



A REACTIVITY MONITOR FOR THE M.I.T. REACTOR

by

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ABSTRACT

As a preamble, the Nuclear Reactor Dynamic Equation is derived and discussed, linearized and converted into a Signal Flow Graph. This is used to point the way towards the construction of a Reactor Simulator, the various implications of which are enlarged upon.

Then various methods of measuring the reactivity of a reactor are set forth, of which one is finally selected while reasons for this choice are also given. The advantages and problems pertaining to this method and possible variations thereof are considered, with particular attention to the photo-neutron effect. Details like scale-changing, stability, accuracy and cost are discussed with some suggestions on the construction of the monitor.

Summarily, the problem of the monitoring of reactor reactivity has been reduced to such a level that actual construction and operation, for any well-moderated reactor, should be more or less routine matters.

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CHAPTER I
INTRODUCTION

As the subject under discussion can most easily be classified under "Reactor Instrumentation" and therefore leans heavily on both electrical and nuclear engineering concepts, and in order to make the treatise more or less self-contained, it was decided to set forth more fully the various techniques and concepts involved than would have been the case if the subject had been in either the one or the other field. At the same time this also aids in pointing out the various approximations and assumptions that were made.

CHAPTER II

REACTIVITY AND THE REACTOR DYNAMIC EQUATION

The concept of reactivity is, though basic and simple, a rather abstract one. Although it lends itself readily to verbal description and elucidation, the fact remains that it is a more purely mathematical, as opposed to physical, quantity than, say, the reactor flux or its gamma spectrum. The continuous monitoring of reactivity thus cannot be solved by measuring the output of some simple transducer, but consists of applying operational techniques, or computer methods, to those parameters of the reactor which do lend themselves readily to direct measurement. To express this reasoning in slightly different words: The concept of reactivity is introduced in the mathematical model that is employed to describe the dynamic behaviour of the reactor. To evaluate the present state of reactivity of a reactor, therefore, it is necessary to reduce the exhibited behaviour of that reactor back to certain factors in that mathematical model.

The extreme importance of the concept of reactivity, and thus also the importance and need of being able to measure it conveniently and continuously, is best brought out by the fact that the effects of both intensive and extensive parameters, like type of fuel, moderator, size, temperature, etc., on the characteristics of a reactor are very conveniently expressed and explained by their effects on the reactivity. The measurement of reactivity thus serves essentially ~~two~~^{three} purposes:

- (i) It tests the extent of the validity of the mathematical model used to describe the reactor.
- (ii) It aids in the evaluation of experimental data.
- (iii) It is of primary importance in understanding reactor behaviour and in predicting the lifetime of the core.

To solve the problem of the continuous measurement of reactivity, it is necessary first to investigate the mathematical model. It is also advantageous to describe qualitatively the events which occur inside a reactor, prior to the derivation of the reactor dynamic equation. To do this, it is convenient to first list and then evaluate the relative effects of the various fates of the neutrons and subatomic particles (and rays of interest) and then to describe the fates of these hypothetical "average" particles.

A thermal neutron enters a nucleus of fissile material (fuel) and fission takes place, i.e., the nucleus disintegrates within an extremely short time into several heavy fragments, with the release of energy. At the moment of fission more than one neutron is also released, these being referred to as "prompt" neutrons, and in addition some "prompt" gamma rays (photons) are created. The interest is confined here mainly to neutrons and photons as these are by far the most important in future fission reactions.

The fission fragments are highly unstable and after several conversions eventually decay to stable end products. Amongst the various forms of radiation emitted during this

decay are again neutrons, hereafter labeled "delayed" neutrons, and gammas similarly called "delayed" photons.. Since these particles are emitted at various stages in various decay-schemes, they also experience different "delays" and can accordingly be classified by various groups, each with a specific "delay". The term "delay" here is, strictly speaking, a misnomer,¹ because from force of habit we tend to associate it with a definite time-interval, as for instance the signal delay in a transmission line. But here the expression "delay" applies to a process of a purely statistic nature, and is governed by an exponential decay law. Perhaps the expression "deferred" particles would have been a better, though not so natural choice.

However, it is the presence of these delayed neutrons which makes reactor control easy, even though they constitute only about $0.67\%²$ of the total released neutrons.

Normally the photons are of little interest in reactivity effects, but in the case of a heavy-water (D_2O) moderated reactor, as for example the M.I.T. installation, these gammas can give rise to new neutrons through a (γ, n) reaction. These neutrons are appropriately called photo-neutrons. The incident photon must have an energy greater than the binding energy of the nucleus for this reaction to take place, and heavy water is the only practical moderator with a binding energy low enough compared to the average gamma spectrum of a reactor to warrant taking these photo neutrons into account. In the case of "intermediate" and "fast" reactors, of course, other considerations might be important, but we are restricting

ourselves here to "slow" or well-moderated reactors.

As the conversion from photon to photo neutron, while being a function of energy and the density of the particles, is yet a practically instantaneous process, we can assign to the photo neutrons the same delay as previously assigned to that particular photon group, because the traverse time of the photons is also negligible. (They propagate with the speed of light).

Although the prompt and delayed fission neutrons, as also the photo-neutrons from the prompt and delayed gammas are born at different energies, according to one-group reactor theory they are all bunched together in one epithermal energy group and are traced as such. This is a valid approximation if the slowing-down time is short compared to the life time of the neutrons, as is the case in well-moderated reactors. During this slowing down to thermal energy neutrons can be lost through various processes, and even of the then remaining neutrons only a certain fraction again cause fission.

It is clear that depending on the production and loss factors, the number of neutrons after one cycle can either have decreased (subcritical), have stayed constant (critical), or have increased (supercritical).

After this preamble, we can now proceed to the derivation of the Reactor Dynamic Equation, keeping in mind the assumptions and approximations that were made. We will furthermore confine our attention to the effects produced by the genuine neutrons and note that the photo-neutrons could be treated exactly in the same manner.

Definition of symbols

- n = neutron density = number of neutron per cm^3 at time t .
- k = ratio of thermal neutrons produced per generation to thermal neutrons captured in the preceding generation.
- δk = reactivity = excess of neutrons in a finite reactor from one generation over the preceding generation per neutron in that previous generation.
- ℓ = mean effective lifetime of a neutron in a finite reactor.
- β = total fraction of the total number of neutrons produced in the fission process that are delayed, or non-instantaneous.
- β_i = fraction of the delayed neutrons in the i^{th} group.
- λ = average decay constant = probability per second per emission.
- λ_i = decay constant for the i^{th} group.
- c = total number of delayed-neutron emitters per cm^3 .
- c_i = number of delayed neutron emitters per cm^3 in the i^{th} group.

The time for each neutron cycle is defined as ℓ . During that time the total number of (delayed) neutrons released by the delayed neutron emitters per $\text{cm}^3 = \sum c_i \lambda_i \ell$

Normalizing with respect to the neutron density, the total number of delayed neutrons released per neutron per $\text{cm}^3 = \ell/n \sum \lambda_i c_i$. Also, per cycle, each neutron produces $k(1 - \beta)$ prompt neutrons.

Thus the total increase in neutron density for one original neutron per cm^3 in the time $\ell = \{k(1-\beta) + \frac{\ell}{n} \sum \lambda_i c_i\} - 1$. For a density of n neutrons/ cm^3 the increase per cycle would clearly be n times as much. We note furthermore that these events take place randomly, not in synchronism, and we can safely view the process as a continuous one. Thus,

$$\frac{dn}{dt} = \frac{n}{\ell} \left[\{k(1 - \beta) + \frac{\ell}{n} \sum \lambda_i c_i\} - 1 \right].$$

We have noted previously that the control of a reactor is facilitated by the fact that a fraction of the fission neutrons is delayed. It is thus understandable that in safe reactor operation the k available at any one time should never exceed 1.0067, although during shutdown it might be considerably less than unity.

$$\text{If } k \approx 1, \quad \delta k \equiv \frac{k - 1}{k} \approx k - 1.$$

$$\text{Also, } k\beta \approx \beta.$$

Thus, we have

$$\frac{dn}{dt} = \frac{\delta k - \beta}{\ell} n + \sum \lambda_i c_i. \quad (2.01)$$

If $k \neq 1$,

$$\frac{dn}{dt} = \frac{k(\delta k - \beta)}{\ell} n + \sum \lambda_i c_i. \quad (2.02)$$

Again, in the interval ℓ , the number of delayed-neutron emitters per cm^3 in the i^{th} group, increases from

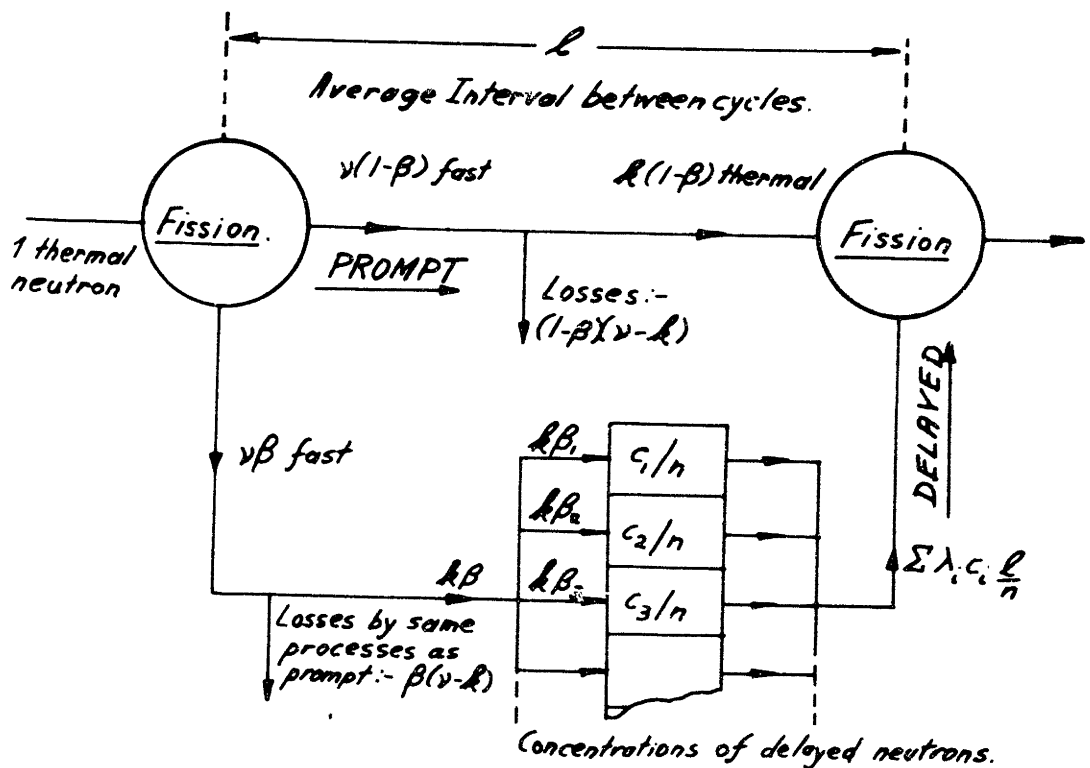
$$\frac{c_i}{n} \text{ to } \left\{ \frac{c_i}{n} + \beta_i - \frac{\ell}{n} \lambda_i c_i \right\} \text{ per neutron/cm}^3.$$

By the same argument as above this leads to

$$\frac{dc_1}{dt} = \frac{\beta_1 n}{\ell} - \lambda_1 c_1. \quad (2.03)$$

Equation (2.01) together with equation (2.03) is hereafter referred to as the Reactor Dynamic Equation. Any extraneous source of neutrons can, of course, be included by the addition of a term on the right hand side of (2.01) and (2.02).

