AN ELECTRONIC COMPUTING DEVICE

by

SIDNEY ALDEN WINGATE

S.B., Massachusetts Institute of Technology

1943

Submitted in Partial Fulfillment of

the Requirement for the Degree of

MASTER OF SCIENCE

from the

Massachusetts Institute of Technology

1946

Signature of Author Signature redacted
Department of Electrical Engineering, June 3, 1946.

Signature of Professor
in Charge of Research Signature redacted

Signature of Chairman of
Department Committee on
Graduate Students Signature redacted

EE
Thesis
1946

An Electronic Computing Device

INDEX

ACKNOWLEDGMENT

For the original suggestion of the subject, acknowledgment is made to Mr. John W. Gray, formerly of the Radiation Laboratory at the Massachusetts Institute of Technology.

For advice and the considerable expenditure of his time on the photographic details of this thesis, acknowledgment is made to Mr. William D. Green, of the M.I.T. Instrumentation Laboratory.

I.

INTRODUCTION

It is often desired that an automatic device perform the fundamental mathematical operations of addition, subtraction, multiplication, and division with accuracy and speed much greater than that possible by a human operator. The purpose of this paper is to describe an all-electronic device which is capable of performing several types of computations, including multiplication, division, and generation of power-law functions. The time required by the device to give the solution is much shorter than is possible with mechanical calculators, thus permitting multiplication of two variables which change rapidly with time.

Because the required operations of this computer represent nonlinear relationships between input and output, the problem is not amenable to the niceties of either the practices or the theory of linear circuits.

For instance, the accuracy of the device cannot be improved by the use of negative feedback. The usual circuit elements - resistance, capacitance, and inductance - are capable of only the linear operations of addition, subtraction, and also differentiation and integration with respect to time. Consequently, a different circuit element with a nonlinear voltage-current relationship is essential to such an electronic calculator. In the problem of this thesis, the non-linear element is a diode operated at retarding plate potentials, which device yields a logarithmic relationship between current through the diode and potential difference across the diode.

The problem of automatic computation is not a new one, and many mechanical, electrical, and combination electro-mechanical systems have been built for the purpose. Some expansive machines costing many thousands of dollars have appeared, but they belong in a category outside of the concern of this paper. Attention will be given to only the low-power, instrument-type of calculator..

In general, there are two classes of computing devices. The first, and probably the most frequent, is the digital-type computer, which counts integral numbers of elementary operations, such as shaft rotations, angular or linear displacements,voltage pulses, etc. An example of this class is the odometer in an automobile. The second type of computer takes smooth or continuous information and gives a continuous answer, such as the total angular rotation of a shaft or the magnitude of a voltage. A slide rule or a speedometer belong to this class. The device herein described belongs to the second, or continuous class, and goes one step further in that the answer appears in the same dimensional quantity as the inputs (all are d-c voltages). In many applications the two classes of computers overlap and may be interchanged, but in some applications (i.e., where the variables are continuously changing with time) the roughness of the digital computer output cannot be tolerated.

Probably the most common method of multiplication (on a small instrument basis) is the potentiometer system in which the current through the resistance and the angular displacement of the potentiometer shaft are made proportional to the two multiplicands. The product appears as the voltage across the potentiometer. This is adequate only when slow-response can be tolerated, and it suffers the disadvantage that only multiplication can be had with one instrument.

An all-electrical analogue of this method involves controlling the plate impedance, transconductance, or amplification factor of a vacuum tube as the nonlinear element. However, these tube characteristics are notoriously unreliable and usually only approximate the desired control function.

Another method of multiplication is accomplished by mixing two voltages of different carrier frequencies and filtering out the product term. This requires modulators and detector with accurately linear characteristics and sharp cutoff filters, to eliminate all undesired mixing components.

Various other methods of multiplication have been suggested and a few have been tried. Some of the more recent attempts have made use of time modulation of pulses and sawtooth voltages, but most of these computers are capable of only one particular operation (say, multiplication), and always compromises have to be made among accuracy, range of operation, response time, and versatility. Any method based on a logarithmic device for its nonlinear element is inherently more versatile in the number of different operations of which it is capable. Furthermore, if the instrument is a continuous type of computer and has sufficiently fast response, it can be used as a nonlinear member in a more complicated computing system.

There have appeared a few logarithmic-type calculators. The first is an a-c amplifier whose steady-state transfer characteristic is made logarithmic by using amplifier tubes with variable grid spacing.[1] This instrument was made for the purpose of recording the logarithm of a function, rather than for use as a multiplier. Another instrument is a ratio meter employed in a Geiger-Muller counter. This device relies on the logarithmic characteristic of a diode with a retarding field at the cathode.[2],[3] Similar devices involving the logarithmic tubes have appeared in specialpurpose instruments.[4],[5] None of these developments were directed towards a versatile calculating device as the end result; the computer was in each case a part of another instrument toward which the major concern was directed.

The fact that a diode displays a logarithmic behavior at very low current levels is explained by the physics of electron emission from a hot cathode.[6],[7] At a temperature of absolute zero, no electrons possess sufficient energy to escape the potential barrier at the surface of the metal, known as the work function. Upon heating the cathode, some of the higher-energy electrons do gain escape energies, and the resulting current density is given by the "Dushman" or "Richardson" equation:

$$J = AT^2 \exp(-\frac{E_w}{E_t}) \ldots\ldots\ldots\ldots\ldots \quad (1)$$

amperes per square centimeter, where

A = a constant depending upon electron charge, electron mass, and Planck's constant,

T = degrees Kelvin,

E_w = work function of the metal,

and E_t = a constant depending upon Boltzman's constant and upon temperature.

If a retarding field is applied by controlling the plate voltage of the diode, less electrons actually reach the plate, and the net effect is simply to add to the work function of the cathode. Hence, the voltage-current relationship for a plane-parallel diode is

$$i = I_{th} \exp(\frac{1}{\alpha} e_d) \quad \ldots\ldots\ldots\ldots\ldots \quad (2)$$

in which I_{th} = the thermal emission current for zero plate voltage,

$\frac{1}{\alpha}$ = a constant depending upon absolute temperature and Boltzman's constant,

e_d = voltage from cathode to plate of the diode.

In the case of a cylindrical anode with a coaxial filament (the emitter), the exponential characteristic is only approximate, but the error is small for fairly large retarding plate potentials.

Rewriting Eq. (2) in the logarithmic form is more convenient for the problem of this paper:

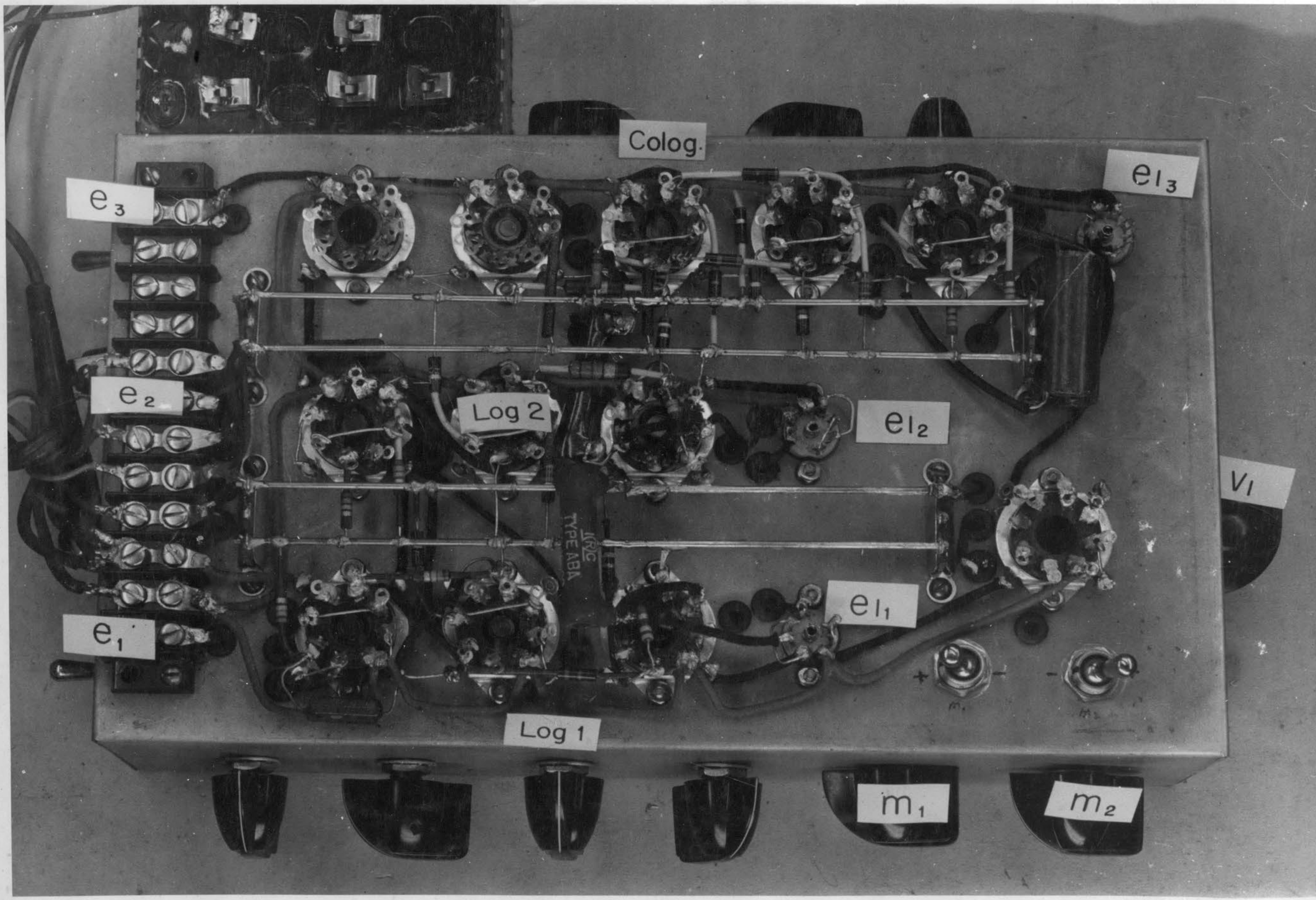
$$e_d = \text{const.} + \alpha \log i \quad \ldots\ldots\ldots\ldots \quad (3)$$

From this it may be seen that a current source connected across the diode will generate a voltage proportional to the logarithm of the current. The voltages from two such diodes may then be added to perform multiplication. Graph I in the Appendix shows the measured characteristic of a 6AL5 diode at negative plate voltages. A very important feature of this curve is that the slope (volts per decade of current) varies imperceptibly from tube to tube. As indicated by Eq. (2) this slope is a function of only the absolute temperature of the emitter and of Boltzman's constant. This is probably the most reliable characteristic of any commercial vacuum tube regardless of type.

The curve of Graph I indicates that multiplication should be possible over a range of more than three decades, which is in fact just what the complete multiplier was capable of. Graphs V and VI give the measured characteristic of the multiplier, adjusted to one volt as the unit. It is seen that the accuracy of multiplication is within 1.5 percent over a range of about 2,000 to 1.0, and over a range of only 100 to 1.0 the error is less than can be measured with an ordinary meter. (The meter used for these measurements was an RCA Voltohmyst which had been carefully calibrated and was accurate to less than one percent.)

Because the core of this computer is a logarithmic device, the computer may be used to generate power-law characteristics by amplifying or attenuating the logarithmic voltages before the cologarithm is generated. Graph VII gives the results of the computer adjusted to generate a square-low, half-power-law, and minus half-power-law output.

Figure I is a photograph of the complete multiplier. The tubes are all mounted upside down in the chassis in order to facilitate work on the circuits. All labels in the picture correspond to the equations and circuits mentioned in the following sections.


