STOCHASTIC RESPONSE OF TALL BUILDINGS
WITH AUXILIARY DAMPERS

by

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Algeria, June 84

SUBMITTED TO THE DEPARTMENT OF CIVIL
ENGINEERING IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 1988

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Submitted to the Department of Civil Engineering on May 13, 1988 in partial fulfillment of the requirements for the degree of Master of Science.

Abstract

The lateral resistance of tall buildings to wind effects is the dominating factor in their design. In recent years auxiliary damping systems have been introduced to improve the damping level of these structures and hence decrease their dynamic response. The increasing number of tall buildings in highly seismic areas raises the concern of assessing the seismic behavior of structures equipped with such mechanisms.

In this thesis, after presenting a review of the various types of vibration reducing devices, we focus on the analysis of the most popular: the tuned mass damper. A review of the relevant literature indicates a wide disagreement concerning the seismic effectiveness of this mechanism. The present study represents our contribution to this highly controversial issue.

To this end, we first developed a stochastic formulation allowing for a parametric study of a number of factors, such as the damper/structure mass ratio, the damping level of the structure and the damper and the ratio of the frequency of the tuned mass damper to the building’s fundamental frequency. The results obtained do not show any detrimental effect of the tuned mass damper on the structure. However, only the participation of the fundamental mode to the total response has been investigated, and relative motions due to a possible contribution of higher modes have been neglected.

Secondly, a new methodology for developing a representative spectrum from an ensemble of ground motion power spectra was incorporated in the formulation. Traditional modeling of the seismic loading by a single design spectrum which is site dependant and for which there is uncertainty as to whether it is adequate for the design under consideration is quite limited. Our results show that this methodology, imported from ocean engineering applications, produces a more realistic spectrum and should be investigated further.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil Engineering
Dedication

A mon jeune frère Réda,
Acknowledgement

I would like to express my profound gratitude to Professor Jerome J. Connor and Professor John Niedzwiecki, for the guidance, help and support they provided particularly during this work and more generally in the course of my interaction with them.

Also, Professor E. N. Lorenz's comments concerning "principal components analysis" are much appreciated.

I would like to thank all my friends for their participation to this work, was it by an encouraging word, by suggesting the appropriate computer command, or in any other way. Especially, I am deeply grateful to Dr. Artur Pais for sharing with me his deep understanding in this area, and giving me priceless advice and practical help. Tommaso Pagnoni for the help he gave me through many fruitful discussions and for reviewing my codes. Laura Demsetz for the very relevant suggestions she gave me about this work, and for her unbounded support. Annette Huber for her valuable suggestions on the writing, and last but not least, Charis Gantes for helping make this thesis look much nicer than it would have, and thanks to whom I made it for this deadline.

I also thank all the members of my family, for their unconditional love and support and for their trust in me, and for giving me the strength to go on...

Finally, I wish to express gratitude to the Ministry of Higher Education of Algeria for financially supporting my stay at M.I.T.
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Chapter 1
Introduction

1.1 Background and Motivation

Earthquakes are one of nature’s most adverse effects faced by man. Despite the tremendous advances of science, especially during the last few decades, scientists are still unable to predict the time, location and intensity of the next earthquake. Engineers nevertheless need, on a regular basis, to design civil structures with a *reasonable* degree of safety and a moderate cost.

To account for lateral seismic load on multistorey structures, building codes specify a static lateral design load, which varies with the site of the construction, according to a probabilistic estimation of the likelihood of the occurrence of a strong earthquake in the given area\(^1\). This design load, for economic reasons, is usually smaller than the load typically experienced by structures during strong motion earthquakes. However, the total collapse of structures is often prevented by the actions of factors that are not ordinarily taken into account during design. These effects are generally due to an increase of damping in the structure caused, for instance, by the non-linear behavior of concrete, and the failure of non-structural elements such as plaster walls.

Manifestly, the energy absorbing capacity of a structure plays a major positive role in its earthquake resistant capacities. Modern

\(^1\)This estimation is generally based on past experiences and seismicity studies.
construction technology however, seems to have a predilection for the use of material with low damping capacities: steel, used for the structural resistance and glass and light weight material with low inherent damping, used for the non-structural elements. The damping level of such structures is generally very low, consequently they often experience undesirable wind-induced oscillations. Even though the structure is resistant enough to withstand these oscillations, its non-structural elements, like partitions, can suffer some damage, and most importantly, the tenants of these buildings experience discomfort which leads to a feeling of unsafety. A few tall structures are required to have their damping increased to meet wind serviceability requirements.

Damping can be increased by means of non-structural mechanisms. Some of these mechanisms are designed as real damping devices so that part of the mechanical energy associated with building motions is converted into heat through the use of either a viscoelastic material or a fluid. These damping devices can be used in trusses, girders or beams where relative displacement usually develop when the structure experiences lateral motion.

A common device currently in use is the tuned mass damper (TMD) system. Tuned mass dampers (or vibration absorbers) are viscous spring-mass units which are added to vibratory systems to reduce their dynamic motion. These devices have proven to be effective in reducing wind-induced vibrations at a cost significantly lower than alternate conventional solutions available [12]. TMD 's have been installed in a number of tall buildings such as the Citicorp Center in New York and the
1.2 Statement of the Problem

Lateral design for tall buildings is generally governed by wind. However, some high risk seismic areas (Los Angeles for instance) have seen an increasing number of tall buildings. Since TMD's have been successfully designed and implemented to reduce wind induced vibrations, the question as to how this system would respond to an earthquake excitation arises. This issue has been investigated by many researchers but remains controversial and open to further discussion. This report contains our contribution to the question.

Another important issue when dealing with seismic design for an elaborate analysis, is the choice of an appropriate model to represent the earthquake ground motion. Our second goal is therefore to test the applicability of a new methodology for developing a "design spectrum" from an ensemble of ground motion power spectra.

1.3 Outline of Thesis

The following chapter presents a review of the literature studying the effectiveness of tuned mass dampers in reducing seismic response of buildings. In Chapter Three, a critical survey of the most prominent devices that are used or are being proposed for use in reducing seismic
building oscillations is presented. Chapter Four concentrates on the
tuned mass damper as an auxiliary damping device and investigates its
effectiveness in reducing seismic motion, based on response spectra and
root mean square / top displacement ratio comparisons. Principal
component analysis is reviewed as a potential alternative method for
defining appropriate earthquake excitation in Chapter five. The
conclusions concerning both this section and the previous one are
summarized in chapter six. The derivation of the equations of motion for
the system analyzed are presented in Appendix A and the formulation in
the frequency domain in Appendix B.
2.1 Overview

The principle of vibration absorbers has been studied extensively in the literature. An early reference to their use dates back to 1909, when Frahm suggested the absorber as a means to reduce the dynamic response of ships.

A vibration absorber limits the oscillations of the primary structure it is attached to through a quasi-resonant behavior. It generally consists of a mass connected to the structure via a spring and a dashpot. The natural frequency of the absorber is tuned to the frequency of the mode of the structure to be damped out, therefore this absorber is also referred to as tuned mass damper (TMD).

Because vibration absorbers could potentially be applied for reducing vibrations in a wide range of mechanical systems, researchers started very soon investigating which characteristics of these systems minimize the response of the principal structure. This problem has been solved by Den Hartog [8] for the case of a linear SDOF system excited by a sinusoidal force. Some attempts have been made to extend the area of applicability of the absorber concept to include random inputs and nonlinear systems.

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3 The spring and the dashpot can be either linear or nonlinear, but for most practical purposes they are assumed to be linear.

4 Single Degree of Freedom
Crandall and Mark [7] present an analysis of a linear SDOF absorber system subjected to a white noise base acceleration\(^5\). This study and others have demonstrated the feasibility of using the linear vibration absorber for random excitations.

Since the early 60's, researchers in the earthquake engineering community started investigating the applicability of the concept of vibration absorbers to buildings for reducing their seismic responses. The principal appeal of this method was that it could provide not only a safer but also a cheaper alternative for aseismic design.

Gupta and Chandrasekaran [16] explored the possibility of using passive\(^6\) TMD's for reducing the structural response of buildings to the irregular motion of earthquakes. They analyzed parametrically a system consisting of a number of absorbers, with various periods and damping coefficients, attached to SDOF primary structures with fundamental periods of 0.5 sec and 1 sec. Their analysis was based on the response of such systems to one earthquake: the S21W component of the Taft earthquake. They observed a reduction of the peak response displacement close to 20% and concluded that vibration absorbers are not as effective in reducing the response of a structure to an earthquake excitation as they are for a sinusoidal input.

The concept of using low-cost motion-reducing devices, not restrained to the tuned mass damper mechanisms only, continued to

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\(^5\) A white noise is an idealized random process with constant power spectral density.

\(^6\) A passive system is a system which does not require any electric power for its functioning.
attract the interest of a number of researchers. Wirsching and Yao [46] studied the seismic behavior of multi-storey buildings, 5- and 10-storeys, with various passive devices and compared their performances. The seismic excitation was provided by a non-stationary stochastic process having statistical characteristics similar to real earthquakes. Only the response contributed by the fundamental mode was taken into account. For the case of a TMD placed on the top floor, a reduction of the mean top displacement of about 40% was achieved.

