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## Restart Could Optimize the Probability of Success in a Bernoulli Trial

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The recently noticed ability of restart to reduce the expected completion time of first-passage processes allows appealing opportunities for performance improvement in a variety of settings. However, complex stochastic processes often exhibit several possible scenarios of completion which are not equally desirable in terms of efficiency. Here we show that restart may have profound consequences on the splitting probabilities of a Bernoulli-like first-passage process, i.e., of a process which can end with one of two outcomes. Particularly intriguing, in this respect, is the class of problems where a carefully adjusted restart mechanism maximizes the probability that the process will complete in a desired way. We reveal the universal aspects of this kind of optimal behavior by applying the general approach recently proposed for the problem of first-passage under restart.

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Stochastic processes subject to restart appear in many disciplines including physics, chemistry, biology, and computer science. Restart means a sudden interruption of a process followed by its starting anew. In some contexts, the restart is an integral part of a phenomenon under study (e.g., substrate unbinding in enzymatic reactions [1] and recovery of RNA polymerase from the backtracked state [2]), while in others it plays the role of an external control tool (e.g., reinitialization of a randomized computer algorithm [3,4] and reduction of growing tumor to its initial size by chemical treatment [5]).

A significant amount of research has been dedicated to the study of the effect of a restart on first-passage properties. The growth of interest in this problem was triggered by the surprising observation that a restart may significantly reduce the mean first-passage time (MFPT). In recent years, it has been demonstrated in a range of diverse examples that a carefully chosen restart rate can bring the MFPT to a minimum [6–14]. Along with the investigation of particular cases, we witness ongoing attempts to reveal the general principles allowing us to navigate in a vast space of first-passage problems under restart. A remarkable result of these attempts is the discovery of universality displayed by all optimally restarted processes [15–17].

To the best of our knowledge, the first-passage process under restart considered so far had only one way of completion. In particular, a diffusion mediated search with stochastic resetting to the initial position [6]—a classic example of a first-passage problem under restart—ends if and only if a searcher finds a target. However, real-life settings often offer a variety of possible ways in which stochastic process can be completed. Plurality of the process outcomes may arise from the competition among several different first-passage phenomena or due to multiple thresholds for one and the same first-passage

mechanism. Assume, for instance, that a gambler stops playing after winning a certain amount of money or getting ruined, whichever happens first [18,19]. In many-target search problems and diffusion-limited reactions, different completion scenarios may correspond to the finding of different targets [20–29]. In search problems with time constraints, a search process can finish either by target detection or by searcher or target death [30–38]. When there are several competing paths of chemical reaction, an individual molecule may be converted into one product or another, depending on which path has been realized [39–41]. Similarly, a biopolymer molecule may fold along one of many possible pathways to one of multiple native states [42–47]. In evolutionary biology and ecology, one could ask if a population goes extinct before its size attains some threshold level [48,49]. Clearly, the immense set of possibilities is not limited to these few examples.

What happens when a first-passage process with several possible outcomes becomes subject to a restart? The main goal of this Letter is to draw attention to a previously unknown type of optimal behavior in first-passage phenomena: a carefully chosen rate of Poisson restart brings the probability of observing a particular completion scenario to a maximum (or minimum). In other words, we argue that a stochastic restart could optimize the so-called *splitting probabilities* [50,51]. The effect is first illustrated in a particular example, and after that, we apply a general framework recently proposed in Ref. [17] by Pal and Reuveni to gain a deeper insight. For the sake of simplicity, we focus on the case where the process has exactly two possible outcomes, but the analysis can be directly extended to a more general situation. We show that the optimality of splitting probabilities always entails an exact match between the unconditional and conditional mean completion times of the process. Looking for further

generalization, we go beyond the assumption of a Poisson restart and demonstrate advantage of the deterministic restart strategy in terms of attaining the most pronounced extrema of splitting probabilities.