Colog.
e3
eI3
e2
Log 2
eI2
V1
IRC
TYPE ABA
e1
eI1
Log 1
m1
m2

II.

THEORY OF THE MULTIPLIER

Before a mathematical analysis of the logarithmic diode multiplier is given, a brief qualitative description of why the device works will be helpful. A functional block diagram of the computer is given in Fig. II. The output voltage e_3 is the product of the two input voltages e_1 and e_2.

First consider the logarithm generator represented by the d-c amplifier of gain $-g_1$, and the associated circuit components r_1 and m_1. Since the gain of the amplifier is negative, the whole system is a negative feedback circuit with the logarithmic diode as the feedback element. With sufficiently high gain in the amplifier, the voltage at its input (the junction of the diode plate and the resistor r_1) will depart very little from ground potential. If the amplifier has infinite input

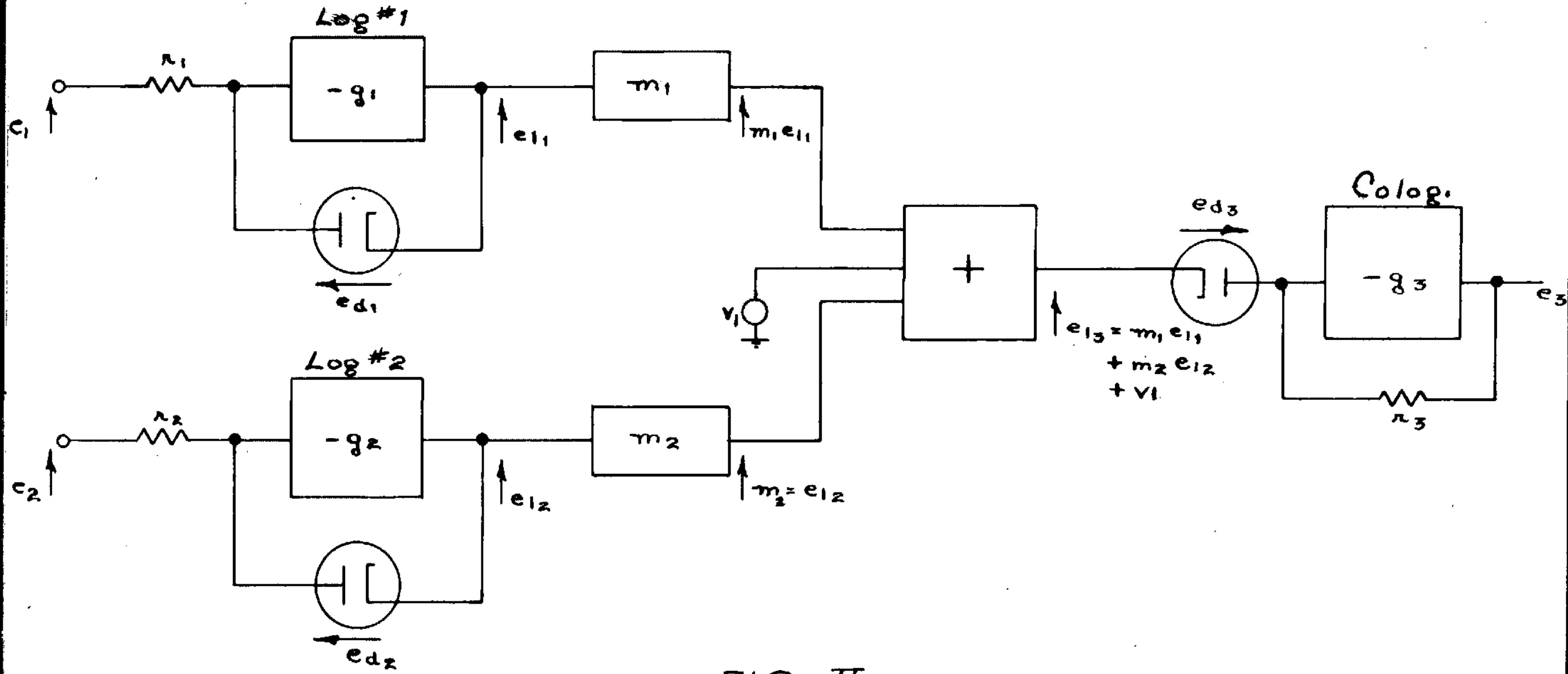


FIG. II

FUNCTIONAL BLOCK DIAGRAM OF MULTIPLIER

-12-

SCALE: ____ X ACTUAL
NOTE: ALL OTHER DECI. DIM. ± .005
ALL OTHER FRAC. DIM. ± 1/64
ALL OTHER RADII
ALL OTHER ANGLES

SIZE | MATERIAL
NUMBER REQUIRED

INSTRUMENT LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE, MASS.

DRAWN DATE FOR

impedance, the same current will flow through both the diode and the resistor r_1. Consequently, the voltage e_1 is a measure of the diode current. Likewise, the voltage $e_{\ell 1}$ is the negative of the diode retarding potential, since the diode plate is maintained at zero potential. The result is that the relationship between $e_{\ell 1}$ and e_1 is just the current-voltage function of the logarithmic diode.

Since the diode current covers a range of 1000 to 1.0, as indicated in Graph I, so will the input voltage vary by 1000 to 1.0. Just for the sake of convenience, it was decided that the input voltage range should be from 0.10 volts to 100 volts. This choice fixes the value of r_1 at 2.5 megohms. From Graph I it can be seen that as the input voltage varies from +0.10 to +100 volts, the logarithmic voltage $e_{\ell 1}$ will travel from about +0.82 to +0.22 volts.

In order to generate the cologarithmic function, the relative positions of the diode and the resistor are reversed, as shown in Fig. II. The input voltage $e_{\ell 3}$ is the potential difference across the diode, and the voltage e_3 tells the diode current.

In both the logarithmic and the ~~colo~~garithmic amplifiers,the error is zero for infinite gain, and can be calculated for a finite value of gain. The circuit analysis which follows is separated into two parts: first the gains of all three amplifiers are assumed infinite, and then the errors incurred by finite gains are examined.

For reasons which will be apparent later, it is convenient to change the form of Eq. (3) for the diode characteristic. Making the substitution

$$\frac{e}{r} = n\, i_u \quad \dots\dots\dots\dots\dots\dots \quad (4)$$

Eq. (3) becomes

$$e_d = +A + \alpha \log n \quad \dots\dots\dots \quad (5)$$

in which $A = (\text{const}) + \alpha \log i_u$,

i_u = the unit current for the multiplier (chosen arbitrarily)

and n = a dimensionless number which equals unity when the diode current equals i_u.

The dimension of A is voltage, and it equals the voltage e_d for n equal to unity. This can be read directly from the diode characteristic for any value of i_u. The sign of A is negative and the sign of α is positive.

From the block diagram of the multiplier, the following circuit equations are now written, making the assumption of infinite gain for each amplifier:

$$e_{\ell 1} = -e_{d1} = -A_1 - \alpha_1 \log n_1 \quad \ldots\ldots\ (6)$$

$$e_{\ell 2} = -e_{d2} = -A_2 - \alpha_2 \log n_2 \quad \ldots\ldots\ (7)$$

$$e_{\ell 3} = -e_{d3} = -A_3 - \alpha_3 \log n_3 \quad \ldots\ldots\ (8)$$

$$e_{\ell 3} = m_1 e_{\ell 1} + m_2 e_{\ell 2} + \mathcal{V}_\ell \quad \ldots\ldots\ldots\ (9)$$

Solution of these equations for the output number n_3 in terms of the input numbers n_1 and n_2 yields

$$n_3 = (n_1)^{m_1 \frac{\alpha_1}{\alpha_3}} (n_2)^{m_2 \frac{\alpha_2}{\alpha_3}} \times K \quad \ldots\ldots\ (10)$$

where $$K = \exp_{10}\left\{\frac{m_1 A_1 + m_2 A_2 - \mathcal{V}_\ell - A_3}{\alpha_3}\right\}.$$

If straight multiplication is desired, the exponents of n_1 and n_2 and the constant K must all be made equal to unity. These conditions are fulfilled if

$$m_1 \frac{\alpha_1}{\alpha_3} = m_2 \frac{\alpha_2}{\alpha_3} = 1 \quad \ldots\ldots\ldots\ldots\ (11)$$

$$\mathcal{V}_\ell = -A_3 + m_1 A_1 + m_2 A_2 \quad \ldots\ldots\ldots\ (12)$$

Regarding the conditions, two points are pertinent. The values of the α's (the slopes of the logarithmic diode characteristics) are all almost exactly equal; consequently m_1 and m_2 may be made close to unity. Also, the sign of η_L will be negative because all the A's are negative and approximately equal. When these conditions are met, Eq. (10) becomes simply

$$n_3 = n_1 \times n_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (13)$$

which are numbers proportional to the magnitudes of the output and input voltages.

The number scale corresponding to the voltages e_1, e_2, and e_3, depends upon the choice of the unit current, or in turn upon the unit voltage obtained from Eq. (4) by letting n equal unity:

$$e_u = r\ i_u \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

For example, if it is desired that the number "one" be the geometric center of the number range,

$$e_u = \sqrt{100 \times 0.10} = 3.16 \text{ volts,}$$

and the numbers consequently range from

$$n_{min} = \frac{0.1}{3.16} = 0.0316$$

$$n_{max} = \frac{100}{3.16} = 31.6 \quad .$$

When the choice of the number scale has been made, it is simply a matter of adjusting the voltage n_{ℓ} to the value given by Eq. (12). It is recalled that the values of the A's are determined by the choice of i_u according to Eq. (5) - that is, A is the diode voltage for i equal to i_u. Actually in operating the computer, one simply adjusts e_1 and e_2 to the unit voltage, and then adjusts the control for n_{ℓ} until e_3 also equals the unit voltage. No measurement of i_u or even of n_{ℓ} is necessary.

The limits on the range of number scales which are possible are set by the extremes of the output voltage range. If on one end the unit voltage were 0.1 volt, the corresponding number range would be from 1.0 to 1000. On the other end of the voltage range, a unit voltage of 100 volts would set the number range from 0.001 to 1.0. It is possible to set n_{ℓ} to go outside of this number range, but the full excursion of the output voltage could not then be achieved.

The next step is to determine the inaccuracy of the multiplier with amplifiers of finite gain. First to be considered is the logarithm generator, for which the circuit equations now become

$$e_\ell = -A - \alpha \log \left(\frac{e - \epsilon_o}{r}\right) + \epsilon_o \quad \ldots\ldots\ldots (15)$$

$$\epsilon_o = -\frac{e_\ell}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (16)$$

in which ϵ_o is the voltage at the input to the amplifier, or the error signal. Substituting Eq. (16) into Eq. (15) gives

$$e_\ell = \left\{ -A - \alpha \log \left(\frac{e + \frac{e_\ell}{g}}{r}\right) \right\} \times \left(\frac{g}{1+g}\right) \quad \ldots (17)$$

The factor $\frac{g}{1+g}$ is nearly unity and introduces zero error since control of both A and α are obtainable with r_ℓ and m. Inside the logarithm brackets, however, the appearance of $\frac{e_\ell}{g}$ does cause an error in the value of the diode current as indicated by the voltage e. This error is greatest when e equals +0.10 volts, at which point e_ℓ equals +0.82 volts. With an amplifier gain of 1,000, the error in diode current, as indicated by voltage e, becomes only 1.4%. For an input voltage e equal to 1.0 volts, the error reduces to 0.07%. It is this figure of gain which was chosen for the design of the logarithmic amplifier.

