Bandwidth Maximization
of a
Single Degree of Freedom Magnetic Suspension System

by
Sanjay Kumar Aggarwal

Submitted to the Department of
Mechanical Engineering
in Partial Fulfillment of the Requirements for the
Degree of
Bachelor of Science in Mechanical Engineering

at the
Massachusetts Institute of Technology

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OF TECHNOLOGY

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Abstract

Bandwidth tests are performed using a single degree of freedom
magnetic suspension system. An analog linear controller is used to
compensate the system and achieve the desired stability margins. To
linearize the plant dynamics of the suspension system, a force bias is used in
the magnetic actuators. The current amplifier used to drive the actuators is
also analyzed and redesigned in order to eliminate oscillations in the original
design of the amplifier.

In trying to maximize the suspension system bandwidth, unexpected
unmodeled poles placed severe limitations on the achievable bandwidth by
dropping the plant phase much lower than expected. However, within the
time constraints of the project, the cause of these unmodeled poles could not
be identified, and therefore they could not be eliminated. Nevertheless, for
the plant transfer function measured, a crossover frequency of 200 Hz with a
phase margin of 15 degrees was achieved. For these values, a closed loop
bandwidth of 400 Hz was measured.

The current amplifier used in the experiment was successfully
redesigned to eliminate loop oscillations. In the new design, the compensator
was implemented which achieved a closed loop bandwidth of 20 kHz.

Faculty Thesis Supervisor: Dr. David Trumper
Title: Professor of Mechanical Engineering
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Chapter 1

Introduction

Magnetic bearings have proven to be an important alternative to standard mechanical bearings for precision positioning applications. The primary advantage of magnetic bearings over mechanical bearings is that they do not contact the workpiece. Thus, the resolution of such systems depends only on the sensors and the control, and not on bearing finishes. Also, because there is no contact, problems with wear and friction are insignificant. A further advantage of magnetic bearings is that they can simultaneously act as both an actuator and a bearing and thus the need for complex bearing/actuator couplings is eliminated. The main drawback to these bearings, however, is that they are open loop unstable and nonlinear. Thus, closed loop control is required.

One of the important considerations for closed loop systems that has not been adequately addressed for magnetic suspension systems is the bandwidth. In particular, the factors affecting the maximum achievable bandwidth of these systems are important to understand. The limits of high frequency control will place real limitations on the applications for magnetic bearings and the achievable performance in the various applications.

This thesis focuses on the high frequency control of a single degree of freedom magnetic bearing system. The present work is an extension of a master’s thesis project completed by Sean Olson in 1994 at the Massachusetts Institute of Technology. Olson’s work focused on a comparison of linear and nonlinear control schemes for a single degree of freedom suspension system designed and built by him. In his research, Olson found that while the nonlinear controller gave consistent performance regardless of the operating
point airgap between the suspended target and the electromagnetic actuators, the linear controller performance degraded as the operating point varied. In this thesis, the limits on the bandwidth of the single degree of freedom system built by Olson are explored. A linear control scheme analogous to the one used by Olson is utilized in this experiment; it is assumed to provide adequate performance, because at the operating point airgap used in this experiment, Olson found no degradation in suspension performance for the linear controller. Also, instead of using a digital controller as Olson did, an analog controller is used in this experiment in order to avoid bandwidth limits caused by sampling frequency in digital controllers.

The following document provides a detailed explanation of the magnetic suspension system used in this experiment and the controller used to drive it. In chapter 2 the hardware for the suspension system is described, and a derivation for the linearized dynamics describing the system is presented. Chapter 3 discusses the current amplifier used to drive the magnetic actuators in the suspension system. In particular, a theoretical model for the amplifier is presented, and a controller design is given which eliminates loop oscillations in the amplifier. Chapter 4 contains the final analog controller design for the suspension system that maximizes the system bandwidth. Finally, chapter 5 provides experimental results for the system and discusses the problems encountered in trying to maximize the bandwidth.
Chapter 2

Mechanical Dynamics

In this chapter, a derivation of the mechanical dynamics of the suspension system is presented in order to help design an appropriate compensator for the system and for understanding the relevant factors limiting the suspension performance. Section 2.1 first gives a brief discussion of the actual suspension system hardware used in the experiment. Next, in section 2.2, the force-current relationship between the magnetic actuators and the target mass is determined both theoretically and experimentally. In section 2.3, the theoretical force-current relationship of section 2.2 is used to derive linearized dynamic equations for the suspension system, assuming a current bias in the actuators. Finally, in section 2.3, the linearized equations are used along with measured system component values to arrive at an expected plant transfer function.

2.1 Suspension System Hardware

The actual suspension system used in this experiment is shown in Figure 2.1. Although a more complete discussion of the hardware and its design can be found in [Olson, 1994], a short description is provided below.

The two magnetic actuators shown in Figure 2.1 are used to control the position of the endpoint target. The target’s motion is constrained by flexures 12 inches away. These flexures constrain the five uncontrolled degrees of freedom while leaving the controlled x-direction free. The composite shaft between the flexures and target was chosen to be sufficiently long so as to allow the target motion to be essentially a linear translation. The capacitive
probe measures the position of the target within a $\pm 50 \, \mu\text{m}$ linear range. The probe output is such that at the equilibrium point ($x=0$), the probe output is zero volts. At $\pm 50 \, \mu\text{m}$ away from equilibrium, the output voltage is $\pm 10 \, \text{V}$, with the voltage proportional to the position within this range. Thus, the voltage-to-position gain of the probe is $200,000 \, \text{V/}\mu\text{m}$.