Concerning the tuned mass damper more specifically, Wirsching and Campbell [47] demonstrated that the absorber is quite effective in reducing first mode response for 5- and 10-storey structures even for small relative values of the absorber's mass. In the analysis, they modeled the structure as a shear building, and the earthquake excitation as a Gaussian white noise acceleration. This model for the earthquake enabled them to use random vibration theory to make a reliability analysis of the structure with and without the damper; they observed a noticeable decrease of the probability of failure for the system with the TMD and concluded that the addition of an absorber with the appropriate characteristics, increases the integrity of the structure. However, they recommended that the effect of the TMD on higher modes be investigated for a more complete assessment of its effectiveness.

In reducing wind-induced vibrations however, the absorber proved its effectiveness not only for hypothetical cases, but also for real buildings: for a number of tall steel buildings, a TMD system has been installed to

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7 The five-storey building has been chosen because its natural frequency, equal to 2 Hz, corresponds to the peak of the power spectrum of the El-Centro earthquake.
mitigate its vibrations. A reduction of 40% of the sway of the top floor of the Citicorp Center, a highrise office building in New York, has been observed. It has been argued that the TMD alternative was much cheaper than stiffening the building or increasing its mass [12]. Currently designers are looking for alternatives to make TMD systems smaller and less heavy without losing any of their effectiveness.

Since TMD's have been implemented in practice, some authors raise the question of the behavior of this compound system in a seismic environment; there has been renewed interest in the investigation of the seismic effectiveness of these absorber systems.

Sladek and Klinger [38] studied a prototype 25-storey building with a fundamental period of 1.90 sec, for which a TMD was installed at the top floor to meet wind serviceability requirements. They analyzed the building both linearly and nonlinearly. The loading was provided by the N-S component of the El-Centro earthquake. Their conclusion was that the TMD did not significantly alter the response of the prototype.

Kaynia et al [25] investigated statistically the effects of one TMD placed at the top floor of a structure by subjecting the system to a population of 48 historical earthquakes. The response of interest was the top displacement contributed by the first mode. Due to the large statistical dispersion about the mean response ratio they observed over the population of earthquakes, they concluded that the seismic effectiveness of tuned mass dampers was very small. They also investigated the effectiveness of TMD's using random vibration theory together with a white noise stochastic model for the earthquake; they concluded, that the results obtained by this approach overestimated the effectiveness of the absorber.
Since the magnitude of the mass was a concern, Villaverde [44] proposed as an alternative the use of small, heavily damped vibration absorbers to reduce seismic response of tall buildings. He tested a 1-storey and a 10-storey building with a small attachment in resonance. The critical damping ratio of the absorber was 40% and 80%\(^8\). The excitation was provided by three earthquake records: El-Centro S00E, Taft N21E, and Paicoma Dam S16E. His conclusion was that the addition of heavily damped absorbers increases the damping of a building and thus reduces appreciably its response to strong ground motions.

Active mass dampers\(^9\) have also been investigated by a number of researchers. These systems offer the advantage of being less heavy, requiring less space and most importantly, they can develop very large forces to stabilize the building's sway. Yang [48] showed analytically how active controllers could reduce significantly the earthquake response of a two-storey building. Yang and Liu [50] demonstrated, also analytically, that under seismic excitation, active mass dampers are effective in reducing building response quantities. In their study, they used a uniformly-modulated non-stationary earthquake ground acceleration with zero mean to model the earthquake acceleration. On an 8-storey building, the root mean square of the base tortional moment was 10 times smaller with the active TMD on top of the building than without. However, because these active systems require electronic or mechanical

\(^8\)In all other reviewed studies, the damping of the absorber was generally in the same order of magnitude as the modal damping of the structure or at most 20%; moreover, most investigators argued that this parameter did not have a dominant effect on the structure's response.

\(^9\)Active systems require electric power to function.
instrumentation and maintenance, they are not generally recommended as reliable means for reducing seismic response.

2.2 Discussion

This brief review of the literature indicates that the seismic effectiveness of TMD's is a controversial issue. Clearly, the conclusions of these researchers are highly dependent on the modeling of the problem, both the modeling of the structural system, and the modeling of the excitation.

Concerning the structure, only the fundamental response has been investigated. This approximation is reasonable when analyzing the earthquake response of buildings of 12 storeys and less. For taller structures however, the contribution of higher modes may be significant, and this procedure will therefore underestimate the response\(^\text{10}\).

Another common assumption in the structural model was that the addition of the damper did not affect the modal characteristics of the building: the absorbers were designed to reduce the first modal response determined for the structure without any attachment and the effect of the absorber on higher modes has been ignored. However, since most investigators made this assumption, despite its possible relevance, it is apparently not the source of their contradictory conclusions and therefore we shall make the same assumption in our analysis.

\(^{10}\)It should be mentioned that the possibility of controlling higher modes has already been considered for the case of wind-induced vibrations. The Center Point Tower (250m) in Sydney, Australia, incorporates two TMD's: one installed at the top of the construction for the control of the vibrations in the first mode, and a second one for monitoring the second mode vibrations, placed at an intermediate level (89m from the base of the tower) [Fig 2-1].
The models of the earthquake excitation chosen by the various investigators cover a broad spectrum of possibilities, varying from the use of a single acceleration record to the idealization of the excitation as a white noise process, by the use of artificially generated accelerogramms.

We believe, that this non-uniformity of representation of the ground motion excitation is certainly responsible for the disagreements in the

**Figure 2-1:** Sydney Tower
reported conclusions. Obviously there is a need for a better representation of the earthquake excitation for design and analysis purposes.

2.3 Proposed Work

Since TMD's have proven useful for reducing wind effects on buildings, one can expect their use to continue; hence we propose to investigate how these devices might affect the seismic behavior of flexible structures and clarify some of the highly controversial findings of previous studies. This research does not emphasize the use of TMD's as aseismic devices, but tries to develop an understanding of the seismic behavior of buildings which have a TMD to accommodate wind effects. To this end, various auxiliary damping devices that could be useful for mitigating the response of buildings to both wind and seismic effects are reviewed. Then the analysis is concentrated on the tuned mass damper because of its current use and the controversial findings of various researchers.

The second issue addressed in this research is the modeling of the excitation. Earthquakes are random phenomena that are difficult to characterize statistically and models using stationary or non stationary white noise are idealistic. Furthermore, one cannot draw conclusions based on the use of one or a few recorded earthquakes. It would thus be very useful in this kind of analysis if one could use a spectrum that captures the characteristics of a large number of time histories or of their corresponding Power Spectra. Our second goal is to apply a new methodology for developing a "design spectrum" from an ensemble of ground motion accelerogrammes or spectral densities.
A statistical method called "Principal Component Analysis" seems to offer some potentialities in this respect. A variant of this method, known under the name "Eigenvector Analysis" has been used successfully in the field of ocean engineering for the characterization of wave spectra. We have attempted to apply this method to earthquake spectra. Mainly due to the computational effort involved when dealing with time histories, we based our analysis on spectral densities. Our results show, that this method is indeed promising, but more analysis needs to be carried out in order to have a better assessment of it.
Chapter 3

Auxiliary Damping Systems

3.1 Introduction

Given its external geometry, which is usually dictated by architectural constraints, the adjustable parameters in a structural system are its mass, stiffness and damping level. Therefore engineers are usually constrained to act upon one or more of these three parameters to control the dynamic response of a given system.

Changes in mass and/or stiffness are rarely practical, unless, in the case of stiffness, it is feasible to change the structural system. However, increase in damping, within reasonable bounds, is always beneficial and can be more easily implemented in practice. Moreover, to resist dynamic loads, a structure needs the ability to dissipate energy, and one sure way to achieve this goal is to augment the damping level of the system. A review of the relevant literature indicates that this objective can be met in two ways: (1) By directly increasing the structural damping of the system, for example by making an appropriate choice of structural material or by including energy dissipating elements within the structural system; (2) By means of external devices that have the overall effect of increasing the damping level of the structure, without being inherently energy dissipating mechanisms; vibration absorbers or vibration isolators can be classified as such systems because, as we shall see later, their controlling function is due primarily to inertial forces or isolation effects and not to their energy dissipation capacities.
Tendon systems offer an additional way to mitigate the dynamic response of a structure by acting as stiffeners in addition to providing another energy dissipating "source".

In what follows, some of the methods of increasing the damping capacity of a structure are reviewed and the focus of the rest of the analysis on the tuned mass damper system is explained.

3.2 Viscoelastic Elements

Viscoelastic elements are layers of viscoelastic types of material which, when constrained within the structural elements of a braced steel frame, directly augment the structural damping level of the entire structure. These elements act analogously to shock absorbers and dissipate sway energy in the form of friction. Owing to this function, viscoelastic layers have been used as passive control devices for wind induced vibrations.

The World Trade Center incorporates some 10,000 such elements in its structural system to decrease perceived wind-induced sway. The heart of these particular viscoelastic damping units is a steel-plastic sandwich [Fig 3-1].

Even though viscoelastic elements appear to be quite effective in increasing the damping level of a structure under wind excitation, their energy dissipation capacity seems to be inadequate for strong ground motion excitation [15]. Moreover, the difficult (not to say impossible) calibration of these devices makes them presently less attractive as control systems.
3.3 Vibration Isolators

A standard protection technique in designing a vibration-sensitive system subjected to an oscillatory base motion is to mount the system on a soft spring. The natural frequency of the resulting system falls below the range of frequencies which dominates the base motion and the structure's response is therefore reduced.

