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# Delay-Aware Wide-Area Control of Power Systems over Sparse Communications with Analytical Guarantees

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**Abstract**—A distributed and sparse control strategy is proposed in this paper for wide-area power system networks. A network control design is proposed that makes use of the notion of communication sparsity and is delay-aware. Communication sparsity is imposed to reduce the cost of the network design, and the delay-aware aspect of the control design accommodates the fact that the sensor measurements utilized may stem from a local generator, generators within an area or those from another area. The latter is addressed using the same notion of sparsity but in a virtual sense by zeroing out the gains that correspond to measurements that are yet to arrive. Both sparsity features are introduced in the control design using the well-known Geromel algorithm. The designs are verified through a simulation study of the IEEE-39 bus, where it is shown that with 90% less communication channels, we can obtain 84% of the performance compared the case when neither of the two sparsity features is present.

**Index Terms**—wide-area control, sparse communication, cyber-physical systems, delays

## I. INTRODUCTION

One of the biggest challenges for a wide-area control infrastructure is the need for a highly robust communication system that works in sync with the control functionalities [1]. Much of the ongoing NASPI-net research is devoted to the hardware and architectural planning aspects of wide-area communication, with little attention to how complicated MIMO control loops, when implemented on top of this communication, may perform under various operating uncertainties. The most critical uncertainty is delay arising due to queuing, scheduling, routing, and computation. With the exponentially increasing number of Phasor Measurement Units (PMUs), producing Terabytes of real-time data that need to be communicated from remote locations to the control centers, the problem of such collective delays is bound to arise and worsen with time. The primary reason why delay is anticipated to become an unavoidable challenge for wide-area control is because utilities are unlikely to establish dedicated communication links for these types of controls, which means that the communication infrastructure must be implemented on top of their existing subnetworks. As a result, PMU data used for control will have to be transported over a shared resource, sharing bandwidth with other ongoing applications, giving rise

to significant delays due to queuing and routing in addition to transport.

In recent literature, several researchers have looked into delay mitigation in wide-area control loops [2]–[5], including the recent seminal work of Vittal and co-authors in [6] where  $\mathcal{H}_\infty$  controllers were designed for redundancy and delay insensitivity. All of these designs are, however, based on worst-case delays, which makes the controller unnecessarily restrictive, and may degrade closed-loop performance. In two of our recent papers [7], [8], we addressed this problem by proposing a delay-aware wide-area controller, where the feedback gain matrix was made an explicit function of delays using ideas from arbitrated network control theory [9]. The controller was implemented using a cloud computing environment, where virtual computers located inside local clouds of each utility company. The limitation of that design, however, was that the gain matrix was allowed to be dense, meaning that the communication network had an all-to-all connection topology between every generator in the system. In reality, however, all-to-all communication can be quite expensive due to the cost of renting communication links in the cloud, if not unnecessary. For example, recent papers such as [10]–[12] have proposed various graph sparsification algorithms based on  $\ell_1$ -optimization to develop wide-area controllers.

In this paper we propose a delay-aware control design that accommodates a reduction in the number of communication links. Unlike the unstructured  $\ell_1$ -optimization approach, we promote sparsity in a structured way, thereby enhancing the efficiency of the closed-loop response. This is accomplished using the concept of Participation Factors (PFs). Communication links between any two generators are retained only if their participation factors for the same mode exceed the chosen threshold, indicating that both generators have high influence in the damping of that mode and removed otherwise. The resulting communication sparsity is suitably imposed on the underlying delay-aware control design, which in this case is chosen as a LQR state-feedback controller. This choice follows from recent LQR designs for wide-area control in [10] and [13].

Similar to [7], the wide-area controller here is considered to be implemented using virtual machines (VMs) in an Internet of cloud. The delays in the feedback paths are categorized into small, medium and large depending on whether they pertain to one specific VM, or two VMs inside the local cloud of the same utility company, or two VMs in two different local clouds belonging to two different utility companies. All three types of delays are assumed to be smaller than the sampling time

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for convenience. The design, however, is not limited by this assumption, and can be easily extended to cases with larger delays, as shown in our recent papers [8], [14]. As the presence of such delays can directly affect control performance, we explicitly accommodate these delays in the control design. The non-arrival of state measurements is captured by introducing sparsity in the corresponding control gains. The novel aspect of our proposed design, therefore, is that it consists of both communication sparsity and a virtual sparsity to accommodate delayed arrival of control messages. The resulting control structure, Geromel's algorithm [15] is used to derive the underlying sparse LQR-controller. The properties and controllability of the underlying system matrices are used to establish closed-loop stability. We illustrate the effectiveness of the overall control architecture using the IEEE 39-bus power system model, and show that a near-optimal closed-loop response can be achieved while promoting as high as 90% sparsity in the underlying communication graph compared to the optimal LQR design.

The remainder of the paper is organized as follows. Section II describes the power system model. It also derives the underlying control problem by suitably representing the feature of multiple message-arrival in the form of a standard sampled-data system model. Section III describes the overall control architecture including communication sparsity and virtual sparsity. The PF-based approach, the notion of virtual sparsity, Geromel's algorithm, and the stability of the overall closed-loop system are all delineated in this section. Simulation results of the IEEE-39 bus are shown in Section IV. Finally, Section V concludes the paper.

## II. STATEMENT OF PROBLEM

### A. Preliminaries

For each matrix  $M = \{m_{ij}\} \in \mathbb{R}^{p \times q}$ ,  $\mathcal{R}(M)$  denotes the range of  $M$ . Where all entries of  $M$  is 1, it is shown by  $I_{p,q}$  and if all of its entries are 0, it is denoted by  $O_{p,q}$ .  $I_p$  is used for the identity  $p \times p$  matrix.  $P = \{p_{ij}\} = M \circ N$  is the Hadamard (element-wise) product and  $p_{ij} = m_{ij}n_{ij}$ .  $\|M\|_2$  is the spectral norm of  $M$ .  $M > 0$  ( $M \geq 0$ ) means that  $M$  is positive (semi-)definite [16]. The notation  $M \in \{0, 1\}^{p \times q}$  implies that the elements of the  $p \times q$  matrix  $M$  are either 0 or 1. The complement of  $M \in \{0, 1\}^{p \times q}$  is defined as  $M^c = I_{p,q} - M$ .

A simple graph  $\mathcal{H}$  is a pair of  $(\mathcal{V}, H)$ , where  $\mathcal{V}$  denotes the set of nodes, i.e.  $\{1, 2, \dots, n\}$ , and  $H = \{h_{ij}\} \in \mathbb{R}^{n \times n}$  is the adjacency matrix whose entries determine how the nodes are connected. If there is an edge between the node  $i$  and  $j$ ,  $h_{ij}$  is 1 and otherwise it is 0. Our graphs are undirectional, i.e.  $h_{ij} = h_{ji}$ , and may have self-loops, which means  $h_{ii}$  might be 1. The terms edge and link are used, interchangeably. If all entries of  $H$  is 1,  $\mathcal{H}$  is called a complete graph [17].

