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A model of comprehensive unification



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ABSTRACT

Comprehensive – that is, gauge *and* family – unification using spinors has many attractive features, but it has been challenged to explain chirality. Here, by combining an orbifold construction with more traditional ideas, we address that difficulty. Our candidate model features three chiral families and leads to an acceptable result for quantitative unification of couplings. A potential target for accelerator and astronomical searches emerges.

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1. Introduction

Our core theory of fundamental physics, based on promoting $SU(3) \times SU(2) \times U(1)$ to a local symmetry, describes a vast range of phenomena precisely and very accurately. In that sense, it is close to Nature's last word. On the other hand, it contains a diversity of interactions and, when we come to the fermions, a plethora of independent elements. It is attractive to imagine that a deeper unity underlies this observed multiplicity. Gauge unification, perhaps most elegantly realized using the group $SO(10)$ and the spinor **16** representation of fermions [1], goes a long way toward that goal. It leaves us with a single interaction (i.e., a simple gauge group) but three fermion families, each embodying a chiral spinor **16** representation. It is then natural to ask, whether one can take that success further, to unite the separate families.

The mathematical properties of spinor representations are suggestive in this regard [2,3]. Specifically, for example, the irreducible spinor **256** representation of $SO(18)$ reduces, upon breaking $SO(18) \rightarrow SO(10) \times SO(8)$, according to $\mathbf{256} \rightarrow (\mathbf{16}, \mathbf{8}) + (\overline{\mathbf{16}}, \mathbf{8}')$, involving spinor representations of the smaller groups (including conjugate and alternate spinors). From the standpoint of $SO(10)$, then, we have eight families and eight mirror families. Notably, there are no problematic exotic color or charge quantum numbers: we get basically the sorts of representations we want, and

no others. Still, there are too many families, and the mirror families carry the “wrong” chirality for low-energy phenomenology [4]. Confinement of some $SO(8)$ quantum numbers, or interaction with condensates, can effectively remove an equal number of families and mirror families, but it seems difficult to change their net balance by those means.

The idea of comprehensive unification has continued to attract attention over the years, both in context of $SO(18)$ and in variant forms [5,6], but the issue of chirality has remained salient.

In this letter we explore a different direction. We use an orbifold construction to break $SO(18) \rightarrow SO(10) \times SO(8)$, with chiral fermion zero modes in $(\mathbf{16}, \mathbf{8})$. In addition we postulate condensates that break $SO(8) \rightarrow SO(5)$ and decompose $\mathbf{8} \rightarrow 3 \times \mathbf{1} + \mathbf{5}$. The $SO(5)$ then becomes strongly coupled and confining at a scale $\mathcal{O}(\text{TeV})$, effectively leaving three chiral spinors of $SO(10)$ at low energies. When one includes contributions from the required Higgs fields, an acceptable fit to gauge coupling unification emerges (despite the absence of low-energy supersymmetry). An interesting consequence of this scheme is the existence of stable $SO(5)$ hyperbaryons, protected by a Z_2 symmetry. Although they annihilate in pairs, a significant relic density emerges from big bang cosmology.

2. Model construction

We will exploit the possibility to obtain chiral fields by imposing appropriate boundary conditions on orbifolds. That mechanism has been used, for example, in a recent higher-dimensional extension of the Standard Model [7].

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Supersymmetry will play no role in our discussion. In the context of warped extra dimensions, a major motivation for supersymmetry is that it avoids Planck scale radiative corrections, that would re-introduce the hierarchy problem, when scalar fields are allowed to propagate in the bulk [8]. Our scalars will be localized on the branes. As will emerge below, it is not implausible that we can fulfill the main quantitative motivation for low-energy supersymmetry – the unification of couplings – in a different way. One can, of course, assume that supersymmetry is present in a more basic underlying theory, but broken at the Planck scale. Here, however, we will not address issues of ultraviolet completion.