If we define ν = number of (fast) neutrons produced per fission, and ignore the fact that losses appear during the entire lifetime of a neutron, now indicating them at arbitrary positions, we can draw the following representation.



NEUTRON REPRODUCTION CYCLE.

The Effect of Photo-Neutrons on the Reactor Dynamic Equation.

Because neutrons are neutrons, whatever their source, and need only be described in terms of their density and energy, we can investigate what the worth of gammas in a reactor is from the neutron point of view, and once this relation is established forget the fact that they are "photo-neutrons". In broad outline, this can be done by listing the energy, intensity and time at which they occur of each gamma that is emitted during and following a fission reaction, and converting them to "equivalent" neutrons by means of an expression for the probability that a (γ, n) reaction will take place. The probability is a function of the energy of the photon and the type and density of the material in which such a reaction can take place. It was pointed out previously, heavy water and beryllium are the only materials with appreciable cross-sections for a (γ, n) reaction in the range of gamma energies found in a reactor.

If there is little or no beryllium present in the core, therefore, the D_2O is the only source of photo neutrons in a reactor where this material is present in sizeable amounts.

The process outlined above has been carried through successfully,³ but the results are not directly applicable to a general reactor, because only the gammas originating directly from the fissile material were taken into account, and semi-infinite size was assumed. Thus one would have to modify the results for a specific reactor, taking into account all possible gamma-sources and the finite dimensions of the

system.⁴ The delay assigned to the various photo-neutron groups is then the delay previously associated with the different gamma groups, and if these conditions are met, it is in order to add the various photo-neutron groups to the "proper" neutron groups in the Dynamic Equation. The generality of the Equation, previously dependent only on the type of fuel and, of course, k , is now lost, and a separate equation has to be set up for each specific reactor.

Although the method outlined is entirely feasible and valid, we shall treat the effects of the photo-neutrons in another way, as will be shown later, also for reasons other than the difficulties encountered in the assessment of the various parameters involved.

CHAPTER III
REACTOR SIMULATORS

That a closer look at reactor simulators at this stage is not a mere digression, but a logical step to take, will hopefully become clear later on. As a further conceptually valuable intermediate step, we shall attempt to convert the time-varying, non-linear reactor equation into a time-independent and linear Signal Flow Graph.⁵

To eliminate the time-dependence, we take the Laplace transform of equations (2.01) and (2.03), denoting the transforms by either capital letters or the symbol $\mathcal{L} [\]$.

We have:

$$sN = \mathcal{L} \left[\frac{\delta k}{\ell} n \right] - \frac{\beta}{\ell} N + \sum \lambda_1 C_1 \quad (3.01)$$

and

$$sC_1 = \frac{\beta_1}{\ell} N - \lambda_1 C_1 \quad (3.02)$$

A point in mind that is usually insufficiently stressed but lightly passed over (e.g., in the derivation of the transfer expression of a reactor in most Control Handbooks), is that

$$\text{Lap} \left[\frac{\delta k}{\ell} n \right] \neq \text{Lap} \left[\frac{\delta k}{\ell} \right] \text{Lap} [n]$$

which would indicate convolution in the time-domain, but is actually the transform of a convolution in the frequency domain.

δk is a variable, the reactivity of the reactor being influenced by a great many internal factors and externally by the operator who moves the control rods. However:

a) Assume that $n(t)$ has initial steady state value n_0 , and that variation is denoted by δn , with corresponding transform $\delta N(s)$, then the transform of $\frac{\delta k}{\ell} (n_0 + \delta n)$ can be approximated by the transform of the term $n_0 \frac{\delta k}{\ell}$, if the product $\frac{\delta k}{\ell} \delta n$ is negligible. The condition

$$\frac{\delta k}{\ell} \delta n \ll \frac{\delta k}{\ell} n_0$$

holds only for small variations of n around n_0 , which is certainly not the case under general operating conditions.

b) Conversely, split δk up in $[(\delta k)_0 + \delta(\delta k)]$. Similarly to the reasoning under a), we find that

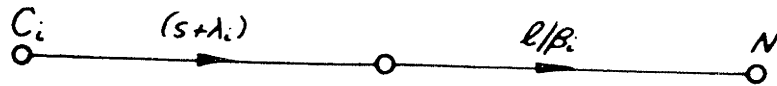
$$\text{Lap}\left(\left[(\delta k)_0 + \delta(\delta k)\right] \frac{n}{\ell}\right) \approx \text{Lap}\left((\delta k)_0 \frac{n}{\ell}\right)$$

$$\text{if } \delta(\delta k) \ll (\delta k)_0.$$

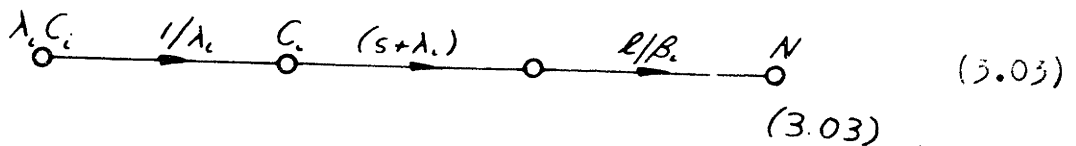
In practical reactor operation, for safety reasons, the time-rates of change of δk and n differ by a large factor, so that it is usually apparent which one to approximate by a constant, under given conditions. For our purposes, however, these limitations are too restrictive, so that we have to "delinearize", i.e., reinstate the basic nonlinearity, after manipulation of the linearized expressions has led to the required results.

The Signal Flow Graph of the Reactor Dynamic Equation.

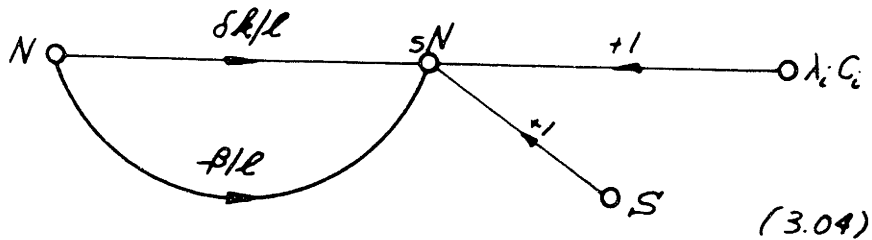
From equation (3.02)



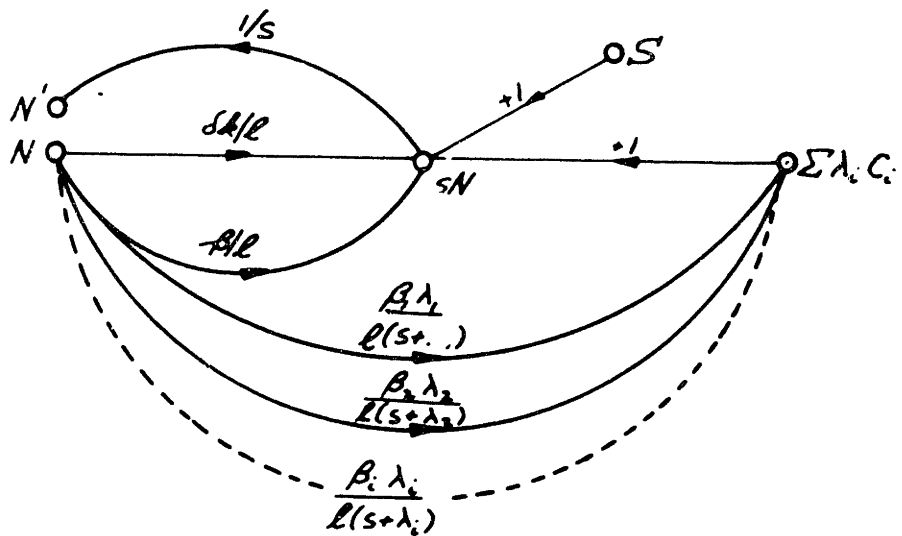
which leads to



From equation (3.01), and adding a possible source S



Since we wish to keep N in (3.04) as source node, for reasons shown later, we invert graph (3.03) and then coalesce (3.03) and (3.04).



(3.05)

— SIGNAL FLOW GRAPH OF REACTOR EQUATION. —

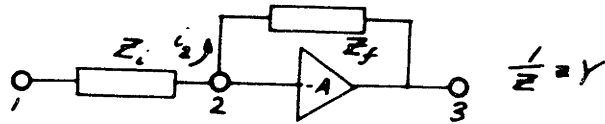
Here δk was assumed constant, and node N has already been split in anticipation of the calculation of loop-transmission, where N is now a dependent source, the dependence on sN being, of course, $\frac{1}{s}$.

$$\text{Loop transmission } \frac{N_1}{N} = \left(\frac{\delta k}{l} - \frac{\beta}{l} + \sum \frac{\beta_i \lambda_i}{l(s + \lambda_i)} \right) \frac{1}{s}$$

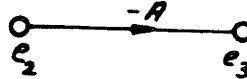
While we could have arrived at this result by very simple algebraic manipulations of equations (3.01) and (3.02), we now have on hand a signal flow graph, which gives us a direct representation of any system that obeys the reactor equation, electrical, mechanical or otherwise. Thus, we can construct a simulator directly from this graph, if we are able to build equivalent branch transmissions.

Some Elementary Computer Building Blocks.