In the actual set-up of the experiment, a foam pad is placed underneath the base of the suspension to isolate the table vibrations from the suspension vibrations, as well as to decouple the suspension from modes of the supporting table. The 180 pound aluminum block on which all the hardware is bolted provides a stiff platform to support the experiment.
Figure 2.1 Photograph of Magnetic Suspension System.
2.2 Actuator Force-Current Relationship

In order to develop dynamic equations for the suspension system, it is first necessary to determine the relationship between input current to the actuators and output force on the target mass. The following derivation of this relationship is based on a similar derivation from [Woodson and Melcher, 1968].

![Diagram of Actuator Force-Current Relationship]

Depth \( h \) perpendicular to the page.
Pole Face Area = \( wh = A \)

Figure 2.2: Schematic of Single Degree of Freedom Magnetic Suspension.

The schematic of Figure 2.2 gives the basic model for the single degree of freedom magnetic suspension system used in this experiment. The suspended mass in this diagram is constrained to move in only the \( x \)-direction. As seen in Figure 2.1, the only difference between this schematic and the actual system is that the actual system has an additional magnet below the mass, and the entire system is rotated by 90 degrees. Thus, the schematic of Figure 2.2 is sufficient for determining the force-current relationship of the actuator if the effect of gravity is neglected.
In order to derive the force-current relationship of the actuator, the following four electromagnetic equations will be used (each of the terms in the equations has its standard electromagnetic significance):

\[ i_c \bar{H} \cdot \bar{d} = i_s \bar{j} \cdot \bar{n} \, d \]  \hspace{1cm} (2.1)
\[ i_s \bar{B} \cdot \bar{n} \, d = 0 \]  \hspace{1cm} (2.2)
\[ i_s \bar{j} \cdot \bar{n} \, d = 0 \]  \hspace{1cm} (2.3)
\[ \bar{B} = \mu \bar{H} \]  \hspace{1cm} (2.4)

In equation 2.4, under the assumption that the material is linear, \( \mu = \mu_0 (1 + \chi_m) \), where \( \chi_m \) is the magnetic susceptibility of the material in which the equation is being applied. First, equation 2.3 requires that the current in the coil is everywhere equal to the current \( i \) shown in Figure 2.2. Next, if we assume that the magnetic material is infinitely permeable (i.e., \( \mu \) is infinite) and that the magnetic flux density everywhere is finite, equation 2.4 requires that \( \bar{H} \) within the magnetic material must be zero. Now, applying equation 2.1 to contour 2 in Figure 2.2, we find

\[ H_3 \, d - H_1 \, d = 0. \]  \hspace{1cm} (2.5)

The term \( d \) in the expression represents the initial air gap between the mass and actuator when \( x = 0 \). Equation 2.5 therefore implies that \( H_3 = H_1 \), or that the magnetic fields are equal in direction and magnitude in the given regions. If we now apply equation 2.1 to contour 1 in Figure 2.2, we find

\[ H_3 \, d + H_2 \, d = N_i, \]  \hspace{1cm} (2.6)

where \( N \) is the number of turns of the wire in the electromagnet. Next, using equation 2.2 on a surface enclosing only the suspended mass gives

\[ \mu_0 H_1 (0.5 \text{wt}) + \mu_0 H_3 (0.5 \text{wt}) - \mu_0 H_2 (\text{wt}) = \mu_0 H_3 (\text{wt}) - \mu_0 H_2 (\text{wt}) = 0. \]  \hspace{1cm} (2.7)

Combining equations 2.6 and 2.7 then implies

\[ H_1 = H_2 = H_3 = \frac{N_i}{2d}, \]  \hspace{1cm} (2.8)
where now \(H_1\) and \(H_3\) are equal in magnitude but opposite in direction to \(H_2\), as shown in Figure 2.2.

Based on the above expressions for the magnetic field, we can now calculate the force on the suspended mass by using the Maxwell stress tensor[Melcher, 1981] specialized to the present case where the gap flux is purely vertical and constant within the gap. That is,

\[
F = \frac{B^2}{2\mu_0} \text{Area}. \quad (2.9)
\]

Thus, using the fact that \(B=\mu_0H\) in free space,

\[
F = \frac{(\mu_0H_1)^2}{2\mu_0} A + \frac{(\mu_0H_2)^2}{2\mu_0} A + \frac{(\mu_0H_3)^2}{2\mu_0} A = \frac{\mu_0AN^2i^2}{4(d-x)^2}, \quad (2.10)
\]

where

\[
A = \text{wh}. \quad (2.11)
\]

Here \(d\) has been replaced by \((d-x)\) to reflect the fact that the mass can move and thus the gap between the target and actuator can change. We can also calculate the magnetic flux(\(\phi\)) through a single turn of the coil:

\[
\phi = BA = \mu_0H_2A = \frac{\mu_0ANi}{2(d-x)}. \quad (2.12)
\]

