Establishing an Unambiguous Connection between Grain Size and Style of Sediment Transport in the Lower Niobrara River, Nebraska, USA

by

Vivian Leung

Submitted to the Department of Earth, Atmospheric and Planetary Sciences

in Partial Fulfillment of the Requirements for the Degree of

Bachelor of Science in Earth, Atmospheric and Planetary Sciences

At the Massachusetts Institute of Technology

June 2, 2006

Author_ Signature redacted ________________________

Department of Earth, Atmospheric and Planetary Sciences

June 2, 2006

Certified by_ Signature redacted ____________________

Signature redacted

David Mohrig

Thesis Advisor

Accepted by_ ________________________

Samuel Bowring

Chair, Committee on Undergraduate Program

Establishing an Unambiguous Connection between Grain Size and Style of Sediment
Transport in the Lower Niobrara River, Nebraska, USA
by
Vivian Leung

Submitted to the
Department of Earth, Atmospheric and Planetary Sciences

June 2, 2006

In Partial Fulfillment of the Requirements for the Degree of
Bachelor of Science in Earth, Atmospheric and Planetary Sciences

ABSTRACT

The transport of sediment is often separated into two components, bedload and suspended load. This division is important because bedload has a dominant control on channel morphology while suspended load dominates the formation of overbank deposits. Experimental data has related the style of sediment transport to mean flow conditions and bed topography. However direct application of this method in natural, sandy rivers is difficult due to large variabilities in flow. We propose a method for determining local flow conditions using the distribution of grain sizes traveling in the water column. Local shear velocity is found by fitting the Rouse equation for suspended sediment transport to measured sediment concentrations. Empirical criteria for distinguishing between suspended load and bedload are used to determine the fraction of sediment traveling in each respective mode. Application of this method to the Niobrara River, Nebraska, shows that ~80 % of the sediment is traveling as suspended load, ~ 20 % is traveling in a transitional mode between bedload and suspended load and less than 1 % is traveling as pure bedload. We establish an unambiguous connection between grain size and the style of sediment transport and highlight the importance of the transitional transport mode in natural systems.

Thesis Supervisor: David Mohrig
Title: Associate Professor

Table of Contents

Figures

Sediment Transport

Rivers are a major conduit for sediment being transported over continents. Sediment is carried by the fluid in the channel and can be moved in several modes of transport.

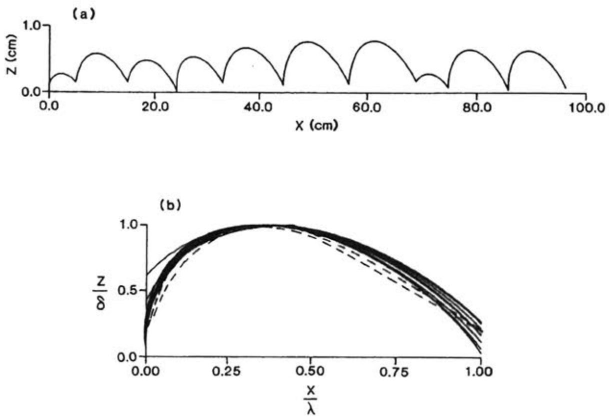


Fig. 1. Predicted saltation trajectories for sand. (a) Sequence of saltation hops predicted for a single grain of sand. (b) Normalized trajectories with δ and λ as the maximum height and length of each individual trajectory. (Wiberg and Smith, 1985)

Bed load transport occurs when the shear stress begins to exceed the critical shear stress for initiation of motion and grains are just beginning to move by sliding, rolling or saltating along the bed. Bedload transport is characterized by the grains staying in very close contact with the bed, with the upper bound for bedload transport being the height of a saltation hop that scales with grain size. Saltation is a form of movement composed of small hops with characteristic trajectories as in Figure 1. Bedload transport has a dominant control on the morphology of channels, including controls on channel width, slope and depth. Instabilities in the bedload transport also act to create bedforms including ripples, dunes and bars that are found in sand-bedded rivers. In bedrock channels bedload transport is the major mechanism for bedrock erosion. Bedload transport also sets the lower boundary condition for sediment transport as suspended load.

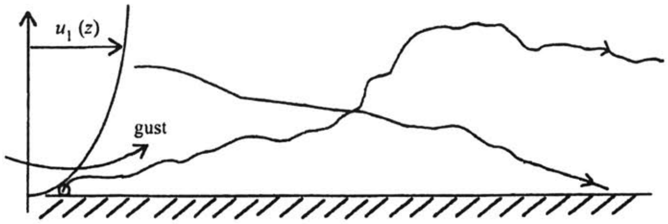


Fig. 2. Idealized transport trajectory for a grain of suspended sediment. (K. Nishimura and J. C. R. Hunt, 2000)

Suspended load transport occurs when grains do not interact as much with the bed and travel higher into the interior flow. Grains are carried by upward directed turbulent eddies resulting in more erratic trajectories. Suspended load transport typically dominates construction of overbank deposits including levees and floodplains.

Field Background

Fig. 3. Location of the field area on the lower Niobrara River in north-eastern Nebraskan. (Bristow, 1999)

The Niobrara River has its headwaters near the town of Lusk in east-central Wyoming. From there it flows generally eastward, cutting through the northern margins of the Nebraska Sand Hills, until turning north in north-eastern Nebraska for its final 10 km before joining the Missouri River (Fig. 3). The Niobrara River is the largest free-flowing river of the Great Plains (Bristow, 1999). The Niobrara River is mostly groundwater fed providing an approximately steady year round base flow with minor overland flow from tributaries (Bristow, 1999). The yearly average discharge for the lower Niobrara River is ~48 m^3/s.