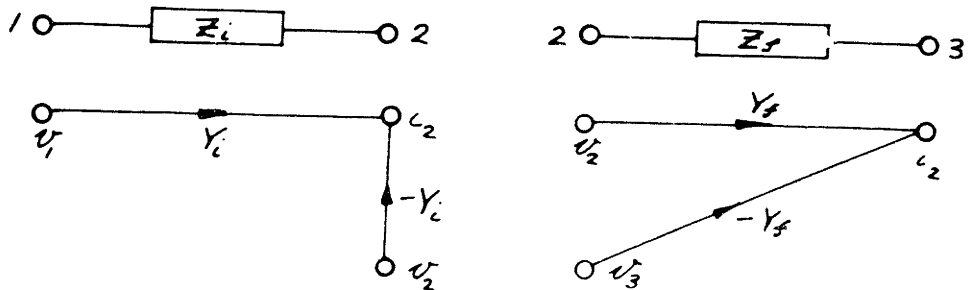
We analyse the general feedback amplifier, with the assumption that the actual voltage amplifier is almost ideal. I.e. $A > 10^6$, Input impedance $\rightarrow \infty$, Output impedance $\rightarrow 0$.



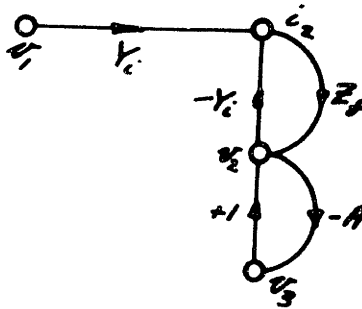
Flow graph of amplifier:



Flow graph of network:



Inverting the flow-graph of Z_f , and coalescing all three gives

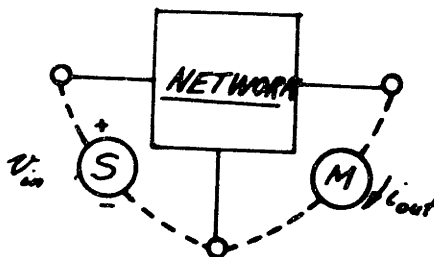


$$\frac{v_3}{v_1} = \frac{-AZ_f Y_i}{1 + (Z_f Y_i + A)}$$

$$= \frac{-Z_f}{\frac{1}{AY_i} + \frac{Z_f}{A} + Z_i}$$

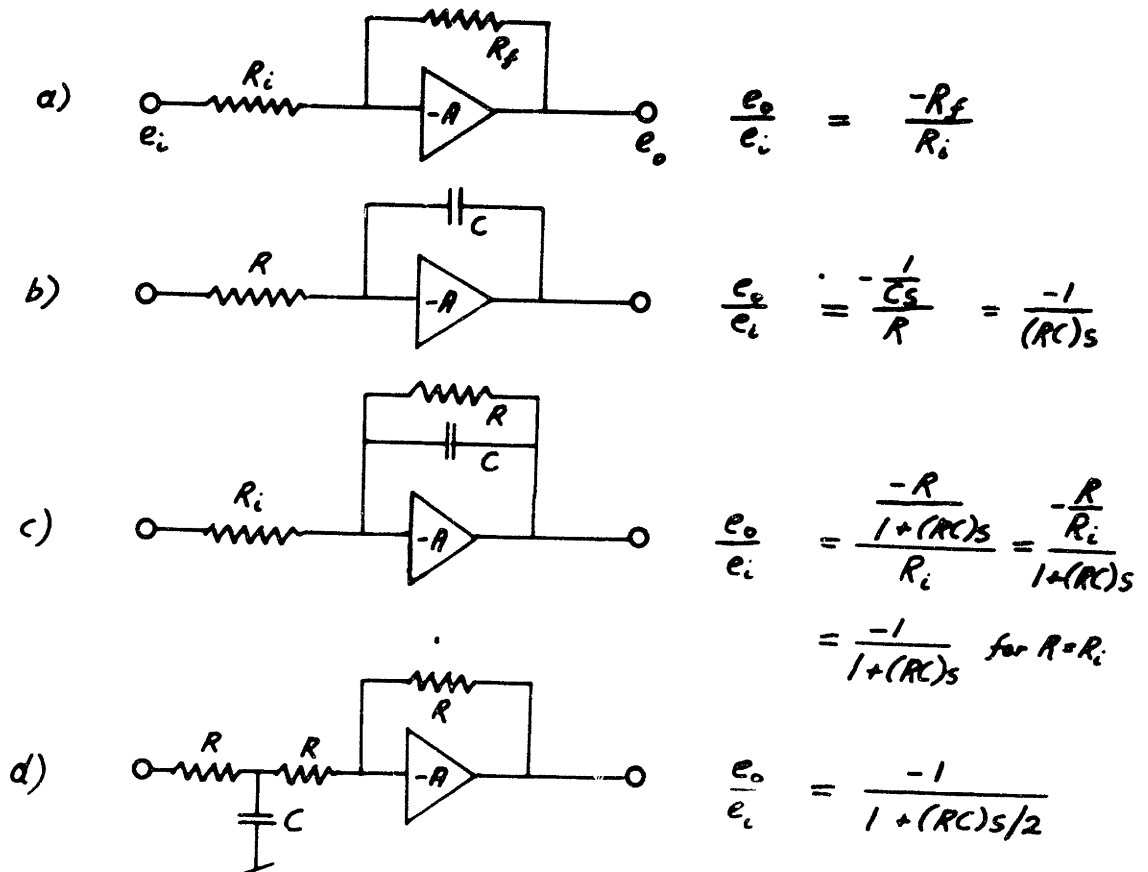
$$\frac{v_3}{v_1} \approx -\frac{Z_f}{Z_i} \quad \text{with } A \gg \gg \gg 1.$$

Usually, Z_f and Z_i , defined as "transfer impedances" can be found by inspection. In the case of more involved circuitry, however, it is very useful to employ topology⁵ to evaluate the "transfer admittance", i.e., the output current due to a unit input voltage, which is the inverse of the transfer impedance.



This holds for both two- and three-terminal networks. (For the case of current-amplifiers, the same analysis is valid, then, of course, in terms of the duals).

Using the technique outlined above, we now have the following building blocks with their respective transmissions:

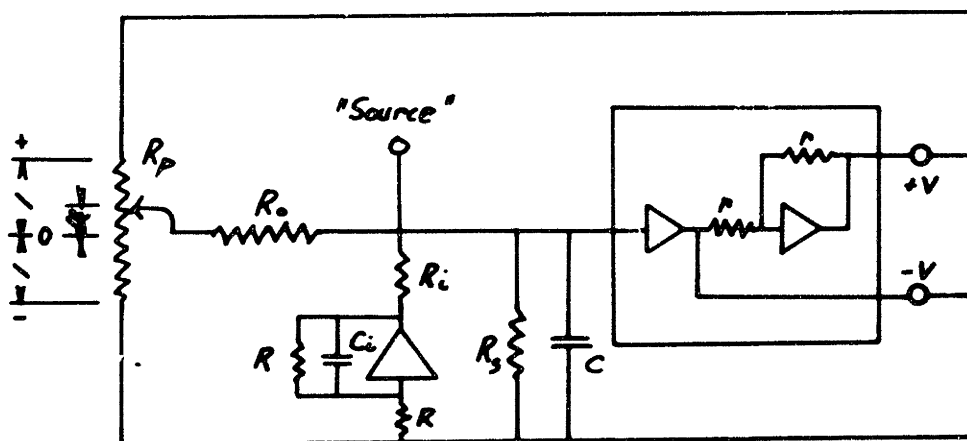


Note that d) is equivalent to a RC network followed by an amplifier of unity gain, and thus could be approximated, apart from the sign change, by a RC network followed by a stable cathode-follower. For economic reasons c) can then be replaced by the similar d).

At last we are in a position to draw the block-diagram of a possible simulator, by using the signal flow graph of the

reactor equation and the transmissions of the various building blocks, while observing the involuntary sign changes encountered. In deciding which analogues to use, and what the scaling factors are, it is helpful to note that the dimensions of time must be preserved; e.g., ℓ in its analogue must still have the dimension "seconds", while λ has the dimension "per second", indicating that a time-constant and the inverse of one, respectively, are to be used.

After considerable, though obvious, simplification and elimination of possible duplication, we have the following block diagram of a reactor simulator, with the conversion factors as indicated.



REACTOR SIMULATOR.

$$\begin{aligned} \ell &= R_0 C & \beta &= R_0 / R_s \\ \beta_1 &= R_0 / R_1 & \lambda_1 &= \frac{1}{RC_1} \\ n &= V \end{aligned}$$

- Note:
- 1) "Multiplication" is achieved by a potentiometer approximation. R_p should be as small as possible in comparison to R_0 as is consistent with the load-characteristics of the amplifiers. Accuracy is sufficient for small excursions of δk , and can be improved for larger changes by calibration.
 - 2) An extraneous neutron source S can be added by injecting a current = $Sx C$ at the point indicated.
 - 3) The number of D.C. amplifiers required is equal to two plus the number of groups whose effect it is desired to include. The higher the long-term accuracy required, the more groups have to be included and the stricter the stability characteristics required of the amplifiers.
 - 4) Due to noise, the range of the simulator is restricted to about 1000:1, apart from linearity and stability considerations.
 - 5) The model for the simulator is only valid for small excursions of δk . While it has been shown that this is the case for positive δk (although somebody might be interested in the behaviour of a reactor beyond prompt criticality), large negative δk is introduced for the purposes of shutdown, and we have to modify the model.

- 6) Within the limitations pointed out under 5), the simulator obeys the reactor equation exactly. The approximations bear only on the signal flow graph, where they were introduced to linearize the equation.
- 7) Instead of using a high-gain amplifier for each group, it is feasible to replace each by the equivalent unity-gain (cathode-follower) circuit, although some accuracy and stability is lost. A cascade arrangement⁶ is an improvement.
- 8) The use of individual amplifiers for each group, while making computations and adjustments very simple, is not really justified, as they are here employed only to obtain a certain transfer function. Instead, each group circuit can be replaced by a simple series RC network, which then, of course, affect (load) each other, and makes computation of the parameters in terms of the scaling factors very involved indeed. Furthermore, adjustment of individual groups is almost impossible because of the interaction with the other groups. This simplification is then only advisable for short-term, low-accuracy and, of course, inexpensive simulators, where for example a three-group approximation is sufficient.

Many of the above remarks are applicable to the reactivity monitor in its variations, and should be kept in mind.

CHAPTER IV
THE MEASUREMENT OF REACTIVITY

In this chapter, reactivity monitors of increasing accuracy and sophistication but, alas, also increasing complexity and cost will be evolved.

