Refinements and Improvements to a Phenomenological Model for the Jet Opening Angles of Gamma-Ray Bursts

by

Yelena Tsitkin

Submitted to the Department of Earth, Atmospheric, and Planetary Sciences
in partial fulfillment of the requirements for the degree of Bachelor of Science in Planetary Science and Astronomy

at the

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Abstract

Long duration gamma-ray bursts (GRBs) are thought to originate from the core collapse of massive, rapidly rotating stars — events called "hypernovae." In this thesis, we improve upon a phenomenological model to determine $\theta$, the jet opening angle of GRBs. We assume that hypernova progenitors are massive stars in binary systems. We calculate $\theta$ by equating two expressions for the probability of a given GRB being detected — one based on the geometry of the beaming model and the other based on the observed and expected rates of long duration GRBs. These expressions give $\theta$ as a function of several key physical parameters. We estimate these parameters, perform a Monte Carlo simulation, and obtain the most probable value of $\theta$ for both single and double jet GRB models. In contrast to previous work, we allow the minimum mass of star-forming galaxies to vary between $10^{6} M_\odot$ and $10^{7} M_\odot$, and we calculate the galactic number density separately for three subtypes of spiral galaxies. For single jet and double jet models, we find that $\theta = 2.8^{+3.2}_{-1.2}$ deg and $\theta = 1.9^{+2.2}_{-0.8}$ deg respectively. These results are somewhat lower than the results obtained in the earlier stages of this project [15, 16], but are in agreement with values inferred from the observed properties of GRBs [4]. Our results therefore support the assumption that massive binary stars are the progenitors of hypernovae that produce long-duration GRBs.

Thesis Supervisor: Paul C. Joss
Title: Professor of Physics
Acknowledgments

Dedicated to my grandfather, Grigory Vladimirovich Vershubsky.

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Chapter 1

Introduction

Since they were first observed in the 1960s, cosmic gamma-ray bursts (GRBs) have poscd an intriguing problem. Under the assumption of isotropic emission, a single observed long-duration burst can be calculated to put out as much as $\sim 2 M_\odot c^2$ worth of energy in gamma rays alone — an incredible amount on such a short timescale ($\sim 10 - 100$ sec). One currently favored model holds that long-duration GRBs are produced by hypernovae — massive, rapidly rotating core-collapse supernovae. During the collapse, hypernovae are thought to emit jets of gamma radiation. The beam- ing of these jets reduces the predicted energy of a GRB to reasonable proportions. In this thesis, we assume that long-duration GRBs are the result of hypernovae that form in massive binary stellar systems [2, 9, 10], and we continue the work of Erica L. McEvoy [15, 16] in using this model to calculate $\theta$, the jet opening angle of GRBs.

As a test of GRB progenitor models, $\theta$ is an extremely important quantity. Whether or not a model predicts a reasonable opening angle has a strong effect on the plausibility of the model. Knowledge of this quantity can also contribute to our understanding of the physics behind the hypernova itself.

1.1 History of GRB observation

The first known GRBs were observed in the 1960s by U.S. Air Force satellites, called "Vela", which were part of a classified research program whose goal was to develop
the technology to detect and localize nuclear tests. The first of the satellites were launched in 1963, shortly following the signing of the Partial Test Ban Treaty. Several years later, Ray Klebesadel and Roy Olson of the Los Alamos National Laboratory found that the Vela 4 satellite had detected a strange burst of gamma rays, lasting approximately six seconds. The technology of the day could not determine where the burst had occurred, and due to suspicions that it was of terrestrial origin, the data was kept classified until 1973 [11]. By that point, it had become clear that this event, and other similar events detected by Vela, were of extraterrestrial origin. The Interplanetary Network (IPN), a set of spacecraft designed to determine the position of GRBs by triangulation, was launched in 1976. By locating bursts to within a few arcminutes, the IPN showed that GRBs were coming from previously unknown sources.

In the past decade and a half, a number of more advanced projects have been launched with the intent of observing and localizing GRBs. Among them was the Compton Gamma-Ray Observatory, launched by NASA in 1991. It included four instruments, including the Burst and Transient Source Experiment (BATSE), which was designed to detect and record bursts as they occurred. Other projects have included the BeppoSAX satellite, an Italian-Dutch collaboration launched in 1996; the High-Energy Transient Burst Explorer (HETE-2), an MIT-built satellite launched by NASA in 2000; INTEGRAL, an ESA project launched in 2002; and Swift, the newest GRB-detecting satellite, launched by NASA in 2004.

