A Modified D Numbers' Integration for Multiple Attributes Decision Making

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A modified D numbers' integration for multiple attributes decision making

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Abstract For multiple attributes decision making (MADM), D numbers theory has been widely used to deal with uncertain and incomplete information. However, the incomplete information is abandoned in the D numbers' integration representation. This results in unreasonable conclusions in some real world applications. To overcome this drawback, this paper proposes an improved D numbers' integration representation method, by effectively allocating the incomplete information into decision making according to the original value of D numbers. The proposed method is applied to assess the performance of different types of motorcycles. The results show that the proposed method can effectively increase both the accuracy and efficiency of assessment when compared with the original D numbers theory.

Keywords Uncertainty · Incompleteness · MADM · D numbers theory · D numbers' Integration representation

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1 Introduction

MADM is used to make the optional decision or rank all the programs under multiple attributes. It is one of the important research topics in the decision analysis and decision making fields [1–9]. During the past decades, various methods have been proposed for MADM, such as Laplace criterion, Hurwicz criterion, Analytic hierarchy process (AHP), Technique for order preference by similarity to ideal solution (TOPSIS) [10–15]. The parameter of each attribute of MADM, the utility function and the preference relation are always required in the application of these methods. Nevertheless, it is difficult to obtain the accurate values of these parametric variations because these variations are usually mutually exclusive or immeasurable, qualitative or quantitative. Meanwhile, decision makers usually provide incomplete information and their subjective judgements are used to be uncertain. As a consequence, how to deal with uncertain and incomplete information is still an open issue in MADM. Many methods have been proposed to handle information of both quantitative and qualitative information with uncertainty, such as fuzzy set theory [16–20], rough set [21, 22], uncertainty theory [23, 24], evidential reason approach [25–27]. These methods have been widely used in many areas, such as supplier selection [28, 29], data fusion [30], risk assessments [31, 32], business decision-making [33, 34] and so on [35–37].

Regarded as the extension of Bayesian theory of probability, Dempster Shafer theory [38, 39] has been widely used in MADM too [40–45]. In Dempster−Shafer theory, the basic probability assignment (BPA) has the ability to represent the information of both certain or uncertain, quantitative or qualitative formation by giving confidence degree to any subsets of it. Furthermore, the Depmster's combination rule can combine multiple pieces of assessments. Hence, Dempster−Shafer theory has been used into many real applications, such as risk assessments [46–48], dependence assessment [49], identifying influential nodes in complex networks [50] and so on [51–54]. However, Dempster−Shafer theory has some drawbacks. The first and foremost, the fame of discernment of evidence theory must be mutually exclusive and collectively exhaustive set. This is difficult to be satisfied in the real applications since some information is represented in the linguistic form. For example, the assessments "bad" and "so bad" are formal assessments in our daily lives [55–57]. Recently, D numbers theory is proposed in Ref.[58] to address the shortcoming of Dempster-Shafer theory. D numbers theory reasonably removes some assumptions made in the Dempster Shafer theory, which makes the D numbers theory a powerful method in dealing with uncertain and incomplete information. D numbers theory has been used into the uncertainty in environmental impact assessment [56, 59], bridge condition assessment [60], failure mode and effects analysis [61], supplier selection [62] and curtain grouting efficiency assessment [63, 64].

As an extension of Dempster-Shafer theory, the elements in D numbers theory do not have to be mutually exclusive and collectively exhaustive. However, the D numbers theory also has some limitations. First, the D numbers combination rule for commutative property is not satisfied, that is, the result of combination is effected by the ranking of combinations. To address this drawback, progress has been made in Refs. [56, 59] to improve the D numbers combination rule. Second, in previous D numbers integration representation, the missing parts of assessment are not considered directly if the information is incomplete. That is, for MADM, the final decision only depends on the current information. However, according to the previous D numbers integration representation, the final decision goes against common sense when the information is incomplete. A numerical example is given in the subsequent section to illustrate this limitation of the D numbers theory. Actually, even if information is incomplete, the missing part of assessment can be allocated into the final decision processing. This allocation is only affected by the original value of D numbers. Inspired by this observation, this paper proposes a modified D numbers' integration representation to deal with both complete and incomplete assessments. The proposed method is then used to MADM, which is assessing the performance of some kinds of motorcycles. The results show the effectiveness of the proposed method in MADM.

The rest of the paper is organized as follows. In section 2, some preliminaries are provided. The proposed method is discussed in section 3. An illustrative numerical example in MADM is presented in section 4 and some conclusions are given in section 5.

2 Preliminaries

2.1 Dempster-Shafer theory

Dempster-Shafer theory also called Dempster-Shafer theory or evidence theory, which is a useful method to deal with uncertainty. Dempster-Shafer theory can represent the uncertainty directly by assessing the probability to any subsets of the set consists of N objects, but not distribute the probability to each element of the set. At the same time, upper and lower bounds are given by the probability for the purpose of measuring the total belief and total plausibility for the elements belong to the subset. Meanwhile, Dempster-Shafer theory can combine multiple pieces of evidence or the belief function. Some basic concepts of Dempster-Shafer theory are introduced as follows [38, 39].

A mutually exclusive and collectively exhaustive set U, which is called frame of discernment, is indicated by $U = \{e_1, e_2 ... e_n\}$.