A. Period-only derived reactivity.

If the effects of the various delayed neutrons and photo-neutrons are ignored, that is, it is assumed that all the neutrons created in the fission process are given off instantly and have a lifetime of ℓ , the reactor dynamic equation simplifies to

$$\frac{dn}{dt} = \frac{\delta k}{\ell} n$$

Integration yields

$$n = n_0 \exp \frac{\delta k}{\ell} t$$

The "time-constant" $\frac{\ell}{\delta k}$ is defined as the period of the reactor, which is the time which the reactor would take to change its level by a factor of $e = 2.716$ ("e-folding time").

Thus the period $T = \frac{n}{\frac{dn}{dt}}$

although the inverse period $T^{-1} = \frac{\frac{dn}{dt}}{n}$ is the quantity usually measured.

Even though the change of the specific flux-level cannot be described by a single exponential, as is shown in the more complete reactor dynamic equation, it is customary to still

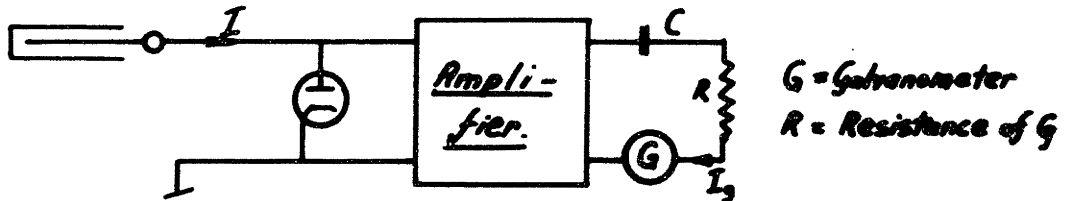
talk of a reactor period, which then has only the meaning of

Period = Ratio of Neutron flux to the time rate of change of the flux.

This quantity is generated in exactly this fashion, and the period meter has a prominent place on the reactor control panel, because the period is a dynamic quantity.

$$\frac{1}{T} = \frac{\delta k}{\ell} = \frac{1}{n} \frac{dn}{dt} = \frac{d(\ln n)}{dt} \quad (4.01)$$

Method 1.



$$I = Dn$$

where D is a constant.

$$V = A \ln Bn$$

where A and B are constants.

$$I_g = \frac{dV}{dt} C$$

when the time-constant RC is small.

$$= CA \frac{d}{dt} \ln Bn$$

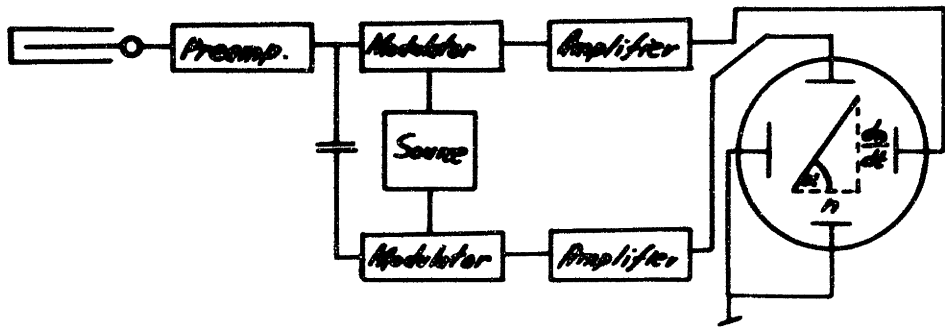
$$\text{Thus } I_g = \frac{E}{T} = E \frac{\delta k}{\ell}$$

where E is a constant

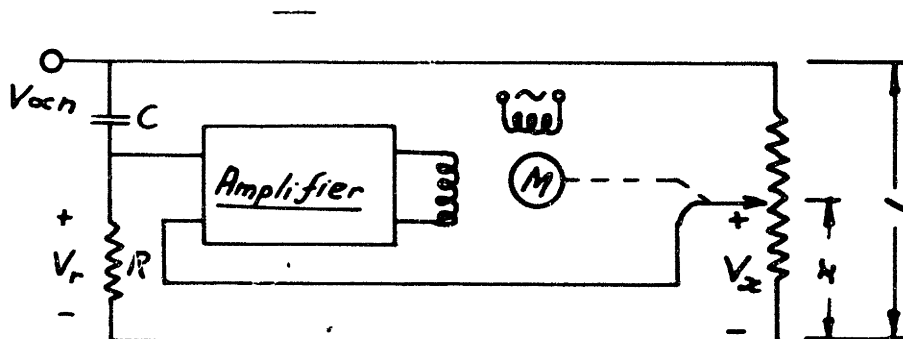
$$\text{or } \delta k = F I_g$$

where F is a constant.

It is thus only necessary to calibrate G in terms of δk .

Method II.

Here components proportional to n and $\frac{dn}{dt}$ are amplified after first being synchronously modulated, ("chopper", vibrating reed, etc.) and are then applied to the horizontal and vertical deflection plates of a cathode ray oscilloscope. If the modulators are exactly in phase, the resultant Lissajous figure is a straight line inclined at an angle α to the horizontal, where $\tan \alpha$ is proportional to δk . If the screen is marked off in a family of straight lines which are calibrated in terms of δk , a very sensitive and elegant monitor results.

Method III.

Here use is made of a bridge-method, where the split-phase motor M keeps the bridge in balance. Because most high-quality strip-chart recorders operate on the same general principle, and thus already incorporate the amplifier, motor, drive and potentiometer, they are easily adapted for this circuit. Reference will be made to this circuit as soon as we wish to incorporate automatic division in a system.

$$V_R \approx RC \frac{dn}{dt}$$

$$V_x = xn$$

In balance, $V_R = V_x$

$$x \propto \frac{1}{n} \frac{dn}{dt} \propto \delta k.$$

Although the above methods do not take into account the presence of delayed neutrons, they do manage to continuously evaluate δk from the flux-change in the reactor. They do this accurately, if the usual definition of δk

$$\delta k = \frac{k - 1}{k}$$

is reformulated to

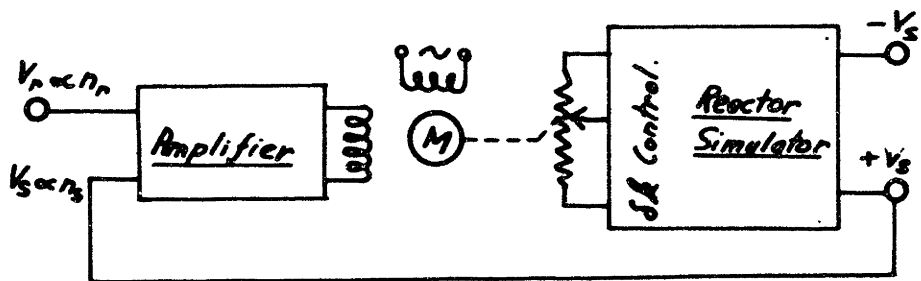
$$\delta k = \frac{\ell}{n} \frac{dn}{dt}$$

Although this may seem very artificial and far-fetched, it makes just as much sense as the assignment of a period or time-constant to a behaviour which can not be described by a single exponential. Furthermore, ℓ can be "adjusted" to give values of δk more nearly consistent with the complete definition.

This can be seen by the fact, that a step-change in reactivity will result in an initial steep rise of the specific flux, which then tapers off due to the effect of the delayed neutrons. The slope of this steady rise is an indication of the reactivity step introduced. In the case of the heavy-water moderated M.I.T.R., because of the large photo-neutron effect, the time elapsed till steady rise is obtained is so long, that the reactivity has changed due to other effects (e.g., temperature), and the methods outlined are ineffective. They are included here, however, because they are usable for other reactors which are not heavy-water moderated. Then they enable the extremely simple construction of reliable, if not instantaneous, reactivity monitors.

B. Synchronous Simulator Method.

In the following, subscript r pertains to reactor, and subscript s to simulator characteristics.



The amplifier feeds a split-phase motor M which adjusts the δk control of the simulator until the input to the amplifier = $(V_r - V_s)$ goes to zero. The sliding tap of the δk control can double as recording pen, as is the case in all potentiometer-type recorders, and thus δk can be recorded and/or displayed continuously.

If a suitably accurate simulator for a reactor is on hand, this can then be converted quickly into a reactivity monitor by the addition of a slightly modified strip-chart recorder of suitable potentiometer branch value. Apart from the presumably very small effects of static friction, this type-one system (one integrator in the feedback loop) will have zero error for inputs of order lower than two, i.e., impulse and step-functions, constant error for order two, i.e., a ramp, and infinite error for all higher order inputs. (The "integrating" action of the simulator is not included because of its inherent non-linearity). During start-up, when the flux increases approximately linearly, the monitor will thus lag behind, although the lag can be made arbitrarily small, subject only to stability considerations.

Inasmuch as the reactor transfer function has already been analysed in great detail,¹⁴ and is used with confidence, the design of this system should not be too difficult, although it is hardly worth the while for reasons to be given shortly. If these objections do not hold, the circuit can readily be analysed, and the whole army of compensation-methods employed to reduce the error.

The one and only drawback to this method is that it is extremely difficult to build a simulator of sufficient range, accuracy and stability, if the system is to be in operation continuously. Especially the effects of the photo-neutrons with their extremely long associated time-constants are difficult to handle by the methods outlined previously.

If, however, a simulator is on hand, the method is close to ideal, because the simulator can at any time be used separately for the purposes it was originally designed for, even though during that period of use the reactivity is no longer monitored.

Extreme care has to be exercised while providing the system with range-switching facilities, if the reactivity is to be monitored over a range of neutron flux which the simulator can no longer encompass. While it is true that if, for instance, the proportionality factor of V_r is changed, V_s will eventually settle down to the same value, and the system from then on will again perform accurately, it must be kept in mind that this settling time will be of the order of hours, especially if the photo-neutrons are included. During that whole interval δk would only be correct within a decreasing fraction of β . This problem could be solved by changing the stored information (the voltages on the capacitors in the delay networks) by a corresponding factor during switch-over. This is clearly not an easy task, especially if it has to be performed automatically.

C. Sampled Data and Digital Computer Methods.⁷

Although these methods are of considerable theoretical interest, the fact that a complete (and fast) computer is used to continuously solve the reactor dynamic equation is akin to shooting at sparrows with cannons. But in principle, at least, it would be possible to continuously sample the neutron flux signal, where the difference between successive signals, being a very good approximation of dn/dt is fed to a computer which then acts on it comparatively to the previously recorded and stored inputs. Since the variations are slow, band-limiting is no problem. Another interesting aspect is, that instead of using the steady signal of an ionization chamber, one could instead directly sample the pulses originating from a counter-type transducer. Within the statistical variation of these signals, which limit the range over which a fixed-position transducer can be employed, this information is directly digestible by a digital computer.