By the early 1990s it was clear that there are two distinct types of GRBs. 75% are “long”, with a mean duration of 20 s, while the other 25% are “short”, with a mean duration of just 0.2 s [14, 7]. It became clear that there must be two separate causes at work, since it was highly unlikely that one type of progenitor could produce two different types of bursts. In this thesis, we discuss only long-duration GRBs.

During its nine-year career, BATSE detected nearly 3000 GRBs [12]. By 1995, it had recorded the angular positions on the sky of over 500 bursts, but could not determine the distances to the sources. However, a plot of all detected bursts showed that they were evenly distributed across the sky, indicating that they were of extra-
galactic origin. This discovery posed a problem — if GRBs originated at cosmological distances, then their extreme energy output became very difficult to explain.

The explanation that we adopt was first proposed by Paczyński in 1998 [17]. He suggested that GRBs might result from hypernova explosions. A hypernova is thought to result from the core collapse of a massive, rapidly rotating star. The collapse creates a Kerr black hole, surrounded by a torus of debris. As debris interacts with the black hole, energy (including gamma rays) is emitted in jets pointed along the axis of rotation, forming a microquasar. These jets are what we observe as GRBs [17]. This model is attractive in part because it does not require the GRB progenitor to be emitting energy isotropically, and therefore reduces the burst energetics to a much more tractable scale.

The massive stars that are thought to be the progenitors of GRBs tend to have very short lifetimes, and are therefore rarely found far from the star-forming regions in which they are born. If the collapsar model is valid, GRBs should likewise occur in active star-forming regions in the spiral arms of galaxies [20]. Bursts have, in fact, been observed to originate in the arms of spiral galaxies.

1.2 Observational properties of GRBs

GRBs are bimodal, forming two distinct groups based on their timescales. Short-duration bursts tend to be fractions of a second long, while long-duration bursts last from a few to hundreds of seconds and emit greater total energies in gamma radiation. The two types of bursts also differ in their peak photon energies, indicating that they have separate causes [16]. GRBs also display a great variety of light curves — from single, sharp peaks to highly complex profiles.

Bursts are often followed by afterglows in other energy bands, such as X-ray, optical, or radio. Optical afterglows are particularly important, because they have allowed us to identify host galaxies and calculate their redshifts. The most distant burst detected to date is GRB 050904, at cosmological redshift $z = 6.295$ [21], and the mean redshift for bursts detected by Swift is 2.8 [8].
1.3 Synopsis of this thesis

In Chapter 2, we improve on McEvoy's phenomenological model for the opening angle $\theta$. In Chapter 3, we perform several Monte Carlo simulations to get a range of probable values for $\theta$, discuss our results, and discuss future areas of research.
Chapter 2

A phenomenological model to determine $\theta$

We assume that GRBs result from hypernovae, which are produced by the core collapse of massive, rapidly rotating stars that end as black holes. We then write two expressions for $P_{\text{detect}}$, the probability that a given burst is detected, and equate them in order to estimate $\theta$.

First, we calculate $P_{\text{detect}}$ purely geometrically. If a GRB occurs at the center of a sphere of radius $r$, then the probability that an observer on the surface of the sphere will detect the burst is the ratio of the solid angle subtended by the GRB beam(s) to that of the entire sphere (Fig. 2-1) [16]. Since we do not know whether a hypernova would produce one jet or two, we do the calculation twice. For a single jet:

$$P_{\text{detect},1} = \frac{2\pi r^2 \left( 1 - \cos \left( \frac{\theta_1}{2} \right) \right)}{4\pi r^2} = \frac{1}{2} \left( 1 - \cos \left( \frac{\theta_1}{2} \right) \right). \quad (2.1)$$

If the GRB emits two jets, then it subtends twice the solid angle (Fig. 2-1):

$$P_{\text{detect},2} = \frac{4\pi r^2 \left( 1 - \cos \left( \frac{\theta_2}{2} \right) \right)}{4\pi r^2} = 1 - \cos \left( \frac{\theta_2}{2} \right). \quad (2.2)$$

In both cases, $P_{\text{detect}}$ is independent of the distance to the observer. Whether a GRB emits one jet or two is still unknown, but it should be noted that the probability
Figure 2-1: The probability of detecting a GRB centered inside a sphere of radius $r$ is given by the ratio of solid angles subtended by the beam(s) of the GRB to that of the entire sphere. A GRB may or may not emit two beams. The dotted lines show a possible second jet.

of detecting a GRB with two jets is twice the probability of detecting a GRB with only one jet [16]. We will carry out calculations for both single and double jet models.

