Predicted and Observed Free-Air Gravity Anomalies for Delamination Models of the Formation of the Siberian Flood Basalts

by

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ABSTRACT

The mechanism responsible for the formation of the Siberian Flood Basalts (SFB) has yet to be discovered and adequately quantified. One theory proposes that thinning of the lithosphere due to delamination triggered the eruption. This model is characterized by a drip of denser material within the mantle, and because it involves a density-driven process, calculations of predicted gravity at the surface can be used to test the model. Temperature, composition, and stress output from the delamination model presented in Elkins-Tanton (2007) were used to calculate predicted gravity measurements at the surface. These predictions were then compared to gravity observations of the SFB, focusing on the potential eruptive center at Noril’sk. Model runs in both Cartesian and axisymmetric coordinates were analyzed, and each run predicted a negative anomaly over the site of the drip with a magnitude ranging from 20 to 50 mGal. In the observations, an average radial gravity profile centered on Noril’sk also contained a slight negative anomaly at the center, suggesting partial agreement with the delamination theory. Because the amplitude of the observed gravity anomaly is substantially smaller than the predicted amplitude, the qualitative agreement is encouraging, but not definitive.
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1 Introduction

The Siberian Flood Basalts (SFB) are among the largest of the igneous provinces on Earth, encompassing a volume of approximately $3 \times 10^6$ km$^3$ (and as much as $3-4 \times 10^7$ km$^3$ including both extruded and intruded material), with a total section thickness of 6.5 km. Emplaced over the one million years spanning 252 to 251 Ma, they formed at a time roughly coincident with the Permian-Triassic Boundary, a transition marked in the geologic record by the most devastating mass extinction in Earth history. It is suspected that the eruptive events released volatiles, such as carbon dioxide, sulfur, and halogens, into the atmosphere in quantities sufficient to overwhelm Earth’s response systems, resulting in drastic, and catastrophic, climate change. A great deal of research is currently being marshalled to address the connection between the eruption of the SFB and the end-Permian extinction, and critical to this enterprise is our understanding of the mechanism responsible for the formation of the SFB, and flood basalts in general (Elkins-Tanton and Hager, 2000; Reichow et al., 2002; Dobretsov and Vernikovsky, 2001).

This study uses a two-stage method to approach and contribute to the understanding of the SFB and their origin. First, we calculate the gravity signatures associated with the results of several runs of the Elkins-Tanton model for lithospheric delamination, a process that may prove to be a key mechanism of flood basalt formation. Second, the modeled gravity signatures are compared to the actual free-air gravity map of the SFB, which has been generated from observations.
1.1 Theories of Flood Basalt Formation

Relatively little is known about the pre-eruptive condition of the area; as yet there is no complete emplacement chronology, and scientists disagree over the tectonic setting of the region at the time the basalts were erupted (Kamo et al., 2003; Elkins-Tanton, 2006). A variety of explanations have been suggested, the most popular of which, at least until recently, has been the mantle plume model. In this scenario, a chunk of hot material rises from the core-mantle boundary through the mantle, melting adiabatically as it moves upwards through the pressure and temperature gradient. When it reaches the base of the lithosphere, the buoyant plume head causes uplift of the surface, creating a dome centered on the temperature anomaly. Because the plume material is at a higher temperature than the ambient mantle, it follows a hotter adiabat and intersects its solidus at a greater depth than does the surrounding mantle material. This shift in the adiabat, along with the additional elevation provided by the uplift lengthens the melting column, providing an opportunity for more melting to occur. Thus, for this model, the large melt volume required to make up the SFB is dependent on the size and temperature of the plume (Elkins-Tanton, 2005).

Some recent studies, however, suggest that this explanation contradicts the geological evidence. Petrologic research suggests an ambient mantle temperature of about 1200-1500°C, while current plume models imply anomalously hot temperatures—from ~1600-1900°C for various models—in the plume head to achieve sufficient melt (Elkins-Tanton, 2005). Elkins-Tanton and Hager (2000) stress that some of the material at the base of the lithosphere needs to be removed in order to make room for the rising and melting mantle material and to account for the vast quantity of lava erupted in flood basalt events. Beneath continents, the lithosphere may be as thick as 150-250 km, while in order to melt adiabatically fertile, non-hydrous mantle
material with a potential temperature of 1,475°C must rise to a depth of 115 km in order to overcome the hydrostatic pressure of the surrounding mantle. Thus, extremely high temperatures would be required to lower the point of intersection of the adiabat and solidus past the bottom of the continental lithosphere. Even with these high temperatures, the plume models almost invariably fail to penetrate the lithosphere, and so they cannot rely on lithospheric removal to lengthen the melting column (Elkins-Tanton, 2005).

Further constraints arise from characterizing the geological setting of the SFB. The flood basalt deposits rest on the sedimentary Tunguska series, separated by an erosional unconformity, which indicates uplift of the surface immediately before the eruption began. The first 1,100 m of the stratigraphy, however, show evidence of marine fossils, pillow basalts, and fossilized tree trunks, suggesting that the region was actually subsiding during the eruption, keeping the craton surface just below sea level for the first kilometer of deposits. Thus the province was initially near sea level, was uplifted and eroded, producing the unconformity, and returned to sea level to subside during the flood basalt emplacement (Elkins-Tanton and Hager, 2000).

Finally, another clue to the formation of the SFB is provided by the uppermost unit of the series, an extremely olivine-rich set of basalts called meimechites. Experimental work suggests that primitive meimechites formed at a depth of 150 km. The continental lithosphere must therefore have been less than 150 km thick, but whether the lithosphere thinned in the process of magma development, or whether it was initially thin has not been determined. Another interesting feature of the meimechites is their composition; originating from hydrous magma (unlike the rest of the SFB), they contain light-rare-element-enriched trace element signatures, indicating that the process of flood basalt production must have shifted somewhat just toward the end of the emplacement (Elkins-Tanton et al., 2006). Any acceptable model for the formation of
the SFB must take into account the depths required for adiabatic melting, the apparent subsidence of the craton during eruption, and the presence of meimechites in the last unit of the basalt deposits. Thus far, the traditional plume model has failed to account for all of these factors, particularly subsidence during eruption (Elkins-Tanton and Hager, 2000).

These geological constraints prompted Elkins-Tanton and Hager (2000) to postulate a new scenario, in which a small initial melt can lead to removal of the lower lithosphere, or delamination, through a Rayleigh-Taylor instability. A weak upwelling or plume creates a small amount of melt, which intrudes the mantle lithosphere and freezes as an eclogite. The latent heat of freezing heats the surrounding material, lowering its viscosity, while the higher density of eclogite with respect to the ambient mantle creates a density contrast of between 1 and 5%. The density anomaly, combined with a lowered viscosity, favors delamination, which as it occurs pulls the lithosphere down to create topographic subsidence, in agreement with the geological evidence discussed above. A dome in the lower lithosphere is formed as the denser, eclogite-rich material is removed. The hot underlying mantle fills the void left by the sinking material, melting adiabatically as it rises in its newly-lengthened melting column.

1.2 Delamination Model Details

Both the Cartesian and the axisymmetric Elkins-Tanton numerical experiments use the two-dimensional finite element model ConMan (King et al., 1990) to solve nondimensional equations of flow including an equation of motion with an incompressibility condition, an advective-diffusion equation for temperature, and an advection equation for composition. Appendix B contains an excerpt from Elkins-Tanton (2007), which lays out in more detail the model specifications. The model box is made up of 10,000 nodes arranged in a 100-by-100
node grid, the left side of which is an axis of symmetry—rotational symmetry in the axisymmetric case and reflection symmetry in the Cartesian case. The top is a free-slip boundary with a temperature set at the nondimensional equivalent of 20°C, while the bottom and right side have flow-through boundary conditions. The experiments include a density distribution modeled by a half cosine wave in x, with a maximum on the left edge of the model box falling to zero to the right, and a half cosine wave in z, with a maximum at the bottom of the mantle lithosphere decreasing upwards to zero. This configuration is analogous to a lithosphere with no root (having a flat lower boundary) into which denser material—representing eclogite or mafic cumulates resulting from magmatic flux—has been emplaced. A complementary error function cooling law was used to create a cooled lithosphere of the desired depth for each model (see Appendix B), and no initial mantle flow field was included.