Definition 1 The power set of U is indicated by 2^U, any element is called a proposition, which belongs to the power set 2^U, where

$$2^U = \{\phi, \{e_1\}, \{e_2\} ... \{e_n\} ... \{e_1, e_2\}, ..., \{e_1, e_2, ... e_n\} ... U\} \quad (1)$$

Definition 2 For a given frame of discernment U, a mass function is a mapping m from the power set 2^U to $[0, 1]$, it is defined as,

$$m : 2^U \to [0, 1], \quad (2)$$

with the following conditions satisfied

$$\begin{cases} m(\emptyset) = 0 \\ \sum_{A \subset 2^U} m(A) = 1, \end{cases} \quad (3)$$

where $\emptyset$ is an empty set and A is a subset of 2^U. The function $m(A)$ is called a BPA of the frame of discernment U, it represents how strongly the evidence supports A.

Definition 3 (Dempster's combination rule) For any given two BPAs m_1 and m_2, the Dempster's rule of combination donated as $m = m_1 \oplus m_2$, which is defined as follows,

$$m(A) = \begin{cases} \frac{1}{1-K} \sum_{B \cap C = A} m_1(B) m_2(C), & A \neq \phi; \\ 0, & A = \phi; \end{cases} \quad (4)$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \quad (5)$$

where A, B and C are the elements of the power set 2^U, K is a normalization constant and shows the conflict coefficient of two BPAs. From Equations 4 and 5, commutative and associative properties are both satisfied in Dempster-Shafer theory.

2.2 D Numbers theory

D numbers theory [58], as the extension of Dempster-Shafer theory, can be perfectly put into usage in the incomplete information. Some details of D numbers theory are introduced as follows [56,58].

Definition 4 Let Ω be a finite nonempty set, D numbers is a mapping

$$D : \Omega \to [0, 1], \tag{6}$$

which satisfies the following conditions,

$$\begin{cases} D(\emptyset) = 0 \\ \sum_{B \subset \Omega} D(B) \leq 1, \end{cases} \tag{7}$$

where $\emptyset$ is an empty set and B is a subset of Ω.

The definition of D numbers theory is similarity to that of Dempster-Shafer theory. Nevertheless, there are many differences between Dempster-Shafer theory of evidence and D numbers theory. The elements of D numbers theory do not require mutually exclusive elements and the sum of the assessment may be less than 1. The information is complete, indeed, which is Dempster-Shafer theory, i. e. $\sum_{B \subset \Omega} D(B) = 1$. Otherwise the information is incomplete. To illustrate the difference between Dempster-Shafer theory and D numbers theory, a numerical example is given as follows. Suppose the value of assessment is mapped to a close interval [0,100]. By Dempster-Shafer theory, an expert gives his evaluation as follows,

$$m(\{a_1\}) = 0.3,$$

$$m(\{a_2\}) = 0.5,$$

$$m(\{a_1, a_2, a_3\}) = 0.2,$$

where $(a_1,\ a_2,\ a_3)$ is the frame of discernment and $a_1 = [1, 30]$, $a_2 = [31, 67]$, $a_3 = [68, 100]$, i.e. $a_i \cap a_j = \phi, (i, j = 1, 2, 3; i \neq j)$. Meanwhile, we have $\sum\limits_{i=1}^{3} m(a_i) = 1$. Nevertheless, as mentioned, the elements are not mutually exclusive for linguistic form. The assessments are incomplete because the lack of professional knowledge in the special fields. Thus, another assessment is given by D numbers as follows,

$$D(\{b_1\}) = 0.3,$$

$$D(\{b_2\}) = 0.5,$$

$$D(\{b_1, b_2, b_3\}) = 0.1,$$

where $b_1 = [1, 30]$, $b_2 = [25, 67]$, $b_3 = [60, 100]$. Comparing the values of $a_i, (i = 1, 2, 3)$, the values b_1, b_2 and b_3 are not mutually exclusive. Meanwhile, the sum of D_i is equal to 0.9, which means the information is incomplete. Thus, a special form of D numbers can be expressed as below,

Definition 5 For a discrete set $\Omega = (b_1, b_2, b_3 ... b_n)$, where $b_i, (i = 1, 2, ...n)$ belongs to N^+ and $b_i \neq b_j$ if $i \neq j$, for any $v_i \geq 0$ and $\sum_{i=1}^{n} v_i \leq 1$, D numbers is denoted by:

$$\begin{aligned} D(\{b_1\}) &= v_1, \\ D(\{b_2\}) &= v_2, \\ D(\{b_3\}) &= v_3, \\ &\cdots \\ D(\{b_n\}) &= v_n. \end{aligned}$$

