



MIT Open Access Articles

Wavemaker theories for acoustic-gravity waves over a finite depth

The MIT Faculty has made this article openly available. **Please share** how this access benefits you. Your story matters.

Citation	Tian, Miao, and Usama Kadri. "Wavemaker Theories for Acoustic-gravity Waves over a Finite Depth." <i>Journal of Engineering Mathematics</i> , vol. 108, no. 1, Feb. 2018, pp. 25–35.
As Published	http://dx.doi.org/10.1007/s10665-017-9902-1
Publisher	Springer Netherlands
Version	Author's final manuscript
Citable link	http://hdl.handle.net/1721.1/115237
Terms of Use	Article is made available in accordance with the publisher's policy and may be subject to US copyright law. Please refer to the publisher's site for terms of use.

1 **Wavemaker theories for acoustic-gravity waves over a finite**
2 **depth**

3 **Miao Tian · Usama Kadri**

4
5 Received: date / Accepted: date

6 **Abstract** Acoustic-gravity waves (hereafter AGWs) in ocean have received much interest
7 recently, mainly with respect to early detection of tsunamis as they travel at near the speed
8 of sound in water which makes them ideal candidates for early detection of tsunamis. While
9 the generation mechanisms of AGWs have been studied from the perspective of vertical os-
10 cillations of seafloor and triad wave-wave interaction, in the current study we are interested
11 in their generation by wave-structure interaction with possible implication to the energy sec-
12 tor. Here, we develop two wavemaker theories to analyse different wave modes generated by
13 impermeable (the classic Havelock's theory) and porous (porous wavemaker theory) plates
14 in weakly compressible fluids. Slight modification has been made to the porous theory so
15 that, unlike the previous theory, the new solution depends on the geometry of the plate. The
16 expressions for three different types of plates (piston, flap, delta-function) are introduced.
17 Analytical solutions are also derived for the potential amplitude of the gravity, acoustic-

Miao Tian

1. Department of Physical Oceanography, Woods Hole Oceanographic Institute, Woods Hole, MA 02543, USA.

2. The Hatter Department of Marine Technologies, University of Haifa, Haifa 3498838, Israel.

3. INTERA Incorporated, Gainesville, FL 32606, USA.

E-mail: mtian@whoi.edu

Usama Kadri

1. The Hatter Department of Marine Technologies, University of Haifa, Haifa 3498838, Israel.

2. School of Mathematics, Cardiff University, Cardiff, CF24 4AG, UK.

3. Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA.

E-mail: ukadri@mit.edu

18 gravity, evanescent waves, as well as the surface elevation, velocity distribution, and pres-
19 sure for AGWs. Both theories reduce to previous results for incompressible flow when the
20 compressibility is neglected. We also show numerical examples for AGWs generated in a
21 wave flume as well as in deep ocean. Our current study sets the theoretical background to-
22 wards remote sensing by AGWs, for optimised deep ocean wave-power harnessing, among
23 others.

24 **Keywords** wavemaker · Havelock's · porous · acoustic-gravity waves

25 1 Introduction

26 Wavemaker theory has received increasing attention not only because its feasibility on gen-
27 erating waves in laboratory experiments, but also due to its application in design of wave-
28 energy harvesting devices [1]. The classic problem of surface waves generated by a wave-
29 maker in infinitely deep ocean was investigated by Havelock, as in Ref. [2], and later ex-
30 tended to the case of finite water depth [3]. The wavemaker was treated as a vertical imper-
31 meable plate which oscillates horizontally and periodically with a small displacement, and
32 the fluid was assumed incompressible. In all these formulations the wave motion was gov-
33 erned by linear wave theory. Extensions to a directional wavemaker problem with slowly-
34 varying depth can be found in [4].

35 The impermeability of the plate is unrealistic for a plate in a wave flume, not to men-
36 tion a landslide in deep ocean. Madsen [5] examined the influence of leakage around the
37 wavemaker on the wave amplitude and concluded that the porous effect can largely reduce
38 the wave amplitude. Therefore it would be more appropriate to take porosity effects into
39 account for many applications.

40 Here, we study the effect of water compressibility and the generation of acoustic-gravity
41 waves (AGWs) by water-structure interactions. AGWs have received much interest recently,
42 as they travel at the speed of sound in water which makes them, among others, ideal pre-
43 cursors of tsunami by employing bottom-pressure recordings [6, 7]. AGWs can interact with
44 continental shelves [8], ice-sheets [9], and might be responsible for deep-ocean water trans-
45 portation and circulation [10]. In contrast to the decaying vertical structure of gravity-wave
46 modes, the wave amplitudes of AGWs exhibit sinusoidal variation in the vertical direction.
47 Therefore wave-energy harnessing devices that are placed in deep water (where the decaying

48 gravity wave modes vanish) can potentially make use of AGWs, whereby the induced mea-
 49 surable pressure signature may reach a maximum at the seabed. While harnessing energy
 50 of AGWs might become possible in the future, e.g. based on a triad interaction mechanism
 51 similar to that proposed by Refs. [11] or [12], a more immediate application is the detection
 52 of sea-state in wave harnessing farms. Here, we show that AGWs radiate by the harnessing
 53 devices, or namely wavemakers, carrying information on their source at the speed of sound
 54 in water. To this end, while the generation mechanisms of AGWs have been studied from
 55 the perspective of vertical oscillations of seafloor [13, 7, 14] and triad wave-wave interaction
 56 [15, 16, 11, 12], here we are particularly interested in their generation by horizontally-moving
 57 wavemakers.

