MIT Libraries | DSpace@MIT

MIT Open Access Articles

Timelike-helicity γ B → ππ form factor from light-cone sum rules with dipion distribution amplitudes

The MIT Faculty has made this article openly available. ***Please share*** how this access benefits you. Your story matters.

Citation: Cheng, Shan, et al. "Timelike-Helicity B → π π Form Factor from Light-Cone Sum Rules with Dipion Distribution Amplitudes." Physical Review D, vol. 96, no. 5, Sept. 2017. © 2017 American Physical Society

As Published: http://dx.doi.org/10.1103/PhysRevD.96.051901

Publisher: American Physical Society

Persistent URL: http://hdl.handle.net/1721.1/113609

Version: Final published version: final published article, as it appeared in a journal, conference proceedings, or other formally published context

Terms of Use: Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.

Massachusetts Institute of Technology

PHYSICAL REVIEW D **96,** 051901(R) (2017)

Timelike-helicity $B \to \pi\pi$ form factor from light-cone sum rules with dipion distribution amplitudes

Shan Cheng,[1] Alexander Khodjamirian,[1] and Javier Virto[2,3]

[1]*Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Straße 3, D-57068 Siegen, Germany*

[2]*Physics Department T31, Technische Universität München, James Frank-Straße 1, D-85748 Garching, Germany*

[3]*Center for Theoretical Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*

(Received 25 August 2017; published 26 September 2017)

We complete the set of QCD light-cone sum rules for $B \to \pi\pi$ transition form factors, deriving a new sum rule for the timelike-helicity form factor F_t in terms of dipion distribution amplitudes. This sum rule, in the leading twist-two approximation, is directly related to the pion vector form factor. Employing a relation between F_t and other $B \to \pi\pi$ form factors, we obtain also the longitudinal-helicity form factor F_0. In this way, all four (axial-)vector $B \to \pi\pi$ form factors are predicted from light-cone sum rules with dipion distribution amplitudes. These results are valid for small dipion masses with large momentum.

DOI: 10.1103/PhysRevD.96.051901

I. INTRODUCTION

The aim of this paper is to present a solution of the problem outlined in Ref. [1], concerning the calculation of the timelike-helicity $\bar{B}^0 \to \pi^+\pi^0$ form factor F_t from light-cone sum rules. In Ref. [1], the method of QCD light-cone sum rules was applied to calculate the $B \to \pi\pi$ transition form factors, starting from a particular correlation function expanded near the light-cone in dipion distribution amplitudes (DAs). The form factor corresponding to the timelike-helicity of the dilepton and denoted as F_t (for a definition and helicity basis of $B \to \pi\pi$ form factors see e.g. Ref. [2]) could not, however, be consistently obtained from that correlation function because of emerging kinematical singularities.

Here we obtain a new light-cone sum rule for F_t employing a modified correlation function,

$$\Pi_5(q,k_1,k_2) = i\int d^4x e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|T\{\bar{u}(x)im_b\gamma_5 b(x),\ \bar{b}(0)im_b\gamma_5 d(0)\}|0\rangle, \quad (1)$$

where, instead of the axial-vector $b \to u$ transition current, the pseudoscalar one is used. The immediate advantage of this choice is that the hadronic matrix element of the pseudoscalar current is solely determined by the timelike-helicity $B \to \pi\pi$ form factor we are interested in:

$$\begin{aligned}-iq^\mu\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu\gamma_5 b|\bar{B}^0(p)\rangle &= \langle\pi^+(k_1)\pi^0(k_2)|\bar{u}im_b\gamma_5 b|\bar{B}^0(p)\rangle\\ &= \sqrt{q^2}F_t(q^2,k^2,\zeta). \end{aligned} \quad (2)$$

Here $k^2 = (k_1+k_2)^2$ is the invariant mass squared of the dipion system, and

$$2\zeta - 1 = \frac{2q\cdot\bar{k}}{\sqrt{\lambda}} = \beta_\pi(k^2)\cos\theta_\pi, \quad (3)$$

where $\lambda \equiv m_B^4 + q^4 + k^4 - 2m_B^2q^2 - 2m_B^2k^2 - 2q^2k^2$ is the Källèn function, $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$, $\bar{k} = k_1 - k_2$, and θ_π corresponds to the angle between the three-momenta of the neutral pion and the B-meson in the dipion center-of-mass frame. Note that we neglect the light u and d quark masses throughout the paper.