The Rayleigh number (1) and the compositional Rayleigh number (2) yield respectively the thermal and compositional contributions to buoyancy.

\[
Ra = \frac{\rho g \alpha \Delta T d^3}{\eta_0 \kappa} \tag{1}
\]

\[
Ra_c = \frac{\Delta \rho g d^3}{\eta_0 \kappa} = Ra \left[ \frac{\Delta \rho}{\rho \alpha \Delta T} \right] \tag{2}
\]

Here, \( \rho \) is the density, \( \Delta \rho \) is the density contrast between the composition of the lower lithosphere and that of the asthenosphere, \( g \) is gravity, \( \alpha \) the thermal expansivity, \( \Delta T \) is the range of temperature across the model box, \( d \) is the height of the model box, \( \eta_0 \) is the reference viscosity, and \( \kappa \) is the thermal diffusivity. For a table of constants used, see Table 3.

Eight model runs with various initial conditions were selected for study, four from the Cartesian calculation and four from the spherically axisymmetric version. The complete list of characteristics for these runs, along with a summary of their behavior, appears in Tables 1 and 2.
<table>
<thead>
<tr>
<th></th>
<th>Lith. (km)</th>
<th>Comp. (km)</th>
<th>E* (kJ mol⁻¹)</th>
<th>S* (Pa s)</th>
<th>Melt Time (Ma)</th>
<th>Topo. (m)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>d2</td>
<td>100</td>
<td>50</td>
<td>15</td>
<td>5</td>
<td>7</td>
<td>6</td>
<td>Rises: -50 to -25</td>
</tr>
<tr>
<td>d6</td>
<td>100</td>
<td>100</td>
<td>15</td>
<td>5</td>
<td>38</td>
<td>5</td>
<td>Rises: -220 to -320</td>
</tr>
<tr>
<td>d8</td>
<td>100</td>
<td>100</td>
<td>15</td>
<td>5</td>
<td>410</td>
<td>4</td>
<td>Sinks then rises: -370 to -460 to -130</td>
</tr>
<tr>
<td>d19</td>
<td>100</td>
<td>100</td>
<td>15</td>
<td>5</td>
<td>1,100</td>
<td>0.4</td>
<td>Sinks then rises: -330 to -350 to -100</td>
</tr>
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</table>

<table>
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<th>Crust (km)</th>
<th>Comp. (km)</th>
<th>η* (s⁻¹)</th>
<th>Melt Time (Ma)</th>
<th>Topo. (m)</th>
<th>Notes</th>
</tr>
</thead>
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<td>d5050</td>
<td>100</td>
<td>50</td>
<td>50</td>
<td>10¹⁹</td>
<td>250</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>d2575D</td>
<td>100</td>
<td>25</td>
<td>100</td>
<td>10¹⁹</td>
<td>250</td>
<td>5</td>
<td>123</td>
</tr>
<tr>
<td>dnc</td>
<td>100</td>
<td>0</td>
<td>50</td>
<td>10¹⁹</td>
<td>250</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>dncD</td>
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<td>0</td>
<td>50</td>
<td>10¹⁹</td>
<td>250</td>
<td>5</td>
<td>200</td>
</tr>
</tbody>
</table>

### 1.3 Gravity Theory

Derived from Newton’s Law of Gravitational Attraction, the gravitational field \( \ddot{g} \) is given by the equation:

\[
\ddot{g} = -\frac{GM}{r^2}\hat{r}
\]  

(3)
where $G$ is the universal gravitational constant, $M$ is the mass of the object generating the field, and $\hat{r}$ is the unit vector in the direction of the distance, $r$, between $M$ and the observation point. The field gives the magnitude of the gravitational acceleration at any given point, where the direction is always towards the center of mass of the object or system. The integral of $\bar{g}$ over all space yields the gravitational potential:

$$U = \frac{GM}{r}$$

(4)

The gravitational potential is a scalar field representing the potential energy per unit mass in the field due to the mass $M$. The gravitational field, therefore, is also given by the gradient of the scalar potential:

$$\bar{g} = \nabla U$$

(5)

Observations of this field above a surface can be interpreted in terms of the interior density structure of the object. Planetary gravitational fields, including the Earth’s, are generated by their three-dimensional internal density distributions. Because rock types have a range of densities, and because pressure and temperature variations form different phase assemblages in the interior, different distributions of these rocks create various patterns in the gravity data. We measure the acceleration due to gravity at the surface of the Earth. All information about the interior mass distribution therefore comes from our interpretation of this measurement, which can be analyzed in terms of solutions to Laplace’s Equation:

$$\nabla^2 U(\hat{r}) = 0$$

(6)

Potential fields obey this equation in the regions outside the sources that generate the field; inside the source region, they obey Poisson’s Equation:

$$\nabla^2 U(\hat{r}) = 4\pi G \rho(\hat{r})$$

(7)
Using measured values of $\tilde{g}$ at Earth’s surface, solutions for Poisson’s Equation can be found for $U(\vec{r})$ by integrating over the contributions from small mass elements within the source region $M$. These solutions take the form of a sum of sines and cosines in Cartesian coordinates, with downward continuation controlled by an exponential term. The solution to Laplace’s Equation in cylindrical coordinates contains the same exponential term, but Bessel functions replace the sines and cosines as the horizontal basis functions. In either case, only the even basis functions are selected for this study, due to the setup of the model box, the left side of which is an axis of symmetry in each version.

To get the total gravity contribution from density contrasts and topography, each relevant quantity (temperature, composition, pressure, and vertical deviatoric stress) is transformed to the frequency domain and combined. The anomalies in the gravitational potential in each coordinate system are given by a combination of these transformed quantities:

$$\Delta U = 2\pi G \sum_{n=0}^{\infty} \cos(k_n x) \left[ \rho \Delta H_{k_n} + \int_0^h \Delta \rho_{k_n}(z)e^{-k_n z}dz \right]$$ (8)

$$\Delta U = 2\pi G \sum_{n=0}^{\infty} J_0(k_n r) \left[ \rho \Delta H_{k_n} + \int_0^h \Delta \rho_{k_n}(z)e^{-k_n z}dz \right]$$ (9)

where $\rho_m$ is the average mantle density, $\Delta H_{k_n}$ is the decomposition of the surface topography (which can be computed from deviatoric vertical stress and pressure), $z$ is the depth, $h$ is the height of the box, and $\Delta \rho_{k_n}$ is the contribution from temperature and pressure variations in the interior:

$$\Delta \rho_{k_n} = \Delta \rho C_{k_n}(z) - \rho_m \alpha T_{k_n}(z)$$ (10)

Here, $\Delta \rho$ is the density contrast between the mantle and the eclogite initially in the lower lithosphere, $C_{k_n}$ is the $n$th harmonic of the composition distribution at depth $z$, $\alpha$ is the thermal
coefficient of expansion, and $T_{k_n}$ is the $n$th harmonic of the temperature field, also at depth $z$.

Due to the attenuation of gravity, each row of data is scaled by its depth and its horizontal wavenumber, and the contributions to gravity from all the rows are summed.

In Cartesian coordinates, the transformation for temperature, for example, takes the following form:

$$T_{k_n} = S_{k_n} \int_0^w T(x) \cos(k_n x) dx \quad (11)$$

where $k_n$ is the horizontal wavenumber:

$$k_n = \frac{2\pi n}{L}, \quad (12)$$

$L/n$ is the wavelength, and $L$ is twice the width of the box, $w$. The normalization constant, $S_{k_n}$, is given by:

$$S_{k_n} = \frac{1}{\int_0^w \cos^2(k_n x) dx} \quad (13)$$

Furthermore, the inverse transform is given by:

$$T(x) = \sum_{n=0}^\infty T_{k_n} \cos(k_n x) \quad (14)$$

In cylindrical coordinates, these equations become

$$T_{k_n} = S_{k_n} \int_0^w T(r) J_0(k_n r) dr \quad (15)$$

$$T(r) = \sum_{n=0}^\infty T_{k_n} J_0(k_n r) \quad (16)$$

$$S_{k_n} = \frac{1}{\int_0^w (J_0(k_n r))^2 dr} \quad (17)$$
If we had chosen to use spherical coordinates instead of cylindrical, the solutions to Poisson’s equations would be given by spherical harmonics, and the downward continuation would be controlled by a power term in $r$ rather than an exponential term. Spherical harmonics were not chosen for this study due to the complicated and implausible boundary conditions required to look only at a small region of a sphere instead of a global phenomena.

The procedure in each coordinate system is the same. The first term of the expansion represents the average, or background, gravity signal—the largest contribution to the gravity field, which is due to the Earth’s dominantly spherical shape and the roughly radially-symmetric distribution of mass in the interior. Taking this out, we see the influence of the heterogeneity of the interior mass distribution. A negative gravity anomaly associated with a particular region indicates a low-density piece of material in that region, and a positive anomaly may be associated with high-density material. With these tools in mind, we may begin to approach the gravity equations from either side, either to calculate in detail the potential and derive the predicted gravity field, or to infer the potential anomalies based on detailed measurements of gravity.

