Labor Demand: Lecture 7

Outline:

Labor demand theory: mostly from Hamermesh's Handbook article

Impact of immigration on wages and employment:	Johnson (ILRR, April 1980)
	Borjas (QJE, Nov 2003)
	Card (JOLE 2001)
	Card (ILR, Mariel Boatlift paper)
Minimum Wage (I won't discuss in class)	Card and Krueger and Neumark, AER

If the supply curve for labor is not completely inelastic (vertical), then labor demand helps determine the equilibrium wage that workers obtain. This theory was developed as far back as Hicks' work in the 1930s, so note there was no data to really examine the initial discussion of labor demand at the time. In many cases, economists are interested in the demand for labor for the sake of knowing the expected response to wages from a change in labor demand from things such as a technology shock, unionization, and business cycle fluctuations.

The theory of two-factor labor demand

Suppose there are two factors used to produce Y. The usual approach is to consider these factors be labor, L, and capital, K, although the analysis also applies when considering high skill versus low skill, old versus young, and immigrant versus non-immigrant workers.

The production function is: $Y = F(L, K)$, and usually we assume: $F_i > 0, F_{ii} < 0$, We also assume constant returns to scale:

$$\delta Y = F(\delta L, \delta K).$$

Firms maximize profit, $\pi = F(L,K) - wL - rK$, by choosing how much of each factor to use. w is the cost of L and r is the cost of K.

The first order conditions are:

$$F_L - \lambda w = 0$$
$$F_K - \lambda r = 0,$$

where λ is the Lagrangean multiplier. The ratio of the two conditions shows the familiar statement that the marginal rate of technical substitution, F_L / F_K, equals the factor-price ratio, w/r, for a profit-maximizing firm.

A crucial parameter of interest in the labor demand framework is the elasticity of substitution between K and L, holding output constant. This is the rate of change in the use of K to L from a change in the relative price of w to r, holding output constant. The definition of this elasticity is:

$$\sigma = \frac{d\ln(K/L)}{d\ln(w/r)}\Big|_{\mathrm{Y=constant}} = \frac{\text{\% change in K/L}}{\text{\% change in w/r}}.$$

Intuitively, this elasticity measures the ease of substituting one input for the other when the firm can only respond to a change in one or both of the input prices by changing the relative use of two factors without changing output.

If σ approaches infinite, the two factors become perfect substitutes, while as σ approaches zero, the two factors cannot be substitutes. A low σ is desirable from a worker's perspective, because it implies a firm cannot replace the worker easily with another factor input.

This elasticity can also be expressed more intuitively as follows:

(1) $$\sigma = \frac{d\ln(K/L)}{d\ln(w/r)}\Big|_{\mathrm{Y=constant}} = \frac{F_L F_K}{Y F_{LK}}$$

Equation (1) shows that σ is always non-negative. The value of F_{LK} depends on the shape of the production function, but is always positive under usual production function assumptions. It is by no means trivial to derive (1). You should go through it at least once. I provide a proof at the end of my notes.

We are also, of course, interested in the straightforward response to the change in demand for labor from a change in its wage. This is the constant output labor demand elasticity:

$$\eta_{LL} = \frac{d \ln L}{d \ln w} |_{\mathrm{Y=constant}} = \frac{\text{\% change in Labor Demand}}{\text{\% change in wage}}.$$

It turns out that this is:

(2) $\eta_{LL} = -(1-s)\sigma < 0$

where s is the share of labor in total revenue: $s = \frac{wL}{Y}$. When output requires substantial amounts of labor for production, the constant output labor demand elasticity will be smaller, because the possible change in spending on other factors is small relative to the amount of labor being used. See proof at the back of the notes.

The constant output cross-elasticity of demand for labor describes the response to labor from a change in the price of the other factor, in this example, capital:

(3)
$$\eta_{LK} = \frac{d \ln L}{d \ln r} |_{\mathrm{Y=constant}} = \frac{\text{\% change in Labor Demand}}{\text{\% change in r}}$$
$$= (1-s)\sigma > 0$$

Finally, we need to take into account the possibility that output will change as a response to a change in the price of labor, and that in turn may affect the overall demand for labor, we can take into account the ‘scale effect’. When the wage rate increases, the cost of producing a given output rises. In a competitive product market, a 1 percent rise in a factor price raises cost, and eventually product price, by that factor’s share. This reduces the quantity of output sold. The scale effect is thus the factor’s share times the product demand elasticity. Thus, the total response from a change in the wage is:

(4) $\eta'_{LL} = -(1-s)\sigma - s\eta$,

where $\eta = \frac{d\ln Y}{d\ln p} = \frac{\%\text{ change in output}}{\%\text{ change in output price}}$.

Equation (4) is the fundamental factor law of demand. It divides the labor demand elasticity into substitution and scale effects. The environment for which this elasticity holds is one with constant returns to scale production, perfect competitive, and every firm faces the same production function and output demand elasticity.

Both (2) and (4) are helpful to try estimate the elasticity of labor demand, depending on the assumptions one wishes to make about the problem under study.

The alternative approach derives the elasticity of labor demand from the cost function: total cost expressed as a function of optimized demand for factors of production. If a firm maximizes profits, this also implies they minimize costs.

A firm chooses L and K to minimize: $C = wL + rK$, subject to output takes on a particular value: $Y = F(L, K)$

After solving for L and K from the first order conditions, we can get express costs that minimize a certain level of production, subject to w, r, and Y:

(5) $C = C(w, r, Y)$.