58 In this paper, we develop Havelock's and porous wavemaker theories for weakly com-
 59 pressible fluids. The paper is organised as follows: the problem is formulated with the gov-
 60 erning equations and boundary conditions in section 2. The general solution is provided in
 61 Section 3, followed by the Havelock's and porous-wavemaker solutions in Section 4. Sec-
 62 tion 5 presents examples for three types of wavemakers placed in a wave flume as well as in
 63 deep ocean. The work is summarised in Section 6.

64 2 Governing equations

65 We take x and z the horizontal and vertical coordinates respectively, and consider a wave-
 66 maker with its plate initially located at $x = 0$. The wavemaker oscillates horizontally along
 67 the x -axis with a displacement s_0 given by

$$s_0(x, z, t) = d(z) \exp(-i\omega t), \quad d \ll h, \quad (1)$$

68 where ω is the radian frequency, $d(z)$ is the maximum amplitude of oscillation, assumed to
 69 be much smaller than the undisturbed fluid depth h , and t is the time. The horizontal velocity
 70 and acceleration of the wavemaker are

$$u_0 = -i\omega d \exp(-i\omega t), \quad a_0 = -\omega^2 d \exp(-i\omega t). \quad (2)$$

71 The equation that governs the irrotational motions of acoustic-gravity waves throughout the
 72 entire water column is

$$\Phi_{tt} = c^2 (\Phi_{xx} + \Phi_{zz}), \quad -h < z < \eta, \quad (3)$$

where Φ is the velocity potential, and c is the speed of sound in water. The linearized kinematic and dynamic conditions at the free surface are

$$\Phi_z = \eta_t, \quad z = \eta, \quad (4)$$

$$\Phi_t + g\eta = 0, \quad z = \eta, \quad (5)$$

73 where η is the free surface elevation. Expanding (4) and (5) at $z = 0$ and eliminating η yield
74 the approximated surface boundary condition as shown in Ref. [17]

$$\Phi_{tt} + g\Phi_z = 0, \quad z = 0. \quad (6)$$

75 Finally the kinematic bottom boundary condition for a flat bottom is given by

$$\Phi_z = 0, \quad z = -h, \quad (7)$$

76 which indicates that the vertical velocity of the fluid must be zero at the bottom.

77 Equations (3), (6), and (7) formulate the linear problem of water wave propagation over a
78 finite depth in a weakly compressible fluid. Appropriate along-channel boundary conditions,
79 depending on the wavemaker type, are included to define the problem completely.

80 For the classic Havelock's wave-maker theory [2, 18, 3], the boundary condition is

$$\Phi_x = u_0, \quad x = 0. \quad (8)$$

81 Here, u_0 is the horizontal velocity of the stroke motion.

82 For a porous-wavemaker problem, the boundary condition at the wavemaker is given by
83 Ref. [19]. The hydrodynamic pressure $p(x, z, t)$ is associated with the velocity potential Φ
84 via the linearised Bernoulli equation as

$$p = -\rho\Phi_t \quad (9)$$

85 in which ρ is the water density.

86 The pressure on the positive and negative sides of the wavemaker are related as

$$p(0, z, t) = p^+(z, t) = -p^-(z, t). \quad (10)$$

87 The normal velocity towards the porous plate is equal to the velocity of the stroke motion
88 u_0 , which is linearly proportional to the pressure difference between the two sides of the
89 wavemaker [20], so that

$$u_0(z, t) = \frac{2b}{\mu} p(0, z, t). \quad (11)$$

90 Here, μ is the dynamic viscosity, and b is the coefficient which represents the width of the
91 plate and has the dimension of length.