2 Methods

The procedure for this study consisted of two parts: the calculation of the gravity signature at the surface from delamination model output, and the comparison of these predictions to gravity observations of the SFB.
2.2 Gravity Code

The gravity model takes the output files from the delamination model and uses them to calculate what we should see in the gravitational field based on the interior structure. We have the two-dimensional mass distribution for a small area, from which we calculate the predicted gravitational field that might be observed at the surface due to that distribution. The basic procedure involves transforming the relevant quantities, combining them in a gravity calculation, and inverse transforming them back into the spatial domain, yielding the gravitational field at the surface. The results of this model are then compared to a gravity map of the Siberian Flood Basalts region, in order to look for similarities or differences that may help determine the origin of the feature.

The model presented here takes output files from the Elkins-Tanton model and calculates the gravitational acceleration at each point along the “surface,” which is taken to be the uppermost row of nodes. These files are large text files containing, for each of several time steps, the temperature, composition, pressure, and stress data for each of 10,000 nodes in a 100-by-100 node array. There are two different versions of the program: one operates in Cartesian coordinates and the other uses cylindrical coordinates to process output from the spherical axisymmetric model. For a table of constants used, see Table 3. Each version of the model reads the relevant data files for the different runs and extracts the appropriate data—temperature, composition number, deviatoric vertical stress, and pressure. It then reshapes each of these from a single column into a 100-by-100 node array. In Cartesian and cylindrical coordinates respectively, $x$ and $r$ increase to the right and $z$ increases upwards.

The reasons for making two versions in different coordinate systems were two-fold. First, the transformation involved in Cartesian coordinates was more familiar and easier to work
with conceptually, while the products of that work have proved directly applicable to the more
elegant axisymmetric version of the model. Second, comparing the results found in each version
allows us to draw conclusions as to the utility of each coordinate system for describing
phenomena of this kind. The results of the Cartesian version of the Elkins-Tanton model form a
coherent set that has never been published, while the results of the axisymmetric version appear

In the Cartesian version, the data is output at evenly spaced points along each row. Each
row of each relevant data type (i.e. temperature, composition, pressure, and deviatoric vertical
stress) is transformed to the frequency domain using Matlab’s discrete cosine transform (DCT).
The cosine transform was chosen based on the setup of the Elkins-Tanton model, in which only
half of the modeled area is shown; the other half is simply a reflection across the z-axis.
Therefore, using only the even (cosine) basis functions is an appropriate approach. The Matlab

<table>
<thead>
<tr>
<th>Parameter and Symbol</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Height and width of model box, ( h ) and ( R )</td>
<td>250 km</td>
</tr>
<tr>
<td>Number of nodes in each dimension of the model box</td>
<td>100</td>
</tr>
<tr>
<td>Height scaling for Cartesian model box</td>
<td>[0,1]</td>
</tr>
<tr>
<td>Height scaling for Axisymmetric model box</td>
<td>[0.96,1]</td>
</tr>
<tr>
<td>Acceleration due to gravity, ( g )</td>
<td>9.81 m s(^{-2})</td>
</tr>
<tr>
<td>Universal gravitational constant, ( G )</td>
<td>(6.67 \times 10^{-11})</td>
</tr>
<tr>
<td>Reference Mantle Density, ( \rho_m )</td>
<td>3,300 kg m(^{-3})</td>
</tr>
<tr>
<td>Density contrast between altered lower lithosphere ( \Delta \rho ) and asthenosphere</td>
<td>1 or 5%</td>
</tr>
<tr>
<td>Thermal expansivity, ( \alpha )</td>
<td>(3 \times 10^{-5}) deg(^{-1})</td>
</tr>
<tr>
<td>Thermal diffusivity, ( \kappa )</td>
<td>(1 \times 10^{-6}) m(^2) s(^{-1})</td>
</tr>
<tr>
<td>Temperature range across model box, ( \Delta T )</td>
<td>1,280°C</td>
</tr>
<tr>
<td>Surface temperature</td>
<td>20°C</td>
</tr>
<tr>
<td>Thermal Buoyancy, ( B_m )</td>
<td>(2 \times 10^5) kg m(^{-2}) s(^{-2}) Cartesian</td>
</tr>
<tr>
<td>Compositional Buoyancy, ( B_c )</td>
<td>(-5 \times 10^5) kg m(^{-2}) s(^{-2}) Cartesian</td>
</tr>
<tr>
<td></td>
<td>(3.3 \times 10^{10}) kg m(^{-2}) s(^{-2}) Axisymmetric</td>
</tr>
<tr>
<td></td>
<td>(-4.2 \times 10^{10}) kg m(^{-2}) s(^{-2}) Axisymmetric</td>
</tr>
</tbody>
</table>
function returns the coefficients for the basis functions \( \cos(k_n x) \), where \( k_n \) is the horizontal wavenumber. Once the data is manipulated in the frequency domain, as described in more detail below, via equations (20)-(23), the inverse transform is applied to find the gravity field at the surface in the spatial domain. The average signal (corresponding to the \( n=0 \) frequency) is taken out to reflect only the horizontal variations in the field.

In the axisymmetric case the process is very similar but with several notable differences. First of all, the model output is in spherical coordinates. Because a spherical decomposition is problematic, due to boundary condition concerns mentioned earlier, we instead stretch the coordinates to form a flat cylinder, instead of a conic section of a spherical shell. The error due to the stretching is minimal, as the shell thickness is small enough that curvature near the base is tiny. The angular range of the data is from 0 to only \( 3.92 \cdot 10^{-4} \) radians, while the normalized radius in true range is from 0.96 to 1. Therefore the approximation of this curved disk to a flat cylinder is at the very least qualitatively, and for all intents and purposes quantitatively as well, sufficient. Coordinates are changed from \( \theta \) and \( r_{\text{spherical}} \) to \( r \) and \( z \) through the following transformation:

\[
    r = \theta \cdot r_{\text{spherical}} \tag{18}
\]
\[
    z = r \tag{19}
\]

Next, the data is prepared for the Hankel transform, the equivalent of the Fourier transform in cylindrical coordinates. The zeroth order Hankel transform is the cylindrical analog of the cosine transform, since all of its basis functions, the \( J_0 \) Bessel functions, are even functions. The MatLab functions BHforward.m and BHinverse.m perform the necessary transform at evenly spaced points along each row of each data field (temperature, composition, stress, and pressure), much like in the Cartesian case. These functions currently use the first
hundred wavenumbers, and, again, the n=0 term is excluded. The gravity calculations in each version of the model are identical. After each time step, the program saves the new data as individual text files for that time step.

There are two main contributions to the gravitational field in these models. First, variations in temperature and composition, scaled by their distance from the surface, directly contribute to the underlying mass distribution and therefore to the gravitational field in the ways discussed above. For a given frequency, the contribution due to temperature and composition is given by:

\[ g_{TC} = \int_0^h (\Delta \rho C(n,z) - \rho_m \alpha T(n,z)) e^{-z/h} dz \]  

(20)

Here, \( \Delta \rho \) is the density contrast between the mantle and the lower lithosphere, \( C(n,z) \) is the composition number at each node, \( \rho_m \) is the average mantle density, \( \alpha \) is the thermal coefficient of expansion, \( T(n,z) \) is the temperature number at each node, \( z \) is the depth, and \( h \) is the height of the model box. The composition number is the fraction of dense material at each node with respect to peridotite, such that a value of zero corresponds to pure peridotite and a value of one to a composition of 5% eclogite. The temperature number is given by:

\[ T = T_{real} \cdot \Delta T + 20, \]  

(21)

where \( T \) is the temperature number, \( T_{real} \) is the scaled temperature, \( \Delta T \) is the range of temperature over the model box, and 20°C is the temperature at the surface. Both composition and temperature numbers are dimensionless. Note that higher composition numbers (higher density) result in higher gravity values, while higher temperature values, due to thermal expansion, result in lower gravity values. Also note that the distance from the surface of any given row scales the contribution to gravity of that row exponentially. After this effect is calculated for each node in the array, it is summed up the columns to produce a function only of \( n \).
Because the boundaries of the box in the model are not allowed to move under stresses, such as those generated by these density differences, we must also calculate the dynamic topography at the top and the bottom of the box. The topography is given by

\[ \Delta h = \frac{\tau_{zz}}{\rho_m g} \tag{22} \]

where \( \tau_{zz} \) is the vertical stress, which can be calculated from pressure and deviatoric vertical stress, both of which are calculated for each time step by the Elkins-Tanton model:

\[ \tau_{zz} = -p + \tau_{zz}^{\text{dev}} \tag{23} \]

This model calculates the effect of dynamic topography in the frequency domain, although it could be added in the spatial domain equivalently. The sign of the contribution at the top of the box is opposite that at the bottom of the box; thus, when stresses are compressive at the surface, the dynamic topography is positive at the top of the box and negative at the bottom of the box to compensate. The contribution to gravity from the topography of the bottom row is much smaller than that of the upper row because it is scaled by an exponential term depending on spatial frequency and distance from the surface.

