A Power Electronic Approach to Improved Dual-Frequency Vibration Energy Harvesting

by

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Submitted to the Department of Electrical Engineering and Computer Science
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Abstract

Vibration energy harvesters may be used as power sources for low-power, self-sufficient, wireless industrial sensors. State-of-the-art vibration energy harvesting uses switching power electronics to synthesize compact conjugate matched loads allowing for maximum harvested power. Previous work demonstrating dual-frequency vibration energy harvesting used a piezoelectric harvester loaded with analog-controlled power electronics but was unable to cancel the parasitic output capacitance typical of piezoelectric harvesters at both frequencies. This thesis addresses the technical challenge of achieving maximum power transfer from multi-frequency vibration energy sources, simultaneously. Improved dual-frequency energy harvesting is demonstrated using a piezoelectric vibration energy harvester loaded with digitally-controlled power electronics. The digital controller - performing fixed-point computations - allows for synthesis of a band-limited negative capacitor needed to improve dual-frequency energy harvesting.

Thesis Supervisor: Jeffrey H. Lang
Title: Professor of Electrical Engineering and Computer Science
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Chapter 1

Introduction

Vibration energy harvesters are bidirectional transducers between electrical and mechanical energy. Motivational applications for vibration energy harvesters include primary or secondary power sources for low-power, self-sufficient, wireless industrial sensors. As a specific example, the company Perpetuum\(^1\) places vibration sensors powered by vibration energy harvesters near the axles of trains. The vibrations produced in this environment are a source of energy and of data - monitoring the vibrations allows for early identification of failures in rotating train components.

Modern vibration energy harvesters - the most popular being piezoelectric, electromagnetic, and electrostatic - can harvest maximum energy from an ideal vibration source of a single, constant-frequency equivalent to the mechanical resonant frequency of the harvester. However, real vibration sources may exhibit frequency shifting and/or contain multiple frequencies. One example is an office window vibrating due to wind. Figures 1-1a and 1-1b show a spectrogram and FFT of such non-ideal vibration conditions [1]. Harvesters may be placed in environments with high-energy, non-ideal vibrations, but unable to efficiently harvest energy because their natural resonant frequency does not match the vibration frequency. The specific technical challenge addressed in this thesis is achieving maximum power transfer from multi-frequency vibration energy sources, simultaneously.

\(^1\)www.perpetuum.com
Figure 1-1: Spectrogram (a) and FFT (b) of non-ideal vibrations recorded off an office window [1].

1.1 Thesis Objective and Contributions

The objective of this research is to demonstrate improved dual-frequency vibration energy harvesting using a piezoelectric vibration energy harvester loaded with digitally controlled power electronics. Building off the work of reference [1], the control algorithms and power electronics are suitable for multi-frequency harvesting using magnetic, electrostatic and piezoelectric based vibration energy harvesting systems. Using a piezoelectric harvester as a working example, the research in this thesis is the first to use digital control to demonstrate improved dual-frequency vibration energy harvesting.
1.2 Thesis Organization

This chapter serves to define a problem with current vibration energy harvesters, motivate the need for improved multi-frequency energy harvesting, and to list the objectives and contributions of this thesis. Chapter 2 introduces the linear modeling and analysis techniques needed to understand the conjugate load matching requirement for maximum energy harvesting from single and multi-resonant vibration sources. In addition, Chapter 2 discusses two design methods for conjugate matched loads and the state-of-the-art power-electronics synthesis of conjugate matched loads. This thesis uses a piezoelectric vibration energy harvester to demonstrate improved dual-frequency energy harvesting. Thus, Chapter 3 details the energy harvester test bench and the equipment and processes used to experimentally determine the model parameters for a cantilever structured piezoelectric harvester. Chapter 4 gives an overview of the complete vibration energy harvesting system - harvester, power-electronics and control - and details the design considerations and hardware choices for the power electronic load and controller. Chapter 5 focuses on the digital controller, detailing the theoretical background and tools used to design the high-speed, fixed-point filters used for multi-frequency vibration energy harvesting. Chapter 6 presents experimental results for dual-frequency vibration energy harvesting and compares the data to predictions using MATLAB, Simulink, and SPICE simulations. Chapter 7 summarizes the thesis, reports conclusions, and suggests areas for future work.
Chapter 2

Background

Vibration energy harvesters are bidirectional transducers between electrical and mechanical energy. Motivational applications for vibration energy harvesters include primary or secondary power sources for low-power, self-sufficient, wireless industrial sensors. Thus, one technical challenge for vibration energy harvesters is getting maximum power transfer from mechanical vibration to electrical loads. Often, ambient vibration is broadband with a non-flat energy spectrum. A second technical challenge for vibration energy harvesters is therefore to harvest energy from multiple, if not all, high-energy vibration sources.

Vibration energy harvesters have different transduction mechanisms; three of the most popular transduction mechanisms are electrostatic, electromagnetic, and piezoelectric. However, from the perspective of linear systems modeling and analysis, these three types of vibration energy harvesters are all under-damped third-order systems and all have Thevenin equivalent circuit representations that must be conjugate matched at their operating frequency for maximum power transfer from source to load. Given a harvester with a fixed resistive load, maximum mechanical to electrical power transfer occurs when the frequency of the vibration equals the resonant frequency of the harvester. Given a harvester with a tunable electrical load, the quality factor and position of the system resonant peak can be adjusted, as has been demonstrated in [3] [4], [5], [6]. Better yet, a tunable electrical load can theoretically add multiple resonant peaks to a vibration energy harvesting system, allowing maximum
energy harvesting simultaneously from multiple vibration frequencies [1]. This thesis builds on the theory and state-of-the-art research summarized above to demonstrate improved dual-frequency vibration energy harvesting using a piezoelectric harvester loaded with digitally-controlled power electronics.

2.1 Piezoelectric Energy Harvester

A piezoelectric vibration energy harvester relies on the electromechanical properties of piezoelectric material to perform the transduction between mechanical and electrical energy. Building on the work in [1], this thesis uses a piezoelectric vibration energy harvester in its experimental test bench, detailed in Chapter 3. The spring-mass-damper and electrical models of a piezoelectric energy harvester are described below.

It should be noted that the three most popular vibration energy harvester transduction mechanisms are electrostatic, electromagnetic, and piezoelectric. However, piezoelectric, electrostatic, electromagnetic vibration energy harvesters are all third-order systems and their electromechanical models will differ only in the mechanical to electrical conversion terms.
2.1.1 Spring-Mass-Damper Model

The spring-mass-damper model of a vibration energy harvester in Figure 2-1 is well known and contains a mass, $M$, spring stiffness, $K$, and damping ratio $B$ [7]. Ambient environmental acceleration with respect to inertial ground is modeled by $a_{\text{environmental}} = \frac{d^2y}{dt^2}$ and causes displacement $y(t)$. Relative displacement of the piezoelectric material, with respect to a reference point on the material, is modeled by $x(t)$. Since the harvester is an electromechanical transducer, the mechanical force $f_{\text{electrical}}$ accounts for general electrical loading of the transducer reflected back into the mechanical domain.

Using Figure 2-1, the following differential equation can be derived:

$$M \frac{d^2x}{dt^2} + B \frac{dx}{dt} + kx = f_{\text{electrical}} - f_{\text{environmental}}$$  \hspace{1cm} (2.1)

Although Equation 2.1 can be used to perform mechanical loading analysis, the work in this thesis relies on electrically loading the piezoelectric harvester since, as shown in the next section, proper electrical loading can actively tune the resonant frequency of a piezoelectric energy harvester. Thus, it is convenient to convert the mechanical model to an electrical model.

![Figure 2-1: Schematic diagram of spring-mass-damper model of a piezoelectric energy harvester.](image)
2.1.2 Electrical Model of Piezoelectric Harvester

Figure 2-2 shows the electromechanical model of a piezoelectric energy harvester, described in [8], while Figure 2-3 is an equivalent reduced electrical model. In the mechanical block of Figure 2-2, the voltage sources \( f_{\text{environmental}} \) and \( Dv \) represent force, and the current \( u \) represents velocity. The resistor, inductor, and capacitor, respectively represent the damper, mass, and spring of Figure 2-1. This mapping follows the impedance analogy between mechanical and electrical variables. The ideal transduction between mechanical and electrical energy is modeled by coupled dependent sources with conversion factor \( D \, [\text{N/V}] \). The capacitance \( C \) in the electrical block models the electric energy storage of the piezoelectric transducer. Capacitance \( C \) is analogous to the magnetizing inductance of a non-ideal electrical transformer.

![Equivalent electromechanical circuit model of a piezoelectric energy harvester.](image)

Figure 2-2: Equivalent electromechanical circuit model of a piezoelectric energy harvester.

![Equivalent electrical circuit model of a piezoelectric harvester without dependent sources.](image)

Figure 2-3: Equivalent electrical circuit model of a piezoelectric harvester without dependent sources [2].
While the electrical model in Figure 2-3 may be derived schematically by eliminating the independent sources of Figure 2-2, the electrical model can also be derived with Equation 2.1, by defining the following relations between general mechanical force \( f \) and velocity \( \frac{dx}{dt} \), and general electrical voltage \( v \) and current \( i \) as

\[
vi = f \frac{dx}{dt}
\]

(2.2)

\[
f = Dv
\]

(2.3)

\[
i = D \frac{dx}{dt}
\]

(2.4)

Substituting Equations 2.2, 2.3, and 2.4 into Equation 2.1 and scaling both sides by \(-1/D\) yields

\[
\frac{M}{D^2} \frac{di}{dt} + \frac{B}{D^2} i + \frac{k}{D^2} \int i dt = \frac{f_{\text{environmental}}}{D} - v_{\text{electrical}}
\]

(2.5)

which is the equation obtained by evaluating a clockwise KVL loop around Figure 2-3 and setting \( v_H = v_{\text{electrical}} \). Under the assumption of sinusoidal steady state with a vibration force of the form \( f = \Re \tilde{F} e^{j\omega t} \), Equations 2.6, 2.7, and 2.8 define the Thevenin equivalent of Figure 2-3 [2].

\[
\tilde{V}_T = \frac{D\tilde{F}/C}{K + D^2/C - \omega^2M + j\omega B}
\]

(2.6)

\[
R_T = \frac{D^2B/C^2}{(K + D^2/C - \omega^2M)^2 + (\omega B)^2}
\]

(2.7)

\[
X_T = \frac{(K + D^2/C - \omega^2M)(K - \omega^2M) + (\omega B)^2}{\omega C((K + D^2/C - \omega^2M)^2 + (\omega B)^2)}
\]

(2.8)
2.2 Load Matching

Multi-frequency energy harvesting suits real-world applications since vibration sources may contain multiple frequency components. The total time-average-power delivered to the conjugate load is the superposition of the time-average-power delivered by each harmonic of a vibration source, since sinusoids with different frequencies are orthogonal if integrated over a sufficiently long time. Piezoelectric, electrostatic, and electromagnetic vibration energy harvesters can be modeled with a Thevenin equivalent circuit, comprised of frequency dependent resistance $R_T(\omega)$, reactance $X_T(\omega)$ and voltage $V_T(\omega)$. Under the assumption of sinusoidal steady state, Figure 2-4 shows a general Thevenin equivalent model of a vibration energy harvester loaded with a general load $Z_L$. Achieving maximum power transfer from any vibration energy harvester requires conjugate load matching. Thus, an alternative to using an array of energy harvesters, where each harvester has a different natural resonant frequency, to harvest energy from $N$ different vibration frequencies is to use a single piezoelectric harvester loaded with a $Z_L$ that conjugate matches $R_T(\omega) + jX_T(\omega)$ for those same $N$ frequencies.

A piezoelectric harvester is used as a working example in this section to introduce switching power electronics that can compactly synthesize a $Z_L$ to conjugate match $R_T(\omega) + jX_T(\omega)$ at multiple frequencies. Two methods of designing $Z_L$ are also discussed.

![Figure 2-4: General Thevenin equivalent circuit of a vibration energy harvested with load $Z_L$.](image)

Figure 2-4: General Thevenin equivalent circuit of a vibration energy harvested with load $Z_L$.  

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2.2.1 State-of-the-Art Loading Implementation

Reference [3] has demonstrated single-frequency vibration energy harvesting using discrete inductors, capacitors and resistors. However, it is unrealistic to implement single-frequency vibration energy harvesting, much less multi-frequency vibration energy harvesting, using discrete elements because: (1) the sizes and losses associated with the inductors are too high; (2) discrete components cannot be easily tuned with frequency shifting vibration sources; and (3) it is desirable to store load energy in a DC reservoir rather than dissipate it in a resistor $R_L$. Switching power electronics such as those shown in Figure 2-5 can compactly synthesize an impedance network needed for single and multi-frequency energy harvesting. When combined with digital control, the power electronics are tunable in real-time. In addition, the power electronics perform AC to DC voltage conversion and store energy in $V_{Batt}$. These switching power electronics, popular in AC/DC rectifiers and power factor correcting applications, have most recently been used as tunable energy harvester loads in [1],[4],[5], and [6]. These same power electronics in combination with mixed-signal hysteresis current control are used in this thesis to implement matched loads for improved dual-frequency energy harvesting.

![Figure 2-5: Switching power electronics used for complex load synthesis.](image-url)
Figure 2-5 behaves as an arbitrary impedance where all net real power absorbed by the impedance is delivered to the battery, ignoring resistive and switching losses. From a systems point of view, the impedance at a port is defined by the ratio of the magnitudes and phase difference between the voltage and current at that port. The power electronics allow one to scale and phase shift $i_L$ as a function of $v_L$. The H-bridge and battery comprise a controllable PWM voltage source with labeled voltage $v_H$ which, when combined with $v_L$, regulates the current $i_L$ through inductor $L_{\text{smooth}}$. Thus Figure 2-5 is a controllable current source. $L_{\text{smooth}}$ and the PWM frequency are chosen to manage the level of the current ripple of $i_L$. Generating the PWM signals for the H-bridge switches requires measuring the error between a desired current $i_{\text{ideal}}$, which is derived from the measured $v_L$, and the measured $i_L$.

Figure 2-6 shows a simplified circuit diagram of a vibration energy harvesting system using the power electronics of Figure 2-5. Figure 2-7 shows typical Simulink waveforms when the controller in Figure 2-6 commands the power electronics to synthesize a band-limited negative capacitor in parallel with a positive resistor. $Y_L$ is the conjugate matched admittance and is derived from the conjugate matched impedance $Z_L$ in Figure 2-4. When the error between $i_{\text{ideal}}$ and $i_L$ is above or below a hysteretic threshold, $S1/S3$ or $S2/S4$ are closed to force $i_L$ to respectively ramp up or down. As the hysteretic threshold is tightened, the allowable error between measured $i_L$ and $i_{\text{ideal}}$ is decreased, allowing for better load matching. However, switching losses are increased as the hysteretic threshold decreases since $\frac{di_L}{dt} = \frac{v_L \pm V_{\text{Batt}}}{L_{\text{smooth}}}$. 