92 3 General Solution

93 Because of the periodic motion of the wavemaker, Φ , η , and p are assumed to be periodic
94 functions in t with a time factor $\exp(-i\omega t)$, i.e.

$$\Phi = \phi(x, z) \exp(-i\omega t), \quad \eta = a(x) \exp(-i\omega t), \quad p = p(x, z) \exp(-i\omega t). \quad (12)$$

95 Using (12), equation (3) reduces to the Helmholtz equation

$$\phi_{xx} + \phi_{zz} + k_c^2 \phi = 0, \quad k_c = \omega/c, \quad (13)$$

96 where k_c is a compressibility coefficient. Similarly, substiting equation (12) into equations
97 (6) and (7) yields the boundary conditions in terms of ϕ ,

$$-\omega^2 \phi + g\phi_z = 0, \quad z = 0; \quad (14)$$

98

$$\phi_z = 0, \quad z = -h. \quad (15)$$

Following similar steps as in Refs. [7] and [14] the solution of equations (13)-(15) is obtained,

$$\begin{aligned} \phi = & A_0 \exp(ik_0 x) \cosh(\lambda_0(z+h)) \\ & + \sum_{n=1}^N A_n \exp(ik_n x) \cos(\lambda_n(z+h)) \\ & + \sum_{n=N+1}^{\infty} B_n \exp(-\kappa_n x) \cos(\lambda_n(z+h)), \end{aligned} \quad (16)$$

where λ_0 and λ_n are real and positive solutions of the following dispersion relationships

$$\omega^2 = g\lambda_0 \tanh(\lambda_0 h); \quad (17)$$

$$\omega^2 = -g\lambda_n \tan(\lambda_n h), \quad n = 1, 2, 3, \dots, \quad (18)$$

99 where λ_n is the n -th eigenvalue and n is the mode number. With specified ω and h , equation
100 (17) has one real solution for λ_0 ; while equation (18) involves infinitely-many different
101 eigenvalues.

102 The parameters k_0 , k_n , κ_n are all real and positive, given by

$$k_0 = \sqrt{k_c^2 + \lambda_0^2}, \quad (19)$$

103

$$k_n = \sqrt{k_c^2 - \lambda_n^2}, \quad n = 1, 2, \dots, N; \quad k_c > \lambda_N, \quad (20)$$

104

$$\kappa_n = \sqrt{\lambda_n^2 - k_c^2}, \quad n = N + 1, \dots; \quad k_c < \lambda_{N+1}, \quad (21)$$

105 where

$$N = \left\lfloor \frac{\omega h}{\pi c} + \frac{1}{2} \right\rfloor \quad (22)$$

106 represents the number of existing AGW modes, and the brackets is the floor function (nearest
 107 integer from below), as in Ref. [8]. The three terms on the right-hand-side of equation (16)
 108 represent the gravity, acoustic-gravity, and evanescent modes, respectively.

109 4 Wavemaker problem

110 4.1 Solution for Havelock's wavemaker

111 Since $\cosh(\lambda_0(z+h))$ and $\cos(\lambda_n(z+h))$ in equation (16) are the eigenfunctions of the
 112 boundary value problem in z , they are orthogonal over the interval from $z = 0$ to $z = -h$
 113 based on the Sturm-Liouville theory. Therefore, we substitute equations (12) and (16) into
 114 equation (8), multiply by $\cosh(\lambda_0(z+h))$ and $\cos(\lambda_n(z+h))$ and integrate over the water
 115 column from $z = -h$ to $z = 0$ so that A_0, A_n, B_n can be calculated as

$$A_0 = \frac{-2\omega}{h\sqrt{k_c^2 + \lambda_0^2}(1 + CQ_0^2)} \int_{-h}^0 d \cosh(\lambda_0(z+h)) dz, \quad (23)$$

$$A_n = \frac{-2\omega}{h\sqrt{k_c^2 - \lambda_n^2}(1 - CQ_n^2)} \int_{-h}^0 d \cos(\lambda_n(z+h)) dz, \quad (24)$$

$$B_n = \frac{2i\omega}{h\sqrt{\lambda_n^2 - k_c^2}(1 - CQ_n^2)} \int_{-h}^0 d \cos(\lambda_n(z+h)) dz, \quad (25)$$

where

$$Q_0 = \sinh \lambda_0 h, \quad Q_n = \sin \lambda_n h, \quad C = \frac{g}{\omega^2 h}. \quad (26)$$

116 In the incompressible case ($c \rightarrow \infty$), equations (23) and (25) reduce to the solutions for
 117 gravity and evanescence modes in [21] (equations (6.21) and (6.22)). The extra term A_n
 118 comes from the newly-generated AGW mode due to the compressibility of the fluid.

119 4.2 Solution for porous wavemaker

Following similar steps using (11) together with (9), (12), and (16) and the orthogonality of $\cosh(\lambda_0(z+h))$ and $\cos(\lambda_n(z+h))$, we can derive expressions of A_0 , A_n , B_n for a porous wavemaker. Chwang [19] described a similar problem for incompressible flow and derived the solutions. Chwang's solution, however, indicates that the produced waves have the same amplitudes regardless to the geometry of the plate. In order to consider different plate types, we modify Chwang's method and derive an alternative solution in a similar form as the Havelock's [21]

$$A_0 = G_0 \frac{-2\omega}{h\sqrt{k_c^2 + \lambda_0^2}(1 + CQ_0^2)} \int_{-h}^0 d \cosh(\lambda_0(z+h)) dz, \quad (27)$$