These two pieces of the gravity, both of them functions of \( n \) alone, are then added together and multiplied by the necessary constants to yield \( G(n) \), the gravitational field in the frequency domain. Using the inverse discrete cosine transform or the inverse Hankel transform, we get \( g(x) \) or \( g(r) \), the gravitational field in the spatial domain. In magnitude, the contributions are comparable, though of opposite sign. This makes sense because, for instance, a high-density region will show up in the density component as a maximum, but it will generate stresses that will pull the dynamic topography downward, resulting in a minimum there.

Because all of the data in the input files is in a dimensionless form, scaling constants must be applied to the calculations so that their physical significance can be interpreted. These
constants are taken from the original input files for the Elkins-Tanton model, which deals purely with dimensionless numbers to facilitate mantle flow calculations (see Appendix A). Thus the temperatures have been multiplied by equation (21), and the stresses have been scaled by

\[ \frac{\rho g c d E T_E}{B_m}, \]

where \( B_m \) is the thermal buoyancy, \( d_E \) is the height of the model box, \( T_E \) is the ambient mantle temperature, 1300°C, and \( \rho \) is the surface layer density. The dimensions of the box in both the Cartesian and the axisymmetric models is 250 km on a side, and all depths are scaled by this height. The horizontal wavenumber \( k_n \) is scaled by the total dimensional width.

2.1 Gravity Data

The gravity data used here were collected by the Topographic Service of the Armed Forces of the Soviet Union between 1950 and 1985, using a combination of satellite and ground-based measurements. The data has a resolution of 10 km and covers the entire area of the former Soviet Union (Kogan et al., 1994). The region of interest for this study, which encompasses all known and suspected outcrops of the SFB, spans the longitudes ranging from 70 to 120°W and latitudes from 55 to 70°N. These data are displayed in map form in Figure 1.

The city of Noril'sk has long been thought to be a key emplacement center for the SFB, due to its abundance of sulfur ores. This point, located at 87.35°W and 69.32°N, along with 24 nearby points spaced 10 km apart, were selected for study (see Table 4 for a list of their locations). For each of these 25 centers, the distance to each data node in the region was calculated using a tangential projection at the center point:

\[ x = 111(\phi - \phi_{center}) \cos\left(\frac{\pi \theta}{180}\right) \]  
\[ y = 111(\theta - \theta_{center}) \]
where $\theta$ is the latitude (measured upward from $0^\circ$ at the Equator) and $\phi$ is the longitude of the node.

Figure 1. Gravity Data for the Siberian flood basalts from Kogan et al. (1994). The magnitudes of gravity anomalies shown span the approximate range from -40 to 40 mGal.

These radii were binned in intervals of 25 km, and the gravity data were averaged within each bin to produce a radially symmetric disc of gravity data positioned over the potential emplacement center. A Hankel transform of the disc allowed us then to attenuate the longer wavelength features, in order to look for features with a wavelength near the size of the features found in the model output, about 50-100 km. Rather than excluding the lower harmonics entirely, which could produce distortions due to the Gibb’s effect, the magnitudes of the longer-wavelength harmonics were therefore scaled using the following sine curve:
where $a_n$ is the new coefficient for each wavelength, $b_n$ is the original coefficient, $k_{\text{min}}$ is the minimum included frequency, corresponding to the first harmonic in this case, and $k_{\text{mod1}}$ is the lowest unscaled frequency, marking the lower boundary of the frequency range of interest. The higher frequencies were also attenuated with a similar function in order to focus on this range.

$$a_n = b_n \sin\left(\frac{k_n - k_{\text{min}}}{k_{\text{mod1}} - k_{\text{min}}} \cdot \frac{\pi}{2}\right),$$

(26)

where $k_{\text{mod2}}$ is the highest unscaled frequency (the upper boundary of the frequency range) and $k_{\text{max}}$ is the largest included frequency. Thus, the magnitudes of the harmonics corresponding to all frequencies between $k_{\text{mod1}}$ and $k_{\text{mod2}}$ are unaffected, but the magnitudes of all other harmonics are diminished in a smooth curve that reaches zero at $k_{\text{min}}$ and $k_{\text{max}}$. Features within the desired range of wavelengths are therefore easier to distinguish and compare to the model output.

The result, an average radial profile blurred over the potential eruptive center at Noril’sk, was then compared to the axisymmetric model predictions. The standard deviation of the average for each bin was computed and added to the plots as vertical error bars. The code used to complete this step is included in Appendix A.4.

3 Results

3.1 Gravity Model Output

The composition and temperature fields for the Cartesian run called d6 are typical of the starting conditions for most of the runs, although some also include a crust that does not affect the calculations. These are shown in Figure 2, along with the final composition and temperature
Figure 2. Initial and final composition (top) and temperature (bottom) fields for a typical Cartesian model run, d6. Composition number ranges from 0 to 1, corresponding to pure peridotite and 5% eclogite, respectively. Temperature ranges from 20°C to 1300°C. Model setup is very similar for both the Cartesian and the axisymmetric case.

distributions. Figure 3 contains two gravity profiles, one from the Cartesian run d2, and one from the axisymmetric run dncD. In each case, the contribution to the gravity from the dynamic topography is greater than that from the interior density variations by about a factor of two and of the opposite sign, such that the initial setup produces a negative anomaly at the left edge of the box that levels off to a slight positive anomaly on the right. The gravity low extends out to about
50 km in the Cartesian case, with a maximum amplitude of about -20 mGal, while the anomaly spans nearly 100 km in the axisymmetric case, with a maximum amplitude of about -50 mGal. The discrepancy in the magnitudes makes sense given the geometry of each coordinate system; the area under the three-dimensional gravity surface represented by each profile should be comparable. Thus the center point in the axisymmetric case has a higher amplitude to make up for its small area in the rotationally symmetric geometry. The model runtimes were scaled to Earth time using the following relation:

$$t_{\text{Earth}} = t_{\text{code}} \left( \sqrt{\frac{h_{\text{Earth}}^2}{h_{\text{code}}^2}} \right) \left( \frac{\kappa_{\text{Earth}}}{250} \right),$$  \hspace{1cm} (28)$$

where $h_{\text{Earth}}$ is the scaled model box height, 250 km, $h_{\text{code}}$ is the unscaled model box height, 1 in the Cartesian models and 0.04 in the axisymmetric models, and $\kappa_{\text{Earth}}$ is the thermal diffusivity. Using this conversion, d8 ran for approximately 193.7 million years, while dncD ran for about 12.4 million years.

![Final Gravity Profiles](image)

**Figure 3.** Final gravity profiles from (a) the Cartesian run d2 and (b) the axisymmetric run dncD. Each predicts a negative anomaly at the axis of symmetry, although they predict a different amplitude and spatial extent.
Table 4. Potential Centers for Gravity Data

<table>
<thead>
<tr>
<th>Longitude</th>
<th>Latitude</th>
<th>Projection Coordinates [x,y] (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>87.35</td>
<td>69.32</td>
<td>[0,0] (Noril’sk)</td>
</tr>
<tr>
<td>87.35</td>
<td>69.4101</td>
<td>[0,10]</td>
</tr>
<tr>
<td>87.35</td>
<td>69.2299</td>
<td>[0,-10]</td>
</tr>
<tr>
<td>87.6051</td>
<td>69.32</td>
<td>[10,0]</td>
</tr>
<tr>
<td>87.0949</td>
<td>69.32</td>
<td>[-10,0]</td>
</tr>
<tr>
<td>87.6062</td>
<td>69.4101</td>
<td>[10,10]</td>
</tr>
<tr>
<td>87.604</td>
<td>69.2299</td>
<td>[10,-10]</td>
</tr>
<tr>
<td>87.0938</td>
<td>69.4101</td>
<td>[-10,10]</td>
</tr>
<tr>
<td>87.096</td>
<td>69.2299</td>
<td>[-10,-10]</td>
</tr>
<tr>
<td>87.35</td>
<td>69.5002</td>
<td>[0,20]</td>
</tr>
<tr>
<td>87.35</td>
<td>69.1398</td>
<td>[0,-20]</td>
</tr>
<tr>
<td>87.8602</td>
<td>69.32</td>
<td>[20,0]</td>
</tr>
<tr>
<td>86.8398</td>
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</tr>
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<td>[10,20]</td>
</tr>
<tr>
<td>87.603</td>
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</tr>
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</tr>
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<td>69.2299</td>
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<td>69.4101</td>
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</tr>
<tr>
<td>87.856</td>
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</tr>
<tr>
<td>86.844</td>
<td>69.1398</td>
<td>[-20,-20]</td>
</tr>
</tbody>
</table>

3.2 Gravity Data Analysis

Locations in latitude and longitude for the 25 centers chosen for analysis are listed in Table 4, along with their xy-coordinates in the Cartesian projection in kilometers. After the data was binned and averaged as mentioned above, we were left with an average gravity profile for
the region, which is plotted in Figure 4a. Figure 4b shows the same profile after all wavelengths longer than 250 km and shorter than 50 km have been attenuated.