$P_{detect}$ can also be calculated by finding the fraction of all GRBs that are actually detected:

$$P_{detect} = \frac{R_{obs}}{R_{exp}}.$$  \hfill (2.3)

$R_{obs}$ is the rate of observed GRBs, and $R_{exp}$ is the expected total rate of GRBs [16].

We can now solve for $\theta$:

$$\theta_1 = 2 \cos^{-1} \left( 1 - 2 \frac{R_{obs}}{R_{exp}} \right)$$  \hfill (2.4)

$$\theta_2 = 2 \cos^{-1} \left( 1 - \frac{R_{obs}}{R_{exp}} \right).$$  \hfill (2.5)
2.1 $R_{\text{obs}}$ — the observed rate of GRBs

For a full and detailed description of the $R_{\text{obs}}$ calculation, we refer you to McEvoy [16]. Here, we will merely summarize the process.

The observed rate of GRBs can be expressed as:

$$R_{\text{obs}} = \frac{\rho_{\text{obs}}}{\Delta T},$$  \hspace{1cm} (2.6)

where $\rho_{\text{obs}}$ is the volume density of the sources of detected bursts, and $\Delta T$ is the total observation time.

Finding the volume density of detected GRBs requires that we know the total volume in which the bursts can be detected. The likelihood of a burst being detected is affected by two factors — the intrinsic brightness of the burst and the distance to the observer. Therefore, for each burst there exists a maximum distance (redshift) at which that burst can be detected. We use this maximum distance $z_{\text{max}}$ to calculate a maximum volume $V_{\text{max}}$ within which the burst can be detected, and use this quantity to calculate $\rho_{\text{obs}}$.

The maximum redshift at which a burst can be detected depends on the brightness of the burst, the actual redshift, and the sensitivity of the detector. By definition, if a burst occurred at $z_{\text{max}}$, it would register at the threshold of the detector. Therefore, given the detector’s threshold and the absolute luminosity of the source, we can find $z_{\text{max}}$. The bursts used in this thesis, all from the BeppoSAX or HETE-2 detectors, are listed in Table 2.1. Unfortunately, data from other instruments are either unavailable or not yet published in sufficient detail.

Taking $\Delta T$ from the histories of the two detectors, we can now calculate the rate of observed GRBs. The equation:

$$R_{\text{obs}} = \left(\frac{\rho_{\text{obs}}}{\Delta T}\right)_{\text{HETE-2}} + \left(\frac{\rho_{\text{obs}}}{\Delta T}\right)_{\text{BeppoSAX}},$$  \hspace{1cm} (2.7)

gives us a total rate of $R_{\text{obs}} = 2.18 \times 10^{-10} \text{ Mpc}^{-3} \text{ yr}^{-1}$. 
<table>
<thead>
<tr>
<th>Burst</th>
<th>Instrument</th>
<th>$z_s$</th>
<th>$z_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>970228</td>
<td>BeppoSAX</td>
<td>0.69</td>
<td>1.98</td>
</tr>
<tr>
<td>970508</td>
<td>BeppoSAX</td>
<td>0.84</td>
<td>1.28</td>
</tr>
<tr>
<td>971214</td>
<td>BeppoSAX</td>
<td>3.42</td>
<td>9.04</td>
</tr>
<tr>
<td>980326</td>
<td>BeppoSAX</td>
<td>1.00</td>
<td>1.12</td>
</tr>
<tr>
<td>980329</td>
<td>BeppoSAX</td>
<td>2.95</td>
<td>19.89</td>
</tr>
<tr>
<td>980613</td>
<td>BeppoSAX</td>
<td>1.10</td>
<td>1.39</td>
</tr>
<tr>
<td>990123</td>
<td>BeppoSAX</td>
<td>1.60</td>
<td>26.26</td>
</tr>
<tr>
<td>990510</td>
<td>BeppoSAX</td>
<td>1.62</td>
<td>7.62</td>
</tr>
<tr>
<td>990705</td>
<td>BeppoSAX</td>
<td>0.84</td>
<td>5.35</td>
</tr>
<tr>
<td>990712</td>
<td>BeppoSAX</td>
<td>0.43</td>
<td>1.04</td>
</tr>
<tr>
<td>000214</td>
<td>BeppoSAX</td>
<td>0.85</td>
<td>1.83</td>
</tr>
<tr>
<td>010222</td>
<td>BeppoSAX</td>
<td>1.48</td>
<td>24.68</td>
</tr>
<tr>
<td>010921</td>
<td>HETE-2</td>
<td>0.45</td>
<td>1.38</td>
</tr>
<tr>
<td>020124</td>
<td>HETE-2</td>
<td>3.20</td>
<td>4.81</td>
</tr>
<tr>
<td>020813</td>
<td>HETE-2</td>
<td>1.25</td>
<td>2.48</td>
</tr>
<tr>
<td>021004</td>
<td>HETE-2</td>
<td>2.34</td>
<td>2.66</td>
</tr>
<tr>
<td>021211</td>
<td>HETE-2</td>
<td>1.01</td>
<td>1.30</td>
</tr>
<tr>
<td>030115</td>
<td>HETE-2</td>
<td>2.20</td>
<td>2.11</td>
</tr>
<tr>
<td>030226</td>
<td>HETE-2</td>
<td>1.99</td>
<td>2.34</td>
</tr>
<tr>
<td>030324</td>
<td>HETE-2</td>
<td>2.70</td>
<td>2.72</td>
</tr>
<tr>
<td>030328</td>
<td>HETE-2</td>
<td>1.52</td>
<td>1.85</td>
</tr>
<tr>
<td>030329</td>
<td>HETE-2</td>
<td>0.17</td>
<td>0.83</td>
</tr>
<tr>
<td>030429</td>
<td>HETE-2</td>
<td>2.66</td>
<td>2.34</td>
</tr>
<tr>
<td>030528</td>
<td>HETE-2</td>
<td>1.00</td>
<td>1.54</td>
</tr>
</tbody>
</table>