Somewhat higher discharges are associated with the spring snowmelt and lower discharges are associated with both winter freeze over and irrigation withdrawal in late summer. At these lower discharges the tops of mid-channel bars within the river become exposed (Ethridge, 1999). Bed composition is mostly medium sand with a very small percentage of pebbles. Sediment sources include Tertiary continental deposits in western Nebraska and Cretaceous Pierre Shale and Niobrara Chalk in eastern Nebraska (Skelly, 2003). Bank vegetation ranges from prairie grasses along most of the river to woody shrubs and scattered trees near its confluence with the Missouri River.

In the past fifty years the lower part of the Niobrara River has undergone large scale changes in channel pattern (Jerolmack et al, 2006). Until the mid-1950s the lower Niobrara River was a wide, braided channel bounded by bedrock valley walls and river terraces that were used for farming (Ethridge, 1999). In 1952 the Fort Randall Dam was closed on the Missouri River, 45 km upstream of the Niobrara confluence, leading to reduced peak and mean discharges on the Missouri River. In 1957 the Gavins Point Dam was closed on the Missouri River, 56 km downstream of the Niobrara confluence, leading to rising water levels on the Missouri River which in turn affected the water elevations of the lower Niobrara River. Since closure of the Gavins Point Dam base level for the lower Niobrara River has risen over 3 meters. At the same time groundwater levels have risen in the Niobrara river valley, changing the vegetation from prairie grasses to dense wetland vegetation. Average channel width has decreased by 45 % and there has been an increase in floodplain area from 8 % to 57 % of the total valley surface since closure of the dam (Ethridge, 1999). The increase in floodplain areas has occurred via sedimentation that has transformed former river terraces into lowland surfaces (Skelly, 2003).

Through sedimentation the bed of the Niobrara River is now perched above its surrounding floodplain on an alluvial ridge (Ethridge, 1999). This superelevation of the channel has led to at least 8 crevasse splays and at least 3 avulsions in the lower Niobrara River between 1938 and 1995 (Ethridge, 1999). Up to 1.8 m of sediment was deposited in one year near an avulsion site (Bristow, 1999). Total bed aggradation in the system extends at least 21 km upstream from the confluence with the Missouri River.

There have been considerable human consequences to the changes in the lower Niobrara River. Farmland has been lost and the headquarters and cabins of Niobrara State Park have been forced to relocate to higher ground. Parts of the roads running beside the river have been moved to higher ground and the Highway 12 bridge has been raised and rebuilt three times since 1960. The town of Niobrara also began experiencing a dramatic increase in flooding following the closure of the two dams on the Missouri River. In 1975 the Army Corps of Engineers began relocating the town of Niobrara to higher ground in response to the flooding. Today the remains of the old town contain only the local high school on the highest land, a golf course, and the old roads. It is wet, frequently floods and wetland vegetation is encroaching.

Field Methods

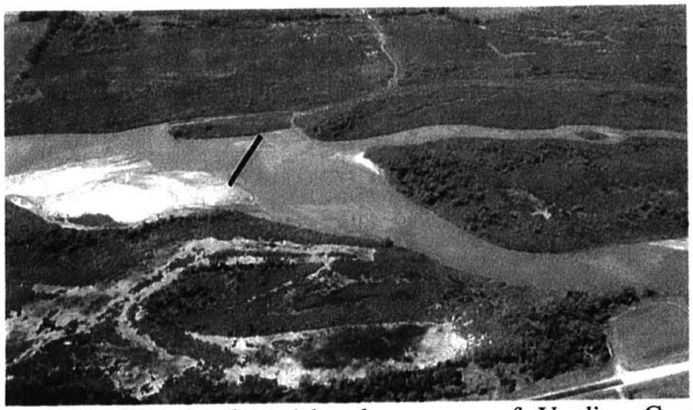


Fig. 4. Oblique aerial view of the field site about 1 km downstream of Verdigre Creek. The black line marks the transect position along which all of the sediment transport data presented here was collected. The transect is about 160 m in length.

The study site is located ~ 4 km upstream of the confluence with the Missouri River. This section of the Niobrara River has a wetted width of ~ 160 m and an average depth of .35 m. The sampling transect across the river is shown in Fig. 4. Measurements were taken every 3 - 8 m along this transect. A Trimble proXH differential GPS was used to record the location of each measurement. Water surface slope, S, was computed from water surface elevations measured using a Leica laser theodolite over 400 m. Water depth, h, was measured using a meter stick. Fluid velocity, u, was measured using a Sontek portable Acoustic Doppler Velocimeter (ADV) which uses transducers to find the water velocity. Average streamwise velocity at each sampling point was estimated by collecting ADV data for 10 s intervals at a flow depth corresponding to $.4h$ above the bed.