This is the cost function, which has several useful properties that are derived from the assumptions about the production function and the firm's optimizing behaviour. Among them, $C_w > 0, C_r > 0, C_{ij} > 0$, and the optimal levels for labor and capital demanded are equal to their respective partial derivatives:

(6) $$\begin{aligned} L^* &= C_w \\ K^* &= C_r \end{aligned}.$$

It turns out that the same constant-output elasticity of substitution can be derived as:

(7) $$\sigma = \frac{CC_{wr}}{C_w C_r},$$

assuming constant returns to scale. Proof at end of notes.

The corresponding factor demand elasticities are:

(8) $$\begin{aligned} \eta_{LL} &= -[1-m]\sigma \\ \eta_{LK} &= [1-m]\sigma \end{aligned},$$

where m is the share of labor in total costs: $m = \frac{wL}{C}$. Equation (8) is equivalent to (2) and (3). Equation (7) is equivalent to equation (1).

Estimating Elasticities of Labor Demand

The game of estimating these elasticities is to propose a production function that ameliorates the estimation process. For example, forget using Cobb-Douglas: the elasticity of substitution is fixed at one. As another example, another production function is the Constant Elasticity of Substitution function (CES), which, as you might guess from the name, the elasticity of labor demand does not depend on current production, or costs. The CES function is:

(9) $$Y = A[\delta L^{-\rho} + (1-\delta)K^{-\rho}]^{-1/\rho}$$

Finding first order conditions, we get

(10) $$\frac{F_K}{F_L} = \frac{1-\delta}{\delta}\left(\frac{L}{K}\right)^{\rho+1}$$

Taking logs and rearranging:

(11) $$\log\left(\frac{L}{K}\right) = \beta_0 + \beta_1\left(\frac{r}{w}\right),$$

where $\beta_1 = \frac{1}{1+\rho}$.

And we can try to estimate this, adding an error term. And estimate of the constant-output elasticity of labor demand is therefore $\hat{\beta}_1$. Unfortunately, this specification seems grossly unrealistic: the elasticity does not depend on the current level of production, or the current relative use of each factor.

Note, if the price of capital is constant, we are in effect estimating a regression equation similar to one we've seen before for labor supply:

$$\log L_i = \delta_0 + \delta_1 w_i + e_i ,$$

But the interpretation of the coefficient is entirely different. This gives us some idea of the complexities of estimating these elasticities. We need some way of determining whether the reason for the wage fluctuations are due to exogenous changes in labor supply, or exogenous changes in labor demand. We also need to assure no omitted variables bias. As you can imagine, the credibility of these estimates depends crucially on the research design of the analysis.

Perhaps the most popular method of estimating the elasticities of labor demand is to use the translog cost function, which is often interpreted as a second-order approximation to an unknown functional form. One way to derive it is as follows: From the unknown function, $C = C(w,r,Y)$, if there is constant returns to scale, then $\frac{C}{Y} = c(w,r)$: Total costs to total output is just a function of the factor prices. Now take logs: $\log\frac{C}{Y} = F(\log w, \log r)$ (with constant returns to scale). Now take the second order Taylor series expansion around the point $w = 1, r = 1$, so that the expanstion point, the log of each variable, is a convenient zero [In practice, analysts sometimes 'normalize' the measured variables by dividing by their respective sample means. The interesting elasticities in this model are unaffected by the normalization".

The translog function is:

$$\log\frac{C}{Y} = C(0,0) + \left(\frac{\partial C}{\partial \log w}\bigg|_{C(0,0)} \log w\right) + \left(\frac{\partial C}{\partial \log r}\bigg|_{C(0,0)} \log r\right) + \frac{1}{2}\left(\frac{\partial^2 C}{\partial^2 \log w}\bigg|_{C(0,0)} (\log w)^2 + \frac{\partial^2 C}{\partial^2 \log r}\bigg|_{C(0,0)} (\log r)^2 + 2\frac{\partial^2 C}{\partial \log w \partial \log r}\bigg|_{C(0,0)} \log w \log r\right) + e$$

Since the function and its derivatives evaluated at the fixed value $F(0,0)$ are constants, we can interpret them as the coefficients and write the estimating linear regression model as:

$$\log\frac{C}{Y} = \beta_0 + \beta_1 \log w + \beta_2 \log r + \beta_3 \frac{1}{2}(\log w)^2 + \beta_4 \frac{1}{2}(\log r)^2 + \beta_5 \log w \log r + e$$

The cost shares, (in the terminology above, the m's), can be calculated as follows from this estimated equation:

$$m = \frac{wL}{Y} = \frac{\partial C(w,r,Y)}{\partial w} = \frac{\partial \log c(w,r)}{\partial \log w} = \beta_1 + \beta_3 \log w + \beta_5 \log r$$

When estimating this equation, theory tells us that it must be true that the sum of the shares must be one, which requires that $\beta_1 + \beta_2 = 1$. So, we can estimate:

$$\log\frac{C}{Y} = \beta_0 + \beta_1 \log w + (1-\beta_1)\log r + \beta_3 \frac{1}{2}(\log w)^2 + \beta_4 \frac{1}{2}(\log r)^2 + \beta_5 \log w \log r + e$$

Finally, using the fact that $\begin{matrix} L^* = C_w \\ K^* = C_r \end{matrix}$, and taking the ratios, it can be shown that:

$$\sigma = \frac{\log\frac{K}{L}}{\partial \log\frac{w}{r}} = \frac{\beta_5 + m(1-m)}{m(1-m)}.$$

We've said nothing here about having to deal with omitted variables bias. For more, see Hamermesh and David Card's notes on static demand.