Our model employs an $S_1/(Z_2 \times Z'_2)$ orbifold. Specifically, we consider a circular fifth dimension of radius $R = 2L/\pi$, with walls at $y = 0, L$ and a warped metric [9]:

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

with

$$\sigma(y) = \sigma(y + 2L) = \sigma(-y) \quad (2)$$

$$\sigma(y) = ky \text{ for } 0 \leq y \leq L.$$

We define the equivalence relations [10]

$$\begin{aligned} \mathbf{P}_0 : y &\sim -y, \\ \mathbf{P}_1 : y' &\sim -y', \end{aligned} \quad (3)$$

where $y' \equiv y + L$. Thus the second relation in Eq. (3) is equivalent to $y \sim y + 2L$. In the standard Randall–Sundrum terminology, we can say that the bulk region, $0 < y < L$, is sandwiched between a Planck brane ($y = 0$) and a IR brane ($y = L$).

The action of these equivalences $\mathbf{P}_0, \mathbf{P}_1$ on matter fields is

$$\begin{aligned} \Phi(x, y) &\sim P_0^\Phi \Phi(x, -y), \\ \Phi(x, y') &\sim P_1^\Phi \Phi(x, -y'), \end{aligned} \quad (4)$$

where P_0^Φ and P_1^Φ are matrices that represent the action of the Z_2 on the bulk fields. We can classify fields by their (P_0^Φ, P_1^Φ) values. It will be convenient to write the orbifold conditions for gauge fields as:

$$\begin{pmatrix} A_\mu \\ A_y \end{pmatrix} (x, y_j - y) \sim P_j^A \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix} (x, y_j + y) (P_j^A)^{-1} \quad (5)$$

where $(y_0, y_1) \equiv (0, L)$. Thus

$$A_M(x, y + 2L) = U A_M(x, y) U^{-1} \quad (6)$$

with $U = P_1^A P_0^A$.

We will choose

$$\begin{aligned} P_0^A &= \text{diag}(\mathbb{I}_{10}, -\mathbb{I}_8), \\ P_1^A &= \text{diag}(\mathbb{I}_{18}), \end{aligned} \quad (7)$$

and the corresponding representation matrices for P_j^Φ . These boundary conditions reduce $SO(18) \rightarrow SO(10) \times SO(8)$.

We can decompose a generic five-dimensional field as:

$$\Phi(x, y) = \frac{1}{\sqrt{L}} \sum_{n=0}^{\infty} \phi^{(n)}(x) f_n(y), \quad (8)$$

where $\phi^{(n)}$ are the Kaluza–Klein (KK) excitations and the KK eigenmodes, $f_n(y)$, obey:

$$\frac{1}{L} \int dy e^{(2-s)\sigma} f_m(y) f_n(y) = \delta_{mn}, \quad (9)$$

where $s = 2, 4, 1$ when the field is a vector field, a scalar or a fermion, respectively [8].

In more detail, according to Eq. (5), the $SO(18)$ gauge adjoint representation will split as

$$\mathbf{153} = (\mathbf{45}, \mathbf{1})^{++} + (\mathbf{1}, \mathbf{28})^{++} + (\mathbf{10}, \mathbf{8})^{-+}, \quad (10)$$

so only adjoint fields corresponding to $SO(10) \times SO(8)$ have zero modes. Because the fifth components, A_y , have opposite boundary condition, they have only Kaluza–Klein modes.

A left-handed fermion field will have a massless zero-mode only when it has Neumann (+) boundary conditions at both Planck and IR branes

$$\phi^{(++)}(x, y) = \frac{1}{\sqrt{L}} (\phi_{++}^{(0)}(x) f(y)^{(0)} + \text{higher modes}). \quad (11)$$

The same occurs with right-handed fields that have Dirichlet (–) boundary conditions at both branes, while fields with (+, –) or (–, +) do not have zero modes regardless of their chirality. The $\phi^{(0)}(x)$ zero mode is a massless field in four dimensions, while the $\phi^{(n)}(x)$ Kaluza–Klein modes have masses of order $\mathcal{O}(1/L)$, and do not appear in the low-energy spectrum of the theory.

For the fermion spinor we have [11]:

$$\mathbf{256} = (\mathbf{16}, \mathbf{8})^{++} + (\overline{\mathbf{16}}, \mathbf{8}')^{-+}. \quad (12)$$

Since only the first of these supports zero modes, the mirror families decouple from low-energy phenomenology.