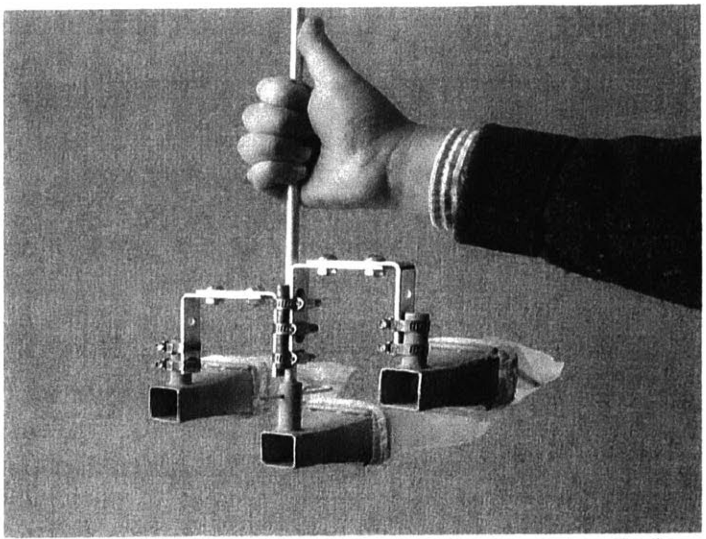


Fig. 5. Three vertically stacked micro-bedload samplers used in this project. Each sampler is a scaled-down version of the Helley-Smith bedload samplers with a 2.54 cm x 2.54 cm orifice.

Sediment flux, q_s, was measured using three scaled-down versions of the traditional Helley-Smith bedload samplers. Each sampler has a 2.54 cm by 2.54 cm orifice. The samplers were stacked vertically in order collect sediment traveling at 0 - 2.54 cm above the bed (Bin B), 2.54 – 5.08 cm above the bed (Bin M) and 5.08 – 7.62 cm above the bed (Bin T). Although Helley-Smith samplers were originally designed to capture only bedload, we used the samplers to collect sediment traveling both as bedload and suspended load in the Niobrara River. The mesh size for the sample bags was 100 μm, so any smaller grain sizes were incompletely captured. Measurements were taken immediately downstream of dune crests with the sampler held in place for 30 s. Any samples taken for longer or shorter time intervals due to the amount of sediment traveling at that location were normalized to a 30 s sampling interval. Water depth was taken as the average water depth at the dune trough and crest.

Lab Methods

Sediment samples were air dried and weighed back at MIT. Some of the samples contained minor amounts of organic debris including leaf fragments, stem fragments and seedpods. Some samples also contained minor amounts of semi-porous precipitates which were mostly larger than the median grain size. The largest fragments of organic matter and precipitates were removed manually before grain size analysis.

Grain size distributions for the sediment samples were measured using a Horiba CAMsizer (Retsch Technology). The CAMsizer has two digital cameras that take photographs of falling grains of sand. An equivalent volume for each grain is calculated by fitting the grain image to an elliptical projection of a particle. The CAMsizer measures most but not all of the particles in a sample. Particle sizes are binned into operator-defined size classes as they are measured. For this study the bin spacing was set at a quarter-phi interval between the sizes of 14 – 4000 μm and was set to a larger spacing for grain diameters from 4000 – 12000 μm.

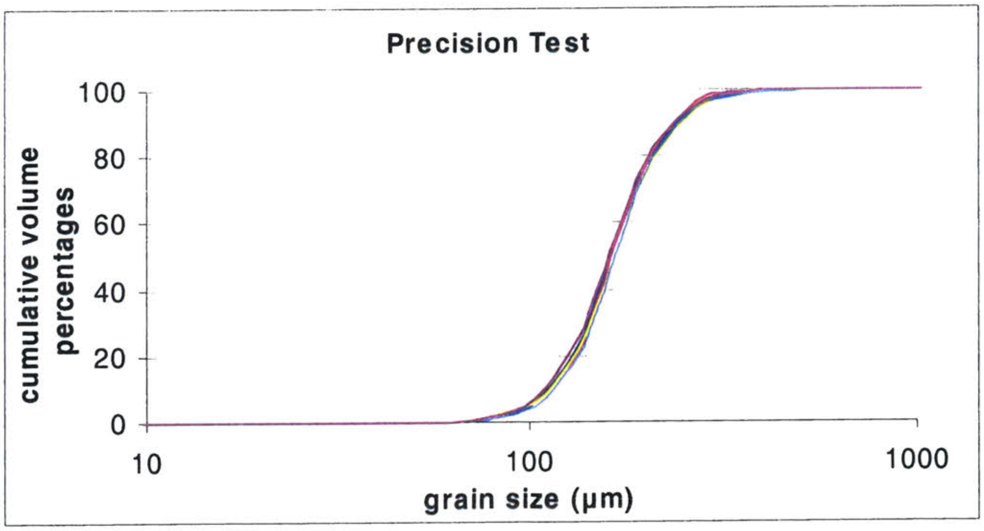


Fig. 6. Reproducibility of CAMsizer results. Five analyses of the same sediment sample demonstrate very good machine precision. Reproducibility is poorest at the coarse tail of the grain size distribution.

Grain size distributions are often represented as cumulative volume percentages. Figure 6 characterizes the precision associated with these distributions for natural sands measured with the CAMsizer. Agreement between the five duplicate samples is very good except for grain sizes above d90. Since the CAMsizer does not measure every particle in a sample and there are usually fewer larger particles in a sample, there is more uncertainty and less precision associated with the coarse tail of the grain size distribution.