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Figure 2-6: Simplified circuit diagram of the vibration energy harvesting system.

Figure 2-7: Typical Simulink waveforms for the vibration energy harvesting system in Figure 2-6. For this simulation, the power electronics are programmed to synthesize a band-limited negative capacitor in parallel with a positive resistor.
2.2.2 Load Design Method 1

Under the assumption of sinusoidal steady state, the Thevenin equivalent of a piezoelectric vibration energy harvester with a general load $Z_L$ is shown in Figure 2-8. Assuming a vibration force of the form $f = \Re \tilde{F} e^{j\omega t}$, Equations 2.6, 2.7, 2.8 respectively define $\tilde{V}_T(\omega)$, $R_T(\omega)$ and $X_T(\omega)$ [2]. Figure 2.2.2 shows plots of frequency dependent $R_T(\omega)$ and $X_T(\omega)$. To harvest maximum energy from $N$ different vibration frequencies using a single piezoelectric harvester, $Z_L = R_T - jX_T$ must hold at those same $N$ frequencies. Method 1 for load design involves conjugate matching $R_T(\omega) + jX_T(\omega)$ and is best understood by studying plots of $-X_T(\omega)$ versus $X_L(\omega)$ and $R_T(\omega)$ versus $R_L(\omega)$. The MATLAB code for all plots in this subsection is given in Appendix A.1.1. The model parameters used for the piezoelectric harvester are $M = 4.164 \times 10^{-3}$ kg, $K = 1.335 \times 10^3$ N/m, $B = 123.3 \times 10^{-3}$ N*s/m, $D = 2.005 \times 10^{-3}$ N/V, and $C = 220$ nF.

![Diagram of electrically loaded piezoelectric harvester](image)

Figure 2-8: Electrically loaded piezoelectric harvester.
Figure 2-9: Plots of $R_T$ (a) and $X_T$ (b) for a piezoelectric harvester. The harvester model parameters used are $M = 4.164e-3$ kg, $K = 1.335e3$ N/m, $B = 123.3e-3$ N*s/m, $D = 2.005e-3$ N/V, and $C = 220$ nF. MATLAB code for the plots is given in Appendix A.1.1
Figure 2-10a shows a passive topology for $Z_L$ which can conjugate match $R_T(\omega) + jX_T(\omega)$ at one frequency while Figure 2-10b shows a passive topology for $Z_L$ which can conjugate match $R_T(\omega) + jX_T(\omega)$ at two frequencies. Since $X_T$ in Figure 2-9b looks capacitive at all frequencies, single frequency vibration energy harvesting is possible by implementing $Z_L$ with a series inductor and resistor. Assuming a piezoelectric harvester is loaded with a circuit of the form in Figure 2-10a, where $L_1 = 12.189 \, \text{H}$ and $R_1 = 112.860 \, \Omega$, Figure 2-12a shows examples of $R_T$ and $R_L$ versus frequency while Figure 2-12b shows examples of $-X_T$ and $X_L$ versus frequency. The intersection of the plotted lines in Figures 2-12a and 2-12b indicate conjugate load matching at 100 Hz. Thus, the time-average-power $<\text{power}>$ delivered to the load from 0.2 g’s of acceleration is maximum at 100 Hz, as shown Figure 2-12c. Although there are two peaks in Figure 2-12c, the smaller peak is a consequence of near conjugate load matching while the larger peak at 100 Hz is a consequence of perfect conjugate load matching. In addition, it’s important to note that the resonant peak is at 100 Hz but the mechanical resonance of the harvester is $1/(2\pi\sqrt{M/K}) = 90.1 \, \text{Hz}$. Thus, a series inductor and resistor can be used to tune the resonance frequency of a loaded piezoelectric harvester to any frequency.

A complex load of the form shown in Figure 2-10b can achieve optimal dual-frequency harvesting at two frequencies $\omega_1$ and $\omega_2$ with the constraint that $R_T(\omega_1) = R_T(\omega_2) = R_L$. $X_L(\omega)$ is implemented with a tank circuit in series with an inductor $L_3$ that conjugate matches $X_T$ at two frequencies. Assuming a piezoelectric vibration harvester loaded with a circuit of the form in Figure 2-10b where $L_1 = 441.5 \, \text{mH}$, $R_1 = 108.7 \, \Omega$, $C_1 = 7.88 \, \mu\text{F}$, and $L_2 = 13.29 \, \text{H}$, Figure 2-13a shows examples of $R_T$ and $R_L$ versus frequency while Figure 2-13b shows examples of $-X_T$ and $X_L$ versus frequency. The vertical red line in Figure 2-13b is a mathematical asymptote to the equation describing $X_L(\omega)$. This asymptote occurs at frequency $1/(2\pi\sqrt{L_1C_1})$. The intersections of the plotted curves in Figures 2-13a and 2-13b indicate conjugate load matching at 80 Hz and 100 Hz. Thus, $<\text{power}>$ delivered to the load from 0.2 g’s of acceleration is maximum at 80 Hz and 100 Hz, as shown Figure 2-13c.
A complex load of the form shown in Figure 2-11 can achieve optimal dual-frequency harvesting at two frequencies without the constraint that $R_T(\omega_1) = R_T(\omega_2)$. The first leg, comprised of $R_2$, $L_2$, $C_2$, $L_3$ is designed to conjugate match $R_T(\omega) + jX_T(\omega)$ at $\omega_1 = 1/\sqrt{L_1C_1}$. The second leg, comprised of $R_1, L_1, C_1$, is designed to conjugate match $R_T(\omega) + jX_T(\omega)$ at $\omega_2 = 1/\sqrt{L_2C_2}$. Although the legs are in parallel, at both $\omega_1$ and $\omega_2$ one of the legs behaves as an open when the respective tank resonates. Assuming a piezoelectric vibration harvester loaded with a circuit of the form in Figure 2-11 - where $R_2 = 671.32 \ \Omega$, $L_2 = 5.1229 \ \text{H}$, $C_2 = 772.59 \ \text{nF}$, $L_3 = 28.047 \ \text{H}$, $R_1 = 108.65 \ \Omega$, $L_1 = 4.6706 \ \text{H}$, $C_1 = 613.77 \ \text{nF}$ - Figure 2-14a shows examples of $R_T$ and $R_L$ versus frequency while Figure 2-14b shows examples of $-X_T$ and $X_L$ versus frequency. The vertical red lines in Figure 2-14b are mathematical asymptotes to the equation describing $X_L(\omega)$. The intersections of the plotted curves in Figures 2-14a and 2-14b indicate conjugate load matching at 80 Hz and 94 Hz. Thus, $\langle \text{power} \rangle$ delivered to the load from 0.2 g’s of acceleration is maximum at 80 Hz and 94 Hz, as shown Figure 2-14c. It is important to note that Figure 2-11 is only a model for a network loading a piezoelectric harvester and is used for easy design of $Y_L(j\omega) = 1/Z_L$ in Figure 2-6. If the load were actually implemented with passive components, power harvested would be burned in both $R_1$ and $R_2$. However, when $Z_L$ is implemented with the power electronics described in Figure 2-6, net harvested power is delivered to a single reservoir $V_{\text{Batt}}$. 
Figure 2-10: (a) A passive topology useful for maximum single-frequency vibration energy harvesting with a piezoelectric harvester. (b) A passive topology useful for maximum dual-frequency harvesting.

\[ Z_L(\omega) = \begin{cases} 
R_2 + j(\omega L_3 + \frac{\omega L_2}{1-\omega^4 L_2 C_2}) & \text{if } \omega = \frac{1}{\sqrt{L_1 C_1}} \\
R_1 + j\frac{\omega L_1}{1-\omega^2 L_1 C_1} & \text{if } \omega = \frac{1}{\sqrt{L_2 C_2}}
\end{cases} \]

Figure 2-11: An improved passive topology useful for maximum dual-frequency harvesting.
Figure 2-12: Example plots showing how a load of the form in Figure 2-10a, with
$L_1 = 12.189 \, \text{H}$ and $R_1 = 112.860 \, \Omega$, conjugate matches $R_T(\omega)$ and $X_T(\omega)$ of a
piezoelectric harvester at $\omega = 2\pi(100 \, \text{Hz})$. (a) shows resistances $R_T$ and $R_L$ versus
frequency. (b) shows reactances $-X_T$ and $X_L$ versus frequency. (c) shows the time-
averaged-power $<\text{Power}>$ delivered to the load from 0.2 g’s of acceleration as the
frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for
the plots is given in Appendix A.1.1
Figure 2-13: Plots showing how a load of the form in Figure 2-10b, with $L_1 = 441.5$ mH, $R_1 = 108.7$ Ω, $C_1 = 7.88 \mu F$, and $L_2 = 13.29$ H, conjugate matches $R_T(\omega)$ and $X_T(\omega)$ of a piezoelectric harvester at $\omega_1 = 2\pi(80 \text{ Hz})$ and $\omega_2 = 2\pi(100 \text{ Hz})$. (a) shows resistances $R_T$ and $R_L$ versus frequency. (b) shows reactances $X_L$ and $-X_T$ versus frequency. (c) shows the $\langle\text{Power}\rangle$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.1.
Figure 2-14: Plots showing how a load of the form in Figure 2-11, with $R_2 = 671.32 \ \Omega$, $L_2 = 5.1229 \ \text{H}$, $C_2 = 772.59 \ \text{nF}$, $L_3 = 28.047 \ \text{H}$, $R_1 = 108.65 \ \Omega$, $L_1 = 4.6706 \ \text{H}$, $C_1 = 613.77 \ \text{nF}$, conjugate matches $R_T(\omega)$ and $X_T(\omega)$ of a piezoelectric harvester at 80 Hz and 94 Hz. (a) shows resistances $R_T$ and $R_L$ versus frequency. (b) shows reactances $X_L$ and $-X_T$ versus frequency. (c) shows the $\langle \text{Power} \rangle$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.1.
2.2.3 Load Design Method 2

In an actual application, it is not only desirable for there to exist a set $W$ of $N$ frequencies where $Z_L$ conjugate matches $R_T(\omega) + jX_T(\omega)$ of Figure 2-4. The closer $Z_L$ comes to conjugate matching $R_T(\omega) + jX_T(\omega)$ at frequencies near those in set $W$, the wider are the bandwidths of the harvesting peaks. Preferably, the bandwidth of the power peaks would be wider than those shown in Figures 2-12c, 2-13c, and 2-14c to accommodate slight shifts in vibration frequency over time. An alternate design method for a conjugate matched load that leads to improved bandwidth in harvesting peaks, used in [1], involves element wise conjugate matching of the discrete components comprising the harvester electrical model in Figure 2-3.

Figure 2-15 shows the electrical model of the piezoelectric harvester with a general complex load. Analysis of the system assuming operation in sinusoidal steady state and a vibration force of the form $f = \Re \tilde{F} e^{j\omega t}$ shows that maximum power transfer (half the source power) from the source to resistive load $R_L$ is achieved when satisfying the following:

\[
X_{L1}(\omega) = -\frac{1}{\omega C} \tag{2.9}
\]
\[
X_{L2}(\omega) = \frac{1 - \omega^2 (M/D^2)(D^2/K)}{\omega(D^2/K)} \tag{2.10}
\]
\[
R_L = \frac{B}{D^2} \tag{2.11}
\]

Figure 2-15: Electrically loaded piezoelectric harvester.
Intuitively, $X_{L1}(\omega)$ is used to cancel the parasitic capacitance $C$ of the piezoelectric harvester, $X_{L2}(\omega)$ is used to conjugate match the remaining reactance $j\omega M/D^2 + K/(j\omega D^2)$ of the harvester, and $R_L$ is required to match the effective resistance $B/D^2$ of the harvester. Unlike the conjugate matched loads described in Section 2.2.2, the resistive term $R_L$ of the matched load is frequency independent. However, conjugate load matching is still frequency dependent since $X_{L1}(\omega)$ and $X_{L2}(\omega)$ are frequency dependent. It is important to realize that the design of $X_{L2}(\omega)$ and $R_L$ as in Equations 2.10 and 2.11 help conjugate match the piezoelectric harvester at N frequencies only when $X_{L1}(\omega) = -1/\omega C$ at those same N frequencies. One way of satisfying Equation 2.9 is to implement $X_{L1}(\omega)$ as a negative capacitor.

Networks for single-frequency harvesting and multi-frequency harvesting that can simultaneously satisfy Equations 2.9, 2.10, and 2.11 are shown in Figure 2-16. For ease of analysis, a wide-band negative capacitor is used to implement $jX_{L1}$ in each load in Figure 2-16. It has been shown in [4] and [5] that a digital controller and switching power electronics can implement $X_{L1}(\omega)$ as a negative capacitor. As a negative capacitor, $X_{L1}(\omega)$ cancels the harvester parasitic output capacitance $C$ at multiple frequencies. Reference [1] cancels $C$ at only one of the two vibration frequencies. In practice, $X_{L1}(\omega)$ is implemented as a band-limited negative capacitor, ensuring that high frequency noise does not cause overflows within a digital controller implementing $Y_L(j\omega)$ in Figure 2-6. Chapter 5 details the implementation of $jX_{L1}$ with a band-limited negative capacitor.

Assuming the parasitic capacitance is completely canceled and $R_L = B/D^2$, single and multi-frequency harvesting requires conjugate requires load matching $-jK/(\omega D^2) + j\omega M/D^2$ with $jX_{L2}$ (satisfying Equation 2.10). $jX_{L2}$ may be implemented as a network of positive inductors and/or capacitors, depending on the number of vibration frequencies from which energy will be harvested. The design of $jX_{L2}$ is best understood by looking at plots of acceptable $X_{L2}$ and $-X_{\text{MECH}} = (-\omega M)/D^2 + K/(\omega D^2)$ versus frequency. All plots assume a piezoelectric harvester with model parameters $M = 4.164\text{e-3 kg}$, $K = 1.335\text{e3 N/m}$, $B = 123.3\text{e-3 N*s/m}$, $D = 2.005\text{e-3 N/V}$, and $C = 220 \text{nF}$. The MATLAB code for all plots in this subsection is given in Appendix.
A.1.2.