The situation for the cologarithm generator is more difficult because the amplifier output voltage reaches much higher values. Eq. (15) still applies to the network, but now

$$\varepsilon_o = \frac{e_3}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (18)$$

and substituting this into Eq. (15) yields

$$e_{\ell 3} = -A_3 - \alpha_3 \log \left(\frac{e_3(1-\frac{1}{g})}{r}\right) + \frac{e_3}{g} \quad \ldots \quad (19)$$

It is obvious that the term in the logarithm brackets can be completely compensated for by an adjustment of the voltage A_{ℓ}, since this error only multiplies the answer by a constant. However, the voltage $e_{\ell 3}$ now differs from the true voltage across the diode by the amount of the last term in Eq. (19). Stated differently,

$$e_{\ell 3} = -e_{d3} + \frac{e_3}{g} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (20)$$

This error is greatest when the output voltage is at its maximum value of 100 volts. Here e_{d3} equals 0.27 volts, and with an amplifier gain of 100,000 the error in e_{d3} becomes 0.001 volts, or 0.37%. The amplifier that was built for the cologarithm generator had an open-cycle gain of about 110,000.

The figures used in the above error calculations are for the particular diode whose characteristic is given in Graph I. Other samples of 6AL5 tubes vary from this characteristic only in voltage level, or the value of the constant A, and not in the slope of the characteristic α . However, the error computations based on the single tube are justified as to order of magnitude, and the errors thus calculated would not vary by more than a factor of 1.3 with different diodes.

There are two other possible sources of error. The first arises from a finite input resistance (or any input grid current) of the d-c amplifier, which would cause a difference between the diode current and that in the resistor. A second error would occur if the voltage e_{d3} did not come from a zero-impedance source; the current drawn by the ~~colo-~~ garithmic diode would cause a voltage drop through any source resistance, and the voltage e_{d3} would not equal the sum of the logarithmic voltages $e_{\ell 1}$, $e_{\ell 2}$, and $\bar{N}_{\ell}$. These two questions are matters of experimental technique, and will be discussed in the next section.

III.

DESIGN AND DEVELOPMENT OF THE MULTIPLIER

1) - Tests on Logarithmic Diodes

As was shown in the last section, the requirements on the d-c amplifiers are determined by the current-voltage characteristic of the logarithmic diodes. For this reason, many tests were carried out on different diodes to determine the most satisfactory and most reliable tubes and operating conditions. It is to be emphasized that this use of the diode is at current levels far below that of the intended purpose of the tube. The tube manufacturers guarantee nothing as to the performance at these levels, which meant that tests of drift, filament voltage effect, and variation from tube to tube had to be made.

The diodes which were examined were all new tubes, and a considerable time drift was observed in the plate voltage for a fixed current. In other words, the value of the constant A decreased continuously over a period of several days. After three days of continuous running, the tubes settled down to a steady level. This aging time seemed to be independent of heater voltage or tube current during aging. After the aging process was completed, some care still had to be taken to prevent any high-current operation of the diode, even for just a few minutes, or else the constant A would jump to a higher value and aging would have to be repeated (for about 12 hours). At all times the fidelity of the logarithmic response remained unchanged (the value of α was constant).

There seemed little choice as to heater voltage for the logarithmic diode. A lower voltage (4.5 volts) would decrease the voltage level for the whole characteristic, but variation with small heater-voltage changes, time required for aging, and the fidelity of the characteristic appeared equally satisfactory. The same was observed for a higher heater voltage (7.5 volts). In order to avoid cathod-to-heater hum in the logarithm generators, (where the diodes are operated with a high impedance between cathode and ground) a storage battery was used as the heater supply. Since the storage battery was necessary for the diode, it was available for the d-c amplifiers and was consequently used for reasons of convenience. This, of course, also simplified the hum problem for the amplifiers.

Three different tube types were tested as logarithmic diodes -- 6AL5, 6J6, and 6AK5. The 6AL5 and 6J6 were nearly equivalent in all respects, but the 6AK5 displayed a poor logarithmic character for currents greater than about three microamperes. Probably the actual range of the logarithmic characteristic for the 6AK5 is as great as for the other tubes, but it would have to be displaced to a much lower current level. The lower current operation would be less convenient for the d-c amplifier. The choice of the 6AL5 instead of the 6J6 was made for the sole reason that the 6AL5 has separate cathodes and therefore can be used as two separate diodes.

2) - <u>Logarithm Generator</u>

Fig. III is the circuit diagram of one of the two logarithm generators, including the addition circuit and the logarithmic voltage source nV_k. Since the two logarithm generators are identical, only one

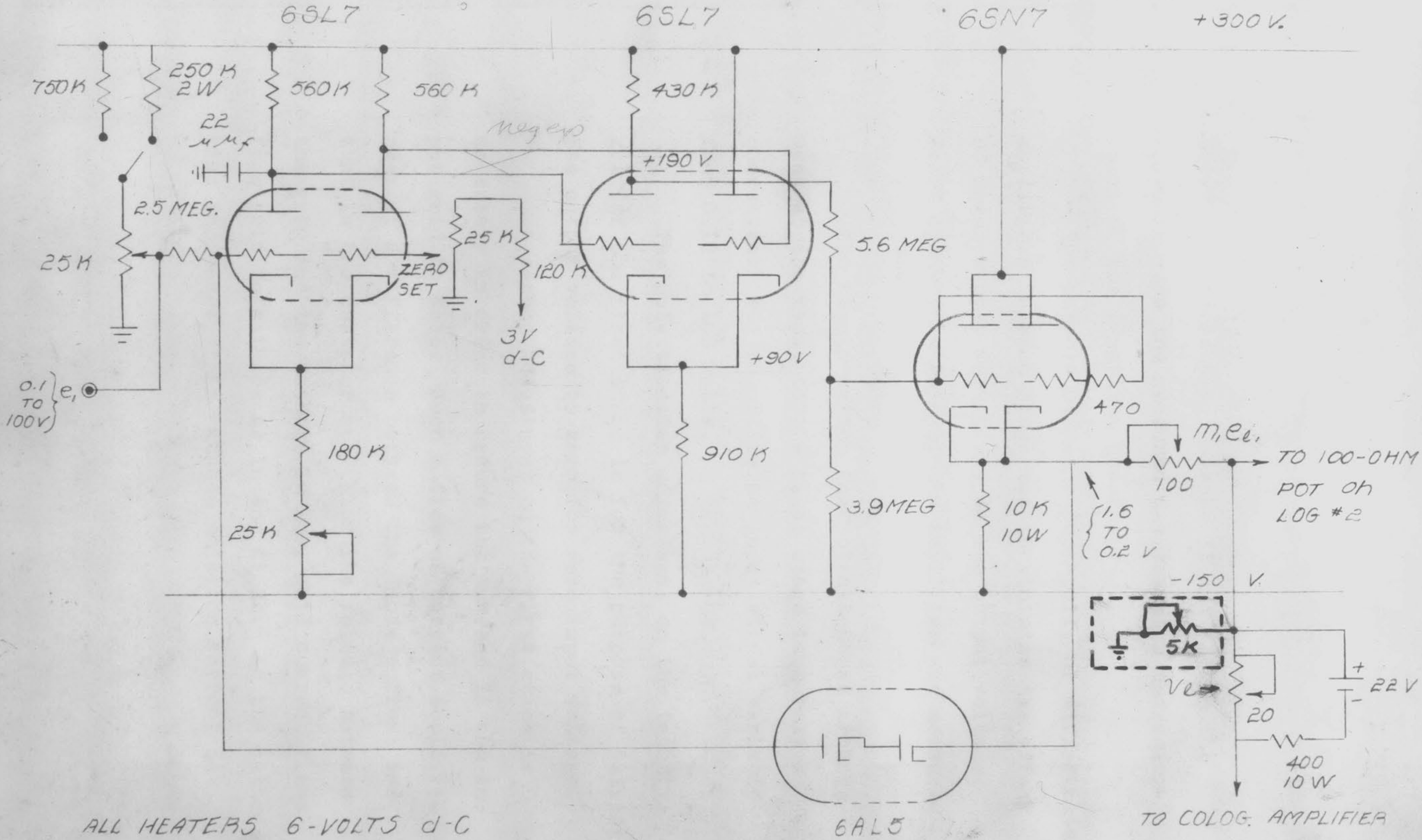


FIG. III

CIRCUIT DIAGRAM OF
LOGARITHMIC AMPLIFIER

diagram is necessary. Graphs II and III in the Appendix show the measured d-c transfer characteristic of the logarithm generators.

The purpose of using push-pull, or differential amplifiers in each stage was to minimize the effect of power supply variations on the output voltage. Total open-cycle gain of the amplifier was measured as 1250.

The toggle switch and potentiometer arrangement at the input to the first stage is merely a convenient way to obtain an input signal variable from 0.10 to 100 volts.

The potentiometer adjustment on the inactive grid of the first stage is for the purpose of setting the output voltage to zero for zero input voltage.

A second adjustment on the first stage is necessary in order to reduce the current in the input grid to zero. Such a flow of current would represent a difference between the diode current and that in the resistor r_1. When the voltage between the grid and the cathode is low, grid current flows from grid to cathode as in any diode. As the cathode bias increases, this cathode emission current decreases rapidly. Upon further increase in the cathode bias, the grid current goes through zero and begins to flow in the opposite direction. This reversed grid current is due to grid emission and to gas current. In order to adjust the first-stage cathode bias to the proper value for zero grid current, a portion of the cathode resistor is made adjustable, as is shown on the diagram. The procedure of adjustment (to be described presently) is sensitive to a flow of less than 10^{-4} microamperes, which is much smaller than the minimum value of logarithmic diode current.

The output cathode-follower must be operated at a sufficiently high current to supply the 200-ohm load of the addition circuit. For this reason, the two sections of a 6SN7 are tied in parallel; the 470 ohms between the grids is to suppress parasitic oscillations. By virtue of the negative feedback over the whole amplifier, the output impedance of the cathode follower is extremely low.

In the feedback link, two diodes are connected in series in order to double the output voltage of each logarithm generator. This is necessary because in adding the voltages $e_{\ell 1}$ and $e_{\ell 2}$ by the resistor circuit, each voltage is divided by two. In other words, the values of m_1 and m_2 in Eq. (10) have a simultaneous maximum of one-half. The only way to restore the exponents of n_1 and n_2 to unity is to double the ratios of $\frac{\alpha_1}{\alpha_3}$ and $\frac{\alpha_2}{\alpha_3}$. The series diodes effectively double the values of α_1 and α_2 .*

It was originally planned that the addition circuit would be an amplifier with adjustable gain in order to control the exponents in the product, but the requirements on stability against drift are too severe for such a d-c amplifier operating with the small signal voltages involved. With the two series diodes some control of the exponents is possible; if a variable power-law generator is required, the two series diodes allow control of one exponent over a range of ± 2.0. More diodes could be added in series in order to obtain a higher power-law transfer characteristic.

---

* There would be nothing to gain by putting two diodes in parallel in the cologarithmic circuit. This would only double the current for the same voltage, and consequently double the output voltage e_3.

An unusual sort of difficulty was encountered in stabilizing the amplifier against oscillation. Since the feedback diode changes its "impedance" very rapidly over the range of operation (from about 2.5 kilohms to about 80 megohms), the stability conditions change tremendously with input voltage. Oscillation would occur at one end of the range, and when corrected there, it would appear at the other end of the range. The simplest sort of stabilizing scheme proved most adequate, and consists of a 22 micromicro-farad condenser connected from the plate of the first tube to ground. With this arrangement the amplifier was perfectly stable at all levels. Confidence in the curative rather than merely palliative benefits of this technique was increased when the same device rendered the second logarithmic generator completely stable.