For short, we write it as follows,

$$D = \{(b_1, v_1), (b_2, v_2), (b_3, v_3) ... (b_n, v_n)\}$$

Definition 6 (D numbers theory combination rule) D_1 and D_2 are two D numbers:

$$D_1 = \{(b_1^1, v_1^1)(b_2^1, v_2^1)(b_3^1, v_3^1) ... (b_n^1, v_n^1)\},$$

$$D_2 = \{(b_1^2, v_1^2)(b_2^2, v_2^2)(b_3^2, v_3^2) ... (b_n^2, v_n^2)\},$$

the combination of D_1 and D_2 denoted as $D = D_1 \oplus D_2$, which is defined as follows,

$$D(\{b\}) = v, \tag{8}$$

where

$$b = \frac{(b_i^1 + b_j^2)}{2} \tag{9}$$

$$v = \frac{(v_i^1 + v_j^2)}{2 \times C} \tag{10}$$

$$C = \begin{cases} \sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_j^2}{2} \right) & \sum\limits_{i=1}^{n} v_i^1 = 1 \ and \ \sum\limits_{i=1}^{m} v_j^1 = 1; \\ \sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum\limits_{j=1}^{m} \left(\frac{v_c^1 + v_j^2}{2} \right); & \\ & \sum\limits_{i=1}^{n} v_i^1 < 1 \ and \ \sum\limits_{i=1}^{m} v_j^1 = 1; \\ \sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_c^2}{2} \right); & \\ & \sum\limits_{i=1}^{n} v_i^1 = 1 \ and \ \sum\limits_{i=1}^{m} v_j^1 < 1; \\ \sum\limits_{j=1}^{m} \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum\limits_{j=1}^{m} \left(\frac{v_c^1 + v_j^2}{2} \right) + \sum\limits_{i=1}^{n} \left(\frac{v_i^1 + v_c^2}{2} \right); & \\ & \sum\limits_{i=1}^{n} v_i^1 < 1 \ and \ \sum\limits_{i=1}^{m} v_j^1 < 1; \end{cases} \tag{11}$$

where $v_c^1 = 1 - \sum_{i=1}^{n} v_i^1$ and $v_c^2 = 1 - \sum_{j=1}^{m} v_j^2$, the superscript in above equation is not the exponent but the order of the D numbers.

Definition 7 (D numbers' Integration)

For a given D numbers, $D = \{(b_1, v_1), ...(b_n, v_n)\}$. The value of overall assessment is denoted by,

$$I(D) = \sum_{i=1}^{n} b_i v_i \tag{12}$$

where $I(D)$ is real number. According to the number of value of $I(D)$, the final decision can be obtained. From Equation 12, the overall assessment includes complete assessments and incomplete assessments.

2.3 Aggregate the Assessment by Stepwise Weighing

In MADM problem, many attributes are considered for the assessments of one object. The main attributes are influenced by an army of sub-attributes. So a hierarchical model for the assessment is built based on this structure firstly [60]. Considering the complexity of handing multiple attribute simultaneously, different attributes play different roles in the decision making, the main attributes must be more important than the others. Thus, it is paramount to get the weights of each attribute secondly. However, experts are always achieving different degree of consistency because of their different knowledge background or experience, so deciding the weight of the attributes may involve in a lot of incomplete and uncertain information. Once the weights and assessments denoted by D numbers are obtained, the stepwise weighting method is adopted to aggregate the assessments. The weights and the assessments of each attribute are represented by ω_i and $I(D_i)$ respectively, then the final overall assessment is given as follows [60],

$$E(D) = \sum_{i=1}^{n} \omega_i I(D_i) \tag{13}$$

3 Limitations and proposed method

3.1 Limitations of previous D numbers' integration representation

In this section, two examples are given to illustrate the lack of D numbers theory. In D numbers' integration, for both complete and incomplete information, we denote that $I(D) = \sum_{i=1}^{n} b_i v_i$. In our opinions, it is not appropriate for the incomplete information. According to the definition, the incompleteness probability is the same as distributing it to the proposition '0'. Thus, one example is shown as follows.

Example 1: an assessment given by D numbers is shown as follows,

$$D = \{(3, 0.7)\}$$

According to Equation 12, it holds that

$$I(D) = 3 \times 0.7 = 2.1$$

That is, the degree of incompleteness, i. e. $1-0.7 = 0.3$, is abandoned or distributed to the assessments '0'. So the assessment is the same with the evaluation of $D = \{(0, 0.3), (3, 0.7)\}$.

Some illogical results are obtained since incomplete information has been abandoned in D numbers' integration for some examples.

Example 2: Suppose the assessments on two cars A and B are conducted and the evaluation system is conducted on the set $\Omega = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, where 1 might means "extremely poor" quality and 9 means "pretty good" quality. Ten experts are invited to evaluate car A, six experts assess the car to be 7, two experts evaluate it to be 8, while the remaining two experts do not give their assessments because of lack of professional knowledge about this kind of car, so the assessment can be represented by D numbers as below,

$$D_A = \{(7, 0.6), (8, 0.2)\}$$

While the same ten experts are invited to evaluate car B, all of them are familiar with car B and all of them evaluate car B to be 6. So this evaluation can be represented by D numbers as below,

$$D_B = \{(6, 1)\}$$

According to D numbers integration, the final score can be calculated. Then, we can make decision according to the final score. According to Equation 12, we have

$$I(D_A) = 7 \times 0.6 + 8 \times 0.2 = 5.8$$

and

$$I(D_B) = 6 \times 1 = 6$$

Note that $6 > 5.8$. Thus the quality of car B is better than that of car A according to the final score. However, the experts who are familiar with car A give the better assessments to car B because of $7 > 6$ and $8 > 6$. That is, the final score is not in consistent with the actual physics. Worse, customers are trend to choose car B because of the better final score. Indeed, the reason of believing car B is better than car A is that two experts don't give their assessments.

From example 1 and example 2, incomplete information cannot be abandoned in the overcall processing of assessments. In next section, the overcall assessment is modified.