Mass Results

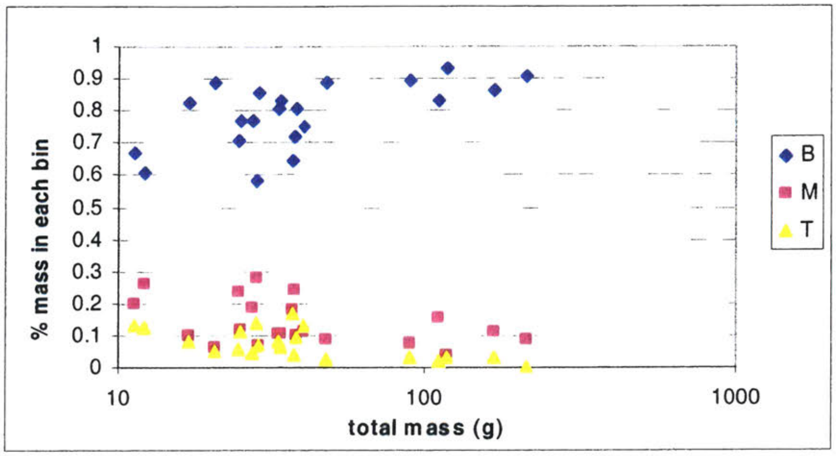


Fig. 7. Fraction of the total sample mass collected in the bottom (B), middle (M) and top (T) sediment sampler. (see Fig. 5)

Figure 7 shows the mass fraction captured in bin B, M or T for each of 21 sediment samples at the Verdigre site (Fig. 4). The mass fraction collected in the bottom bin increases as the total mass of the sample increases. The majority of the sediment, over 60 %, was captured in the bottom bin (B) with a lesser amount, 10 % to 30 %, captured in the middle bin (M) and very little, less than 10%, captured in the top bin (T). Most of the sediment moving in the water column is traveling within 2.54 cm of the bed.

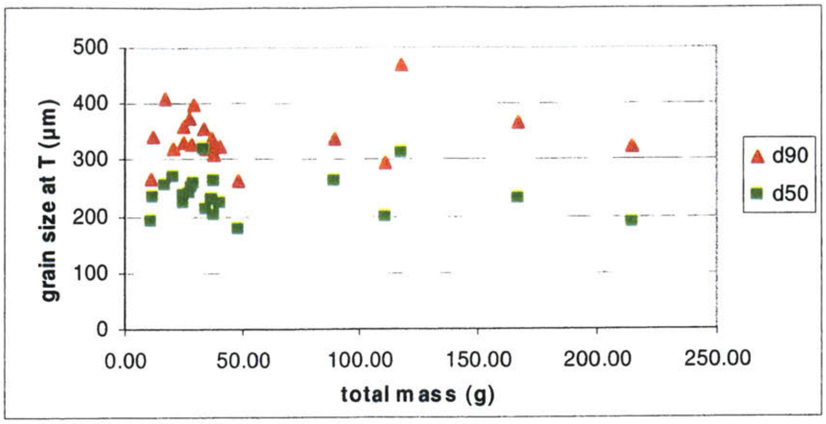


Fig. 8. Size of particles traveling between 5.08 cm and 7.62 cm above the bed at the Verdigre transect (Fig.4) d50 is the median grain size collected in bin T and d90 is nominal grain diameter associated with the ninetieth percentile collected in bin T.

Figure 8 shows the variability in grain sizes traveling in the top bin, between 5.08 cm and 7.62 cm above the bed as a function of total sample mass. The d90 and the d50 for top bin show a considerable amount of scatter but there is no trend in size as the total mass captured increases. The variability in grain sizes shown in Figure 8 is probably due to the variability inherent in the uneven bed topography and multiple thalwegs of a sandy braided river.

Rouse Profiles

A theoretical model for characterizing the suspended sediment concentration in a river is the Rouse profile:

$$\frac{\varepsilon_s}{1-\varepsilon_s} = \left(\frac{\varepsilon_s}{1-\varepsilon_s}\right)_{z=a} \left(\frac{z_a}{z}\right)^p \qquad (1)$$

At low concentrations equation 1 can be simplified to:

$$\frac{\varepsilon_s}{\varepsilon_R} = \left(\frac{z_R}{z}\right)^p \qquad (2)$$

where z refers to the elevation above the bed and ε_s is the concentration of suspended sediment at that elevation. z_R and ε_R refer the elevation and suspended sediment concentration at a reference level, usually the top boundary of the bedload transport layer. P is the Rouse number:

$$p = \frac{w_s}{\kappa u_*} \tag{3}$$

The Rouse number is a ratio that balances the terminal or settling velocity, w_s, against the upward-directed drag forces exerted on the particle by fluid turbulence: κ is von Karmen's constant which is equal to 0.407 and u_* is shear velocity, the square root of shear stress divided by fluid density, which scales to the size of turbulent eddies. For this study the settling velocity, the rate at which a particle falls in a water column due solely to the force of gravity was calculated using the Dietrich method (Dietrich, 1982) and standard grain properties for natural siliciclastic sand.

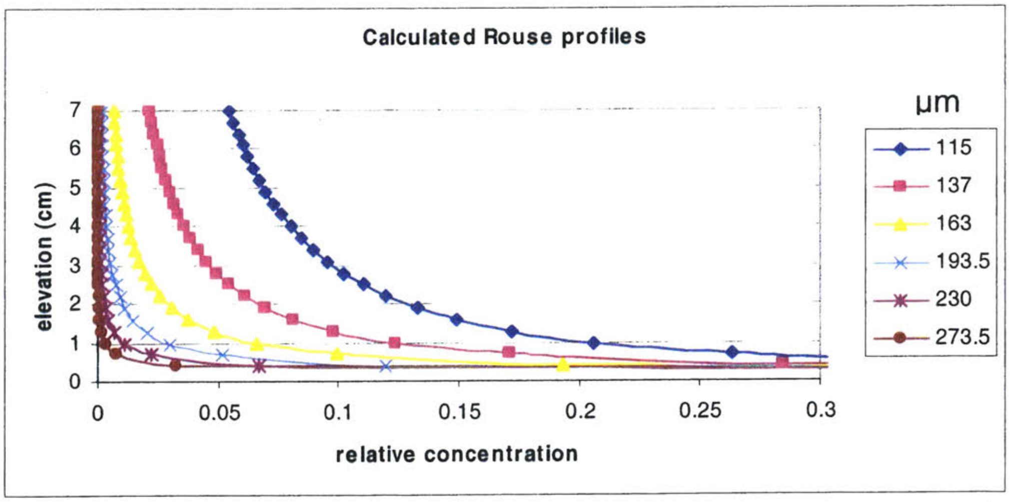


Fig. 9. Near bed suspended sediment profiles for a range of grain sizes, calculated using the approximate Rouse equation (eq. 2). For this calculation the reference elevation is set at 0.1 cm.