While a system of this sort possesses none of the drawbacks of stability and time-variation considerations, a thorough analysis is not attempted here because of the reason given above.

D. Direct Analogue Computation Method.

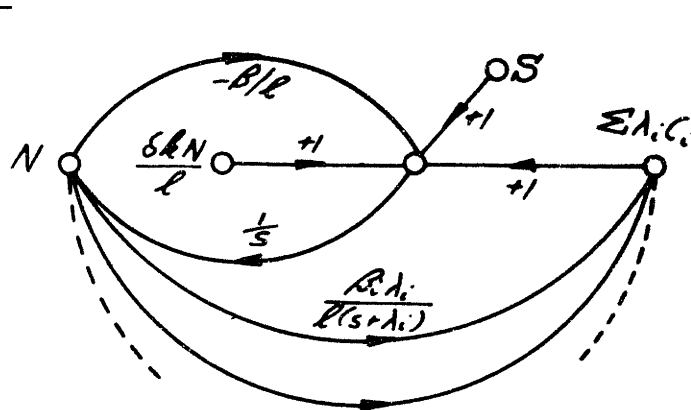
Here the Dynamic Equation is rearranged, and simulated by an analogous electrical system. This method seems to be the one most suitable for the semi-continuous monitoring of reactivity. It is semi-continuous because provision has to be made for periodic re-adjustment. Because of the inherent

integrating action of the device, the attempt to set up a really continuous device is similar to the attempt to measure (or integrate) time by a more or less crude clock, without re-adjustment of the latter. This is so because an integrator is essentially an unstable system, (a bounded input produces an unbounded output), and any arbitrarily small error-input will result in an error-output that grows out of all bounds with increasing time. An attempt can, of course, be made to increase the time interval between adjustments. So, for instance, a "D.C." system with its problems of time-varying, non-correlatable offset (lack of stability, drift), can be converted to a system modulated by a crystal-controlled frequency. This obviously introduces major complexities, and because the M.I.T. reactor usually is shutdown for a 36 to 60 hour period during each week, a time-span of one week between re-adjustments seems to be indicated, this being a sufficiently short period to be handled by stabilized D.C. equipment.

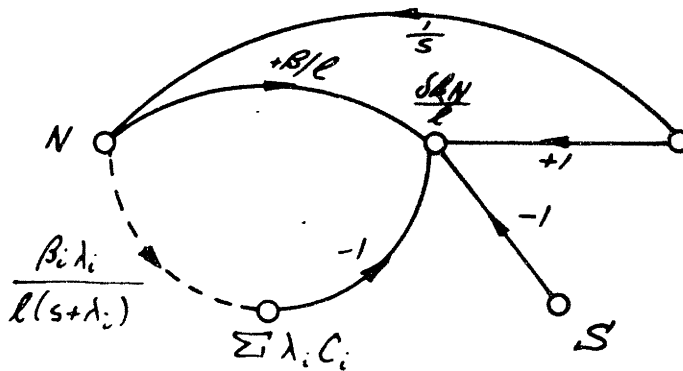
It is again stressed at this point, that the photo-neutrons will not be handled by an analogue operation on the neutron flux derived signal, primarily because of the difficulties introduced by the sundry extraordinarily long time-constants involved, and also because of the difficulty encountered during the assessment of the various parameters with any measure of accuracy. Instead, we shall show that the effects of the photo-neutrons can be treated as those of an equivalent, variable source of neutrons. An attempt will also be made not to include pure integrators (root at $s = 0$) in the system.

(i) Elementary Block Diagram.

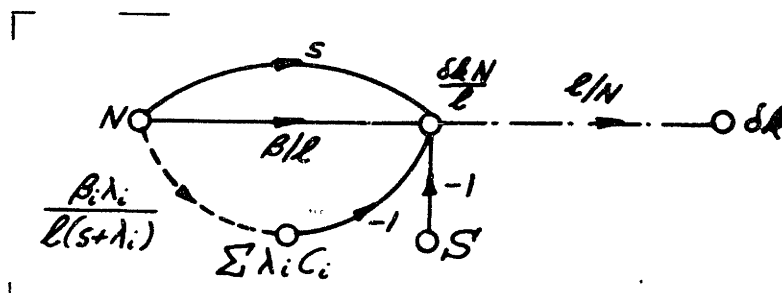
We had (3.05):



which results after inversion in

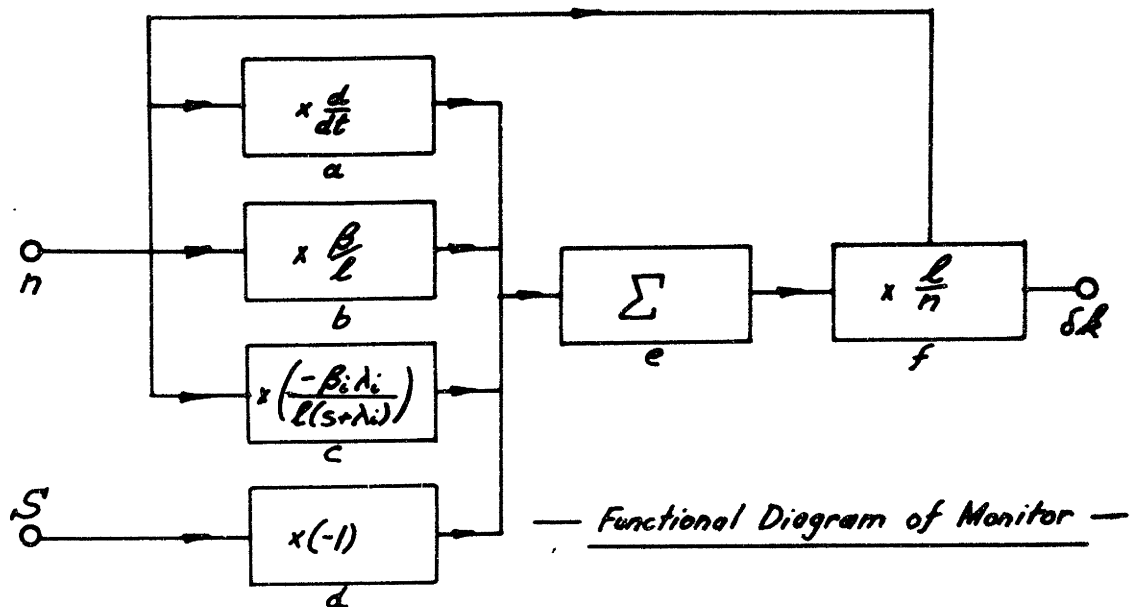


This leads to



Note that again the product $5kN$ has been somewhat loosely dealt with, but that the final process will again be exact, similar to the case of the derivation of the simulator block-diagram.

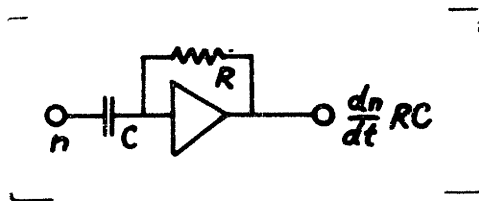
To an intermediate² step we translate the signal flow graph into a block-diagram, and then show what each block consists of.



Here the expression $x \left(- \frac{\beta_1 \lambda_1}{l(s + \lambda_1)} \right)$ is used, although the operation now takes place in the time-domain, and is only loosely used to describe the actual performance of the various delay-channels.

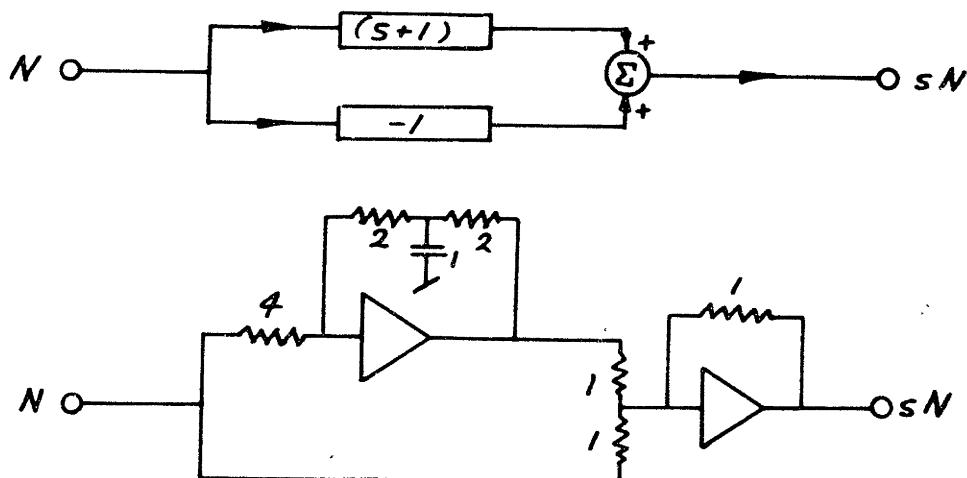
(ii) Building Blocks

a) For the differentiation network, within a change of sign, one could use



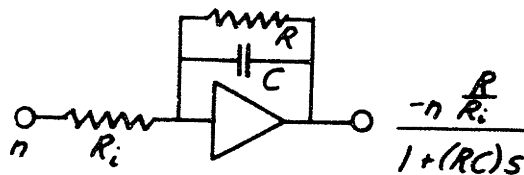
This circuit is not recommended, however, because of noise considerations. Analysis of a noise equivalent circuit shows that it is desirable to keep the input impedance Z_i as high as possible. Since a capacitor's impedance reduces with increasing frequency, and since noise is of higher frequencies, this condition is not fully met. Similar considerations do not apply to the impedance Z_f , because the negative feedback causes a cancellation of the noise effects.

Thus the following circuit is recommended, where the summing circuit can be included in the main adder. The accompanying block diagram shows the operations involved.



- b d). The $x\beta/\ell$ and $x(-1)$ operations are performed by putting $Z_1 = 1$, and $Z_f = \beta/\ell$ and 1, respectively, around two amplifiers. Algebraic signs will be cleared up later on, as also the fact that multiplication by a constant can actually be included in the main adder e), whose circuit is very straightforward and will be shown in detail later.
- f). The multiplication by $\frac{\ell}{n}$ or the equivalent division by n/ℓ will be done by the now familiar bridge-method using a modified potentiometer-type recorder.
- c). The delay-channels present the thorniest problem from an economic point of view (there are 6 "proper" delayed neutron groups) and the various solutions are presented in some detail.