The application of this concept, known as base isolation, to earthquake problems leads to a design strategy in which an isolation system is used to decouple a structure from the ground. In general, an isolation system both reduces the dynamic response of the main structure by shifting its natural frequency, and serves as an energy-absorbing
device, which not only attenuates the transmission of energy into the building, but also reduces its displacements relative to the ground [Fig 3-2].

Figure 3-2: Schematic Representation of a Multi-storey Shear Building Equipped with a Base Isolation System

Furthermore, it has been shown [29] that it is possible to construct isolation systems that allow the structure proper [ie the structure above the base-foundation interface] to remain in the elastic range during very strong ground motions.

This concept of base isolation has been studied by many researchers. Wirshing and Yao [46], in their comparative study of passive devices, which when added to the structure can effectively reduce its seismic structural response, found the isolation system to be very effective, compared to a vibration absorber. However, their studies were
based on the response of five- and ten-storey buildings for which wind effects are not dominating in the design process.

Figure 3-3: Possible Foundation Arrangement with Base Isolation

Figure 3-3 is a schematic representation of a possible foundation arrangement of a base isolation system. The rubber pads are mounted on removable wedges to allow for pad elevation adjustment or pad testing. The hysteretic dampers are located in a space conveniently accessible in case there is a need to correct any residual plastic offset following an earthquake. The design of these elements has been thoroughly discussed in the literature.

Vibration isolators are not only a topic for mere research, but are actually implemented in practice. A four-storey reinforced concrete building was constructed in New-Zealand in 1979 on a system of lead-
rubber hysteretic bearings. Another device utilizing laminated rubber pads was employed earlier to support the Koeberg Nuclear Power Plant in South Africa. More recently, a five-storey steel building near the San Andreas fault in California was equipped with a base isolation system.

Although these systems are effective passive control devices for seismic loading, and have been implemented in practice with reasonable success, they will not be the subject of further analysis in this report; their primary application is for short, massive structures, for which wind serviceability requirements are not a major issue.

3.4 Soft First Storey Systems

Another way of isolating the building from ground motion is to design a building with a first storey much less stiff than the rest of the structure. Since the earthquake loading is transmitted to a structure through its basis, and since the distribution of forces directly depends upon the stiffness properties of the structure, if the first storey has a low stiffness, then the forces transmitted to the higher storeys can be appreciably reduced.

This concept of insulating the upper storeys of a building from the earthquake input has attracted the interest of structural designers for several decades. In 1935, N. B. Green proposed that buildings be constructed with a flexible first storey. Fintel and Kahn [14] have reported evidence that buildings with soft first storeys without shear walls suffered less damage than those with shear walls in both the Skopje, Yugoslavia (1963) and Caracas, Venezuela (1964) earthquakes.
However, soft first storey systems require a thorough and sophisticated analysis because of the complicated amplification mechanisms and dynamic earthquake response of a non-uniform multi-storey building [5]. Also, since slender structures have very heavy wind resistance and serviceability requirements, this mechanism will not be considered any further.

3.5 Tendon Systems

Tendon systems consist of an ensemble of cables running along the frames in a way similar to lateral bracings as illustrated in figure 3-4. Tendons have been used successfully to control\textsuperscript{11} vibrations of bridges, but no tendon systems have been implemented in buildings, although they have been the subject for extensive studies [48].

Tendons can be either active or passive. Passive systems respond mechanically to the motion of the building and tend to mitigate it by stiffening the structure [Fig 3-5].

Active systems also bathe the building response by stiffening it. They are not however actuated mechanically, but through a sophisticated control system that senses the deformations of the building and subsequently generates control forces in the tendons. Because the active tendon system forces are generated through a "controllable" system, these forces can be much larger than the ones generated through mechanical interaction alone under the same global conditions.

\textsuperscript{11}The objective of controlling a system is to make it operate in a more desirable way.
Figure 3-4: Schematic Tendon System

Analytical studies show that tendon systems, both active and passive, are, in general, very effective in controlling the response of buildings under seismic loadings, but active systems provide larger, more effective control forces than passive ones. Concerning the use of active systems for seismic control of buildings, many authors raise the question of reliability of the electronic controlling system during an earthquake strike.

Small-scale testing of tendon systems as a means of reducing dynamic frame responses have been undertaken recently at the University of Buffalo in the state of New York. The initial results of these tests appear to be positive, but more extensive testing needs to be carried out before these systems can be implemented in practice.
3.6 Tuned Mass Damper or Vibration Absorber

The tuned mass damper (TMD), also referred to as auxiliary mass damper or dynamic vibration absorber, is a device that consists of a mass attached to a building or a structure in such a way that it vibrates at the same frequency of the structure but with a phase shift. The mass is attached to the structure by a spring-dashpot system and energy is dissipated by the dashpot as relative motion develops between the mass and the structure. Figure 3-6 is a schematic representation of the tuned mass damper system designed by MTS for the Citicop Center in New York City.

The effect of the damper on the response of the structure is to split the original single mode into two [Fig 3-7].
Such devices have been installed successfully in a number of structures to mitigate wind-induced vibrations: the John Hancock Building in Boston, the Citicorp Building in New York, the CN Tower in Toronto and the Centerpoint Tower in Sydney.

Tuned mass dampers can be either active or passive. Passive devices respond directly to the motion of the building in a mechanical manner (law of action and reaction). On the other hand active TMD's respond to sensors placed throughout the structure, which continuously send signals to an electronic system. When a motion is detected, the
Figure 3-7: Effect of Tuned Mass Damper on Response

system activates the TMD by giving it an acceleration such that the motion of the mass counteracts that of the building. These systems are becoming very popular; their main advantage over the passive systems is the fact that the TMD can be designed with a much smaller mass for the same effect; also, the stoke length of the mass\textsuperscript{12} gets appreciably reduced. Analytical results show a great effectiveness of these devices in controlling seismic effects in addition to wind effects. However, since

\textsuperscript{12}Stoke length is the relative displacement of the mass with respect to the floor of the building where it is attached.
active systems require electronic or mechanical instrumentation and maintenance, they are not generally recommended as a means of reducing seismic responses because of the likelihood of electrical failure during an earthquake stroke.

The passive TMD's have little or no dependence on electronic sensors, mechanical or motor failure etc... hence, their effect during an earthquake (if any) would be more reliable.

Because of the implementation of such devices in actual tall buildings, to reduce their wind oscillations, we propose to investigate the seismic behavior of structures equipped with TMD's in the next chapter.
Chapter 4
Effect of the Tuned-Mass-Damper on the Structural Response

4.1 Structural System

The selection and formulation of an appropriate structural model is one of the most important parts of any analysis. If the model is too complicated, the analyst generally loses perspective on the problem and the interpretation of the results becomes a burdensome task; in addition the cost of the analysis increases exponentially with the degree of complexity. If on the other hand the model is too simple, one can miss important features of the behavior of the real structure and can be lead to grossly erroneous results and interpretations.

The structural model selected for this study consists of a linear elastic shear building with a linear elastic tuned mass damper at its top level. The idealization of the structure as a shear building has the advantage of a simple formulation, and has proven to be applicable to a wide range of building sizes [3].

As mentioned previously, it does not seem that the origin of the conflicting findings reported in the literature concerning the seismic effectiveness of TMD's lies primarily in the choice of the structural model. However, for the sake of completeness, a more general model of the system (structure and TMD's) has been developed in this research. The important features of this model are as follows:
1. It can allow for the inclusion of more than one TMD in the structure (but no more than one per floor).

2. The position of the TMD's is variable and can be optimized by performing an appropriate analysis.

3. Direct solution of the equations of motion can be performed using direct integration schemes, and thus no prior assumptions on the mode shapes need to be made and no consequent error is introduced.

A more detailed description of this model together with the explicit formulation of the equations of motion can be found in Appendix A.

Although the above-mentioned model has been formulated and implemented in a computer program and was found to give satisfactory results for simple cases, it has not been used in further analysis for the following reasons:

1. The solution procedure is iterative and sensitive to the values of input variables making a parametric study of the problem very complicated and computationally involved.

2. The model is based on a time domain formulation, and in the rest of the analysis the excitation and the probabilistic responses of interest are expressed in the frequency domain.

3. The second objective of this work is to investigate a new approach for defining the ground motion excitation. To this end, a simple model is more appropriate.

Mainly for this last reason, the multimodality of the response will not be addressed, and, in compliance with other works, the response of the building will be assumed to be mainly contributed by its fundamental mode. This assumption will allow us to study the building without a damper and the building with a damper as, respectively, a single degree-of-freedom [Fig4-1] and a two-degrees-of-freedom system [Fig4-2].
Figure 4-1: Equivalent Single-Degree-of-Freedom System

\[ M_1 = \Phi_1^T M \Phi_1 \]

Figure 4-2: Equivalent Two-Degrees-of-Freedom System

\[ M_1 = \Phi_1^T M \Phi_1 \]
4.1.1 Equilibrium Equations

The equations of motion of the building-TMD system, illustrated in Figure 4-3 has been derived in Appendix B. The main assumptions made while deriving these equations are linear behavior and viscous damping. Decoupling of the equations is possible thanks to these assumptions which imply the existing of normal modes.