Figure 9 shows the relative suspended sediment profiles for a range of grain sizes, calculated using Equation 2. For a constant value of shear velocity, u_*, a greater fraction of the smaller particles occur at all elevations throughout the water column while significant numbers for the larger particles are restricted to very near the bed.

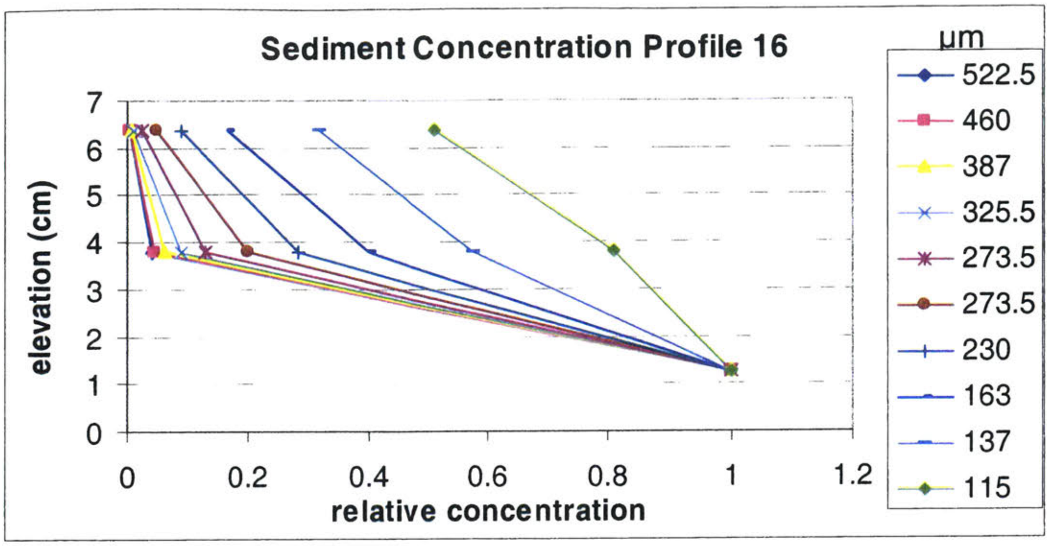


Fig. 10. Vertical profiles of relative sediment concentration for representative grain size within sample 16. The reference elevation is set at 1.27 cm.

Figure 10 shows a example of how particles of different sizes are distributed within the water column directly overlying the bed in the Niobrara river at the Verdigre site. Sediment mass for each grain size class is determined by multiplying its volume fraction by the total sediment mass collected in that particular sampler. This size-class mass is converted to a value for relative volume concentration by dividing by the mass associated with the same size-class collected in the bottom sampler. It is unnecessary to find the water mass flux associated with each sample as this flux is canceled out when local sediment concentration is normalized against the sediment concentration of the bottom bin. The reference elevation for suspended sediment calculations is set at 1.27 cm, the midpoint of the bottom bin. It is worth noting that this elevation is greater than the top of the bedload layer which is $\leq 10 \times$ d50 (Wiberg and Smith, 1985). Given the constraints of bedload traveling very close to the bed and the size of the bedload samplers, the bottom bin is taken to represent conditions at the top of the bedload layer and is therefore used in all following calculations of suspended sediment transport. This approximation is supported by the similarity observed between predicted and measured suspended sediment profiles in Figures 9 and 10, respectively. The sediment profiles in Figure 10 also has small particles occurring at higher concentrations throughout the water column and larger particles at low concentrations everywhere except very near the bed. Particles of all sizes decrease in concentration as elevation from the bed increases.

Grain Size Distributions

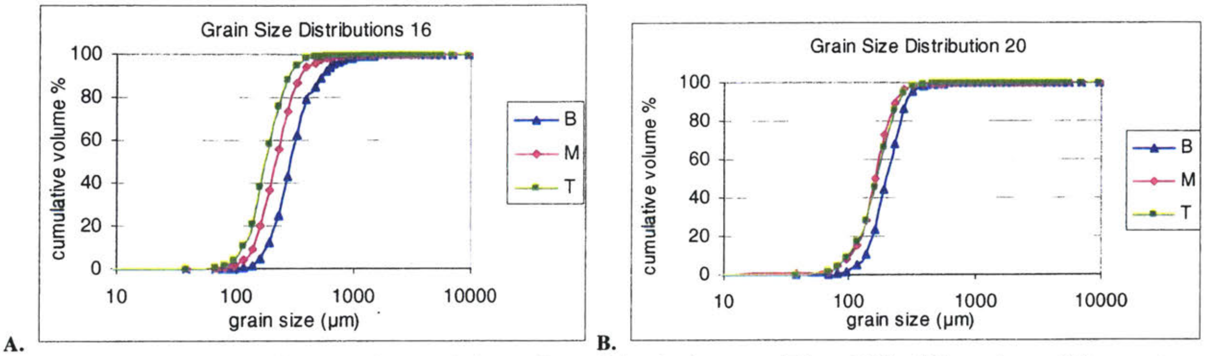


Fig. 11. Grain size distributions for particles collected in the bottom (T), middle (M) and top (T) samplers at locations 16 and 20 at the Verdigre site. These shows representative grain size distributions at the Verdigre site.