Table 2.1: Measured redshifts ($z_s$) and maximum redshifts ($z_{\text{max}}$) for bursts detected by BeppoSAX and HETE-2 [16].

### 2.2 Estimating $R_{\text{exp}}$, the expected rate of GRBs

We define a hypernova as a rapidly rotating core collapse supernova that ends in the formation of a black hole. The rate of occurrence of hypernovae can be estimated as [16]:

$$R_{\text{exp}} = r_p \cdot \beta \cdot f \cdot \rho,$$

(2.8)

where $r_p$ is the Galactic rate of core collapse supernovae (SNe) that produce pulsars (the pulsar birthrate); $\beta$ is the ratio of SNe in the Galaxy that produce black holes to those that produce pulsars; $f$ is the fraction of core collapse SNe in the Galaxy with cores that are rotating rapidly enough to produce microquasars; and $\rho$ is the number
density of Milky Way equivalent galaxies in the universe. By Milky Way equivalent galaxies we mean ones with massive star formation rates equal to the current massive star formation rate in the Galaxy [16]. Each of these variables has a range which will be used in a Monte Carlo simulation to calculate the most probable value of $\theta$.

Since it is difficult to directly count the rate of core-collapse SNe, we instead use the birth rate of their end products. The Galactic pulsar birthrate, $r_p$, is much better known than the black hole birthrate [16]. A pulsar is the end result of the collapse of a moderately massive star. The lower mass limit for a pulsar progenitor is approximately $8M_{\odot}$, while the upper limit is between $20M_{\odot}$ and $30M_{\odot}$ [5]. The precise limit between pulsar and black hole progenitors may depend on factors other than the mass of the star. Heger et al. [5] suggest that metallicity could be a major factor, and that black holes do not form at metallicities significantly above solar. However, since the precise effect of metallicity has not been calculated, we do not take it into account. Akiyama and Wheeler [1] assert that the cutoff also depends on the speed of rotation of the progenitor, but what exactly this dependence entails is not yet known. We take $r_p$ to be between 0.004 and 0.05, with a best-estimate value of 0.008 [16].

To deduce the black hole birthrate we calculate $\beta$, the ratio of black hole to pulsar birthrates, using the Salpeter initial mass function. We take $8 - 11M_{\odot}$ as a lower mass limit for pulsar progenitors [16], and choose $150M_{\odot}$ as an upper mass limit for stars in general [22]:

$$\beta = \frac{\int_{150M_{\odot}}^{10-30M_{\odot}} M^{-2.35} dM}{\int_{8-11M_{\odot}}^{20-30M_{\odot}} M^{-2.35} dM}. \quad (2.9)$$

To determine $f$, the fraction of rapidly rotating core collapse SNe, we must ask what distinguishes a SN progenitor that is rotating sufficiently rapidly. We argue that only stars that are members of close binary systems can acquire and sustain the necessary rotational velocity, since a single star would rapidly lose angular momentum via its own stellar wind [2, 9, 10]. Using estimates published by Podsiadlowski et al. [18], we take the percentage of core collapse SNe that occur in binary systems to be between 2.4% and 6.4%.
We now have $r_p \cdot \beta \cdot f$, an estimate for the rate of GRBs in our Galaxy. The only factor left to calculate is $\rho$, the number density of Milky Way equivalent galaxies.