Figure 4-3: Shear Building-TMD System
Since we are only considering the contribution of the fundamental mode to the response, the system of equations to be solved reduces to:

\begin{align*}
\ddot{y}_1 + 2\xi_1 \omega_1 \dot{y}_1 + \omega_1^2 y_1 - \mu_1 \phi_1 (2\xi_2 \omega_2 \dot{x} + \omega_2^2 x) &= -P \ddot{u}_n \quad (4.1) \\
\ddot{x} + 2\xi_2 \omega_2 \dot{x} + \omega_2^2 x &= -\ddot{u}_g - \ddot{u}_n \quad (4.2)
\end{align*}

where

- $y_1$ Modal coordinate, equal in our case to the top displacement $u_n$.
- $x$ Relative displacement of the TMD with respect to the $n^{th}$ floor.
- $\omega_1$ Natural frequency of the fundamental mode.
- $\omega_2$ Natural frequency of TMD.
- $\xi_1$ Damping ratio of first mode.
- $\xi_2$ Damping ratio of TMD.
- $P_i$ Participation factor.
- $\mu_1$ Modal mass ratio.
- $\ddot{u}_g$ Ground acceleration.

The analysis of the effectiveness of the TMD will be based on comparisons of the magnitudes of the top displacement of the building with and without the absorber.

The top displacement $u_n$ of the structure can be written:

$$u_n = \phi_1^T \cdot y$$

Where:

- $\phi_1$ Vector of the fundamental mode shape.
Vector of modal coordinates.

Since we are neglecting higher modes participation, $u_n$ can be approximated by:

$$u_n = \phi_{n1} y_1$$

(4.4)

Where:

$\phi_{n1}$  
the $n^{th}$ entry in the first eigenvector.

$y_1$  
the first modal coordinate.

The modal matrix can be normalized in any convenient way. We shall normalize it such that $\phi_{n1} = 1$. This way, the top displacement is equal to the modal variable $y_1$.

Mainly because of the uncertainty attached to the loading and subsequently to the response, we shall represent our response of interest [ie the top displacement of the structure] as a random variable. The response will be characterized by the maximum expected values (which is the most meaningful quantity to the designer), and also in terms of root-mean-square displacements. In this stochastic formulation, the ground acceleration is represented by a Power Spectral Density, and the transfer functions corresponding to the building without TMD [ie 1DOF system] and the building with TMD [ie 2DOF system] are respectively, as derived in Appendix B:

$$H_{z1}(\omega) = \frac{-1}{\omega_1^2 \omega^2 + 2\xi_1 \omega_1 \omega}$$

(4.5)
\[ H_{y_1}(\omega) = \frac{N(\omega)}{D(\omega)} \]

\[ N(\omega) = P_2 \omega^2 - 12 \omega \xi_2 \omega_2 (P_1 + \mu_1) - \omega_2^2 (P_1 + \mu_1) \]

\[ D(\omega) = \omega^4 - 12 \omega^3 (\omega_1 \xi_1 + \omega_2 \xi_2 + \mu_1 \omega_1 \xi_1) \]

\[ + \omega^2 (\omega_1^2 + \omega_2^2 + 4 \xi_1 \xi_2 + \mu_1 \omega_1 \omega_2) \]

\[ + 12 \omega (\xi_1 \omega_1^2 + \xi_2 \omega_2^2) \]

\[ + \omega_1^2 \omega_2^2 \]

and the response power spectra are given by:

\[ S_{y_1} = \left| H_{y_1} \right|^2 S_{u_g} \]  \hspace{1cm} (4.7)

\[ S_{z_1} = \left| H_{z_1} \right|^2 S_{u_g} \]  \hspace{1cm} (4.8)

where \( S_{u_g} \) is the power spectra of the ground acceleration.
4.2 Response Spectra

The Response Spectra corresponds to the maximum observable response (either displacement or force or any other quantity of interest) of a 1DOF oscillator with a prescribed natural frequency and damping ratio, subjected to the motion considered.

When the analysis focuses on the response to a stochastic process $x(t)$, then the maximum response observed on a 1DOF oscillator is a random variable and will change for each realization of the process. In such cases, a Response Spectra can be obtained by computing the expected value of the maximum response, $X$. If the stochastic process is assumed to be stationary (which simplifies the probabilistic manipulations) then the maximum response is a monotonic function growing with time; to limit this maximum, the duration of the motion has to be fixed to some realistic value $\tau$, which can be thought of as the duration of the earthquake.

Vanmarcke [42] derived a probability distribution for the time to first-exceedance of a given level "b" by the absolute value of the response $|x(t)|$, which is applied when the stochastic process is assumed to have a Normal (Gaussian) distribution in space for a given time.

This distribution of the first-exceedance time takes into account the period the response is above the limit "b" and assumes an exponential distribution of the time between a down-crossing of "b" by the envelop of the process $x(t)$ and the next upcrossing.

Kiureghian [27] derived some approximate formulae to compute the maximum expected response for a duration $\tau$, $X$, according to Vanmarcke's distribution:
\[ \bar{X}_T = (\sqrt{2} \ln(\frac{\nu e}{\nu T}) + \frac{0.5772}{\sqrt{2} \ln(\frac{\nu e}{\nu T})}) \sigma_x \]  
\[ (4.9) \]

where

\[ \nu_e = \begin{cases} 
(1.63 \delta 0.45 - 0.38) \nu & \delta < 0.69 \\
\nu & \delta \geq 0.69 
\end{cases} \]  
\[ (4.10) \]

and

\[ \delta = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0^2}} \]  
\[ (4.11) \]

\( \lambda_1, \sigma_x \) and \( \nu \) are given by

\[ \nu = \frac{\sigma_x}{\pi \sigma_x} = \frac{1}{\pi} \sqrt{\frac{\lambda_2}{\lambda_0}} \]  
\[ (4.12) \]

\[ \sigma_x = \sqrt{\lambda_0} \quad \text{and} \quad \lambda_i = \int_0^\infty \omega^i \ S_X(\omega) \ d\omega \]  
\[ (4.13) \]

\( S_x \) is a one-sided spectral density.

### 4.3 Ground Motion Representation

The Power Spectral Density function of the ground acceleration chosen for the analysis is the one proposed by Tajimi and Kanai and given by:
\[ S_{u_g u_g} (\omega) = \frac{1 + 4 \left( \frac{\zeta_g}{\omega_g} \right)^2 \left[ \frac{\omega}{\omega_g} \right]^2}{\left[ 1 - \left( \frac{\omega}{\omega_g} \right)^2 \right]^2 + 4 \left( \frac{\zeta_g}{\omega_g} \right)^2 \left[ \frac{\omega}{\omega_g} \right]^2} S_o \]  

(4.14)

The parameters \( \omega_g \) and \( \zeta_g \) appearing in this equation can be thought of as some characteristic ground frequency and characteristic damping ratio respectively, and the parameter \( S_o \) as a measure of the amplitude of the function. Figure 4-4 is a normalized representation of this function indicating the influence of these three parameters.

With very low frequencies however, some problems may arise with this representation of the ground motion. In fact, since the displacement power spectrum is obtained from the acceleration power spectrum by dividing its values by \( \omega^4 \), the resulting values for zero frequency become unbounded. To avoid this problem, we introduced a local variation for very low frequencies compatible with realistic motions. An example of power spectrum is shown in Figure 4-5 and is given by the following equation:

\[ S_{u_g} = \begin{cases} 
\frac{f^4}{0.5 + f^4} \left( 1 - f^2/64 \right) S_o & f \leq 8 \text{ Hz} \\
0 & f > 3 \text{ Hz}
\end{cases} \]  

(4.15)

Where \( f \) represents the frequency in Hz.
The Tajimi-Kanai spectrum has been corrected for frequencies lower than .25 Hz by introducing a fourth order polynomial variation with the frequency. The resulting spectrum is shown in Figure 4-6

4.4 Parametric study

In the present formulation, the parameters are: the modal mass ratio, the participation factor of the fundamental mode, the TMD natural frequency, the modal damping and the damping ratio of the TMD. These parameters will be varied within some reasonable ranges, and their influence on the system response will be investigated. The various comparisons will be based on response spectra values, or on root-mean-square displacement ratios of the system without TMD to the system with TMD.

The range of frequencies covered is from 0.05 Hz to 10 HZ, to include flexible structures as well. Since all response spectra have been obtained by the same basic shape of the ground acceleration Power Spectral Density, the curves obtained have the same characteristics. We observe a first peak from the left of the graph, at 0.25 Hz due to the change in spectrum [see section 4.3], then another peak for the frequency corresponding to $\omega_g$.

In what follows, we will investigate the effect of the previously mentioned parameters on the top displacement response ratios.
Figure 4-4: Normalized Tajimi-Kanai Power Spectrum for $\omega_g=28.393 \text{rd/sec}$ and $\xi_g=31.7\%$
4.4.1 Mass Ratio

The mass ratio that appears in the equation is not the actual ratio of the mass of the TMD to that of the structure, but the modal mass ratio which is defined as:

$$\mu_1 = \frac{m_a}{\frac{\sum_j m_j \phi_j^2}{n}}$$  \hspace{1cm} (4.16)

An approximate value of this quantity is obtained by assuming a linear variation of the mode shape with the height as follows:

$$\mu_1 = \frac{m_a}{\frac{\sum_j m_j \phi_j^2}{n}} = \rho \frac{6n^2}{(n+1)(2n+1)}$$  \hspace{1cm} (4.17)

where $\rho$ is the actual mass ratio (ratio of the mass of the damper to the total mass of the building).
Figure 4-6: Spectral Density for Response Spectra

We did not observe a large variation in the maximal response throughout the range of frequencies considered [ie .05 hz to 10 hz] varying μ from 0.5% to 10%. The effectiveness of the TMD increases with the mass ratio, but the difference, at least within reasonable ranges, is not very pronounced [see Fig 4-7 to 4-10].