Figure 11 shows representative grain size distributions for the sediment traveling at different distances above the bed at the Verdigre field site. These sediment samples are relatively well sorted so cumulative volume percentage curves defining the grain size distributions are relatively steep. It is common to find that the grain size distribution curves for the bottom, middle and top bins are either evenly spaced (Fig. 11.A), or that the distribution curves for the middle and top bins are nearly the same while the grains in the bottom bin are measurably coarser (Fig. 11.B). At high transport stages there is more of a change in the sediment composition and mass collected in the middle and top bins while at lower transport stages the sediment transport properties in the middle and top bins are very similar.

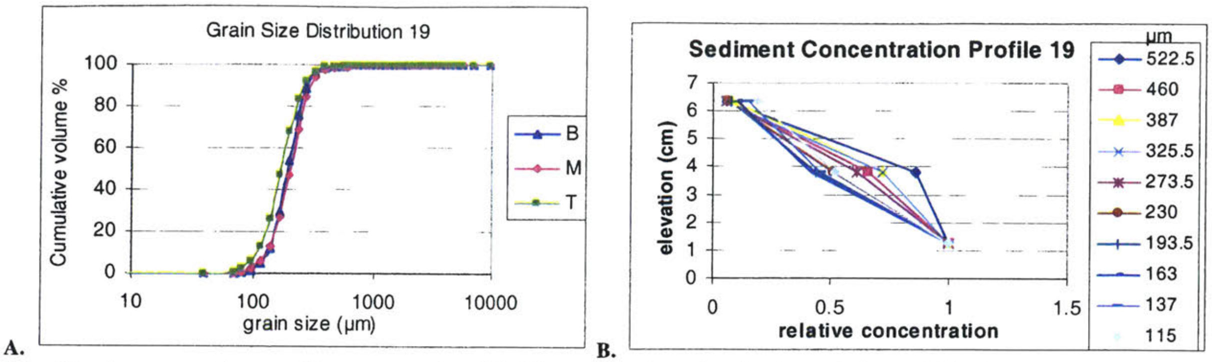


Fig. 12. An anomalous sediment sample. (A) The grain size distribution of the middle bin is coarser than the grain size distribution in the bottom bin. (B) Resulting sediment concentration profiles overlap for different grain sizes. Similarly anomalous samples captured high percentages of sediment, over 30 %, in the middle bin.

Four samples were collected that had anomalous grain size distributions and sediment concentrations profiles. These samples had grain size distributions in which the sediment from the middle bin was coarser than that from the bottom bin (Fig. 12.A), or the sediment from the top bin was coarser than that from the middle bin. For these cases, the sediment concentration profiles overlapped, criss-crossing each other for different grain sizes (Fig. 12.B). All of these samples captured high percentages of sediment, over 30 % by mass, in the middle sampler. These anomalous samples may be the product of tilting of the handheld instrument during sampling so that the middle bin was at a lower elevation than expected and captured material traveling closer to the bed. There may also have been a transient in sediment transport produced by higher dunes adjacent to the sampling location that lead to the higher bins capturing more of the coarser sediment.

Shear Velocity

The shear velocity, u_*, for a stretch of river can be calculated by combining:

$$\tau = \rho g h S \tag{4}$$

$$\tau = \rho u_*^{\,2} \tag{5}$$

Equation 4 can be used to estimate the reach-averaged boundary shear stress; where τ is the boundary shear stress, ρ is the water density, h is the water depth and S is the water surface slope. For the Niobrara River u_* is 6.42 cm/s. This shear velocity includes both the skin friction that drives the sediment transport in the system and the form drag associated with all scales of bed irregularity that extract momentum from the flow. It is difficult to correct for the form drag due to the bed topography in a system, although it can be approximated using a friction factor, c_f. Here we have chosen to directly estimate local values for the skin friction component of the total boundary shear stress through analysis of the sediment transport samples collected.

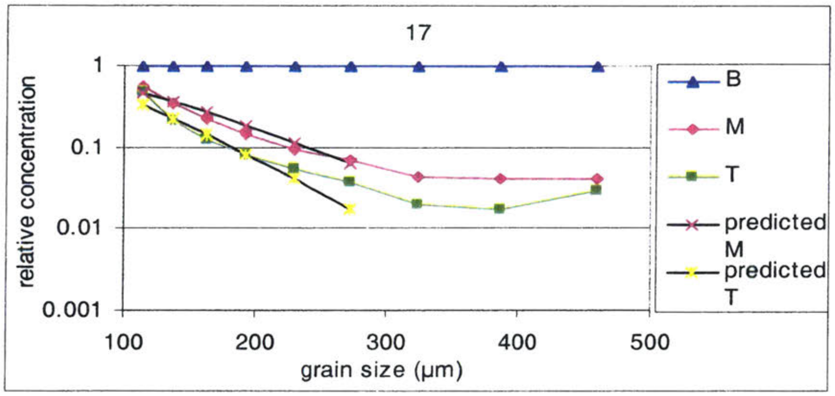


Fig. 13. Relative concentration plotted against grain size for the bottom, middle and top bins of the sampler and the best fit concentrations for the middle and top bins predicted using Eq. 2.