At the mechanical resonance frequency $1/\sqrt{M/K}$ of the harvester, effective capacitance $D^2/K$ and effective inductance $M/D^2$ act as a short. Thus, $X_{L2}$ can be implemented with a short as shown in Figure 2-16a. Figure 2-17 shows plots of $-X_{\text{MECH}} = -\omega M/D^2 + K/(\omega D^2)$ and $X_{L2}$ versus frequency. In Figure 2-17a, $-X_{\text{MECH}}$ crosses zero at frequency $1/(2\pi \sqrt{M/K})$ and is the frequency at which Figure 2-17b is maximum. Above and below the mechanical resonance frequency $1/(2\pi \sqrt{M/K})$, Equation 2.10 may be satisfied at single frequencies by implementing $X_{L2}(\omega)$ as either a capacitor or an inductor.

$-X_{\text{MECH}}$ is negative (capacitive) at frequencies below $1/(2\pi \sqrt{M/K})$ and positive (inductive) at frequencies above $1/(2\pi \sqrt{M/K})$. Assuming a piezoelectric vibration harvester loaded with a circuit of the form in Figure 2-16b, Figure 2-18a shows examples of $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. Assuming a piezoelectric vibration harvester loaded with a circuit of the form in Figure 2-16c, Figure 2-19a shows examples of $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. The frequencies where the red and blue curves intersect in both figures correspond to the frequencies at which Figures 2-18b and 2-19b are maximum.

The complex load in Figure 2-16d has $jX_{L2}(\omega)$ implemented with a tank circuit and is an example of a load that conjugate matches $j\omega M/D^2 - jK/(\omega D^2)$ at two frequencies. Figure 2-20a shows how the reactance $X_{L2}$ is positive (inductive) at frequencies below $1/(2\pi \sqrt{L_2C_2})$ and negative (capacitive) at frequencies above $1/(2\pi \sqrt{L_2C_2})$. Note that the vertical red line in Figure 2-20a is a mathematical asymptote to the equation describing $X_{L2}$ and occurs at frequency $1/(2\pi \sqrt{L_2C_2})$. Ignoring the asymptotes, the frequencies where the red and blue curves intersect in Figure 2-20a correspond to the frequencies at which Figure 2-20b is maximum.

The complex load in Figure 2-16e, with $jX_{L2}(\omega)$ implemented with two series tanks, is an example of a load that conjugate matches $j\omega M/D^2 - jK/(\omega D^2)$ at three frequencies. Figure 2-21a shows examples of $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. Again, the two vertical lines are mathematical asymptotes to the equation describing $X_{L2}$ are present at frequencies $1/(2\pi \sqrt{L_1C_1})$ and at $1/(2\pi \sqrt{L_2C_2})$. Ignoring the
asymptotes, the three intersections of the two curves represent the three frequencies at which Figure 2-21b is maximum. In general, assuming the parasitic capacitance $C$ of the piezoelectric harvester is canceled and the effective resistance $B/D^2$ is matched, conjugate load matching at $N$ harmonics of a vibration source is achieved by implementing $jX_L$ with $(N-1)$ LC tanks in series.
Figure 2-16: Passive loads (a), (b), (c) allow for optimal single-frequency vibration energy harvesting at, below, and above a piezoelectric harvester’s mechanical resonant frequency \(\frac{1}{\sqrt{M/K}}\), respectively. Passive load (d) allows for dual-frequency vibration energy harvesting. Passive load (e) allows for triple-frequency vibration energy harvesting.
Figure 2-17: Example plots showing how a load of the form in Figure 2-16a with $X_{L2} = 0$ conjugate matches $X_{\text{MECH}} = \omega M/D^2 - K/(\omega D^2)$ of a piezoelectric harvester at $\omega = 1/\sqrt{M/K}$. (a) shows reactances $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. (b) shows the $<\text{Power}>$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.2.
Figure 2-18: Example plots showing how a load of the form in Figure 2-16b, with $L_1 = 278.07\, \text{H}$, conjugate matches $X_{\text{MECH}} = \omega M/D^2 - K/(\omega D^2)$ of a piezoelectric harvester at $\omega = 2\pi(80\, \text{Hz})$. (a) shows reactances $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. (b) shows the $<\text{Power}>$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.2.
Figure 2-19: Example plots showing how a load of the form in Figure 2-16c, with $C_1 = 12.993 \text{nF}$, conjugate matches $X_{\text{MECH}} = \omega M/D^2 - K/(\omega D^2)$ of a piezoelectric harvester at $\omega = 2\pi(100 \text{ Hz})$. (a) shows reactances $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. (b) shows the $\langle \text{Power} \rangle$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.2.
Figure 2-20: Example plots showing how a load of the form in Figure 2-16d, with $L_1 = 52.33 \text{ H}$ and $C_1 = 61.4 \text{ nH}$, conjugate matches $X_{\text{MECH}} = \omega M/D^2 - K/(\omega D^2)$ of a piezoelectric harvester at $\omega = 2\pi(80 \text{ Hz})$ and $\omega = 2\pi(100 \text{ Hz})$. (a) shows reactances $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. (b) shows the $<\text{Power}>$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 60 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.2.
Figure 2-21: Example plot showing how a load of the form in Figure 2-16e, with $L_1 = 223.09 \, \text{H}, \, C_1 = 15.47 \, \text{nF}, \, L_2 = 186.98 \, \text{H},$ and $C_2 = 7.7096 \, \text{nF}$, conjugate matches $X_{\text{MECH}} = (-\omega M/D^2 - K/(\omega D^2))$ at three frequencies. (a) shows reactances $-X_{\text{MECH}}$ and $X_{L2}$ versus frequency. (b) shows the $\langle \text{Power} \rangle$ delivered to the load from 0.2 g’s of acceleration as the frequency of the vibration input is swept from 0 Hz to 120 Hz. MATLAB code for the plots is given in Appendix A.1.2.
2.3 Improving Multi-Frequency Energy Harvesting

The demonstration of dual-frequency harvesting, the use of non-linear hysteresis control, and the use of a piezoelectric vibration energy harvester as a working example differentiate this thesis from work in [4], [5], and [6]. This thesis uses the theoretical analysis and electrical load design methodology developed in [1] - explained in Section 2.2.3 - for optimal multi-frequency vibration energy harvesting but adds digital control to demonstrate improved dual-frequency energy harvesting.

Although the power electronics of Figure 2-6 are capable of synthesizing loads needed for multi-frequency vibration energy harvesting, [4], [5], and [6] only demonstrate improvements in single-frequency vibration energy harvesting. [1] is the most recent work that demonstrates dual-frequency vibration energy harvesting using a piezoelectric harvester. Although [1] demonstrates dual-frequency harvesting, $R_T + jX_T$ is not perfectly conjugate matched at both frequencies. [1] used analog control which cannot be easily modified and which physically scales with the complexity of the load being synthesized. In this thesis, detailed in Chapter 4, the admittance block of Figure 2-6 is implemented in a digital signal controller while the non-linear hysteresic control is implemented with analog comparators. The digital controller allows for synthesis of a band-limited negative capacitor needed for improved dual-frequency vibration energy harvesting.
2.4 Summary

Using the piezoelectric vibration energy harvester as a working example, this chapter introduced the linear modeling and analysis techniques needed to understand the conjugate load matching requirement for maximum energy harvesting from single and multi-resonant vibration energy sources. State-of-the-art impedance syntheses using a tunable electrical load was discussed along with two methods for designing the conjugate matched loads to be synthesized. This thesis uses Load Design Method 2 described in Section 2.2.3 and the power electronics shown in Figure 2-5, building on the work of [1], to demonstrate improved dual-frequency harvesting using a piezoelectric harvester.
Chapter 3

Energy Harvester Test Bench

This chapter details the energy harvester test bench and processes used to experimentally determine the electromechanical model parameters for a cantilever-structured piezoelectric vibration energy harvester. The test bench, automated data collection process, and harvester characterization are discussed.

3.1 Shaker Table

The core of the energy harvester test bench is a custom shaker table, Figure 3-1, and a Mide V25W piezoelectric vibration harvester Figure 3-2. A calibrated shaker table is needed to provide controlled energy transfer to the piezoelectric harvester. The shaker table was built from a sub woofer speaker and used in [1]. A custom shaker table is used due to the unavailability of a commercial shaker table and is acceptable due to the light mass of the V25W harvester and the accelerations less than 1 g being used in the final experiments [1].

An accelerometer (ADXL103CE), custom mounting bracket for the piezoelectric harvester, and a sub woofer comprise the shaker table, Figure 3-1. The sub woofer generates vibrations and the accelerometer is used to ensure that constant, vertical acceleration is applied to the piezoelectric harvester during experiments. The custom mounting bracket ensures the unloaded piezoelectric harvester exhibits a third-order response within the frequency and acceleration limits of performed experiments [1].
Figure 3-1: Vibration energy harvester test bench. The sub woofer serves as a shaker and the custom mounting bracket secures the piezoelectric device and accelerometer to the speaker cone.

Figure 3-2: Mide V25W cantilever-structured vibration energy harvester.
3.2 Automated Data Collection

To easily and repeatedly characterize the piezoelectric energy harvester, an automated data collection system was used. Characterization involves supplying a constant acceleration to the harvester as the frequency of the vibration is swept first while measuring the output voltage magnitude of the harvester and second while measuring the short circuit current magnitude of the harvester. The short circuit current $i_{SC}$ measurement is performed using the OpAmp circuit shown in Figure 3-3. For the frequency sweep experiments, a National Instruments USB Device (NI-DAQ 6215) interfaced with the MATLAB Data Acquisition Toolbox controls the shaker table and records corresponding accelerometer output voltage magnitude, harvester output voltage magnitude, and the magnitude and frequency of the voltage driving the speaker. While the NI-DAQ drives the speaker with a user programmable voltage, the NI-DAQ measures the accelerometer output. MATLAB code then compares the measured acceleration to a programmable acceleration and scales the speaker drive voltage magnitude as needed to maintain the desired acceleration. Once the desired acceleration is measured from the accelerometer, data triplets of the form (accelerometer output, harvester output, speaker drive) are stored, where the harvester output is either the open circuit voltage magnitude or the short circuit current magnitude.

![OpAmp Circuit](image)

**Figure 3-3**: OpAmp Circuit used to measure piezoelectric harvester short circuit current $i_{SC}$. 
3.3 Piezoelectric harvester Characterization

Characterization of a piezoelectric harvester is needed to accurately conjugate load match the harvester output in an effort to achieve maximum output power. As described in Section 2.1, the piezoelectric harvester is a third order electromechanical system. It has an effective mass $M$, damping factor $B$, spring constant $K$, conversion ratio $D$ and parasitic capacitance $C$. Figure 3-4 shows the complete electromechanical model of the harvester driven by a force $f_{\text{environmental}}$ and with dependent sources. Figure 3-5 shows an equivalent electrical model with the dependent sources eliminated [2]. Table 3.1 summarizes the electromechanical model parameters in Figures 3-4 and 3-5. It should be noted that the V25W has two pairs of output terminals, which may be connected in either series or parallel. In order to limit the output voltages from the harvester, a parallel rather than a series combination was used for experiments in this thesis. The model parameters in Table 3.1 are for a parallel connection of the V25W output terminals.

![Figure 3-4: Equivalent Electromechanical Circuit Model of Piezoelectric Harvester](image)

![Figure 3-5: Equivalent Electrical Circuit Model of Piezoelectric Harvester without Dependent Sources](image)
Table 3.1: Energy Harvester Model Parameters

<table>
<thead>
<tr>
<th>Model Parameters</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Mass ( M )</td>
<td>4.165e-3</td>
<td>[kg]</td>
</tr>
<tr>
<td>Spring Constant ( K )</td>
<td>1.335e3</td>
<td>[N/m]</td>
</tr>
<tr>
<td>Damping Factor ( B )</td>
<td>123.3e-3</td>
<td>[N*s/m]</td>
</tr>
<tr>
<td>Conversion Ratio ( D )</td>
<td>2.005e-3</td>
<td>[N/V]</td>
</tr>
<tr>
<td>Parasitic Capacitance ( C )</td>
<td>220e-9</td>
<td>[F]</td>
</tr>
<tr>
<td>Force ( f_{\text{environmental}} )</td>
<td>independent</td>
<td>[N]</td>
</tr>
</tbody>
</table>

The harvester characterization involves running two separate frequency sweep experiments. In both experiments, the frequency of the vibration is swept while the acceleration is held constant. One experiment measures the open circuit voltage magnitude while the second experiment measures the short circuit current magnitude. The Thevenin voltage of the piezoelectric harvester has been derived in [2] to be

\[
\tilde{V}_T = \frac{D\tilde{A}/C}{K/M + D^2/(CM) - \omega^2 + j\omega B/M}
\]  

(3.1)

where \( \tilde{A} \) is the complex acceleration amplitude. Thus, the open circuit voltage measurements of the harvester can be fit to Equation (3.1) by solving for the macro variables \( N_1 = D/C \), \( N_2 = K/M + D^2/(CM) \), \( N_3 = B/M \). Within these macro variables are 5 unknowns but only 3 equations. Thus, the short circuit current measurements are used to measure the mechanical resonant frequency \( 1/(2\pi\sqrt{M/K}) \) and an impedance analyzer is used to measure \( C \).