However, below the previously mentioned range, as μ gets smaller, the effectiveness of the TMD clearly decreases as can be seen from Figure 4-11. On the analysis of the other parameters, we adopted 2% as a reasonable value for μ except when the combined effect of the mass ratio together with some other parameter is being investigated.
Figure 4-7: Effect of the Mass Ratio on the Response Spectra: $\mu=10\%$
Figure 4-8: Effect of the Mass Ratio on the Response Spectra: μ=2%
Figure 4-9: Effect of the Mass Ratio on the Response Spectra: μ=1%
Figure 4-10: Effect of the Mass Ratio on the Response Spectra: $\mu=0.5\%$
Figure 4-11: Effect of the Mass Ratio on the Response Spectra: $\mu = 0.1\%$
4.4.2 Modal Damping

The initial damping of the structure is certainly an important factor to consider if the incorporating a TMD is thought of as a means to increase the overall damping of the system.

Since the values for the first modal damping coefficient $\xi_1$ in concrete buildings and steel buildings are taken in practice, to be respectively equal to 5% and 2%, we decided to restrict this analysis to these two values of the modal damping coefficient.

Interestingly enough, we notice from Figures 4-14, 4-15, and 4-16 that the TMD has little effect on the expected maximum top displacement of a concrete structure. One can notice though that the response decreases as the weight of the TMD increases [Fig 4-16]. However, since these values are given in percentage of the total mass, and since concrete structures are heavier than steel ones, the masses that would result in a decrease of the top displacement response would be most probably of an impractical magnitude. Table 4-I indicates that for a practical range of frequencies the RMS displacement is reduced by approximately 15%. For practical purposes, this attenuation is very little and the TMD should not be used in general for this type of structures. Moreover, this conclusion did not take into account relative displacements which could become unacceptable due to a possibly high contribution of the higher modes of vibration of the structure.

For steel buildings on the other hand, there is a noticeable decrease in both the maximum responses [Fig 4-8] and average responses [See Fig 4-13 and Table 4-II] which for the range of frequencies varying from .10 Hz to 4 Hz, is close to 35%. 
R.M.S Displacement Ratios for:

- Participation Factor = 1.00
- Mass Ratio = 0.02
- Modal Damping = 0.05
- TMD Damping = 0.05

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**Table 4-I:** Effect of the Structure's Damping

$\zeta_1 = 5\%$
R.M.S Displacement Ratios for:

- Participation Factor = 1.00
- Mass Ratio = 0.02
- Modal Damping = 0.02
- TMD Damping = 0.05

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Table 4-II: Effect of the Structure's Damping
\(\xi_1=2\%\)
Figure 4-12: Effect of the Modal Damping on the RMS Displacement Response $\xi_1=5\%$
Figure 4-13: Effect of the Modal Damping on the RMS Displacement Response $\xi_1 = 2\%$
A look at the transfer functions of the 1DOF and 2DOF systems for both \( \xi_1 \) equal 2\% [Fig 4-18] and 5\% [Fig 4-19], more specifically a quantification of the relative changes in the areas under these curves for each case justifies these differences in behavior.

### 4.4.3 Frequency Ratio

The optimal tuning of the TMD is when the ratio of the frequency of the absorber to that of the building is close to 1 (more specifically .98). However this would assume perfect tuning and perfect elastic behavior of the building. Since the world is not perfect, we decided to investigate the importance of this parameter by varying it in the neighborhood of 1. The results are illustrated on Figures 4-20 to 4-24. We observe that the effectiveness decreases as the frequency ratio moves away from the optimal value. However, the response is more sensitive to an increase than to a decrease in the frequency ratio; the lack of symmetry of the transfer function is responsible for this behavior.

### 4.4.4 Damping of the Damper

Most investigators argue that the TMD’s damping only affects the motion of the absorber and that its influence on the building’s response is negligible. However, Figures 4-25 to 4-28 show that the effectiveness of the absorber decreases as the TMD damping increases.

### 4.4.5 Participation Factor

The participation factor for the fundamental mode is given by:
Figure 4-14: Effect of the Structure's Damping: $\xi_1 = 5\%$
for $\mu = 1\%$
Figure 4-15: Effect of the Structure's Damping: $\xi_1 = 5\%$
for $\mu = 2\%$
Figure 4-16: Effect of the Structure's Damping: $\xi_1=5\%$
for $\mu=10\%$
Figure 4-17: Effect of the Structure's Damping: $\xi_1 = 5\%$
for $\mu = 0.5\%$
Figure 4-18: Transfer Functions for $\xi_1=2\%$
TRANSFER FUNCT. for the 1DOF and 2DOF
LINEAR SYSTEMS with Modal Damping = .05

Figure 4-19: Transfer Function for $\xi_1=5\%$
Figure 4-20: Effect of the Frequency Ratio
\[ \frac{\omega_2}{\omega_1} = 0.98 \]
Figure 4-21: Effect of the Frequency Ratio
\( \omega_2/\omega_1 = 1.01 \)
Figure 4-22: Effect of the Frequency Ratio
\[ \frac{\omega_2}{\omega_1} = 1.1 \]
Figure 4-23: Effect of the Frequency Ratio

\( \omega_2/\omega_1 = 0.95 \)
Figure 4-24: Effect of the Frequency Ratio

\[ \omega_2 / \omega_1 = 0.85 \]
Figure 4-25: Effect of TMD Damper: $\xi_2 = 10\%$
Figure 4-26: Effect of TMD Damper: $\xi_2 = 40\%$
Figure 4-27: Effect of TMD Damper: $\xi_2=60\%$
Figure 4-28: Effect of TMD Damper: $\xi_2=80\%$
Approximating the modal shape by a linear curve and assuming uniformity of masses and inter-storey heights, one gets:

\[
P_1 = \frac{\sum_{j=1}^{n} m_j \phi_j^1}{\sum_{j=1}^{n} m_j \phi_j^2} = \frac{3n}{2n+1} \quad (4.19)
\]

One can show that for regular shear buildings, the fundamental mode participation factor lies between 1 and 1.5. In their analysis, Kaynia et al. [24] assumed that this factor did not affect the response and set it equal to 1. On the contrary, our results show a notable influence of this parameter on the output maximum expected value: the reduction of response is lightly noticeable even after parameters like the mass ratio or the modal damping promote an appreciable decrease of the response; for some frequency ranges, this value even increases. Figures 4-29 to 4-32 illustrate these effects. The decrease of the average response due to the addition of the TMD is so light [see Table 4-III] that it is meaningless.

Hence, it seems that the influence of this parameter is so important that it cancels out or at least heavily attenuates any beneficial effect of the other parameters. One should note however that since the summation
Figure 4-29: Effect of the Participation Factor
P=1.2 μ=10%
Figure 4-30: Effect of the Participation Factor

P=1.5 μ=2%
Figure 4-31: Effect of the Participation Factor
P=1.5 μ=5%
Figure 4-32: Effect of the Participation Factor
P=1.5 μ=10%
R.M.S Displacement Ratios for:

- Participation Factor = 1.50
- Mass Ratio = 0.02
- Modal Damping = 0.02
- TMD Damping = 0.05

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Period [sec]</th>
<th>r.m.s ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>20.00</td>
<td>1.08</td>
</tr>
<tr>
<td>0.06</td>
<td>16.76</td>
<td>1.06</td>
</tr>
<tr>
<td>0.07</td>
<td>14.05</td>
<td>1.04</td>
</tr>
<tr>
<td>0.08</td>
<td>11.77</td>
<td>1.02</td>
</tr>
<tr>
<td>0.10</td>
<td>9.87</td>
<td>1.00</td>
</tr>
<tr>
<td>0.12</td>
<td>8.27</td>
<td>0.99</td>
</tr>
<tr>
<td>0.14</td>
<td>6.93</td>
<td>0.98</td>
</tr>
<tr>
<td>0.17</td>
<td>5.81</td>
<td>0.97</td>
</tr>
<tr>
<td>0.21</td>
<td>4.87</td>
<td>0.95</td>
</tr>
<tr>
<td>0.25</td>
<td>4.08</td>
<td>0.90</td>
</tr>
<tr>
<td>0.29</td>
<td>3.42</td>
<td>0.96</td>
</tr>
<tr>
<td>0.35</td>
<td>2.87</td>
<td>0.97</td>
</tr>
<tr>
<td>0.42</td>
<td>2.40</td>
<td>0.97</td>
</tr>
<tr>
<td>0.50</td>
<td>2.01</td>
<td>0.97</td>
</tr>
<tr>
<td>0.59</td>
<td>1.69</td>
<td>0.97</td>
</tr>
<tr>
<td>0.71</td>
<td>1.41</td>
<td>0.97</td>
</tr>
<tr>
<td>0.84</td>
<td>1.19</td>
<td>0.98</td>
</tr>
<tr>
<td>1.01</td>
<td>0.99</td>
<td>0.98</td>
</tr>
<tr>
<td>1.20</td>
<td>0.83</td>
<td>0.98</td>
</tr>
<tr>
<td>1.43</td>
<td>0.70</td>
<td>0.98</td>
</tr>
<tr>
<td>1.71</td>
<td>0.58</td>
<td>0.97</td>
</tr>
<tr>
<td>2.04</td>
<td>0.49</td>
<td>0.97</td>
</tr>
<tr>
<td>2.43</td>
<td>0.41</td>
<td>0.97</td>
</tr>
<tr>
<td>2.90</td>
<td>0.34</td>
<td>0.96</td>
</tr>
<tr>
<td>3.47</td>
<td>0.29</td>
<td>0.95</td>
</tr>
<tr>
<td>4.14</td>
<td>0.24</td>
<td>0.94</td>
</tr>
<tr>
<td>4.93</td>
<td>0.20</td>
<td>0.98</td>
</tr>
<tr>
<td>5.89</td>
<td>0.17</td>
<td>1.06</td>
</tr>
<tr>
<td>7.02</td>
<td>0.14</td>
<td>1.10</td>
</tr>
<tr>
<td>8.38</td>
<td>0.12</td>
<td>1.13</td>
</tr>
<tr>
<td>10.00</td>
<td>0.10</td>
<td>1.31</td>
</tr>
</tbody>
</table>