### B. Power System Model

Consider a power system network with  $n$  synchronous generators. Any conventional generator model may be considered for our design. For example, one may use a flux-decay model assuming that the time constants of the  $d$ - and  $q$ -axis flux are

fast enough to neglect their dynamics, that the rotor frequency is around the normalized constant synchronous speed, and that the amortisseur effects are negligible. The model of the  $i^{\text{th}}$  generator is then written as [18]:

$$\dot{\delta}_i = \omega_i - \omega_s \quad (1)$$

$$M_i \dot{\omega}_i = P_{mi} - (V_i I_{qi} \cos(\theta_i - \delta_i) + V_i I_{di} \sin(\delta_i - \theta_i)) - d_i(\omega_i - \omega_s) \quad (2)$$

$$T_{qi} \dot{E}'_{qi} = -E'_{qi} + (x_{di} - x'_{di})I_{di} + E_{fdi} \quad (3)$$

$$T_{di} \dot{E}'_{di} = -E'_{di} + (x_{qi} - x'_{qi})I_{qi} \quad (4)$$

$$T_{Ai} \dot{E}_{fdi} = -E_{fdi} + K_{Ai}(V_{ref,i} - V_i) + u_i(t). \quad (5)$$

for  $i = 1, \dots, n$ . Equations (1)-(2) are referred to as the swing equations while (3)-(5) as the excitation equations. The states  $\delta_i$ ,  $\omega_i$ ,  $E'_{qi}$ ,  $E'_{di}$ , and  $E_{fdi}$  respectively denote the generator phase angle (radians), rotor velocity, the quadrature-axis internal emf, the direct-axis internal emf, and the field excitation voltage. The voltage at the generator terminal bus is denoted in the polar representation as  $\tilde{V}_i(t) = V_i(t) \angle \theta_i(t)$ .  $V_{ref,i}$  is the constant setpoint for  $V_i$ . The generator current in complex phasor form is written as  $I_{di} + jI_{qi} = I_i \angle \phi_i$ .  $\omega_s$  is the synchronous frequency, which is equal to  $120\pi$  rad/sec for a 60-Hz power system.  $M_i$  is the generator inertia,  $d_i$  is the generator damping, and  $P_{mi}$  is the mechanical power input from the  $i^{\text{th}}$  turbine, all of which are considered to be constant.  $T_{di}$ ,  $T_{qi}$ , and  $T_{Ai}$  are the excitation time constants;  $K_{Ai}$  is the constant voltage regulator gain;  $x_{di}$ ,  $x'_{di}$ ,  $x_{qi}$  and  $x'_{qi}$  are the direct-axis and quadrature-axis salient reactances and transient reactances, respectively. We assume that all of these constant model parameters are known. All variables, except for the phase angles (radians), are expressed in per unit. Equations (1)-(5) can be written in a compact form as

$$\dot{x}_i(t) = g(x_i(t), z_i(t), u_i(t), \alpha_i) \quad (6)$$

where  $x_i = [\delta_i \ \omega_i \ E'_{qi} \ E'_{di} \ E_{fdi}]' \in \mathbb{R}^5$  denotes the vector of state variables,  $z_i = [V_i \ \theta_i \ I_{di} \ I_{qi}]' \in \mathbb{R}^4$  denotes the vector of algebraic variables,  $u_i \in \mathbb{R}$  is the control input, and  $\alpha_i$  is the vector of the constant parameters  $P_{mi}$ ,  $\omega_s$ ,  $d_i$ ,  $T_{qi}$ ,  $T_{di}$ ,  $T_{Ai}$ ,  $M_i$ ,  $K_{Ai}$ ,  $V_{ref,i}$ ,  $x_{di}$ ,  $x_{qi}$ ,  $x'_{di}$ , and  $x'_{qi}$ , all of which are assumed to be known. The definition of the nonlinear function  $g(\cdot)$  follows from (1)-(5).

The model (6) is completely decentralized since it is driven by variables belonging to the  $i^{\text{th}}$  generator only. It is, however, not a state-space model as it contains the auxiliary variables  $z_i$ . The states  $x_i$  can be estimated in a decentralized way if one has access to  $z_i(t)$  at every instant of time. This can be assured by placing PMUs within each utility area such that the generator buses inside that area becomes geometrically observable, measuring the voltage and currents at the PMU buses, sending these measurements at every time to compute the generator bus voltage  $V_i \angle \theta_i$  and current  $I_i \angle \phi_i$  (or equivalently,  $I_{di}$  and  $I_{qi}$ ) from those measurements. As the PMU measurements will be corrupted with noise, the estimates  $\hat{z}_i(t)$  of  $z_i(t)$  can be expressed as  $\hat{z}_i(t) = z_i(t) + n_i(t)$ , where  $n_i(t)$  is a Gaussian noise. An unscented Kalman-filter is next designed as

$$\dot{\hat{x}}_i(t) = g(\hat{x}_i(t), \hat{z}_i(t), u_i(t), \alpha_i), \quad \hat{x}_i(0) = \hat{x}_{i0}$$

producing the state estimates  $\hat{x}_i(t)$  for the  $i^{\text{th}}$  generator at any instant of time  $t$ . This state estimator can be installed directly at the generation site to minimize the communication of signals, and run continuously before and after any disturbance. The reader is referred to [19] for details.

The network equations that couple  $(x_i, z_i)$  of the  $i^{\text{th}}$  generator in (6) to the rest of the network can be written as

$$\begin{aligned} 0 &= I_{di}(t)V_i(t) \sin(\delta_i(t) - \theta_i(t)) + I_{qi}(t)V_i(t) \\ &\times \cos(\delta_i(t) - \theta_i(t)) + P_{Li}(t) \\ &- \sum_{j=1}^m V_i(t)V_j(t)(G_{ij} \cos(\theta_{ij}(t)) + B_{ij} \sin(\theta_{ij}(t))) \end{aligned} \quad (7)$$

$$\begin{aligned} 0 &= I_{di}(t)V_i(t) \cos(\delta_i(t) - \theta_i(t)) - I_{qi}(t)V_i(t) \\ &\times \sin(\delta_i(t) - \theta_i(t)) + Q_{Li}(t) \\ &- \sum_{j=1}^m V_i(t)V_j(t)(G_{ij} \cos(\theta_{ij}(t)) - B_{ij} \sin(\theta_{ij}(t))) \end{aligned} \quad (8)$$

where  $m$  is the total number of buses in the network. Here,  $\theta_{ij} = \theta_i - \theta_j$ ,  $P_{Li}$  and  $Q_{Li}$  are the active and reactive power load demand at bus  $i$ , and  $G_{ij}$  and  $B_{ij}$  are the conductance and susceptance of the line joining buses  $i$  and  $j$ . As shown in [18], the variables  $z_i(t)$  in (6) can be eliminated using (7)-(8) by employing Kron reduction. The resulting dynamic model is linearized about a given operating point, and the small-signal model for the power system is written as