An Application Using the Estimated Elasticity of Demand for High/Low Skilled Workers

The rise in relative wage inequality in the United States, beginning in the late 1970s, seems to match the pattern on the rise in the college premium (dummy variable for college, or rise in the return to education). This rapid increase in the college premium is widely interpreted as evidence that labor market forces were driving up the price of skills. The argument is reinforced by the fact that the increase in the college premium was also accompanied by a rise in the relative college labor force. Ceteris paribus, the increase in the supply of college graduates should have led to a reduction in their relative wages. Ergo, a shift in the demand for college graduates seems likely.

One obvious explanation for what caused a shift in the relative demand for college graduates is a change in productivity. A Skill Biased Technological Shock to a particular labor group's level of productivity can raise their relative wages, thus producing predicted changes to the labor market which match what is observed empirically.

Theory:

Setup, Consider a firm's production function that is CES:

$$Y_t = \left[\theta_{s,t}(L_{s,t})^{\frac{\sigma-1}{\sigma}} + \theta_{u,t}(L_{u,t})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

$Y_t \equiv$ Output at time t
$\theta_{s,t} \equiv$ productivity parameter for labor group i at t, i = skilled, unskilled
$L_{s,t} \equiv$ Labor used of group i at time t
$\sigma \equiv$ elasticity of substitution between the services of skilled and unskilled labor, holding output constant

The representative firm's cost function is:

(2) $$C_t = w_{s,t}L_{s,t} + w_{u,t}L_{u,t}$$

Firm minimizes (2) subject to (1). Note we have essentially assumed away any institutional influences. From this we derive the labor demand functions:

$$L_{j,t} = \theta_{j,t} w_{j,t}^{-\sigma} Y \left[\theta_{s,t}(L_{s,t})^{\frac{\sigma-1}{\sigma}} + \theta_{u,t}(L_{u,t})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}-1},$$

for j=s,u. Dividing $L_{s,t}$ by $L_{u,t}$ and taking logs,

(4) $$\log \frac{L_{s,t}}{L_{u,t}} = \log \frac{\theta_{s,t}}{\theta_{u,t}} - \sigma \log \frac{w_{s,t}}{w_{u,t}}$$

Assume now that there is full employment so that labor demand equals labor supply. Then labor in this above equation becomes exogenous, and we can solve for relative wages. Rearranging (4) we get:

$$\log\frac{w_{s,t}}{w_{u,t}} = \frac{1}{\sigma}\left[\log\frac{\theta_{s,t}}{\theta_{u,t}} - \log\frac{L_{s,t}}{L_{u,t}}\right]$$

This is the same equation used in Johnson (p. 44 JEP) and by many others for discussion and estimation purposes. The first expression in the brackets on the right hand side loosely covers the demand side influences of relative wages and the second expression covers supply factors. A few comments for field exam purposes before going on: There are a couple of variants to the production function used above which lead to slightly different expressions for (5). For example, $Y_t = \left[(\theta_{s,t}L_{s,t})^{\frac{\sigma-1}{\sigma}} + (\theta_{u,t}L_{u,t})^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$. Labor demand is usually derived from applying Sheppard's Lemma to the cost function: $L_j = \partial C / \partial w_j$. For estimation purposes of σ, usually you must derive labor demand from $w_j / p = \partial Y / \partial L_j$ and thus assume the additional restriction of perfect competition.

OK, moving on. Empirically, we know that relative labor supply of college graduates has risen, while relative wages for college grads has also increased. If we substitute skilled labor for college labor, and unskilled labor for high school labor, then from (5), we see that for this relationship to hold, there must have been a productivity shock for college grads (σ is almost always assumed greater than one, based on econometric estimation – see Hamermesh's book on labor demand, chapter three). It is perhaps better to speak of college vs. high school rather than skilled vs. unskilled. This helps us understand exactly what kind of technological shock we need to explain the data – a kind which raises the relative productivity for college graduates compared to those of high school graduates.

The rise in the rate of return to education is compelling evidence that there was indeed such a productivity shock. This is what led many economists to investigate the possibility of skill biased technological change (defined as $\frac{\theta_{s,t}}{\theta_{u,t}}$ increasing over time) as an explanation for the rise in wage inequality. One crucial problem to keep in mind with this research is that we are dealing with two vague concepts: changes in skill and changes in technology, both of which are hard to measure and hard to clearly define. In order to assess the SBTC hypothesis, we need to quantify these concepts, which is easier said than done. There are what I would call three approaches to empirically investigating whether SBTC was the main contributor for rising wage inequality: the residual approach, the case study approach, and the 'it must be the computer' approach.

The residual approach

From 1979 to 1989, $\frac{w_{s,t}}{w_{u,t}}$ increased by about 1.3 percent per annum, and $\frac{L_{s,t}}{L_{u,t}}$ increased by 2.7 percent per annum. Assuming an elasticity of substitution of 1.5, the change in SBTC can be calculated as a residual from equation (5). $\frac{\theta_{s,t}}{\theta_{u,t}}$ is estimated to have risen by about 4.7 percent per annum (Johnson JEP 1997). This value is considered quite large. Note that most estimates for σ run between 1 and 2. Some particular forms of the CES production function require $\sigma > 1$ for SBTC to exist. Also, the value of σ is subject to some debate.

This approach has been applied using more empirically intensive methods. The most commonly refered to papers in this area are Berman, Eli, Jon Bound, and Zvi Griliches. 1994 "Changes in the Demand for Skilled Labor within U.S. Manufacturing Industries: Evidence from the Annual Survey of Manufacturing." Quarterly Journal of Economics (May): 367-397, Bound and Johnson (AER, 92), and Katz and Murphy (QJE 92). In very general terms, these papers apply a similar 'residual' approach, but for finer data, looking at a larger number of worker group classes, and for different industries. The finding that no factor outside of SBTC seems to have changed enough to observe the magnitude of change in relative wages, however, is consistent among all three. They then conclude that some sort of technological shock must be the culprit.