Table 4-III: Effect of the Participation Factor on the RMS Displacement Ratio
of all participation factors should equal to 1, when the fundamental mode participation factor is 1.5 for example, this implies that there is a heavy participation of higher modes, and thus other effects that have not been taken into account now would need to be included in the analysis.

4.4.6 Input Spectrum

The results presented up to now were obtained through the use of a Tajimi-Kanai Power Spectral Density function with parameters $\xi_1=0.317$ and $\omega_1=28.394\text{rd/sec}$. However, the use of the same type of Spectral Density with different parameters showed the same general tendencies as illustrated by Figures 4-33 and 4-34. Also, the use of a white noise spectrum for frequencies higher than .25 Hz show the same overall effects [Fig 4-35].
Figure 4-33: Effect of the Input Spectrum

$\omega_g = 6.375 \text{ rd/sec}$ and $\xi_g = 17.6\%$
Figure 4-34: Effect of the Input Spectrum

$\omega_g = 11.183 \text{ rd/sec and } \xi_g = 24.5\%$
Figure 4-35: Effect of the Input Spectrum
White Noise for .25Hz < f < 10Hz
Chapter 5

Treatment of the Earthquake Excitation

5.1 Introduction

The definition of an appropriate ground motion time history is the most difficult and uncertain phase when predicting structural responses to earthquakes. Generally, earthquake ground motions are expressed in terms of three components of translational accelerations: two horizontal and one vertical. In the hypothesis of linear systems, the standard analytical problem is reduced to the evaluation of the structural response to a single component of support translation since the response of any linear system to these three components of input can be computed by superposing the responses calculated separately for each of these components. Nowadays, a deterministic analysis of the response of a building to a specified earthquake motion can be carried out quite accurately, but there still remains the question as to how to interpret the obtained results: what will happen when an earthquake will hit the building, knowing that there will be, in the best possible case, very little correlation or similarities between the actual earthquake and the one used during the design phase?

Seismic design requirements therefore can often be described more effectively in probabilistic terms than by a single accelerogram and nondeterministic earthquake-response analysis procedures are required.

Furthermore, despite the uncertainty concerning the mechanisms
that produce tectonic motions, it is widely accepted that seismic waves are initiated by irregular slippage along faults. This, together with the fact that these waves are followed by numerous random reflections, and attenuations within the complex ground formations through which they pass, render the stochastic modeling of strong motions appropriate.

5.2 Principal Component Analysis

In any natural system, there exists modes of organization which account for persistent spacial and temporal patterns. The predictability of the associated phenomena results because they are governed by physical laws that presumably do not change with time. These laws, even if unknown to us do exist and are contained implicitly in the observation of these phenomena. Generally, this information is collected and gathered in the form of measurements.

To gain more understanding of the phenomenon and be able to predict its future manifestations one has only these measurements to interpret. Because of the complexity of natural phenomena, the use of multivariate statistics seems to be adequate for this purpose. A technique that allows for a consistent processing of the data by taking away redundancies from the measurements and capturing the basic independent patterns of behavior in the phenomenon would be most appropriate. More specifically, the technique should provide new functions that more efficiently represent the organization of the sample. Such functions would then allow for a reliable prediction of the phenomenon.
This task is rendered more difficult by the fact that the observations (generally gathered as measurements) cannot be regarded as completely accurate. Moreover, most natural systems interact with some other processes and this creates deviations from the basic behavioral patterns.

Such a technique would find wide acceptance and applicability in areas where large bodies of measurements can be available such as meteorology, acoustics, coastal engineering etc...

In 1956, in the field of meteorology, Dr. Lorenz seemed to have found in "Principal Component Analysis" such a technique. This method, seeks the Standard Linear Combination of the original variables which has maximal variance. More generally, P.C.A looks for a few linear combinations which can be used to summarize the data, losing in the process as little information as possible.

Principal component analysis is a multivariate statistical technique developed by Hotelling(1933) after its origin by Karl Pearson(1901). It is basically an orthogonal transformation which transforms any set of variables into a set of new variables that are uncorrelated with each other.

More recently, in 1977, the same basic technique had been successfully used for the treatment and processing of wave spectra characteristics. In coastal engineering, one has to deal with a tremendous amount of wave spectra. The general problem of treating spectral characteristics is one of isolating a small number of non-redundant parameters which describe the spectrum to some prescribed tolerance.

A variant of Principal Component Analysis (P.C.A) termed
Eigenvector Analysis has been used by US Army engineers in 1977 [45] to characterize wave spectra. The technique is essentially the same, but is applied to spectral quantities instead of raw data. The method separates out the time varying characteristics of the process into orthogonal functions. These functions are generally termed Empirical Orthogonal Functions. In coastal engineering they are called Characteristic Spectral Functions (CSF) or Eingenfuctions.

The CSF are empirically derived; they are a natural basis for representing spectral data in the sense that they are optimally arranged to maximize the variance explained. Furthermore, the shapes of the major functions result from physical rather than circumstancial reasons.

The mechanics of this technique is illustrated in Figures 5-1 and 5-2. The CSF are the eigenvectors of the covariance matrix, and their associated eigenfunctions represent the amount of variance of the sample captured by the vector. Each of the original sample functions can be expressed as a linear combination of these vectors. Moreover, since the CSF are arranged such that their associated variances vary monotonically in a decreasing order, the original sample data can be reconstructed using fewer eigenvectors unless the elements of the original sample were already totally uncorrelated.

This technique has been used in practice in a few sites in Canada [45]. An effective condensation of the data had been achieved: ten of the CSF reproduced more than 90% of the variation in climatological records of about three thousands wave spectra. Moreover, some physical interpretation could be attached to the most prominent components. Hence, beyond the capability to represent the spectra, these
Method Description

1. Sample Set

\[ \omega_m = \frac{i}{m} \sum_{j} \omega_j(t) \]

2. Mean

\[ M = \frac{1}{m} \sum_{j} \omega_j(t) \]

3. Deviate Matrix

\[ F = X - M \]

Figure 5-1: Eigenfunction Parametrization

Page 1
4. Covariance Matrix $R$

$$R = \frac{1}{m} F^T F$$

5. $R = \Phi^T \lambda \Phi$

6. Reconstruction of the original set

$$G_{i,j} = M_j + \sum_{k=1}^{n} \omega_{ij} \Phi_{jk}$$

$$G_{i,j} \approx M_j + \sum_{k=1}^{p} \omega_{ij} \Phi_{jk} \quad \text{with} \quad p < n$$

Figure 5-2: Eigenfunction Parametrization
eigenfunctions provide a unique means of investigating the significance of multimodality at the given site.

In light of the success that this method gained in these fields, we propose to investigate its applicability in the field of earthquake engineering, as a means for identifying and constructing some meaningful design spectra for a given site.

5.3 Principal Components and Earthquake Spectra

An important shortcoming one encounters when dealing with strong motion records is their scarcity. To test the applicability of Principal Component Analysis to this field, we chose to work with 45 smoothed power spectra of real earthquakes. These non-constant spectral densities which are frequently used in modeling ground acceleration have been proposed by Tajimi and Kanai and have the general form:

\[
S_{u_g} (\omega) = \frac{S_0 \left[1 + 4 \left(\frac{\zeta_g}{\omega_g}\right)^2 \left(\frac{\omega}{\omega_g}\right)^2 \right]}{1 - \left(\frac{\omega}{\omega_g}\right)^2 + 4 \left(\frac{\zeta_g}{\omega_g}\right)^2 \left(\frac{\omega}{\omega_g}\right)^2}
\]

with parameters \(\omega_g, \zeta_g\) and \(S_0\).

We carried out some numerical experimentations by varying the parameters of spectral density. Table 5-I summarizes our results.

