Bound acoustic modes in the radiation continuum in isotropic layered systems without periodic structures

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We study the existence of guided acoustic modes in layered structures whose phase velocity is higher than that of bulk waves in a solid substrate or an adjacent fluid half space, which belong to the class of bound states in the radiation continuum (BICs). We demonstrate that in contrast to the electromagnetic case, non-symmetry-protected BICs exist in isotropic layered systems without periodic structures. Two systems supporting non-symmetry-protected sagittally polarized BICs have been identified: (i) a supported solid layer yields BICs whose phase velocity is higher than the transverse velocity of the substrate but lower than the longitudinal velocity; (ii) a supported solid layer loaded by a fluid half space supports BICs whose velocity is higher that the bulk velocity of the fluid but lower than acoustic velocities of the substrate. The latter case is a unique example of BICs in the sense that it does not involve an evanescent field in the fluid half space providing the radiation continuum. In either case, BICs are represented by isolated points in the dispersion relations located within "leaky" branches. We show that these BICs are robust with respect to small perturbations of the system parameters. Numerical results are provided for realistic materials combinations. We also show that no BICs exist in all-fluid layered structures, whereas in solid layered structures there are no shear horizontal BICs and no sagittally polarized BICs whose velocity exceeds the longitudinal velocity of the substrate.

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I. INTRODUCTION

Guided waves, such as light in an optical fiber or a surface acoustic wave (SAW) in a solid, are generally slower than bulk waves in an adjacent unbounded medium; otherwise they become leaky due to the radiation of energy into the bulk. However, there are isolated nonradiating guided modes exemplifying embedded or bound states in the radiation continuum (BICs). The existence of BICs was originally demonstrated by Wigner and von Neumann for a model quantum-mechanical system [1], but to date, while there have been some reports of realistic quantum-mechanical models that exhibit BICs, see, e.g., Refs [2–4], it is in classical systems, primarily in optics and acoustics, where they have generally been encountered (see Ref. [5] for a recent review). Oftentimes BICs are protected by symmetry: for example, SAW in the [110] direction on the basal plane of cubic crystals such as silicon and germanium propagates faster that the bulk slow transverse mode but cannot radiate because in the SAW particles move in ellipses contained in the vertical (sagittal) plane, whereas the slow transverse wave is polarized in the horizontal direction orthogonal to that symmetry plane [6]. As soon as the wave vector deviates from [110], symmetry incompatibility is lifted and the mode becomes leaky. Such symmetry-protected BICs are trivial and not considered in this paper. Much more interesting is the existence of non-symmetry-protected BICs encountered at inconspicuous combinations of system parameters yielding an "accidental" cancellation of the radiation loss. The recent discovery of such non-symmetry-protected robust embedded guided modes in photonic crystal slabs [7,8] has stimulated renewed interest in the subject. Embedded guided modes associated with periodic structures have also been identified in solid-state acoustics [9,10], in a discrete mechanical system [11], and with water waves [12].

In solid-state acoustics, non-symmetry-protected embedded guided modes exist in many anisotropic solids without periodic structures. For example, the same leaky SAW branch that yields a symmetry-protected SAW in the [110] direction on the basal plane of many cubic crystals often contains another isolated embedded mode in a totally inconspicuous direction [13], or sometimes two such modes [14]. These non-symmetry-protected supersonic SAWs are of substantial practical importance as they are employed in SAW filters fabricated on rotated Y-Z cuts of the crystals $LiTaO_3$ and $LiNbO_3$ [15,16], which are widely used in telecommunications devices. Non-symmetry-protected BICs have also been identified in film-substrate systems, both for isotropic films on anisotropic substrates [16,17] and for anisotropic films on isotropic substrates [18,19].

Thus the presence of either periodic structures or elastic anisotropy might appear to be essential for the existence of guided acoustic modes embedded in the radiation continuum [20]. Indeed, in optics it was found that a periodic structure (photonic crystal) is necessary for the existence of BICs in a layered system [8]. The purpose of the present paper is to challenge this premature conclusion and demonstrate that in contrast to the optical case, non-symmetry-protected acoustic BICs do exist in layered structures without periodicity or anisotropy. We start with a discussion of control parameters which are necessary but not sufficient for the existence of non-symmetry-protected BICs. We show that where a BIC is found in a given system, an analysis of control parameters and radiation channels allows us to determine whether or

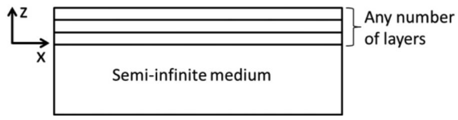

FIG. 1. The physical system under consideration.