The rest of the derivation follows the procedure explained in detail in Ref. [1]. First we calculate the correlation function (1) to the leading order (zeroth order in α_s) and in the lowest twist-two approximation. To this end, we contract the b-quark fields in the free propagator and perform the integration in Eq. (1). The result for the correlation function (1), which is by itself an invariant amplitude, reads

$$\Pi_5(p^2,q^2,k^2,\zeta) = \sqrt{2}m_b^2\int_0^1 du\frac{q\cdot k + uk^2}{(q+uk)^2 - m_b^2}\Phi_\parallel^{I=1}(u,\zeta,k^2), \quad (4)$$

in terms of the isospin-one dipion DAs introduced and defined in [3–6]. Importantly, the above expression—as opposed to the correlation function considered in Ref. [1]—is free from kinematical singularities. Also important is that Eq. (4) only depends on the chiral-even dipion DA:

$$\langle\pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu\int_0^1 du e^{iu(k\cdot x)}\Phi_\parallel^{I=1}(u,\zeta,k^2), \quad (5)$$

which is normalized to the pion vector form factor in the timelike region:

 © 2017 American Physical Society

RAPID COMMUNICATIONS

$$\int_0^1 du \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi(k^2). \quad (6)$$

We also use the double expansion of this DA in partial waves and Gegenbauer polynomials [4]:

$$\Phi_{\parallel}^{I=1}(u,\zeta,k^2) = 6u\bar{u}\sum_{n=0,2,\cdots}^{\infty}\sum_{\ell=1,3,\cdots}^{n+1} B_{n\ell}^{\parallel}(k^2)C_n^{3/2} \times (u-\bar{u})\beta_\pi(k^2)P_\ell^{(0)}(\cos\theta_\pi), \quad (7)$$

where $\bar{u} \equiv 1-u$, $P_\ell^{(m)}$ are the associated Legendre polynomials, and the normalization of the DA in Eq. (6) fixes the coefficient $B_{01}^{\parallel}(k^2) = F_\pi(k^2)$.

We now insert the B-meson ground state in the correlation function (1) and, using the definition of F_t in Eq. (2), write down the hadronic dispersion relation in the variable $p^2 = (q+k)^2$, the square of the momentum transferred to the B-meson interpolating current,

$$\Pi_5(p^2,q^2,k^2,\zeta) = \frac{f_B m_B^2 \sqrt{q^2}F_t(q^2,k^2,\zeta)}{m_B^2-p^2} + \cdots, \quad (8)$$

where f_B is the B-meson decay constant and the ellipses denote the contributions of radially excited and continuum states with B-meson quantum numbers. The two remaining steps in the derivation of the light-cone sum rule involve (1) employing the quark-hadron duality approximation with a threshold parameter s_0^B and (2) applying the Borel transformation in the variable p^2. The resulting sum rule reads

$$\sqrt{q^2}F_t(q^2,k^2,\zeta) = -\frac{m_b^2}{\sqrt{2}f_B m_B^2}\int_{u_0}^1 \frac{du}{u^2} e^{\frac{m_B^2-s(u)}{M^2}} \times (m_b^2-q^2+u^2k^2)\Phi_{\parallel}^{I=1}(u,\zeta,k^2), \quad (9)$$

where $s(u) = (m_b^2 - \bar{u}q^2 + u\bar{u}k^2)/u$, and u_0 is the solution to $s(u_0) = s_0^B$.

Using the above LCSR for the form factor F_t, and the sum rule for the form factor $F_\parallel$ obtained in Ref. [1], together with the relation between three form factors valid to the same twist-two accuracy, we calculate the longitudinal-helicity form factor:

$$\sqrt{q^2}F_0(q^2,k^2,\zeta) = \frac{1}{m_B^2-q^2-k^2}[\sqrt{\lambda}\sqrt{q^2}F_t(q^2,k^2,\zeta) + 2\sqrt{k^2}q^2(2\zeta-1)F_\parallel(q^2,k^2,\zeta)]. \quad (10)$$

Thus, all four $B \to \pi\pi$ form factors in the region of small and intermediate q^2 and small k^2 are now accessible from LCSRs with dipion DAs.