4 Discussion and Conclusions

The average gravity profile from the gravity observations (Figure 4b) contains a negative anomaly at the center, with a magnitude of about 8 mGal. Thus, while the profile may agree with the model qualitatively, there remains a large discrepancy in magnitude between the predictions and the observations. This discrepancy could be due to a variety of factors. First, the model does not currently account for elastic plate flexure of the lithosphere, which should affect features on the small wavelengths we are interested in here. This effect would damp the dynamic topography, effectively subduing the magnitude of the gravity anomaly. Alternatively, the small magnitude of the gravity signature could be the result of subsequent magma or sediment deposition in the basin produced by the drip. The large predicted anomaly could also be the result of the short runtime for the axisymmetric model (only ~12.4 million years); while the amplitude of the anomaly would have changed very slowly with time after the initial sinking event, a longer runtime could show further attenuation of the signal, if the dense material continued to drain.

The gravity data from Kogan et al., (1994) shows that if any signature is left in the gravity field over the SFB from a delamination event 250 million years ago, it is small. However, the average gravity profile centered at Noril’sk suggests that there may be slight negative anomaly that agrees with the predictions of the gravity model. This outcome is encouraging for the delamination theory of flood basalt formation, but certainly not conclusive.
5 Future Work and Applications

5.1 Next Steps

Several steps could be taken to improve the gravity model and the gravity data analysis techniques. First, the runtime of the axisymmetric model could be increased in order to compare the gravity data to the predicted signature for 250 Ma. Changes to the model that would allow it to take into account elastic plate flexure would also produce a more accurate prediction. Finally, the gravity model could be modified to include contributions to gravity from known geologic events on the Siberian craton, particularly material filling in the depression since the emplacement of the SFB. This correction could be made, alternatively, in the gravity data analysis by attempting to remove gravity anomalies due to these events.

Furthermore, the averaging process that produces the gravity profile for a given point could be improved to better account for the structure of the Siberian basalt field, and additional likely eruptive centers could be identified and processed to compare to the results at Noril’sk.

5.2 Application of Axisymmetric Code to the Moon

Finally, the cylindrical version of the program developed for this project may also prove useful in examining in the production of mare basalts on the moon, triggered by large impacts. One recent model proposes a two-stage mechanism for this process, in which the first stage consists of in-situ decompression melting induced by crater excavation. Convection currents generated by the impact, in which material moves upward during rebound at the impact site, out along the sides of the dome, and down at the edges, inducing adiabatic melting that can persist for hundreds of millions of years after the initial impact (Elkins-Tanton et al., 2004). The movement of material in the second stage is calculated using a spherically-axisymmetric, two-
dimensional version of ConMan, the same finite element program that is used for the
delamination models in *Elkins-Tanton* (2007). Analyzing the predicted gravity signatures from
this model’s output and comparing them to gravity data obtained from lunar missions may
provide useful insights into the process of impact-induced melt accumulation.
References


Elkins-Tanton, L. T., D. Draper, C. Agee, J. Jewell, A. Thorpe, and P. Hess (2007), The last lavas erupted during the main phase of the Siberian flood volcanic province: results from experimental petrology, Contributions to Mineralogy and Petrology, 153(2), 191-209


Appendix A: Codes

A.1 Gravity Code: Cartesian Coordinates

%%%cartd8.m
%%%This code takes output files str.d8 and field.d8 from the Cartesian
%%%version of the Elkins-Tanton delamination model and calculates the
%%%predicted gravity signature at the surface.

%%Define constants
alpha = 3e-5; % per degree
rhoM = 3300; % kg/m^3
gr = 9.81; % m/s^2
Gr = 6.67e-11; % N*m^2/kg^2
d_E = 6.375e6; %m
width = .25e6; % Both the height and width of the box
B_m = 1e5; % kg/[m^2 s^2]
dT = 1280; % deg. C
drho = 5*rhoM*alpha*dT; % kg/m^3

%%%Open output files
fid = fopen('str.d8','r');
fid3 = fopen('field.d8','r');

%%%Note: the field and str files should have the same number of nodes,
%%%the same time increments, and the same number of runs.

%%%In this case, the output files contain data for 11 different times, the
%%%first of which represents the initial conditions.

for m = 0:10

%Pull out the appropriate columns from each file, extract number of
%points and time headers

Cstr = fscanf(fid,'%g',7);
points = Cstr(2);
time = Cstr(6);
s_c_time = time*(width^2/1e-6)/3.15576e7

Cfield = fscanf(fid3,'%g',8);

numstr = textscan(fid,'%s',7);
labelstr = numstr{1};
labelstr = labelstr';

numfield = textscan(fid3,'%s',8);
labelfield = numfield{1};
labelfield = labelfield';

%Reshape columns into arrays (size = 100 x 100)
Dstr = fscanf(fid,'%g',[7,points]);
\( \text{TauDev0} = \text{reshape}(\text{Dstr}(4,:)',100,100) \)
\( \text{xO} = \text{reshape}(\text{Dstr}(2,:)',100,100); \)
\( \text{z} = \text{reshape}(\text{Dstr}(3,:)',100,100); \)
\( \text{P0} = \text{reshape}(\text{Dstr}(6,:)',100,100); \)
\( \text{P0} = -\text{P0}; \)

\begin{verbatim}
for j = 1:100
    zE(j,:) = z(j,:)*width + d_E - width;
end
\end{verbatim}

%%%Interpolate the data at evenly-spaced points
\begin{verbatim}
\text{x} = [ 0:0.010101:1.000]';
\text{xE} = \text{x}*width;
\text{TauDev} = \text{interp1q}(\text{xO}(1,:)',\text{TauDev0}',\text{x}');
\text{TauDevE} = \text{TauDev}*rhoM*alpha*gr*width*dT/B_m;
\text{P} = \text{interp1q}(\text{xO}(1,:)',\text{P0}',\text{x}');
\end{verbatim}

\( \text{Dfield} = \text{fscanf(fid3,'\%g',[8,points]);} \)
\( \text{node} = \text{reshape}(\text{Dfield}(1,:)',100,100); \)
\( \text{T0} = \text{reshape}(\text{Dfield}(6,:)',100,100); \)
\( \text{CO} = \text{reshape}(\text{Dfield}(7,:)',100,100); \)

\( \text{T} = \text{interp1q}(\text{xO}(1,:)',\text{T0}',\text{x}'); \)
\( \text{TE} = 20+\text{T}*dT; \)
\( \text{C} = \text{interp1q}(\text{xO}(1,:)',\text{CO}',\text{x}'); \)

%%%Create the transformed version of T, C, TauDev, and P
%%%Also create arrays for n, kn, and dz.
\begin{verbatim}
for j = 1:100
    \text{Tn}(j,:) = \text{dct}(\text{TE}(j,:));
    \text{Cn}(j,:) = \text{dct}(\text{C}(j,:));
    \text{TauDevn}(j,:) = \text{dct}(\text{TauDevE}(j,:));
    \text{Pn}(j,:) = \text{dct}(\text{P}(j,:));

    for i = 1:100
        \text{n}(j,i) = i;
        \text{kn}(j,i) = pi*i/width;
    end
end

for j = 1:99
    for i = 1:100
        \text{dz}(j,i) = z(j+1,i) - z(j,i);
    end
end
\text{dz}(100,:) = 0;
\text{dzE} = \text{dz}*width;
\end{verbatim}