Together with the bulk spinor and gauge fields, we will incorporate brane-localized scalars which implement spontaneous symmetry breaking by condensation (Higgs mechanism). Further breaking to the Standard Model might proceed through intermediate steps associated with either a Pati–Salam [12] or left-right symmetric [13] stage. However, here we assume just the simplest case of direct breaking by Higgs fields in the representations

$$(\mathbf{210}, \mathbf{1}) + (\mathbf{126}, \mathbf{1}) + (\mathbf{10}, \mathbf{1}). \quad (13)$$

While the scalars $(\mathbf{210}, \mathbf{1})$ and $(\mathbf{126}, \mathbf{1})$ are localized at the Planck brane, the $(\mathbf{10}, \mathbf{1})$ is confined to the IR brane. Quantitative unification of couplings roughly supports this simplest choice *a posteriori*, as will appear. The $(\mathbf{10}, \mathbf{1})$ lies at the TeV scale and drives electroweak breaking. Planck brane scalars naturally acquire large masses, thanks to the warp factor.

A special feature of $SO(8)$ is the existence of three different 8-dimensional representations: vector, spinor, and alternate spinor. They are equivalent to one another under a symmetric S_3 “triality” group of outer automorphisms. For our purposes, it may be simplest to regard the spinor $\mathbf{8}$ of our fermions as an equivalent vector, and break $SO(8) \rightarrow SO(5)$ by means of an adjoint, or three vectors. Alternatively, we might take the spinor as it comes, and note that it decomposes as $\mathbf{8} \rightarrow 2 \times \mathbf{1} + \mathbf{6}$ under the natural $SU(4)$ subgroup of $SO(8)$. We can break to that using a spinor. Then exploiting the isomorphism $SU(4) \rightarrow SO(6)$, we break down to $SO(5)$ using a vector of $SO(6)$. In either case, we have $\mathbf{8} \rightarrow 3 \times \mathbf{1} + \mathbf{5}$ under $SO(8) \rightarrow SO(5)$. Assuming that this breaking occurs through $SO(10)$ singlet scalars localized on the Planck brane, the details do not influence low energy phenomenology.

The upshot is that our low-energy fermions transforms as $3 \times (\mathbf{16}, \mathbf{1}) + (\mathbf{16}, \mathbf{5})$ under $SO(10) \times SO(5)$. Running of couplings down to low energies suggests that the $SO(5)$ becomes strongly interacting at $\mathcal{O}(\text{TeV})$.¹ Thus the $\mathbf{5}$ will be confined, and at low energies we arrive at just three chiral spinor families of $SO(10)$, as desired. (The mechanism of “heavy color confinement” has a long history in this context, see Refs. [2,14,15].)

¹ Note that if confinement takes place above EW scale the only allowed condensate is formed by the SM singlet contained in the $(\mathbf{16}, \mathbf{5})$.

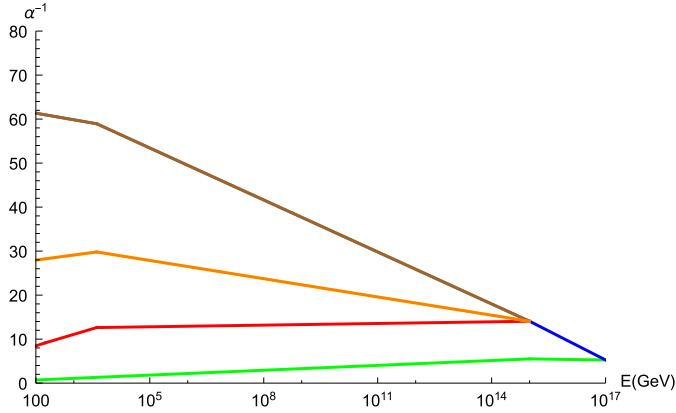


Fig. 1. (Color online) Running of gauge couplings (top-down approach): below the $SO(10)$ scale we have the $SO(5)$ gauge coupling (green line) in addition to the Standard Model couplings (red, orange and brown lines). See text.

Proton decay is potentially very rapid, if the scale of the IR brane is low. The simplest solution is to make that scale large, e.g. associated with conventional unification or with gravitational physics. In this scenario, we are using the extra dimension to address chirality, rather than the hierarchy problem. Other solutions may be possible [16,17].