Applying the partial wave expansion to the two form factors under consideration, we write

$$F_{0,t}(q^2,k^2,\zeta) = \sum_{\ell=0}^{\infty}\sqrt{2\ell+1}\,F_{0,t}^{(\ell)}(q^2,k^2)P_\ell^{(0)}(\cos\theta_\pi). \quad (11)$$

Substituting Eqs. (7) and (11) into (9) and (10), multiplying both sides by $P_{\ell'}^{(0)}(\cos\theta_\pi)$ and integrating over $\cos\theta_\pi$, we obtain the ℓth partial wave contribution to the $\bar{B}^0 \to \pi^+\pi^0$ form factor (note that only odd partial waves $\ell = 1,3,5,\ldots$ contribute for the isovector dipion state):

$$\sqrt{q^2}F_t^{(\ell)}(q^2,k^2) = -\frac{6m_b^2}{\sqrt{2}f_B m_B^2}\frac{\beta_\pi(k^2)}{\sqrt{2\ell+1}} \times \sum_{\substack{n=\ell-1\\ n\,\mathrm{even}}}^{\infty} B_{n\ell}^{\parallel}(k^2)\int_{u_0}^1 \frac{du}{u}\bar{u}e^{\frac{m_B^2-s(u)}{M^2}}(m_b^2-q^2+u^2k^2) \times C_n^{3/2}(u-\bar{u}), \quad (12)$$

$$\sqrt{q^2}F_0^{(\ell)}(q^2,k^2) = \frac{\sqrt{\lambda}\sqrt{q^2}}{m_B^2-q^2-k^2}F_t^{(\ell)}(q^2,k^2) + \frac{\sqrt{k^2}q^2\beta_\pi(k^2)}{m_B^2-q^2-k^2}\sum_{\ell'=1}^{\infty} I_{\ell\ell'}F_\parallel^{(\ell')}(q^2,k^2), \quad (13)$$

where

$$I_{\ell\ell'} = \sqrt{2\ell+1}\sqrt{2\ell'+1} \times \int_{-1}^{1} d\cos\theta_\pi \frac{\cos\theta_\pi}{\sin\theta_\pi}P_\ell^{(0)}(\cos\theta_\pi)P_{\ell'}^{(1)}(\cos\theta_\pi). \quad (14)$$

Equations (12) and (13) complement the ones for the partial waves of the form factors $F_{\parallel,\perp}^{(\ell)}$ obtained in Ref [1].

II. RESONANCE CONTRIBUTION

To assess the dominant ρ-resonance contribution to the P-wave of $B \to \pi\pi$ timelike-helicity form factor, we follow Refs. [1,7] and relate the form factor F_t to the corresponding $B \to \rho$ form factor $A_0(q^2)$ (defining the $B \to \rho$ form factors as in Refs. [8,9]) by means of a resonance model,

$$\sqrt{q^2}F_t^{(\ell=1)}(q^2,k^2) = -\frac{\sqrt{\lambda}\beta_\pi(k^2)}{\sqrt{3}}\frac{g_{\rho\pi\pi}m_\rho A_0(q^2)}{[m_\rho^2-k^2-i\sqrt{k^2}\Gamma_\rho(k^2)]} + \cdots, \quad (15)$$

where the ellipsis denotes the contributions of excited resonances, and the energy-dependent width $\Gamma_\rho(k^2)$ (see definition in Eq. (36) of Ref [1]) effectively takes into account the two-pion mixing with the ρ. For $A_0(q^2)$ we derive a LCSR in terms of the ρ-meson DA (in the zero-width approximation) taking for consistency the twist-two