%%%Calculation of gravity produced by dynamic topography for top and bottom
%%%rows
\begin{verbatim}
for i = 2:100
    \text{DTop}(i) = ((\text{TauDevn}(1,i) - \text{Pn}(1,i)))*exp(-\text{kn}(100,i-1)*width)...
\end{verbatim}
(TauDevn(100,i) - Pn(100,i))/(gr);

%%%Calculation of gravity due to interior density variations
for j = 1:100
    for i = 2:100
        dg(j,i) = (drho*Cn(j,i) - rhoM*alpha*Tn(j,i)+...+
                   exp(kn(j,i-1)*(zE(j,i)-d_E))*dzE(j,i);  
    end
end
gsum = sum(dg,1);  
G = 2*pi*Gr*(DTop+gsum);  
G(1)=0;

%%%Inverse cosine transform to get g(x)
g = idct(G);  
xlabel('x');  
ylabel('Gravitational Signal (mGal)');  
plot(xE,g);  
title(time);  
Framesd8(:,m+1) = getframe;

%%%Write out files: one set containing all transformed data, and one
%%%containing all gravity results. Separate files for each time
%%%period.

name2 = strcat('d',num2str(m),'.d8');  
name4 = strcat('g',num2str(m),'.d8');  
nodecol = reshape(node,10000,1);  
kncol = reshape(kn,10000,1);  
zcol = reshape(z,10000,1);  
cmp = reshape(n,10000,1);  
Tncol = reshape(Tn,10000,1);  
Cncol = reshape(Cn,10000,1);  
TauDevncol = reshape(TauDevn,10000,1);  
Pncol = reshape(Pn,10000,1);  
dzcol = reshape(dz,10000,1);  
allout = [ nodecol,ncmp,kncol,zcol,Tncol,...
          Cncol,TauDevncol,Pncol,dzcol];  

fid2 = fopen(name2,'w');  
fprintf(fid2,' node  
');  
fprintf(fid2,'n  
');  
fprintf(fid2,'kn  
');  
fprintf(fid2,'z  
');  
fprintf(fid2,'Tn  
');  
fprintf(fid2,'Cn  
');  
fprintf(fid2,'TauDevn  
');  
fprintf(fid2,'Pn  
');  
fprintf(fid2,'dz  
');  
fprintf(fid2,'%+6.3e %+6.3e %+6.3e %+6.3e %+6.3e %+6.3e...
fclose(fid2);

fid4 = fopen(name4,'w');
fprintf(fid4,' n G(n) x g(x)\n');
fprintf(fid4,'%+6.3e %+6.3e %+6.3e %+6.3e\n',n,G',x,g');
fclose(fid4);

m = m + 1
end

%% Close output files
fclose(fid3);
fclose(fid);

%% Play back movie frames.
title('d8 Gravity');
movie(Framesd8,1,2);
A.2 Gravity Code: Axisymmetric Coordinates

%%%cylndncD.m
%%%This code takes output files str.dncD and fld.dncD from the axisymmetric
%%%version of the Elkins-Tanton delamination model and calculates the
%%%predicted gravity signature at the surface.

%%%Define constants
alpha = 3e-5; % per degree
rhoM = 3300; % kg/m^3
gr = 9.81; % m/s^2
Gr = 6.67e-11; % N*m^2/kg^2
d_E = 6.375e6;
height = .25e6;
B_m = 3.3e10;
dT = 1280;

%%%Calculate drho

addpath /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/SSAXC/dncD;
addpath /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis;
format short e;

%%%Open output files

fidmen = fopen('men.dncD','r');
fidfld = fopen('fld.dncD','r');

%%%In this case, the output files contain data for 10 time steps, and the
%%%first one represents the model box after the first time step has passed.

for m = 1:10

%%%Extract headers from files

    Fmen = fscanf(fidmen,'%g',5);
    Ffld = fscanf(fidfld,'%g');
    points = Ffld(4);
    time = Ffld(6);
    sc_time = time*((250000/.04)^2/le-6)/3.15576e7;

    nummen = textscan(fidmen,'%s',8);
    labelmen = nummen{1};
    labelmen = labelmen';

    numfld = textscan(fidfld,'%s',6);
    labelfld = numfld{1};
    labelfld = labelfld';

%Reshape columns into arrays (size = 100 x 100)

    Dmen = fscanf(fidmen,'%g',[8,points]);
    theta = reshape(Dmen(2,:)',100,100);
    z = reshape(Dmen(3,:)',100,100);
\[ zE = z^d_E; \]
\[ \text{TauDevO} = \text{reshape}(\text{Dmen}(4,:)',100,100); \]
\[ \text{P0} = \text{reshape}(\text{Dmen}(7,:)',100,100); \]
\[ \text{eta} = \text{reshape}(\text{Dmen}(8,:)',100,100); \]

%%%Correct the pressure data for first and last rows by scaling by the viscosity.
\[ \text{P0}(100,:) = 1e5*\text{P0}(100,:); \]
for \( i = 1:100 \)
\[ \% \text{P1top}(i) = \text{eta}(100,i)*\text{P0}(100,i); \]
\[ \text{P0}(1,i) = \text{eta}(1,i)*\text{P0}(1,i); \]
end
\[ \text{P0} = -\text{P0}; \]

\[ \text{Dfld} = \text{fscanf}(\text{fidfld,'\%g',[7,points]}); \]
\[ \text{node} = \text{reshape}(\text{Dfld}(1,:)',100,100); \]
\[ \text{T0} = \text{reshape}(\text{Dfld}(6,:)',100,100); \]
\[ \text{C0} = \text{reshape}(\text{Dfld}(7,:)',100,100); \]

for \( j = 2:100 \)
for \( i = 1:99 \)
\[ \text{dz}(j,i) = z(j,i) - z(j-1,i); \]
end
dz(1,:) = 0;
dzE = dz*d_E;
\[ \text{R} = 250e3; \]
\[ \text{width} = \text{R}; \]
\[ \text{dr}=\text{R}/99.; \]
\[ \text{r} = 0:dr:\text{R}; \]
for \( i = 1:100 \)
\[ \text{xE}(i) = zE(i,1)*\theta(100,i); \]
end
\[ \text{TauDevE} = \text{TauDevO}*\rho_0*\alpha*\text{gr}^*\text{height}^*dT/(B_m^*.04); \]
\[ \text{TE} = 20+T0*dT; \]

%%%Subtract average value from each row of each data field to make transform more efficient.
\[ \text{avgT} = \text{mean}(\text{TE},2); \]
\[ \text{avgC} = \text{mean}(\text{C0},2); \]
\[ \text{avgTau} = \text{mean}(\text{TauDevE},2); \]
\[ \text{avgP} = \text{mean}(\text{P0},2); \]
for \( j = 1:100 \)
\[ \text{T}(j,:) = \text{TE}(j,:) - \text{avgT}(j); \]
\[ \text{C}(j,:) = \text{C0}(j,:) - \text{avgC}(j); \]
\[ \text{TauDev}(j,:) = \text{TauDevE}(j,:) - \text{avgTau}(j); \]
\[ \text{P}(j,:) = \text{P0}(j,:) - \text{avgP}(j); \]
%%%Perform transform

for j = 1:100
    [Tf,Tf2,Tfex] = BHforward2(T(j,:)',R);
    Tn(j,:) = Tf2';
    [Cf,Cf2,Cfex] = BHforward2(C(j,:)',R);
    Cn(j,:) = Cf2';
    [Tauf,Tauf2,Taufex] = BHforward2(TauDev(j,:)',R);
    TauDevn(j,:) = Tauf2';
    [Pf,Pf2,Pfex] = BHforward2(P(j,:)',R);
    Pn(j,:) = Pf2';
end

load /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/JlzeroslOO.dat;

for j = 1:100
    for i = 1:100
        n(j,i) = i;
        kn(j,i) = Jlzeros100(i)/width;
    end
end

%%%Dynamic Topography calculation for top and bottom rows

for i = 1:100
    DTop(i) = ((TauDevn(1,i) - Pn(1,i))*exp(-kn(1,i)*height)...
               - (TauDevn(100,i) - Pn(100,i)))/(gr);
end

%%%Scale DTop coefficients to avoid ringing from high frequencies.

for i = 1:100
    if i <= 20
        dtop(i) = DTop(i);
    end
    if i > 20
        dtop(i) = DTop(i)*exp((20-i)/25);
    end
end