$$A_n = G_n \frac{-2\omega}{h\sqrt{k_c^2 - \lambda_n^2}(1 - CQ_n^2)} \int_{-h}^0 d \cos(\lambda_n(z+h)) dz, \quad (28)$$

$$B_n = H_n \frac{2i\omega}{h\sqrt{\lambda_n^2 - k_c^2}(1 - CQ_n^2)} \int_{-h}^0 d \cos(\lambda_n(z+h)) dz, \quad (29)$$

where the porous factors are

$$G_0 = \frac{\mu\sqrt{k_c^2 + \lambda_0^2}}{2\omega\rho b}, \quad G_n = \frac{\mu\sqrt{k_c^2 - \lambda_n^2}}{2\omega\rho b}, \quad n = 1, 2, \dots, N, \quad (30)$$

$$H_n = \frac{-i\mu\sqrt{\lambda_n^2 - k_c^2}}{2\omega\rho b} \quad n = N+1, N+2, \dots$$

120 We focus on the porous factor G_n that is associated with AGW modes. As pointed out
 121 in [19], the reciprocal of G_n in equation (30) can be understood as a Reynolds number for
 122 the flow passing through the porous wavemaker, while G_n also measures the porosity. For
 123 example, $G_n = 0$ (or equivalently, $\mu = 0$) corresponding to a wavemaker that is completely
 124 permeable. Obviously, the expressions reduce to the Havelock's solution when the porous
 125 factors, G_0 , G_n , H_n , are unity. Moreover, as λ_n increases with the mode number n , the porous
 126 factor G_n decreases for higher AGW modes, meaning that the porous media dissipates more
 127 energy from shorter waves (lower modes). Specifying values for the porous factor of the
 128 gravity mode G_0 (0.1, 0.2, 0.5, etc.), Chwang [19] presented the surface elevation of gravity
 129 waves produced by the wavemaker. In this study, the porous factor for the first AGW mode
 130 G_1 will be set to 0.5 for illustration purposes, while G_n ($n = 2, 3, \dots$) can be determined
 131 accordingly.

The amplitudes of the surface elevation, horizontal velocity, and pressure at AGW mode n can also be given employing equations (5), (9), (12), (16)

$$a_n = i \frac{\omega}{g} A_n \exp\left(i \sqrt{k_c^2 - \lambda_n^2} x\right) \cos(\lambda_n h); \quad (31)$$

$$u_n = i \sqrt{k_c^2 - \lambda_n^2} A_n \exp\left(i \sqrt{k_c^2 - \lambda_n^2} x\right) \cos(\lambda_n (z + h)); \quad (32)$$

$$p_n = i \omega \rho A_n \exp\left(i \sqrt{k_c^2 - \lambda_n^2} x\right) \cos(\lambda_n (z + h)); \quad (33)$$

132 where A_n is defined in equation (28).

133 4.3 Different types of plates

134 We focus on the AGWs term in equations (24) and (28) and derive the explicit form based on
 135 different types of plates. Piston and flap motions [21] are commonly used for wave flumes in
 136 laboratory experiments, while a wavemaker of δ -function type is considered for deep ocean
 137 [18]. Therefore the function $d(z)$ that describes the piston motion in equation (1) has been
 138 assumed to be

$$d(z) = \begin{cases} D, & \text{piston plate;} \\ D(1 + z/h), & \text{flap plate;} \\ D\delta(z + h_0), & \delta\text{-function plate.} \end{cases} \quad (34)$$

139 Here, D is the horizontal amplitude of the stroke motion. The δ -function type wavemaker is
 140 located at $z = -h_0$.

Substituting equation (34) into the Havelock's solution (24), the expression for the amplitude A_n of the velocity potential can be readily obtained in the form

$$A_n = \begin{cases} M_n \sin(\lambda_n h), & \text{piston plate;} \\ \frac{M_n}{\lambda_n h} [\lambda_n h \sin(\lambda_n h) + \cos(\lambda_n h) - 1], & \text{flap plate;} \\ M_n \cos(\lambda_n (h - h_0)), & \delta\text{-function plate.} \end{cases} \quad (35)$$

141 in which

$$M_n = \frac{-2\omega D}{\lambda_n h \sqrt{k_c^2 - \lambda_n^2} (1 - C Q_n^2)}. \quad (36)$$

142 A comparison of the normalised velocity potential amplitude in equation (35) is given
 143 in Figure 1. For illustration, the δ -function in (34) is assumed to be located at the depth
 144 $z = -h_0 = -h/2$. Apparently, a flap wavemaker produces the largest first-mode AGW for