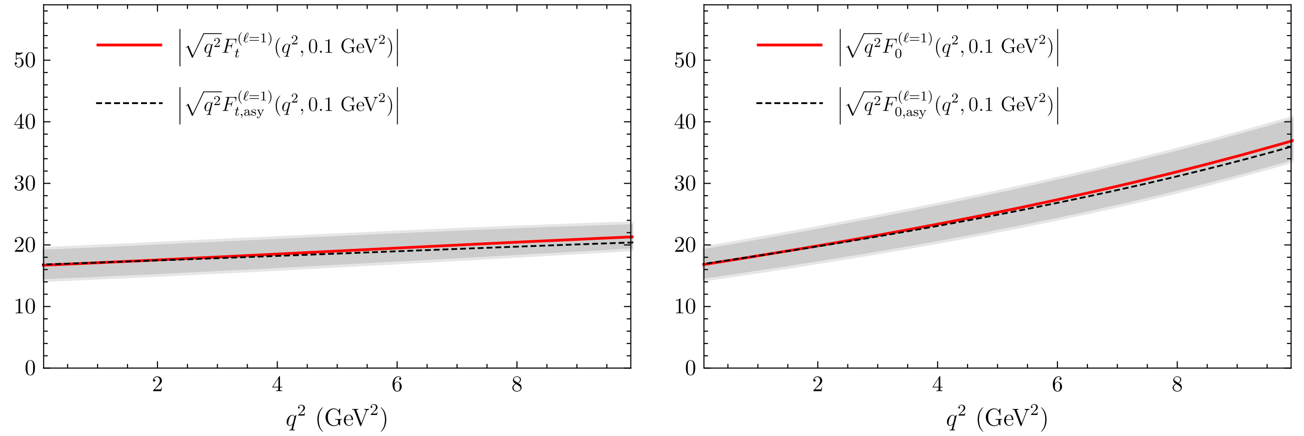

FIG. 1. LCSR predictions for the absolute values of the timelike- and longitudinal-helicity P-wave $B \to \pi\pi$ form factors $\sqrt{q^2}F_{t,0}^{(\ell=1)}(q^2, k^2 = 0.1\ \mathrm{GeV}^2)$. Solid (dashed) curves are the central values with the nonasymptotic (asymptotic) dipion DAs. Shaded bands show the estimated theoretical uncertainties.

LO approximation. This sum rule is similar to the sum rules derived in Refs. [8,9], where one can also find the necessary details on the ρ-meson DAs. We find

$$A_0(q^2) = \frac{m_b^2 f_\rho}{2 f_B m_B^2} \times \int_{u_0}^1 \frac{du}{u} \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}q^2 + u\bar{u}m_\rho^2}{uM^2}\right)\phi_\parallel^\rho(u), \tag{16}$$

containing the chiral-even ρ-meson DA $\phi_\parallel^\rho(u)$, normalized to the ρ-meson decay constant f_ρ.

Note that if we substitute the LCSRs (12) for $F_t^{(\ell=1)}(q^2, k^2)$ and (16) for $A_0(q^2)$ to the left-hand and right-hand sides of the one-resonance approximation (15), respectively, and use for simplicity the asymptotic two-pion and ρ-meson DAs, that is: $B_{n>0,\ell} = 0$ and $\phi_\parallel^\rho(u) = 6u(1-u)$, the resulting relation will restore $B_{01}(k^2) = F_\pi(k^2)$ in the form of the ρ-meson contribution to the pion form factor (e.g., Eq. (25) in Ref. [7]). However, this is valid only up to power corrections of $\mathcal{O}(q^2/m_b^2)$, $\mathcal{O}(k^2/m_b^2)$ and $\mathcal{O}(\Delta/m_b)$, where we rescale the effective threshold as $s_0^B = (m_b + \Delta)^2$. The latter correction originates from a global factor of $1/u$ in the integrand.

III. NUMERICAL ANALYSIS

For the numerical analysis of the new sum rule (9), we adopt the same input as in Ref. [1] in particular: the b-quark mass, decay constant of B, Borel parameter range and the duality threshold. The nonperturbative universal input encoded in the functions $B_{n\ell}^\parallel(k^2)$ entering the Gegenbauer expansion of the DA are taken from the instanton model used in Ref. [4] and are listed in Eqs. (6.7)–(6.12) there. These estimates are only valid near the dipion threshold

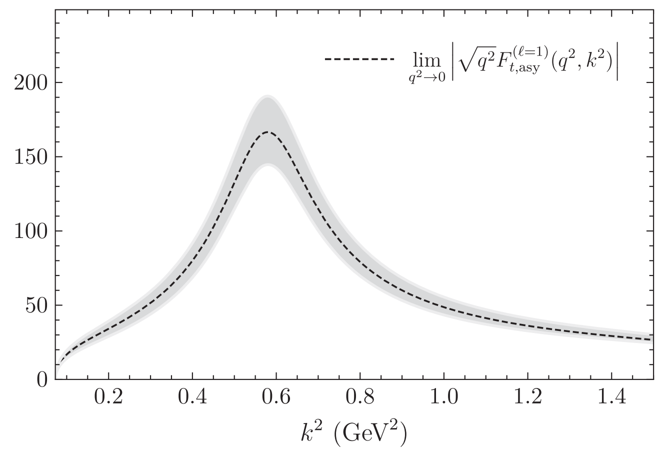

FIG. 2. Absolute value of the timelike-helicity P-wave $B \to \pi\pi$ form factor $\sqrt{q^2}F_t^{(\ell=1)}(q^2, k^2)$ at $q^2 = 0$ obtained from LCSR with the asymptotic dipion DA, using the data [10] on the pion form factor.

$k^2 \sim 4m_\pi^2$.[1] Hence, we are able to predict the q^2-dependence of the form factors $F_{t,0}^{(\ell)}(q^2, k^2 \sim 4m_\pi^2)$ in the region of large recoil $0 \le q^2 \le 10$–$12\ \mathrm{GeV}^2$, where one can trust the light-cone expansion of the correlation function. The results for the P-wave form factors are plotted in Fig. 1, where only the uncertainties from the sum rule parameters s_0^B and M^2 are shown.