%%%Gravity from interior density variations

for j = 1:100
    for i = 1:100
        dg(j,i) = (drho*Cn(j,i) - rhoM*alpha*Tn(j,i))*...
                   exp(kn(j,i)*(zE(j,1)-d_E))*dzE(j,1);
    end
end

gsum = sum(dg,1);
G = 2*pi*Gr*(dtop+gsum);

%%%inverse hankel transform to get g(x)
\[ G_{\text{fretrieved}} = B H_{\text{inverse}}(G_{\text{comb}}, R) \]
\[ g = G_{\text{fretrieved}}' \]

xlabel('r');
ylabel('Gravitational Signal (mGal)');
title(time);
plot(r, g);

Frames(:, m) = getframe;

\% write out files: one set containing all transformed data, and one
\% containing all gravity results. Separate files for each time
\% period.

name2 = strcat('d', num2str(m), '.dncD');
name4 = strcat('g', num2str(m), '.dncD');

nodecol = reshape(node, 10000, 1);
kncol = reshape(kn, 10000, 1);
zcol = reshape(z, 10000, 1);
thetacol = reshape(theta, 10000, 1);
ncol = reshape(n, 10000, 1);
Tncol = reshape(Tn, 10000, 1);
Cncol = reshape(Cn, 10000, 1);
TauDevncol = reshape(TauDevn, 10000, 1);
Pncol = reshape(Pn, 10000, 1);
dzcol = reshape(dz, 10000, 1);

allout = [nodecol, ncol, kncol, zcol, thetacol, Tncol, ... 
          Cncol, TauDevncol, Pncol, dzcol];

fid2 = fopen(name2, 'w');
fprintf(fid2, ' node
');
fprintf(fid2, 'n
');
fprintf(fid2, 'kn
');
fprintf(fid2, 'z
');
fprintf(fid2, 'theta
');
fprintf(fid2, 'Tn
');
fprintf(fid2, 'Cn
');
fprintf(fid2, 'TauDevn
');
fprintf(fid2, 'Pn
');
fprintf(fid2, 'dz \n');
fprintf(fid2, '%+6.3e %+6.3e %+6.3e %+6.3e %+6.3e %+6.3e... 
               %+6.3e %+6.3e %+6.3e\n', allout');
fclose(fid2);

fid4 = fopen(name4, 'w');
fprintf(fid4, ' n G(n) x g(x)\n');
fprintf(fid4, '%+6.3e %+6.3e %+6.3e %+6.3e\n', n, G', x, g');
fclose(fid4);

m = m + 1
end

title('dncD Gravity');

%%%Play movie frames.
movie(Frames,1,2);

%%%Close output files.
fclose(fidmen);
fclose(fidfld);
A.3 Hankel Transform and Inverse

%%%BHforward.m is Brad's Hankel transform for domain bounded by zeros of J1, %%%modified by Meg Rosenberg to be used as a function and to use the first %%%100 wavenumbers.

%%%Inputs: f is the function to be transformed, in this case output from the %%%delamination models; R is the outer radius.

%%%Output: c is the input function transformed into the frequency domain, %%%and fex is the product of the inverse transform.

function [f,c,fex] = BHforward(f,R)

%%% read in the first 100 zeros of J1
load /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/Jlzeros100.dat
for i = 1:100
    k(i) = Jlzeros100(i)/R;
end

%%% Calculate the position vector r:
   dr=R/99.;
r = 0:dr:R;
for ik = 1:10
    for ir = 1:100
        j0(ik, ir) = besselj(0,k(ik)*r(ir));
    end
end

%%%Perform the transform:
for ik = 1:10
    c(ik)=0.;
    d(ik)=0.;
    for ir = 1:100
        c(ik) = c(ik)+f(ir)*j0(ik, ir)*r(ir)*dr;
        d(ik) = d(ik)+j0(ik, ir)*j0(ik, ir)*r(ir)*dr;
    end
end

%% Normalize
   c = c./d;

%% reexpand
   fex = 0.*r;
for ik = 1:10
    fex = fex + c(ik)*j0(ik,:);
end
%% BHinverse.m is the inverse transform.

%% Inputs: c is the data in the frequency domain to be put through the
%% inverse transform, and R is the outer radius.

%% Output: fex is the inverse transform of c.

function [fex] = BHinverse(c,R)

%% read in the first 100 zeros of J1
load /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/Jlzeros100.dat

%% find the first 100 wavenumbers
for i = 1:100
    k(i) = Jlzeros100(i)/R;
end

%% Calculate position vector r.

dr=R/99.;

r = 0:dr:R;

for ik = 1:100
    for ir = 1:100
        j0(ik, ir) = besselj(0,k(ik)*r(ir));
    end
end

%% Reexpand

fex = 0.*r;

for ik = 1:100
    fex = fex + c(ik)*j0(ik,:);
end
A.4 Gravity Analysis Code

%%%This program reads in the gravity data from the file fa_nsib.xyz and
%%%calculates an average gravity profile for a given center.
addpath /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/GravData;
addpath /afs/athena.mit.edu/user/m/e/meg3l7/Desktop/Thesis/;

%%%Read in data from file and extract information.

fidgrav = fopen('fa_nsib.xyz','r');
GravPoints = fscanf (fidgrav,'%g',[3,220974]);
long = GravPoints(1,:);'
lat = GravPoints(2,:);'
grav = GravPoints(3,:);'

%%%Choose a center for the average gravity profile.
cenlat = 69.32;
cenlong = 87.35;

%%%Choose the 24 surrounding centers, spaced 10 km apart.

xcoords = [ 0 0; 0 10e3; 0 -10e3; 10e3 0; -10e3 0; 0 20e3; 0 -20e3; 20e3 0; -20e3 0; 10e3 -10e3; -10e3 -10e3; 10e3 10e3; -10e3 10e3; 20e3 20e3; 20e3 -20e3; -20e3 20e3; -20e3 -20e3; 20e3 10e3; 10e3 20e3; 20e3 -10e3; 10e3 -20e3; -10e3 -20e3; -20e3 10e3; -20e3 -10e3; 20e3 10e3]

%%%Calculate coordinates in [long, lat] for all 25 centers.

for i = 1:length(xcoords)
    thcoords(i,1) = xcoords(i,2)/111e3 + cenlat;
    thcoords(i,2) = xcoords(i,1)/(111e3*cos((xcoords(i,2)/111e3 + cenlat)*pi/180)) + cenlong;
end

%%%For each of the 25 centers, find the distance to each gravity data point,
%%%sort by distance and separate into bins. Then average within the bins.

for cens = 1:length(thcoords)
    for i = 1:length(lat)
        x(i) = 111e3*(long(i)-thcoords(cens,2))*cos(lat(i)*pi/180);
        y(i) = 111e3*(lat(i)-thcoords(cens,1));
        dist(i) = sqrt(x(i)^2+y(i)^2);
        disp(i);
        end
    dist = dist';

%%%Fix maximum distance and bin size, and find number of bins.

    R = 10000e3;
    if mod(R,25e3) ~= 0
numbins = fix(R/25e3) + 1;

else
    numbins = R/25e3;
end

%%%Find appropriate bin for each data point and put all information into an
%%%array to be sorted.
for i = 1:length(lat)
    diff(i) = fix(dist(i)/25e3)+1;
    all(i,1) = i;
    all(i,2) = diff(i);
    all(i,3) = dist(i);
    all(i,4) = long(i);
    all(i,5) = lat(i);
    all(i,6) = grav(i);
    disp(i);
end

%%%Sort array by bin number and find bin sizes.
sorted = sortrows(all,2);

count = 0;
cutoffs=[1];
for i = 2:length(sorted)
    if (sorted(i,2)-sorted(i-1,2)) == 1
        count = count + 1;
        cutoffs = [cutoffs,i];
        %disp(i);
    end
end
cutoffs = [cutoffs,[length(lat)+1]];
cutoffs = cutoffs';

%%%Check that bin sizes add up to the number of data points.
for i = 2:numbins
    binsizes(i-1) = (cutoffs(i) - cutoffs(i-1));
end
pointnum = sum(binsizes);

%%%Average within each bin and store the average profile in an array called
%%%avgs that will contain all 25 average profiles. Find also the variance
%%%for each average.
for j = 1:numbins
    avgs(cens,j) = mean(sorted(cutoffs(j):(cutoffs(j+1)-1),6));
    vars(cens,j) = (std(sorted(cutoffs(j):(cutoffs(j+1)-1),6)))^2;
end

%%%End of loop over the centers.
%%%Average the 25 profiles and find new standard deviation for each point.
centeravg = sum(avgs,1)/25;
errZ = sqrt(sum(vars,1))/25;