not the BIC is robust, i.e., structurally stable with respect to small perturbations of the system. We then consider fluid, solid, and solid-fluid layered systems and set out either to prove that BICs do not exist or to demonstrate examples of their existence in each specific case. We identify two cases when non-symmetry-protected BICs do exist, both involving sagittally polarized waves: (i) solid layer-substrate system, with BIC phase velocity between transverse and longitudinal velocities of the substrate; (ii) fluid-loaded solid layer-substrate system, with BIC phase velocity between the sound velocity in the fluid and the transverse velocity in the substrate. In both cases BICs are found to be robust to perturbations of the system parameters. We will also show that non-symmetry-protected BICs in a solid plate immersed in a fluid have been in fact known [21] even if not reported as such.

II. CONTROL PARAMETERS AND RADIATION CHANNELS

The system we consider consists of a semi-infinite medium occupying a half space $z < 0$, with a layer of different material, or any number of layers, on top of it, as shown in Fig. 1. Above the top layer there can be vacuum or a rigid wall. We will also consider a solid layered structure on a solid substrate loaded by a fluid half space. (However, only one semi-infinite medium is needed to provide a radiation continuum of bulk waves.) BICs in our system are guided modes propagating in the x direction whose phase velocity is larger than the lowest bulk wave velocity in the half space (in a liquid medium there is only one bulk velocity for pressure waves; in a solid medium the transverse velocity is the lowest). Since in an isotropic system the xz plane is a symmetry plane, the waves propagating in the layered structure can be separated into modes polarized in the xz plane (sagittal plane) and modes polarized along y, which can only propagate in a solid medium and are often referred to as shear horizontal (SH) waves. A trivial example of a symmetry-protected BIC is a SH wave in a solid plate immersed in a nonviscous fluid: it is totally decoupled from bulk waves in the fluid even if its phase velocity is higher than the speed of sound in the fluid.

Non-symmetry-protected BICs in planar structures are parametric [5]; that is, the cancellation of radiation losses occurs at a certain combination of control parameters such as the acoustic wave vector, material properties, and layer thickness. The availability of control parameters is a necessary but not sufficient condition for the existence of BICs. To find out the required number of control parameters we need to determine the number of radiation channels. A guided mode in a planar structure attached to a half space with a given in-plane wave vector can radiate into a single bulk (pressure) wave in a fluid half space or into up to two bulk waves in a solid half space: A sagittally polarized mode can radiate both longitudinal and transverse waves, whereas a SH wave can only radiate into a single transverse mode [22].

The amplitude of a radiated bulk wave is generally a complex quantity and it takes two control parameters to make it vanish. However, it has been shown [8] that if the system conforms to C_2 rotational symmetry with respect to the z axis (which is always the case for isotropic materials), then the amplitude of a radiated bulk wave can be considered a real quantity. Therefore, a BIC within a leaky branch radiating onto N bulk waves requires N control parameters. For example, a system comprising a fluid layer on a fluid half space has one radiation channel and three control parameters, i.e., the ratios of the densities and the speeds of sound of the layer and the half space as well as the layer thickness (or the acoustic wave vector, as the acoustic mode pattern is entirely determined by the product of the wave vector and thickness kh). For a solid layer on a solid half space the number of control parameters increases to five (Poisson's ratios of both media should be added) while the number of radiation channels does not exceed two. Thus, even for simple layered systems we will typically have more than enough control parameters to make the existence of BICs a plausible possibility.

The next question is whether BICs, if found, will be robust, i.e., structurally stable with respect to small perturbations of the system. For example, non-symmetry-protected supersonic SAWs on the basal plane of cubic crystals are robust to perturbations of elastic constants [14] but not robust with respect to perturbations of the surface orientation [23,24]. In order to determine whether a BIC will be robust, we need to distinguish between control parameters perturbing the physical system (such as material properties and layer thicknesses) and nonperturbing parameters, i.e., the direction and magnitude of the wave vector. In the vicinity of a BIC, the amplitudes of radiated bulk waves are given by

$$A_n = \sum_{i=1}^{M} c_{ni}(\alpha_i - \alpha_{i0}) + \sum_{i=1}^{L} d_{nj}(\beta_j - \beta_{j0}), \tag{1}$$

where α_i are nonperturbing parameters and β_j are perturbing parameters, and subscript "0" refers to the parameter values yielding the BIC. One can see that the BIC is robust as long as the number of nonperturbing parameters M is equal or greater than the number of the radiation channels N: In this case, for any small change of a perturbing parameter there is a combination of nonperturbing parameters that makes the amplitudes of the radiated waves vanish.