We find that the higher partial waves are also strongly suppressed in F_t as in the other form factors considered in Ref. [1]. For example, the ratio of $\ell = 3$ to $\ell = 1$ contributions to $F_t(q^2, 4m_\pi^2)$ is smaller than 5% at all

[1]For numerical illustration we will take $k^2 = 0.1\ \mathrm{GeV}^2$, slightly above the two-pion threshold, since the phase-space factor $\beta_\pi(k^2)$ in Eq. (12) makes the form factor vanish at threshold.

accessible q^2. The contribution of nonasymptotic terms is also small, as can bee seen by setting to zero all $B^{\parallel}_{n\ell}$ except $B^{\parallel}_{01}$ (see the dashed curves in Fig. 1). This allows us to predict the P-wave form factor also at larger k^2, including the ρ-resonance region and even beyond, provided we use for $B^{\parallel}_{01}(k^2)$ the accurate data for the pion form factor $F_\pi(k^2)$ provided by the Belle collaboration [10]. Our result for $\lim_{q^2\to 0}\sqrt{q^2}F_t^{(\ell=1)}(q^2,k^2)$, as a function of dipion invariant mass squared is shown in Fig. 2.

We have also calculated the $B\to\rho$ contribution to the form factor $F_t^{(\ell=1)}(q^2,k^2)$ using Eq. (15) and find that the remaining resonant and continuum contributions to this form factor can amount up to 20% in the small k^2 region, in agreement with the findings of Refs. [1,7] for the other $B\to\pi\pi$ form factors.

IV. CONCLUSION

In conclusion, we have completed the set of LCSRs with dipion DAs for the $B\to\pi\pi$ form factors. These sum rules are complementary to the ones derived in Ref. [7] in terms of B-meson DAs and complementary to the derivations from dispersion theory [11] and to the calculations at large k^2 [12]. The form factor F_t obtained here, while not contributing to the semileptonic $B\to\pi\pi\ell\nu$ rate in the massless lepton approximation, plays an important role in the factorization formula for nonleptonic $B\to\pi\pi\pi$ decays [13–15]. Further improvements of the sum rules presented here and in Refs. [1,7] require the inclusion of higher twists and NLO corrections, but most importantly—for the ones derived here and in Ref. [1]—a better knowledge of dipion DAs and their Gegenbauer coefficients $B_{n\ell}$.

ACKNOWLEDGMENTS

This work is supported by the DFG Research Unit FOR 1873 "Quark Flavour Physics and Effective Theories," Contract No. KH 205/2-2. J. V. acknowledges funding from the Swiss National Science Foundation, from Explora Project No. FPA2014-61478-EXP, and from the European Union's Horizon 2020 research and innovation program under the Marie Sklodowska-Curie Grant Agreement No. 700525 NIOBE.

[1] C. Hambrock and A. Khodjamirian, Nucl. Phys. **B905**, 373 (2016).
[2] S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel, and D. van Dyk, Phys. Rev. D **89**, 014015 (2014).
[3] M. Diehl, T. Gousset, B. Pire, and O. Teryaev, Phys. Rev. Lett. **81**, 1782 (1998).
[4] M. V. Polyakov, Nucl. Phys. **B555**, 231 (1999).
[5] D. Müller, D. Robaschik, B. Geyer, F. M. Dittes, and J. Hořejši, Fortschr. Phys. **42**, 101 (1994).
[6] M. V. Polyakov and C. Weiss, Phys. Rev. D **59**, 091502 (1999).
[7] S. Cheng, A. Khodjamirian, and J. Virto, J. High Energy Phys. 05 (2017) 157.
[8] P. Ball and V. M. Braun, Phys. Rev. D **55**, 5561 (1997).
[9] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014029 (2005).
[10] M. Fujikawa *et al.* (Belle Collaboration), Phys. Rev. D **78**, 072006 (2008).
[11] X. W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, Phys. Rev. D **89**, 053015 (2014).
[12] P. Böer, T. Feldmann, and D. van Dyk, J. High Energy Phys. 02 (2017) 133.
[13] S. Kränkl, T. Mannel, and J. Virto, Nucl. Phys. **B899**, 247 (2015).
[14] J. Virto, *Proc. Sci.*, FPCP 2016 (2017) 007 [arXiv:1609.07430].
[15] R. Klein, T. Mannel, J. Virto, and K. K. Vos, arXiv: 1708.02047.