3.2 Proposed method

As explained above, the existing D numbers' integration is not appropriate for problems with incomplete information. The main reason for the circumstance is the incomplete information has been abandoned. Actually, it should be allocated to the evaluated grades according to the information we have obtained. Firstly, we may obtain wrong results if the incomplete information is abandoned. The numerical examples have illustrated that in the last section. Secondly, the incomplete information can be allocated into other assessments. For a given D number, $D = \{(b_1, v_1), (b_2, v_2), ...(b_i, v_i), ...(b_n, v_n)\}$, $v_i, (i = 1, 2..., n)$ is the confidence degree about the assessment grade $b_i, (i = 1, 2, ..., n)$. $\frac{v_i}{\sum\limits_i v_i}$ is proportion of the confidence degree for the assessment grade $b_i, (i = 1, 2, ...n)$ in all assessments. Hence, the previous overall assessment is modified as follows.

Definition 8 For $D = \{(b_1, v_1), ...(b_n, v_n)\}$. The overall assessment is denoted by,

$$I^{'}(D) = \sum_{i=1}^{n}\{b_i \times [v_i + (1 - \sum_{i=1} v_i) \times \frac{v_i}{\sum_{i=1} v_i}]\} \quad (14)$$

When the information is complete, i.e. $\sum_{i=1}(v_i) = 1$, Equation 14 is the same as Equation 12, then the proposed method is degenerated as the traditional D numbers' integration. If the information is incomplete, i.e. $\sum_{i=1}(v_i) < 1$, the incomplete information is separated to the assessments by the confidence to their assessments.

Using our proposed method, assessing two cars A and B in section 3.1 is considered. The assessments on car B is represented by D numbers as $D_B = \{(6, 1)\}$. According to Equation 14, the overcall assessment is calculated as follows,

$$I^{'}(D_B) = 6 \times 1 = 6$$

It is the same as the original D numbers' integration. If the given information is incomplete, for instance, D numbers of the assessments of car A in section 3.1 is given by $D_A = \{(7, 0.6), (8, 0.2)\}$. That is, the incomplete probability is 0.2, i. e. $1 - 0.6 - 0.2 = 0.2$. Due to our method, 0.2 should be allocated into the assessment grades "7" and "8". According to Equation 14, we have

$$I^{'}(D_A) = 7 \times (0.6 + 0.2 \times \frac{0.6}{0.8}) + 8 \times (0.2 + 0.2 \times \frac{0.2}{0.8}) = \frac{29}{4}$$

Note that $\frac{29}{4} > 6$, i. e. the value of final assessment of car A is more than that of car B. Thus, car A is trended to be chosen according to experts' assessments. It is reasonable because the experts who are acquainted with those cars give car A better assessments.

Accordingly, our proposed D numbers theory has four steps for handing MADM.

1). All the attributes of MADM problem are evaluated. Usually, the evaluation is represented using linguistic formation and is given by some experts.

2). All the linguistic evaluations are translated into D numbers. For example, ten experts are invited for the assessment of a attribute in MADM. Five experts think that is good, three experts think that is bad, while the remaining experts don't give their opinions because of lack of professional knowledge about that attribute. Suppose the linguistic formation are made of "bad", "average" and "good", the value of them are "5","7" and "9", respectively. Then, the assessment of attribute can be represented by D numbers, where $D = ((5, 0.3), (9.0.5))$.

3). Although there has incomplete information, i. e. $1 - 0.3 - 0.5 = 0.2$, the incomplete information is allocated according to Equation 14. Thus, the final overcall assessments of all attributes are calculated.

4). Aggregate the assessment by stepwise weighing is considered. All selections of MADM are ranked.

4 Applications in MADM

4.1 An assessment of motorcycles in MADM

In this section, a tutorial example in MADM in Refs.[65, 66] is used to illustrate the effectiveness of the modified D numbers theory. The MADM problem is about assessing the performance of four kinds of motorcycles, which include Kawasaki, Yamaha, Honda, and BMW. For MADM, usually the overall assessment includes many attributes. For the assessment of motorcycles, the overall assessments of each motorcycle are evaluated with three main attributes, namely, the quality of engine, operation, and general finish. These attributes are difficult to be assessed directly. So these three main attributes are influenced by several sub-attributes. For example, when assess the attribute "operation", handing, transmission and the brakes are three main sub-attributes for the upper attribute, each sub-attribute is correspondingly decomposed into several detailed sub-attributes, which are shown in Fig.1 [65,66]. From Fig.1, twenty basic attributes are chosen for the assessments of the motorcycles and the hierarchical structure in Figure 1 can often be expanded for general applications in MADM.