As the most obvious and least subtle approach, we could use the following circuit for each channel:



The transfer function can be rearranged

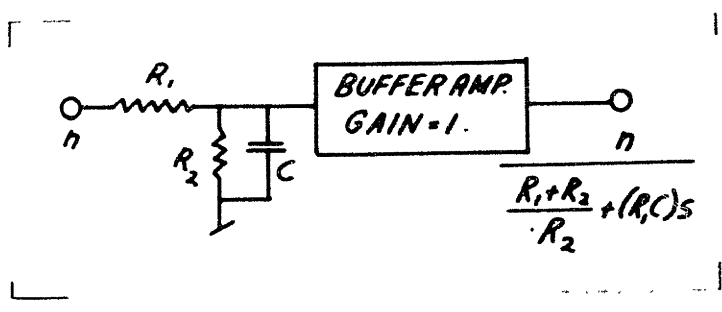
$$- \frac{\frac{R}{R_i} CR}{s + \frac{1}{RC}} = \frac{\beta_1 \lambda_1}{s + \lambda_1} \ell$$

Thus

$$\frac{1}{RC} = \lambda_1 \quad \frac{R}{R_1} = \frac{\beta_1}{\ell}$$

This enables us to determine two of the circuit elements in terms of the conveniently chosen third one.

Another variation would be the following circuit built around a less costly amplifier, but also one which would be more difficult to stabilize (cathode-follower).



We have

$$\frac{\frac{R_1 + R_2}{R_1 R_2 C} \times \frac{R_2}{R_1 + R_2}}{s + \frac{R_1 + R_2}{R_1 R_2 C}} = \frac{\lambda_1 \beta_1}{s + \lambda_1}$$

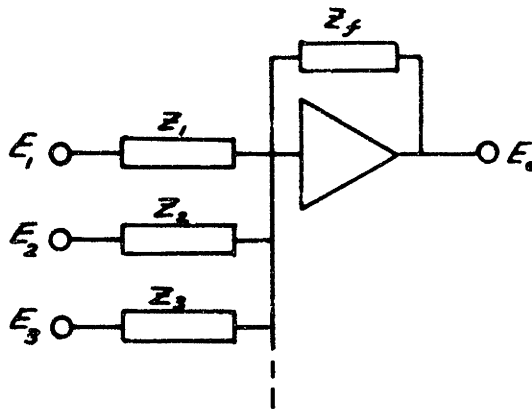
giving

$$\frac{R_1 + R_2}{R_1 R_2 C} = \lambda_1 \quad \frac{R_2}{R_1 + R_2} = \frac{\beta_1}{\ell}$$

The same comment as for the previous model applies. The buffer amplifiers are necessary to prevent the interaction between channels (loading).

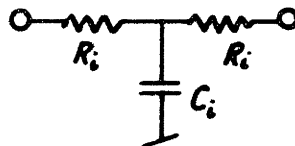
What happens if we do allow interaction to take place? Although we do not expect to get such neat, easily evaluable equivalences as before, the computational effort and the

resulting odd values for the circuit-elements are more than compensated for by the fact that we would save six amplifiers. This effort, fortunately, is not necessary, because the adder is capable of more than just straight summation. This is brought to mind by the following:



$$-E_o = Z_f \left(\frac{E_1}{Z_1} + \frac{E_2}{Z_2} + \frac{E_3}{Z_3} + \dots \right)$$

If we make Z_f a resistor of value unity, connect the inputs together to n , and replace the Z_1 by the following



$$Z_i = 2R_i \left(\frac{R_i C_i s}{2} + 1 \right)$$

Then

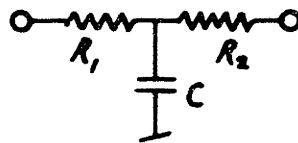
$$\frac{1}{Z_1} = \frac{\frac{\beta_1 \lambda_1}{\ell}}{s + \lambda_1}$$

$$Z_1 = \frac{\ell}{\beta_1} \left(\frac{s}{\lambda_1} + 1 \right).$$

Thus

$$\frac{R_1 C_1}{2} = \frac{1}{\lambda_1} \quad 2 R_1 = \frac{\ell}{\beta_1} \quad (4.02)$$

An additional degree of freedom is introduced when the following Z_1 is used:



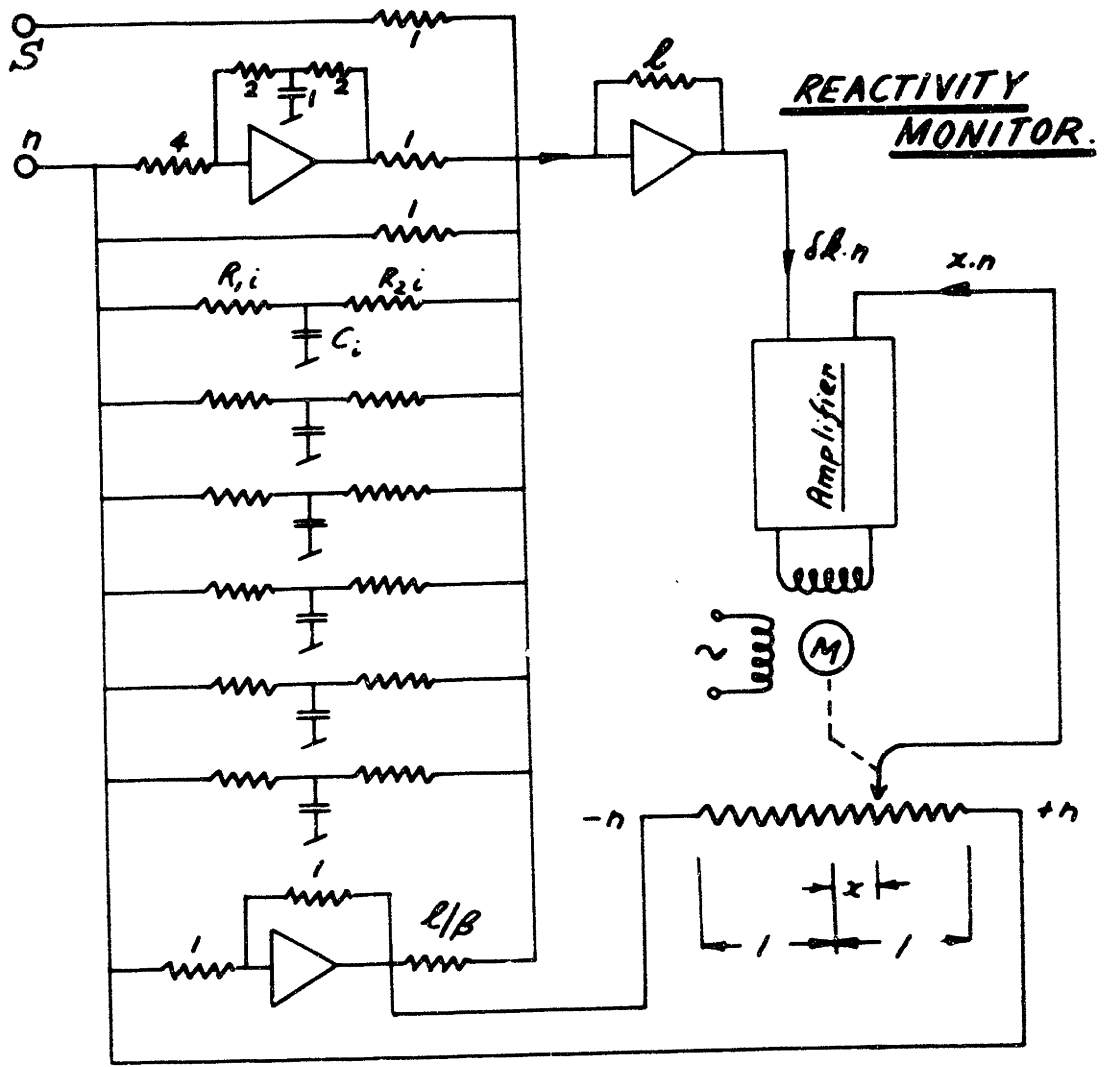
$$Z_i = (R_1 R_2 C s + R_1 + R_2)_i$$

Then

$$\left(\frac{R_1 R_2 C}{R_1 + R_2} \right)_1 = \frac{1}{\lambda_1} \quad (R_1 + R_2)_1 = \frac{\ell}{\beta_1} \quad (4.03)$$

This enables one to first choose C at a convenient value, and then determine R_1 and R_2 . Care must be taken to choose C large enough to ensure positive R_1 and R_2 .

(iii) Complete Block Diagram.



Here the adder has been further modified to multiply $5k.n/l$ by l , and division by n is done by a modified strip-chart recorder, which then automatically records $5k$, because in balance

$$5k.n = x.n$$

$$5k = x.$$

The relative magnitude of the circuit elements is given, where R_{11} , R_{21} and C_1 can be evaluated from (4.03) or (4.02).

(iv) The Photo-Neutron Source.

It is required to determine the neutron-equivalence of the gamma-flux in the reactor. This will evidently be a function of the number of photons, the number of D-atoms, the flux-shape, the photon energy distribution (gamma-spectrum) and the energy-dependent cross-section for the (γ , n) reaction. Of these, the number of D-atoms and the flux-shape are constants, and the number of neutrons is related to the number of photons of a specific energy. As we are interested only in thermal neutrons, another function is introduced pertaining to the losses of neutrons during slowing down. At a fixed point, however, we can write

$$n_{\text{thermal}} = K \int \Gamma(E) \sigma(E) dE \quad (4.04)$$

where

K = a constant, to be determined by calibration

$\Gamma(E)$ = gamma spectrum

$\sigma(E)$ = cross section for (γ , n) reaction.

As soon as we change our observation point, K will change. This is important in view of possible scale-changing.

$\sigma(E)$ is known,¹⁰ and if $\Gamma(E)$ were constant, we could measure it once, and thus determine the integral. In fact, then K and the integral could be combined into a new constant, evaluable by calibration. $\Gamma(E)$ is not fixed, however,

because the spectrum hardens after start-up, and conversely softens with time after shutdown. This is a consequence of the Geiger-Nutall law. We thus have to build a device which will continuously evaluate the integral, within a constant factor.