The harvester model parameters are verified using SPICE and MATLAB simulations. Figure 3-6a compares the open circuit voltage experimental measurements to those predicted in SPICE and MATLAB using the fitted electromechanical parameter values in Table 3.1. Figure 3-6b peaks at frequency \( 1/(2\pi\sqrt{M/K}) \). The values plotted are for an acceleration of 0.2 g’s being applied to the piezoelectric harvester. It’s important to notice that the MATLAB and SPICE models match the experimental data for the majority of the resonant peak.
Figure 3-6: Piezoelectric harvester characterization: (a) open circuit voltage and (b) short circuit current of the piezoelectric harvester measured experimentally with 0.2 g’s of acceleration exciting the harvester and modeled by SPICE.
3.4 Chapter Summary

This chapter detailed the energy harvester test bench and the equipment and processes used to experimentally determine the electromechanical model parameters for the cantilever-structured piezoelectric vibration energy harvester introduced in Chapter 2. The test bench, automated data collection process, and harvester characterization were discussed. Table 3.1 and Figure 3-6 summarize the harvester model parameters used for plots in Chapter 2 and 4.
Chapter 4

System Architecture

Figure 4-1 shows a simplified circuit diagram of the vibration energy harvesting system. The admittance block (red) and hysteresis control block (yellow) comprise the mixed-signal control of the power electronics (blue) loading the piezoelectric vibration energy harvester (grey). The admittance block is implemented using a fixed-point digital filter on a digital signal controller. The hysteresis control is implemented using analog comparators. The power electronics are those used in [1] and similar to those used in [4], [5], [6]. The control inputs are load voltage \( v_{\text{LOAD}} \) and load current \( i_{\text{LOAD}} \), and two control outputs each drive a pair of switches within the power electronics. The control system does the following: (1) samples the load voltage \( v_{\text{LOAD}} \); (2) digitally processes this load voltage to calculate the ideal current for the system; (3) outputs acceptable upper and lower threshold values for the ideal current; (4) compares the measured load current \( i_{\text{LOAD}} \) to the threshold values using analog comparators; (5) drives the transistor switching network; (6) temporarily latches the comparator outputs to prevent false comparisons in the next cycle. Power going into the power electronics is calculated from recorded measurements of \( v_{\text{LOAD}} \) and \( i_{\text{LOAD}} \). In the end, ignoring \( R_{\text{sense}} \) and any series resistance in \( L_{\text{smooth}} \), the switching-network shapes the measured load current \( i_{\text{LOAD}} \) to the ideal load current by making \( i_{\text{LOAD}} \) ramp up and down with a slope approximated by

\[
\frac{di_{\text{LOAD}}(t)}{dt} \approx \frac{v_{\text{LOAD}} \pm V_{\text{Batt}}}{L_{\text{smooth}}}. \tag{4.1}
\]
4.1 Design Considerations

The demonstration of optimum multi-frequency and single-frequency vibration energy harvesting becomes more difficult as the distance between the frequencies of the vibrations and natural resonance of the harvester increases. Considering single-frequency vibration energy harvesting, the size of the reactive components needed to conjugate load match the harvester must increase as the resonant frequency of a piezoelectric harvester is tuned further away from its natural resonance. A larger reactive load implies that more reactive power is exchanged between the harvester and load. This reactive power raises the magnitudes of the load voltages in the system.
Figure 4-2 shows the piezoelectric harvester loaded with a general load. Assuming that an input with constant acceleration is swept in frequency and that $X_{L1}(w)$ and $X_{L2}(w)$ are chosen to respectively conjugate match $-j\frac{1}{\omega C}$ and $j\omega\frac{M}{D} - j\frac{K}{\omega D^2}$, Figure 4-3 shows plots of $|v_{LOAD}|$ versus Resonant Frequency. These plots are computed assuming the use of a piezoelectric harvester with model parameters $M = 4.164e^{-3}$ kg, $K = 1.335e3$ N/m, $B = 123.3e-3$ N*s/m, $D = 2.005e-3$ N/V, and $C = 220$ nF. The MATLAB code for this plot is in Appendix A.2. The lowest $|v_{LOAD}|$ occurs at the natural resonant frequency of the harvester, when the reactive load components are at their minimum. While this plot shows the $|v_{LOAD}|$ expected for single-frequency harvesting, the $|v_{LOAD}|$ for multi-frequency harvesting increase linearly with the number of frequencies.

The characteristic displayed in Figure 4-3 reveals the requirement for the analog instrumentation amplifiers and power electronics shown in Figure 4-1 to handle increased voltage stresses while tuning the resonant frequency of the harvester away from its natural resonance. Proper hysteresis control of $i_{LOAD}$ is achieved when $\frac{di_{LOAD}}{dt} = \frac{v_{LOAD}(t)+V_{Batt}}{L_{\text{smooth}}} > 0$ and $\frac{di_{LOAD}}{dt} = \frac{v_{LOAD}(t)-V_{Batt}}{L_{\text{smooth}}} < 0$, where the choice of the sum or difference of $v_{LOAD}(t)$ and $V_{Batt}$ is performed by switching the power electronics. This requires $V_{Batt}$ to be greater than the magnitude of $v_{LOAD}$. Thus, as the resonant frequency of the harvester is tuned further away from its natural resonance and as the number of vibration frequencies from which energy is simultaneously harvested increases, drain-to-source voltage stresses on the MOSFETs increase. The MOSFET switches require high reverse breakdown voltages to withstand the necessary $V_{Batt}$. Furthermore, instrumentation amplifiers require high power supply and common-mode voltage ratings since $V_{Batt}$ is presented to amplifiers $A_V$ and $A_I$ as a common-mode.

The system shown in Figure 4-1 is ultimately limited by the instrumentation amplifiers implementing $A_V$ and $A_I$. For the experiments in this thesis, these amplifiers are powered by $\pm 16V$ power supplies. $\pm 16V$ power supplies and the requirement that $V_{Batt} > V_{LOAD}$ require $|V_{LOAD}| < 8$ V, since the sum of the common-mode voltage and load voltage must not exceed the power supply voltage. Analyzing Figure
4-3, maintaining $|V_{\text{LOAD}}| < 8\text{ V}$ with 0.2 g's of acceleration limits the demonstration of maximum single-frequency energy harvesting to vibrations frequencies within ±8 Hz of the harvester’s mechanical resonant frequency $1/(2\pi\sqrt{M/K})$. These limits are more stringent for the demonstration of dual-frequency energy harvesting since each frequency component of a vibration source will cause the load voltage $V_{\text{LOAD}}$ to linearly increase.
Figure 4-2: Electrically loaded piezoelectric harvester. When $X_{L2}(\omega)$ conjugate matches $j\omega \frac{M}{D^2} - j\frac{K}{\omega D^2}$ across a range of frequencies, $v_{LOAD}$ has magnitude plots as shown in Figure 4-3.

Figure 4-3: $|v_{LOAD}|$ versus Resonance Frequency when a piezoelectric harvester, modeled by parameters given in Table 3.1, has its resonant peak tuned to frequencies away from its natural resonant frequency and is harvesting energy from 0.1 g and 0.2 g vibrations at that tuned resonant frequency. The MATLAB code for this plot is in Appendix A.2.
4.2 Controller

A controller for the power electronics of Figure 2-6 may be implemented with pure analog control as in [1] or pure digital control, using a linear PID controller, as in [4], [5], and [6]. In this thesis, the admittance block used to calculate \( i_{\text{desired}} \) from \( v_L \) is implemented in a digital signal controller while the non-linear hysteretic control is implemented with analog comparators. The admittance block is implemented digitally because an analog implementation cannot be easily modified and the area of control circuitry scales with the complexity of the load being synthesized. The hysteresis control is implemented with analog comparators primarily for speed.

4.2.1 Admittance Block

The admittance block in Figure 4-1 is a synchronous block controlled by a dsPIC CE512MU810 digital signal controller. The harvester output voltage \( v_{\text{LOAD}} \), measured by \( A_V \) and amplified by \( B_V \), is continuously sampled at 61kHz by a 16BIT ADC. Immediately following, a fixed-point filter, coded in the dsPIC, computes the ideal load current and the corresponding hysteresis thresholds \( i_{\text{IDEAL}_{\text{LOWER}}} \) and \( i_{\text{IDEAL}_{\text{UPPER}}} \). The details of the fixed-point filter are given in Chapter 5. A dsPIC CE512MU810 running at 60 MIPS and 16-Bit ADC’s and DAC’s are primarily chosen to provide flexibility and ease in the implementation of digital control for the power electronics. Future work concerned with the power consumption of the energy harvester control electronics would need micro-controller choice, instruction speed, and converter resolution to be optimized. An AD8421 instrumentation amplifier measuring \( v_{\text{LOAD}} \), represented by \( A_V \), was chosen to handle the steps between the common-mode voltage of \( V_{\text{Batt}} \) and system ground. Two digitally programmable LTC6912 amplifiers and an LT1050 connected in series implement \( B_V \), adding variable amplification of \( v_{\text{LOAD}} \).

4.2.2 Hysteresis Control

For speed and experimental flexibility, the comparisons between \( i_{\text{SENSE}} \) and the hysteresis thresholds are performed by dual analog comparators, an LTC1712. The use
of dual comparators allows adjustment of the upper and lower hysteresis thresholds. For the experiments in this thesis, the hysteresis thresholds were set to be a fraction of the theoretical magnitude of $i_{LOAD}$ predicted by SPICE. Adjustable thresholds are desired since $|i_{LOAD}|$ varies when synthesizing different loads. Other major components of the hysteresis control include $A_I$ which is implemented by an AD8429 instrumentation amplifier and $B_I$ which is implemented by two digitally programmable LTC6912 amplifiers connected in series. An SN74ALS109 flip-flop is used to store the state of the previous hysteresis comparison.

The LTC1712 contains latch enabled dual comparators, allowing input blanking to be performed on the hysteresis control block. After every switching event of the power electronics, a step between $V_{Batt}$ and system ground occurs in the AD8429 common-mode voltage. The common-mode voltage step causes the AD8429 output to ring and hence risks false switching of the power electronics. However, the amplifier ringing settles to 0.01% in $640\,\text{ns}$, which is less than the amount of time needed for experimental $i_{LOAD}$ to traverse the hysteresis band. The logic block shown in Figure 4-1 instructs the digital signal controller to temporarily latch the output of the comparators immediately after the signal switching the power electronics is sent, preventing any ringing at the output of the AD8429 from changing the comparator output.

### 4.3 Power Electronics

The transistor switching network and energy storage blocks were created exactly as in [1], primarily to produce comparable data and focus on the control rather than the design of the power electronics. Four FDV301N n-channel MOSFETS comprise the H-Bridge and are driven by two LTC4449 gate drivers. Experimentally, $V_{Batt}$ is implemented with a power supply shunted with a resistor and large capacitor. Thus in theory all the power harvested is burned in this resistor, but this would not be the case for a real application.

For experiments in this thesis, time-average-power $<\text{Power}>$ going into the power
electronics is calculated from recorded measurements of $v_{LOAD}$ and $i_{LOAD}$; $< \text{Power} > = < v_{LOAD} * i_{LOAD} >$. An attempt to measure the time-average-power going into $V_{Batt}$ was performed using a high-side sense resistor placed in series with $V_{Batt}$. However, a leakage current of up to 300$\mu$A - on the order of the experimental peak $i_{SENSE}$ currents - was found to be flowing out of $V_{Batt}$ and into the gate drivers. The leakage, along with the quick current transients caused by the charging of transistor output capacitance, made it difficult to get accurate measurements of the current flowing directly into $V_{Batt}$.

In future iterations of energy harvesting systems, the transistors, FET drivers, and possibly the transistor switching schemes will need to be updated. This hypothesis is generated from analysis of Figure 4-3 and the requirement that $V_{Batt} > V_{LOAD}$. Since the FDV301N transistors have a 25 V reverse breakdown voltage, $V_{Batt} < 25$ V is an upper bound for any system using these same power electronic. Multilevel switching topologies such as those used in [9] would be needed to handle the high voltages present when harvesting farther away from the mechanical resonant frequency of a piezoelectric harvester and when harvesting from multi-frequency vibration sources. A multilevel switching topology would still need to synthesize the same loads as the single level switching topology used in this thesis, but it may allow one to push the limits of multi-frequency vibration energy harvesting using a single piezoelectric device.

### 4.4 Power Supplies

The system in Figure 4-1 requires 6 different supply rails to power the control and power electronics. The control blocks use a 3.5V supply for the digital signal controller, $\pm16$V supplies for the AD8429 and AD8421 instrumentation amplifiers, and $\pm5$V supplies for everything else. The $\pm16$V for the instrumentation amplifiers are needed to handle both the measured and common-mode voltages without saturating. The gate drives of the power electronics also use a $+5$V supply. $V_{Batt}$ is a variable supply that is adjusted on an experiment by experiment basis.
4.5 Conclusion

This chapter detailed the design considerations and hardware choices for the admittance block, hysteresis control, and power electronics comprising the vibration energy harvesting system in Figure 4-1. Chapter 5 details the digital filter design for the admittance block. It is shown in Figure 4-3 that the load voltage $v_{\text{Load}}$ during harvesting increases as the resonant frequency of a piezoelectric harvester is placed farther away from the mechanical resonant frequency $1/(2\pi\sqrt{M/K})$. The output voltage swing and common-mode voltage ratings of the instrumentation amplifiers measuring $v_{\text{LOAD}}$ and $i_{\text{LOAD}}$ limit the frequencies from which maximum power may be harvested. For maximum energy harvesting from a single frequency vibration source with 0.2 g’s of acceleration, the limit is within $\pm8$ Hz of $1/(2\pi\sqrt{M/K})$. The limit becomes more stringent for dual-frequency energy harvesting since each frequency component of a vibration source will cause the load voltage $V_{\text{LOAD}}$ to linearly increase.
Chapter 5

Fixed Point Filter Design

The digital signal controller in the admittance block of Figure 4-1 is programmed with a fixed-point filter with a transfer function \( Y_{fp}(z) \). The power electronics in Figure 4-1 synthesize an impedance, referred to as \( Z_{synth} \), that is meant to conjugate load match the piezoelectric harvester, allowing for maximum power transfer from the piezoelectric harvester to the reservoir \( V_{Batt} \). Due to the chopping of the power electronics, \( Z_{synth} \) is time varying, but on average \( Z_{synth} \) is equivalent to the impedance \( Z(s) = 1/Y(s) \) of a passive impedance network. Thus, the design flow for fixed-point filter \( Y_{fp}(z) \) includes: design of a continuous time filter \( Y(s) \) with analog circuit components, discretization of \( Y(s) \) via the bilinear transform to obtain \( Y(z) \), factoring of \( Y(z) \) into second order sections, and quantization of \( Y(z) \) to obtain \( Y_{fp}(z) \). The MATLAB Signal Processing Tool Box and DSP System Tool Box aid in the filter design process. The MATLAB Filter Design and Analysis Tool allow for visual inspection of the filter quantization error. In addition, the Simulink Simscape-Power-Systems model library and Fixed-Point Tool are used to perform verification of the mixed-signal piezoelectric harvesting system shown in Figure 4-1. The Fixed-Point Tool allows for signal comparisons between systems with floating-point and fixed-point admittance filters. This Chapter details the theoretical background and tools used to design the fixed-point filters needed to perform the single-frequency and dual-frequency vibration energy harvesting experiments summarized in Chapter 6.
5.1 Analog Design and Discretization

The first step in designing the fixed-point filter for the admittance block in Figure 4-1 is to design the optimum analog load needed for single-frequency and dual-frequency vibration energy harvesting. Once designed, the admittance \( Y(s) \) for each load is the ideal admittance filter for the admittance control block of Figure 4-1. In MATLAB, the bilinear transform is performed on \( Y(s) \) to obtain \( Y(z) \). \( Y(z) \) is then quantized to obtain \( Y_{fp}(z) \), which is programmed on the digital signal controller.