$$\dot{x}(t) = A_c x(t) + B_c u(t) \quad (9)$$

$$y(t) := \Delta\omega(t) = C x(t). \quad (10)$$

In this model  $x(t) = [x_1(t), \dots, x_n(t)]^T \in \mathbb{R}^N$  which corresponds to the state variables of the generators around their equilibrium values, and  $u(t) \in \mathbb{R}^n$  which corresponds to the deviation of  $u_i$ ,  $i = 1, \dots, n$  around their equilibrium values, and  $N = \sum_{i=1}^n n_i$ . As stated above, our control design can handle any generic model of a synchronous generator as long as that model captures the electro-mechanical damping dynamics. Thus  $N$  can be any arbitrary positive integer. For the flux-decay model in (6) we have  $N = 5n$ . We assume that  $y$  is a vector of the small-signal generator frequency  $\Delta\omega(t)$  which often is measurable, and is an effective indicator of the underlying damping in the dynamic system. The input  $u(t)$  is commonly used for designing feedback controllers such as Power System Stabilizers (PSS), which takes local feedback from the generator speed, and passes it through a lead-lag controller for producing damping effects on the oscillations in phase angle and frequency. PSSs, however, are most effective in adding damping to the fast oscillation modes in the system, and perform poorly in adding damping to the slow or inter-area oscillation modes [1]. In this paper, our goal is to design a supplementary controller  $u(t)$  for the model (9)-(10) in addition to the local PSS by using a state-feedback from selected sets of other generators. We refer to this controller as a wide-area controller. This selected set will be decided based on the relative modal participation factors of  $y(t)$  corresponding to the inter-area oscillation modes. Since feedback between distant generators will invariably introduce communication delays, the design for  $u(t)$  will also be made

delay-aware. Considering the KF dynamics to be fast, from here onwards we will assume  $\text{xhat}=x$ , i.e., full state availability for state feedback.

Let the power system network be divided into  $p$  non-overlapping areas belonging to  $p$  respective utility companies. The wide-area control architecture is realized using a cloud-in-the-loop cyber-physical architecture that was recently proposed in [7]. This architecture is briefly shown in Fig. 1. In this architecture every utility is assumed to own a local cloud network, where virtual computer, referred to as virtual machines (VMs) can be created on-demand for computing control signals. Every generator in a utility area sends its estimated state vector at every time  $t$  to a designated VM. Upon receiving these estimates the VMs communicate with each other, through a communication graph  $\mathcal{H}$ , and share their respective state vectors. After that each VM computes the control input for its respective generator by using the block row of the state-feedback gain matrix  $K$  which is pre-embedded in that VM. This control signal finally goes back from the local cloud to the corresponding generator and its KF. The problem in this implementation arises due to two factors. First, if the gain matrix  $K$  is dense that would imply that every VM must communicate with every other VM, or equivalently  $\mathcal{H}$  is a complete graph, which may lead to unnecessary data flooding. One of our goals in this paper is to circumvent this problem by finding a tractable way for sparsifying the matrix  $K$ . The second issue is that VM-to-VM communication, whether inside a given local cloud or across two different local clouds, would entail delays due to queuing, transport, and routing. We group the delays into three categories. We denote  $\tau_s$  as the self-delay for every VM, which may be attributed to delays due to computation. This delay is typically close to zero. We denote  $\tau_m$  as the VM-to-VM communication delay inside the same local cloud. And finally, we denote  $\tau_\ell$  as the VM-VM communication delay across two different local clouds. For convenience, each category of delays is assumed to be of the same value, which can be easily accommodated by considering the maximum value of each. Quite naturally,  $\tau_s < \tau_m < \tau_\ell$ .

The control design that we propose is based on a sampled-data representation of (9). For this purpose, a sampling period of  $h$  is chosen such that Nyquist sampling conditions are satisfied. In this paper, we assume that this sampling is such that  $\tau_\ell < h$ .

### C. A Sampled-Data Power System Model with Delayed Inputs

Given that the goal is the control of (9) using input at discrete instants, we convert (9) into a zero-order sampled-data model as follows.

$$x[k+1] = Ax[k] + Bu[k], \quad (11)$$

where

$$A = e^{A_c h} \quad \text{and} \quad B = \int_0^h e^{A_c s} B_c ds. \quad (12)$$

However, with consideration of the three delays  $\tau_s, \tau_m, \tau_\ell$ , over every interval  $[kh, (k+1)h)$ , new information arrives from various state measurements at an area  $j$  at three different

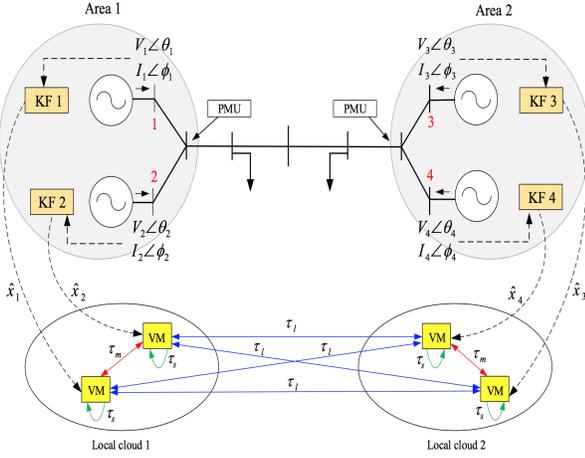


Fig. 1: Control of WAMS via Internet of Clouds

intervals. As will be seen later, this information is judiciously used in the control input. Let  $D^i = \{\tau_{ij}\}$  denote the set of delays that corresponds to the computation of the control input of the  $i$ -th generator using measurements from generator  $j$ . Let  $g(i)$  denote the cardinality of the set  $D^i$ , where  $\sum_{i=1}^n g(i) = \mathcal{G}$ . Then the input of generator  $i$  is given by

$$U_i(t) = \begin{cases} u_{ij}[k] & \text{if } t - kh \in [\tau_{ij}, \tau_{i(j+1)}], j < g(i), \\ u_{ig(i)}[k] & \text{if } t - kh \in [\tau_{ig(i)}, h), j = g(i), \\ u_{ig(i)}[k-1] & \text{if } t - kh \in [0, \tau_{i1}), \end{cases} \quad (13)$$

where  $\tau_{ij}$  is the  $j$ -th member of the sequence of delays of all elements in  $D^i$  in an ascending order. That is,  $u_{ij}[k] \in \mathbb{R}$  is the control of the  $i$ -th generator adjusted using the measurements of the  $j$ -th arrival of new information at time  $k$ . To have a better understanding of the problem, we provide an example:

**Example 1.** Assume that generators 1 and 2 are in area 1 and generator 3 and generator 4 in area 2 ( $p = 2$ ). The control input of generator 1, for example, is obtained in the interval  $[kh + \tau_m, kh + \tau_\ell)$  using  $x_1[k]$  and  $x_2[k]$ , where  $x_i[k] \in \mathbb{R}^{n_i}$  is the vector of all state variables of generator  $i$ . The control inputs  $u_{1j}[k]$ ,  $j = 1, 2, 3$ , are applied after each time new measurements arrive at the VM of area 1 which correspond to the three ticks shown in Fig. 2 between  $kh$  and  $(k+1)h$ .