The procedure for setting the input current and the output voltage to zero was developed as follows. The first step is to disconnect the diode plate and set the input voltage e_1 to zero. Adjustment of the zero-set potentiometer (see Fig. III) is then made to reduce the output voltage to zero. Without the negative feedback, the output will probably drift up and down a few tenths of a volt, but this is small when referred to the amplifier input. Next the voltage at the input grid is measured with a high-impedance voltmeter. This voltage may be positive or negative by about 0.05 volts, depending upon the direction of grid current in the 2.5 megohm resistor. Reducing this grid voltage to zero by adjustment of the 25K potentiometer in the cathode will move the output voltage away from zero. A second adjustment of the zero-set control is then made to return the output to ground potential. These adjustments having been made, the change in output voltage caused by shorting the input grid to ground, indicates just what the input grid voltage is, and hence the value of the grid current flowing through the 2.5 megohm resistor r_1. By successively adjusting the zero-set control and the cathode resistor, the grid current can be reduced to practically zero. When adjusting the cathode potentiometer, the direction of the correction should be such as to change the output voltage in the same direction as it moves when the grid is grounded. It is possible to zero both the output voltage and the input grid current so that the output changes only 0.10 volts when the input grid is grounded. This corresponds to a grid current of

$$i_g = \frac{0.10}{1250} \times \frac{1}{2.5 \times 10^{-6}}$$

$$\approx 4 \times 10^{-5} \text{ microamperes,}$$

which is 0.1% of the minimum diode current. Actually the whole process of zero-setting requires much less time than is necessary to read this explanation of it. A satisfactory adjustment may be had in about one minute.

Addition of the voltage v_E to the sum of the two logarithmic voltages is made as shown in the diagram. It is a simple series addition with a source impedance of about 20 ohms.

3) - Cologarithm Generator

The circuit diagram of the cologarithm generator is shown in Figure IV, and the measured trans-

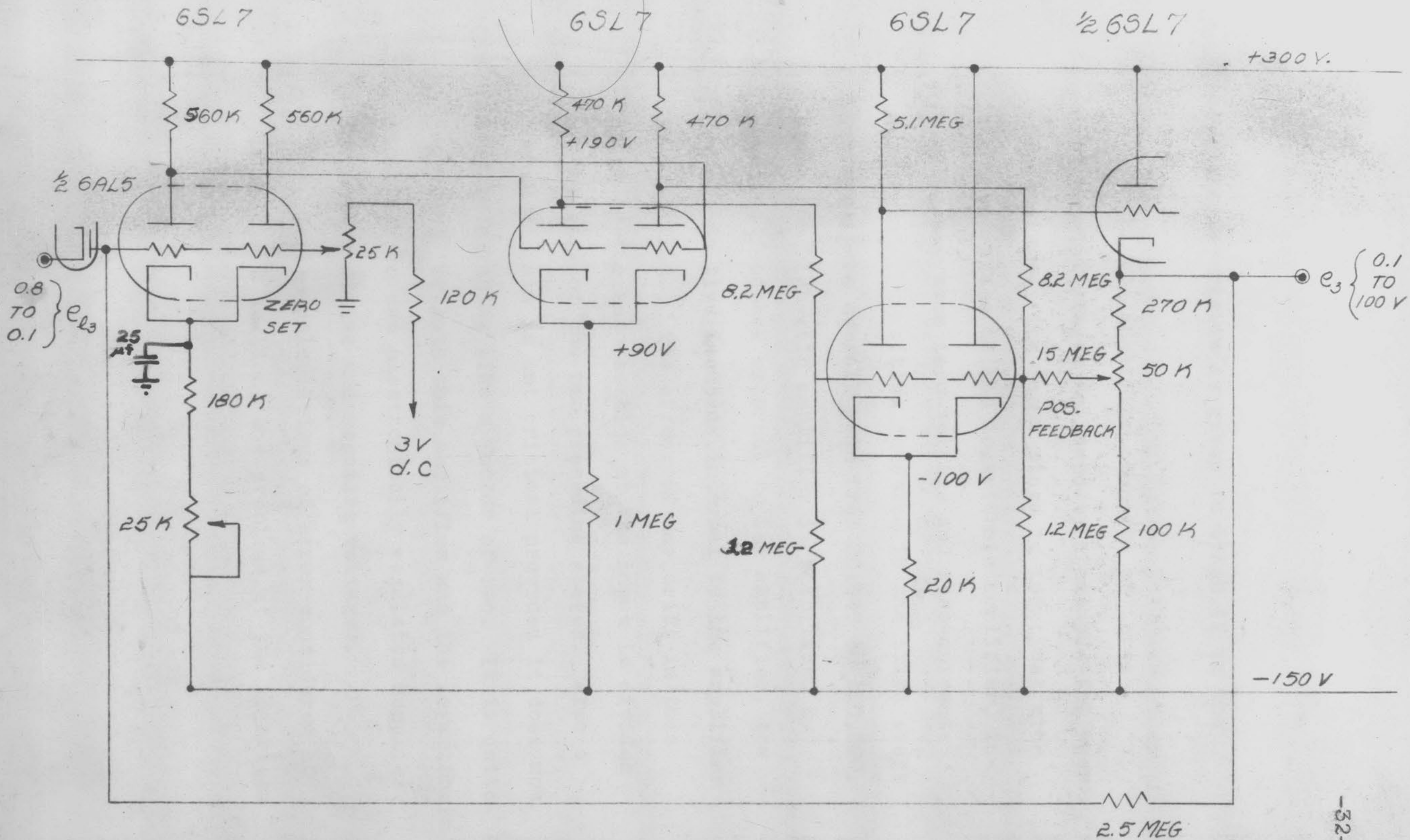


FIG. IV
COLOGARITHMIC AMPLIFIER

fer characteristic is given in Graph IV in the Appendix.

The d-c amplifier consists of three differential stages followed by a cathode-follower output stage. The input stage is identical with the input stage of the logarithmic amplifier, including the same zero-voltage and zero-current adjustments. Positive feedback in the third stage increases the overall gain from 20,000 to 110,000. Since the high-gain amplifier is stabilized by negative feedback external to the amplifier, the use of positive feedback internal to the amplifier is justified. The effect of any drift in this stage on the zero setting at the input is divided by the gain of the two preceding stages. Also a change of gain is not critical provided it does not drop by anything like a factor of two. It is quite important for both this amplifier and the logarithmic amplifier to use electronically regulated supplies for both positive and negative voltages.

In the calculations of error considered in the analysis section, the problems of the logarithm generator and the cologarithm generator were considered separately. It was assumed that no error would be involved in adding the two logarithmic voltages $e_{\ell 1}$ and $e_{\ell 2}$ to N_ℓ and applying the sum to the input of the cologarithm generator. If the addition circuit has appreciable impedance, then the cologarithmic diode current will cause a voltage drop in this source impedance. Consequently, it is necessary to calculate the error in the output e_3 due to an inaccuracy in the voltage $e_{\ell 3}$. Eq. (8) is repeated here for convenience:

$$e_{\ell 3} = -A_3 - \alpha_3 \log n_3 \quad \ldots\ldots\ldots \quad (8)$$

Solving the equation for n_3 in terms of $e_{\ell 3}$ yields

$$n_3 = C \exp\left\{ - \frac{e_{\ell 3}}{\alpha_3} \right\} \quad \ldots\ldots\ldots \quad (20)$$

C = a constant,

from which the percentage error in n_3 for small errors $e_{\ell 3}$ is obtained:

$$\frac{dn_3}{n_3} = - \frac{C}{\alpha_3} de_{\ell 3} \quad \ldots\ldots\ldots\ldots\ldots (21)$$

This equation states that the percentage error in n_3 is linearly related to the voltage error in $e_{\ell 3}$.

The constant can be determined from the cologarithm generator characteristic given in Graph IV. An error of 0.01 volts in $e_{\ell 3}$ corresponds to an error of 10% in n_3.

Now it is apparent how severe the drift and hum requirements would be on a d-c amplifier to perform the functions of m_1 and m_2. Actually a single-stage amplifier was tried for this purpose, but the resulting output of the cologarithm generator drifted far too much and too rapidly.

The greatest error in $e_{\ell 3}$ caused by the voltage drop in the source impedance of the addition circuit occurs when e_3 and consequently i_3 are maximum. At this point, i_3 equals 40 microamperes, which produces a voltage drop through the source resistance (about 60 ohms) of

$$40 \times 10^{-6} \times 60 = 2.4 \times 10^{-3} \text{ volts.}$$

According to the analysis given above, this error in the voltage $e_{\ell 3}$ corresponds to an error of 2.4% in e_3. Of course this error decreases rapidly as the voltage e_3 decreases. When e_3 equals ten volts, this error is 0.24%.

The adjustment procedure for the cologarithm generator is somewhat different ~~than~~ from that for the logarithm generator, although the end result is still a zero-output-voltage and zero-input-current condition. The first step is to disconnect the positive feedback, ground the input grid, and adjust the output voltage to zero by means of the zero-set potentiometer. Before executing the next step, the voltage $e_{\ell 3}$ should be turned up to about +1.2 volts, or more, by adjustment of either r_ℓ or the logarithmic generator inputs. This will effectively turn the cologarithmic diode off. The grounding short on the input grid is next removed, and the output voltage e_3 will assume a value about ± 0.2 volts. This output voltage is then adjusted to exactly zero by turning the potentiometer in the cathode of the input circuit, not by resetting the zero-set control. Next the voltage at the input grid is measured and set to zero by adjusting the zero-set control, and not by adjustment of the cathode resistor. Note that this is opposite to the technique for zero-setting the logarithmic amplifier. Two or three successive adjustments of the zero-set control and the cathode resistor will bring both e_3 and the input grid to zero volts. It is important to have a high-impedance, vacuum-tube voltmeter with at least a 3-inch 3-volt scale to perform these adjustments.

Finally the potentiometer in the cathode of the output stage has to be adjusted to give the same voltage as that of the third stage grid, and the positive feedback resistor is then connected. This may upset the zero setting slightly, but one adjustment of the zero-set control and the input-stage cathode resistor control should return the amplifier to the proper quiescent condition described above. The reason why the positive feedback cannot be connected from the start is that with no external negative feedback (i.e., when the input grid is grounded) the amplifier tends to act like a d-c multivibrator, or "flip-flop" circuit.

4) - Multiplier Operation

The results of straight multiplication are given in Graphs V and VI in the Appendix. The control of N_k was set to make the unit voltage one volt. The process by which this is done is to set

$$e_1 = e_2 = 1 \text{ volt},$$

and adjust N_k until e_3 also equals one volt. The actual value of N_k is not important. Some adjustment of m_1 and m_2 was found necessary in order to make the exponents exactly unity. The addition circuit used is rather awkward because adjustment of each potentiometer affects both m_1 and m_2 and successive adjustments of all three controls (the two 100-ohm and one 5,000-ohm potentiometers) are necessary. Once adjusted, however, these controls need not be touched as long as straight multiplication is desired.