Here we use the fact that the magnetic flux density is constant throughout the core, and it is equal to the flux density in the gap. Taking the total flux through a surface bounded by the N turn coil then gives a value of \(N\phi\). This total flux is also called the flux linkage(\(\lambda\)), and we can determine the coil voltage(\(v_c\)) if we use the fact that

\[
v_c = \frac{d\lambda}{dt}. \quad (2.13)
\]

Thus, we find
\[ v_c = \frac{\mu_0 A N^2}{2(d - x)} \frac{di}{dt} + \frac{\mu_0 A N^2}{2(d - x)^2} \frac{dx}{dt} + iR. \] (2.14)

In this expression, the first term represents the voltage due to the coil inductance, the second term represents the back emf, and the \( iR \) term has been added to represent the voltage due to the coil resistance. Therefore, equation 2.14 shows us that the coil inductance \( L \) is

\[ L = \frac{\mu_0 A N^2}{2(d - x)}. \] (2.15)

Finally, equations 2.10 and 2.15, the most important of the above equations, can be rewritten with all the constant terms grouped in a single parameter \( C \):

\[ F = C\left(\frac{i}{(d - x)}\right)^2 \] (2.16)

\[ L = \frac{2C}{(d - x)} \] (2.17)

\[ C = \frac{\mu_0 A N^2}{4} \] (2.18)

The actuators used in this experiment were found to have force-current relationships similar to relationship predicted by 2.16. In this experiment, the actuators have 230 turns of wire and a pole face area of 1.613 cm\(^2\). Thus, the predicted value of \( C \) is

\[ C = 2.7 \times 10^{-6} \text{Nm}^2/A^2. \] (2.19)

The experimental value was determined by [Olson, 1994] using a least squares approximation on the data in Figure 2.3. The experimental value is

\[ C = 3.8 \times 10^{-6} \text{Nm}^2/A^2. \] (2.20)

The data in Figure 2.3 was measured by [Poovey, Holmes, and Trumper, 1994] using magnetic actuators with the same dimensions and number of wire
turns as those used in this experiment. The calibration fixture they designed and used measures the force exerted by a magnetic actuator as a function of coil current and air gap.

The discrepancy between measured and theoretical values of C were attributed by [Olson, 1994] to several possible causes. First, fringing fields that were neglected in the analysis would have effectively increased the pole face area, and thus increased C. Second, the data in Figure 2.3 is not from the identical actuators used in this experiment and on which the theoretical relationship is based. Finally, the calibration fixture itself may have been bending during the testing, so the air gap data would have been incorrect. Ultimately, though, the theory and the experiment are within 40% of one another, so the discrepancy is not completely implausible, and we do not investigate this discrepancy further in this thesis.

![Graph showing force vs. current and airgap](image)

Figure 2.3: Magnetic Actuator Data obtained using Calibration Fixture.
Finally, it is important to note that the physics of the magnetic actuators do place important limitations on the suspension performance. First, as seen in Figure 2.3, the actuators can saturate, thus leaving the relationship of equation 2.16 invalid. Therefore, it is important to operate at airgaps and currents outside of the saturation region. Second, eddy currents in the actuators can cause problems. Because of eddy currents which oppose changes in the magnetic field, there is a finite time required for magnetic field changes to propagate through the magnetic core. These eddy currents thus increase the time required for the field to fully permeate the actuators. The end result is a limitation on the actuator bandwidth. In the present actuators, laminated cores are used to limit the effects of eddy currents. With the laminated cores, the frequencies at which the eddy currents become significant are pushed upward.

2.3 Linearized System Dynamics

A schematic of the actual suspension system used in this experiment is shown in Figure 2.4. Here again, the system is actually rotated by 90 degrees, so gravity can be neglected. The actuators are now placed in push-pull opposition so that the suspended mass can be actively forced in either the positive or negative x-direction. At the equilibrium position, an air gap of \( \delta \) exists between each of the actuators and the suspended mass. The basic idea for linearizing the dynamics is to use a constant current bias in each of the actuators, and then send an incremental current to each of the actuators such that the incremental currents are equal in magnitude but opposite in sign. In other words,

\[
i_1 = I + \tilde{i} \tag{2.21}
\]

and

\[
i_2 = I - \tilde{i}, \tag{2.22}
\]

where \( I \) is the constant current bias and \( \tilde{i} \) is the incremental current. Based on these currents and equation 2.16, the net force on the mass becomes
\[ F_{\text{net}} = C\left(\frac{1 + \tilde{I}}{d-x}\right)^2 - C\left(\frac{1 - \tilde{I}}{d+x}\right)^2. \quad (2.23) \]

The net force is in the positive x-direction. Now doing a Taylor expansion of equation 2.23 around \( x = 0 \) and \( \tilde{I} = 0 \) and keeping only first order terms gives

\[ F_{\text{net}} \approx k_i \tilde{I} + k_x x, \quad (2.24) \]

where

\[ k_i = \frac{4CI}{d^2} \quad (2.25) \]

and

\[ k_x = \frac{4CI^2}{d^3}. \quad (2.26) \]

In this model, the dynamics of the suspension system have been linearized. A more detailed analysis also shows that for \( x = 0 \), equation 2.24 is exact. Besides being useful for linearizing the dynamic equations, the current bias is also useful for increasing the force-to-current gain. As seen in Figure 2.3, for currents near zero, the slope of the force curve is very small, so larger current changes are required for achieving even moderate force changes. By using a current bias, the actuators move into a region of the force-current curve where the slope is larger, and thus larger force changes can be achieved for smaller current changes.