In general, the in-plane wave vector in a planar waveguide structure supplies two nonperturbing parameters, the wave-vector direction and magnitude; all other control parameters are perturbing. However, in the isotropic case the wave-vector direction is irrelevant; therefore, we have a single nonperturbing control parameter, i.e., the wave-vector magnitude, for a structure involving at least one layer of finite thickness, or no nonperturbing parameters otherwise (e.g., for a single half space or two half spaces in contact). Thus, the answer to the robustness question is simple: In the typical case when at least one layer of a finite thickness is present, a BIC, if found, will be robust if there is only one radiation channel, and not robust in the presence of two or more radiation channels. All BICs

TABLE I. The existence of non-symmetry-protected BICs in various classes of layered systems.

System	Existence of non-symmetry-protected BICs
Fluid	No
Solid, Love (SH) waves	No
Solid, Rayleigh-Lamb waves	Yes, between the transverse and longitudinal velocities of the substrate
Fluid-loaded solid, Rayleigh-Lamb waves	Yes

demonstrated below involve a single radiation channel and, consequently, are robust.

III. EXISTENCE OF BICS IN LAYERED STRUCTURES

The availability of control parameters is a necessary but not a sufficient condition for the existence of BICs. There are structures in which many control parameters are available yet BICs do not exist. Consider, for example, SH waves in a structure comprising any number of solid layers on a semi-infinite solid half space. A solid layer on a substrate may support guided SH waves, termed Love wave [25], whose phase velocity is lower than the transverse acoustic velocity of the substrate c_{t0}. A guided mode with a phase velocity above c_{t0} would be a BIC embedded in the continuum of bulk modes in the substrate. We will now show that such a BIC cannot exist in a structure with an arbitrary number of layers, even though the number of control parameters (i.e., thicknesses and elastic properties of the layers) is unlimited. In a guided SH mode with wave vector k directed along x, the displacement field is given by $u_y(z)\exp(ikx - i\omega t)$,where the substrate is assumed to occupy the half space $z > 0$, and $u_y(z) \to 0$ at $z \to \infty$. From the elasticity equations for an isotropic material, we find that the displacement field in each layer and in the substrate should take the form

$$u_y = (A_i e^{-i\gamma_i z} + B_i e^{i\gamma_i z})e^{ikx - i\omega t}, \quad \gamma_i = \sqrt{\omega^2/c_{ti}^2 - k^2}, \tag{2}$$

where c_{ti} is the transverse velocity in the ith layer. γ_i can be either real or imaginary depending on whether the phase velocity ω/k exceeds the transverse velocity in a given layer. In the substrate, however, γ_0 must be real for the guided wave to be a BIC. Consequently, the condition of vanishing $u_y(z)$ at infinity can be satisfied only if $A_0 = B_0 = 0$, i.e., when the displacement field in the substrate is identically zero. Let us now consider the boundary conditions at the interface between the substrate and the first layer, requiring the displacement u_y and the stress component σ_{yz} be continuous across the interface. It is easy to see that if the field in the substrate is zero, boundary conditions require that the displacement field in the first layer be identically zero as well. Applying boundary conditions at the next interface and so on, we find that the field must be identically zero in every layer; consequently, a shear horizontal BIC in a layered structure cannot exist.

A similar consideration shows that BICs cannot exist in a system of liquid layers bounded by a liquid half space [26]. Acoustic waves of sagittal polarization in a solid layered structure (guided waves of this kind are often referred to as Rayleigh-Lamb waves) present a more interesting case. Now the acoustic field in each layer comprises four partial waves, two longitudinal and two transverse. If the phase velocity of a guided wave ω/k exceeds the longitudinal velocity of the substrate c_{l0}, then all partial waves in the substrate have real wave vectors, and the only way to make the field in the substrate vanish at infinity is to require that it be identically zero. Then, by applying boundary conditions at interfaces we can prove that such a guided wave cannot exist. However, if the phase velocity lies between transverse and longitudinal velocities of the substrate, $c_{t0} < \omega/k < c_{l0}$, one of transverse partial waves in the substrate becomes evanescent, with a z dependence in the form of $\exp[-(k^2 - \omega^2/c_{t0}^2)^{1/2}z]$. Now the "nonexistence proof" used above does not apply because a BIC can have an evanescent field in the substrate that helps satisfy the boundary conditions.