The key properties of a first passage under restart have originally been learned from the one dimensional diffusion process [6]. We will use the same exemplary case to demonstrate the ability of a restart to optimize the splitting probabilities. Specifically, let us consider a mortal Brownian searcher with the diffusion constant  $D$  and the mortality rate  $\alpha$  that starts from the initial position  $x_0 \geq 0$ . The search process ends either when the searcher dies or when it finds the immobile target located at  $x = 0$ . It is shown in Ref. [32] (see also [37]) that target detection occurs with the probability  $p = e^{-\sqrt{\alpha x_0^2/D}}$ . Assume now that the process is stochastically restarted; i.e., the searcher is returned to its initial position  $x_0$  at some constant rate  $r$  [52]. What is the detection probability  $p_r$  in the presence of a restart? The exact solution of the initial-boundary value problem for the probability density of the searcher's position yields (see Supplemental Material [53])

$$p_r = \frac{r + \alpha}{\alpha e \sqrt{(r+\alpha)/D x_0} + r}. \quad (1)$$

Analyzing Eq. (1), one can readily see that if  $\alpha \geq \alpha_0 = (z^*)^2 D/x_0^2$ , where  $z^* \approx 1.59362\dots$  is the solution to  $z/2 = 1 - e^{-z}$ , then  $p_r$  monotonically decreases as  $r$  increases from zero to infinity. Otherwise, when  $\alpha < \alpha_0$ , the probability  $p_r$  takes its maximum at the nonvanishing restart rate  $r_0 = \alpha_0 - \alpha$ . In Fig. 1 we plot  $p_r$  as a function of  $r/\alpha_0$  for different  $\alpha/\alpha_0$ .

Let us give a qualitative explanation for the observed behavior of  $p_r$ . If  $\alpha$  is large compared to  $D/x_0^2$ , then the typical size of the region explored by the searcher during its lifespan is less than the initial distance to the target, and thus, a nonvanishing restart inevitably leads to a reduction of the search efficiency. Otherwise, when  $\alpha$  is small in

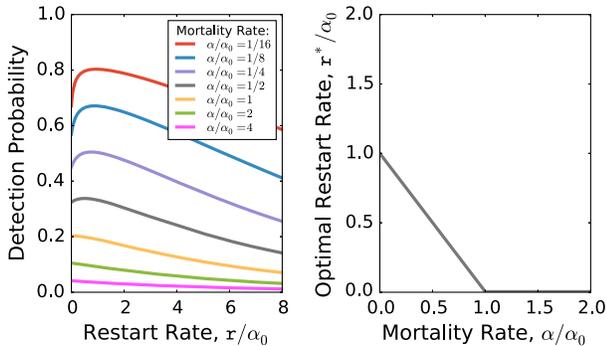


FIG. 1. (Left) The probability of target detection  $p_r$  versus the rate  $r$  of Poisson restart for different values of the searcher's decay constant  $\alpha$ . (Right) Optimal restart rate as a function of the decay constant.

comparison with  $D/x_0^2$ , the searcher lives long enough to be able to reach the target via a typical diffusive path, but it is also able to execute a distant excursion in empty areas of the search space. These excursions prolong the search process and typically end with searcher death. Then, the nonvanishing restart rate censors the fatal paths and increases chances to find the target. On the other hand, too large of a restart rate hinders target detection, since the searcher has less time between restarts to reach the origin under the same mortality rate. This is why a nonvanishing optimal restart rate  $r^*$  exists, which brings the probability that the searcher will find the target before dying to a maximum.

Having examined the exemplary case, we now turn to a more general setting. Let us consider a generic stochastic process that can end in two different ways and is subject to a generic restart mechanism. For the sake of convenience we will call one of the two possible outcomes “success” and the other “failure.” Thus, the problem can be viewed as kind of a Bernoulli experiment, see Fig. 2(a). In the example discussed above, detection of the target naturally corresponds to success, while the searcher's death is interpreted as failure. Obviously, in other contexts, these conventional terms may not have any real meaning.