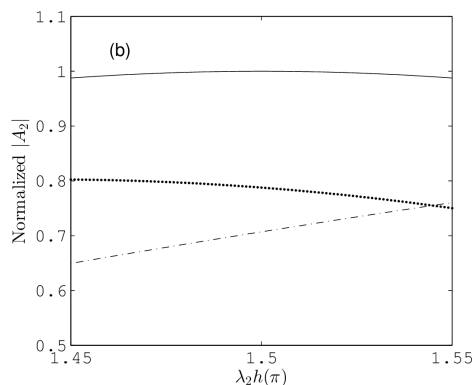


Fig. 1 normalised AGW velocity potential amplitudes $|A_1|$ (a) and $|A_2|$ (b) by a factor of M_n ($n = 1$ and 2 , respectively) for three types of plate as a function of $\lambda_n h$, which is chosen to vary between $[0.45, 0.55]n\pi$. The δ -function type wavemaker is assumed to be located at $z = -h/2$ for illustration purposes. Solid: piston-type plate; dotted: flap-type plate; dashed: δ -function type plate.

145 the same plate-motion amplitude D . For a flap plate, equation (35) also indicates that the nor-
 146 malised AGW amplitudes are inversely proportional to $\lambda_n h$, therefore higher AGW modes
 147 must have smaller normalised amplitudes.

148 5 Examples

149 5.1 Acoustic-gravity waves in a wave flume

150 The number of existing AGW modes associated with specific frequency and depth are cal-
 151 culated by (22). This relation shows that more AGW modes can be generated at a higher
 152 frequency ω or impractically deep water. Therefore, generating AGWs in the laboratory is
 153 not an easy task. In order to obey AGW theory (with the absence of bottom elasticity, e.g.
 154 see Ref. [22] for the detailed analysis) and create AGWs in the laboratory experiments we
 155 need to operate at relatively very high frequencies. For example, there are three AGW modes
 156 corresponding to a 5 kHz wavemaker in a 0.5 m wave flume. Although working with a 5
 157 kHz source introduces some real difficulties, we can still have feasible experiments with
 158 piezoelectric membranes to validate the proposed theory (an on-going research effort). Al-
 159 ternatively, one needs to carry out an experiment in the deep ocean, which is by no means
 160 easier to perform. Due to this conflicting choice of an appropriate experimental environ-

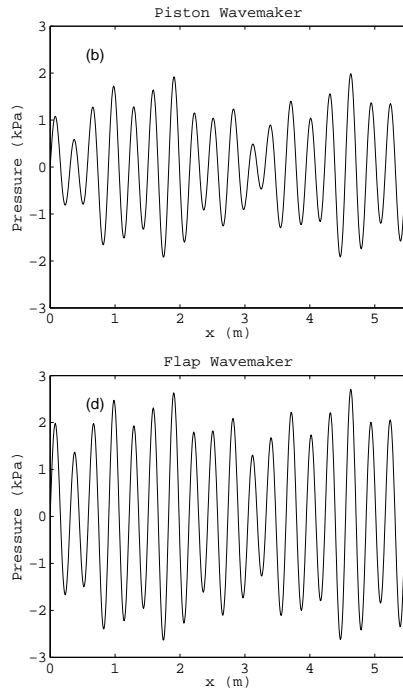


Fig. 2 Acoustic-gravity waves generated by a wavemaker in a wave flume for $f = 5$ kHz based on the Havelock's wavemaker theory; $L = 5.5$ m, $h = 0.5$ m, $b = 1$ m, $c = 1500$ m/s. The motion of the plate is limited by the constraint that, the horizontal movement ≤ 2.1 m, the horizontal velocity ≤ 3.8 m/s, horizontal acceleration ≤ 19.6 m/s² (parameters come from the unidirectional wavemaker in O.H. Hinsdale Wave Research Laboratory, Oregon State University). (a), (c) Vertical distribution of the velocity amplitude at $x = 0.04$ m, 0.39 m, and 0.79 m away from the wavemaker. (b), (d) Horizontal distribution of the pressure amplitude at the bottom of the flume. (a), (b) piston wavemaker; (c), (d) flap wavemaker.

161 ment, we dedicate this section and the following to the disparate wave flume and deep ocean
 162 systems, respectively.

163 Examples of AGWs generated in a wave flume by piston- and flap-type plates are shown
 164 in Figures 2 and 3 based on Havelock's and porous wavemaker theories, respectively. Notice
 165 that the stroke motion is not only limited by its maximum stroke distance, but also the
 166 maximum velocity and acceleration. Here, we assume that the wavemaker has the same
 167 constraints as the unidirectional wavemaker of the O.H. Hinsdale Wave Research Laboratory
 168 in Oregon State University (for example the laboratory experiment presented in Ref. [23]); a
 169 simple calculation using equations (2) shows that the stroke amplitude of a wavemaker with
 170 $f = 5$ kHz is in the order of 10^{-8} m, which requires a very careful experiment.