%%%Perform a Hankel transform on the average profile.
[avgf,avgf2,avgfex] = BHforwardavg3(centeravg,R,numbins);

%%%Read in first 100 zeroes of J1 and define k.
load /afs/athena.mit.edu/user/m/e/meg37/Desktop/Thesis/J1zeros100.dat
for i = 1:100
    k(i) = J1zeros100(i)/R;
end

%%%Define the frequency range of interest
kmod1 = k(8);
kmod2 = k(13);
kmin = k(1);
kmax = k(40);

%%%Scale the coefficients in avgf2 to focus on the range of interest.
%%%Numbers here based on numbins = 40.
for i = 1:40
    if i < 8
        avgf2b(i) = avgf2(i)*sin(((k(i)-kmin)/(kmod1-kmin))*pi/2);
    end

    if (i >= 8) && (i <= 13)
        avgf2b(i) = avgf2(i);
    end

    if i > 13
        avgf2b(i) = avgf2(i)*sin(((k(i)-kmax)/(kmod2-kmax))*pi/2);
    end
end

%%%Inverse transform:
[Gfret] = BHinverseavg(avgf2,R,numbins);
gret = Gfret';

%%%Write to file.
fidwrite = fopen('/afs/athena.mit.edu/user/m/e/meg37/Desktop/Thesis/GravData/averages.dat','w');
fprintf(fidwrite,'gret
');
fprintf(fidwrite,'%g
',gret);
fprintf(fidwrite,'centeravg
');
fprintf(fidwrite,'%g
',centeravg');
fprintf(fidwrite,'errZ
');
fprintf(fidwrite,'%g
',errZ');
close(fidwrite);
close(fidgrav);
Appendix B: Delamination Model Details, from *Elkins-Tanton (2007)*, pp. 2-4

2. Numerical Experiments

[9] The models include a lithosphere with a flat lower boundary (no lithospheric root) into which a region of higher-density material has been injected. This denser material is an analog of magmatic flux that has frozen as an eclogite, or has left behind dense mafic cumulates. This material is therefore both denser and warmer than the surrounding lithosphere, and it has a different composition. The denser composition is modeled as a half cosine wave in $x$ with a maximum at the left side of the axisymmetric model box, falling to zero in the positive $x$ direction, and a half cosine wave in $z$, with a maximum at the bottom of the mantle lithosphere, falling to zero vertically in $z$. The width of the dense regions is twice the height, except as noted in Table 1. Temperature is also added to the lithosphere in the same pattern, bringing the region of intrusion closer to the mantle temperature beneath and correspondingly lowering its viscosity. In some numerical experiments the density and temperature is added throughout the thickness of the lithosphere, and in others through only half the thickness of the lithosphere.

[10] The effect of a buoyant crust is examined by varying the relative thicknesses of a crustal layer and its underlying mantle lithosphere [see also Neil and Houseman, 1999; Houseman and Hoogenboom, 2006]. The crust differs from the mantle lithosphere in these models only in its buoyancy. The modeled crust and mantle lithospheric material both possess the same temperature dependence of viscosity and the same thermal diffusivity as the underlying convecting mantle, though the cooler temperatures in the lithosphere and crust produce higher viscosities.

[11] In some models a weak lower crustal layer is added, modeled after example H1 of Ranalli and Murphy [1986], which uses a hot geotherm in a granitic crust and shows a steep decrease in strength at 30 km depth, interpreted as the Moho. Lamontagne and Ranalli [1995] further show that depending upon its composition and the composition of the surrounding materials, warm lower crust can be up to 3 orders of magnitude lower in viscosity than the cool upper crust and lower lithosphere which surround it. When a weak lower crustal layer is added in these models, it has an initial viscosity 3 orders of magnitude lower than the background value. Starting conditions for several numerical experiments are shown in Figure 1. For a list of values used in the experiments, see Table 2, and for a list of all models, see Table 1.

[12] This series of numerical experiments was conducted using the axisymmetric two-dimensional finite element numerical code ConMan [King et al., 1990]. The model box consists of a 100 by 100 grid of nodes. The left-hand side of the model box is an axis of symmetry. The bottom and right sides have flow-through boundary conditions, and the top is a free-slip boundary with its temperature set to the nondimensional equivalent of 20 dimensional equations for flow as given by van Keken et al. [1997], composed of an equation of motion,

$$\nabla \cdot (\eta \mathbf{\varepsilon}) - \nabla P = (Ra_T - Ra_G) \hat{z},$$

a statement of incompressibility,

$$\nabla \cdot \mathbf{v} = 0,$$

an advection-diffusion equation for temperature,

$$\frac{\partial T}{\partial t} + (\mathbf{v} \cdot \nabla)T = \nabla^2 T,$$

and an advection equation for composition,

$$\frac{\partial C}{\partial t} + (\mathbf{v} \cdot \nabla)C = 0$$

depending upon viscosity ($\eta$), the deviatoric strain rate tensor ($\varepsilon$), dynamic pressure ($P$), thermal and compositional Rayleigh numbers ($Ra_T$ and $Ra_G$, given below), composition ($C$), velocity ($\mathbf{v}$), temperature ($T$), time ($t$), and the unit vector in the direction of gravity ($\hat{z}$).

[13] Both temperature and composition contribute to buoyancy. Thermal buoyancy is determined by the Raleigh number, containing terms for density ($\rho$), gravity ($g$), thermal expansivity ($\alpha$), temperature range across the model
box (ΔT), height of the model box (h), reference viscosity (η₀), and thermal diffusivity (κ):

\[ Ra = \frac{\rho g a \Delta Th^3}{\eta_0 \kappa} = 2 \times 10^6. \]

Both the negative buoyancy of the dense lower lithosphere and the positive buoyancy of the crust are measured by the compositional Rayleigh number,

\[ Ra_\Delta = \frac{\Delta \rho gh^3}{\eta_0 \kappa} = Ra \left[ \frac{\Delta \rho}{\rho_0 \Delta T} \right]. \]

in which Dr is the difference between the densities of the lower lithospheric composition and the adjoining asthenosphere. The dense material in the lithosphere is set to either a 1 or 5% density increase over adjacent mantle lithosphere, and its maximum temperature addition is 10% above the ambient asthenospheric temperature, consistent with heats of fusion. The buoyant crust is set to a 12% density decrease when compared to reference mantle density.

[14] Viscosity is calculated in most experiments using the following Newtonian law:

\[ \eta_{\text{Newtonian}} = \eta_0 \exp \left( \frac{E + V z}{T + T_0} - \frac{E + V z_0}{T + T_0} \right), \]

where \( \eta_0, z_0, \) and \( T_0 \) are reference values for viscosity, depth, and temperature, respectively; \( E \) is the activation energy, and \( V \) is the reference volume (Table 2). The importance of the temperature dependence of viscosity is investigated by using values for the activation energy \( E \) equivalent to either 250 or 500 kJ mol\(^{-1}\). The reference mantle temperature \( T_0 \) is 1300°C.

[15] A starting condition for each numerical experiment was created by using a complementary error function cooling law to make a cooled lithosphere of the desired depth, employing the temperature at the surface \( (T_s) \), the temperature of the convecting mantle \( (T_m) \), thermal diffusivity \( (\kappa) \), and the time period of thermal diffusion \( (\tau) \) [
Turcotte and Schubert, 2002]:

\[ T(z) = (T_s - T_{m}) \text{erfc} \left[ \frac{z}{2(\kappa \tau)^{1/2}} \right] + T_{m}, \]

to create a mantle lithosphere of 100, or in one case, 50 km thickness. There is no imposed initial mantle flow field.

[16] Any melt produced by dry adiabatic melting in convection currents associated with the gravitational instability is calculated with a postprocessor routine using the parameters listed in Table 2. This postprocessor routine uses the mantle flow fields to calculate the volume of asthenospheric material moving above its solidus during each time step of the numerical calculations. Melt is produced at a rate of 0.1% per degree rise above the solidus, which is fitted to experimental data on the continental fertile peridotite KLB-1 from Herzberg et al. [2000] and Takahashi et al. [1993]. The surface topography resulting from stresses imposed on the lithosphere by the density anomaly and its subsequent gravitational instability are calculated from differential stress values at the surface produced by the numerical calculations. The magnitude of topography calculated from deviatoric stress output from numerical models is highly dependent upon the scaling parameters used. Here I use the same scaling parameters used for the numerical experiments themselves. If other scaling rules are used, the magnitude of the topographic expression will change, but not its sense.