The original process is characterized by a random completion time  $T$  having the probability distribution  $P(T)$ . The later can be decomposed into a sum  $P(T) = P^s(T) + P^f(T)$ , where  $P^s(T)$  and  $P^f(T)$  are the probability densities of successful and failed trials, respectively. Normalization of the probability density  $P^s(T)$  defines the “unperturbed” probability  $p$  of success:  $p = \int_0^\infty P^s(T) dT$ . We will also utilize the trivial fact that the ratio  $P^s(T)/P(T)$  gives the probability of success in a trial with the completion time  $T$  [54].

Being subject to restart, the process can be interrupted at a random time  $R$ , characterized by a proper probability distribution  $P^r(R)$ , and started again. The probability  $p_r$  of success for the restarted process can be computed as the expectation of a binary random variable  $x$ , which takes the value 1 if the process is successfully completed and is equal

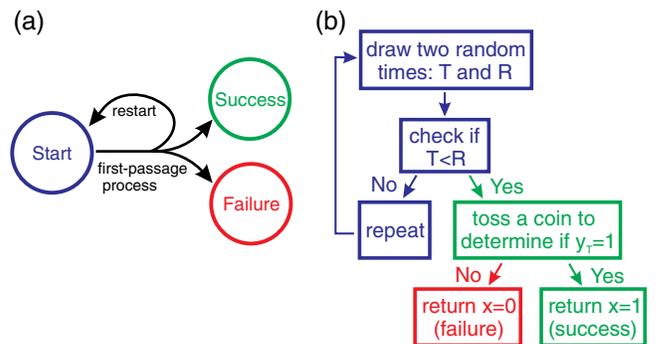


FIG. 2. (a) Bernoulli-like first-passage process under restart. (b) Pseudocode representation of Eq. (2).

to 0 in the case of failure. This variable obeys the following renewal equation:

$$x = I(T < R)y_T + I(T \geq R)x', \quad (2)$$

where  $I(T < R) = 1 - I(T \geq R)$  is an indicator random variable that is equal to unity when  $T < R$  and is zero otherwise;  $x'$  is an independent and identically distributed copy of  $x$ , and  $y_T$  is an auxiliary binary variable that takes the value one with the probability  $P^s(T)/P(T)$ .

The intuition behind Eq. (2) is very simple. Imagine that we run a computer simulation designed to reproduce behavior of the random variable  $x$ , see Fig. 2(b). At the first step, we should choose two random times from the distributions  $P(T)$  and  $P^r(R)$  and decide which of the two, restart or completion, happened first. If  $T < R$ , then the process is completed prior to restart. To determine whether the process ends in success or failure, we toss a coin with the probability of success  $P^s(T)/P(T)$  and assign the outcome to the variable  $x$ . Otherwise, if  $T \geq R$ , the process begins completely anew, and we should repeat the procedure until the process reaches completion.

After averaging the statistics of the underlying process and random restart events, Eq. (2) yields

$$p_r = \langle x \rangle = \frac{\langle I(T < R)y_T \rangle}{\langle I(T < R) \rangle}. \quad (3)$$

Once the probability density functions  $P^s(T)$  and  $P^r(R)$  are known, one can readily compute  $\langle I(T < R)y_T \rangle = \int_0^\infty \int_0^R P^r(R)P^s(T)dRdT$  and  $\langle I(T < R) \rangle = \int_0^\infty \int_0^R P^r(R)P(T)dRdT$ . When restart events come from Poisson statistics with a constant rate parameter  $r$ , the restart time  $R$  has an exponential distribution  $P^r(R) = re^{-rR}$  and Eq. (3) reduces to

$$p_r = \frac{\tilde{P}^s(r)}{\tilde{P}(r)}, \quad (4)$$

where  $\tilde{P}^s(r)$  and  $\tilde{P}(r)$  denote the Laplace transforms of, respectively,  $P^s(T)$  and  $P(T)$  evaluated at  $r$ . Note that for the above problem of diffusion mediated search  $P^s(T) = \sqrt{x_0^2/4\pi DT^3} e^{-\alpha T - x_0^2/4DT}$  and  $P^f(T) = \alpha e^{-\alpha T} \text{erf}(\sqrt{x_0^2/4DT})$  [37]. It is straight forward to show then that Eq. (4) reproduces Eq. (1) previously obtained through the less generic method (see SM [53]).