With the piecewise constant control in (13), we integrate (9) by partitioning  $[kh, (k+1)h)$  into  $(g(i) + 1)$  sub-intervals and obtain the discrete-time model

$$x[k+1] = Ax[k] + B_1U[k] + B_2U[k-1], \quad (14)$$

where

$$B_{j1}^i = \begin{cases} \int_0^{h-\tau_{ij}} e^{A_c s} ds B_c^i & \text{if } j = g(i), \quad (15a) \\ \int_{h-\tau_{i(j+1)}}^{h-\tau_{ij}} e^{A_c s} ds B_c^i, & \text{if } j \neq g(i), \quad (15b) \end{cases}$$

$$B_{g(i)2}^i = \int_{h-\tau_{i1}}^h e^{A_c s} ds B_c^i, \quad (16)$$

$B_c^i$  is the  $i$ th column vector of  $B_c \in \mathbb{R}^{N \times n}$ , and  $B_{j1}^i \in \mathbb{R}^{N \times 1}$ ,  $j = 1, \dots, g(i)$ , are the coefficients of  $u_{ij}[k]$  and

$B_{g(i)2}^i \in \mathbb{R}^{N \times 1}$  is the coefficient of  $u_{ig(i)}[k-1]$  in (13) with measurements arriving at different instances,  $n$  expanded to  $\mathcal{G}$  with

$$U[k] = [u_{11}[k] \ \dots \ u_{1g(1)}[k] \ u_{21}[k] \ \dots \ u_{ng(n)}[k]]^T. \quad (17)$$

In an ideal case, where all measurements arrive with negligible delay, we note that  $B_2 = 0$  and  $U[k]$  coincides with a sampled value  $u[k]$  of dimension  $n$  as (11). Likewise, the matrices  $B_1$  and  $B_2$  can be written in the following forms:

$$B_1 = [B_{11}^1 \ \dots \ B_{g(1)1}^1 \ B_{11}^2 \ \dots \ B_{g(n)1}^n] \quad (18)$$

and

$$B_2 = [0 \ \dots \ 0 \ B_{g(1)2}^1 \ 0 \ \dots \ 0 \ B_{g(2)2}^2 \ \dots \ B_{g(n)2}^n]. \quad (19)$$

We note that Eq. (14) can be viewed as a power system model of the wide-area measurement system which is delay-aware. That is, each control input  $u_{ij}[k]$  in  $U[k]$  corresponds to an action that can be taken using new information that becomes available during an interval  $[kh, (k+1)h)$ , and affected by delays in measurements. Such a delay-aware dynamic model has been utilized previously in [7], [20], [21].

#### D. Problem Statement

The starting point for the control design is the delay-aware power system model in (14) with the goal of designing  $U[k] = [U_1[k]^T \ U_2[k]^T \ \dots \ U_n[k]^T]^T$  so that the states  $x[k]$  tend to zero for any initial conditions. The structure of the power system model in (14) suggests state-feedback based control design that optimizes a suitable quadratic cost function:

$$U[k] = Kx[k] + GU[k-1], \quad (20)$$

where  $K$  and  $G$  are control gain matrices. However, implementation of (20) implies that at  $k$ ,  $x[k]$  and  $U[k-1]$  must be fully available at all generator inputs. However, not all elements of  $x[k]$  are available at all  $U[k]$ . Returning to Example 1, we note that  $u_{11}[k]$ , the first element of  $U[k]$ , can be computed using (20) as  $u_{11}[k] = K_1x[k] + G_1U[k]$ , where  $K_1$  and  $G_1$  are the first row-vectors of  $K$  and  $G$  respectively. However, as shown in Fig. 1, at  $kh + \tau_s$ , none of the state measurements  $x_2, x_3$ , or  $x_4$  are available. The same is true for  $U_2[k-1]$ ,  $U_3[k-1]$  or  $U_4[k-1]$ , as they correspond to control inputs at those locations, and will arrive at VM 1 with delays  $\tau_m$  or  $\tau_\ell$ . Thus (20) is not implementable.

A fix to this problem was suggested in [7] in the following way. Suppose every VM has a local copy of the matrices  $A$ ,  $B_1$ ,  $B_2$ , and cost function parameters. Then every VM can compute the entire state vector  $x[k]$  and  $u[k]$  using (14) and (16). This is highly undesirable because of loss of privacy and because of the increased computational burden at each VM. In this paper, we completely avoid this state estimation and propose an alternate control design instead of (20). This is one of our main contributions.

The second contribution of this paper, as described in the introduction, is sparsification of the communication network through which the wide-area design is meant to be implemented. We will show that this design is also delay-aware, and is implementable in a distributed manner. We will show

that the proposed control method will decrease the number of links in the VM-network as much as possible.

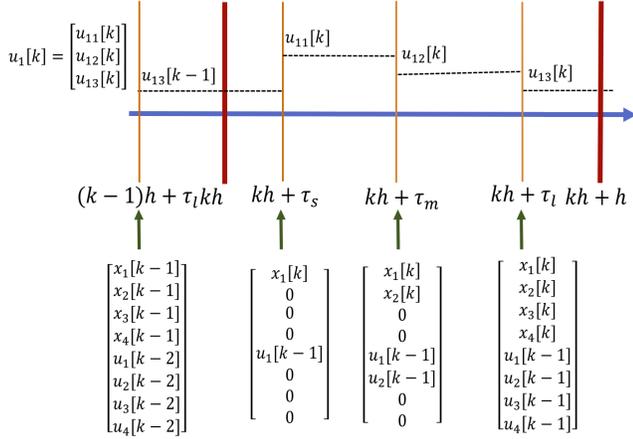


Fig. 2: A schematic on distributed ANCS with  $\tau_\ell < h$

### III. A SPARSE DELAY-AWARE CONTROL ARCHITECTURE

Our overall goal is to derive a control architecture that is capable of damping small-signal power system model (14) using a sparse, delay-aware, implementable design. Towards this end, we first address sparsity promotion in Section III-A. An approach based on Participation Factors (PF) [22] is used for this purpose. The adjacency matrix that is derived from PF is then used to determine a sparse control structure. In Section III-B, a novel delay-aware control design is proposed by viewing the absence of arrival of new information from a measurement as being *virtually sparse*, and is shown to be stable.

#### A. A Sparse Control Architecture

*Participation Factors:* We begin with a brief description of PF.

**Definition 1.** [22] For a given matrix  $M \in \mathbb{R}^{s \times s}$ , the matrix  $\mathcal{P} = p_{ki} \in \mathbb{R}^{s \times s}$  is defined as

$$p_{ki} = \frac{|\nu_{ik}| |\eta_{ki}|}{\sum_{k=1}^n |\nu_{ik}| |\eta_{ki}|},$$

where  $\lambda_i = \sigma_i + j\beta_i$ ,  $\nu_i = [\nu_{ik}]$ , and  $\eta_i = [\eta_{ki}]$  are, respectively, the  $i$ -th eigenvalue, right eigenvector, and left normalized eigenvectors of the matrix  $M = m_{ij} \in \mathbb{R}^{s \times s}$ , where  $i = 1, \dots, s$ , i.e.  $M\nu_i = \lambda_i\nu_i$ , and  $\eta_i M = \lambda_i\eta_i$ .