### 2.2.1 Calculating $\rho$, the number density of Milky Way equivalent galaxies

We define Milky Way equivalent galaxies to be galaxies that contain regions of active star formation. It is believed that such regions occur only in spiral galaxies and irregular [16]. We therefore use the galaxy luminosity function, $\Phi(L)$, to calculate the number density of spiral galaxies as a function of luminosity. Integrating $L \Phi(L) dL$, the luminosity density, and dividing by $L_{MW}$, the luminosity of the Milky Way, gives us:

$$
\rho = \frac{\int_{0}^{\infty} L \Phi(L) dL}{L_{MW}}.
$$

The Schechter approximation makes it possible to solve for $\rho$ analytically [16]:

$$
\Phi(L) dL = \phi_* \left( \frac{L}{L_*} \right)^\alpha e^{-L/L_*} \frac{dL}{L_*}.
$$

Here, $\phi_*$, $L_*$, and $\alpha$ are empirical constants that vary with galaxy morphology. $\phi_*$ is a normalization factor, $L_*$ is a characteristic galaxy luminosity, and $\alpha$ is the slope of the luminosity function at $L << L_*$ [16]. These parameters, as calculated by Marinoni et al. [13], can be found in Table 2.2. Note that the characteristic magnitude $Mag_*$ was calculated for the $B$ band of the optical spectrum. $Mag_*$ is converted to $L_*$ with the following equation, using $Mag_{\odot} = 4.7896$ [16]:

$$
\frac{L_*}{L_{\odot}} = 10^{-0.4(Mag_* - Mag_{\odot})}.
$$

McEvoy integrated the galaxy luminosity function for all spiral galaxies, but a more accurate approximation is to calculate $\rho$ separately for each morphological subtype of spirals. The main difference between galaxy types, for our purposes, is their mass-to-light ratio $<M/L>$. Portinari et al. [19] calculated this ratio in the $I$ band (near infrared) for the Sa/Sab, Sb, and Sbc/Sc Hubble types, using several different
Table 2.2: Measured parameters of the Schechter luminosity function, for each of three galaxy types. The parameters were calculated using the multi-attractor velocity model. $L_*$, the characteristic luminosity, is easily calculated from the characteristic magnitude $Mag_*$. From Marinoni et al. [13].

<table>
<thead>
<tr>
<th>Type</th>
<th>$\alpha$</th>
<th>$Mag_B$</th>
<th>$\phi^*(10^{-3}\text{Mpc}^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sa-Sb</td>
<td>-0.62±0.11</td>
<td>-20.51±0.12</td>
<td>2.20±0.46</td>
</tr>
<tr>
<td>Sc-Sd</td>
<td>-0.89±0.10</td>
<td>-20.39±0.11</td>
<td>3.12±0.59</td>
</tr>
<tr>
<td>Sm-Im</td>
<td>-2.41±0.28</td>
<td>-21.11±0.72</td>
<td>0.07±0.07</td>
</tr>
</tbody>
</table>

initial mass functions. We will use the Kennicutt and Chabrier IMFs.

Since the mass-to-light ratios are in the $I$ band and the Schechter function parameters are in the $B$ band, the ratios must be converted into the appropriate band with the following equation [19]:

$$
\log \left( \frac{M}{L_x} \right) = s_x [(B - V) - 0.6] + q_x. 
$$

(2.13)

Here $B - V$ is a measure of the galaxy’s color, the subscript $x$ indicates a choice of bands, and $s_x$ and $q_x$ are the slope and zero point of the relationship, respectively. Given $<M/L>_I$, we calculate the color, and use color to calculate $<M/L>_B$. Note that the choice of color is entirely arbitrary, as it is merely an intermediate step. The parameters for this equation can be found in Appendix B of Portinari et al. [19]. The mass-to-light ratios for each Hubble type are in Table 2.3.

Table 2.3: Mass-to-light ratios in the $I$ and $B$ bands for each of three Hubble galaxy types, using two different initial mass functions [19].