Then the qualitative attributes are all assessed on five same basic evaluation grades, which are usually de-

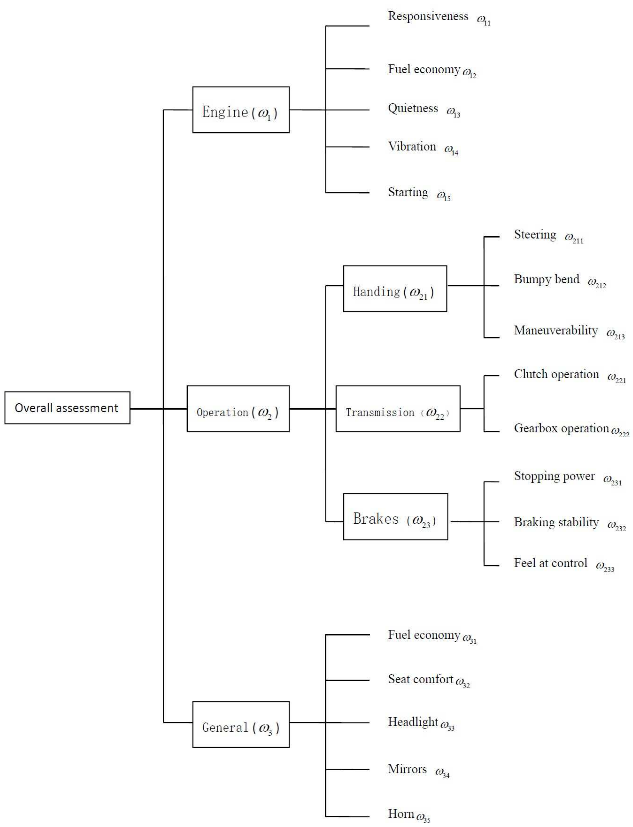

Fig. 1 Evaluation hierarchy for motorcycle performance assessment[65, 66]

fined as "poor" "indifferent", "average", "good" and "excellent", P, I, A, G and E are used for the representation for short respectively [65, 66]. Then imprecise and precise assessments are involved in the decision matrix. Absence and incomplete evaluation also exist in the assessments. The assessment data for the motorcycles selection problem is shown in Table 1.

Furthermore, all relevant attributes are set to be of equal relative importance, that is, all the lower sub-attributes are of the same weights to the upper level [66]. We have

$$\omega_1 = \omega_2 = \omega_3 = \frac{1}{3}$$

$$\omega_{11} = \omega_{12} = \omega_{13} = \omega_{14} = \omega_{15} = \frac{1}{5}$$

$$\omega_{21} = \omega_{22} = \omega_{23} = \frac{1}{3}$$

$$\omega_{211} = \omega_{212} = \omega_{213} = \omega_{214} = \frac{1}{4}$$

$$\omega_{221} = \omega_{222} = \frac{1}{2}$$

$$\omega_{231} = \omega_{232} = \omega_{233} = \frac{1}{3}$$

$$\omega_{31} = \omega_{32} = \omega_{33} = \omega_{34} = \omega_{35} = \frac{1}{5}$$

where ω_i, ω_{ij} and ω_{ijk} are the weights of the main attributes, the sub-attributes and the basic attributes in Fig. 1 respectively.

4.2 Applications in the assessment of motorcycles

In Table 1, some values are special since their values of assessment are incomplete, such as $\{(E, 0.8)\}$ and $\{(G, 0.3)(E, 0.6)\}$ in the second line. In this case, we have $0.8 < 1$ and $0.3 + 0.6 < 1$. Indeed, in practice, ten experts are invited for the assessments of the four kinds of motorcycles, six experts evaluate the motorcycle average, the other four experts evaluate it to be good, so the final assessments can be represented as $\{(A, 0.6)(G, 0.4)\}$. Some evaluations and random numbers are incomplete in the sense the overall degree of belief in an assessment is not summed to one, if six experts assess the motorcycle to be average and two of them evaluate it to be good while the left experts don't give their opinions because of lack of professional knowledge about that motorcycles, then the evaluation can be represent as $\{(A, 0.6)(G, 0, 2)\}$, where the total belief degree is $0.2 + 0.6 < 1$.

In a word, there are incomplete assessment in real MADM example. According to the definition of D numbers theory, all these assessments are represented by D numbers. To change all the assessments represented by D numbers in linguistic structures into numerical formations. The utility function $\mu' \to [0, 1]$ defined in Ref. [66] is used, the definition of the utility function is detailed shown as follows,

$$\begin{cases} \mu'(P) = 0; \quad \mu'(I) = 0.35; \\ \mu'(A) = 0.55; \mu'(G) = 0.85; \mu'(E) = 1; \end{cases} \tag{15}$$

According to Equation 15, we can change all the assessments represented by D numbers in linguistic structures into numerical formations. For example, if the assessment grade is evaluated to be excellent (E) by all the experts, then the utility function is 1 corresponding, and the D numbers is $\{(1, 1)\}$.

Thus, according to previous methods, all elements of Table 1 given by D numbers are shown in Table 2. From Table 2, some incomplete assessment are drawn by red color. That is, $\sum_{\text{i}} v_i < 1$.