3. Gauge coupling evolution

One can write the running of the gauge coupling constants in the four dimensional unified gauge theory

$$\alpha_i^{-1}(M_Z) = \alpha_{GUT}^{-1} + \frac{b_i}{2\pi} \log \frac{M_{GUT}}{M_Z} + \Delta_i, \quad (14)$$

where Δ_i denote threshold corrections. Within a five dimensional warped space-time one should take into account contributions from the Kaluza–Klein modes, as well. In [18] it was argued that warped extra dimensions, unlike flat extra dimensions, lead to logarithmic running of couplings. Indeed, an equation similar to Eq. (14) holds, with the b_i given as [16,18]:

$$b_i^{RS} = \frac{1}{3} [-C_2(G)(11I^{1,0}(\Lambda) - \frac{1}{2}I^{1,i}(\Lambda)) + 2I^{1/2,0}(\Lambda)T_f(R) + I^{2,0}(\Lambda)T_s(R)]. \quad (15)$$

We take the cut-off scale to be $\Lambda \sim k$, which implies the numerical values [18]:

$$\begin{aligned} I^{1,0} &= 1.024, \\ I^{1,i} &= 0.147, \\ I^{1/2,0} &= 1.009, \\ I^{2,0} &= 1.005. \end{aligned} \quad (16)$$

For scalars localized on branes, we just change $I^{2,0}(\Lambda) \rightarrow 1$. In Fig. 1 we fix, for definiteness, the unification scale at 10^{15} GeV, and perform a first estimate of the electroweak mixing angle within a top-down approach. We find $\sin^2 \theta_w \approx 0.215$, to be compared with the observed value 0.22. Given our neglect of (inherently uncertain) threshold corrections and higher order renormalization, this seems an acceptable result (see below).

Note that in this simplest case one breaks the $SO(10)$ directly to $SU(3) \times SU(2) \times U(1)$. One finds that the $SO(5)$ coupling reaches non-perturbative values at $\mathcal{O}(\text{TeV})$ (green curve). This fact is reflected into a kink in the evolution of the Standard Model couplings at this value. Thanks to the large number of “active” flavors,

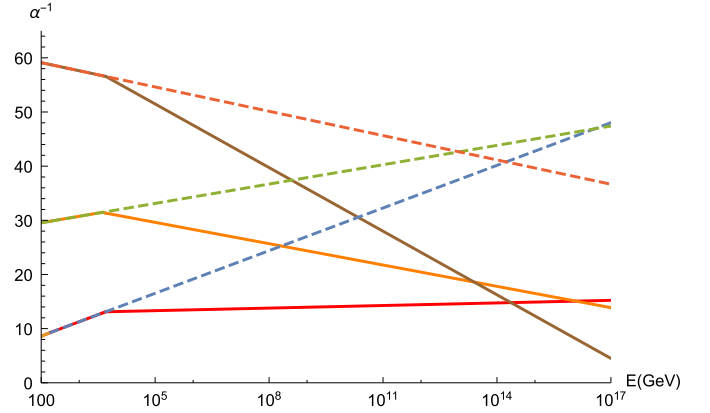


Fig. 2. (Color online) Running of gauge couplings below the $SO(10)$ scale compared with the SM (dashed lines). Bottom-up approach.

the evolution of g_3 is nearly flat all the way from few TeV up to the GUT scale (see red curve). Above the GUT scale α_{10} (blue curve) rises again due to the large Higgs boson multiplets.

In Fig. 2 we compare the bottom-up running at one loop compared with a similar Standard Model extrapolation. One sees that our simple unification scenario gives a marginal improvement with respect to the minimal Standard Model case. However, these results come from a rough estimate, taking renormalization group evolution to first order and neglecting threshold corrections.

Charged fermion masses arise from the $((\mathbf{10}, \mathbf{1}))$ vacuum expectation value,² while neutrino masses can be induced by the conventional (high scale) seesaw mechanism [14,19–22,13]. Note also that the doublet-triplet splitting problem may be solved with a generalization of the Dimopoulos–Wilczek mechanism [23] for $SO(18)$, using a heavy bulk scalar that leaves the $SU(2)$ doublet massless.

As a final comment we note that the breaking of the $SO(3)$ subgroup of $SO(8)$ will be important in connection with the flavor puzzle, and could lead to new ways of addressing details of the family mass hierarchy and mixing pattern. The implementation of specific mechanisms, however, lies beyond the scope of our minimal scenario.