Application 1: The effect of immigration on native wages and employment

One of the more interesting applications to labor demand theory is to examine the comparative statics of equilibrium wages and employment after a change in immigration. George Johnson (ILRR, 1980) provides a nice model that describes the main predictions, which end up depending crucially on the elasticities of both supply and demand.

Johnson considers an effect of immigration as an increase in the labor supply of low skilled workers. Define the total employment of low skilled labor E_1, as:

$$E_1 = E_{1d} + E_{1m},$$

where E_{1d} and E_{1m} are native and immigrant employment respectively. How does an increase in E_{1m} affect w_1 and E_{1d}?

Set up the market: Labor demand for unskilled workers must equal labor demand:

$$D(w_1) = E_{1d} + E_{1m}$$

Since we are focussing on natives, from a change in immigrants, let the labor supply of immigrants be perfectly inelastic (given), and the labor supply of natives by

$$E_{1d} = h(w_1).$$

Define the elasticities:

$\varepsilon = \dfrac{d \log E_{1d}}{d \log w_1} = \dfrac{w_1}{E_{1d}} \dfrac{dE_{1d}}{dw_1} = \dfrac{w_1 h'(w_1)}{E_{1d}}$ is the elasticity of labor supply for natives

$\eta = -\dfrac{d \log D(w_1)}{d \log w_1} = -\dfrac{w_1}{D(w_1)} \dfrac{dD(w_1)}{dw_1} = \dfrac{D'(w_1) w_1}{D(w_1)}$ is the elasticity of labor demand for unskilled workers

$\mathrm{f} = \dfrac{E_{1m}}{E_1}$ is the fraction of immigrants

Totally differentiate equilibrium condition:

$$D'(w_1)dw_1 = dE_{1d} + dE_{1m}$$

$$\frac{D'(w_1)}{D(w_1)}\frac{w_1}{w_1}dw_1 = \frac{dE_{1d}}{E_1} + \frac{dE_{1m}}{E_1}$$

$$-\eta\frac{dw_1}{w_1} = \frac{h'(w_1)dw_1}{E_1} + \frac{dE_{1m}}{E_1}$$

$$-\eta\frac{dw_1}{w_1} = \frac{h'(w_1)w_1}{E_{1d}}\frac{E_{1d}}{E_1}\frac{dw_1}{w_1} + \frac{dE_{1m}}{E_1}$$

$$-\eta\frac{dw_1}{w_1} = \varepsilon(1-\mathrm{f})\frac{dw_1}{w_1} + \frac{dE_{1m}}{E_1}$$

$$-d\log w_1(\eta + \varepsilon(1-\mathrm{f})) = \frac{dE_{1m}}{E_1}$$

$$-d\log w_1(\eta + \varepsilon(1-\mathrm{f})) = \frac{dE_{1m}}{E_{1m}}\mathrm{f}$$

$$\frac{d\log w_1}{d\log E_{1m}} = \frac{-\mathrm{f}}{(\eta + \varepsilon(1-\mathrm{f}))} < 0$$

When labor demand for unskilled workers more elastic (can substitute other inputs more easily with unskilled labor), wages will less. This is just from a shift in the overall labor supply curve when labor demand curve is flat (draw on board). Also, if labor supply more elastic (work less if wage increases), the wage will change less. This is because the firm has less ability to adjust wages without losing workers that are currently at the firm.

Recall that $\varepsilon = \frac{d\log E_{1d}}{d\log w_1}$

$$\frac{d\log E_{1d}}{d\log E_{1n}} = \varepsilon\frac{d\log w_1}{d\log E_{1n}} = \frac{-\varepsilon \text{f}}{(\eta+\varepsilon(1-\text{f}))} < 0$$

$$\frac{dE_{1d}}{dE_{1n}} = \frac{-\varepsilon(1-\text{f})}{(\eta+\varepsilon(1-\text{f}))}$$

Immigrants affect the labor market of natives only to the extent that they affect wages.

If labor demand for unskilled perfectly elastic, no displacement. Firms are use all new immigrants plus all old natives.

If labor demand perfectly inelastic, perfect displacement. E.g. firms must use X unskilled workers. Immigrants have inelastic labor supply and are willing to work at any wage. Wage adjust downwards as dE_{1m} are used to replace natives.

Perfect displacement also if labor supply of native unskilled workers perfectly elastic. Natives respond to any change in wage by a big fall in employment (they all stop working in this extreme case).

Assumptions of model:

1) natives and immigrants perfect substitutes
2) natives and immigrants get same wage
3) All immigrants work
4) No interaction with other inputs (no complemtarity)
5) Immigrants don't buy anything (no demand effects)

Relaxing this last assumption can change the predictions dramatically. If demand for goods within a city increases the same rate as immigration, constant returns to scale implies no impact from immigration (Card calls this 'the Krueger conjecture'). We may be more interested, however, in the short run impact, which may still last some time before demand effects mitigate the wage impact (and perhaps only partially).

For a model that considers demand effects, see Altonji and Card, "The effects of immigration on th elabor market outcomes of natives'.

Empirical Evidence of Effects of Immigration

A. Mariel boatlift

Summer of 1980, Castro let people leave for a few months: let prisoners, hospital patients, any 'scum' who wants to leave Cuba to go or forever hold their peace. (See Scarface)

More than 125,000 Cubans arrived to the port of Mariel, and most settled in Miami. Increased the labor force (adult working pop) by 7%!, unskilled working pop increased more), Increased Cuban population by 20%.

What's nice is we have an exogenous shock to labor supply not likely do to response in local demand factors.