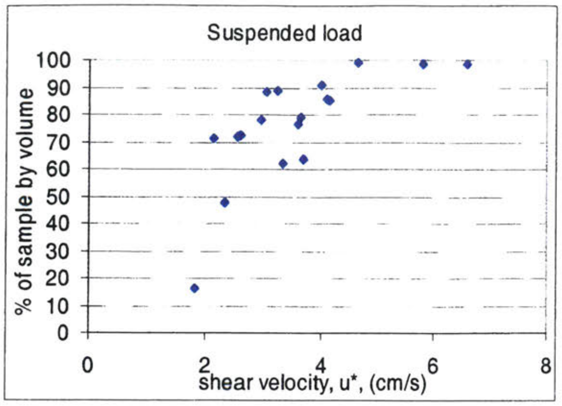
Figure 13 shows relative concentration plotted against grain size for the bottom, middle and top bins of a single sample. Relative concentration can be predicted for each grain size by combining equation 2, equation 3 and the Dietrich method for determining particle settling velocity. This set of equations can be used to determine a best fit to the measured relative concentrations for both the middle bin and top bin of a sample by simply adjusting $u_{*\ skin\ friction}$ until the root mean square of differences between calculated and observed values is minimized. This allows for a determination of the skin friction component of instantaneous

u_* at each sample location. The equation set is only compared to the grain size classes where at least 0.1 g of sediment was collected in the top bin of a sample.

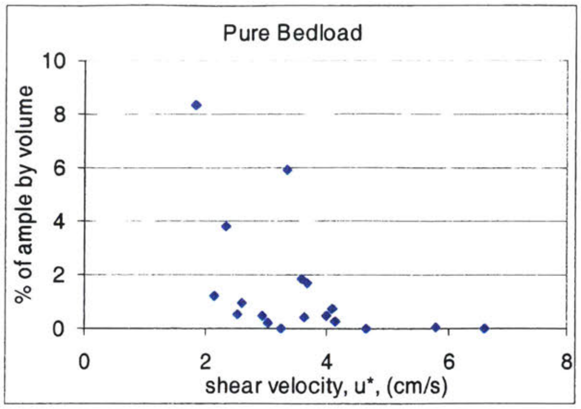
For grain sizes classes where less than 0.1 g of particles was collected in the top bin, the relative concentrations associated with the middle and top bins often remain constant instead of decreasing as grain size increases. This may be due to larger upstream turbulent boils that locally increase the amount of larger particles moving as suspended sediment in the water column.

Suspended Load and Bedload Transport

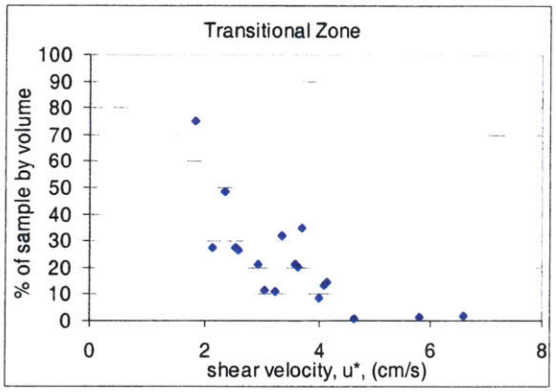
Experimental data has led to empirical criteria for distinguishing between suspended load transport and bedload transport. Development of a measurable characteristic suspended sediment concentration profile occurs when $w_s/u_* \leq 1$ (Bagnold, 1966) and pure bedload transport occurs when $w_s/u_* > 3$ (Nino et al, 2003). By combining these criteria with the skin friction u_* for each sample it is possible to directly estimate which grain sizes are traveling as suspended load versus pure bedload at each location. Using the measured grain size distributions it is then possible to calculate how much moving sediment is traveling as suspended load, bedload or in a transitional zone between suspended load and pure bedload at each sampling locality.



A.



B.



C.

Fig. 14. Percentage of sediment traveling as (A) suspended load, (B) pure bedload and (C) in the transitional zone in between suspension and pure bedload. The majority of the sediment is traveling as suspended load with a lesser amount traveling in the transitional zone and very little traveling as pure bedload. As u_* increases an increasing percentage of the sediment travels as suspended load.

The percentage of sediment traveling in suspension ranges from 16 % to 99 % with a median value of 79 % for all sediment-transport samples at the Verdigre site (Fig. 14.A). The percentage of sediment traveling in the transitional zone ranges from 1 % to 75 % with a median value of 21 % (Fig. 14.C). The percentage of sediment traveling as bedload ranges from 0 % to 8 % with a median value of .5 % for all sediment-transport samples (Fig. 14.B). While most of the sediment is traveling as suspended load a significant percentage is traveling in the transitional zone and very little is traveling as pure bedload.

Close to the bed there is a greater percentage of sediment traveling as bedload and in the transitional zone than in the whole flow. There is an approximately 20 % increase in the amount of sediment that is traveling as bedload and in the transitional zone in the bottom bin compared to the entire sample. There is also an approximately 3 % decrease in the amount of suspended sediment traveling in the bottom bin compared to the entire sample.

