). Comparison to values of \( \rho_{\text{inj}} \) obtained from energization data give a difference that is within a factor of 2 to 5, which may well be within the combined experimental error of the two experiments.

5.3 Recommendations for Future Work

The present work has demonstrated the viability of the Couette electrification system as a sophisticated device for measurements of both equilibrium and non-equilibrium charge generation phenomena. There are, however, some aspects of its performance which need to be improved and/or better understood before all parameters associated with transformer charging can be precisely detailed. These recommendations are divided into two groups: improvements of theoretical models and modifications of the experimental apparatus.

Suggested advances in theoretical understanding:
The theoretical models for both energized and unenergized generation could be further refined. The adaptation of Sonin's work for currents due to rotation could be made rigorous by the exact solution of conservation of charge in cylindrical geometry. Here, radial currents due to turbulent diffusion and charge relaxation would be balanced based on the best available turbulent diffusion coefficient profile.

Improvements in the charge injection model could include a more sophisticated injection boundary condition (i.e. a charge density at the wall that is not constant, perhaps proportional to electric field strength), but more importantly should establish a more realistic view of the turbulence. It is likely that accounting for the effects of turbulence inside the "laminar" sub-layer would resolve many of the present inconsistencies.

Suggest experimental modifications:

There are several experimental alterations which could provide data that is more easily correlated to theoretical models. Perhaps the most significant of these would involve eliminating the cellular convection flow regime and replacing it with typical laminar flow. This could be achieved in the Couette configuration by rotating the outer cylinder instead of the inner, and would provide flow conditions that are much better understood-- in particular it would produce a large laminar region in which many of the ideas dependent on turbulence could be tested.

The present sampling method could be improved by using a probe that extends into the central part of the flow to sample only fluid from the core. The use of such a probe is now being explored, and should remove the ambiguities with sampling the core and the sub-layers, especially under turbulent energization conditions.

In order to more thoroughly validate the sampling approach in experiments where currents are solely caused by rotation, experiments analogous to those performed in the presence of an external field should be performed in which the dependence of the measured currents on flow rate is investigated. These results will indicate what range of flow rates can indeed be considered linear.

Finally, refined experiments yielding improved values of the charge density in the interstitial regions of the paper are required to provide a solid check on values of $\rho_{\text{inj}}$ inferred from energization data.
Appendix A
Moisture Control

Two systems were developed to control the moisture content of the materials in the experiment. The first is an oil degassing chamber, shown in Figure A-1. Wet oil is introduced into this chamber, which is then evacuated. The combination of the vacuum and the stirring caused by the magnetic stir bar in the bottom drive moisture out of the oil to be collected by a trap in the vacuum system. When the oil is drawn into the chamber under vacuum it immediately gives up a large fraction of its water content. Very wet oils, however, require many hours (usually overnight) of evacuation and stirring to reduce their moisture content to levels on the order of 10 ppm. Upon completion of the drying process, the oil can be directly pumped into the fluid reservoir for use in the experiment.

The second system, illustrated in Figure A-2, is a makeshift vacuum oven inside which the paper can be dried prior to use in the flow system. An aluminum vacuum-tight chamber holds the paper-wrapped Couette mixer, and can be inserted into an oven. Connections through the top of the oven allow the heated chamber to be evacuated. After evacuation for roughly 16 hours at 90°C, dry oil is drawn into the Couette mixer and allowed to sit for 2-4 hours. This process first dries the paper, and then impregnates it with properly conditioned oil in a manner analogous to that used in treating the insulation inside an actual transformer.

Once the impregnation process is finished, the sealed vacuum chamber can be removed from the oven and placed inside a glovebox filled with dry nitrogen. The dry atmosphere in this box allows the Couette device to be reassembled (e.g. the top cap put on) without exposure to humid room air.
Figure A-1: Oil degassing facility.
Figure A-2: Vacuum oven arrangement for drying the paper.
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