<table>
<thead>
<tr>
<th>IMF</th>
<th>Hubble Type</th>
<th>$\frac{M}{L_I}$</th>
<th>$\frac{M}{L_B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kennicutt</td>
<td>Sa/Sab</td>
<td>1.45</td>
<td>2.34</td>
</tr>
<tr>
<td></td>
<td>Sb</td>
<td>1.22</td>
<td>1.72</td>
</tr>
<tr>
<td></td>
<td>Sbc/Sc</td>
<td>0.97</td>
<td>1.16</td>
</tr>
<tr>
<td>Chabrier</td>
<td>Sa/Sab</td>
<td>1.97</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>Sb</td>
<td>1.61</td>
<td>2.24</td>
</tr>
<tr>
<td></td>
<td>Sbc/Sc</td>
<td>1.24</td>
<td>1.41</td>
</tr>
</tbody>
</table>

Using these mass-to-light ratios, we can now state the number density of each
spiral galaxy subtype as:

\[ \rho = \frac{\phi_* M_* \int_{y_0}^{\infty} y^{\alpha+1} e^{-y} dy}{M_{MW}}, \]  

(2.14)

where \( y = M/M_* \) and \( y_0 = M_0/M_* \). We take \( M_{MW} = 1.5 \times 10^{12} M_\odot \) [3].

The one remaining factor to be considered is \( M_0 \), the lower limit for galaxy mass. In order to have star-forming regions, a galaxy must contain hot gas. It must therefore be at least large enough that the gas in its HII regions does not escape — the root mean square velocity of the hot gas in the HII regions must not exceed the escape velocity of the galaxy. Setting \( v_{rms} \) and \( v_{esc} \) equal, we carry out the following calculation:

\[ v_{rms} = v_{esc}, \]  

(2.15)

\[ \left( \frac{3RT}{M_m} \right)^{\frac{1}{2}} = \left( \frac{2GM_0}{r} \right)^{\frac{1}{2}}, \]  

(2.16)

\[ \frac{M_0}{r} = \frac{3RT}{2GM_m}. \]  

(2.17)

Here, \( r \) is the radius of the galaxy, \( M_m \) is the molar mass of hydrogen, \( T \) is the temperature of the HII region, and \( R \) is the universal gas constant. Taking \( T = 10^4 \text{ K} \), the ratio of mass to radius becomes: \( \frac{M_0}{r} = 1.869 \times 10^{18} \). We will refer to this ratio as \( \alpha \).

The mass of the galaxy is proportional to \( r^\alpha \), where \( 2 \leq \alpha \leq 3 \), depending on the shape of the galaxy. We will calculate the mass for the two extreme values, \( M \propto r^2 \) and \( M \propto r^3 \). If galactic mass goes as the square of the radius, the following calculation holds:

\[ M = K_a r^2 \]  

(2.18)

\[ M = \alpha r \]  

(2.19)

\[ M = \frac{\alpha^2}{K_a}. \]  

(2.20)

If galactic mass goes as the cube of the radius, then the following is true:

\[ M = K_b r^3 \]  

(2.21)
In these equations, $K_a$ and $K_b$ are constants of proportionality which must be calculated empirically. To that end, we take a sample of fifteen galaxies, including the Magellanic Clouds and thirteen blue compact dwarf galaxies (BCDs), and calculate $K_a$ and $K_b$ for each one (see Table 2.4). On average, $K_a = 1.05$ and $K_b = 2.5 \times 10^{-20}$. We find that $M_0 \approx 10^6 M_\odot$ if mass goes as radius squared, and $M_0 \approx 10^7 M_\odot$ if mass goes as radius cubed.