The most direct device would consist of a neutron-sensitive detector (gamma-compensated, boron activated chamber or counter) placed in a large volume of D_2O , which is surrounded by a cadmium foil to keep out slow neutrons. This device, if placed where only well-thermalized neutrons are present, and if able to see a representative sample of the γ -flux, would give out either a current or a pulse-rate proportional to the integral. The disadvantage of this method is that a large volume of D_2O is required to keep the intrinsic efficiency approximately constant, and this with its cadmium shield has to be placed in a very high gamma-flux to achieve any significant output currents or count-rates with a sufficiently small statistical spread, respectively. This means then that we have placed a large absorber of neutrons into the reactor with resulting loss in neutron economy, and flux depression. Where this is no objection (the device can for instance be used as a rough control rod), the method is recommended.

Another device could be built up around an energy-dependent gamma-detector (e.g., scintillation counter), on the output of which more or less sophisticated electrical operations are performed. Although a monoenergetic beam of gammas

impinging on, say, the thallium activated NaI crystal of a scintillation counter does not result in an output of pulses of constant height, there is a pronounced peak in the pulse spectrum, and we shall thus assume that the output pulse height is a function of energy.

First Approximation.

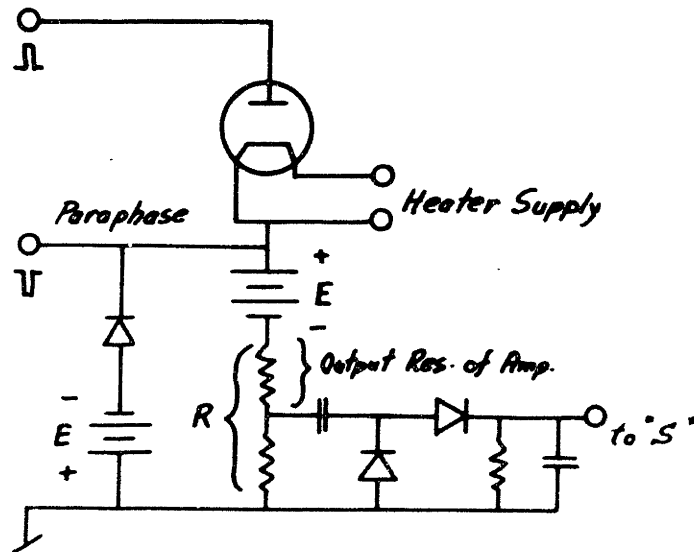
Here the energy dependence of σ is approximated by a step function occurring at the threshold. It is thus only necessary to discriminate between gammas of energy higher or lower than the threshold energy. This is easily accomplished by feeding the linearly amplified pulses into a Schmitt-trigger circuit and applying the resultant shaped output, representing the (fixed) neutron equivalence per gamma, to a standard diode-pump rate circuit,⁸ the output of which then is the equivalent photo-neutron source of dimensions neutrons per second. Apart from range-switching, and the overall evaluation of K by calibration, only one adjustment is necessary and that is the threshold equivalent setting of the Schmitt trigger.

Second Approximation.

The (i, v) curve of a directly heated, current-limited and pre-biased vacuum diode⁹ is similar in shape to that of σ versus energy.¹⁰ While the space-charge limited part of the curve deviates slightly from the cross-section behaviour around threshold, the region of pronounced Schottky effect is a near perfect approximation of the behaviour of σ beyond 2.3 MeV. A voltage proportional to the current is applied to a standard rate circuit by the resistor R, which should include

the output resistance of the preceding linear pulse amplifier. The threshold voltage is represented by battery E, which could, of course, be replaced by a fixed current through R, externally applied. To prevent a pulse with fast leading edge from passing through the equivalent capacitance of the diode, even though its height is not sufficient to cause conduction, a paraphase signal is injected. This is an out-of-phase signal, equal in magnitude to the signal proper, but not exceeding the blocking voltage. Through variation of R a good approximation to $S \times \sigma$ is obtained for the output to input voltage characteristic, where S is the energy proportionality constant for the scintillation counter. Because the device is very sensitive to variation of cathode temperature, the heater supply should be well regulated (and adjustable).

With a more sophisticated type of rate circuit, the system is capable of speeds in excess of 100,000 p.p.s., where the pulse rate is limited mainly in the linear amplifier and, of course, the rate meter. The system is more accurate than the previous one and is not more involved.



Third Approximation.

Let the output of a specific scintillation counter due to a uniform gamma spectrum be described by $M(E)$. It is necessary that this be accurately known in the energy band of interest, i.e., from threshold to about 8 MeV. Let the cross-section be described by $\sigma(E)$. We can then determine $F(E) \equiv \sigma(E) M(E)$. This we attempt to represent, within a constant factor, by a piece-wise linear approximation (diode circuits),¹¹ and fail to do so because $\partial^2 F / \partial E^2 < 0$. This can be seen by the fact that $\sigma(E)$ has a negative second derivative, as will $M(E)$ in all probability, because the intrinsic efficiency of the counter decreases with energy, unless a very large crystal is used. We can, however, reproduce $F^{-1}(E)$ to any reasonable degree of accuracy, because $\partial^2 F^{-1} / \partial E^2 > 0$.

If this "energy" (voltage) dependent circuit is now applied as feedback circuit to the formerly linear pulse amplifier, the resultant output will give the neutron equivalence per gamma to the degree of accuracy of the piece-wise linear approximation. This is readily seen either by flow-graph analysis, or the fact that

$$\text{if } i(v) = F^{-1}$$

$$\text{then } Z_f \equiv v(i) = F.$$

Thus

$$\frac{e_{\text{out}}}{e_{\text{in}}} = - \frac{Z_f}{Z_1} = B F(E) = B \sigma(E)M(E).$$

where B is a constant.

Care has to be taken to build up the feedback network with high-speed switching diodes, else the maximum pulse rate of the system is severely limited.

The output is once again applied to a rate circuit, where the previously rendered remarks apply.

This system is by far the most accurate, and could be used to test a theory of photo delayed neutrons, where the system would give the actual photo-neutron source, and our analogue simulator would give the photo neutron source according to theory. Since this experiment could be done in a relatively short time span, the simulator need not be extremely stable and the necessary very long time-constants can be achieved by substituting operational amplifiers (Miller effect) for the capacitances. The design can then proceed exactly

along the lines sketched for the "short" time-constant proper neutron delayed groups.

In all the three methods the overall gain (constant K) can be adjusted by varying the input resistance of the source input to the main summing amplifier. If it should be too bothersome to change the position of the scintillation counter to cope with the extreme variations of gamma flux during reactor operation, then at least the tank circuit of the rate meter should be switched so as not to overload it (the voltage across the tank circuit should be small compared to the average height of the incoming pulses), and also to make selection of a proper time-constant possible, to achieve a more or less uniform statistical variation. This will then, of course, change K, and the input resistance has to be switched synchronously.

(v) Scale Changing.

As can be seen from the block-diagram, the gains of the three operational amplifiers are all low, so that the system can accept input variations of 1000:1 with ease without either overloading the amplifiers or having signals that are not at least two orders of magnitude greater than the inherent noise-level. Of course, from practical considerations, one would also be able to read the result with ease and accuracy. This is readily achieved by the amplifier scale switches already built into most servo-controlled recorders, and, of course, by switching in equal resistors on the ends of the divider-potentiometer so as always to have a readable deflection. It is seen that the system is fully adapted to cope with signal

ratios of 1000:1. This is also approximately the range over which the present fixed linear neutron channel of the M.I.T. reactor gives usable outputs, and the range over which the photo-neutron source will be able to operate with efficiency. The only addition to the instrumentation of the reactor, apart from the monitor and its photo-neutron source, is a high-grade current amplifier to supply a suitable voltage to the n-input, because the existing amplifier employed to drive the linear-channel recorder has to be switched to cover the complete range. Since chopper-stabilized amplifiers,¹⁵ that are proposed for the computer amplifiers, are not very suitable for the amplification of the low chamber-currents encountered, a vibrating-reed electrometer is to be preferred.

Why scale-changing is to be avoided, and to what extent, will become apparent as we investigate the possibilities of extending the range of inputs. Various difficulties are encountered.

- (a) Either the position of the scintillation counter for the photo-neutron source has to be switched (commendable), or the rate-circuit parameters have to be changed. An upper limit to the fixed-position design is set by the fastest pulse-rate that the system will digest, and a lower limit by the tolerable statistical fluctuations as opposed to the speed of response of the whole system. A range of 100,000:1 would just barely be practical for the fixed position type, while any desirable variation is possible for the movable position type. For both systems

the variation of parameter K are accommodated by synchronous switching of the source channel input resistor. This is not objectionable.

- (b) For the best results, the neutron chamber should also be movable so as to yield constantly a near-maximum signal, with resultant less stringent requirements of the current amplifier. But, however, an optimum compromise of accuracy versus involvedness is reached, the fact remains that as soon as the 1000:1 limit, previously mentioned, is exceeded, a scale-changed input will be presented to, and required by, the neutron channel. If all the voltages in the circuit are changed by the same factor, the output will not change. (Range-switching of the servo-amplifier and the potentiometer need only accommodate the variation of $5k$). During switch over the differentiation circuit will be temporarily overloaded, and a pronounced excursion of short duration by the recording pen will result. This can serve as marker for the switching time, or if objectionable, can be reduced by temporarily opening the output of the differentiation channel. Far more objectionable is the fact that the time-delay circuits will first have to settle down before the output is once again reliable. As the longest time-constant involved is about 80 s, a period of say 400 s has to elapse. If it is agreed that scale-switching will take place at approximately the same

upper or lower limit of a certain neutron-signal range, the divergence time can be appreciably shortened by changing, during switch over, the calculable steady-state voltage across the capacitors by the new range-constant. This is best done by temporarily connecting the capacitors to pre-set voltage sources of low-internal impedance. Of course, this method is only approximately correct, as there is no predicting what the actual voltages will be, when the system signal is not steady. But because of the slow variation of the signal an improvement is achieved as now the circuits will not have to change by the full scale factor, but rather by the error-factor. - - - Another way out would be given by an automatic system that, during switch over, scans the voltages, multiplies them by the range-constant (holding circuit) and applies these new voltages in turn to the capacitors. This method, besides being mechanically involved (e.g., motor-operated switches) would decrease the error time-span not very significantly over that of the previous one, while both systems have the disadvantage that undue stresses are put on the capacitors, and that the remanence of the dielectric will result in a gradual increase of voltage after a discharge. - - -

The simplest method is to have an extra bank of R-C-R circuits, short-circuited at the output, and fed by a voltage source proportional to the actual

input voltage by the expected next scale-changing factor. As the span of time between range switching is much longer than the changing time of the slowest circuit, the bank has ample time to prepare itself for the next switch-over, when it is connected to the system, while the original bank is prepared for the consecutive switch. This presupposes knowledge of the operator to which scale he will have to change next, higher or lower, because we cannot build a prophetic system (non-realizable). The method then requires only an additional operational amplifier to pre-change the delay channels, an extra delay channel set, and some switches.