The down side of using the current bias to linearize the equations of motion is that the operating range is very limited. As found by [Olson, 1994], for large deviations from the equilibrium center position, the suspension performance begins to deteriorate if the linearized equations derived above are used in a linear control scheme. For such cases, a nonlinear controller may be required, as described in [Olson, 1994].
Depth $h$ perpendicular to the page.
Pole Face Area = $wh = A$

Figure 2.4: Magnetic Bearing with Actuators in Pairwise Opposition.
2.4 Plant Transfer Function

Based on equation 2.24, the linearized dynamic equation of motion, the expected plant transfer function can be determined. An equilibrium gap size of 200 μm and a current bias of 0.5 A are used in our experiment for the following reasons: to ensure that the actuator would not saturate for the current range used, to ensure an adequate force range, and to achieve a reasonable force-to-current gain. Based on these values, we find

\[ k_i = 190 \frac{N}{A} \quad (2.27) \]

and

\[ k_x = 475 \frac{N}{mm} \quad (2.28) \]

suspended mass can be modeled as a pure inertial element whose transfer function from force to position is

\[ \frac{x(s)}{F(s)} = \frac{1}{Ms^2} \quad (2.29) \]

The value of M was measured by [Olson, 1994] to be 0.53 kg. Thus, the block diagram from the input current to the output mass position is given in Figure 2.5.

![Figure 2.5: Magnetic Suspension Plant Block Diagram.](image)

Based on the block diagram in Figure 2.5, the expected transfer function from current to position for the suspension system is
\[ \frac{x(s)}{i(s)} = \frac{k_i}{s^2 - k_x/M}. \quad (2.30) \]

As expected for magnetic suspension systems, this plant is unstable with poles at \( s = \pm \sqrt{k_x/M} = \pm 950 \text{ rad/sec (±150 Hz)}. \) A bode diagram for the system is shown in Figure 2.6. Besides the terms in equation 2.30, this plot also includes a gain of 200,000 to represent the capacitance probe gain and a gain of 0.05 to represent the current amplifier gain. These terms are included because in the experiment, neither the position nor the current can be measured directly. Instead, the capacitance probe voltage must be used to measure position, and the input voltage to the current amplifier must be used to measure current. Thus, Figure 2.6 is directly comparable to the actual experimental measurement. From the bode diagram, we see that the suspension system should have a constant phase of -180 degrees, and the magnitude should be similar to a critically damped second-order system with a DC gain of 4 and a bandwidth of 150 Hz.

![Bode Diagram](image)

**Figure 2.6:** Expected Plant Bode Diagram of Magnetic Suspension System.
Chapter 3

Current Amplifier

In this chapter, we discuss the design of the current amplifier used to drive the electromagnetic actuators of the suspension system. In section 3.1, we present the characteristics of the amplifier which make it particularly well suited for magnetic bearing applications. Next, in section 3.2, a dynamic analysis of the amplifier is performed in order to better understand the operation of the amplifier. This analysis is important because the initial design of the circuit exhibited an unexpected oscillation. Finally, section 3.3 presents a compensator design which eliminates the circuit oscillation, and the experimental transfer function and step response of the amplifier is given.

3.1 Amplifier Design

The initial design for the amplifier is shown in Figure 3.1. It is identical to the amplifier used by [Olson, 1994] and is borrowed from a design presented in [Poovey, Holmes, and Trumper, 1994]. The following discussion of the amplifier design is based on the descriptions in those two papers. The amplifier, which was designed especially for magnetic bearing applications, has several desirable characteristics. First, it can drive the electromagnetic actuators into saturation. Thus, the full capability of the actuators can be exploited. Second, the circuit has a high negative current slew rate, so it can change the force exerted by the actuators very rapidly. Next, the circuit exhibits low current noise, and finally, the flyback network and fuse F1 provide circuit protection. The fuse prevents the coil from
overheating by limiting the coil current, and the flyback network protects FET Q2 from excessive drain to source voltages if Q2 is suddenly turned off by allowing current to continually flow through the coil.

![Figure 3.1: Initial Current Amplifier Design.](image)

The key advantage of this circuit topology, though, is the high negative slew rate capability that results from the flyback network. The combination of D1, D2, R4, and Q1 acts as a 40 V zener. This high voltage zener then allows for large flyback voltages and thus high negative current slew rates. A further advantage of the flyback network used is the ability of the network to dissipate large amounts of heat. In particular, most of the energy is dissipated through Q1, which is a power transistor capable of dissipating up to 100 Watts. In previous amplifier designs of [Trumper, et al, 1991], a flyback network consisting of simply a diode was used. With this system, insufficient negative current slew rates through the coil were found to cause a degradation in the suspension performance. The low negative slew rates were due to the relatively small flyback voltages.
Besides the high negative current slew rates made possible by the flyback network, this network also allows the amplifier to function as a bipolar amplifier for coil voltages of ±40 V. For positive coil voltages up to 40 V, the drain voltage of Q2 is smaller than the 40 V source, and the flyback network is inactive. In this case, diode D3 prevents current from flowing through the flyback network, and the current through the coil is linearly related to the input voltage of the amplifier. Beyond +40 V, Q2 saturates because the drain voltages tries to become negative. For negative coil voltages, the drain voltage is greater than the 40 V source. In this case, the 40 V zener allows the drain voltage to rise as high as 40 V above the 40 V source. Here, for coil voltages up to -40 V the current in the coil is again linearly related to the input voltage to the amplifier. Thus, the amplifier is linear for coil voltages of ±40 V.