We note that participation factor  $p_{ki}$  of  $A_c$  is a dimensionless value for the contribution of  $k$ -th state variable on  $i$ -th eigenvalue. Each row of a  $\mathcal{P}$  is representative of each state variable of the system and each column of  $\mathcal{P}$  associates with each eigenvalue of the system. For more details about PF and their properties, the reader may refer to [22], [23]. For damping the inter-area electro-mechanical oscillation modes (whose frequency,  $f_i$ , typically lies between 0.1 Hz and 2 Hz), such modes which have lower damping ratios,  $\zeta_i$ , have significant effects on the system and have to be taken care of, where  $\zeta_i = -\frac{\sigma_i}{\sqrt{\sigma_i^2 + \beta_i^2}}$ , and  $f_i = \frac{\beta_i}{2\pi}$ . For this purpose, we need to characterize (i) which eigenvalues are critical in

damping and (ii) which generators have to be influenced by each other to influence those eigenvalues.

*Sparsification Algorithm:* We now describe an algorithm on how the PF entries can be utilized to determine a sparse communication structure among the VMs. In particular, an adjacency matrix  $H_\mu, \mu \in [0, 1]$  is proposed, where  $\mu$  is proportional to the level of sparsity.

#### Algorithm 1 Sparsification Algorithm

- 1: **procedure** PF-BASED SPARSIFICATION
- 2: Pick a  $\mu \in [0, 1]$ .
- 3: Determine participation factors table,  $\mathcal{P}$  of  $A_c$ .
- 4: Copy only those rows of  $\mathcal{P}$  that are associated with  $\Delta\omega_s$  and only those columns of  $\mathcal{P}$  associated with inter-area modes to the matrix  $\mathcal{P}^f = \{p_{ij}^f\} \in \mathbb{R}^{n \times n_\lambda}$ , where  $n_\lambda$  is the number of inter-area modes.
- 5: Given  $\mu$ , the  $\mu$ -cut binary matrix,  $\mathcal{P}_\mu = p_{ij}^\mu \in \mathbb{R}^{n \times n_\lambda}$  is obtained as follows

$$p_{ij}^\mu = \begin{cases} 1 & \text{if } p_{ij}^f \geq \mu, \\ 0 & \text{if } p_{ij}^f < \mu, \end{cases}$$

- 6: Determine the adjacency matrix  $H_\mu \in \mathbb{R}^{n \times n}$

$$H_\mu = \{h_{ij}^\mu\} = \begin{cases} 1 & \text{if } i = j, \\ 1 & \text{if } \exists s = 1, \dots, n_\lambda; p_{is}^\mu = p_{js}^\mu, \\ 0 & \text{if } p_{ij}^\mu = 0, \end{cases}$$

- 7: **end procedure**

In order to connect the sparse communication structure captured in  $H_\mu$  to the control architecture, we introduce two matrices  $\mathbb{I}_K^\mu \in \{0, 1\}^{\mathcal{G} \times N}$  and  $\mathbb{I}_G^\mu \in \{0, 1\}^{\mathcal{G} \times \mathcal{G}}$  given by

$$\mathbb{I}_K^\mu = \begin{bmatrix} \mathbb{I}_{1,1}^\mu & \cdots & \mathbb{I}_{1,n}^\mu \\ \vdots & \ddots & \vdots \\ \mathbb{I}_{n,1}^\mu & \cdots & \mathbb{I}_{n,n}^\mu \end{bmatrix}, \text{ and } \mathbb{I}_G^\mu = \begin{bmatrix} \mathbb{G}_{1,1}^\mu & \cdots & \mathbb{G}_{1,n}^\mu \\ \vdots & \ddots & \vdots \\ \mathbb{G}_{n,1}^\mu & \cdots & \mathbb{G}_{n,n}^\mu \end{bmatrix},$$

where  $\mathbb{I}_{i,j}^\mu \in \{0, 1\}^{g(i) \times g(j)}$  and  $\mathbb{I}_{i,j}^\mu \in \{0, 1\}^{g(i) \times n_j}$ . Suppose that we choose the elements of  $\mathbb{I}_K^\mu$  and  $\mathbb{I}_G^\mu$  as

$$\mathbb{I}_{i,j}^\mu = h_{ij}^\mu \mathbb{I}_{g(i), n_j} \text{ and } \mathbb{G}_{i,j}^\mu = h_{ij}^\mu \mathbb{I}_{g(i), g(j)}, \quad (23)$$

we note that resulting  $\mathbb{I}_K^\mu$  and  $\mathbb{I}_G^\mu$  will have the same sparsity pattern as in  $H_\mu$ . If we now choose  $K_s$  and  $G_s$  to have the same sparsity structure of  $\mathbb{I}_K^\mu$  and  $\mathbb{I}_G^\mu$  by choosing

$$K_s = K \circ \mathbb{I}_K^\mu \text{ and } G_s = G \circ \mathbb{I}_G^\mu, \quad (24)$$

we will have a control design in (20), where  $K$  and  $G$  are replaced by  $K_s$  and  $G_s$ , respectively, that is sparse. One needs to ensure of course that the non-zero elements of  $K_s$  and  $G_s$  are chosen so as to guarantee stability.

#### B. Sparse Delay-Aware Control

While Algorithm 1 provides an approach to sparsify the communications among VMs, the absence of information due to the delays is yet to be included in the design, which is addressed in this section. For this purpose, we return to Example 1, and note that to compute  $u_{11}[k]$ , states and inputs

from VM 2, 3, and 4 have not yet arrived. Rather than using state estimation as in [7], we will accommodate this non-arrival by simply introducing zeros in the corresponding entries of the  $K_s$  and  $G_s$  matrices. For example, the lack of arrival of VMs 2, 3, and 4 at  $u_{11}[k]$  can be accommodated by setting the  $k_{12}$ ,  $k_{13}$ , and  $k_{14}$  element of  $K_s$  to be zero. In other words, appropriate entries of  $K_s$  and  $G_s$  are set to zero whenever the associated states and inputs are not available. And this is done even if an actual VM link exists in  $\mathcal{H}$ , and hence can be viewed as *virtual sparsity*. This is described more formally below.