4. Hypercolor and hyperbaryons

The evolution of each of the $SO(10)$ and $SO(8)$ coupling constants can be computed imposing the initial unification condition

$$g_{10}(M_{18}) = g_8(M_{18}), \quad (17)$$

at some scale $M_{18} \lesssim M_P$ where gauge couplings meet. (In our concrete estimates we set M_{18} , the scale which breaks $SO(8)$ to $SO(5)$, at $\approx 10^{17}$ GeV.) The value of $g_{10}(M_{18})$ can be inferred from the observed value of Standard Model couplings. The largest Standard Model coupling at low energies is the g_3 of strong $SU(3)$. Being a larger gauge symmetry, our $SO(5)$ is “more asymptotically free” than $SU(3)$, and we expect that its coupling becomes confining at a larger mass scale. This is confirmed by our numerical estimates. We infer a confinement scale around $\mathcal{O}(\text{TeV})$, in order of magnitude. We will refer to $SO(5)$ as hypercolor, and the $SO(5)$ vector fermions as hyperquarks.

$SO(5)$ supports a Z_2 conserved quantum number, which counts the number of vector indices [24]. It is analogous to quark num-

² The $\mathbf{10}$ scalar belongs to a $\mathbf{18}$ localized at the IR brane, where the $SO(18)$ is not broken by boundary conditions. When orbifold breaking takes place this scalar splits as $\mathbf{18} \rightarrow \mathbf{10} + \mathbf{8}$, and the $\mathbf{8}$ can be decoupled using a generalized Dimopoulos–Wilczek mechanism.

ber (or baryon number) in QCD, but of course the distinction between Z_2 and conventional, additive baryon number has major physical consequences. The lightest unconfined Z_2 odd ($SO(5)$ singlet) states are hyperbaryons. In quark model language, they are formed from 5 hyperquarks; in operator language, the lowest mass dimension operator that creates them involves the product of 5 hyperquark fields. Although they are highly stable individually, hyperbaryons can annihilate into ordinary matter in pairs. Conversely, they might be pair-produced in high energy collisions.

At high enough temperatures in the early universe, $T \gg 10$ TeV, hyperbaryons would be in thermal equilibrium and their number density will be comparable to the photon number density. As the temperature cools below their mass $M \sim 10$ TeV, their equilibrium abundance will diminish, until they become so rare that annihilation cannot keep up with the expansion of the universe, and a residual abundance freezes out. This scenario has a long history in cosmology.

The ratio of the residual number density of hyperbaryons to photons is of order $\sim M/M_{\text{Planck}}$, and the freezeout temperature is parametrically less than M by a logarithmic factor, roughly $\ln M/M_{\text{Planck}}$. A more careful calculation, following [25], gives

$$\Omega_\chi h^2 \approx 10^{-5} (M/\text{TeV})^2 \quad (18)$$

Thus for $M \lesssim 10$ TeV the relic hyperbaryons contribute only a small fraction of the mass density of the universe. In consequence, though the current hyperbaryon relic abundance presents no obvious phenomenological catastrophe, the relic hyperbaryons might conceivably be detectable. One may also envisage that the lightest hyperbaryon might contribute to the dark matter density, as suggested in Ref. [26].

It is noteworthy that this cosmological mass bound ensures that if they exist at all, hyperbaryons are not far beyond the reach of high-energy accelerators currently under discussion.

5. Summary and outlook

We have presented a model of comprehensive unification, bringing together both gauge and family structure, with several attractive features. Within this approach, the existence of multiple fermion families and the fact that they appear in spinor representations of $SO(10)$ are intimately connected. By combining orbifold projection, Higgs symmetry breaking, and hypercolor confinement in a reasonably simple way we can obtain just three chiral families, as is observed. An interesting consequence is the emergence of highly stable hyperbaryons, with mass ~ 10 TeV, protected by a discrete Z_2 symmetry associated with the $SO(5)$ hypercolor group.

They provide an attractive target for accelerator and astrophysical searches. Finally, let us mention that one might attempt to pursue spinor unification further, to bring in the space–time spinor structure, as recently discussed in Ref. [27].

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