Compare with similar cities: Atlanta, Los Angelas, Houston, and Tampa-St. Petersburg, who also have large Black and Hispanic population.

Provides a simple but clear diff in diff study. What happened to wages and employment in Miami, relative to 'counterfactual' control group where supply shock did not occur.

Figure 1 in Angrist and Krueger

General results suggest large increase in labor supply from immigrants had minimal impact on wages and labor supply of natives.

Concerns with diff in diff strategy: control groups are not the same as Miami.
Total population in city may have changed: natives may have moved out.

Most native population don't compete with natives.

B. Card (JoLE 2001)

Another approach to test the effects of immigration is to treat cities as separate economies and examine the correlation between native wages and the fraction of immigrants in a city.

$$\log y_{cn} = X_{cn}\beta + f_c\delta + e_{cn}$$

where y_{cn} is the mean outcome for native group N in city c. Clearly there is an omitted variables bias concern if the fraction of immigrants is somehow correlated with unobservables related to city wages. Just considering demand factors along, we would expect immigrants to move to cities that pay relatively more, which would bias the elasticity estimate upwards.

Some have tried differencing over two time periods:

$$\Delta \log y_{cn} = \Delta X_{cn}\beta + \Delta f_c\delta + \Delta e_{cn}$$

with the idea being that the first differences approach removes city specific factors that are constant over time. Transitory effects will still lead to bias.

One approach has been to instrument Δf_c with the fraction of immigrants in the city at the start of the initial period. The motivation is that immigrants are mainly attracted to cities with large concentrations of previous immigrants from the same country. The first difference approach is also more likely to capture short run effects.

The coefficient for δ with this approach is about -.1. A 10 percent point increase in the fraction of immigrants in a city decreases wages by 1 percent. This evidence seems to coincide with the mariel boatlift paper suggesting the labor market impact of immigration is small.

2 major concerns with the cross-city approach:

1) natives may move out, and this is not reflected in estimates: labor supply may not be changing (denominator in f is getting smaller).

2) upward bias may still remain if instrument (initial fraction of immigrants in city) correlated with demand shock trends or other factors.

Card proposes to re-examine the issue by looking within cities, by occupation. Consider his model, where each city produces one output. The city production function is:

$$Y_c = F(K_c, L_c)$$

K_c is non-labor inputs.

L_c is CES aggregate of labor types. Let N_{jc} be number employed of skill-type (occupation) j in city c:

$$L_c = \left[\sum_j (e_{jc} N_{jc})^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

σ is the elasticity of substitution between occupation groups. e_{jc} is a city-occupation augmentation factor. Since wage is equal to marginal value product in equilibrium,

$$w_{jc} = q \frac{1}{\sigma-1} F_{L_c} L_c^{\frac{1}{\sigma}} \frac{\sigma-1}{\sigma} N^{\frac{-1}{\sigma}}{}_{jc} e^{\frac{\sigma-1}{\sigma}}{}_{jc}$$

rearranging:

$$\log N_{jc} = \theta_c + (\sigma-1)\log e_{jc} - \sigma \log w_{jc}$$

This is not a proper labor demand function, because we have not solved for F_{L_c}. Nevertheless, we've expressed employment as a function of city effects, city/occupation effects, and wages.

Let P_{jc} represent the total population of individuals in occupation j in city c. Think of the unemployed here as people with skills associated with the occupation, but who are not working. We'll see how Card measures this in a moment. Assume the labor supply function is:

$$\log(N_{jc} / P_{jc}) = \varepsilon \log w_{jc} ,$$

so ε is the elasticity of labor supply, which Card assumes positive.

$$\log w_{jc} = \frac{1}{\varepsilon + \sigma}\left[(\theta_c - \log P_c) + (\sigma - 1)\log e_{jc} - \log(\frac{P_{jc}}{P_c}) \right]$$

where P_c is total city population.

$$\log(N_{jc} / P_{jc}) = \frac{\varepsilon}{\varepsilon + \sigma}\left[(\theta_c - \log P_c) + (\sigma - 1)\log e_{jc} - \log(\frac{P_{jc}}{P_c}) \right]$$

The productivity augmentation factor is assumed to have a common occupation effect, a city effect, and a occupation-city specific term:

$$\log e_{jc} = e_j + e_c + e_{jc}$$

From this the regression equations are:

$$\log(w_{jc}) = u_j + u_c + d_1 \log f_{jc} + u_{jc}$$
$$\log(N_{jc} / P_{jc}) = v_j + v_c + d_2 \log f_{jc} + v_{jc}$$

This specification allows for city fixed effects, which absorb citywide variables that might otherwise influence levels of wages and employment, and occupation fixed effects Still have to worry about u_{jc}: occupation specific local shocks. The bias is still likely upwards to the extent that local productivity shocks raise wages and increase in the population of a particular occupation group.

Major assumptions:

One output, produced and consumed within cities. No demand shocks

Relative Wage only depends on population share in occupation.

$$\log(\frac{w_{jc}}{w_{kc}}) = \frac{\varepsilon}{\varepsilon+\sigma}\left[(\sigma-1)\log\frac{e_{jc}}{e_{kc}} - \log(\frac{f_{jc}}{f_{kc}})\right]$$

Notice, we haven't yet brought immigrants into the picture. A exogenous change in the city-occupation labor supply has the same effect, whether driven by immigration or from something else.

Card tries to instrument the population shares with recent inflows of immigrants…

Data from 1990 census. Mean and women 16 to 68 with at least 1 year of potential experience. Total annual earnings, along with weeks worked and hours per week.

Defines cities as MSAs (324) Focus on largest 175. Separate out immigrants (foreign born) within last 5 years and longer.