1) *Virtual Sparsity*: For ease of exposition, we set  $g(i) = 3$ , i.e. during each sampling interval new states and inputs arrive three times (which corresponds to the three ticks shown in Fig. 2) for  $i = 1, \dots, n$ . The following transformation matrices are defined:

$$T_{i,j \neq i}^{u-s} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \quad T_{i,j}^{u-d} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}, \quad \text{and} \\ T_{i,i}^{u-s} = I_{3,3} \quad (25)$$

The zero entries of  $T_{i,j \neq i}^{u-s}$  correspond to the information from inputs from other generators in the same area. Since at the first tick, inputs from intra-area generators have not yet arrived, the first row of  $T_{i,j \neq i}^{u-s}$  is zero. Similarly, in  $T_{i,j}^{u-d}$ , the first and second row are zero as they correspond to the lack of arrivals of inputs from inter-area generators at both the first tick and second tick. Likewise, we introduce two more matrices to correspond to information from states in both intra-area and inter-area generators as

$$T_{i,j \neq i}^{x-s} = \begin{bmatrix} 0 & \dots & 0 \\ 1 & \dots & 1 \\ 1 & \dots & 1 \end{bmatrix}, \quad T_{i,j}^{x-d} = \begin{bmatrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 1 & \dots & 1 \end{bmatrix}, \quad \text{and} \\ T_{i,i}^{x-s} = I_{g(i),n_i} \quad (26)$$

As above, here too, the zero entries correspond to lack of arrival of information, with the first and second row corresponding to the first and second ticks respectively. All of the above matrices are defined for  $i, j = 1, 2, \dots, n$ . Then, the delay-aware sparsity patterns are defined as below:

$$\mathbb{K}_{i,j}^\mu = \begin{cases} h_{ij}^\mu T_{i,j}^{x-s} & \text{if } i \text{ and } j \text{ are in the same area,} \\ h_{ij}^\mu T_{i,j}^{x-d} & \text{if } i \text{ and } j \text{ are not in the same area,} \end{cases} \quad (27)$$

and

$$\mathbb{G}_{i,j}^\mu = \begin{cases} h_{ij}^\mu T_{i,j}^{u-s} & \text{if } i \text{ and } j \text{ are in the same area,} \\ h_{ij}^\mu T_{i,j}^{u-d} & \text{if } i \text{ and } j \text{ are not in the same area,} \end{cases} \quad (28)$$

Using these definitions we then choose the individual entries of  $\mathbb{K}_K^\mu$  and  $\mathbb{K}_G^\mu$  as in (27)-(28) and the resulting  $K_s$  and  $G_s$  as in (24). It should be noted that the final  $K_s$  and  $G_s$  obtained includes both the communication sparsity as well as the virtual sparsity which accommodates a delay-aware feature, leading to a sparse delay-aware control architecture. In particular, step 6 in Algorithm 1 introduces communication sparsity, whereas the choices of the transformation matrices

in (25) and (26) introduce virtual sparsity to incorporate the delay-aware feature. One can view the multiplication of the adjacency terms by the transformation matrices as punching "holes" in the corresponding entries in  $K$  and  $G$  through (24). What remains to be shown is how the non-zero gains of  $K_s$  and  $G_s$  are chosen to guarantee a stable control design.

2) *Stable Control Design*: We start with the underlying power system model in (14) and rewrite it in an extended form [7] as is in the following form

$$W_h[k+1] = A_h W_h[k] + B_h U[k], \quad (29)$$

where

$$W_h[k] = \begin{bmatrix} x[k] \\ U[k-1] \end{bmatrix}, \quad B_h = \begin{bmatrix} B_1 \\ I_G \end{bmatrix},$$

and

$$A_h = \begin{bmatrix} A & B_2 \\ 0 & 0 \end{bmatrix}.$$

We now propose the use of Geromel's algorithm in [15] in order to determine a stable control design for (30). The idea in [15] is to come up with an iterative procedure for determining the weight matrix  $Q$  in the standard quadratic cost function  $\mathcal{J} = \sum_0^\infty W_h[k]^T Q W_h[k] + U[k]^T R U[k]$  so as to have the solution using the Riccati equation to converge to the control gain  $\mathcal{K}_s = [K_s \ G_s]$  of the desired sparsity  $\mathbb{K}_\mu = [\mathbb{K}_K^\mu \ \mathbb{K}_G^\mu]$ .

---

#### Algorithm 2 Delay-Aware Geromel Algorithm

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- 1: **procedure** DELAY-AWARE GEROMEL
  - 2:   Get  $0 < \epsilon < 1$ ,  $j = 0$ , and  $Q_j = Q$ .
  - 3:   Solve the full Discrete Algebraic Riccati Equation (DARE):  $P_j = A_h' P_j A_h - A_h' P_j B_h (R + B_h' P_j B_h)^{-1} B_h' P_j A_h + Q_j$
  - 4:    $L_\mu^{j+1} = -(R + B_h' P_j B_h)^{-1} B_h' P_j A_h \circ \mathbb{K}_\mu^c$ .
  - 5:    $Q_{j+1} = Q + (L_\mu^{j+1})'(R + B_h' P_j B_h) L_\mu^{j+1}$
  - 6:    $j = j + 1$  and go to 3
  - 7:   While  $\frac{\|P_j - P_{j-1}\|_2}{\|P_0\|_2} > \epsilon$ , repeat 3-5.
  - 8:    $K^j = -(R + B_h' P_j B_h)^{-1} B_h' P_j A_h$  and  $\mathcal{K}_s^j = K^j - L_\mu^{j+1}$ .
  - 9: **end procedure**
- 

Denoting the value that  $\mathcal{K}_s^j$  converges to as  $\mathcal{K}_s^* = [K_s^* \ G_s^*]$ , it follows that the complete sparse, delay-aware control design is given by

$$U[k] = K_s^* x[k] + G_s^* U[k-1]. \quad (30)$$

It should be noted that the only centralized component of the complete control design is in the calculation of  $K_s^*$  and  $G_s^*$  as specified in Algorithm 2. Once the control gains are computed, the implementation of  $U[k]$  occurs in an entirely distributed manner, with each  $u_{ij}[k]$  implemented at generator  $i$ , for  $j = 1, \dots, g(i)$ , using only the corresponding rows of  $K$  and  $G$ . These control gains utilize all of the communication sparsity using PF and all of the delay-aware aspects using the virtual sparsity notion. This is the central contribution of this paper.

3) *Stability and Controllability Analysis*: The following proposition verifies the stability of the closed-loop system with the sparse gain matrices.

**Proposition 1.** The closed-loop system  $W_h[k+1] = (A_h + B_h K_s^*)W_h[k]$  is stable if  $(A_h, B_h)$  is controllable.

We omit the proof of Proposition 1 as it follows along the lines of that in [15]. In order to satisfy the controllability condition in Proposition 1, the following definition is needed.

**Definition 2.** The operator  $\odot$  is defined on three input matrices  $M = [m_1, \dots, m_s]$ ,  $N = [n_1, \dots, n_s]$ , and  $R = [r_s, \dots, r_s]$ , all in  $\mathbb{R}^{\ell \times s}$  as below:

$$M \odot N \odot R = [m_1 \ n_1 \ r_1 \ m_2 \ n_2 \ r_2 \ \dots \ m_s \ n_s \ r_s].$$

Before providing the controllability result, we represent  $B_1$  in (18), as  $B_1 = B_1^1 \odot B_2^1 \odot B_3^1$ , where

$$B_i^1 = [B_{i1}^1 \ B_{i2}^1 \ \dots \ B_{in}^1], \quad (31)$$

for  $i = 1, 2, 3$ .

The following lemma connects controllability of (14) to that of the continuous-time system in (9). Proofs of all lemmas and theorems can be found in the Appendix.