For the selected 45 spectra, a complete analysis has been done. With the ten first eigenvectors, 99% of the variability in the sample was recovered.
<table>
<thead>
<tr>
<th>Varying Parameters</th>
<th>Variance (% of total)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Component</td>
</tr>
<tr>
<td>$v$</td>
<td>10.0</td>
</tr>
<tr>
<td>$i$</td>
<td>90.0</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>35.1</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>80.0</td>
</tr>
</tbody>
</table>

Table 5-I: Numerical Experimentation
Figure 5-3 is an envelop of the weights associated with each component (eigenvector) in the reconstruction of the original spectra. It clearly shows that as the degree of the component increases [i.e., as the amount of variance explained by the vector decreases], its weight and thus importance in the sample decreases.

![Graph showing envelop of the weights with the degree of eigenvectors.](image)

**Figure 5-3**: Envelop of the Variation of the Weights with the Degree of the Eigenvectors and their associated Eigenvalues

Figure 5-4 shows the fraction of variance explained as a function of the number of Empirical Orthogonal Functions (or eigenvectors) considered.

Figures 5-5 to 5-8 show for the 45 Tajimi-Kanai type spectra used, the mean spectrum obtained as well as the first, second and third eigenvector.

This decomposition of a sample into orthogonal components that reduce the dimensionality of the problem is analogous to the normal
modes decomposition of the dynamic response of a structure and the assumption that only the few first modes contribute to the response.
Figure 5-6: First Empirical Orthogonal Function

Figure 5-7: Empirical orthogonal Function # 2
Figure 5-8: Empirical orthogonal Function # 3
Chapter 6

Summary and Conclusions

The design of tall buildings for lateral loads is governed by wind effects. Because of the technological advances in the fields of construction materials and construction techniques, modern tall buildings have very low damping capacities. Often designers remedy to this shortcoming by incorporating in the building auxiliary damping devices that increase the damping of the structure and hence decrease its dynamic responses.

One of these devices, the tuned mass damper, has proven very effective in reducing wind induced vibrations. Because of the increasing number of tall buildings in seismic areas, we proposed to investigate the behavior of a structure equipped with a TMD under a seismic load. The performance index chosen was the ratio of maximum expected values of the top displacement of the building with and without the damper. Since only the fundamental mode was considered, the models for these cases were 1DOF and 2DOF systems. The various parameters that affect the design have been studied. The mass ratio was found to have little effects when varied within the acceptable range. The damping of the structure on the contrary can be very important. Since concrete structures have relatively high damping ratios, the tuned mass damper does not seem to be an appropriate motion reducing device. The effectiveness of the damper did not seem to be highly dependent on the accuracy of the tuning, and a reasonable degree of fluctuation around the optimal value is possible, without having important consequences for the effectiveness of the TMD.
The participation factor measuring the participation of the mode considered on the total response, seems however to have more importance than previously suggested by other researchers. The effectiveness of the damper seems to decrease as the participation factor increases, since for these cases the fundamental mode contributes more to the response and in this mode less energy can be dissipated than in higher modes.

Another important issue we dealt with in this study is the definition of a meaningful ground acceleration or spectral density function for the design. Principal Component Analysis is a statistical method that extracts from a ensemble of data, the most important features in them (the ones that represent most of the variability of the group). We applied this method to an ensemble of ground Power Spectra and found the results we obtained quite satisfactory: a fair reduction of the dimensionality of the sample has been achieved, with very little loss on information on the variability. We would recommend the application of the method on real Power Spectra, in contrast with the smooth analytical ones used during this work. We believe, that interesting findings might result from such an investigation, especially if an extensive collection of ground motion records is available.
Appendix A

Derivation of the Equilibrium Equations for the System

A.1 2-Storey Building

Let us first consider the simple 2-storey building with 2 tuned mass dampers shown in Fig A-1.

Figure A-1: 2-Storey Building With 2 TMD's

We assume that the mass dampers move with the structure when the base is accelerated, i.e. there is an inertia force due to the ground acceleration for each damper. The equations of motion will be expressed in terms of the displacements of the dampers relatively to the ground and
the displacements of the dampers relatively to the floor on which they are attached.

Fig A-2 shows the notation we are using on an equivalent, more illustrative model. This notation is as follows:

Figure A-2: Equivalent Mechanical System for a 2-Storey Building With 2 TMD's

- $m_i$: mass of floor $i$.
- $k_i$: stiffness of floor $i$.
- $c_i$: damping coefficient of floor $i$.
- $u_i$: horizontal displacement of floor $i$ measured with respect to the base of the building.
- $f_i$: additional (non-seismical) horizontal force on floor $i$.
- $a_i$: mass of damper attached on floor $i$. 
s_i  stiffness of damper attached on floor i.
d_i  damping coefficient of damper attached on floor i.
x_i  horizontal displacement of damper attached on floor i measured with respect to this floor.
h_i  additional (non-seismical) horizontal force on damper attached on floor i.

The equations of motion of the system are the following:

\[ m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 - c_2 (\ddot{x}_2 - \dot{x}_1) - k_2 (x_2 - x_1) - d_1 x_1 - s_1 x_1 = f_1(t) \]  
\[ m_2 \ddot{x}_2 + c_2 (\ddot{x}_2 - \dot{x}_1) + k_2 (x_2 - x_1) - d_2 x_2 - s_2 x_2 = f_2(t) \]

\[ a_1 (\ddot{x}_1 + \dot{x}_1 + x_1) + d_1 \dot{x}_1 + s_1 x_1 = h_1(t) \]
\[ a_2 (\ddot{x}_2 + \dot{x}_2 + x_2) + d_2 \dot{x}_2 + s_2 x_2 = h_2(t) \]

These equations can be written in matrix form as follows:
\[
\begin{bmatrix}
  k_1 + k_2 & -k_2 & -s_1 & 0 \\
  -k_2 & k_2 & 0 & -s_2 \\
  0 & 0 & s_1 & 0 \\
  0 & 0 & 0 & s_2 \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
  x_1 \\
  x_2 \\
\end{bmatrix}
= 
\begin{bmatrix}
  f_1 \\
  f_2 \\
  h_1 \\
  h_2 \\
\end{bmatrix}
\]  

(a.5)

Our goal is to have all terms involving the properties of the structure on one side and all other terms related to the forcing functions, the ground acceleration and the tuned mass dampers on the other side.

Using partitioning of the matrices in the above equation, we obtain:

\[
\begin{bmatrix}
  m_1 & 0 \\
  0 & m_2 \\
\end{bmatrix}
\begin{bmatrix}
  \ddot{u}_1 + \ddot{u}_g \\
  \ddot{u}_2 + \ddot{u}_g \\
\end{bmatrix}
+ 
\begin{bmatrix}
  c_1 + c_2 & -c_2 \\
  -c_2 & c_2 \\
\end{bmatrix}
\begin{bmatrix}
  u_1 \\
  u_2 \\
\end{bmatrix}
+ 
\begin{bmatrix}
  -d_1 & 0 \\
  0 & -d_2 \\
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
\end{bmatrix}
\]  

(a.6)

and

\[
\begin{bmatrix}
  a_1 & 0 \\
  0 & a_2 \\
\end{bmatrix}
\begin{bmatrix}
  \ddot{x}_1 \\
  \ddot{x}_2 \\
\end{bmatrix}
+ 
\begin{bmatrix}
  a_1 & 0 \\
  0 & a_2 \\
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
\end{bmatrix}
\]  

(a.7)
This system of equations can also be written as:

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
c_1 + c_2 & -c_2 \\
-c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2
\end{bmatrix}
= \begin{bmatrix}
f_1 \\
f_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 \\
\ddot{u}_2
\end{bmatrix}
+ \begin{bmatrix}
d_1 & 0 \\
0 & d_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
s_1 & 0 \\
0 & s_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
h_1 \\
h_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
d_1 & 0 \\
0 & d_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix}
+ \begin{bmatrix}
s_1 & 0 \\
0 & s_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
h_1 \\
h_2
\end{bmatrix}
\]

\[
\begin{bmatrix}
a_1 & 0 \\
0 & a_2
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_1 + \ddot{u}_2 \\
\ddot{u}_2 + \ddot{u}_1
\end{bmatrix}
- \begin{bmatrix}
a_1 & 0 \\
0 & a_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
= \begin{bmatrix}
h_1 \\
h_2
\end{bmatrix}
\]

This 2*2 system can be solved with an iterative procedure: We assume an initial \( u \), solve (a.9) for \( x \), substitute into (a.8), solve for \( u \), etc.
A.2 n-Storey Building

Let us now consider the general case of an n-storey building with n tuned mass dampers, one at each floor, shown in Fig A-3. An equivalent model is shown in Fig A-4.