Summarily it can be said that range switching presents no real problem from the view-of-point of the monitor proper, but introduces complexities into the signal system where positioning devices are then required for far-extended ranges.

(vi) Calibration.

The monitor proper requires no calibration. All ratios are calculable directly from the block-diagram and equations (4.02) and (4.03). The reading-out scale factor is also directly calculable if the obvious extension is made that "1" is not just the half-value of the dividing potentiometer, but the half-value plus the switchable external resistor, that is used for changing that scale factor. This resistor is not shown in the block-diagram, but of course equal resistors are required, one at each end of the potentiometer.

The one undertermined factor is K in equation (4.04). Assume that the reactor has been shut down for a sufficiently long time so that the multiplication factor (k less than unity) has settled down to $1/(1 - k)$. Then the neutron-flux due to a steady source S would be $S/(1 - k)$. k is known with sufficient accuracy from the design-considerations and other measurements on the reactor, and although S is no longer a steady source, but also declines slowly with time, this effect is negligible if sufficient time has elapsed so that only the dominant time-constant in the photo-neutron theory remains. The neutron-signal is also known, therefore, S is determined, and K , though not evaluated explicitly can be adjusted to its proper value by adjustment of the input resistance of the S channel. (We know what S should be for unity input resistance - this follows from the analogue and the calculation carried out above. If the real S is different, we simply adjust the input resistance to $S_{\text{real}}/S_{\text{calculated}}$).

This calibration has to be done only once for each detector position, although it is, of course, advisable to check it from time to time. It is also recommended that the amplifiers be compensated for drift, which will not be necessary more than at the most once a week, for the type of amplifier envisaged. This is readily done by shorting all inputs, waiting for a sufficient time, and adjusting for zero output. After reconnection, ample time has to be allowed for the system to settle down. It is thus advisable that this adjustment be done over the week-end.

(vii) Accuracies.

In the following discussion an attempt is made to separate the error contributions into

- (a) Deficiency of the mathematical model.
- (b) Uncertainty in the factors of the model.
- (c) Instrument errors.

(a) Mathematical model.

The monitor was evolved from equations (2.01) and (2.03). There the assumption was made that $k \approx 1$. This makes good sense, for the concept of δk was presumably originally introduced to facilitate calculations in the vicinity of $k = 1$, which is the region of prime interest, and it seems artificial to extend the concept outside that region. However, once the monitor has been modified to encompass large values of δk (additional ranges added to the read-out section), some interesting aspects are revealed by the following reasoning.

Let δk = true reactivity

$$k = \text{true value} = (1 - \delta k)^{-1}$$

δk_1 = indicated reactivity

$$k_1 = \text{indicate value} = (1 - \delta k_1)^{-1}.$$

At a certain time, from equations (2.01) and (2.02),

$$\frac{\delta k}{\delta k_1} = \left\{ \frac{\frac{dn}{dt} - \sum \lambda_1 c_1 - S + \frac{k\beta n}{\ell}}{\frac{dn}{dt} - \sum \lambda_1 c_1 - S + \frac{\beta n}{\ell}} \right\} \frac{1}{k}.$$

This is of the form

$$\frac{\delta k}{\delta k_1} = \left(\frac{x + ky}{x + y} \right) \frac{1}{k} = \frac{1}{k} + \left(\frac{y}{x + y} \right) \left(\frac{k - 1}{k} \right) \quad (4.05)$$

For $k \approx 1$, the correction factor is approximately $\frac{1}{k}$, i.e., $(1 - \delta k)$, which can then be approximated by $(1 - \delta k_1)$. It is apparent that this process can be continued, δk_1 being corrected by an approximate correction factor, this yielding a new δk_1 , which gives rise to a new correction factor, etc.

For the case that k deviates significantly from unity, the second term in equation (4.05) becomes important. At the expense of a separate summing amplifier, the x and y terms can be separated and continuously indicated in the circuit. If then k_1 is evaluated from the δk_1 , and used in the right hand side of the correction factor expression, we once again get an approximate correction factor, which can be used to modify k_1 , etc. The evaluation of k has thus been achieved, correct to any desired value of mathematical accuracy, and δk_1 can correspondingly be evaluated even where $k \neq 1$. The true k , however, is usually known, for various reactor conditions, within about 10%, and this information could be used in the evaluation of the correction factor. This fact also reveals the philosophical inconsistency of extending the concept of δk beyond the region of $k \approx 1$. Here δk was introduced as a small variation of k , and we are attempting to measure δk so as to accurately determine k , where the deviation in the measurement of δk will then be a second order effect. As soon as δk is comparable to k , however, it does not make sense to measure it with an accuracy of, say, 10% when we already know

k to within that range. Yet it is interesting to know that we could, in fact, measure k by the method outlined above, which could even be automatically performed by a computer.

(b) Uncertainty of factors.

Statistical fluctuations are presumed to be small relative to the uncertainties of the factors. The delayed neutron data have quoted standard deviations of between 5 and 10%.⁴ The errors in the S and n signals, though actually instrument induced, are also included, as they are terms of the dynamic equation. Unluckily, little can be said a priori about the value of these errors, but with the most accurate circuitry feasible, both S and n can be contained within standard deviations of 3%. An error of similar size is attributed to ℓ .¹²

(c) Instrument errors.

By "instrument" we mean to indicate the monitor proper, or the computing section. Drift, having a non-Gaussian distribution, is not readily described in terms of a standard deviation of the amplifier output. For the amplifiers in mind, drift is negligible after a length of time, and the only errors introduced are those of the feedback-circuit elements. Without undue difficulty these can be accurate to within 1%, so that we may arbitrarily ascribe an accuracy of 1% to the computing part. The possible error due to time-lag is not considered, as the variation of n and S is slow compared to the speed of response of the circuit. It is also assumed that n

and S be voltage sources of sufficiently small internal resistance, or else that these be included in the design of the computing section.

If the above errors are summed according to the accumulation law of uncertainties,¹³ we arrive at a standard deviation very close to that of the delayed neutron data. This is as expected, because n is only indirectly involved, S's contribution is small, and ℓ is coupled to the β term. An a posteriori measurement can be made by comparing the indicated δk to the reactivity change produced by a known change of the control-rod position, which are calibrated in terms of their "worth". If this calibration, which was carried out for a cold, clean core, is no longer trusted, it can be repeated. This seems hardly necessary, however.

(viii) Suggestions and Cost-Estimate.

In view of the continuity of reactivity measurement, it is imperative that the operational amplifiers exhibit extremely low drift. Except for the possibly required pre-amplification of the source-signals, which has already been discussed, the chopper-stabilized D.C. amplifier is admirably suited for our purposes. One commercial example of this type is Philbrick's model UPA-2 at about \$150. It is questionable whether home construction would be more economic.

Varian Associates produces a servo-type recorder, which is adaptable to our purposes, and is economical at some \$500.

The precision resistors are readily available at moderate cost, and odd values can be built up. This is not the case

for the capacitors, which are relatively expensive if ordered with a guaranteed accuracy. It is, therefore, suggested that the approximate value of C be calculated, rounded up to the nearest commercial value, accurately measured in practice, and R_1 and R_2 in (4.03) then calculated accordingly. The capacitors should, of course, be stable and have high leakage resistance, although that could be incorporated in a correction to R_2 . The various measuring techniques for the determination of capacitance with 1% accuracy are fully described in pertinent literature.

The cost of the photo-neutron source, using commercial components wherever feasible and economic, is about \$800, and a power-supply for the amplifiers would cost \$200. Excluding labour and a possible preamplifier for the neutron flux signal, the total cost should, therefore, not exceed \$2000, if provision is made for range-switching, but with fixed-position scintillation counter and neutron chamber.

CHAPTER V

CONCLUSION

A practical and economical solution to the problem of the continuous measurement of reactivity has been set forth. Economy has been achieved by drastically reducing the number of operational amplifiers needed, a step which also contributes to the stability of the system. Stability was further enhanced by the choice of chopper-stabilized systems and by insisting that no pure integration take place. The output is directly recorded at almost no extra expense because a recorder is used as basis of a division circuit, where other solutions are almost as costly. The mathematical model employed rests on well-known parameters, while less familiar quantities were eliminated by the introduction of the photo-neutrons as a separate source. This step, forced on us mainly by electrical engineering exigencies, now provides a method to test the theoretical analysis of that effect. The possibilities and difficulties of range coverage were explained, while the predicted accuracy of the monitor was seen to be influenced mainly by the deviations of the nuclear parameters employed. The reader's indulgence is asked for the fact that "neutron density" and "neutron flux" were used interchangeably, although the one differs from the other by the factor of the velocity of the neutrons. This is perhaps excusable because no ambiguity is likely to arise, and because this exchange carries the sanction of usage.

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Although the ~~first~~^{final} result is extremely simple, it is pointed out that this very simplicity was one of the prime aims of this investigation. That the proposed solution is by no means obvious is amply demonstrated by the fact that at least one other monitor is in use costing 2500% more, while not possessing any significant advantages.

BIBLIOGRAPHY

1. Chamber's Etymological Dictionary.
2. Keepin, Wimmert and Zeigler. J. Nucl. Energy 6, 1-21 (1957).
3. Bernstein, et al. Phys. Rev. 72, 575 (1955).
4. Lewins. M.I.T.R. Memorandum JL2 (1958).
5. Mason and Zimmermann. "Electronic Circuits, Signals and Systems". Wiley, (1960).
6. Martin. British Sc. Instr. Research Asso. Report R226.
7. Wilts. "Principles of Feedback Control". Addison-Wesley, (1960).
8. Price. "Nuclear Radiation Detection". McGraw-Hill, (1958).
9. Gray. "Applied Electronics". Wiley, (1958).
10. Mobley and Laubenstein. Phys. Rev. 80, 509 (1950).
Snell, Parker and Sternberg. Phys. Rev. 80, 637 (1950).
11. Zimmermann and Mason. "Electronic Circuit Theory". Wiley, (1960).
12. Lewins. Unpublished.
13. Braae. "M^{ee}tteorie". Stellenbosch, (1957).
14. Schultz. "Control of Nuclear Reactors and Power Plants". McGraw-Hill, (1955).
15. Goldberg. R.C.A. Rev. 11, June 1950.