**Lemma 1.**  $(A, B_1^1)$  is controllable if  $(A_c, B_c)$  is controllable and for all eigenvalues of  $A_c$  with non-zero imaginary parts  $\beta_i$ ,  $\tau_m - \tau_s \neq \frac{2m\pi}{\beta_i}$  for all integer numbers  $m \geq 1$ .

We prove the main result in Theorem 1 that the control design (30) leads to a closed-loop stable system. The following lemma is essential to prove Theorem 1.

**Lemma 2.** If any arbitrary linear system  $(A, B)$  is controllable, for any  $\tilde{B}$ , such that  $\mathcal{R}(B) \subset \mathcal{R}(\tilde{B})$ ,  $(A, \tilde{B})$  is also controllable [16].

**Theorem 1.** If  $(A_c, B_c)$  in (9) is controllable, the control design in (30) guarantees that the closed-loop system corresponding to the underlying sampled-data system in (14) is stable.

The proof of Theorem 1 follows from the fact that the controllability of  $(A_h, B_h)$  follows from that of  $(A_c, B_c)$ . This controllability property together with Geromel's algorithm ensures the stability of the control design in (30). Details of the proof are provided in the Appendix.

#### IV. SIMULATION RESULTS

The IEEE 39-bus New England network is used to validate the sparse control design in (30). Recalling that  $x$  corresponds to the deviation of  $\hat{x}$  from its equilibrium value, we assume in the simulations that the time to compute  $\hat{x}$  is negligible compared to the sampling period. This network includes 10 generators with a total of 130 state variables ( $n_i = 13$ ) which include rotor phase angle and frequency, excitation voltage, d-axis sub-transient flux, exciter states, power system stabilizer states, turbine/governor states, active-and reactive load modulation states, and states of Static Var Compensators, and FACTS devices [24]. Generator 1 is in area 1. Generators 2 and 3 belong to area 2. Generators 4, 5, 6, and 7 are located

in area 3, and finally area 4 includes generators 8, 9, and 10. The delays and sampling time are as following:

$$\tau_s = 1 \text{ ms}, \quad \tau_m = 10 \text{ ms}, \quad \tau_\ell = 30 \text{ ms}, \quad h = 33 \text{ ms}.$$

We examine the validity of the results using different control designs with different levels of sparsity. The following definitions are required to understand Table I.

**Definition 3.** If  $\mathcal{H}$  is complete, or equivalently only virtual sparsity is present so that all structurally-zero entries of  $K_s$  and  $G_s$  are due to delays, then the design is denoted by F-D.

**Definition 4.** If  $\mathcal{H}$  is not complete, i.e. both communication sparsity and virtual sparsity is present, or equivalently if some VMs do not communicate with other VMs, and the design is delay-aware, the design is called S-D.

**Definition 5.** If  $\mathcal{H}$  is complete, and no virtual sparsity is present, and the control design (30) is implemented, we denote the corresponding control design as F-C, which coincides with the method presented in [7].

The letters  $D$ ,  $C$ ,  $F$ , and  $S$  stand for *centralized*, *distributed*, *full*, and *sparse*, respectively.

**Definition 6.** (i) Sparsity quotient  $\psi$ : The ratio of the number of zero entries to the total number of entries in  $[K^* \ G^*]$ . (ii) Level of marginality  $\lambda_*$ : Number of eigenvalues of the closed-loop system  $(A_h, B_h)$  whose magnitude is larger than 0.99.

The results obtained using S-D, F-D, and F-C are summarized in Table I, which clearly show that our proposed design of S-D can achieve 90% sparsity with only a 16% reduction in the quadratic cost with the original optimal  $Q$  (sixth row in Table I). We note that any further increase in sparsity causes a large drop in performance (see Figure 5). It is especially noteworthy, as can be gathered from lines 6 and 7, that by simply adding one extra link between 1 and 8, we get a 5% improvement in performance compared to using only local controllers.

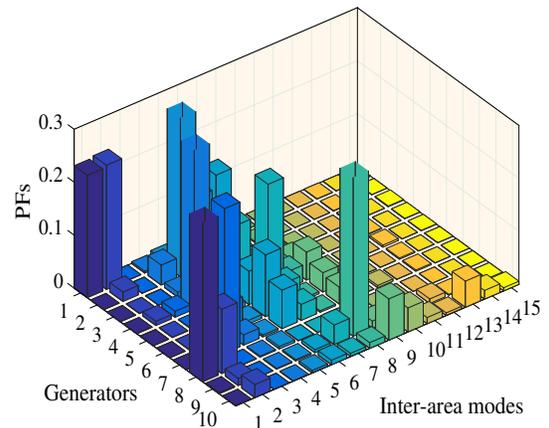


Fig. 3: Participation factors of inter-area modes

Design	$\lambda_*$	$\psi$	$\mu$	comment	$\frac{\mathcal{J} - \mathcal{J}^*}{\mathcal{J}^*}$
F-C	13	0	0	The design in [7]	0%
F-D	13	52%	0	Virtual Sparsity Only	12%
S-D	13	79%	0.05		16%
S-D	13	87%	0.1		16%
S-D	13	88%	0.15	1 ↔ 8, 6 ↔ 7, 2 ↔ 3	16%
<b>S-D</b>	<b>13</b>	<b>90%</b>	<b>0.2</b>	1 ↔ 8	<b>16%</b>
S-D	13	91%	0.25	only local feedback	21%
Open-Loop	15	100%	1	$U[k] = O_{\mathcal{G},1}$	380%

TABLE I: A comparison of different designs, showing that the proposed controller results in a maximum of 16% reduction in the overall performance with 90% sparsity.

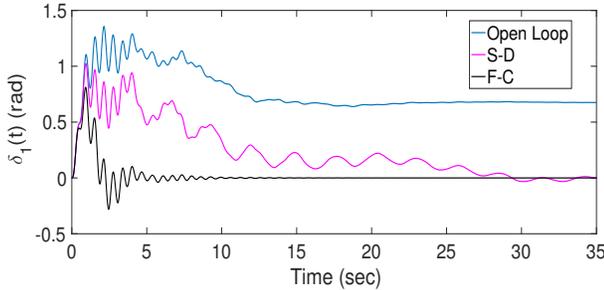


Fig. 4:  $\delta_1(t)$  :Open-Loop, with S-D with 90% sparsity, and with F-C. The responses show that the performance in comparison with open-loop is improved for all cases.

## V. CONCLUSIONS

In this paper, we have proposed a distributed and sparse delay-aware wide-area control of power system networks. A sparse control design that accommodates both communication sparsity and delay-induced sparsity is proposed. Simulation results on a 39-bus New England network verified the proposed approaches. In future work, we intend to investigate the effect of cyber-physical attacks on such designs and for the case when the inter-area delay is significantly larger.

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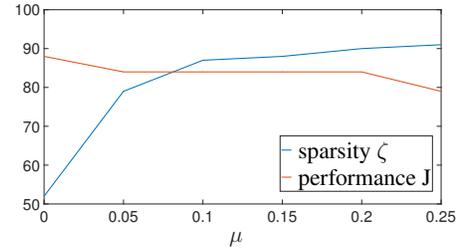


Fig. 5: Change in performance and sparsity as a function of the parameter  $\mu$ , which is the threshold that determines the level of sparsity. This figure clearly indicates the trade-off between sparsifying the control design and closed-loop performance.