What might be a very important feature of this multiplier is the possibility of generating power-law functions, as well as simply multiplying. This was achieved with an accuracy at least as great as the meter used, and probably even greater. The process involves only one logarithm generator (with its 100-ohm potentiometer connected between output and ground) and one cologarithm generator. With only two diodes in series in the feedback link of the logarithm generator, the highest power obtainable is square-law. The inclusion of more diodes would permit higher powers, but the amplifier gain would have to be increased in order to maintain the same accuracy. Graph VII in the Appendix gives the measured characteristic of the multiplier used as a power-law generator for the functions:

$$e_3 = (e_2)^2$$

$$e_3 = (e_2)^{1/2}$$

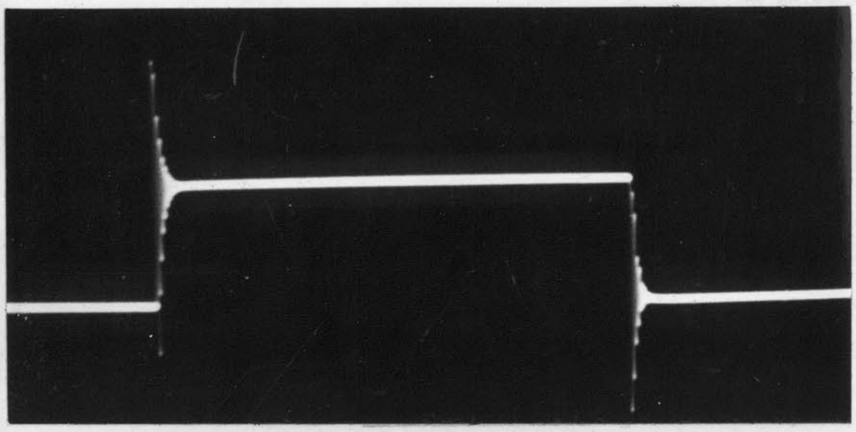
$$e_3 = \frac{1}{\sqrt{e_2}}$$

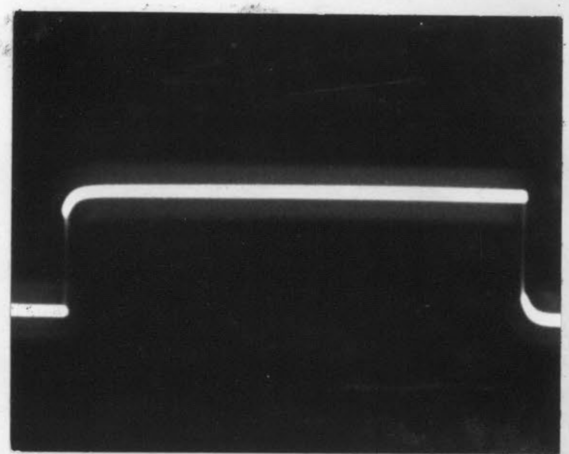
In order to obtain the negative exponent, the connections to the feedback diodes (in the logarithm generator) were reversed, and the input voltage was made negative. The purpose of the negative input signal was to maintain a negative feedback system. Actually it is not necessary to use a negative input signal; the amplifier gain may be reversed in sign by switching the leads between the plates of the first stage and the grids of the second stage. If a very flexible computer were desired, both the diode reversing and amplifier phase reversal could be accomplished by a single switch.

The generation of negative exponents indicates the practicality of using the multiplier to perform division; this, however, has not yet been tried.

It was stated earlier in this report that one of the objectives of this computer problem was to perform computations much faster than is possible with a mechanical calculator. Consequently, a study of the transient response of the device was made by injecting a square wave into one of the inputs and photographing the response as viewed on a cathode-ray-oscilloscope. These photographs are given in Figure V.

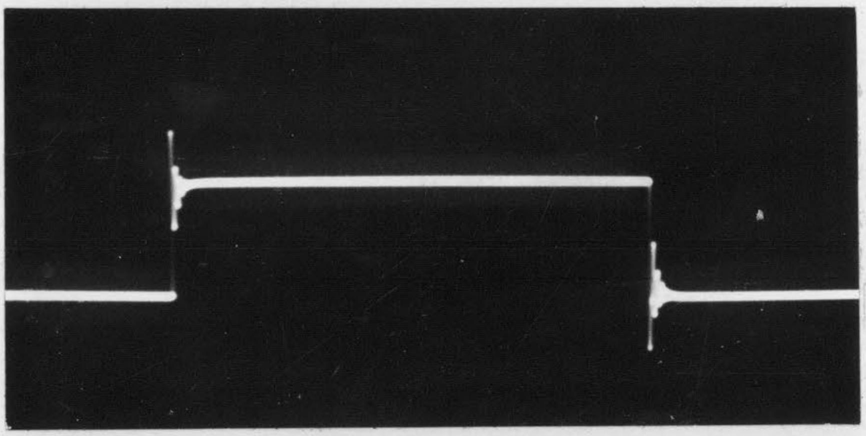
The square wave used was at a fundamental frequency of 150 cycles-per-second, and was of





Top: - Input Signal

Top-Left: - $e_1 = 2.0$ v.



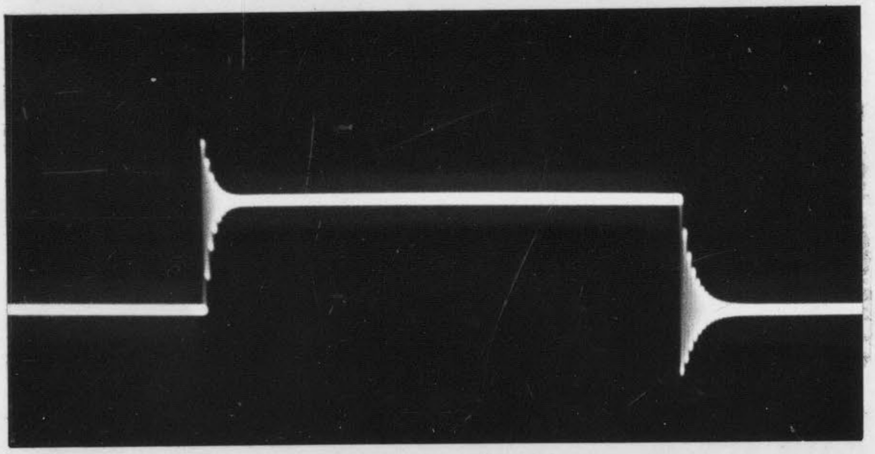
$e_1 = 5.0$ v.

Fig. V

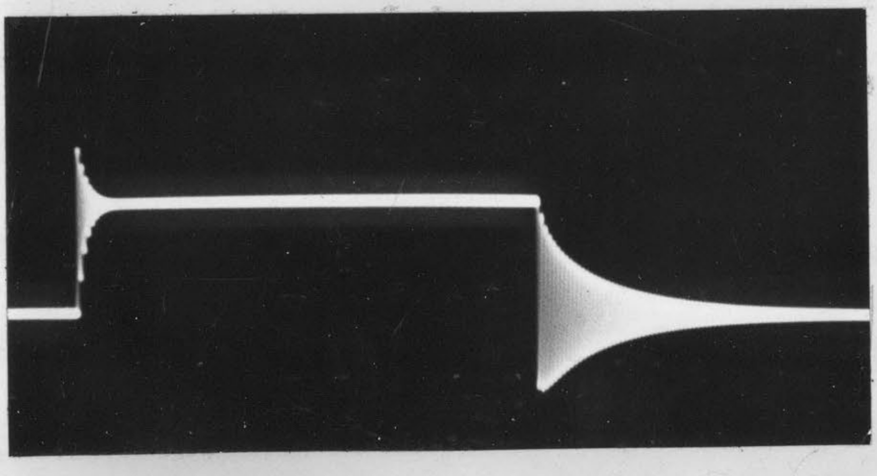
Oscillograph pictures of the output voltage e_3, for

$$e_2 = \begin{cases} 1.25 \text{ v.} & t < 0 \\ 1.75 \text{ v.} & t \geq 0, \end{cases}$$

and e_1 as indicated.



$e_1 = 10$ v.



$e_1 = 15$ v.

amplitude ± 0.25 volts. This signal was superposed on a d-c voltage of 1.5 volts, so the input signal e_2 moved from 1.25 to 1.75 volts. Of course the oscilloscope amplifier removed the component of d-c voltage in the output wave. Without changing the square-wave input, the other input signal at e_1 was set at 2.0, 5.0, 10, and 15 volts, and pictures of the output were taken. By no means are these results presented as a complete transient study of the multiplier; it would be necessary to take a great many pictures for all different input conditions to cover the field of possibilities. Such is the price of a highly nonlinear device.

Attention is called to some very interesting facts derived from the transient response pictures. It is immediately apparent that the system is nonlinear from several observations: In the first place the response at the end of the pulse differs from that at the beginning, even though the front-edge transients have completely died out. Secondly, the response is very much dependent upon the amplitude of the inputs. With input e_1 turned up higher than

15 volts, the duration of the overshoots at the tail end of the pulse becomes very long -- too long to permit usable reproduction of a 150-cycle square wave.

The only attempt at explaining or interpreting the response is made in terms of the change in the "impedance" of the diodes at different levels. The higher-voltage signals drive more current through the diodes, and consequently, there are moments when the diodes have lower "instantaneous resistance". In the logarithmic amplifiers, the greater negative feedback at these instants would tend to increase these overshoots. Consequently, it is at least reasonable that the transient response follows the pattern indicated in the pictures.

IV.

RECOMMENDATIONS AND CONCLUSIONS

There are a number of questions concerning the computer which still must be answered before the degree of practicality of the method can be completely decided. Before the data were taken for the curves given in the Appendix, all amplifiers were carefully zeroed, and during the tests care was taken to assure no change in the heater voltage. The use of conventional regulated power supplies for the positive 300-volt source and also for the negative 150-volt supply was certainly necessary, but probably adequate. Without having made conclusive tests on the dependence of the device upon heater-voltage variation, it is the opinion of the author that this supply should also be regulated.

It was found that over a period of one day, the d-c amplifier had to be reset for zero output voltage and zero input grid current, but the amount of the readjustment was not great. Possibly with properly aged tubes (those used in the circuits were new), such readjustments would be less frequent.

If it is desired to have a very flexible computer, capable of many different types of operations, it would be worthwhile to spend some time designing a stable, drift-free amplifier to perform the functions of m_1 and m_2. Possibly the same result could be obtained by using as many as four diodes in series in the logarithm generators, with a less awkward resistance network for controlling m_1 and m_2. In such a computer it would be useful to have a switch to reverse the diodes and reverse the phase of the logarithmic amplifier in order to include negative exponents.

Further work should be spent on the study and improvement of the transient response of the computer.

One very useful modification of this computer would be an arrangement by which inputs of both signs could be handled. Figure VI shows the way in which