Let us now consider a solid layered structure bounded by a nonviscous fluid half space. Pressure waves in the fluid cannot be excited by SH waves in the solid due to symmetry constrains. Consequently, shear horizontal Love waves with a phase velocity higher than the speed of sound in liquid cannot radiate acoustic energy into the fluid thus representing symmetry-protected BICs. A more interesting situation arises for Rayleigh-Lamb waves in this system, which can couple to bulk acoustic waves in the fluid. A sagittally polarized BIC whose phase velocity exceeds the speed of sound in the fluid requires, again, that the acoustic field in the fluid be identically zero. There are three boundary conditions at the solid-liquid interface: the vertical displacement u_z and stress component σ_{zz} are continuous across the interface while the stress component σ_{xz} should be zero. If the field in the fluid is zero, the three boundary conditions do not suffice to determine the amplitudes of the four partial waves in the solid layer adjacent to the liquid half space. Hence it is not required that the acoustic field in the layer be zero: It might be possible to satisfy the boundary conditions with a nonzero field, but the displacement in the solid at $z = 0$ must be horizontal. Thus in this case we cannot prove that a BIC does not exist.

Table I summarizes our findings. In a number of cases we can prove that BICs do not exist. In two cases we were not able to prove this: for Rayleigh-Lamb waves in a solid layered structure with a phase velocity between the transverse and longitudinal velocities of the substrate, and in the case of a liquid-loaded solid structure. In the following two sections we will show that in these two cases BICs are indeed found to exist.

IV. BICS IN A SUPPORTED SOLID LAYER

We consider the simplest layered structure, i.e., an isotropic elastic layer on an isotropic elastic half space. If the acoustic

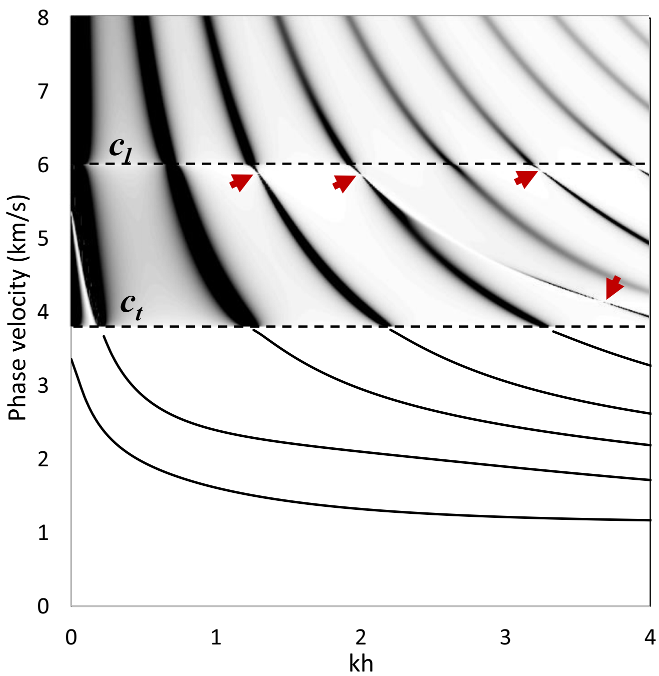

FIG. 2. Gray-scale map of ImG_{11} for a gold layer on a fused silica substrate. Below the transverse threshold c_{t0} of the substrate, ImG_{11} is identically zero except for delta functions corresponding to the discrete spectrum of guided modes; these are shown by solid lines. Above the transverse threshold, leaky modes yield resonances of a finite width except at isolated points marked by arrows where leaky modes transform into BICs.