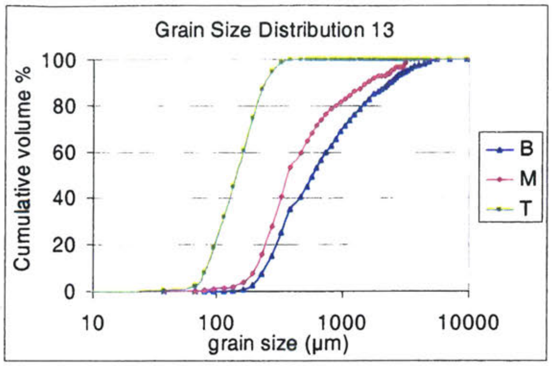


Fig. 15. Grain size distribution for the sample collected at the greatest water depth, 1.09 m. For this sample 53 % of the sediment was traveling as pure bedload, 40 % was traveling in the transitional zone and 7 % was traveling as suspended load.

The sample collected in the deepest water, 1.09 m, had a very different distribution of grain sizes from all others presented here and was not included in creating Figure 14. Using the previously described method it was estimated that for this case 53 % of the sediment was traveling as pure bedload, 40 % was traveling in the transitional zone and only 7 % was traveling as suspended load. The grain size distributions for the three bin show that while the top bin has a typical distribution curve, the bottom and middle bins are poorly sorted and much coarser than other samples (Fig. 15). Since this location had a high percentage of sediment traveling as very coarse bedload, using the bottom bin as the reference concentration is not a good approximation affecting the calculation of skin friction shear velocity. The shear velocity calculated at this location, 2.25 cm/s, was below the median shear velocity of 3.48 cm/s.

Conclusions

By measuring the local sediment transport at different elevations above the bed in the Niobrara River and determining their grain size distributions associated with each of these samples it is possible to produce a set of vertical sediment concentration profiles associated with the river flow at a particular point in space and time. Fitting a Rouse profile to the profiles of relative sediment concentration at each location produces a best estimate for the local, skin friction shear velocity. Grain sizes traveling as pure bedload and suspended load at each location are then estimated from empirical ratios of settling velocity and shear velocity., making it possible to determine how much sediment is traveling as pure bedload, suspended load or in a transitional zone between bedload and suspended load.

The methodology developed here allows for a direct estimate of the skin friction and form drag components of shear velocity at arbitrary locations. The shear velocity derived from analysis of the measured grain size distributions can be compared against the shear velocity derived from measurements of instantaneous water velocity. If more data is assembled that defines how much of each transport mode is trapped in bedforms compared to traveling in the interior of the flow then it will help to improve the accuracy associated with paleoflow inversions from the analysis of ancient sediment deposits.

There have been few studies in sandy rivers that specifically identify how much sediment is being transported in each transport mode in natural systems. Determining how much and which sizes of sand are moving as bedload or suspended load is important as it constrains the amount and type of sediment available to build the alluvial channel form itself and to build the surrounding floodplain. On the lower Niobrara River approximately 70 % of the sediment is moving as suspended load and 20 % as a transitional load. It had been anticipated that a lot of sediment was moving as suspended load but the actual percentage found to be traveling in suspension was higher than expected. Similarly the amount of sediment traveling in the transitional zone between bedload and suspended load was higher than expected. There is a need for a better understanding of how sediment is transported in natural systems and specifically a better understanding of the transitional behavior of sand in many rivers.

Acknowledgements

Special thanks to my advisor David Mohrig and to Douglas Jerolmack for their support, time and effort in explaining concepts concerning my research, collecting data in the field and advice on the structure and writing of my thesis. Thanks to Kyle Straub and Joel Johnson for their help in the field, general support and answering my many questions. Thanks also to Paul Heller, Heather Jones, Ranie Lynds, Laura Vietti, Neil Humphrey and Margaret Wilson for their support and company in the field. Thanks to my mother, father, grandmother, Janet Leung, Victoria Chang and Christopher Follett for moral support, happiness and many distractions. This work was supported in part by the MIT UROP Office and in part by the STC Program of the National Science Foundation via the National Center for Earth-surface Dynamics under Agreement Number EAR-0120914.

References

Bagnold, R., 1966, An approach to the sediment transport problem from general physics: U.S. Geological Survey Professional Paper 422-1.

Bristow, C.S., Skelly, R.L., and Ethridge, F.G., 1999, Crevasse splays from the rapidly aggrading, sand-bed, braided Niobrara River, Nebraska: effect of base-level rise: Sedimentology, v. 46, p. 1029-1047.

Dietrich, W. E., 1982, Settling velocity of natural particles: Water Resources Research, v. 18(6), p. 1615-1626.

Ethridge, F.G., Skelly, R.L., and Bristow, C.S., 1999, Avulsion and crevassing in the sandy, braided Niobrara River: Complex response to base-level rise and aggradation. In: Smith, J., and Rogers, J. (Eds.), Fluvial Sedimentology VI. International Association of Sedimentologists, Special Publication 28. Blackwell Science, London, p. 179-191.

Jerolmack, D. J., Mohrig, D., Straub, K., Leung, V., 2006 The interplay of channel pattern, avulsions and sediment transport in a sandy river: in review.

Nino, Y., Lopez, F., and Garcia, M., 2003, Threshold for particle entrainment into suspension: Sedimentology, v. 50, p. 247-263.

Nishimura, K. and Hunt, J.C.R., 2000, Saltation and incipient suspension above a flat particle bed below a turbulent boundary layer: Journal of Fluid Mechanics, v. 417, p. 77-102.

Skelly, R.L., Bristow, C.S., and Ethridge, F.G., 2003, Architecture of channel-belt deposits in an aggrading shallow sandbed braided river: the lower Niobrara River, northeast Nebraska: Sedimentary Geology, v. 158, p. 249-270.

Wiberg, P.L., and Smith, J.D., 1985, A theoretical model for saltating grains in water: Journal of Geophysical Research, v.90(C4), p.7341-7354.