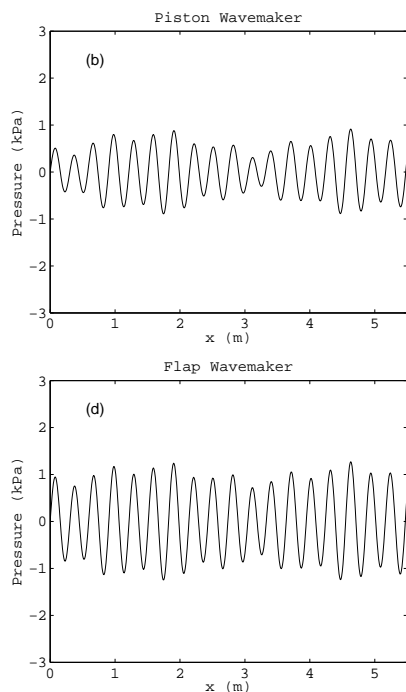


Fig. 3 Acoustic-gravity waves generated by a wavemaker in a wave flume for $f = 5$ kHz based on the porous wavemaker theory; $L = 5.5$ m, $h = 0.5$ m, $b = 1$ m, $c = 1500$ m/s. The motion of the plate is limited by the constraint that, the horizontal movement ≤ 2.1 m, the horizontal velocity ≤ 3.8 m/s, horizontal acceleration ≤ 19.6 m/s² (parameters come from the unidirectional wavemaker in O.H. Hinsdale Wave Research Laboratory, Oregon State University). (a), (c) Vertical distribution of the velocity amplitude at $x = 0.04$ m, 0.39 m, and 0.79 m away from the wavemaker. (b), (d) Horizontal distribution of the pressure amplitude at the bottom of the flume. (a), (b) piston wavemaker; (c), (d) flap wavemaker.

171 It is difficult to measure AGW surface elevations directly due to their small amplitudes
 172 (not shown in the figure), whereas the horizontal velocity component which is in the order
 173 of 10^{-3} m/s (Figure 2 (a) and (c)), is detectable using a particle image velocimetry (PIV)
 174 system. It is also worth mentioning that, unlike gravity waves, the AGW velocity ampli-
 175 tude oscillates vertically and leaves a distinct pressure signature throughout the entire water
 176 column, and mainly at the bottom.

177 The time series of the pressure at the bottom (Figure 2 (b) and (d)) behaves in a similar
 178 way to the surface elevation, although measurable by a wired pressure sensor. Therefore, in
 179 spite of the small amplitude of their surface elevation, AGWs are expected to be detectable
 180 on bottom-pressure records or PIV velocity measurement in a laboratory study. On the other

181 hand the flap wavemaker is able to produce larger waves compared to the piston wavemaker
182 as shown in Figure 2 (c) and (d). Figure 3 presents AGW-induced pressure and velocities
183 produced by porous wavemakers. Due to the porosity effect, the velocity and pressure am-
184 plitudes are generally smaller than those of the Havelock's theory.

185 5.2 Acoustic-gravity waves in deep ocean

186 We treat the problem of a wavemaker plate in deep ocean as a point source in deep water
187 (similar to the ocean acoustics problem in Ref. [24]), and consider it as a δ -function. An
188 example of a δ -function wavemaker located at $z = -12.5$ m in deep ocean with $f = 1$ Hz
189 and $h = 4000$ m is presented in Figure 4 for both Havelock's and porous wavemaker theo-
190 ries. The AGWs produced by an impermeable wavemaker ($G = 0$) have larger amplitudes
191 (Figure 4 (a) and (b)), whereas the wave amplitude decreases as the wavemaker becomes
192 porous ($G \neq 0$). The surface elevation increases to 10^{-3} m (not shown) compared to those in
193 the laboratory example, although still hard to be distinguished from that of surface gravity
194 waves. The velocity amplitudes are almost zero at the surface; they reach a maximum at
195 about $z = 500$ m, and oscillate across the water column in the z -direction. Although AGWs
196 have frequencies similar to that of the gravity mode, their distributions are periodic through-
197 out the water column (i.e. do not decay with depth). Therefore, they can be distinguished
198 from the decaying gravity waves. The order of magnitude of the velocity reaches 10^{-2} m/s,
199 which is measurable by standard instruments such as the ADCP (Acoustic Doppler Current
200 Profiler), PCADP (Pulse-coherent Acoustic Doppler Profiler). It is thus suggested to employ
201 pressure sensors at the seabed, or deep below the free surface, where surface-wave signa-
202 tures are negligible. The AGW signal, however, is in the order of 10 kPa, which is easy to
203 measure (e.g., the MODE experiment that measures the pressure fluctuation on the deep-
204 sea floor by Ref. [25]). This simple example shows that AGWs may be responsible for the
205 low-frequency oceanic noise on the seabed [26].