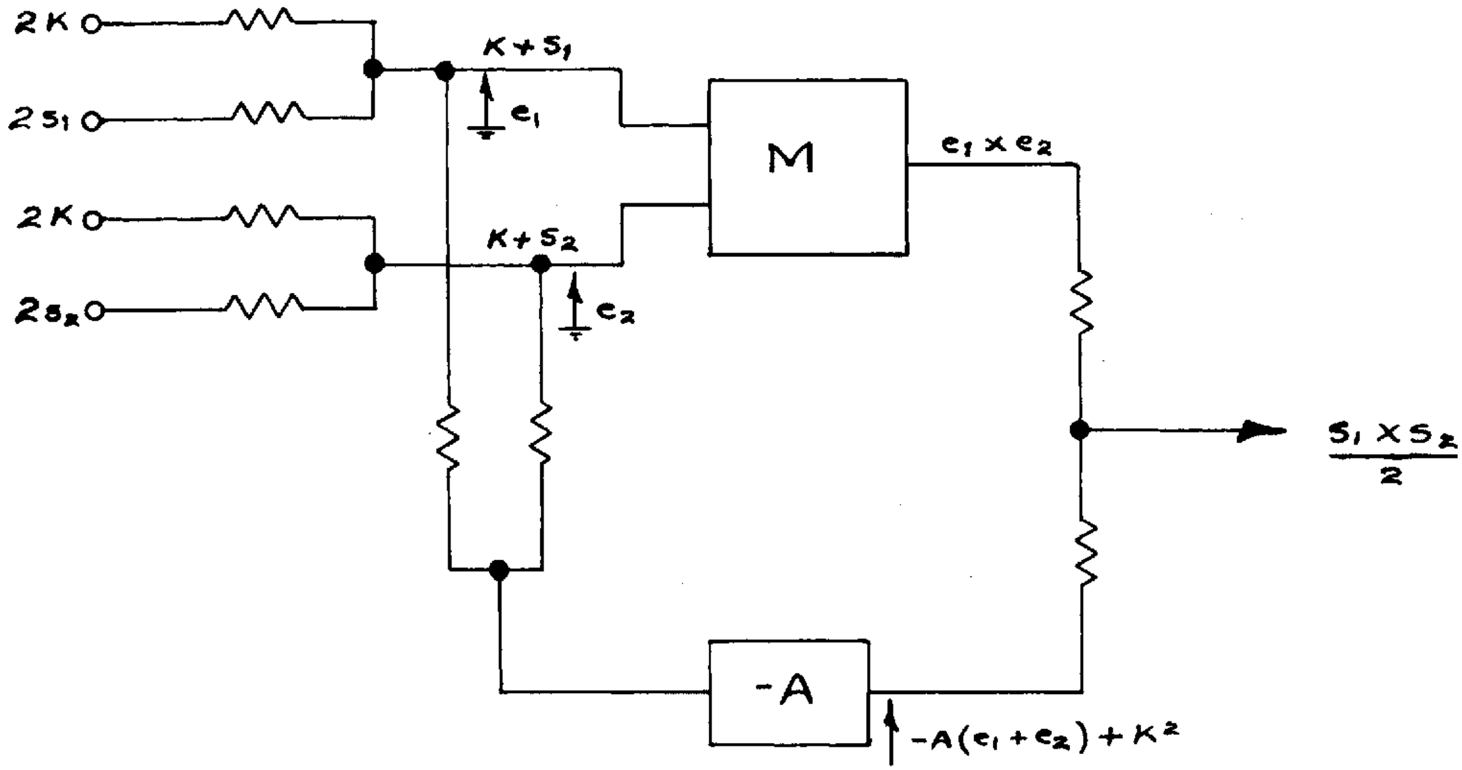


FIG. VI

WAVEFORM MULTIPLICATION

$$e_1 \times e_2 = S_1 S_2 + K(e_1+e_2) - K^2$$

$$A = K$$

SCALE: _____ X ACTUAL

NOTE: ALL OTHER DECI. DIM. ± .005
ALL OTHER FRAC. DIM. ± 1/64
ALL OTHER RADII
ALL OTHER ANGLES

DATE	CHANGE	BY	APP'D	SIZE / NUMBER REQUIRED / FINISH	MATERIAL

INSTRUMENT LABORATORY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
CAMBRIDGE. MASS.

DRAWN DATE FOR
CHECKED DATE APPROVED DATE
DRAWING NUMBER

this might be accomplished. If a constant voltage K were added to the two waveforms to be multiplied (s_1 and s_2) such that the total voltage at each input was always positive, the output of the multiplier would be

$$e_1 \times e_2 = (K+s_1)(K+s_2) = s_1 s_2 + K(e_1+e_2)-K^2 \quad .. \ (22)$$

The first term in the right-hand member is the desired product, and by the method shown in the diagram, the remaining terms could be subtracted out. Because the input signals now represent only a portion of the total input voltage, the range of numbers is greatly reduced. The best that can be done in this respect is to fix N_l such that the unit voltage equals the minimum value within the voltage range, or 0.10 volts. Here the number range of the multiplier itself is from 1 to 1,000. Since the greatest input signals will be equal to 2K, the maximum multiplier output will be $4K^2$. Setting

$$4K^2 = 1000$$

gives $$K = 15.8,$$

which is consequently the maximum value of each s_1 and s_2. This assumes that s_1 and s_2 will reach negative values of no greater magnitude than their positive maxima.

Assuming that one has a rather versatile computer of this sort, there are some very interesting possibilities for the device. Figure VII indicates a method by which power-law generators might be used to solve a polynomial for its roots. The network equations for this circuit are:

$$e = F(e_o)^f \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

$$e_o = Ae^a + Be^b + Ce^c + \ldots, \quad \ldots\ldots \quad (24)$$

in which A,B,C,F are constants (obtained by adjusting N_L)

and a,b,c,f are the exponents of the power-law generators.

By letting $D = -F^{-\frac{1}{f}}$ and $d = 1/f$,

the solution of the system yields

$$Ae^a + Be^b + Ce^c + De^d + \ldots = 0 \quad ..(25)$$

Consequently, the voltage e will assume the value

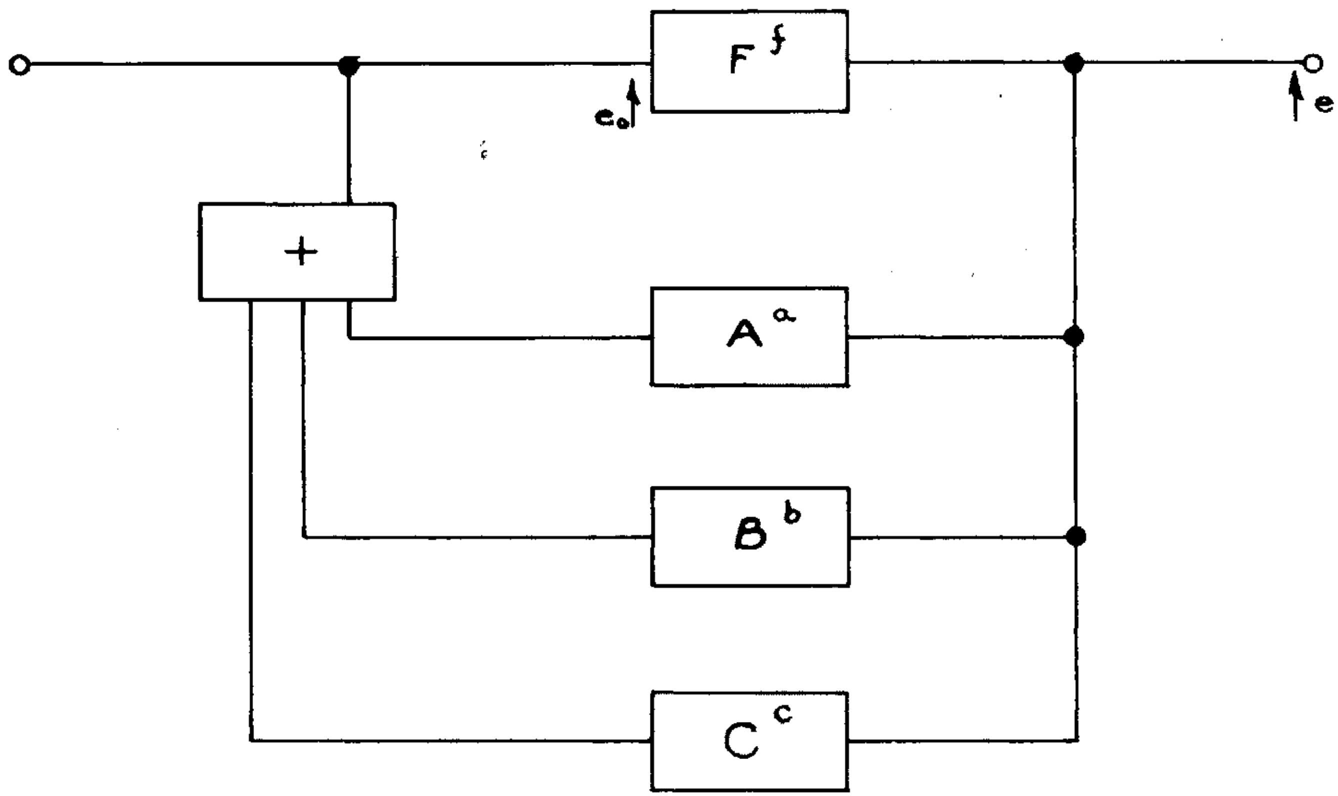


FIG. VII

USE OF MULTIPLIERS FOR THE SOLUTION OF A POLYNOMIAL FOR ITS REAL ROOTS.

SCALE: ____ X ACTUAL

NOTE: ALL OTHER DECI. DIM. ± .005

ALL OTHER FRAC. DIM. ± 1/64

ALL OTHER RADII

ALL OTHER ANGLES

INSTRUMENT LABORATORY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

CAMBRIDGE, MASS.

ATE	CHANGE	BY	APP'D	SIZE	MATERIAL
				NUMBER REQUIRED	
				FINISH	

DRAWN DATE FOR

CHECKED DATE APPROVED DATE

of one of the roots, and will be stable only at these voltages. Some means of forcing e from one stable root to another would be necessary in order to obtain all the roots. One caution to be recognized here is that this analysis is based solely on a sort of "amplitude" analysis, and the possibility of the whole system oscillating due to "phase shifts"* is probably quite high, in view of the observed transient response of the multiplier. There is little doubt however, that the circuits could be loaded with capacity in order to slow down the response and stabilize the system.

Another possibility for the power-law generator is in matching power-series representations of empirical functions. In other words, a test instrument could be devised to generate, or indicate, the terms of a power series, similar to the role of a harmonic analyzer or wave analyzer in determining the components of a Fourier series.

---

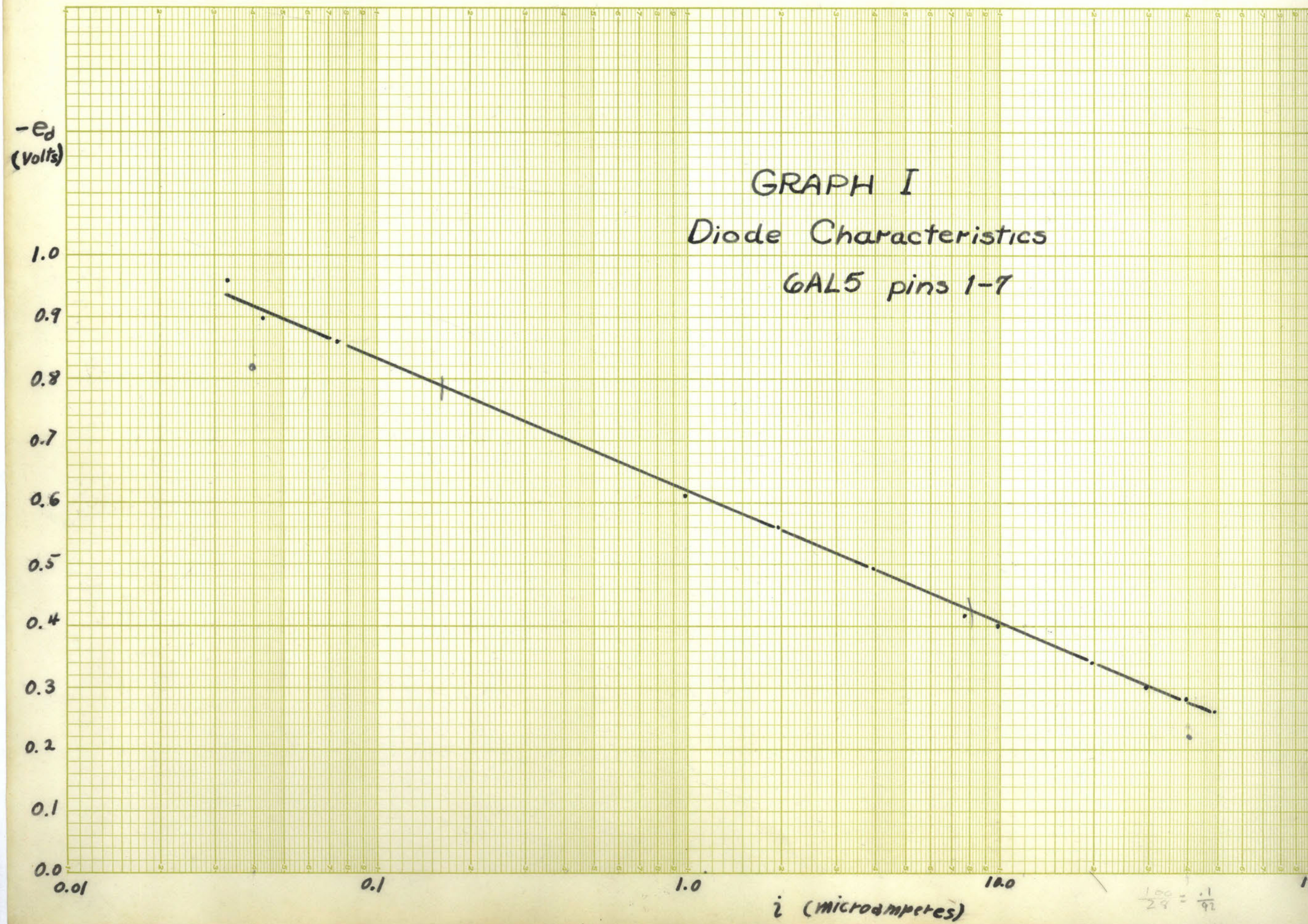
* The terms "amplitude" and "phase" are not correctly used here, since the system is non-linear. Words such as "magnitude" and "lag" with reference to step voltages might be better.