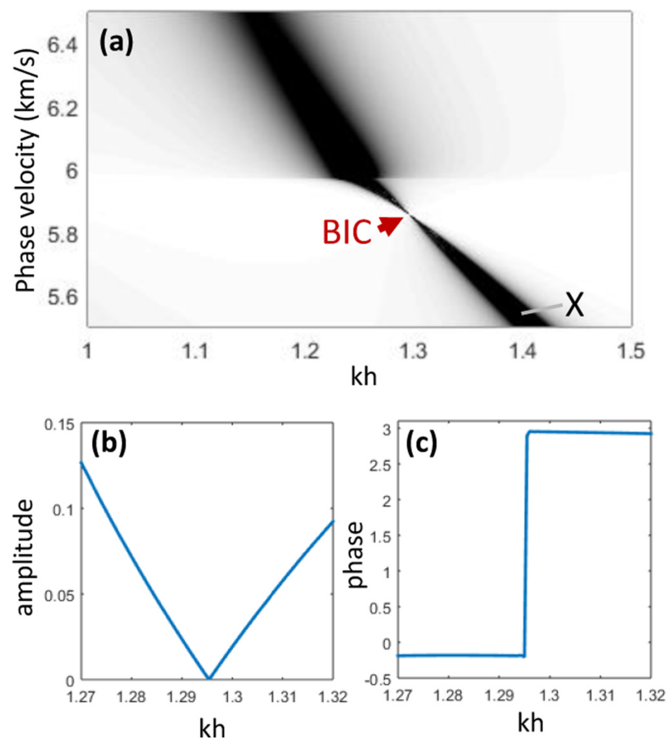

FIG. 3. (a) Gray-scale map of ImG_{11} in the vicinity of one of the BICs from Fig. 2. (b) Displacement amplitude in the transverse wave radiated into the substrate normalized to the horizontal displacement amplitude at the surface in the leaky wave in the vicinity of the BIC point. (c) Phase of the radiated transverse wave.

velocities in the layer are smaller than those in the substrate (slow-on-fast system), it supports an infinite number of guided Rayleigh-Lamb modes whose phase velocities are below the transverse velocity of the substrate [27]. Above the substrate transverse velocity threshold, these modes become leaky, i.e., they radiate acoustic energy into the substrate and are no longer true guided modes. If we consider the in-plane wave vector k (or the dimensionless product kh, where h is the layer thickness) as a control parameter, then, according to the analysis in Secs. II and III, leaky branches might contain isolated points at certain values of kh, where the leakage vanishes and the leaky mode becomes a BIC. However, our analysis has given us no hints as to how to look for those isolated points other than that they can only be encountered in the range $c_{t0} < \omega/k < c_{l0}$. Hence, we set out to investigate leaky branches of slow-on-fast systems using the Green's functions method [28] in a hope to stumble upon a BIC.

We found that BICs are routinely encountered for many common materials combinations. For example, Fig. 2 shows dispersion of guided and leaky modes in a gold layer ($\rho_1 = 19\,300\,\mathrm{kg/m^3}$, $c_{t1} = 1200\,\mathrm{m/s}$; $c_{l1} = 3240\,\mathrm{m/s}$) on a fused silica substrate ($\rho_0 = 2200\,\mathrm{kg/m^3}$, $c_{t0} = 3764\,\mathrm{m/s}$; $c_{l0} = 5.968$). We are plotting the imaginary part of surface Green's function ImG_{11} as a function of the dimensionless wave vector kh and the phase velocity ω/k. The imaginary part of Green's function yields the power dissipated by a horizontal force acting on the surface of the gold layer and having a spatiotemporal distribution $\exp(ikx - i\omega t)$. (ImG_{33} representing the power dissipated by a vertical force could be used as well.) Our model has no material losses; therefore, energy can be dissipated only by radiating acoustic waves into the substrate. Below the transverse threshold $\omega/k < c_{t0}$ radiation into the substrate cannot occur; hence, ImG_{11} is identically zero everywhere except at guided Rayleigh-Lamb modes where it yields delta functions. In Fig. 1(a), these delta functions are represented by solid lines. Above the transverse threshold of the substrate, the modes become leaky and delta functions are replaced by resonances of a finite width. However, at several isolated points between the transverse and longitudinal thresholds, marked by arrows in the figure, the width of the resonance becomes infinitely small (as far as can be assessed within the machine precision) and the leakage disappears, indicating that these points correspond to BICs [29]. As one can see in Fig. 2, BICs are encountered in many but not all leaky branches between transverse and longitudinal thresholds. As expected, no BICs are observed above the longitudinal threshold.

We now consider a leaky branch containing one such BIC point shown in Fig. 3(a) and calculate the displacement field pattern in the leaky wave produced by the source used for the Green's function calculation. Figures 3(b) and 3(c) show the amplitude and phase of the transverse wave radiated into the substrate as a function of kh along the leaky branch (with the amplitude normalized to the horizontal displacement amplitude at the surface, equal to the modulus of G_{11}). One can see that at the BIC point the radiated wave vanishes while its phase jumps by π. Figure 4 shows the full displacement