Choose 6 occupation categories. Want to avoid allowing movement across occupations (then occupations are perfectly substitutable – we would expect no effect from immigration).

Estimating occupation populations. For the sample working, estimate probability of being in occupation group j based on underlying characteristics: age, education, race, gender, national origin, and length of time in the country. Use coefficients to assign a probability for every individual in the sample of being in occupation j.

The predicted population in occupation j, within a city, gender, or education group, is just the sum of the probabilities

Card also uses these probabilities to compute predicted
<draw graph of

Empirical Application to Labor Demand: The Effect of Raising the Minimum Wage on Employment

Most of us learn in 1st year undergraduate economics the theoretical implications of a minimum wage in a perfectly competitive setting. Graphically, it's easy to describe the argument: if authorities impose a minimum wage for workers above the market clearing wage, competitive firms will choose to hire less labor. How large the response will depend on the elasticity of demand for labor: if the elasticity is high, firms will substitute away from low skilled / low paid workers for other factors to produce Y.

In early 1990, the State of New Jersey passed a bill to raise the state minimum wage from $4.25 to $5.05. The law change was not to happen until 1992, two year later. David Card and Alan Krueger decided to test the theory of minimum wages and measure the response in employment from this change. A few months before the change, they telephoned managers and assistant managers at hundreds of fast-food chains in New Jersey. They asked about the wages paid to employees, and how many each restaurant employed (number of full-time and part-time workers). They picked the fast food industry because these stores obviously pay employers low-wages, they comply with minimum wage regulations, and the set of skills, and tasks of non-manger employees are similar.

One possible empirical strategy is to simply compare the average employment rates before and after the change. Consider the following regression analysis

$$H_{gt} = \beta_0 + \beta_1 T_{gt} + v_{gt}$$

Where H_{gt} is the average labor supply for group g, in period t, $T_{gt}=1$ if the minimum wage changed for group 1, zero otherwise. The analysis is over time. Let t=1 the year=1991 (before the change) and t=2 if the year = 1993 (after the change). Note the authors have not control over how large or small the change is: they are only able to examine the effect from the change in New Jersey.

Key here is, what is the counterfactual?? We'd like to know the treatment effect relative to a similar group that was not eligible.

The estimate of B_1 from the above equation is equivalent to taking the difference between H before and after the change. What is our estimate for B_1? Well, the above equation just has two observations for G=1 (the group of all fast food chains). The OLS estimate for B_1 is equal to the difference between H before and after the change:

$$H_{12}-H_{11}=\beta_1+(v_{12}-v_{11})$$

By construction, v_{gt} = 0 over both periods combined, but this does not mean that v_{12} or v_{11} = 0. Notice we are comparing means over two different time periods. Any underlying trends in labor force participation or hours of work between 1991 and 1993, or any economic shocks that affect labor market outcomes will affect H differently over the time frame examined. In other words, we attribute any difference over this 2 year time period to the change in the minimum wage, but any effect to labor supply over the same time period examined cannot be separated by the minimum wage's effect.

To get around this, Card and Krueger take what is know as a 'difference in differences' strategy. They try to find a 'control' group that was not affected by the minimum wage shock being examined, but was affected by other shocks or trends that are not controlled for.

Card and Krueger decide also to collect data for fast-food employment in the neighboring state, Pennsylvania. Pennsylvania did not experience a change in the minimum wage that New Jersey did between these periods, so T = 0 in both periods for them. Thus $\beta_1 = 0$ for this group. Let this group be G=2. Card and Krueger carry out the following regression:

$$H_{gt} = \beta_0 + \beta_1 T_{gt} + v_g + v_t + v_{gt}$$

v_g and v_t are 'fixed effects'. This is just a fancy word for dummy variables. We include a dummy variable for whether the individual is from group 1 or 2, to control for the <u>time invariant</u> mean difference in H between the two groups, and the <u>group invariant</u> mean difference in H between the two periods. Including these dummy variables is equivalent to estimating B_1 from the difference in differences of H:

$$(H_{12} - H_{11}) - (H_{22} - H_{21}) = \beta_1 + (v_{12} - v_{11}) - (v_{22} - v_{21})$$

The estimate of B1 is unbiased if $(v_{12} - v_{11}) - (v_{22} - v_{21})$ is equal to zero. If there are other factors that affect the two groups over the time period the same way, then taking the difference between the two groups will absorb those shocks.

The difference in differences strategy makes 2 crucial assumptions: 1) the time effects are common between the groups, 2) the composition of both groups remains stable before and after the policy change.

The smaller the time range examined, the less likely other factors will explain the differences.

Note, essentially the way I've described this analysis, there are only 4 observations: the mean labor supply for the 2 groups, before and after the change. If we can observe other factors for individuals that could affect labor supply (that could change between periods), we may be able to get more efficient estimates by controlling for these observables and working at a smaller level of data than means.

Findings:

Table 3, AER 94: Data from telephone interviews

Table 4, Neumark and Wascher, AER 2000 (implied elasticity calculations from 18% change in minimum wage): data from requesting from owners by letters payroll data

Figure 2: Card and Kruger, AER 2000, Data from administrative Bureau of Labor Statistics data.

Baker, Benjamin, Stanger examine minimum wage changes in Canada, across provinces, between 1975 and 1993. Substantial variation in the level and timing of changes in the minimum wage. Large number of teenagers affected: between 8% and 30% of jobs held by teenagers pay within 5 cents of the adult minimum wage. More able to look account for long-run adjustments because longer bandwidth. Card and Krueger only look at the year before and after. If adjustments take time, they may miss this. Short differences may prematurely censor the adjustment in employment. BBS suggest examining changes over a 4 year period at least. BBS find in Table 1 a significant and clear minimum wage effect, lowering teen employment with province and year fixed effects. The independent variable is the minimum wage divided by the average provincial wage. Table 4 shows that as the time difference increases, the estimate of the negative elasticity becomes more clear.