The only problem found by [Poovey, Holmes, and Trumper, 1994] in the amplifier design of Figure 3.1 is an oscillation in the circuit. In the initial design, capacitor C7 was put in to damp out the oscillation as much as possible. Its value was chosen by trial and error. Unfortunately, even with this capacitor, they found a small amplitude oscillation of approximately 30 kHz still present in the circuit.

3.2 Amplifier Dynamics

In order to better understand the cause of the 30 kHz oscillation, the small signal model for the amplifier was derived. In particular, a transfer function from the gate of FET Q2 to the source of FET Q2 was determined, and this transfer function was then used as the feedback load of the 741 op-amp. The small signal model used in the analysis is shown in Figure 3.2 and is taken from [Gray and Meyer, 1993]. The values used for the various components in the model were taken from the data sheet of the IRF 530 MOSFET, assuming an operating point coil current of 0.5 A, an operating point drain voltage of 40 V, and an operating point air gap of 200 μm between the actuator face and the target. The following values were used:

\[
\begin{align*}
C_{gs} &= 600 \text{ pF} \\
C_{gd} &= 25 \text{ pF} \\
C_{ds} &= 150 \text{ pF}
\end{align*}
\]
\[ gm = 6 \]  
\[ Ro = 300 \, \Omega \]  
\[ L = 38 \, \text{mH} \]  

\[ (3.4) \]  
\[ (3.5) \]  
\[ (3.6) \]  

Figure 3.2: Small Signal Model for Power MOSFET in Current Amplifier.
Based on the model in Figure 3.2, the following transfer function was obtained between the gate voltage and the source voltage:

\[
\frac{V_S}{V_G} = \frac{gmRsRo}{gmRsRo + Rs + Ro} * \\
\left[ \left( \frac{L}{gm} \right)(CgsCgd + CoCgs + CgdCds + CgsCds) \right] s^3 + \\
\left[ \left( \frac{L}{gm} \right)(CgsCgd + CoCgs + CgdCds + CdsCo) \right] s^2 + \left( \frac{Cgs}{gm} \right) s + 1 \\
\left[ \left( \frac{LRsRo}{gmRsRo + Rs + Ro} \right)(CgsCgd + CoCgs + CgdCds + CgsCds) \right] s^3 + \\
\left( \frac{L}{gm} \right)(CgsRs + LdsRo) \right] s^2 + \left( \frac{L + CgsRsRo + CdsRsRo}{gmRsRo + Rs + Ro} \right) s + 1
\]

(3.7)

A root locus plot and a bode diagram based on equation 3.7 and the values given above are shown in Figures 3.3 and 3.4. In the root locus plot, a pole at the origin that represents the op-amp pole has been added to the poles and zeros from equation 3.7 to more clearly illustrate the circuit instability. Figures 3.5 and 3.6 are also included in order to show the corresponding root locus and bode plots when the 0.01 \(\mu\)F capacitor used to damp out circuit oscillations is removed. Again, a pole at the origin has been added in the root locus plot.
Figure 3.3: Theoretical Root Locus Plot of Loop Transmission of Current Amplifier with C7 in place.
Figure 3.4: Theoretical Bode Plot of FET Transfer Function with C7 in place.
Figure 3.5: Theoretical Root Locus Plot of Loop Transmission of Current Amplifier without C7.
From the figures above, it appears that for sufficiently high gain, the circuit becomes unstable. This is apparent in each of the root locus plots because the zeros are in the right half plane. In the bode plots, a similar result can also be seen if the additional 90 degrees of phase drop from the op-amp integrator is considered. For a sufficiently large op-amp gain, cross-over will occur in a region with negative phase margin, and thus the loop will be unstable. Interestingly, in the bode plots, the loop goes unstable at a lower frequency with the 0.01 μF capacitor than without it. This seems to contradict the observation by [Poovey, Holmes, and Trumper, 1994] that this capacitor improved the circuit stability. Nevertheless, the plots above imply that stability can be achieved by using a sufficiently low crossover frequency, either with or without the 0.01 μF capacitor.
3.3 Amplifier Compensation

In order to design an appropriate compensator to eliminate all oscillations of the circuit, the actual transfer function from the gate to the source was first measured with the 0.01 μF capacitor removed from the circuit shown in Figure 3.1. The capacitor was removed because the instability observed by [Poovey, Holmes, and Trumper, 1994] in the amplifier with the 0.01 μF capacitor removed was not observed in this experiment. The transfer functions obtained with zero air gap between the actuator and the target (near-infinite coil inductance) and an air gap of 1500 μm (inductance = 5 mH) are shown in Figures 3.7 and 3.8. In both cases, a coil bias current of 0.5 A was used. These air gaps were assumed to represent the whole range of likely operating conditions for the suspension system, and thus both of these transfer functions could be used to design a compensator which ensured stability for all operating conditions.