This thesis uses a load design methodology involving element wise conjugate matching of the discrete components comprising the harvester electrical model. This methodology was used in [1] and is detailed in section 2.2.3. The generic complex load used is shown in Figure 5-1. Figure 5-2 shows passive impedance networks used for single-frequency and dual-frequency vibration energy harvesting experiments summarized in Chapter 6. Figure 5-2a allows for optimum single-frequency vibration harvesting at the mechanical resonant frequency \( 1/\sqrt{M/K} \) of the piezoelectric harvester. Experimental element values are \(-C_1 = -220 \text{nF}, -R_1 = -361.7 \Omega, -L_1 = -115.1 \text{mH}\) and \( R_L = B/D^2 = 30.676 \text{k}\Omega \). Figure 5-2b allows for dual-frequency vibration energy harvesting and the same \( C_1, R_1, L_1, \) and \( R_L \) values as in Figure 5-2a are used; the LC tank is designed with \( C_2 = 1.045 \mu \text{F}, L_2 = 3.071 \text{H} \).

\( R_L \) in Figure 5-1 and 5-2 is meant to match \( B/D^2 \). \( X_{L2} \) in Figure 5-1 is meant to conjugate match \( j\omega M/D^2 - jK/(\omega D^2) \) at one or more frequencies, and is implemented by the \( C_2, L_2 \) tank of 5-2. \( X_{L1} \) in Figure 5-1 is meant to cancel shunt capacitance \( C \) of the piezoelectric harvester and is implemented by a band-limited negative capacitor comprised of \(-C_1, -R_1, \) and \(-L_1 \) in Figure 5-2. It’s important to note that the use of positive instead of negative resistance and inductance \( R_1 \) and \( L_1 \) would make \( Y(s) \) for each load of Figure 5-2 unstable. Furthermore, since the band-limited negative capacitor amplifies high frequencies, it may cause the digital signal controller in 4-1 to saturate or overflow if not designed properly.
5.1.1 Band-limited Negative Capacitor

The conjugate load matching requirement for optimal vibration energy harvesting, whether single-frequency or multi-frequency, under specifies the design of a load. For both an analog load and power electronic load implementation, design considerations which limit the set of viable loads include the desired bandwidth for the resonant peaks of time-averaged-power versus frequency and the precision to which a load may be synthesized. References [4] and [5] have shown that a negative capacitor may be
implemented to cancel $C$, which is especially useful for dual-frequency vibration energy harvesting. However, the limited dynamic range of the digital signal controller, and the stability of the mixed-signal system in Figure 4-1 with non-zero controller delays must be considered when synthesizing the negative capacitor. Due to the non-linear nature of the hysteresis controller and the time-varying hysteresis thresholds, a mixture of empirical and analytical methods was used to design the band-limited negative capacitor - shown in Figure 5-3 with $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, and $-L_1 = -123.6 \text{mH}$.

Preventing overflow or saturation in the digital signal controller caused by noise present at the input of the admittance block requires implementing a band-limited negative capacitor, rather than a wide-band negative capacitor. For $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, and $-L_1 = -123.6 \text{mH}$, Figure 5-4 shows a bode plot of the admittance $Y$ for a band-limited negative capacitor shown in Figure 5-3. $|Y|$ of Figure 5-4a peaks at $1/(2\pi\sqrt{L_1C_1})$. At frequencies well below $1/(2\pi\sqrt{L_1C_1})$, the system approximates an ideal differentiator. At frequencies well above $1/(2\pi\sqrt{L_1C_1})$, the system approximates an ideal integrator which attenuates high frequency noise and prevents the digital signal controller from saturating. Ideally $1/\sqrt{L_1C_1}$ is designed high enough such that the phase at frequencies of interest is $-90^\circ$ but low enough (below $1/2$ the controller sampling frequency) to prevent aliasing and overflow or saturation in the digital signal controller.

Even if the digital signal controller does not overflow or saturate, it’s possible for the system in Figure 4-1 to limit-cycle. Due to the nonlinear nature of the hysteresis controller and the time-varying hysteresis thresholds, finding an analytical solution to system stability is difficult. However, laboratory experiments revealed that $1/(2\pi\sqrt{L_1C_1})$ had to be designed below $1.6 \text{kHz}$, the quality factor of the system in Figure 5-3 had to be kept below 2, and $0 \geq -C_1 \geq -205 \text{nF}$ to prevent the energy harvesting system from limit-cycling.

In conclusion, it’s important to note that the band-limited negative capacitor shown in Figure 5-3, with $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, and $-L_1 = -123.6 \text{mH}$, does not allow for perfect dual-frequency vibration energy harvesting and is therefore
a good place for improvement in future work. Figure 5-5 illustrates this point with plots of time-averaged power $< P_{\text{Load}} >$ versus frequency delivered from a piezoelectric vibration energy harvester excited by 0.2 g’s of acceleration to three different analog loads of the form in Figure 5-2b. Each curve is obtained assuming a piezoelectric harvester with parameters $M = 4.164\text{e-3 kg}$, $K = 1.335\text{e3 N/m}$, $B = 123.3\text{e-3 N*s/m}$, $D = 2.005\text{e-3 N/V}$, and $C = 220 \text{nF}$. The analog load for each curve has $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, $-L_1 = -128.6 \text{mH}$, and $R_L = 30.676 \text{k}\Omega$. The values for $L_2$ and $C_2$ differ for each load. The red curve is obtained using $L_2 = 2.045 \text{H}$ and $C_2 = 1.534 \text{\mu F}$. The green curve is obtained using $L_2 = 8.217 \text{H}$ and $C_2 = 382.93 \text{nF}$. The red curve is obtained using $L_2 = 18.575 \text{H}$ and $C_2 = 170.24 \text{nF}$. Together the three curves show that dual-frequency energy harvesting worsens as one attempts to move the peaks of power curves further away from the mechanical resonance $1/\sqrt{M/K} = 90.1 \text{Hz}$ of the harvester. The left-hand peaks fall slightly and the right-hand peaks suffer the most. This is a consequence of the imperfect cancellation of the piezoelectric output capacitance. The MATLAB code for these plots is given in Appendix A.3.

![Analog load diagram](image)

Figure 5-3: Analog load used to implement a band-limited negative capacitor and corresponding admittance function. For experiments in this thesis, $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, and $-L_1 = -123.6 \text{mH}$.
Figure 5-4: Magnitude (a) and Phase (b) plots of the $Y(s)$ in Figure 5-3 with $-C_1 = -220 \text{nF}$, $-R_1 = -388.2 \Omega$, and $-L_1 = -123.6 \text{mH}$. 
Figure 5-5: Plots of time-averaged power $<P_{\text{Load}}>$ versus frequency delivered to various analog loads of the form in Figure 5-2b from a piezoelectric vibration energy harvester excited by 0.2 g’s of acceleration. The three curves show that dual-frequency energy harvesting worsens as one attempts to move the peaks of power curves further away from the mechanical resonance $1/(2\pi \sqrt{M/K}) = 90.1Hz$ of the harvester. This is a consequence of the design of the band-limited negative capacitance. Each curve is obtained assuming a piezoelectric harvester with parameters $M = 4.164e-3$ kg, $K = 1.335e3$ N/m, $B = 123.3e-3$ N*s/m, $D = 2.005e-3$ N/V, and $C = 220$ nF. The analog load for each curve has $-C_1 = -220$ nF, $-R_1 = -388.2$ Ω, $-L_1 = -123.6$ mH, and $R_L = 30.676$ kΩ. The values for $L_2$ and $C_2$ differ for each load. The MATLAB code for these plots is given in Appendix A.3.
5.1.2 Descretization

The admittance \( Y(s) \) for each analog load is the ideal admittance filter for the admittance control block of Figure 4-1. To help minimize computational error, the bilinear transform, rather than a simpler zero-order hold computation, is used to obtain \( Y(z) \) from \( Y(s) \). Despite the use of the bilinear transform to obtain the coefficients of \( Y(z) \), quantization of these coefficients still limits how precise the passive impedance networks in Figure 5-2 can be synthesized. From a circuits perspective, quantization error on the values \( L_2 \) and \( C_2 \) effectively limits the precision to which the resonant power peaks for the energy harvesting system in Figure 4-1 can be tuned. In addition, quantization error on the values of \( C_1, R_1 \), and \( L_1 \) limits the percentage of parasitic capacitance \( C \) that can be canceled in the absence of over cancellation. Even worse, from a systems perspective, quantization error in the coefficients of \( Y_{fp}(z) \) can move the poles of \( Y(z) \) outside the unit circle. Mitigating quantization error requires careful design of the quantized filter structure.

5.2 Filter Structure

Fixed-Point implementations of digital filters are affected by arithmetic round-off, overflows, and coefficient quantization [10]. The effects of arithmetic round-off include decreased output signal-to-noise-ratio and zero-input limit cycles. Overflows due to finite accumulator length can also cause limit cycling. Filter coefficient quantization places effective poles and zeros in different locations than the the nominal poles and zeros. For the experiments run in this thesis, coefficient quantization poses the biggest threat because it effectively changes the admittance block of Figure 4-1, causing the power electronics of Figure 4-1 to synthesize an impedance that does not accurately conjugate match the output impedance of the piezoelectric vibration energy harvester. Although state-space filter structures can be designed to simultaneously minimize round-off noise and coefficient quantization error as well as eliminate overflow oscillations and zero input limit cycling [11], such filter structure optimization techniques were not needed for the experiments in this thesis. A simple cascade
of second-order Direct Form 1 sections in Figure 5-6 with quantization parameters in Table 5.1 was enough to hide the effects of coefficient quantization and arithmetic round-off.

A load of the form in Figure 5-2b - which is needed for dual frequency vibration energy harvesting at frequencies not equal to the mechanical resonant frequency of the harvester - has an admittance $Y(s)$ that is 4th order with closely clustered poles and zeros. When using a single Direct Form structure for a fixed-point implementation of $Y_{fp}(z)$, increasing the word length for each coefficient is one solution to reduce coefficient sensitivity to quantization. A solution which allows for smaller word length, is decomposing a high order Direct Form filter into a cascade of second-order and first-order sections via factorization, or into a parallel set of second-order and first-order sections via partial fraction expansion [10]. This design technique works because closely clustered roots of a polynomial are more sensitive to changes in the coefficients of the polynomial, especially as the order of the polynomial increases [10]. Since filter transfer functions are ratios of polynomials, high order filters with closely clustered poles and zeros are most sensitive to coefficient quantization. However, each second-order and first order section of a decomposed filter realization has coefficients that are less sensitive to quantization.

Figure 5-6: A general second-order-section structure used for single frequency and multi-frequency vibration energy harvesting. Each second-order section has a Direct Form 1 structure.
Table 5.1: Fixed-Point Filter Quantization Parameters

<table>
<thead>
<tr>
<th>Filter Parameter</th>
<th>Quantization Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input,Output</td>
<td>Q 0.15</td>
</tr>
<tr>
<td>Coefficients</td>
<td>Q 7.24</td>
</tr>
<tr>
<td>Product</td>
<td>Q 15.48</td>
</tr>
<tr>
<td>Accumulator</td>
<td>Q 7.24</td>
</tr>
</tbody>
</table>

5.3 Filter Design and Verification Tools

The design flow for fixed-point filter $Y_{fp}(z)$ includes: design of a continuous time filter $Y(s)$, discretization of the factored $Y(s)$ via the bilinear transform to obtain $Y(z)$, factoring of $Y(s)$ into second order sections, and quantization of $Y(z)$ to obtain $Y_{fp}(z)$. The factorization of $Y(z)$ into a cascaded structure of second-order sections is done with the MATLAB function $tf2sos$. As mentioned in [10], there is an algorithm for pole and zero pairing that helps reduce round-off noise and filter coefficient sensitivity to quantization. The MATLAB $tf2sos$ automatically performs this algorithm during the decomposition. Each coefficient in the filter may be quantized using the $fi$ function from the MATLAB Fixed-Point Designer or by using the MATLAB Filter Design and Analysis Tool (FDATool). The FDATool - part of the DSP System Toolbox - aids in visual inspection of the quantization error between $Y(z)$ and $Y_{fp}(z)$. Through a single GUI, a designer can import the discrete time filter $Y(z)$ coefficients into the FDATool, set the filter structure for each second order section (a Direct Form 1 structure is used in this thesis), input quantization parameters (5.1 shows parameters used in this thesis), and view pole-zero and bode plots for the quantized $Y_{fp}(z)$. Each plot overlays the plots for the quantized filter $Y_{fp}(z)$ with the reference plots for the floating-point precision filter $Y(z)$.

Although useful, pole-zero and bode plots for the quantized $Y_{fp}(z)$ do not directly show how much vibration energy will be harvested when the fixed-point filter is used
in the mixed-signal system of Figure 4-1. A mixed-signal simulator such as Simulink is needed to verify the operation of the system. The Simulink Simscape-Power-Systems model library and Fixed-Point Tool was used to perform verification of the mixed-signal piezoelectric harvesting system shown in Figure 4-1. The library had the necessary electrical components needed to model the piezoelectric harvester loaded with ideal power electronics. The Fixed-Point Tool allowed for visual comparison of the Simulink system signals when using floating-point and fixed-point admittance filter implementations.

5.4 Conclusion

This Chapter detailed the theoretical background and tools used to design the fixed-point filters used for single-frequency and dual-frequency vibration energy harvesting experiments. The most important design considerations were the digital signal controller dynamic range, bandwidth over which a negative capacitor could be implemented, and the structure for the fixed-point filters. The theory and design and verification tools presented in this chapter are useful in optimizing the digital computations in future work involving digitally controlled loads for piezoelectric, electrostatic, and electromagnetic vibration energy harvesters.
Chapter 6

Harvesting Experiments and Simulations

This Chapter presents two experiments, one demonstrating single-frequency vibration energy harvesting and a second demonstrating dual-frequency harvesting. Experiments are conducted using a piezoelectric vibration energy harvester loaded with digitally-controlled power electronics. Simulations are performed using the piezoelectric vibration energy harvester model, of the form in Figure 3-5, loaded with passive or power electronic loads. In all experiments, each frequency component of the vibration source chosen to excite the harvester maintains a constant acceleration of 0.2 g’s.

The single-frequency harvesting experiment is used to confirm the operation of the band-limited negative capacitor used to cancel the piezoelectric harvester output capacitance. The dual-frequency harvesting experiment picks up where [1] left off and demonstrates improved dual-frequency energy harvesting. For each experiment, a conjugate matched load ($Z_L$ in Figure 2-15) is initially designed using Design Method 2 described in Section 2.2.3. The admittance block $Y_L = 1/Z_L$ in Figure 2-6 is implemented as described in Chapter 5.