Proof 1: Elasticity of substitution: $\sigma = \dfrac{d\ln(K/L)}{d\ln(w/r)} = \dfrac{F_L F_K}{Y F_{LK}}$

(1) $$\sigma = \frac{d\ln(K/L)}{d\ln(w/r)} = \frac{d\left(\frac{K}{L}\right)}{d\left(\frac{F_L}{F_K}\right)} \frac{\left(\frac{F_L}{F_K}\right)}{\left(\frac{K}{L}\right)}$$

Total differentiate $\frac{F_L}{F_K}$ with respect to K and L and note that:

(2) $$d\left(\frac{F_L}{F_K}\right) = \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} dK + \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L} dL$$

Totally differentiate Y with respect to L and K, holding Y constant (this is the slope of the isoquant line):

$$\begin{aligned} Y &= F(L,K) \\ 0 &= F_L dL + F_K dK \end{aligned}$$, which means:

(3) $$dL = -\left(\frac{F_K}{F_L}\right) dK .$$

Substitute (3) into (2) to get:

(4)

$$d\left(\frac{F_L}{F_K}\right) = \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K}dK - \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L}\left(\frac{F_K}{F_L}\right)dK$$
$$= \left[F_L\frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} - F_K\frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L}\right]\frac{dK}{F_L}$$

OK, now let's look at $d\left(\frac{K}{L}\right)$: totally differentiate:

$$d\left(\frac{K}{L}\right) = \frac{1}{L}dK - \frac{L}{L^2}dL$$
$$= \frac{LdK - KdL}{L^2}$$

Using (3) again:

(5)

$$d\left(\frac{K}{L}\right) = \frac{LdK - K\left(\frac{F_K}{F_L}\right)dK}{L^2}.$$
$$= \left(LF_L + KF_K\right)\frac{dK}{F_L L^2}$$

Substituting (4) and (5) into (1), we get:

$$\sigma = \frac{d\left(\frac{K}{L}\right)}{d\left(\frac{F_L}{F_K}\right)} \frac{\left(\frac{F_L}{F_K}\right)}{\left(\frac{K}{L}\right)}$$

$$= \frac{(LF_L + KF_K)\frac{dK}{F_L L^2}}{\left[F_L \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} - F_K \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L}\right]\frac{dK}{F_L}} \frac{\left(\frac{F_L}{F_K}\right)}{\left(\frac{K}{L}\right)}$$

$$= \frac{(LF_L + KF_K)\frac{F_L}{F_K LK}}{\left[F_L \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} - F_K \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L}\right]}$$

Note that

$$\frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} = \frac{F_{LK}}{F_K} - \frac{F_L}{F_K^2} F_{KK} = \frac{(F_K F_{LK} - F_L F_{KK})}{F_K^2}$$

$$\frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L} = \frac{F_{LL}}{F_K} - \frac{F_L}{F_K^2} F_{LK} = \frac{(F_{LL} F_K - F_L F_{LK})}{F_K^2}$$

Substituting these in:

$$\sigma = \frac{(LF_L + KF_K)\frac{F_L}{F_K LK}}{\left[F_L \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial K} - F_K \frac{\partial\left(\frac{F_L}{F_K}\right)}{\partial L} \right]}$$

$$= \frac{(LF_L + KF_K)\frac{F_L}{F_K LK}}{\left[F_L \frac{(F_K F_{LK} - F_L F_{KK})}{F_K^2} - F_K \frac{(F_{LL} F_K - F_L F_{LK})}{F_K^2} \right]}$$

$$= \frac{(LF_L + KF_K)\frac{F_L}{F_K LK}}{\frac{(2F_L F_K F_{LK} - F_L^2 F_{KK} - F_K^2 F_{LL})}{F_K^2}}$$

$$= \frac{(LF_L + KF_K) F_K F_L}{LK(2F_L F_K F_{LK} - F_L^2 F_{KK} - F_K^2 F_{LL})}$$

Now, let's use some properties of constant returns to scale: $\delta Y = F(\delta L, \delta K)$. Totally differentiate δY with respect to δ to get Euler's Theorem: $Y = LF_L + KF_K$

Note, there is a corollary to Euler's Theorem: Since $Y = F(L, K) = LF_L + KF_K$,

$F_L = LF_{LL} + F_L + KF_{KL}$, and we note that the same term on the left hand side is on the right hand side. We can cancel and solve:

$$F_{LL} = -\frac{K}{L} F_{KL}$$

Similarly $F_{KK} = -LF_{KL}$

Let's substitute these equations into the elasticity equation:

$$
\begin{aligned}
\sigma &= \frac{(LF_L + KF_K)F_K F_L}{LK\left(2F_L F_K F_{LK} - F_L^2 F_{KK} - F_K^2 F_{LL}\right)} \\
&= \frac{(LF_L + KF_K)F_K F_L}{LK\left(2F_L F_K F_{LK} + F_L^2 \frac{L}{K} F_{KL} + F_K^2 \frac{K}{L} F_{KL}\right)} \\
&= \frac{(LF_L + KF_K)F_K F_L}{\left(F_L^2 L^2 + 2F_L F_K LK + F_K^2 K^2\right)F_{LK}} \\
&= \frac{F_K F_L}{(LF_L + KF_K)^2 F_{LK}} \\
&= \frac{(LF_L + KF_K)F_K F_L}{(LF_L + KF_K)^2 F_{LK}} \\
&= \frac{YF_K F_L}{Y^2 F_{LK}} \\
&= \frac{F_K F_L}{YF_{LK}}
\end{aligned}
$$