![Graph of Voltage Gain vs. Frequency](image)

![Graph of Phase vs. Frequency](image)

Figure 3.7: Measured Transfer Function from Gate to Source of MOSFET with Zero Air Gap (0.01 μF Capacitor Removed and 0.5 A Bias Current in Coil).
Figures 3.7 and 3.8 are expected to be approximately the same as Figure 3.6. The only difference is in the value of the load inductance, but this should not affect the form of the curve. The main difference between the two sets of Figures is that the theory predicts the DC gain to be approximately 0.85, whereas the measured DC gain is approximately 0.5. The other important difference is that the theory predicts the magnitude to start rolling off significantly by about 10 kHz, however in the measurement, the roll off does not begin until about 20 kHz. Despite these differences, the theory and experiment are close enough that the theory seems to be reasonably accurate. Also, it is important to note that the transistor component values used in Figure 3.6 were only approximations, so some discrepancy is expected. Furthermore, a more complete comparison of the measured and theoretical plots is not possible because the HP35665A dynamic analyzer used to measure Figures 3.7 and 3.8 was limited to measurements below 50 kHz.
In order to design a compensator, Figures 3.7 and 3.8 can be used directly as the loop transmission that must be compensated for. Figure 3.9 shows the final compensated amplifier design. There are several key features of this design. First, the 741 op-amp of the original design was replaced by a 356 op-amp. The reason for this is that the circuit oscillated when the 741 was used, whereas with the 356, a similar problem was not observed. The proposed explanation is that the capacitive load from the gate of the MOSFET somehow interacted with the 741 output stage to cause some instability. This hypothesis was not investigated more thoroughly, however. Second, the 0.01 \( \mu \text{F} \) capacitor was removed. As mentioned above, the circuit was not found to oscillate without this capacitor, so it was removed. Finally, to compensate the amplifier, the op-amp internal integrator was replaced by the integrator formed by the feedback capacitor(C8) and the feedback resistor(R5). The combination of these two components effectively replaced the op-amp gain and integrator with a \(-1/RC_s\) transfer function. Thus, the bandwidth could be reliably set by selecting appropriate values for C8 and R5 and therefore changing the open loop unity gain crossover frequency.

For the purpose of this experiment, a 10 kHz bandwidth was designed for. From Figures 3.7 and 3.8, the FET transfer function of the amplifier was found to have a closed loop gain of about 0.5 at 10 kHz. Thus, the values of C8 and R5 were be chosen so that at 10 kHz, \(|1/RC_s|\) was approximately 2. In this way, unity gain crossover of the open loop system would occur at 10 kHz, and the closed loop bandwidth would be approximately 10 kHz. The values selected were \(C8 = 0.1 \text{ nF}\) and \(R5 = 56 \text{ k}\Omega\). The resulting measured amplifier transfer function is shown in Figure 3.10, and the step response is shown in Figure 3.11. From Figure 3.10, the actual bandwidth of the amplifier is about 20 kHz rather than 10 kHz. This difference is not unreasonable considering that the bandwidth is very sensitive to small changes in the gate to source gain of the MOSFET. In the step response, the distinctive feature is the vertical jump at the beginning of the response. The reason for this jump is that there is an additional zero in the compensator caused by driving the system at the non-inverting terminal of the op-amp. An additional RC network at the non-inverting terminal could be used to cancel this zero. However, it was not considered to be necessary because it does not significantly affect the bandwidth. In any case, the rise time in the step response is consistent with the 20 kHz bandwidth.
A final interesting point about the compensation is that the circuit layout is critical to the amplifier performance. In particular, we observed experimentally that parasitic capacitance between the high voltage drain of the MOSFET and the inverting op-amp terminal can cause instabilities in the circuit. Thus, it is important to connect the feedback resistor and capacitor of the compensator at a point well-shielded from the drain of the MOSFET. Besides the compensation, some other changes in the final amplifier design are also worth noting. First, the voltage reference and potentiometer were added at the input terminal in order to set the bias current. The various resistor values were chosen in order to be able to set the bias up to 0.5 A. Second, the attenuation factor from Vref to the output current was set to about 20:1. This value was chosen so that for the ±10 V output swing of the capacitive probe, the current could be driven from 0 A to 1 A. Thus, the full capability of the current driver could be exploited.
Figure 3.9: Final Compensated Current Amplifier Design.
Figure 3.10: Measured Amplifier Transfer Function.

Figure 3.11: Measured Amplifier Step Response.
Chapter 4

Compensator Design

This chapter discusses the design of the compensator used to stabilize the magnetic suspension system studied in this experiment. In section 4.1, the general design objectives for the controller are given. Section 4.2 provides the measured bode plot of the open loop suspension system. This bode plot is compared to the expected transfer function given in chapter 2, and the particular characteristics of the measured transfer function and their implications for the controller design are presented. Finally, in section 4.3 the final analog controller design and implementation are discussed.