Then, according the proposed method, the incomplete information (i.e. $1 - \sum_{\text{i}} v_i$) is allocated to each v_i. By using Equation 14, for example, in Table 2, the

Table 1 Assessment data for the motorcycle selection problems [65, 66]

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Responsiveness	{(E,0.8)}	{(G,0.3)(E,0.6)}	{(G,1.0)}	{(I,1.0)}
Fuel economy	{(A,1.0)}	{(I,1.0)}	{(I,0.5)(A,0.5)}	{(E,1.0)}
Quietness	{(I,0.5)(A,0.5)}	{(A,1.0)}	{(G,0.5)(E,0.3)}	{(E,1.0)}
Vibrating	{(G,1.0)}	{(I,1.0)}	{(G,0.5)(E,0.5)}	{(P,1.0)}
Starting	{(G,1.0)}	{(A,0.6)(G,0.3)}	{(G,1.0)}	{(A,1.0)}
Steering	{(E,0.9)}	{(G,1.0)}	{(A,1.0)}	{(A,0.6)}
Bumpybenda	{(A,0.5)(G,0.5)}	{(G,1.0)}	{(G,0.8)(E,0.1)}	{(P,0.5)(I,0.5)}
Maneuverability	{(A,1.0)}	{(E,0.9)}	{(I,1.0)}	{(P,1.0)}
Top speed stability	{(E,1.0)}	{(G,1.0)}	{(G,1.0)}	{(G,0.6)(E,0.4)}
Clutchopeartion	{(A,0.8)}	{(G,1.0)}	{(A,0.5)(G,0.5)}	{(I,0.2)(A,0.8)}
Gearbox opeartion	{(A,0.5)(G,0.5)}	{(I,0.5)(A,0.5)}	{(E,1.0)}	{(P,1.0)}
Stoppingpower	{(G,1.0)}	{(A,0.3)(G,0.6)}	{(G,0.6)}	{(E,1.0)}
Brakibg stability	{(G,0.5)(E,0.5)}	{(G,1.0)}	{(A,0.5)(G,0.5)}	{(E,1.0)}
Feelatcontrol	{(P,1.0)}	{(G,0.5)(E,0.5)}	{(G,1.0)}	{(G,0.5)(E,0.5)}
Qality of finish	{(P,0.5)(I,0.5)}	{(G,1.0)}	{(E,1.0)}	{(G,0.5)(E,0.5)}
Seat comfort	{(G,1.0)}	{(G,0.5)(E,0.5)}	{(G,0.6)}	{(E,1.0)}
Headlight	{(G,1.0)}	{(A,1.0)}	{(E,1.0)}	{(G,0.5)(E,0.5)}
Mirrors	{(A,0.5)(G,0.5)}	{(G,0.5)(E,0.5)}	{(E,1.0)}	{(G,1.0)}
Horn	{(A,1.0)}	{(G,1.0)}	{(G,0.5)(E,0.5)}	{(E,1.0)}

Table 2 Assessment data for the motorcycle selection problem represented by D numbers

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Responsiveness	{(1,0.8)}	{(0.85,0.3)(1,0.6)}	{(0.85,1.0)}	{(0.35,1.0)}
Fuel economy	{(0.55,1.0)}	{(0.35,1.0)}	{(0.35,0.5)(0.55,0.5)}	{(1,1.0)}
Quietness	{(0.35,0.5)(0.55,0.5)}	{(0.55,1.0)}	{(0.85,0.5)(1,0.3)}	{(1,1.0)}
Vibrating	{(0.85,1.0)}	{(0.35,1.0)}	{(0.85,0.5)(1,0.5)}	{(0,1.0)}
Starting	{(0.85,1.0)}	{(0.55,0.6)(0.85,0.3)}	{(0.85,1.0)}	{(0.55,1.0)}
Steering	{(1,0.9)}	{(0.85,1.0)}	{(0.55,1.0)}	{(0.55,0.6)}
Bumpybenda	{(0.55,0.5)(0.85,0.5)}	{(0.85,1.0)}	{(0.85,0.8)(1,0.1)}	{(0,0.5)(0.35,0.5)}
Maneuverability	{(0.55,1.0)}	{(1,0.9)}	{(0.35,1.0)}	{(0,1.0)}
Top speed stability	{(1,1.0)}	{(0.85,1.0)}	{(0.85,1.0)}	{(0.85,0.6)(1,0.4)}
Clutchopeartion	{(0.55,0.8)}	{(0.85,1.0)}	{(0.55,0.5)(0.85,0.5)}	{(0.35,0.2)(0.55,0.8)}
Gearbox opeartion	{(0.55,0.5)(0.85,0.5)}	{(0.35,0.5)(0.55,0.5)}	{(1,1.0)}	{(0,1.0)}
Stoppingpower	{(0.85,1.0)}	{(0.55,0.3)(0.85,0.6)}	{(0.85,0.6)}	{(1,1.0)}
Brakibg stability	{(0.85,0.5),(1,0.5)}	{(0.85,1.0)}	{(0.55,0.5)(0.85,0.5)}	{(1,1.0)}
Feelatcontrol	{(0,1.0)}	{(0.85,0.5)(1,0.5)}	{(0.85,1.0)}	{(0.85,0.5)(1,0.5)}
Qality of finish	{(0,0.5)(0.35,0.5)}	{(0.85,1.0)}	{(1,1.0)}	{(0.85,0.5)(1,0.5)}
Seat comfort	{(0.85,1.0)}	{(0.85,0.5)(1,0.5)}	{(0.85,0.6)}	{(1,1.0)}
Headlight	{(0.85,1.0)}	{(0.55,1.0)}	{(1,1.0)}	{(0.85,0.5)(1,0.5)}
Mirrors	{(0.55,0.5)(0.85,0.5)}	{(0.85,0.5)(1,0.5)}	{(1,1.0)}	{(0.85,1.0)}
Horn	{(0.55,1.0)}	{(0.85,1.0)}	{(0.85,0.5)(1,0.5)}	{(1,1.0)}

assessments about the sub-attribute responsiveness of the motorcycle "Honda", the assessment is {(0.85,0.3), (1,0.6)}, we have

$$\begin{aligned} I^{'}(D) &= 0.85 \times (0.3 + (1 - 0.3 - 0.6) \times \frac{0.3}{0.9}) \\ &+ 1 \times (0.6 + (1 - 0.3 - 0.6) \times \frac{0.6}{0.9}) \\ &= 0.95 \end{aligned} \tag{16}$$

Similarly, the results of integrating the assessments of all the bottom sub-factors are calculated and shown in Table 3.