Proof 2: Elasticity of substitution in terms of cost function $\sigma = \frac{CC_{wr}}{C_w C_r}$

The cost function is: $C(w,r,Y)$. From Shephard's lemma, $C_w = L$ and $C_r = K$

(*) $\log\frac{K}{L} = \log C_r - \log C_w$

Input demands are homogeneous of degree 0 (see above), so:

$$C_w(w,r,Y) = C_w(\frac{w}{r},1,Y)$$

differentiate with respect to w:

$$C_{ww}(w,r,Y)=\frac{1}{r}C_{ww}(\frac{w}{r},1,Y)$$

Likewise,

$$C_r(w,r,Y)=C_r(\frac{w}{r},1,Y)$$
$$C_{rw}(w,r,Y)=\frac{1}{r}C_{wr}(\frac{w}{r},1,Y)$$

rewrite (*) as $\log\frac{K}{L}=\log C_w(\frac{w}{r},1,Y)-\log C_r(\frac{w}{r},1,Y)$. Thus

$$\frac{d\log\frac{K}{L}}{d\frac{w}{r}}=\frac{1}{C_w}C_{ww}(\frac{w}{r},1,Y)-\frac{1}{C_r}C_{rw}(1,\frac{r}{w},Y)$$
$$=\frac{1}{C_w}rC_{ww}(w,r,Y)-\frac{1}{C_r}rC_{rw}(w,r,Y)$$

$$\frac{d\log\frac{K}{L}}{d\log\frac{w}{r}}=\frac{w}{r}\left[\frac{1}{C_w}rC_{ww}(w,r,Y)-\frac{1}{C_r}rC_{rw}(w,r,Y)\right]$$

One of the properties of the cost function is that it is homogeneous of degree 0. This implies: $wC_{ww}+rC_{wr}=0$

$$\frac{w}{r}=-\frac{C_{wr}}{C_{ww}}$$
$$wC_{ww}=-rC_{wr}$$

Subing in: $\frac{d\log\frac{K}{L}}{d\log\frac{w}{r}}=\frac{C_{wr}}{C_{ww}}\left[\frac{1}{C_w}rC_{ww}+\frac{1}{C_r}wC_{ww}\right]$

$$\frac{d\log\frac{K}{L}}{d\log\frac{w}{r}} = \frac{C_{wr}}{C_{ww}}\left[\frac{1}{C_w} rC_{ww} + \frac{1}{C_r} wC_{ww}\right]$$

$$= \frac{C_{wr}}{C_{ww}}\left[\frac{rC_{ww}C_r + wC_{ww}C_w}{C_r C_w}\right]$$

$$= \frac{C_{wr}}{}\left[\frac{wC_w + rC_r}{C_r C_w}\right]$$

$$= \frac{C_{rw}}{}\left[\frac{C}{C_r C_w}\right]$$

Proof 3: Derived Demand: $\eta_{LL} = -(1-s)\sigma < 0$

If we have constant returns to scale, the cost function can be expressed as: $C(w,r,Y) = Y\gamma(w,r)$, where $\gamma(w,r)$ is the unit cost function. In a competitive market, the price of the output will be equal to marginal cost

$$p = \gamma(w,r)$$

And we can close the model by assuming a demand function for output: $Y = D(p)$, with elasticity $\frac{d\log Y}{d\log p} = n$.

Now, $\log L = \log\frac{\partial C(w,r)}{\partial w} = \log Y + \log \gamma_w$

Differentiating with respect to w,

$$\frac{\partial \log L}{\partial w} = \frac{\partial \log Y}{\partial \log p}\frac{\partial \log p}{\partial w} + \frac{\gamma_{ww}}{\gamma_w}$$

$$= -n\frac{\partial \log \gamma}{\partial w} + \frac{\gamma_{ww}}{\gamma_w}$$

$$= -n\frac{\gamma_w}{\gamma} + \frac{\gamma_{ww} Y C_r p Y}{\gamma_w Y C_r p Y}$$

$$= -n\frac{\gamma_w}{\gamma} + \frac{\gamma_{ww} Y C_r \gamma Y}{C_w C_r \gamma Y}$$

$$= -n\frac{\gamma_w}{\gamma} + \frac{\gamma_{ww} Y C_r C}{C_w C_r C}$$

$$\frac{\partial \log L}{\partial \log w} = -n\frac{Y\gamma_w w}{\gamma Y} + \frac{\gamma_{ww} YKCw}{C_w C_r C}$$

$$= -n\frac{C_w w}{\gamma Y} + \frac{C_{ww} CKw}{C_w C_r C}$$

The cost function is homogeneous of degree 0, which implies $wC_{ww} + rC_{wr} = 0$

$$\frac{\partial \log L}{\partial \log w} = -n\frac{C_w w}{\gamma Y} + \frac{C_{ww} CKw}{C_w C_r C}$$

$$= -n\frac{C_w w}{\gamma Y} + -\frac{r}{w}\frac{C_{wr} CKw}{C_w C_r C}$$

$$= -n\frac{Lw}{C} + -\frac{w}{w}\sigma\frac{Kr}{C}$$

$$= -n\theta_L - \sigma\theta_K$$

$$= -n\theta_L - \sigma(1-\theta_L)$$

$$= -[\sigma(1-\theta_L) + n\theta_L]$$

where $\theta_K = \frac{Kr}{C}$ is K's share of cost.

And if we hold output constant:

$$\eta_{LL} = -(1-s)\sigma < 0$$