4.1 Control System Objectives

The basic objective of this study is to determine the maximum bandwidth that can be achieved from magnetic suspension systems. Thus, this is the first and most important criterion for the controller design. However, a second important criterion to consider is achieving a small steady state error. A small error is critical because magnetic suspension systems are most applicable in precision positioning systems.

In order to satisfy the two criteria given above, a lead-lag controller will be used. The lead controller will provide an adequate phase margin at unity gain crossover to ensure stability, and the lag controller will be used to increase the low frequency gain and thus improve disturbance rejection. The overall closed loop block diagram with the compensator is given in Figure 4.1. The suspension system plant is the same as was given in chapter 2.
4.2 Loop Transmission of Suspension System

Figure 4.2 shows the measured bode diagram of the suspension system. It represents the transfer function from the input voltage to the current amplifier to the output voltage of the capacitance probe. Thus, just as in Figure 2.6, the amplifier gain of approximately 0.05 and the capacitance probe gain of 200,000 are included with the suspension dynamics. Figure 4.3 was measured in a stable closed loop configuration which was designed using the theoretical plant transfer function with a bandwidth of 200 Hz.

From the measured bode plot of the suspension system, the accuracy of the theoretical model developed in chapter 2 can be verified. First, in the measured plot, the DC gain is approximately 3.8. This compares favorably to the theoretical value of 4 from Figure 2.6. The small discrepancy is likely due to small inaccuracies in the values used to obtain Figure 2.6. The measured bandwidth of about 100 Hz also compares well to the theoretical value of 150 Hz. The key difference between the measured and the theoretical responses is that the measured plot has a roll off with a -3 slope, whereas the theoretical plot rolls off with a -2 slope. Correspondingly, the phase of the measured transfer function drops well below the constant -180 degree phase of the theoretical responses.

Unfortunately, within the time limits of this project, the cause of the discrepancy between the measured and theoretical transfer functions could not be identified, though two possibilities are suggested. First, the unmodeled poles in the measured transfer function could be the result of an unexpectedly
low bandwidth for the capacitance probe. The probe has a specified bandwidth of 10 kHz, in which case the probe would have no effect on the measured transfer function up to the frequencies investigated. However, this value has not been verified experimentally. Second, as described in chapter 2, eddy currents can limit the actuator bandwidth and thus may have caused the unmodeled poles in Figure 4.2. A measurement was conducted using the calibration fixture described in chapter 2 in order to test for eddy current effects in an actuator very similar to that used in this experiment. In that test, eddy currents appeared to be insignificant up to about 1 kHz, which was the limiting frequency for the calibration fixture. However, the actual actuators used in this experiment were not tested, so there is a possibility that they are affected by eddy currents.

![Voltage Gain vs Frequency](image)

![Phase Shift vs Frequency](image)

Figure 4.2: Measured Plant Bode Diagram of Magnetic Suspension System.

Ultimately, the discrepancy has severe implications for the achievable bandwidth of the suspension system. For a simple lead-lag compensator, the
maximum achievable positive phase shift is less than 90 degrees. Thus, for the measured plant transfer a function, a stable crossover of no more than 100 or 200 Hertz is possible.

4.3 Controller Design

As discussed in section 4.1, a lead-lag compensator will be used to achieve the desired closed loop suspension characteristics. The overall analog controller used in this experiment is shown in Figure 4.3. An analog controller is used instead of a digital controller in order to avoid bandwidth limitations caused by sampling frequency in a digital controller.

In Figure 4.3, op-amp U1 functions as a difference junction by subtracting Vin from Vcap. Vin is subtracted from Vcap instead of the other way around because an additional inversion occurs at the lead lag compensator from op-amp U2. Thus, this double inversion results in the block diagram shown in Figure 4.1. The lead-lag transfer function from op-amp U2 is as follows (the additional inversion is neglected):

\[
\frac{R_3C_2s + 1}{R_2C_2s} \frac{(R_1 + R_2)C_1s + 1}{R_1C_1s + 1}.
\]  

(4.1)

Here, the first term constitutes the lag compensator and the second term constitutes the lead compensator. For the lag, the low frequency gain is infinite since the lag pole is at \( s = 0 \), and the high frequency gain is \( R_3/R_2 \). The integrator in the lag should drive the steady state error of the system to zero. For the lead, the pole zero separation factor(\( \alpha \)) is given by

\[
\alpha = \frac{R_1 + R_2}{R_1},
\]  

(4.2)

and the time constant(\( \tau \)) is given by

\[
\tau = R_1C_1.
\]  

(4.3)

Based on these terms, the maximum positive phase shift(\( \phi_m \)) of the lead is
\[ \phi_m = \sin^{-1}\left(\frac{\alpha - 1}{\alpha + 1}\right), \quad (4.4) \]

and it occurs at a frequency \(\omega_m\) of

\[ \omega_m = \frac{1}{\sqrt{\alpha \tau}}. \quad (4.5) \]

Finally, op-amp U3 is connected as a follower and op-amp U4 is an inverter. Thus, these two op-amps send equal positive and negative voltages to the current amplifiers, which then run the electromagnetic actuators at incrementally positive and negative currents, as is required to linearize the plant dynamics.

![Lead-Lag Compensator Circuit Diagram](image.png)

Figure 4.3: Lead-Lag Compensator Circuit Diagram.