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>r (in kpc)</th>
<th>M (in $10^8 M_\odot$)</th>
<th>$K_1$</th>
<th>$K_2 (\times 10^{-22})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMC</td>
<td>4.6</td>
<td>200</td>
<td>2.04</td>
<td>146</td>
</tr>
<tr>
<td>SMC</td>
<td>3.8</td>
<td>60</td>
<td>0.83</td>
<td>69.4</td>
</tr>
<tr>
<td>Mrk1416</td>
<td>2</td>
<td>5.4</td>
<td>0.29</td>
<td>46.2</td>
</tr>
<tr>
<td>SBS0926+606B</td>
<td>5</td>
<td>12.9</td>
<td>0.12</td>
<td>7.70</td>
</tr>
<tr>
<td>SBS0943+563B</td>
<td>4.1</td>
<td>254</td>
<td>3.02</td>
<td>232</td>
</tr>
<tr>
<td>Mrk1460</td>
<td>0.5</td>
<td>0.5</td>
<td>0.44</td>
<td>293</td>
</tr>
<tr>
<td>SBS1057+511B</td>
<td>0.9</td>
<td>14.5</td>
<td>3.70</td>
<td>1320</td>
</tr>
<tr>
<td>Mrk1335</td>
<td>1.1</td>
<td>2.5</td>
<td>0.43</td>
<td>127</td>
</tr>
<tr>
<td>Mrk1499</td>
<td>2.9</td>
<td>42.8</td>
<td>1.07</td>
<td>121</td>
</tr>
<tr>
<td>Mrk826</td>
<td>1.1</td>
<td>1.7</td>
<td>0.29</td>
<td>86.5</td>
</tr>
<tr>
<td>SBS1430+526</td>
<td>3.6</td>
<td>37</td>
<td>0.61</td>
<td>55.6</td>
</tr>
<tr>
<td>SBS1415+437</td>
<td>1.0</td>
<td>1.6</td>
<td>0.33</td>
<td>107</td>
</tr>
<tr>
<td>SBS1205+557</td>
<td>1.0</td>
<td>4.6</td>
<td>0.95</td>
<td>305</td>
</tr>
<tr>
<td>Mrk1446</td>
<td>3.2</td>
<td>19.7</td>
<td>0.40</td>
<td>40.2</td>
</tr>
<tr>
<td>SBS1011+601</td>
<td>1.1</td>
<td>7.3</td>
<td>1.30</td>
<td>382</td>
</tr>
</tbody>
</table>

Table 2.4: Radii, masses, and proportionality constants for a sample of fifteen galaxies. Data on BCD galaxies was taken from [6].

We now have all of the ingredients necessary to calculate $R_{exp}$. For each variable we have either a value or a range of possible values that we can use in a Monte Carlo simulation. The results of this simulation will be discussed in the next chapter.
Chapter 3

Results, conclusions, and future work

3.1 Estimating $\theta$

Having calculated values for all of the relevant parameters, we can now estimate $\theta$. We express $\rho$ as $G/M_{MW}$, where $G$ is the mass density of galaxies, and include the density for each Hubble type.

$$\theta_1 = 2 \cos^{-1} \left( 1 - \frac{2R_{obs}M_{MW}}{r_p \beta f(G_{(Sa-Sb)} + G_{(Sc-Sd)} + G_{(S_m-I_m)})} \right) \quad (3.1)$$

$$\theta_2 = 2 \cos^{-1} \left( 1 - \frac{R_{obs}M_{MW}}{r_p \beta f(G_{(Sa-Sb)} + G_{(Sc-Sd)} + G_{(S_m-I_m)})} \right). \quad (3.2)$$

Many of these factors have a range of possible values, given in Tables 3.1 and 3.2. In our Monte Carlo simulation, we choose a value at random within the possible range and calculate $\theta$. The calculation is repeated 10,000 times, and the resulting frequency distribution of $\theta$ is plotted as a histogram (Figure 3-1).

There are three factors of which we are not certain. We do not know whether a GRB source emits one jet or two; whether the Kennicutt or Chabrier IMF is more suitable for the $< M/L >$ calculation; and whether the minimum galaxy mass is $10^6 M_\odot$ or $10^7 M_\odot$. We therefore do eight separate Monte Carlo plots, taking all of the
possibilities into account. The most probable values of $\theta$ for each set of conditions can be found in Table 3.3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{obs}$</td>
<td>$2.18 \times 10^{-10} \text{yr}^{-1} \text{Mpc}^{-3}$</td>
</tr>
<tr>
<td>$M_{MW}$</td>
<td>$1.5 \times 10^{12} \bigodot$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>[8,11] $\bigodot$</td>
</tr>
<tr>
<td>$M_b$</td>
<td>[20,30] $\bigodot$</td>
</tr>
<tr>
<td>$M_c$</td>
<td>150 $\bigodot$</td>
</tr>
<tr>
<td>$r_p$</td>
<td>$0.005 \times 10^{[0.1]} \text{yr}^{-1}$</td>
</tr>
<tr>
<td>$f$</td>
<td>[0.024,0.064]</td>
</tr>
</tbody>
</table>

Table 3.1: Estimated values and ranges for the parameters necessary for the calculation of $R_{\exp}$. $M_a$ and $M_b$ are the lower and upper mass limits for pulsar progenitors, and $M_c$ is the upper mass limit on stars in general (see Section 2.2).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{(S_a-S_b)}(\times 10^7 \bigodot \text{Mpc}^{-3})$</td>
<td>[4.4,8.5] [6.0,11.6] [4.4,8.5] [6.0,11.6]</td>
</tr>
<tr>
<td>$G_{(S_c-S_d)}(\times 10^7 \bigodot \text{Mpc}^{-3})$</td>
<td>[3.0,5.7] [3.6,7.0] [3.0,5.7] [3.6,7.0]</td>
</tr>
<tr>
<td>$G_{(S_m-I_m)}(\times 10^7 \bigodot \text{Mpc}^{-3})$</td>
<td>[0.075,2600] [0.11,4370] [0.050,527] [0.071,885]</td>
</tr>
</tbody>
</table>