206 6 Conclusion

207 Without overlooking the slight compressibility of water, as is usually assumed, we present
208 Havelock's and porous wavemaker theories to analyse different modes of water waves fol-
209 lowing Refs. [21] and [19], with a focus on AGW modes. These theories may have important

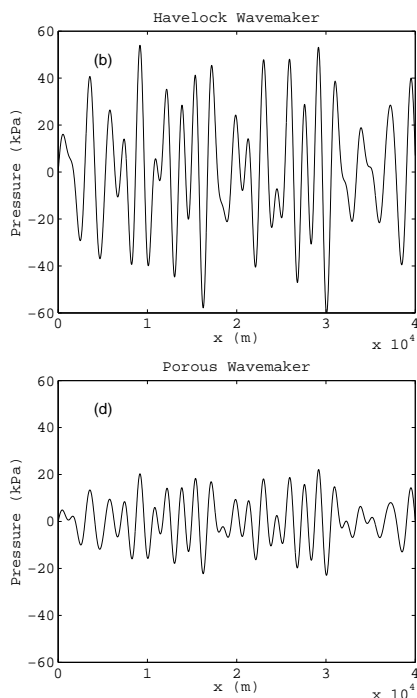


Fig. 4 Acoustic-gravity waves generated by a δ -function wavemaker in the ocean placed at $z = -12.5$ m for $f = 1$ Hz. The ocean is $h = 4000$ m deep and the speed of sound is $c = 1500$ m/s. (a), (c) Vertical distribution of the velocity amplitude at $x = 15$ m, 9525 m, and 14985 m away from the wavemaker. (b), (d) Horizontal distribution of the pressure amplitude at the bottom of the flume. (a), (b) Havelock's wavemaker theory; (c), (d) porous wavemaker theory.

210 implications in the study of surface waves in flume experiment [18] or tsunamis caused by
 211 landslides during earthquakes in deep ocean [8, 7, 14]. Moreover, the generation of AGWs
 212 can be attributed to wave-structure interaction [18], therefore another implication is where
 213 the efficiency of wave-energy harnessing devices is of interest, with the wavemaker being
 214 subjected to some form of wave energy converter, e.g., a flap gate [27]. Another and prob-
 215 ably a more immediate implication is the remote detection of the wavy sea-state which can
 216 help tuning the surface wave energy converters for maximum efficiency. These are left for
 217 future studies, and we hope this work will motivate scientists and engineers to look into
 218 these important implications.

219 Both Havelock's and porous wavemaker solutions reduce to previous theories [19, 21]
 220 for incompressible flow when the compressibility coefficient k_c in equation (13) tends to
 221 zero. The solutions for three types of plates as well as the spatial distribution of the AGW

222 components are presented. For the same horizontal plate displacements, a flap wavemaker
223 is capable of making larger waves than piston and δ -function-type wavemakers. The spatial
224 distribution of the amplitude of the surface elevation, horizontal velocity, and bottom pres-
225 sure due to both theories shows that the porous wavemaker generally results in smaller waves
226 than those produced by Havelock's theory due to the porosity factor G_n in equation (30). The
227 calculations reveal that the surface elevation of AGWs in the current lab experimental set-
228 tings is in the order of 10^{-9} m, and can reach 10^{-3} m in deep ocean. Consequently, surface
229 elevation of AGWs is hard to measure; while velocity amplitude suggests that AGWs can
230 be detected by a particle image velocimetry (PIV) system in the laboratory experiment. Fi-
231 nally, the pressure distributions show that AGW signals are significantly large at the bottom
232 of a wave flume and deep ocean, to be captured by a standard pressure sensor. This study
233 motivates further laboratory studies and field measurements on deep ocean as it predicts
234 the characteristics of the generated waves, and provides insights on how to carry out di-
235 rect measurements. It also sheds some light on the development of tsunami early-detection
236 systems from the perspective of describing AGWs near the epicentre during earthquakes.
237 Finally, the porous wavemaker theory can potentially contribute to the study of deep-ocean
238 energy-harvest devices where the porous plates can be treated as an energy absorber.

239 **Acknowledgements** The first author acknowledges the Postdoctoral Scholar Program at the University of
240 Haifa in collaboration with Woods Hole Oceanographic Institution.

241 References

- 242 1. Mei C.C., Hydrodynamic principles of wave power extraction, *Phil. Trans. R. Soc. A* 370 , 208-234
243 doi:10.1098/rsta.2011.0178, 2012.
- 244 2. Havelock T., Forced surface-waves on water, *The London, Edinburgh, and Dublin Philosophical Magazine*
245 *and Journal of Science*, 8(51), 569-576, 1929.
- 246 3. Ursell F., Dean R.G., Yu Y.S., Forced small-amplitude water waves: a comparison of theory and experi-
247 ment, *J. Fluid Mech.*, 7, 33-52, 1960.
- 248 4. Dalrymple R.A., Directional wavemaker theory with sidewall reflection, *J. Hydraul. Res.*, 27:1, 23-34,
249 1989.
- 250 5. Madsen O.S., Waves generated by a piston-type wavemaker, In *Proc. 12th Coastal Engng Conf.*, 589-607,
251 ASCE, 1970.
- 252 6. Hendin G., Stiassnie M., Tsunami and acoustic-gravity waves in water of constant depth, *Phys. Fluids*,
253 25, 086103, 2013.