In the design of the compensator, a value of \(\alpha = 10\) will be used for the lead, and the lag zero will be placed a factor of 10 below crossover. By using \(\alpha = 10\), the maximum positive phase of the lead is 55 degrees. Although a larger positive phase could be achieved with larger \(\alpha\), the increase in phase is only marginal for increasing \(\alpha\). Also, increasing \(\alpha\) places increasing demands on the current from the current amplifiers. For example, in a step response, the value of \(\alpha\) is proportional to the jump in current that occurs at the edge of the step. Using \(\alpha = 10\) is considered a reasonable middle ground between the benefits of using either a larger or smaller value.

By placing the lag zero a factor of 10 below crossover, the residual negative phase at crossover due to the lag is -5.7 degrees. This negative phase
could be decreased by moving the lag zero further below crossover, but as with the lead, there is a cost. First, the decrease in negative phase is only marginal as the zero location is moved downward. Second, the closer the zero is to \( s = 0 \), the longer the time constant will be for the compensator. As a result, the step response for the system will exhibit a very long settling time. Thus, placing the lag zero a factor of 10 below crossover is considered the lowest acceptable zero location.

For the suspension system, a crossover frequency of 200 Hz will be designed for. As discussed above, this is about the highest crossover frequency that can be achieved from the measured plant transfer function while still maintaining a reasonable level of stability. To achieve the maximum phase margin at this frequency, \( \omega_m \) of the lead should be placed at 200 Hz. Thus, since \( \alpha = 10 \), \( \tau = 0.25 \) msec. Furthermore, in order to force the magnitude at 200 Hz to be unity, the compensator gain must be 1.25 since the plant magnitude is 0.8. At \( \omega_m \), the lead magnitude is \( \sqrt{\alpha} \). Therefore, the lag magnitude, which is approximately \( R_3/R_2 \), must be 0.4. Finally, as described above, the lag zero should be at 20 Hz. In order to meet these criteria, the following resistor and capacitor values from Figure 4.3 were used:

\[
\begin{align*}
R_1 &= 11.4 \text{k}\Omega \\
R_2 &= 103 \text{k}\Omega \\
R_3 &= 40.7 \text{k}\Omega \\
C_1 &= 0.022 \mu\text{F} \\
C_2 &= 0.22 \mu\text{F}
\end{align*}
\]

(4.6)  \hspace{1cm} (4.7)  \hspace{1cm} (4.8)  \hspace{1cm} (4.9)  \hspace{1cm} (4.10)

Based on these values, the predicted phase margin at the 200 Hz crossover is 15 degrees. This in due to the -215 degree phase of the plant at 200 Hz and the predicted 50 degree phase of the compensator at 200 Hz.
Chapter 5

Results and Conclusions

This chapter gives the experimental results of the controller design presented in chapter 4. Section 5.1 contains the measured bode plots of the closed loop system. The bandwidth and stability attained in this plot are also compared to the expected values from chapter 4. Section 5.2 discusses the results of this thesis, focusing on the amplifier design and the suspension performance. Suggestions for further research are also given.

5.1 Experimental Results

For the compensator design described in chapter 4, the experimental loop transmission and the experimental closed loop response are shown in Figures 5.1 and 5.2 respectively. From Figure 5.1, we see that the achieved crossover frequency is 200 Hz, and the phase margin is 15 degrees, as was designed for. Figure 5.2 shows that the actual -3 dB bandwidth is about 400 Hz. From Figure 5.2, we can also see that the closed loop DC gain is unity, indicating that the integrator did indeed drive the steady state error to zero. Also, the peak closed loop magnitude of 4.5 is consistent with the phase margin of 15 degrees.
Figure 5.1: Measured Loop Transmission of Magnetic Suspension System.
5.2 Conclusions and Suggestions for Future Work

The primary accomplishment of this thesis is in the compensator design for the current amplifiers used to drive the electromagnets for the suspension system. The oscillation encountered in the original design of the amplifier was successfully eliminated, and a compensator was implemented that achieved an amplifier bandwidth of 20 kHz. This design has promise for future magnetic bearing applications, and it should help improve the performance of such systems since it does eliminate the oscillation of the previous design.

In terms of the suspension bandwidth, the unexplained unmodeled poles in the plant transfer function severely limited the achievable bandwidth. As described above, possible explanations for the unmodeled poles are low capacitance probe bandwidth and low actuator bandwidth due to eddy current effects. Both of these possible problems require further study, however, if their significance in this application is to be determined.
Nevertheless, for the plant transfer function measured, a crossover frequency of 200 Hz and a phase margin of 15 degrees was achieved. For these values, the resulting bandwidth was approximately 400 Hz.

Finally, a good deal of further research can be done on this project. Once the source of the unmodeled poles has been identified and eliminated, the linear controller discussed in this thesis can be redesigned to achieve a higher bandwidth. It might also be interesting to use a nonlinear controller to test the large-signal bandwidth limits of the suspension because the linear controller places important limits on the large-signal operating range of the suspension system. The nonlinear controller could be used in a much wider range of applications, and it could possibly push the large-signal bandwidth even higher than what could be achieved with the linear controller.
Chapter 6

References