Table 3.2: Values of $G$, the mass density of galaxies, for each of three galaxy types. For each type we calculate four values, using $M_0 = 10^6 \bigodot$, $M_0 = 10^7 \bigodot$, and the Kennicutt and Chabrier initial mass functions [19].

### 3.2 Discussion

We find that the average value of $\theta$ is $2.8^{+3.2}_{-1.2}$ deg for a single jet model, and $1.9^{+2.2}_{-0.8}$ deg for a double jet model. McEvoy [16] estimated $\theta$ at $8^{+6}_{-3}$ for a single jet model and $5.5^{+3}_{-1.5}$ for a double jet model. Our values are somewhat lower than hers, but with smaller FWHM error values. In 2001, Frail et al. [4] estimated $\theta$ for a set of GRBs with known redshifts, based on the physical properties of the jet and assuming a single jet model. They concluded that, in most cases, $3^\circ < \theta < 25^\circ$, with a significant concentration at $\sim 4^\circ$. Our results are at the low end of their range, but are still
Most probable values of $\theta$

<table>
<thead>
<tr>
<th></th>
<th>1 jet</th>
<th>2 jets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6 M_\odot$, Kennicutt</td>
<td>$2.3^{+2.7}_{-1.0}$</td>
<td>$1.5^{+1.6}_{-0.7}$</td>
</tr>
<tr>
<td>$10^6 M_\odot$, Chabrier</td>
<td>$2.6^{+3.4}_{-1.3}$</td>
<td>$1.8^{+2.3}_{-0.9}$</td>
</tr>
<tr>
<td>$10^7 M_\odot$, Kennicutt</td>
<td>$3.5^{+3.6}_{-1.4}$</td>
<td>$2.4^{+2.6}_{-0.9}$</td>
</tr>
<tr>
<td>$10^7 M_\odot$, Chabrier</td>
<td>$2.9^{+2.9}_{-1.1}$</td>
<td>$2.0^{+2.2}_{-0.8}$</td>
</tr>
</tbody>
</table>

Table 3.3: The most probable values and FWHM of $\theta$ for each set of parameters.

consistent, especially considering that they did not take into account the possibility of a second jet. We therefore conclude, just as McEvoy [16] did, that it is very plausible that the binary-star hypernova population and the GRB population are one and the same. Our results also uphold the hypothesis that the origin of hypernovae in massive interacting binary systems is consistent with the observational results of Frail et al. [4].

3.3 Future work

There are two main ways in which this work could be improved. First, there are a number of approximations that can be refined. For example, our calculation for $\rho$ assumes that galactic number density does not vary with redshift. If this assumption is not valid, then cosmological redshift needs to be taken into account. We assume that the cutoff between neutron star and black hole progenitors depends only on mass, but Heger [5] suggests that stars with high metallicity cannot form black holes, and Akiyama and Wheeler [1] argue that the progenitor’s speed of rotation should play a role. If and when more specific analyses of these topics are published, they will need to be taken into account as well. Several parameters, including the minimum galaxy mass, $M_0$, and the mass-to-light ratios, $<M/L>$, should also be included in the Monte Carlo simulation.

Second, the analysis could be much improved by including data on more bursts. The Swift mission has measured the redshifts of a large number of GRBs, and including these bursts would greatly increase the sample size for the $R_{\text{obs}}$ calculation.
We could also use these data to carry out another $V/V_{\text{max}}$ test [16] to determine the spatial distribution of GRBs. Unfortunately, the Swift data that are needed for our analysis have not yet been published.
Figure 3-1: The Monte Carlo distribution of possible values of $\theta$. (A,B) $M_0 = 10^6 M_\odot$, Kennicutt IMF; (C,D) $M_0 = 10^6 M_\odot$, Chabrier IMF; (E,F) $M_0 = 10^7 M_\odot$, Kennicutt IMF; (G,H) $M_0 = 10^7 M_\odot$, Chabrier IMF. Here $M_0$ is the minimum mass of a galaxy capable of star formation (see Sec. 2.2.1)
Bibliography