Primarily for the accuracy of the power measurements, time-averaged-power $<\text{Power}>$ was measured going into the entire power electronic load in Figure 4-1; $<\text{Power}> = <i_{\text{LOAD}}*v_{\text{LOAD}}> = <i_L*v_L>$. Waveform data of $i_L(t)$ and $v_L(t)$ were captured from
an oscilloscope and MATLAB was used to compute the time-average-power using the data. Due to the finite sampling frequency of the oscilloscope, only 1.25 to 3 periods of \( v_L \) and \( i_L \) were captured. This ensured the highest data capture resolution. The time average of \( i_L(t) \cdot v_L(t) \) was computed by performing a running windowed-integral on the product (the length of the window was equivalent to the period of \( v_L \)), scaling each integral by one over the period, and averaging the time-average-power computations. This method helped mitigate the error in time-average-power computations present when integrating over a few cycles of \( i_L(t) \cdot v_L(t) \).

For each experiment, a prediction of the \(<\text{Power}>\) delivered to the load is performed using MATLAB, Simulink, and SPICE. In the MATLAB prediction, the harvester is assumed to be loaded with passive components. In the Simulink prediction, the harvester is loaded with ideal power electronics and controlled by a mixed-signal controller performing fixed-point computations and with modeled control loop delay. SPICE simulations account for system delay but use a behavioral analog controller and a first-order approximation of a transistor - an ideal switch in parallel with an ideal diode and capacitor. SPICE simulations are primarily used to predict the amount of power that would be stored in the battery in Figure 4-1 - after accounting for power path losses due to series resistance and drain-to-source switching losses of the power electronics. SPICE predictions for \(<\text{Power}>\) delivered to the load and to the battery are computed for a good comparison of power losses.

### 6.1 Single-Frequency Harvesting

Figure 6-1 shows the electrical model of the piezoelectric harvester loaded with a conjugate matched load needed for single-frequency harvesting at the harvester mechanical resonant frequency \( 1/\sqrt{M/K} \). In this experiment, the harvester is modeled by \( M = 4.164e-3 \text{ kg}, K = 1.335e3 \text{ N/m}, B = 123.3e3 \text{ N*s/m}, D = 2.005e-3 \text{ N/V}, \) and \( C = 220 \text{ nF} \). The load design choices are \( R_L = B/D^2 = 30.68 \text{ kΩ}, C_1 = 205 \text{ nF}, L_1 = 123.6 \text{ mH}, R_1 = 388.18 \text{ Ω} \). The band-limited negative capacitor is designed with the considerations described in Section 5.1.1. During the experiment, the harvester is
excited by a constant acceleration of 0.2 g and loaded with digitally controlled power electronics.

Figure 6-2 plots time-averaged-power <Power> versus frequency delivered to passive and synthesized versions of the load in Figure 6-1. Experimental results are plotted with "+" (red). Predictions of <Power> delivered to an ideal passive load are plotted in black. Simulink predictions, which assume ideal power electronics and account for fixed-point computations of the controller and control loop delay, are plotted with "X" (blue). SPICE predictions plotted with "O" (green) are <Power> delivered to a power electronic load. SPICE predictions plotted with "O"(purple) are <Power> delivered to the battery in Figure 4-1 - in the presence of power path losses due to series resistance and MOSFET drain-source switching losses. SPICE simulations account for system delay but use a behavioral analog controller and a first-order approximation of a transistor - an ideal switch in parallel with an ideal diode and capacitor.

Figure 6-3 shows experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for the piezoelectric harvester when harvesting energy from a constant acceleration 0.2 g at 90.5 Hz. The lower plot of Figure 6-3 is a zoomed in version of the upper plot and shows the ripple current. The transients in the current ripple are a consequence of the instrumentation amplifier used to measure $i_L$ reacting to the stepping common-mode.

It’s important to note that although the resonant peak shown in Figure 6-2 could be shifted to the left or right of the natural mechanical resonant frequency $1/(2\pi\sqrt{M/K}) = 90.1$ Hz, the load voltage $v_L$ will increase. Considering the predicted load voltage levels in Figure 4-3 and the common-mode and output voltage swing ratings of the instrumentation amplifiers in the energy harvesting system, the peaks shown in Figure 6-2 could theoretically only be shifted with ±8 Hz of the mechanical resonant frequency 90.1 Hz - this analysis assumes 0.2 g’s of acceleration will be exciting a conjugate load matched harvester for each experiment. Going past these limits would cause the instrumentation amplifiers to saturate and system instability. In addition, since the output capacitance $C$ of the piezoelectric harvester is not perfectly canceled, pushing the resonance of the harvester farther from the mechanical
resonant frequency will result in less than optimal vibration energy harvesting (as shown in Figure 5-4).

In conclusion, this experiment confirms that the power electronics and mixed-signal controller of Figure 4-1 can synthesize a band-limited negative capacitance needed to perform experiments demonstrating dual-frequency vibration energy harvesting. In the end, 93.2% of the piezoelectric harvester parasitic output capacitance was successfully canceled over a band of roughly 95 Hz. Important engineering leading to this success included the careful design of the band-limited negative capacitor (as detailed in Section 5.1.1), the design of fixed-point filters with limited quantization error (as detailed in Chapter 5), and the implementation of blanking in the hysteresis controller (as described in Section 4.2.2).

![Figure 6-1: Model of a piezoelectric harvester loaded with a band-limited negative capacitor in parallel with a resistor. For the single-frequency harvesting experiment, $M = 4.164e^{-3}$ kg, $K = 1.335e3$ N/m, $B = 123.3e^{-3}$ N*s/m, $D = 2.005e^{-3}$ N/V, and $C = 220$ nF. $R_L = B/D^2 = 30.68$ kΩ, $C_1 = 205$ nF, $L_1 = 123.6$ mH, $R_1 = 388.18$ Ω.](image)
Figure 6-2: Simulated and experimental results of time-averaged-power \(<\text{Power}>\) versus frequency delivered from a single vibration frequency with constant 0.2 g acceleration to a load of the form in Figure 6-1. Experimental results are plotted with "+" (red). Predictions of \(<\text{Power}>\) delivered to an ideal passive load are plotted in black. Simulink predictions, which assume ideal power electronics and account for fixed-point computations of the controller and control loop delay, are plotted with "X" (blue). SPICE predictions plotted with "O" (green) are \(<\text{Power}>\) delivered to a power electronic load. SPICE predictions plotted with "O" (purple) are \(<\text{Power}>\) delivered to the battery in Figure 4-1 - in the presence of power path losses due to series resistance and MOSFET drain-source switching losses. SPICE simulations account for system delay but use a behavioral analog controller and a first-order approximation of a transistor - an ideal switch in parallel with an ideal diode and capacitor.
Figure 6-3: Experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for a piezoelectric harvester loaded with power electronics synthesizing a load of the form in Figure 6-1. A constant acceleration of 0.2 g at 90.5 Hz excited the harvester while the waveform data was captured from the oscilloscope. The lower plot of Figure 6-3 is a zoomed in version of the upper plot and shows the ripple current. The transients in the current ripple are a consequence of the instrumentation amplifier used to measure $i_L$ reacting to the stepping common-mode.
6.2 Dual Frequency Harvesting

Figure 6-4 shows the electrical model of the harvester loaded with a conjugate matched load needed for dual-frequency harvesting. In this experiment, the harvester is modeled by $M = 4.164 \times 10^{-3}$ kg, $K = 1.335 \times 10^3$ N/m, $B = 123.3 \times 10^{-3}$ N*s/m, $D = 2.005 \times 10^{-3}$ N/V, and $C = 220$ nF and is loaded with digitally-controlled power electronics. In all experiments, each frequency component of the vibration source chosen to excite the harvester maintains a constant acceleration of 0.2 g. The load design choices are $R_L = \frac{B}{D^2} = 30.676$ kΩ, $C_1 = 205$ nF, $L_1 = 123.6$ mH, $R_1 = 388.18$ Ω as in 6-1 while $C_2 = 1.045 \mu$F, and $L_2 = 3.071$ H. The band-limited negative capacitor is designed with the considerations described in Section 5.1.1.

Figure 6-5 plots time-averaged-power $<\text{Power}>$ versus frequency delivered to passive and synthesized versions of the load when a single vibration source with 0.2 g of acceleration is swept in frequency. Experimental results are plotted with "+" (red). Predictions of $<\text{Power}>$ delivered to an ideal passive load are plotted in black. Simulink predictions, which assume ideal power electronics and account for fixed-point computations of the controller and control loop delay, are plotted with "X" (blue). SPICE predictions plotted with "O" (green) are $<\text{Power}>$ delivered to a power electronic load. SPICE predictions plotted with "O"(purple) are $<\text{Power}>$ delivered to the battery in Figure 4-1 - in the presence of power path losses due to series resistance and MOSFET drain-source switching losses. SPICE simulations account for system delay but use a behavioral analog controller and a first-order approximation of a transistor - an ideal switch in parallel with an ideal diode and capacitor.

The discrepancy between the experimental data and the Simulink predictions on the second peak of Figure 6-5 is suspected to be caused by non-ideal mounting of the piezoelectric harvester to the custom shaker table and hence a time-varying model of the piezoelectric harvester. It’s important to note that although the experiments summarized by Figures 6-2 and 6-5 were performed back to back, periodic frequency sweep experiments used to characterize the piezoelectric harvester - described in Chapter 3 - resulted in a harvester model that changed slightly over time. The importance of
this observation is that, since the circuit in Figure 6-4 has a high quality factor, slight changes in the harvester model can result in significant changes in the predictions.

Figure 6-4: Harvester loaded with a band-limited negative capacitor in parallel with a series LC tank and resistor. \( R_L = \frac{B}{D^2} = 30.676 \text{k}\Omega \), \( C_1 = 205 \text{nF} \), \( L_1 = 123.6 \text{mH} \), \( R_1 = 388.18 \Omega \) as in 6-1 while \( C_2 = 1.045 \mu\text{F} \), and \( L_2 = 3.071 \text{H} \). For experiments, the harvester is modeled by \( M = 4.164e-3 \) kg, \( K = 1.335e3 \text{N/m} \), \( B = 123.3e-3 \text{ N*s/m} \), \( D = 2.005e-3 \text{ N/V} \), and \( C = 220 \text{nF} \).
Figure 6-5: Simulated and experimental results of time-averaged-power $\langle \text{Power} \rangle$ versus frequency delivered from a single vibration source with constant 0.2 g acceleration to a load of the form in Figure 6-4. Experimental results are plotted with "+" (red). Predictions of $\langle \text{Power} \rangle$ delivered to an ideal passive load are plotted in black. Simulink predictions, which assume ideal power electronics and account for fixed-point computations of the controller and control loop delay, are plotted with "X" (blue). SPICE predictions plotted with "O" (green) are $\langle \text{Power} \rangle$ delivered to a power electronic load. SPICE predictions plotted with "O" (purple) are $\langle \text{Power} \rangle$ delivered to the battery in Figure 4-1 - in the presence of power path losses due to series resistance and MOSFET drain-source switching losses. SPICE simulations account for system delay but use a behavioral analog controller and a first-order approximation of a transistor - an ideal switch in parallel with an ideal diode and capacitor.
It’s important to note that although the resonant peaks shown in Figure 6-2 could be shifted out to the left and right of the natural mechanical resonant frequency $1/(2\pi \sqrt{M/K}) = 90.03$ Hz, pushing the resonance peaks of the harvester farther from the mechanical resonant frequency will result in less than optimal vibration energy harvesting (as shown in Figure 5-4). This is due to the imperfect band-limited negative capacitor and is especially true for peaks above $1/(2\pi \sqrt{M/K})$. However, even if a perfect band-limited capacitor was synthesized to cancel all of the piezoelectric harvester output capacitance, shifting of the resonant peaks in Figure 6-2 is still limited by the increasing load voltage $v_L$. Assuming a single 0.2 g vibration input and considering the predicted load voltage levels in Figure 4-3 and the common-mode and output voltage swing ratings of the instrumentation amplifiers in the energy harvesting system, either peak shown in Figure 6-2 can theoretically only be shifted within ±8 Hz of the mechanical resonant frequency 90.03 Hz. Going past these limits risks causing amplifier saturation and system instability.

The dual-harvesting ability of the system can be seen when exciting the harvester with a superposition of two vibration sources, one with an acceleration of 0.2 g at 86.1 Hz and a second with an acceleration of 0.2 g at 92.8 Hz. These two frequencies correspond to the peaks in the experimental <Power> versus Frequency data in Figure 6-5. Table 6.1 shows that the total <Power> delivered to the power electronic load is the superposition of the <Power> delivered by each harmonic of a vibration source, within reasonable error.

Figures 6-6 and 6-7 show experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for the piezoelectric harvester when harvesting energy from a constant acceleration of 0.2 g’s at 86.1 Hz and 0.2 g’s at 92.8 Hz, respectively. The lower plots in each figure are zoomed in versions of the upper plots and show the ripple current. The transients in the current ripple are a consequence of the instrumentation amplifier used to measure $i_L$ reacting to the stepping common-mode. It’s important to note that even though similar amounts of <Power> are being harvested at 86.1 Hz for the experiment of Figure 6-6 and at 90.5 Hz for the experiment of Figure 6-3, the positive voltage peak in Figure 6-6 is +3.25 V and is greater than the +2.25 V peak in Figure
The positive voltage peak in Figure 6-7 is +2.5 V and is also greater than that in Figure 6-3. This is despite the fact that the experiment of Figure 6-3 harvests about 15 μW more than the experiment in Figure 6-7, when comparing Table 6.1 and Figure 6-2. This follows the expected trend introduced in Figure 4-3: as the resonant frequency of the piezoelectric harvester is tuned farther from the mechanical resonant frequency $1/(2\pi \sqrt{M/K})$, the load voltages will rise when energy is harvested from vibrations of constant acceleration and frequencies equal to the system resonant frequencies.

Despite the system limitations, the data in Figure 6-5 still confirms that the power electronics and mixed-signal controller of Figure 4-1 can synthesize a band-limited negative capacitance and LC tank needed to perform dual-frequency vibration energy harvesting. The data in Figure 6.1 confirms that harvesting from two vibrations simultaneously is possible. In the end, 93.2% of the piezoelectric harvester parasitic output capacitance was successfully canceled over a band of roughly 95 Hz. Important engineering leading to these successes included the careful design of the band-limited negative capacitor (as detailed in Section 5.1.1), the design of fixed-point filters with limited quantization error (as detailed in Chapter 5), and the implementation of blanking in the hysteresis controller (as described in Section 4.2.2).