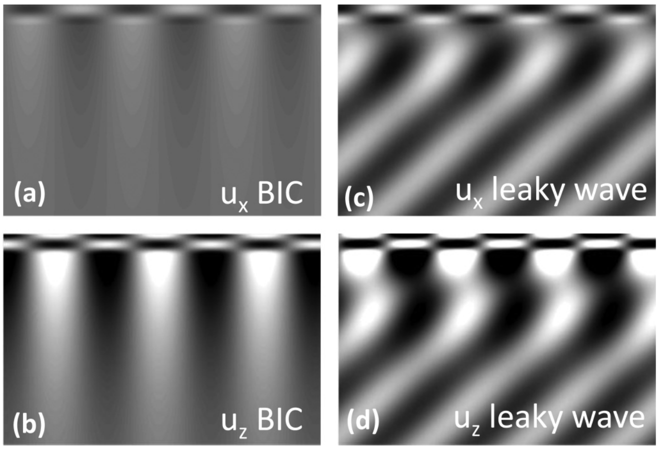

FIG. 4. Gray-scale maps of horizontal (a), (c) and vertical (b), (d) displacement components in the BIC (a), (b) and at the point X (c), (d) in the leaky branch indicated in Fig. 3(a).

field pattern at the BIC point and at an arbitrarily selected point X in the leaky branch indicated in Fig. 3(a). In the latter case one can clearly see the transverse wave radiated into the substrate, whereas at the BIC point only the evanescent field in the substrate is present.

We find that the BIC points are indeed robust: When elastic properties of the layer or the substrate are varied, a BIC moves along a leaky branch. However, a BIC can disappear at the transverse or longitudinal threshold. We also observe that two BICs can "annihilate," as was previously found for electromagnetic BICs in photonic crystal slabs [30]. Figure 5 shows an example of such annihilation for a light layer (with properties typical for a polymer material) on a gold substrate [31]. Since the substrate is much denser than the layer, the leaky modes shown in Figs. 5(a)–5(c) resemble the two lowest guided modes of a layer on a rigid substrate (i.e., a clamped-free layer) whose dispersion is shown in Fig. 5(d) for a reference. A leaky branch in Fig. 5(a) contains two BICs marked by red arrows. As the longitudinal velocity of the layer increases, the two BICs move closer together and annihilate, with no BICs present in Fig. 5(c). Interestingly, the annihilation of BICs seems to be accompanied by a bifurcation of the leaky branches. We note that the notion of BIC's topological charge invoked in Ref. [30] is not strictly required to explain BIC's robustness and annihilation: in our case the wave-vector space is one-dimensional, since there is no dependence on the direction of the wave vector.

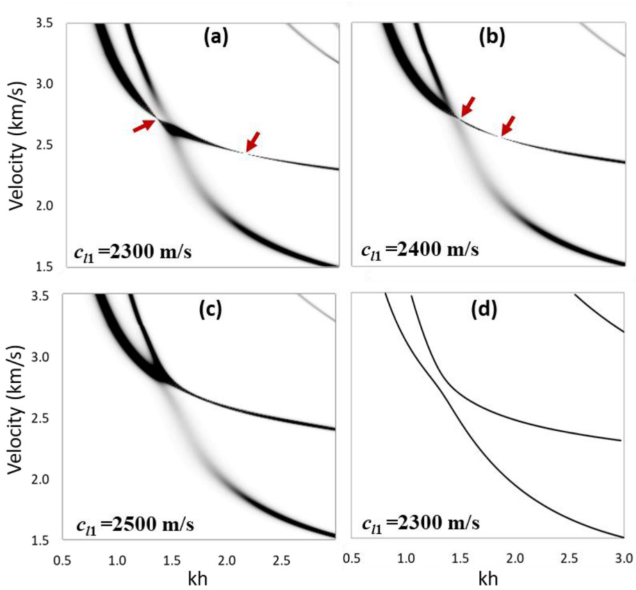

FIG. 5. (a)–(c) Annihilation of two BICs (marked by arrows) occurring in a light layer (with properties typical for a polymer material) on a gold substrate as the longitudinal velocity of the layer varies from 2300 to 2500 m/s. Other material properties of the layer: $\rho_1 = 1000\,\mathrm{kg/m^3}$, $c_{t1} = 1400\,\mathrm{m/s}$. Plotted is a gray-scale map of $(\mathrm{Im}G_{11})^2 + (\mathrm{Im}G_{33})^2$. (d) Dispersion curves of guided modes for a layer with the same properties as in (a) on a rigid substrate.