Noteworthy, at  $r \rightarrow 0$ , Eq. (4) yields  $p_r \approx p + p(\langle T \rangle - \langle T^s \rangle)r$ , where  $\langle T \rangle = \int_0^\infty P(T)TdT$  and  $\langle T^s \rangle = p^{-1} \int_0^\infty P^s(T)TdT$  represent, correspondingly, the unconditional MFPT of the original process and its MFPT conditional to success (assuming that these expectations as well as the high-order moments exist). Thus, the inequality  $\langle T^s \rangle < \langle T \rangle$  gives a simple criteria of whether the introduction of a Poisson restart with an infinitesimally

small rate will increase the success probability. In particular, for the mortal Brownian searcher we have  $\langle T \rangle = (1 - e^{-\sqrt{x_0^2\alpha D}})/\alpha$  and  $\langle T^s \rangle = x_0/(2\sqrt{\alpha D})$  so that this criteria is satisfied only when  $\alpha < \alpha_0$  in accord with the above analysis. Let us also stress that in some cases, applying restart at a high enough rate may be beneficial even when restart with a small rate decreases the chances of success. Indeed, one finds from Eq. (4) that at  $r \rightarrow \infty$  the success probability becomes  $p_r = \lim_{T \rightarrow 0} P^s(T)/P(T)$ , and thus, it can take any value in the interval from zero to unity depending on the behavior of  $P^s(T)$  and  $P(T)$  at  $T \rightarrow 0$ .

We are mostly interested in the class of problems where the probability  $p_r$  is maximized for some optimal rate  $r^*$ . What do these problems all have in common? To address this question, let us take a look at the first-passage-time properties of the process illustrated in Fig. 2(a). As was shown in [17], the completion time  $T_r$  of a generic first-passage process under a generic restart mechanism obeys the following identity

$$T_r = I(T \geq R)(R + T'_r) + I(T < R)T, \quad (5)$$

in which  $T'_r$  is an independent and identically distributed copy of  $T_r$ . Equation (5) allows one to express the MFPT as  $\langle T_r \rangle = \langle \min(T, R) \rangle / \langle I(T < R) \rangle$ , where  $\min(T, R)$  is the minimum of  $T$  and  $R$ . Next, one could ask also how to compute the MFPT  $\langle T_r^s \rangle$  conditional to success, which is simply the average completion time of successful trials. By virtue of its definition, this quantity can be written as  $\langle T_r^s \rangle = \langle xT_r \rangle / \langle x \rangle$ . Substituting Eqs. (2) and (5) into this relation results in

$$\langle T_r^s \rangle = \frac{\langle I(T > R)R \rangle}{\langle I(T < R) \rangle} + \frac{\langle I(T < R)y_T T \rangle}{\langle I(T < R)y_T \rangle}. \quad (6)$$

For an exponentially distributed restart, Eq. (6) takes a particularly simple form (see SM [53])

$$\langle T_r^s \rangle = \langle T_r \rangle - \frac{d \ln p_r}{dr}, \quad (7)$$

where  $p_r$  is given by Eq. (4) and  $\langle T_r \rangle = r^{-1}(1 - \tilde{P}(r))/\tilde{P}(r)$  [16]. Equation (7) tells us that the response of the success probability to the change of the restart rate is determined by the difference between the unconditional and the conditional MFPTs; thus, generalizing the result of the asymptotic analysis. To demonstrate the general validity of Eq. (7), we numerically checked it in four different settings, see Fig. 3. Importantly, if the success probability  $p_r$  of the restarted process attains a maximum for some  $r^*$ , the second term in the right hand side of Eq. (7) vanishes and we get

$$\langle T_{r^*}^s \rangle = \langle T_{r^*} \rangle. \quad (8)$$

Also, since  $p_r \langle T_r^s \rangle + (1 - p_r) \langle T_r^f \rangle = \langle T_r \rangle$ , a similar identity holds true for the mean completion time of failed trials:

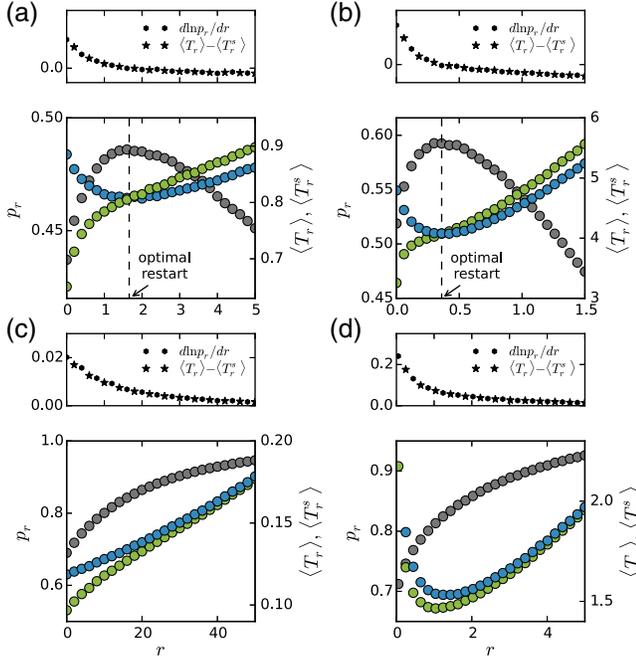


FIG. 3. The probability of success  $p_r$  (gray circles) and the MFPTs,  $\langle T_r \rangle$  (blue circles) and  $\langle T_r^s \rangle$  (green circles), versus the rate  $r$  of a Poisson restart obtained from the numerical simulations of a Bernoulli experiment in various settings: mortal Brownian searcher (a), mortal run-and-tumbling searcher (b), Brownian searcher and two competing targets (c), and two Brownian searchers competing for a target (d). As we see, the relation  $(d \ln p_r / dr) = \langle T_r \rangle - \langle T_r^s \rangle$  [a simple rearrangement of Eq. (7)] is generally valid. The details of simulations can be found in [53].

$\langle T_{r^*}^f \rangle = \langle T_{r^*} \rangle$ . We thus conclude that when the rate of a Poisson restart is optimal, in the sense that it maximizes or minimizes the probability to observe specific outcome, the unconditional MFPT is equal to the conditional MFPT in this outcome. This universal feature is shared by all optimally restarted processes irrespective on their fine details [see Figs. 3(a) and 3(b)].

The surprising simplicity of Eq. (8) calls for its intuitive explanation. To provide such an explanation, let us assume that one starts to observe a first-passage process, which is allowed to repeat itself over and over, at a random moment of time. What is then the *expected probability*  $p^{\text{exp}}$  of getting success in the next outcome? It can be shown that this probability is given by  $p^{\text{exp}} = p \langle T^s \rangle / \langle T \rangle$  (see SM [53]). Obviously, applying a Poisson restart with an infinitesimally small rate  $\delta r$  will increase the chances of success whenever  $p^{\text{exp}} < p$ , while at  $p^{\text{exp}} > p$  the effect will be opposite. At the same time, if the process is already restarted at the optimal rate  $r^*$ , then  $[dp_r / dr]_{r^*} = 0$  and a small additional correction  $\delta r$  to  $r^*$  does not change the probability of success  $p_{r^*}$  in the leading order approximation. Therefore, for the optimally restarted process,  $p_{r^*}$  must be equal to  $p_{r^*}^{\text{exp}} = p_{r^*} \langle T_{r^*}^s \rangle / \langle T_{r^*} \rangle$  that immediately leads to Eq. (8).

Interestingly, the match of unconditional and conditional MFPTs is an inherent property of some two-thresholds first-passage processes relevant to kinetics of enzyme reactions [55,56], motor proteins dynamics [57], entropy-production fluctuations [58], and decision making [59]. For all these processes the splitting probabilities coincide with the corresponding expected splitting probabilities.