At last, the overall evaluations of these four motorcycles can be obtained by stepwise weighing from sub-attributes to the three main attributes. Table 4 shows the results of weighing aggregation on sub-attributes.

Table 5 gives the results of weighing aggregation on components for each main attribute.

To summarize, the proposed method is applied to MADM to deal with uncertain and incomplete information. Usually, it contains four steps, which are drawn as the last section. The modified D numbers' integration representation makes great difference to the final decision in step three and step four.

Table 3 Integration of the assessment results for bottom factors

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Engine$(\frac{1}{3})$				
Responsiveness	1.00	0.95	0.85	0.35
Fuel economy	0.55	0.35	0.45	1.00
Quietness	0.45	0.55	0.91	1.00
Vibrating	0.85	0.35	0.925	0
Starting	0.85	0.65	0.85	0.55
Opeartion$(\frac{1}{3})$				
Handing				
Steering	1.00	0.85	0.55	0.55
Bumpybenda	0.70	0.85	0.87	0.175
Maneuverability	0.55	1.00	0.35	0
Top speed stability	1.00	0.85	0.85	0.91
Transmission				
Clutchopeartion	0.55	0.85	0.70	0.51
Gearbox opeartion	0.70	0.45	1.00	0
Brakes				
Stoppingpower	0.85	0.75	0.85	1.00
Brakibg stability	0.925	0.85	0.70	1.00
Feelatcontrol	0	0.925	0.85	0.925
General$(\frac{1}{3})$				
Qality of finish	0.175	0.85	1.00	0.925
Seat comfort	0.85	0.925	0.85	1.00
Headlight	0.85	0.55	1.00	0.925
Mirrors	0.70	0.925	1.00	0.85
Horn	0.55	0.85	0.925	1.00

Table 4 Weighing aggregation on sub-factors.

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Engine$(\frac{1}{3})$				
Responsiveness	0.333	0.317	0.283	0.117
Fuel economy	0.183	0.117	0.150	0.333
Quietness	0.150	0.183	0.303	0.333
Vibrating	0.283	0.117	0.308	0
Starting	0.283	0.217	0.283	0.183
Opeartion$(\frac{1}{3})$				
Handing$(\frac{1}{4})$				
Steering	0.333	0.283	0.183	0.183
Bumpybenda	0.233	0.283	0.290	0.058
Maneuverability	0.183	0.333	0.117	0
Top speed stability	0.333	0.283	0.283	0.303
Transmission$(\frac{1}{2})$				
Clutchopeartion	0.275	0.425	0.350	0.255
Gearbox opeartion	0.350	0.225	0.500	0
Brakes$(\frac{1}{3})$				
Stoppingpower	0.283	0.250	0.283	0.333
Brakibg stability	0.308	0.283	0.233	0.333
Feelatcontrol	0	0.308	0.283	0.308
General$(\frac{1}{5})$				
Qality of finish	0.058	0.283	0.333	0.308
Seat comfort	0.283	0.308	0.283	0.333
Headlight	0.283	0.183	0.333	0.308
Mirrors	0.233	0.308	0.333	0.283
Horn	0.183	0.283	0.308	0.333

Table 5 Weighting aggregation on three main attributes assessing motorcycles

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Engine$(\frac{1}{3})$	0.740	0.570	0.796	0.580
Opeartion$(\frac{1}{3})$	0.676	0.793	0.768	0.546
General$(\frac{1}{3})$	0.625	0.820	0.955	0.940

4.3 Analyzing the results

From Equation 16, the value of $I^{'}(D)$ is 0.95. However, the value of $I(D) = 0.85 \times 0.3 + 1 \times 0.6 = 0.855$ according to Equation 12. The result indicates that there is different between the classical D numbers theory and the proposed method.

Note that the value of difference between the classical D numbers theory and the proposed method is small (i.e. $0.95 - 0.855 = 0.095$). In practice, the small value does not affect the decision-making. For example, we compare the ranking results of five methods, which are modified evidential reasoning methods (shortly, YMER method) [65], the method of the weighted sum aggregation scheme (shortly, WS method) [65], the modified Yager's combination method (shortly, MYMER method) [66], the original D numbers method (shortly, original method) and our proposed D numbers method (shortly, proposed method) are shown in Table 6.