<table>
<thead>
<tr>
<th>Vibration Frequency [Hz]</th>
<th>Harvester Output Power [μW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>86.1</td>
<td>64.5</td>
</tr>
<tr>
<td>92.8</td>
<td>49.75</td>
</tr>
<tr>
<td>86.1 + 92.8</td>
<td>114.7</td>
</tr>
</tbody>
</table>
Figure 6-6: Experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for a piezoelectric harvester loaded with power electronics synthesizing a load of the form in Figure 6-4. A constant acceleration of 0.2 g, at 86.1 Hz excites the harvester while the waveform data is captured from the oscilloscope. The lower plot is a zoomed in version of the upper plot and shows the ripple current. The transients in the current ripple are a consequence of the instrumentation amplifier used to measure $i_L$ reacting to the stepping common-mode.
Figure 6-7: Experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for a piezoelectric harvester loaded with power electronics synthesizing a load of the form in Figure 6-4. A constant acceleration of 0.2 g at 92.8 Hz excited the harvester while the waveform data was captured from the oscilloscope. The lower plot is a zoomed in version of the upper plot and shows the ripple current. The transients in the current ripple are a consequence of the instrumentation amplifier used to measure $i_L$ reacting to the stepping common-mode.
Figure 6-8: Experimental results of load voltage $v_L(t)$ and load current $i_L(t)$ for a piezoelectric harvester loaded with power electronics synthesizing a load of the form in Figure 6-4. A superposition of a constant acceleration of 0.2 g at 86.1 Hz and 0.2 g at 92.8 Hz excited the harvester while the waveform data was captured from the oscilloscope.
6.3 Summary

This Chapter presented two experiments, the first demonstrating single frequency vibration energy harvesting and the second demonstrating dual frequency vibration energy harvesting. Both experiments are conducted using a piezoelectric vibration energy harvester loaded with digitally-controlled power electronics. In all experiments, each frequency component of the vibration source chosen to excite the harvester maintains a constant acceleration of 0.2 g. Experimental power data is summarized in Figures 6-2, 6-5, and Table 6.1. In the end, 93.2% of the piezoelectric harvester parasitic output capacitance was successfully canceled over a band of roughly 95 Hz to allow for improved dual-frequency vibration energy harvesting from that reported in [1] - the most recent work demonstrating dual-frequency energy harvesting. Important engineering leading to these successes included the careful design of the band-limited negative capacitor (as detailed in Section 5.1.1), the design of fixed-point filters with limited quantization error (as detailed in Chapter 5), and the implementation of blanking in the hysteresis controller (as described in Section 4.2.2).
Chapter 7

Summary, Conclusion, and Future Work

7.1 Summary and Conclusions

The specific technical challenge addressed in this thesis is achieving maximum power transfer from multi-frequency vibration energy sources, simultaneously. Modern vibration energy harvesters - the most popular being piezoelectric, electromagnetic, and electrostatic - can easily harvest maximum energy from an ideal vibration source of a single, constant-frequency equivalent to the mechanical resonant frequency of the harvester. However, real vibration sources may exhibit frequency shifting and/or contain multiple frequencies. The objective of this research is to demonstrate improved dual-frequency energy harvesting using a piezoelectric energy harvester loaded with digitally-controlled power electronics.

In general, optimum multi-frequency vibration energy harvesting requires a load which conjugate matches the output impedance of a vibration energy harvester - whether electromagnetic, electromagnetic, or piezoelectric - at multiple frequencies. This thesis uses digitally-controlled power electronics to synthesize such a load for a piezoelectric harvester. Building primarily off the work of [1], the loads are designed via element wise conjugate matching of the discrete components comprising the piezoelectric harvester electrical model. In conclusion, this thesis demonstrates improved
dual frequency vibration energy harvesting; however, perfect conjugate matching of the piezoelectric harvester parasitic output capacitance is not achieved. In addition to the increasing voltage levels associated with optimum vibration energy harvesting at frequencies distant from the mechanical resonant frequency of the harvester, imperfect conjugate matching of the piezoelectric harvester output capacitance greatly limits the frequency range and number of frequencies over which useful multi-frequency vibration energy harvesting may be achieved. These two limits are technical challenges worthy of future research. The summary of work useful in understanding this conclusion is given below.

Chapter 2 introduces the linear modeling and analysis techniques needed to understand the conjugate load matching requirement for maximum energy harvesting from single and multi-frequency vibration sources. An electrical model for a piezoelectric energy harvester is derived from the spring-mass-damper model. In addition, Chapter 2 discusses two design methods for conjugate matched loads and the state-of-the-art power-electronics synthesis of conjugate matched loads. This thesis chooses the load design method, used in [1], that involves element wise conjugate matching of the discrete components comprising the harvester electrical model in Figure 2-3, leading to improved bandwidth in harvesting peaks.

This thesis uses a piezoelectric vibration energy harvester to demonstrate improved dual-frequency energy harvesting. Thus, Chapter 3 details the energy harvester test bench and the equipment and processes used to experimentally determine the model parameters for a cantilever-structured piezoelectric harvester. The core of the test bench is a custom shaker comprised of a woofer speaker, a MIDE V25W, and an accelerometer. The automated data collection system, which allows for easy and repeated characterization of the piezoelectric energy harvester, is described.

Chapter 4 gives an overview of the complete vibration energy harvesting system of Figure 4-1 - harvester, power-electronics and control - and details the design considerations and hardware choices for the power electronic load and controller. The controller is a mixed signal controller with a digital impedance block and an analog hysteresis control block. The power electronics are those used in [1]. The biggest
consideration when designing the controller and power electronics is the requirement for the analog amplifiers and power electronics to handle increased harvester output voltage levels while tuning the resonant frequency of the harvester away from its mechanical resonant frequency \(1/(2\pi \sqrt{M/K})\). It is shown in Figure 4-3 that the load voltage \(v_{\text{Load}}\) during harvesting increases as the resonant frequency of the harvester is placed farther away from the mechanical resonant frequency \(1/(2\pi \sqrt{M/K})\) of a piezoelectric harvester. For optimum energy harvesting from a single frequency vibration source with 0.2 g’s of acceleration, the limit is within ±8 Hz of \(1/(2\pi \sqrt{M/K})\). Thus, the frequencies and the acceleration levels of vibrations chosen for laboratory experiments in Chapter 6 had to be carefully chosen to ensure system operation. Each frequency component of the experimental vibration sources had 0.2 g’s of acceleration and the peaks of the dual-frequency harvesting experiment were placed at 86.1 Hz and 92.8 Hz - this is within ±8 Hz of the harvester mechanical resonant frequency \(1/(2\pi \sqrt{M/K})\).

Chapter 5 focuses on the digital controller, detailing the theoretical background and tools used to design the low-power, high-speed, fixed-point filters needed for dual-frequency vibration energy harvesting. The design flow for the fixed-point filter includes: design of a continuous time filter \(Y(s)\), discretization of \(Y(s)\) via the bilinear transform, design of the filter structure, and quantization of the filter structure. Out of all the steps, the implementation of a second order section filter structure was the most important since it helped minimize quantization error and allowed for stable synthesis of the complex load used for dual-frequency energy harvesting experiments. Useful tools which aid in the filter design and verification process - the MATLAB Signal Processing Tool Box, DSP System Tool Box, MATLAB Filter Design and Analysis Tool, Simulink Simscape-Power-Systems model library, and Fixed-Point Tool - are also discussed. The design and verification tools presented in this chapter are useful in optimizing the digital computations in future work involving digitally-controlled loads for piezoelectric, electrostatic, and electromagnetic harvesters. Most importantly, Section 5.1.1 discusses the design considerations and performance limits of the band-limited negative capacitor used for dual-frequency energy harvesting experi-
iments in this thesis.

Chapter 6 presents two experiments, the first demonstrating single-frequency vibration energy harvesting and the second demonstrating dual-frequency vibration energy harvesting. In both experiments, each frequency component of the vibration source exciting the harvester maintains a constant acceleration of 0.2 g. The experimental data is plotted against simulated data. The first experiment is used to confirm the operation of the band-limited negative capacitor which cancels out 93.2% of the piezoelectric harvester output capacitance while maintaining system stability. The second experiment picks up where reference [1] left off and demonstrates improved dual-frequency energy, made possible by the cancellation of 93.2% of the piezoelectric harvester output capacitance. Important engineering leading to these successes included the careful design of the band-limited negative capacitor (as detailed in Section 5.1.1), the design of fixed-point filters with limited quantization error (as detailed in Chapter 5), and the implementation of blanking in the hysteresis controller (as described in Section 4.2.2).

7.2 Future Work

Although this thesis demonstrates improved dual frequency vibration energy harvesting it also points out two major limits to the range and number of frequencies over which maximum multi-frequency vibration energy harvesting may be achieved using a piezoelectric harvester. The first limit is the increasing voltage levels associated with optimum vibration energy harvesting at frequencies distant from the mechanical resonant frequency of the harvester - as described in Section 4.1. The second limit is imperfect conjugate matching of the piezoelectric harvester output capacitance, as discussed in Section 5.1.1. For the piezoelectric harvester case, if one wants to push these limits, two promising areas for future research include evaluation of different power electronic switching topologies and the evaluation of a linear controller rather than a non-linear hysteresis controller for the power electronics. However, the most interesting area for future research is to discover what major implementation difficul-
ties exist for multi-frequency vibration energy harvesting using electromagnetic and electrostatic harvesters loaded with digitally controlled power electronics.

The power electronics shown in Figure 2-5 are only one of many switching topologies. As explained in Chapter 2, the switches allow one to generate an effective PWM voltage source with labeled voltage $v_H$. However, when performing optimum harvesting power from a piezoelectric harvester, $v_H$ and $v_L$ of Figure 2-5 increase at frequencies distant from the mechanical resonant frequency of the harvester - as shown in Figure 4-3. This PWM voltage source only has two levels, $\pm V_{\text{Batt}}$. There are multi-level switching topologies, such as those in [9], which could allow for multiple levels, such as $\pm V_{\text{Batt}}$, $\pm V_{\text{Batt}}/2$, and 0. Multi-level switching topologies require more transistors and thus more complex control but have two major benefits. The first benefit is that the maximum $v_H$ is no longer the breakdown voltage of a single transistor but rather the sum of the breakdown voltages in stacked transistors. A multi-level switching topology could thus increase the frequency range over which a single piezoelectric harvester could deliver maximum power to a power electronic load. The second benefit of multi-level switching is that the average number of switching events during the period of the vibration waveform is decreased. Thus the switching losses can be potentially decreased.

For experiments in this thesis, 93.2% of the piezoelectric harvester parasitic output capacitance was successfully canceled over a band of roughly 95 Hz, allowing for improved dual-frequency vibration energy harvesting. Although enough for the experiments performed in this thesis, the imperfect cancellation of the harvester output capacitance limits the frequency range over which impressive dual-frequency harvesting can be demonstrated, as shown in Figure 5-5. Simulations and laboratory experiments were used to determine how much of the piezoelectric output capacitance could be canceled while maintaining system stability, rather than mathematical analysis. Future work should explore the possibility of using a linear PID controller, rather than a non-linear controller, for improved cancellation of the piezoelectric output capacitance. Benefits of linear control include the ability to predict the stability of the system in the design stage, especially when designing a controller for an energy
harvesting system for vibration sources which change in frequency and amplitude. Although it is possible to linearize a hysteresis block using a describing function, the hysteresis controller in this thesis is more complex since the hysteresis thresholds are time varying. Linear PID controllers are easier to analyze.
Appendices
Appendix A

MATLAB Code Scripts

All MATLAB computations given assume the following parameters for the Piezoelectric vibration energy harvester and vibration source:

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Harvester Electromechanical Model Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
M = 4.165e-3;   %[kg]
B = 123.3e-3;   %[N*s/m]
K = 1.335e3;   %[N/m]
D = 2.005e-3;   %[N/V]
C = 220e-9;    %[F]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Vibration Source Model
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
omega = 2*pi*linspace(0,1000,10001);   %[rad/sec]
a = 0.2*9.8067;   %[m/s/s]
force = M*a ;   %[N]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Harvester Electrical Model Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
L_eff = M/D/D;
C_eff = (1/K)*D*D;
R_eff = B/D/D;
Vsource = force/D;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Harvester Thevenin Equivalent Model Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
R_T = (D*D*B/C/C)./((K+D*D/C - M*omega.^2).^2 + (omega*B).^2);
X_T = -(K + D*D/C - M*omega.^2).*((K - M*omega.^2) + (omega*B).^2)./((omega*C.*((K+D*D/C - M*omega.^2).^2 + (omega*B).^2)));
V_T = D*M*a/C*(sqrt((K+D*D/C - M*omega.*omega).^2 + (B*omega).^2 ));
```
A.1 Load Matching

The plots shown in Sections 2.2.2 and 2.2.2 were given as illustrative explanations of how loads in Figures 2-10 and ?? that conjugate load match a piezoelectric vibration energy harvester. Subsection A.1.1 contains the MATLAB code for plots in Section 2.2.2 while Subsection A.1.2 contains the MATLAB code for plots in Section subsec:method1.

A.1.1 Load Design Method 1

```matlab
%! Begin Plots: Thevenin Impedance: Figure 2-9
%! %R\_T vs Frequency
figure()
plot(omega/2/pi, R_T*10^-3,'LineWidth',2,'color', 'b');
xlabel('Frequency [Hz]'); ylabel('Resistance [k\Omega]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('R_{T}'); l.Location = 'northeast'; l.FontSize = 14;
axis([20 120 0 2.5]); axis square; grid on; grid minor;

%! %X\_T vs Frequency
figure()
plot(omega/2/pi, X_T*10^-3,'LineWidth',2,'color', 'b');
xlabel('Frequency [Hz]'); ylabel('Reactance [k\Omega]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('X_{T}'); l.Location = 'northeast'; l.FontSize = 14;
axis([20 120 -40 10]); axis square;
grid on;
grid minor;