V. BICS IN A FLUID-LOADED SUPPORTED LAYER

Rayleigh-Lamb waves in solid systems are normally faster than the speed of sound in a typical liquid. Then, if a solid is loaded by a fluid half space, Rayleigh-Lamb waves radiate acoustic energy into the fluid and become leaky. Unlike in the previous section, in this case we know how to look for BICs within these leaky branches: We should look for isolated

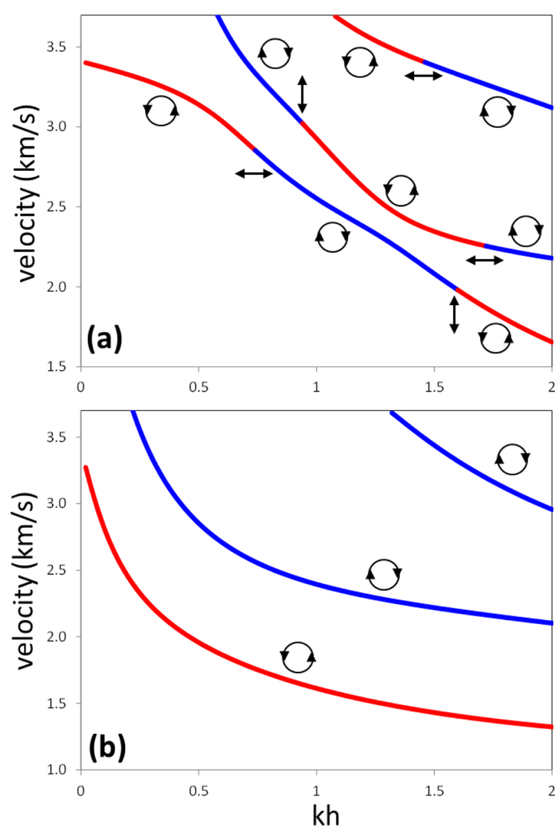

FIG. 6. (a) Dispersion curves of Rayleigh-Lamb waves for a polystyrene layer on fused silica with regions of different chirality of the surface motion indicated for each mode; these regions are separated by points at which the polarization is either pure vertical or pure horizontal. (b) A similar plot for gold on silica, no change in chirality within a single mode.

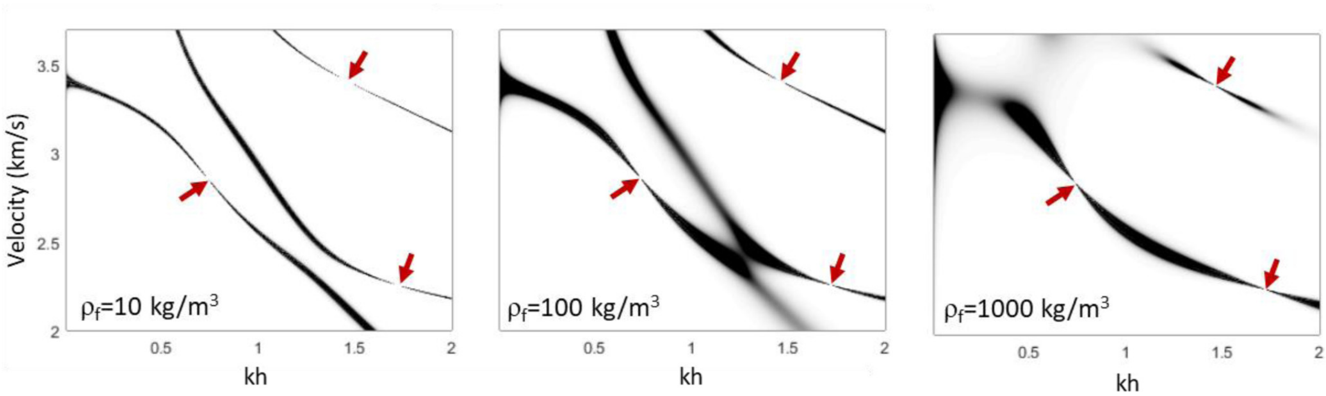

FIG. 7. Maps of $(\mathrm{Im}G_{11})^2 + (\mathrm{Im}G_{33})^2$ for a polystyrene layer on fused silica loaded by a fluid half-space with different fluid densities. BICs are marked by arrows.

point in the Rayleigh-Lamb dispersion curves when the vertical component of the surface displacement vanishes. The ratio of the horizontal to vertical displacement amplitudes is termed the ellipticity of the Rayleigh-Lamb wave. This quantity plays an important role in seismology and has been investigated in that context [32,33].