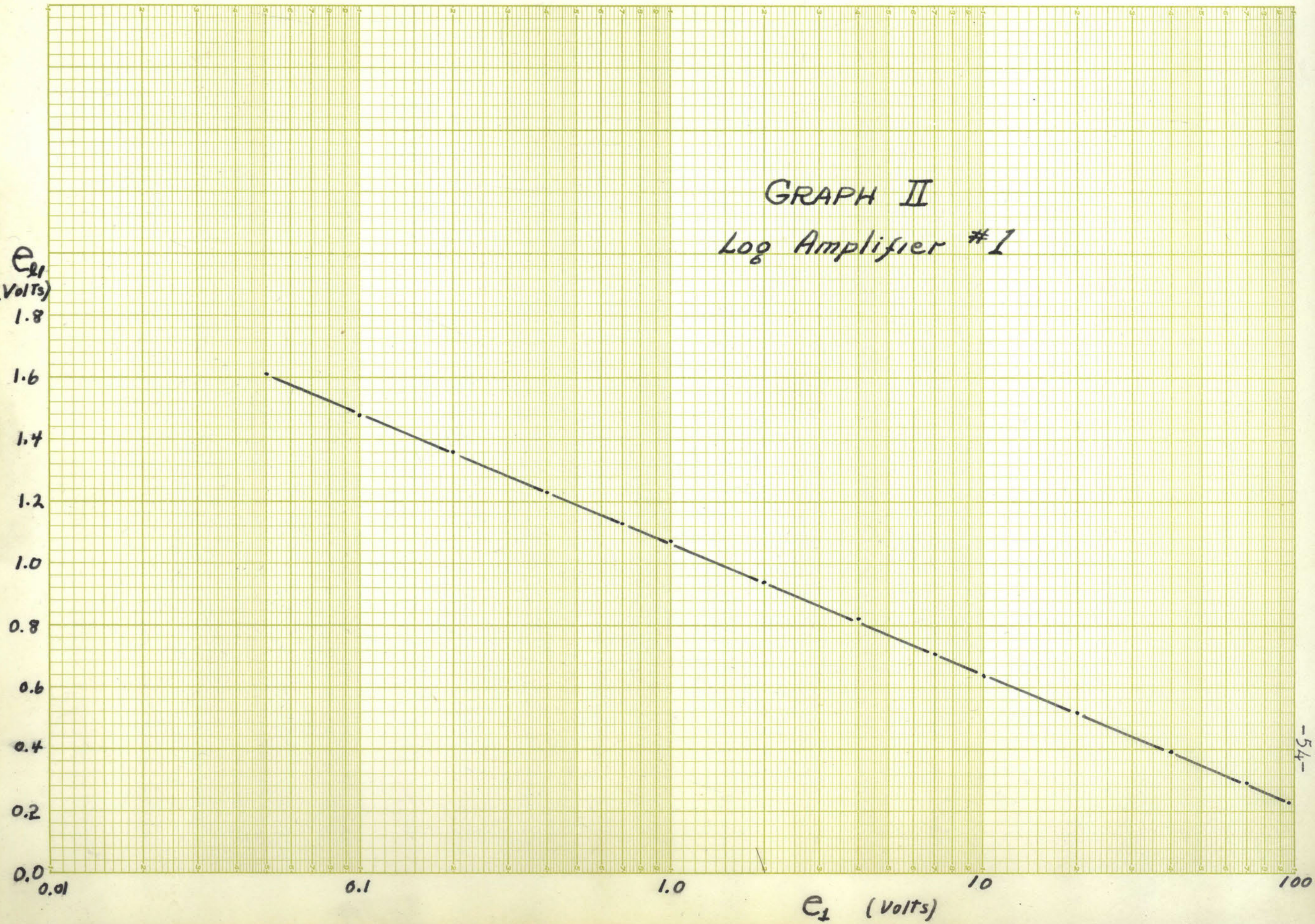
Probably the most valuable use of this computer would be as the nonlinear element in an all-electronic differential analyzer, or any electrical-analogue device. An accurately controllable nonlinear element, such as is possible with this sort of device, could play an extremely useful role in a larger calculating system including differentiators and integrators. Of course, the speed of the transients in the larger computer would have to be slow compared with the maximum response time of any of its elementary computers, including the multiplier (or power-low generators).

V.

APPENDIX

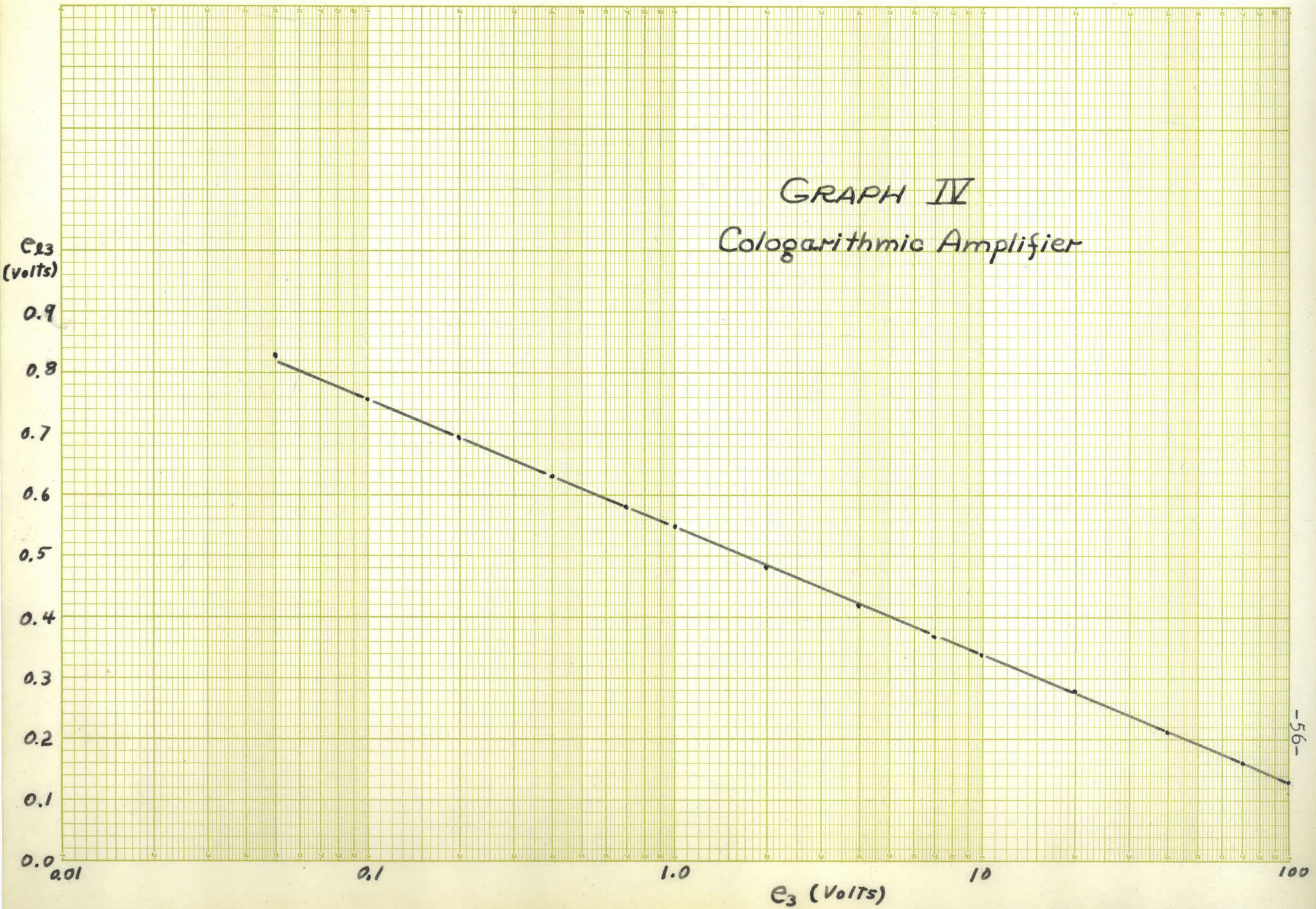
The following graphs are plots of the data taken on the computer described in this report. All of these data, including all laboratory notes taken during the development of the computer, are recorded in the M.I.T. Instrumentation Laboratory notebook which is under the name of the author.

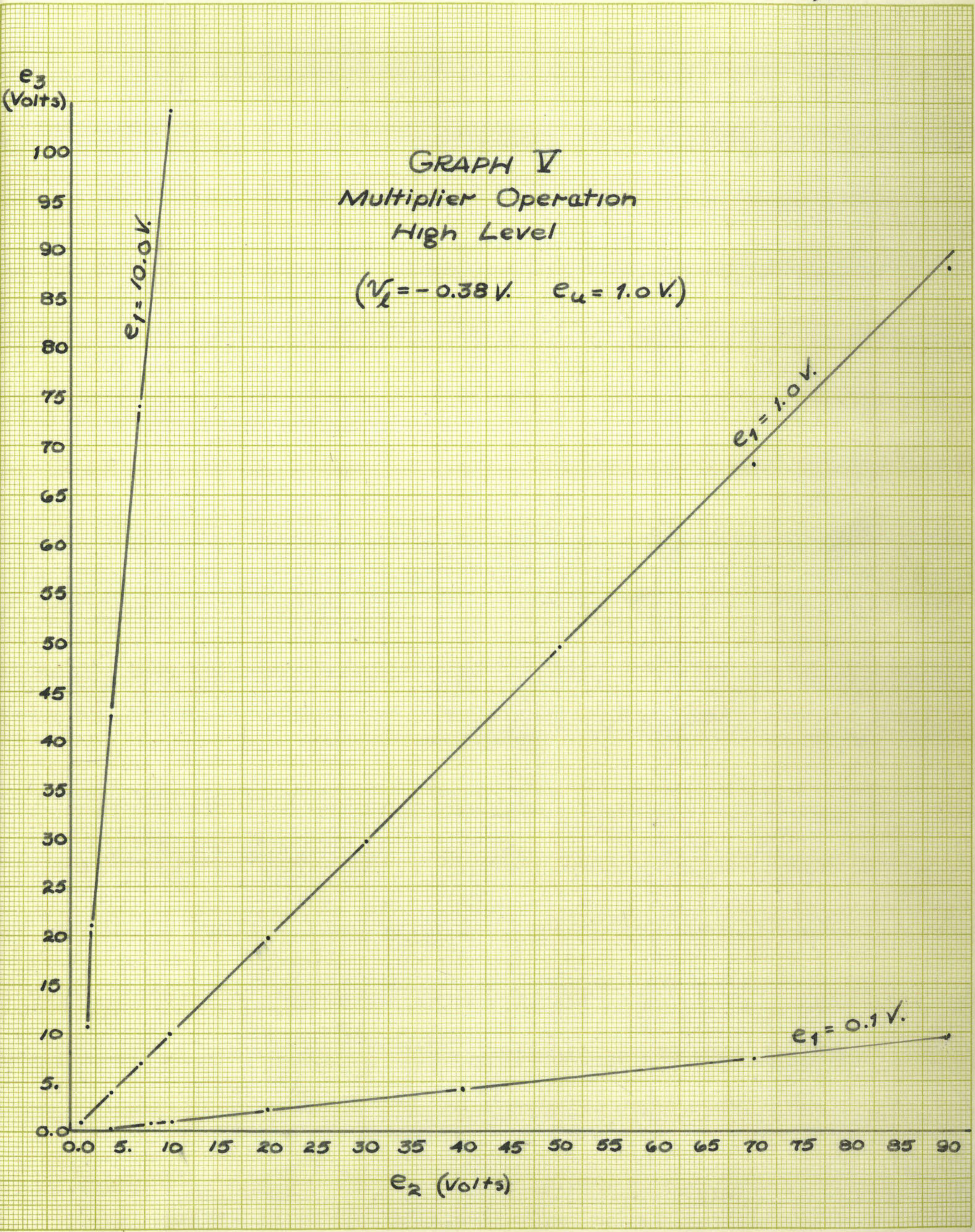

GRAPH I
Diode Characteristics
6AL5 pins 1-7
$-e_d$ (volts)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.01
0.1
1.0
10.0
100
i (microamperes)