%! % End Plots: Thevenin Impedance: Figure 2-9
```

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% Parameters for load in Figure 2-10a
R1 = 112.860; %[Ohms]
L1 = 12.189; %[H]

% Load Resistance
R_L = R1*ones(1,length(omega)); %[Ohms]

% Load Reactance
X_L = L1*omega; %[Ohms]

% Time average power delivered to load from 0.2g of vibration force
Pavg_load = ...
(0.5*R_L).*(V_T.^2)./...
(R_T + R_L).^2 + (X_T + X_L).^2);

% R_L, R_T vs Frequency
figure()
plot(omega/2/pi, R_T*(10^-3),'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, R_L*(10^-3),'LineWidth',2,'color', 'r');
ylabel('Resistance [k\Omega]'); xlabel('Frequency [Hz]');
axis([60 120 0 2.5]);
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('R_{T}','R_{L}'); l.Location = 'northeast'; l.FontSize = 14;
axis([60 120 0 2.5]); axis square;
grd on; grid minor;
hold off;

% X_L, -1*X_T vs Frequency
figure()
plot(omega/2/pi, -1*X_T*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, X_L*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
xlim([60, 120]);
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('\textbf{-X_T}','\textbf{X_L}'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60, 120]); axis square;
grd on; grid minor;
hold off;

%<Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\muW]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_L>'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60,120]); axis square;
grd on; grid minor;
hold off;
% Parameters for load in Figure 2-10b
R1 = 108.7; % [Ohms]
L1 = 441.5e-3; % [H]
C1 = 7.880e-6; % [F]
L2 = 13.29; % [H]

% Load Resistance
R_L = R1*ones(1,length(omega)); % [Ohms]

% Load Reactance
X_L = omega*L2 + omega*L1./(1 - (omega.^2)*L1*C1); % [Ohms]

% Time Average Power Delivered to Load from 0.2g of Vibration Force
Pavg_load = ...
    (0.5*R_L).*(V_T.^2)./
    (R_T + R_L).^2 + (X_T + X_L).^2

% R_L, R_T vs Frequency
figure()
plot(omega/2/pi, R_T*(10^-3),'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, R_L*(10^-3),'LineWidth',2,'color', 'r');
ylabel('Resistance [kΩ]'); xlabel('Frequency [Hz]');

% X_L, -1*X_T vs Frequency
figure()
plot(omega/2/pi, -1*X_T*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, X_L*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [kΩ]'); xlabel('Frequency [Hz]');

% <Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('Power [uW]'); xlabel('Frequency [Hz]');
% Begin Plots: Method 1 Dual-Frequency Harvesting: Figure 2-14

% Parameters for load in Figure 2-11
R1 = 108.65; % [Ohms]
L1 = 4.6706; % [H]
C1 = 613.77e-9; % [F]
R2 = 671.32; % [Ohms]
L2 = 5.1229; % [H]
C2 = 772.59e-9; % [F]
L3 = 28.0465; % [H]

% State Space Model for Load in Figure 2-11
Ass = [-1/(C1)*(1/R1), 0, -1/C1, 0, 0; 0, 0, 0, -1/C2, 1/C2; 1/L1, 0, 0, 0, 0; 0, 1/L2, 0, 0, 0; 0, -1/L3, 0, 0, -R2/L3];
Bss = [1/C1/R1; 0; 0; 0; 1/L3];
Css = [-1/R1, 0, 0, 0, 1];
Dss = [1/R1];

% Transfer Function
[num, den] = ss2tf(Ass, Bss,Css,Dss);
H = freqs(den, num, omega);

% Load Resistance
R_L = real(H); % [Ohms]

% Load Reactance
X_L = imag(H); % [Ohms]

% Time Average Power Delivered to Load from 0.2g of Vibration Force
Pavg_load = ...
( (0.5*R_L).*(V_T.^2))./((R_T + R_L).^2 + (X_T + X_L).^2);

% R_L, R_T vs Frequency
figure()
plot(omega/2/pi, R_T*(10^-3), 'LineWidth', 2, 'color', 'b'); hold on;
plot(omega/2/pi, R_L*(10^-3), 'LineWidth', 2, 'color', 'r');
ylabel('Resistance [k\Omega]'); xlabel('Frequency [Hz]');
axis([60 120 0 2.5]);
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('R_{T}','R_{L}'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60, 120]); axis square; grid on; grid minor;
hold off;

% X_L, -1*X_T vs Frequency
figure()
plot(omega/2/pi, -1*X_T*10^-3, 'LineWidth', 2, 'color', 'b'); hold on;
plot(omega/2/pi, X_L*10^-3, 'LineWidth', 2, 'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{T}','X_{L}'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60, 120]); axis square; grid on; grid minor;
hold off;

%<Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\mu W]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60,120]); axis square;
grid on; grid minor;
hold off;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% End Plots: Method 1 Dual-Frequency Harvesting: Figure 2-14
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
A.1.2 Load Design Method 2

% Parameters for Load in Figure 2-16a
R1 = 30.676e3; % [H]
R_L = R1*ones(1,length(omega)); % [Ohms]
X_L2 = zeros(1,length(omega)); % [Ohms]
Pavg_load = ...
((0.5*R_L).*(V_FORCE.^2))./...(R_MECH + R_L).^2 + (X_MECH + X_L2).^2);

% Figure 2-17
figure()
plot(omega/2/pi,-1*X_MECH*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi,X_L2*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{MECH} = -(\omega M/D^{2} - K/(\omega D^{2}))', 'X_{L2}');
l.Location = 'northeast'; l.FontSize = 14;
hold off;

figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\mu W]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>'); l.Location = 'northeast'; l.FontSize = 14;
hold off;

% End Plots: Method 2 Single-Frequency Harvesting at Mechanical Resonance
% Figure 2-17
% Begin Plots: Method 2 Single-Frequency Harvesting below Mechanical Resonance
% Figure 2–18
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Parameters for Load in Figure 2–16b
R1 = 30.676e3;  % [Ohms]
L1 = 278.07;    % [H]

% Effective Load Resistance
R_L = R2*ones(1,length(omega));  % [Ohms]

% Effective Load Reactance
X_L2 = omega*L1;  % [Ohms]

% Time Average Power Delivered to Load from 0.2G of Vibration Force
Pavg_load = ...
   ((0.5*R_L).*V_FORCE.^2)./...
   (R_MECH + R_L).^2 + (X_MECH + X_L2).^2);

% X_L2, -1*X_MECH vs Frequency
figure()
plot(omega/2/pi, -1*X_MECH*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, X_L2*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{MECH} = -\omega M/D^2 - K/\omega D^2','X_{L2}');
l.Location = 'northeast'; l.FontSize = 14;
xlim([60, 120]); axis square;
grid on; grid minor;
hold off;

% <Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\muW]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>'); l.Location = 'northeast'; l.FontSize = 14;
xlim([60,120]); axis square;
grid on; grid minor;
hold off;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% End Plots: Method 2 Single-Frequency Harvesting below Mechanical Resonance
% Figure 2–18
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Parameters for Load in Figure 2-16c
R1 = 30.676e3; %[Ohms]
C1 = 12.993e-9; %[F]

% Effective Load Resistance
R_L = R1*ones(1,length(omega)); %[Ohms]

% Effective Load Reactance
X_L2 = -1./omega./C1; %[Ohms]

% Time Average Power Delivered to Load from 0.2G of Vibration Force
Pavg_load = ...
    (0.5*R_L)*(V_FORCE.^2)./...
    ((R_MECH + R_L).^2 + (X_MECH + X_L2).^2);

% X_L2, -1*X_MECH vs Frequency
figure()
plot(omega/2/pi, -1*X_MECH*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, X_L2*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{MECH} = -(\omega MM^2 - K/\omega D^2)', 'X_{L2}');
l.Location = 'northeast'; l.FontSize = 14;
xlim([60, 120]); axis square;
grid on; grid minor;
hold off;

% <Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\muW]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>');
l.Location = 'northeast'; l.FontSize = 14;
xlim([60,120]); axis square;
grid on; grid minor;
hold off;
% Begin Plots: Method 2 Dual-Frequency Harvesting
% Figure 2-20

% Parameters for Load in Figure 2-16d
R1 = 30.676e3; % [Ohms]
L1 = 52.33; % [H]
C1 = 61.40e-9; % [F]

% Effective Load Resistance
R_L = R1*ones(1,length(omega)); % [Ohms]

% Effective Load Reactance
X_L2 = omega*L1./(1 - (omega.^2)*L1*C1); % [Ohms]

% Time Average Power Delivered to Load from 0.2G of Vibration Force
Pavg_load = ...
    ((0.5*R_L).*(V_FORCE.^2))./...
    ((R_MECH + R_L).^2 + (X_MECH + X_L2).^2);

% X_L2, -1*X_MECH vs Frequency
figure()
plot(omega/2/pi, -1*X_MECH*10^-3,'LineWidth',2,'color', 'b'); hold on;
plot(omega/2/pi, X_L2*10^-3,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{MECH} = -\left(\omega M/D^{2} - K/\left(\omega D^{2}\right)\right)', 'X_{L2}');
l.Location = 'northeast'; l.FontSize = 14;
axis([60 120 -1000 1000]); axis square;
grid on; grid minor;
hold off;

% <Power> vs Frequency
figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\muW]'); xlabel('Frequency [Hz]');
xlim([60,120]);
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>');
l.Location = 'northeast'; l.FontSize = 14;
axis square;
grid on; grid minor;
hold off;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% End Plots: Method 2 Dual-Frequency Harvesting
% Figure 2-20
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Parameters for Load in Figure 2-16e
R1 = 30.676e3;  %[Ohms]
L1 = 223.09;  %[H]
C1 = 15.474e-9;  %[F]
L2 = 186.98;  %[H]
C2 = 7.7096e-9;  %[F]
%
R_L = R1*ones(1,length(omega));  %[Ohms]
X_L2 = omega*L2./(1-(omega.^2)*L2*C2) + omega*L1./(1-(omega.^2)*L1*C1);  %[Ohms]
%
%
Time Average Power Delivered to Load from 0.2G of Vibration Force
Pavg_load = ...
((0.5*R_L).*(V_FORCE.^2))./((R_MECH + R_L).^2 + (X_MECH + X_L2).^2);

figure()
plot(omega/2/pi,-1*X_MECH*10^-6,'LineWidth',2,'color', 'b');
hold on;
plot(omega/2/pi, X_L2*10^-6,'LineWidth',2,'color', 'r');
ylabel('Reactance [k\Omega]');
xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('-X_{MECH} = -(\omega M/D^2 - K/(\omega D^2))', 'X_{L2}');
l.Location = 'northeast';  l.FontSize = 14;
axis([0 200 -30 30]); axis square;
grid on; grid minor;
hold off;

figure()
plot(omega/2/pi, Pavg_load*10^6,'LineWidth',2,'color', 'r');
ylabel('<Power> [\muW]');
xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{L}>');
l.Location = 'northeast';  l.FontSize = 14;
xlim([0,200]); axis square;
grid on; grid minor;
hold off;

% End Plots: Method 2 Triple-Frequency Harvesting
% Figure 2-21

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A.2 System Stresses Versus Resonant Frequency

Figure 4-3 - code given below - reveals the requirement for the analog amplifiers, hysteresis control, and power electronics shown in Figure 4-1 to handle increased voltage stresses while tuning the resonant frequency of the harvester away from its natural resonance.

```
s = tf('s');
R_L = B/D^2; %[ohms]
Z_L1 = -1/s/(C); %[ohms]
Z_L2 = -1/s/(D^2/K) - s*M/D^2; %[ohms]

% Computation
% Comput v_LOAD via Impedance Analysis
v_LOAD = (force/D)*(R_L + Z_L2)/(R_L + B/D^2);

% Extract Magnitude and Phase
[mag,phase] = bode(v_LOAD,omega);
v_LOAD_MAG = zeros(1,length(omega)); v_LOAD_MAG(1,:) = mag(1,1,:);

% Plot Magnitude
figure() %Plot |v_LOAD|
plot(omega/2/pi, v_LOAD_MAG, 'LineWidth',3,'color', 'r'); hold on;
plot(omega/2/pi, 2*v_LOAD_MAG, 'LineWidth',3,'color', 'b');
xlabel('Resonant Frequency [Hz]'); ylabel('|v_{LOAD}| [V]');
l=legend('0.1g','0.2g'); l.Location = 'northeastoutside';
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
axis([80 100 0 20]); axis square;
grid on; grid minor;
hold off;
```
A.3 Limits of Band-Limited Negative Capacitor

Section 5.1.1 notes that the bad-limited negative capacitor shown in Figure 5-3, with 
\(-C_1 = -220nF\), \(-R_1 = -388.2\Omega\), and \(-L_1 = -123.6mH\), does not allow for perfect 
dual-frequency vibration energy harvesting and is therefore a good place for improve-
ment in future work. Figure 5-5, the code shown below, illustrates this point.

```matlab
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Load Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Band Limited Negative Capacitor Parameters
L1 = -123.6e-3;  %[H]
C1 = -205e-9;  %[F]
R1 = -388.18;  %[Ohms]
%Resistance to Conjugate Match R_eff=B/D/D
R2 = 30.676e3  %[Ohms]
%Multiple Values of LC Tank Pairs
%L2_vector(i) is used with C2_vector(i) to perform dual frequency
%harvesting at the i'th pair of frequencies
L2_vector = [2.045; 8.2170; 18.575]  %[H]
C2_vector = [1.534e-6; 382.93e-9; 170.24e-9]  %[F]
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%% Perform Calculations of <Power_{LOAD}> using each LC tank pair
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
omega = 2*pi*linspace(0,1000,10001);  %[rad/sec]
%Create a vector to hold Pavg_load Computations
Pavg_load_vector = zeros(3,length(omega));
for index=1:1:3
L2 = L2_vector(index);
C2 = C2_vector(index);
%State Space Matrices for Load in Figure 5
%Matrices Constructed under following assumptions
%Input: Voltage Source
%Output: Current from input Voltage Source
Ass = [ 0, 0, 1/C1, 0;...  
0, -1/C2/R2, 0, -1/C2;...  
-1/L1, 0, -R1/L1, 0;...  
0, 1/L2, 0, 0];
Bss = [0;1/C2/R2;1/L1;0];
Css = [0,-1/R2,1,0];
Dss = [1/R2];
%Transfer Function
[num,den] = ss2tf(Ass,Bss,Css,Dss);
H = freqs(den,num,omega);  %Impedance Transfer Function
%Load Resistance
R_L = real(H);  %[Ohms]
%Load Reactance
X_L = imag(H);  %[Ohms]
%Time Average Power Delivered to Load from 0.2 g of Acceleration
```

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Pavg_load = ...
  ((0.5*R_L).*((V_T).^2))./...
  ((R_T + R_L).^2 + (X_T + X_L).^2 )
  Pavg_load_vector(index,:) = Pavg_load;
end
figure()
plot(omega/2/pi, Pavg_load_vector(1,:)*10^6,'LineWidth',2,'color', 'r'); hold on;
plot(omega/2/pi, Pavg_load_vector(2,:)*10^6,'LineWidth',2,'color', 'g');
plot(omega/2/pi, Pavg_load_vector(3,:)*10^6,'LineWidth',2,'color', 'b');
ylabel('<Power> [\mu W]'); xlabel('Frequency [Hz]');
set(gca, 'FontSize', 18, 'FontWeight', 'bold', 'LineWidth', 1);
l=legend('<P_{Load1}>','<P_{Load2}>','<P_{Load3}>');
l.Location = 'northeast'; l.FontSize = 14;
xlim([70,110]); axis square;
grid on; grid minor;
hold off;
Bibliography