Table 6 Comparison of raking of four motorcycles

YMERmethod	$Honda > Yamaha > BMW > Kawasaki$
WS method	$Honda > Yamaha > BMW > Kawasaki$
MYMERmethod	$Honda > BMW > Yamaha > Kawasaki$
Original method	$Honda > Yamaha > BMW > Kawasaki$
Proposed method	$Honda > Yamaha > BMW > Kawasaki$

From Table 6, the ranking results of proposed method is completely consistent with that of YMER method, WS method, and even original D numbers method. Only the result of MYMER method is different with others. The results illustrate the proposed method is efficient in dealing with MADM as other methods. Meanwhile, the results also indicate there may be same in real MADM even using different methods. In YMER method and WS method, incomplete information such as $\{(E, 0.8)\}$ and $\{(G, 0.3)(E, 0.6)\}$, are not considered. As Jianbo Yang said in ref [65]: "Real-world decision problems are complex. It should also be noted that there may be overlap in utility intervals if there is greater incompleteness in original assessments. In such circumstances, it may be necessary to improve the quality of the original information to achieve a reliable ranking". That is, it is very important to improve the quality of the original information.

However, there are always some incomplete information in real-world decision problems. In this case, degrees of distinction of our method is better than that of original D numbers method. From Equations (12),(13) and (14), the overall assessments and the ranking of

Table 7 The overall assessments and the ranking of the four motorcycles by proposed method and Original method, respectively.

Basic attribute	*Kawasaki*	*Yamaha*	*Honda*	*BMW*
Proposed method				
Assessments	0.680	0.728	0.839	0.687
Ranking	4	2	1	3
Original method				
Assessments	0.658	0.712	0.770	0.696
Ranking	4	2	1	3

the four motorcycles of the proposed method and the original method are shown in Table 7.

From Table 7, in our proposed method, the value of assessments of "Kawasaki", "Yamaha", "Honda" and "BMW" are "0.680", "0.728", "0.839", and "0.687", respectively. In original D numbers method, the value of assessments of "Kawasaki", "Yamaha", "Honda" and "BMW" are "0.658", "0.712", "0.770", and "0.696", respectively. In our method, the difference between maximum and minimum is 15.9%. However, in the original method, the difference between maximum and minimum is 11.2%. That is, our method increases the accuracy of distinguishing by 4.7%.

The precision of them is shown in Fig. 2. From Fig. 2, the solid line is more flat than dotted line, the dotted line (our proposed method) shows a much more clear ranking results, we can easily find that Honda ranks the first and Yamaha ranks the second, Kawasaki and BMW are of worsest quality. However, the change rate by stepwise weighting with the original D numbers' integration is not obvious, the evaluation results of the four motorcycles is nearly at the same level.

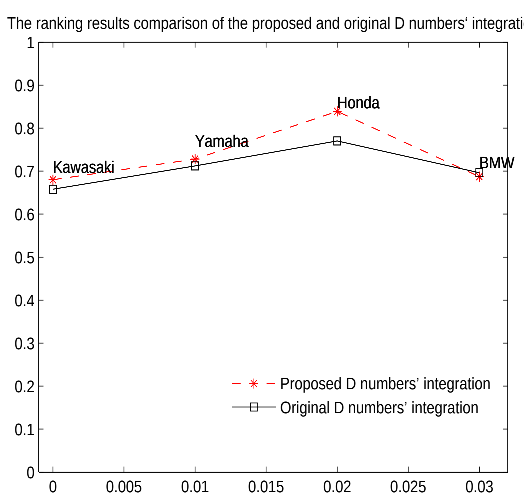

Fig. 2 The ranking result comparsion

5 Conclusion and discussion

In MADM, decision makers are hard to make a decision because both complete and incomplete, certain and uncertain, quantitative and qualitative assessments need to be handled together in a reasonable, systematic and reliable way. D numbers theory provides a pragmatic way to hand these information in MADM. The frame of discernment must be mutually exclusive and collectively exhaustive set in evidence theory no longer needs to be satisfied in D numbers theory. The key issue of D number theory is to deal with incomplete information. In combination of several D numbers, incomplete information have been consider fully by Equation 11. However, in D numbers' integration (Equation 12), incomplete information is overlooked. Irrationality of D numbers' integration is given by some numerical cases (e.g. examples 1 and 2 in section 3).

In this paper, the D numbers' integration is modified. The incomplete part is separated proportionally according to the information we got. Two major contributions of the proposed method are drawn as follows.

1) The incomplete information are not only considered in combine processing but in the rule of D numbers' integration. For real decision-making problem, the final result is important. This result is given by D numbers' integration in D number theory. Thus, it has important significance for modifying D numbers' integration.

2)In proposed method, the modified D numbers' integration only depends on the values of D numbers themselves. We separate incomplete part into D numbers' integration according to the proportionally of themselves.

For real decision-making problems in uncertainty environment, we have usually two ways to improve the accuracy of decision. One is to make the value of incomplete information as small as possible. In this case, the values of incomplete part are usually small. That is, the value of $(1-\sum_{i=1} v_i)$ is small. Thus, the difference between original D numbers' integration and modified D numbers' integration is not obvious. That is, the value of between Equation 12 and Equation 14 is small.

Another is improving the accuracy of theory model. There is no denying that proposed method of this paper has theory significance and practical value. For instance, an real example about assessing four different motorcycles is adopted to illustrate efficiency of the proposed method. The accuracy of distinguishing is increased by 4.7% for the studied four motorcycles. Meanwhile, the value of weight of each attribute is equal in this paper. In the future, different value of weights of attribute will be considered by using the modified D numbers theory for MADM. Meanwhile, the D numbers theory will be developed to denote more incomplete and uncertain information in MADM and MCDM, such as the degree for each linguistic value assigned by decision makers to each criterion in the hierarchical structure..

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