- 254 7. Stiassnie M., Tsunamis and acoustic-gravity waves from underwater earthquakes, *J. Eng. Math.*, 67, 23-
255 32, doi:10.1007/s10665-009-9323-x, 2010.
- 256 8. Kadri U., Stiassnie M., Acoustic-gravity waves interacting with the shelf break, *J. Geophys. Res.*, 117(C3),
257 C03035, 2012.
- 258 9. Kadri U., Generation of Hydroacoustic Waves by an Oscillating Ice Block in Arctic Zones, *Advances in*
259 *Acoustics and Vibration*, 8076108, <http://dx.doi.org/10.1155/2016/8076108>, 2016.
- 260 10. Kadri U., Deep ocean water transport by acoustic-gravity waves, *J. Geophys. Res.: Oceans*, 119(11),
261 7925-7930, 2014.
- 262 11. Kadri U., Triad resonance between a surface gravity wave and two high frequency hydro-acoustic waves,
263 *Eur J Mech B-Fluid*, 55, 1, 157-161, 2016.
- 264 12. Kadri U., Akylas T.R., On resonant triad interactions of acoustic-gravity waves, *J. Fluid Mech.*, 788:
265 R1(12 pages), doi:10.1017/jfm.2015.721, 2016.
- 266 13. Oliveira T.C.A. and Kadri U., Pressure field induced in the water column by acoustic-gravity
267 waves generated from sea bottom motion. *J. Geophys. Res. Oceans*. Accepted Author Manuscript.
268 doi:10.1002/2016JC011742, 2016.
- 269 14. Yamamoto T., Gravity waves and acoustic waves generated by sub-marine earthquakes, *Soil Dyn. Earth-*
270 *quake Eng.*, 1, 75-82, 1982.
- 271 15. Kadri U., Stiassnie M., Generation of an acoustic-gravity wave by two gravity waves, and their subse-
272 quent mutual interaction, *J. Fluid Mech.*, 735, R61-R69, 2013.
- 273 16. Kadri U., Wave motion in a heavy compressible fluid: Revisited, *Eur J Mech B-Fluid*, 49(A), 50-57,
274 2015.
- 275 17. Lamb, H., *Hydrodynamics*, Cambridge: at the University Press, 4th edition, 352, 1916
- 276 18. Stuhlmeier R., Stiassnie M., Adapting Havelock's wave-maker theorem to acoustic-gravity waves, *IMA*
277 *J. Appl. Math.*, accepted.
- 278 19. Chwang A.T., A porous-wavemaker theory, *J. Fluid Mech.*, 132, 395-406, 1983.
- 279 20. Taylor G.I., Fluid flow in regions bounded by porous surfaces, *Proc. R. Soc. Lond. A*234, 45-75, 1956.
- 280 21. Dean R.G., Dalrymple R.A., *Water Wave Mechanics for Engineers and Scientists*, Advanced Series on
281 *Ocean Engineering*, 2, 174, World Scientific, 176, 1991.
- 282 22. Eyov E., Klar A., Kadri U., and Stiassnie M., Progressive waves in a compressible ocean with elastic
283 bottom, *Wave Motion* 50, 929-939. doi: 10.1016/j.wavemoti.2013.03.003, 2013.
- 284 23. Tian M., Sheremet A., Kaihatu J.M., Ma G-F, On the Shoaling of Solitary Waves in the Presence of Short
285 Random Waves, *J. Phys. Oceanogr.*, 45, 3, 792-806, 2015.
- 286 24. Jensen F.B., Kuperman W.A., Porter M.B., and Schmidt, H., *Computational Ocean Acoustics*, Modern
287 *Acoustics and Signal Processing*, ISBN 978-1-4419-8678-8, Springer, 2011.
- 288 25. Brown W., Munk W., Snodgrass F., Mofjeld H., Zetler B., MODE bottom experiment, *J. Phys. Oceanogr.*,
289 5, 75-85, 1975.
- 290 26. Ardhuin F., Lavanant T., Obrebski M., Marie L., Royer J.-Y., D'Eu J.-F., Howe B. M., Lukas R., Aucan
291 J., A numerical model for ocean ultra-low frequency noise: wave-generated acoustic-gravity and Rayleigh
292 modes., *J. Acoust. Soc. Am.*, 134(4), 3242-59, 2013.
- 293 27. Sammarco P., Michele S., d'Errico M., Flap gate farm: From Venice lagoon defense to resonating wave
294 energy production. Part 1: Natural modes, *Appl. Ocean Res.*, 43, 206-213, 2013.