Anticipating that optimization is not an exclusive prerogative of a Poisson restart, it is natural to ask how to choose a restart time distribution  $P^r(R)$  that provides the maximum probability of success  $p_r$  for a given first-passage process. Recently, it was proven that a deterministic restart [i.e.,  $P^r(R) = \delta(R - t)$ ] always outperforms stochastic restart strategies in terms of attaining the lowest MFPT [17]. Arguments similar to those used in [17] allow us to conclude that a deterministic restart is also universally preferable when one needs to optimize the splitting probabilities. It can be shown that if there exists such  $t^*$  that a deterministic restart with a restart time distribution  $P^r(R) = \delta(R - t^*)$  brings the probability of success to a maximum  $p_{r^*}$ , then the value  $p_{r^*}$  cannot be exceeded by stochastic restart strategies (see SM [53]).

Equation (8) is no longer valid when restart events have non-Poisson statistics. Instead, the conditional and unconditional mean first-passage times of a process undergoing optimally tuned deterministic restart obey the universal inequality constraint

$$\langle T_{t^*}^s \rangle \geq \langle T_{t^*} \rangle. \quad (9)$$

To prove Eq. (9), let us assume that the process, which is being restarted deterministically in an optimal way, becomes subject to an additional Poisson restart with an infinitesimally small rate  $\delta r$ . That produces a differential correction  $\delta p$  to the probability of success  $p_{r^*}$  attained by deterministic restart. Equation (7) allows us to write  $\langle T_{t^*} \rangle - \langle T_{t^*}^s \rangle = \delta p / (p_{r^*} \delta r)$ . Because of the dominance of a deterministic restart over other restart strategies, one can be sure that  $\delta p \leq 0$ , and therefore,  $\langle T_{t^*}^s \rangle \geq \langle T_{t^*} \rangle$ . Note also that the MFPT of failed trials satisfies the opposite inequality:  $\langle T_{t^*}^f \rangle \leq \langle T_{t^*} \rangle$ .

*Conclusion.*—When the different outcomes of a first-passage process are not equally valuable, the splitting probabilities may come to the fore as a crucial measure of efficiency and reliability [36–41,59] In this Letter, we applied a general theoretical approach to describe the effect of restart on the splitting probabilities of a process with exactly two possible completion scenarios. It was shown that a carefully chosen rate of Poisson restart could maximize (minimize) the probability that the process will complete in the desirable (undesirable) way. Whenever it is the case, the conditional and unconditional mean completion times are equal to each other. We also established the global dominance of a deterministic restart in the entire space of restart strategies—further evidence of the great optimization potential of deterministic restart in

first-passage problems [12,17]. Note that these conclusions are robust to appearance of a generally distributed random time penalty for restart (see SM [53]). Thus, our work adds to the collection of universal results in the field of first-passage phenomena [16,17,60–63].

Of many implications of the above results, let us emphasize the issue relevant to chemical kinetics. The two fundamental problems of chemistry are control over the reaction rate [64] and the product selectivity [39–41]. As we know, thanks to the recent study of enzymatic reactions [1], the restart of a catalytic step can potentially accelerate the rate of product formation. The results of the present work lead to the complementary conclusion that when competing pathways of a chemical reaction end up with different products, the introduction of a restart mechanism may allow us to have control over the product ratio.

A particularly interesting line of future research concerns the so-called deadline meeting problem. As was noticed in computer science, a restart could help to increase the chances that a randomized search algorithm will find the solution before a prescribed deadline passes [65–67]. One could try to extend the analysis presented here to address the issue of universality of the optimal restart in this setting [68].

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- [53] See Supplemental Material at <http://link.aps.org/supplemental/10.1103/PhysRevLett.120.080601>, which includes Ref. [15] and Ref. [26], for derivations of Eqs. (1) and (7), a derivation of the expected probability of success, proof of the superiority of deterministic restart, a generalization of analysis that accounts random time penalty for restart, and details of numerical simulations.
- [54] Clearly,  $P^i(T)$ , where  $i$  is either “s” (success) or “f” (failure), represents the joint probability distribution of the process outcome and completion time, while  $p(s|T) = P^s(T)/P(T)$  is the conditional probability of success given the completion time  $T$ .
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