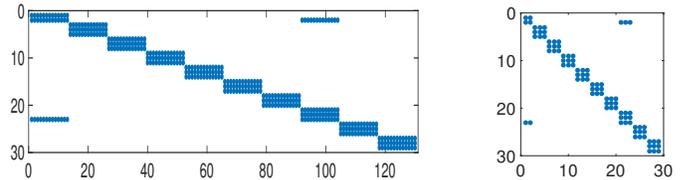


Fig. 6: Non-zero elements of  $K_s^*$  (left) and  $G_s^*$  (right) with 90% sparsity. Only VMs 1 and 8 need to communicate among all inter- and intra- areas, with all local feedbacks on, i.e. the diagonal blocks are all non-zero. The dimensions of  $K_s^*$  and  $G_s^*$  are  $29 \times 130$  and  $29 \times 29$ , respectively, where the number 130 corresponds to the total number of state variables for 10 generators, each generator containing 13 state variables and 29 is the total number of control inputs: Generator 1 is alone in area 1 and does not have intra-area delays but all other generators have three delays (and therefore produce three updates for each during each sampling interval).

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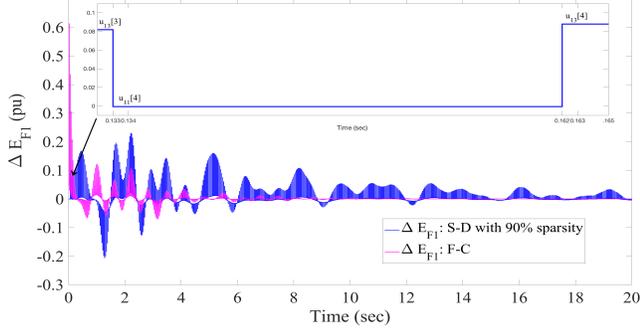


Fig. 7:  $U_1(t)$ : Delay-aware control input with 90% sparsity vs. Delay-aware without sparsity. It shows that the overall amplitude range of the two inputs are comparable, with the sparsification requiring a longer time-duration of the input. The zoomed-in S-D plot around  $k = 4$  corresponds to the time interval  $[0.132, 0.165]$ . The two ticks of updates is seen (since only generator 1 is present in area 1, and hence no inputs arrive with a time delay of  $\tau_m$ ). It is worth noting that  $u_{13}[4]$ , which uses information from generator 8, is much larger than  $u_{11}[4]$  which uses only its own information.

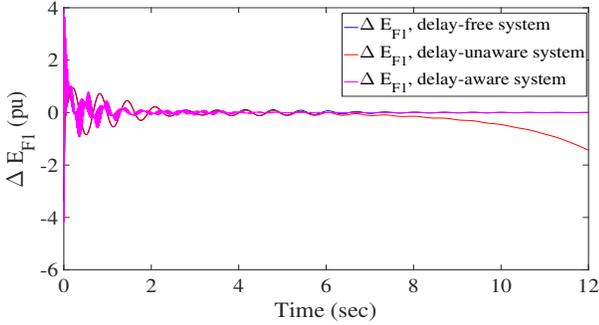


Fig. 8: Comparison of delay-aware, delay-free, and delay-unaware control inputs with 90% sparsity. The delay-free case is associated with the designed control input for (11). However, the delay-unaware case is using the delay-free control inputs on delayed dynamic (14).

## APPENDIX

### A. Proof of Lemma 1

We know that an LTI system is controllable if and only if its Jordan form is controllable. Writing  $(A, B_1^1)$  in Jordan form as

$$z[k+1] = Jz[k] + \Psi\Gamma B_c u[k], \quad (32)$$

where  $\Psi$  is a non-singular matrix,  $J = \Psi A \Psi^{-1}$ ,  $\Gamma = \int_{h-\tau_m}^{h-\tau_s} e^{A_c s} ds$ , and  $z = \Psi x$ . We assume, without loss of generality, that  $A_c$  is in Jordan form, with  $\lambda_i$ s as distinct eigenvalues. The diagonal entry  $\gamma_{ii}$  of  $\Gamma$  then is given by  $\gamma_{ii} = \int_{h-\tau_m}^{h-\tau_s} e^{\lambda_i s} ds$ . It follows then  $\gamma_{ii} = \tau_m - \tau_s$  if  $\lambda_i = 0$  and  $\gamma_{ii} = \frac{1}{\lambda_i} (e^{\lambda_i(h-\tau_s)} - e^{\lambda_i(h-\tau_m)})$ , otherwise. It can be provided that  $\gamma_{ii} \neq 0$  as follows: Assume that  $\gamma_{ii} = 0$ . Then, we consider two cases: (i)  $\lambda_i = 0$  which implies that  $\tau_s = \tau_m$ . (ii) If  $\lambda_i \neq 0$ , then it follows that  $e^{\lambda_i(h-\tau_s)} = e^{\lambda_i(h-\tau_m)}$ ; If  $\lambda_i$  is real, it follows once again that  $\tau_s = \tau_m$ . But if  $\lambda_i = \sigma_i + j\beta_i$  then  $e^{j\beta_i(h-\tau_s)} = e^{j\beta_i(h-\tau_m)}$  because  $e^{\sigma_i}$  is non-zero. The last equation in turn implies that  $\beta_i(h-\tau_s) - \beta_i(h-\tau_m) = 2m\pi$  or  $\tau_m - \tau_s = \frac{2m\pi}{\beta_i}$ , which establishes that  $\gamma_{ii} \neq 0$ . This in turn proves that  $\Gamma$  is full rank. Since  $\Psi$  is also nonsingular,  $\Psi\Gamma B_c$  and  $B_c$  share their rank properties. We also know that  $J = \Psi A \Psi^{-1}$  shares the ranks properties with  $A$ , which establishes that  $\gamma_{ii} \neq 0$ . From  $A = e^{A_c h}$  being full rank [8],  $\mathcal{C}(A_c, B_c)$  and  $\mathcal{C}(A, B_1^1)$  have the same row rank, where  $\mathcal{C}(M_A, M_B)$  is the controllability matrix of the system  $x[k+1] = M_A x[k] + M_B u[k]$ , proving Lemma 1.  $\square$

### B. Proof of Theorem 1

The first step is to show that  $(A_h, B_h)$  is controllable. Lemma 1 implies that  $(A, B_{11})$  is controllable if  $(A_c, B_c)$  is controllable. Defining  $\overline{B}_1 = B_1^1 \odot B_2^1 \odot O_{N,n}$ , it follows that if we choose  $u_{ig(i)}[k] = 0$ , then

$$B_1 U[k] + B_2 U[k-1] = \overline{B}_1 U[k]$$

Since  $\mathcal{R}(B_1^1) \subset \mathcal{R}(\overline{B}_1)$ , it follows therefore that  $(A, \overline{B}_1)$  is controllable and this proves the theorem.  $\square$

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