For a Rayleigh wave in a homogeneous isotropic elastic half space, the ellipticity varies between about 0.5 and 0.95 depending on the Poisson ratio [32]. Thus, in this case the surface displacement always contains a vertical component; consequently a fluid-loaded solid half space supports no BICs. However, in Rayleigh-Lamb waves in a layer-substrate system the ellipticity may become both infinite or zero [32,33]. As an example, Fig. 6(a) presents dispersion curves of guided modes for a polystyrene layer ($\rho_1 = 1050\,\mathrm{kg/m^3}$, $c_{l1} = 2350\,\mathrm{m/s}$, $c_{t1} = 1120\,\mathrm{m/s}$) on a fused silica substrate, with indicated sections of clockwise and counterclockwise sagittal particle motion at the surface. We observe that in each mode the motion may be clockwise or counterclockwise depending on the wave vector; at the points where the chirality of the surface motion changes, the polarization is either pure horizontal or pure vertical, indicated by vertical and horizontal arrows in the figure. For comparison, in Fig. 6(b) we show dispersion curves for gold on fused silica (the same dispersion curves as in Fig. 2 below the transverse threshold of the substrate); in this case there is no change of chirality within a single mode and the points of pure vertical or horizontal polarization are absent.

Now if the polystyrene layer on silica is loaded by a fluid half space, we expect that the points of the pure horizontal polarization will yield BICs, and this indeed turns out to be the case. Figure 7 shows maps of $(\mathrm{Im}G_{11})^2 + (\mathrm{Im}G_{33})^2$ for fluids having the acoustic velocity of water (1484 m/s) and different densities, from a very low density to that of water. In the lowest fluid density case, the leaky modes closely resemble those shown in Fig. 4(a), and one can see that at the points of the pure horizontal polarization the resonances narrow down to a zero width, indicating that leaky modes transform into BICs. As the fluid density increases, the leaky mode resonances broaden and the dispersion curves change shape and even bifurcate, but they remain pinned at the BIC points which are not affected by the fluid loading.

Another example of BICs in a fluid-loaded system is provided by Lamb waves in a plate immersed in a nonviscous fluid. It is known that the normal component of the surface displacement in Lamb modes may vanish [34,35]. This happens, for example, when dispersion curves of symmetric Lamb modes cross the line corresponding to the longitudinal velocity [34]. Freedman [21] has shown that for a plate immersed in a lossless fluid, the resonance width associated with Lamb waves becomes zero at these isolated points. Thus, Lamb wave BICs in immersed plates have been in effect known even if not reported as such and not appreciated by other researchers working on BICs in optics and acoustics [5,36].

VI. CONCLUSIONS

We have presented a general analysis of existence criteria for non-symmetry-protected acoustic BICs in layered isotropic structures and provided numerical examples of their occurrence. We have demonstrated that in acoustics, in contrast to optics, BICs can exist in simple structures without periodicity such as a solid layer on a solid substrate. There are at least two important differences between the acoustic and electromagnetic systems: (i) in solid-state acoustics, there are transverse and longitudinal waves. Consequently, a BIC with a phase velocity between the transverse and longitudinal velocities of the substrate may involve an evanescent longitudinal field in the substrate, which makes it possible for a BIC to satisfy the boundary conditions. (ii) The special case of acoustic boundary conditions between a solid and a fluid has no analogs in optics. These boundary conditions can be satisfied even if the acoustic field in the fluid is identically zero, provided that the displacement at the solid surface is purely horizontal. This enables a unique kind of BICs which do not involve an evanescent field in the fluid half space that provides the radiation continuum.

For the two situations listed above, BICs are found in simple layered systems involving commonly encountered materials combinations. We have shown that the BICs we identified are robust with respect to small perturbations of material properties or the layer thickness. We have also identified cases such as shear horizontal waves in a solid layered structure or pressure waves in a fluid layered structure, in which BICs do not exist similarly to the optical case. The absence of sagittally polarized BICs whose velocity exceeds the longitudinal velocity of the substrate has been proved as well.

Acoustic BICs are not just a peculiarity attracting the attention of theoreticians. Surface acoustic BICs found on anisotropic substrates are already widely used in SAW devices [15,16]. There, the control parameters that are used to eliminate the acoustic radiation into the substrate are the surface orientation and the wave-vector direction [23]. The use of BICs in layered structures will create more options for the design of SAW devices. BICs in fluid-loaded systems have implications for acoustic microscopy: Unlike regular Rayleigh-Lamb waves, BICs cannot be probed by the $V(z)$ method [37]. At the same time, the existence of Rayleigh-Lamb modes not subject to the radiative loss in the fluid environment may be of interest for the designers of SAW sensors.

Finally, we anticipate that an approach similar to that developed in this work may lead to the discovery of optical BICs without photonic crystals in a layered structure on an *optically anisotropic* substrate, in which case the existence of two bulk eigenmodes with different phase velocities may enable a BIC between these two velocity values.

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