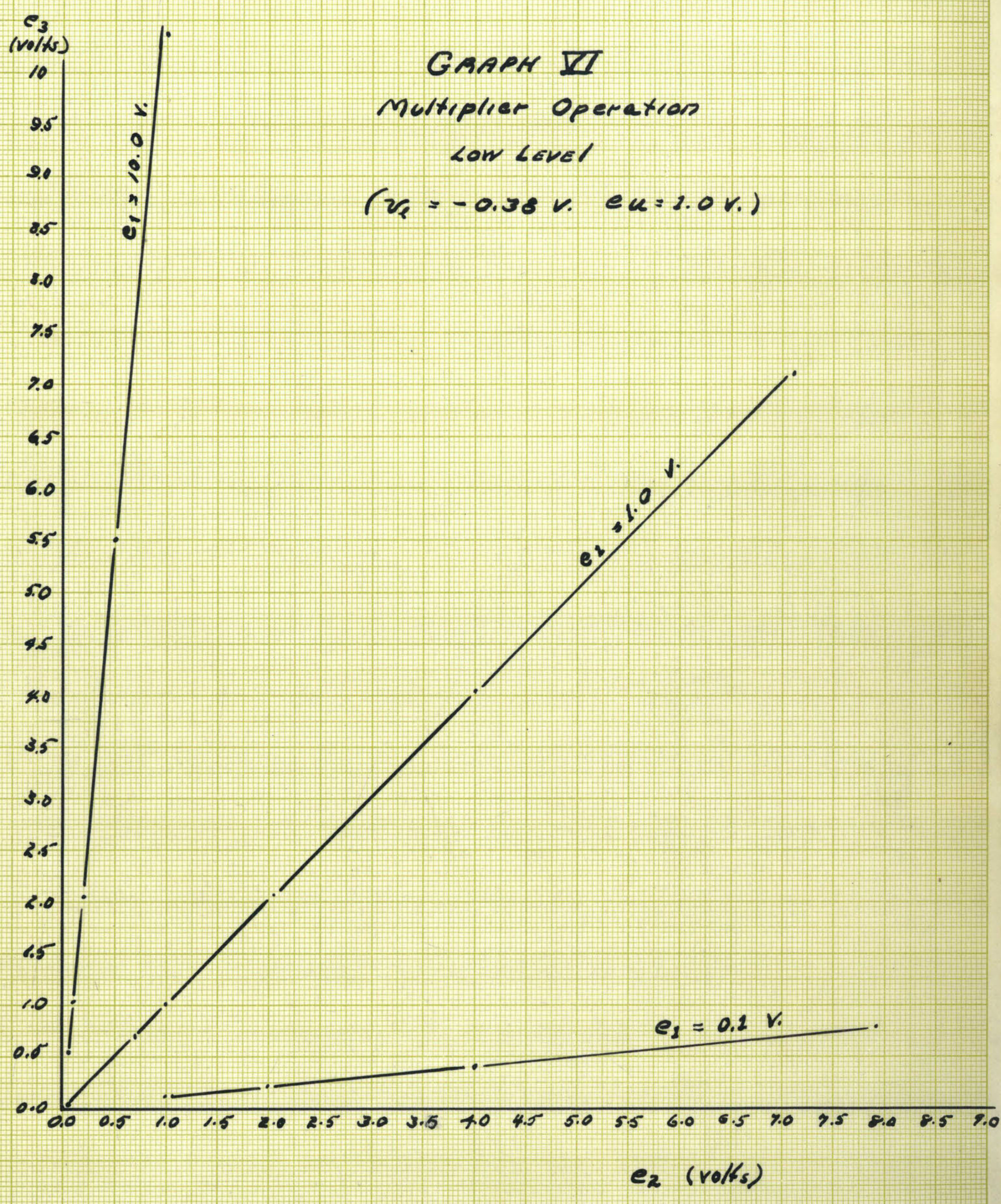


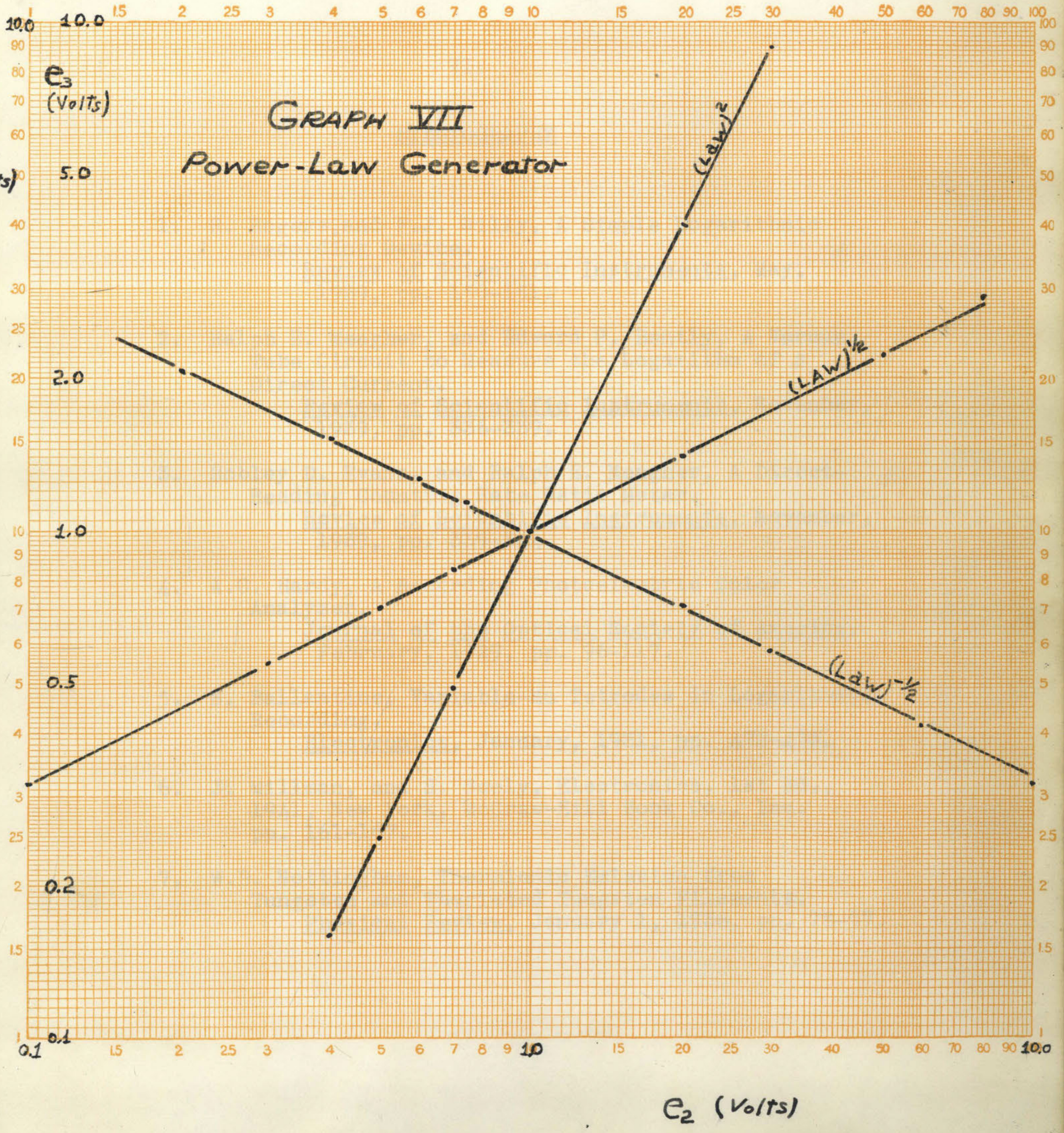
GRAPH II
Log Amplifier #1
e_{21} (Volts)
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.1
1.0
10
100
e_1 (volts)


GRAPH III
Log Amplifier #2
e_{l2} (volts)
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.01
0.1
1.0
10
100
e_2 (volts)


GRAPH IV
Cologarithmic Amplifier
e_{23} (volts)
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.01
0.1
1.0
10
100
e_3 (VOLTS)


GRAPH V
Multiplier Operation
High Level
(V_l = - 0.38 V. e_u = 1.0 V.)
e_3 (Volts)
e_1 = 10.0 V.
e_1 = 1.0 V.
e_1 = 0.1 V.
0.0
5.
10
15
20
25
30
35
40
45
50
55
60
65
70
75
80
85
90
95
100
e_2 (Volts)


GRAPH VI
Multiplier Operation
Low Level
(v_c = -0.38 V. e_u = 1.0 V.)
e_3 (volts)
e_1 = 10.0 V.
e_1 = 1.0 V.
e_1 = 0.1 V.
e_2 (volts)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10


GRAPH VII
Power-Law Generator
e_3 (Volts)
e_2 (Volts)
$(Law)^2$
$(LAW)^{1/2}$
$(Law)^{-1/2}$
10.0
5.0
2.0
1.0
0.5
0.2
0.1
10.0

VI.

BIBLIOGRAPHY

1. R. Rogers,and F.J. Willy, A Simple Logarithmic Recording Device.
Review of Scientific Instruments, May, 1939, pp. 150-151.

2. Ralph E. Meagher, and Edward P. Bently, A Vacuum Tube Circuit to Measure the Logarithm of a Direct Current.
Review of Scientific Instruments, November, 1939, pp. 336-339.

3. Robley D. Evans, and Ralph E. Meagher, A Direct-Reading Counting Rate Ratio Meter.
Review of Scientific Instruments, November, 1939, pp. 339-344.

4. A.C. Hardy, A Recording Photoelectric Color Analyzer.
Journal of the Optical Society of America, February, 1929, pp. 96-117.

5. S. Ballantine, Variable-Mu Tetrodes in Logarithmic Recording.
Electronics, January, 1931, pp. 472-473.

6. J. Millman, and S. Seely, Electronics, 1st Ed., 1941, New York, McGraw-Hill Book Co., Inc. pp. 161-167.

7. W.B. Nottingham, Thermionic Emission from Tungsten and Thoriated Tungsten Filaments.
Physical Review, January 1, 1936, pp. 78-97.