Equation (a.10) can be easily generalized and leads to the following general equations of motion for the n-storey building with tuned mass dampers attached at all floors:

\[
\begin{bmatrix}
    m_1 \\
    m_2 \\
    \vdots \\
    m_n
\end{bmatrix}
\begin{bmatrix}
    \ddot{u}_1 \\
    \ddot{u}_2 \\
    \ddots \\
    \ddot{u}_n
\end{bmatrix} +
\begin{bmatrix}
    c_1 & c_2 & -c_2 & 0 & \cdots & 0 \\
    -c_2 & c_2 + c_3 & -c_3 & 0 & \cdots & 0 \\
    -c_3 & c_3 + c_4 & -c_4 & \ddots & \ddots & \ddots \\
    \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    -c_i & c_i + c_{i+1} & -c_{i+1} & \ddots & \ddots & \ddots \\
    \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
    -c_n & c_n & 0 & \cdots & \cdots & \ddots
\end{bmatrix}
\begin{bmatrix}
    u_1 \\
    u_2 \\
    \ddots \\
    \ddots \\
    u_i \\
    \ddots \\
    u_n
\end{bmatrix}
\]
\[
\begin{bmatrix}
  d_1 \\
  d_2 \\
  \vdots \\
  \vdots \\
  d_n
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \vdots \\
  \vdots \\
  \dot{x}_n
\end{bmatrix} + 
\begin{bmatrix}
  s_1 \\
  s_2 \\
  \vdots \\
  \vdots \\
  s_n
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_1 \\
  \dot{x}_2 \\
  \vdots \\
  \vdots \\
  \dot{x}_n
\end{bmatrix}
\]

(a10)
\[
\begin{bmatrix}
    s_1 \\
    s_2 \\
    \vdots \\
    s_n
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2 \\
    \vdots \\
    x_n
\end{bmatrix}
= 
\begin{bmatrix}
    h_1 \\
    h_2 \\
    \vdots \\
    h_n
\end{bmatrix}
\begin{bmatrix}
    a_1 \\
    a_2 \\
    \vdots \\
    a_n
\end{bmatrix}
\begin{bmatrix}
    \bar{u}_1 + \bar{u}_g \\
    \bar{u}_2 + \bar{u}_g \\
    \vdots \\
    \bar{u}_i + \bar{u}_g \\
    \bar{u}_n + \bar{u}_g
\end{bmatrix}
\]
Figure A-3: n-Storey Building with n TMD's
Figure A-4: Equivalent Mechanical System for n-Storey Building With n TMD's
Appendix B
Frequency Domain Formulation of the Equations of Motion

B.1 Systems without Tuned Mass Damper

The equations of motion of a shear building without vibration absorber under a ground motion acceleration can be written as:

\[ M \ddot{U} + C \dot{U} + K U = - M \dot{u}_g \]  \hspace{1cm} (b.1)

where

- \( M \) is the mass matrix of the system.
- \( C \) is the damping matrix of the system.
- \( K \) is the stiffness matrix of the system.
- \( U \) is the relative displacements with respect to the base.
- \( \dot{u}_g \) is the ground acceleration.

Assuming that the damping is proportional, the equations of motion can be expressed in terms of normal mode coordinates. The displacements can be written as:

\[ U = \Phi Z \]  \hspace{1cm} (b.2)

where

- \( \Phi \) is the modal matrix.
- \( Z \) is the modal coordinates.

If we substitute equation (b.2) into (b.1) we obtain:

\[ M \ddot{Z} + C \dot{Z} + K Z = - M \dot{u}_g \]  \hspace{1cm} (b.3)

and premultiplying by the transpose of the modal matrix

\[ \Phi^T M \ddot{Z} + \Phi^T C \dot{Z} + \Phi^T K Z = - \Phi^T \dot{u}_g \]  \hspace{1cm} (b.4)
Hence, we obtain \( n \) decoupled equations of the form:

\[
\ddot{z}_i + 2\xi_i \omega_i \dot{z}_i + \omega_i^2 z_i = -\frac{\phi_i^T M \dot{u}_i}{\phi_i^T M \phi_i} (1)
\]

where

\( \omega_i \) \hspace{1cm} \text{natural frequency corresponding to mode} \ i.
\( \xi_i \) \hspace{1cm} \text{damping ratio associated with mode} \ i.
\( \phi_i \) \hspace{1cm} \text{mode shape for mode} \ i.

The transfer function for the fundamental mode of this 1-degree-of-freedom system is given by:

\[
H_1(\omega) = \frac{-1}{\omega_1^2 - \omega^2 + 2\xi_1 \omega_1 \omega} (b.6)
\]

**B.2 Systems with a Tuned Mass Damper attached at the top floor**

In this case the equations of motion are:

\[
M \dddot{u} + C \ddot{u} + K u = -M \ddot{u}_s + F(t) (b.7)
\]

where
where \( k_a \) and \( c_a \) are the stiffness and the damping coefficient of the TMD respectively.

The displacements can be written as:

\[
U = \Phi Y
\]

Hence, the equations of motion become

\[
\phi^T M \ddot{y} + \phi^T \Phi \dot{Y} + \phi^T K \phi \dot{y} = -\phi^T M(1) \ddot{u} + \phi_n (k_a x + c_a \dot{x})
\]

or

\[
y + 2\xi \omega y + \omega^2 y = -\frac{\phi^T M \Phi (1)}{\phi^T M \phi} + \frac{\phi_n}{\phi^T M \phi} (k_a x + c_a \dot{x})
\]

Equilibrium considerations for the absorber lead to:

\[
m_a x + c_a x + k_a x = -m_a (\ddot{u} + \ddot{u}_n)
\]

or

\[
\ddot{x} + 2\xi_2 \omega_2 x + \omega_2^2 x = -\ddot{u}_n - \ddot{u}_n
\]
where we define the fundamental frequency and the damping ratio of the damper respectively as

$$\omega_2^2 = \frac{k_2}{m_2}$$  \hspace{1cm} (b.14)

$$\xi_2 = \frac{c_2}{2 \omega_2 m_2}$$  \hspace{1cm} (b.15)

The acceleration of the $n^{th}$ floor can be approximated as:

$$\ddot{u}_n = \sum_{j=1}^{n} \phi_j \dot{y}_j = \phi_1 y_1$$  \hspace{1cm} (b.16)

Now we can define the modal mass ratio as

$$\mu_1 = \frac{m_1}{\sum_{j=1}^{n} \phi_j^2 \omega_j}$$  \hspace{1cm} (b.17)

and the participation factor of the $i^{th}$ mode as

$$P_i = \frac{\phi_i^T M(1) \phi_i}{\sum_{j=1}^{n} \phi_j^T m \phi_j}$$  \hspace{1cm} (b.18)

Hence, the equations of motion can be written as:

$$y_i + 2 \xi_2 \omega_2 y_i + \omega_2^2 y_i = -P_i \ddot{u}_i + \mu_i \phi_i (2 \xi_2 \omega_2 x + \omega_2^2 x)$$  \hspace{1cm} (b.19)

For the absorber we obtain:

$$x + 2 \xi_2 \omega_2 x + \omega_2^2 x = -\ddot{u}_n - \phi_n \dot{y}_1$$  \hspace{1cm} (b.20)

Equations (b.18) and (b.19) can be rewritten as
\[ \begin{align*}
y_1 + 2\xi \omega y_1 + \omega^2 y_1 \cdot \mu \phi \left( 2\xi \omega x_2 + \omega^2 x_2 \right) &= -P_1 \ddot{y} \\
x_2 + 2\xi \omega x_2 + \omega^2 x_2 \cdot \phi \cdot \frac{n_1}{1} x_1 &= -\ddot{x}
\end{align*} \tag{b.21} \]

The displacements \( y \) and \( x \) can be written in terms of their Fourier transforms as

\[ \begin{align*}
y &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} Y(\omega) e^{i\omega t} d\omega \\
x &= \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{i\omega t} d\omega
\end{align*} \tag{b.23} \]

The complex frequency response can be computed from:

\[ \begin{align*}
-\omega^2 Y(\omega) + 2i\omega \xi \omega Y(\omega) + \omega^2 Y(\omega) \cdot \mu \phi \left[ i2\xi \omega X(\omega) + \omega^2 X(\omega) \right] &= -P_1 \ddot{y} \\
-\omega^2 X(\omega) + i2\xi \omega X(\omega) + \omega^2 X(\omega) \cdot \omega^2 \phi \cdot \frac{n_1}{1} Y(\omega) &= -\ddot{x}
\end{align*} \tag{b.25} \]

and in matrix form

\[ \begin{bmatrix}
-\omega^2 + 2i\omega \xi \omega + \omega^2 \\
-\omega^2 \phi \cdot \frac{n_1}{1}
\end{bmatrix}
\begin{bmatrix}
-\mu \phi \left( i2\xi \omega x_2 + \omega^2 x_2 \right)
-\omega^2 + 2i\omega \xi \omega + \omega^2 x_2
\end{bmatrix}
\begin{bmatrix}
Y(\omega)
X(\omega)
\end{bmatrix}
= \begin{bmatrix}
P_1 \ddot{y}
\ddot{x}
\end{bmatrix} \tag{b.27} \]
where \( u_g \) is the Fourier transform of \( u_g \). The above system can be solved for \( X \) and \( Y \). The complex frequency response function or transfer function is defined by

\[
Y(\omega) = H(\omega)u_g
\]  

and is given by

\[
H(\omega) = \frac{-N(\omega)}{D(\omega)}
\]  

where, for \( i = 1 \),

\[
N(\omega) = P_1\omega^2 - 12\omega\xi_2\omega_2(P_1 + \mu_1) - \omega^2(P_1 + \mu_1)
\]  

\[
D(\omega) = \omega^4 - 12\omega^3(\omega_2\xi_2 + \omega_1\xi_1 + \mu_2\omega_1\xi_1)
\]

\[
-\omega^2(\omega_2^2 + \omega_1^2 + 4\xi_1\xi_2\omega_2\omega_1 + \mu_1\omega_1^2)
\]

\[
+12\omega(\xi_1\omega_2^2 + \xi_2\omega_2\omega_1^2)
\]

\[
+\omega_1^2\omega_2^2
\]  

(b.28)  

(b.29)  

(b.30)  

(b.31)
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